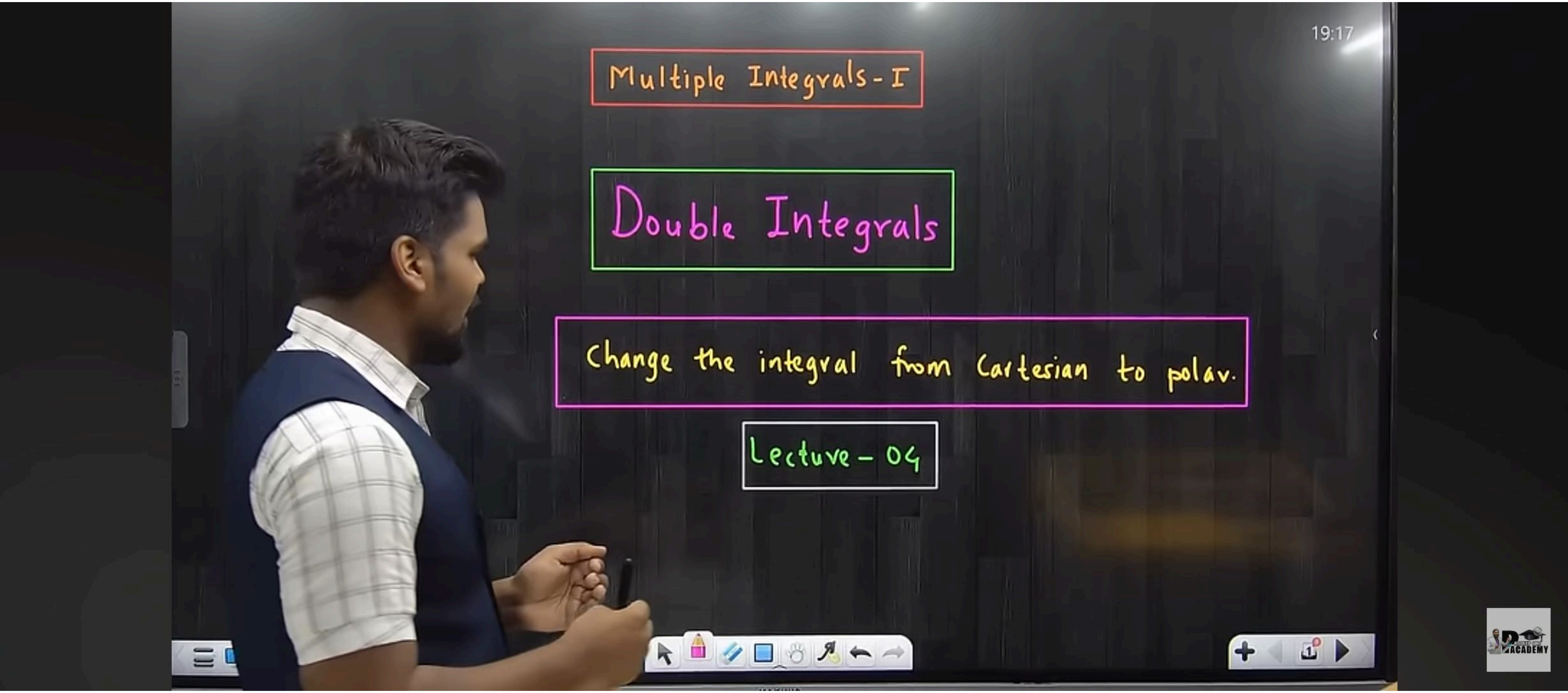
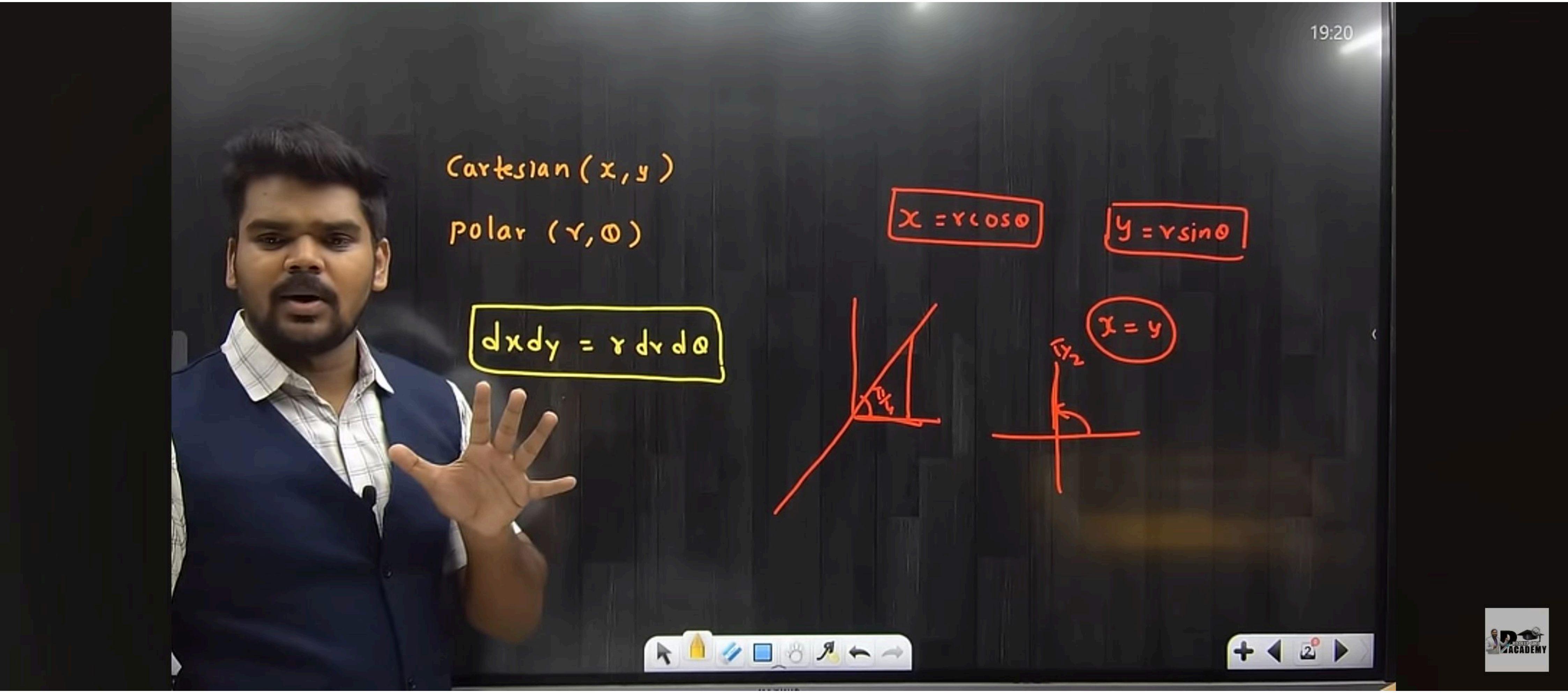
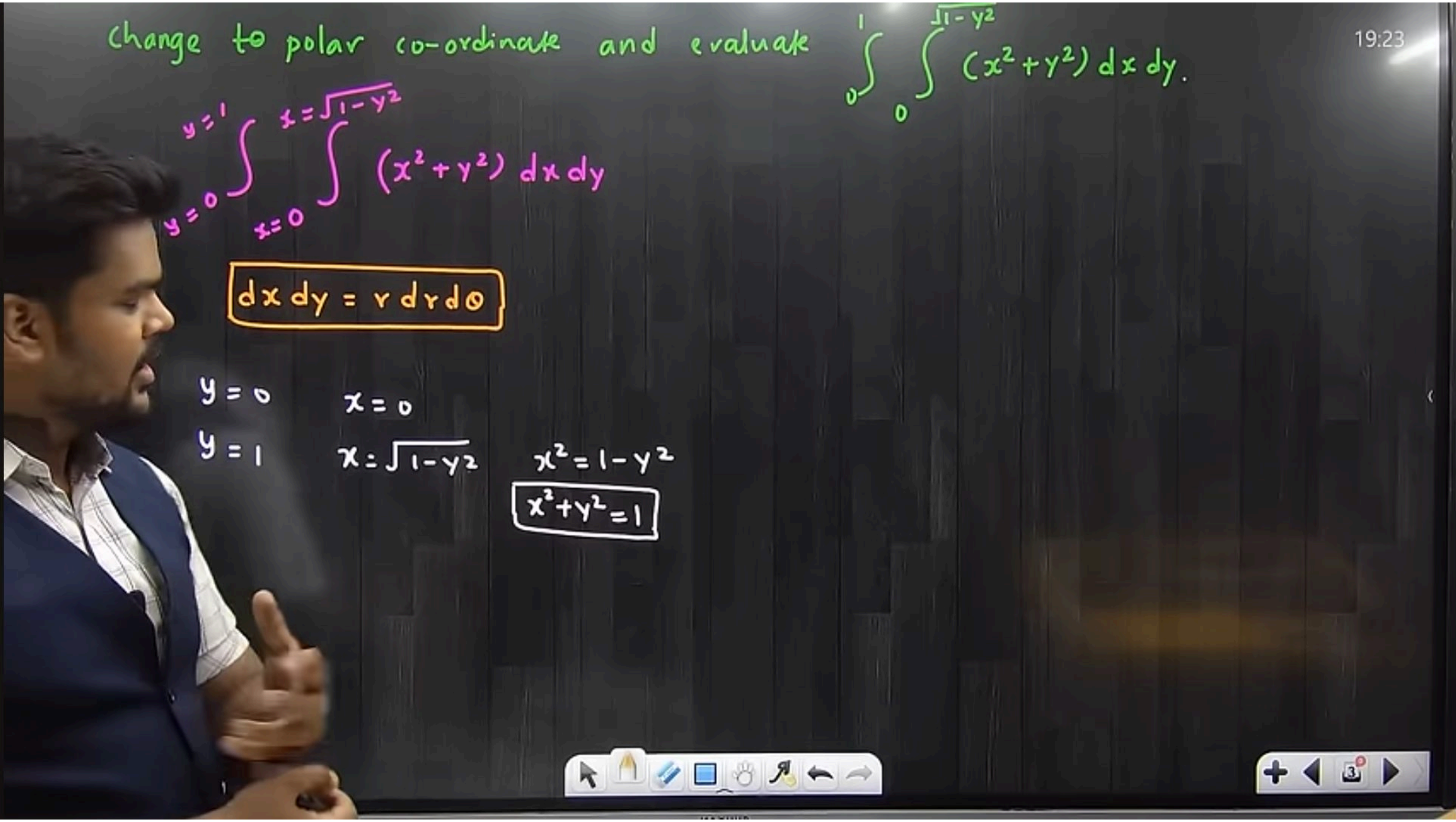
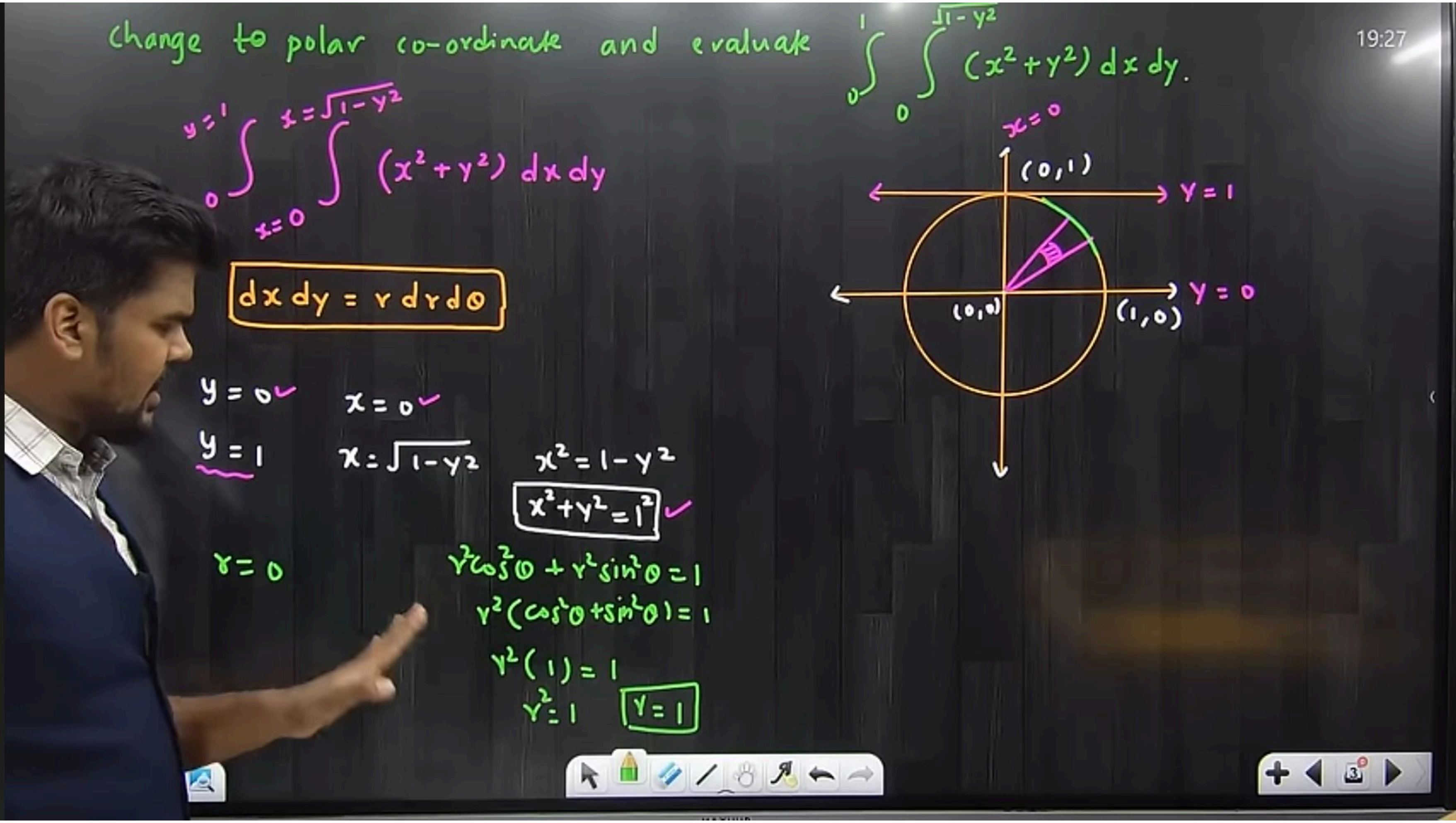

