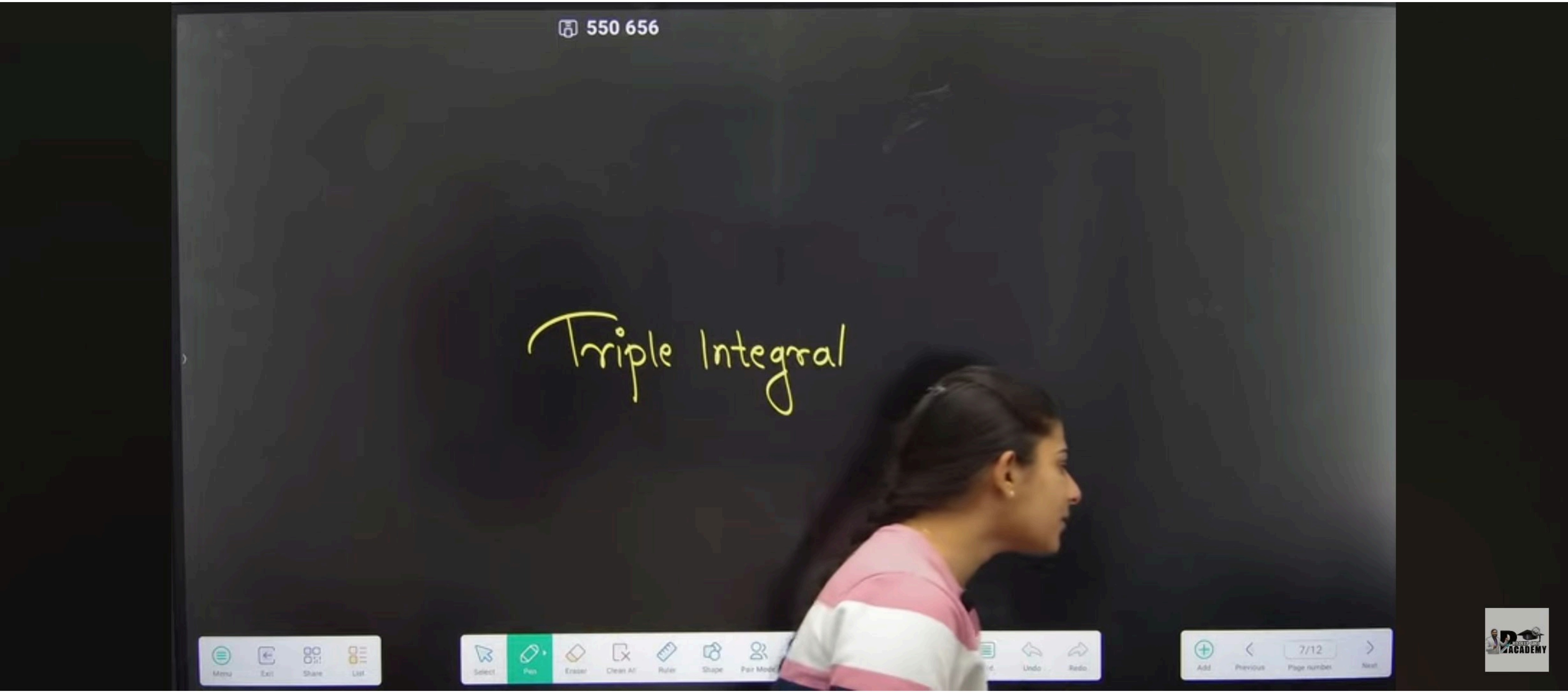

550 656
Triple Integral
7/12
ACADEMY

When the region of integration is bounded by plane

(1) Evaluate $\iiint \frac{dx\,dy\,dz}{(1+x+y+z)^3}$ over the volume of the tetrahedron $x=0, y=0, z=0, x+y+z=1$

To find limits we need to change order to dzdydx

550 656 $\iint (1+x+y+z)^3$

tetrahedron $x=0, y=0, z=0, x+y+z=1$

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} (1+x+y+z)^{-3} \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^{1-x} \left[\frac{(1+x+y+z)^{-2}}{-2} \right]_0^{1-x-y} dy\, dx$$

$$= \int_0^1 \int_0^{1-x} \left(\frac{(1+x+y+1-x-y)^{-2}}{-2} \right) - \left(\frac{(1+x+y)^{-2}}{-2} \right) dy\, dx$$

