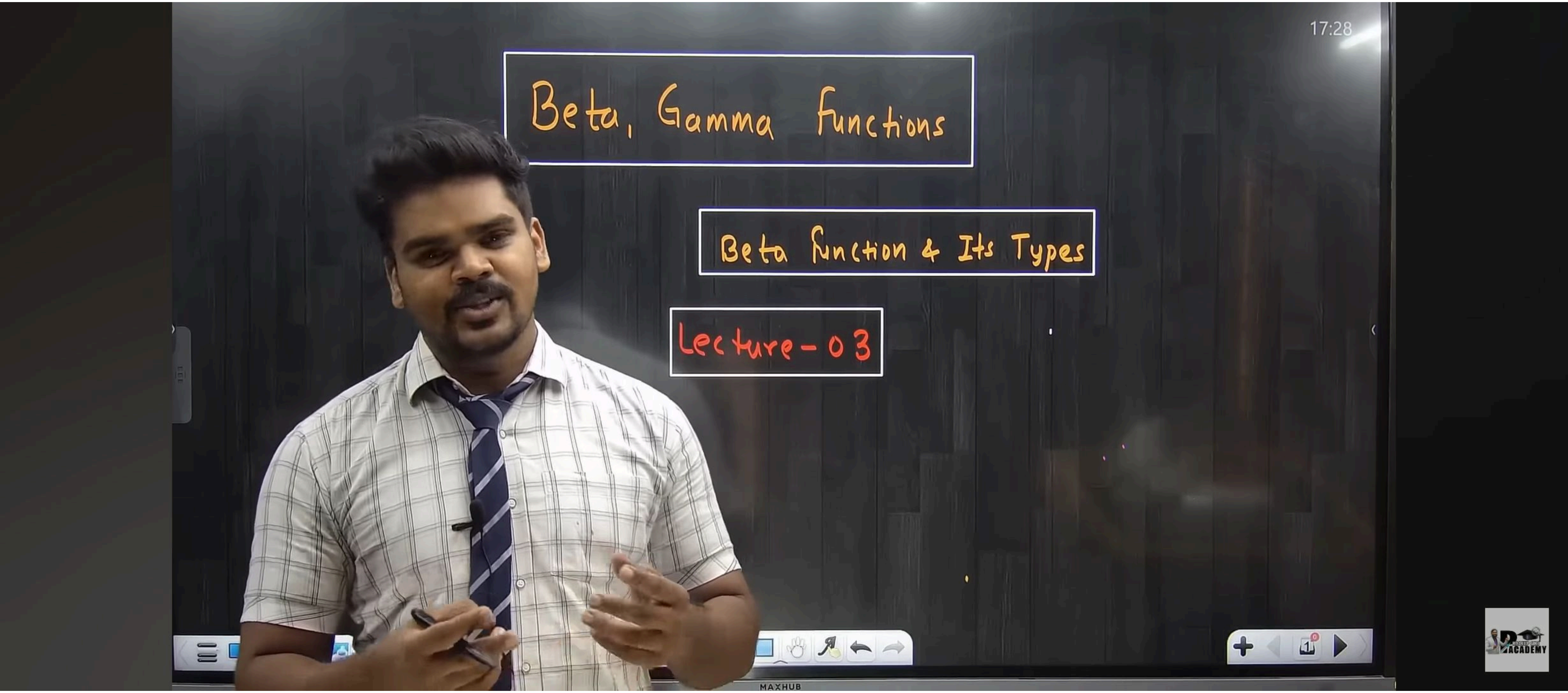
17:28
Beta, Gamma functions
Beta function & Its Types
Lecture-03
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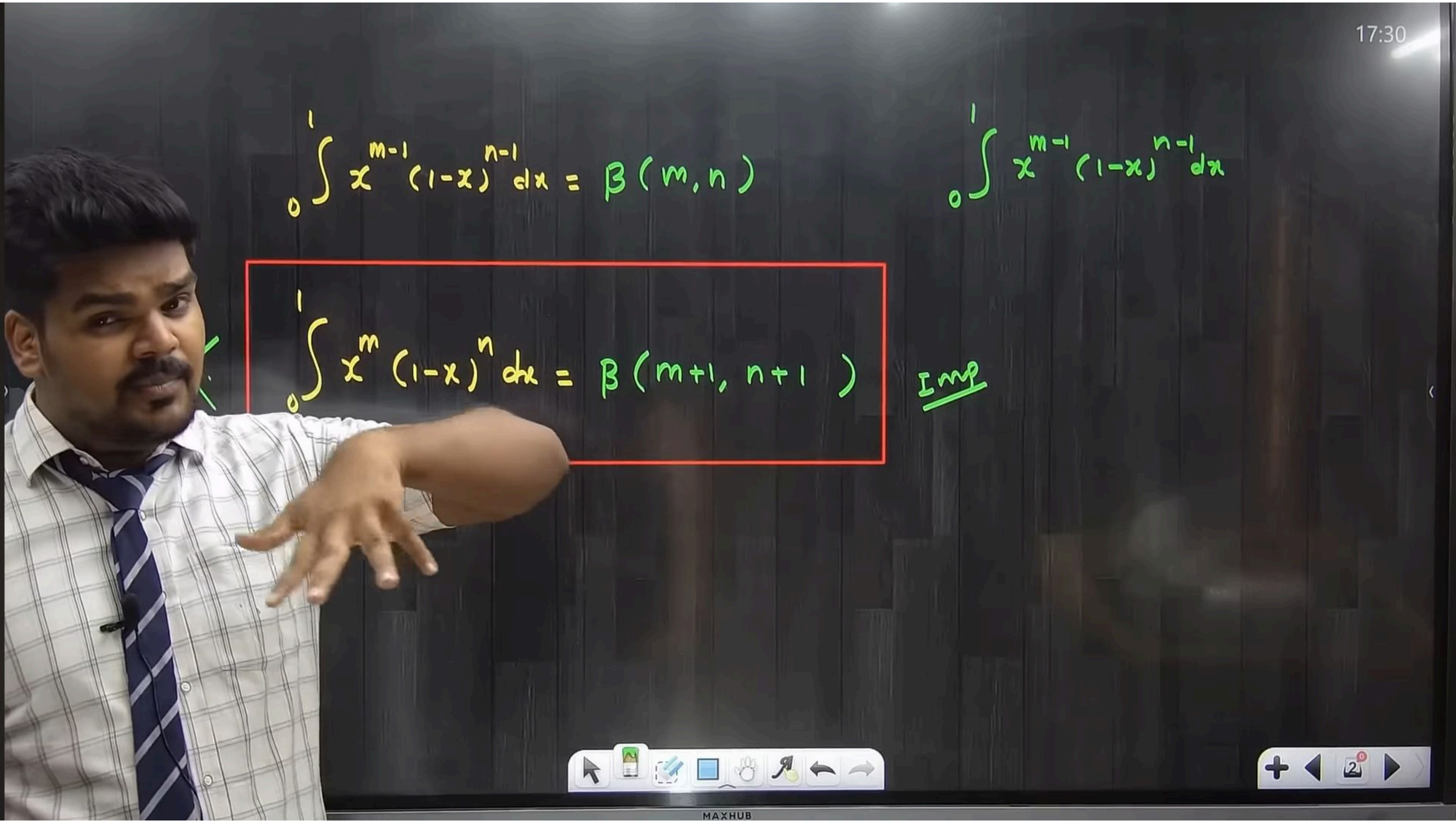
ACADEMY


