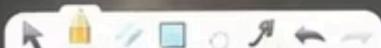


13:19

# Double Integrals

To change the order of Integration

Lecture-02



$y$ -axis

$x$ -axis

$x=0$

$y=0$

$x=5$

$x=5$

parallel to  $y$ -axis

$y=2$

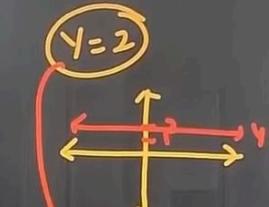
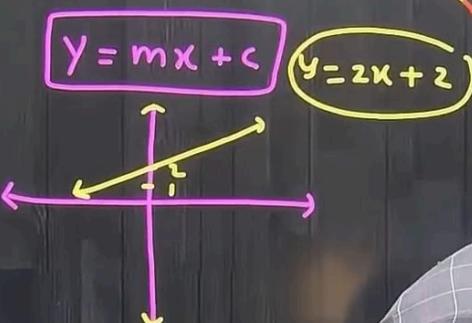
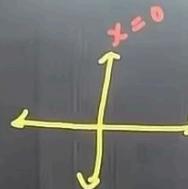
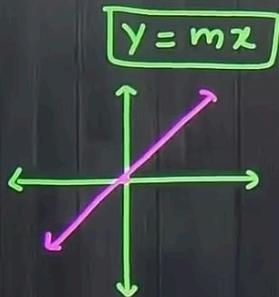
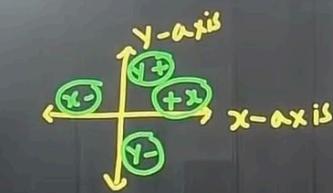
$y=2$

parallel to  $x$ -axis

13:22



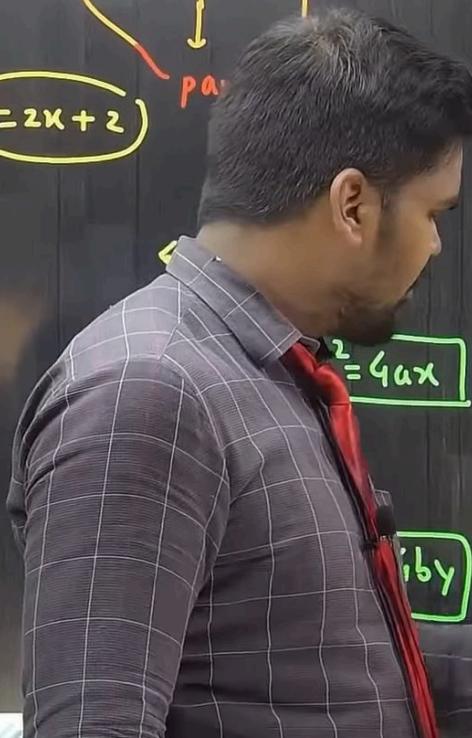
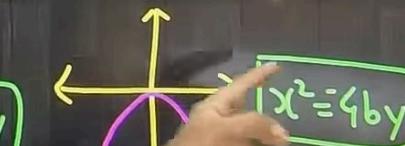
13:26



parallel to x-axis

$$\frac{y^2}{4} = 4ax$$

$$y^2 = -4ax$$



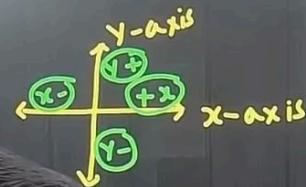
A teacher stands in front of a chalkboard filled with hand-drawn diagrams and equations illustrating different types of Cartesian coordinate systems.

The chalkboard contains the following content:

