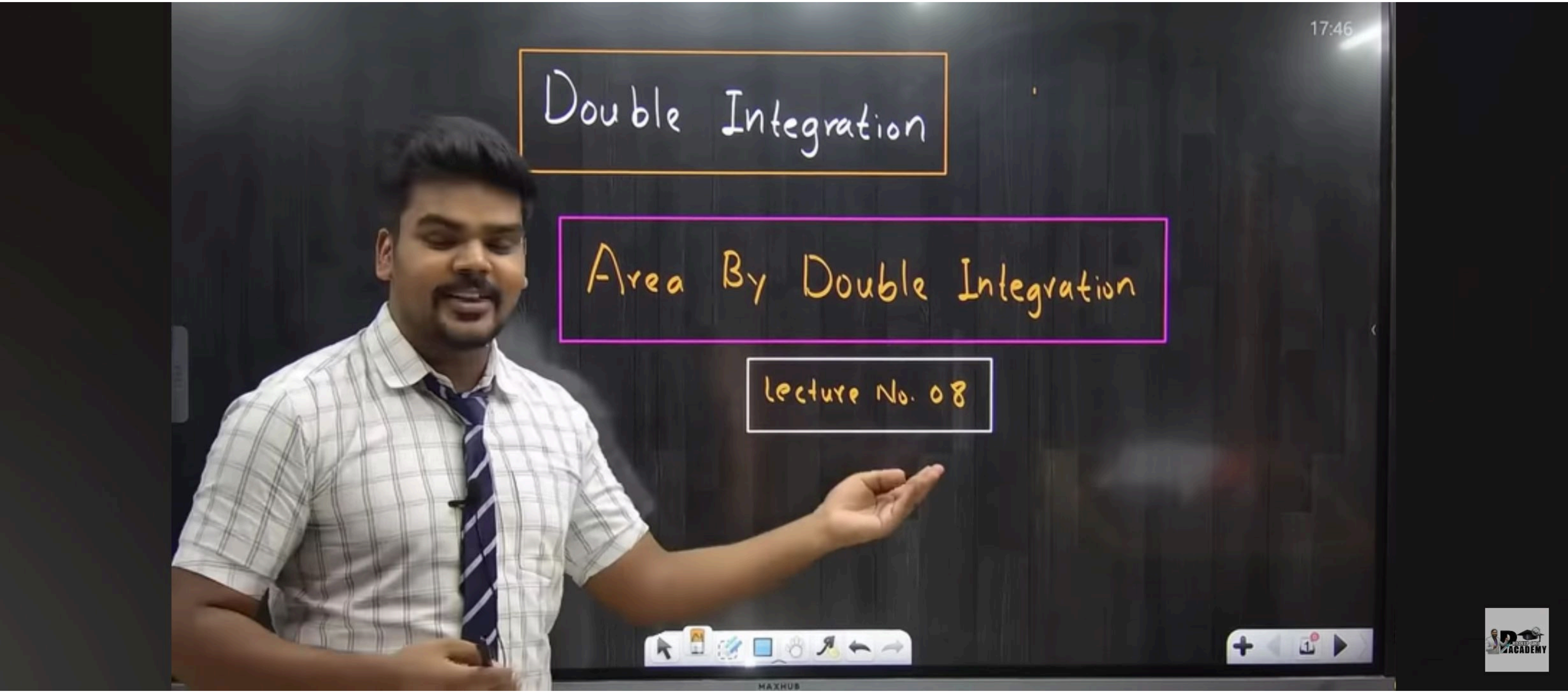

17:46
Double Integration
Area By Double Integration
lecture No. 08

Find by double integration the area betn the curve $y^2=4x$ and $2x-3y+4=0$

17:51

$$y^2=4x$$

$$\left(\frac{2x+4}{3}\right)^2=4x$$

$$\frac{(2x+4)^2}{9}=4x$$

$$(2x)^2+2(2x)(4)+4^2=36x$$

$$4x^2+16x+16-36x=0$$

$$4x^2-20x+16=0$$

$$x^2-5x+4=0 \qquad \div 4$$

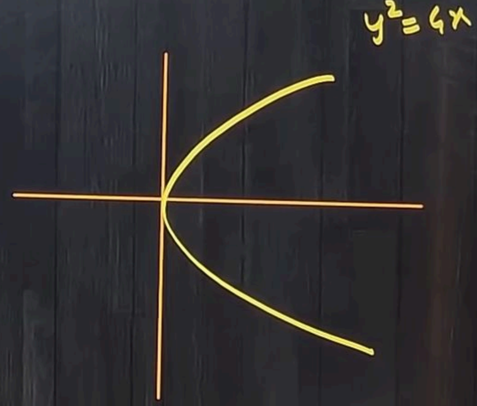
$$(x-4)(x-1)=0$$

$$\boxed{x=4} \qquad \boxed{x=1}$$

$$2x-3y+4=0$$

$$2x+4=3y$$

$$\boxed{\frac{2x+4}{3}=y}$$




MAXHUB

Find by double integration the area betⁿ the curve $y^2 = 4x$ and $2x - 3y + 4 = 0$

