Rule:

If $f(x, \alpha)$ is the continuous function of x & α is the parameter & if $\frac{\partial f}{\partial \alpha}$ is the continuous function of x & α together throughout the interval $[a, b]$ where a & b are constant & independent of α & if

$I(\alpha)$

x & α together throughout the interval $[a,b]$ where

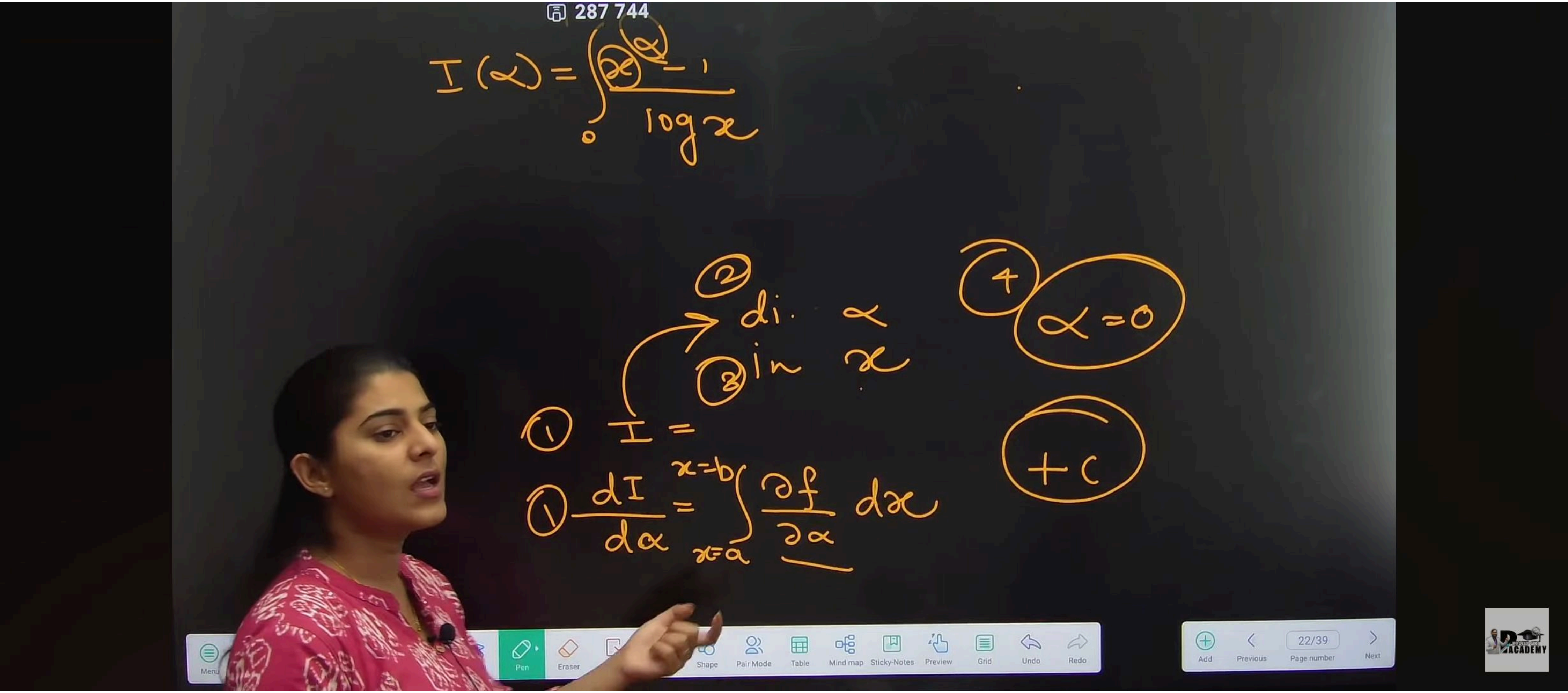
a & b are constant & independent of α & if

$$\boxed{I(\alpha) = \int_a^b f(x,a)\,dx \quad \text{then} \quad \frac{dI}{d\alpha} = \int_a^b \frac{\partial I}{\partial \alpha}\,dx}$$

$$I(\alpha) = \int_0 \frac{x^{\alpha} - 1}{\log x}$$

① $I =$

$$\frac{dI}{d\alpha} = \int_a^b \frac{\partial f}{\partial \alpha}\, dx$$


287 744
I(α) = ∫ (x^α − 1) / log x
0
2 di. α
3 in x
4
α = 0
1 I =
1 dI/dα = ∫ from x=a to x=b ∂f/∂α dx
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Type A) with one parameter