19:17
Multiple Integrals - I
Double Integrals
Change the integral from Cartesian to polar.
Lecture - 04
ACADEMY


19:20
Cartesian (x, y)
polar (r, θ)
dxdy = r dr dθ
x = rcosθ
y = rsinθ
x = y
π/4
π/2
ACADEMY

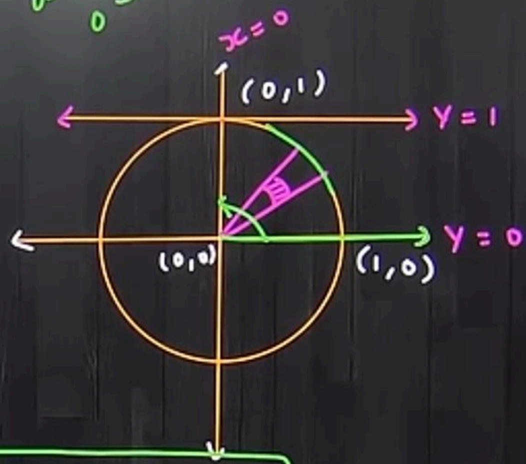

Change to polar co-ordinate and evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$.
$\int_{y=0}^{y=1} \int_{x=0}^{x=\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$
$\boxed{dx\,dy = r\,dr\,d\theta}$
$y=0$ $x=0$
$y=1$ $x=\sqrt{1-y^2}$ $x^2=1-y^2$
$\boxed{x^2+y^2=1}$
19:23


