12.6 Graphical Analysis

The motion of a particle along a straight path can be represented by motion curves. Common motion curves are position–time (x – t), velocity–time (v – t), acceleration–time (a – t) and velocity–position (v – x) curves.

1) position – time (x – t) curve

This is drawn with position on the ordinate and time on abscissa.

Since $v = \frac{dx}{dt}$ at any instant of time the slope of x – t curve gives the velocity of the particle at that instant.

$\therefore$ $\boxed{v = (\text{slope } x - t \text{ curve})}$ ......[12.17]

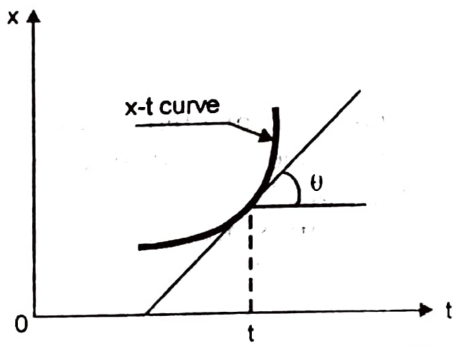


Fig. 12.10

2) velocity – time (v – t) curve

This is drawn with velocity on the ordinate and time on abscissa

since $a = \frac{dv}{dt}$,

At any instant of time the slope of v – t curve gives the acceleration of the particle at that instant.

$\therefore$ $\boxed{a = (\text{slope } v - t \text{ curve})}$ ......[12.18]

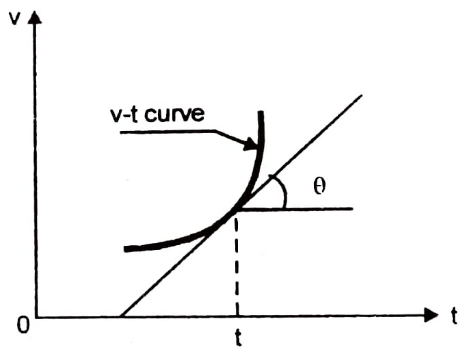


Fig. 12.11

Now, $v = \frac{dx}{dt}$

$\therefore$ $dx = v dt$

or $\int dx = \int v dt$

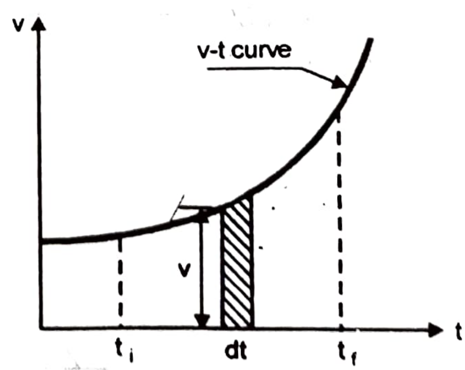


Fig. 12.12

Let the particle's position be x_i at time t_i and its position change to x_f at time t_f.

If an elemental strip of width dt is taken between t_i and t_f, then the area of this strip is vdt.

$\therefore \quad \int v dt$ represents the entire area under the v – t curve between t_i and t_f

$\therefore \quad \int_{x_i}^{x_f} dx$ = area under v – t curve

$\therefore \quad x_f - x_i$ = area under v – t curve

or $\quad \boxed{x_f = x_i + [\text{area under v – t curve}]}$ ......[12.19]

3) **acceleration – time (a – t) curve**

This is drawn with acceleration on the ordinate and time on the abscissa.

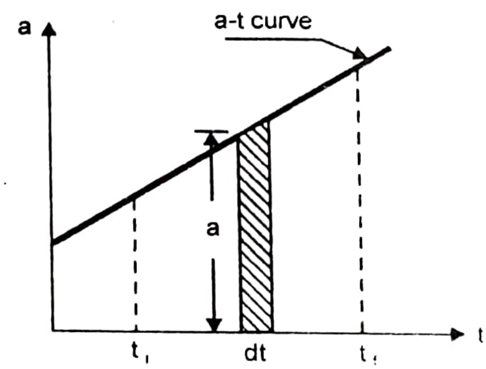


Fig. 12.13

we know, $\quad a = \frac{dv}{dt}$

$\therefore \quad dv = adt$

or $\quad \int dv = \int adt$

Let the particle's velocity be v_i at the time t_i and its velocity be v_f at time t_f.
If an elemental strip of width dt is taken between t_i and t_f, then the area of this strip is adt

$\therefore \quad \int adt$ represents the entire area under the a – t curve between t_i and t_f.

$\int_{v_i}^{v_f} dv$ = area under a – t curve

$\therefore \quad v_f - v_i$ = area under a – t curve

or $\quad \boxed{v_f = v_i + [\text{area under a – t curve}]}$ ....[12.20]

From an a – t curve the particle's position x_f can also be known at any instant t_f, knowing the particles position x_i and velocity v_i at an prior instant t_i, using the *area moment method* formula given below.

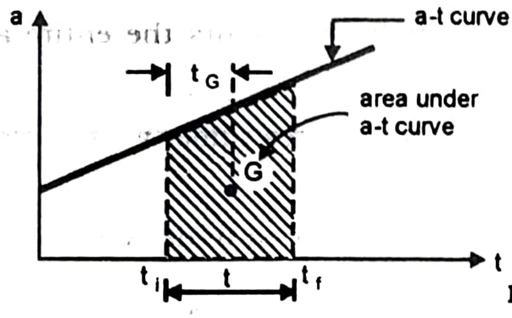


Fig. 12.14

$$\boxed{x_f = x_i + v_i t + (\text{area under } a - t \text{ curve})(t - t_G)} \quad ......[12.21]$$

here $t = t_f - t_i$ and t_G is the abscissa of the centroid G of the area under a – t curve.

4) **velocity – position (v – x) curve**

This is drawn with velocity on the ordinate and position on abscissa.

We know, $a = \dfrac{vdv}{dx}$

$$\therefore \quad \boxed{a = v \times [\text{slope } v - x \text{ curve}]} \quad ......[12.22]$$

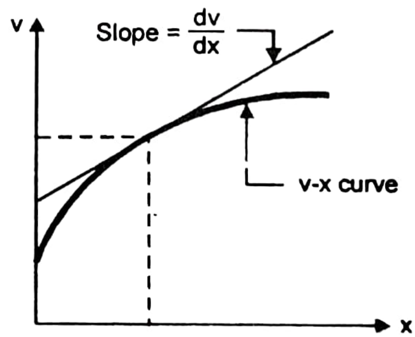


Fig. 12.15

From v - x curve we can find the particle's velocity v and the corresponding slope at a given position x and hence using equation 12.22 we can find the particles acceleration.

