$\frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Similar matrices

$7-15$

$A=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $P=\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$, $P^{-1}=\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

$$P^{-1}AP=\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}} = \underbrace{\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}} = \begin{bmatrix} -1 & -8 \\ 1 & 5 \end{bmatrix} = B$$

$$P^{-1}AP=\begin{bmatrix} -1 & -8 \\ 1 & 5 \end{bmatrix} = B$$

$|A| = \underline{3}$ $|B| = -5+8 = \underline{3}$

Similar Matrices

$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ $\qquad \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$7-15$

$$P^{-1}AP = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}} = \underbrace{\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}} = \begin{bmatrix} -1 & -8 \\ 1 & 5 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} -1 & -8 \\ 1 & 5 \end{bmatrix} = B$$

$|A| = \underline{3} \qquad |B| = -5+8 = \underline{3}$ —(i)

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 1 = 4 - 4\lambda + \lambda^2 - 1 = \underline{\lambda^2 - 4\lambda + 3}$$

$$|B-\lambda I| = \begin{vmatrix} -1-\lambda & -8 \\ 1 & 5-\lambda \end{vmatrix} = (-1-\lambda)(5-\lambda) + 8 = -5 + \lambda - 5\lambda + \lambda^2 + 8$$

$$= \underline{\lambda^2 - 4\lambda + 3}$$

$$= \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & 5 \end{bmatrix} = B$$

$$P^{-1}AP = \begin{bmatrix} -1 & -8 \\ 1 & 5 \end{bmatrix} = B$$

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$$|B-\lambda I| = \begin{vmatrix} -1-\lambda & -8 \\ 1 & 5-\lambda \end{vmatrix} = (-1-\lambda)(5-\lambda) + 8 = -5 + \lambda - 5\lambda + \lambda^2 + 8$$

$$= \underline{\lambda^2 - 4\lambda + 3}$$

$A, B \rightarrow$ Similar matrices.

A matrix $\underline{B}$ is said to be similar $\underline{A}$, if $\exists$ a non-singular matrix P s.t $B = P^{-1}AP$.

To find similar matrix
Trace (sum of diagonal) should be equal
Determinant should be equal

$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 6 & 3 \\ -5 & -2 \end{bmatrix}$ $-12 + 15 = 3$

$|A| = 3$ $|B| = -12 + 15 = 3$

$$|B - \lambda I| = \begin{vmatrix} 6-\lambda & 3 \\ -5 & -2-\lambda \end{vmatrix} = (6-\lambda)(-2-\lambda) + 15 = -12 - \underline{6\lambda} + \underline{2\lambda} + \lambda^2 + 1$$

$$= \lambda^2 - 4\lambda + 3$$

$A=\begin{bmatrix}2&1\\1&2\end{bmatrix}$ $\quad B=\begin{bmatrix}6&3\\-5&-2\end{bmatrix}$ $\quad -12+15=3$

$|A|=3$ $\quad |B|=-12+15=3$

$$|B-\lambda I| = \begin{vmatrix}6-\lambda&3\\-5&-2-\lambda\end{vmatrix} = (6-\lambda)(-2-\lambda)+15 = -12-\underline{6\lambda}+\underline{2\lambda}+\lambda^2+\underline{15}$$

