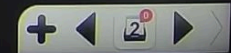
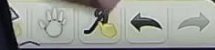
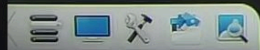


18:39

# TAYLOR'S & MACLAURIN'S THEOREM



Expand  $x^3 + 7x^2 + x - 6$  in power of  $(x-3)$ .

18:42

$$f(x+h) = f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \dots$$

Expand  $x^3 + 7x^2 + x - 6$  in power of  $(x-3)$ .

$$\text{let } f(x) = x^3 + 7x^2 + x - 6$$

$$f(x-3+3)$$

$$x-5$$

$$x+7$$

$$f(x-5+5)$$

$$f(x+7-7)$$

$$x+6$$

$$f(x+6-6)$$

Expand  $x^3 + 7x^2 + x - 6$  in power of  $(x-3)$ .

let  $f(x) = x^3 + 7x^2 + x - 6$

$$3! = 3 \times 2 \times 1$$

$$2! = 2 \times 1$$

$$f(\underbrace{x-3}_x + \underbrace{3}_h) =$$

$$f(x+h) = f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \dots$$

$$f(\underbrace{x-3}_{x} + \underbrace{3}_{h}) =$$

$$2! = 2 \times 1$$

18:46

$$f(x+h) = f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \dots$$

$$f(x-3+3) = f(3) + (x-3) f'(3) + \frac{(x-3)^2 f''(3)}{2!} + \frac{(x-3)^3 f'''(3)}{3!} + \dots \quad \text{--- (I)}$$



$$\text{let } f(x) = x^3 + 7x^2 + x - 6$$

$$f(3) = 3^3 + 7(3)^2 + 3 - 6 = 87 \quad 18:52$$

$$f(\underbrace{x-3}_{x} + \underbrace{3}_{h}) =$$

$$f'(x) = 3x^2 + 14x + 1$$

$$f'(3) = 3(3)^2 + 14(3) + 1 = 70$$

$$f''(x) = 6x + 14$$

$$f''(3) = 6(3) + 14 = 32$$

$$f'''(x) = 6$$

$$f'''(3) = 6$$

$$f(x+h) = f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \dots$$

$$f(x-3+3) = f(3) + (x-3) f'(3) + \frac{(x-3)^2 f''(3)}{2!} + \frac{(x-3)^3 f'''(3)}{3!} + \dots \quad \text{--- (I)}$$

$$\text{let } f(x) = x^3 + 7x^2 + x - 6$$

$$f(3) = 3^3 + 7(3)^2 + 3 - 6 = 87 \quad 18:53$$

$$f(\underbrace{x-3}_{x} + \underbrace{3}_{h}) =$$

$$f'(x) = 3x^2 + 14x + 1$$

$$f'(3) = 3(3)^2 + 14(3) + 1 = 70$$

$$f''(x) = 6x + 14$$

$$f''(3) = 6(3) + 14 = 32$$

$$f'''(x) = 6$$

$$f'''(3) = 6$$

$$f(h) = f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \dots$$

$$f(x-3) = f(3) + (x-3) f'(3) + \frac{(x-3)^2 f''(3)}{2!} + \frac{(x-3)^3 f'''(3)}{3!} + \dots \quad \text{--- (I)}$$

$$= 87 + (x-3) 70 + \frac{(x-3)^2}{2} \times 32 + \frac{(x-3)^3}{6} \times 6 + \dots$$

$$= 87 + (x-3) 70 + 16(x-3)^2 + (x-3)^3 + \dots$$

Max power of  $x$  times  
differentiate Krna hai



Expand  $3x^3 - 2x^2 + x - 4$  in power of  $(x+2)$ .

