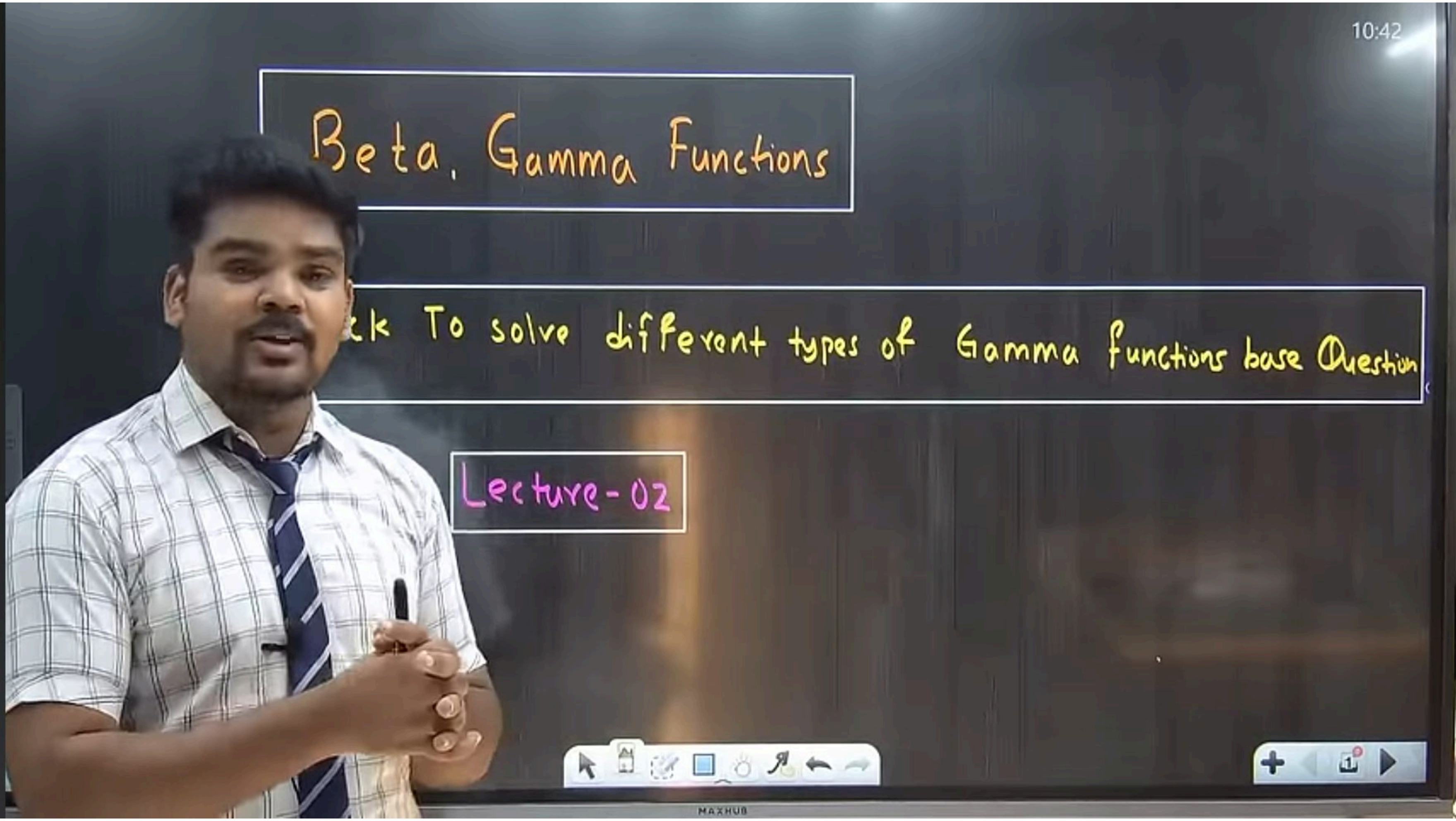
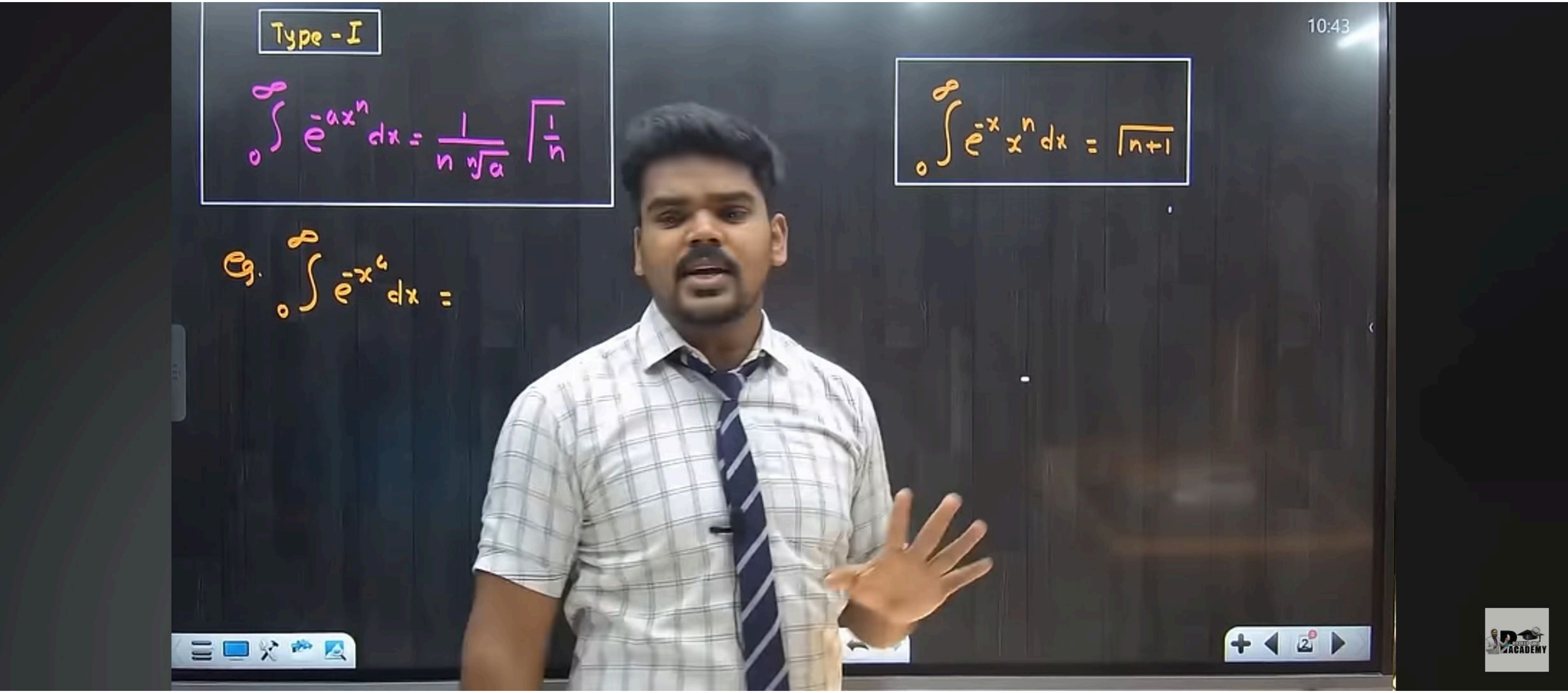

10:42
Beta, Gamma Functions
To solve different types of Gamma functions base Question
Lecture-02
MAXHUB
ACADEMY



Type - I
$\int_0^\infty e^{-ax^n}\,dx = \frac{1}{n\sqrt[n]{a}}\,\Gamma\left(\frac{1}{n}\right)$
eg. $\int_0^\infty e^{-x^4}\,dx =$
$\int_0^\infty e^{-x}x^n\,dx = \Gamma(n+1)$
10:43

10:49

Type - I

$$\int_0^\infty e^{-ax^n}\,dx = \frac{1}{n\sqrt[n]{a}}\,\Gamma\left(\frac{1}{n}\right)$$