17:30
$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx = \beta(m,n)$
$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$
$\boxed{\int_0^1 x^{m}(1-x)^{n}\,dx = \beta(m+1,\ n+1)}$
Imp
MAXHUB

Properties of Beta functions

$\Gamma 5$

1) $\beta(m,n) = \beta(n,m)$

2) $$\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma (m+n)} = \frac{(m-1)!\,(n-1)!}{(m+n-1)!}$$

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find : $\Gamma\frac{1}{4}\,\Gamma\frac{3}{4}$

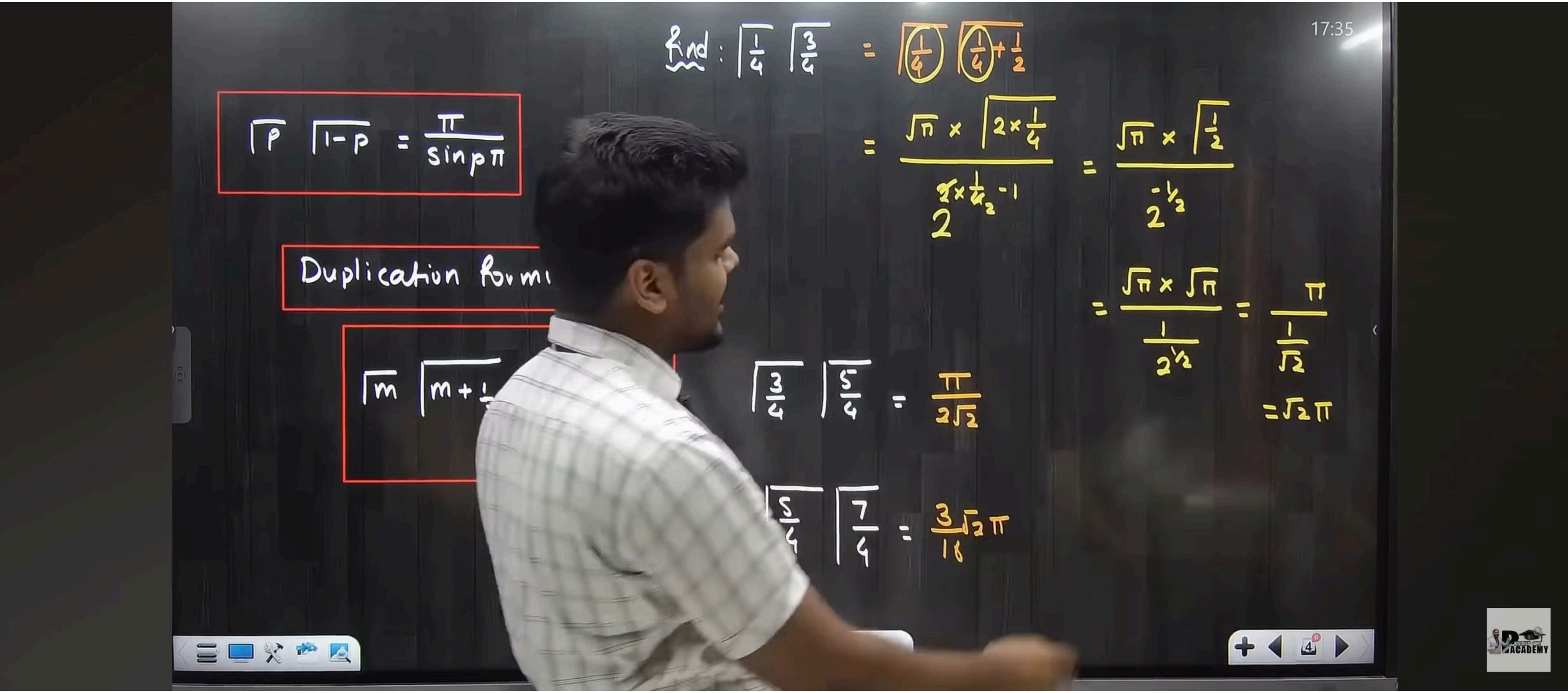
$$\boxed{\Gamma p\,\Gamma(1-p) = \frac{\pi}{\sin p\pi}}$$