17:52

$y^2 = 4x$

$\left(\frac{\ x+4}{3}\right)^2 = 4x$

$\frac{\ +4)^2}{9} = 4x$

$+ 2(2x)(4) + 4^2 = 36x$

$4x^2 + 16x + 16 - 36x = 0$

$^2 - 20x + 16 = 0$

$- 5x + 4 = 0 \quad \div 4$

$)(x - 1) = 0$

$2x - 3y + 4 = 0$

$2x + 4 = 3y$

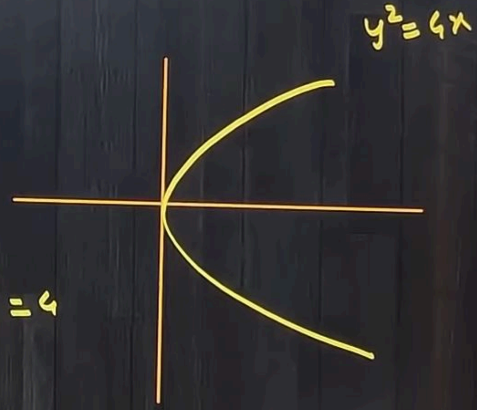
$\boxed{\frac{2x+4}{3} = y}$

$x = 4 \quad y = \frac{2(4)+4}{3} = \frac{12}{3} = 4$

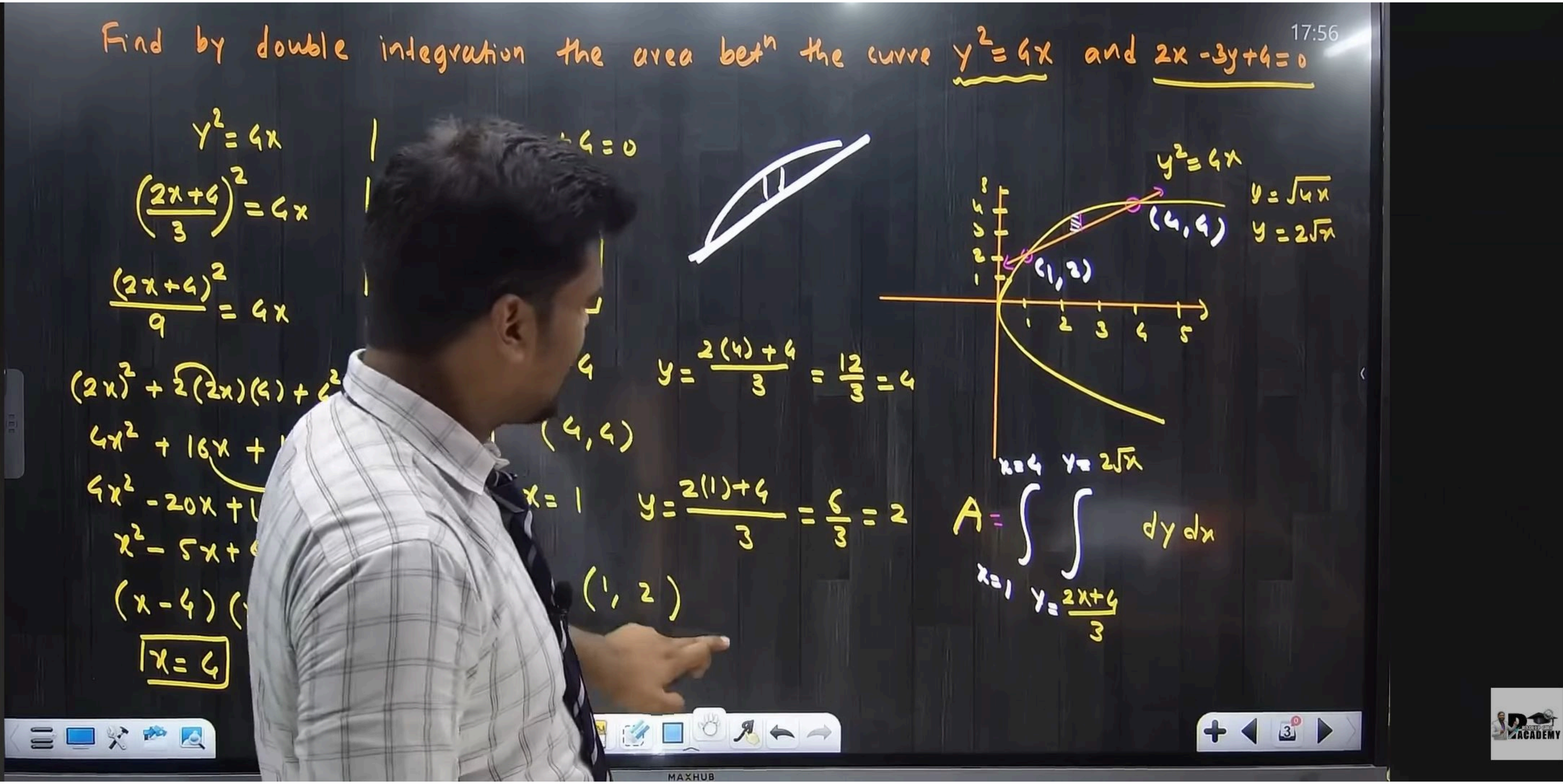
$(4, 4)$

$x = 1 \quad y = \frac{2(1)+4}{3} = \frac{6}{3} = 2$

$(1, 2)$



Strip is parallel to y axis therefore we will take $y =$ for the inner integral


Find by double integration the area betn the curve y2 = 4x and 2x - 3y + 4 = 0
17:56
y2 = 4x
((2x+4)/3)2 = 4x
(2x+4)2/9 = 4x
(2x)2 + 2(2x)(4) + 4
4x2 + 16x +
4x2 - 20x +
x2 - 5x +
(x - 4) (
x = 4
+4 = 0
(4, 4)
x = 1
(1, 2)
y = (2(4) + 4)/3 = 12/3 = 4
y = (2(1)+4)/3 = 6/3 = 2
y2 = 4x
y = √4x
y = 2√x
(4, 4)
(1, 2)
