

Type IV

$$\int_0^{\pi/2} \sin^p \theta \cdot \cos^q \theta \, d\theta = \frac{1}{2} B\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$$

$$\int_0^{\pi/2} \sin^n x \, dx =$$

=

$$\int_0^{\pi/2} \sin^4 x \, dx =$$

$$\int_0^{\pi/2}$$

$$\int_0^{\pi/2} \sin^m x \cdot \cos^n x \, dx$$

=

$$\cos^5 x \, dx =$$

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{1}{2} B\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$$

$$\int_0^{\pi/2} \sin^n x dx =$$

=

$$\left(\frac{1}{2}\right) \left(\frac{2}{3}\right)$$

$$\int_0^{\pi/2} \sin^4 x dx = \frac{3}{4} \cdot \frac{1}{2} \times \frac{\pi}{2}$$

$$\int_0^{\pi/2} \cos^n x dx =$$

=

$$\int_0^{\pi/2} \cos^6 x dx = \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2}$$

$$\int_0^{\pi/2} \sin^m x \cos^n x dx$$

=

$$\int_0^{\pi/2} \sin^4 x \cos^5 x dx =$$

$$\int_0^{\pi/2} \sin \theta \cdot \cos \theta \, d\theta = \frac{1}{2} \ln \left(\frac{1}{2} + \frac{1}{2} \right)$$

$$\int_0^{\pi/2} \sin^n x \, dx =$$

=

$$\int_0^{\pi/2} \sin^4 x \, dx$$

$$\int_0^{\pi/2} \cos^n x \, dx =$$

$$\int_0^{\pi/2} \cos^5 \theta \, d\theta = \frac{4}{5} \cdot \frac{2}{3}$$

$$\int_0^{\pi/2} \cos^6 \theta \, d\theta = \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \times \frac{\pi}{2}$$

$$\int_0^{\pi/2} \sin^m x \cdot \cos^n x \, dx$$

=

$$\int_0^{\pi/2} \sin^4 x \cdot \cos^5 x \, dx =$$

$$\int \sin^4 x \, dx = \frac{3}{4} \cdot \frac{1}{2} \times \frac{\pi}{2}$$

$$\left(\frac{1}{2}\right) \left(\frac{2}{3}\right)$$

$$\int_0^{\pi/2} \cos^5 \theta \, d\theta = \frac{4}{5} \cdot \frac{2}{3}$$

$$\int_0^{\pi/2} \cos^5 \theta \, d\theta = \frac{4}{5} \cdot \frac{2}{3}$$

$$\int_0^{\pi/2} \sin^4 x \cdot \cos^5 x \, dx =$$

$$= \frac{3 \times 1 \times 4 \times 2}{9 \times 7 \times 5 \times 3 \times 1}$$

$$= \frac{[(m-1)(m-3)\dots]^{2 \text{ or } 1} [(n-1)(n-3)\dots]^{2 \text{ or } 1}}{[(m+n)(m+n-2)]}$$

$$\int_0^a (a^2 - x^2)^{5/2} dx$$

put $x = a \sin \theta$

$$(1 - x^2)$$

$$\frac{dx}{d\theta} = a \cdot (\cos \theta)$$

$$dx = a(\cos \theta) d\theta$$

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$$\int_0^a (a^2 - x^2)^{1/2} dx$$

$$\sin^{-1}\left(\frac{x}{a}\right) = \theta$$

$$x = 0$$

$$\theta = 0$$

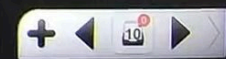
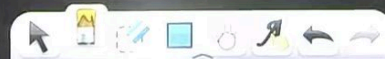
$$x = a$$

$$\theta = \frac{\pi}{2}$$

$$a \sin \theta$$

$$= a \cdot (\cos \theta)$$

$$dx = a(\cos \theta) d\theta$$



$$\frac{dr}{d\theta} = a \cdot (\cos \theta)$$

$$dr = a(\cos \theta) d\theta$$

$$= \int_0^{\pi/2} (a^2 - a^2 \sin^2 \theta)^{1/2} a(\cos \theta) d\theta$$

$$= a \int_0^{\pi/2} (a^2)^{1/2} (1 - \sin^2 \theta)^{1/2} (\cos \theta) d\theta$$