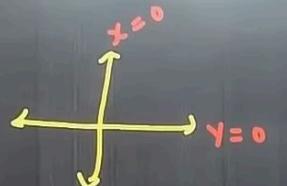
- Top Left:** A diagram showing a coordinate system with a horizontal axis labeled "x-axis" and a vertical axis labeled "y-axis". The origin is marked with a green circle containing a "1". Arrows point from the origin to the positive x and y directions. Labels "x<0" and "y<0" are shown in green.
- Top Middle:** A coordinate system with a horizontal axis labeled "x=0" and a vertical axis labeled "y=0".
- Top Right:** A coordinate system with a horizontal axis labeled "x=5" and a vertical axis labeled "y=5". The x-axis is marked with tick marks at 1, 2, 3, 4, and 5. The y-axis is marked with tick marks at 1, 2, 3, 4, and 5. Labels "parallel to y-axis" and "parallel to x-axis" are written in red.
- Middle Left:** A coordinate system with a diagonal line passing through the origin. The equation  $y = mx + c$  is written above it, and the specific equation  $y = 2x + 2$  is written below it. The axes are labeled "x" and "y".
- Middle Right:** A coordinate system with a parabola opening upwards. The equation  $y^2 = 4ax$  is written in green.
- Bottom Left:** A coordinate system with a parabola opening to the left. The equation  $x^2 = 4by$  is written in green.
- Bottom Right:** A coordinate system with a parabola opening downwards. The equation  $y^2 = -4ax$  is written in green.

The time "13:26" is visible in the top right corner of the chalkboard. A small watermark for "D2 Academy" is in the bottom right corner of the image frame.

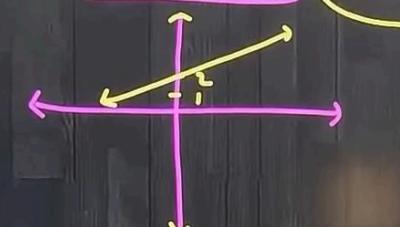
13:28



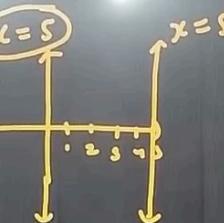
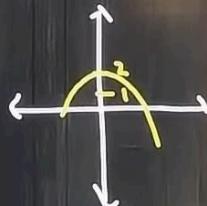
$$y = mx$$



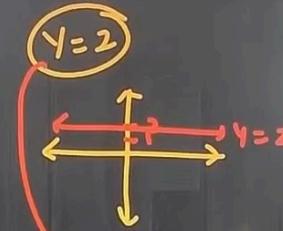
$$y = mx + c$$



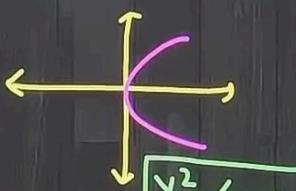
$$x^2 = -y + 2$$



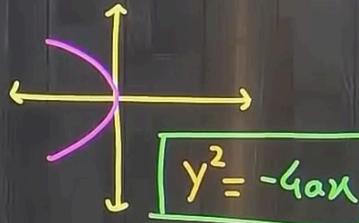
parallel to y-axis



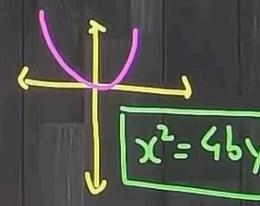
parallel to x-axis



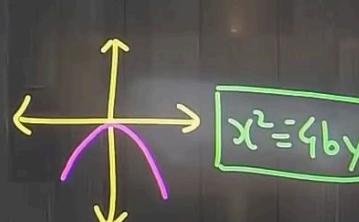
$$y^2 = 4ax$$



$$y^2 = -4ax$$



$$x^2 = 4by$$



$$x^2 = 4by$$

13:29

A man with dark hair and a beard, wearing a grey checkered shirt and a red tie, is pointing towards a chalkboard. On the chalkboard, there is a red circle centered at the origin of a coordinate system with red axes. Two points on the circle are labeled:  $(1, 0)$  and  $(\sqrt{3}, 0)$ . To the right of the circle, several mathematical expressions are written in yellow:

- $r^2$
- $(\sqrt{7})^2$
- $x^2 + y^2 = 1$  (inside a yellow oval)
- $y = \sqrt{1 - x^2}$
- $x = \sqrt{1 - y^2}$

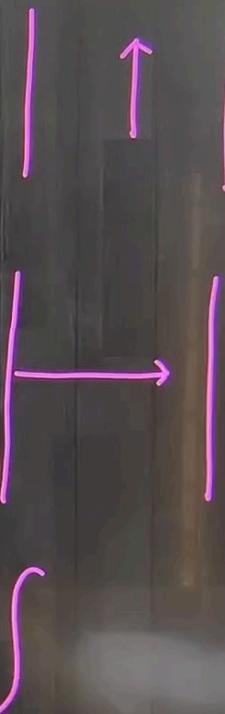
The chalkboard has a dark background with vertical wood paneling. The time "13:29" is in the top right corner. A toolbar with various icons is at the bottom, and a small logo for "Dynamilis ACADEMY" is in the bottom right corner.