A = ∫ ∫ dy dx
x=1 to x=4
y = (2x+4)/3 to y = 2√x
MAXHUB
ACADEMY

$\boxed{x=4}$ $\boxed{x=1}$

$$A = \int_1^4 \left[\, y \,\right]_{\frac{2x+4}{3}}^{2\sqrt{x}} dx$$

$$A = \int_1^4 \left[2\sqrt{x} - \left(\frac{2x+4}{3}\right)\right] dx$$

$$A = \int_1^4 \left[\frac{6\sqrt{x} - 2x - 4}{3}\right] dx$$

$$A = \frac{1}{3}\int_1^4 \left[6\sqrt{x} - 2x - 4\right] dx$$

$$A = \frac{1}{3}\left[\cancel{6}^{2} \cdot \frac{2}{\cancel{3}} x^{3/2} - \cancel{2} \cdot \frac{x^2}{\cancel{2}} - 4x\right]_1^4$$

$$A = \frac{1}{3}\left[4 \cdot x^{3/2} - x^2 - 4x\right]_1^4$$

$$4^{3/2} = 2^{\cancel{2} \times \frac{3}{\cancel{2}}}$$

$$= 2^3$$

$$= 8$$

$$A = \frac{1}{3}\left[4 \cdot \right.$$

18:00

$4^{3/2} = 2^{\cancel{2}\times\frac{3}{\cancel{2}}}$

$$A = \frac{1}{3}\left[\cancel{6}^{2}\cdot\frac{2}{\cancel{3}}x^{3/2} - \cancel{2}\cdot\frac{x^2}{\cancel{2}} - 4x\right]_1^4$$