Duplication formula

$$\boxed{\Gamma m\,\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}\,\Gamma 2m}{2^{2m-1}}}$$

$$\Gamma\frac{3}{4}\,\Gamma\frac{5}{4} = \frac{\pi}{2\sqrt{2}}$$

$$\Gamma\frac{5}{4}\,\Gamma\frac{7}{4} = \frac{3}{16}\sqrt{2}\,\pi$$


17:35
find : $\Gamma\frac{1}{4}\,\Gamma\frac{3}{4} = \Gamma\frac{1}{4}\,\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)$
$= \frac{\sqrt{\pi} \times \Gamma\left(2\times\frac{1}{4}\right)}{2^{2\times\frac{1}{4}-1}} = \frac{\sqrt{\pi}\times\Gamma\frac{1}{2}}{2^{-1/2}}$
$= \frac{\sqrt{\pi}\times\sqrt{\pi}}{\frac{1}{2^{1/2}}} = \frac{\pi}{\frac{1}{\sqrt{2}}}$
$= \sqrt{2}\,\pi$
$\Gamma p\,\Gamma(1-p) = \frac{\pi}{\sin p\pi}$
Duplication formu
$\Gamma\frac{3}{4}\,\Gamma\frac{5}{4} = \frac{\pi}{2\sqrt{2}}$
$\Gamma\frac{5}{4}\,\Gamma\frac{7}{4} = \frac{3}{16}\sqrt{2}\,\pi$
ACADEMY

Type - I

$$\int_0^a x^m (a-x)^n \, dx = a^{m+n+1} \beta(m+1, n+1)$$

eg. $\int_0^4 \sqrt{x} \, (4-x)^{3/2} \, dx$

put $x = 4t$

Type - I

$$\int_0^a x^m (a-x)^n \, dx = a^{m+n+1} \beta(m+1, n+1)$$

$$\int_0^1$$

eg. $\int_0^4 \sqrt{x}\,(4-x)^{3/2} \, dx$

put $x = 4t$ | $x = 0 \quad t = 0$

$\frac{dx}{dt} = 4(1)$ | $x = 4 \quad t = 1$

$\boxed{dx = 4\,dt}$

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Type - I

$$\int_0^a x^m (a-x)^n \, dx = a^{m+n+1} \beta(m+1, n+1)$$

$$\int_0^4 \sqrt{x} \, (4-x)^{3/2} \, dx$$

put $x = 4t$

$\frac{dx}{dt} = 4(1)$

$\boxed{dx = \text{[illegible]}}$

$$\int_0^1 \sqrt{4t} \, (4-4t)^{3/2} \, 4 \, dt$$

$$= \int_0^1 2 \, t^{1/2} \cdot 4^{3/2} (1-t)^{3/2} \cdot 4 \, dt$$

$$= 2 \cdot 4^{3/2} \cdot 4 \int_0^1 t^{1/2} (1-t)^{3/2} \, dt$$

$$= 2 \times 2^{\cancel{2} \times \frac{3}{\cancel{2}}} \times 4 \; \beta\left(\frac{1}{2}+1, \frac{3}{2}+1\right)$$

$$\boxed{dx = 4\,dt}$$

$$= 2 \times 2^{\cancel{2}\times\frac{3}{\cancel{2}}} \times 4\ \beta\left(\frac{1}{2}+1, \frac{3}{2}+1\right)$$

$$= 2 \times 8 \times 4\ \beta\left(\frac{3}{2}, \frac{5}{2}\right)$$

$$= 64\ \beta\left(\frac{3}{2}, \frac{5}{2}\right)$$

$$= 64 \times \frac{\Gamma\frac{3}{2}\ \Gamma\frac{5}{2}}{\Gamma\left(\frac{3}{2}+\frac{5}{2}\right)}$$

