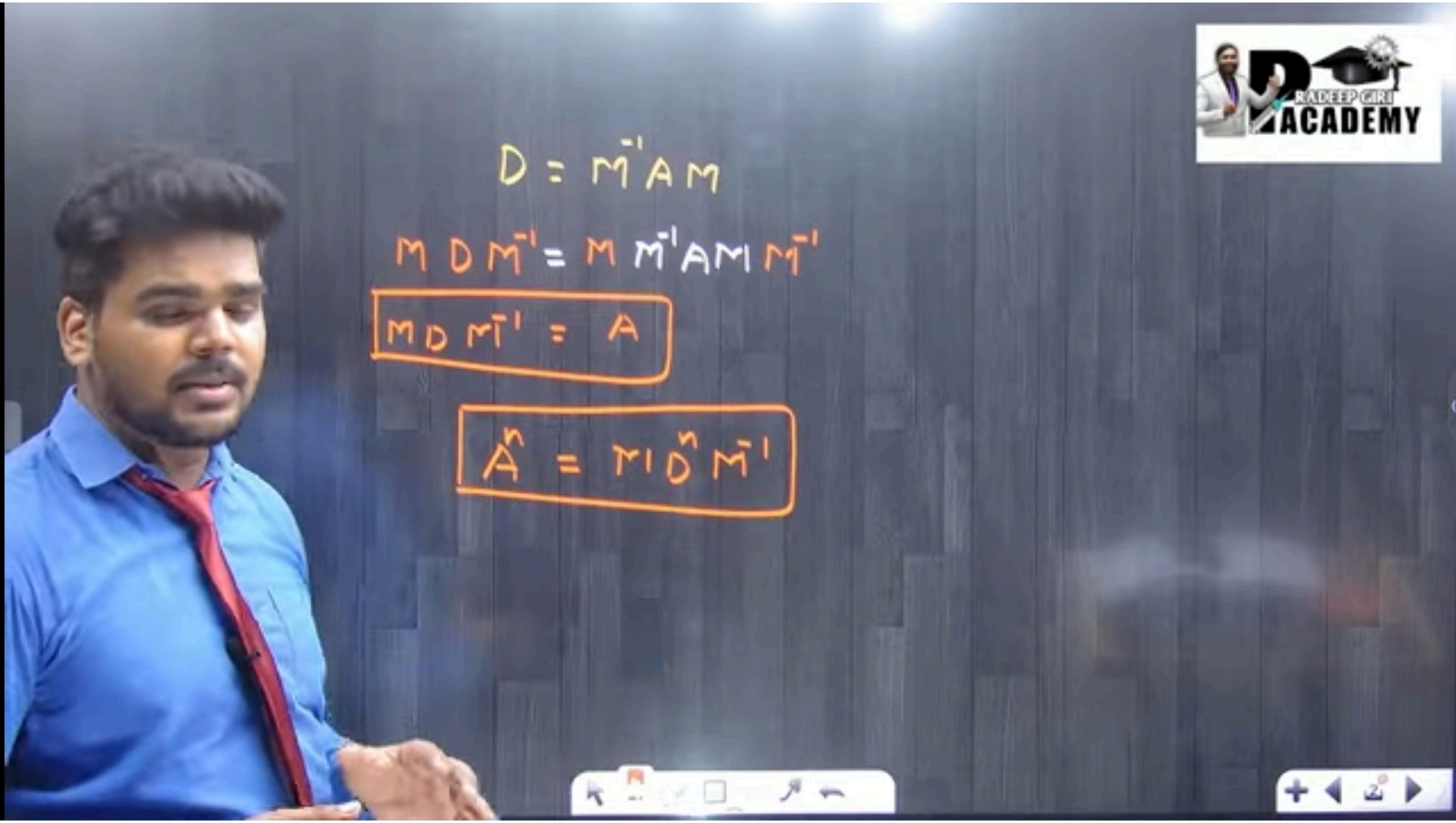
Functions of square matrix


ACADEMY
$D = M^{-1} A M$
$M D M^{-1} = M M^{-1} A M M^{-1}$
$\boxed{M D M^{-1} = A}$
$\boxed{A^n = M D^n M^{-1}}$

If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ find A^{50}

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - S_1 \lambda + |A| = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda = 3, 1$$

$\lambda = 3$

$$[A - \lambda I] X = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 + R_1$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x_1 + x_2 = 0$

let $\boxed{x_2 = t}$

$-x_1 + t = 0$

$-x_1 = -t$

$\boxed{x_1 = t}$

For $\lambda = 3$, Eigen vector

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} t \\ t \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t$$

$$\boxed{X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$\underset{\sim\sim\sim}{\lambda = 1}$

$$[A - \lambda I][X] = [0]$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 - 1R_1$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