eg. $\int_0^\infty e^{-x^4}\,dx =$

put $x^4 = t$

$x = t^{1/4}$

$\frac{dx}{dt} = \frac{1}{4} t^{-3/4}$

$$\boxed{dx = \frac{1}{4} t^{-3/4}\,dt}$$

$x = 0$

$x = \infty$

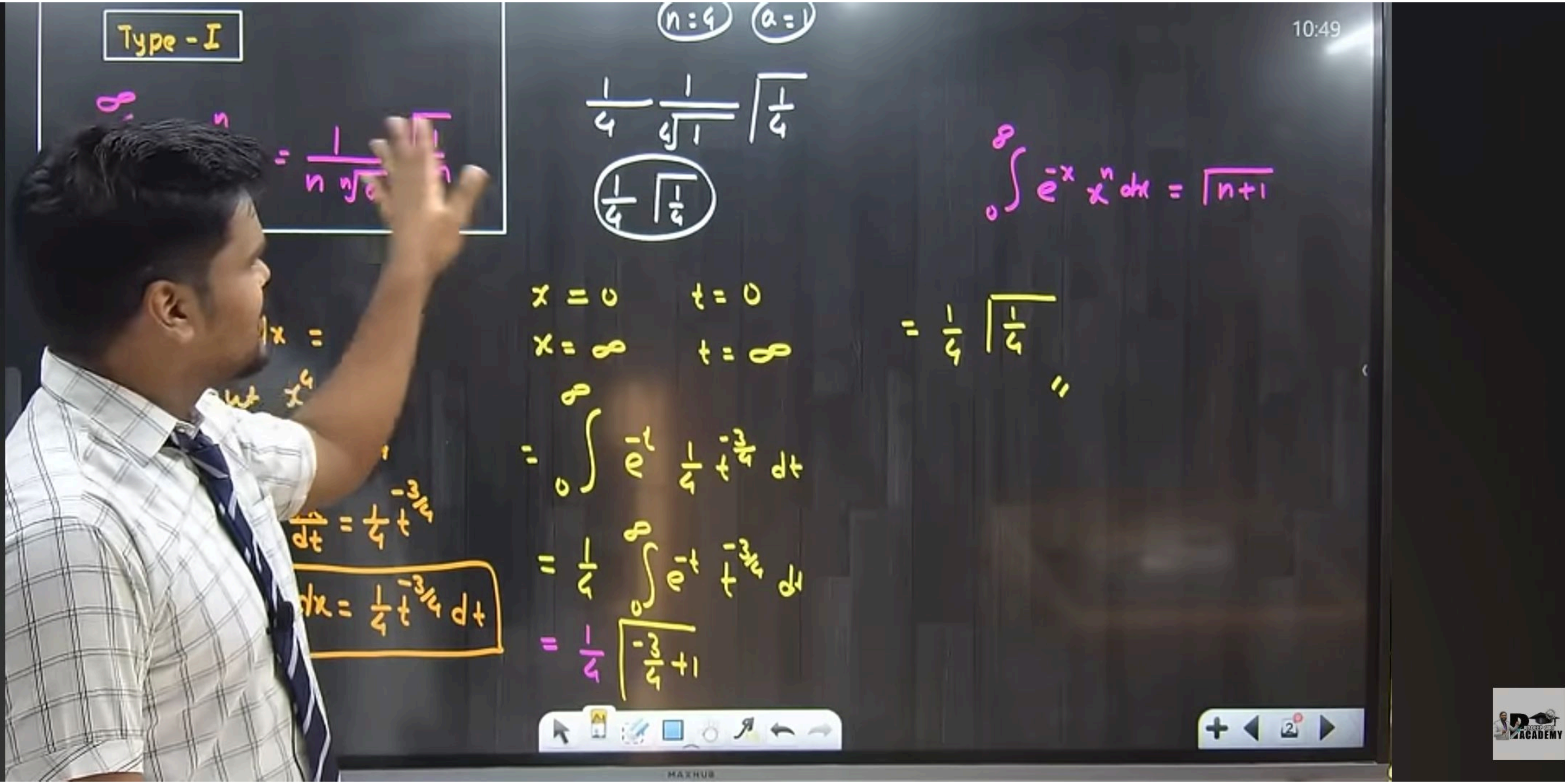
∞

$n = 4$ $a = 1$

$\frac{1}{4}\,\frac{1}{4\sqrt{1}}\,\Gamma\frac{1}{4}$

$\frac{1}{4}\,\Gamma$

$\int_0^\infty e$



Type - I
$n:4$ $a:1$
$\frac{1}{4}\frac{1}{\sqrt[4]{1}}\Gamma\frac{1}{4}$
$\frac{1}{4}\Gamma\frac{1}{4}$
$\int_0^\infty e^{-x} x^n dx = \Gamma n+1$
$x=0$ $t=0$
$x=\infty$ $t=\infty$
$= \frac{1}{4}\Gamma\frac{1}{4}$
$= \int_0^\infty e^{-t}\,\frac{1}{4}\,t^{-3/4}\,dt$
$= \frac{1}{4}\int_0^\infty e^{-t}\,t^{-3/4}\,dt$
$= \frac{1}{4}\Gamma\left(-\frac{3}{4}+1\right)$
$= \frac{1}{4}t^{-3/4}$
$dx = \frac{1}{4}t^{-3/4}\,dt$
10:49
MAXHUB
ACADEMY

10:53

Type - II

$$\int_0^\infty x^m \cdot \boxed{e^{-ax^n}}\, dx = \frac{1}{n} \frac{1}{a^{\frac{m+1}{n}}} \Gamma\left(\frac{m+1}{n}\right)$$

$$a^m \times a^n = a^{m+n}$$

$$t^{1/4} \times t^{-3/4} = t^{\frac{1}{4} + \left(-\frac{3}{4}\right)}$$

$$= t^{\frac{1-3}{4}} = t^{\frac{-2}{4}} = t^{-1/2}$$

eg. $\int_0^\infty x \cdot e^{-x^4} dx$

put $x^4 = t$

$x = t^{1/4}$

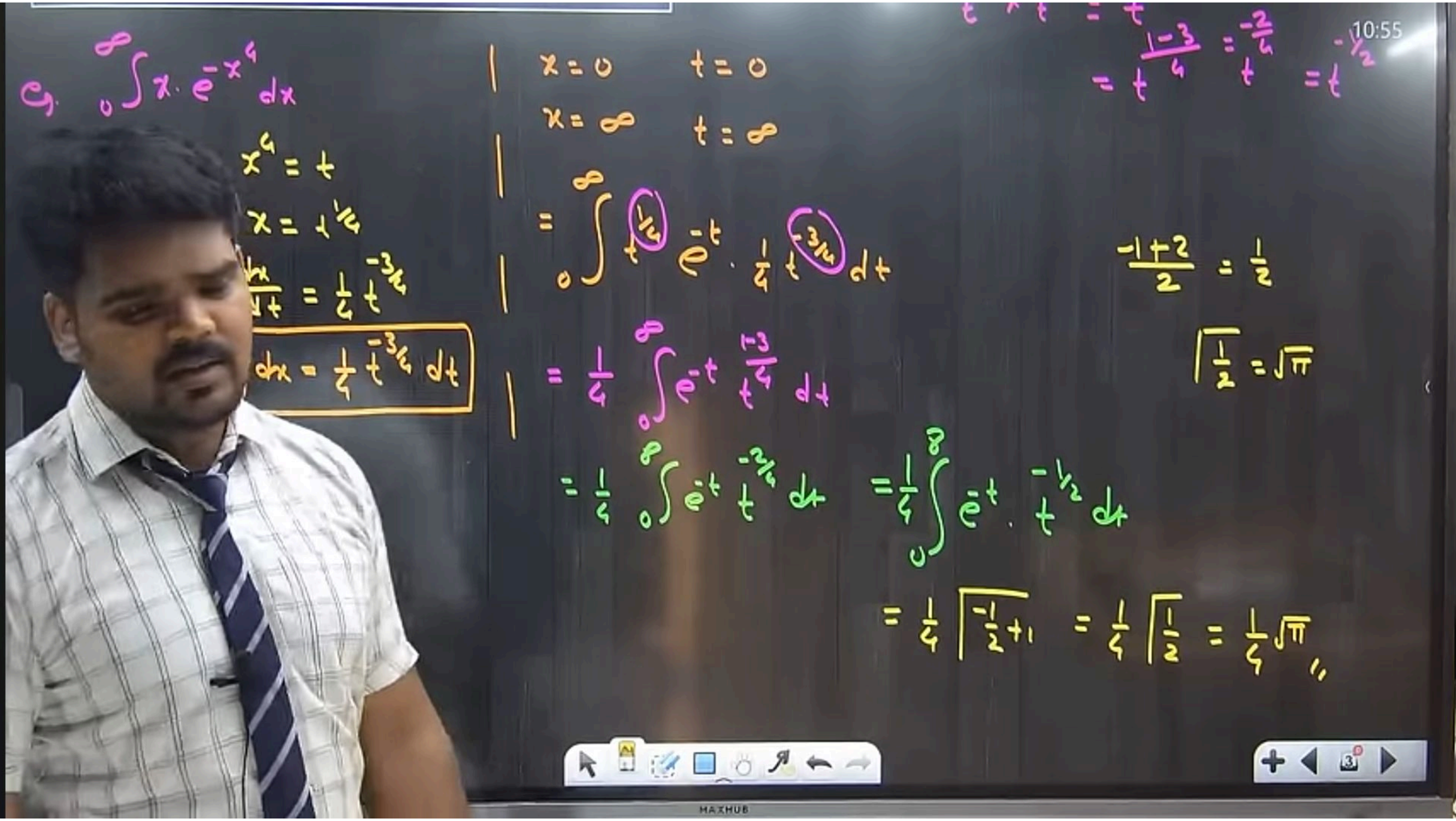
$\frac{dx}{dt} = \frac{1}{4} t^{-3/4}$

$\boxed{dx = \frac{1}{4} t^{-3/4} dt}$

$x = 0$

$x = \infty$

$= \int^\infty \ldots dt$


10:55
eg. $\int_0^\infty x \cdot e^{-x^4} dx$
$x^4 = t$
$x = t^{1/4}$
$\frac{dx}{dt} = \frac{1}{4} t^{-3/4}$
$\boxed{dx = \frac{1}{4} t^{-3/4} dt}$
$x = 0 \quad t = 0$
$x = \infty \quad t = \infty$
$= \int_0^\infty t^{1/4} e^{-t} \cdot \frac{1}{4} t^{-3/4} dt$
$= \frac{1}{4} \int_0^\infty e^{-t} t^{\frac{1-3}{4}} dt$
$= \frac{1}{4} \int_0^\infty e^{-t} t^{-2/4} dt = \frac{1}{4} \int_0^\infty e^{-t} \cdot t^{-1/2} dt$
$= \frac{1}{4} \Gamma\left(-\frac{1}{2}+1\right) = \frac{1}{4} \Gamma\frac{1}{2} = \frac{1}{4}\sqrt{\pi}$
$= t^{\frac{1-3}{4}} = t^{\frac{-2}{4}} = t^{-1/2}$
$\frac{-1+2}{2} = \frac{1}{2}$
$\Gamma\frac{1}{2} = \sqrt{\pi}$
ACADEMY
MAXHUB

10:56

S.T $\underbrace{\int_0^\infty x \cdot e^{-x^8} dx}_{I_1} \cdot \underbrace{\int_0^\infty x^2 \cdot e^{-x^4} dx}_{I_2} = \frac{\pi}{16\sqrt{2}}$

$I_1 \cdot I_2$

$I_1 = \int_0^\infty x \cdot e^{-x^8} dx$

$I_2 = \int_0^\infty x^2 \cdot e^{-x^4} dx$

S.T $\int_0^\infty x \cdot e^{-x^8} dx \cdot \int_0^\infty x^2 \cdot e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$