① Prove that $\displaystyle\int_0^1 \frac{x^\alpha - 1}{\log x}\,dx = \log(1+\alpha)$, $\alpha \geq 0$

Hence evaluate $\displaystyle\int_0^1 \frac{x^7 - 1}{\log x}\,dx$

$$I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x}\,dx$$

① Prove that $\int_0^1 \frac{x^\alpha - 1}{\log x}\, dx = \log(1+\alpha)$, $\alpha \geq 0$

Hence evaluate $\int_0^1 \frac{x^{7} - 1}{\log x}\, dx$ $\quad 7 \to \alpha$

$\alpha = 7$

log 8.

$$I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x}\, dx$$

$$\int_0 \frac{\quad}{\log x}\, dx$$

$$\boxed{\frac{dI}{d\alpha} = \int_0^1 \frac{\partial f}{\partial \alpha}\, dx}$$

$$\frac{d}{d\alpha} x^\alpha = x^\alpha \log x$$

$$\frac{d}{dx} a^x = a^x \log a$$

(1) D..... w.r.t α

$$\frac{dI}{d\alpha} = \int_0^1 \frac{x^\alpha \cancel{\log x}}{\cancel{\log x}}\, dx$$

$$=$$

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① D..... w.r.t α

$$\frac{d}{dx} a^x = a^x \log a$$

$$\frac{dI}{d\alpha} = \int_0^1 \frac{x^\alpha \cancel{\log x}}{\cancel{\log x}} dx$$