Let us tabulate the uses of various motion curves

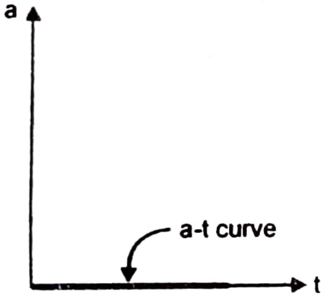
No.	Motion curve	Use	Formula
1	x – t	Slope of x – t curve gives velocity	v = (slope x – t curve)
2	v – t	a) Slope of v – t curve gives acceleration b) Area under v – t curve gives change in position and hence the new position.	A = (slope v – t curve) $x_f = x_i$ + (area under v – t curve)
3	a – t	a) Area under a – t curve gives change in velocity and hence the new velocity b) Area under a – t curve also helps in finding the particle's position	$v_f = v_i$ + (area under a – t curve) $x_f = x_i + v_i \times t$ + (area under a – t curve) $(t - t_G)$
4	v – x	Slope of v – x curve helps in finding the particle's acceleration	a = v × (slope v – x curve)

12.6.1 Standard Motion Curves

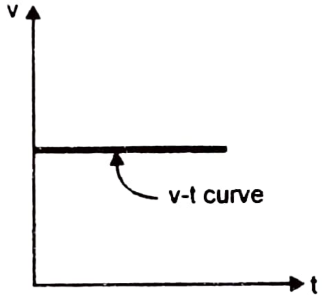
We know that rectilinear moving particles either move with uniform velocity or uniform acceleration or may move with variable acceleration. The general or standard motion curves (viz. a – t, v – t, x – t) for these different rectilinear motions are given here.

The particle's motion may be easily identified if we relate the given motion curve problem with these standard curves. For example, if the particle's a – t curve is parallel to acceleration axis, the particle is in uniform acceleration motion. Similarly if the x – t curve has a cubic equation it indicates variable acceleration motion.

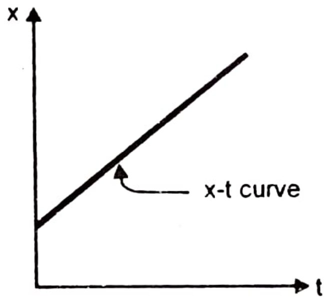
a) Uniform Velocity Motion curves



Straight horizontal curve on the acceleration axis

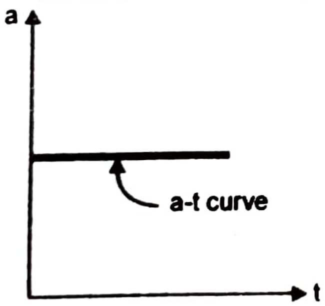


Straight horizontal curve parallel to velocity axis

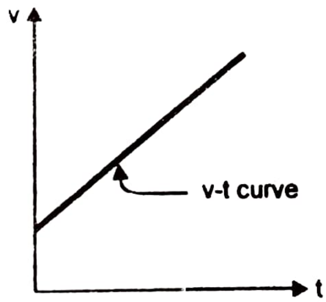


Straight inclined curve

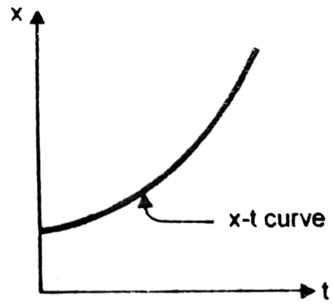
b) Uniform Acceleration Motion curves



Straight horizontal curve parallel to acceleration axis

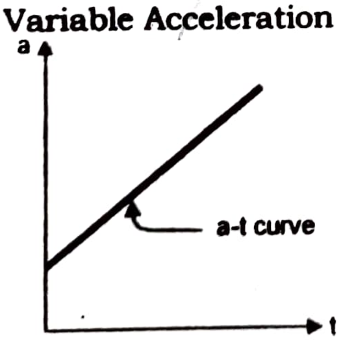


Straight inclined curve

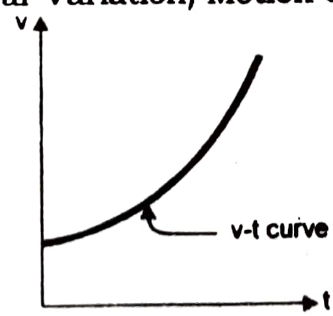


Second degree curve

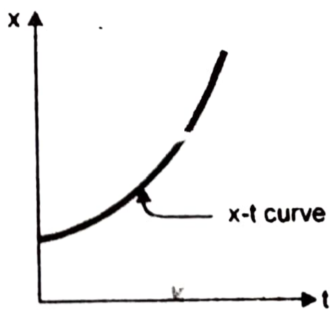
c) Variable Acceleration (Linear Variation) Motion curves



Straight inclined curve

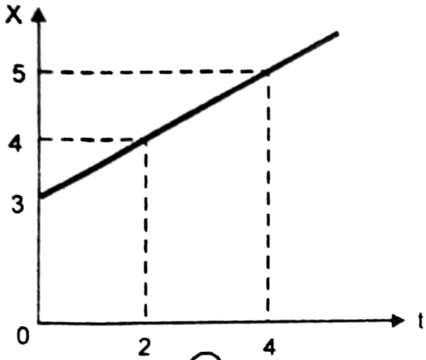


Second degree curve

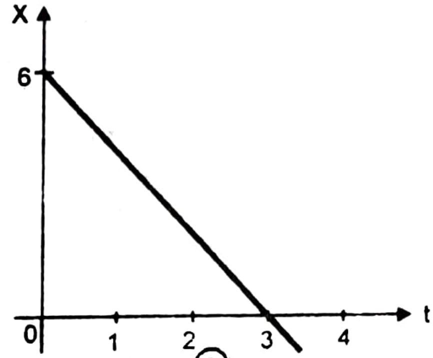


Third degree curve

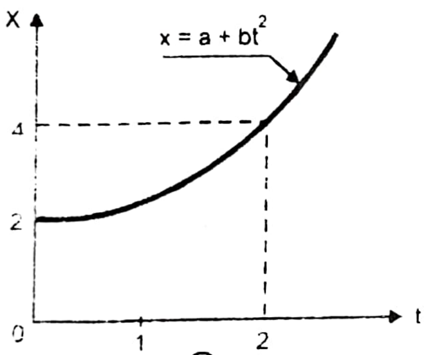
Ex. 12.24 The following graphs depict the kinematics relations. For each graph determine the particle's velocity at t =4 sec.
Take v = 2 m/s at t = 0 for graphs 4, 5 and 6.



