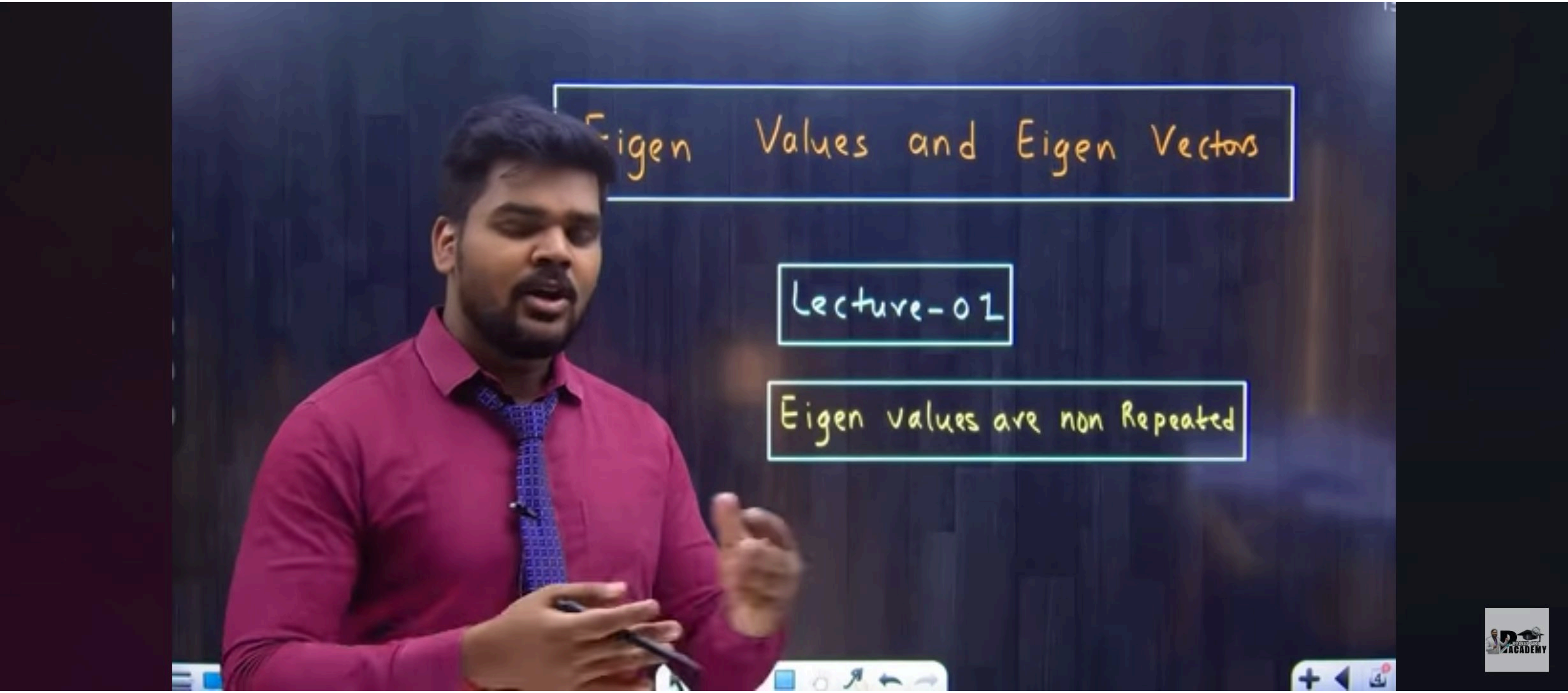
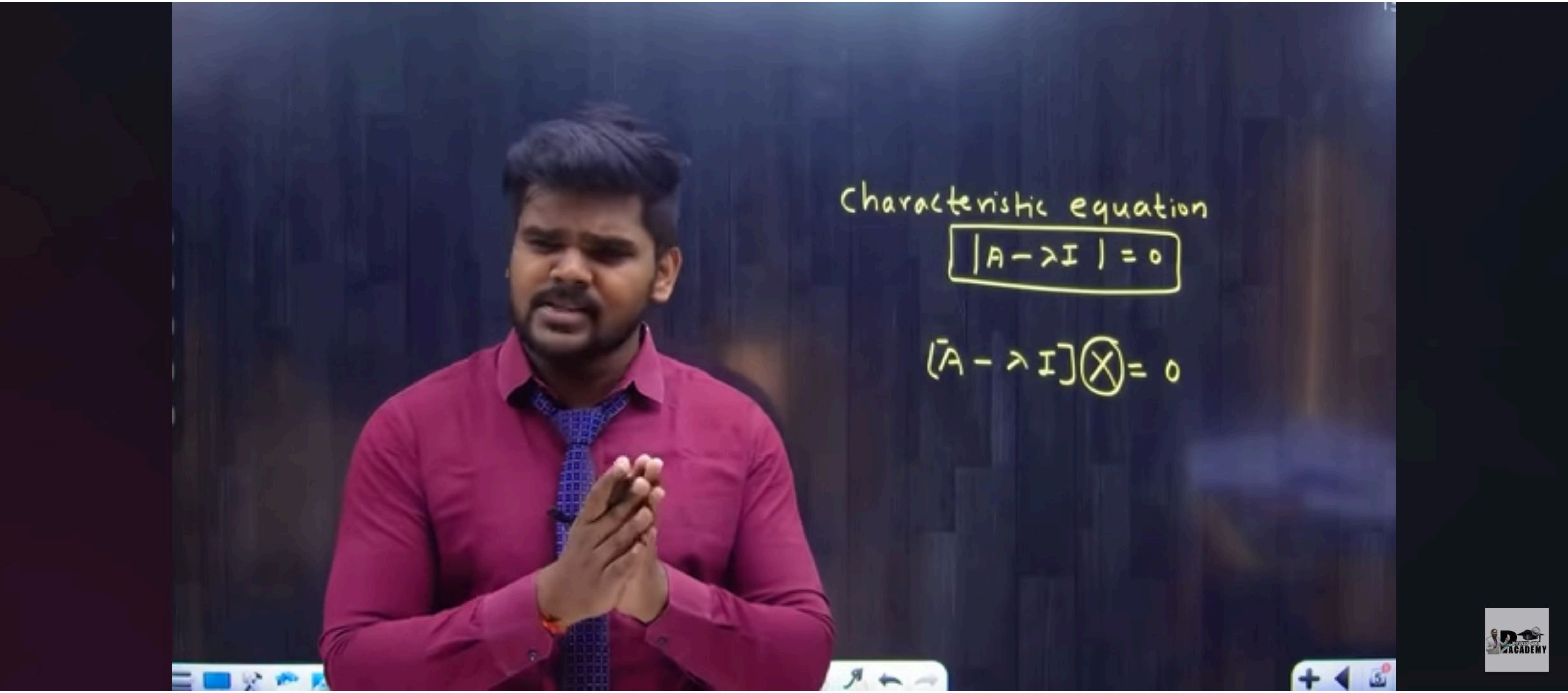

Eigen Values and Eigen Vectors
Lecture-02
Eigen values are non Repeated
ACADEMY


Characteristic equation
|A − λI| = 0
[A − λI] X = 0

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 6 & 9 & 1 \\ 5 & 4 & 3 \end{bmatrix}$$

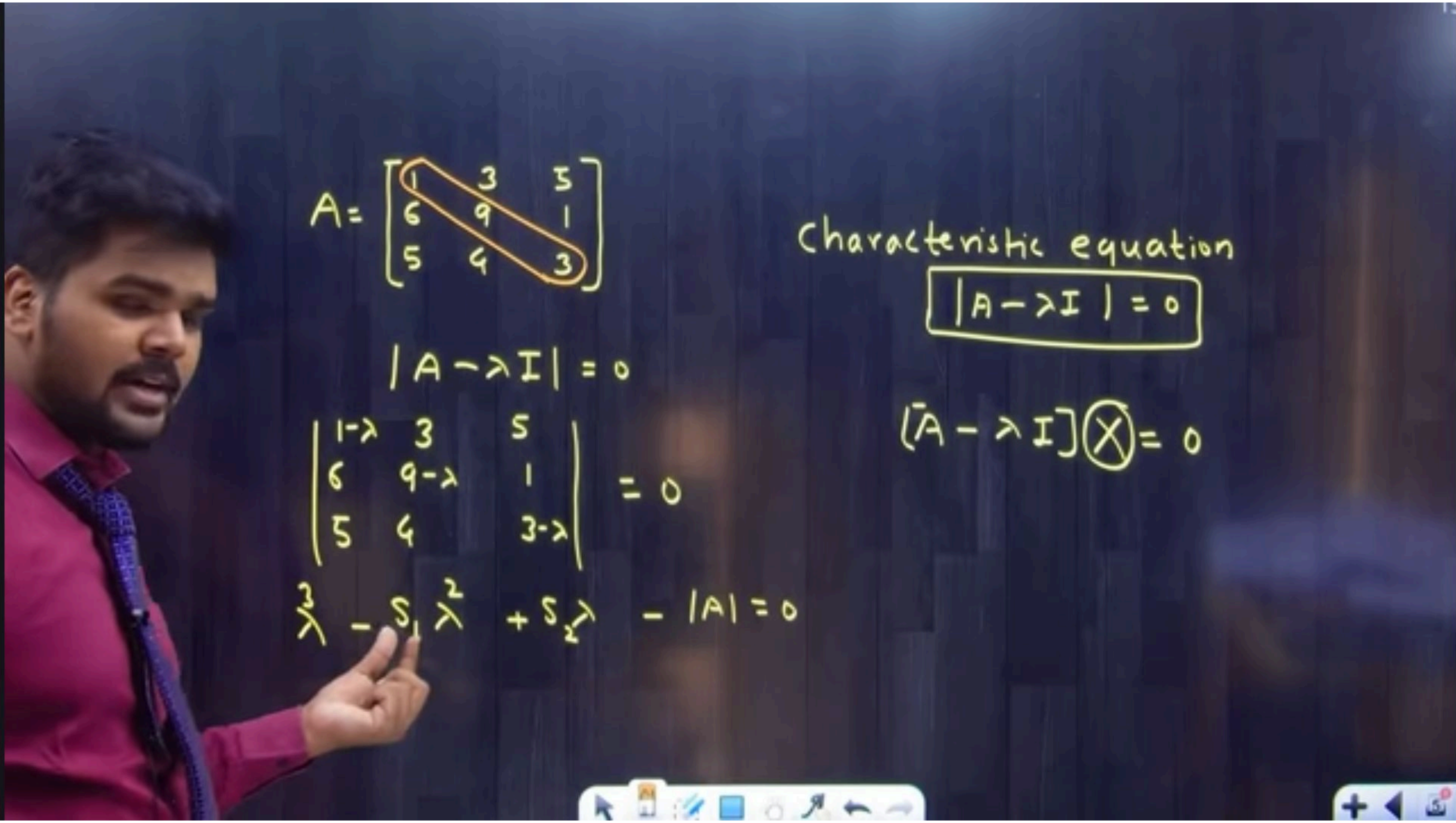
$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 3 & 5 \\ 6 & 9-\lambda & 1 \\ 5 & 4 & 3-\lambda \end{vmatrix} = 0$$