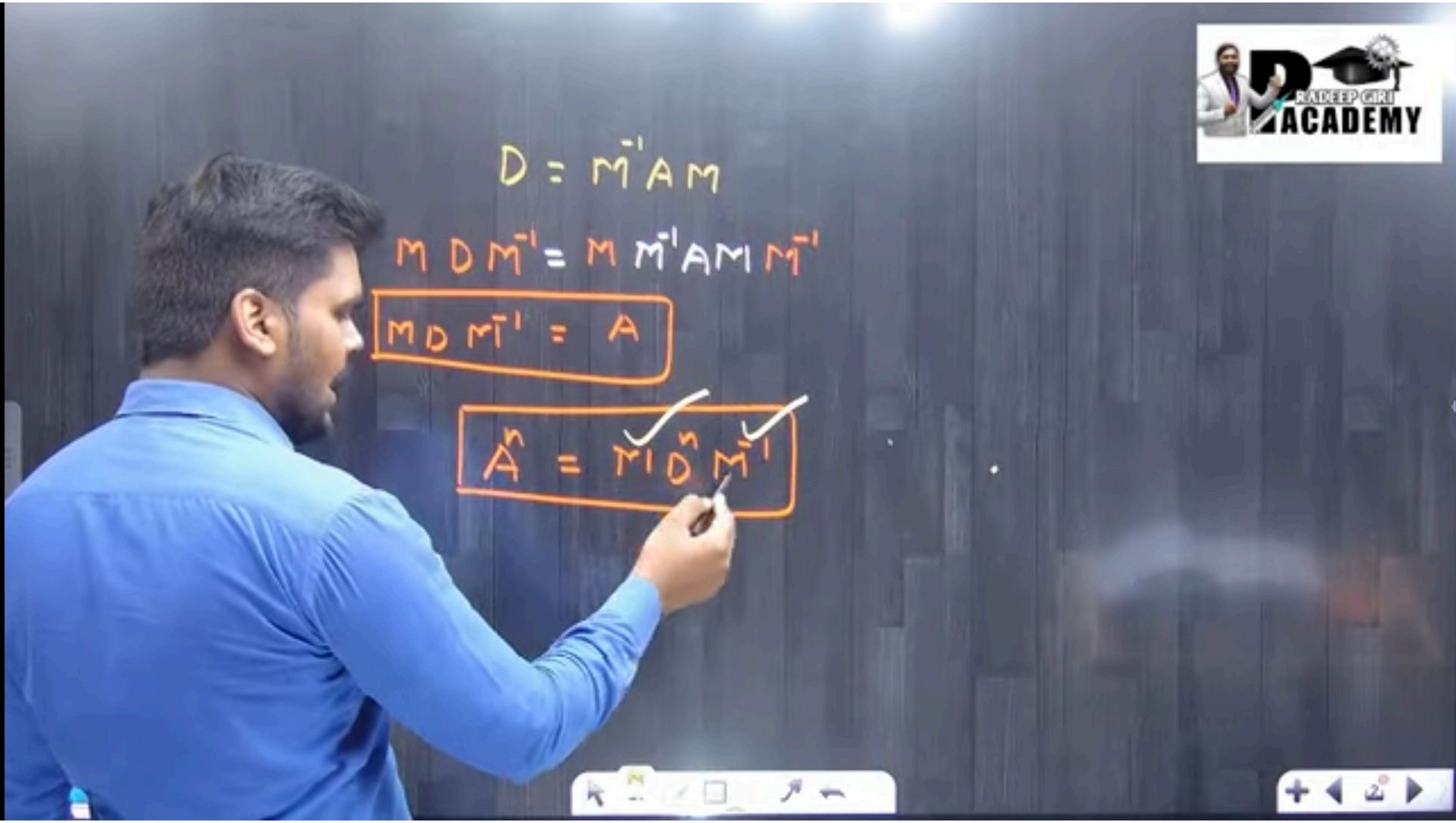
$x_1 + x_2 = 0$

put $x_2 = -t$

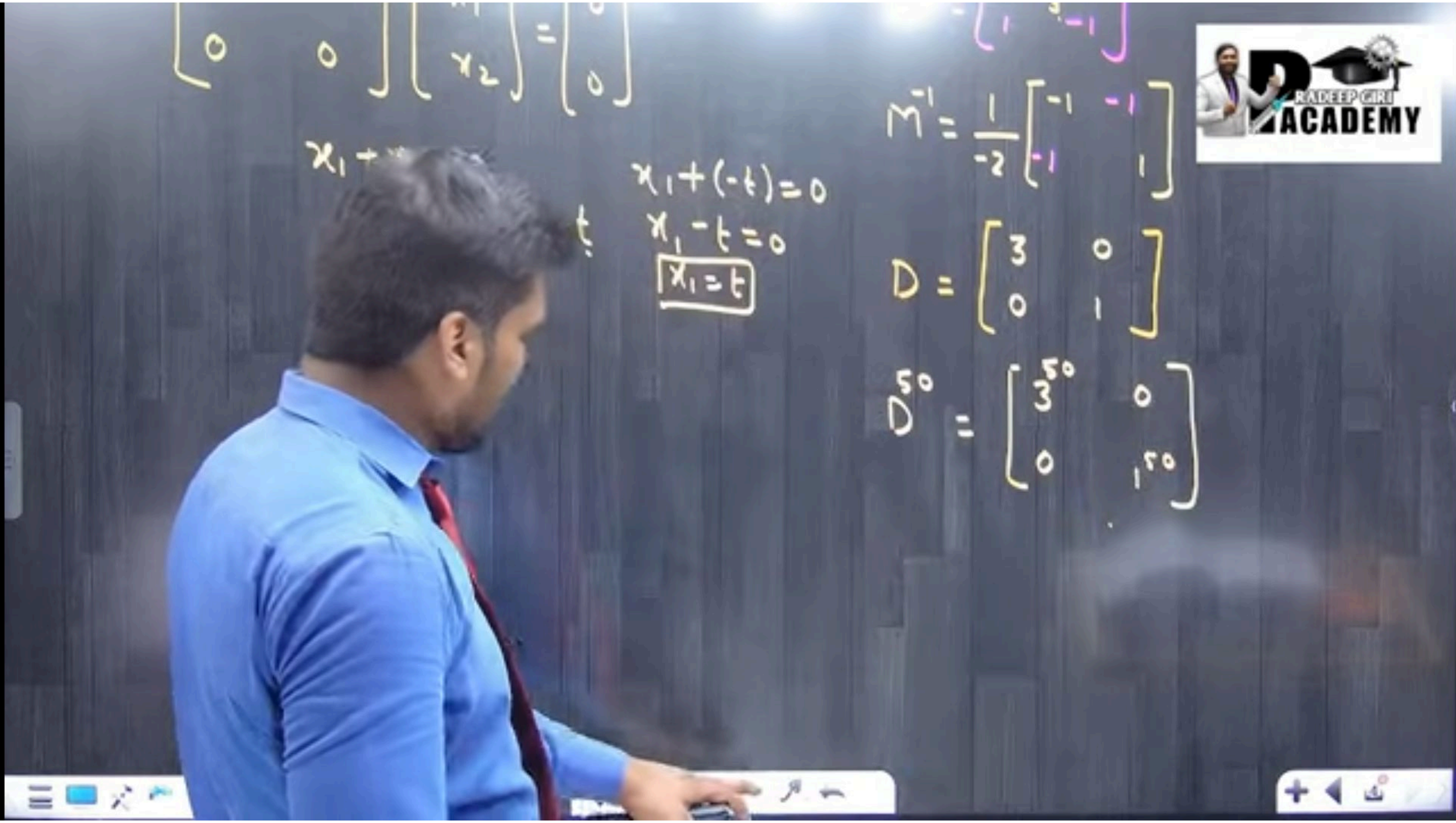
$x_1 + (-t) = 0$

$x_1 - t = 0$

$\boxed{x_1 = t}$



PRADEEP GIRI ACADEMY
$D = M^{-1}AM$
$MDM^{-1} = MM^{-1}AMM^{-1}$
$\boxed{MDM^{-1} = A}$
$\boxed{A^n = MD^nM^{-1}}$



PRADEEP GIRI ACADEMY
$x_1 + (-t) = 0$
$x_1 - t = 0$
$\boxed{x_1 = t}$
$M^{-1} = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$
$D = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$
$D^{50} = \begin{bmatrix} 3^{50} & 0 \\ 0 & 1^{50} \end{bmatrix}$

$x_1 + x_2 = 0$ $\qquad$ $x_1 + (-t) = 0$

put $x_2 = -t$ $\qquad$ $x_1 - t = 0$

$\boxed{x_1 = t}$

$$D = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D^{50} = \begin{bmatrix} 3^{50} & 0 \\ 0 & 1^{50} \end{bmatrix}$$

$$A^n = M D^n M^{-1}$$

$$A^{50} = M D^{50} M^{-1}$$

$$A^{50} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3^{50} & 0 \\ 0 & 1^{50} \end{bmatrix} \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^{50} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3^{50} & 0 \\ 0 & 1^{50} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A^{50} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3^{50} & 3^{50} \\ 1^{50} & -1^{50} \end{bmatrix}$$

$$A^{50} = \frac{1}{2}\begin{bmatrix} 3^{50}+1^{50} & 3^{50}-1^{50} \\ 3^{50}-1^{50} & 3^{50}+1^{50} \end{bmatrix}$$