$$A = \frac{1}{3}\left[4\,x^{3/2} - x^2 - 4x\right]_1^4$$

3-4

$$A = \frac{1}{3}\left[(4\cdot 8 - 16 - 16) - (4 - 1 - 4)\right]$$

$$A = \frac{1}{3}\left[(32 - 32) - (-1)\right]$$

$$A = \frac{1}{3}\left[0 + 1\right] = \frac{1}{3}$$

$\frac{2x+4}{3}$

$$A = \int_1^4 \left[2\sqrt{x} - \left(\frac{2x+4}{3}\right)\right] dx$$

$$A = \int_1^4 \left[\frac{6\sqrt{x} - 2x - 4}{3}\right] dx$$

$$A = \frac{1}{3}\int_1^4 \left[6\sqrt{x} - 2x - 4\right] dx$$

Find by double integration the area betⁿ the parabola $y = x^2 - 6x + 3$ and the line $y = 2x - 9$.

$$y = x^2 - 6x + 3$$

$$y = \underbrace{x^2 - 2(x)(3) + 3^2}_{} + 3 - 3^2$$

$$y = (x - 3)^2 + 3 - 9$$

$$y = (x-3)^2 - 6$$

$$\boxed{y + 6 = (x-3)^2} \qquad \boxed{(3, -6)}$$

18:27

and the line $y = 2x - 9$.

18:30

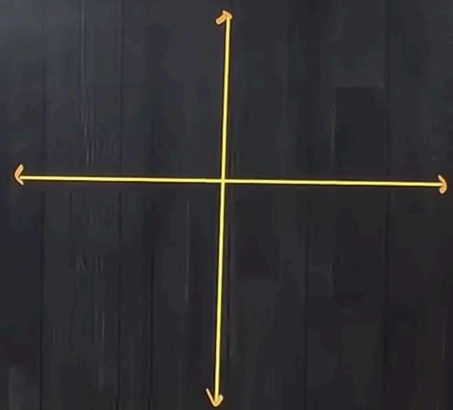
$y = x^2 - 6x + 3$

$y = x^2 - 2(x)(3) + 3^2 + 3 - 3^2$

$y = (x - 3)^2 + 3 - 9$

$y = (x - 3)^2 - 6$

$\boxed{y + 6 = (x - 3)^2}$ $\quad$ $\boxed{(3, -6)}$



$y = x^2 - 6x + 3$

$2x - 9 = x^2 - 6x + 3$

$0 = x^2 - 6x - 2x + 3 + 9$

$0 = x^2 - 8x + 12$

$0 = (x - 6)(x - 2)$

$\boxed{x = 6}$ $\boxed{y = 3}$ $(6, 3)$

$\boxed{x = 2}$ $\boxed{y = -5}$ $(2, -5)$

and the line $y = \underline{2x - 9}$.