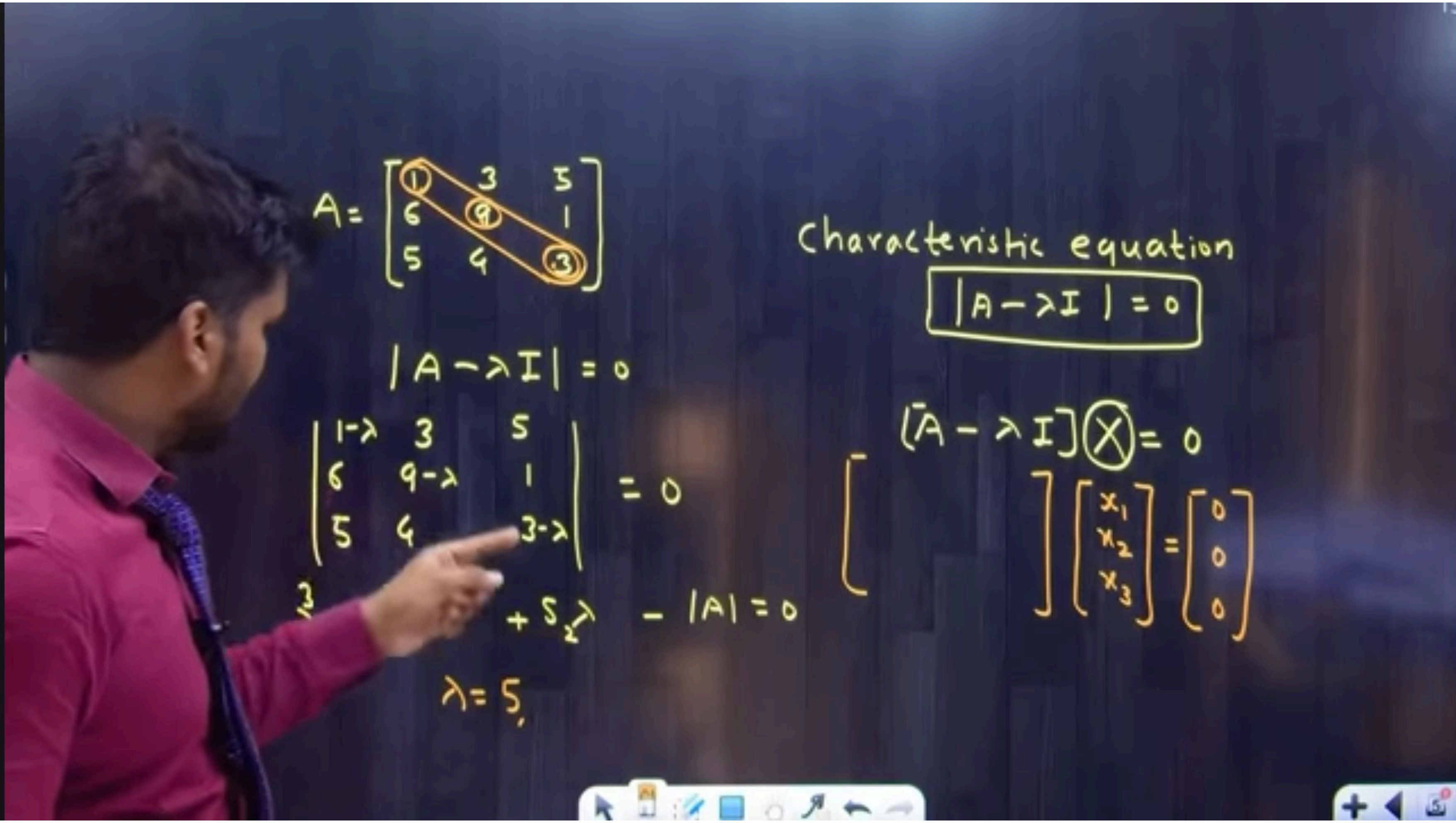
Characteristic equation

$$\boxed{|A - \lambda I| = 0}$$

$$[A - \lambda I] X = 0$$


$A = \begin{bmatrix} 1 & 3 & 5 \\ 6 & 9 & 1 \\ 5 & 4 & 3 \end{bmatrix}$
$|A - \lambda I| = 0$
$\begin{vmatrix} 1-\lambda & 3 & 5 \\ 6 & 9-\lambda & 1 \\ 5 & 4 & 3-\lambda \end{vmatrix} = 0$
$\lambda^3 - S_1 \lambda^2 + S_2 \lambda - |A| = 0$
Characteristic equation
$\boxed{|A - \lambda I| = 0}$
$[A - \lambda I] X = 0$
ACADEMY

S1 - all the positive
diagonal elements
S2 - the minors of
diagonal elements
(+ve)



$A = \begin{bmatrix} 1 & 3 & 5 \\ 6 & 9 & 1 \\ 5 & 4 & 3 \end{bmatrix}$
Characteristic equation
$\boxed{|A - \lambda I| = 0}$
$|A - \lambda I| = 0$
$\begin{vmatrix} 1-\lambda & 3 & 5 \\ 6 & 9-\lambda & 1 \\ 5 & 4 & 3-\lambda \end{vmatrix} = 0$
$[A - \lambda I] X = 0$
$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
$+ S_2 \lambda - |A| = 0$
$\lambda = 5$
ACADEMY

Example 2 : Find the eigenvalues and eigenvectors of the following matrix.

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

Characteristic equation.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$

xample 2 : Find the eigenvalues and eigenvectors of the following matrix.

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \boxed{8} & -8 & -2 \\ 4 & -3 & -2 \\ 3 & & 1 \end{bmatrix}$$

Character equation.