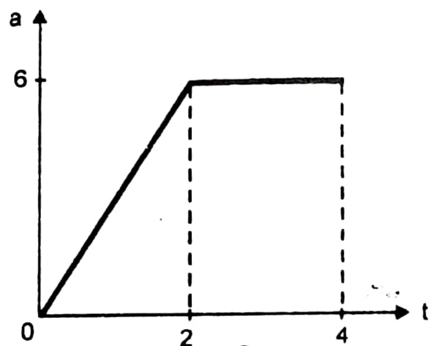
Graph 1



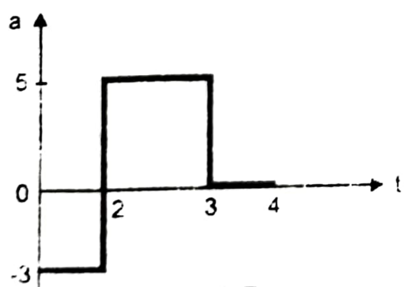
Graph 2



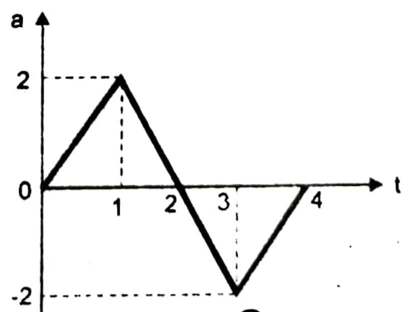
Graph 3



Graph 4



Graph 5



Graph 6

Solution: Graph (1) is a straight x – t curve.

We know slope of x – t curve gives the velocity

$$v = (\text{slope } x-t \text{ curve}) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{4-0} = \frac{1}{2}$$

$\therefore$ v = 0.5 m/s .... **Ans.**

Graph (2) is a straight line x – t curve indicating uniform velocity motion.

$$(\text{slope } x-t \text{ curve}) = \frac{0-6}{3-0} = \frac{-6}{3} = -2 \text{ m/s}$$

$\therefore$ velocity is constant at v = – 2 m/s .... **Ans.**

Graph (3) is a parabolic x – t curve indicating uniform acceleration motion.

Given $x = a + b.t^2$

From graph at t = 0, x = 2 and at t = 2 sec, x = 4, substituting we get a = 2, b = 0.5

$\therefore$ $x = 2 + 0.5t^2$

$$v = (\text{slope } x-t \text{ curve}) = \frac{dx}{dt} = t$$

$\therefore$ at t = 4 sec, v = 4 m/s ...... **Ans.**

Graph (4) is an a – t curve

Using $v_f = v_i + [\text{ area under } a-t \text{ curve}]$

$$v_{t=4} = 2 + [\text{ area under } a-t \text{ curve}]_{0-4}$$

$$= 2 + [\, ½ \times 2 \times 6 + 2 \times 6\,]$$

$$= 20 \text{ m/s}$$ ...... **Ans.**

Graph (5) is an a – t curve

Using $v_f = v_i + [\text{ area under } a-t \text{ curve }]$

$$v_{t=4} = 2 + [\text{ area under } a-t \text{ curve }]_{0-4}$$

$$= 2 + [-2 \times 3 + 1 \times 5\,]$$

$$= 1 \text{ m/s}$$ ...... **Ans.**

Graph (6) is an a – t curve

Using $v_f = v_i + [\text{ area under } a-t \text{ curve }]$

$$v_{t=4} = 2 + [\text{ area under } a-t \text{ curve }]_{0-4}$$

$$= 2 + [\, ½ \times 2 \times 2 - ½ \times 2 \times 2\,]$$

$$= 2 \text{ m/s}$$ .... **Ans.**

Ex. 12.25 The position of a particle is defined by $x = 2t^3 - 24t + 6$ m. Draw x – t, v – t and a – t curves from t = 0 to t = 4 sec.

Solution: Given $x = 2t^3 - 24t + 6$ m .... (1)

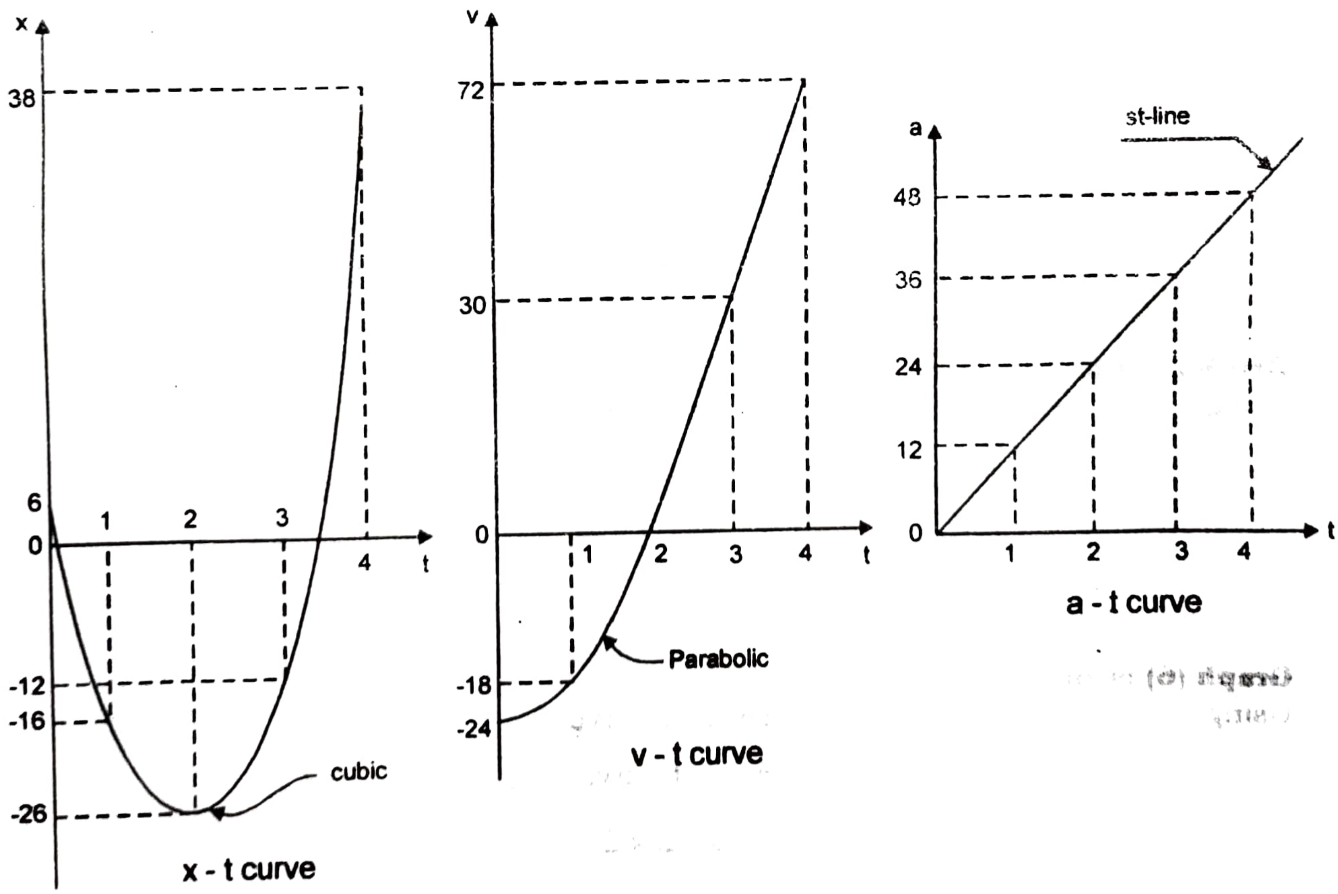
$$v = \frac{dx}{dt} = 6t^2 - 24 \text{ m/s} \quad \text{..........(2)}$$