$y = x^2 - 6x + 3$

$y = \underline{x^2 - 2(x)(3) + 3^2} + 3 \quad - 3^2$

$y = (x - 3)^2 + 3 - 9$

$y = (x-3)^2 - 6$

$\boxed{y + 6 = (x-3)^2}$ $\quad \boxed{(3, -6)}$

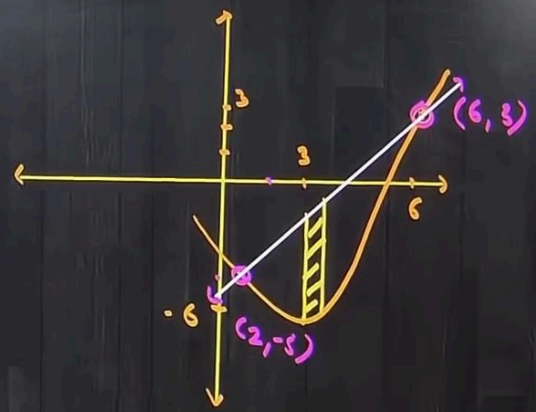
$y = x^2 - 6x + 3$

$2x - 9 = x^2 - 6x + 3$

$0 = x^2 - 6x - 2x + 3 + 9$

$0 = x^2 - 8x + 12$

$0 = (x - 6)(x - 2$



18:35

$y = x^2 - 6x + 3$

$2x - 9 = x^2 - 6x + 3$

$0 = x^2 - 6x - 2x + 3 + 9$

$0 = x^2 - 8x + 12$

$0 = (x-6)(x-2)$

$\boxed{x = 6}$ $\boxed{y = 3}$ $(6, 3)$

$\boxed{x = 2}$ $\boxed{y = -5}$ $(2, -5)$

$$A = \int_2^6 \int_{x^2-6x+3}^{2x-9} dy\,dx$$

$$= \int_2^6 \Big[\, y \,\Big]_{x^2-6x+3}^{2x-9} dx$$

$$= \int_2^6 \Big[(2x-9) - (x^2-6x+3)\Big]\,dx$$

$$= \int_2^6 \Big[2x - 9 - x^2 + 6x - 3\Big]\,dx$$

$0 = (x-6)(x-2)$

$$A = \int_2^6 \left[8x - x^2 - 12\right] dx$$

$$A = \left[\cancel{8}^{4} x \frac{x^2}{\cancel{2}} - \frac{x^3}{3} - 12x\right]_2^6$$

$A -$

$$= \int_2^6 \Big[ y \Big]_{x^2-6x+3} dx$$

$$= \int_2^6 \left[(2x-9) - (x^2 - 6x + 3)\right] dx$$

$$= \int_2^6 \left[2x - 9 - x^2 + 6x - 3\right] dx$$

18:36

MAXHUB

ACADEMY

$$A = \left[4\cancel{8}\frac{x^2}{\cancel{2}} - \frac{x^3}{3} - 12x\right]_2$$

$$A = \left[4x^2 - \frac{x^3}{3} - 12x\right]_2^6$$

$$A = \left[4\times 36 - \frac{216}{3} - 12\times 6\right] - \left[4(4) - \frac{8}{3} - 12(2)\right]$$

$$A = \left[144 - \frac{216}{3} - 72\right] - \left[16 - \frac{8}{3} - 24\right]$$

$$A = [$$

$$A = \left[4\times 36 - \frac{216}{3} - 12\times 6\right] - \left[4(4) - \frac{8}{3} - 12(2)\right]$$

$$A = \left[144 - \frac{216}{3} - 72\right] - \left[16 - \frac{8}{3} - 24\right]$$

$$A = \left[72 - \frac{216}{3}\right] - \left[-8 - \frac{8}{3}\right]$$

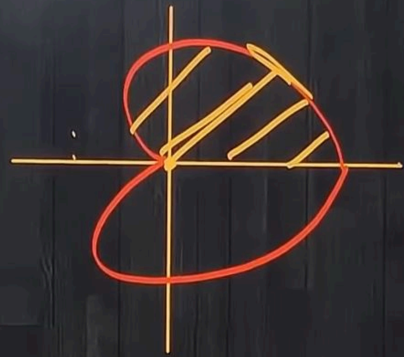
$$A = \left[\cancel{72} - \cancel{72} + 8 + \frac{8}{3}\right]$$

$$A = \frac{24+8}{3} = \frac{32}{3}$$

21:22

Find by double integration the area of the cardiod $r = a(1+\cos\theta)$.

