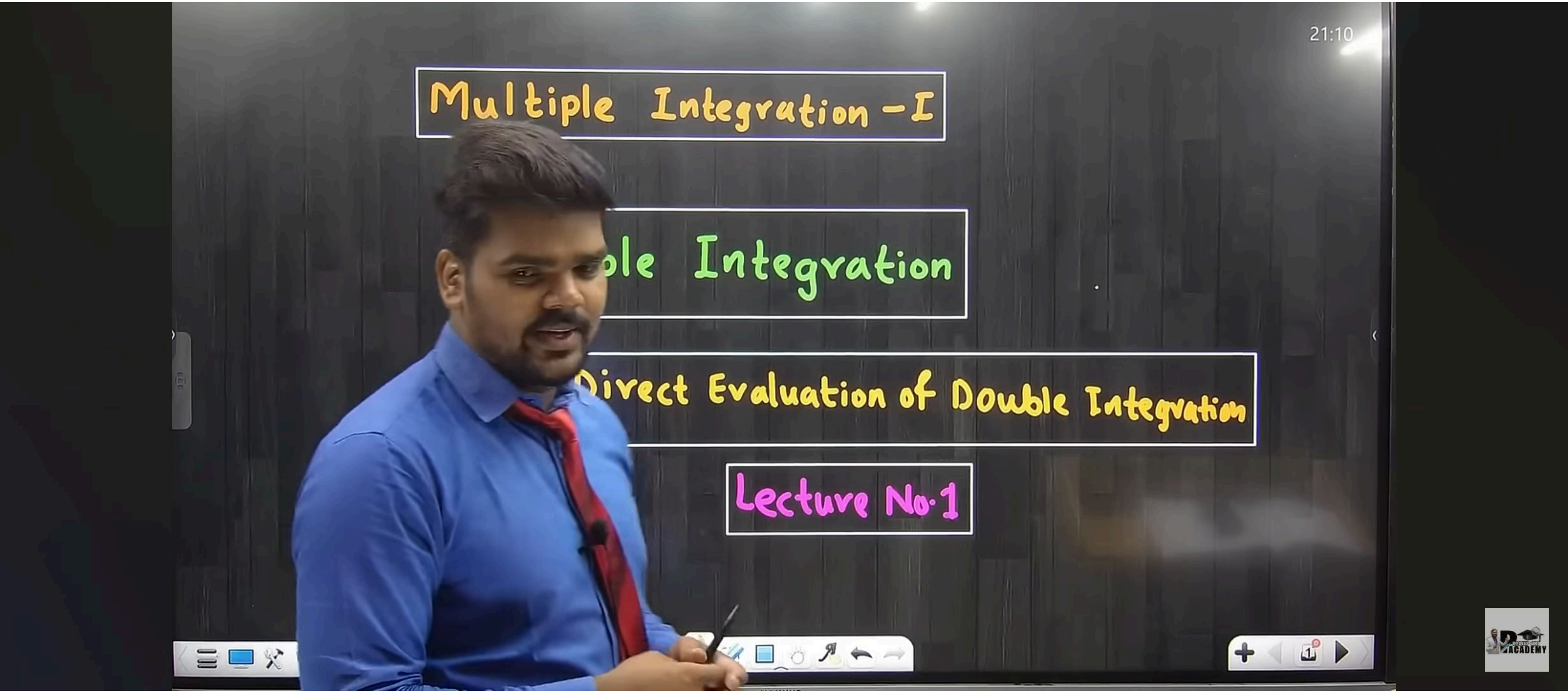
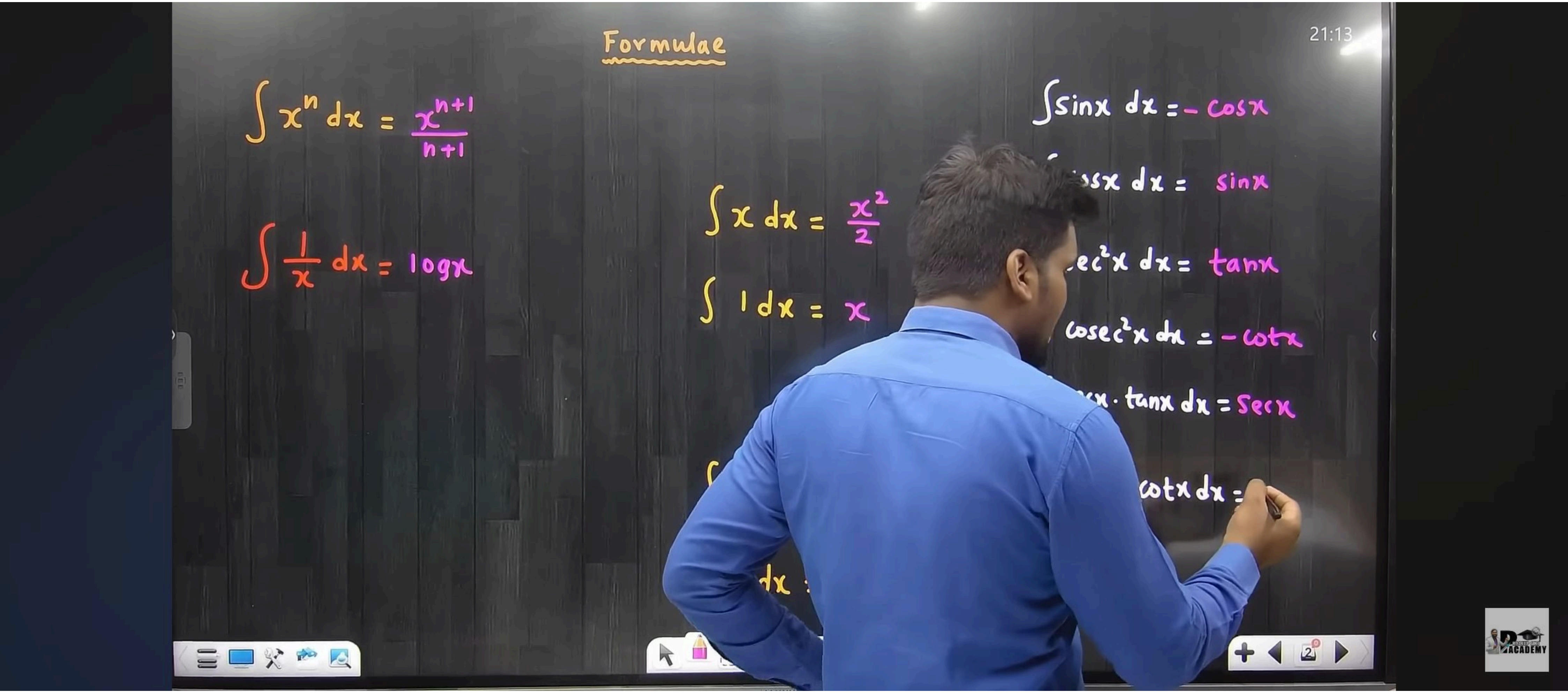
