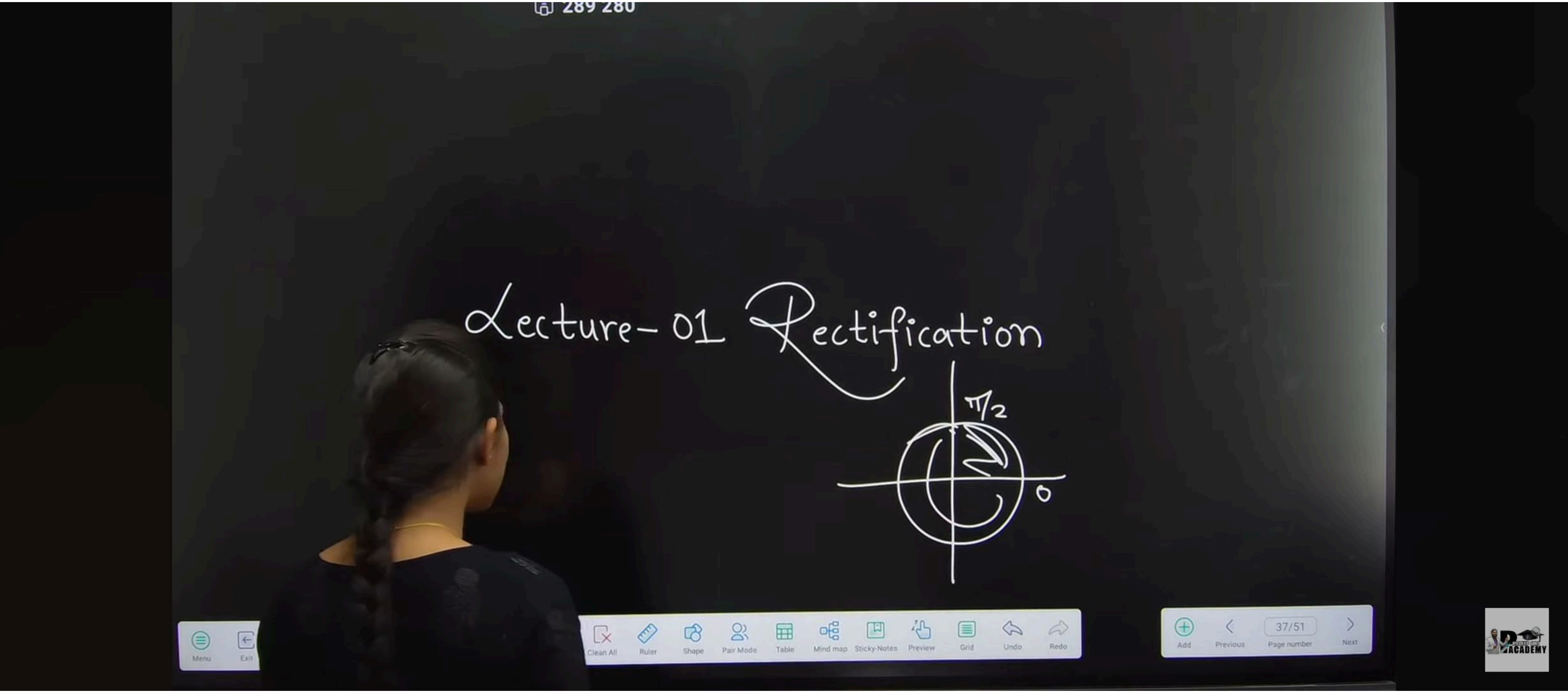
289 280
Lecture-01 Rectification
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Rectification ⇒ to find length of the given plane curves

We will study to find

① Length of the arc of curve given by $y=f(x)$

② Length of the loop

③ Length of the arc of curve given by $r=f(\theta)$

Length of arc of a curve given by $y = f(x)$

$$S = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$\Rightarrow$ If $x = f(y)$