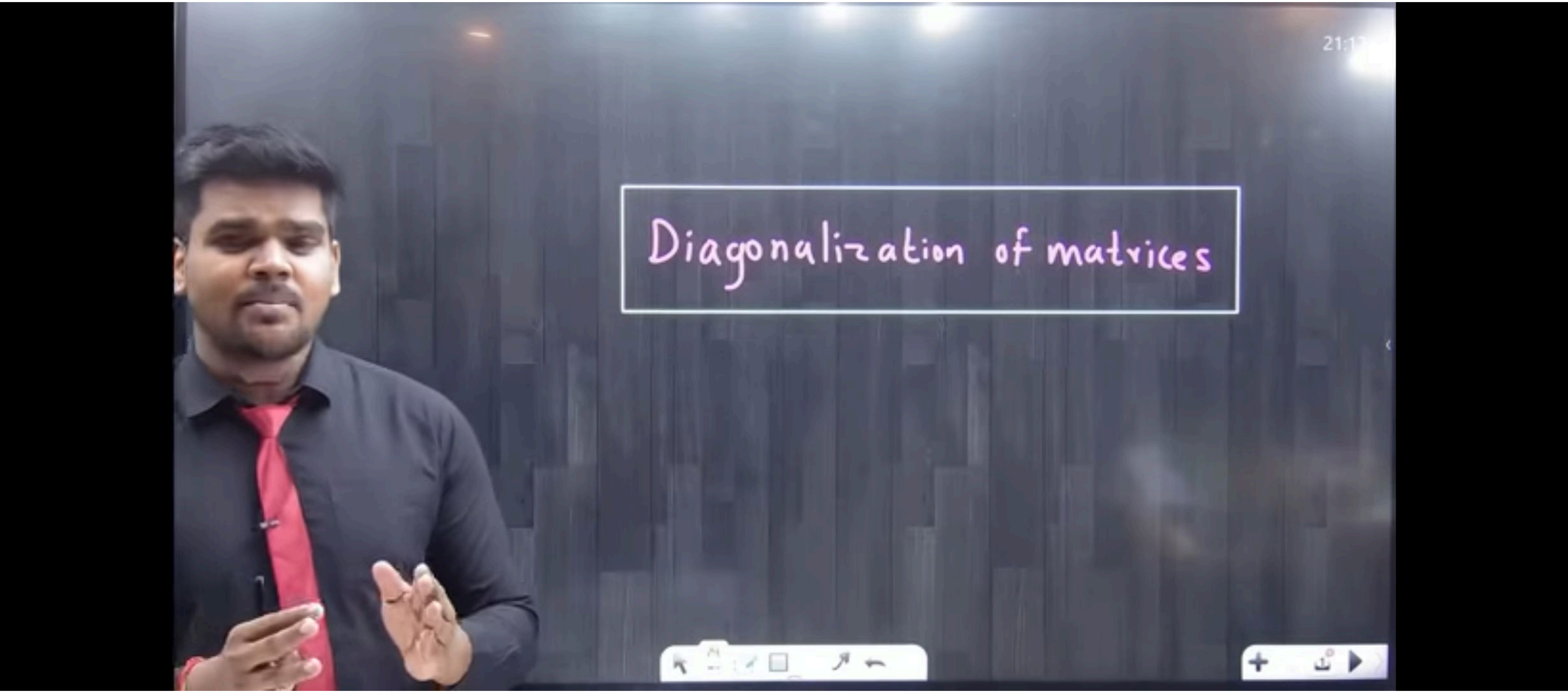
$$= \lambda^2-4\lambda+3$$

$A=\begin{bmatrix}2&1\\0&2\end{bmatrix}$ $\quad |A|=4$ $\quad \lambda=2,2$ $\quad B=\begin{bmatrix}2&0\\0&2\end{bmatrix}$ $\quad |B|=4$

$A\not\sim B$ $\quad \lambda=2,2$

$$P^{-1}BP = P^{-1}\begin{bmatrix}2&0\\0&2\end{bmatrix}P = P^{-1}(2)IP$$

$$=2P^{-1}P = 2I = \begin{bmatrix}2&0\\0&2\end{bmatrix}$$


21:1
Diagonalization of matrices

Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. Find the transforming matrix and Diagonal matrix.

$|A - \lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix} = 0$$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