18:58

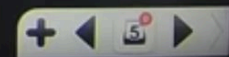
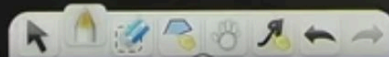
$$f(x) = 3x^3 - 2x^2 + x - 4 \quad f(\quad) =$$

$$f'(x) = 9x^2 - 4x + 1 \quad f'(\quad) =$$

$$f''(x) = 12x - 4 \quad f''(\quad) =$$

$$f'''(x) = 12 \quad f'''(\quad) =$$

$$(x+2-2)$$



$$f'(x) = 9x^2 - 4x + 1$$

$$f'(\quad) =$$

$$f''(x) = 12x - 4$$

$$f''(\quad) =$$

$$f'''(x) = 12$$

$$f'''(\quad) =$$

$$f(\underbrace{x+2}_{\downarrow x} \underbrace{-2}_{\downarrow h})$$

$$f(x+h) = f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \dots$$

$$f(x) = 3x^3 - 2x^2 + x - 4 \quad f(-2) = -38$$

$$f'(x) = 9x^2 - 4x + 1 \quad f'(-2) = 45$$

$$f''(x) = 18x - 4 \quad f''(-2) = -40$$

$$f'''(x) = 18 \quad f'''(-2) = 18$$

$$\begin{aligned} & \underline{2} \quad \underline{-2} \\ & \quad \downarrow \\ & \quad h \end{aligned}$$

$$f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \dots$$

$$(x+2) + \frac{(x+2)^2 f''(-2)}{2!} + \frac{(x+2)^3 f'''(-2)}{3!} + \dots$$

Expand  $\log \cos x$  about  $\frac{\pi}{3}$  using Taylor's Expansion.

$$f(x) = \log \cos x$$

$$x = \frac{\pi}{3}$$

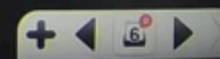
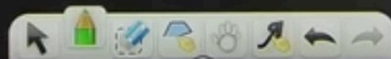
$$f'(x) = \frac{1}{\cos x} \times -\sin x = -\tan x \quad \left(x - \frac{\pi}{3}\right)$$

$$f''(x) = -\sec^2 x$$

$$f'''(x) = -2(\sec x)^{2-1} \times \sec x \cdot \tan x.$$

$$f'''(x) = -2 \sec x \cdot \sec x \cdot \tan x$$

$$f'''(x) = -2 \sec^2 x \cdot \tan x.$$



$$f'(x) = \frac{1}{\cos x} \times -\sin x = -\tan x \quad \left(x - \frac{\pi}{3}\right)$$

$$f''(x) = -\sec^2 x$$

$$f'''(x) = -2(\sec x)^{2-1} \times \sec x \cdot \tan x.$$

$$f'''(x) = -2 \sec x \cdot \sec x \cdot \tan x$$

$$f'''(x) = -2 \sec^2 x \cdot \tan x.$$

$$\underbrace{-\frac{\pi}{3}}_x + \underbrace{\frac{\pi}{3}}_h = f\left(\frac{\pi}{3}\right) + \left(x - \frac{\pi}{3}\right) f'\left(\frac{\pi}{3}\right)$$



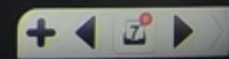
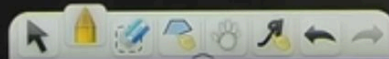
Using Taylor's Theorem express.

Type-II

19:08

$7 + (x+2) + 3(x+2)^3 + (x+2)^4$  in ascending power of  $x$ .

$$f(x) = 7 +$$



$7 + (x+2) + 3(x+2)^3 + (x+2)^4$  in ascending power of  $x$ .