Minimal Polynomial

Minimal Polynomial

$|A-\lambda I| = 0$

$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2}$ $\quad (2-\lambda)^2 = 0$

$\underbrace{(\lambda-2)^2 = 0}_{\text{Cayley-Hamilton thm}} \Rightarrow \lambda = 2, 2$

$\lambda^2 - 4\lambda + 4 = 0$

$A^2 - 4A + 4I = 0$

$(\lambda - 2) = 0$

$-2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\lambda^2 - 5\lambda + 6 = 0$

$A^2 - 5A + 6I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^2 - 5\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} +$

Minimal Polynomial

$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2}$ $\quad (2-\lambda)^2 = 0 \quad |A - \lambda I| = 0$

$\underline{(\lambda-2)^2 = 0} \Rightarrow \lambda = 2, 2$

Cayley-Hamilton thm $\quad \lambda^2 - 4\lambda + 4 = 0$

$A^2 - 4A + 4I = 0$

$(\lambda - 2) = 0$

$$A - 2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (A - 2I) \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} [\ \ ]$$

$\lambda^2 - 5\lambda + 6 = 0 \quad (\lambda-2)(\lambda-3) = 0$

$$A^2 - 5A + 6I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^2 - 5\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2×2

$\underbrace{(\lambda-2)^2=0}$ $\Rightarrow$ $\lambda=2,2$

Cayley-Hamilton thm $\quad \lambda^2-4\lambda+4=0$

$A^2-4A+4I=0$

$\underline{(\lambda-2)}=0$

$2I=\begin{bmatrix}2&0\\0&2\end{bmatrix}-2\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$, $(A-2I)$ $\begin{bmatrix}0&0\\0&0\end{bmatrix}[\;]$

$\lambda^2-5\lambda+6=0 \quad (\lambda-2)(\lambda-3)=0$

$A^2-5A+6I=\begin{bmatrix}2&0\\0&2\end{bmatrix}^2-5\begin{bmatrix}2&0\\0&2\end{bmatrix}+6\begin{bmatrix}1&0\\0&1\end{bmatrix}$

$=\begin{bmatrix}4&0\\0&4\end{bmatrix}-\begin{bmatrix}10&0\\0&10\end{bmatrix}+\begin{bmatrix}6&0\\0&6\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$\lambda-2 \rightarrow$ minimal polynomial.

Let T be a linear operator on a finite dimensional vector space (F)

T : V ⟶ V is linear transformation.

Then a unique polynomial $m(x) \in F[x]$ is called a minimal polynomial if

(i) $m(x)$ is a monic polynomial

(ii) $m(x)$ annihilates T i.e $m(T) = 0$ $\qquad m(A) = 0$

(iii) Among all the polynomials which annihilate T, $m(x)$ is of lowest degree

Then a unique polynomial $m(x) \in F[x]$ is called a minimal polynomial if

(i) $m(x)$ is a monic polynomial

(ii) $m(x)$ annihilates T i.e $m(T)=0$ $\quad m(A)=0$

Among all the polynomials which annihilate T,

$m(x)$ is of lowest degree

$m(x) \in$ minimal polynomial.

char. eqn & minimal po eqn roots are

Same

$$A = \begin{bmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}_{3\times 3} \quad 12-6$$

$-12+(12)$

$|A - \lambda I| = 0$

$A_{11} + A_{22} + A_{33}$

$\lambda^3 - \lambda^2(\text{trace of } A) + \lambda(\text{Sum of cofactors of Diagonal ele of } A) - |A| =$

$\lambda^3 - \lambda^2(4) + \lambda(-1+6+0) - 2 = 0$

$-4+2(6)+2(-3)$

$-4+12-6$

$+2$

$\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$

$(\lambda-2)(\lambda-1)^2 = 0$

$\lambda = 2, 1, 1$ $\qquad$ $2, 1$

minimal polynomial

$(\lambda-2)(\lambda-1)$

1	1	-4	5	-2
	–	1	-3	2
1	1	-3	2	0
	–	1	-2	
2	1	-2	0	
	–	2		
	1	0		

$(\lambda-2)(\lambda-1) \rightarrow$ minimal polynomial $-?$

$$(\lambda-2)(\lambda-1)=0$$

$$(A-2I)(A-I)$$

$$\begin{bmatrix} 2 & -2 & 2 \\ 6 & -5 & 4 \\ 3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & -2 & 2 \\ 6 & -4 & 4 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 6-12+6 & -4+8-4 & 4-8+4 \\ 18-30+12 & -12+20-8 & 12-20+ \\ 9-12+3 & -6+8-2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Minimal Polynomial