$S_1 = 6$

$S_2 = -11 + 14 + 8 = 11$

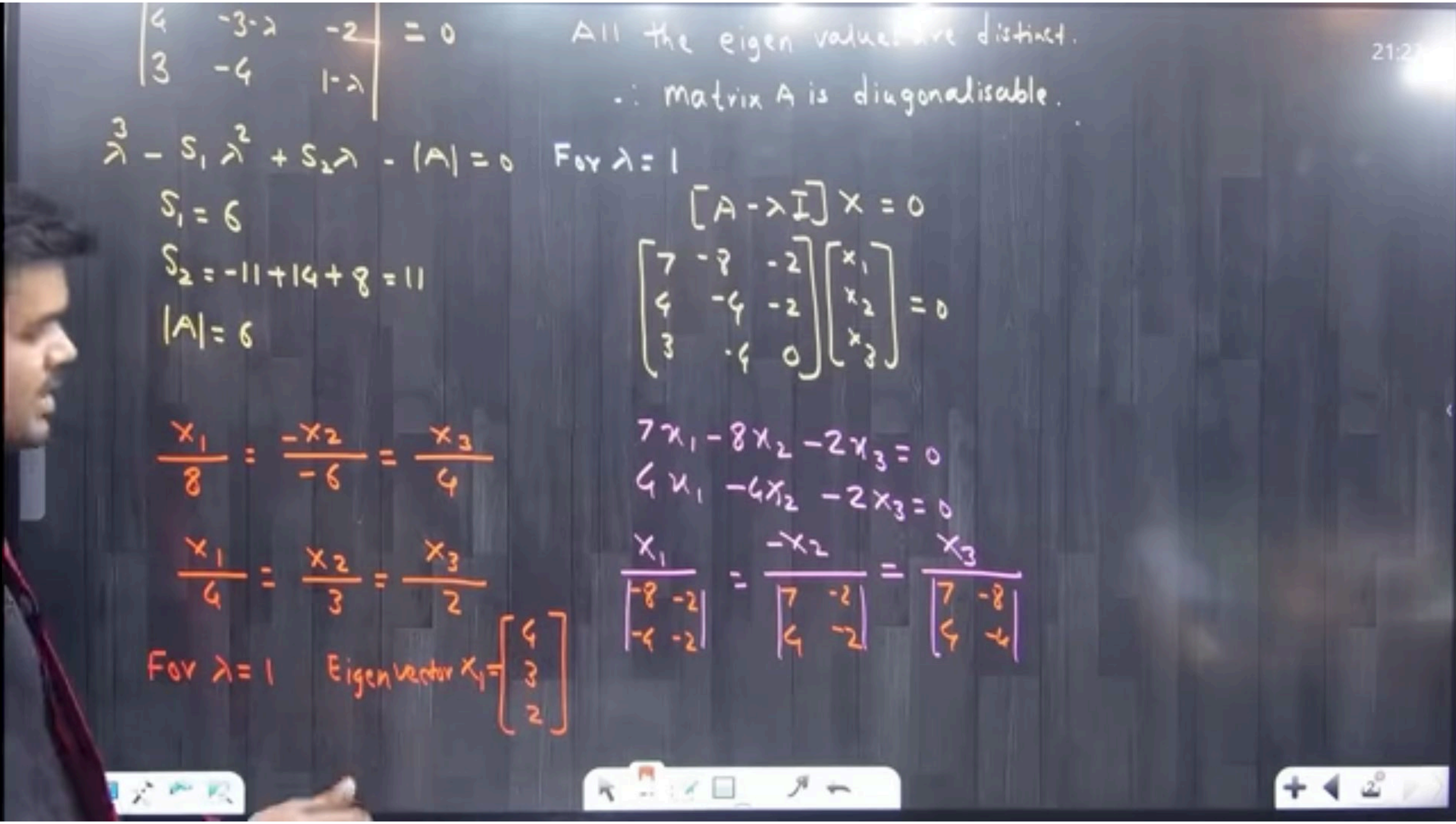
$|A| = 6$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