$$= a \cdot a \int_0^{\pi/2} (\cos^2 \theta)^{1/2} \cos \theta d\theta$$

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$$= a \int_0^{\pi/2} (a^2)^2 (1 - \sin^2 \theta)^2 (\cos \theta) d\theta$$

$$= a \cdot a^5 \int_0^{\pi/2} (\cos^2 \theta)^2 \cdot \cos \theta d\theta$$

$$= a^6 \int_0^{\pi/2} \cos^5 \theta \cdot \cos \theta d\theta$$

$$= a^6 \int_0^{\pi/2} \cos^6 \theta d\theta$$

$$= a^6 \cdot \frac{5}{8} \cdot \frac{3}{4} \cdot \frac{1}{2} \times \frac{\pi}{2}$$

Type - V

$$\int_0^{\infty} \frac{x^m}{(1+x^2)^n} dx = \frac{1}{2} B\left(\frac{m+1}{2}, \frac{2n-m-1}{2}\right)$$

eg. $\int_0^{\infty} \frac{dx}{(1+x^2)^{3/2}}$

Type - VI



$$\int_0^{\infty} \frac{x^m}{(1+x)^n} = B(m+1, n-m-1)$$

eg. $\int_0^{\infty} \frac{x^5(1+x^4)}{(1+x)^{16}} dx$

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Type - V

$$\int_0^{\infty} \frac{x^m}{(1+x^2)^n} dx = \frac{1}{2} B\left(\frac{m+1}{2}, \frac{2n-m-1}{2}\right)$$

$$\int_0^{\infty} \frac{dx}{(1+x^2)^{3/2}}$$

$$x=0 \quad \theta=0$$

$$x=\infty \quad \theta=\frac{\pi}{2}$$

$$\text{put } x = \tan \theta$$

$$\frac{dx}{d\theta} = \sec^2 \theta \quad dx = \sec^2 \theta d\theta$$

$$= \int_0^{\pi/2} \frac{1}{(1+\tan^2 \theta)^{3/2}} \sec^2 \theta d\theta$$

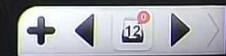
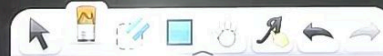
Type - VI



$$\int_0^{\infty} \frac{x^m}{(1+x)^n} = B(m+1, n-m-1)$$

$$\text{eg. } \int_0^{\infty} \frac{x^5 (1+x^4)}{(1+x)^6} dx$$

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$$\text{put } x = \tan \theta$$

$$\frac{dx}{d\theta} = \sec^2 \theta \quad dx = \sec^2 \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{(1+\tan^2 \theta)^{3/2}} \sec^2 \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \cos^5 \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{(\sec^2 \theta)^{3/2}} \sec^2 \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{(\sec \theta)^3} \times \sec^2 \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{\sec \theta} d\theta$$

In sum consisting of
both sin and cos, if
both are even only
then multiply $\pi/2$
else no

$$\left(\frac{1}{2}\right) \left(\frac{2}{3}\right)$$

$$\int \sin^4 x \, dx = \frac{3}{4} \cdot \frac{1}{2} \times \frac{\pi}{2}$$

$$\int_0^{\pi/2} \cos^5 \theta \, d\theta$$

$$\int_0^{\pi/2} \cos^5 \theta \, d\theta = \frac{4}{3} \cdot \frac{2}{3}$$

$$\int_0^{\pi/2} \sin^4 x \cdot \cos^3 x \, dx =$$

$$= \frac{3 \times 1 \times 4 \times 2}{9 \times 7 \times 5 \times 3 \times 1}$$

$$\frac{[(m-1)(m-3)\dots]^{2 \text{ or } 1} [(n-1)(n-3)\dots]^{2 \text{ or } 1}}{[(m+n)(m+n-2)]}$$