19:11

$$f(x) = 7 + x + 3x^3 + x^4$$

$$f'(x) = 1 + 9x^2 + 4x^3$$

$$f''(x) = 18x + 12x^2$$

$$f'''(x) = 18 + 24x$$

$$f^{iv}(x) = 24.$$

**Bracket hi expansion  
hai**

$$f''(x) = 18x + 12x^2$$

$$f'''(x) = 18 + 24x$$

$$f^{iv}(x) = 24.$$

$$f(x+h) = f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \frac{x^4 f^{iv}(h)}{4!} + \dots$$

$$f(\underbrace{x+2}_{\substack{\uparrow \\ x}}) = f(\underbrace{2}_{\substack{\uparrow \\ h}}) + x \cdot f'(2) + \frac{x^2 \cdot f''(2)}{2!} + \frac{x^3 \cdot f'''(2)}{3!} + \frac{x^4 \cdot f^{iv}(2)}{4!} + \dots$$

$1 + (x+2) + 3(x+2)^2 + (x+2)^4$  in ascending power of  $x$ .

19:15

$$f(x) = 7 + x + 3x^3 + x^4 \quad f(2) = 49$$

$$f'(x) = 1 + 9x^2 + 4x^3 \quad f'(2) = 69$$

$$f''(x) = 18x + 12x^2 \quad f''(2) = 84$$

$$f'''(x) = 18 + 24x \quad f'''(2) = 66$$

$$f^{iv}(x) = 24 \quad f^{iv}(2) = 24$$

$$f(x) = f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \frac{x^4 f^{iv}(h)}{4!} + \dots$$

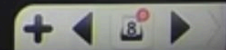
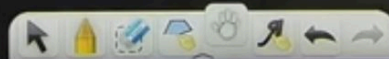
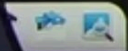
$$= f(2) + \frac{x f'(2)}{1!} + \frac{x^2 f''(2)}{2!} + \frac{x^3 f'''(2)}{3!} + \frac{x^4 f^{iv}(2)}{4!} + \dots$$



Using Taylor's theorem express

$(x-2)^4 - 3(x-2)^3 + 4(x-2)^2 + 5$  in power of  $x$ .

19:17



Using Taylor's theorem express

$(x-2)^4 - 3(x-2)^3 + 4(x-2)^2 + 5$  in power of  $x$ .

$$f(x) = x^4 - 3x^3 + 4x^2 + 5$$

$$f(\underbrace{x}_{\substack{\uparrow \\ x}} - \underbrace{2}_{\substack{\downarrow \\ h}}) =$$

Using Taylor's Theorem to obtain approximate value of the following upto  
Four decimal places.

$$\sqrt{25.15} = ?$$

$$f(x) = \sqrt{25.15}$$

$$\sqrt{x+h}$$

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$(25) + (0.15)$$

$$9.12 = 9 + 0.12$$

$$16.05 = 16 + 0.05$$

$$\sqrt{10} \quad 9$$

Four decimal places.

$$\sqrt{25.15} = ?$$

$$f(x) = \sqrt{25.15}$$

$$f(x+h) = \sqrt{x+h}$$

$$f(x+h) = \sqrt{25+0.15}$$

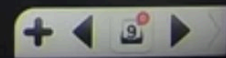
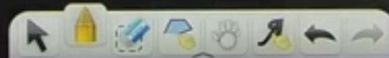
$$\boxed{x=25} \quad \boxed{h=0.15}$$

$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2x^{1/2}}$$

$$f''(x) = \frac{1}{2} \left(-\frac{1}{2}\right) x^{-3/2} = \frac{-1}{4x^{3/2}}$$

19:23



$$\sqrt{25.15} = ?$$

$$f(x) = \sqrt{x}$$

$$f(x+h) = \sqrt{x+h}$$

$$f(x+h) = \sqrt{25+0.15}$$

$$\boxed{x=25} \quad \boxed{h=0.15}$$

$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2x^{1/2}}$$

$$f''(x) = \frac{1}{2} \left(-\frac{1}{2}\right) x^{-3/2} = \frac{-1}{4x^{3/2}}$$

$$f'''(x) = \frac{-1}{4} \left(-\frac{3}{2}\right) x^{-5/2} = \frac{3}{8x^{5/2}}$$