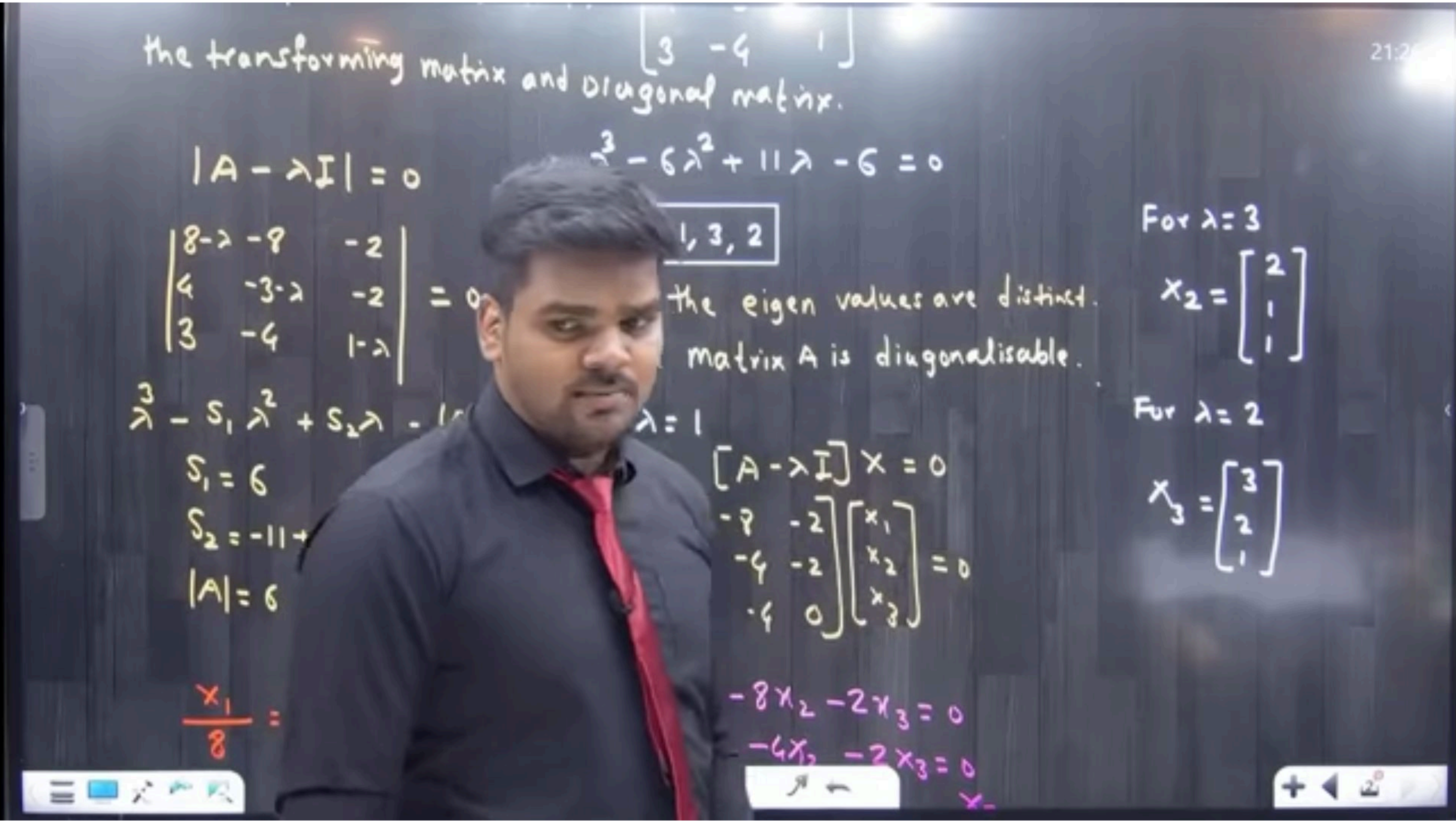
$\boxed{\lambda = 1, 3, 2}$

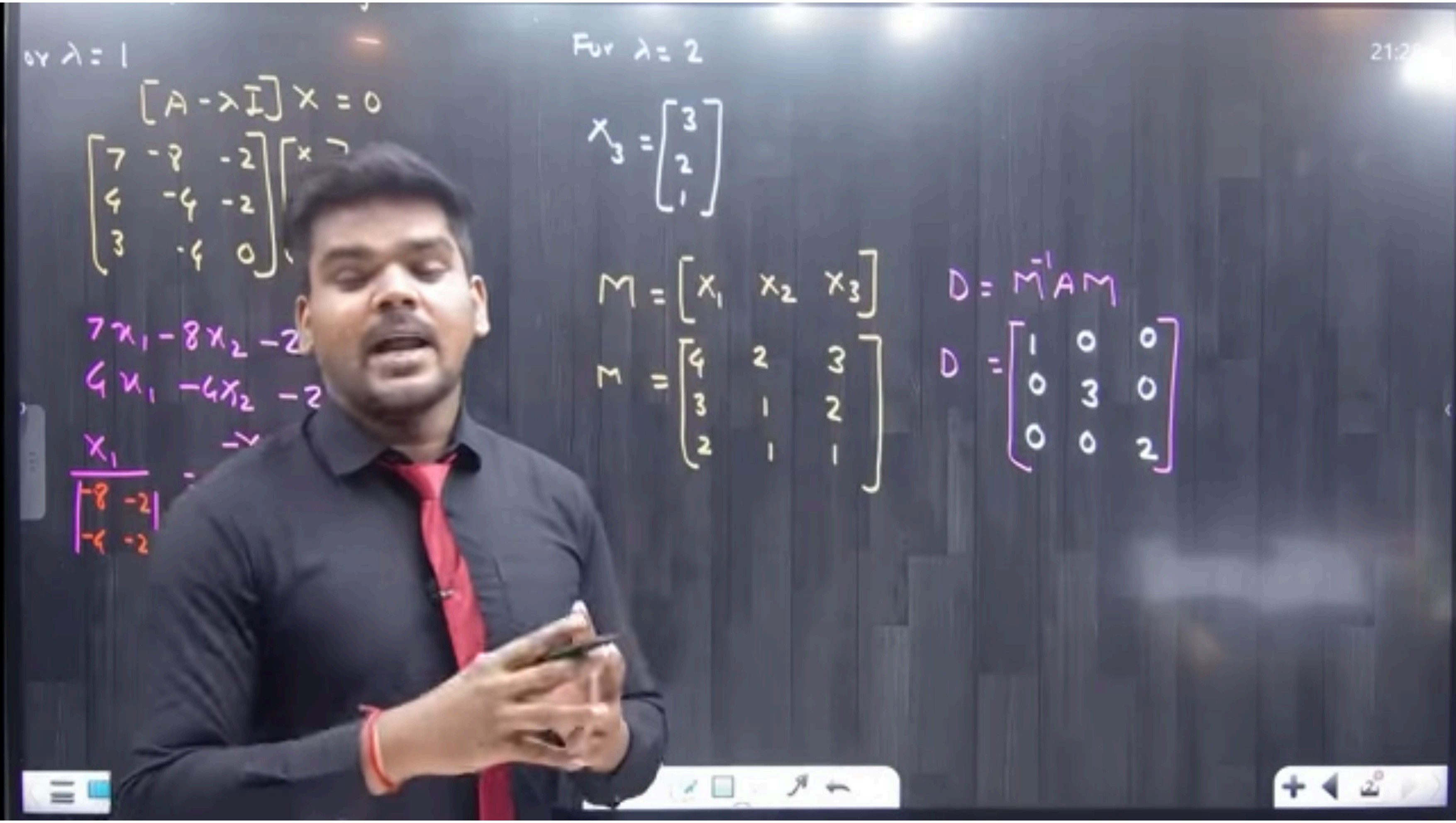
All the eigen values are distinct.