13:31



13:31

$$\begin{aligned}x &= s \\y &= 0 \\y &= \sqrt{1-x^2}\end{aligned}$$

 $dy$  $dy$ 

change the order of integration and evaluate the integral.

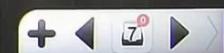
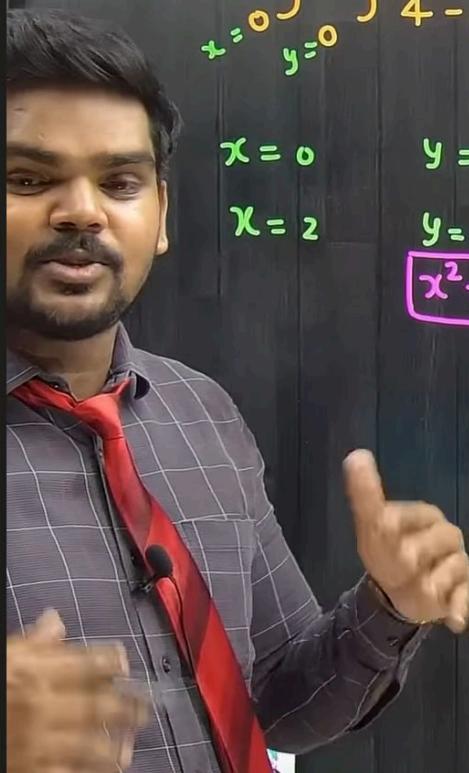
13:33

$$\int_{x=0}^{x=2} \int_{y=0}^{y=4-x^2} \frac{x \cdot e^{2y}}{4-y} dy dx$$

$$x = 0 \quad y = 0$$

$$x = 2 \quad y = 4 - x^2$$

$$x^2 = -y + 4$$



13:40

change the order of integration and evaluate the integral.

$$\int_{-2}^2 \int_{y=4-x^2}^{y=4} \frac{x \cdot e^{2y}}{4-y} dy dx$$

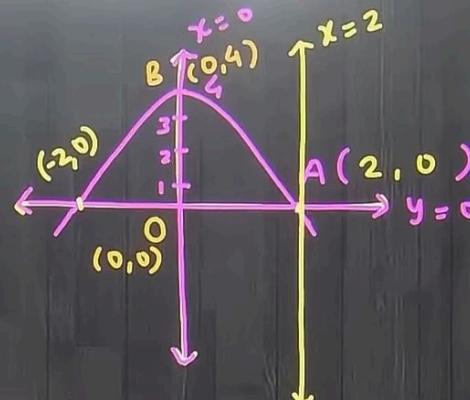
$$x = 0$$

$$y = 0$$

$$x = 2$$

$$y = 4 - x^2$$

$$x^2 = -y + 4$$



To solve Find co-ordinates of A  $\equiv ( )$

$$x^2 = -y + 4$$

$$y = 0$$

$$x^2 = -0 + 4$$

$$x^2 = 4 \quad x = \sqrt{4}$$

$$(x = \pm 2)$$

$$x^2 = -y + 4$$

$$y = 0$$

$$x^2 = -0 + 4$$

$$x^2 = 4$$

$$x = \pm 2$$

13:41

change the order of integration and evaluate the integral.