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$$\text{put } x = \tan \theta$$

$$\frac{dx}{d\theta} = \sec^2 \theta \quad dx = \sec^2 \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{(1-x^2)^{3/2}} dx$$

$$= \int_0^{\frac{\pi}{2}} \cos^5 \theta d\theta$$

$$= \frac{4}{5} \frac{2}{3}$$

$$= \int_0^{\frac{\pi}{2}} \cos^5 \theta d\theta$$

$$d\theta$$



$$\int_0^{\infty} \frac{x^m}{(1+x^2)^n} dx = \frac{1}{2} B\left(\frac{m+1}{2}, \frac{2n-m-1}{2}\right)$$

$$\int_0^{\infty} \frac{x^m}{(1+x)^n} dx = B(m+1, n-m-1)$$

$$9. \int_0^{\infty} \frac{dx}{(1+x^2)^{3/2}}$$

$$x=0$$

$$x=\infty$$

$$\text{put } x = \tan \theta$$

$$\frac{dx}{d\theta} = \sec^2 \theta \quad dx = \sec^2 \theta d\theta$$

$$= \int_0^{\pi/2} \frac{1}{(1+\tan^2 \theta)^{3/2}} \sec^2 \theta d\theta$$

$$= \int_0^{\pi/2} \frac{1}{(\sec^2 \theta)^{3/2}} \sec^2 \theta d\theta$$

$$9. \int_0^{\infty} \frac{x^5 (1+x^4)}{(1+x)^{16}} dx = \int_0^{\infty} \frac{(x^5 + x^5 \cdot x^4)}{(1+x)^{16}} dx$$

$$= \int_0^{\infty} \frac{x^5 + x^9}{(1+x)^{16}} dx$$

eg. $\int_0^{\infty} \frac{dx}{(1+x^2)^{3/2}}$

$x=0 \quad \theta=0$

$x=\infty \quad \theta=\frac{\pi}{2}$

put $x = \tan \theta$

$\frac{dx}{d\theta} = \sec^2 \theta$

$= \int_0^{\pi/2} \frac{1}{(1+\tan^2 \theta)^{3/2}} \sec^2 \theta d\theta$

$= \int_0^{\pi/2} \frac{1}{(\sec^2 \theta)^{3/2}} d\theta$

$= \int_0^{\pi/2} \frac{1}{\sec^3 \theta} d\theta$

$= \int_0^{\pi/2} \cos^3 \theta d\theta$

eg. $\int_0^{\infty} \frac{x^5 (1+x^4)}{(1+x)^{16}} dx = \int_0^{\infty} \frac{(x^5 + x^5 \cdot x^4)}{(1+x)^{16}} dx$

$= \int_0^{\infty} \frac{x^5 + x^9}{(1+x)^{16}} dx$

$= \int_0^{\infty} \frac{x^5}{(1+x)^{16}} dx + \int_0^{\infty} \frac{x^9}{(1+x)^{16}} dx$

$= B(6, 10) + B(10, 6)$

$= 2 B(6, 10)$

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Type - IX

$$\textcircled{a} \int_a^b \frac{(x-a)^m}{(x-L)^m} \cdot \frac{(b-x)^n}{(b-L-x)^n} dx = (b-a)^{m+n+1} \cdot \beta(m+1, n+1)$$

eg. $\int_7^{11} \sqrt[4]{(x-7)(11-x)} \, dx.$

eg. $\int_3^7 (x-3)^{\frac{1}{4}} \cdot (7-x)^{\frac{1}{4}} \, dx.$

Ex. $\int_7^{11} \sqrt[4]{(x-7)(11-x)} \, dx$

" $\int (x-7)^{\frac{1}{4}} (11-x)^{\frac{1}{4}} \, dx$

$\frac{1}{4} + \frac{1}{4} + 1$

$a=7 \quad b=11 \quad m=n=\frac{1}{4}$

$\frac{2}{4} + 1$

$\frac{1}{2} + 1$

$= 4^{\frac{3}{2}} \beta\left(\frac{1}{4}+1, \frac{1}{4}+1\right)$

$= 2^{3\frac{3}{2}} \beta\left(\frac{5}{4}, \frac{5}{4}\right)$

$= 8 \beta\left(\frac{5}{4}, \frac{5}{4}\right)$