$$\frac{d^2y}{dx^2} \neq \left(\frac{dy}{dx}\right)^2$$

$\Rightarrow$ If $x = f(y)$

$$S = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

551 424

Type 1) Using $\frac{dy}{dx}$

$$S = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

① Find the total length of curve $x^{2/3}+y^{2/3}=a^{2/3}$

$$x^{2/3}+y^{2/3}=a^{2/3}$$

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ACADEMY

551 424

Type 1) Using $\frac{dy}{dx}$

$$S = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

① Find the total length of curve $x^{2/3}+y^{2/3}=a^{2/3}$

$$x^{2/3}+y^{2/3}=a^{2/3}$$

$\frac{2}{3}-1$

x^n

$$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\frac{dy}{dx}=0$$

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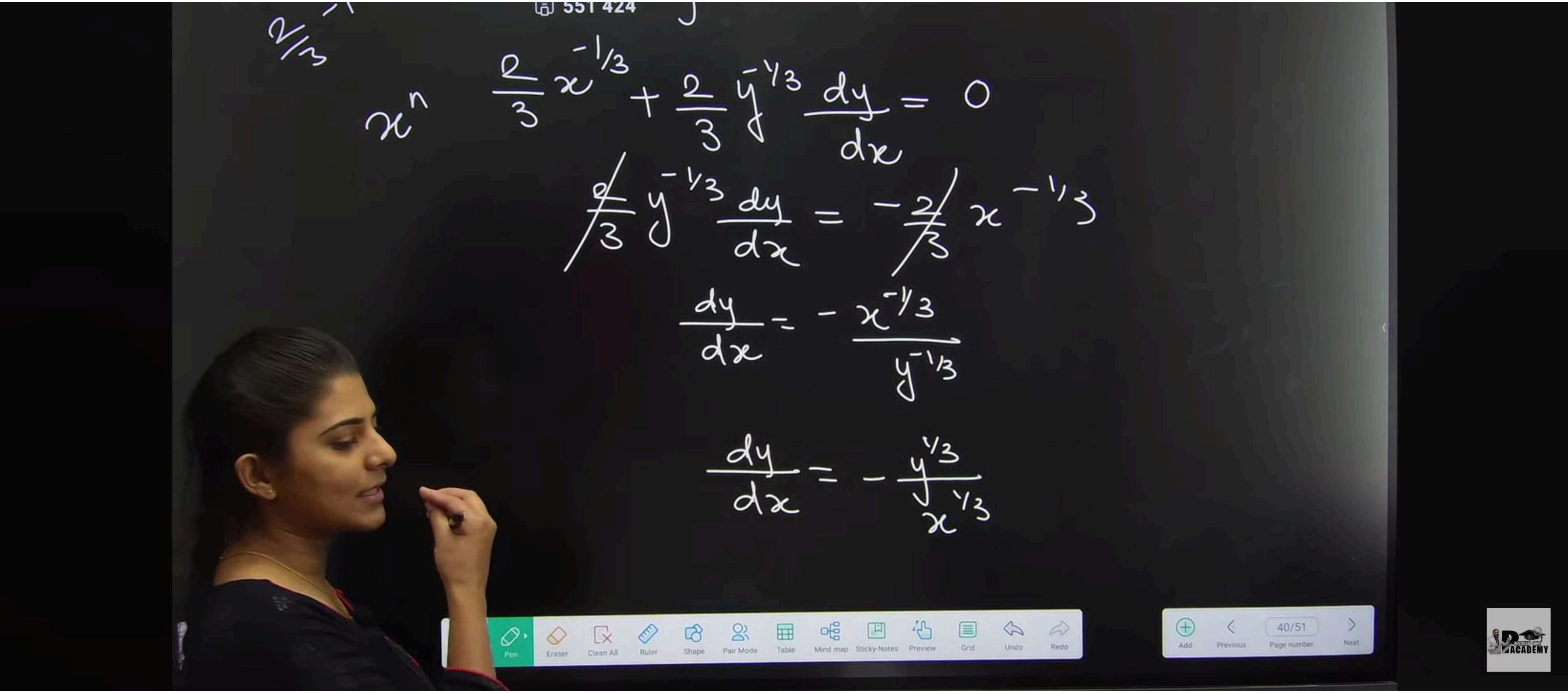
551 424
x^n
$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$
$\cancel{\frac{2}{3}}y^{-1/3}\frac{dy}{dx} = -\cancel{\frac{2}{3}}x^{-1/3}$
$\frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}}$
$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$
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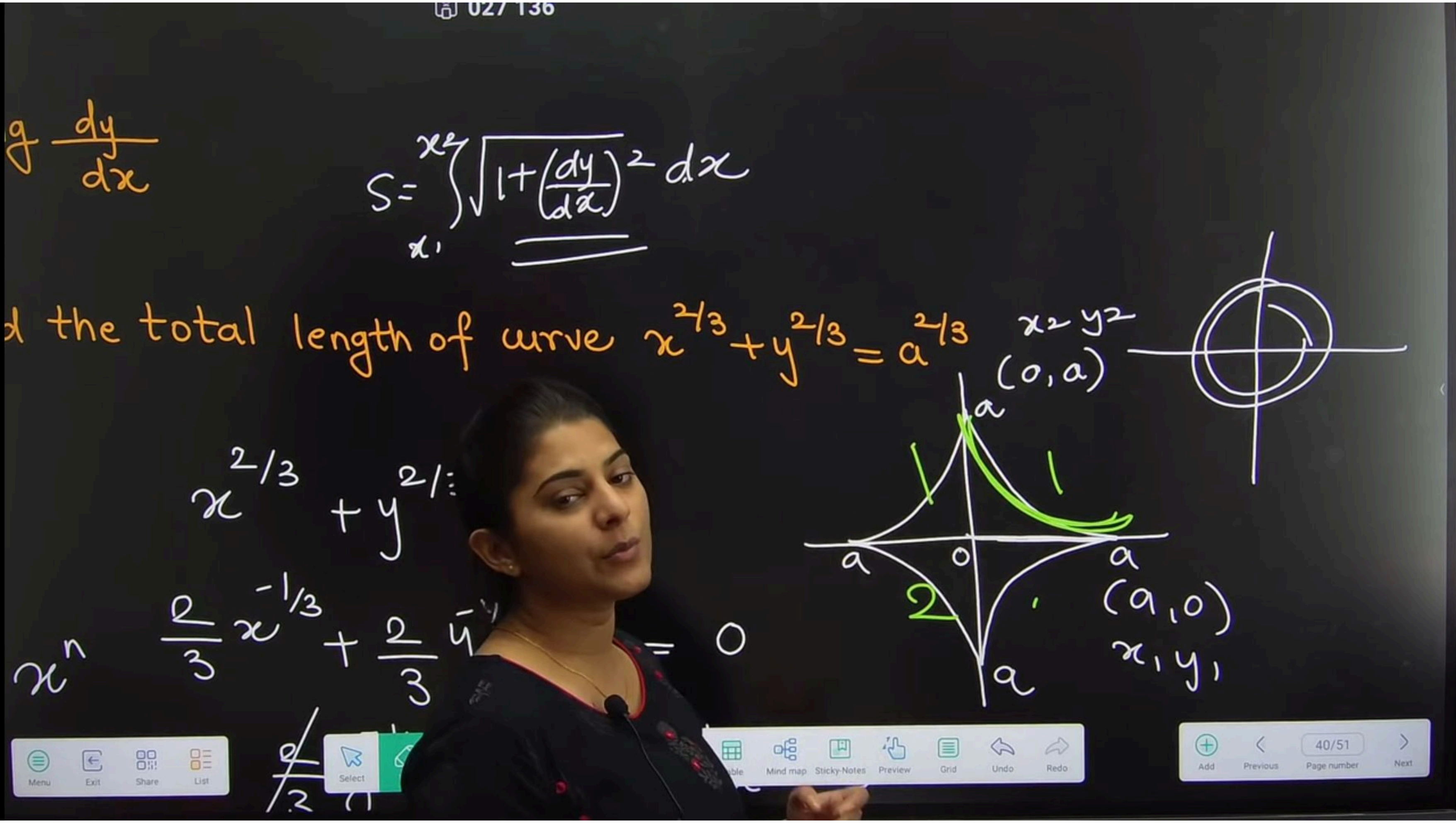
027 136
dy/dx
S = ∫ from x1 to x2 √(1 + (dy/dx)²) dx
d the total length of curve x^{2/3} + y^{2/3} = a^{2/3}
x2 y2
(0, a)
a
a
0
a
(a, 0)
x1, y1
a
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$$y^{-1/3}$$

$$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$$

$$S = \int_a^0 \sqrt{1 + \left(\frac{y^{1/3}}{x^{1/3}}\right)^2}\, dx$$

$$= \int_a^0 \sqrt{1 + \frac{y^{2/3}}{x^{2/3}}}\, dx$$

$$= \int_a^0 \sqrt{1 + \frac{y^{2/3}}{x^{2/3}}}\, dx$$

$$= \int_a^0 \sqrt{\frac{x^{2/3} + y^{2/3}}{x^{2/3}}}\, dx$$

$$= \int_a^0 \sqrt{\frac{\left(x^{1/3} + y^{1/3}\right)^2}{\left(x^{1/3}\right)^2}}\, dx$$