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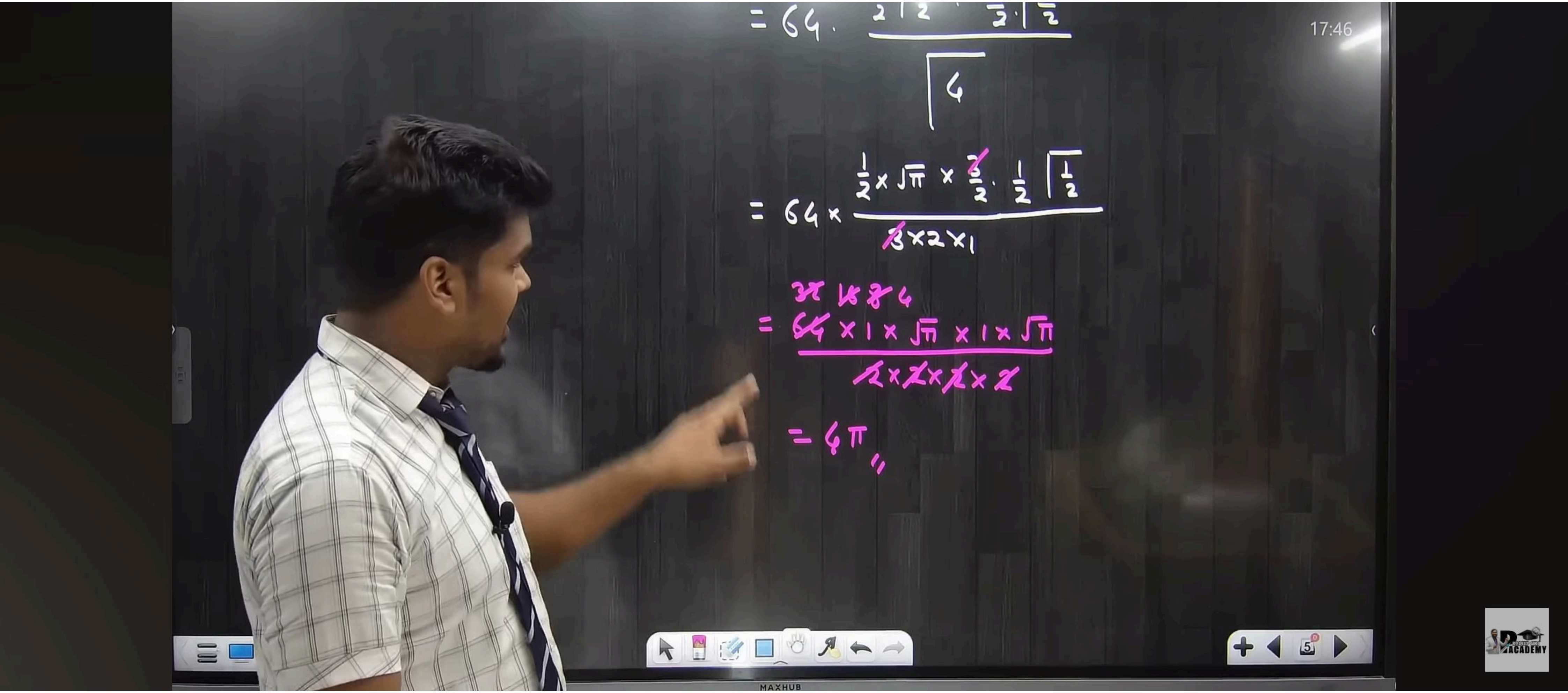
$$= 64 \times \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{3}{2}+\frac{5}{2}\right)}$$

$$= 64 \cdot \frac{\frac{1}{2}\Gamma\left(\frac{1}{2}\right) \cdot \frac{3}{2} \cdot \Gamma\left(\frac{3}{2}\right)}{\Gamma(4)}$$

$$= 64 \times \frac{\frac{1}{2} \times \sqrt{\pi} \times \frac{3}{2} \cdot \frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{3 \times 2 \times 1}$$



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= 64 × (1/2 × √π × 3/2 · 1/2 Γ(1/2)) / (3 × 2 × 1)
= (64 × 1 × √π × 1 × √π) / (2 × 2 × 2 × 2)
= 4π
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$\boxed{\text{Type - II}}$

$$\int_0^1 x^m (1-x^n)^p \, dx = \frac{1}{n} \beta\left(\frac{m+1}{n}, p+1\right)$$

eg. $\int_0^1 x^6 (1-x^2)^{1/2} \, dx$

put $x^2 = t$

$x = t^{1/2}$

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Type-II

$$\int_0^1 x^m (1-x^n)^p \, dx = \frac{1}{n} \beta\left(\frac{m+1}{n}, p+1\right)$$