$I_1 \cdot I_2$

$I_1 = \int_0^\infty x \cdot e^{-x^8} dx$

put $x^8 = t$ $\quad x = t^{1/8}$

$\frac{dx}{dt} = \frac{1}{8} t^{-7/8}$

$\boxed{dx = \frac{1}{8} t^{-7/8} dt}$

$I_1 = \int_0^\infty t^{1/8} \cdot$

$I_2 = \int_0^\infty x^2 \cdot e^{-x^4} dx$

$I_1 = \int_0 x \cdot e \; dx$

$I_2 = \int_0 x^2 \cdot e^{x} \; dx$

10:58

put $x^8 = t$ $\quad x = t^{1/8}$

$\frac{dx}{dt} = \frac{1}{8} t^{-7/8}$

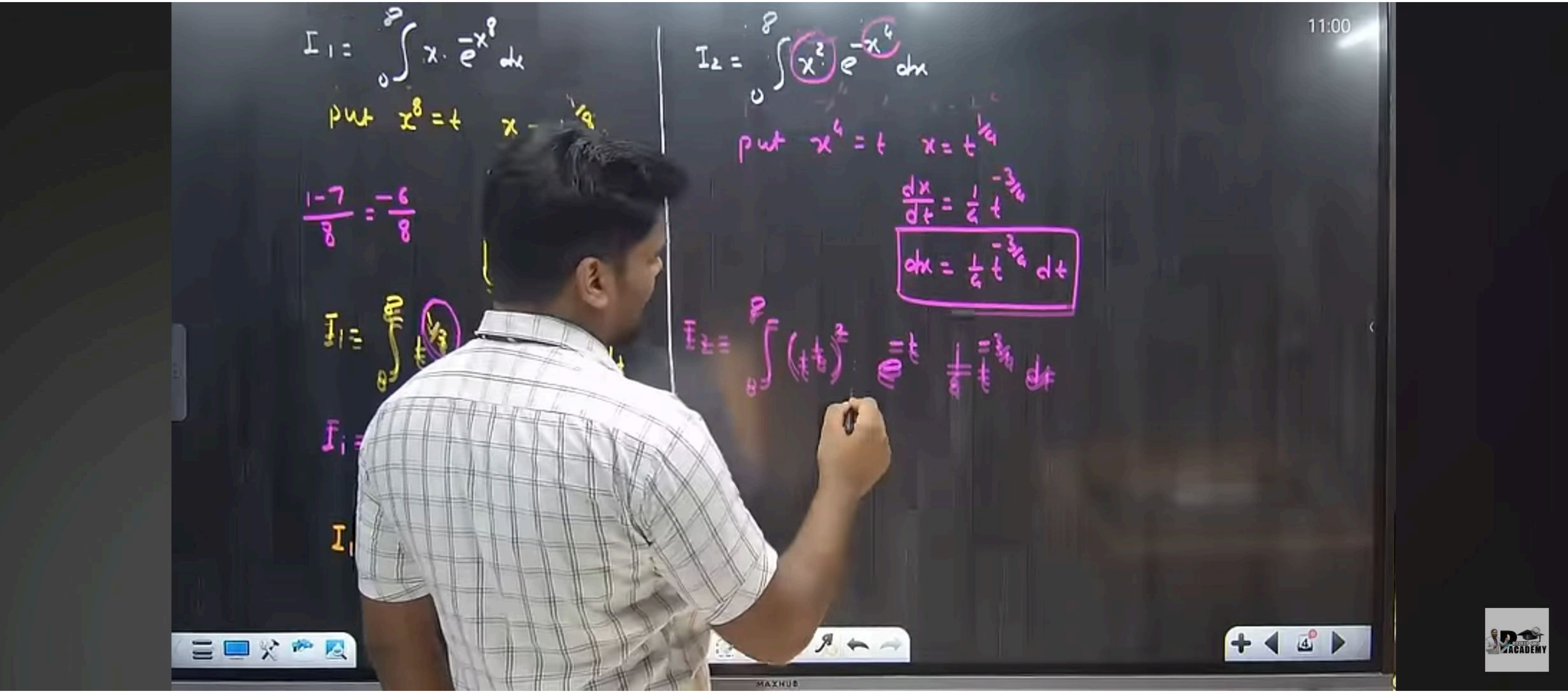
$\frac{1-7}{8} = \frac{-6}{8}$

$$\boxed{dx = \frac{1}{8} t^{-7/8} dt}$$

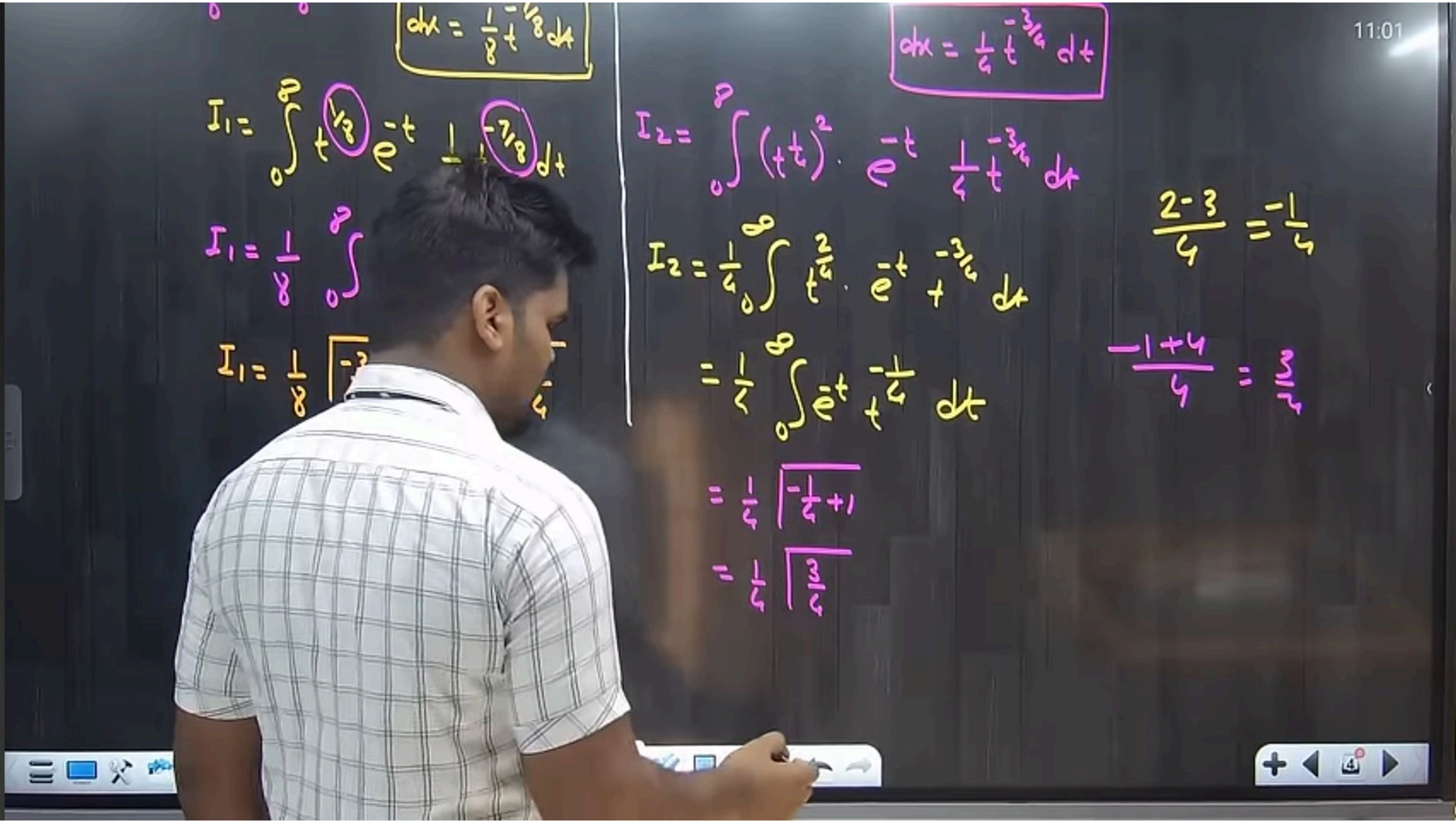
$$I_1 = \int_0^\infty t^{1/8} e^{-t} \frac{1}{8} t^{-7/8} dt$$

$$I_1 = \frac{1}{8} \int_0^\infty e^{-t} t^{-3/4} dt$$

$$I_1 = \frac{1}{8} \Gamma\left(\frac{-3}{4}+1\right) = \frac{1}{8} \Gamma\left(\frac{1}{4}\right)$$



$I_1 = \int_0^\infty x \cdot e^{-x^8} dx$
put $x^8 = t$
$\frac{1-7}{8} = \frac{-6}{8}$
$I_2 = \int_0^\infty x^2 e^{-x^4} dx$
put $x^4 = t$ $x = t^{1/4}$
$\frac{dx}{dt} = \frac{1}{4} t^{-3/4}$
$\boxed{dx = \frac{1}{4} t^{-3/4} dt}$
$I_2 = \int_0^\infty (t^{1/4})^2 e^{-t} \frac{1}{4} t^{-3/4} dt$
11:00
MAXHUB
ACADEMY


11:01
$dx = \frac{1}{8} t^{-1/8} dt$
$I_1 = \int_0^\infty t^{1/8} e^{-t} \frac{1}{8} t^{-7/8} dt$
$I_1 = \frac{1}{8} \int_0^\infty$
$I_1 = \frac{1}{8} \Gamma$
$dx = \frac{1}{4} t^{-3/4} dt$
$I_2 = \int_0^\infty (t^{1/4})^2 \cdot e^{-t} \frac{1}{4} t^{-3/4} dt$
$I_2 = \frac{1}{4} \int_0^\infty t^{2/4} \cdot e^{-t} t^{-3/4} dt$
$= \frac{1}{4} \int_0^\infty e^{-t} t^{-1/4} dt$
$= \frac{1}{4} \Gamma\left(-\frac{1}{4}+1\right)$
$= \frac{1}{4} \Gamma\left(\frac{3}{4}\right)$
$\frac{2-3}{4} = \frac{-1}{4}$
$\frac{-1+4}{4} = \frac{3}{4}$
ACADEMY

$$= \frac{1}{4}\,\Gamma\left(\frac{3}{4}\right)$$

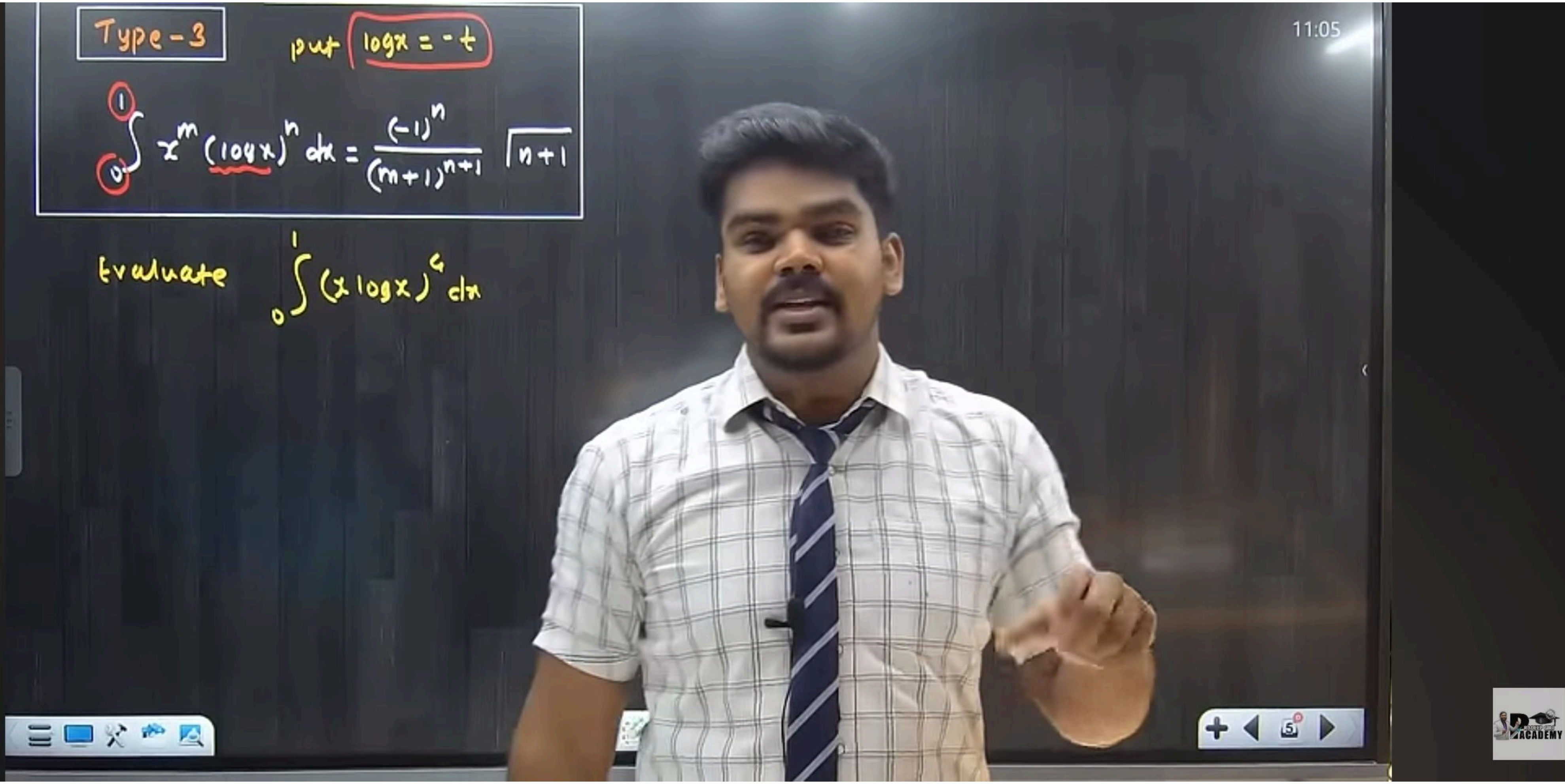
$$\therefore\ I_2 = \frac{1}{8}\,\Gamma\left(\frac{1}{4}\right) \times \frac{1}{4}\,\Gamma\left(\frac{3}{4}\right)$$