$$= \int_0^1 x^\alpha \, dx$$

$$= \left[\frac{x^{\alpha+1}}{\alpha+1}\right]_0^1$$

$$x^n \quad \frac{x^{n+1}}{n+1}$$

$$= \left[\frac{1^{\alpha+1}}{\alpha+1} - \frac{0^{\alpha+1}}{\alpha+1}\right]$$

549 888

$$= \int_0 x^\alpha \, dx$$

$$= \left[\frac{x^{\alpha+1}}{\alpha+1}\right]_0^1$$

$$= \left[\frac{1^{\alpha+1}}{\alpha+1}\right]$$

$$\frac{dI}{d\alpha} = \frac{1}{\alpha+1}$$

$$x^n \qquad \frac{x^{n+1}}{n+1}$$

$$1^{3+}$$

549 888

$$\left[\frac{\quad}{\alpha+1}\right]_0 \qquad x \quad \frac{x}{n+1}$$

$$= \left[\frac{1^{\alpha+1}}{\alpha+1}\right] \qquad 1^{3+\alpha}$$

$$\frac{dI}{d\alpha} = \frac{1}{\alpha+1}$$

$$I = \int \frac{1}{\alpha+1}\, d\alpha$$

$$\int \frac{1}{x}\, dx = \log x$$

$$I = \log(\alpha+1) + c$$

$$I = \int \frac{\quad}{\alpha+1}$$

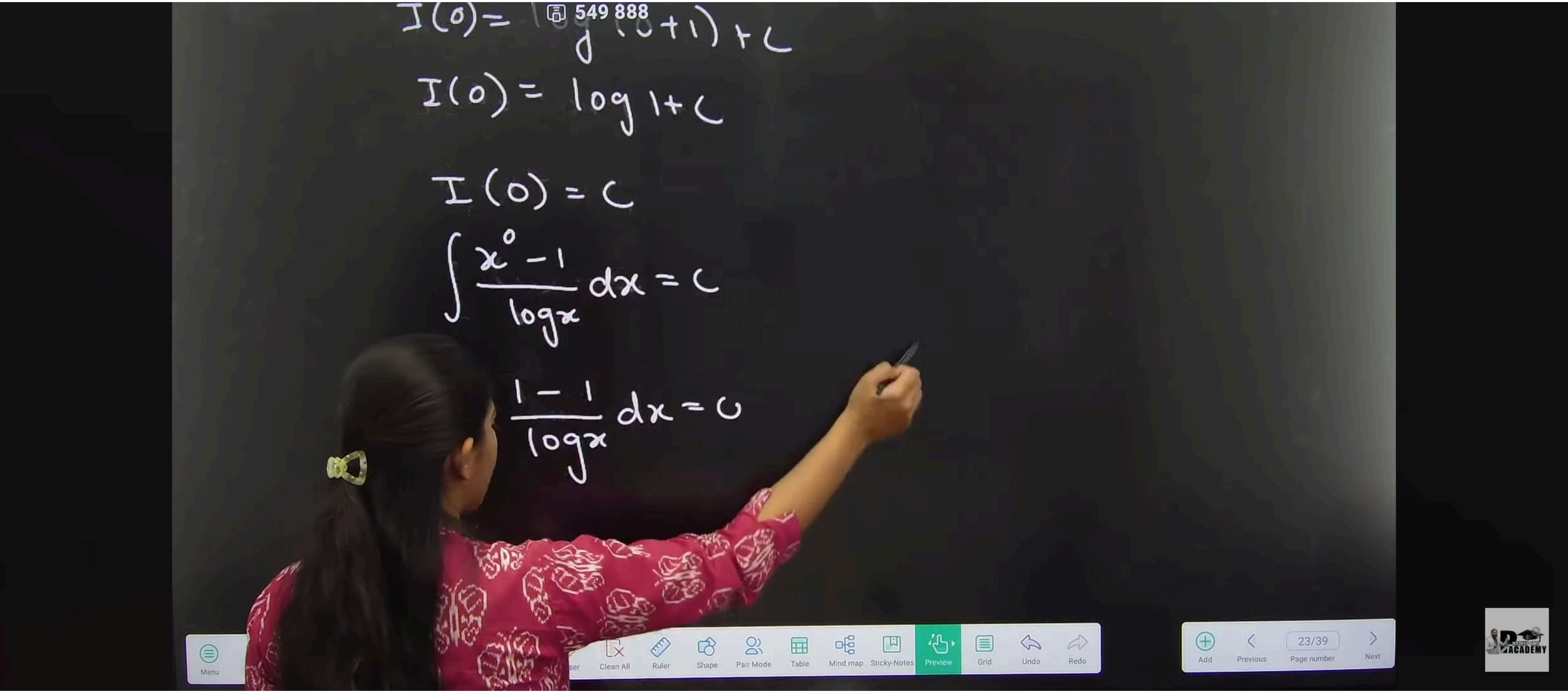
$$\int \frac{1}{x}\,dx = \log x$$

$$I = \log(\alpha+1) + c$$

Put $\alpha = 0$

$$I(0) = \log(0+1) + c$$

$$I(0) = \log 1 + c$$


549 888
$I(0) = \log 1 + c$
$I(0) = c$
$\int \frac{x^0 - 1}{\log x} dx = c$
$\frac{1-1}{\log x} dx = c$
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$$I(0) = C$$

$$\int_0^1 \frac{x^0 - 1}{\log x}\,dx = C$$

$$\int_0^1 \frac{1 - 1}{\log x}\,dx = C$$

$$\int_0^1 0\,dx = C$$

$$\boxed{C = 0}$$

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$$\boxed{C = 0}$$

$$I(\alpha) = \log(1+\alpha) + C$$

$$I(\alpha) = \log(1+\alpha)$$

$$\int_0^1 \frac{x^7 - 1}{\log 7} = \log(1+7)$$

$$\alpha = 7 \qquad = \log 8$$

Corollary : Prove that $\int_0^1 \frac{x^\alpha - x^\beta}{\log x}\,dx = \log\left(\frac{1+\alpha}{1+\beta}\right)$

$= \log(1+\alpha) - \log(1+\beta)$

Hence Evaluate $\int_0^1 \frac{x^7 - x^3}{\log x}\,dx$

025 600

Hence Evaluate $\int_0^1 \frac{x^\alpha - x^\beta}{\log x}\, dx$

$$\log(\alpha+1) = \int_0^1 \frac{x^\alpha - 1}{\log x} \qquad \log(\beta+1) = \int_0^1 \frac{x^\beta - 1}{\log x}$$

$$\log(1+\alpha) - \log(1+\beta) = \int_0^1 \frac{x^\alpha - \cancel{1} - x^\beta + \cancel{1}}{\log x}$$

$$\log\left(\frac{1+\alpha}{1+\beta}\right) = \int_0^1 \frac{x^\alpha - x^\beta}{\log x}$$