Change to polar co-ordinate and evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$.
$\int_{y=0}^{y=1} \int_{x=0}^{x=\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$
$dx\,dy = r\,dr\,d\theta$
$y=0$ ✓ $x=0$ ✓
$y=1$ $x=\sqrt{1-y^2}$ $x^2=1-y^2$
$\boxed{x^2+y^2=1^2}$ ✓
$r=0$
$r^2\cos^2\theta + r^2\sin^2\theta = 1$
$r^2(\cos^2\theta+\sin^2\theta)=1$
$r^2(1)=1$
$r^2=1$ $\boxed{r=1}$
$x=0$
$(0,1)$
$y=1$
$\theta$
$(0,0)$
$(1,0)$
$y=0$
19:27
ACADEMY

Change to polar co-ordinate and evaluate $\int_0 \int_0 (x^2+y^2)\,dx\,dy.$

$$\int_{y=0}^{y=1}\int_{x=0}^{x=\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$$

$$\boxed{dx\,dy = r\,dr\,d\theta}$$




$y = 0$ ✓ $\quad x = 0$ ✓

$y = 1 \quad x = \sqrt{1-y^2} \quad x^2 = 1 - y^2$

$$\boxed{x^2+y^2 = 1^2}$$ ✓

$$\begin{aligned} x^2+y^2 &= (r\cos\theta)^2 + (r\sin\theta)^2 \\ &= r^2\cos^2\theta + r^2\sin^2\theta \\ &= r^2(\cos^2\theta + \sin^2\theta) \\ &= r^2(1) \end{aligned}$$

$$\boxed{x^2+y^2 = r^2}$$

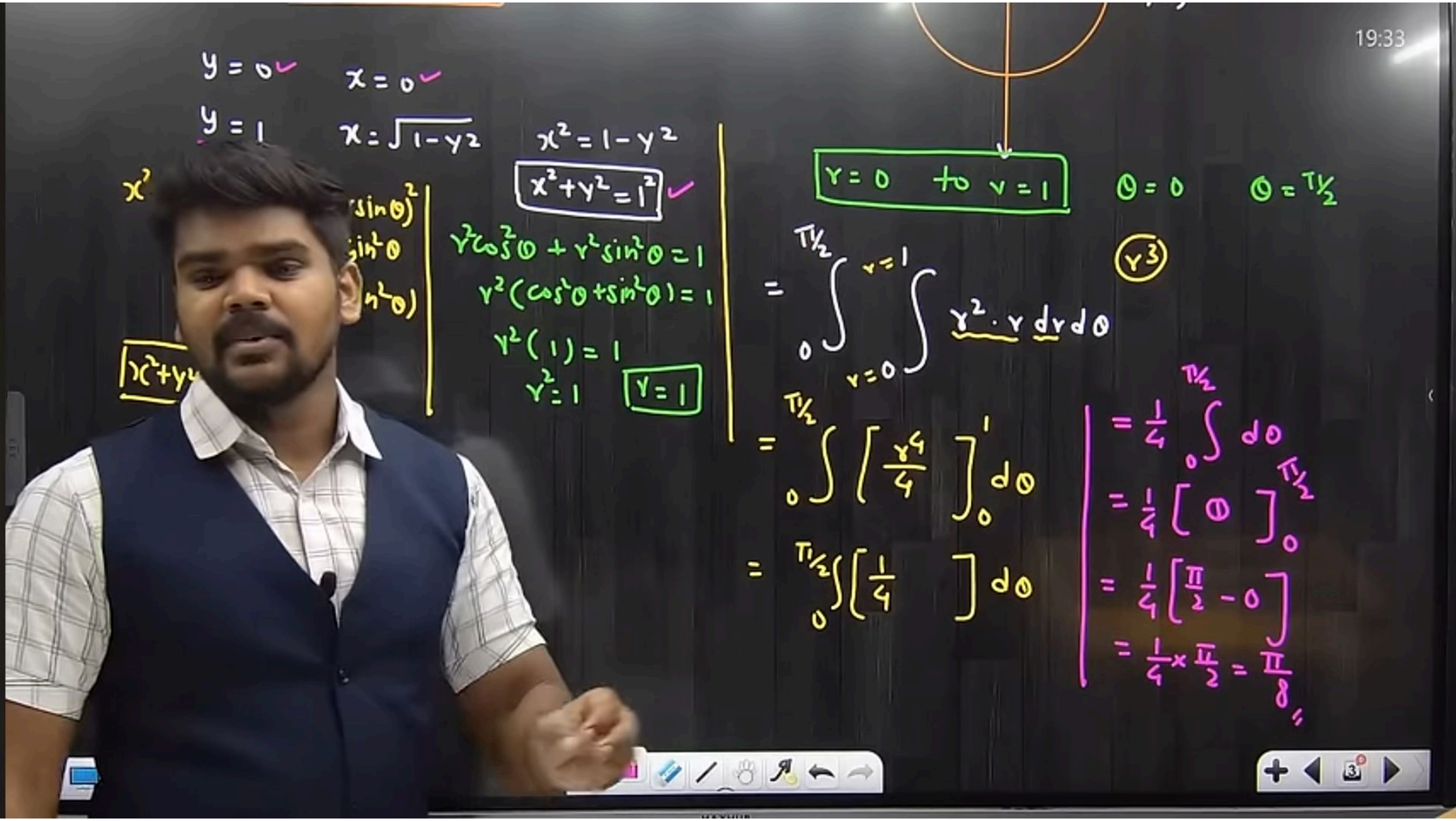
$$r^2\cos^2\theta + r^2\sin^2\theta = 1$$

$$r^2(\cos^2\theta + \sin^2\theta) = 1$$

$$r^2(1) = 1$$

$$r^2 = 1 \quad \boxed{r = 1}$$

$$\boxed{r = 0 \text{ to } r = 1} \qquad \theta = 0 \qquad \theta = \pi/2$$

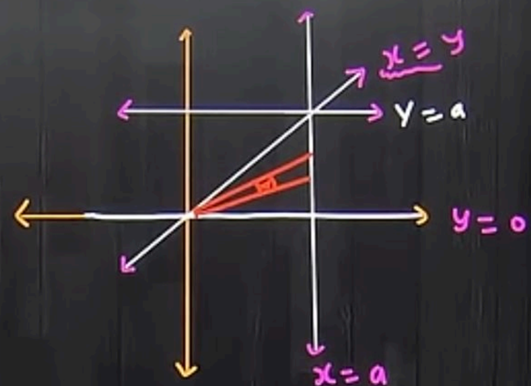


19:33
y = 0
x = 0
y = 1
x = √(1 - y²)
x² = 1 - y²
x² + y² = 1²
r²cos²θ + r²sin²θ = 1
r²(cos²θ + sin²θ) = 1
r²(1) = 1
r² = 1
r = 1
r = 0 to r = 1
θ = 0
θ = π/2
r³
= ∫₀^{π/2} ∫_{r=0}^{r=1} r² · r dr dθ
= ∫₀^{π/2} [r⁴/4]₀¹ dθ
= ∫₀^{π/2} [1/4] dθ
= 1/4 ∫₀^{π/2} dθ
= 1/4 [θ]₀^{π/2}
= 1/4 [π/2 - 0]
= 1/4 × π/2 = π/8

