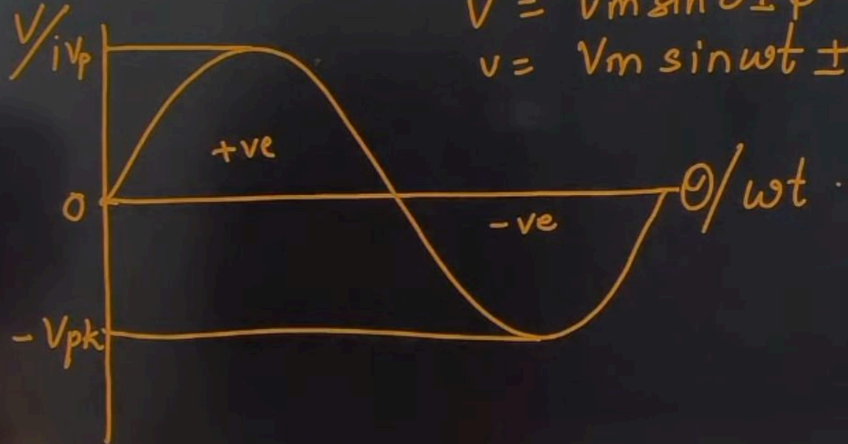
Phase Angle ϕ and Phasor.

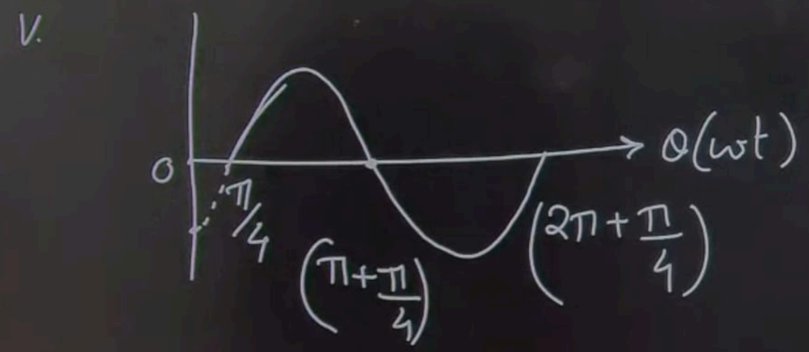
$$V = V_m \sin\theta \pm \phi$$
$$v = V_m \sin\omega t \pm \phi$$

$$i = I_m \sin\theta \pm \phi$$
$$i = I_m \sin\omega t \pm \phi.$$



0 - Origin → Reference point.

Case 1. The following represents graph of alternating sinusoidal voltage.



$$v = V_m \sin\theta \pm \phi$$

$$\phi = \frac{\pi}{4}$$

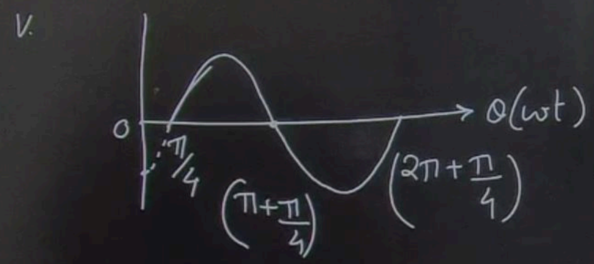
$$v = V_m \sin\left(\theta \pm \frac{\pi}{4}\right)$$

$$= V_m \sin\left(\theta - \left(+\frac{\pi}{4}\right)\right)$$

$$= V_m \sin(\theta - \pi/4) = V_m \sin(\theta - 45).$$

The voltage here is <u>lagging</u>.

Case 1. The following represents graph of alternating sinusoidal voltage.



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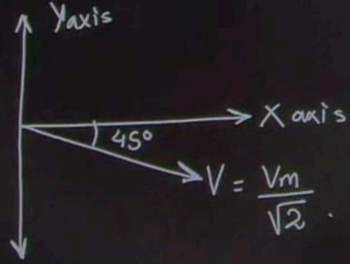
The voltage here is <u>lagging</u>.

