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	Course : AM	- T
	Experiment / as	signment / tutorial No9
١	Grade:	Signature of the Faculty with date

	Tutorial 9: Complex Numbers
Q1)	If α, β are the roots of the equation $\alpha^2 - 2x + 2 = 0$, prove that $\alpha^2 + \beta^2 = 2 \cdot 2^{1/2} \cos n \pi / 4$.
	$\chi^2 - 2\chi + 2 = 0$ prove that $\chi^n + \beta^n = 2 \cdot 2^{n/2} \cos n\pi I/4$, Hence deduce $\chi^8 + \beta^8 = 32$
. Sol°	$\chi^2 - 2\chi + 2 = 0$
	$\frac{1}{2} = 1 + i ; 1 - i$
98	$ \text{Let}, \propto = 1 + i ; \beta = 1 - i$
	a & B can be written as
	$\alpha = \sqrt{2} \cos \Pi + i\sqrt{2} \sin \Pi$
	$B = \sqrt{2} \cos \Pi - i\sqrt{2} \sin \Pi$
	4
	Hence, for $tx^n + \beta^n = \left[\sqrt{2} \cos II + i\sqrt{2} \sin II \right]^n + \left[\sqrt{2} \cos II - i\sqrt{2} \sin II \right]^n$
	By solving further,
	$= \sqrt{2} \cos \pi n + i \sqrt{2} \sin \pi n + \sqrt{2} \cos \pi n - i \sqrt{2} \sin \pi n$
	$= 2\left(\overline{J_2}^{\circ} \cos n\overline{I}\right)$
	$\alpha^{n} + \beta^{n} = 2 \cdot 2^{n/2} \cdot \cos n \mathbb{I}$
	$\alpha + \beta = 2.2 \cdot \cos \beta$
, = -	For, $\alpha^8 + \beta^8 = 2 \cdot 2^{\cdot 8/2} \cos 8\pi$
	4
	= 2.2" cas 211
	$\left[\begin{array}{ccc} \alpha^{3} + \beta^{3} & = 32 \end{array} \right] \qquad \left[\begin{array}{ccc} \cos 2\Pi = 1 \end{array} \right]$
	0 - 23/1
92)	Find all the values of (1+i)3/4 and find
	continued product of all the roots.
Sal"	$(1+i)^{3/4} = (\sqrt{2} \cos T + i \sqrt{2} \sin T)^{3/4}$
	4 4
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	3/4 5
	$= (\sqrt{2})^{3/4} \left[\cos T + i \sin T \right]^{3/4}$
	$= (\sqrt{2})^{3/4} \left[\cos \left(2K\Pi + \Pi \right) + i \sin \left(2K\Pi + \Pi \right) \right]^{3/4}$
	$= (\sqrt{5})^{8/4} \left[\cos \left(\frac{8K\Pi + \Pi}{4} \right) + i \sin \left(\frac{8K\Pi + \Pi}{4} \right) \right]^{3/4}$
	$= (\sqrt{2})^{3/4} \left[\cos \left(-94 \text{KTI} + 3 \text{TI}^{*} \right) + i \sin \left(-24 \text{KTI} + 3 \text{TI} \right) \right]$
	16)
•	
	For the values, $K = 0 \rightarrow x_0 = (\sqrt{2})^{3/4} \left[\cos 3TI + i \sin 3TI\right]$
	L 16 16 16 J
	$K = 1 \longrightarrow \chi_1 = (\sqrt{2})^{3/4} \left[\cos 2711 + i \sin 2711 \right]$ 16 16
, ,	$K = 2 \rightarrow \chi_2 = (\sqrt{2})^{3/4} \left[\cos 51 \pi + i \sin 51 \pi \right]$
	[16]
	$K = 3 \rightarrow \chi_3 = (\sqrt{2})^{3/4} \left[\cos \frac{751}{16} + i \sin \frac{751}{16} \right]$
*	
	$\therefore \chi_{6} \chi_{1} \chi_{2} \chi_{3} = (\sqrt{3})^{3} \cdot \cos(3\Pi + 27\Pi + 5\Pi\Pi + 75\Pi) +$
	(sin (3T+27T+51T+75T)
	16
	$ \cdot , \chi_0 \chi_1 \chi_2 \chi_3 = 2.\sqrt{2} \left[\cos \left(\frac{156 \pi}{16} \right) + i \sin \left(\frac{156 \pi}{16} \right) \right]$
1	
03)	$ \frac{Tf}{u} = \log \tan \left(\frac{T}{4} + 0 \right) \text{Prove that} $
	(i) coshu = sec 0 (ii) sinhy = tan 0
A	(iii) $tanhu = sin 0$ (iv) $tanhu = tan 0$