025 600

Corollary : Prove that $\int_0^1 \frac{x^{\alpha} - x^{\beta}}{\log x}\, dx = \log\left(\frac{1+\alpha}{1+\beta}\right)$

$= \log(1+\alpha) - \log(1+\beta)$

Hence Evaluate $\int_0^1 \frac{x^{7} - x^{3}}{\log x}\, dx$

$\alpha = 7 \qquad \beta = 3$

$\log(\alpha+1) = \int_0^1 \frac{x^{\alpha} - 1}{\log x} \qquad \log(\beta+1) = \int_0^1 \frac{x^{\beta} - 1}{\log x}$

$\log(1+\alpha) - \log(1+\beta) \ \int^1 x^{\alpha} \ \ \beta$

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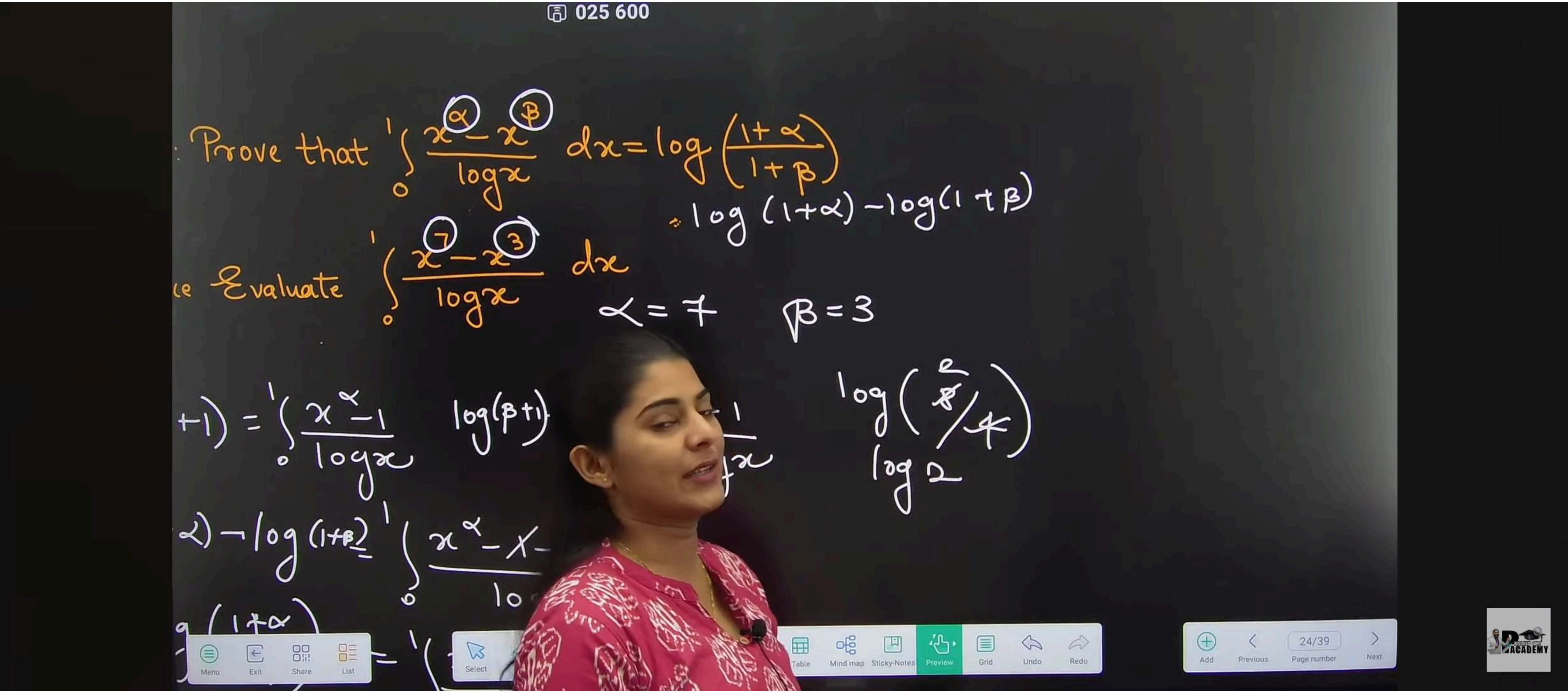
025 600
Prove that $\int_0^1 \frac{x^{\alpha} - x^{\beta}}{\log x}\, dx = \log\left(\frac{1+\alpha}{1+\beta}\right)$
$= \log(1+\alpha) - \log(1+\beta)$
Evaluate $\int_0^1 \frac{x^{7} - x^{3}}{\log x}\, dx$
$\alpha = 7 \quad \beta = 3$
$+1) = \int_0^1 \frac{x^{\alpha} - 1}{\log x}$
$\log(\beta+1)$
$\log\left(\cancel{8}/\cancel{4}\right)$
$\log 2$
$\alpha) - \log(1+\beta)$
$\int_0^1 \frac{x^{\alpha} - x}{}$
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② Show that $\displaystyle\int_0^\infty \frac{\log(1+ax^2)}{x^2}\,dx = \pi\sqrt{a}$ $(a>0)$