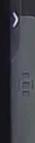
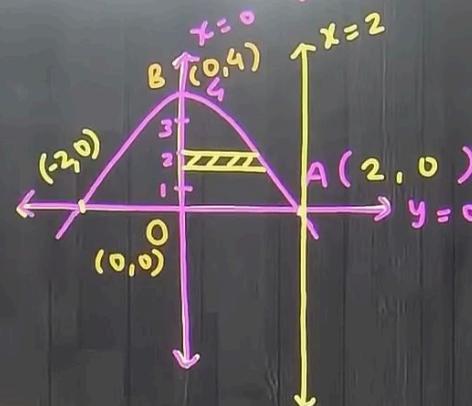
$$x=0 \int_{y=0}^{y=4-x^2} \int_{x=0}^{x=2} \frac{x \cdot e^{2y}}{4-y} dy dx$$

$$\boxed{x=0}$$

$$\boxed{x=2}$$

$$\boxed{y}$$

$$\boxed{y}$$



Because limit me  $4-x^2$   
hai isliye strip will be  
parallel to x

Agar y ka term aata toh  
parallel to y

change the order of integration and evaluate the integral.

$$x=2 \int_{y=0}^{y=4-x^2} \int \frac{x \cdot e^{2y}}{4-y} dy dx$$

$$y = 0 \quad \checkmark$$

$$y = 4 - x^2$$

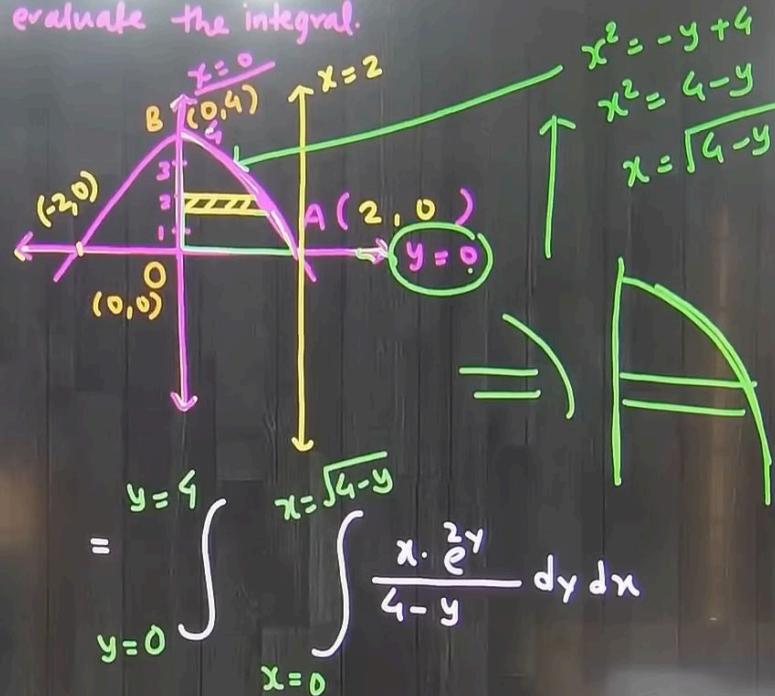
$$x^2 = -y + 4 \quad \checkmark$$

(order of integration of A = ( ))

$$x^2 = -y + 4$$

$$y = 0$$

$x=2$  ↗



13:45

To solve find co-ordinates of A = ( )

$$x^2 = -y + 4$$

$$Y=0$$

$$x^2 = -0 + 4$$

$$x^2 = 4$$

$$x^2 = -y + 4$$

$$y=0$$

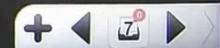
$$= \int_{y=0}^{4-y} \int_{x=0}^{x^2} \frac{x \cdot e^{2y}}{4-y} dy dx$$

13:48

$$= \int_0^4 \left[ \int_0^{\sqrt{4-y}} \frac{x \cdot e^{2y}}{4-y} dx \right] dy$$

$$= \int_0^4 \left[ \frac{e^{2y}}{4-y} \cdot \frac{x^2}{2} \right]_0^{\sqrt{4-y}} dy$$

$$= \int_0^4 \left[ \frac{e^{2y}}{4-y} \cdot \frac{(\sqrt{4-y})^2}{2} \right] dy$$



13:52

$$\begin{aligned}
 &= \frac{1}{2} \left[ \frac{e^{2y}}{2} \right]_0^4 \\
 &= \frac{1}{4} [e^{2y}]_0^4 \\
 &= \frac{1}{4} [e^{2 \cdot 4} - e^{2 \cdot 0}] \\
 &= \frac{1}{4} [e^8 - 1]
 \end{aligned}$$

$$\begin{aligned}
 &\int_0^4 \left[ \frac{e^{2y}}{4-y} \cdot \frac{x^2}{2} \right] dy \\
 &= \int_0^4 \left[ \frac{e^{2y}}{4-y} \cdot \frac{(\sqrt{4-y})^2}{2} \right] dy \\
 &= \int_0^4 \left[ \frac{e^{2y}}{4-y} \cdot \frac{(4-y)}{2} \right] dy \\
 &= \frac{1}{2} \int_0^4 e^{2y} dy
 \end{aligned}$$