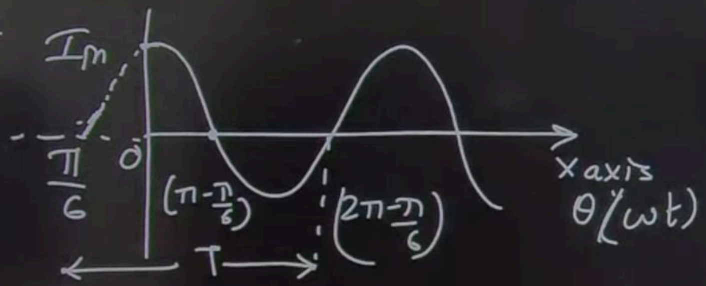
<u>Phasor diagram</u>



Lagging --> -ve
Leading --> +ve

Case 2. The following graph represents alternating sinusoidal current.

I.

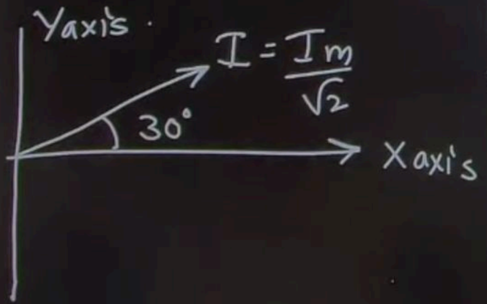


$$i = I_m \sin(\theta \pm \phi)$$

$$i = I_m \sin\left(\theta + \frac{\pi}{6}\right).$$

$$\phi = \frac{\pi}{6}$$

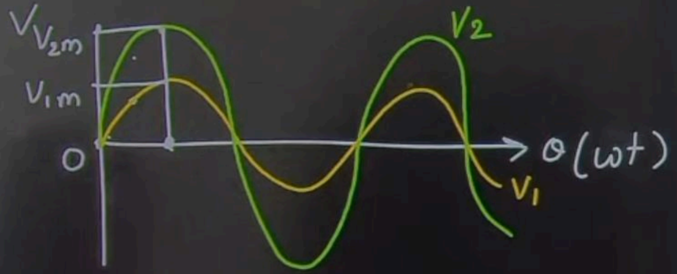
[Phasor diagram.]



leading The current is leading.

026 624

In Phase

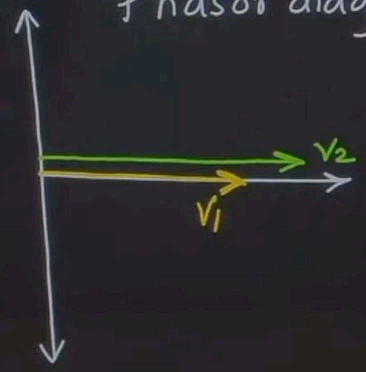


Voltage V_1 & V_2 are in phase.

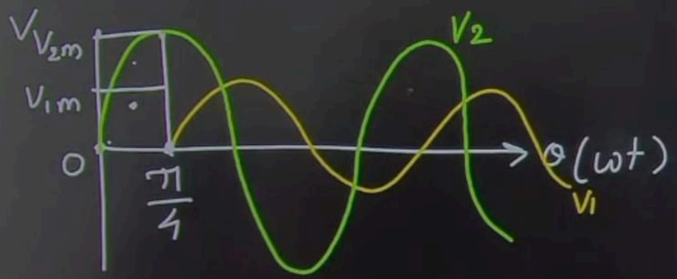
$V_1 = V_m \sin\theta$

$V_2 = V_m \sin\theta$.

Phasor diagram.



In Phase / Out phase.

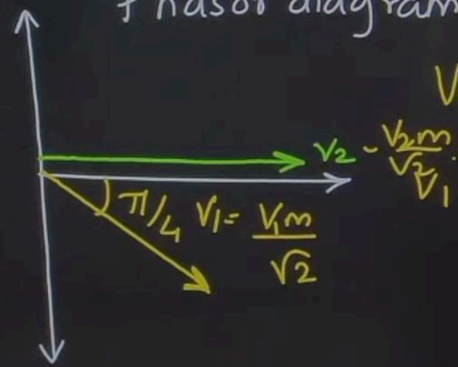


Voltage V_1 & V_2 are In phase.

$$V_1 = V_m \sin\left(\theta - \frac{\pi}{4}\right)$$

$$V_2 = V_m \sin\theta .$$

Phasor diagram.



