Example 1 : Verify Cayley-Hamilton Theorem for the matrix A and hence, find A^{-1}, A^{-2} and A^4 where

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

that $A^{-1} = A^2 - 5A + 9I$.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$S_1 = 5$

$S_2 = 3 + 1 + 5 = 9$

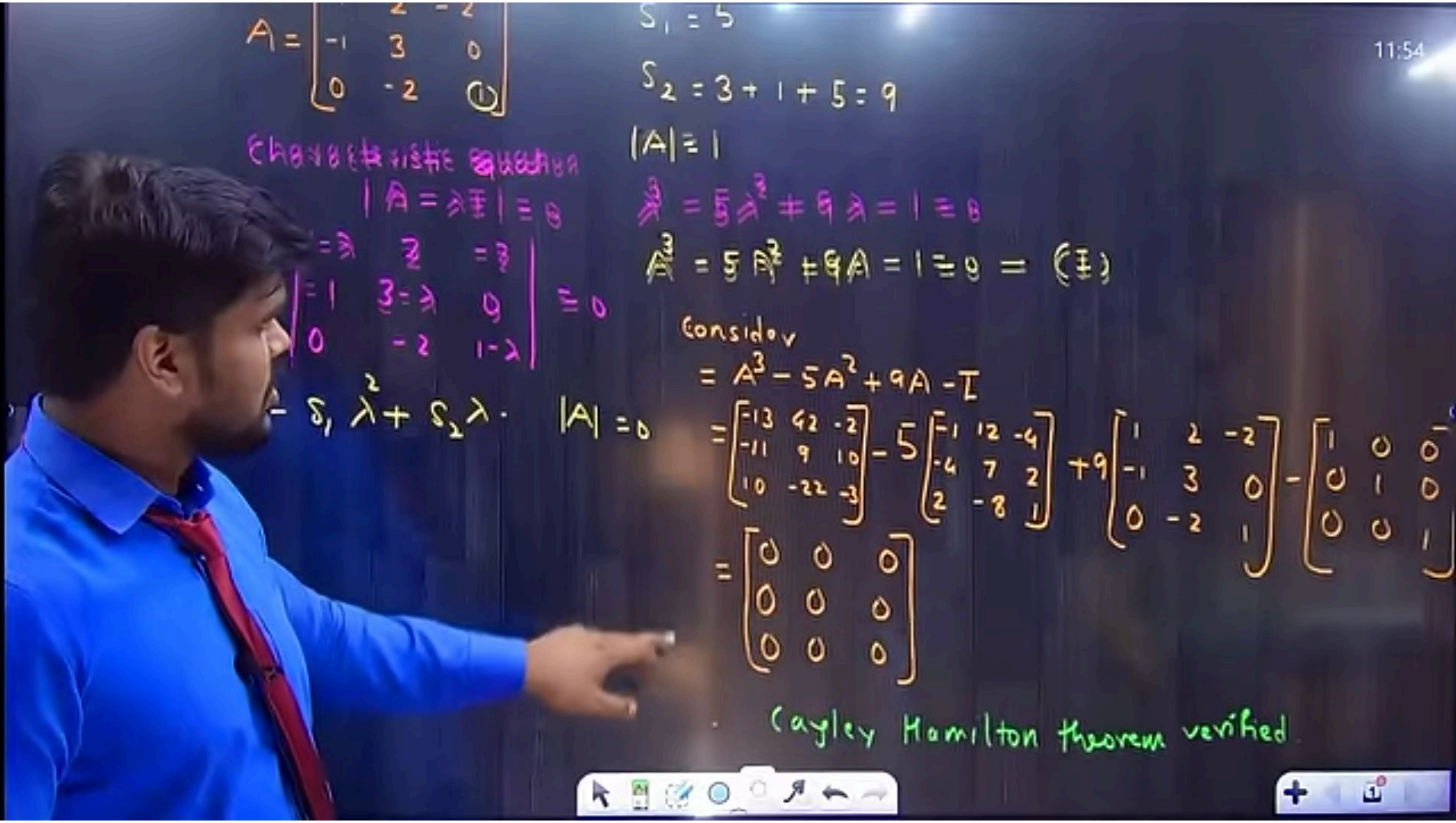
$|A| = 1$

Characteristic equation.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & -2 \\ -1 & 3-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - S_1 \lambda^2 + S_2 \lambda - |A| = 0$$