550 656

$$= \int_0^1 \int_0^{1-x} \left[\frac{(1+x+y+z)^{-2}}{-2} \right]_0^{1-x-y} dy\,dx$$

$$= \int_0^1 \int_0^{1-x} \left(\frac{(1+\cancel{x}+\cancel{y}+1-\cancel{x}-\cancel{y})^{-2}}{-2} \right) - \left(\frac{(1+x+y)^{-2}}{-2} \right) dy\,dx$$

$$= -\frac{1}{2} \int_0^1 \int_0^{1-x} 2^{-2} - (1+x+y)^{-2}$$

550 656

$$= \int_0^1 \int_0^{1-x} \left[\frac{(1+x+y+z)^{-2}}{-2} \right]_0^{1-x-y} dy\,dx$$

$$= \int_0^1 \int_0^{1-x} \left(\frac{(1+\cancel{x}+\cancel{y}+1-\cancel{x}-\cancel{y})^{-2}}{-2} \right) - \left(\frac{(1+x+y)^{-2}}{-2} \right) dy\,dx$$

$$= -\frac{1}{2} \int_0^1 \int_0^{1-x} \frac{1}{4} - (1+x+y)^{-2}\; dy\,dx$$

$$= \int_0^1 \int_0^{1-x} \left(\frac{(1+\cancel{x}+\cancel{y}+1-\cancel{x}-\cancel{y})^{-2}}{-2}\right) - \left(\frac{(1+x+y)^{-2}}{-2}\right) dy\,dx$$

$$= -\frac{1}{2} \int_0^1 \int_0^{1-x} \frac{1}{4} - (1+x+y)^{-2} \, dy\,dx$$

$$\frac{1}{2} \int_0^1 \left[\frac{y}{4} + (1+x+y)^{-1} \right]_0^{1-x} dx$$

$$\frac{1-x}{4} + (2)^{-1} - \qquad (1+x)^{-1}$$

$$= -\frac{1}{2}\int_0 \left[\frac{y}{4} + \ldots(\ldots+y)\right]_0 dx$$

$$= -\frac{1}{2}\int_0^1 \left(\frac{1-x}{4} + \frac{1}{2} - \frac{1}{1+x}\right) dx$$

$$= -\frac{1}{2}\left[\frac{(1-x)^2}{8} + \frac{x}{2} - \log(1+x)\right]_0^1$$

$$= -\frac{1}{2}\int_0 \left[\frac{y}{4} + \ldots + y)\right]_0 \, dx$$

$$= -\frac{1}{2}\int_0^1 \left(\frac{1-x}{4} + \frac{1}{2} - \frac{1}{1+x}\right) dx$$

$$= -\frac{1}{2}\left[\frac{(1-x)^2}{8} + \frac{x}{2} - \log(1+x)\right]_0^1$$

$$= -\frac{1}{2}\left(\frac{1}{2} - \log 2 - \frac{1}{8} + \log 1\right)$$

026 368

$$= \frac{-1}{2}\int_{0} \left(\frac{1-x}{4} + \frac{1}{2} - \frac{1}{1+x}\right) dx$$

$$= \frac{-1}{2}\left[\frac{(1-x)^2}{8} - \frac{x}{2} - \log(1+x)\right]_0^1$$

$$= +\frac{1}{2}\left(-\frac{1\times 4}{2\times 4} + \log 2 - \frac{1}{8}\right)$$

$$= \frac{1}{2}\left[\log 2 - \frac{5}{8}\right]$$

026 368

(2) Evaluate $\iiint (x+y+z)\,dx\,dy\,dz$ over the tetrahedron bounded by $x=0, y=0, z=0, x+y+z=1$

Menu Exit Share List

Select Pen Eraser Clean All Ruler Shape Pair Mode Table Mind map Sticky Notes Preview Grid Undo Redo

