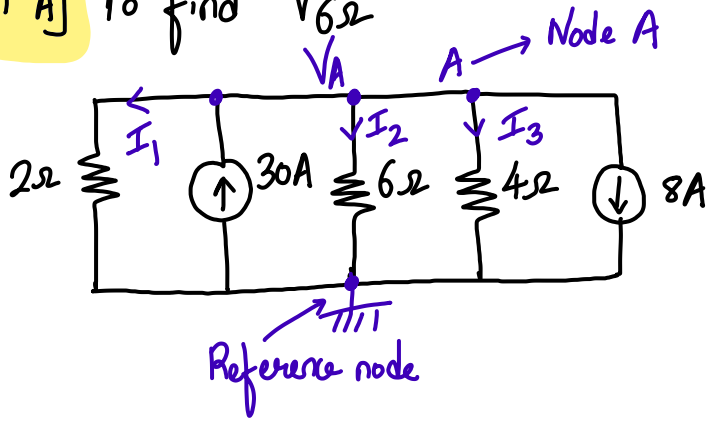
EEEE ISE Solution 19/10/23

Q1 A] To find $V_{6\Omega}$



Using Nodal analysis,

KCL at node A,

$8 + I_1 + I_2 + I_3 = 30$

i.e $8 + \frac{V_A}{2} + \frac{V_A}{6} + \frac{V_A}{4} = 30$

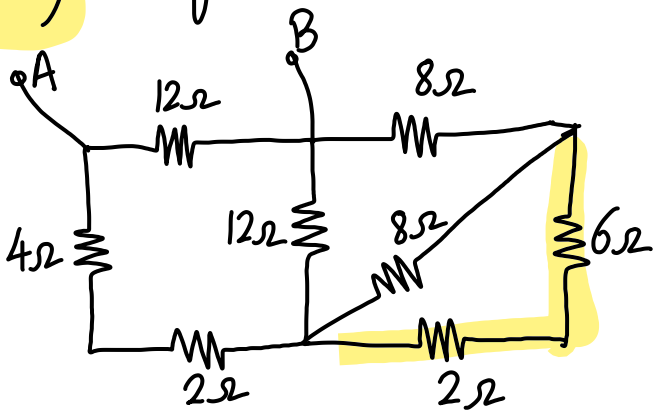
i.e $0.5V_A + 0.16667V_A + 0.25V_A = 22$

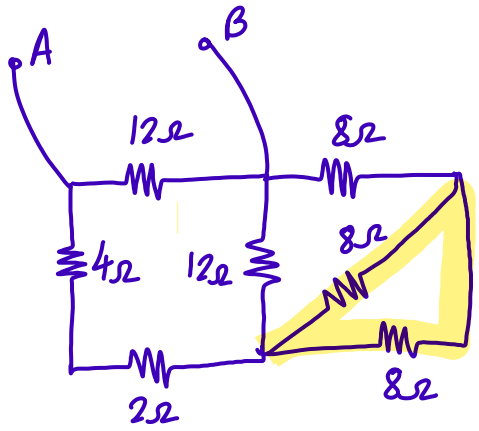
i.e $0.91667\,V_A = 22$

i.e $V_A = 23.9999\,V$

i.e $\boxed{V_A = 24V}$

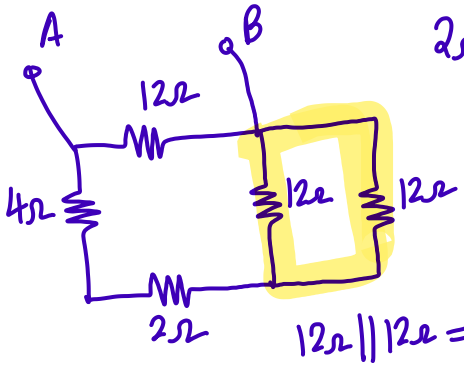
(Q1 b) To find R_{AB} = ?

