Chapter 6
Centroid and Centre of Gravity

6.1 Introduction

We know every body is attracted to the centre of the earth by a force of attraction, known as the weight of the body.

The weight being a force acts through a point known as the centre of gravity of the body. In Chapter 2 we have emphasised in article 2.2 that the point of application of the force is one of the necessary data to define a force. Hence the location of centre of gravity becomes important while dealing with the weight force.

In this chapter we will learn to find the centre of gravity of bodies, plane areas and lines. We will also study the approach using *integration method* to find the centre of gravity of figures bounded by curve. Finally we will study the application of the location of centre of gravity to certain engineering problems.

6.2 Centroids and Centre of Gravity defined

6.2.1 *Centre of Gravity*

It is defined as a point through which the whole weight of the body is assumed to act. It is a term used for all actual physical bodies of any size, shape or dimensions e.g. book, cupboard, human beings, dam, car, etc.

6.2.2 *Centroid*

The significance of centroid is same as centre of gravity. It is a term used for centre of gravity of all plane geometrical figures. For example, two dimensional figures (Areas) like a triangle, rectangle, circle, and trapezium or for one dimensional figures (Lines) like circular arc, straight lines, bent up wires, etc.

6.3 Relation for Centre of Gravity

Consider a body of weight W whose centre of gravity is located at G $(\overline{X}, \overline{Y})$ as shown. If the body is split in n parts, each part will have its elemental weight W_i acting through its centre of gravity located at G_i (x_i, y_i). Refer Fig. 6.1

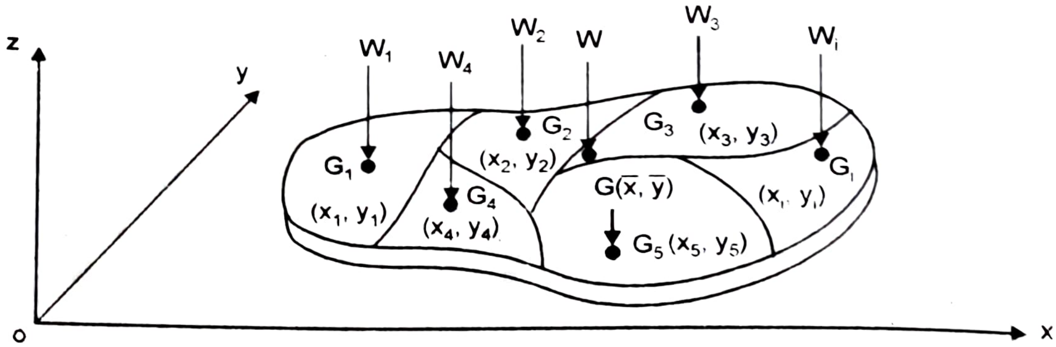


Fig. 6.1

The individual weights W_1, W_2, W_3, ... W_i .... W_n form a system of parallel forces. The resultant weight of the body would then be

$$W = W_1 + W_2 + W_3 \text{ ....} + W_i \text{ ....} W_n$$

$$= \Sigma W_i$$

To locate the point of application of the resultant weight force W using Varignon's theorem (discussed earlier in Chapter 2).

Taking moments about y axis

Moments of individual weights about y axis	=	Moment of the total weight about y axis
$W_1 \times x_1 + W_2 \times x_2 + \text{....} + W_i \times x_i \text{.......} + W_n \times x_n$	=	$W \times \bar{X}$
$\Sigma W_i x_i$	=	$W \times \bar{X}$

$$\bar{X} = \frac{\Sigma W_i x_i}{W} = \frac{\Sigma W_i x_i}{\Sigma W_i} \quad \text{......... 6.1 (a)}$$

Similarly if the moments are taken about x axis

Moments of individual weights about x axis	=	Moment of the total weight about x axis
$W_1 \times y_1 + W_2 \times y_2 + \text{....} + W_i \times y_i \text{.......} + W_n \times y_n$	=	$W \times \bar{Y}$
$\Sigma W_i y_i$	=	$W \times \bar{Y}$

$$\bar{Y} = \frac{\Sigma W_i y_i}{W} = \frac{\Sigma W_i y_i}{\Sigma W_i} \quad \text{......... 6.1 (b)}$$

Using the relation 6.1 (a) and 6.1 (b), the centre of gravity G of a body having co-ordinates $(\bar{X}, \bar{Y})$ can be located.

6.3.1 Relation for Centroid

We recall that weight = mass × g

= (Density × Volume) × g

= (Density × Area × Thickness) × g

$W = \rho \times A \times t \times g$

$= (\rho \times t \times g)\, A$

For uniform bodies i.e. of same density and thickness throughout the body, we get,

$$\bar{X} = \frac{\Sigma(\rho \times t \times g) A_i x_i}{\Sigma(\rho \times t \times g) A_i} = \frac{\Sigma A_i x_i}{\Sigma A_i} \qquad \text{......... 6.2 (a)}$$

Similarly,

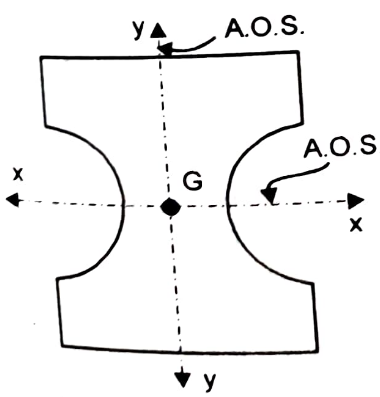
$$\bar{Y} = \frac{\Sigma(\rho \times t \times g) A_i y_i}{\Sigma(\rho \times t \times g) A_i} = \frac{\Sigma A_i y_i}{\Sigma A_i} \qquad \text{......... 6.2 (b)}$$

Using the relation 6.2 (a) and 6.2 (b), the centroid G having co-ordinates $(\bar{X}, \bar{Y})$ of a plane area can be located.

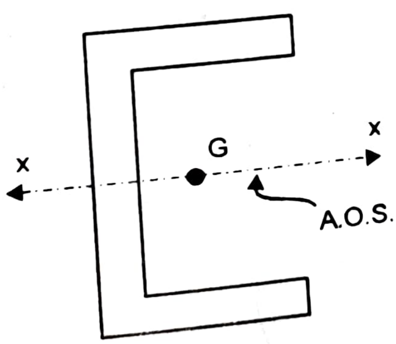
6.4 Axis Of Symmetry (A. O. S.)

Axis Of Symmetry is defined as the line which divides the figure into two equal parts such that each part is a mirror image of the other.