Add Previous 9/12 Page number Next

ACADEMY

$\iiint$ 026 368 $z)\,dx\,dy\,dz$ over the tetrahedron

bounded by $x=0, y=0, z=0, x+y+z=1$

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} (x+y+z)\,dz\,dy\,dx$$

[

Menu Exit Share List

Select Pen Eraser Clean All Ruler Shape Pair Mode Table Mind map Sticky-Notes Preview Grid Undo Redo

Add Previous 9/12 Page number Next

ACADEMY

026 368

$$\int_0^1\int_0^{1-x}\left[\frac{(x+y+z)^2}{2}\right]_0^{1-x-y} dy\, dx$$

$$\frac{\cancel{x}+\cancel{y}+1-x-y}{2} - \frac{(x+y)}{2}$$

$$\frac{1}{2}-\frac{x+y}{2}$$

$$\left(\frac{\cancel{x}+\cancel{y}+1-x-y}{2}\right)^2-\left(\frac{x+y}{2}\right)^2$$

$$=\frac{1}{2}\int_0^1\int_0^{1-x}(1)^2-(x+y)^2\,dy\,dx$$

$$=\frac{1}{2}\int_0^1\left[y-\frac{(x+y)^3}{3}\right]_0^{1-x}dx$$

$$= \frac{1}{2}\int_0 \left[y - \frac{(\ldots)}{3}\right]_0 dx$$

$$= \frac{1}{2}\int_0^1 \left(\frac{2}{3} - x + \frac{x^3}{3}\right) dx$$

$$= \frac{1}{2}\left[\frac{2}{3}x - \frac{x^2}{2} + \frac{x^4}{12}\right]_0^1$$

$$= \frac{1}{2}\left[\frac{2\times 4}{3\times 4} - \frac{1}{2} + \frac{1}{12}\right]$$

$$= \frac{1}{2}\times \frac{1}{4} = \frac{1}{8}$$

288 512

When the region of integration is sphere

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$dx\,dy\,dz = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$\Rightarrow$ whol