Hence evaluate $\displaystyle\int_0^\infty \frac{\log(1+x^2)}{x^2}\,dx$

025 600

(2) Show that $\displaystyle\int_0^\infty \frac{\log(1+ax^2)}{x^2}\,dx = \pi\sqrt{a} \quad (a>0)$

1

Hence evaluate $\displaystyle\int_0^\infty \frac{\log(1+x^2)}{x^2}\,dx = \pi$

1

Hence evaluate $\int_0^\infty \frac{\log(1+x^2)}{x^2}\,dx = \pi$

$$I(\alpha) = \int_0^\infty \frac{\log(1+ax^2)}{x^2}\,dx$$

$$\frac{dI}{da} = \int_0^1 \frac{\partial f}{\partial a}\,dx$$

$$\frac{dI}{da} = \int_0^\infty \frac{1}{\cancel{x^2}(1+ax^2)}\cancel{x^2}\,dx$$

025 600

$$\frac{dI}{da} = \int_0^\infty \frac{1}{\cancel{x^2}(1+ax^2)}\cancel{x^2}\,dx$$

$$\frac{dI}{da} = \int_0^\infty \frac{1}{1+ax^2}\,dx$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$$

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$$\frac{\quad}{da} \quad \int_0 \frac{\quad}{x^2(1+ax^2)}$$

$$\frac{dI}{da} = \int_0^\infty \frac{1}{1^2+(\sqrt{a}x)^2}\,dx$$

$$2 = \sqrt{2}\cdot\sqrt{2}$$

$$(\sqrt{a})^2 \qquad 2 = (\sqrt{2})^2$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$(\sqrt{2})^2 = 2$$

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$(\sqrt{a})^2$

$$(\sqrt{2})^2 = 2$$

$$\int_0^\infty \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$= \frac{1}{1}\tan^{-1}\left(\frac{\sqrt{a}x}{1}\right) + c$$