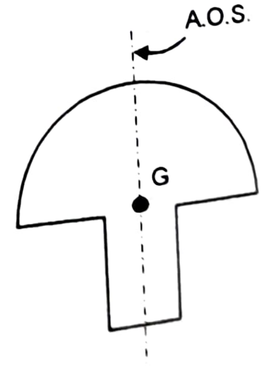
If the geometrical figure whose centroid has to be located is a symmetrical figure, then the centroid will lie on the axis of symmetry (A.O.S.). If the figure has more than one axis of symmetry, the centroid will lie on the intersection of the axis of symmetry. Fig 6.2 below shows the importance of identifying the axis of symmetry.



(a) Fig. having more than one A.O.S., hence centroid lies on their intersection



(b) Fig. is symmetrical about a horizontal axis, hence centroid lies on the horizontal axis.

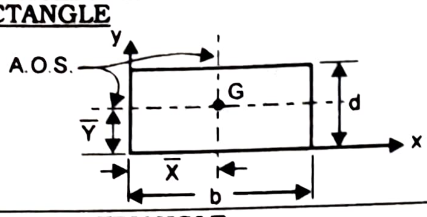
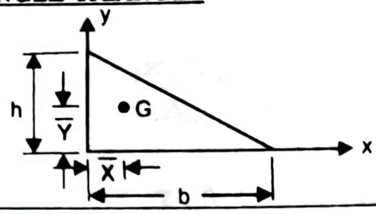
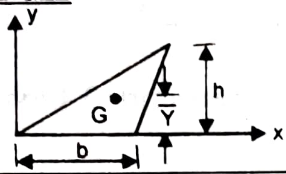
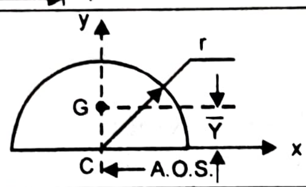
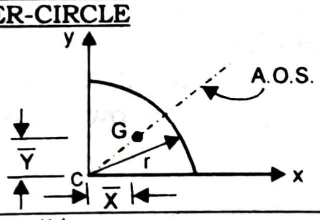
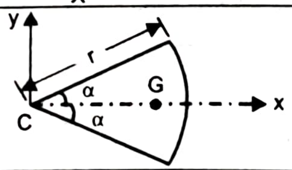
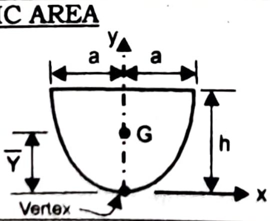
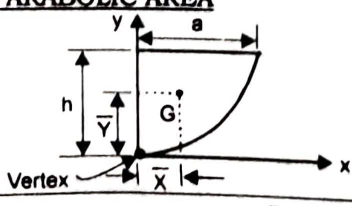


(c) Fig. is symmetrical about a vertical axis, hence centroid lies on vertical axis.

Fig. 6.2

6.5 Centroids of Regular Plane Areas

Table 6.1 shows the centroids of regular plane areas. The co-ordinates $(\bar{X}, \bar{Y})$ of the centroid 'G' are with respect to the axis shown in the figure.

SR. NO.	FIGURE	AREA	$\bar{X}$	$\bar{Y}$
1.	RECTANGLE 	bd	$\frac{b}{2}$	$\frac{d}{2}$
2.	RT. ANGLE TRIANGLE 	½ bh	$\frac{b}{3}$	$\frac{h}{3}$
3.	ANY TRIANGLE 	½ bh	–	$\frac{h}{3}$
4.	SEMI-CIRCLE 	$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$
5.	QUARTER-CIRCLE 	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
6.	SECTOR 	$r^2 \alpha$ *	$\frac{2r\sin\alpha}{3\alpha^{\#}}$	0
7.	PARABOLIC AREA 	$\frac{4ah}{3}$	0	$\frac{3h}{5}$
8.	SEMI PARABOLIC AREA 	$\frac{2ah}{3}$	$\frac{3a}{8}$	$\frac{3h}{5}$

* α is in radians

Table 6.1

α in the denominator is in radians

6.6 Centroid of Composite Area

An area made up of number of regular plane areas is known as a Composite Area.

To locate the centroid of a composite area, adopt the following procedure.

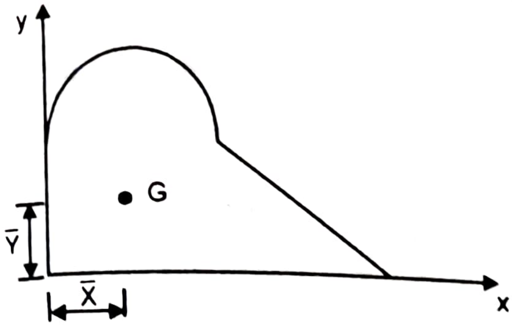


Fig. 6.3 (a)

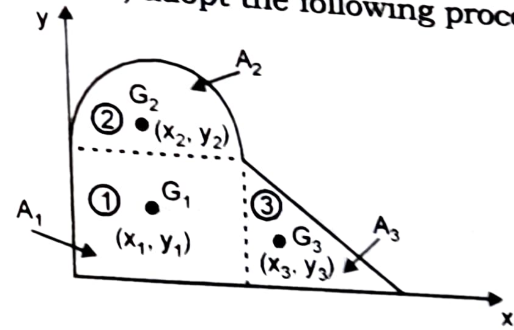


Fig. 6.3 (b)

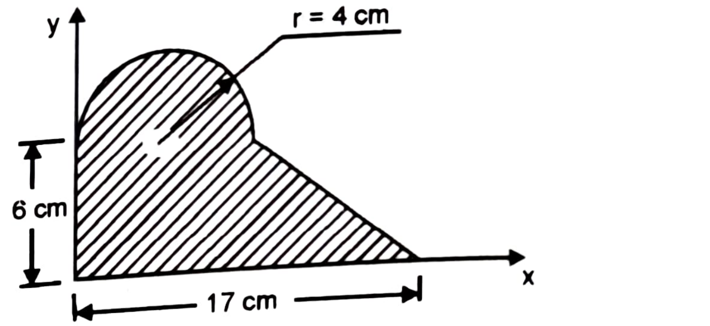
PART	AREA A_i	CO-ORDINATES		$A_i.X_i$	$A_i.Y_i$
		X_i	Y_i		
1.RECTANGLE	A_1	X_1	Y_1	$A_1.X_1$	$A_1.Y_1$
2. SEMI-CIRCLE	A_2	X_2	Y_2	$A_2.X_2$	$A_2.Y_2$
3.RT. ANGLE TRIANGLE	A_3	X_3	Y_3	$A_3.X_3$	$A_3.Y_3$
	ΣA_i			$\Sigma A_i.X_i$	$\Sigma A_i.Y_i$