13:56

$$\int_{y=0}^{1} \int_{x=4y}^{4} e^{x^2} dx dy$$

$$y = 0 \quad \boxed{x = 4y}$$

$$y = 1 \quad x = 4$$

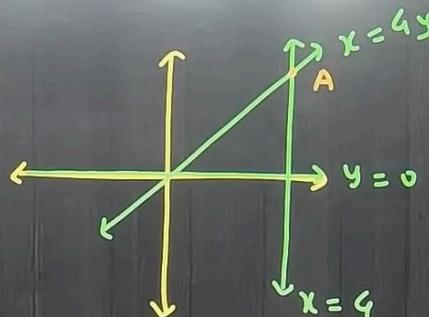
$$\text{Find } A \equiv (4, 1)$$

$$\boxed{x = 4} \text{ & } x = 4y$$

$$4 = 4y$$

$$\frac{4}{4} = y$$

$$\boxed{1 = y}$$



$$y=0 \int_0^1 \int_0^{4y} e^{x^2} dx dy$$

$$y=0$$

$$y=1$$

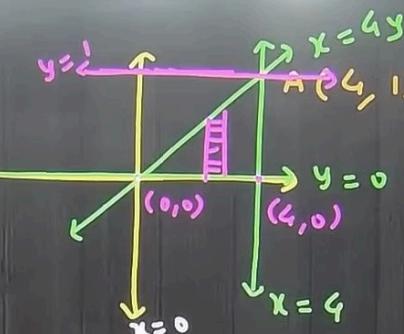
$$\int_0^1 \int_0^{4y} e^{x^2} dx dy$$

$$x=0 \quad x=4y$$

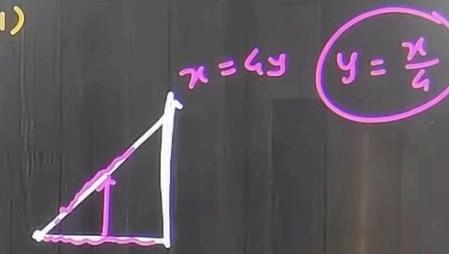
$$x=0 \quad x=4y$$

$$x=0 \quad y=0$$

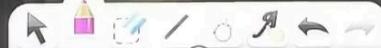
$$x=0 \quad y=0$$



$$x=0 \int_0^4 y = \frac{x}{4}$$



13:59



$$y = x = \frac{xy}{4}$$

$$y = 0$$

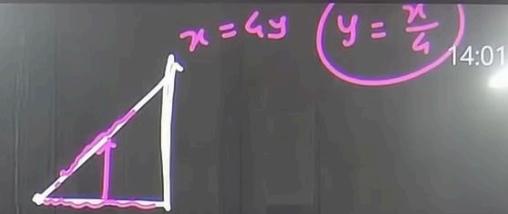
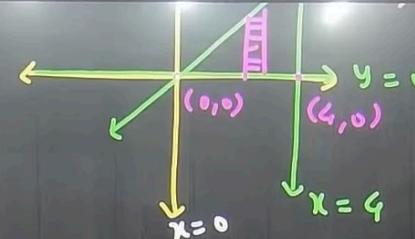
$$x = 4y$$