eg. $\int_0^1 x^6 (1-x^2)^{1/2} \, dx$

put x^2 ... $t=0$

$t=1$

$$= \int_0^1 (t^{1/2})^6 (1-t)^{1/2} \cdot \frac{1}{2} t^{-1/2} \, dt$$

$$= \frac{1}{2} \int_0^1 t^{6/2} \cdot (1-t)^{1/2} \cdot t^{-1/2} \, dt$$

$$= \frac{1}{2} \int_0^1 t^{5/2} \cdot (1-t)^{1/2} \, dt$$

$$= \frac{1}{2} \beta\left(\frac{5}{2}+1, \frac{1}{2}+1\right)$$

17:50

eg. $(x^2)^{1/2}\,dx$

$x^2 = t$

$x = t^{1/2}$

$\boxed{\dfrac{dx}{dt} = \dfrac{1}{2}t^{-1/2}}$

$^{-1/2}\,dt$

$x = 0 \quad t = 0$

$x = 1 \quad t = 1$

$$= \frac{1}{2}\int_0^1 t^{6/2}\cdot(1-t)^{1/2}\cdot t^{-1/2}\,dt$$

$$= \frac{1}{2}\int_0^1 t^{5/2}\cdot(1-t)^{1/2}\,dt$$

$$= \frac{1}{2}\beta\left(\frac{5}{2}+1,\ \frac{1}{2}+1\right)$$

$$= \frac{1}{2}\beta\left(\frac{7}{2},\ \frac{3}{2}\right)$$

ACADEMY

17:51

P.T $\int_0^3 \frac{x^{3/2}}{\sqrt{3-x}}\,dx \cdot \int_0^1 \frac{dx}{\sqrt{1-x^{1/4}}} = \frac{632}{35}\pi$

$I_1 . \quad I_2 =$

Let $I_1 = \int_0^3 \frac{x^{3/2}}{\sqrt{3-x}}\,dx$

$I_2 = \int_0^1 \frac{dx}{\sqrt{1-x^{1/4}}}$

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let $I_1 = \int_0^3 \frac{x^{3/2}}{\sqrt{3-x}}\,dx$