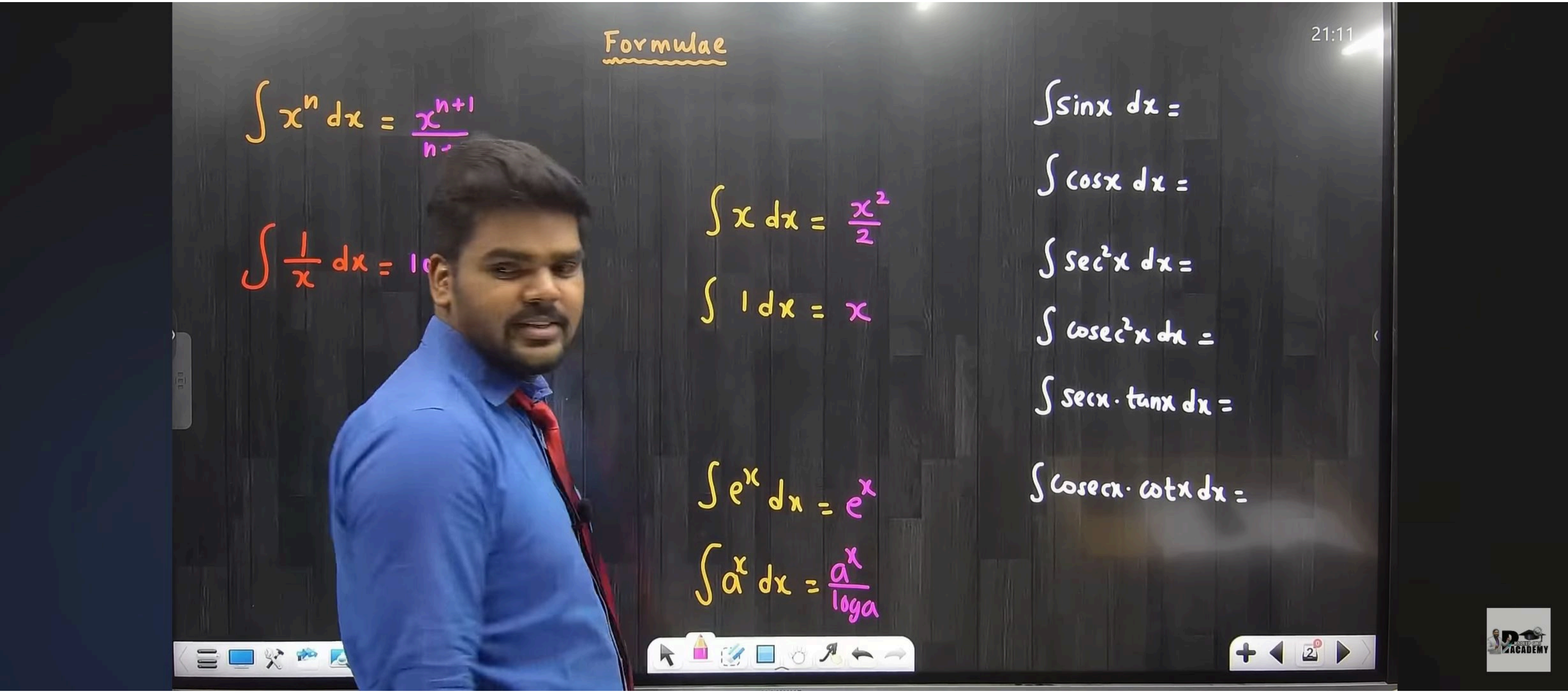
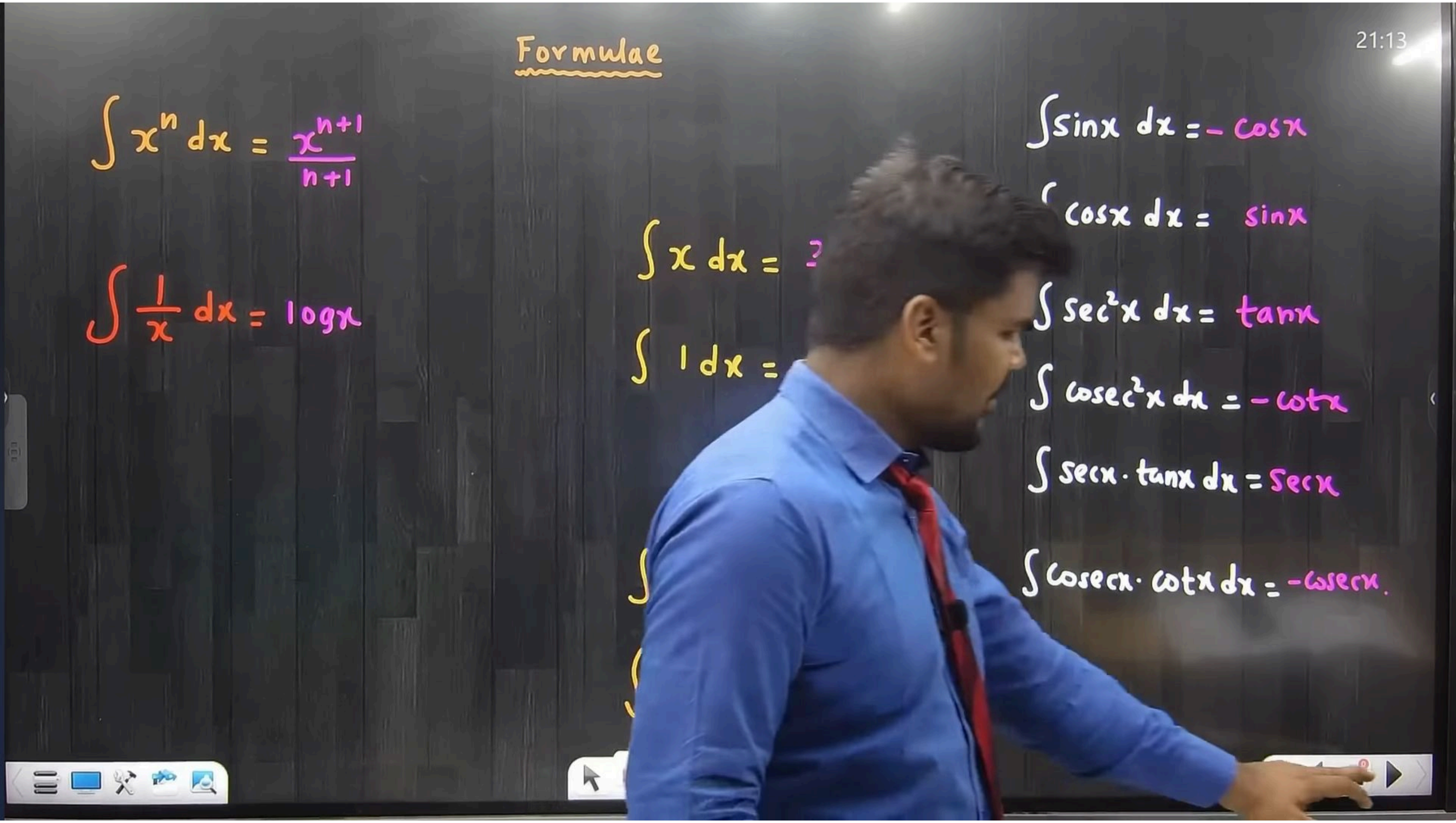