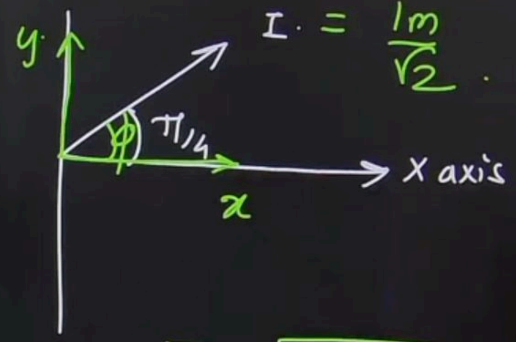
V_2 leads V_1 by $\pi/4$

V_1 lags V_2 by $\pi/4$.

Phasor Algebra.

① Complex / Rectangular form.

② Polar form.



$I = \frac{I_m}{\sqrt{2}}$.

$I_m = I \times \sqrt{2}$

$I = x \pm yj$

$I = \sqrt{x^2 + y^2}$

$\bar{I} = \left(\frac{I_m}{\sqrt{2}} \angle 45 \right)$

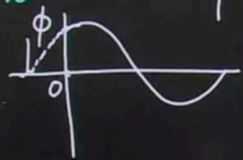
$\phi = \tan^{-1}\left(\frac{y}{x}\right)$.

Leading circuits

→ Phase angle is +ve.

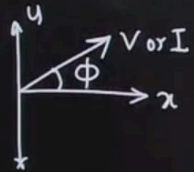
$\therefore \sin(\theta + \phi)$.

$\sin(\omega t + \phi)$



ϕ

→ In Phasor diagram the current or voltage lie above x axis.

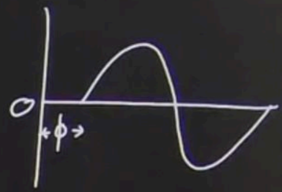


Lagging circuits

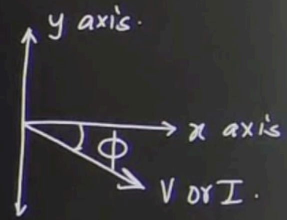
→ Phase angle is −ve

$\sin(\theta - \phi)$

$\sin(\omega t - \phi)$.



→ In Phasor diagram the current or voltage lie below x axis.



Complex $I = x + yj$ Rectangular Complex $I = x - yj$

Polar $\bar{I} = (I\angle\phi)$. $\frac{I_m}{\sqrt{2}}$. $I = (I\angle{-\phi})$.

550 912

The instantaneous values of two alternating voltages are represented by $v_1 = 60 \sin\theta$ & $v_2 = 40 \sin(\theta + \pi/3)$. find the sum of voltages and its differences.

$V_1 = 60 \sin\theta$.

Standard form $v = V_m \sin(\theta \pm \phi)$.

$V_m = 60 \quad \phi = 0$.

Polar form $= \left(\frac{V_m}{\sqrt{2}}, \angle\phi\right)$

$= (42.426, \angle 0)$.

$V_2 = 40 \sin(\theta + \pi/3)$.

$V_m = 40 \quad \phi = \pi/3 \quad \frac{\pi}{3} = \frac{180}{3} = 60$

Polar form $= \left(\frac{V_m}{\sqrt{2}}, \angle\phi\right)$

$= (28.28, \angle 60)$

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To find rectangular form using Calci

Shift then -ve(rec) Then put the value and angle separated by Shift+)

550 912

The instantaneous values of two alternating voltages are represented by $v_1 = 60\sin\theta$ & $v_2 = 40\sin(\theta + \pi/3)$. find the sum of voltages and its differences.

$v_1 = 60\sin\theta$.

$$\boxed{\text{Standard form } v = V_m \sin(\theta \pm \phi)}$$

$V_m = 60 \quad \phi = 0$.

Polar form $= \left(\frac{V_m}{\sqrt{2}}, \angle\phi\right)$

$= (42.426, \angle 0)$.

Rectangularform $= x + jy$

$= 42.426 + 0j$

$V_2 = 40\sin(\theta + \pi/3)$.