$I_2 = \int_0^1 \frac{dx}{\sqrt{1-x^{1/4}}}$

$I_1 = \int_0^3 x^{3/2}\cdot(3-x)^{-1/2}\,dx$

put $x = 3t$ $\quad x = 0 \quad t = 0$

$\frac{dx}{dt} = 3 \quad x = 3 \quad t = 1$

$\boxed{dx = 3\,dt}$

$I_1 = \int_0^1$

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ACADEMY

put $x = 3t$ $\quad x = 0 \quad t = 0$

$\frac{dx}{dt} = 3 \quad x = 3 \quad t = 1$

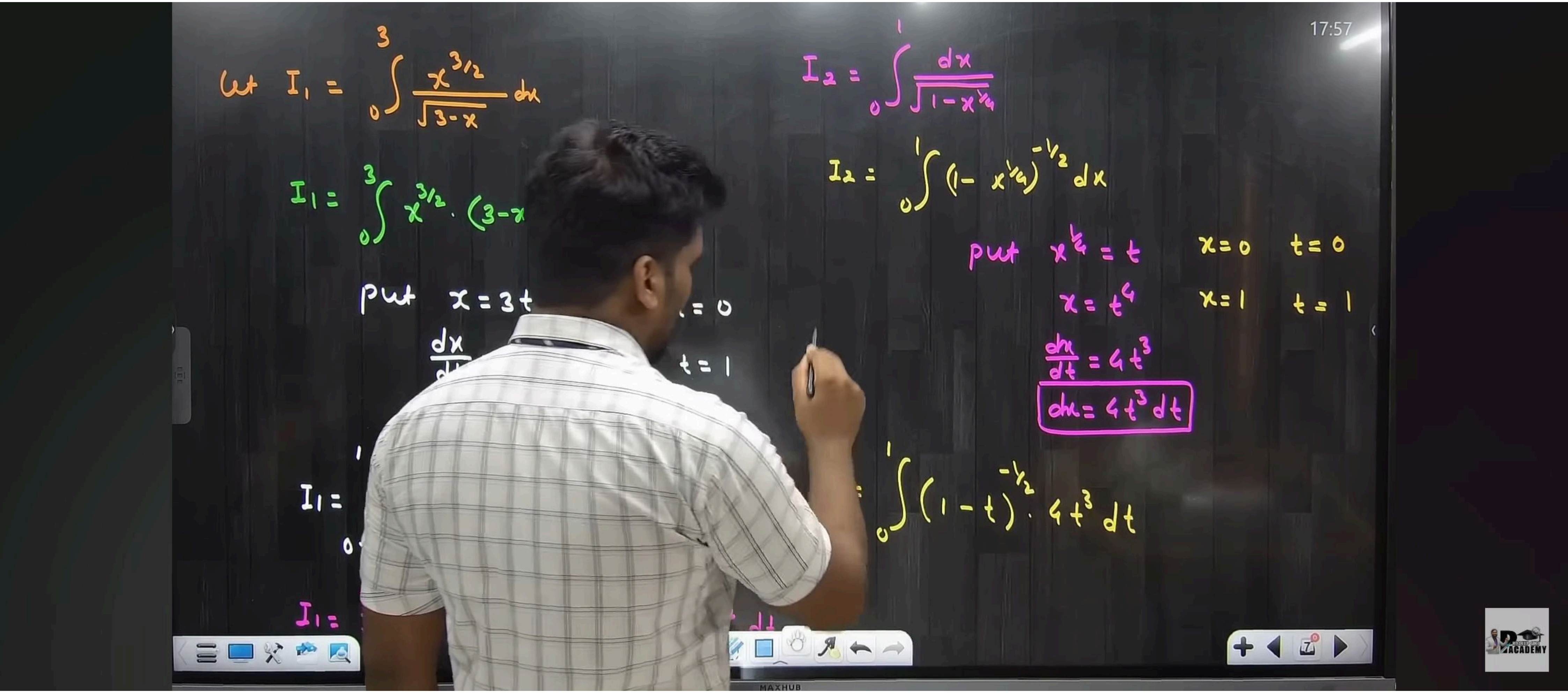
$$\boxed{dx = 3\,dt}$$

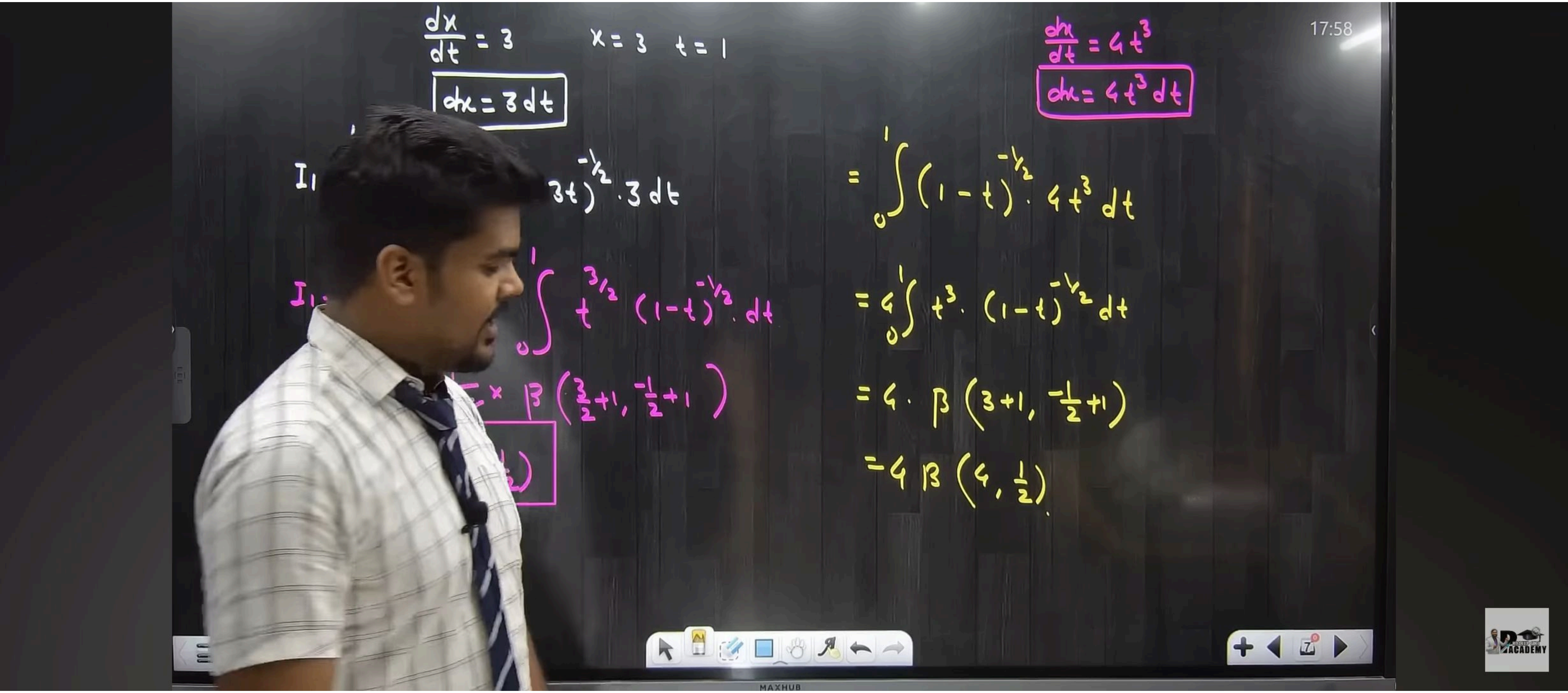
$$I_1 = \int_0^1 (3t)^{3/2} \cdot (3 - 3t)^{-1/2} \cdot 3\,dt$$

$$I_1 = 3^{3/2} \cdot 3 \cdot 3^{-1/2} \int_0^1 t^{3/2} (1-t)^{-1/2} \cdot dt$$

$$I_1 = 3\cancel{\sqrt{3}} \times 3 \times \frac{1}{\cancel{\sqrt{3}}} \times \beta\left(\frac{3}{2}+1, \frac{-1}{2}+1\right)$$

$$\boxed{I_1 = 9 \times \beta\left(\frac{5}{2}, \frac{1}{2}\right)}$$


17:57
Let $I_1 = \int_0^3 \frac{x^{3/2}}{\sqrt{3-x}} dx$
$I_1 = \int_0^3 x^{3/2} \cdot (3-x$
put $x = 3t$
$t = 0$
$\frac{dx}{dt}$
$t = 1$
$I_1 =$
$I_1 =$
$I_2 = \int_0^1 \frac{dx}{\sqrt{1-x^{1/4}}}$
$I_2 = \int_0^1 (1-x^{1/4})^{-1/2} dx$
put $x^{1/4} = t$
$x = 0$ $t = 0$
$x = t^4$
$x = 1$ $t = 1$
$\frac{dx}{dt} = 4t^3$
$\boxed{dx = 4t^3 dt}$
$\int_0^1 (1-t)^{-1/2} \cdot 4t^3 dt$
ACADEMY
MAXHUB