Table 6.2

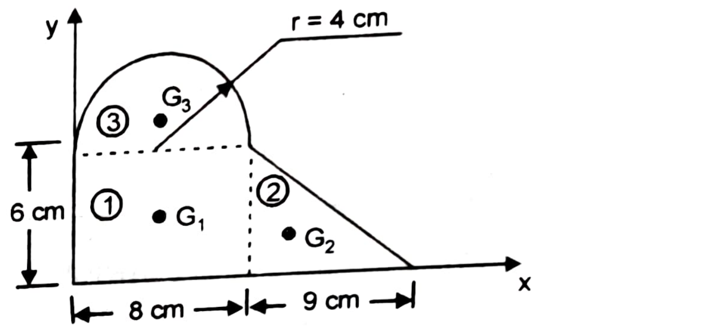
1. Divide the composite area into regular areas as in Fig. 6.3 (b)
2. Mark the centroids G_1, G_2, G_3, ... on the composite figure as shown in Fig. 6.3 (b) and find their co-ordinates w. r. t. the given axis. Let the area of a regular part be A_i and the co-ordinates be X_i and Y_i.
3. Prepare a table as shown (Table 6.2)
4. a) Add up the areas of the different parts to get ΣA_i
   b) Add up the product of area and x co-ordinate of different parts to get $\Sigma A_i.X_i$
   c) Add up the product of area and y co-ordinate of different parts to get $\Sigma A_i.Y_i$
5. The co-ordinates of the centroid of the composite figure are obtained by using relations 6.2 (a) and 6.2 (b) viz.

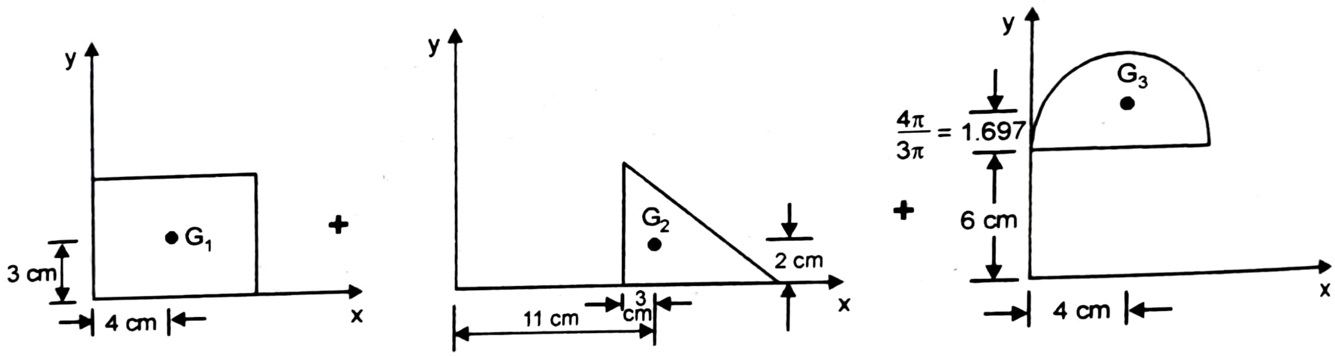
$$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} \quad \text{and} \quad \bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i}$$

Ex. 6.1 Find the centroid of the shaded area shown.



Solution: The composite area can be obtained by adding a rectangle, a rt-angle triangle and a semi-circle. Mark the points G_1, G_2, and G_3 as shown in figure. The areas and the co-ordinates are entered in the table.





PART	AREA A_i cm^2	Co-ordinates		$A_i.X_i$ cm^3	$A_i.Y_i$ cm^3
		X_i cm	Y_i cm		
1. Rectangle	$8 \times 6 = 48$	4	3	192	144
2. Rt. Angled triangle	$\frac{1}{2} \times 9 \times 6 = 27$	11	2	297	54
3. Semi-circle	$\frac{1}{2} \pi (4)^2 = 25.13$	4	7.697	100.53	193.44
	$\Sigma A_i = 100.13$			$\Sigma A_i.X_i$ = 589.53	$\Sigma A_i.Y_i$ = 391.44

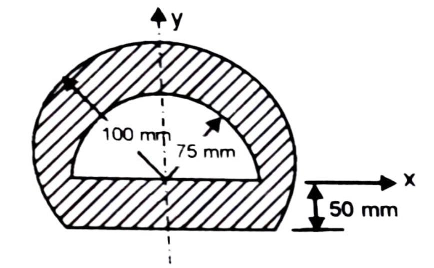
Using $\overline{X} = \dfrac{\Sigma A_i X_i}{\Sigma A_i} = \dfrac{589.53}{100.13} = 5.88$ cm

$$\overline{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{391.44}{100.13} = 3.91 \text{ cm}$$

$\therefore \quad (\overline{X}, \overline{Y}) = (5.88, 3.91)$ cm

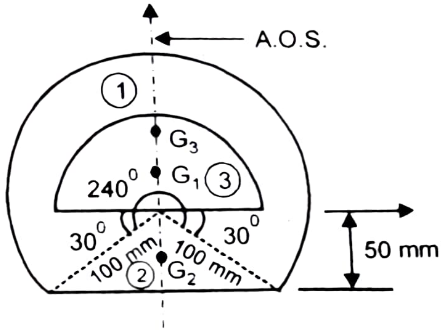
.... **Ans.**

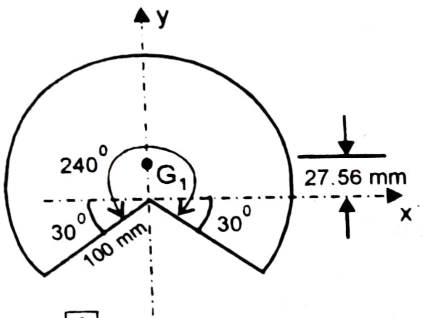
Ex. 6.2 A semi-circular section is removed from the plane area as shown. Find centroid of the remaining shaded area.



Solution: The given shaded area can be obtained by adding a sector, a triangle and subtracting a semi-circle. Marking the centroid of these three parts G_1, G_2 and G_3 as shown in figure. The area and the co-ordinates are entered in the table.