21:10
Multiple Integration - I
Integration
Direct Evaluation of Double Integration
Lecture No. 1

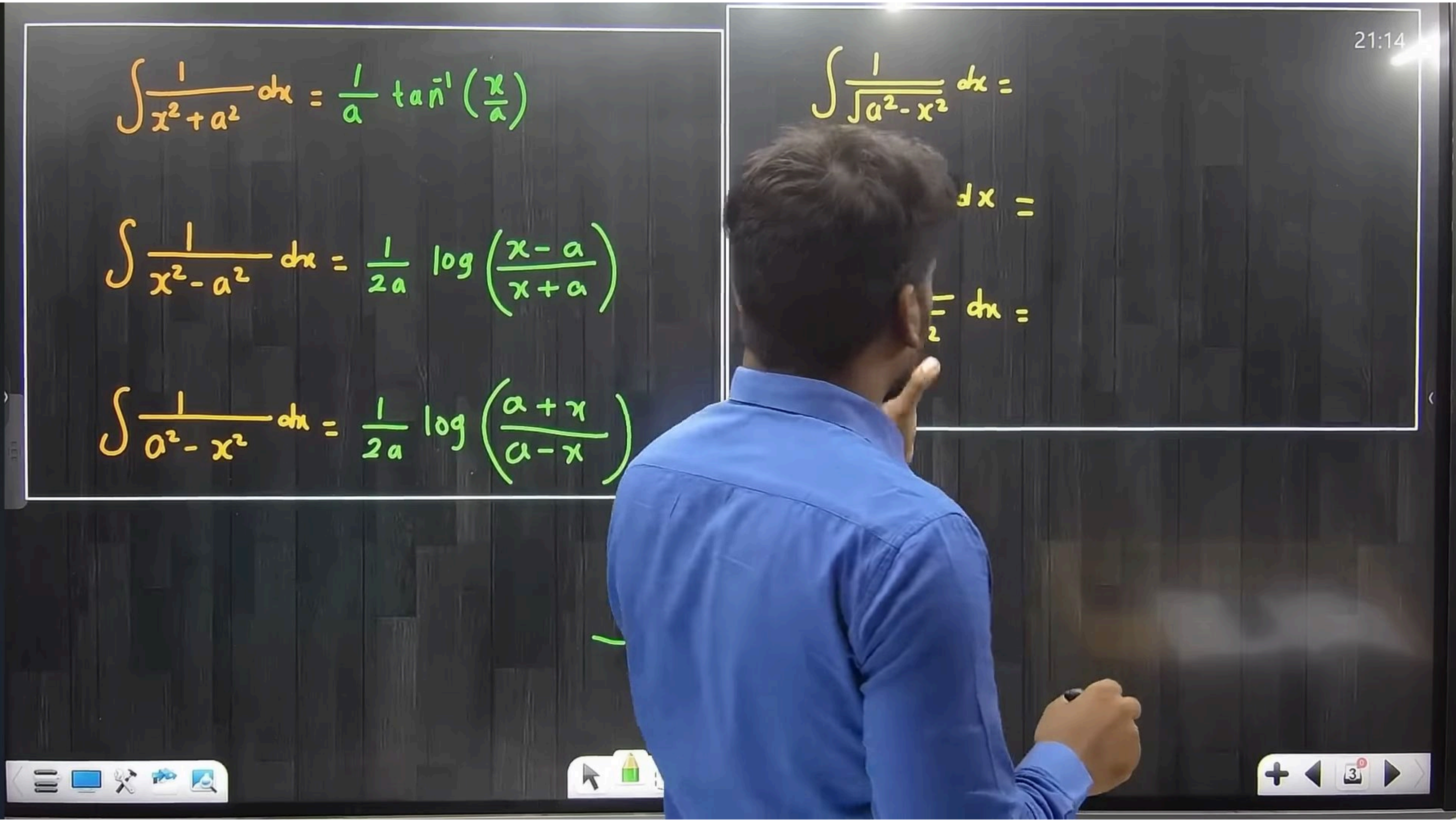


Formulae
21:13
$\int x^n \, dx = \frac{x^{n+1}}{n+1}$
$\int \frac{1}{x} \, dx = \log x$
$\int x \, dx = \frac{x^2}{2}$
$\int 1 \, dx = x$
$\int \sin x \, dx = -\cos x$
$dx = \sin x$
$x \, dx = \tan x$
$\mathrm{cosec}^2 x \, dx = -\cot x$
$\tan x \, dx = \sec x$
$\cot x \, dx =$
ACADEMY

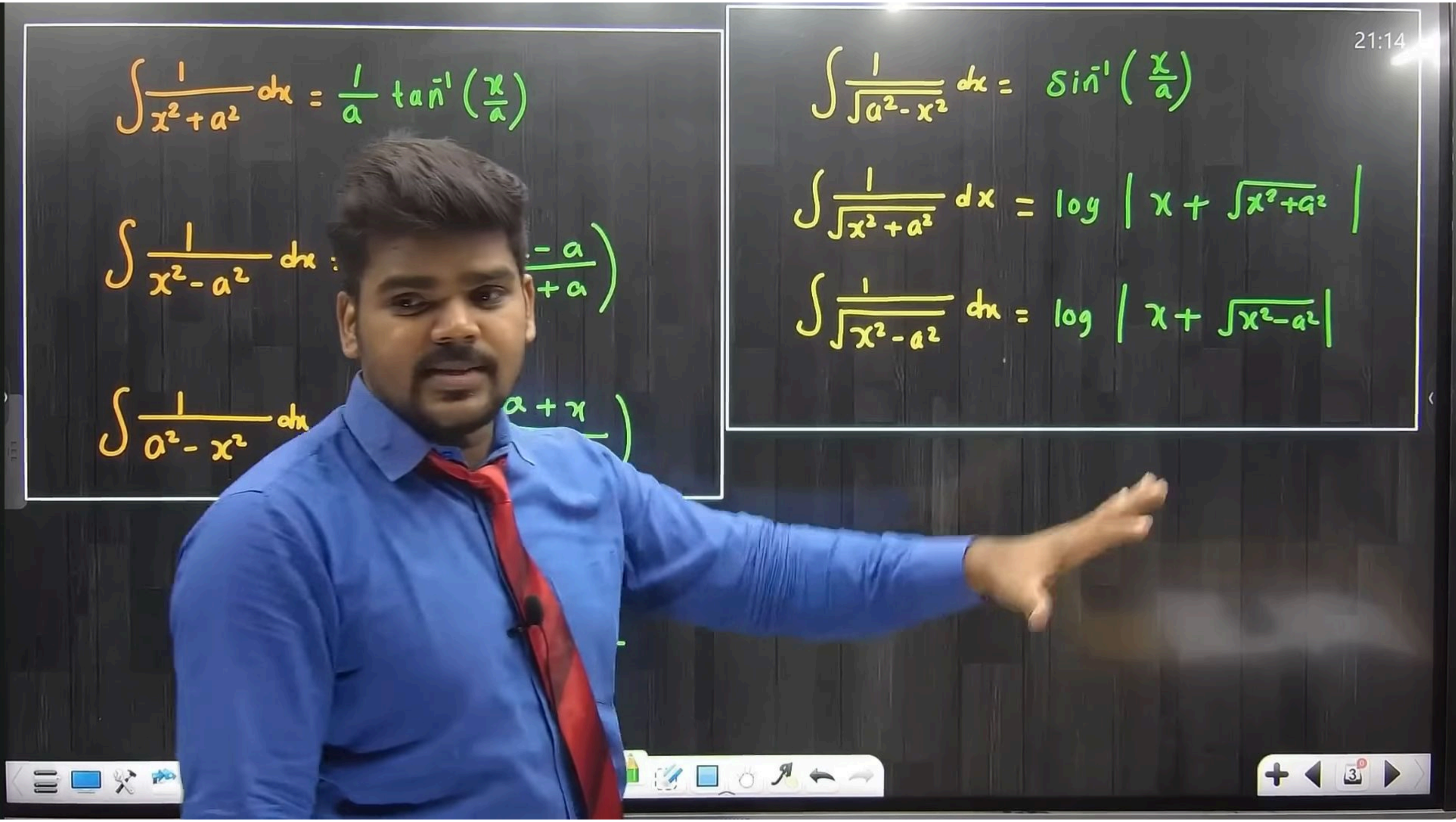

21:11
Formulae
$\int x^n \, dx = \frac{x^{n+1}}{n}$
$\int \frac{1}{x} \, dx = l$
$\int x \, dx = \frac{x^2}{2}$
$\int 1 \, dx = x$
$\int e^x \, dx = e^x$
$\int a^x \, dx = \frac{a^x}{\log a}$
$\int \sin x \, dx =$
$\int \cos x \, dx =$
$\int \sec^2 x \, dx =$
$\int \mathrm{cosec}^2 x \, dx =$
$\int \sec x \cdot \tan x \, dx =$
$\int \mathrm{cosec}\, x \cdot \cot x \, dx =$
ACADEMY