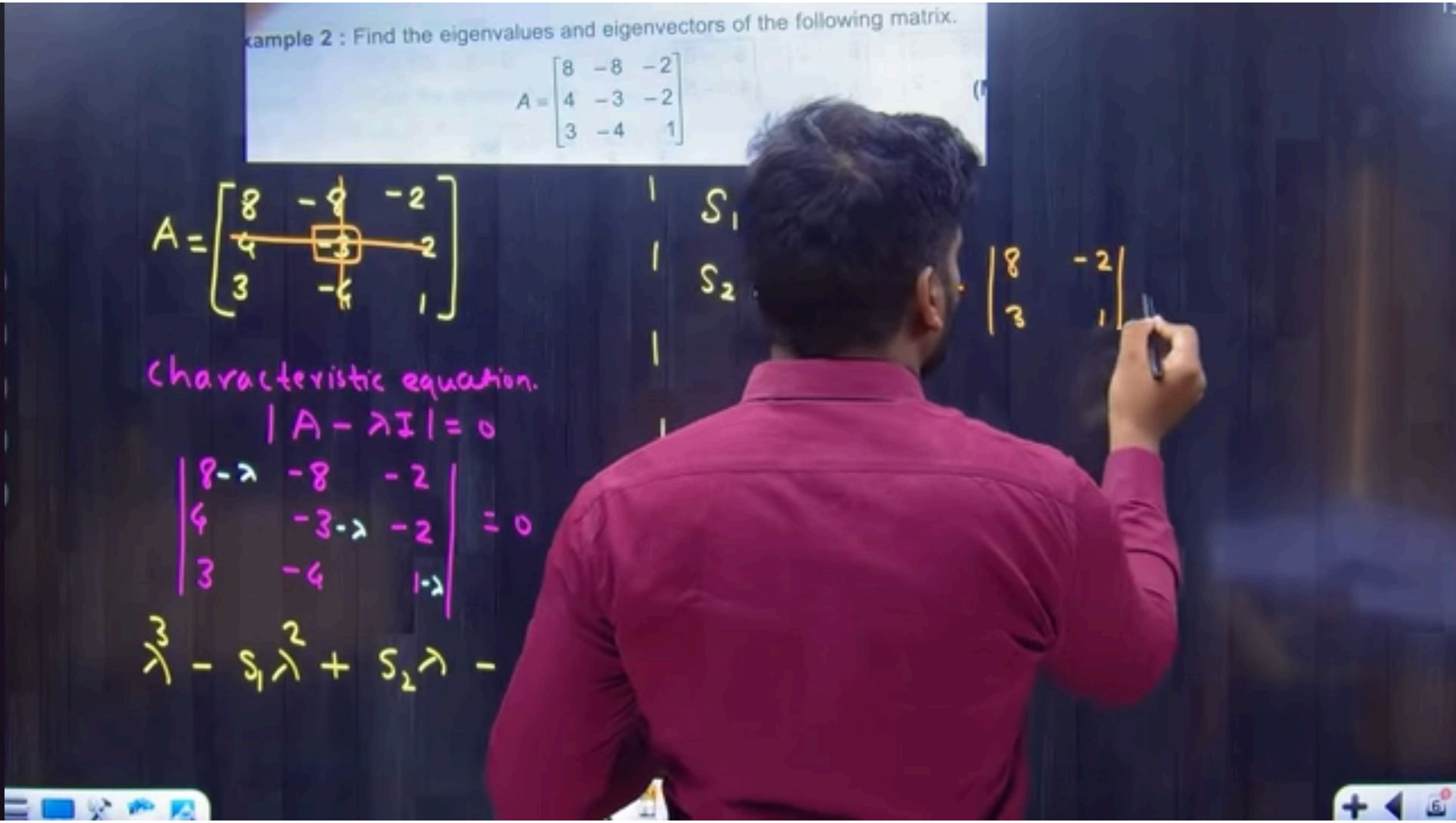
$$|A - \lambda I| = 0$$

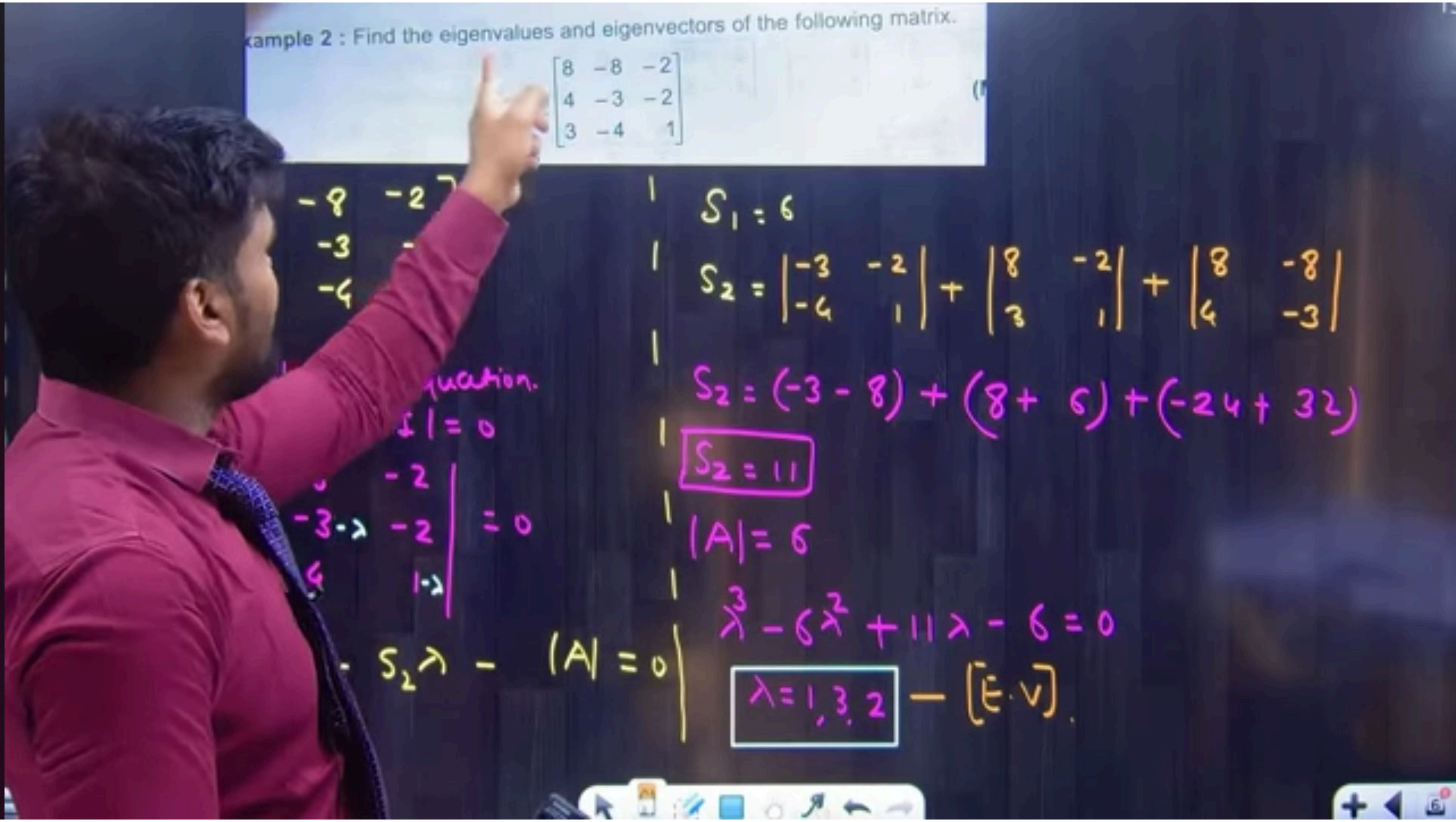
$$\begin{vmatrix} 8-\lambda & - & -2 \\ & & -2 \\ & & 1-\lambda \end{vmatrix} = 0$$

$$\lambda - S_1\lambda + S_2\lambda - |A| = 0$$

$S_1 = 6$

$$S_2 = \begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix} + \begin{vmatrix} & \\ & \end{vmatrix}$$


xample 2 : Find the eigenvalues and eigenvectors of the following matrix.
A = [8 −8 −2; 4 −3 −2; 3 −4 1]
A = [8 −8 −2; 4 −3 −2; 3 −4 1]
characteristic equation.
|A − λI| = 0
|8−λ −8 −2; 4 −3−λ −2; 3 −4 1−λ| = 0
λ³ − S₁λ² + S₂λ −
S₁
S₂
|8 −2; 3 1|
ACADEMY


xample 2 : Find the eigenvalues and eigenvectors of the following matrix.
8 −8 −2
4 −3 −2
3 −4 1
S1 = 6
S2 = |−3 −2; −4 1| + |8 −2; 3 1| + |8 −8; 4 −3|
S2 = (−3 − 8) + (8 + 6) + (−24 + 32)
S2 = 11
|A| = 6
λ³ − 6λ² + 11λ − 6 = 0
λ = 1, 3, 2 — [E.V]
uation.
S2λ − |A| = 0
ACADEMY