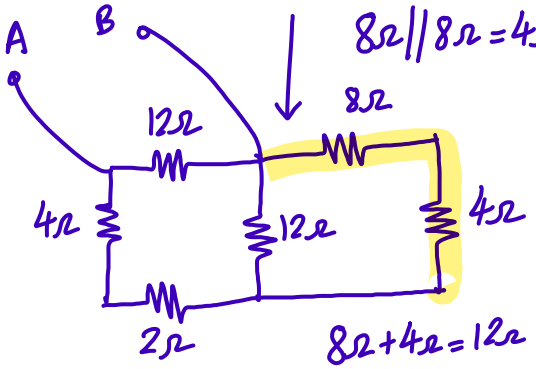




$2\Omega + 6\Omega = 8\Omega$

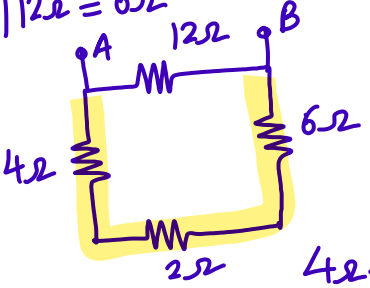
$8\Omega \| 8\Omega = 4\Omega$

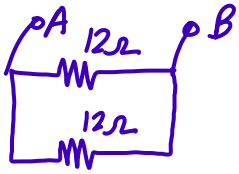




$8\Omega + 4\Omega = 12\Omega$

$12\Omega \| 12\Omega = 6\Omega$



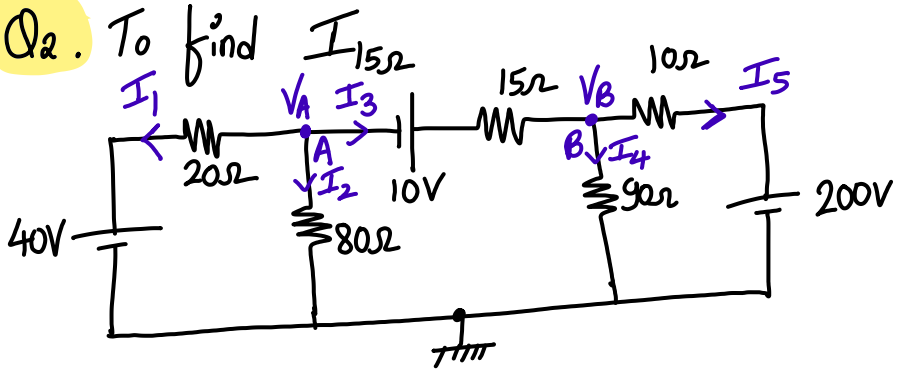


$4\Omega + 2\Omega + 6\Omega = 12\Omega$

$R_{AB} = 12\Omega \| 12\Omega$

$\boxed{R_{AB} = 6\Omega}$

Q2. To find $I_{15\Omega}$



Using Nodal analysis,

KCL at node A,

$I_1 + I_2 + I_3 = 0$

i.e $\frac{V_A - 40}{20} + \frac{V_A}{80} + \frac{V_A + 10 - V_B}{15} = 0$

i.e $0.05V_A - 2 + 0.0125V_A + 0.0667V_A + 0.6667 - 0.0667V_B = 0$

i.e $0.1292V_A - 0.0667V_B = 1.3333$ — ①

KCL at node B,

$I_3 = I_4 + I_5$

$\frac{V_A + 10 - V_B}{15} = \frac{V_B}{90} + \frac{V_B - 200}{10}$

i.e $0.0667V_A + 0.6667 - 0.0667V_B = 0.01111V_B + 0.1V_B - 20$

i.e $0.0667V_A - 0.1778V_B = -20.6667$ — ②

Solving ① & ②, we get $\begin{bmatrix} 0.1292 & -0.0667 \\ 0.0667 & -0.1778 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 1.3333 \\ -20.6667 \end{bmatrix}$