The given composite area is symmetrical about the y-axis. Knowing centroid lies on the A.O.S., which is the y-axis, we have $\overline{X} = 0$





Sector

here $\alpha = \dfrac{240}{2} = 120° = 2.094$ radians

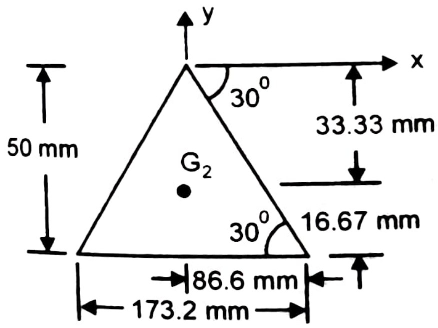
$$\frac{2}{3} \times \frac{r \sin \alpha}{\alpha}$$

$$\frac{2}{3} \times \frac{100 \sin 120}{2.094} = 27.56 \text{ mm}$$

$$\text{Area} = A_1 = r^2 \pi = (100)^2 \times 2.094$$

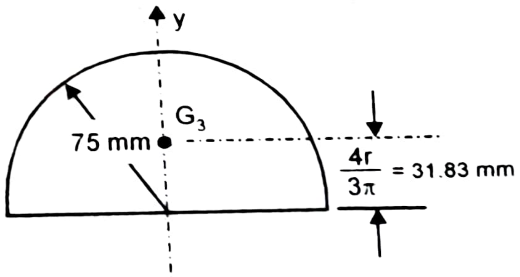
$$= 20944 \text{ mm}^2$$

$\boxed{+}$



Triangle

$\boxed{-}$



Semicircle

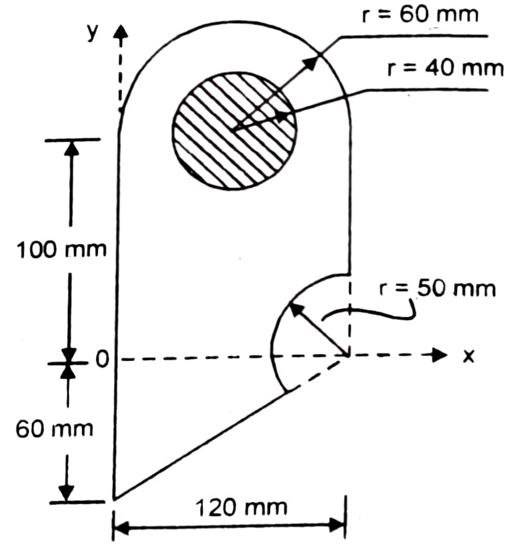
PART	AREA A_i, mm^2	Y_i mm	A_iY_i mm^3
1. SECTOR	20944	27.56	577217
2. TRIANGLE	$\frac{1}{2} \times 173.2 \times 50 = 4330$	– 33.33	– 144319
3. SEMI-CIRCLE	$-\frac{1}{2}\pi(75)^2 = -8835.7$	31.83	– 281241
	$\Sigma A_i = 16438.3$		$\Sigma A_i.Y_i = 151657$

Using

$$\overline{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{151657}{16438.3} = 9.22 \text{ mm}$$

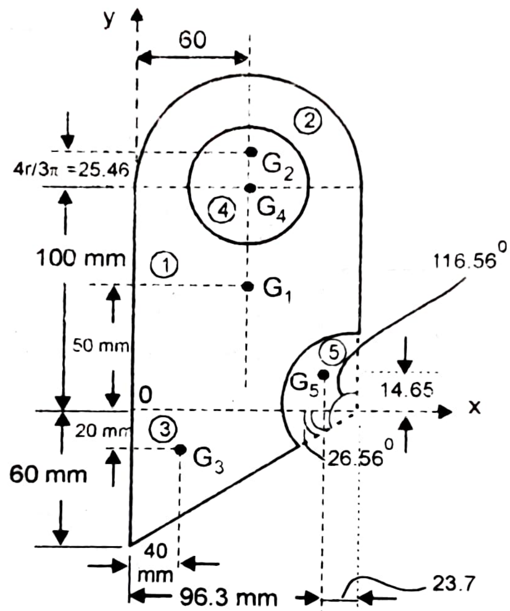
$\therefore$ $(\overline{X}, \overline{Y}) = (0, 9.22)$ mm .... **Ans.**

Ex. 6.3 Determine the centroid of the plane lamina. Shaded portion is removed.



Solution: The given plane lamina can be obtained by adding a rectangle, a semi-circle, a rt-angle triangle and subtracting a circle and a sector.

Let us mark G_1, G_2, G_3, G_4 and G_5 the centroid of the five parts as shown in figure. Entering the areas and the co-ordinates in the table below.



$$\alpha = \frac{116.56}{2} = 58.28° = 1.017\,\text{rad}$$

$$\frac{2}{3} \times \frac{r \sin\alpha}{\alpha} = \frac{2 \times 50 \sin 58.28}{3 \times 1.017}$$

$$= 27.87\text{ mm}$$

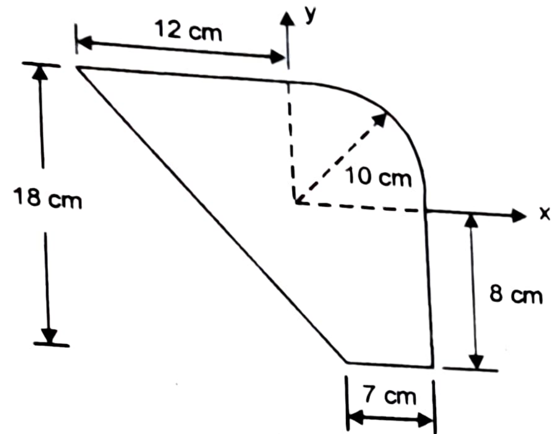
All dimensions are in mm

PART	AREA A_i, mm^2	Co-ordinates		A_iX_i mm^3	A_iY_i mm^3
		X_i mm	Y_i mm		
1. Rectangle	$100 \times 120 = 12000$	60	50	720000	600000
2. Semi-Circle	$\frac{1}{2}\pi (60)^2 = 5654.8$	60	125.46	339288	709478
3. Rt. Angled Δ	$\frac{1}{2} \times 120 \times 60 = 3600$	40	– 20	144000	– 72000
4. Circle	$-\pi (40)^2 = -5026.5$	60	100	– 301593	– 502650
5. Sector	$-(50)^2 \times 1.017 = -2543$	96.5	14.65	– 244891	– 37255
	$\Sigma A_i = 13685.3$			$\Sigma A_iX_i = 656804$	$\Sigma A_iY_i = 697573$

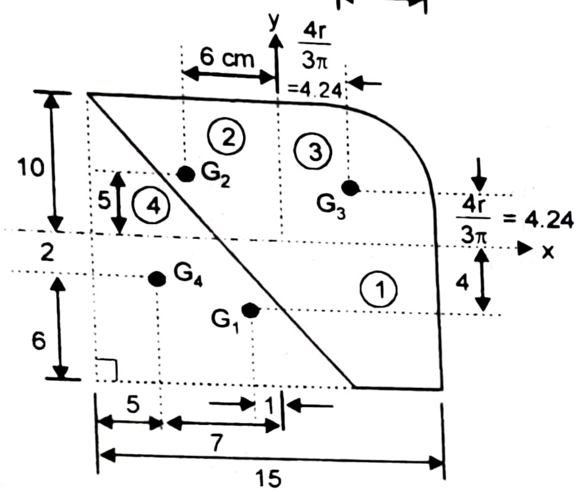
Using $\bar{X} = \frac{\Sigma A_i X_i}{\Sigma A_i} = \frac{656804}{13685.3} = 47.99$ mm and $\bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{697573}{13685.3} = 50.97$ mm

$\therefore$ $(\bar{X}, \bar{Y}) = (47.99, 50.97)$ mm .... **Ans.**

Ex. 6.4 Determine the centroid of the plane lamina shown.