288 512

$\Rightarrow$ whole sphere $x^2+y^2+z^2=a^2$ then

r varies from 0 to a

θ varies from 0 to π

ϕ varies from 0 to 2π

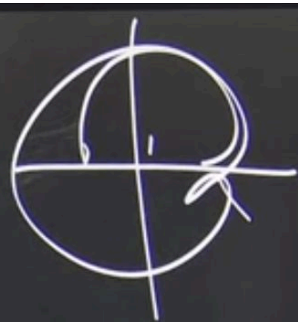
$\Rightarrow$ hemisphere

r varies from 0 to a

θ varies from 0 to $\pi/$

ϕ

ϕ varies from 0 to 2π



$\Rightarrow$ hemisphere

r varies from 0 to a

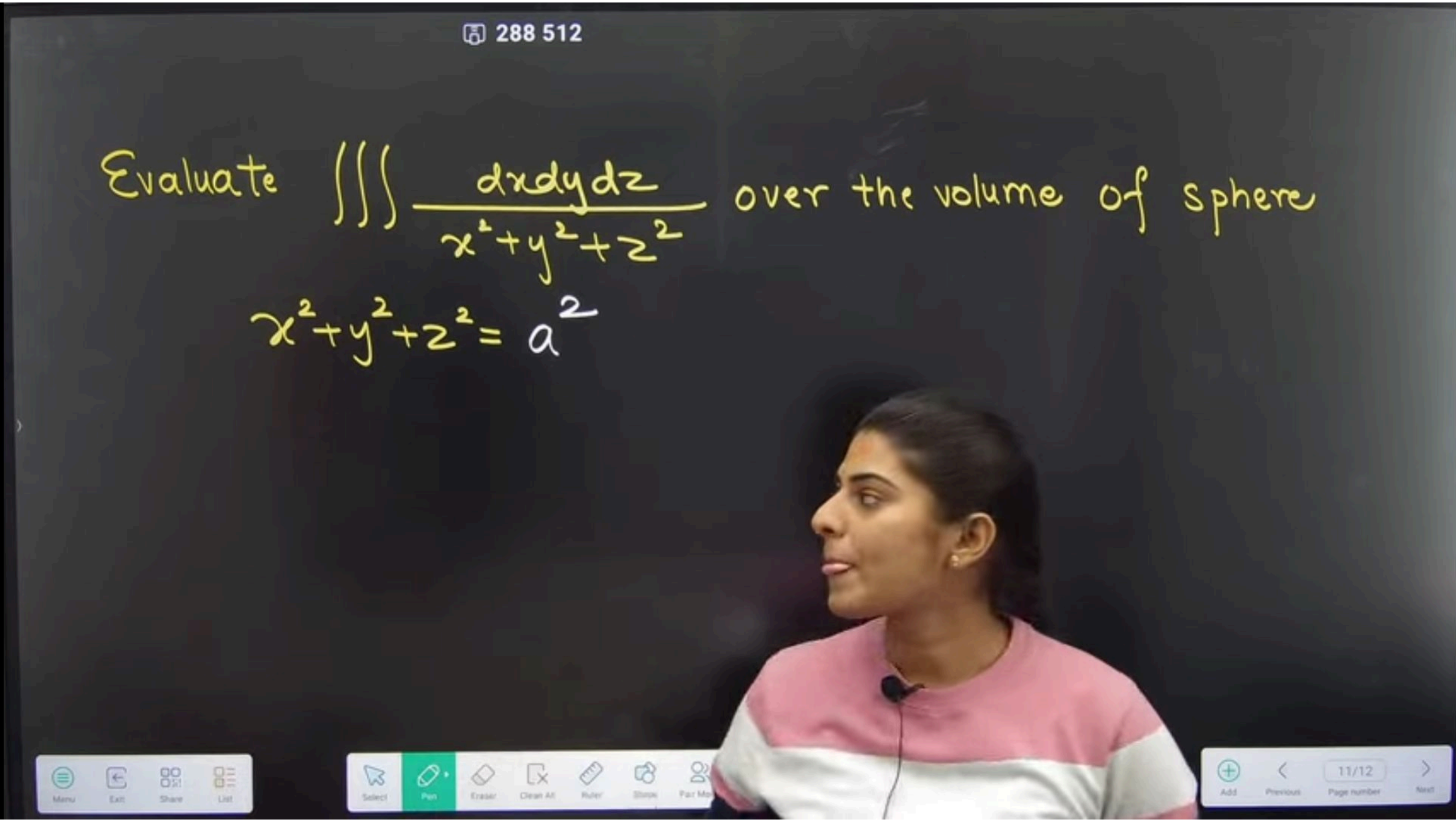
θ varies from 0 to $\pi/2$

ϕ varies from 0 to 2π

$\Rightarrow$ first octant

r varies from 0 to a

θ v


288 512
Evaluate $\iiint \frac{dx\,dy\,dz}{x^2+y^2+z^2}$ over the volume of sphere
$x^2+y^2+z^2=a^2$
11/12

288 512

$$\iiint \frac{dx\,dy\,dz}{x^2+y^2+z^2}$$ over the volume of sphere

$$x^2+y^2+z^2 = a^2$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$dx\,dy\,dz = r^2\sin\theta\,dr\,d\theta\,d\phi$$