$$\int_{y=0}^{y=a}\int_{x=y}^{x=a} x\,dx\,dy.$$

$y = 0$ $\quad$ $y = a$

$x = y$ $\quad$ $x = a$



19:38

$$\int_{y=0}^{y=a}\int_{x=y}^{x=a} x\,dx\,dy.$$

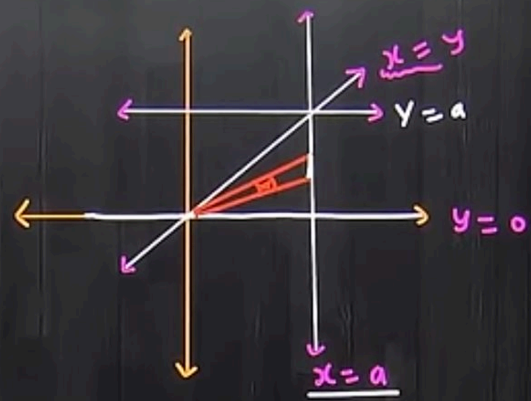
$y=0$ $\quad$ $y=a$

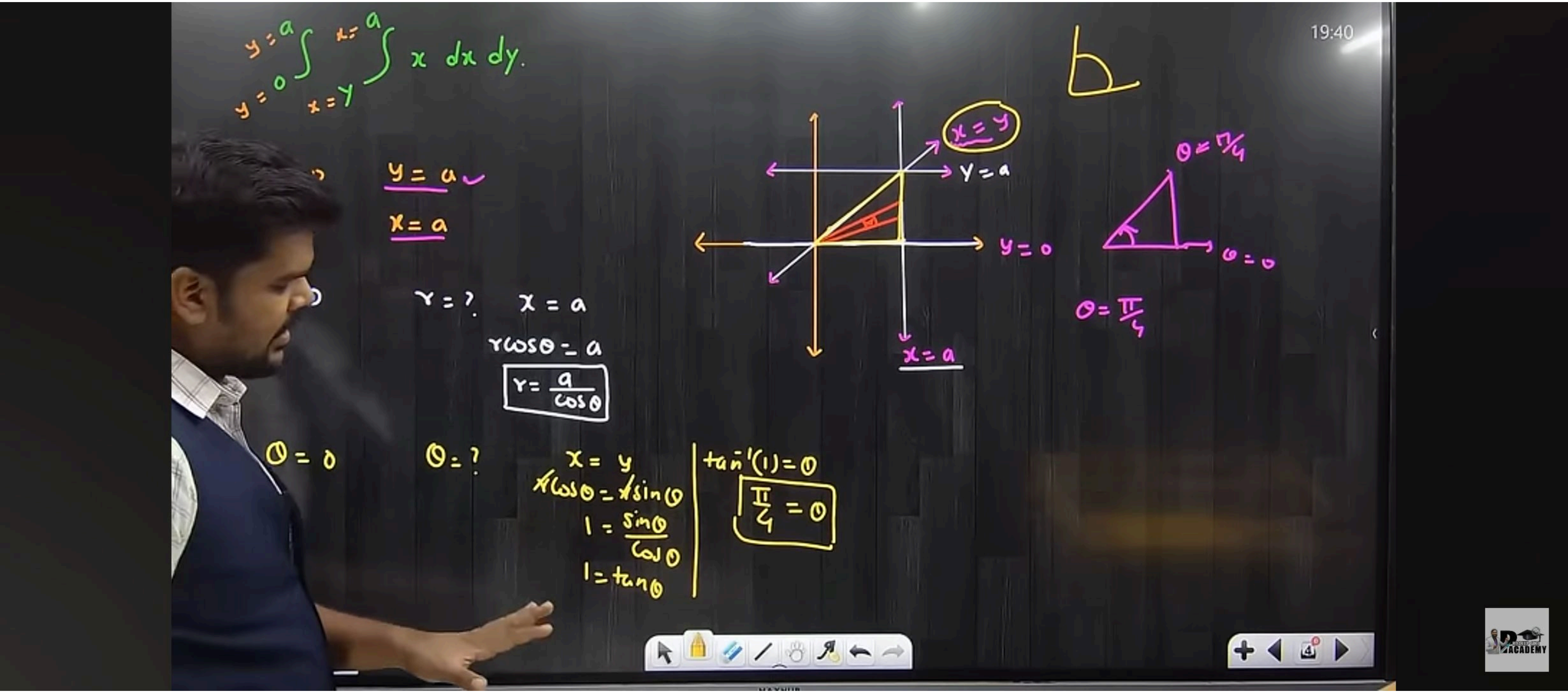
$x=y$ $\quad$ $x=a$

$r=0$ $\quad$ $r=?$ $\quad$ $x=a$

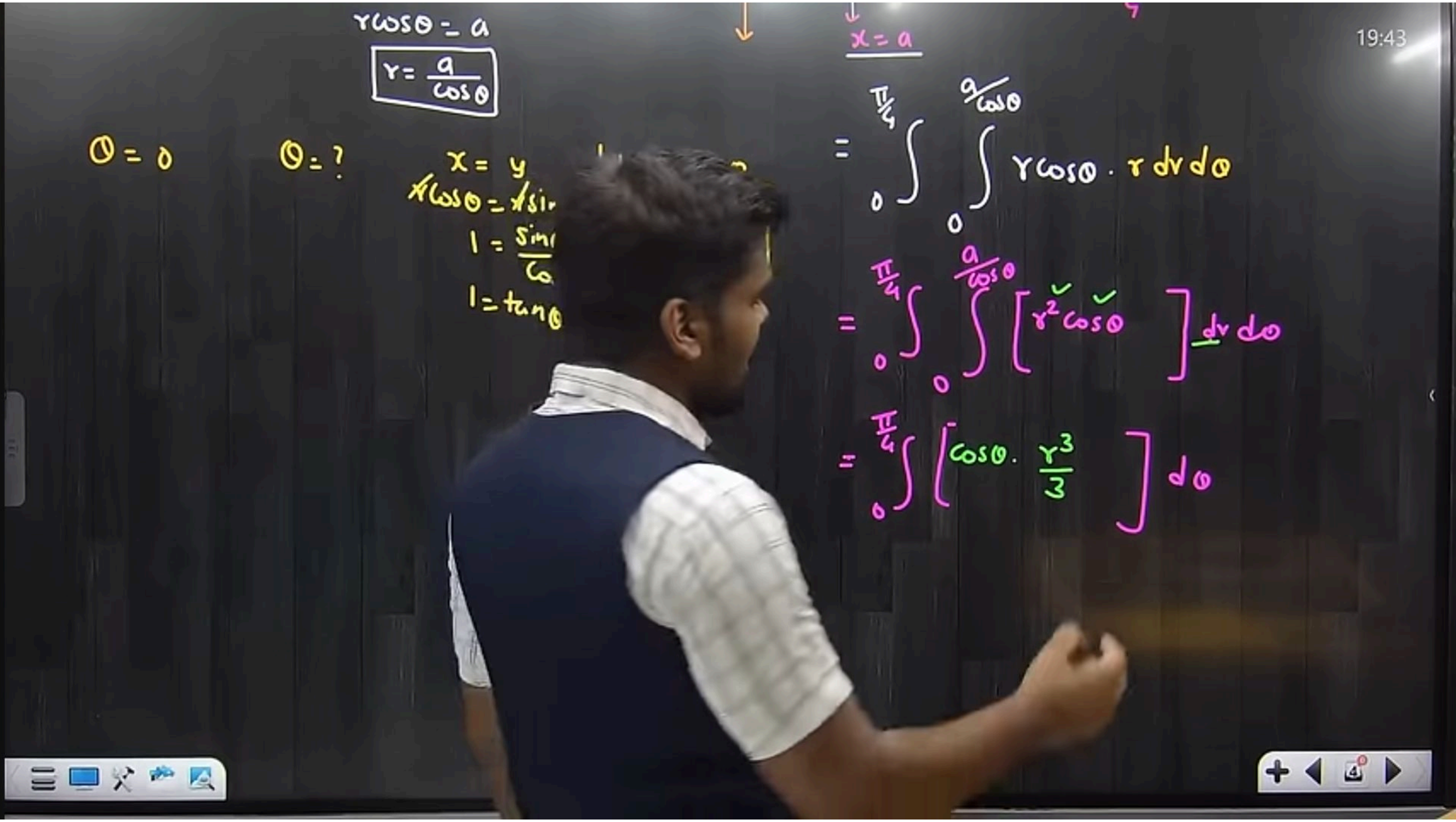