$$y = 1$$

$$x = 4$$

Find

$$x = 4$$

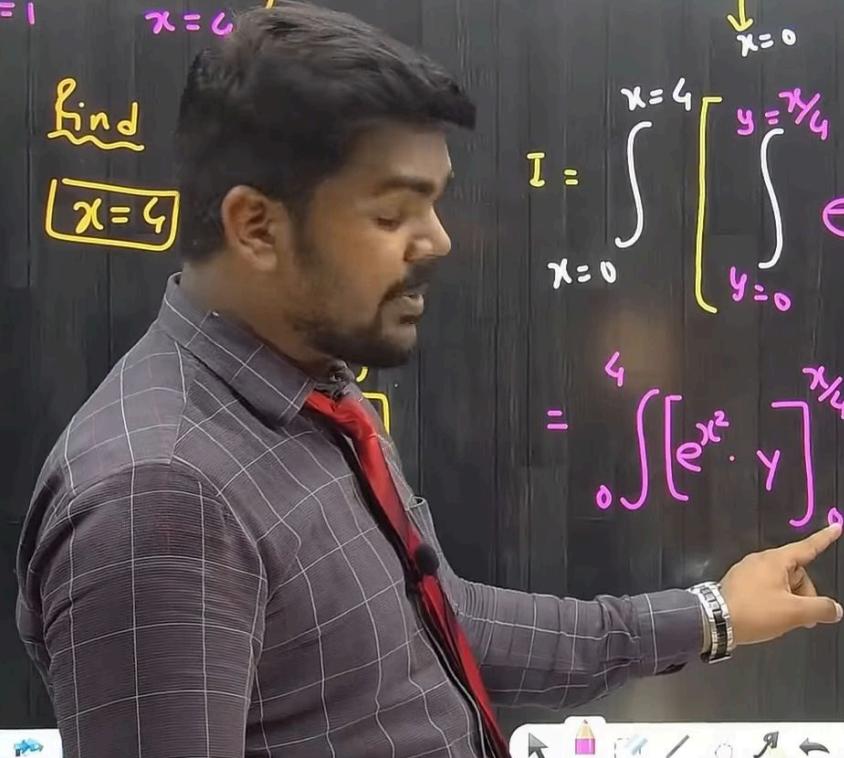


14:01

$$I = \int_{x=0}^{x=4} \left[ \int_{y=0}^{y=\frac{x}{4}} e^{x^2} dy \right] dx$$

$$= \int_0^4 \left[ e^{x^2} \cdot y \right]_0^{\frac{x}{4}} dx$$

$$= \int_0^4 \left[ e^{x^2} \cdot \frac{x}{4} - \right]$$



$x^2 + y^2 = 16$

$$y = 4x$$

$$\frac{y}{x} = 4$$

$$\sqrt{1+16x^2}$$

$$x=0 \quad \left[ y=0 \right] \quad \text{---} \quad \text{---}$$

14:03

$$= \int_0^4 \left[ e^{x^2} \cdot y \right]_{0}^{x/4} dx = \int_0^4 \left[ e^{x^2} \cdot \frac{x}{4} \right] dx$$

$$= \frac{1}{4} \int_0^4 e^{x^2} \cdot x dx$$

$$\text{put } x^2 = t$$

$$2x = \frac{dt}{dx}$$

$$x dx = \frac{dt}{2}$$

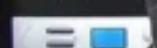
$$x=0$$

$$t=x^2 \quad t=0$$

$$x=4$$

$$t=x^2 \quad t=16$$

$$(t=x^2)$$



$$\int | = y$$

$$= \int_0^4 \left[ e^{x^2} \cdot y \right]_{\frac{x}{4}}^{x^2} dx = \int_0^4 \left[ e^{x^2} \cdot \frac{x}{4} \right] dx$$

$$= \frac{1}{4} \int_0^{16} e^t \cdot \frac{dt}{2}$$

$$= \frac{1}{8} \int_0^{16} e^t dt$$

$$= \frac{1}{8} \left[ e^t \right]_0^{16}$$

$$= \frac{1}{8} [e^{16} - e^0] = \frac{1}{8} [e^{16} - 1]$$

$$= \frac{1}{4} \int_0^4 e^t dt$$

$$t = x^2$$



14:07

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y + 1) \sqrt{1-x^2-y^2}} dy dx$$

$$x = 0 \quad y = 0$$

$$x = 1 \quad y = \sqrt{1-x^2}$$

$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$



$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y + 1) \sqrt{1-x^2-y^2}} dy dx$$