For $\lambda = 1$

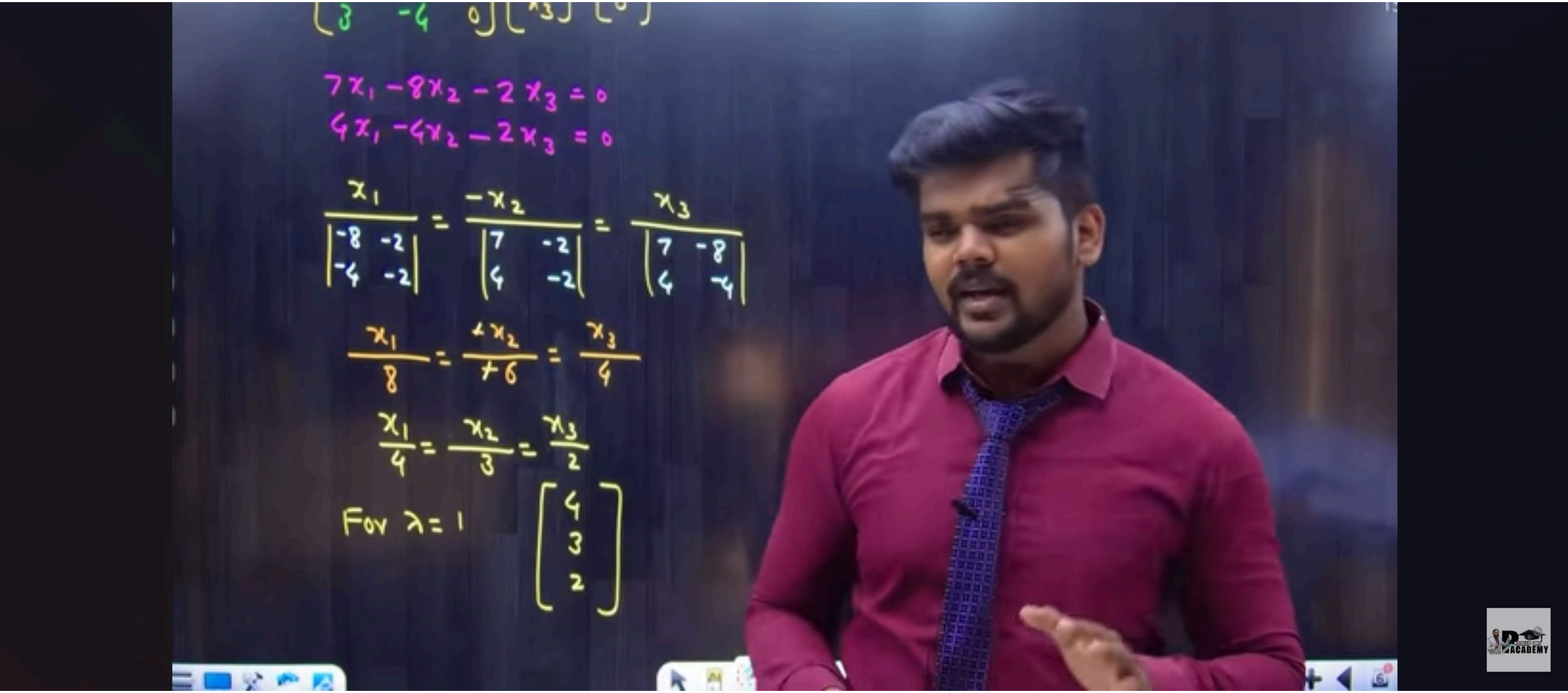
$$[A - \lambda I]X = 0$$

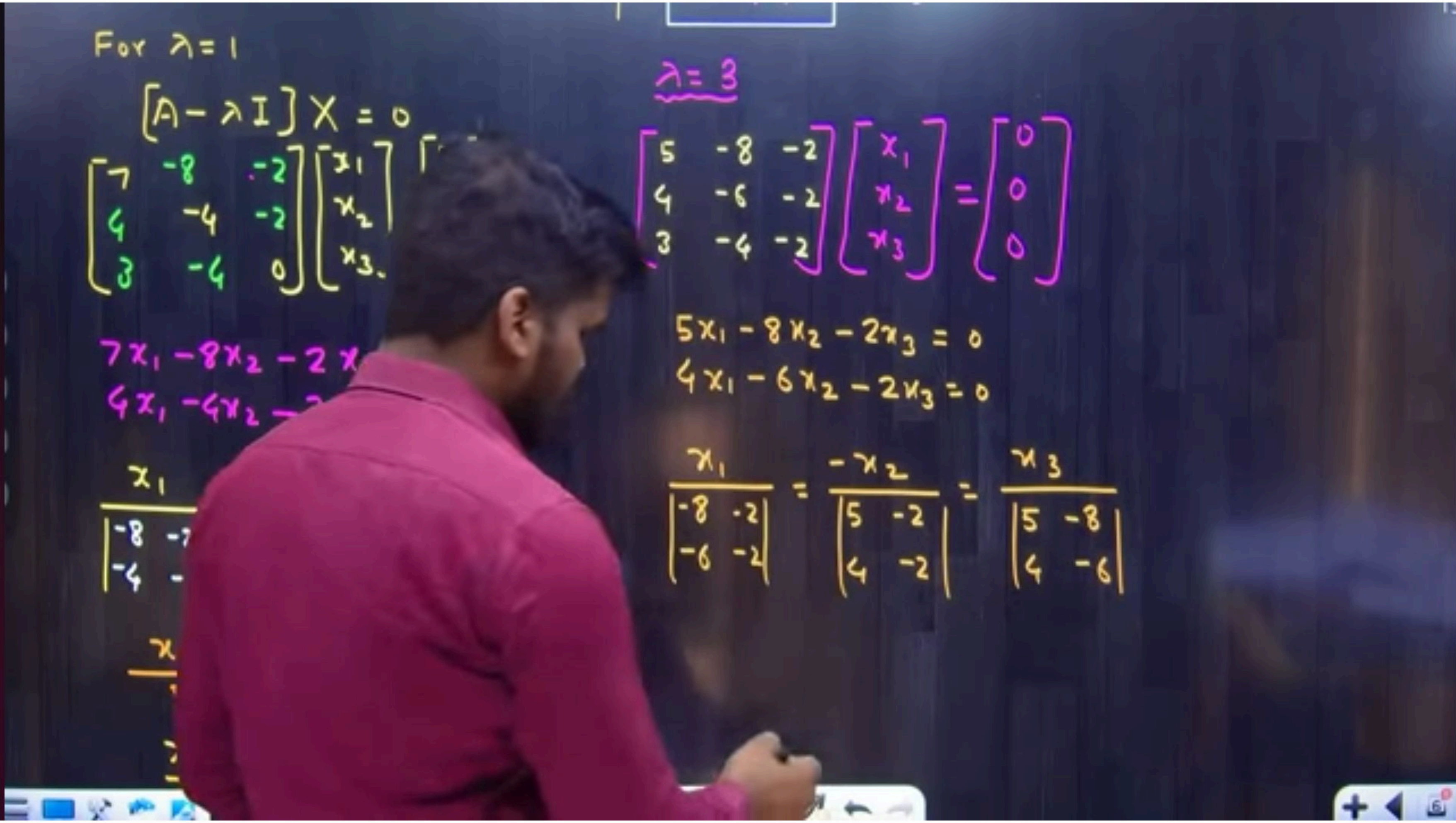
$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$7x_1 - 8x_2 - 2x_3 = 0$$

$$4x_1 - 4x_2 - 2x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$$


$7x_1 - 8x_2 - 2x_3 = 0$
$4x_1 - 4x_2 - 2x_3 = 0$
$\frac{x_1}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$
$\frac{x_1}{8} = \frac{x_2}{+6} = \frac{x_3}{4}$
$\frac{x_1}{4} = \frac{x_2}{3} = \frac{x_3}{2}$
For $\lambda = 1$ $\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$


For λ=1
[A−λI] X = 0
λ=3
5 −8 −2; 4 −6 −2; 3 −4 −2 · x1, x2, x3 = 0, 0, 0
5x1 − 8x2 − 2x3 = 0
4x1 − 6x2 − 2x3 = 0
x1 / |−8 −2; −6 −2| = −x2 / |5 −2; 4 −2| = x3 / |5 −8; 4 −6|
ACADEMY

$$7x_1 - 8x_2 - 2x_3 = 0$$
$$4x_1 - 4x_2 - 2x \ldots = 0$$

$$\frac{x_1}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \ldots$$

$$\frac{x_1}{8} = \ldots$$

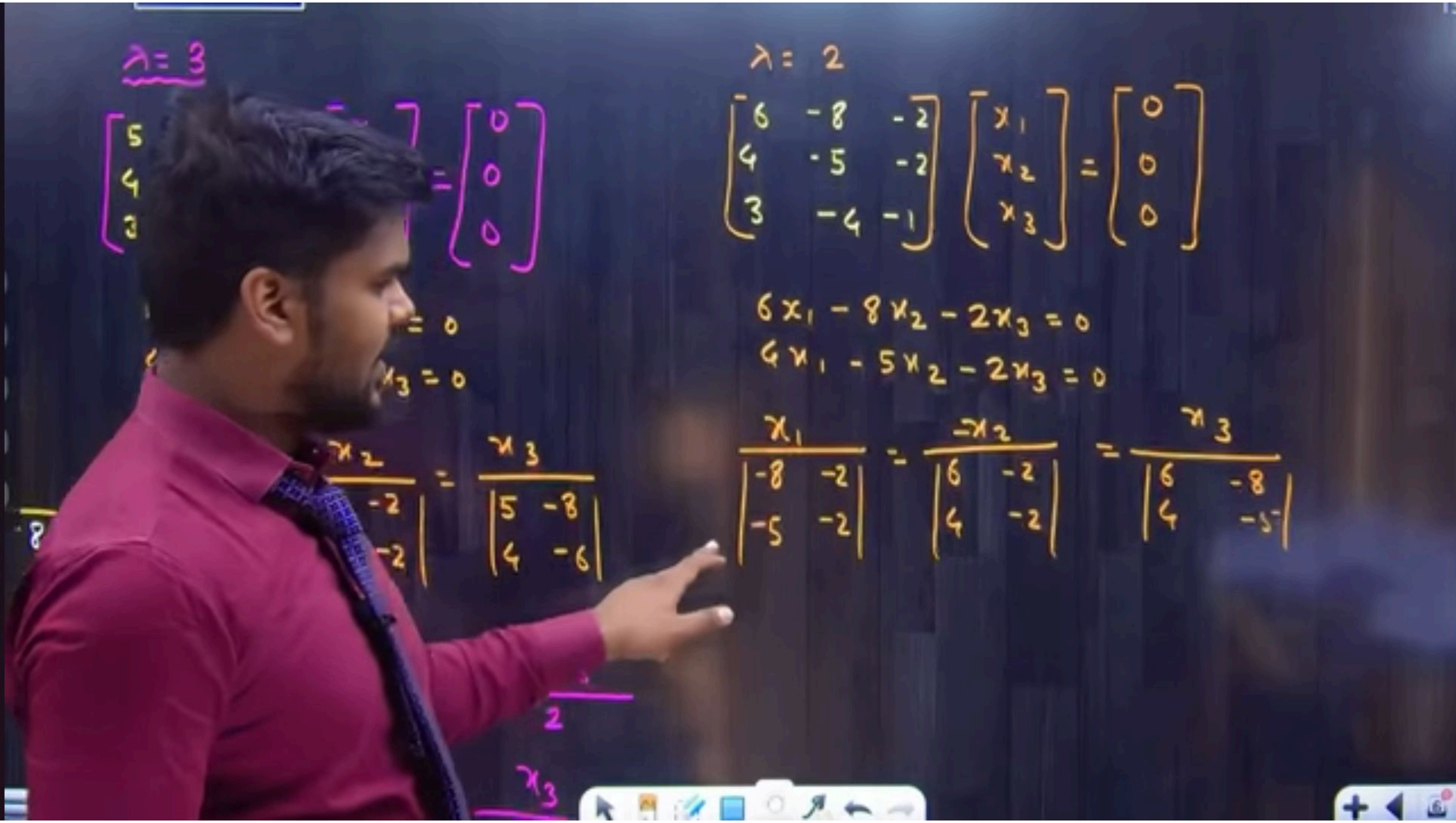
$$5x_1 - 8x_2 - 2x_3 = 0$$
$$4x_1 - 6x_2 - 2x_3 = 0$$