$$r\cos\theta = a$$

$$\boxed{r=\frac{a}{\cos\theta}}$$

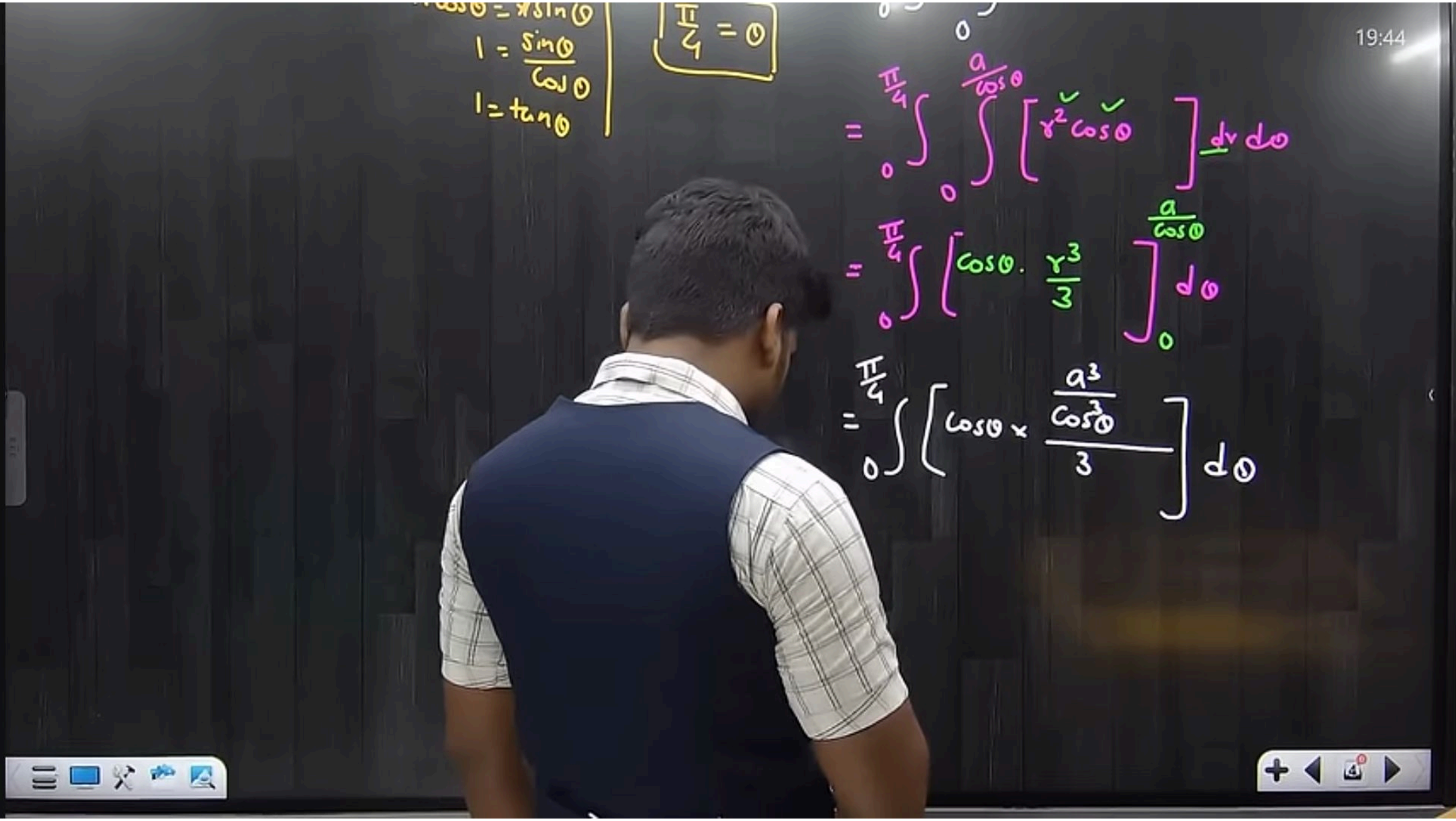




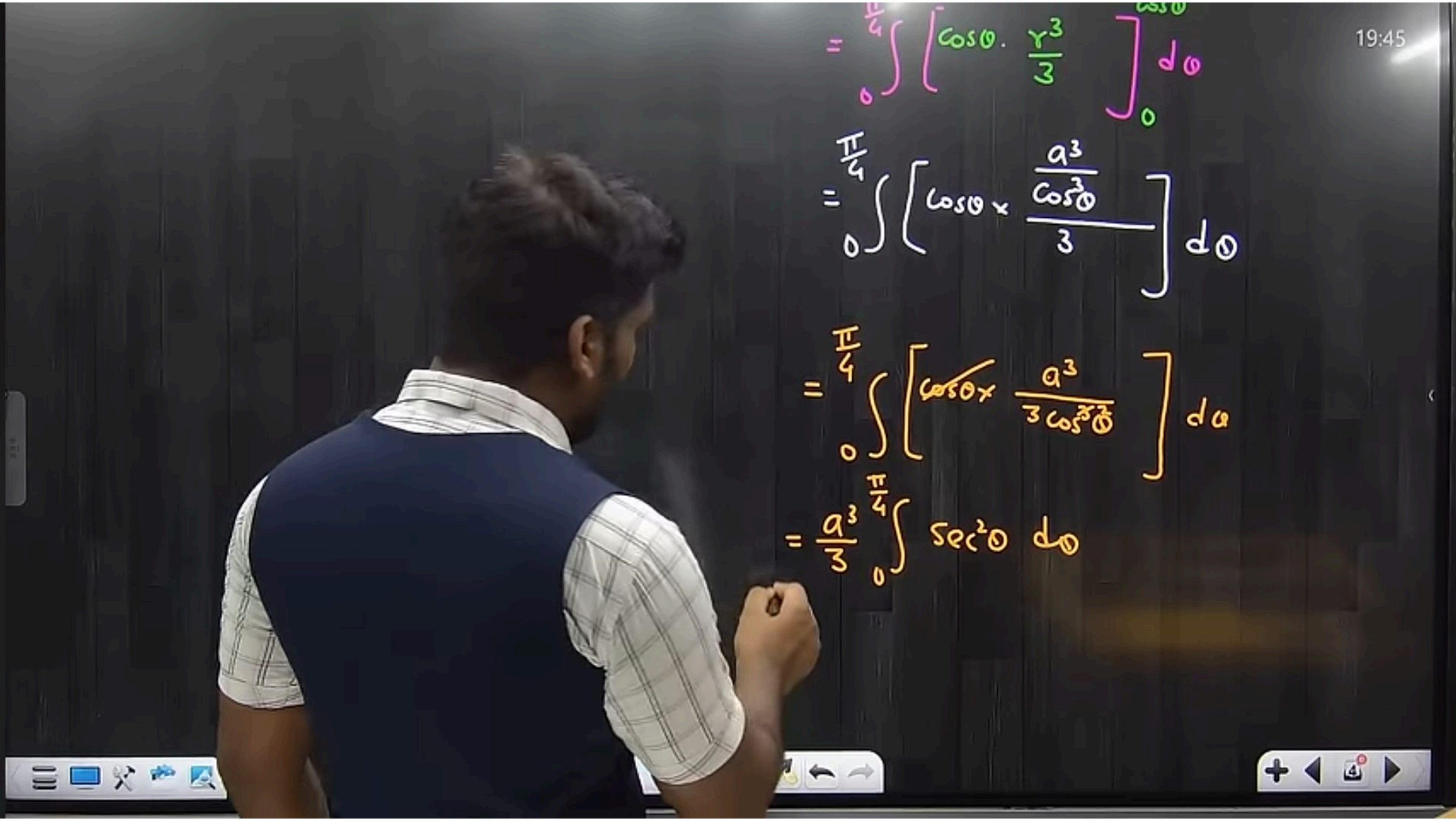
19:40
y = a ∫ x = a ∫ x dx dy.
y = 0 x = y
y = a
x = a
x = y
y = a
y = 0
x = a
θ = π/4
θ = 0
θ = π/4
r = ? x = a
r cos θ = a
r = a / cos θ
θ = 0
θ = ?
x = y
r cos θ = r sin θ
1 = sin θ / cos θ
1 = tan θ
tan⁻¹(1) = θ
π/4 = θ
ACADEMY