$A=\begin{bmatrix}2&0\\0&2\end{bmatrix}_{2\times 2}$ $\quad (2-\lambda)^2=0 \quad |A-\lambda I|=0 \quad (\lambda-2)(\lambda-2)$

$\underbrace{(\lambda-2)^2=0}$ $\Rightarrow \lambda=2,2$ $\quad\downarrow\quad (\lambda-2)=0$

Cayley-Hamilton thm $\quad \lambda^2-4\lambda+4=0$

$A^2-4A+4I=0$

$(\lambda-2)=0$

$A-2I=\begin{bmatrix}2&0\\0&2\end{bmatrix}-2\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$ $\quad (A-2I)\begin{bmatrix}0&0\\0&0\end{bmatrix}[\ \ ]$

$(\lambda-2)(\lambda-3)=0$

$\lambda^2-5\lambda+6=0$

$A^2-5A+6I=\begin{bmatrix}2&0\\0&2\end{bmatrix}^2-5\begin{bmatrix}2&0\\0&2\end{bmatrix}+6\begin{bmatrix}1&0\\0&1\end{bmatrix}$

$\begin{bmatrix}4&0\\0&4\end{bmatrix}-\begin{bmatrix}10&0\\0&10\end{bmatrix}+\begin{bmatrix}6&0\\0&6\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$\lambda-2 \rightarrow$ minimal polynomial.

$A=\begin{bmatrix}2&1\\0&2\end{bmatrix}$

$(\lambda-2)^2=0$

$(\lambda-2)=0$

$A-2I=\begin{bmatrix}2&1\\0&2\end{bmatrix}-\begin{bmatrix}2&0\\0&2\end{bmatrix}$

$=\begin{bmatrix}0&1\\0&0\end{bmatrix}$

Minimal Polynomial

$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2}$ $\quad (2-\lambda)^2 = 0 \quad |A-\lambda I| = 0$

$\underbrace{(\lambda-2)^2 = 0} \Rightarrow \lambda = 2, 2$

Cayley-Hamilton thm $\quad \lambda^2 - 4\lambda + 4 = 0$

$A^2 - 4A + 4I = 0$

$(\lambda - 2) = 0$

$-2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (A-2I) \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} [\;\;]$

$(\lambda-2)(\lambda-3) = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$- 5A + 6I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^2 - 5\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} \;\; & 0 \\ \;\; & 4 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\lambda - 2 \rightarrow$ minimal polynomial.