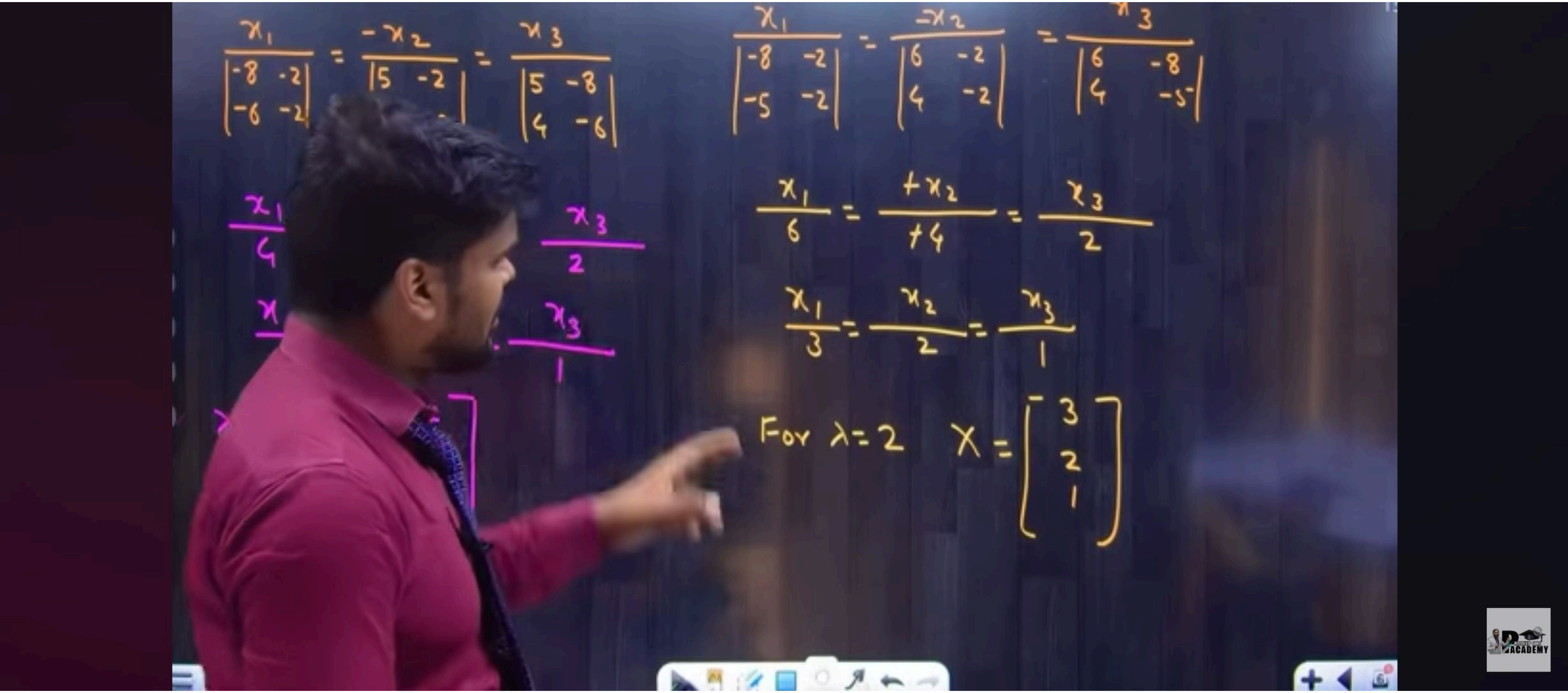
$$\frac{x_1}{\begin{vmatrix} -8 & -2 \\ -6 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -8 \\ 4 & -6 \end{vmatrix}}$$

$$\frac{x_1}{4} = \frac{\cancel{-}x_2}{\cancel{-}2} = \frac{x_3}{2}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{1}$$

For $\lambda = 3$ $\quad X = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$


λ = 3
λ = 2
6 -8 -2
4 -5 -2
3 -4 -1
x1
x2
x3
0
0
0
6x1 - 8x2 - 2x3 = 0
4x1 - 5x2 - 2x3 = 0
x1 / |-8 -2; -5 -2| = -x2 / |6 -2; 4 -2| = x3 / |6 -8; 4 -5|



$\frac{x_1}{\begin{vmatrix}-8 & -2\\ -5 & -2\end{vmatrix}} = \frac{-x_2}{\begin{vmatrix}6 & -2\\ 4 & -2\end{vmatrix}} = \frac{x_3}{\begin{vmatrix}6 & -8\\ 4 & -5\end{vmatrix}}$
$\frac{x_1}{6} = \frac{+x_2}{+4} = \frac{x_3}{2}$
$\frac{x_1}{3} = \frac{x_2}{2} = \frac{x_3}{1}$
For $\lambda = 2$ $X = \begin{bmatrix}3\\2\\1\end{bmatrix}$
ACADEMY

Find the eigenvalues and eigenvectors of the following matrix.

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

characteristic equation.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - S_1 \lambda^2 + S_2 \lambda - |A| = 0$$

Find the eigenvalues and eigenvectors of the following matrix.

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

characteristic equation.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$

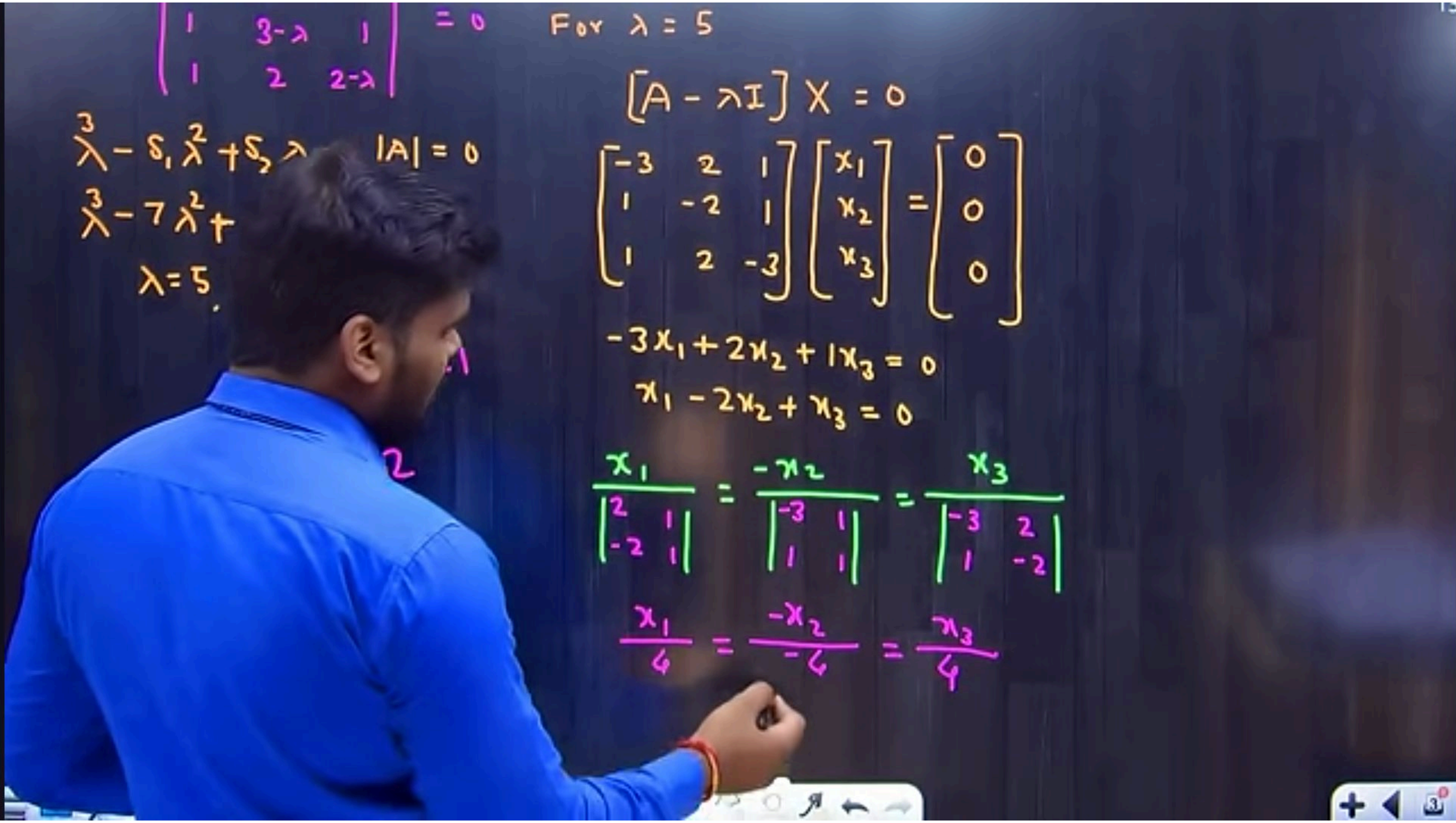
$$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$$

$$\lambda = 5, 1, 1$$

$S_1 = 7$

$S_2 = 4 + 3 + 4 = 11$

$|A| = 5$



$\begin{vmatrix} 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$
For $\lambda = 5$
$[A - \lambda I]X = 0$
$\lambda^3 - S_1\lambda^2 + S_2\lambda$
$|A| = 0$
$\lambda^3 - 7\lambda^2 +$
$\lambda = 5$
$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
$-3x_1 + 2x_2 + 1x_3 = 0$
$x_1 - 2x_2 + x_3 = 0$
$\frac{x_1}{\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -3 & 2 \\ 1 & -2 \end{vmatrix}}$
$\frac{x_1}{4} = \frac{-x_2}{-4} = \frac{x_3}{4}$