$V_m = 40 \quad \phi = \pi/3 \quad \frac{\pi}{3} = \frac{180}{3} = 60$

Polarform $= \left(\frac{V_m}{2}, \angle\phi\right)$

$= (28.28, \angle 60)$

Rectangularform $= 14.14 + 24.49j$

$Vm = 60 \quad \phi =$ 550 912

Polar form $= \left(\frac{Vm}{\sqrt{2}}, \angle\phi\right)$

$= (42.426, \angle 0)$

Rectangular form $= x + jy$

$= 42.426 + 0j$

Polar form $= \left(\frac{Vm}{2}, \angle\phi\right)$

$= (28.28, \angle 60)$

Rectangular form $= 14.14 + 24.49j$

Sum of $V_s = \bar{V}_1 + \bar{V}_2 = (42.426 + 0j) + (14.14 + 24.49)j$

$V_s = 56.56 + 24.49j$

$V_s = (\underline{61.63} \angle 23.412)$

$Vm = V \times \sqrt{2}$

$= 61.63 \times \sqrt{2}$

$= 87.157$

$V = Vm \sin(\theta + \phi)$

$= 87.157 \sin(\theta + 23.412)$

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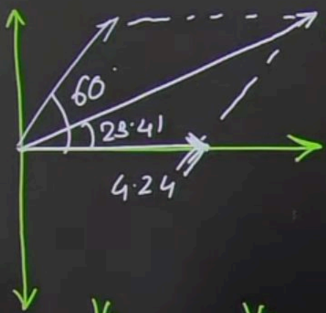
ACADEMY

026 624

$V_s = 87.157 \sin(\theta + 23.412)$



Scale $1\,cm = 10V$.

V_1	V_2	V_S
42.426	28.28	61.63
4.24	2.8	6.1.
$\angle 0$		

Rectangular form: $x + jy$ 288 768

$= 42.426 + 0j$

Sum of $V_s = \bar{V}_1 + \bar{V}_2 = (42.426 + 0j) + (14.14 + 24.49)j$

$V_s = 56.56 + 24.49j$

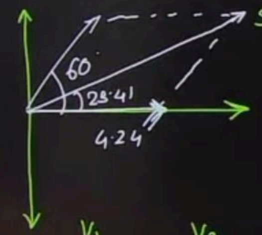
$V_m = V \times \sqrt{2}$ $\quad V_s = (\underline{61}.63 \angle 23.412)$

$= 61.63 \times \sqrt{2}$

$= 87.157$ $\quad V = V_m \sin(\theta + \phi)$

$V_s = 87.157 \sin(\theta + 23.412)$

Scale 1 cm = 10 V.



V_1	V_2	V_s	V_d
42.426	28.28	61.63	37.417
4.24	2.8	6.1	3.7
$\angle 0$			

Diff $= \bar{V}_1 - \bar{V}_2 = (42.426 + 0j) - (14.14 + 24.49j)$

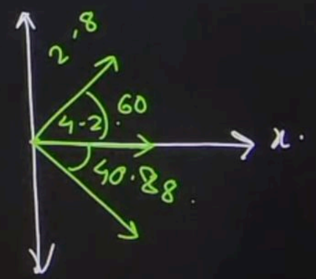
$V_d = (28.29 - 24.49j)$

$V_d = (37.417 \angle -40.88)$

$V = V_m \sin(\theta \pm \phi)$.

$V = 52.91 \sin(\theta - 40.88)$

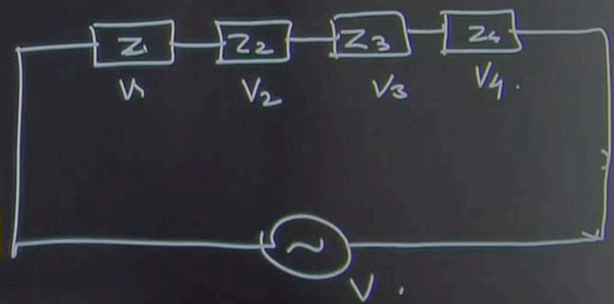
Phasor diagram.



Q. The voltage drop across the four series connected Impedances are

$v_1 = 60\sin\left(\omega t + \frac{\pi}{6}\right)$, $v_2 = 75\sin\left(\omega t - \frac{5}{6}\pi\right)$, $v_3 = 100\cos\left(\omega t + \frac{\pi}{4}\right)$, $v_4 = V_{m4}\sin(\omega t + \phi$

Calculate the values of V_{m4} & ϕ_4 if applied voltage across the series circuit is $140\sin\left(\omega t + \frac{3\pi}{5}\right)$.