$(\lambda-2)(\lambda-2)$

$\downarrow$

$(\lambda-2) = 0$

$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ minimal polynomial

$(\lambda-2)^2 = 0$

$(\lambda - 2) = 0$

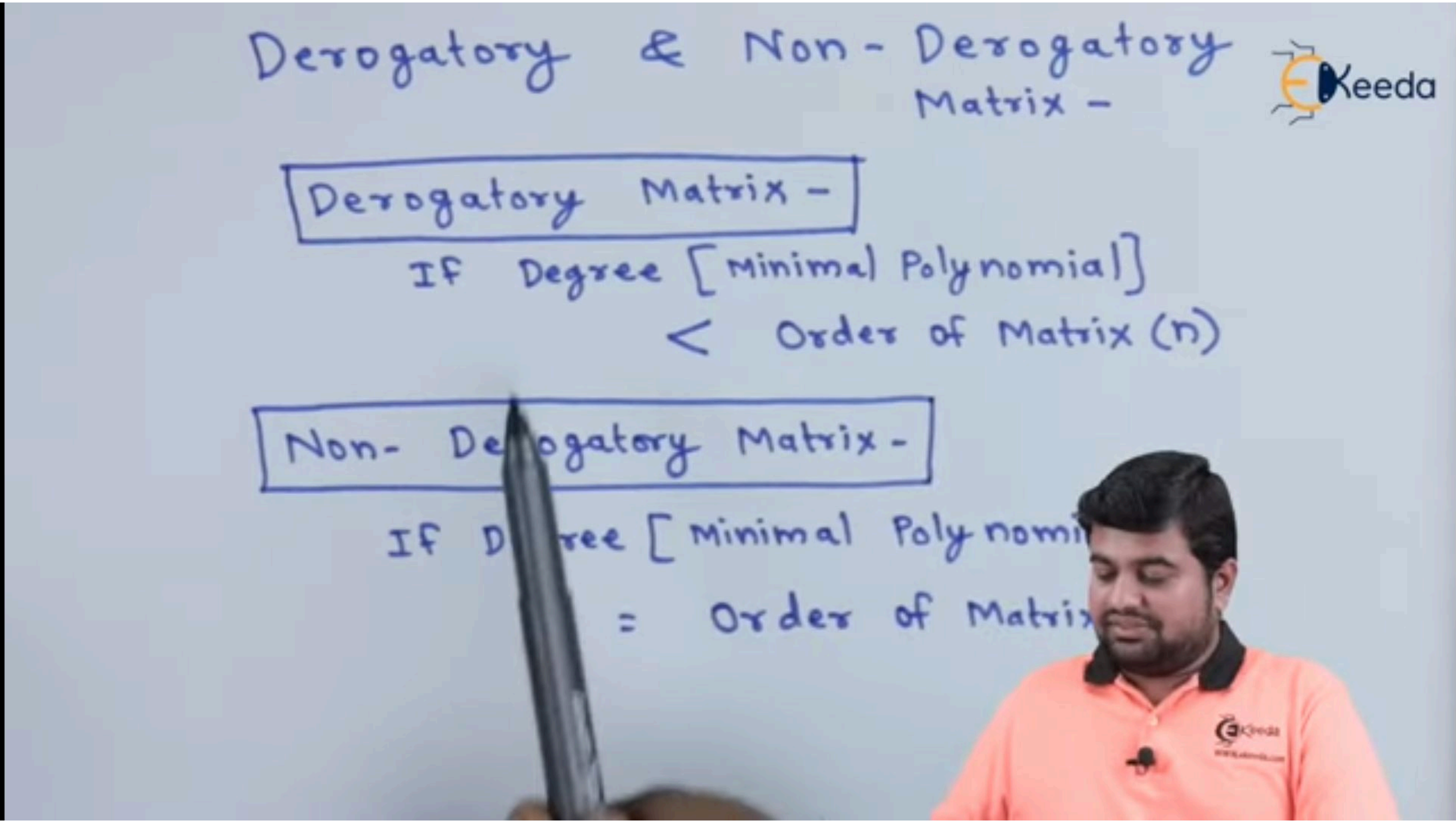
$A - 2I = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$(\lambda-2)^2$

In the previous case the characteristic polynomial is its minimal polynomial, because uss se chota polynomial is not satisfied by A

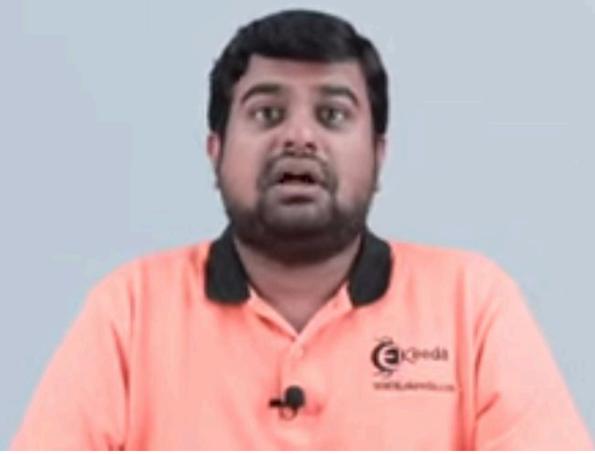
Derogatory & Non-derogatory Matrix


Derogatory & Non-Derogatory Matrix -
Derogatory Matrix -
If Degree [Minimal Polynomial]
< Order of Matrix (n)
Non- De ogatory Matrix -
If D ree [Minimal Polynomi
= Order of Matri

Check whether matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$

is Derogatory or Not?

Sol^n :-



Check whether matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is Derogatory or Not?

Keeda

Soln:- The characteristic Equation $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 5-\lambda & -6 & -6 \\ -1 & 4-\lambda & 2 \\ 3 & -6 & -4-\lambda \end{vmatrix} = 0$$

$\therefore \lambda^3 - [\text{SUM of D-Elements}]\lambda^2 + \lambda[\text{SUM of Minors of D-Elements}] - |A| = 0$

$\therefore \lambda^3 - [5 + 4 - 4]\lambda^2 + [-4 + (-2) + 14]\lambda - 4 = 0$

$\therefore \quad \lambda^3 - 5\lambda^2 +$

Check whether matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is Derogatory or Not?