$$\int_a \sqrt{\frac{\cdots + y \cdots}{x^{2/3}}}\, dx$$

$$= \int_a^0 \sqrt{\frac{(a^{1/3})^2}{(x^{1/3})^2}}\, dx$$

$$= \int_a^0 \frac{a^{1/3}}{x^{1/3}}\, dx \qquad x^{2/3} + y^{2/3} = a^{2/3}$$

$$\int_a^0 \sqrt{\frac{\cdots}{x^{2/3}}}\,dx$$

$$= \int_a^0 \sqrt{\frac{(a^{1/3})^2}{(x^{1/3})^2}}\,dx$$

$$= \int_a^0 a^{1/3}\, x^{-1/3}\,dx \qquad x^{2/3}+y^{2/3}=a^{2/3}$$

$$= a^{1/3}\int_a^0 x^{-1/3}\,dx \qquad \frac{x^{n+1}}{n+1}$$

$$= a^{1/3}\left[\frac{x^{2/3}}{2/3}\right] \qquad -\frac{1}{3}+1$$

$$\int_a \sqrt{\frac{\quad}{x^{2/3}}}\, dx$$

$$= \int_a^0 \sqrt{\frac{(a^{1/3})^2}{(x^{1/3})^2}}\, dx$$

$$= \int_a^0 a^{1/3} x^{-1/3}\, dx \qquad x^{2/3} + y^{2/3} = a^{2/3}$$

$$= a^{1/3} \int_a^0 x^{-1/3}\, dx \qquad \frac{x^{n+1}}{n+1}$$

$$\int x^n \qquad = \frac{3}{2} a^{1/3} \left[x^{2/3} \right]_a^0 \qquad -\frac{1}{3}+1$$

$$= a^{-1/3} \int_a^0 x^{-1/3} \, dx \qquad \frac{x^{n+1}}{n+1}$$

$$= \frac{3}{2} a^{-1/3} \left[x^{2/3} \right]_a^0 \qquad -\frac{1}{3}+1$$

$$= \frac{3}{2} a^{-1/3} \left[-a^{2/3} \right]$$

$$= -\frac{3}{2} \, a^{-1/3 + 2/3}$$

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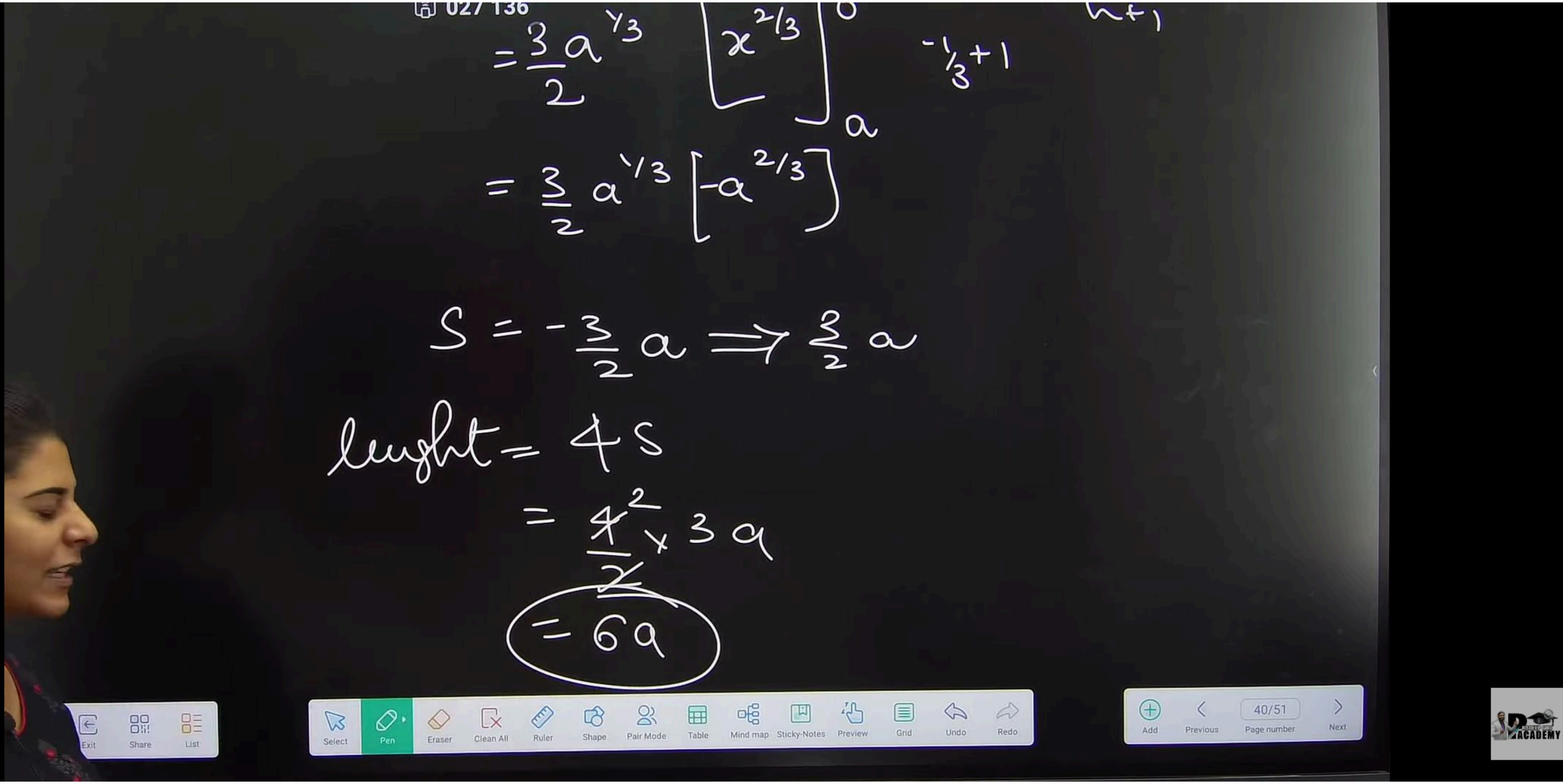
027 136
= 3/2 a^{1/3} [x^{2/3}] 0 a
-1/3+1
= 3/2 a^{1/3} [-a^{2/3}]
S = -3/2 a ⟹ 3/2 a
lenght = 4 S
= 4^2/2 × 3 a
= 6a
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Type 2) using $\frac{dx}{dy}$

$$S = \int_{y_1=1}^{y_2=2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

① Find the length of the curve $x = \frac{y^4}{4} + \frac{1}{8y^2}$ from

to $y = 2$.