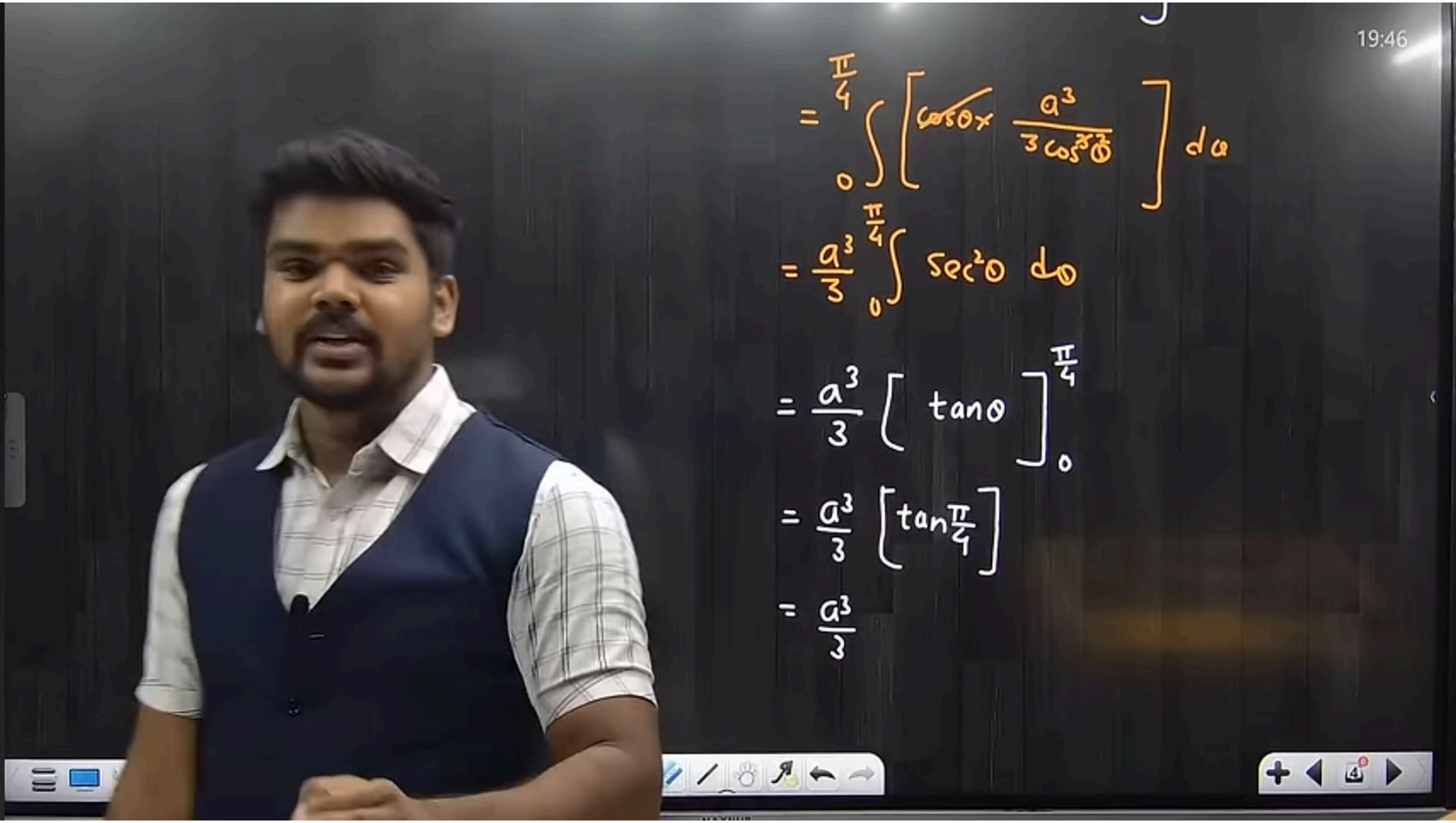
$r\cos\theta = a$
$r = \frac{a}{\cos\theta}$
$\theta = 0$
$\theta = ?$
$x = y$
$1 = \tan\theta$
19:43
$x = a$
$= \int_0^{\pi/4} \int_0^{a/\cos\theta} r\cos\theta \cdot r\,dr\,d\theta$
$= \int_0^{\pi/4} \int_0^{a/\cos\theta} \left[ r^2 \cos\theta \right] dr\,d\theta$
$= \int_0^{\pi/4} \left[ \cos\theta \cdot \frac{r^3}{3} \right] d\theta$
ACADEMY



$r\cos\theta = r\sin\theta$
$1 = \frac{\sin\theta}{\cos\theta}$
$1 = \tan\theta$
$\boxed{\frac{\pi}{4} = \theta}$
$= \int_0^{\frac{\pi}{4}} \int_0^{\frac{a}{\cos\theta}} \left[ r^2 \cos\theta \right] dr\, d\theta$
$= \int_0^{\frac{\pi}{4}} \left[ \cos\theta \cdot \frac{r^3}{3} \right]_0^{\frac{a}{\cos\theta}} d\theta$
$= \int_0^{\frac{\pi}{4}} \left[ \cos\theta \times \frac{\frac{a^3}{\cos^3\theta}}{3} \right] d\theta$
19:44



$= \int_0^{\pi/4} \left[ \cos\theta \cdot \frac{r^3}{3} \right]_0^{\ldots\cos\theta} d\theta$
$= \int_0^{\pi/4} \left[ \cos\theta \times \frac{\frac{a^3}{\cos^3\theta}}{3} \right] d\theta$
$= \int_0^{\pi/4} \left[ \cancel{\cos\theta} \times \frac{a^3}{3\cos^3\theta} \right] da$
$= \frac{a^3}{3} \int_0^{\pi/4} \sec^2\theta \, d\theta$


19:46
$= \int_0^{\frac{\pi}{4}} \left[ \cancel{\cos\theta} \times \frac{a^3}{3\cos^{\cancel{3}\,2}\theta} \right] d\theta$
$= \frac{a^3}{3} \int_0^{\frac{\pi}{4}} \sec^2\theta \, d\theta$
$= \frac{a^3}{3} \left[ \tan\theta \right]_0^{\frac{\pi}{4}}$
$= \frac{a^3}{3} \left[ \tan\frac{\pi}{4} \right]$
$= \frac{a^3}{3}$

$$\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2+y^2)\,dy\,dx$$

$x = 0 \qquad y = x$

$x = 1 \qquad y = \sqrt{2x - x^2}$

$$y^2 = 2x - x^2$$

$$x^2 \underline{-2x} + y^2 = 0$$

$$x^2 - 2(x)(1) + 1^2 - 1^2 + y^2 = 0$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(\quad) = x^2 - 2(x)(1) + \bigcirc^2$$

If there is any variable
with number then it
is not standard circle
equation
Therefore, we need to
convert it to standard
circle equation