$|A| = 5$

For $\lambda = 5$

$[A - \lambda I]$

For $\lambda = 1$

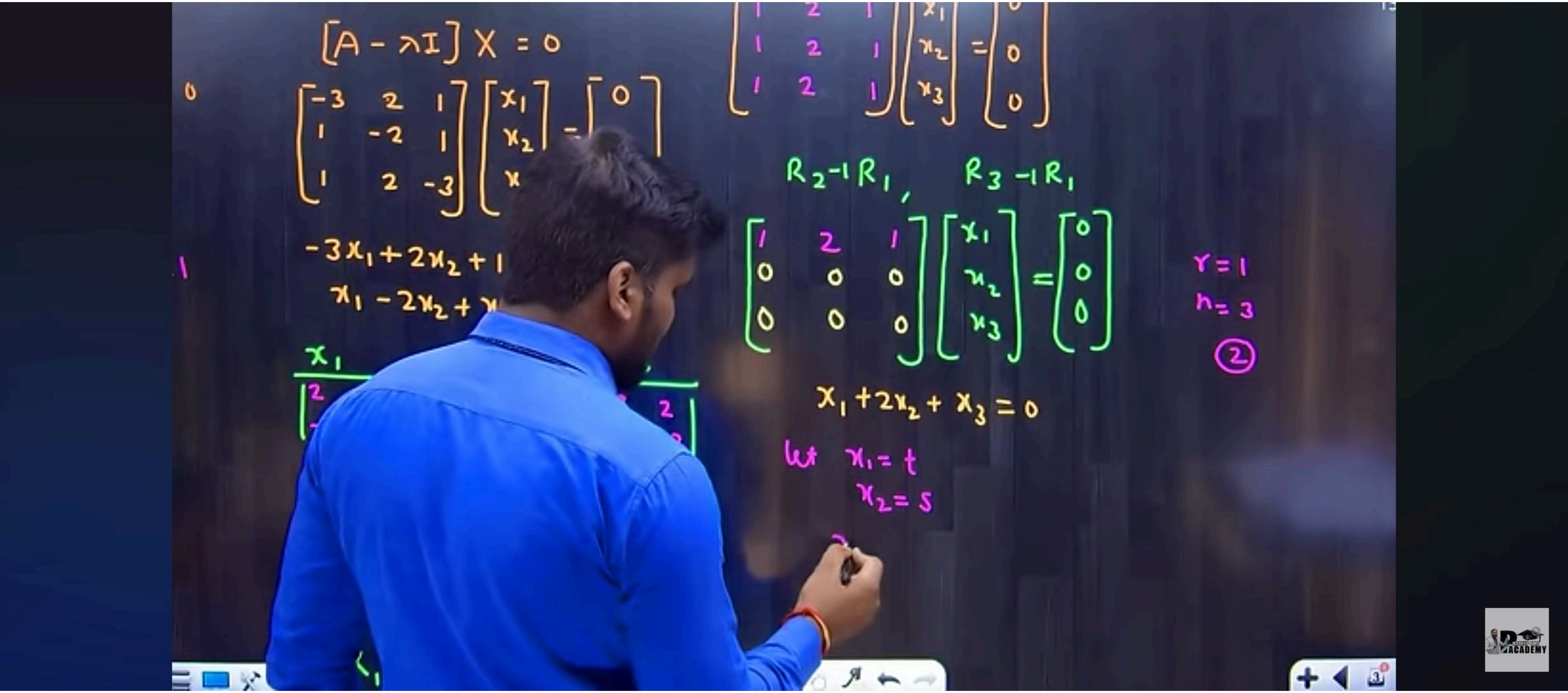
$$[A - \lambda I] X = 0$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 - 1R_1$, $R_3 - 1R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 + x_3 = 0$$


[A - λI] X = 0
-3x1 + 2x2 + 1
x1 - 2x2 + x
R2-1R1, R3-1R1
x1 + 2x2 + x3 = 0
let x1 = t
x2 = s
r = 1
n = 3
2

$$\begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix} \qquad \begin{vmatrix} -3 & 2 \\ 1 & -2 \end{vmatrix}$$

$$= \frac{-x_2}{-6} = \frac{x_3}{4}$$

$$= \frac{x_2}{1} = \frac{x_3}{1}$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$x_1 + 2x_2 + x_3 = 0$$

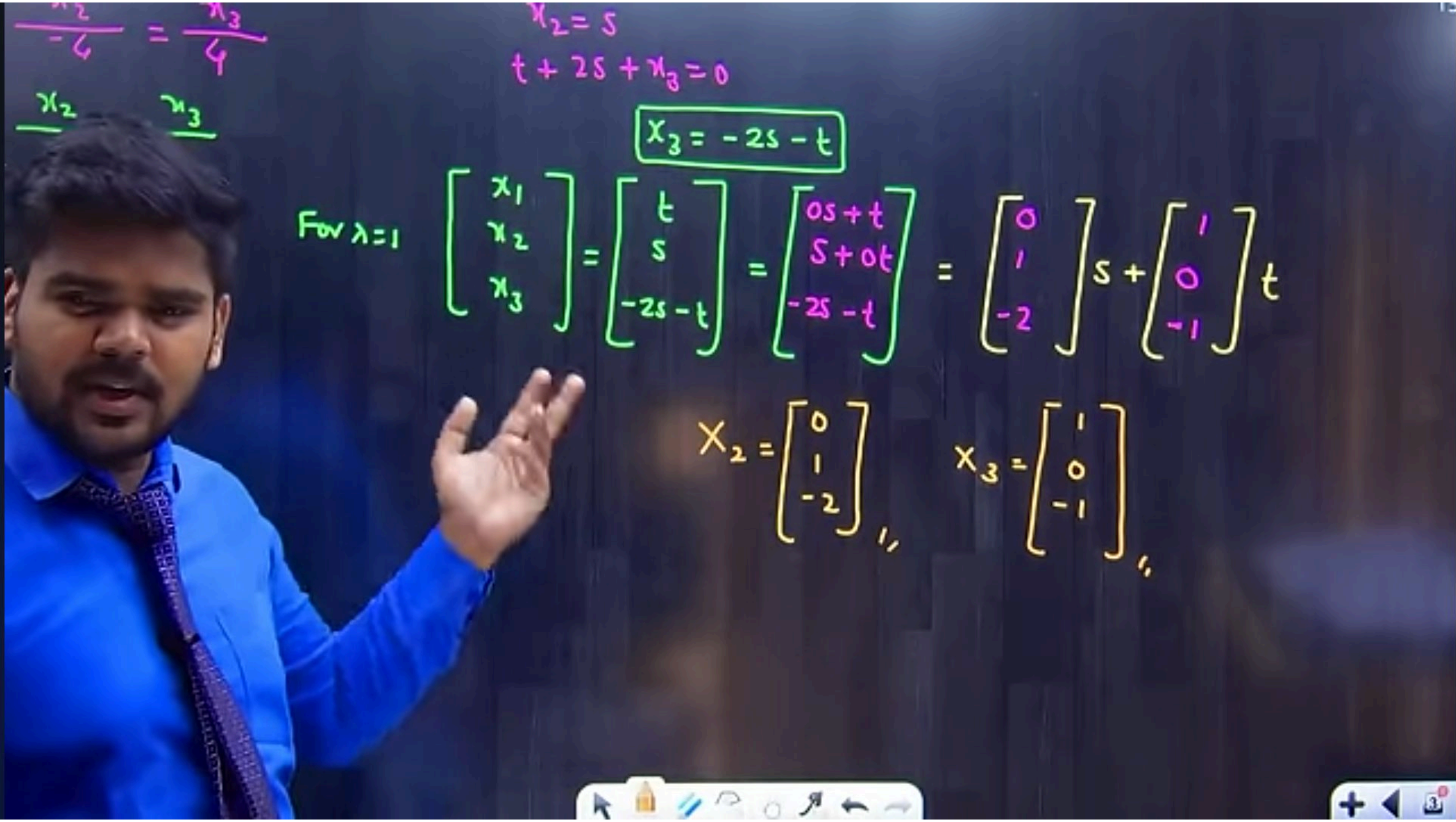
let $x_1 = t$

$x_2 = s$

$$t + 2s + x_3 = 0$$

$$\boxed{x_3 = -2s - t}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ s \\ -2s - t \end{bmatrix} = \begin{bmatrix} 0s + t \\ s + 0t \\ -2s - t \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} s + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} t$$


$\frac{x_2}{-4} = \frac{x_3}{4}$
$x_2 = s$
$t + 2s + x_3 = 0$
$\boxed{x_3 = -2s - t}$
For $\lambda = 1$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ s \\ -2s-t \end{bmatrix} = \begin{bmatrix} 0s+t \\ s+0t \\ -2s-t \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} s + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} t$
$X_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$, $X_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$
ACADEMY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If $A=\begin{bmatrix}1 & 0\\ 2 & 4\end{bmatrix}$ then find Eigen values of $4A^{-1}+3A+2I$.

$$A=\begin{bmatrix}1 & 0\\ 2 & 4\end{bmatrix}$$

12:49

If $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ then find Eigen values of $4A^{-1} + 3A + 2I$