11:54
A = -1 3 0
0 -2 1
S1 = 5
S2 = 3 + 1 + 5 = 9
|A| = 1
|A| = 0
Consider
= A3 - 5A2 + 9A - I
= -13 42 -2 / -11 9 10 / 10 -22 -3 - 5 -1 12 -4 / -4 7 2 / 2 -8 1 + 9 1 2 -2 / -1 3 0 / 0 -2 1 - 1 0 0 / 0 1 0 / 0 0 1
= 0 0 0 / 0 0 0 / 0 0 0
Cayley Hamilton theorem verified

Multiply eqn (I) by A^{-1}

$$A^2 - 5A + 9I - A^{-1} = 0$$

$$A^2 - 5A + 9I = A^{-1}$$

$$\begin{bmatrix} [illegible] & 12 & -6 \\ [illegible] & 7 & 2 \\ 2 & -8 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A^{-1}$$

$$\boxed{\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = A^{-1}}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

∴ Cayley Hamilton theorem verified

$$\begin{vmatrix} 0 & -2 & 1-\lambda \end{vmatrix}$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda \cdot |A| = 0$$

M ... (I) by A^{-1}

$$\ldots - A^{-1} = 0 \quad — (II)$$

$$\ldots 9I = A^{-1}$$

$$\ldots \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A^{-1}$$

$$\boxed{\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = A^{-1}}$$

Consider

$$= A^3 - 5A^2 + 9A - I$$

$$= \begin{bmatrix} -13 & 42 & -2 \\ -11 & 9 & 10 \\ 10 & -22 & -3 \end{bmatrix} - 5\begin{bmatrix} -1 & 12 & -4 \\ -4 & 7 & 2 \\ 2 & -8 & 1 \end{bmatrix} + 9\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