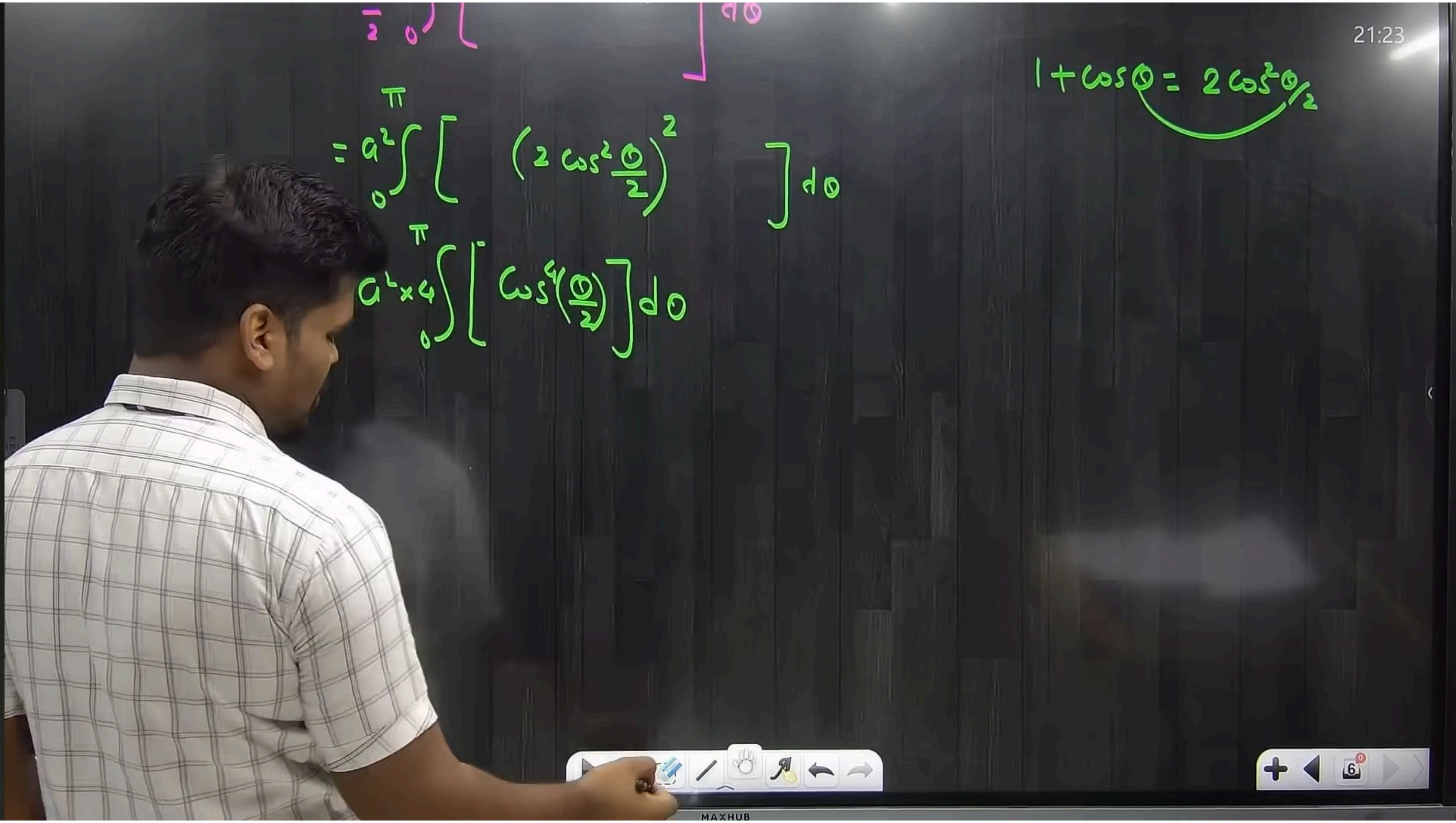
$$= 2\times\int_0^{\pi}\int_0^{a(1+\cos\theta)} r\,dr\,d\theta$$



$$2\int_0^{\pi}\left[\frac{r^2}{2}\right]_0^{a(1+\cos\theta)} d\theta$$

$$= \frac{2}{2}\int_0^{\pi}\left[a^2(1+\cos\theta)^2\right]$$

MAXHUB



21:23
$= a^2 \int_0^{\pi} \left[ \left(2\cos^2\frac{\theta}{2}\right)^2 \right] d\theta$
$= a^2 \times 4 \int_0^{\pi} \left[ \cos^4\left(\frac{\theta}{2}\right) \right] d\theta$
$1 + \cos\theta = 2\cos^2\theta/2$
MAXHUB
ACADEMY