$$x=0 \checkmark \quad y=0 \checkmark$$

$$x=1 \checkmark \quad y=\sqrt{1-x^2}$$

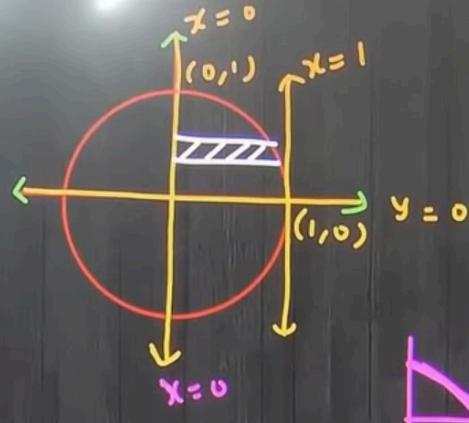
$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$

$$x^2 = 1 - y^2$$

$$x = \sqrt{1-y^2}$$

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y + 1) \sqrt{1-x^2-y^2}} dy dx$$



14:10

$$x=0 \int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{e^y}{(e^y+1) \sqrt{1-x^2-y^2}} dy dx$$

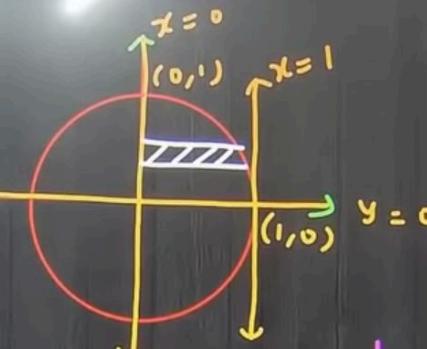
$$Y = 0$$

$$Y = \sqrt{1-x^2}$$

$$Y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$

$$= \int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1) \sqrt{1-x^2-y^2}} dy dx$$



14:11

14:14

$$x=0 \checkmark \quad y=0 \checkmark$$

$$x=1$$

$$y=0$$

$$y=\sqrt{1-x^2}$$

$$y^2=1-x^2$$

$$x^2$$

$$x$$

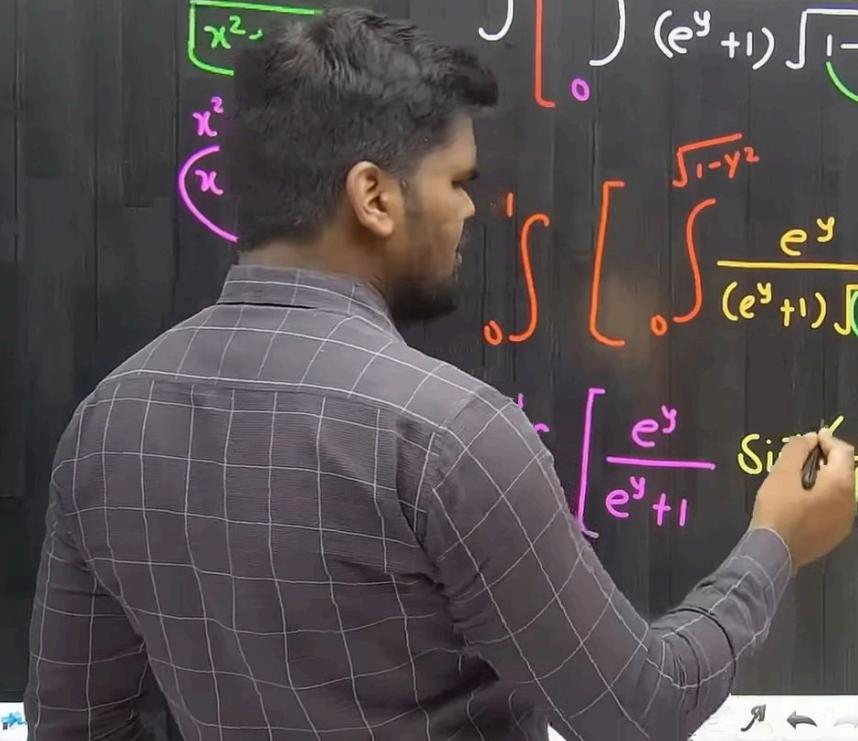
$$= \int_0^1 \left[ \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1) \sqrt{1-x^2-y^2}} dx \right] dy$$

$$= \int_0^1 \left[ \int_0^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1) \sqrt{(\sqrt{1-y^2})^2-x^2}} dx \right] dy$$

$$= \int_0^1 \left[ \frac{e^y}{e^y+1} \sin^{-1} \left( \frac{x}{\sqrt{1-y^2}} \right) \right]_{0}^{\sqrt{1-y^2}} dy$$