$y = 1$

(1) Find the length of the curve $x = \frac{y^4}{4} + \frac{1}{8y^2}$ from $y=1$ to $y=2$.

$$x = \frac{y^4}{4} + \frac{1}{8y^2}$$

$y=1$ to $y=2$.

$$x = \frac{y^4}{4} + \frac{1}{8y^2} \qquad y^{-2-1}$$

$$\frac{dx}{dy} = \frac{\cancel{4}y^3}{\cancel{4}} + \frac{1}{\cancel{8}\,4}\left(-\cancel{2}\,1\right)y^{-3}$$

$$= y^3 - y^{-3}/4$$

$$= y^3 - \frac{1}{4y^3}$$

$$\frac{dx}{dy} = \frac{\cancel{4}y^3}{\cancel{4}} + \frac{1}{\cancel{8}_4}(-\cancel{2})y^{-3}$$

$$= y^3 - y^{-3}/4$$

$$\left(\frac{dx}{dy}\right)^2 = \left(y^3 - \frac{1}{4y^3}\right)^2$$

$$\sqrt{1+\left(\frac{dx}{dy}\right)^2}$$

$$= y^3 - y^{-3}/4$$

$$\left(\frac{dx}{dy}\right)^2 = \left(y^3 - \frac{1}{4y^3}\right)^2$$

$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \left(y^3 - \frac{1}{4y^3}\right)^2$$

$$(a-b)^2 = a^2 - \ldots$$

$$= (\ldots 3)\left(\frac{1}{4y^3}\right) + \left(\frac{1}{4\ldots}\right.$$

$$\left(\frac{dx}{dy}\right)^2 = 1 + \left(y^3 - \frac{1}{4y^3}\right)^2$$

$$\frac{1}{(y^3)^2} \quad \frac{1}{y^6}$$

$$= a^2 - 2ab + b^2$$

$$= (y^3)^2 - \overset{1}{\cancel{2}}(\cancel{y^3})\left(\frac{1}{\underset{2}{\cancel{4}}\cancel{y^3}}\right) + \left(\frac{1}{4y^3}\right)^2$$

$$= y^6 - \frac{1}{2} + \frac{1}{16y^6}$$

$$= 1 -$$

$$= a^2 - 2ab + b^2$$

$$= (y^3)^2 - \cancel{2}^{\,1}(\cancel{y^3})\left(\frac{1}{\cancel{4}_{2}\cancel{y^3}}\right) + \left(\frac{1}{4y^3}\right)^2$$

$$= y^6 - \frac{1}{2} + \frac{1}{16y^6}$$

$$= 1 + y^6 - 1/2 + 1/16y^6$$

$$= y^6 + \frac{1}{2} + \frac{1}{16y^6}$$

$$(a+b)^2$$

$$= (y^3)^2 + \overset{2ab}{\frac{1}{2}} + (1/4y)^2$$

$$= 1 + y^6 - \frac{1}{2} + \frac{1}{16y^6}$$

$$= y^6 + \frac{1}{2} + \frac{1}{16y^6} \qquad (a+b)^2$$

$$= (y^3)^2 + \overset{2ab}{\frac{1}{2}} + \left(\frac{1}{4y^3}\right)^2$$

$$1 + \left(\frac{dx}{dy}\right)^2 = \left(y^3 + \frac{1}{4y^3}\right)^2$$

$$= (y^3)^2 + 2 \cdot \ldots + \left(\frac{1}{4y^3}\right)^2$$

$$1 + \left(\frac{dx}{dy}\right)^2 = \left(y^3 + \frac{1}{4y^3}\right)^2$$

$$S = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\left(y^3 + \frac{1}{4y^3}\right)^{\cancel{2}}}\, dy \qquad \frac{1}{\cancel{2}} \times \cancel{2}$$

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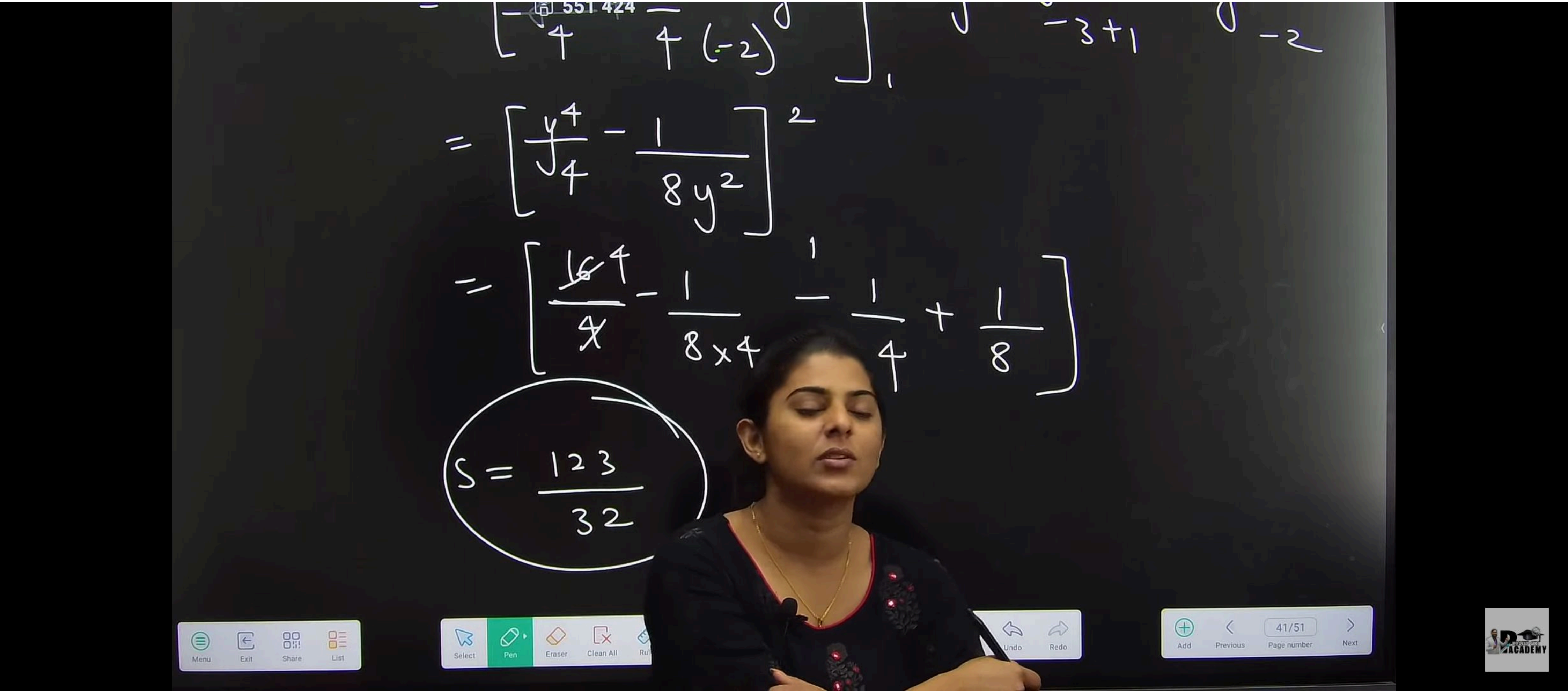
$$\int_1 \sqrt{\left(\quad \frac{}{4y^3}\right)} \, dy$$

$$= 2\int_1 y^3 + \frac{1}{4y^3} \, dy$$

$$= \left[\frac{y^4}{4} + \frac{1}{4(-2)} y^{-2}\right]_1^2 \qquad \frac{1}{y^3} \qquad \frac{y^{-3+1}}{-3+1} \qquad \frac{y^{-2}}{-2}$$

$$= \left[\frac{y^4}{4} - \frac{1}{8y^2}\right]_1^2$$



551 424
= [y^4/4 − 1/(8y^2)]^2
= [16/4 − 1/(8×4) − 1/4 + 1/8]
S = 123/32
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ACADEMY