$$\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2+y^2)\,dy\,dx$$

$x = 0 \qquad y = x$

$x = 1 \qquad y = \sqrt{2x - x^2}$

$y^2 = 2x - x^2$

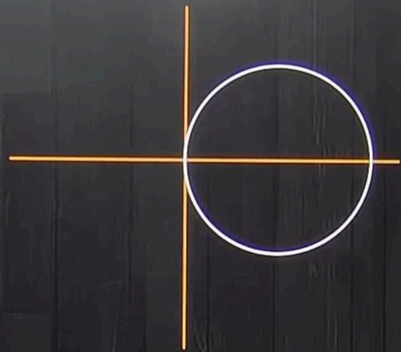
$x^2 - 2x + y^2 = 0$

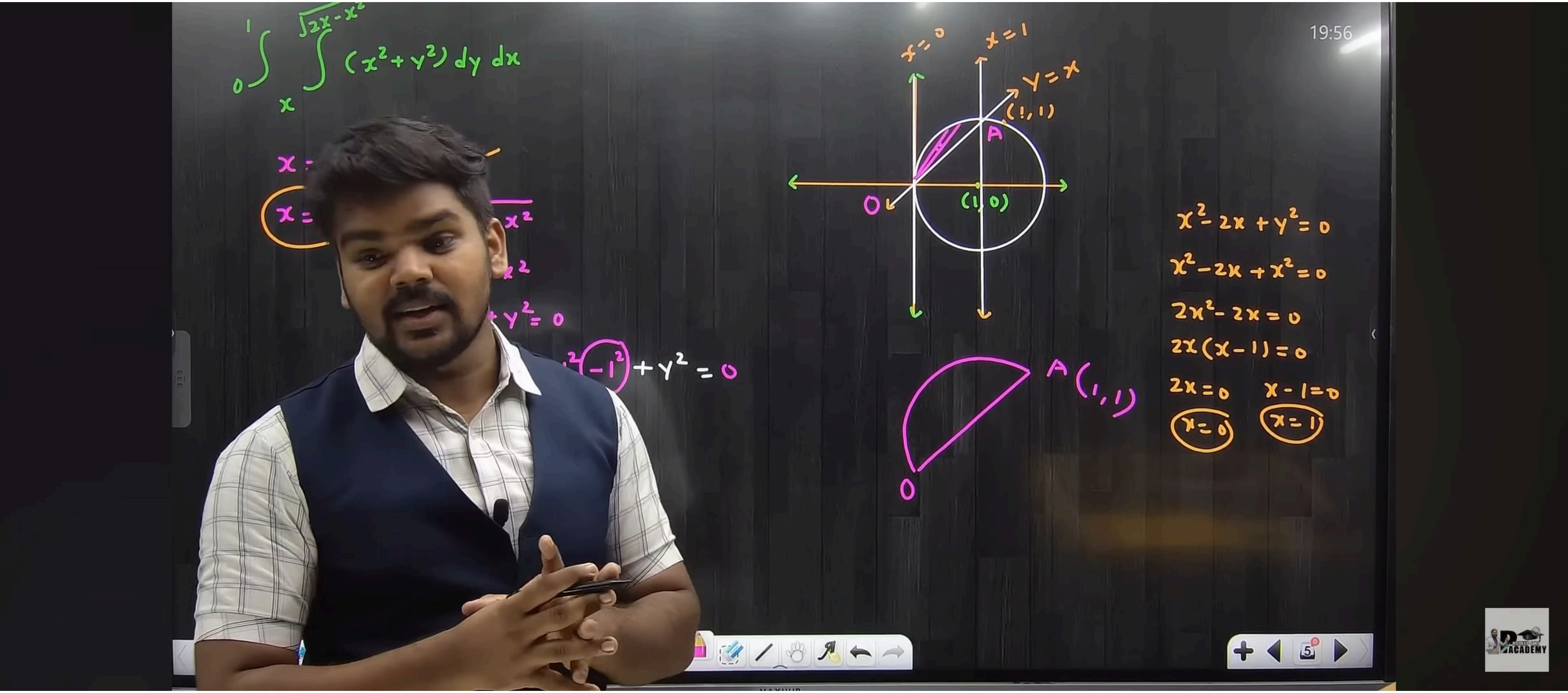
$x^2 - 2(x)(1) + 1^2 - 1^2 + y^2 = 0$

$x^2 - 2x + 1 + y^2 = 1$

$$\boxed{(x-1)^2 + y^2 = 1}$$

Centre $\equiv (1, 0)$





$\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2+y^2)\,dy\,dx$
19:56
$x=0$
$x=1$
$y=x$
$(1,1)$
A
O
$(1,0)$
A $(1,1)$
O
$x^2-2x+y^2=0$
$x^2-2x+x^2=0$
$2x^2-2x=0$
$2x(x-1)=0$
$2x=0$ $x-1=0$
$x=0$ $x=1$
$+y^2=0$
$-1^2+y^2=0$
ACADEMY

19:58

$$\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2+y^2)\,dy\,dx$$

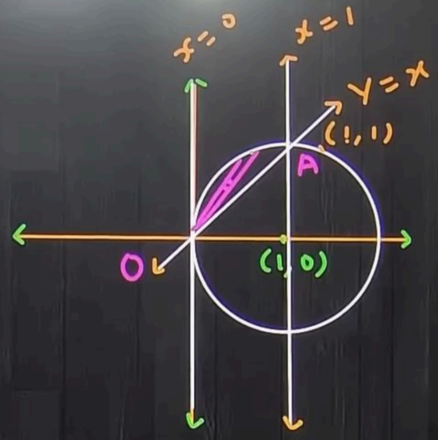
$x = 0$

$x = 1$

$= 0$

$(1) + 1^2 - 1^2 + y^2 = 0$

$y^2 = 1$




$r = 0$

$y^2 = 2x - x^2$

$y^2 + x^2 = 2x$

$r^2 \sin^2\theta + r^2\cos^2\theta = 2r\cos\theta$

$r^2(\sin^2\theta + \cos^2\theta) = 2r\cos\theta$

$r^{\cancel{2}}(1) = 2\cancel{r}\cos\theta$

$\boxed{r = 2\cos\theta}$

$x^2 - 2x + y^2 = 0$

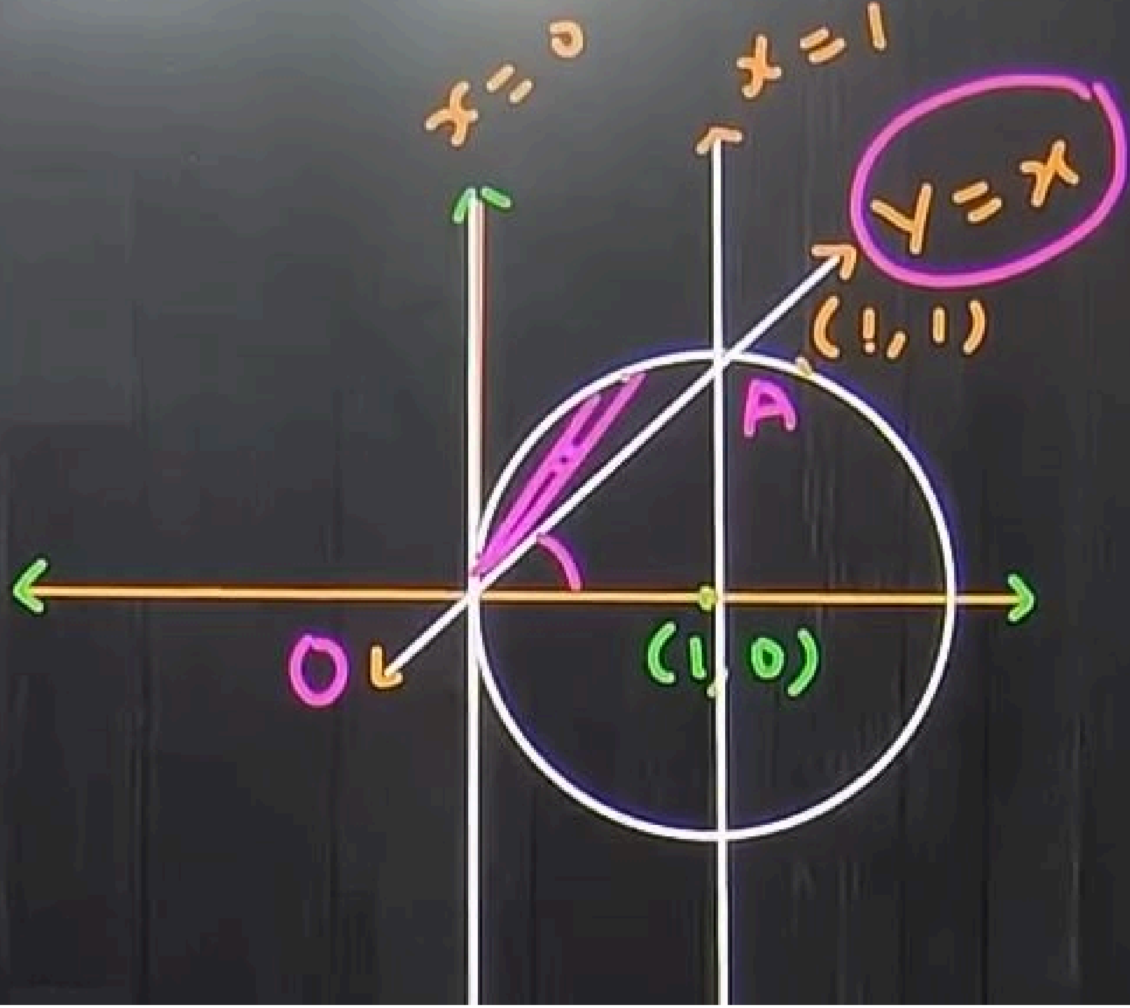
$x^2 - 2x + x^2 = 0$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$2x = 0 \qquad x - 1 = 0$

$x = 0 \qquad x = 1$




$\theta = \frac{\pi}{4}$ $\theta = \frac{\pi}{2}$

$x^2 - 2x + y^2 = 0$

$x^2 - 2x + x^2 = 0$

$$\boxed{(x-1)^2 + y^2 = 1}$$

Centre $\equiv (1, 0)$

$$\int_{\pi/4}^{\pi/2}\int_{0}^{2\cos\theta} r^2 \cdot r \, dr \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \left[\frac{r^4}{4}\right]_0^{2\cos\theta} d\theta$$