$V_A = 87.218V$

$V_B = 148.95V$

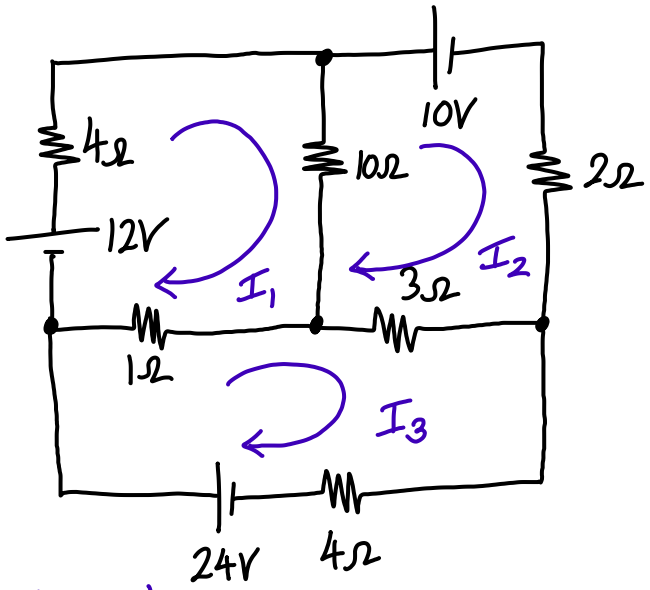
$I_{15\Omega} = I_3 = \frac{V_A + 10 - V_B}{15} = \frac{87.218 + 10 - 148.95}{15}$

$I_{15\Omega} = -3.4488\ A \rightarrow$

$I_{15\Omega} = 3.4488\ A \leftarrow$

OR

Q2. To find $I_{1\Omega}$ using mesh analysis



KVL to mesh 1,

$12 - 4I_1 - 10(I_1 - I_2) - (I_1 - I_3) = 0$

i.e $-15I_1 + 10I_2 + I_3 = -12$ — ①

KVL to mesh 2,

$-10(I_2 - I_1) - 10 - 2I_2 - 3(I_2 - I_3) = 0$

i.e $10I_1 - 15I_2 + 3I_3 = 10$ — ②

KVL to mesh 3,

$24 - (I_3 - I_1) - 3(I_3 - I_2) - 4I_3 = 0$

i.e $I_1 + 3I_2 - 8I_3 = -24 - (3)$

$$\begin{bmatrix} -15 & 10 & 1 \\ 10 & -15 & 3 \\ 1 & 3 & -8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 10 \\ -24 \end{bmatrix}$$