Solution : The given plane area can be obtained by adding two rectangles, quarter circle and removing a rt-angle triangle as shown in figure. Marking the centroid of the four parts as G_1, G_2, G_3 and G_4 on the figure. The areas of the different parts and the co-ordinates of their centroids are entered in the table.



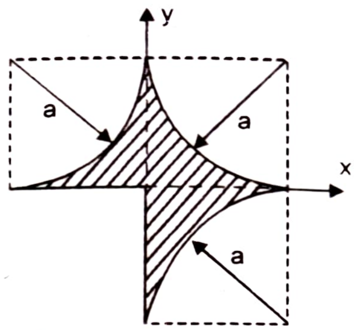
All dimensions are in cm

PART	AREA A_i, cm^2	Co-ordinates (cm) X_i	Y_i	A_iX_i cm^3	A_iY_i cm^3
1. RECTANGLE	$22 \times 8 = 176$	– 1	– 4	– 176	– 704
2. RECTANGLE	$12 \times 10 = 120$	– 6	5	– 720	600
3. QUARTER-CIRCLE	$\frac{\pi (10)^2}{4} = 78.54$	4.24	4.24	333	333
4. RT-ANGLE TRIANGLE	$\frac{-15 \times 18}{2} = -135$	– 7	– 2	945	270
	$\Sigma A_i = 239.54$			$\Sigma A_iX_i = 382$	$\Sigma A_iY_i = 499$

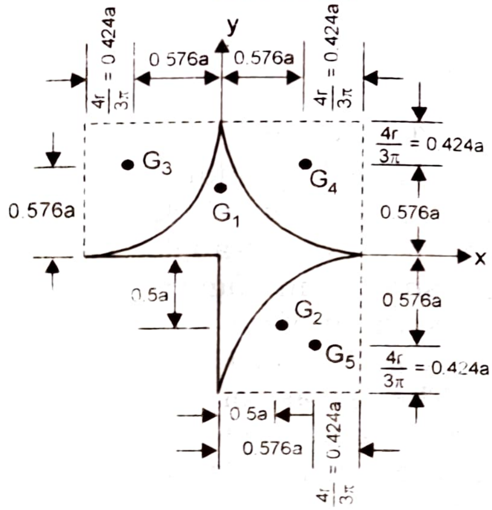
Using $\overline{X} = \frac{\Sigma A_i X_i}{\Sigma A_i} = \frac{382}{239.54} = 1.59$ cm and $\overline{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{499}{239.54} = 2.08$ cm

∴ Centroid 'G' of the plane area has co-ordinates,($\overline{X}$, $\overline{Y}$) = (1.59, 2.08) cm.... **Ans.**

Ex. 6.5 Determine the centroid of the shaded plane area shown.



Solution: The shaded plane area can be obtained by adding a rectangle of dimensions 2a × a, adding a square of dimensions a × a, subtracting three quarter circles of radius 'a'. Let us mark the centroid of the five parts as G_1, G_2, G_3, G_4 and G_5 on the figure. The areas and the co-ordinates are entered in the table.

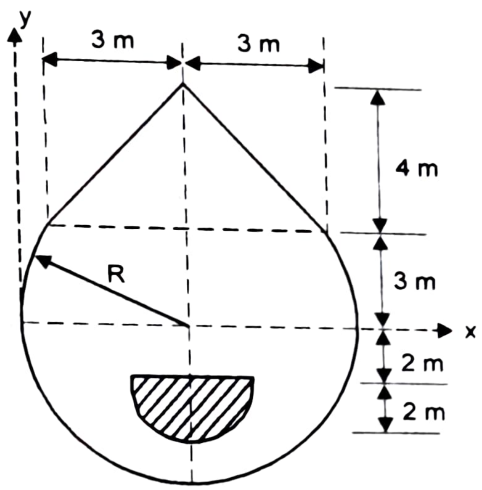


PART	AREA A_i,	Co-ordinates		A_iX_i	A_iY_i
		X_i	Y_i		
1. RECTANGLE	$2a \times a = 2a^2$	0	0.5a	0	a^3
2. SQUARE	$a \times a = a^2$	0.5a	– 0.5a	$0.5a^3$	$-0.5a^3$
3. QUARTER-CIRCLE	$-\frac{\pi(a)^2}{4} = -0.785a^2$	– 0.576a	0.576a	$0.452a^3$	$-0.452a^3$
4. QUARTER CIRCLE	$-\frac{\pi(a)^2}{4} = -0.785a^2$	0.576a	0.576a	$-0.452a^3$	$-0.452a^3$
5. QUARTER CIRCLE	$-\frac{\pi(a)^2}{4} = -0.785a^2$	0.576a	–0.576a	$-0.452a^3$	$0.452a^3$
	$\Sigma A_i = 0.645a^2$			$\Sigma A_iX_i = 0.048a^3$	$\Sigma A_iY_i = 0.048a^3$

Using $\overline{X} = \frac{\Sigma A_i X_i}{\Sigma A_i} = \frac{0.048a^3}{0.645a^2} = 0.074a$ and $\overline{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{0.048a^3}{0.645a^2} = 0.074a$

∴ the centroid 'G' has co-ordinates $(\overline{X}, \overline{Y}) = (0.074a, 0.074a)$ units ....**Ans.**

Ex. 6.6 Find co-ordinates of centre of gravity for the following lamina. Note that the shaded area is an opening in the lamina.