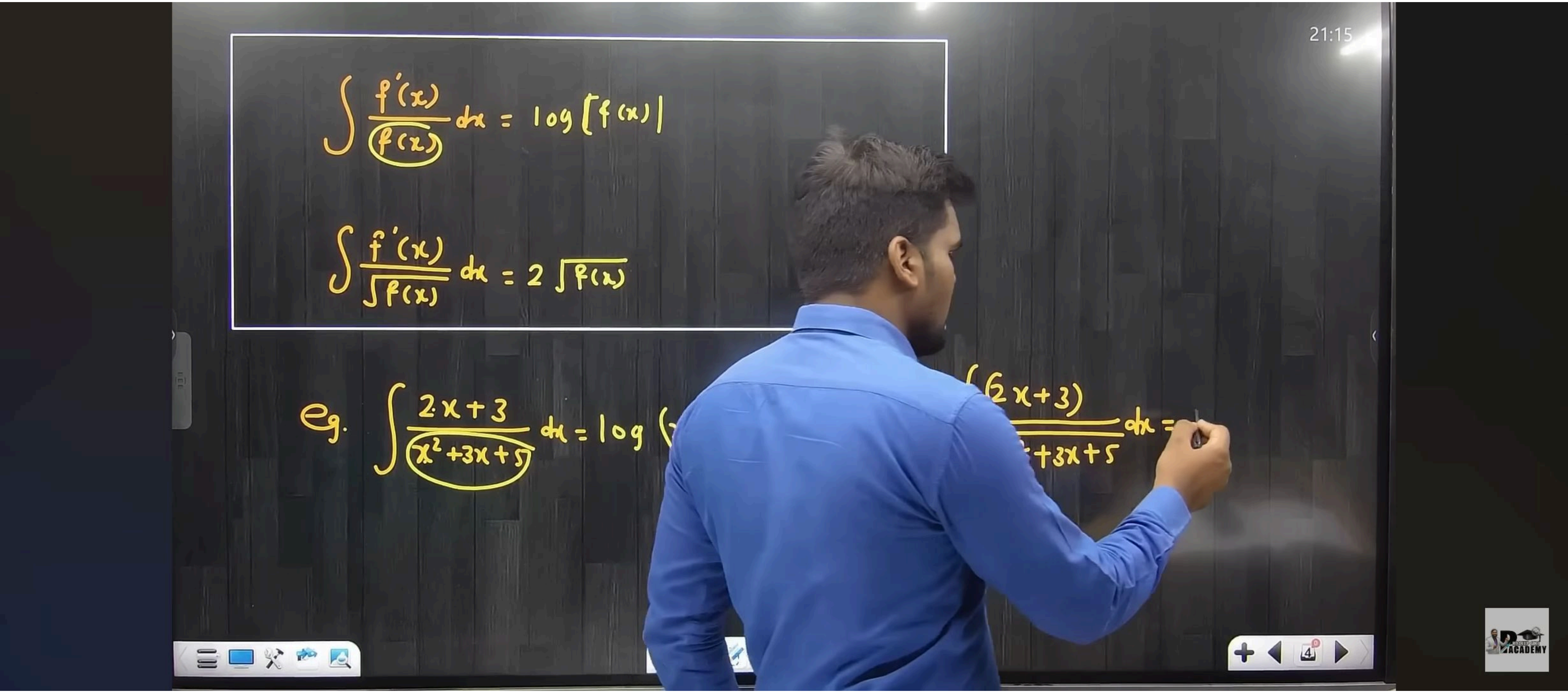


Formulae
$\int x^n\,dx = \frac{x^{n+1}}{n+1}$
$\int \frac{1}{x}\,dx = \log x$
$\int x\,dx =$
$\int 1\,dx =$
$\int \sin x\,dx = -\cos x$
$\int \cos x\,dx = \sin x$
$\int \sec^2 x\,dx = \tan x$
$\int \mathrm{cosec}^2 x\,dx = -\cot x$
$\int \sec x \cdot \tan x\,dx = \sec x$
$\int \mathrm{cosec}\, x \cdot \cot x\,dx = -\mathrm{cosec}\, x$
21:13