$$\frac{dI}{da} = \frac{\tan^{-1}(\sqrt{a}x) + c}{\sqrt{a}}$$

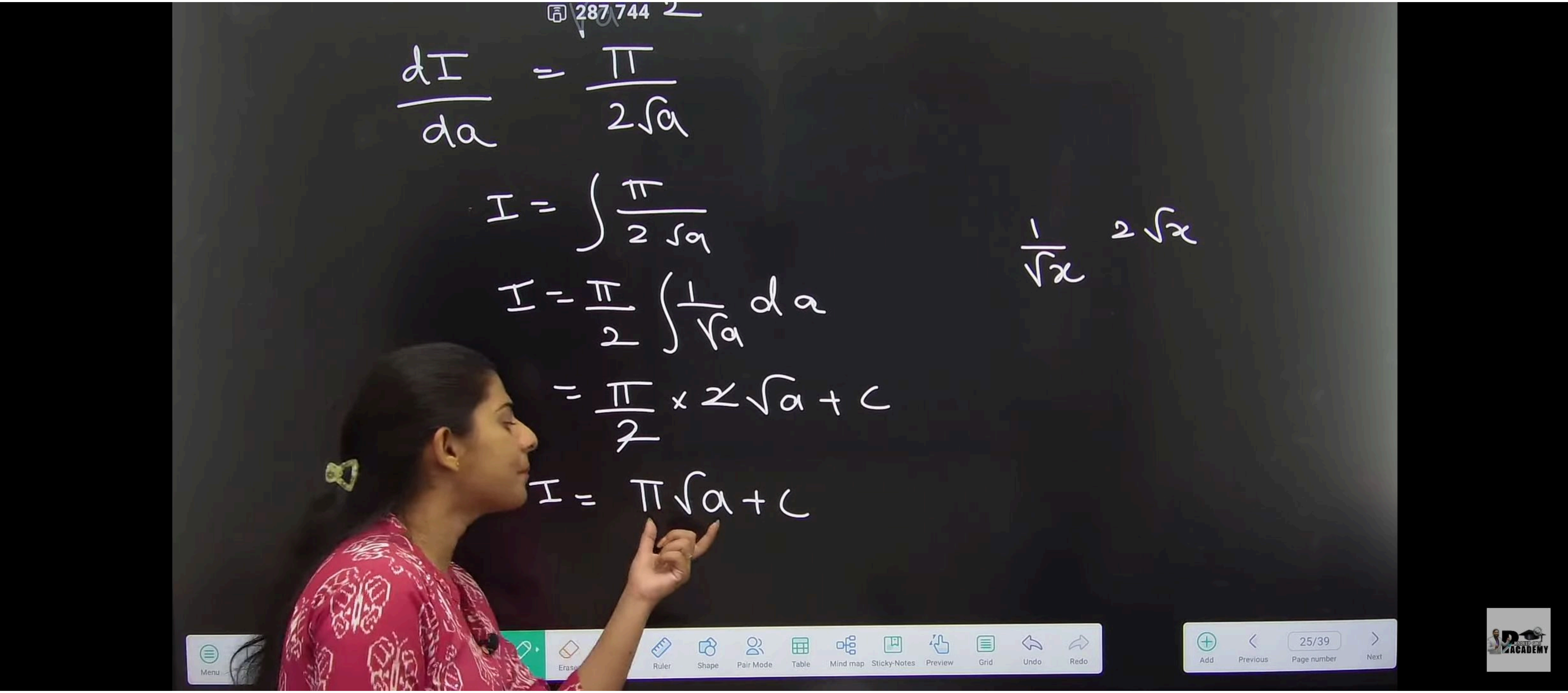
$$= \frac{1}{\sqrt{a}}\tan^{-1}(\sqrt{a}x) + c$$

$$= \frac{1}{1}\tan^{-1}\left(\frac{\sqrt{a}x}{1}\right) + c$$

$$\frac{dI}{da} = \left[\frac{\tan^{-1}(\sqrt{a}x)}{\sqrt{a}}\right]_0^{\infty} + c$$

$$= \left[\frac{1}{\sqrt{a}}\tan^{-1}(\sqrt{a}x)\right]_0^{\infty} + c$$

$$= \frac{1}{\sqrt{a}}\,\frac{\pi}{2}$$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$\frac{dI}{da} = \frac{\pi}{2\sqrt{a}}$
$I = \int \frac{\pi}{2\sqrt{a}}$
$I = \frac{\pi}{2} \int \frac{1}{\sqrt{a}} \, da$
$= \frac{\pi}{\cancel{2}} \times \cancel{2}\sqrt{a} + c$
$I = \pi\sqrt{a} + c$
$\frac{1}{\sqrt{x}}$ $2\sqrt{x}$
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$$I = \int \frac{\quad}{2\sqrt{a}}$$