Keeda

<u>Solⁿ:-</u> The characteristic Equation $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 5-\lambda & -6 & -6 \\ -1 & 4-\lambda & 2 \\ 3 & -6 & -4-\lambda \end{vmatrix} = 0$$

$\therefore \lambda^3 - [\text{SUM of D-Elements}]\lambda^2 + \lambda[\text{SUM of Minors of D-Elements}] - |A| = 0$

$\therefore \lambda^3 - [5 + 4 - 4]\lambda^2 + [-4 + (-2) + 14]\lambda - 4 = 0$

$\therefore \lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0 \qquad \therefore \lambda = 2, 2, 1$

$\therefore \lambda = 2, 2, 1 \qquad \therefore f(\lambda) = (\lambda-2)(\lambda-2)(\lambda-1)$

$\therefore f(x) = (x-2)(x-1) = x^2 - 3x + 2$

$\therefore f(A) = A^2 - 3A + 2I = 0$

Degree 2 is less than the order of the matrix 3

$\therefore \lambda = 2, 2, 1 \qquad \therefore f(\lambda) = (\lambda-2)(\lambda-2)(\lambda-1)$

$\therefore f(x) = (x-2)(x-1) = x^2 - 3x + 2$

$\therefore f(A) = A^2 - 3A + 2I = 0$

$$A^2 = A \cdot A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix} \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix} = \begin{bmatrix} 13 & -18 & -18 \\ -3 & 10 & 6 \\ 9 & -18 & -14 \end{bmatrix}$$

$$\therefore A^2 - 3A + 2I = \begin{bmatrix} 13 & -18 & -18 \\ -3 & 10 & 6 \\ 9 & -18 & -14 \end{bmatrix} - 3\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

$$+ 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0_{(3\times 3)}$$

$\therefore$ Matrix (A) is Derogatory $\because (2 < 3)$

#Trick : If Eigen value is repeated then it is derogatory
If not repeated then non-derogatory.

$$\lambda^3 - 18\lambda^2 + 81\lambda - 108 = 0$$

$$\lambda = 3, 3, 12$$

degr

1, 2, 3

m

$$f(x) = (x-3)(x-12)$$

$(x-1)(x-2)(x-3)$

x^{3}

$$x^2 - 12x - 3x + 36$$

$$A^2 - 15A + 36I$$

$$\begin{bmatrix} 69 & 60 & -15 \\ 60 & 69 & -15 \\ -60 & -60 & 24 \end{bmatrix} - 15\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix} + 36\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Minimal polynomial extra question

8. Find the minimal polynomial for the matrix

$$A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$

Soln.

The characteristic equation of A is –

$$\begin{vmatrix} 7-x & 4 & -1 \end{vmatrix}$$

Solⁿ.

The characteristic equation of A is -

$$|A - xI| = \begin{vmatrix} 7-x & 4 & -1 \\ 4 & 7-x & -1 \\ -4 & -4 & 4-x \end{vmatrix} = 0$$

expanding we get -

expanding we get –

$(7-x)[(7-x)(4-x)-4]-4[16-4x-4]-1$

$(-16+28-4x)=0$

$(7-x)[(x-8)(x-3)]+20(x-3)=0$

$(x-3)[(7-x)(x-8)+20]=0$

$(x-3)(x-12)(x-3)=0$

$$(7-x)\left[(7-x)(4-x)-4\right]-4\left[16-4x-4\right]-1$$

$$(-16+28-4x)=0$$

$$(7-x)\left[(x-8)(x-3)\right]+20(x-3)=0$$

$$(x-3)\left[(7-x)(x-8)+20\right]=0$$

$$(x-3)(x-12)(x-3)=0$$

$$\boxed{x=3,3,12}$$

SUBSCRIBE

$\Rightarrow$ Characteristic roots of A are 3, 3, 12.

Now

$$f(x) = (x-3)^2(x-12)$$

$$\Rightarrow \quad g(x) = (x-3)(x-12)$$

By Cayley-Hamilton theorem -

every square matrix satisfies its

...ristic equation.

Now

$$f(x) = (x-3)^2 (x-12)$$

$\Rightarrow \quad g(x) = (x-3)(x-12)$

By Cayley-Hamilton theorem -

every square matrix satisfies its characteristic equation.

ie $\quad f(x) = 0$

&

every square matrix satisfies its characteristic equation.

i.e $\quad f(x) = 0$

&

$f(A) = 0.$

Let us verify that $g(A)$ is zero or not?