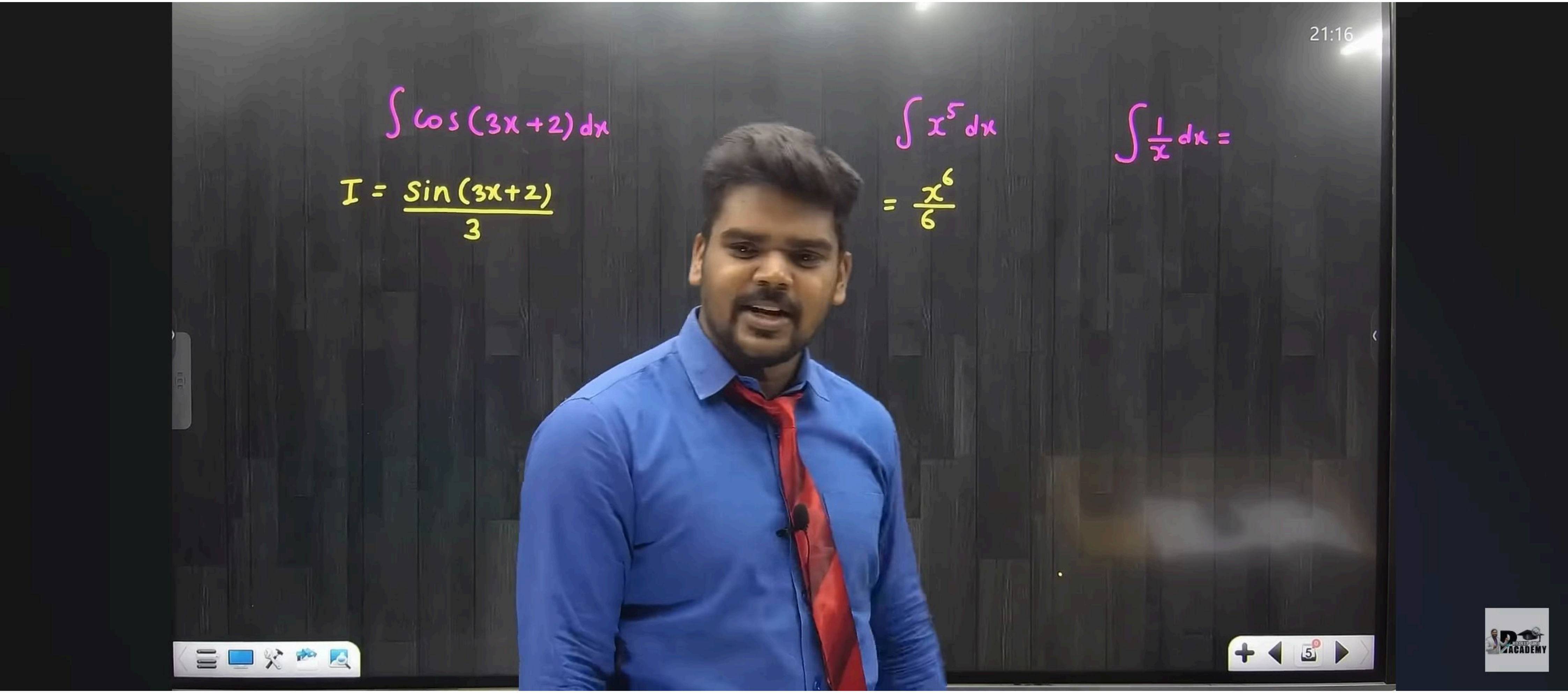


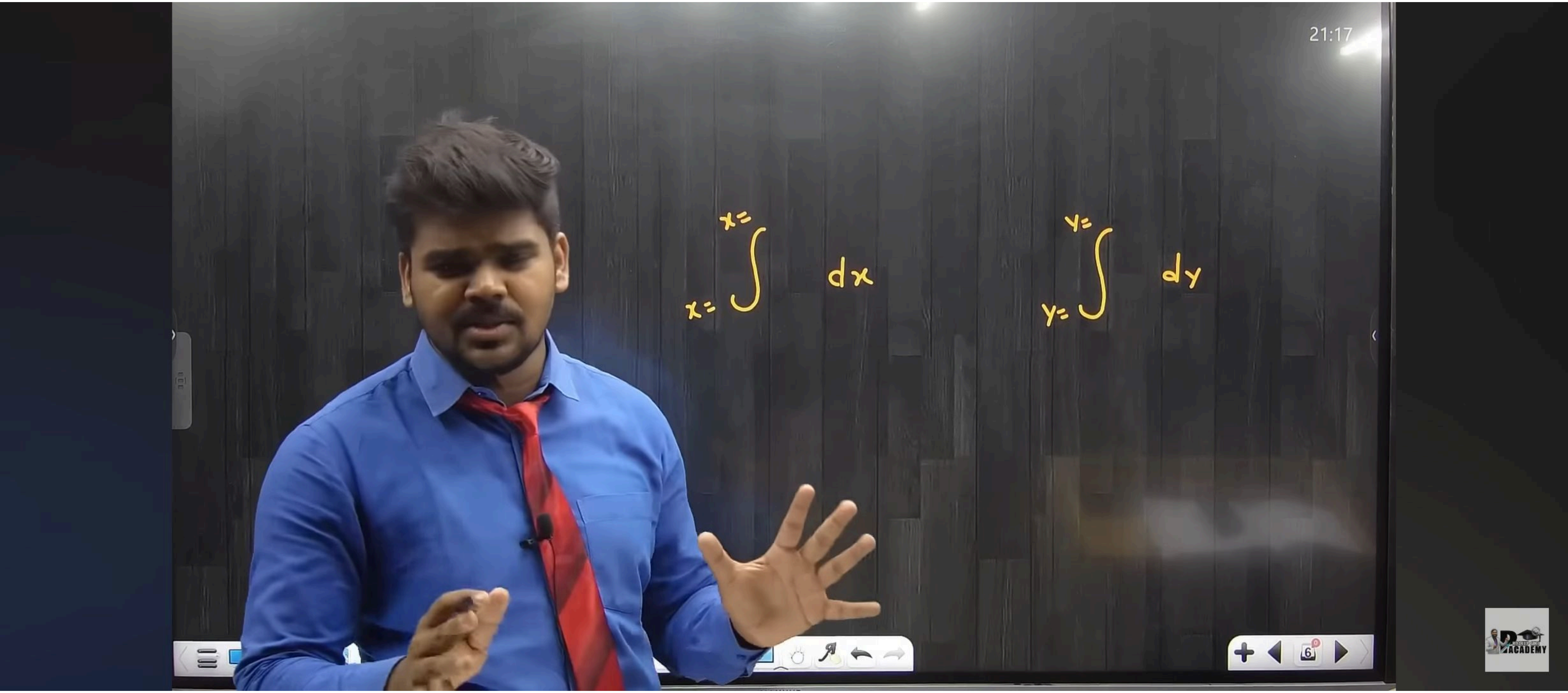
$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$
$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)$
21:14
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx =$
$dx =$
$dx =$
ACADEMY


$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\int \frac{1}{x^2-a^2}dx$
$\int \frac{1}{a^2-x^2}dx$
$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)$
$\int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|$
$\int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|$
21:14

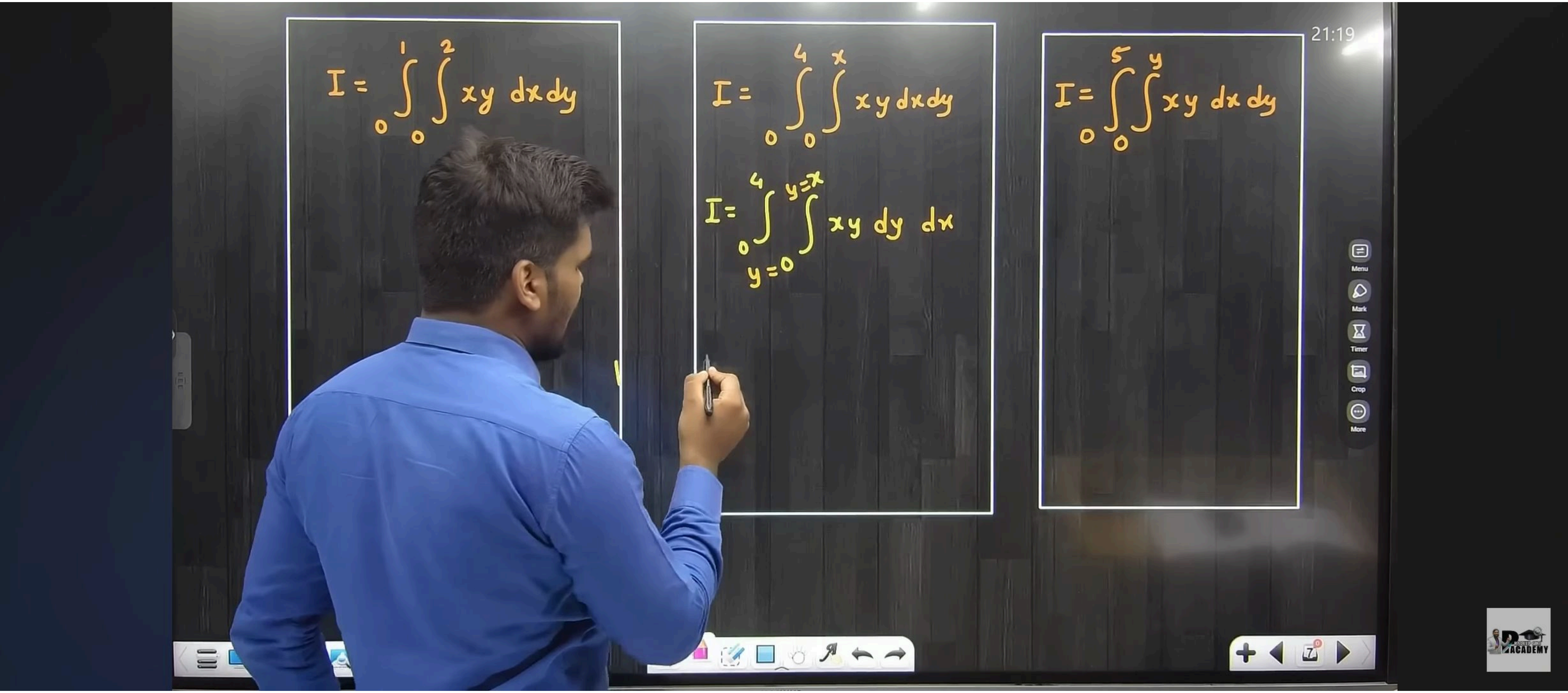


21:15
$\int \frac{f'(x)}{f(x)} dx = \log|f(x)|$
$\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)}$
eg. $\int \frac{2x+3}{x^2+3x+5} dx = \log ($
$\frac{(2x+3)}{+3x+5} dx =$
ACADEMY


21:16
$\int \cos(3x+2)\,dx$
$I = \frac{\sin(3x+2)}{3}$
$\int x^5\,dx$
$= \frac{x^6}{6}$
$\int \frac{1}{x}\,dx =$
ACADEMY