$V_T = \boxed{140\sin\left(\omega t + \frac{3\pi}{5}\right)}$ (Standard form)

Polar form $= \left(\frac{V_m}{\sqrt{2}}, \angle\phi\right) = \left(\frac{140}{\sqrt{2}}, \angle 108\right) = \boxed{(99, \angle 108)}$

Complex form $= \boxed{-30.59 + 94.154j}$

$v_1 = V_m \sin(\omega t + \phi)$ 550 912 $\bar{V} = \left(\frac{V_m}{\sqrt{2}}, \angle\phi\right)$ $\bar{V} = x + yj$

$v_1 = 60 \sin\left(\omega t + \frac{\pi}{6}\right)$ $\bar{V}_1 = (42.42, \angle 30)$ $\bar{V}_1 = 36.73 + 21.21j$

$v_2 = 75 \sin\left(\omega t - \frac{5\pi}{6}\right)$ $\bar{V}_2 = (53.03, -\angle 150)$ $\bar{V}_2 = -45.92 - 26.515j$

$v_3 = 100 \sin\left(\omega t + \frac{3\pi}{4}\right)$ $\bar{V}_3 = (70.71, \angle 135)$ $\bar{V}_3 = -50 + 50j$

$v_T = 140 \sin\left(\omega t + \frac{3\pi}{5}\right)$ $\bar{V}_T = (99, \angle 108)$ $\bar{V}_T = (-30.59 + \hat{j}\, 94.15)$

The impedances are connected in series

$$\therefore \bar{V}_T = \bar{V}_1 + \bar{V}_2 + \bar{V}_3 + \bar{V}_4$$

$$V_4 = \bar{V}_T - (\bar{V}_1 + \bar{V}_2 + \bar{V}_3)$$

$$= (-30.59 + 94.15j) - (36.73 + 21.21j - 45.92 - 26.515j - 50 + 50j)$$

$$= (\underline{-30.59} + \underline{94.15j}) - (-\underline{59.18} + \underline{44.7j})$$

$$= (28.59 + 49.45j)$$

$V_3 = 100\sin(\omega t$ 550 912 $V_3 = (70.71, \angle 135)$ $V_3 = -50 + 50j$

$V_4 = 140\sin(\omega t + \frac{3\pi}{5})$ $\bar{V}_T = (99, \angle 108)$ $\bar{V}_T = (-30.59 + \hat{j}\,94.15)$

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$$V_4 = \bar{V}_T - (\bar{V}_1 + \bar{V}_2 + \bar{V}_3)$$

$$= (-30.59 + 94.15j) - (36.73 + 21.21j - 45.92 - 26.515j - 50 + 50j)$$

$$= (-30.59 + 94.15j) - (-59.18 + 44.7j)$$

$$= (28.59 + 49.45j)$$

$V_m = 57.119 \times \sqrt{2}$

$$\bar{V}_4 = (57.119, \angle 60).$$

$$V_4 = 80.77\sin(\omega t + \frac{\pi}{3}).$$

$$\boxed{V_{m_4} = 80.77 \quad \phi_4 = \frac{\pi}{3}}$$

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ACADEMY

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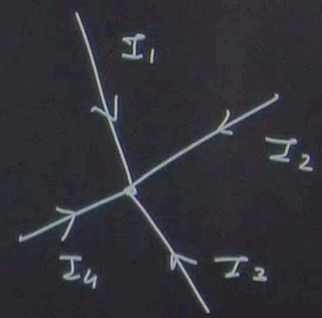
Q. Four wires p, q, r and s are connected to a common point. The currents in lines p, q & r are $6\sin\left(\omega t + \frac{\pi}{3}\right)$, $5\cos\left(\omega t + \frac{\pi}{3}\right)$ & $3\cos\left(\omega t + \frac{\pi}{3}\right)$.

Find the current in wire s.



026 624

Q. Four wires p, q, r and s are connected to a common point. The currents in lines p, q & r are $6\sin\left(\omega t + \frac{\pi}{3}\right)$, $5\cos\left(\omega t + \frac{\pi}{3}\right)$ & $3\cos\left(\omega t + \frac{\pi}{3}\right)$.
Find the current in wire s.