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$$

$$S_1 = 1 + 4 = 5$$

$$|A| = 4 - 0 = 4$$

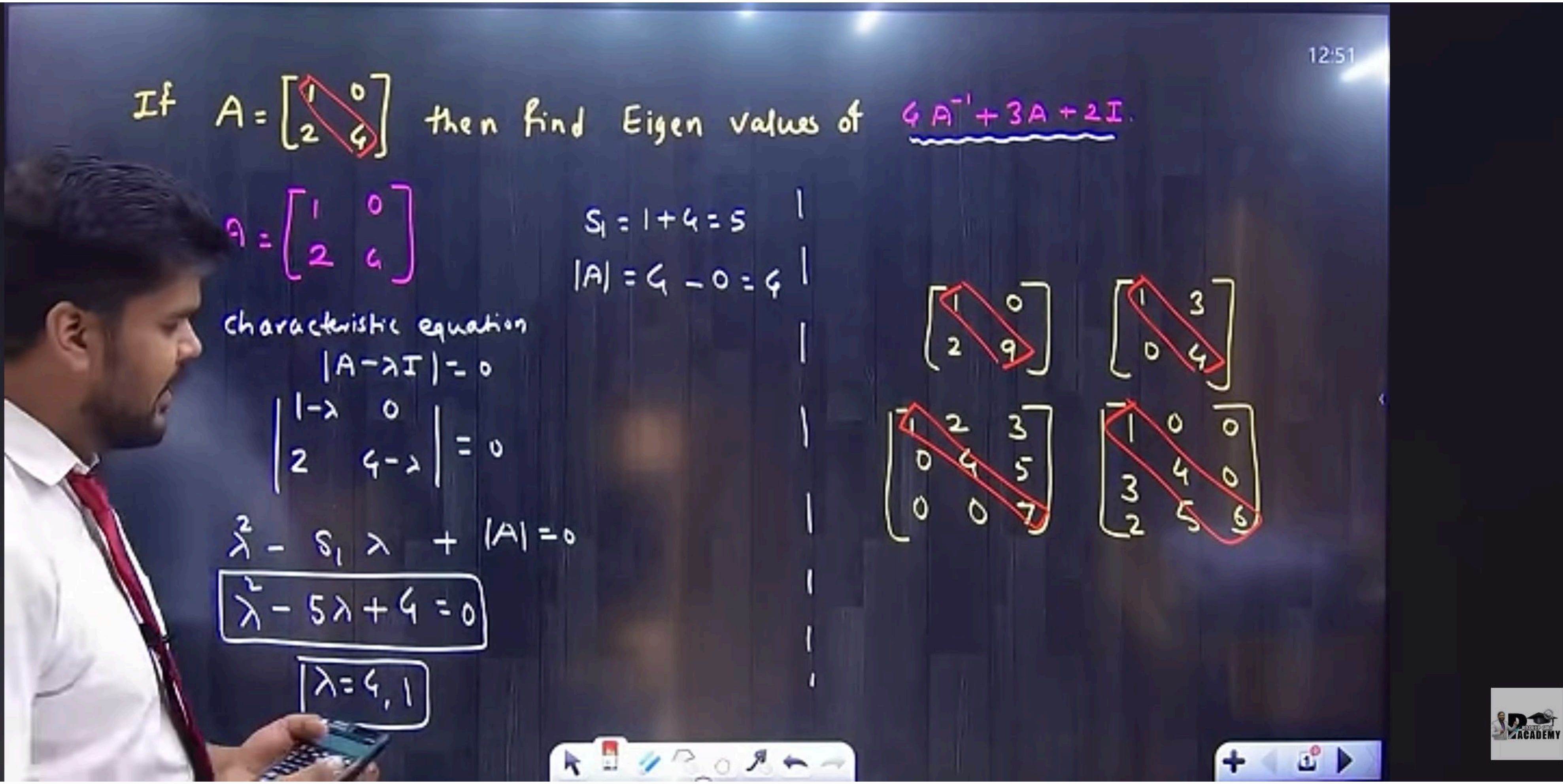
characteristic equation

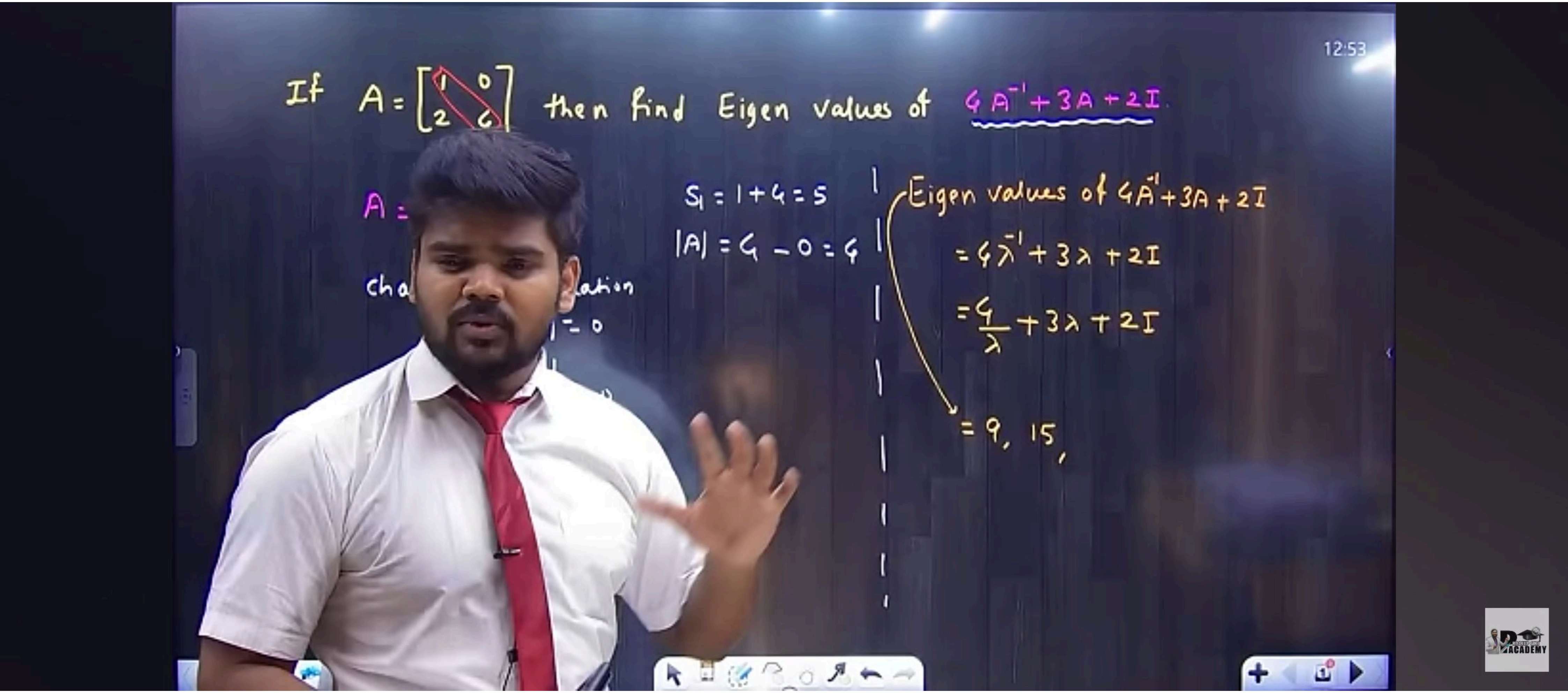
$$|A - \lambda I| = 0$$

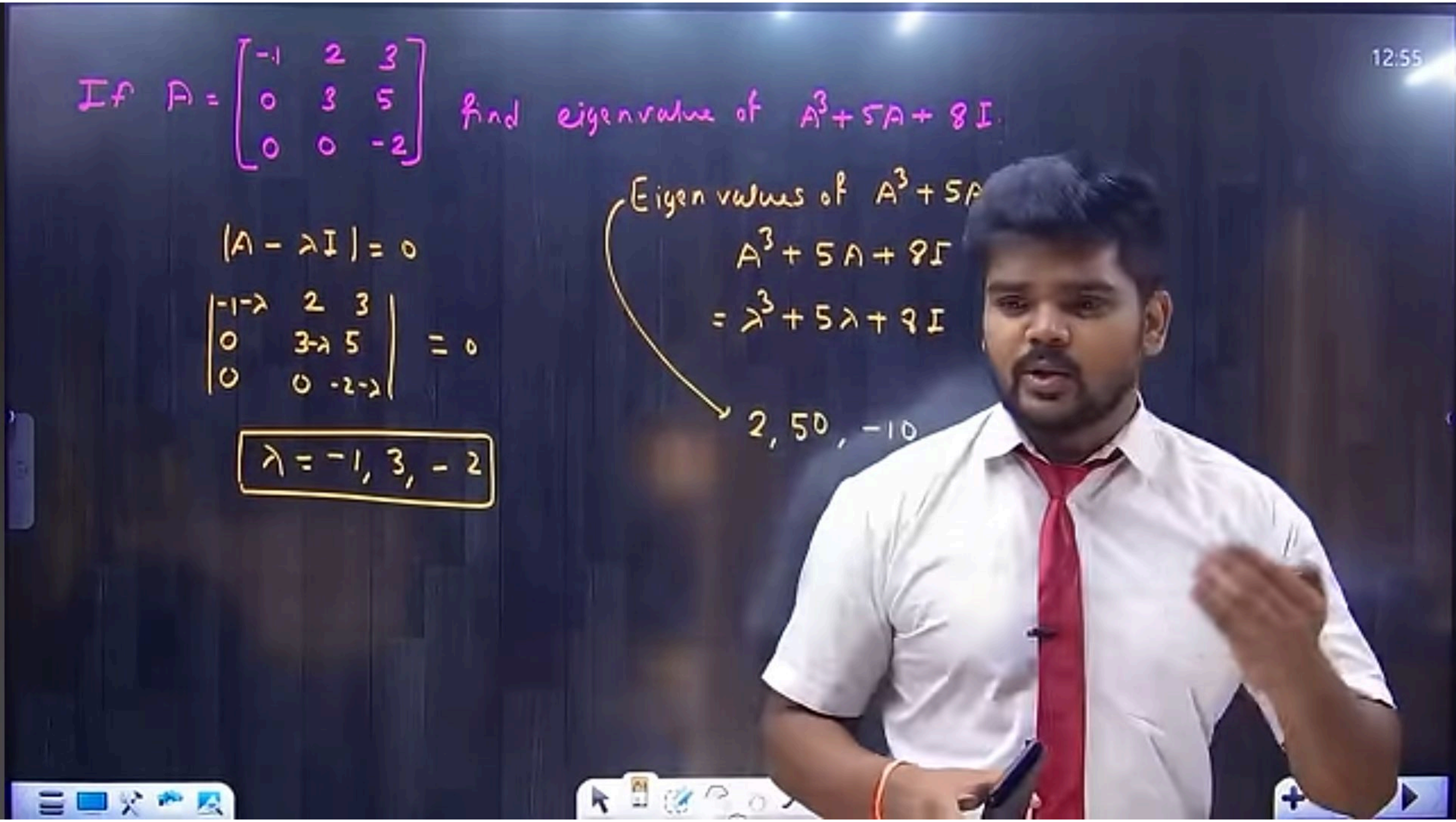
$$\begin{vmatrix} 1-\lambda & 0 \\ 2 & 4-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - S_1 \lambda + |A| = 0$$