21:17
$\int_{x=}^{x=} dx$
$\int_{y=}^{y=} dy$
ACADEMY


$I = \int_0^1 \int_0^2 xy \, dx \, dy$
$I = \int_0^4 \int_0^x xy \, dx \, dy$
$I = \int_0^4 \int_{y=0}^{y=x} xy \, dy \, dx$
$I = \int_0^5 \int_0^y xy \, dx \, dy$
21:19
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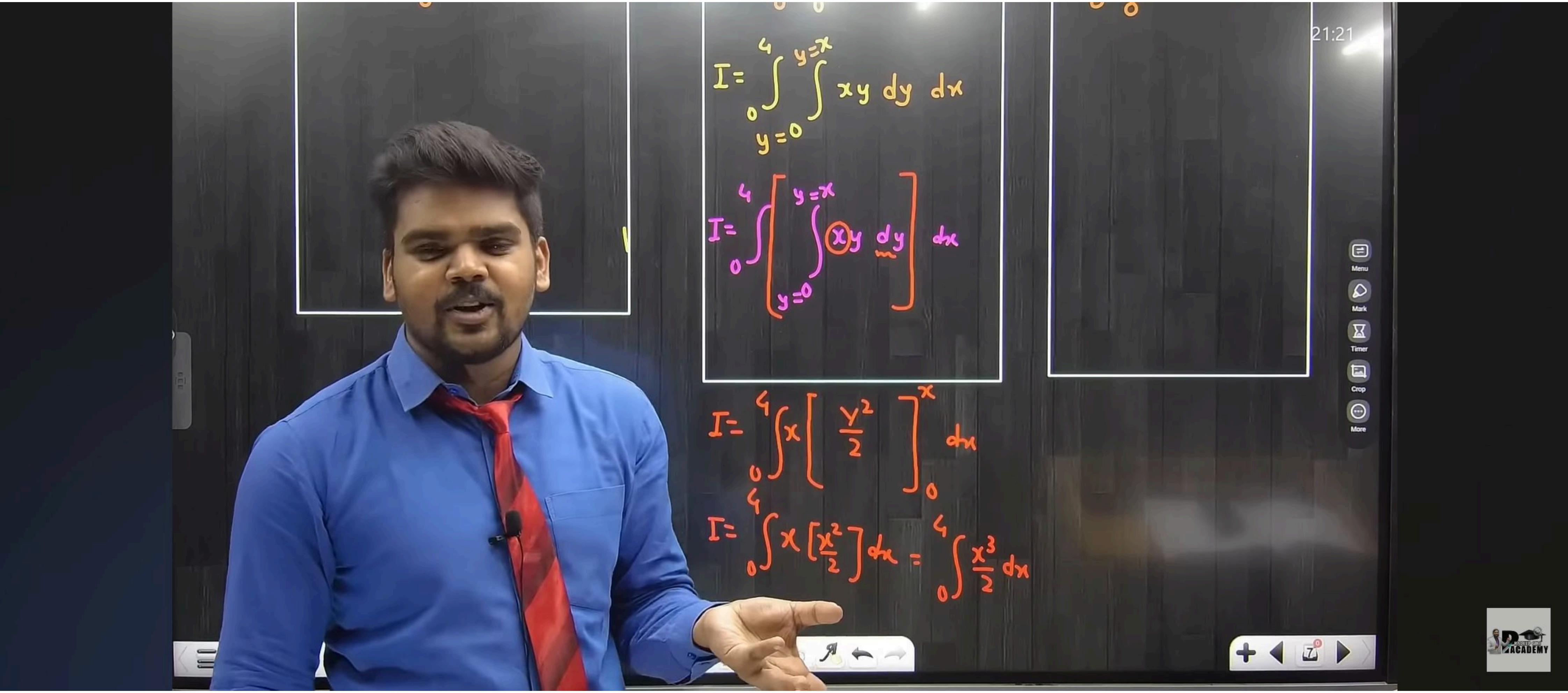

$I = \int_{0}^{4} \int_{y=0}^{y=x} xy \, dy \, dx$
$I = \int_{0}^{4} \left[\int_{y=0}^{y=x} x y \, dy \right] dx$
$I = \int_{0}^{4} x \left[\frac{y^2}{2} \right]_{0}^{x} dx$
$I = \int_{0}^{4} x \left[\frac{x^2}{2} \right] dx = \int_{0}^{4} \frac{x^3}{2} dx$
21:21
Menu
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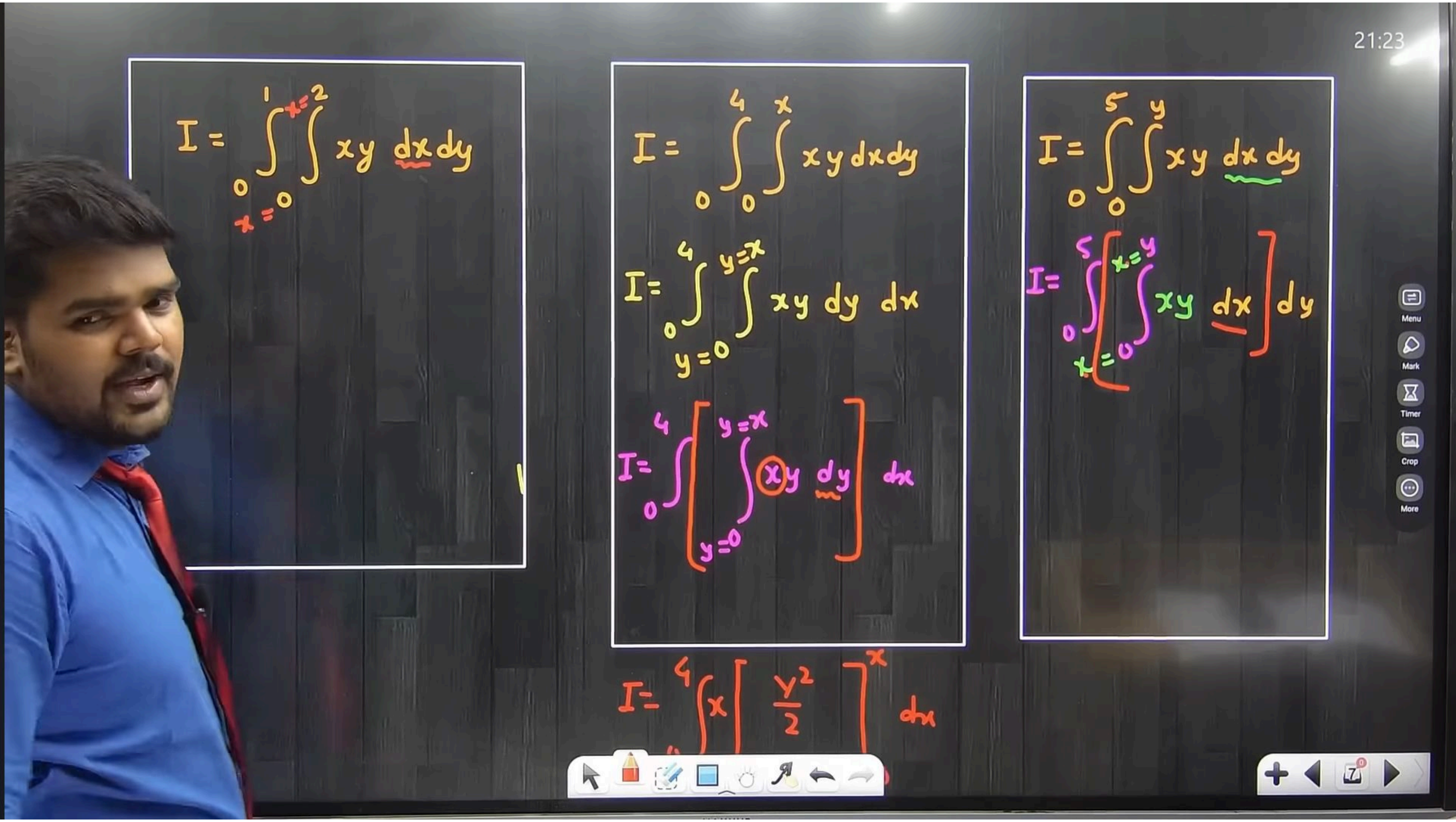
Crop
More
ACADEMY