$$f(x+h) = \sqrt{x+h}$$

$$f(x+h) = \sqrt{25+0.15}$$

$$\boxed{x=25} \quad \boxed{h=0.15}$$

$$f(x)$$

$$f'(x) =$$

$$f''(x) =$$

$$f(25) = \sqrt{25} = 5$$

$$f'(25) = 0.1$$

$$f''(25) = -\frac{1}{500}$$

$$f'''(25) = \frac{3}{25000}$$

Interchange  $x$  and  
 $h$  in the formula for  
type 3

$$f(x) = \sqrt{x}$$

$$f(25) = \sqrt{25} = 5$$

$$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2x^{1/2}}$$

$$f'(25) = 0.1$$

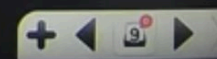
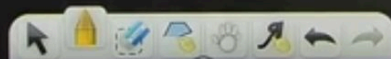
$$f''(x) = \frac{1}{2} \left(-\frac{1}{2}\right) x^{-3/2} = \frac{-1}{4x^{3/2}}$$

$$f''(25) = -\frac{1}{500}$$

$$f'''(x) = \frac{-1}{4} \left(-\frac{3}{2}\right) x^{-5/2} = \frac{3}{8x^{5/2}}$$

$$f'''(25) = \frac{3}{25000}$$

$$\begin{aligned} f(x+h) &= f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \dots \\ &= f(25) + 0.15 f'(25) + \frac{(0.15)^2}{2} f''(25) + \frac{(0.15)^3}{6} f'''(25) \end{aligned}$$



$$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2x^{1/2}}$$

$$f'(25) = 0.1$$

$$f''(x) = \frac{1 \cdot (-\frac{1}{2})}{2x^{3/2}} = \frac{-1}{4x^{3/2}}$$

$$f''(25) = -\frac{1}{500}$$

$$f'''(x) = \frac{-1 \cdot (-\frac{3}{2})}{4x^{5/2}} = \frac{3}{8x^{5/2}}$$

$$f'''(25) = \frac{3}{25000}$$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \dots$$

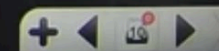
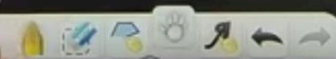
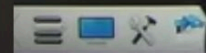
$$= f(25) + 0.15 f'(25) + \frac{(0.15)^2}{2!} f''(25) + \frac{(0.15)^3}{3!} f'''(25)$$

$$= 5 + 0.15 \times 0.1 + \frac{(0.15)^2 \cdot (-1)}{500 \times 2} + \frac{(0.15)^3 \times 3}{25000 \times 6}$$

$$= 5.0149775$$

19:38

# MACLAURIN'S Theorem



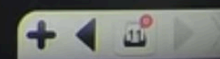
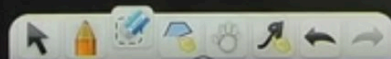


Expand  $\log(1+e^x)$  by maclaurin's theorem upto  $x^3$ .

19:40

$$f(x+h) = f(h) + x f'(h) + \frac{x^2 f''(h)}{2!} + \frac{x^3 f'''(h)}{3!} + \dots$$

$$f(x+0) = f(0) + x \cdot f'(0) + \frac{x^2 \cdot f''(0)}{2!} + \frac{x^3 \cdot f'''(0)}{3!} + \dots$$



Expand  $\log(1+e^x)$  by maclaurin's theorem upto  $x^3$ .