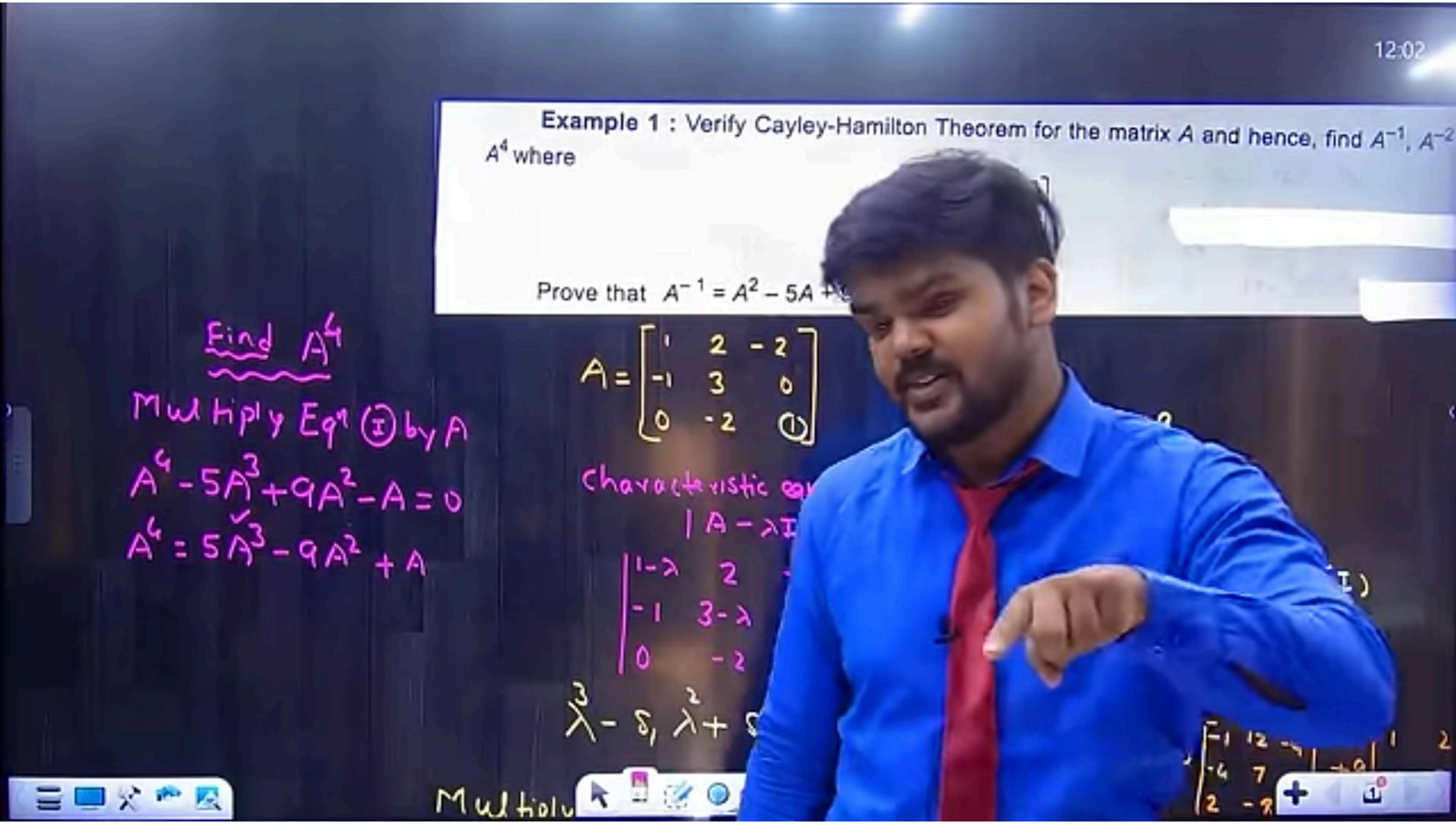
$\therefore$ Cayley Hamilton theorem verified.

Find A^{-2}

Multiply eq^n (II) by A^{-1}

$$A - 5I + 9A^{-1} - A^{-2} = 0$$

$$A - 5I + 9A^{-1} = A^{-2}$$


12:02
Example 1 : Verify Cayley-Hamilton Theorem for the matrix A and hence, find A⁻¹, A⁻²
A⁴ where
Prove that A⁻¹ = A² − 5A +
Find A⁴
Multiply Eqⁿ (I) by A
A⁴ − 5A³ + 9A² − A = 0
A⁴ = 5A³ − 9A² + A
A = [1 2 −2; −1 3 0; 0 −2 1]
Characteristic eq
|A − λI
λ³ − S₁ λ² +
Multiply

[redacted] Verify Cayley-Hamilton theorem and hence, find the matrix represented by

$A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I$, where A is $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ [redacted]