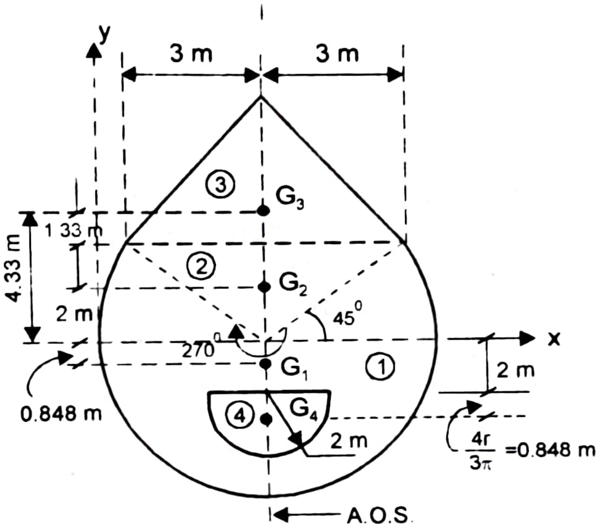
Solution: We note that the value of radius R is not given, but can be calculated from the dimensions given as

$$R^2 = 3^2 + 3^2$$

or $\quad R = 4.24$ m

We also note that the area is symmetrical about an axis parallel to the y-axis at a distance of 4.24 m

Hence, $\bar{X} = 4.24$ m



The composite plane area can be obtained by

adding a sector of radius 4.24 m,

adding a triangle of base 6 m and height 3 m,

adding another triangle of base 6 m and height 4 m,

subtracting a semi-circle of radius 2 m.

Let us mark the centroids of the four parts as G_1, G_2, G_3 and G_4 on the figure. The areas and the co-ordinates are entered in the table.

For the sector $2\alpha = 270^o$ $\quad \therefore \alpha = 135° = 2.356$ radians.

Area of sector $= r^2 \alpha = (4.24)^2 \times 2.356 = 42.36 \text{ m}^2$

The centroid G_1 of the sector lies on the A.O.S. at a distance from the centre

$$= \frac{2 r \sin\alpha}{3\alpha} = \frac{2 \times 4.24 \sin 135}{3 \times 2.356} = 0.848 \text{ m}$$

PART	AREA A_i, m^2	Y_i m	A_iY_i m^3
1. SECTOR	42.36	– 0.848	– 35.92
2. TRIANGLE	½ × 6 × 3 = 9	2	18
3. TRIANGLE	½ × 6 × 4 = 12	4.33	51.96
4. SEMI-CIRCLE	– ½ × π (2)² = – 6.28	– 2.848	17.89
	$\Sigma A_i = 57.08$		$\Sigma A_iY_i = 51.93$

Using

$$\bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{51.93}{57.08} = 0.909 \text{ m}$$

∴ the centroid 'G' of the area has co-ordinates $(\bar{X}, \bar{Y}) = (4.24, 0.909)$ m ....**Ans.**

6.7 Centroid of Lines

So far we have studied to locate the centroid of plane areas. Now let us learn to find out centroid of lines which are also geometrical figures.

Lines are one dimensional figures whose length (L) is prominent than its thickness (b). The thickness also is uniform throughout the length. We therefore modify relation 6.2 (a) and (b) to obtain the relation of centroid of lines.

We recall $$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} \qquad \text{......... 6.2 (a)}$$

$$\therefore \quad \bar{X} = \frac{\Sigma (L \times b)_i x_i}{\Sigma (L \times b)_i} = \frac{\Sigma L_i x_i}{\Sigma L_i} \qquad \text{......... 6.3 (a)}$$

also $$\bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i} \qquad \text{......... 6.2 (b)}$$

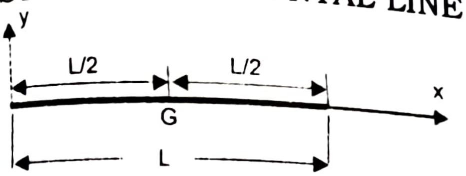
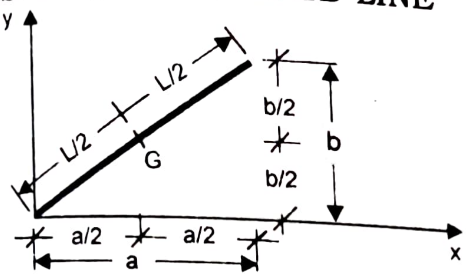
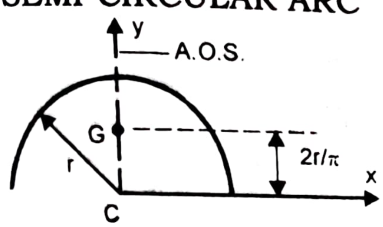
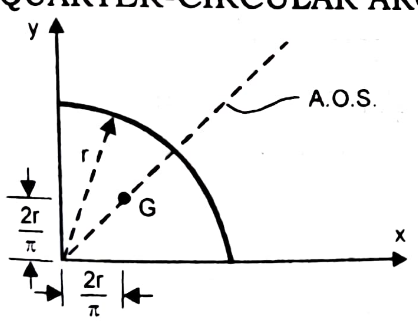
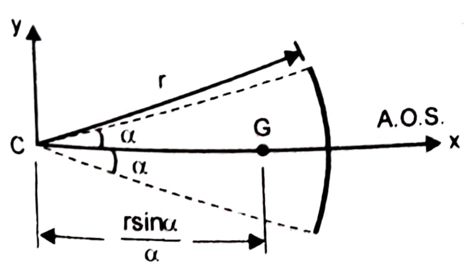
$$\therefore \quad \bar{Y} = \frac{\Sigma (L \times b)_i y_i}{\Sigma (L \times b)_i} = \frac{\Sigma L_i y_i}{\Sigma L_i} \qquad \text{......... 6.3 (b)}$$

Note that thickness b cancels out since it is uniform throughout the length

The physical bodies which are equivalent to lines are bent up wires, pipe lines etc. If these bodies are uniform throughout their length, then the centroid would coincide with the centre of gravity.

6.7.1 Centroids of Regular Lines

Table 6.3 shows the centroid of regular lines. The co-ordinates $(\bar{X}, \bar{Y})$ of the centroid 'G' are with respect to the axis shown in the figure.