$$f(x) = \log(1+e^x)$$

$$f'(x) = \frac{1}{1+e^x} \times e^x = \frac{e^x}{1+e^x}$$

$$f''(x) = \frac{(1+e^x) \cdot e^x - e^x \cdot (e^x)}{(1+e^x)^2}$$

$$f''(x) = \frac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$$

$$f'(x) = \frac{1}{1+e^x} \times e^x = \frac{e^x}{1+e^x}$$

$$(\quad)^2 = (x^n)$$

$$f''(x) = \frac{(1+e^x) \cdot e^x - e^x \cdot (e^x)}{(1+e^x)^2}$$

$$f''(x) = \frac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$$

$$f'''(x) = \frac{(1+e^x)^2 e^x - e^x \cdot 2(1+e^x) \cdot e^x}{(1+e^x)^4}$$

$$f'''(x) = \frac{(1+e^x)^2 \cdot e^x - 2e^x(1+e^x) \cdot e^x}{(1+e^x)^4}$$

$$\frac{(1+e^x)^2}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$$

$$f'''(x) = \frac{(1+e^x)^2 e^x - e^x \cdot 2(1+e^x)' \cdot e^x}{(1+e^x)^4}$$

$$f'''(x) = \frac{(1+e^x)^2 \cdot e^x - 2e^x(1+e^x) \cdot e^x}{(1+e^x)^4}$$

$$f'''(x) = \frac{e^x(1+e^x)[1+e^x-2e^x]}{(1+e^x)^4} \quad 1-2$$

$$f'''(x) = \frac{e^x[1-e^x]}{(1+e^x)^3}$$

$$\frac{(1+e^x)^2}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$$

$$f'''(x) = \frac{(1+e^x)^2 e^x - e^x \cdot 2(1+e^x)' \cdot e^x}{(1+e^x)^4}$$

$$f'''(x) = \frac{(1+e^x)^2 \cdot e^x - 2e^x(1+e^x) \cdot e^x}{(1+e^x)^4}$$

$$f'''(x) = \frac{e^x(1+e^x)[1+e^x-2e^x]}{(1+e^x)^4} \quad 1-2$$

$$f'''(x) = \frac{e^x[1-e^x]}{(1+e^x)^3}$$

$$f'''(x) = \frac{e^x - e^{2x}}{(1+e^x)^3}$$



Expand  $\log(1+e^x)$  by maclaurin's theorem upto  $x^3$ .

$$f(x) = \log(1+e^x)$$

$$f'(x) = \frac{1}{1+e^x} \times e^x = \frac{e^x}{1+e^x}$$

$$f''(x) = \frac{(1+e^x) \cdot e^x - e^x \cdot (e^x)}{(1+e^x)^2}$$

$$= \frac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$$

$$f'''(x) = \frac{(1+e^x)^2 \cdot e^x - e^x \cdot 2(1+e^x) \cdot e^x}{(1+e^x)^4}$$

$$= \frac{e^{3x} + 2e^{4x} - 2e^{4x} - e^{4x}}{(1+e^x)^3} = \frac{-e^{4x}}{(1+e^x)^3}$$

$$f(0) = \log 2$$

$$f'(0) = \frac{1}{2}$$

$$f''(0) = \frac{1}{4}$$

$$f'''(0) =$$



$$f'''(0) = 0$$

$$f'''(x) = \frac{(1+e^x)^2 e^x - e^x \cdot 2(1+e^x) \cdot e^x}{(1+e^x)^4}$$

$$f'''(x) = \frac{(1+e^x)^2 e^x - 2e^{2x}(1+e^x)}{(1+e^x)^4}$$

$$f'''(x) = \frac{e^x(1+e^x - 2e^x)}{(1+e^x)^4} \quad 1-2$$

$$f'''(x) = \frac{e^x(1-e^x)}{(1+e^x)^4}$$

$$f'''(x)$$

$$f(x) = \frac{\ln(1+e^x)}{(1+e^x)^3}$$

$$f'''(x) = \frac{e^x - e^{2x}}{(1+e^x)^3}$$

$$\begin{aligned} f(x) &= f(0) + x \cdot f'(0) + \frac{x^2 \cdot f''(0)}{2!} + \frac{x^3 \cdot f'''(0)}{3!} + \dots \\ &= \log 2 + x \cdot \frac{1}{2} + \frac{x^2 \cdot \frac{1}{4}}{2} + \frac{x^3 \cdot 0}{3!} + \dots \\ &= \log 2 + \frac{x}{2} + \frac{x^2}{8} + 0 \end{aligned}$$