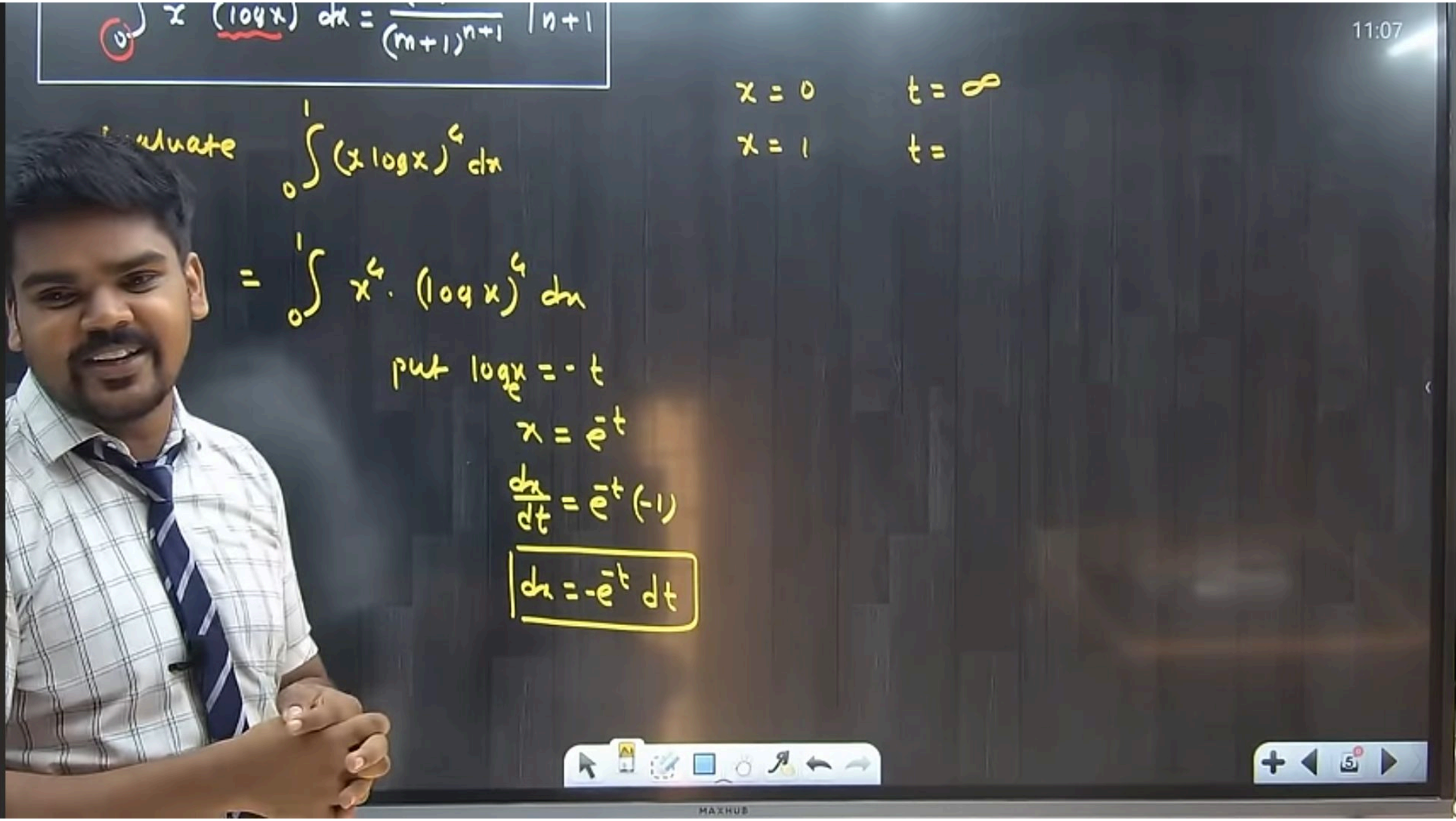
$$= \frac{1}{32}\,\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$$

$$= \frac{1}{32} \times \sqrt{2}\,\pi$$

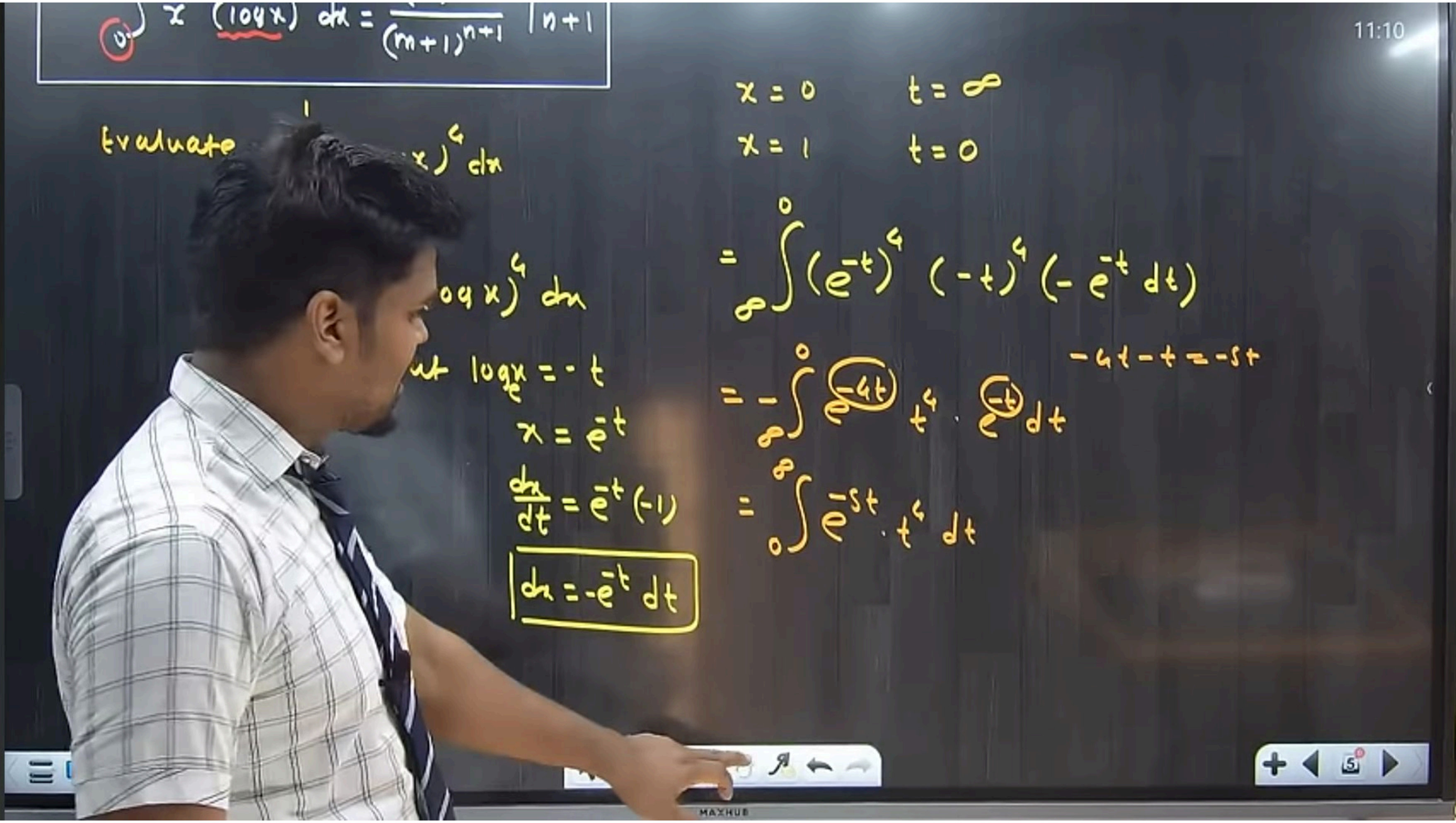
$$= \frac{\sqrt{2}\,\pi}{32} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\cancel{2}\,\pi}{\cancel{32}_{16}\sqrt{2}} = \frac{\pi}{16\sqrt{2}}$$


Type-3
put $\log x = -t$
$\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n}{(m+1)^{n+1}} \Gamma(n+1)$
Evaluate $\int_0^1 (x \log x)^4 \, dx$
11:05
ACADEMY



$x\ (\log x)\ dx = \frac{}{(m+1)^{n+1}}\ |n+1$
11:07
$x = 0 \quad t = \infty$
$x = 1 \quad t =$
luate $\int_0^1 (x \log x)^4 dx$
$= \int_0^1 x^4 \cdot (\log x)^4 dx$
put $\log_e x = -t$
$x = e^{-t}$
$\frac{dx}{dt} = e^{-t}(-1)$
$\boxed{dx = -e^{-t} dt}$
MAXHUB