$\frac{dx}{dt} = 3$
$x = 3 \quad t = 1$
$\boxed{dx = 3dt}$
$3t)^{-1/2} \cdot 3\,dt$
$\int_0^1 t^{3/2} (1-t)^{-1/2} \cdot dt$
$\beta\left(\frac{3}{2}+1, -\frac{1}{2}+1\right)$
$\frac{dx}{dt} = 4t^3$
$\boxed{dx = 4t^3 dt}$
$= \int_0^1 (1-t)^{-1/2} \cdot 4t^3 dt$
$= 4\int_0^1 t^3 \cdot (1-t)^{-1/2} dt$
$= 4 \cdot \beta\left(3+1, -\frac{1}{2}+1\right)$
$= 4\, \beta\left(4, \frac{1}{2}\right)$
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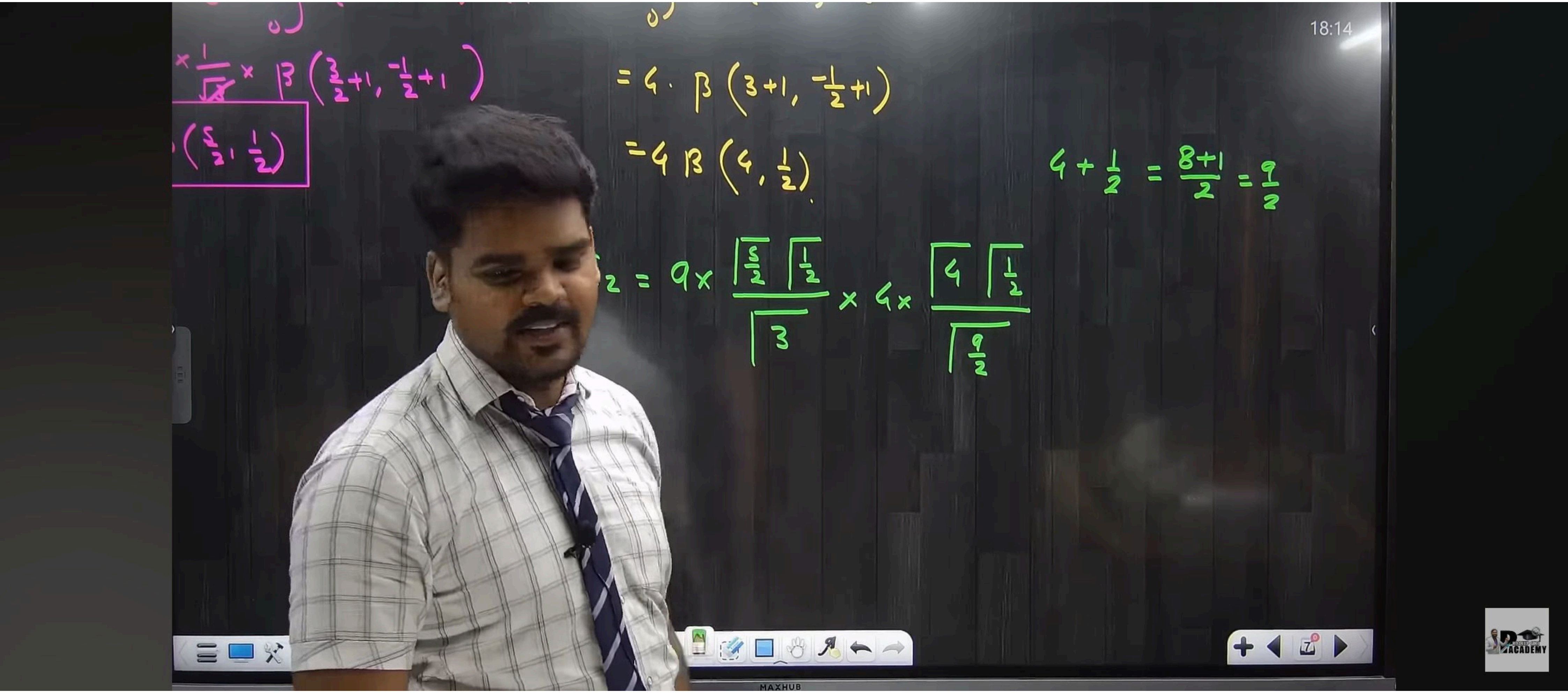
18:14
$\times \frac{1}{\sqrt{2}} \times \beta\left(\frac{3}{2}+1, -\frac{1}{2}+1\right)$
$\left(\frac{5}{2}, \frac{1}{2}\right)$
$= 4 \cdot \beta\left(3+1, -\frac{1}{2}+1\right)$
$= 4\,\beta\left(4, \frac{1}{2}\right)$
$4 + \frac{1}{2} = \frac{8+1}{2} = \frac{9}{2}$
$2 = 9 \times \frac{\Gamma\frac{5}{2}\,\Gamma\frac{1}{2}}{\Gamma 3} \times 4 \times \frac{\Gamma 4\,\Gamma\frac{1}{2}}{\Gamma\frac{9}{2}}$
MAXHUB
ACADEMY