21:25

$$= a^2 \times 4 \int_0^{\pi} \left[ \cos^4\left(\frac{\theta}{2}\right) \right] d\theta$$

put $\frac{\theta}{2} = t$ $\qquad \theta = 0 \qquad t = 0$

$\theta = 2t$ $\qquad \theta = \pi \qquad t = \frac{\pi}{2}$

$\frac{d\theta}{dt} = 2(1)$

$\boxed{d\theta = 2dt}$

$$\int^{\pi/2} \cos^4 t \,(2dt)$$

MAXHUB

21:26

$\frac{\theta}{2} = t$

$\theta = 2t$ $\qquad$ $\theta = \pi$ $\qquad$ $t = \frac{\pi}{2}$

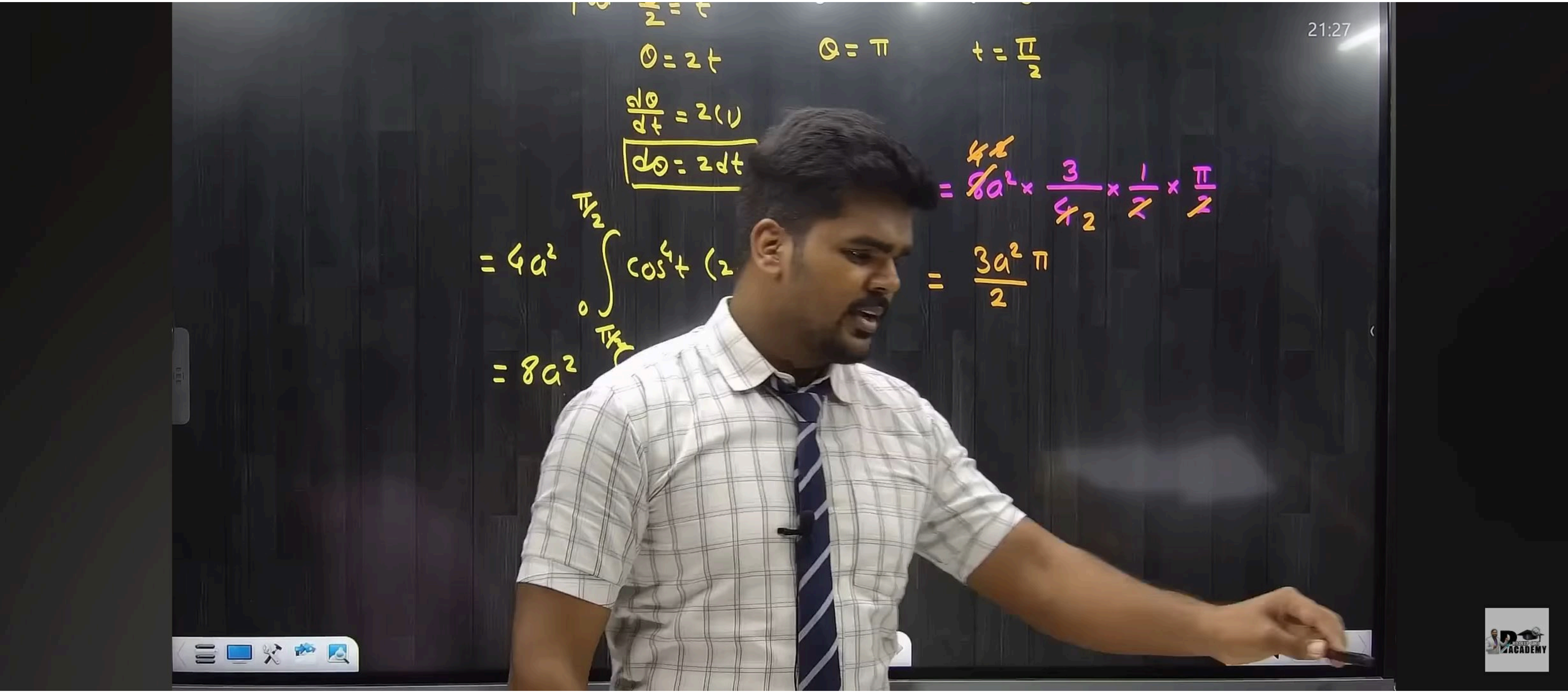
$\frac{d\theta}{dt} = 2(1)$

$\boxed{d\theta = 2dt}$

$$= 4a^2 \int_0^{\pi/2} \cos^4 t \,(2dt)$$

$$= 8a^2 \int_0^{\pi/2} \cos^4 t \, dt$$

$\times \frac{1}{2} \times$



21:27
θ = 2t
θ = π
t = π/2
dθ/dt = 2(1)
dθ = 2dt
= 8a² × 3/4 × 1/2 × π/2
= 4a² ∫ cos⁴t (2
= 3a²π/2
= 8a²
ACADEMY