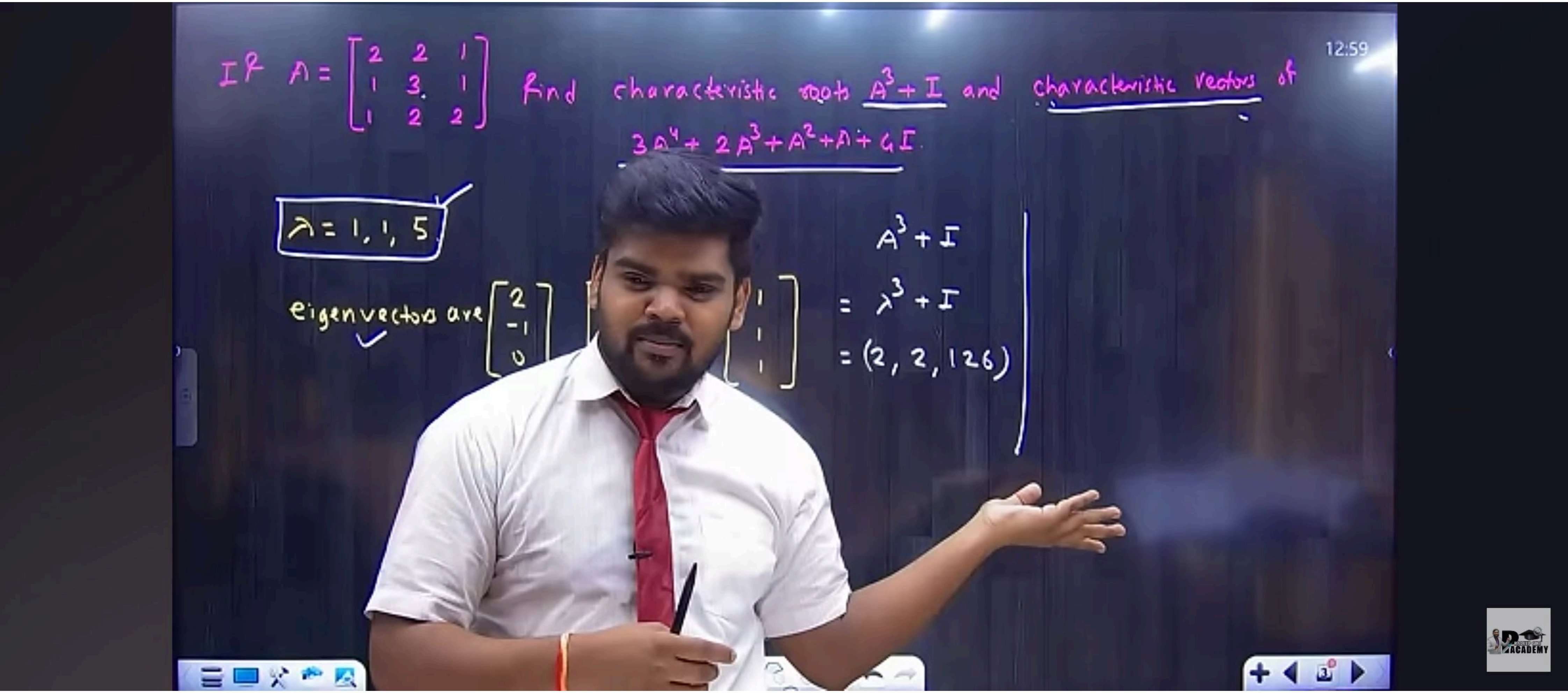
$$\boxed{\lambda^2 - 5\lambda + 4 = 0}$$


12:51
If A = [1 0; 2 4] then find Eigen values of 4A^-1 + 3A + 2I.
A = [1 0; 2 4]
S1 = 1+4 = 5
|A| = 4 - 0 = 4
characteristic equation
|A - λI| = 0
|1-λ 0; 2 4-λ| = 0
λ^2 - S1 λ + |A| = 0
λ^2 - 5λ + 4 = 0
λ = 4, 1
[1 0; 2 9]
[1 3; 0 4]
[1 2 3; 0 4 5; 0 0 7]
[1 0 0; 4 0; 3 ; 2 5 6]
ACADEMY


12:53
If A = [1 0; 2 4] then find Eigen values of 4A⁻¹ + 3A + 2I
A =
S1 = 1 + 4 = 5
|A| = 4 − 0 = 4
Eigen values of 4A⁻¹ + 3A + 2I
= 4λ⁻¹ + 3λ + 2I
= 4/λ + 3λ + 2I
= 9, 15,
ACADEMY


12:55
If A = [-1 2 3; 0 3 5; 0 0 -2] find eigenvalue of A^3 + 5A + 8I.
|A - λI| = 0
|-1-λ 2 3; 0 3-λ 5; 0 0 -2-λ| = 0
λ = -1, 3, -2
Eigen values of A^3 + 5A
A^3 + 5A + 8I
= λ^3 + 5λ + 8I
2, 50, -10
ACADEMY

12:59
If $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ find characteristic roots $A^3 + I$ and characteristic vectors of
$3A^4 + 2A^3 + A^2 + A + 6I$.
$\boxed{\lambda = 1, 1, 5}$
eigenvectors are $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ =
ACADEMY


12:59
If A = [2 2 1; 1 3 1; 1 2 2] find characteristic roots A^3 + I and characteristic vectors of
3A^4 + 2A^3 + A^2 + A + 4I.
λ = 1, 1, 5
eigenvectors are [2; -1; 0]
A^3 + I
= λ^3 + I
= (2, 2, 126)

The characteristic vector of this equation will be same as the vector of the matrix

Find eigen value and eigen vectors

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Are they $\boxed{\text{orthogonal?}}$

$\lambda = 0$ $\quad x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

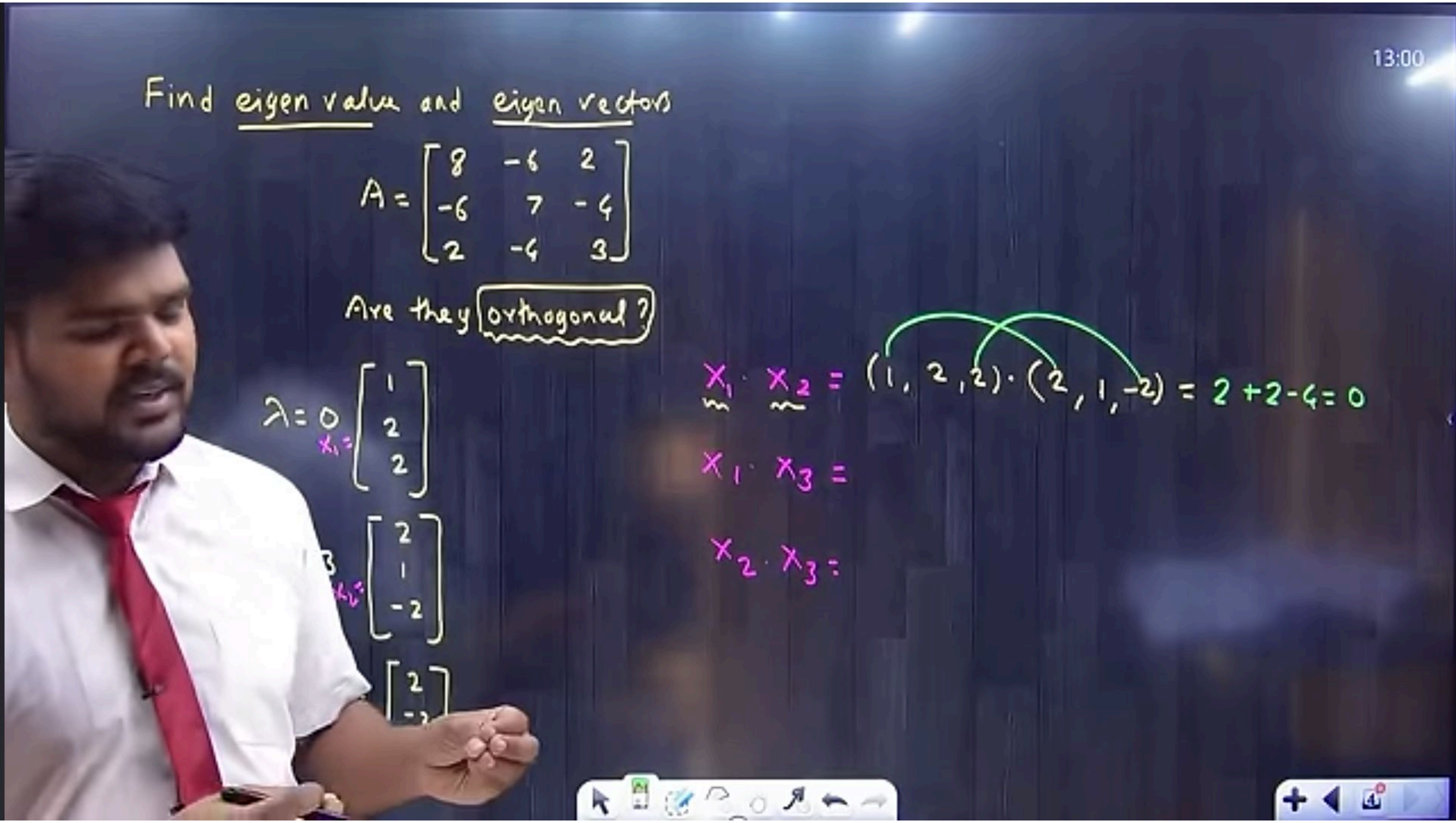
$\lambda = 3$ $\quad x_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

$\lambda = 15$ $\quad x_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$X_1 \cdot X_2 =$

$X_1 \cdot X_3 =$

$X_2 \cdot X_3 =$


13:00
Find eigen value and eigen vectors
A = [8 -6 2; -6 7 -4; 2 -4 3]
Are they orthogonal?
λ = 0
x1 = [1; 2; 2]
x2 = [2; 1; -2]
X1 · X2 = (1, 2, 2) · (2, 1, -2) = 2 + 2 - 4 = 0
X1 · X3 =
X2 · X3 =
ACADEMY

If all the three are zero then it is orthogonal