$$a = \frac{dv}{dt} = 12t \text{ m/s}^2 \quad \text{..........(3)}$$

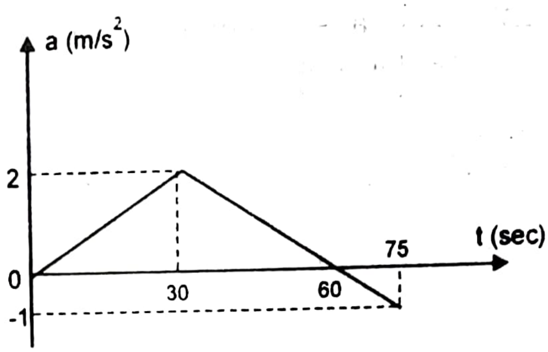
From equations (1), (2) and (3), finding out the values for 0 – 4 sec interval and tabulating we get,

t	x (m)	v (m/s)	a (m/s²)
0	6	–24	0
1	–16	–18	12
2	– 26	0	24
3	– 12	30	36
4	38	72	48

Now plotting the (x – t), (v – t) and (a – t) curves



Ex. 12.26 Figure shows (a – t) diagram for particle moving along a straight path for a time interval 0 – 75 sec. Plot (v – t) and (x – t) diagrams and hence find the maximum speed attained by the particle. The particle started from rest from origin.

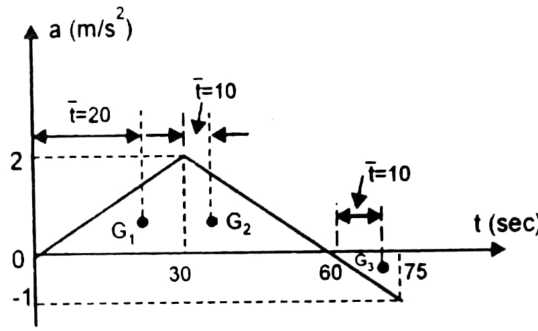


Solution: Area under a – t curve

$0 - 30$ sec $= \frac{1}{2} \times 30 \times 2 = 30$

$30 - 60$ sec $= \frac{1}{2} \times 30 \times 2 = 30$

$60 - 75$ sec $= -\frac{1}{2} \times 15 \times 1 = -7.5$



Velocity calculations

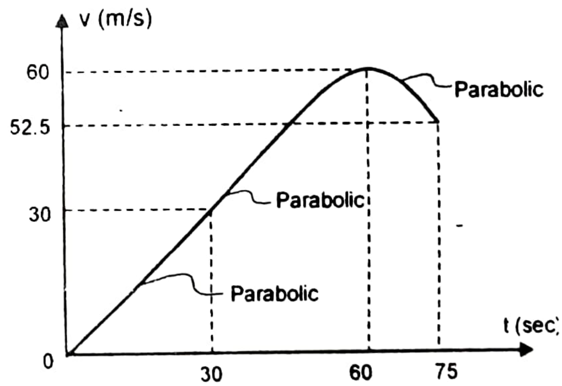
Since a – t diagram is given, velocity can be found out using

$$v_f = v_i + (\text{area under a - t curve})$$

$$v_{30} = v_o + (\text{area under a - t curve})_{0\text{-}30}$$
$$= 0 + 30 = 30 \text{ m/s}$$

$$v_{60} = v_{30} + (\text{area under a - t curve})_{30\text{-}60}$$
$$= 30 + 30 = 60 \text{ m/s}$$

$$v_{75} = v_{60} + (\text{area under a - t curve})_{60\text{-}75}$$
$$= 60 + (-7.5) = 52.5 \text{ m/s}$$



Position calculations

From a – t curve position can be calculated using

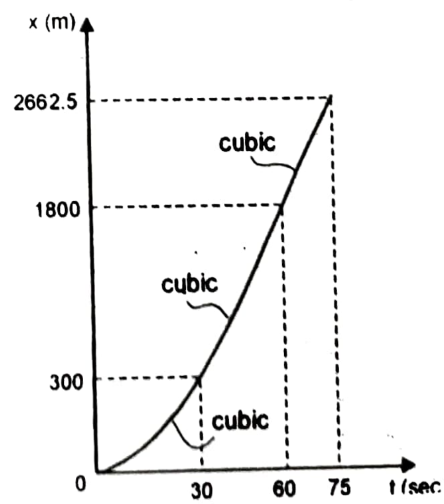
$$x_f = x_i + v_i \times t + (\text{area under a – t curve})\ (t - t_G)$$

Given $\quad x_o = 0$ and $v_o = 0$

$$x_{30} = x_o + v_o \times t + (\text{area under a - t curve})_{0\text{-}30}\ (t - t_G)$$
$$= 0 + 0 + 30\,(30 - 20)$$
$$= 300 \text{ m}$$

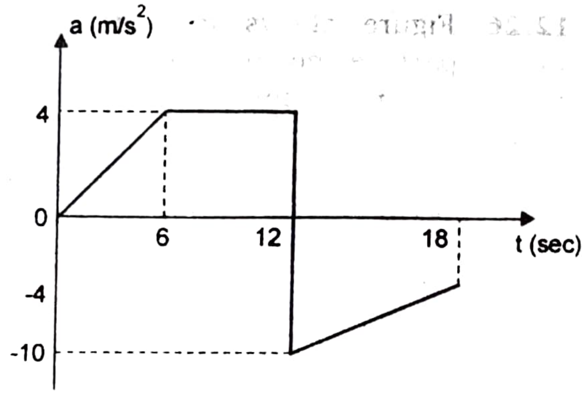
$$x_{60} = x_{30} + v_{30} \times t + (\text{area under a - t curve})_{30\text{-}60}\ (t - t_G)$$
$$= 300 + 30 \times 30 + 30\,(30 - 10)$$
$$= 1800 \text{ m}$$

$$x_{75} = x_{60} + v_{60} \times t + (\text{area under a - t curve})_{60\text{-}75}\ (t - t_G)$$
$$= 1800 + 60 \times 15 + (-7.5)(15 - 10)$$
$$= 2662.5 \text{ m}$$