Standard form	Polar form	Complex
$p = i_1 = 6\sin\left(\omega t + \frac{\pi}{3}\right)$	$\bar{I}_1 = (4.24, \angle 60)$	$\bar{I}_1 = 2.12 + 3.67j$
$q = i_2 = 5\cos\left(\omega t + \frac{\pi}{3}\right)$ $= 5\sin\left(\omega t + \frac{5\pi}{6}\right)$	$\bar{I}_2 = (3.53, \angle 150)$	$\bar{I}_2 = -3.05 + 1.76j$
$r = i_3 = 3\cos\left(\omega t + \frac{\pi}{3}\right)$ $= 3\sin\left(\omega t + \frac{5\pi}{6}\right)$	$\bar{I}_3 = (2.12, \angle 150)$	$\bar{I}_3 = -1.83 + 1.06j$

288 768

$q = i_2 = 5\cos\left(\omega t + \frac{\pi}{3}\right)$

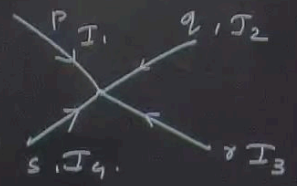
$= 5\sin\left(\omega t + \frac{5}{6}\pi\right)$

$r = i_3 = 3\cos\left(\omega t + \frac{\pi}{3}\right)$

$= 3\sin\left(\omega t + \frac{5\pi}{6}\right)$

$\bar{I}_2 = (3.53, \angle 150)$ $\quad \bar{I}_2 = -3.05 + 1.76j$

$\bar{I}_3 = (2.12, \angle 150)$ $\quad \bar{I}_3 = -1.83 + 1.06j$



According to Kirchhoff's current law.

$$\bar{I}_1 + \bar{I}_2 + \bar{I}_3 + \bar{I}_4 = 0$$

$$\bar{I}_4 = -(\bar{I}_1 + \bar{I}_2 + \bar{I}_3)$$

$$= -(2.12 + 3.67j) + (-3.707 + 1.77j) + (-1.84 + 1.06j)$$

$$= -(-2.79 + 6.5j)$$

$$\bar{I}_4 = 2.79 - 6.5j)$$

Polar form $\bar{I}_4 = (\underline{7.07}, -\angle 66.76)$

$\underline{Im} = 7.07 \times \sqrt{2}$

$Im = 9.998 \approx 10.$

$\phi = -66.76.$

$s = i_4 = 10\sin(\omega t - 66.76).$

288 768

Q. Three coils are connected in series. Each of them generates an emf of 230V. The emf of the second coil leads that of the first coil by 120°, and the emf of third coil lags behind that of the first by same angle. What is the resultant emf across the series combination of the coils.

let E_1, E_2 & E_3 be the emf generated by 3 coils. Thus, polarform of E_1 is

$\bar{E}_1 = (230, \angle 0)$. $\quad \bar{E}_1 = 230 + j0$

$\bar{E}_2 = (230, +\angle 120)$ $\quad \bar{E}_2 = -115 + 199.18j$

$\bar{E}_3 = (230, -\angle 120)$ $\quad \bar{E}_3 = -115 - 199.18j$

Menu Exit Share List Select Pen Eraser Clean All Ruler Shape Pair Mode Table Mind map Sticky-Notes Preview Grid Undo Redo Add Previous 22/22 Page number Next combination of the coils

288 768

let E_1, E_2 & E_3 be the emf generated by 3 coils. Thus, polarform of E_1 is

$\bar{E}_1 = (230, \angle 0)$. $\quad \bar{E}_1 = 230 + j0$

$\bar{E}_2 = (230, +\angle 120)$ $\quad \bar{E}_2 = -115 + 199.18j$

$\bar{E}_3 = (230, -\angle 120)$ $\quad \bar{E}_3 = -115 - 199.18j$

To calculate Resultant emf

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3$$

$$= (230 + j0) + (-115 + 199.18j) + (-115 - 199.18j)$$

$$\bar{E} = 0\,V$$

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ACADEMY

$$\cos \rightarrow \sin\left(+ \frac{\pi}{2} \right)$$

$$-\sin \rightarrow \sin(+ \pi)$$