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Batch:	Roll No.:
Name :	
Course :	
Experiment / a	essignment / tutorial No
Grade:	Signature of the Faculty with date

	,
Sol	$u = \log \tan \left(\frac{1}{4} + 0 \right)$
	4.2
	$e^{4} = \tan\left(\frac{1}{4} + 0\right)$ (Property of log)
	4 2
	$e^{\alpha} = 1 + \tan^{\theta/2} - 0$
	1 - tan 0/2
	$e^{-q} = 1 - \tan \theta/2$ — 2
	1 + tan 0/2
	let $\cosh u = e^{u} + e^{-u}$
€	2
	$= \left(1 + \tan^{0} / 2 + 1 - \tan^{0} / 2\right) \left[\text{from } 0 \& 0\right]$
	1-tan 0/2 1+tan 0/2
	$= (1 + \tan^{0}/2)^{2} + (1 - \tan^{0}/2)^{2}$
	$\frac{2-2\left(\tan^2\theta\right)}{2}$
	$= (1+2\tan \theta/2 + \tan^2 \theta/2 + 1 - 2 \tan \theta/2 + \tan^2 \theta/2)$
	$2 - 2 tan^2 0/2$
	$= 2 + 2 \tan^2 \theta / 2$
	2 - 2 ton 2 0/2
1	cosh u =
	cos O
	We get, cosh u = sec 0 — 3
	We know that.
	$\sin hu = \sqrt{\cosh^2 u - 1}$
	$\frac{\sinh u = \sqrt{\sec^2 0 - 1} \left[\text{from } 3 \right]}{\sin h u} = \sqrt{\frac{20}{3}}$
	$\sin h u = \sqrt{\tan^2 \theta}$
	$\sin h u = \tan \theta - G$
	tanh u = sinh u
- 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1	cosh u
- 1	(05)1 4

	= tan 0 [from 3 & 9]
	sec 0
	= sin 0.cos 0
	cos O
	tanh u = sin 0 - 6
	considering,
	tanh u = sinh u/2
	COS h 12
	Multiplying by 2 cosh 4/2
	$=$ $\sin h u$
	2 cash 2 4/2
	= sin hy
	1+ cosh u
	= tan 0 [from 3 & 4]
	1+sec0
	$= \sin \theta / \cos \theta$
	(cos0+1) / cos0 =_sin0
	$= 2\sin^{\theta/2}\cos^{\theta/2}$
	2611 12 (08 12
	$= \sin \theta/2$
	cos 9/2
	tan y = tan 0 - 6
	2 2
	Hence Proved
04)	If $cos(x+iy) = cos x + i sin x$, prove that
	(i) $\sin \alpha = \pm \sin^2 x = \pm \sin h^2 y$
	(ii) $\cos 2x + \cosh 2y = 2$



Batch:	Roll No.:	PAZ.
Name :		4
Course :		T. 4.
Experiment / ass	ignment / tutorial No	25
Grade:	Signature of the Faculty	