From graph $\quad v_{max} = 60$ m/s $\quad$ at $\quad t = 60$ sec .... **Ans.**

Ex. 12.27 The a – t curve for a particle performing rectilinear motion is shown. At t = 0 the particle's velocity is 5 m/s and the particle is located at 25 m to the left of the origin. Determine the velocity and position values at 6, 12 and 18 sec.



Solution:

Velocity calculations

From a – t curve velocity can be calculated using

$$v_f = v_i + (\text{area under a – t curve})$$

knowing $x_o = -25$ m and $v_o = 5$ m/s

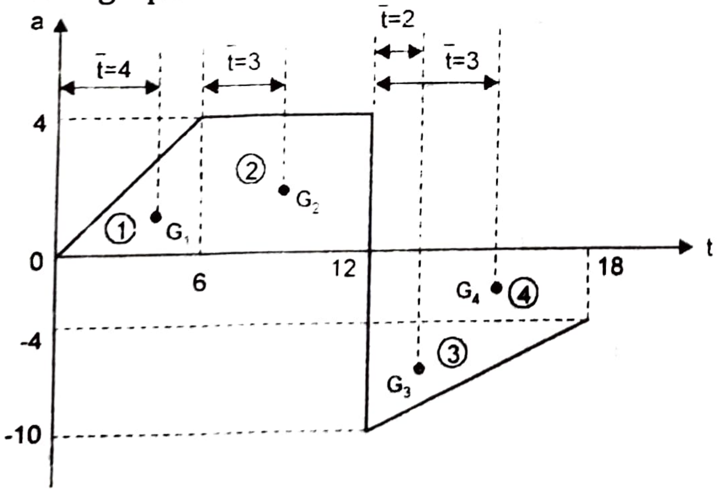
$$v_6 = v_o + (\text{area under a - t curve})_{0-6}$$
$$= 5 + 12$$
$$= 17 \text{ m/s} \quad \text{....... Ans.}$$

$$v_{12} = v_6 + (\text{area under a - t curve})_{6-12}$$
$$= 17 + 24$$
$$= 41 \text{ m/s} \quad \text{....... Ans.}$$

$$v_{18} = v_{12} + (\text{area under a - t curve})_{12-18}$$
$$= 41 + [-(18 + 24)] \quad \text{- ve sign} \because \text{area lies below t axis}$$
$$= -1 \text{ m/s} \quad \text{...... Ans.}$$

Position calculations

Refer graph



From Graph

Area 0 – 6 sec = 1/2 × 6 × 4 = 12

Area 0 – 12 sec = 6 × 4 = 24

Area 12 – 18 sec = 1/2 × 6 × 6 + 6 × 4

= 18 + 24 = 42

From a – t curve, position can be calculated using

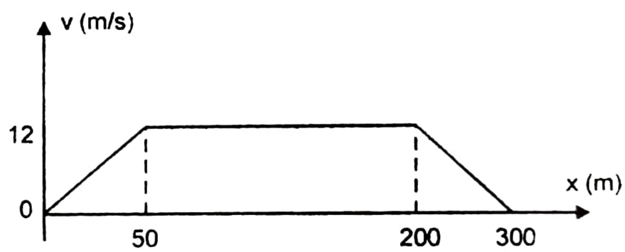
$$x_f = x_i + v_i \times t + (\text{area under a – t curve})(t - t_G)$$

$$x_6 = x_o + v_o \times t + (\text{area under a - t curve})_{0-6}\,(t - t_G)$$
$$= -25 + 5 \times 6 + 12\,(6 - 4)$$
$$= 29 \text{ m}$$ ..... **Ans.**

$$x_{12} = x_6 + v_6 \times t + (\text{area under a - t curve})_{6-12}\,(t - t_G)$$
$$= 29 + 17 \times 6 + 24\,(6 - 3)$$
$$= 203 \text{ m}$$ ..... **Ans.**

$$x_{18} = x_{12} + v_{12} \times t + (\text{area under a - t curve})_{12-18}\,(t - t_G)$$
$$= 203 + 41 \times 6 + [-18\,(6 - 2) - 24\,(6 - 3)]$$
$$= 305 \text{ m}$$ ...... **Ans.**

Ex. 12.28 The v – x graph of a car traveling on a straight road is shown. Determine the acceleration of the car at x = 25 m, x = 100 m and at x = 225 m.



Solution: From (v – x) graph, the acceleration can be found out using

$$a = v \times (\text{slope v – x curve})$$

From graph at $x = 25$ m, $v = 6$ m/s

also (slope v – x curve) $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{12 - 0}{50 - 0} = 0.24$

$\therefore$ $a_{x=25m} = 6 \times 0.24 = 1.44 \text{ m/s}^2$ ...... **Ans.**

From graph at $x = 100$ m, $v = 12$ m/s

also (slope v – x curve) = 0

$\therefore$ $a_{x=100m} = 12 \times 0$
$= 0$ .... **Ans.**

From graph at $x = 225$ m, $v = 9$ m/s

also (slope v – x curve) $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - 12}{300 - 200} = -0.12$

$\therefore$ $a_{x=225m} = 9 \times (-0.12) = -1.08 \text{ m/s}^2$ ...... **Ans.**

Exercise 12.5

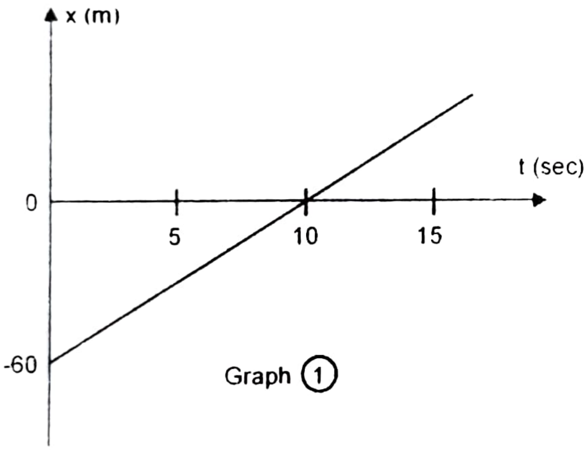
P1. From graph (1) find position and velocity at t = 4 sec.