$$I_1 = \frac{791}{395} A$$

$$I_2 = \frac{563}{395} A$$

$$I_3 = \frac{299}{79} A$$

$$I_{1\Omega} = I_1 - I_3 = \frac{791}{395} - \frac{299}{79}$$

$I_{1\Omega} = -1.782\ A \leftarrow$

$I_{1\Omega} = 1.782\ A \rightarrow$

Q3 A] $v = 100 \sin(200\pi t)$

$v = V_m \sin(\omega t + \phi)$

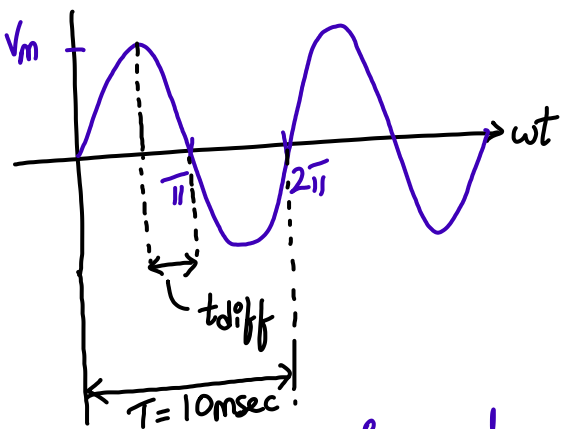
i.e $V_m = 100V$ — maximum voltage

$V_{avg} = 0.637 V_m = 0.637 \times 100$

$V_{avg} = 63.7V$ — Average voltage

$V_{rms} = 0.707 V_m = 0.707 \times 100$

$V_{rms} = 70.7V$ — RMS voltage



$\omega = 2\pi f$

i.e $200\pi = 2\pi f \rightarrow f = 100\ Hz$

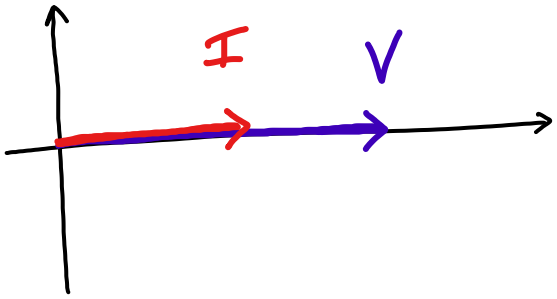
$T = \frac{1}{f} = \frac{1}{100} = 10msec$

$t_{diff} = \frac{T}{4} = \frac{10ms}{4} = 2.5ms$ → time difference between maximum value & next zero value

