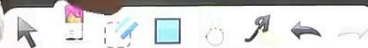
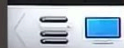


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# Beta, Gamma Functions

## Property of Gamma function

Lecture - 01



$$\int_0^{\infty} e^{-x} x^{n-1} dx = \Gamma(n)$$

$$\int_0^{\infty} e^{-x} x^n dx =$$

$$= 3 \times 2 \times 1$$

$$4! = 4 \times 3 \times 2 \times 1$$

$$\Gamma(n) = (n-1)! \quad \text{eg. } \Gamma(5) = 4!$$

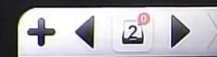
$$\Gamma(n) = \frac{\Gamma(n+1)}{n}$$

$$\Gamma(n) = (n-1) \Gamma(n-1)$$

$$\Gamma(0) = \infty$$

$$\Gamma(1) = 1$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$



Find  $n$

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i) If  $n$  is positive integer

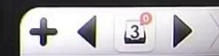
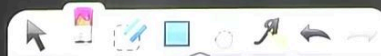
$$\text{eg. } \sqrt{5} = 4! = 4 \times 3 \times 2 \times 1 = 24$$

$$\sqrt{6} = 5! = 5 \times 4 \times 3 \times 2 \times 1 =$$

$$\sqrt{7} =$$

ii) If  $n$  is negative integer.

$$\text{eg. } \sqrt{-4} =$$





Find  $\Gamma n$

Imp

$$\Gamma n = \frac{\Gamma n+1}{n}$$

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i) If  $n$  is a positive integer

eg.  $\Gamma 4 = 3 \times 2 \times 1 = 24$

$2 \times 2 \times 1 =$

ii) If  $n$  is a negative integer.

$$\begin{aligned} \text{eg. } \Gamma -4 &= \frac{\Gamma -4+1}{-4} = \frac{\Gamma -3}{-4} = \frac{1 \cdot \Gamma -3+1}{-4(-3)} \\ &= \frac{\Gamma -2}{12} = \frac{\Gamma -2+1}{12 \times (-2)} = \frac{\Gamma -1}{-24} \\ &= \frac{\Gamma -1+1}{-24 \times (-1)} = \frac{\Gamma 0}{24} = \infty \end{aligned}$$

$$\sqrt[n]{n} = (n-1) \sqrt[n-1]{n-1}$$

If  $n$  is positive fraction:

$$\begin{aligned} \text{eg. } \sqrt{\frac{5}{2}} &= \left(\frac{5}{2} - 1\right) \sqrt{\frac{5}{2} - 1} \\ &= \frac{3}{2} \sqrt{\frac{3}{2}} \\ &= \frac{3}{2} \left(\frac{3}{2} - 1\right) \sqrt{\frac{3}{2} - 1} \\ &= \frac{3}{2} \times \frac{1}{2} \sqrt{\frac{1}{2}} \\ &= \frac{3}{4} \sqrt{\frac{1}{2}} \end{aligned}$$

If  $n$  is negative fraction:

=

$$\sqrt{n} = (n-1)\sqrt{n-1}$$

$$\sqrt{n} = \frac{|n+1|}{n}$$

If n is positive fraction:

$$\text{eg. } \sqrt{\frac{5}{2}} = \left( \frac{5}{2} - 1 \right) \sqrt{\frac{5}{2} - 1}$$

$$= \left( \frac{3}{2} \right) \sqrt{\frac{3}{2}}$$

If n is negative fraction:

$$\text{eg. } \sqrt{-\frac{5}{3}} = \frac{\sqrt{-\frac{5}{3} + 1}}{-\frac{5}{3}} = \frac{\sqrt{-\frac{2}{3}}}{-\frac{5}{3}} = \frac{\sqrt{-\frac{2}{3} + 1}}{\left(-\frac{5}{3}\right)\left(-\frac{2}{3}\right)}$$

$$= \frac{\sqrt{-\frac{1}{3}}}{\frac{10}{9}}$$



$$\int_0^{\infty} e^{-x} x^{n-1} dx = \Gamma(n)$$

$$\int_0^{\infty} e^{-x} x^n dx = \Gamma(n+1)$$

$$3! = 3 \times 2 \times 1$$

$$4! = 4 \times 3 \times 2 \times 1$$

$$\Gamma(n) = (n-1)! \quad \text{eg. } \Gamma(5) = 4!$$

$$\Gamma(n) = \frac{\Gamma(n+1)}{n}$$

$$\Gamma(n) = (n-1) \Gamma(n-1)$$

$$\Gamma(0) = \infty$$

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$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$