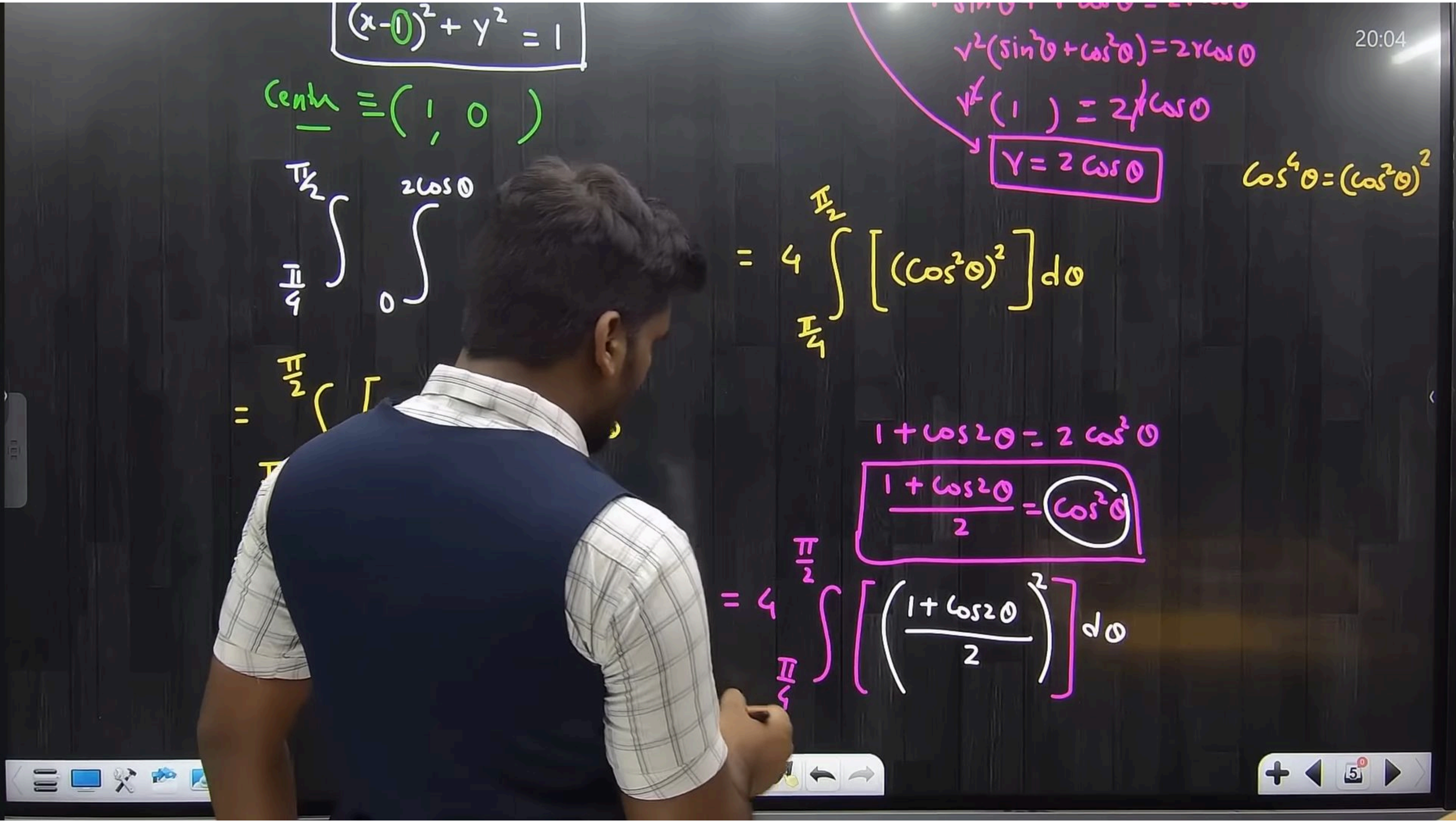
$$= \int_{\pi/2}^{\pi/2} \left[\frac{16(\cos\theta)^4}{4}\right] d\theta$$

$$r^2(\sin^2\theta + \cos^2\theta) = 2r\cos\theta$$

$$r^{\cancel{2}}(1) = 2\cancel{r}\cos\theta$$

$$\boxed{r = 2\cos\theta}$$

20:01



$(x-1)^2 + y^2 = 1$
Centre $\equiv (1, 0)$
$r^2(\sin^2\theta + \cos^2\theta) = 2r\cos\theta$
$r^2(1) = 2r\cos\theta$
$r = 2\cos\theta$
$\cos^4\theta = (\cos^2\theta)^2$
$\int_{\pi/4}^{\pi/2} \int_{0}^{2\cos\theta}$
$= 4 \int_{\pi/4}^{\pi/2} \left[ (\cos^2\theta)^2 \right] d\theta$
$1 + \cos 2\theta = 2\cos^2\theta$
$\frac{1+\cos 2\theta}{2} = \cos^2\theta$
$= 4 \int_{\pi/4}^{\pi/2} \left[ \left( \frac{1+\cos 2\theta}{2} \right)^2 \right] d\theta$
20:04

$$\int_{\frac{\pi}{4}} \left[ \quad 4 \quad \right]$$

$$= 4 \int_{\frac{\pi}{4}} \left[ \left( \frac{1+\cos 2\theta}{2} \right)^2 \right] d\theta$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(1+\cos 2\theta)^2 = 1 + 2(1)\cos 2\theta + (\cos 2\theta)^2$$

$$= \cancel{4} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[ \frac{(1+\cos 2\theta)^2}{\cancel{4}} \right] d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[ 1 + 2\cos 2\theta + (\cos 2\theta)^2 \right] d\theta$$

$\frac{\pi}{4} \int \left[ \quad 4 \quad \right] d\theta$

$= 4 \int_{\frac{\pi}{4}} \left[ \left( \frac{1+\cos 2\theta}{2} \right)^2 \right] d\theta$

20:07

$(a+b)^2 = a^2 + 2ab + b^2$

$(1+\cos 2\theta)^2 = 1 + 2(1)\cos 2\theta + (\cos 2\theta)^2$

$= \cancel{4} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[ \frac{(1+\cos 2\theta)^2}{\cancel{4}} \right] d\theta$

$\frac{1+\cos 2\theta}{2} = \cos^2\theta$

$\frac{1+\cos 4\theta}{2} = \cos^2 2\theta$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[ 1 + 2\cos 2\theta + (\cos 2\theta)^2 \right] d\theta$

$\frac{1}{2} + \frac{\cos 4\theta}{2}$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[ 1 + 2\cos 2\theta + \qquad \right]$

ACADEMY

2

$$\frac{1+\cos 4\theta}{2} = \cos^2 2\theta$$

$$\frac{1}{2} + \frac{\cos 4\theta}{2}$$

$$= \int_{\frac{\pi}{4}} \left[1 + 2\cos 2\theta + (\cos 2\theta)^2\right] d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[1 + 2\cos 2\theta + \frac{1}{2} + \frac{\cos 4\theta}{2}\right] d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\frac{3}{2} + 2\cos 2\theta + \frac{\cos 4\theta}{2}\right] d\theta$$

$$= \left[\frac{3}{2}\theta + 2\cdot\frac{\sin 2\theta}{2} + \frac{\sin 4\theta}{2\times 4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \int_{\pi/6}^{\pi/2} \left[\frac{3}{2} + 2\cos 2\theta + \frac{\cos 4\theta}{2}\right] d\theta$$

$$= \left[\frac{3}{2}\theta + \cancel{2}\cdot\frac{\sin 2\theta}{\cancel{2}} + \frac{\sin 4\theta}{2\times 4}\right]_{\pi/4}^{\pi/2}$$

$$= \left[\frac{3}{2}\theta + \sin 2\theta + \frac{\sin 4\theta}{8}\right]_{\pi/4}^{\pi/2}$$

20:11

$$= \left[\frac{3}{2}\theta + \cancel{2}\cdot\frac{\sin 2\theta}{\cancel{2}} + \frac{\sin 4\theta}{2\times 4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\frac{3}{2}\theta + \sin 2\theta + \frac{\sin 4\theta}{8}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\frac{3}{2}\cdot\frac{\pi}{2} + \sin 2\times\frac{\pi}{2} + \frac{\sin 4\times\frac{\pi}{2}}{8} - \left(\frac{3}{2}\frac{\pi}{4} + \sin 2\times\frac{\pi}{4} + \frac{\sin 4\times\frac{\pi}{4}}{8}\right)\right]$$