	Signature of the Faculty with date
Sol	$0 \cos (x + iu) = \cos x \cos iu = \sin x \sin iu$
	$\frac{1}{\cos \alpha + i \sin \alpha} = \cos \alpha \cos i y - \sin \alpha \sin i y$ $\cos \alpha + i \sin \alpha = \cos \alpha \cos y - i \sin \alpha \sin y$
	separate,
	$\cos \alpha = \cos x \cosh y$
	sio x = rsio x six h
	sin x = -sin x sin hy Square & add,
	$\frac{1}{12} \frac{1}{12} \frac$
	$\frac{\cos^2 x + \sin^2 x}{\cos^2 x + \sin^2 x} = \cos^2 x + \sin^2 x + \sin^$
	$= \frac{\cos^2 x \cosh^2 y + \sin^2 x \sinh^2 y}{\cos^2 x \cosh^2 y}$
	$= (1+\sinh^2 y)(1-\sinh^2 y) + \sin^2 x \cdot \sinh^2 y$ $= 1-\sin^2 x + \sin h^2 y - \sin h^2 y \cdot \sin^2 y + \sin^2 x \sin h^2 y$
	= 1-sin x + sin h y -sin h y . sin 2 y + sin 2 x sin h y
	$\frac{0}{1-\sin^2 x} = -\sin^2 x$
	$= 1 - \sin^2 x + \sinh^2 y - \sinh^2 y \cdot \sin^2 y + \sin^2 x \sinh^2 y$ $0 = -\sinh^2 y - \sin^2 x$ $\therefore \pm \sin h y = \pm \sin^2 x - 0$ Hence Proved $= -\sin^2 x + \sinh^2 y - \sinh^2 y \cdot \sin^2 y + \sin^2 x \sinh^2 y$
	Hence Proved
	2) $\cos 2x + \cosh^2 y = (1 - \sin^2 x \cdot 2) + (1 + 2 \sinh^2 y)$
	$= 1 - 2 \sin^2 x + 1 + 2 \sinh^2 y$
	but, $\sin^2 x = \sinh^2 y$ [from 0]
	$\frac{(2) \cos 2x + \cosh^{2}y = (1 - \sin^{2}x \cdot 2) + (1 + 2 \sinh^{2}y)}{= 1 - 2 \sin^{2}x + 1 + 2 \sinh^{2}y}$ $= 1 - 2 \sin^{2}x + 1 + 2 \sinh^{2}y$ but, $\sin^{2}x = \sinh^{2}y$ [from 0] $\cos 2x + \cosh^{2}y = 2$
05)	Prove that $\cosh^{-1}(\sqrt{1+x^2}) = \tanh^{-1}(x)$
*	$\sqrt{1+\chi^2}$
sol"	consider
	$L-H-S = \cosh^{-1}\sqrt{1+x^2} = a$
	$\sqrt{1+x^2} = \cosh a$
a a	$1 + \chi^2 = \cosh^2 a$
	But, $1 + \sinh^2 a = \cosh^2 a$
	$\therefore x = sinha - 0$
	(onsider,
	$x = \sin ha$
	$\sqrt{1+x^2}$ cosha
	: sin ha = tan ha
	cos ha
	200 110

	-
	$\therefore \tan^{-1}(\sin ha) = a$
	cos ha
	$\frac{1}{1+x^2} = a$
Α	$\sqrt{1+x^2}$
	$\frac{1}{12} \left(\frac{\cosh^{-1}(\sqrt{1+x^2})}{1+x^2} \right) = \frac{\tanh^{-1}(x)}{1+x^2}$
	(11+x²)
	LHS = RHS
	Hence Proved
06)	Find the value of log [sin (x+iy)]
C 10	
Sal	We know that,
	$\sin(x+iy) = \sin x \cos iy + \cos x \sin iy$
-	$= \sin x \cosh y + i \cos x \sinh y$
	i. log (sin x cosh y + i cos x sin hy) We know,
	$\log \left(x + \delta y\right) = 1 \log \left(\alpha^2 + y^2\right) + i + \alpha^{-1} \left(y\right)$
	We know, $\log (x+iy) = 1 \log (x^2+y^2) + i \tan^{-1}(y)$
	$\frac{1}{2} \log \left(\frac{\sin x \cosh y + i \cos x \sinh y}{\cos x \sinh y} \right)$ $= \frac{1}{2} \log \left(\frac{\cosh^2 y \sin^2 x + \cos^2 x \sinh^2 y}{\cos x \sinh y} \right) + i \tan^2 y \sin x \cosh y}{\cos x \sinh y}$
	= 1 log (cosh 2 ysin 2 + cos 2 x sinh 2 y] + i tan - / sin x coshy)
	2 U (cosx sinhy)
	= 1 log (cosh 2y -cos2x .cosh2x +cos2x cosh2y -cos2x)+i tan-1(cotx.t
	2
	$= \frac{1}{2} \log \left[\frac{1 + \cos 2x}{2} - \frac{1 + \cos 2x}{2} \right] + i \tan^{-1}(\cot x \cdot \tanh y)$
	= $\frac{1 \log \left[\frac{1}{2} \left(\cosh 2y - \cos 2x \right) \right] + \left[\tan^{-1} \left(\cot x \cdot \tan hy \right) \right]}{2}$
	2 12
	Hence,
	$\log (\sin (\alpha + iy)) =$
	1 log [1 cos h2y -cos 2x] + i tan-1 (cotx .tan xhy)