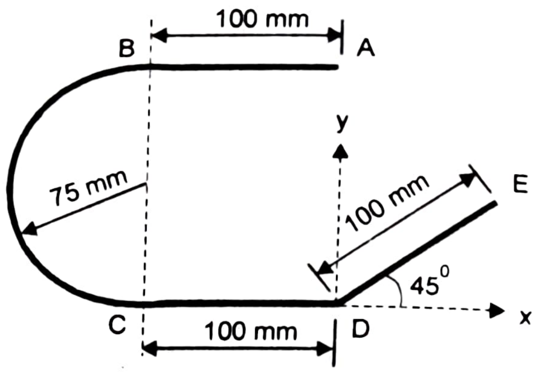
SR. NO	FIGURE	LENGTH	Co-ordinates X	Co-ordinates Y
1.	STRAIGHT HORIZONTAL LINE 	L	$\frac{L}{2}$	0
2.	STRAIGHT INCLINED LINE 	L	$\frac{a}{2}$	$\frac{b}{2}$
3.	SEMI-CIRCULAR ARC 	πr	0	$\frac{2r}{\pi}$
4.	QUARTER-CIRCULAR ARC 	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
5.	CIRCULAR ARC 	$2 r \alpha^{*}$	$\frac{r \sin\alpha}{\alpha^{\#}}$	0

* α is in radians

\# α in the denominator is in radians

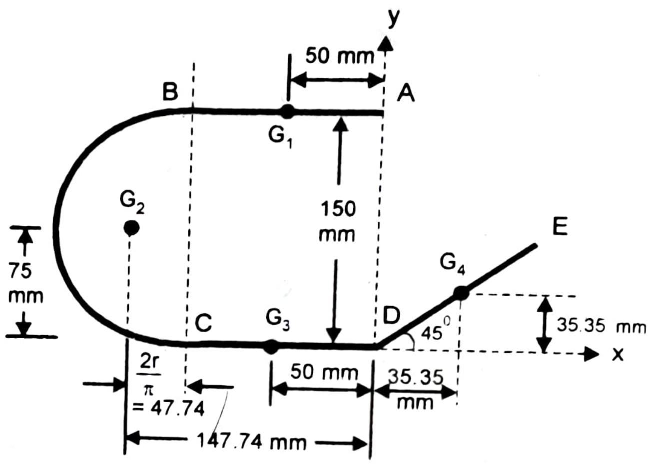
Table 6.3

Ex. 6.7 A uniform wire is bent into a shape shown. Calculate position of C.G. of the wire.



Solution : The given bent up wire can be obtained by adding two straight portions AB and CD, adding a semi-circular portion BC and adding a straight inclined portion DE.

Let us mark the centroids G_1, G_2, G_3 and G_4 of the four parts on the figure. The lengths and co-ordinates of centroids of the different parts are entered in the table.



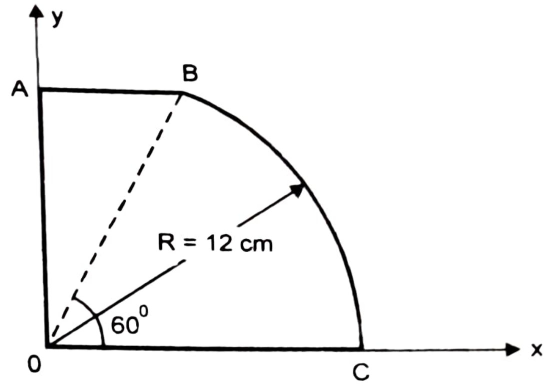
PART	LENGTH L_i, mm	Co-ordinates		$L_i.X_i$ mm²	$L_i.Y_i$ mm²
		X_i (mm)	Y_i (mm)		
AB St. horizontal	100	– 50	150	– 5000	15000
BC Semi circular arc	$\pi \times 75 = 235.62$	– 147.74	75	– 34812	17671
CD St. horizontal	100	– 50	0	– 5000	0
DE St. inclined	100	35.35	35.35	3535	3535
	$\Sigma L_i = 535.62$			$\Sigma L_i.X_i =$ – 41277	$\Sigma L_i.Y_i =$ 36206

Using $\overline{X} = \dfrac{\Sigma L_i X_i}{\Sigma L_i} = \dfrac{-41277}{535.62} = -77.06$ mm and $\overline{Y} = \dfrac{\Sigma L_i Y_i}{\Sigma L_i} = \dfrac{36206}{535.62} = 67.59$ mm

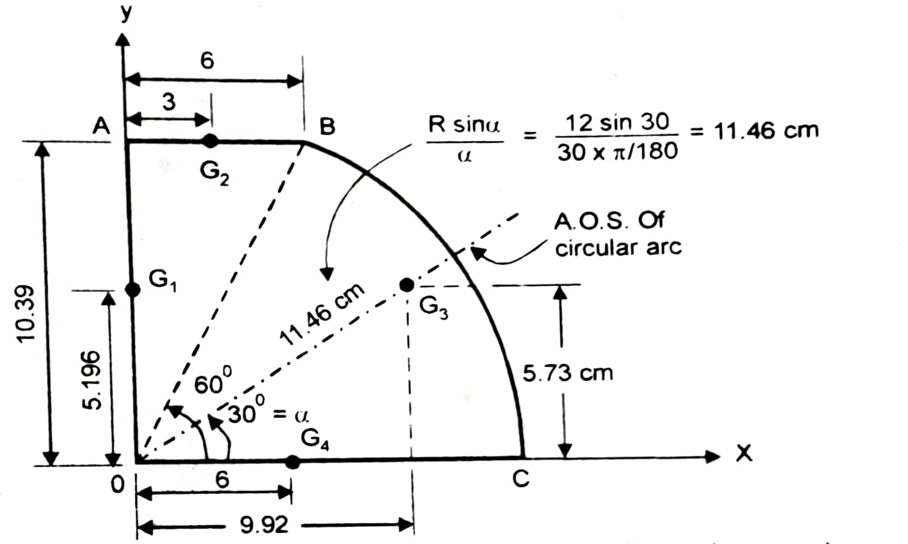
∴ the centroid of the bent up wire has co-ordinates,

$(\overline{X}, \overline{Y}) = (-77.06, 67.59)$ mm ……….**Ans.**

Ex. 6.8 A thin homogeneous wire of uniform cross-section is bent into shape OABCO as shown. Find its centroid.



Solution: The given bent up wire can be obtained by adding three straight portions OA, AB and CO and adding a circular arc.
Let us mark the centroids G_1, G_2, G_3 and G_4 of the four parts on the figure. The lengths and co-ordinates of centroids of the different parts are entered in the table.