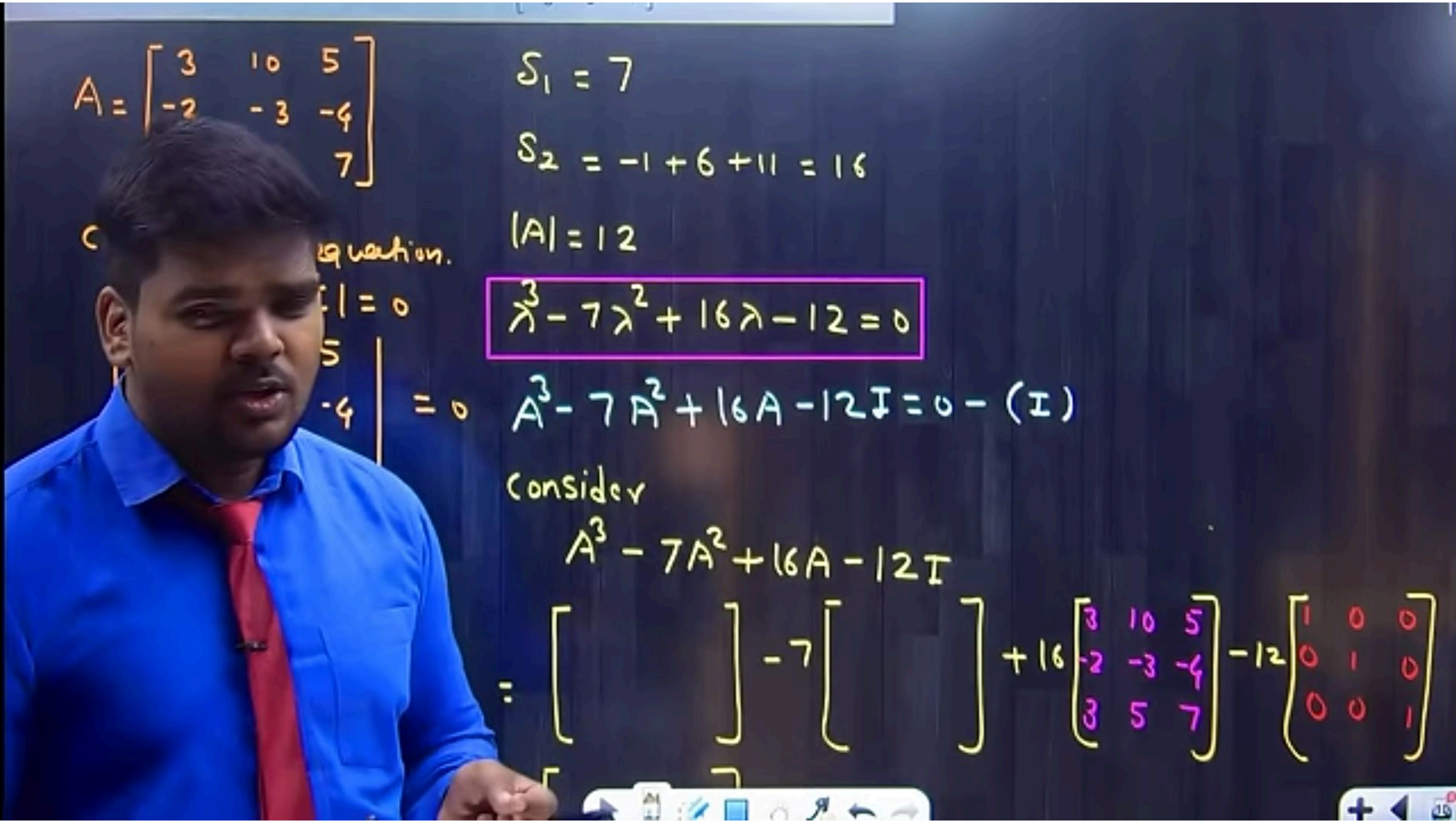
$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

Characteristic equation.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 10 & 5 \\ -2 & -3-\lambda & -4 \\ 3 & 5 & 7-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$



A = [3 10 5; -2 -3 -4; 7]
S1 = 7
S2 = -1 + 6 + 11 = 16
|A| = 12
λ³ - 7λ² + 16λ - 12 = 0
A³ - 7A² + 16A - 12I = 0 - (I)
consider
A³ - 7A² + 16A - 12I
= [ ] - 7 [ ] + 16 [3 10 5; -2 -3 -4; 3 5 7] - 12 [1 0 0; 0 1 0; 0 0 1]

$$\begin{vmatrix} 3 & 5 & 7-\lambda \end{vmatrix}$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$