$\therefore$ matrix A is diagonalisable.



$\begin{vmatrix} 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix} = 0$
All the eigen values are distinct.
∴ matrix A is diagonalisable.
$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$
$S_1 = 6$
$S_2 = -11 + 14 + 8 = 11$
$|A| = 6$
For $\lambda = 1$
$[A - \lambda I]X = 0$
$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$
$7x_1 - 8x_2 - 2x_3 = 0$
$4x_1 - 4x_2 - 2x_3 = 0$
$\frac{x_1}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$
$\frac{x_1}{8} = \frac{-x_2}{-6} = \frac{x_3}{4}$
$\frac{x_1}{4} = \frac{x_2}{3} = \frac{x_3}{2}$
For $\lambda = 1$ Eigen vector $X_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$
21:2


the transforming matrix and Diagonal matrix.
|A - λI| = 0
λ³ - 6λ² + 11λ - 6 = 0
1, 3, 2
8-λ -8 -2
4 -3-λ -2
3 -4 1-λ
= 0
the eigen values are distinct.
matrix A is diagonalisable.
λ³ - S₁λ² + S₂λ
S₁ = 6
S₂ = -11
|A| = 6
λ = 1
[A - λI] X = 0
= 0
-8x₂ - 2x₃ = 0
-4x₂ - 2x₃ = 0
x₁/8 =
For λ = 3
x₂ = [2 1 1]
For λ = 2
x₃ = [3 2 1]



For λ = 1
[A - λI] X = 0
7x₁ - 8x₂ - 2
4x₁ - 4x₂ - 2
For λ = 2
X₃ = [3 2 1]
M = [X₁ X₂ X₃]
M = [4 2 3; 3 1 2; 2 1 1]
D = M⁻¹AM
D = [1 0 0; 0 3 0; 0 0 2]

Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find Diagonal form D and the diagonalising matrix M.

Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find Diagonal form D and the diagonalising matrix M.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -9-\lambda & 4 & 4 \\ -8 & 3-\lambda & 4 \\ \text{[illegible]} & 8 & 7-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$

$S_1 = 1$

$S_2 = -11+1+5 = -5$

$|A| = 3$

$$\lambda^3 - \lambda^2 + (-5)\lambda - 3 = 0 \qquad \boxed{\lambda = 3, -1, -1}$$

For $\lambda = 3$

$$[A - \lambda I]X = 0$$

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find Diagonal form D ... diagonalising matrix M.

$|A - \lambda I| =$

$\begin{vmatrix} -9-\lambda & 4 & 4 \\ -8 & 3-\lambda & 4 \\ -16 & 8 & \ldots \end{vmatrix}$