11:10
$x = 0 \quad t = \infty$
$x = 1 \quad t = 0$
Evaluate
$x)^4\, dx$
$= \int_{\infty}^{0} (e^{-t})^4 (-t)^4 (-e^{-t}\, dt)$
$-4t - t = -5t$
$\log_e x = -t$
$= -\int_{\infty}^{0} e^{-4t}\, t^4 \cdot e^{-t}\, dt$
$x = e^{-t}$
$\frac{dx}{dt} = e^{-t}(-1)$
$= \int_{0}^{\infty} e^{-5t} \cdot t^4\, dt$
$dx = -e^{-t}\, dt$
MAXHUB
ACADEMY

put $\log_e x = -t$

$x = e^{-t}$

$\frac{dx}{dt} = e^{-t}(-1)$

$$= -\int_{\infty}^{0} e^{-4t}\, t^4 \cdot e^{-t}\, dt$$

$-4t - t = -5t$

$$= \int_{0}^{\infty} e^{-5t} \cdot t^4\, dt$$

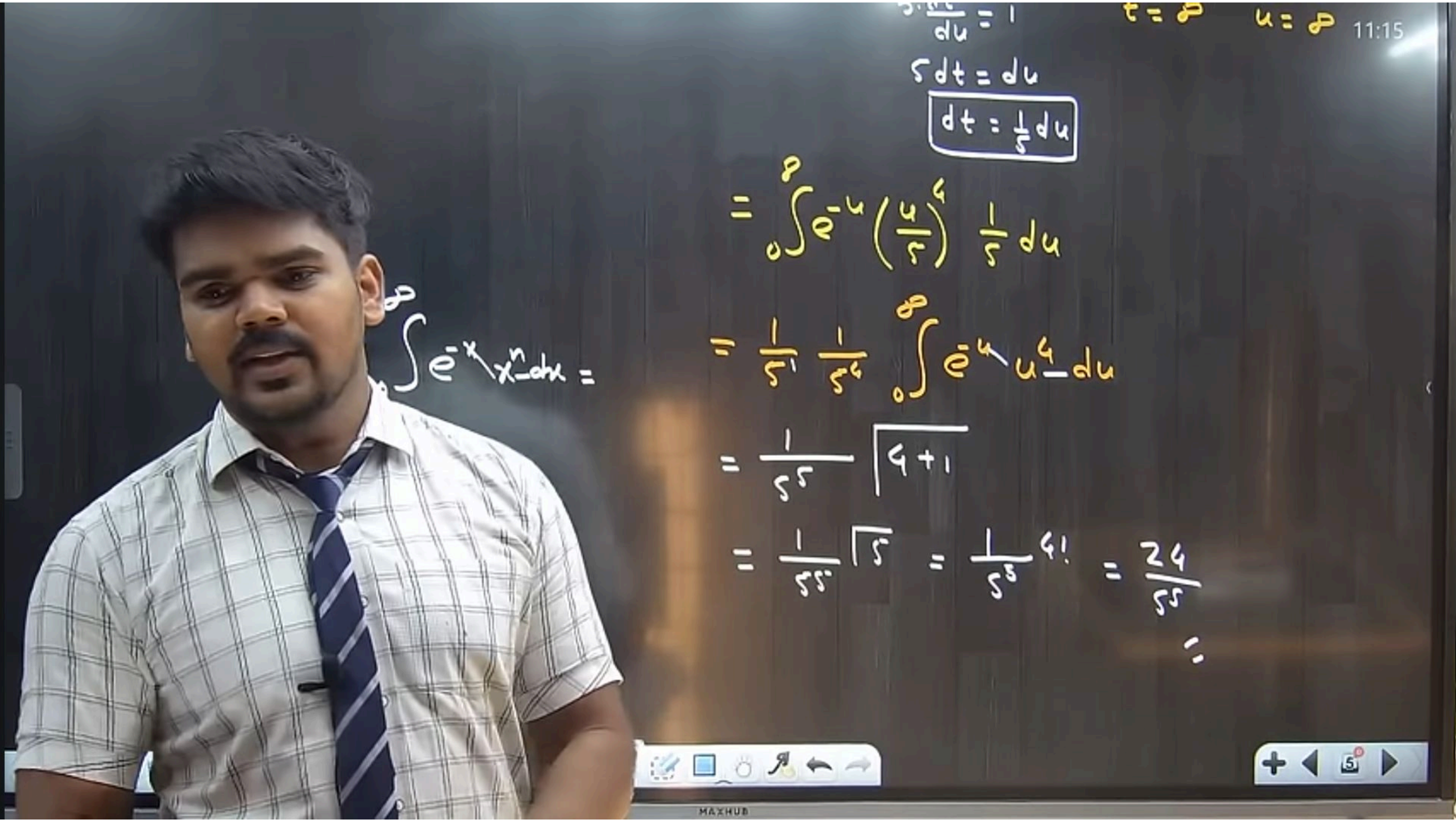
put $5t = u$ $\boxed{t = \frac{u}{5}}$ $t = 0$ $u = 0$