$g(x) = (x-3)(x-12)$

Let us verify that $g(A)$ is zero or not?

$$\because \quad g(x) = (x-3)(x-12)$$

$$= x^2 - 15x + 36.$$

$$g(A) = A^2 - 15A + 36I$$

$$= \begin{bmatrix} 69 & 60 & -15 \\ 60 & 69 & -15 \\ -60 & -60 & 24 \end{bmatrix} - \begin{bmatrix} 105 & 60 & -15 \\ 60 & 105 & -15 \\ -60 & -60 & 60 \end{bmatrix} + \begin{bmatrix} 36 & 0 & 0 \\ 0 & 36 & 0 \\ 0 & 0 & 36 \end{bmatrix}$$

$A^2 - 15A + 36I = 0.$

Also:

$h(x) = (x-12)$, $A - 12I \neq 0$

$p(x) = (x-3)$, $A - 3I \neq 0$

$\therefore$ $g(x)$ is the monic polynomial of lowest degree

s.t $g(A) = 0.$

$h(x) = (x-12)$, $A - 12I \neq 0$

$p(x) = (x-3)$, $A - 3I \neq 0$

$\therefore$ $g(x)$ is the monic polynomial of lowest degree

s.t $g(A) = 0$.

$\Rightarrow$ $g(x) = (x-3)(x-12)$ is minimal polynomial.

Minimal Polynomial Example 2

Q. Let $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$, then find.

[1]. $c(x)$ = Characteristic polynomial

[2]. $m(x)$ = Minimal polynomial.

[3]. Is A Diagonalizable.

[4]. Five Different Annihilating polynomial of A.

Sol:

Characteristic polynomial $= |A - \lambda I|$

$\Rightarrow (4-\lambda)[\lambda^2-7\lambda+12]-3+2\lambda=0$

$\Rightarrow 4\lambda^2-28\lambda+48-\lambda^3+7\lambda^2-12\lambda$

$-3+2\lambda=0$

$\Rightarrow -\lambda^3+11\lambda^2-38\lambda+45=0$

OR

$\lambda^3-11\lambda^2+38\lambda-45=0$

$\because \lambda=5$ satisfied the above eqn.

$\therefore$ ch. polynomial $= (\lambda-3)^2(\lambda-5)$ Ans.

$\Rightarrow$ $\boxed{\text{ch. roots} = 3, 3, 5}$ An

For minimal polynomial -

$\because$ Distinct ch. roots of A

the minimal equati

For minimal polynomial -

$\because$ Distinct ch. roots of A are also roots of the minimal equation of A.

So 3, 5 are roots of the minimal eqn of A.

$\therefore$ The minimal polynomial of A is either

$$(\lambda-3)(\lambda-5)$$

OR

$$(\lambda-3)^2(\lambda-5)$$

ie $\lambda^2-8\lambda+15$

So 3, 5 are roots of

∴ The minimal polynomial of A is either

$$(\lambda-3)(\lambda-5)$$

OR

$$(\lambda-3)^2(\lambda-5)$$

ie $\lambda^2-8\lambda+15$

OR

$$\lambda^3-11\lambda^2+38\lambda-45=0.$$

- Fist check $\lambda^2-8\lambda+15$ is a minimal poly

$\therefore A^2 - 8A + 15I$

$$\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}^2 - 8\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix} + 15\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

——— (1)

$A^2 = \begin{bmatrix} 4 & & \\ 2 & 5 & \end{bmatrix} \begin{bmatrix} 4 & 1 & -1 \\ & & -2 \end{bmatrix}$

$$\begin{bmatrix} 47 & 8 & -8 \\ 16 & 55 & -16 \\ 8 & 8 & 31 \end{bmatrix} - \begin{bmatrix} 32 & 8 & -8 \\ 16 & 40 & -16 \\ 8 & 8 & 16 \end{bmatrix} + \begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 47 & 8 & -8 \\ 16 & 55 & -16 \\ 8 & 8 & 31 \end{bmatrix} - \begin{bmatrix} 47 & 8 & -8 \\ 16 & 55 & -16 \\ 8 & 8 & 31 \end{bmatrix}$$

$$\begin{bmatrix} 47 & 8 & -8 \\ 16 & 55 & -16 \\ 8 & 8 & 31 \end{bmatrix} - \begin{bmatrix} 47 & 8 & -8 \\ 16 & 55 & -16 \\ 8 & 8 & 31 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ minimal polynomial $= \lambda^2 - 8\lambda + 15$

$(\lambda-3)(\lambda-5)$

$$\Rightarrow m(x) = (x-3)(x-5)$$

Ans.

Result.

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ minimal polynomial $= \lambda^2 - 8\lambda + 15$

$(\lambda-3)(\lambda-5)$

$\Rightarrow m(x) = (x-3)(x-5)$ Ans.

Result:

(3). A matrix is diagonalizable iff its minimal polynomial has no repeated linear factor.