OR

Q2A] $v = V_m \sin \omega t$

i) Phasor diagram of pure resistor R

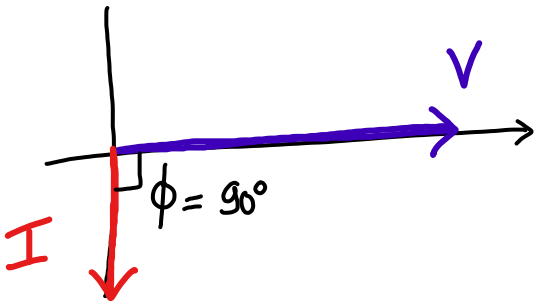


$\phi = 0$

p.f = unity

Taking voltage V as reference

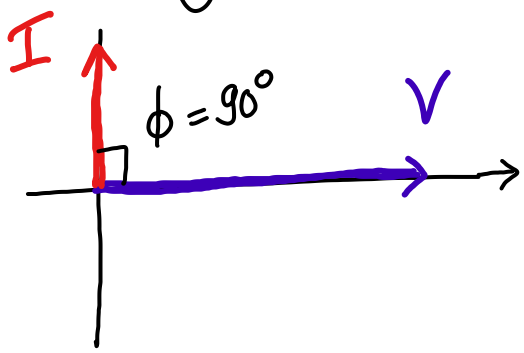
Phasor diagram of pure inductor L



$\phi = 90°$

Taking voltage V as reference

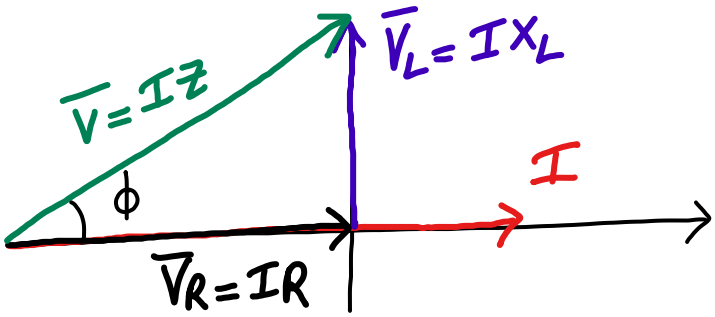
Phasor diagram of pure capacitor C



$\phi = 90°$

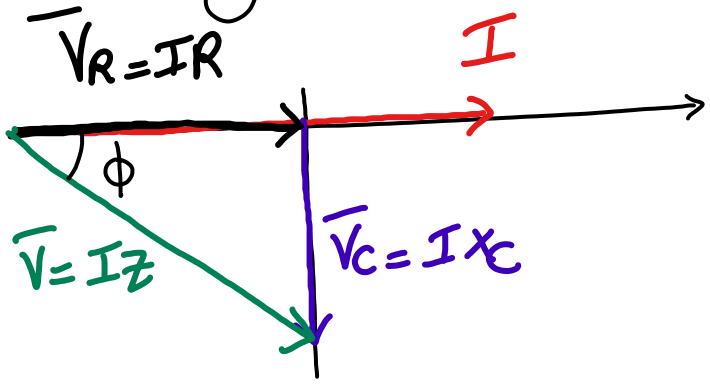
Taking voltage V as reference

Phasor diagram of RL series



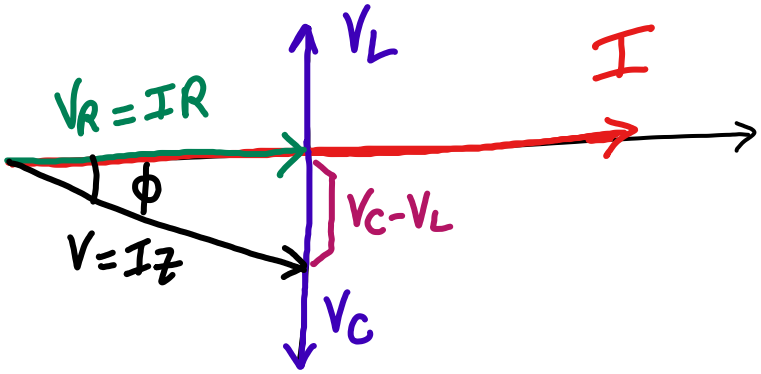
Taking current I as reference

Phasor diagram of RC series



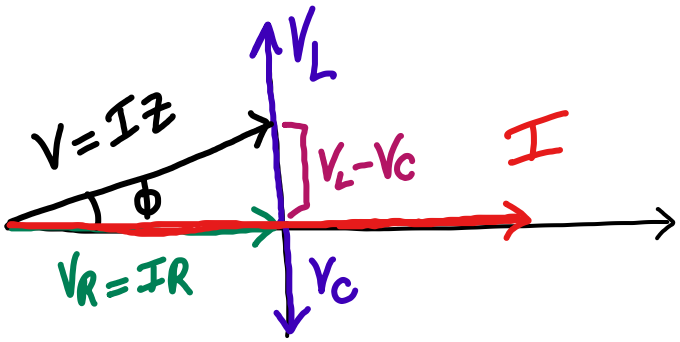
Taking current I as reference

Phasor diagram of RLC series



Taking current I as reference

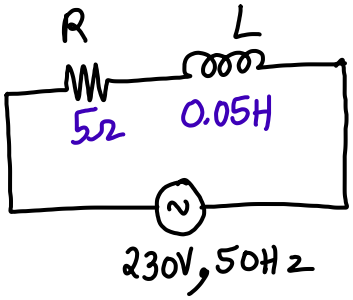
$(X_C > X_L)$



Taking current I as reference

$(X_L > X_C)$

(Q3 B) Given: $R = 5\Omega$, $L = 0.05H$, $V_{rms} = 230V$, $f = 50Hz$



To find: Z_{coil}, I, p.f

Solution: i) $Z_{coil} = R + jX_L$

ii) $X_L = 2\pi fL = 2\pi \times 50 \times 0.05$

$X_L = 15.7079\Omega$

i.e $Z_{coil} = 5 + j15.7079\Omega$ → Rectangular form

iii) $Z_{coil} = 16.4845 \angle 72.34^\circ \ \Omega$ → Polar form

iv) $I = \frac{V}{Z_{coil}} = \frac{230 \angle 0^\circ}{16.4845 \angle 72.34^\circ}$

$$I = 13.9525 \angle -72.34^\circ \ A$$

v) $p.f = \cos\phi = \cos(72.34^\circ)$

$$p.f = 0.3034 \quad \text{lagging}$$