$5 \cdot \frac{dt}{du} = 1$ $t = \infty$ $u = \infty$

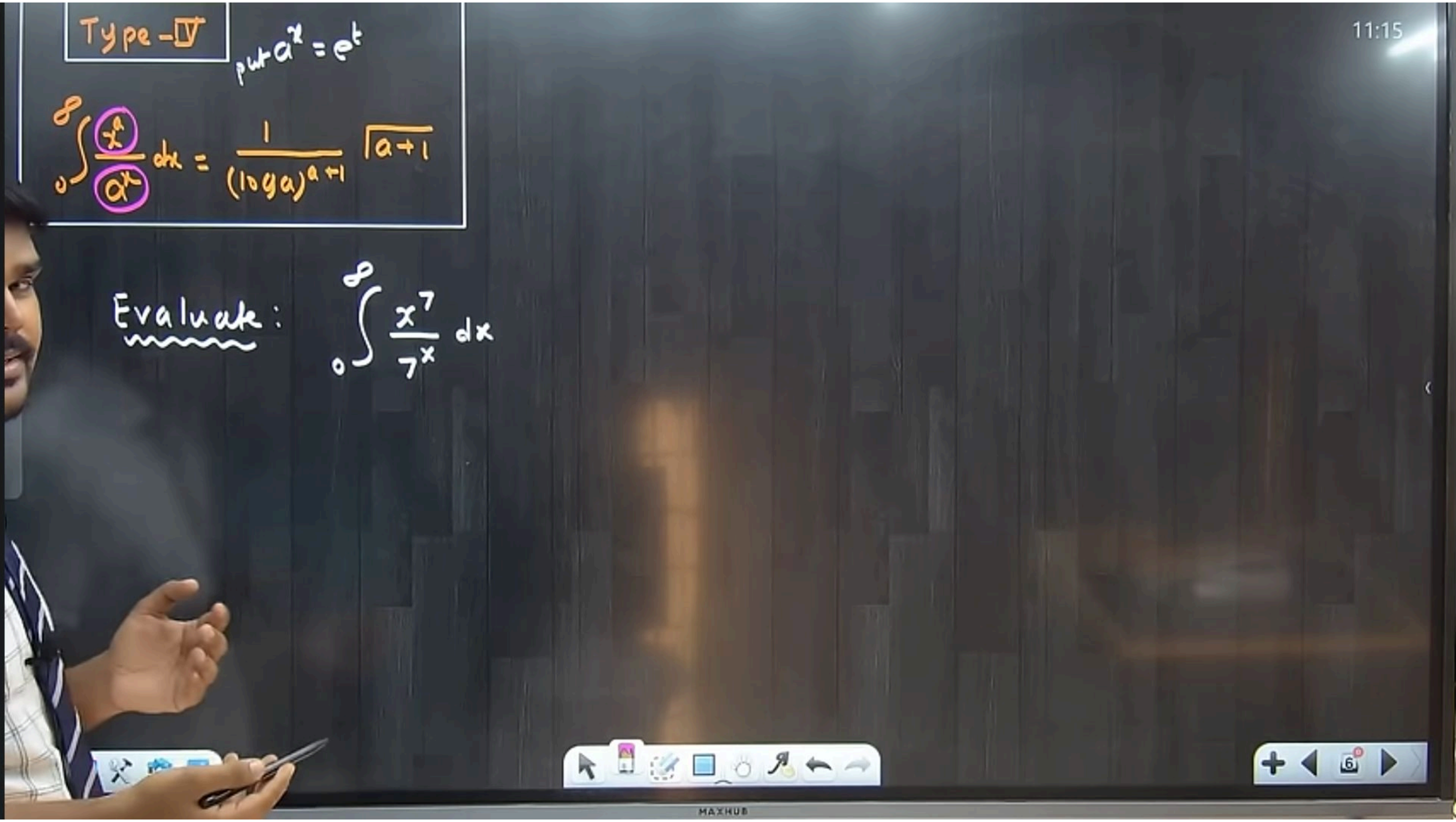
$5\,dt = du$

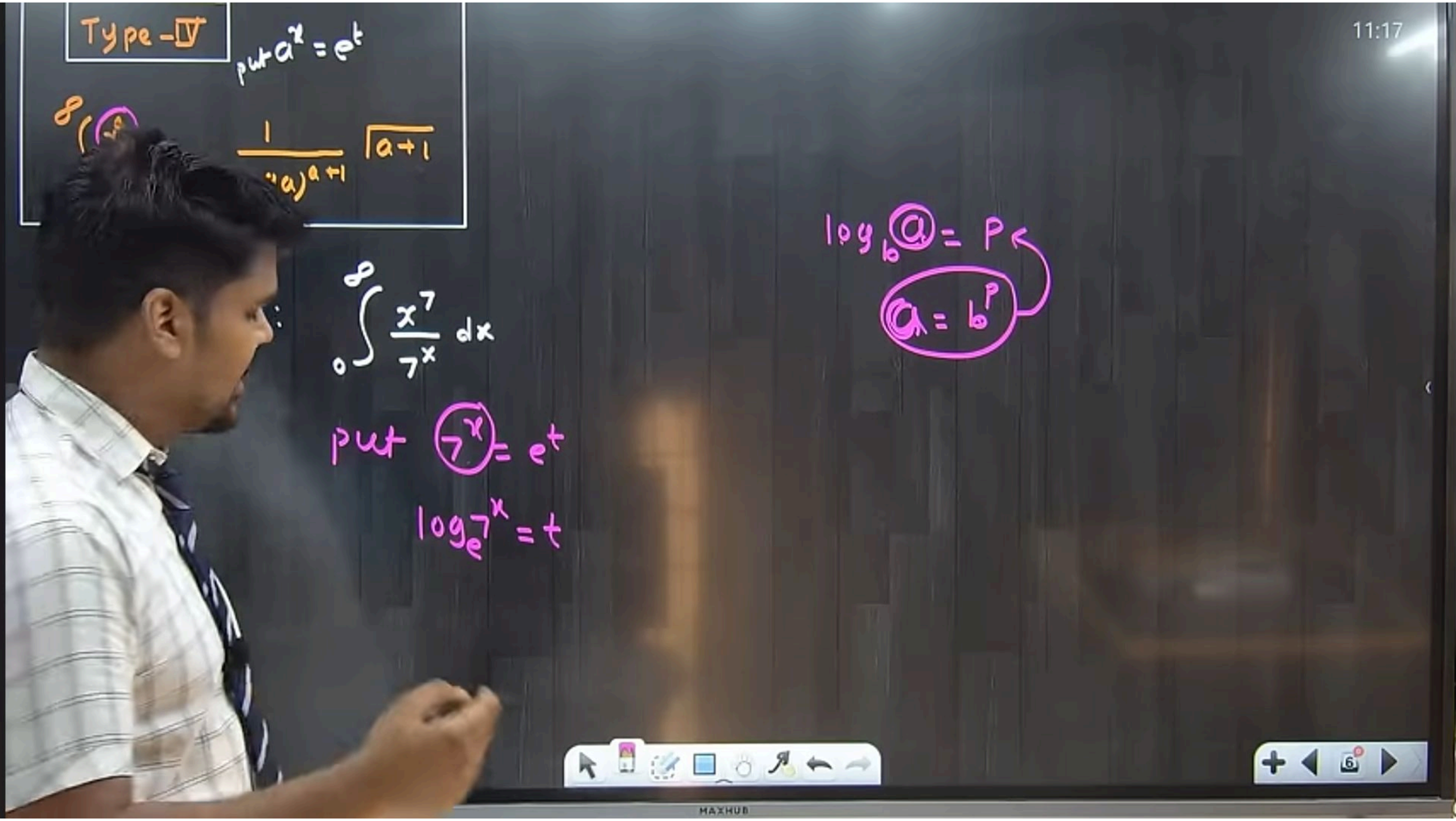
$\boxed{dt = \frac{1}{5}\,du}$

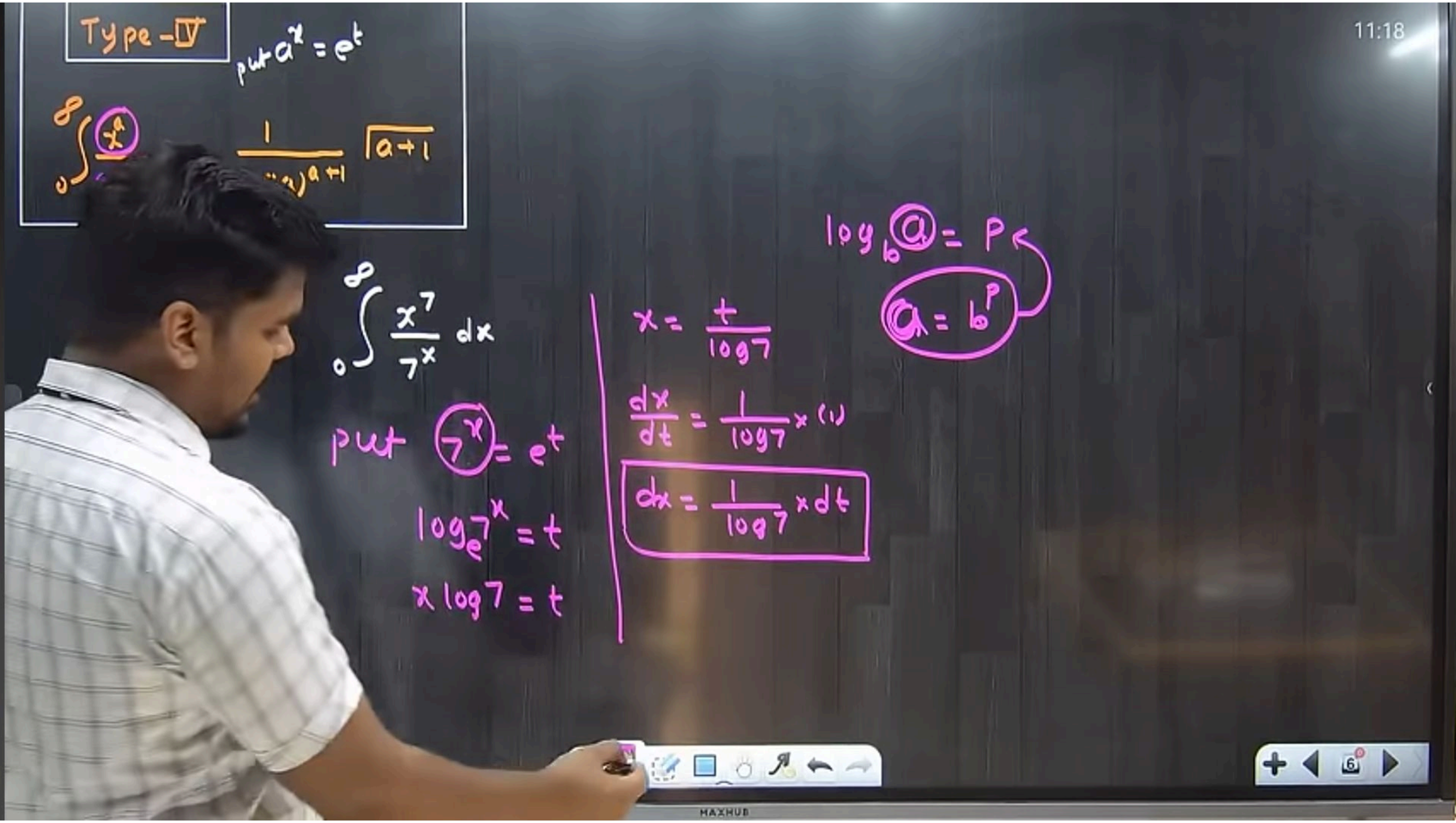
$$= \int_{0}^{\infty} e^{-u} \left(\frac{u}{5}\right)^4 \frac{1}{5}\, du$$

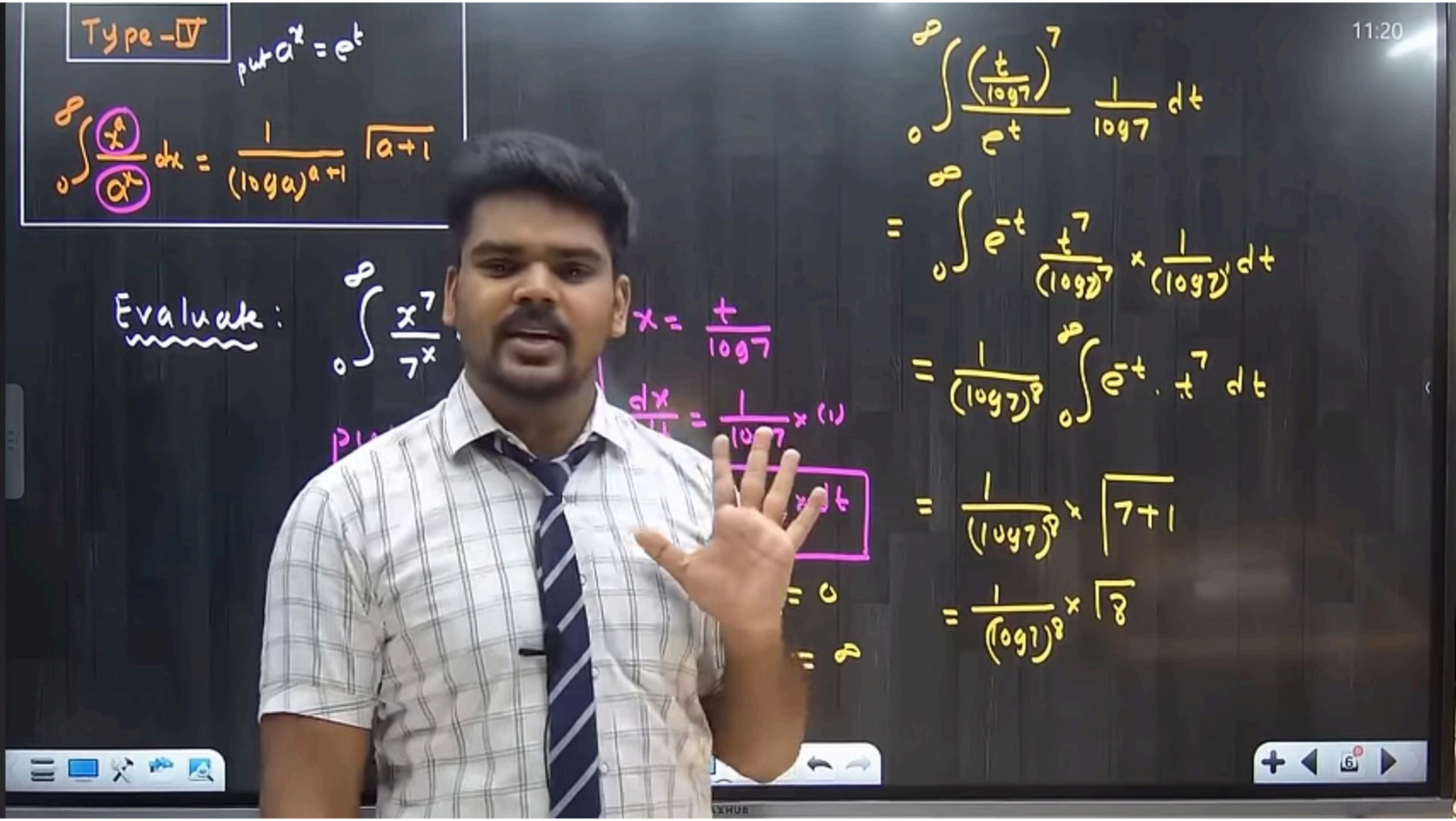


t = ∞
u = ∞
11:15
5 dt = du
dt = 1/5 du
= ∫₀^∞ e^-u (u/5)^4 1/5 du
= 1/5^1 · 1/5^4 ∫₀^∞ e^-u u^4 du
= 1/5^5 Γ(4+1)
= 1/5^5 Γ5 = 1/5^5 4! = 24/5^5
∫₀^∞ e^-x x^n dx =
MAXHUB
ACADEMY


Type - IV
put $a^x = e^t$
$\int_0^\infty \frac{x^a}{a^x}\,dx = \frac{1}{(\log a)^{a+1}}\,\Gamma(a+1)$
Evaluate: $\int_0^\infty \frac{x^7}{7^x}\,dx$
11:15
MAXHUB


Type - IV
put a^x = e^t
11:17
log_b a = P
a = b^P
∫_0^∞ x^7 / 7^x dx
put 7^x = e^t
log_e 7^x = t
MAXHUB


Type - IV
put $a^x = e^t$
$\int_0^\infty \frac{x^a}{\dots} = \frac{1}{(\dots)^{a+1}} \Gamma(a+1)$
$\int_0^\infty \frac{x^7}{7^x} dx$
put $7^x = e^t$
$\log_e 7^x = t$
$x \log 7 = t$
$x = \frac{t}{\log 7}$
$\frac{dx}{dt} = \frac{1}{\log 7} \times (1)$
$dx = \frac{1}{\log 7} \times dt$
$\log_b a = p$
$a = b^p$
11:18
MAXHUB


Type - IV
put $a^x = e^t$
$\int_0^\infty \frac{x^a}{a^x}\,dx = \frac{1}{(\log a)^{a+1}}\,\overline{|a+1}$
Evaluate :
$\int_0^\infty \frac{x^7}{7^x}$
$x = \frac{t}{\log 7}$
$\int_0^\infty \frac{\left(\frac{t}{\log 7}\right)^7}{e^t}\,\frac{1}{\log 7}\,dt$
$= \int_0^\infty e^{-t}\,\frac{t^7}{(\log 7)^7} \times \frac{1}{(\log 7)^1}\,dt$
$= \frac{1}{(\log 7)^8}\int_0^\infty e^{-t}\cdot t^7\,dt$
$= \frac{1}{(\log 7)^8} \times \overline{|7+1}$
$= \frac{1}{(\log 7)^8} \times \overline{|8}$
11:20
ACADEMY