$$\frac{1}{\sqrt{x}} \quad 2\sqrt{x}$$

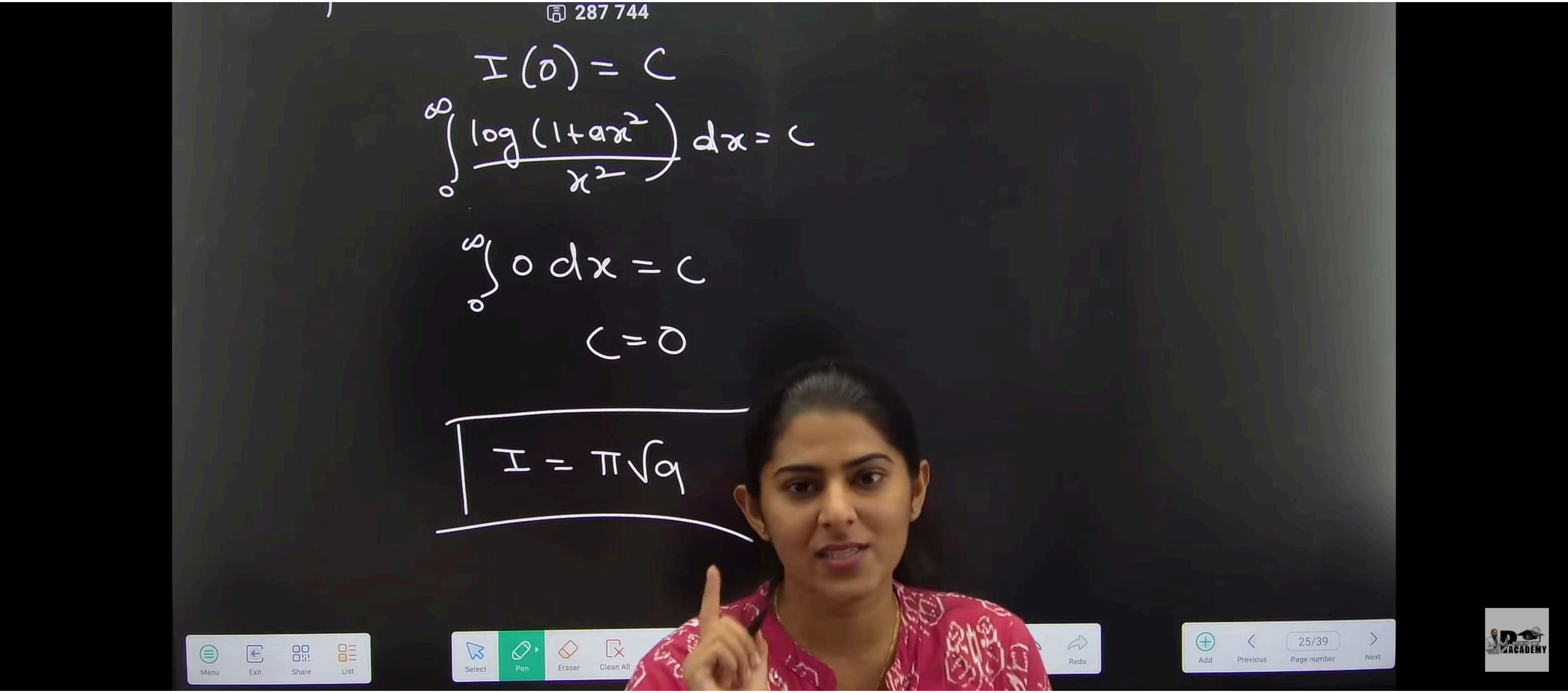
$$I = \frac{\pi}{2}\int \frac{1}{\sqrt{a}}\,da$$

$$= \frac{\pi}{\cancel{2}} \times \cancel{2}\sqrt{a} + c$$

$$I = \pi\sqrt{a} + c$$

Put $\Rightarrow a = 0$

$$I(0) = C$$



287 744
$I(0) = C$
$\int_0^\infty \left(\frac{\log(1+ax^2)}{x^2} \right) dx = c$
$\int_0^\infty 0 \, dx = c$
$c = 0$
$\boxed{I = \pi\sqrt{a}}$
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Show that $\int_0^\infty \frac{\log(1+ax^2)}{x^2}\,dx = \pi\sqrt{a}$ $\quad (a>0)$ $\quad a=1$

1

Hence evaluate $\int_0^\infty \frac{\log(1+x^2)}{x^2}\,dx = \pi$

$$I(\alpha) = \int^{\infty} \log(1+ax^2)\,dx$$