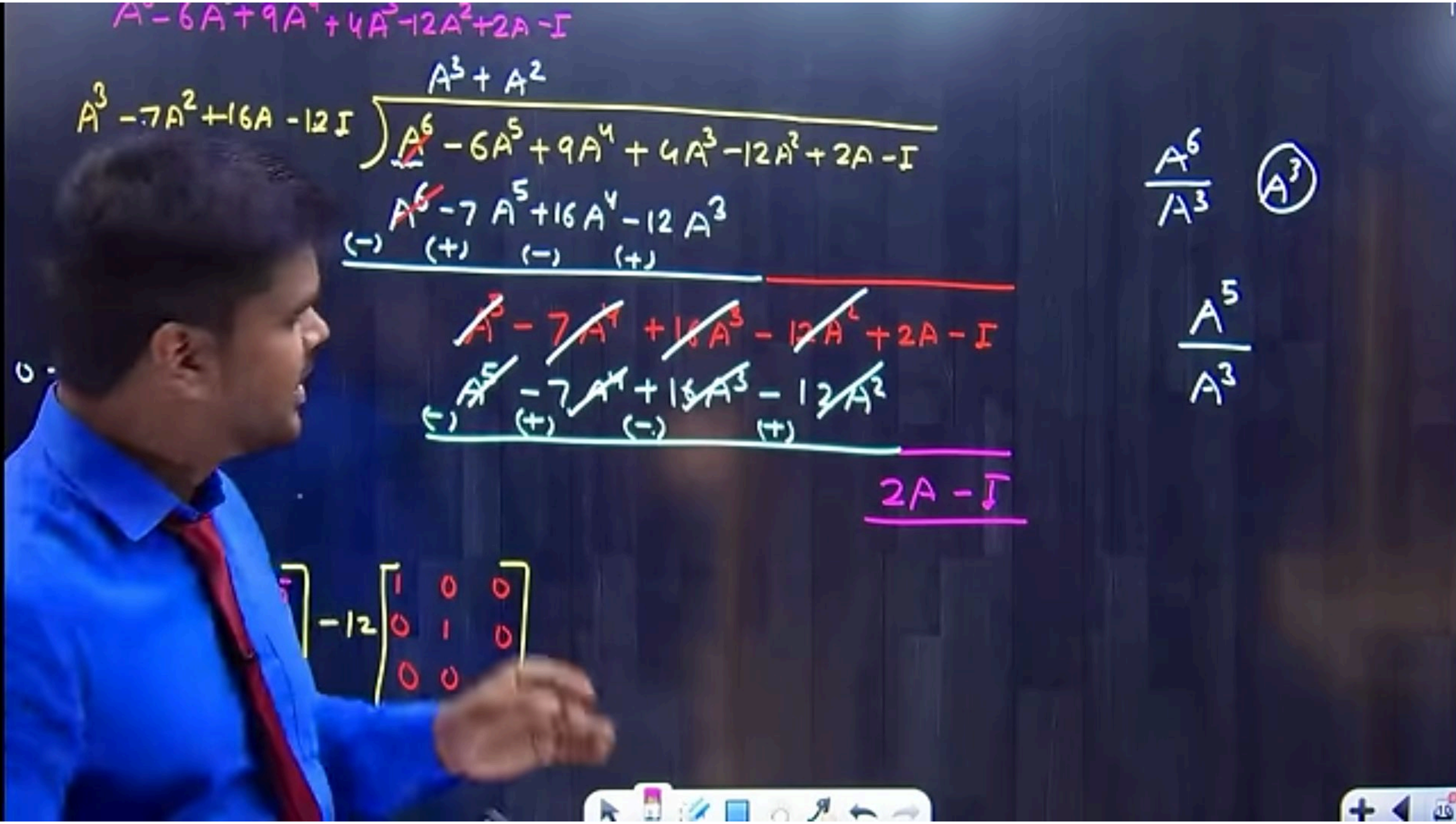
consider

$$A^3 - 7A^2 + 16A - 12I$$

$$= \begin{bmatrix} -8 & 15 & -10 \\ -52 & -157 & -118 \\ 92 & 270 & 208 \end{bmatrix} - 7\begin{bmatrix} 4 & 25 & 10 \\ -12 & -31 & -26 \\ 20 & 50 & 44 \end{bmatrix} + 16\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix} - 12\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Cayley Hamilton theorem verified,,