$\lambda^3 - S_1 \ldots = 0$

$S_1 =$

$S_2 =$

$|A| :$

$\lambda^3 - \lambda^2 \ldots$

For $\lambda = 3$

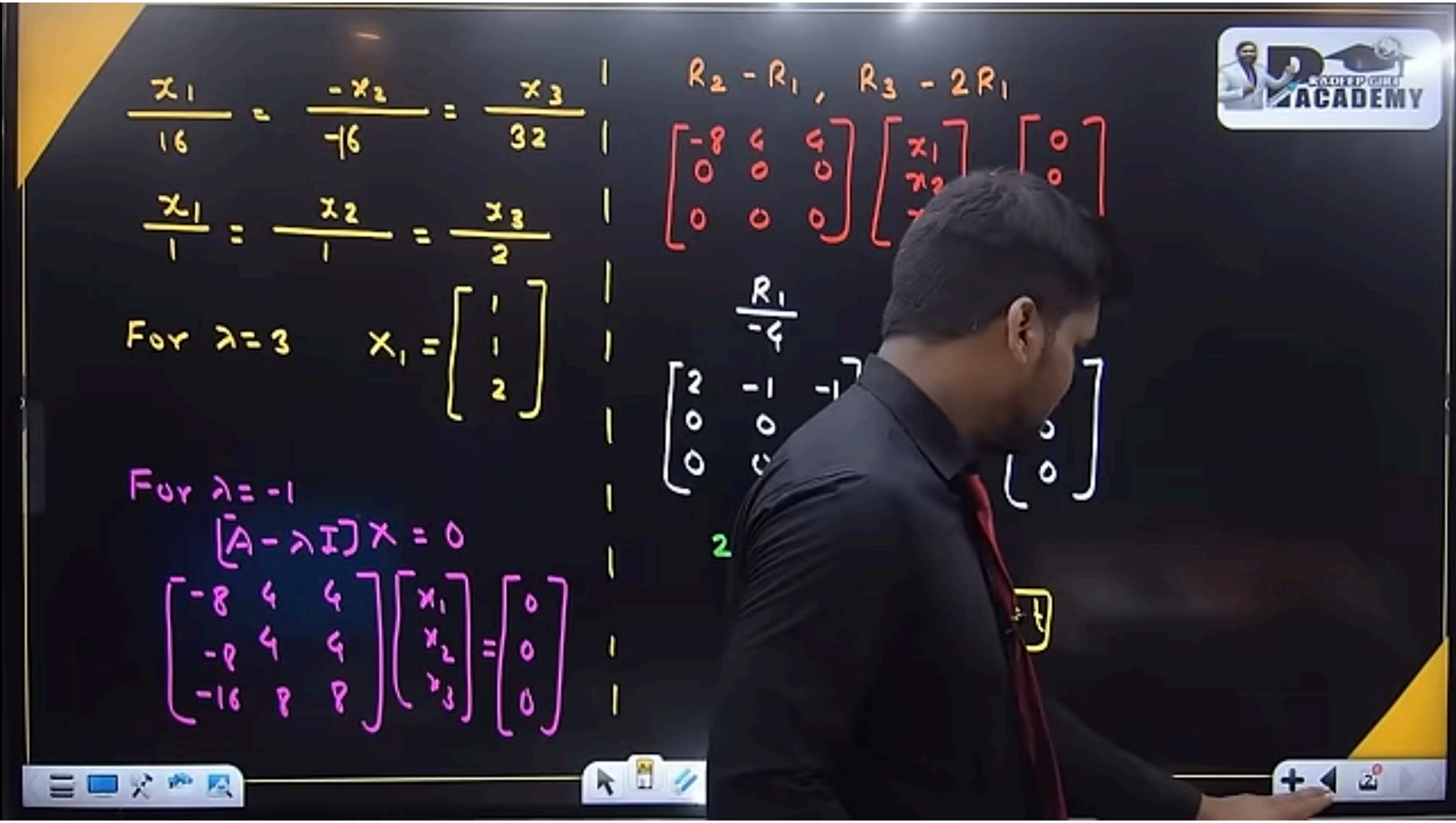
$[A - \lambda I] X = 0$

$$\begin{bmatrix} -12 & 4 & 4 \\ -8 & 0 & 4 \\ -16 & 8 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-12x_1 + 4x_2 + 4x_3 = 0$

$-8x_1 + 0x_2 + 4x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 4 & 4 \\ 0 & 4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -12 & 4 \\ -8 & 4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -12 & 4 \\ -8 & 0 \end{vmatrix}}$$



x1/16 = -x2/-16 = x3/32
x1/1 = x2/1 = x3/2
For λ=3 X1 = [1 1 2]
For λ=-1
[A - λI] X = 0
[-8 4 4; -8 4 4; -16 8 8] [x1 x2 x3] = [0 0 0]
R2 - R1, R3 - 2R1
[-8 4 4; 0 0 0; 0 0 0]
R1/-4
[2 -1 -1; 0 0; 0]

$$\frac{x_1}{16} = \frac{-x_2}{-16} = \frac{x_3}{32}$$

$$\frac{x_1}{1} = \frac{x_2}{1} =$$

For $\lambda = 3$ x

For λ

$R_2 - R_1,\ R_3 - 2R_1$

$$\begin{bmatrix} -8 & 4 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{R_1}{-4}$$