21:23
$I = \int_0^1 \int_{x=0}^{x=2} xy \, dx \, dy$
$I = \int_0^4 \int_0^x xy \, dx \, dy$
$I = \int_0^4 \int_{y=0}^{y=x} xy \, dy \, dx$
$I = \int_0^4 \left[\int_{y=0}^{y=x} xy \, dy \right] dx$
$I = \int_0^4 x \left[\frac{y^2}{2} \right]^x dx$
$I = \int_0^5 \int_0^y xy \, dx \, dy$
$I = \int_0^5 \left[\int_{x=0}^{x=y} xy \, dx \right] dy$
Menu
Mark
Timer
Crop
More
ACADEMY

$$\int_0^1 \int_0^x xy \, dy \, dx$$

$$I = \int_0^1 \int_{y=0}^{y=x} xy \, dy \, dx$$

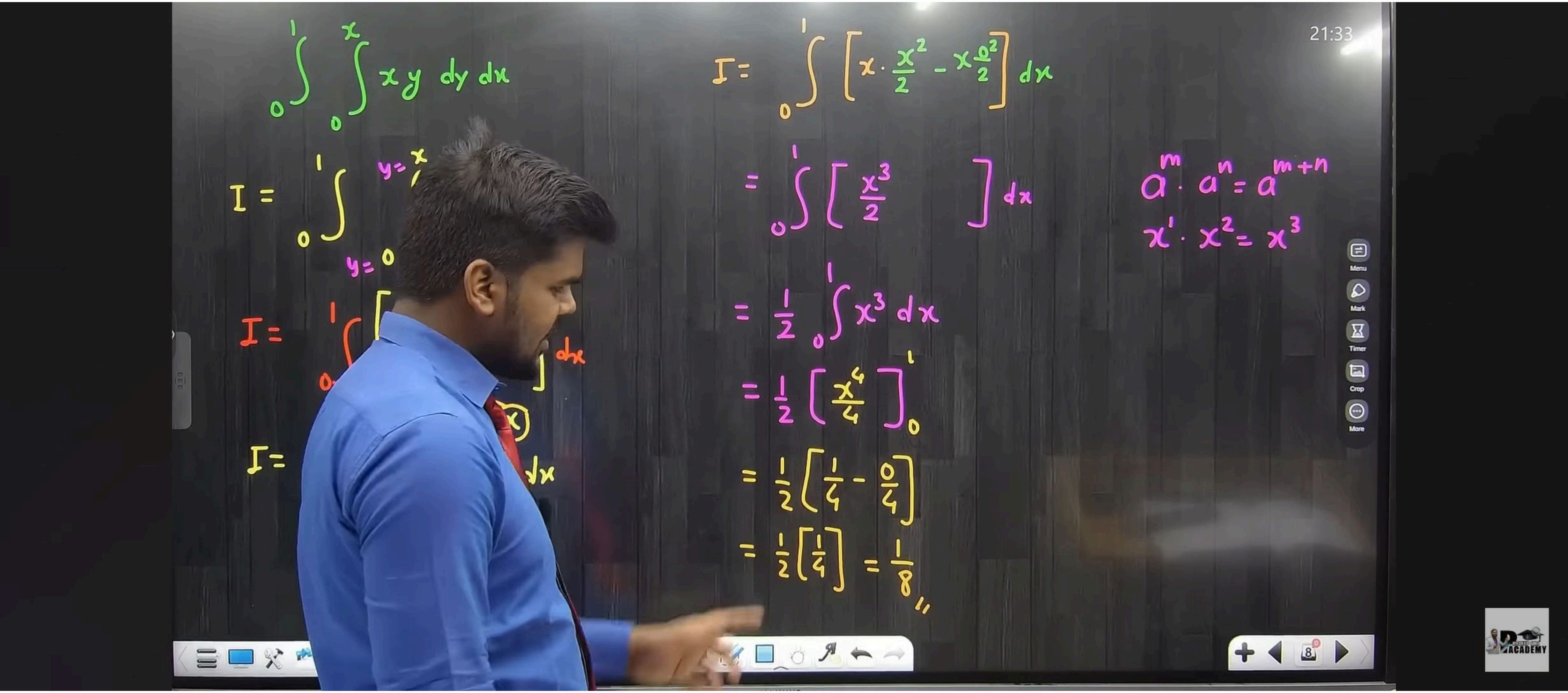
$$I = \int_0^1 \left[\int_0^x xy \, dy \right] dx$$

$$I = \int_0^1 \left[x \cdot \frac{y^2}{2} \right]_0^x dx$$

$$I = \int_0^1 \left[x \cdot \frac{x^2}{2} - x \frac{0^2}{2} \right] dx$$

$$= \int_0^1 \Big[\quad x$$

a



21:33
$\int_0^1 \int_0^x xy \, dy \, dx$
$I = \int_0^1 \left[x \cdot \frac{x^2}{2} - x \frac{0^2}{2} \right] dx$
$= \int_0^1 \left[\frac{x^3}{2} \right] dx$
$a^m \cdot a^n = a^{m+n}$
$x^1 \cdot x^2 = x^3$
$= \frac{1}{2} \int_0^1 x^3 \, dx$
$= \frac{1}{2} \left[\frac{x^4}{4} \right]_0^1$
$= \frac{1}{2} \left[\frac{1}{4} - \frac{0}{4} \right]$
$= \frac{1}{2} \left[\frac{1}{4} \right] = \frac{1}{8}$
Menu
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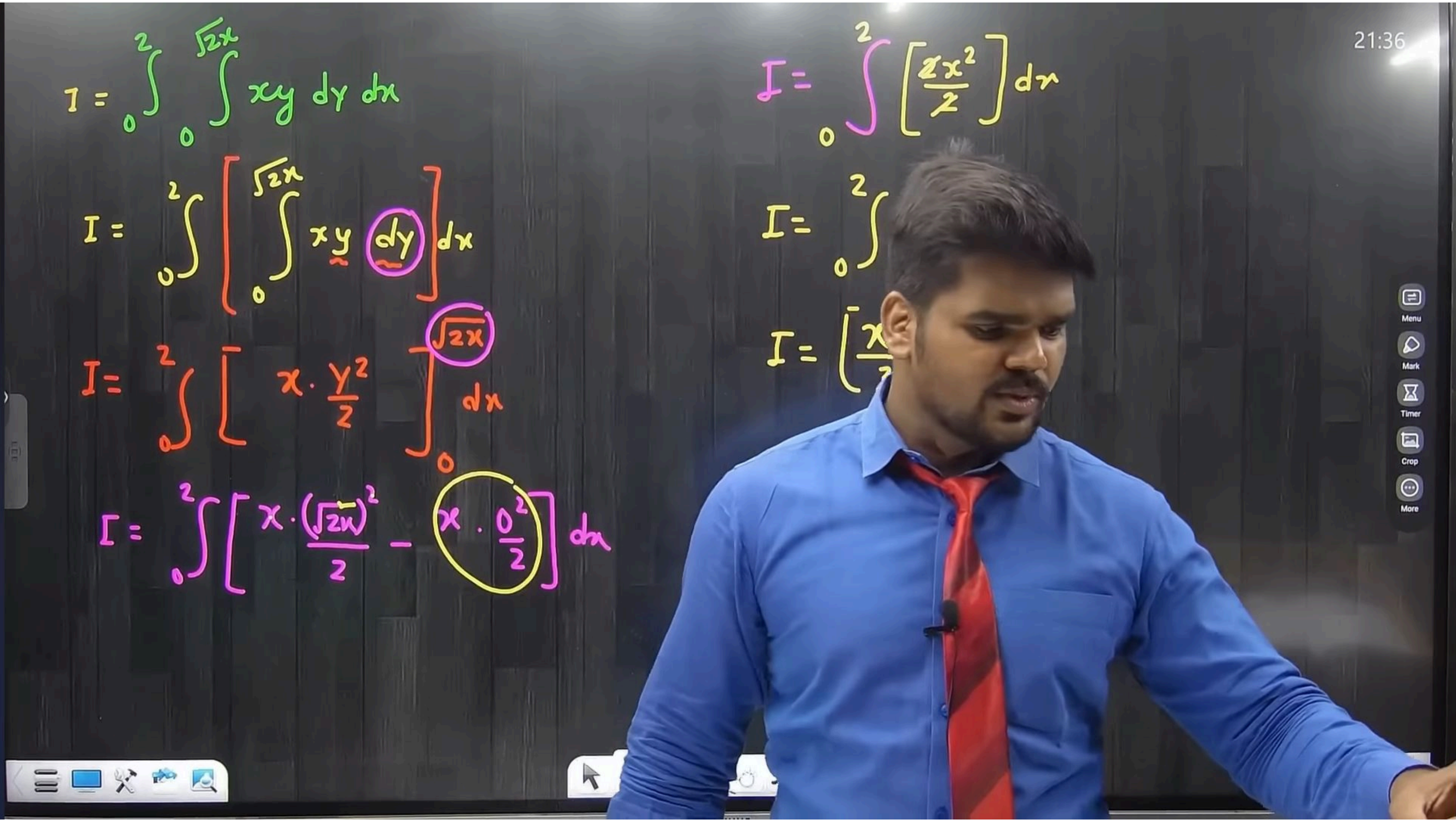
ACADEMY