L2 : length of loops

551 424

To find the length of loop

① Find the total length of the loop of the curve

$$9y^2 = (x+7)(x+4)^2$$

551 424

① Find the total length of the loop of the curve

$$9y^2 = (x+7)(x+4)^2$$

If $y = 0$

$$0 = (x+7)(x+4)^2$$

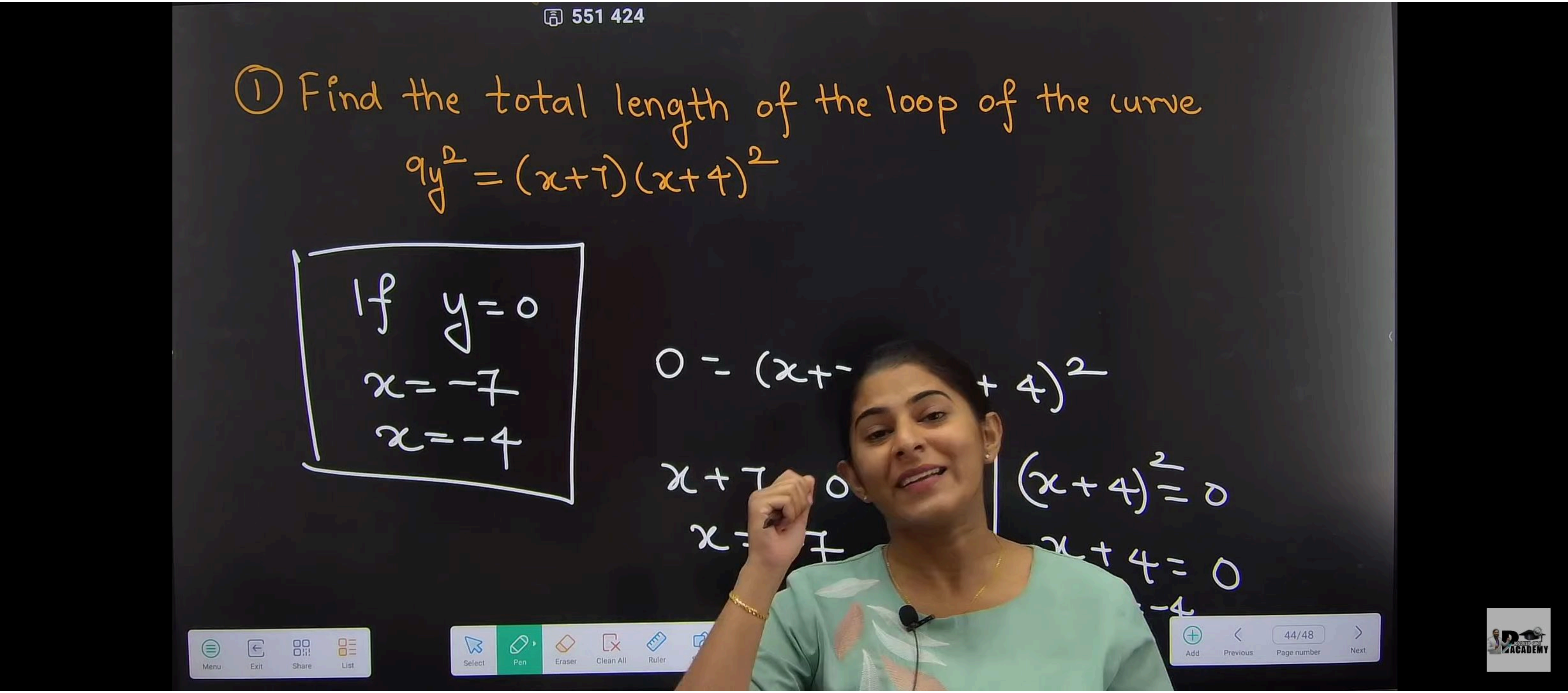
$x + 7 = 0$ | $(x+4)^2 = 0$

$x = -7$ | $x + 4 = 0$

| $x = -4$

7

4



551 424
① Find the total length of the loop of the curve
$9y^2 = (x+7)(x+4)^2$
If $y=0$
$x=-7$
$x=-4$
$(x+4)^2 = 0$
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ACADEMY

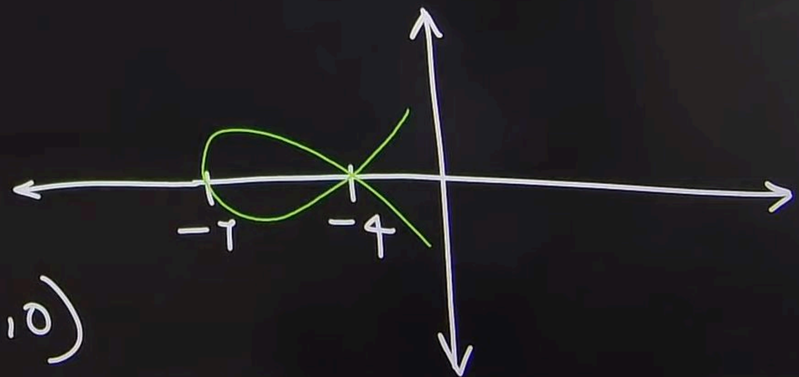
$$\boxed{\begin{aligned} x &= -7 \\ x &= -4 \end{aligned}}$$

$$0 = (x+7)(x+4)^2$$

$$x+7=0$$
$$x=-7$$

$$(x+4)^2=0$$
$$x+4=0$$
$$x=-4$$

$$y=0$$
$$(-7,0),\ (-4,0)$$



027 136

$(-7, 0), (-4, 0)$

$$S = 2\int_{-7}^{-4} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$9y^2 = (x+7)(x+4)^2$$