P2. Refer graph (2). Find velocity at t = 2, 4 and 6 sec.

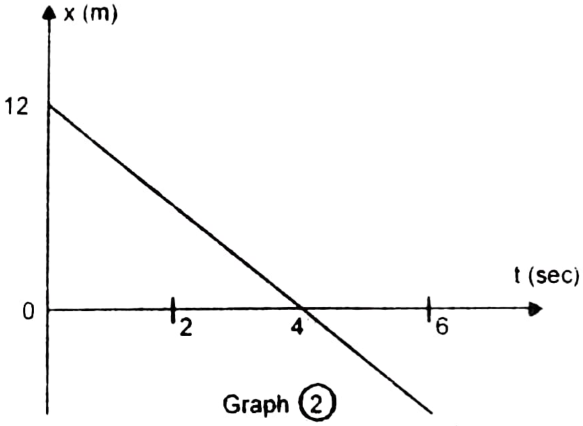
P3. Refer graph (3). Find acceleration at t =10, 30, 45 and 50 sec.

P4. From graph (3) find position at t = 20, 30, 40, 50 and 60 sec knowing $x_o = 0$.

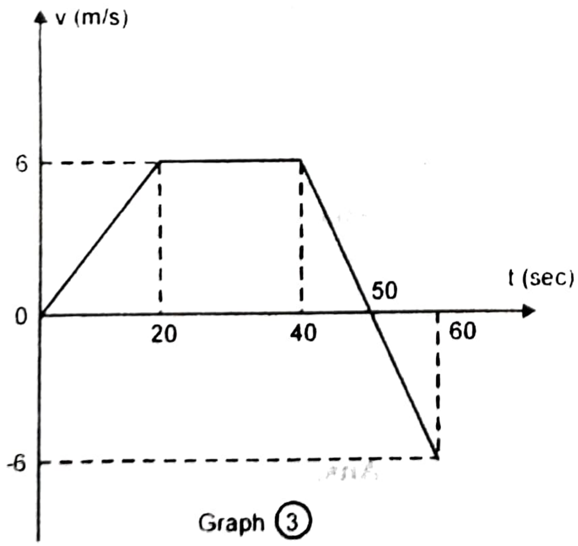
P5. From graph (4) find position and velocity at t = 6, 12 and 18 sec knowing $x_o = 10$ m, $v_o = 5$ m/s.



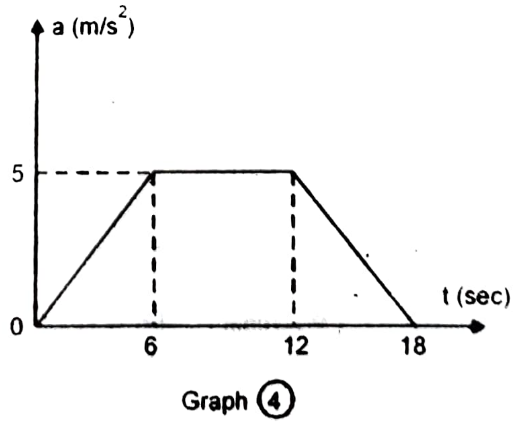
Graph (1)



Graph (2)

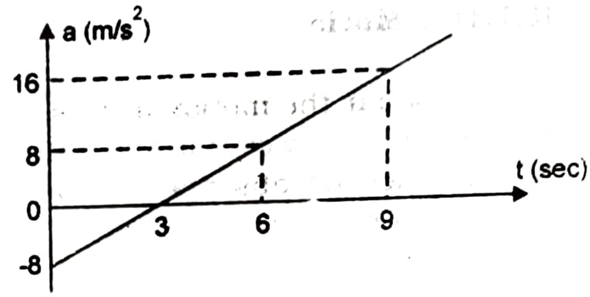


Graph (3)

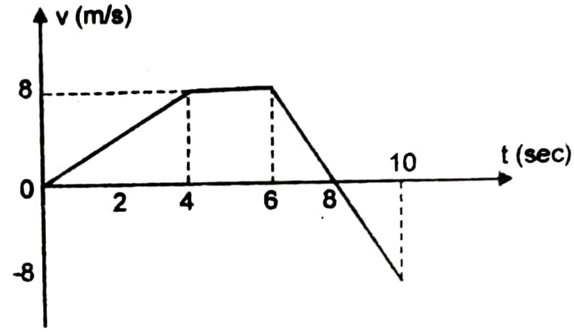


Graph (4)

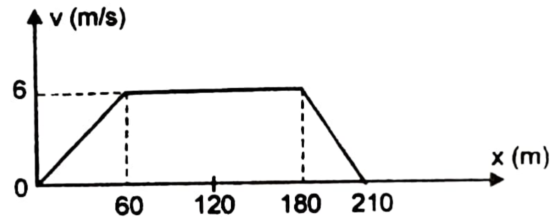
P6. Figure shows a – t curve for a particle performing rectilinear motion. Knowing that at t = 0, v = 4 m/s and x = 20 m, find graphically the velocity and position of the particle at t = 9 sec.



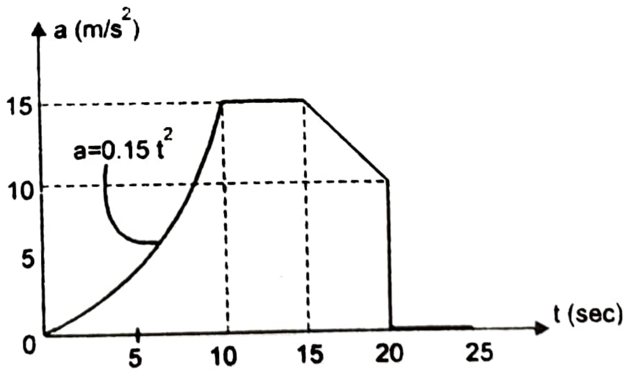
P7. For a particle performing rectilinear motion, the v – t diagram is shown. Draw a – t and x – t diagrams for the motion if at t = 2 sec, x = 20 m. What is the displacement during 6 – 10 sec ? Also find the total distance traveled.