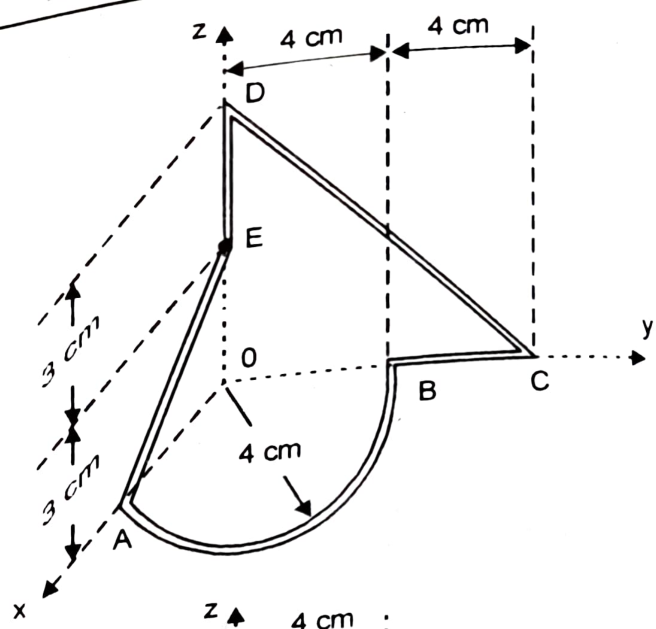
All dimensions are in cm

PART	LENGTH L_i, cm	Co-ordinates (cm) X_i	Y_i	$L_i.X_i$ cm^2	$L_i.Y_i$ cm^2
OA St. vertical	10.39	0	5.196	0	53.98
AB St. horizontal	6	3	10.39	18	62.34
BC Circular arc	$2r\alpha = 2\times12\left[30\times\frac{\pi}{180}\right]$ = 12.56	9.92	5.73	124.6	71.96
CO St. horizontal	12	6	0	72	0
	$\Sigma L_i = 40.95$			$\Sigma L_i.X_i$ = 214.6	$\Sigma L_i.Y_i$ = 188.28

Using $\overline{X} = \dfrac{\Sigma L_i X_i}{\Sigma L_i} = \dfrac{214.6}{40.95} = 5.24$ cm and $\overline{Y} = \dfrac{\Sigma L_i Y_i}{\Sigma L_i} = \dfrac{188.28}{40.95} = 4.59$ cm

∴ the co-ordinates of centroid of the bent up wire are, $(\overline{X}, \overline{Y})$ = (5.24, 4.59) cm ...**Ans.**

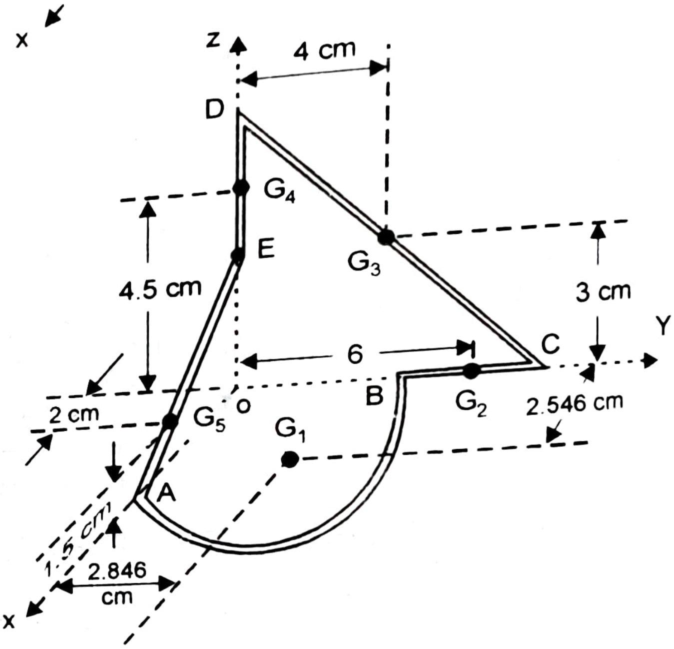
Ex. 6.9 A thin uniform wire ABCDEA is bent into the shape as shown. Determine the centre of gravity of the same. Arc AB of radius 4 cm lies in the xoy plane



Solution: The given bent up wire can be obtained by adding two straight portions BC and DE, adding two straight inclined portions CD and EA and adding a quarter circular arc AB

Let us mark the centroids G_1, G_2, G_3, G_4 and G_5 of the five parts on the figure.

The lengths and the co-ordinates of centroid of the different parts are entered in the table.



PART	Length L_i cm	Co-ordinates			L_iX_i cm^2	L_iY_i cm^2	L_iZ_i cm^2
		X_i cm	Y_i cm	Z_i cm			
AB Quarter-circular arc	$\frac{\pi r}{2} = 6.28$	2.546	2.546	0	16	16	0
BC St-horizontal	4	0	6	0	0	24	0
CD St-inclined	10	0	4	3	0	40	30
DE St-vertical	3	0	0	4.5	0	0	13.5
EA St-inclined	5	2	0	1.5	10	0	7.5
	$\Sigma_i L_i = 28.28$				$\Sigma L_i.X_i = 26$	$\Sigma L_i.Y_i = 80$	$\Sigma L_i.Z_i = 51$

Using $\overline{X} = \dfrac{\Sigma L_i X_i}{\Sigma L_i} = \dfrac{26}{28.28} = 0.919$ cm

$$\overline{Y} = \frac{\Sigma L_i Y_i}{\Sigma L_i} = \frac{80}{28.28} = 2.828 \text{ cm}$$

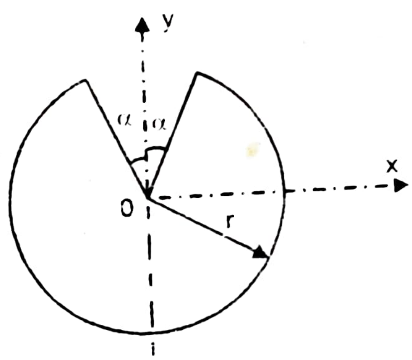
$$\overline{Z} = \frac{\Sigma L_i Z_i}{\Sigma L_i} = \frac{51}{28.28} = 1.803 \text{ cm}$$

$\therefore$ the centroid of the bent up wire has co-ordinates,

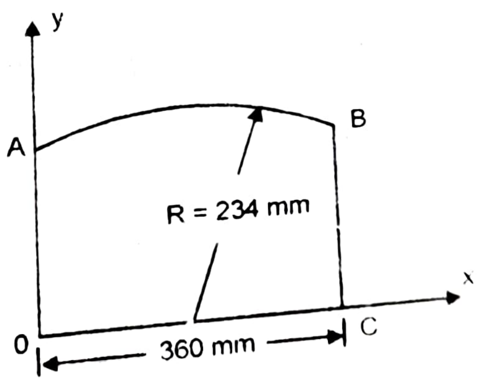
$(\overline{X}, \overline{Y}, \overline{Z}) =$ (0.919, 2.828 , 1.803) cm .... **Ans**

Excercise 6.2

P1. A thin homogeneous wire is built into a shape as shown. Determine the position of centre of gravity of the wire. Take $\alpha = 30^\circ$



P2. A symmetrically shaped bent up wire ABCOA is shown. Find its centre of gravity



P3. A bent up wire ABCDE is as shown. Locate its centre of gravity.