Type - III

$$\boxed{\int_0^1 \left(1-\sqrt[n]{x}\right)^m dx = \frac{m!\,n!}{(m+n)!}}$$

Evaluate $\displaystyle\int_0^1 \left(1-\sqrt[3]{x}\right)^{11/2} dx$

put $\sqrt[3]{x} = t$

$x = t^3$

$\dfrac{dx}{dt} = 3t^2$

$\boxed{dx = 3t^2\,dt}$

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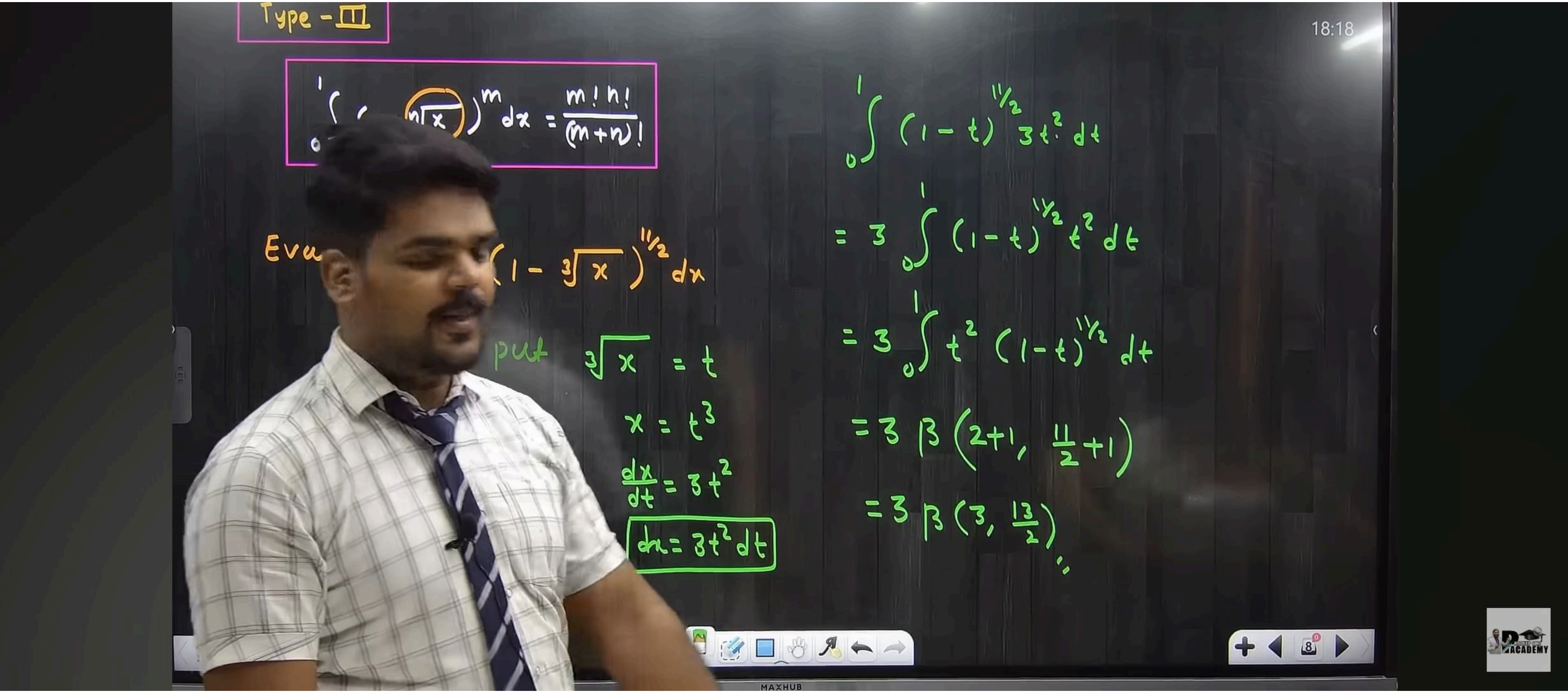
Type - III
$\frac{m!\,n!}{(m+n)!}$
$(1-\sqrt[3]{x})^{11/2}\,dx$
put $\sqrt[3]{x} = t$
$x = t^3$
$\frac{dx}{dt} = 3t^2$
$\boxed{dx = 3t^2\,dt}$
$\int_0^1 (1-t)^{11/2}\, 3t^2\, dt$
$= 3\int_0^1 (1-t)^{11/2}\, t^2\, dt$
$= 3\int_0^1 t^2 (1-t)^{11/2}\, dt$
$= 3\,\beta\left(2+1, \frac{11}{2}+1\right)$
$= 3\,\beta\left(3, \frac{13}{2}\right)$
18:18
MAXHUB
ACADEMY