11:23

$$\int_0^{\infty} \sqrt[4]{x}\; e^{-\sqrt{x}}\, dx$$

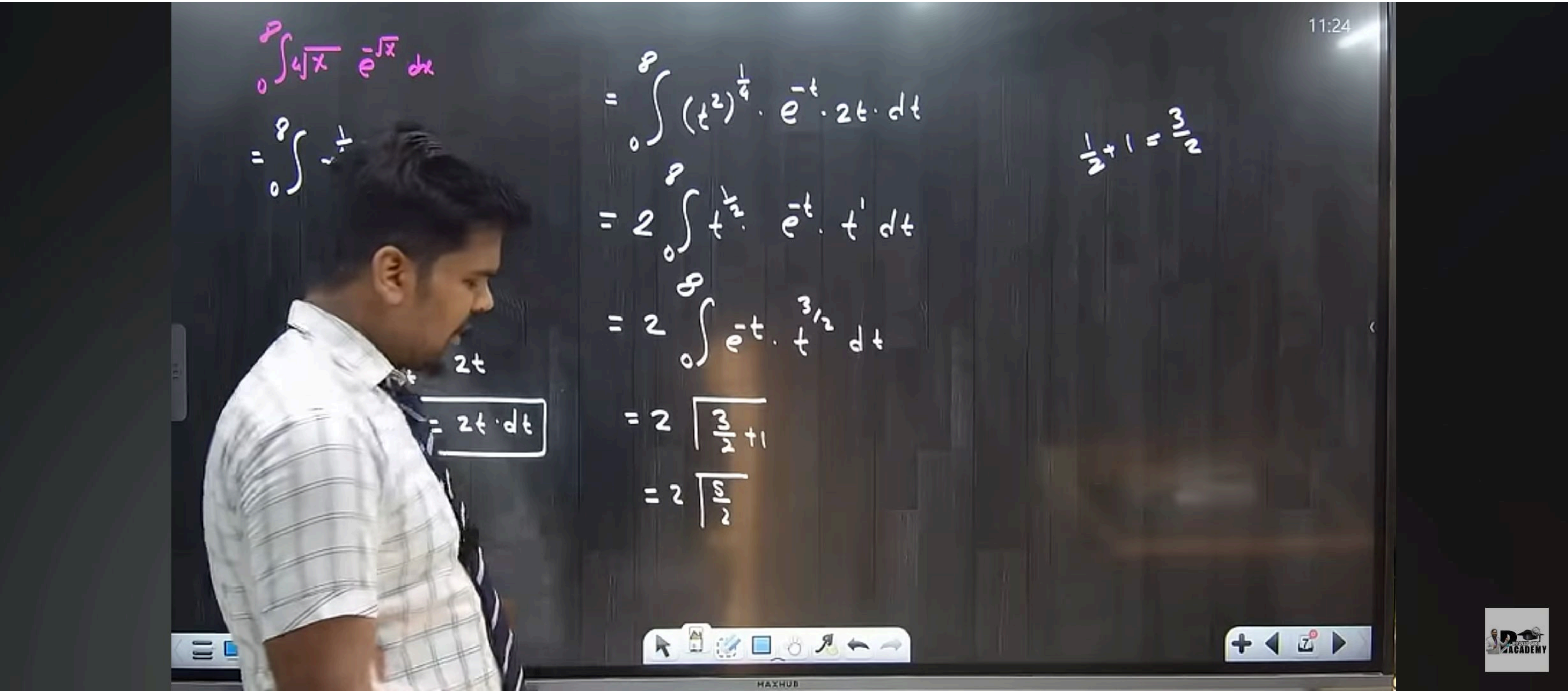
$$= \int_0^{\infty} x^{\frac{1}{4}}\; e^{-\sqrt{x}}\, dt$$

$$\sqrt{x} = t$$

$$x = t^2$$

$$\frac{dx}{dt} = 2t$$

$$\boxed{dx = 2t \cdot dt}$$


11:24
$\int_0^\infty 4\sqrt{x}\, e^{-\sqrt{x}}\, dx$
$= \int_0^\infty (t^2)^{\frac{1}{4}} \cdot e^{-t} \cdot 2t \cdot dt$
$= 2\int_0^\infty t^{\frac{1}{2}} \cdot e^{-t} \cdot t^1\, dt$
$= 2\int_0^\infty e^{-t} \cdot t^{3/2}\, dt$
$= 2\,\Gamma\left(\frac{3}{2}+1\right)$
$= 2\,\Gamma\left(\frac{5}{2}\right)$
$\frac{1}{2}+1 = \frac{3}{2}$
$2t$
$= 2t \cdot dt$
MAXHUB
ACADEMY

$$\int_0^{\infty} \frac{dx}{3^{4x^2}}$$

$$3^{4x^2} = e^t$$

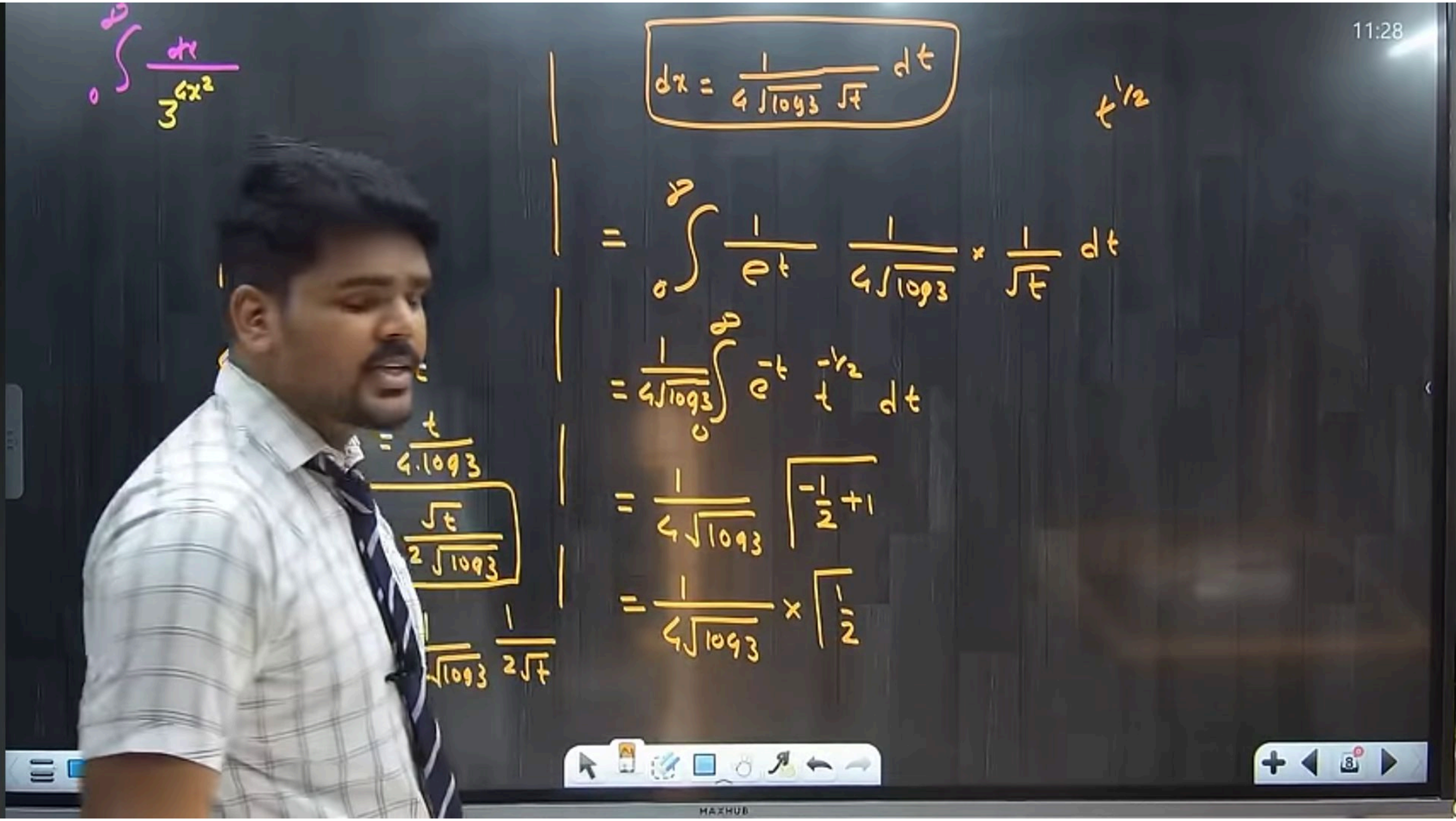
$$\log_e 3^{4x^2} = t$$

$$4x^2 \cdot \log 3 = t$$

$$x^2 = \frac{t}{4 \cdot \log 3}$$

$$\boxed{x = \frac{\sqrt{t}}{2\sqrt{\log 3}}}$$

$$\frac{dx}{dt} = \frac{1}{2\sqrt{\log 3}} \frac{1}{2\sqrt{t}}$$



11:28
$\int_0^\infty \frac{dx}{3^{4x^2}}$
$\boxed{dx = \frac{1}{4\sqrt{\log 3}\,\sqrt{t}}\,dt}$
$t^{1/2}$
$= \int_0^\infty \frac{1}{e^t}\,\frac{1}{4\sqrt{\log 3}} \times \frac{1}{\sqrt{t}}\,dt$
$= \frac{1}{4\sqrt{\log 3}}\int_0^\infty e^{-t}\,t^{-1/2}\,dt$
$= \frac{1}{4\sqrt{\log 3}}\,\Gamma\left(-\frac{1}{2}+1\right)$
$= \frac{1}{4\sqrt{\log 3}} \times \Gamma\left(\frac{1}{2}\right)$
$= \frac{t}{4.\log 3}$
$\boxed{\frac{\sqrt{t}}{2\sqrt{\log 3}}}$
$\frac{1}{2\sqrt{t}}$
MAXHUB