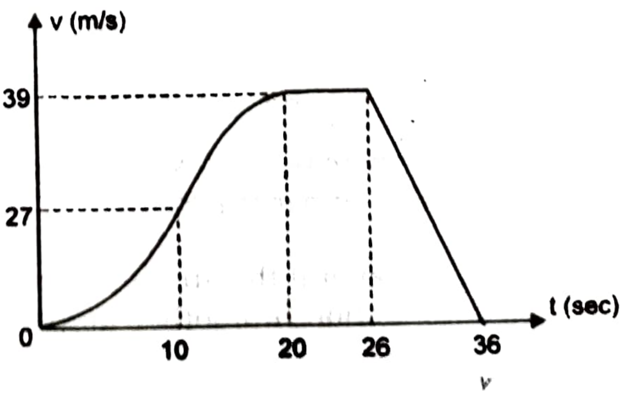
P8. The v – x graph of a rectilinear moving particle is shown. Find acceleration of the particle at 20 m, 80 m and 200 m.



P9. The a – t graph of a train starting from rest along a straight track is shown. Find the position and velocity of the train at the end of 25 sec.



P10. Figure shows v – t curve for a particle starting from rest and finally coming to stop after 36 sec. Find the total distance traveled by the particle. The v – t diagram (0 – 10) sec and (10 – 20) sec are parabolic curves with zero slope at t = 0 and at t = 20 sec.
Hint- (area of semi-parabola of base a and height h is $A = {}^2/_3\, a \times h$)



12.7 Relative Motion

So far the motion analysis was done from a fixed frame of reference, fixed to the earth. However if the motion analysis is undertaken from a moving frame of reference i.e observations by a person also in motion, then such analysis comes under relative motion.

Few examples of situations where relative motion analysis is done are

i) A person in a moving train observes another moving train on a parallel track.

ii) A captain of a moving ship observes the motion of other ships close to it.

iii) The pilot of a fighter plane observing a moving target before striking.

Figure shows two particles A and B moving independent of each other. Let $\bar{r}_A$ and $\bar{r}_B$ be their positions defined from a fixed frame of reference xoy, fixed at o.

If now A observes B, then A will find B to be occupying the position $\bar{r}_{B/A}$. This position is measured from a moving reference located at A. From the vector triangle so formed, we may write

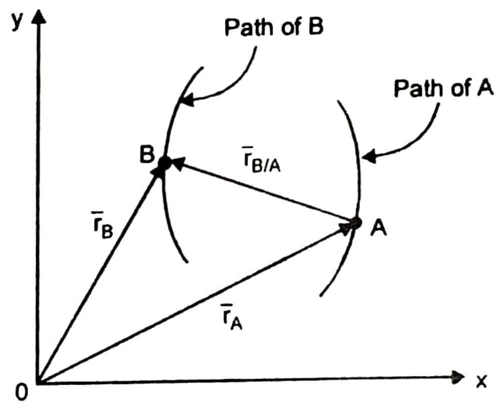


Fig. 12.16

$$\bar{r}_B = \bar{r}_A + \bar{r}_{B/A}$$

or $\bar{r}_{B/A} = \bar{r}_B - \bar{r}_A$ Relative position relation ....[12.23]

Differentiating the above relation w.r.t time, we have

$\bar{v}_{B/A} = \bar{v}_B - \bar{v}_A$ Relative velocity relation ....[12.24]

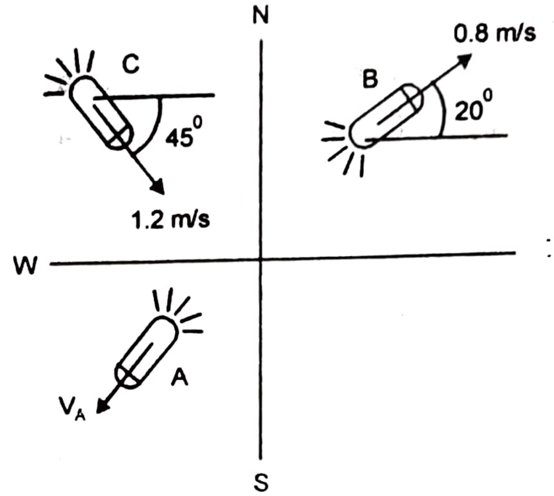
Further differentiating w.r.t time, we get

$\bar{a}_{B/A} = \bar{a}_B - \bar{a}_A$ Relative acceleration relation ....[12.25]

Here $\bar{r}_{B/A}$, $\bar{v}_{B/A}$ and $\bar{a}_{B/A}$ are the relative position, velocity and acceleration of B w.r.t A whereas $\bar{r}_B$, $\bar{v}_B$ and $\bar{a}_B$ are the absolute position, velocity and acceleration of particle B.

In general the absolute motion of any moving particle say B is the sum of absolute motion of another moving particle say A and the relative motion of B w.r.t A

Ex. 12.29 Three ships sail in different directions as shown. If the captain of ship C observes ship A, he finds ship A sailing at 3 m/s at $\theta = 60°$. Find a) true velocity of ship A b) velocity of B as observed by A c) velocity of C as observed by B.



Solution:

a) Given: $v_B = 0.8$ m/s, $\theta = 20°$ $\therefore \bar{v}_B = 0.752\,i + 0.274\,j$ m/s

$v_C = 1.2$ m/s, $\theta = 45°$ $\therefore \bar{v}_C = 0.848\,i - 0.848\,j$ m/s

$v_{A/C} = 3$ m/s, $\theta = 60°$ $\therefore \bar{v}_{A/C} = -1.5\,i - 2.6\,j$ m/s

$$\bar{v}_{A/C} = \bar{v}_A - \bar{v}_C$$

$$-1.5\,i - 2.6\,j = \bar{v}_A - (0.848\,i - 0.848\,j)$$

$$\bar{v}_A = -0.652\,i - 3.45\,j \text{ m/s}$$

$$v_A = 3.51 \text{ m/s}, \ \theta = 79.3°$$ .... **Ans.**

b)

$$\bar{v}_{B/A} = \bar{v}_B - \bar{v}_A$$

$$= (0.752\,i + 0.274\,j) - (-0.652\,i - 3.45\,j)$$

$$= 1.404\,i - 3.724\,j$$

$$\bar{v}_{B/A} = 3.98 \text{ m/s}, \ \theta = 69.34°$$ .... **Ans.**

c)

$$\bar{v}_{C/B} = \bar{v}_C - \bar{v}_B$$

$$= (0.848\,i - 0.848\,j) - (0.752\,i + 0.274\,j)$$

$$= 0.096\,i - 1.122 \text{ m/s}$$

$$\bar{v}_{C/B} = 1.126 \text{ m/s}, \ \theta = 85.1°$$ .... **Ans.**

Ex. 12.30 A trailer traveling to the right on a straight road starts from rest and accelerates uniformly at 1 m/s². A loose container, kept at 12 m from the rear end on the horizontal platform of the trailor, starts slipping backwards with an acceleration of 0.2 m/s² relative to the trailer. Determine

a) **The absolute acceleration of the container**
b) **Velocity of the trailer and the true velocity of container at t = 8 sec.**
c) **Time when the container falls from the trailer.**
d) **True velocity of the container just as it falls from the trailer.**
e) **Distance traveled by the trailer as container falls off.**