$$18y\frac{dy}{dx} = (x+7)\,2(x+4) + (x+2)^2$$

$$9y^2 = (x+7)(x+4)^2$$

$$18y\frac{dy}{dx} = (x+7)\,2(x+4) + (x+4)^2$$

$$\frac{dy}{dx} = \frac{(x+4)\left[2x+14+x+4\right]}{18y}$$

$$= \frac{(x+4)(3x+18)}{18y}$$

$$\frac{dy}{dx} = \frac{(x+4)\left[2x+14+x+4\right]}{18y}$$

$$= \frac{(x+4)\,\overset{1}{\cancel{3}}(x+6)}{\underset{6}{\cancel{18}}y}$$

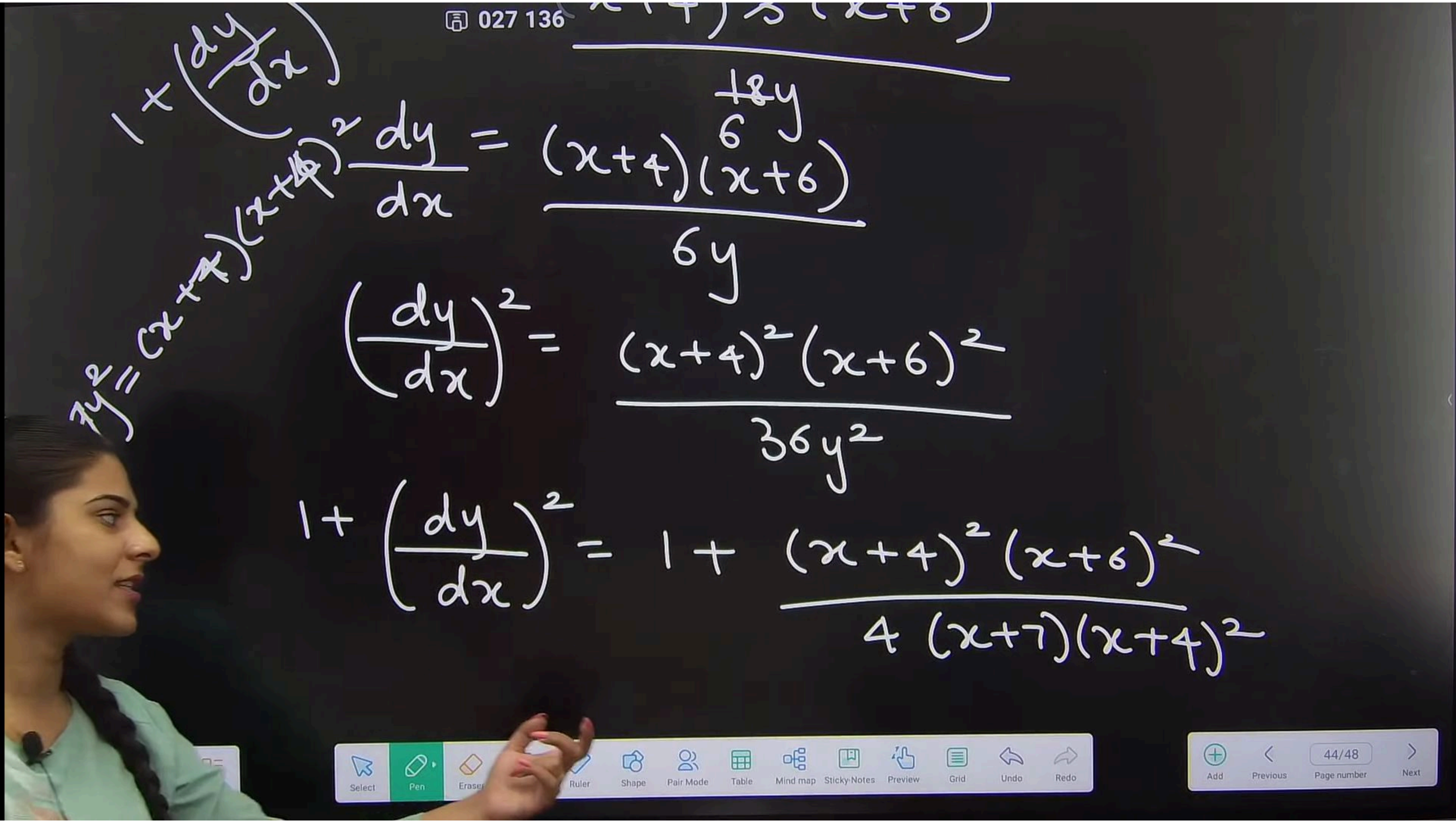
$$\frac{dy}{dx} = \frac{(x+4)(x+6)}{6y}$$

$$1+\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{(x+4)(x+6)}{6y}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{(x+4)^2(x+6)^2}{36y^2}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1 + \frac{(x+4)^2(x+6)^2}{36y^2}$$


027 136
$1+\left(\frac{dy}{dx}\right)^2$
$\frac{dy}{dx} = \frac{(x+4)(x+6)}{6y}$
$\left(\frac{dy}{dx}\right)^2 = \frac{(x+4)^2(x+6)^2}{36y^2}$
$1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{(x+4)^2(x+6)^2}{4(x+7)(x+4)^2}$
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$$9y^2 = (x+4)(x+\ldots$$

$$\frac{dy}{dx} = \frac{\ldots(x+6)}{6y}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{(x+4)^2(x+6)^2}{36y^2}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{(x+4)^2(x+6)^2}{4(x+7)(x+4)^2}$$

$$= \frac{4(x+7)(x+4)^2+(x+4)^2(x+6)^2}{4(x+7)(x+4)^2}$$

$4(x+7)(x+4)^2$

$$= \frac{4(x+7)(x+4)^2 + (x+4)^2(x+6)^2}{4(x+7)(x+4)^2}$$

$$= \frac{\cancel{(x+4)^2}\left(4x+28+x^2+12x+36\right)}{4(x+7)\cancel{(x+4)^2}}$$