$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \left( \frac{x}{a} \right)$$



$$x^2 + y^2 = 1$$

$$x^2 = 1 - y^2$$

$$x = \sqrt{1 - y^2}$$

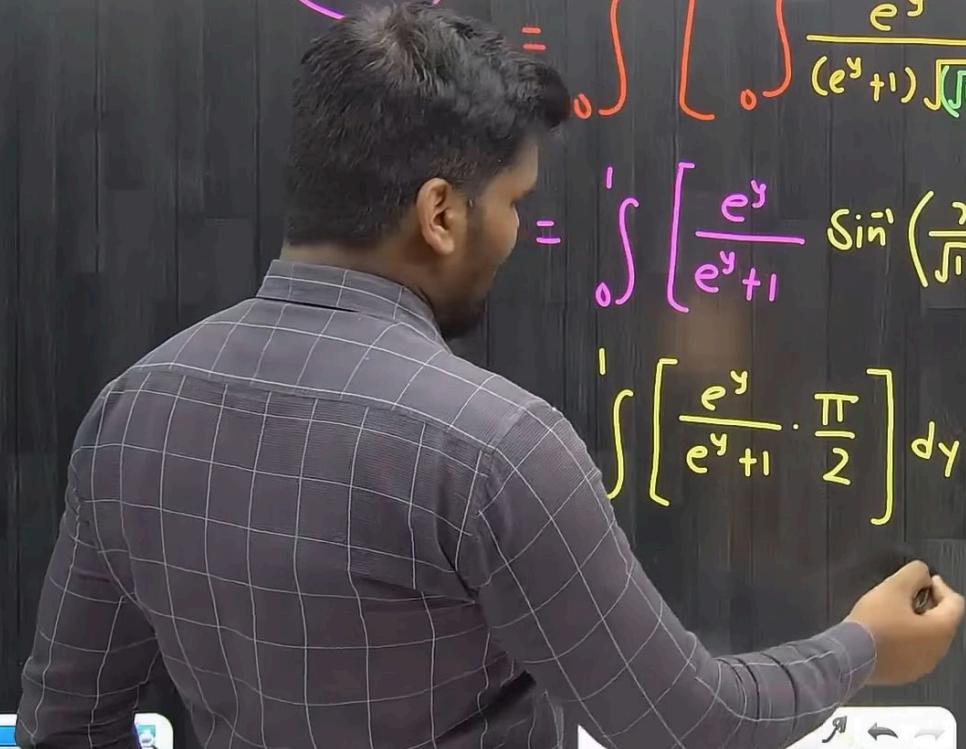
$$= \int_0^\pi \left[ \int_0^{(e^y+1)} \frac{(e^y+1)}{\sqrt{1-x^2}} dx \right] dy$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right)$$

$$= \int_0^\pi \left[ \int_0^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1)\sqrt{(1-y^2)^2 - x^2}} dx \right] dy$$

$$= \int_0^\pi \left[ \frac{e^y}{e^y+1} \sin^{-1}\left(\frac{x}{\sqrt{1-y^2}}\right) \right]_0^{\sqrt{1-y^2}} dy$$

$$= \int_0^\pi \left[ \frac{e^y}{e^y+1} \cdot \frac{\pi}{2} \right] dy$$



$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) \quad 14:17$$

$$= \frac{\pi}{2} \int_0^1 \frac{e^y}{e^y + 1} dy$$

$$= \frac{\pi}{2} \left[ \ln(e^y + 1) \right]_0^1$$

$$= \frac{\pi}{2} \left[ \ln(e^1 + 1) - \ln(e^0 + 1) \right]$$

$$= \frac{\pi}{2} \left[ \ln(e+1) - \ln(\underline{e} + \underline{1}) \right]$$

$$\log m - \log n = \log \left( \frac{m}{n} \right)$$

$$= \frac{\pi}{2} \log \left( \frac{e+1}{2} \right)$$