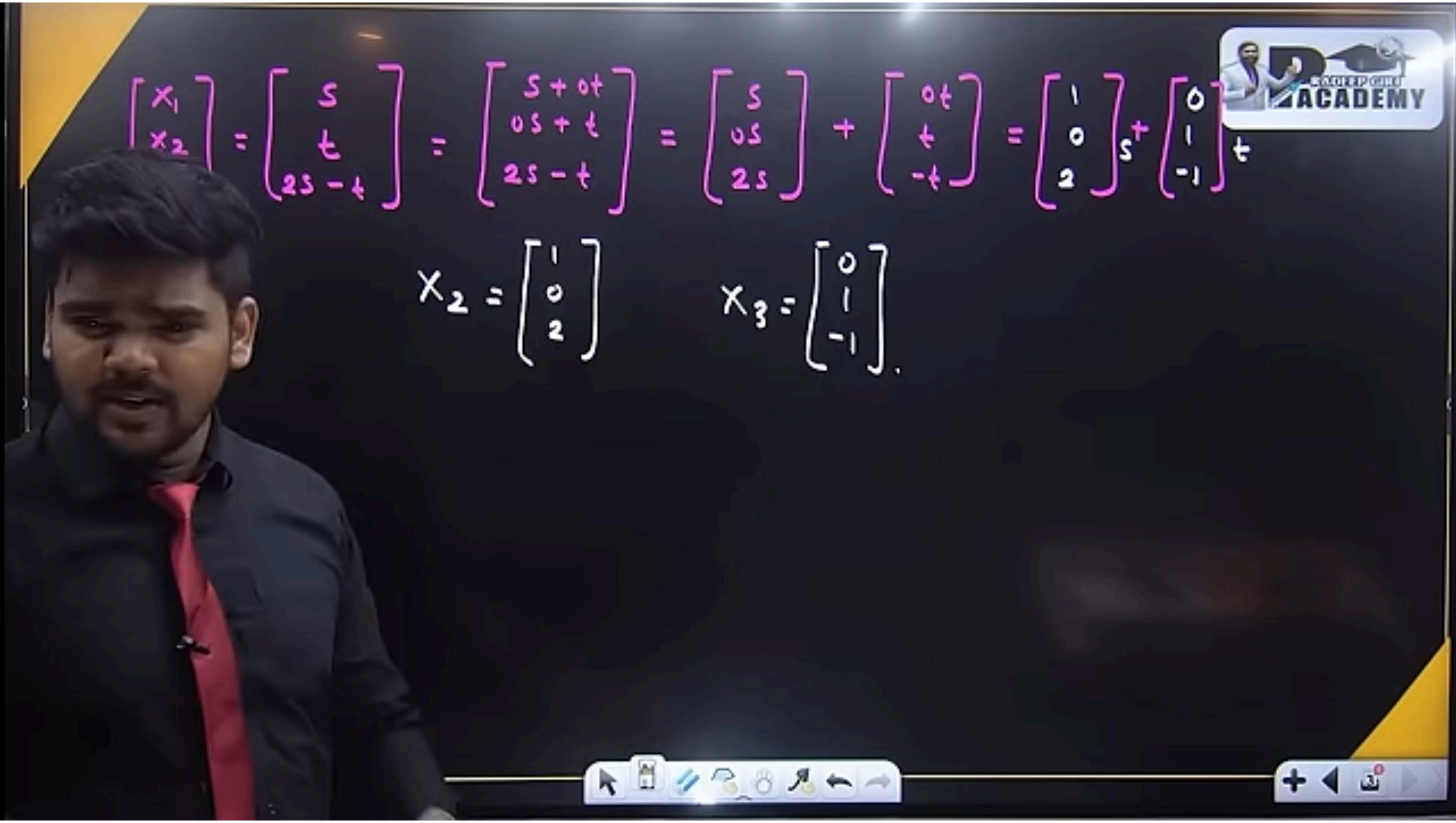
$$\begin{bmatrix} 2 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 - x_2 - x_3 = 0$$