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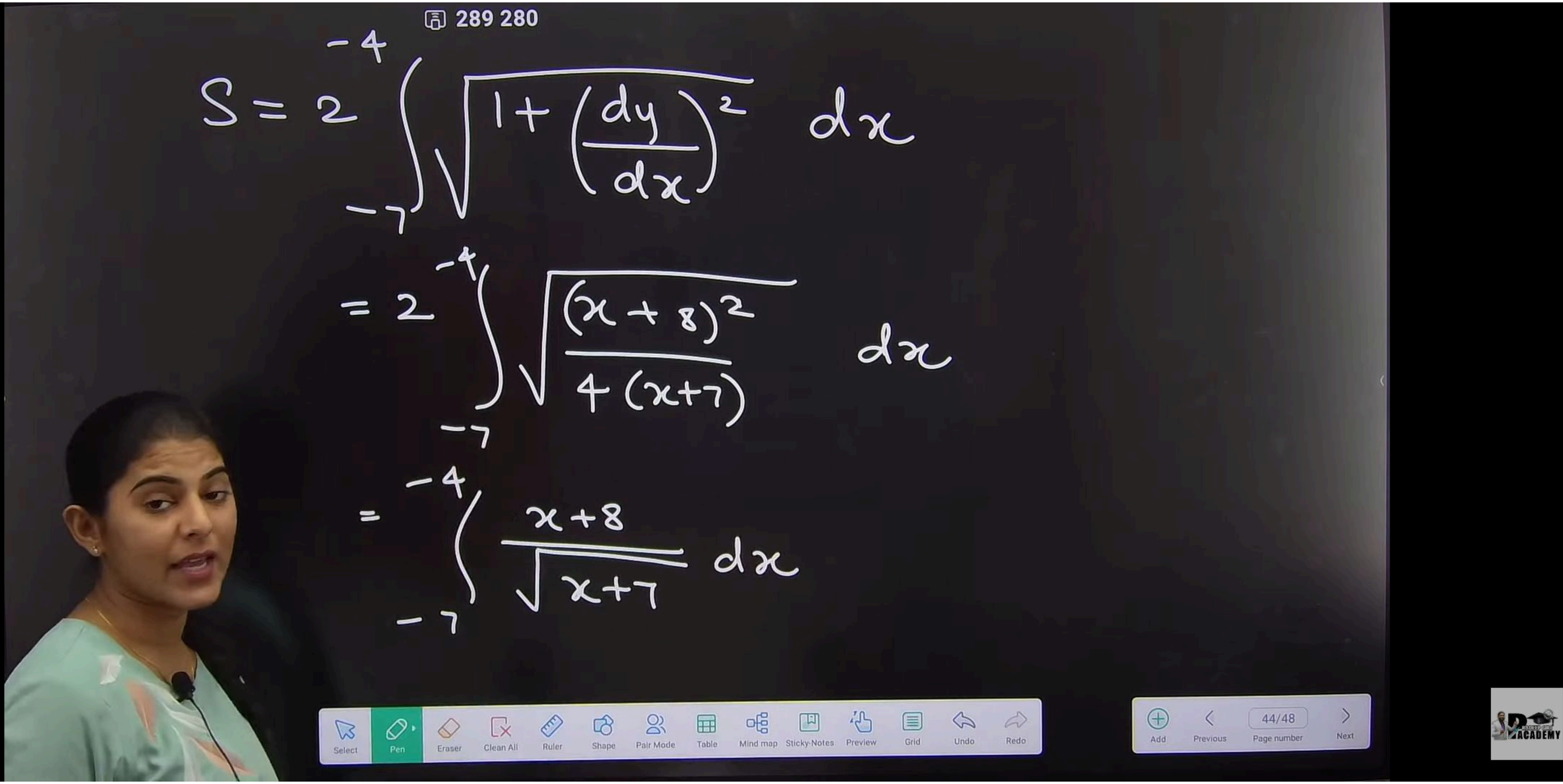
$$= \frac{x^2 + 16x + 64}{4(x+7)} \qquad 64 \to 8^2$$

$$(a+b)^2 = a^2 + 2ab +$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{(x+8)^2}{4(x+7)}$$


289 280
$S = 2\int_{-7}^{-4} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$
$= 2\int_{-7}^{-4} \sqrt{\frac{(x+8)^2}{4(x+7)}}\, dx$
$= \int_{-7}^{-4} \frac{x+8}{\sqrt{x+7}}\, dx$
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$$= \int_{-7}^{-4} \frac{x+8}{\sqrt{x+7}}\, dx$$

Put $\sqrt{x+7} = t$

$$x+7 = t^2$$

$$\frac{dx}{dt} = 2t$$

$$dx = 2t\ dt$$

$$= \int_{-7}^{-4} \frac{x+8}{\sqrt{x+7}}\, dx$$

$$x+8$$

$$\underline{x+7}+1$$

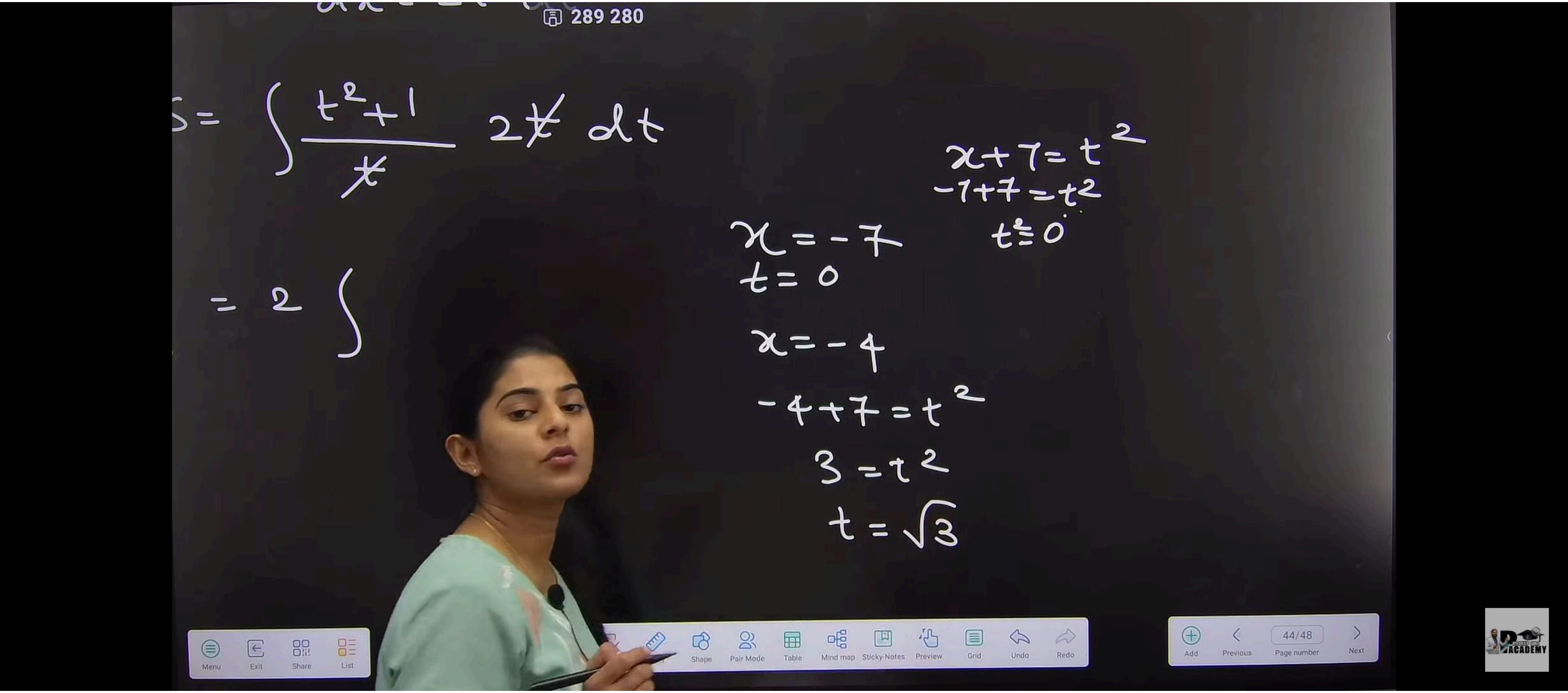
$$t^2+1$$

Put $\sqrt{x+7} = t$

$$\ldots 7 = t^2$$

$$\ldots = 2t$$

$$dx = 2t\ dt$$


289 280
$= \int \frac{t^2+1}{\cancel{t}} \, 2\cancel{t} \, dt$
$= 2 \int$
$x + 7 = t^2$
$-7+7 = t^2$
$t^2 = 0$
$x = -7$
$t = 0$
$x = -4$
$-4+7 = t^2$
$3 = t^2$
$t = \sqrt{3}$
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ACADEMY