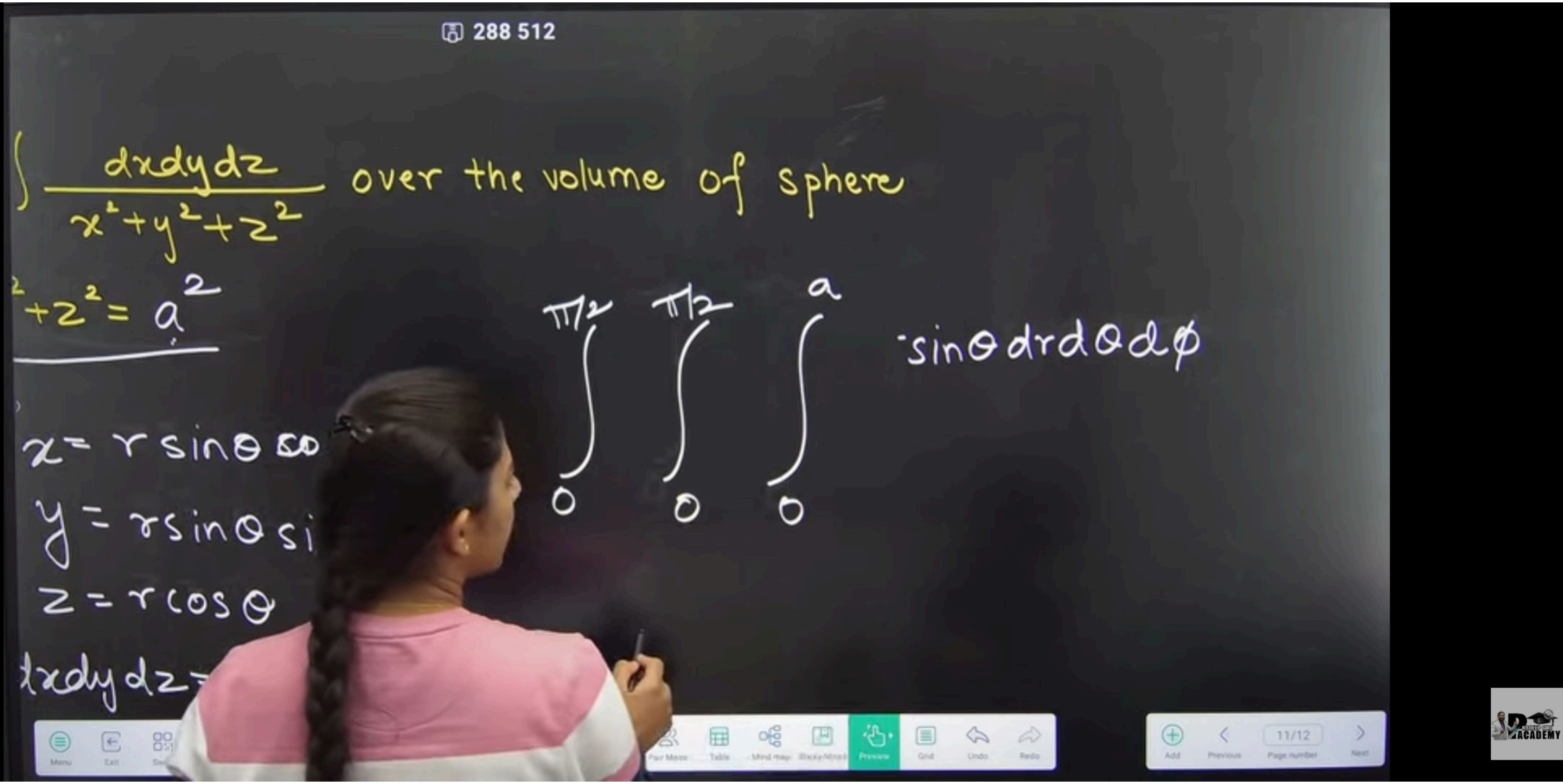
$$\int\int\int \frac{r^2\sin\theta\,dr\,d\theta\,d\phi}{r^2}$$

Menu Exit Share List

Select Pen Eraser Clean All Ruler Shape Pair Mode Table Mind map Sticky Notes Preview Grid Undo Redo

Add Previous 11/12 Page number Next

ACADEMY



288 512
∫ dxdydz / (x²+y²+z²) over the volume of sphere
+z² = a²
∫₀^{π/2} ∫₀^{π/2} ∫₀^{a} sinθ dr dθ dφ
x = r sinθ co
y = r sinθ si
z = r cos θ
11/12
ACADEMY

550 656

$\theta \cos\phi$

$\theta \sin\phi$

θ

$r^2 \sin\theta \, dr \, d\theta \, d\phi$

$$\int_0^{\frac{\pi}{2}} d\phi \cdot \int_0^{\frac{\pi}{2}} \sin\theta \, d\theta \cdot \int_0^{a} dr$$

$$[\phi]_0^{\pi/2} \cdot [-\cos\theta]_0^{\pi/2} \cdot [r]_0^{a}$$

$$= \frac{\pi}{2} \cdot 1 \cdot a$$

$$= \frac{\pi a}{2}$$

1) Sphere

r from 0 to a

θ from 0 to π

ϕ from 0 to 2π

2) Hemisphere

r from 0 to a

θ from 0 to $\pi/2$

ϕ from 0 to 2π

3) first octant

r from 0 to a

θ from 0 to $\pi/2$

ϕ from 0 to $\pi/2$