Solution:

a) $a_T = 1 \text{ m/s}^2 \rightarrow$ $\therefore \bar{a}_T = 1\,i \text{ m/s}^2$

$a_{C/T} = 0.2 \text{ m/s}^2 \leftarrow$ $\therefore \bar{a}_{C/T} = -0.2\,i \text{ m/s}^2$

$$\bar{a}_{C/T} = \bar{a}_C + \bar{a}_T$$

$$\therefore \quad \bar{a}_C = \bar{a}_{C/T} - \bar{a}_T$$
$$= -0.2\,i + 1i$$
$$= 0.8\,i \text{ m/s}^2$$
$$= 0.8 \text{ m/s}^2 \rightarrow$$

$\therefore$ the absolute acceleration of the container is $0.8 \text{ m/s}^2 \rightarrow$ ... **Ans.**

b) Motion of trailer

$u = 0, v = ?, \quad a = 1 \text{ m/s}^2, t = 8$ sec.

using $v = u + at$
$= 0 + 1 \times 8$
$= 8 \text{ m/s} \rightarrow$

$\therefore$ the velocity of trailer at $t = 8$ sec is $8 \text{ m/s} \rightarrow$ ... **Ans.**

Absolute Motion of container

$u = 0, \quad v = ?, \quad a = 0.8 \text{ m/s}^2, \quad t = 8$ sec

using $v = u + at$
$= 0 + 0.8 \times 8$
$= 6.4 \text{ m/s} \rightarrow$

$\therefore$ the true velocity of container at $t = 8$ sec is $6.4 \text{ m/s} \rightarrow$ ... **Ans.**

c) Relative Motion of container

$u = 0, \quad s = 12 \text{ m}, \quad a = 0.2 \text{ m/s}^2, \quad t = ?$

using $s = ut + \frac{1}{2} at^2$

$12 = 0 + \frac{1}{2} (0.2) (t)^2$

$t = 10.95$ sec

$\therefore$ the container falls off from the trailer at $t = 10.95$ sec ... **Ans.**

d) Absolute Motion of container

$u = 0, \quad v = ?, \quad a = 0.8 \text{ m/s}^2, \quad t = 10.95$ sec.

using $v = u + at$
$= 0 + 0.8 \times 10.95$
$= 8.76 \text{ m/s} \rightarrow$

$\therefore$ the velocity of the container as it falls from the trailer is $8.76 \text{ m/s} \rightarrow$ ... **Ans.**

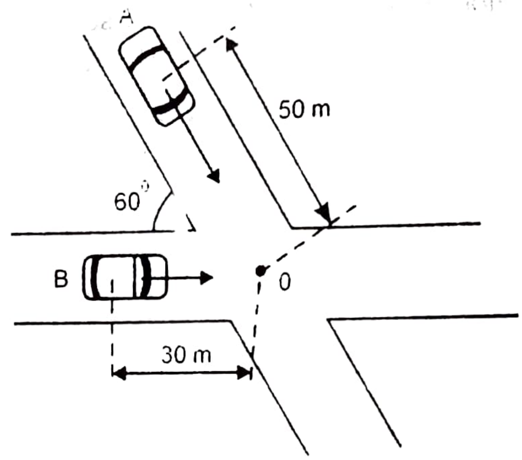
e) Motion of trailer

$u = 0, \quad a = 1 \text{ m/s}^2, \quad t = 10.95$ sec, $\quad s = ?$

using $s = ut + \frac{1}{2} at^2$
$= 0 + \frac{1}{2} \times 1 \times 10.95^2$
$= 59.95$ m

$\therefore$ the trailer has moved 59.95 m just when the container falls from it ... **Ans.**

Ex. 12.31 Figure shows the location of cars A and B at t = 0. Car A starts from rest and travels towards the intersection at a uniform rate of 2 m/s^2. Car B travels towards the intersection at a constant speed of 8 m/s. Determine relative position, velocity and acceleration of car B w.r.t car A. at t = 6 sec.



Solution:

Motion of Car A

Uniform acceleration
u = 0
v = ?
s = ?
a = 2 m/s^2
t = 6 sec
Using v = ut + at
= 0 + 2 × 6
= 12 m/s

Using s = ut + ½ at^2
= 0 + ½ × 2 × $(6)^2$
= 36 m

Motion of car B

Uniform velocity
v = 8 m/s
s = ?
t = 6 sec.

Using v = s/t
8 = s/6
∴ s = 48 m

Relative position of B w.r.t A at t = 6 sec
car A is 50 – 36 = 14 m away from the origin.

r_A = 14 m at θ = 60°

∴ $\bar{r}_A = -7\,i + 12.12\,j$ m

r_B = 48 – 30 = 18 m to the right from the origin

∴ $\bar{r}_B = 18\,i$ m

Now $\bar{r}_{B/A} = \bar{r}_B - \bar{r}_A$

$= 18i - (-7i + 12.12j)$

$= 25i - 12.12j$

$\bar{r}_{B/A} = 27.78\,m, \quad \theta = 25.86°$ ....**Ans.**

Relative velocity of B w.r.t A at t = 6 sec

$$v_A = 12\ m/s,\ \theta = 60^\circ \searrow$$

$$\therefore \quad \bar{v}_A = 6i - 10.39\,j\ m/s$$

$$v_B = 8\ m/s \rightarrow$$

$$\therefore \quad \bar{v}_B = 8i\ m/s$$

Now $\bar{v}_{B/A} = \bar{v}_B - \bar{v}_A$

$$= 8i - (6i - 10.39\,j)$$

$$= 2i + 10.39\,j$$

$\therefore \quad v_{B/A} = 10.58\ m/s, \quad \theta = 79.1^\circ \nearrow$ ....**Ans.**

Relative acceleration of B w.r.t A at t = 6 sec.

$$a_A = 2\,m/s^2, \quad \theta = 60^\circ \searrow$$

$$\therefore \quad \bar{a}_A = 1i - 1.732\,j\ m/s^2$$

also $a_B = 0$

Now $\bar{a}_{B/A} = \bar{a}_B - \bar{a}_A$

$$\therefore \quad \bar{a}_{B/A} = 0 - (1i - 1.732\,j)$$

$$= -1i + 1.732\,j$$

$\therefore \quad a_{B/A} = 2\,m/s^2,\ \theta = 60^\circ \nwarrow$ ......**Ans.**