$$\frac{dx}{dt} = 2t$$

$$dx = 2t\,dt$$

$$S = \int_0^{\sqrt{3}} \frac{t^2+1}{\cancel{t}}\, 2\cancel{t}\, dt$$

$$= 2\int_0^{\sqrt{3}} t^2+1\, dt$$

$x = -7$

$t = 0$

$x = -4$

$-4+7 = t^2$

$3 = t^2$

$$= 2\int_0^{\sqrt{3}} t^2 + 1 \, dt$$

$$= 2\left[\frac{t^3}{3} + t\right]_0^{\sqrt{3}}$$

$$= 2\left[\frac{(\sqrt{3})^3}{3} + \sqrt{3}\right]$$

$$= 2\left[\sqrt{3} + \sqrt{3}\right]$$

$$= 2\left[2\sqrt{3}\right]$$

$t = 0$

$x = -4$

$-4 + 7 = t^2$

$3 = t^2$

$t = \sqrt{3}$

3

$$\frac{\cancel{\sqrt{3}}\,\cancel{\sqrt{3}}\,\sqrt{3}}{\cancel{\sqrt{3}}\,\cancel{\sqrt{3}}}$$

289 280

$$= 2\left[\frac{(\sqrt{3})^3}{3} + \sqrt{3}\right]$$

$$= 2\left[\sqrt{3} + \sqrt{3}\right]$$

$$= 2\left[2\sqrt{3}\right]$$

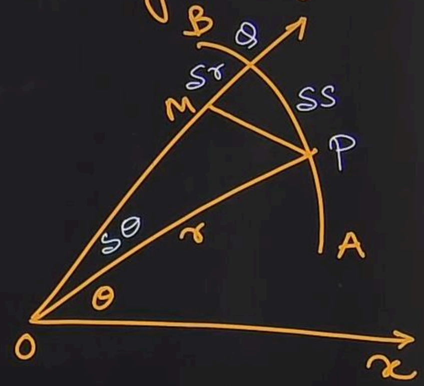
$$\boxed{S = 4\sqrt{3}}$$

$t = \sqrt{3}$

3

$$\frac{\cancel{\sqrt{3}}\,\cancel{\sqrt{3}}\,\sqrt{3}}{\cancel{\sqrt{3}}\,\cancel{\sqrt{3}}}$$

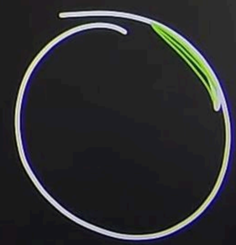
Length of the arc of a curve given by $r=f(\theta)$



$$S=\int_{\theta_1}^{\theta_2}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

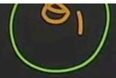
$x=f(y)$

$y=f(x)$



$\Rightarrow$ If $\theta=f(r)$

$$S=\int_{r_1}^{r_2}\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}\,dr$$



551 424

$r = f(\theta)$

$\theta = f(r)$

$x = f(y)$

$y = f(x)$

$\Rightarrow$ If $\theta = f(r)$

$$S = \int_{r_1}^{r_2} \sqrt{1 + r^2 \left(\frac{d\theta}{dr}\right)^2}\, dr$$

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(2) Find the length of arc of the curve $r = a\sin^2\left(\frac{\theta}{2}\right)$ from $\theta = 0$ to any point $P(\theta)$

$$r = a\sin^2(\theta/2)$$

$$S = \int_0^\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

θ

0

$$\frac{dr}{d\theta} = a\sin\theta/2 \cdot \cos\theta/2$$

$$s = \int_0 \sqrt{\;\;+\left(\frac{dr}{d\theta}\right)^2}\; d\theta$$

$$\left(\frac{dr}{d\theta}\right)^2 = a^2 \sin^2\theta/2 \cdot \cos^2\theta/2$$

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = r^2 + a^2 \sin^2\theta/2 \cdot \cos^2\theta/2$$

$$= a \sin^2\theta/2 + a^2 \sin^2\theta/2 \cdot \cos^2\theta/2$$

027 136

$$s = \int_0 \sqrt{\quad + \left(\frac{dr}{d\theta}\right)^2}\; d\theta$$

$$\left(\frac{dr}{d\theta}\right)^2 = a^2 \sin^2 \theta/2 \cdot \cos^2 \theta/2$$

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = r^2 + a^2 \sin^2 \theta/2 \cdot \cos^2 \theta/2$$

$$= a^2 \sin^4 \theta/2 + a^2 \sin^2 \theta/2 \cdot \cos^2 \theta/2$$

027 136

$$= a^2 \sin^4 \theta/2 + a^2 \sin^2 \theta/2 \cdot \cos^2 \theta/2$$

$$= a^2 \sin^2 \theta/2 \left(\underbrace{\sin^2 \theta/2 + \cos^2 \theta/2}_{1} \right)$$

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = a^2 \sin^2 \theta/2$$

027 136

$$= a^2 \sin^4 \theta/2 + a^2 \sin^2 \theta/2 \cdot \cos^2 \theta/2$$

$$= a^2 \sin^2 \theta/2 \left(\underbrace{\sin^2 \theta/2 + \cos^2 \theta/2}_{1} \right)$$

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = a^2 \sin^2 \theta/2$$

$$s = \int_0^\theta \sqrt{a^2 \sin^2 \theta/2}\; d\theta$$

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$$= a^2 \sin^4 \theta/2 + a^2 \sin^2 \theta/2 \cdot \cos^2 \theta/2$$

$$= a^2 \sin^2 \theta/2 \left(\underbrace{\sin^2 \theta/2 + \cos^2 \theta/2}_{1} \right)$$

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = a^2 \sin^2 \theta/2$$

$$s = \int_0^{\theta} a \sin \theta/2 \, d\theta$$

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$$\left(\frac{\sin^2\theta/2+\cos^2\theta/2}{1}\right)$$

$$r^2+\left(\frac{dr}{d\theta}\right)^2=a^2\sin^2\theta/2$$

$$S=a\int_0^\theta \sin\theta/2\,d\theta$$

$$S=2a\left[-\cos\theta/2\right]_0^\theta$$

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$$S = a\int_0^{\theta} \sin \theta/2 \, d\theta$$

$$S = 2a\left[-\cos \theta/2\right]_0^{\theta}$$

$$= 2a\left[-\cos \theta/2 + 1\right]$$

$$= 2a\left[1 - \cos \theta/2\right]$$

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$$= 2a\left[-\cos\theta/2 + 1\right]$$

$$= 2a\left[1-\cos\theta/2\right]$$

$$= 2a\left(2\sin^2\theta/4\right)$$

$$= 4a\sin^2(\theta/4)$$

$$1-\cos\theta/2$$

$$= 2\sin\theta/4$$

$$\theta = 1-2$$