A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I
A^3 + A^2
A^3 - 7A^2 + 16A - 12I) A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I
A^6 - 7A^5 + 16A^4 - 12A^3
(-) (+) (-) (+)
A^5 - 7A^4 + 16A^3 - 12A^2 + 2A - I
A^5 - 7A^4 + 16A^3 - 12A^2
(-) (+) (-) (+)
2A - I
A^6 / A^3
A^3
A^5 / A^3
-12

A^3 se chhota ho jayega phir division nahi karna

$A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I$

$D = D \times Q + R$

$$\begin{array}{r l}
 & A^3 + A^2 \\
A^3 - 7A^2 + 16A - 12I \,\big) & \cancel{A^6} - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I \\
 & \cancel{A^6} - 7A^5 + 16A^4 - 12A^3 \\
 & (-) \quad (+) \quad (-) \quad (+) \\
\hline
 & \cancel{A^5} - \cancel{7A^4} + \cancel{16A^3} - \cancel{12A^2} + 2A - I \\
 & \cancel{A^5} - \cancel{7A^4} + \cancel{16A^3} - \cancel{12A^2} \\
 & (-) \quad (+) \quad (-) \quad (+) \\
\hline
 & 2A - I
\end{array}$$

$\frac{A^6}{A^3}$ A^3

$\frac{A^5}{A^3}$

$16\begin{bmatrix}3 & 10 & 5\\ -2 & -3 & -4\\ 3 & 5 & 7\end{bmatrix} - 12\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I$

$= (A^3 - 7A^2 + 16A - 12I)(A^3 + A^2) + 2A - I$

$\underline{2A - I}$

$$\ldots 6\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ & 5 & 7 \end{bmatrix} - 12\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

...rified,,

$$A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I$$

$$= \underline{(A^3 - 7A^2 + 16A - 12I)(A^3 + A^2)} + 2A - I$$

$$= 2A - I$$

$$= 2\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 20 & 10 \\ -4 & -7 & -8 \\ 6 & 10 & 13 \end{bmatrix}$$

Ex. Use Cayley Hamilton theorem to find $A^7 - 9A^2 + I$.

Where $A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$.

The characteristic equation is given by $|A - \lambda I| = 0$

$\therefore \begin{vmatrix} 1-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix} = 0$

$\therefore (1-\lambda)^2 - 4 = 0$

$\therefore f(\lambda) = \lambda^2 - 2\lambda - 3 = 0$: The characteristic equation

$$\lambda = -1, 3$$

By Cayley-Hamilton Theorem, A satisfies the characteristic equation.

$$\therefore f(A) = A^2 - 2A - 3I = 0$$

Let $g(A) = A^7 - 9A^2 + I$

As coefficients of are $f(A)$ and $g(A)$ are not same/similar, we use division algorithm

$g(A) = f(A).q(A) + r(A)$; where $degree\ of\ f(A) < degree\ of\ r(A)$.

$\therefore g(A) = 0.q(A) + a_0A + a_1I$

$A^7 - 9A^2 + I = a_0A + a_1I$ 1

Eigenvalues of A satisfies this equation.

$\therefore \lambda^7 - 9\lambda^2 + I = a_0\,\lambda + a_1\,I$

For $\lambda = -1$

$-1 - 9 + 1 = 2107 = -a_0 + a_1$.....................2

For $\lambda = 3$, $2187 - 9(9) + 1 = 2107 = 3a_0 + a_1$............3

Solving (2) & (3),

We get, $a_0 = 529$ & $a_1 = 520$

By (1),

$$g(A) = A^7 - 9A^2 + I = 529A + 520I$$

13-14/15