11:34

$$\int_0^\infty x^9 \cdot e^{-2x^2}\,dx$$

put $2x^2 = t$

$$x^2 = \frac{t}{2}$$

$$x = \frac{\sqrt{t}}{\sqrt{2}}$$

$$\frac{dx}{dt} = \frac{1}{\sqrt{2}} \times \frac{1}{2\sqrt{t}}$$

$$\boxed{dx = \frac{1}{2\sqrt{2}\,\sqrt{t}}\,dt}$$

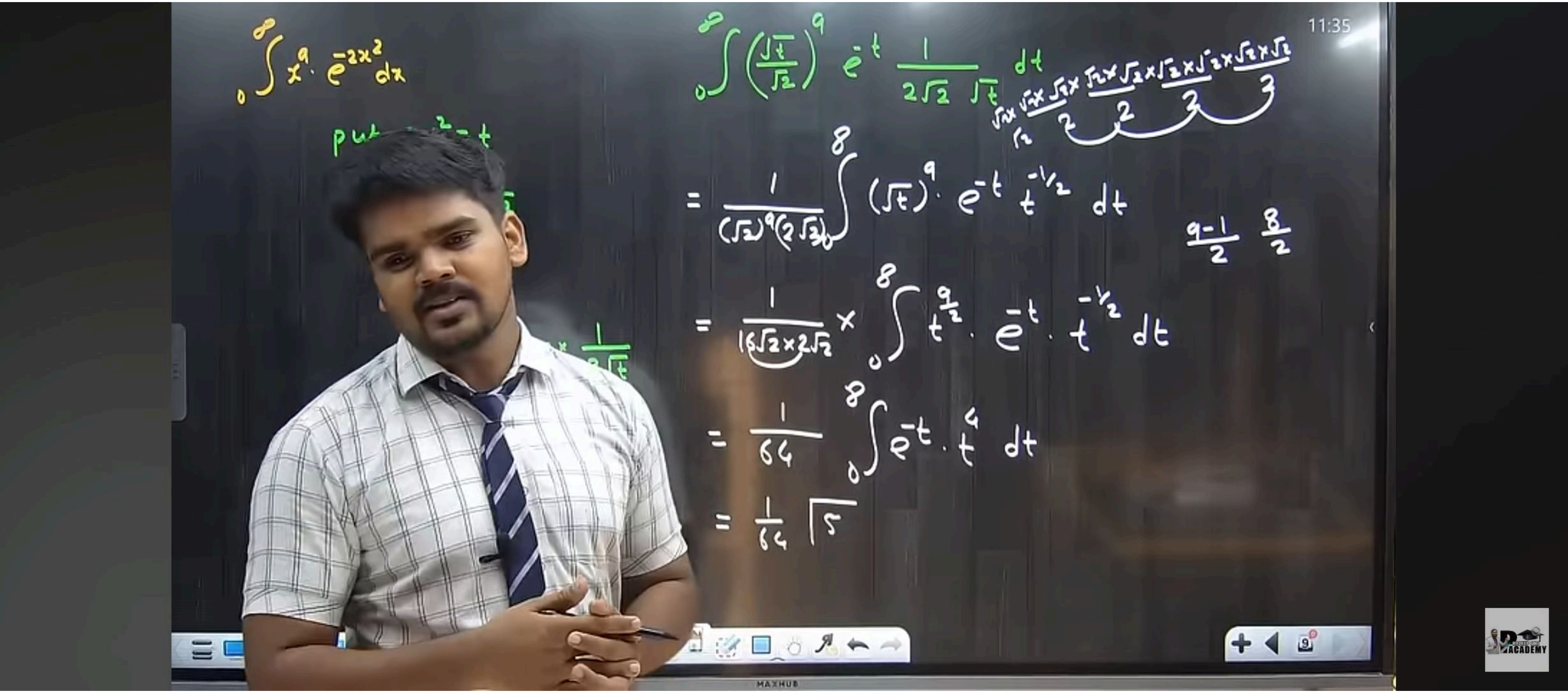
$$\int_0^\infty \left(\frac{\sqrt{t}}{\sqrt{2}}\right)^9 e^{-t}\,\frac{1}{2\sqrt{2}\,\sqrt{t}}\,dt$$

$$\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$$

$$= \frac{1}{(\sqrt{2})^9(2\ldots} \ldots e^{-t}\, t^{-1/2}\,dt$$

$$= \frac{1}{16\ldots} \ldots e^{-t} \cdot t^{-1/2}\,dt$$

ACADEMY



11:35
$\int_0^\infty x^9 \cdot e^{-2x^2} dx$
put
$\int_0^\infty \left(\frac{\sqrt{t}}{\sqrt{2}}\right)^9 e^{-t} \frac{1}{2\sqrt{2}\sqrt{t}} dt$
$= \frac{1}{(\sqrt{2})^9 (2\sqrt{2})} \int_0^\infty (\sqrt{t})^9 \cdot e^{-t} t^{-1/2} dt$
$\frac{9-1}{2}$ $\frac{8}{2}$
$= \frac{1}{16\sqrt{2} \times 2\sqrt{2}} \times \int_0^\infty t^{9/2} \cdot e^{-t} \cdot t^{-1/2} dt$
$= \frac{1}{64} \int_0^\infty e^{-t} \cdot t^4 dt$
$= \frac{1}{64} \Gamma(5)$
MAXHUB
ACADEMY

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}}$$

put $\cos^2\theta = \sqrt{t}$

$$\cos^2\theta = t^{1/2}$$

$$\cos\theta = t^{1/4}$$

$$\theta = \cos^{-1}\left(t^{1/4}\right)$$

$$\frac{d\theta}{dt} = \frac{-1}{\sqrt{1-\left(t^{1/4}\right)^2}} \times \frac{1}{4}\, t^{-3/4}$$

$$\boxed{d\theta = \frac{-1}{\sqrt{1-\sqrt{t}}}\, \frac{1}{4}\, t^{-3/4}}$$

$$\sin^2\theta = 1-\cos^2\theta$$

$$\boxed{\sin^2\theta = 1-\sqrt{t}}$$

18:58

ACADEMY

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}}$$

$\cos^2\theta = \sqrt{t}$

$\cos^2\theta = t^{1/2}$

$\cos\theta = t^{1/4}$

$= \ldots \frac{\ }{(1-\sqrt{t})} \times \frac{-1}{\sqrt{1-\sqrt{t}}} \times t^{-\frac{3}{4}}\,dt$

$\theta = \cos^{-1}\left(t^{1/4}\right)$

$$\frac{d\theta}{dt} = \frac{-1}{\sqrt{1-\left(t^{1/4}\right)^2}} \times \frac{1}{4}\, t^{-\frac{3}{4}}$$

$$\boxed{d\theta = \frac{-1}{\sqrt{1-\sqrt{t}}}\,\frac{1}{4}\, t^{-\frac{3}{4}}\,dt}$$

$\sin^2\theta = 1-\cos^2\theta$

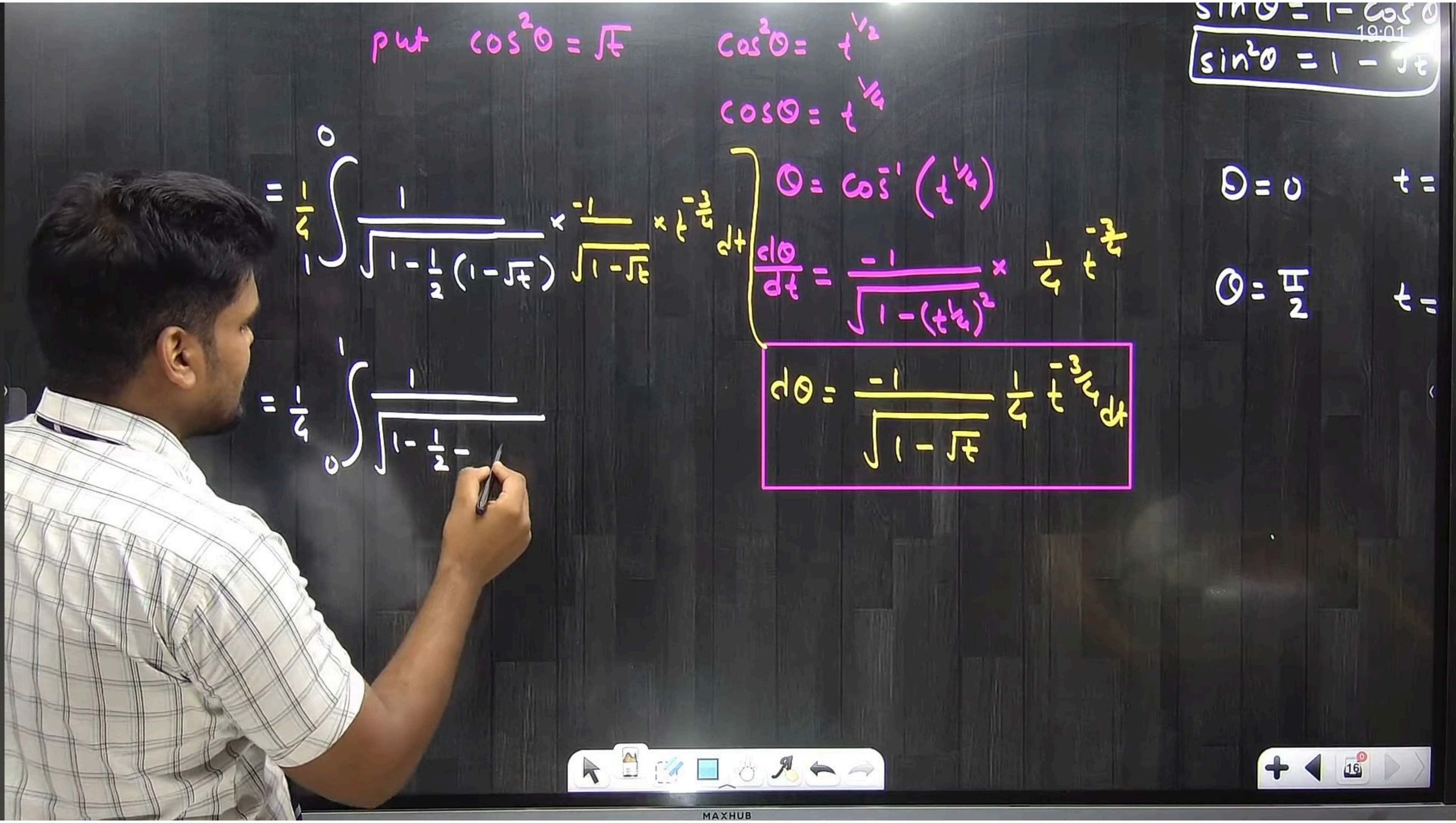
$\boxed{\sin^2\theta = 1-\sqrt{t}}$

$\theta = 0 \quad t = 1$

$\theta = \frac{\pi}{2} \quad t = 0$

19:00

MAXHUB


put cos²θ = √t
cos²θ = t^(1/2)
cosθ = t^(1/4)
sin²θ = 1 − √t
θ = cos⁻¹(t^(1/4))
dθ/dt = −1/√(1 − (t^(1/4))²) × 1/4 t^(−3/4)
dθ = −1/√(1 − √t) · 1/4 t^(−3/4) dt
θ = 0   t =
θ = π/2   t =
= 1/4 ∫ from 1 to 0 of 1/√(1 − 1/2(1 − √t)) × −1/√(1 − √t) × t^(−3/4) dt
= 1/4 ∫ from 0 to 1 of 1/√(1 − 1/2 −
19:01
MAXHUB
ACADEMY

19:03

$$= \frac{1}{4} \int_0^1 \frac{1}{\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{t}}} \times \frac{1}{\sqrt{1-\sqrt{t}}}\, t^{-3/4}\, dt$$

$$= \frac{1}{4} \int_0^1 \frac{1}{\sqrt{\frac{1}{2}(1-\sqrt{t})}} \times \frac{1}{\sqrt{1-\sqrt{t}}} \times t^{-3/4}\, dt$$

$$= \frac{1}{4} \times \int_0^1 \frac{1}{\frac{1}{\sqrt{2}}\sqrt{1-\sqrt{t}}}$$

$$= \frac{\sqrt{2}}{4} \int_0^1 \frac{1}{\sqrt{(1+\sqrt{t})(1-\sqrt{t})}}\, t^{-3/4}\, dt$$

$$= \frac{\sqrt{2}}{4} \times \int_0^1 \frac{1}{\sqrt{1^2-(\sqrt{t})^2}}\, t^{-3/4}\, dt$$

$$= \frac{\sqrt{2}}{4} \times \int_0^1 \frac{\quad}{\quad -t}\, t^{-3/4}\, dt$$

19:05

$$= \frac{\sqrt{2}}{4} \times \int_0^1 \frac{1}{\sqrt{1^2 - (\sqrt{t})^2}} t^{-3/4} \, dt$$

$$= \frac{\sqrt{2}}{4} \times \int_0^1 \frac{1}{\sqrt{1 - t}} t^{-3/4} \, dt$$

$$= \frac{\sqrt{2}}{4} \int_0^1 t^{-3/4} (1-t)^{-1/2} \, dt$$

$$= \frac{\sqrt{2}}{4} \times \beta\left(-\frac{3}{4}+1, -\frac{1}{2}+1\right)$$