Let $\boxed{x_1 = s}$ $\boxed{x_2 = t}$

$$2s - t - x_3 = 0$$

$$\boxed{2s - t = x_3}$$

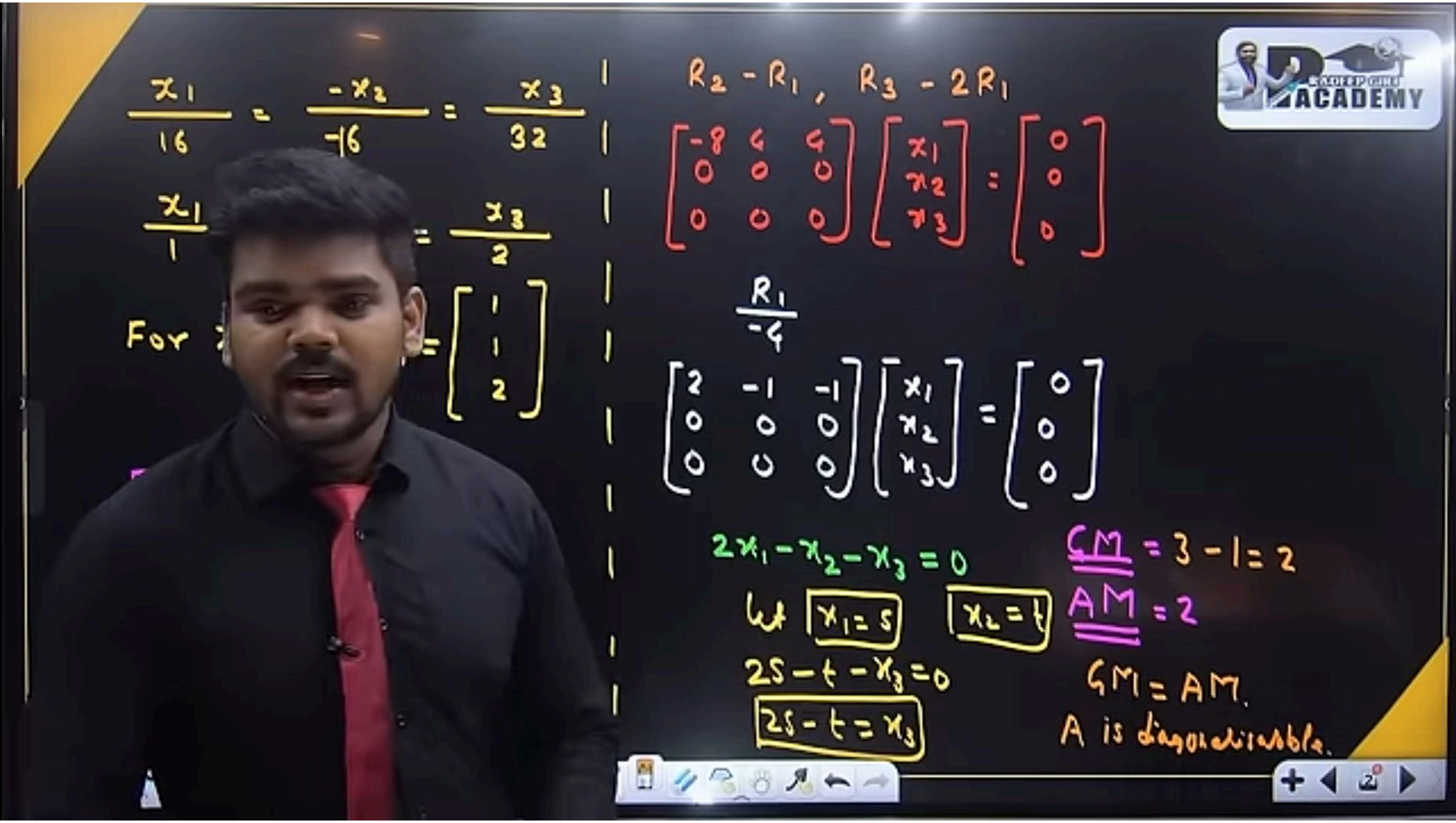

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} s \\ t \\ 2s-t \end{bmatrix} = \begin{bmatrix} s+0t \\ 0s+t \\ 2s-t \end{bmatrix} = \begin{bmatrix} s \\ 0s \\ 2s \end{bmatrix} + \begin{bmatrix} 0t \\ t \\ -t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} s + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} t$
$X_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$
$X_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

GM-Geometric multiplicity

AM-Algebraic Multiplicity

AM=how many times Eigen value is repeated

GM=n-r



x1/16 = -x2/-16 = x3/32
x1/1 = ... = x3/2
For ... = [1 1 2]
R2 - R1 , R3 - 2R1
[-8 4 4; 0 0 0; 0 0 0] [x1; x2; x3] = [0; 0; 0]
R1/-4
[2 -1 -1; 0 0 0; 0 0 0] [x1; x2; x3] = [0; 0; 0]
2x1 - x2 - x3 = 0
let x1 = s x2 = t
2s - t - x3 = 0
2s - t = x3
GM = 3 - 1 = 2
AM = 2
GM = AM.
A is diagonalisable.
ACADEMY

$$X_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \qquad X_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 2 & -1 \end{bmatrix}$$

$$D = M^{-1}AM$$

$$= \begin{bmatrix} -2 & 1 & 1 \\ 3 & -1 & -1 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 2 & -1 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$