$I = \int_0^2 \int_0^{\sqrt{2x}} xy\, dy\, dx$
$I = \int_0^2 \left[\int_0^{\sqrt{2x}} xy\, dy \right] dx$
$I = \int_0^2 \left[x \cdot \frac{y^2}{2} \right]_0^{\sqrt{2x}} dx$
$I = \int_0^2 \left[x \cdot \frac{(\sqrt{2x})^2}{2} - x \cdot \frac{0^2}{2} \right] dx$
$I = \int_0^2 \left[\frac{\cancel{2}x^2}{\cancel{2}} \right] dx$
$I = \int_0^2$
21:36
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ACADEMY

$$I = \int_0^2 \int_0^{\sqrt{2x}} xy \, dy \, dx$$

$$I = \int_0^2 \left[\int_0^{\sqrt{2x}} xy \, dy \right] dx$$

$$I = \int_0^2 \left[x \cdot \frac{y^2}{2} \right.$$

$$I = \int_0^2 \left[x \cdot (\right.$$

$$I = \int_0^2 \left[\frac{\cancel{2}x^2}{\cancel{2}} \right] dx$$

$$I = \int_0^2 x^2 \, dx$$

$$I = \left[\frac{x^3}{3} \right]_0^2$$

$$I = \left[\frac{2^3}{3} - \frac{0^3}{3} \right]$$

$$I = \left[\frac{8}{3} - 0 \right] = \frac{8}{3}$$

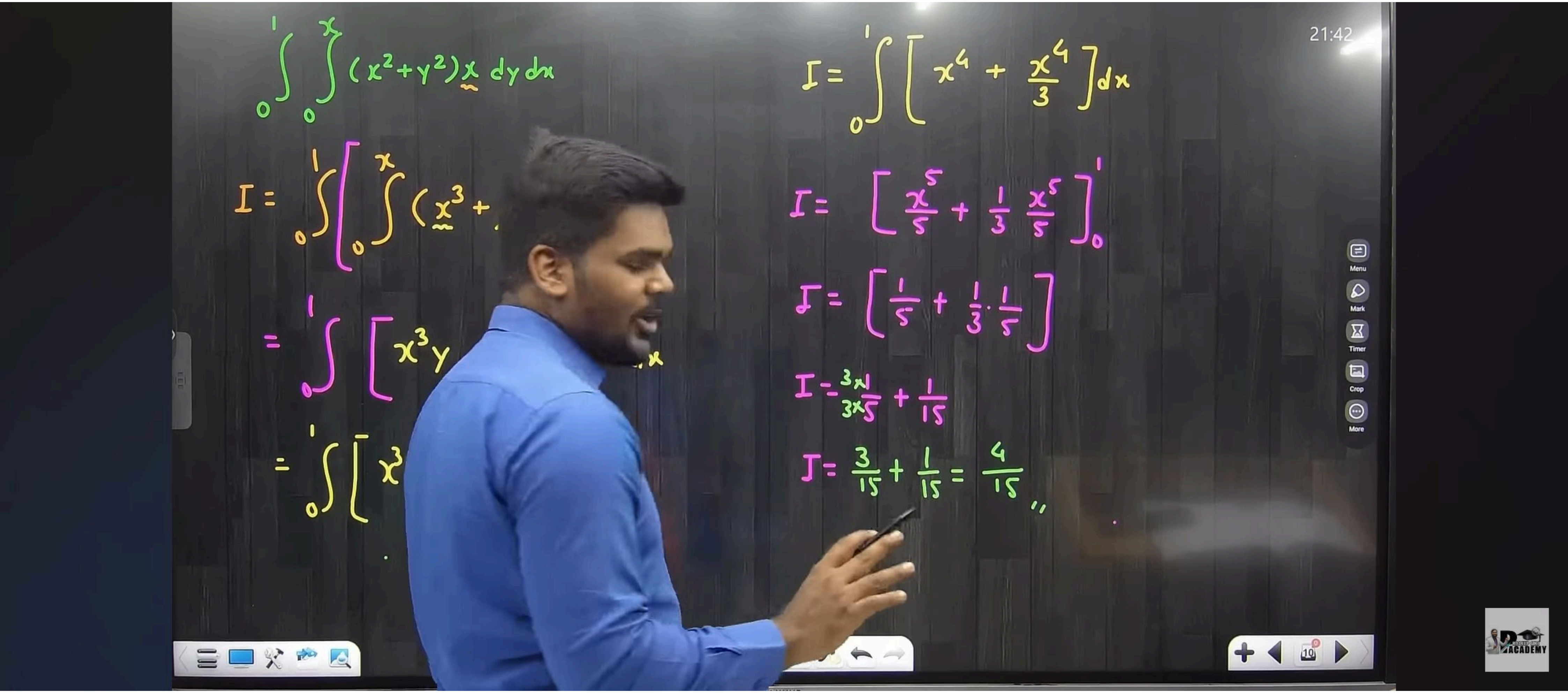
$$\int_0^1 \int_0^x (x^2+y^2)\,x \, dy\, dx$$

$$I = \int_0^1 \left[\int_0^x (x^3 + xy^2)\, dy \right] dx$$

$$= \int_0^1 \left[x^3 y + x \cdot \frac{y^3}{3} \right]_0^x dx$$

$$= \int_0^1 \left[x^3 \cdot x + x \cdot \frac{x^3}{3} \right] dx$$

21:40



$\int_0^1 \int_0^x (x^2+y^2)x \, dy \, dx$
$I = \int_0^1 \left[\int_0^x (x^3 +$
$= \int_0^1 \left[x^3 y \right.$
$= \int_0^1 \left[x^3 \right.$
21:42
$I = \int_0^1 \left[x^4 + \frac{x^4}{3} \right] dx$
$I = \left[\frac{x^5}{5} + \frac{1}{3} \frac{x^5}{5} \right]_0^1$
$I = \left[\frac{1}{5} + \frac{1}{3} \cdot \frac{1}{5} \right]$
$I = \frac{3 \times 1}{3 \times 5} + \frac{1}{15}$
$I = \frac{3}{15} + \frac{1}{15} = \frac{4}{15}$
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ACADEMY

288 768

Find $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dy\, dx.$

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{y^2+\left(\sqrt{1+x^2}\right)^2}\, dy \quad dx$$

$$x^2 + a^2$$

$(\sqrt{x})^2 = x$

$(\sqrt{5})^2$

$$\int \frac{1}{x^2 + \boxed{a^2}}\, dx$$

$$= \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

550 912

Find $\displaystyle\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\,dy\,dx.$

$$\int_0^1 \left[\int_0^{\sqrt{1+x^2}} \frac{1}{y^2+\left(\sqrt{1+x^2}\right)^2}\,dy \right] dx$$

$$x^2 + a^2$$

$$\int_0^1 \left[\frac{1}{\sqrt{1+x^2}} \tan^{-1}\left(\frac{y}{\sqrt{1+x^2}} \right) \right]_0^{\sqrt{1+x^2}} dx$$

$$\int_0^1 \left[\frac{1}{\sqrt{1+x^2}} \tan^{-1}\left(\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} \right) \right] dx$$

$$\left(\sqrt{x}\right)^2 = x$$

$$\left(\sqrt{5}\right)^2$$

$$\int \frac{1}{x^2 + \boxed{a^2}}\,dx$$

$$= \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

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550 912

Find $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dy\, dx.$

$I = \int_0^1 \left[\int_0^{\sqrt{1+x^2}} \frac{1}{y^2+(\ldots} \right] dx$

$x^2 + (\ldots$

$I = \int_0^1 \left[\ldots \right]_0^{\sqrt{1+x^2}} dx$

$I = \int_0^1 \ldots$

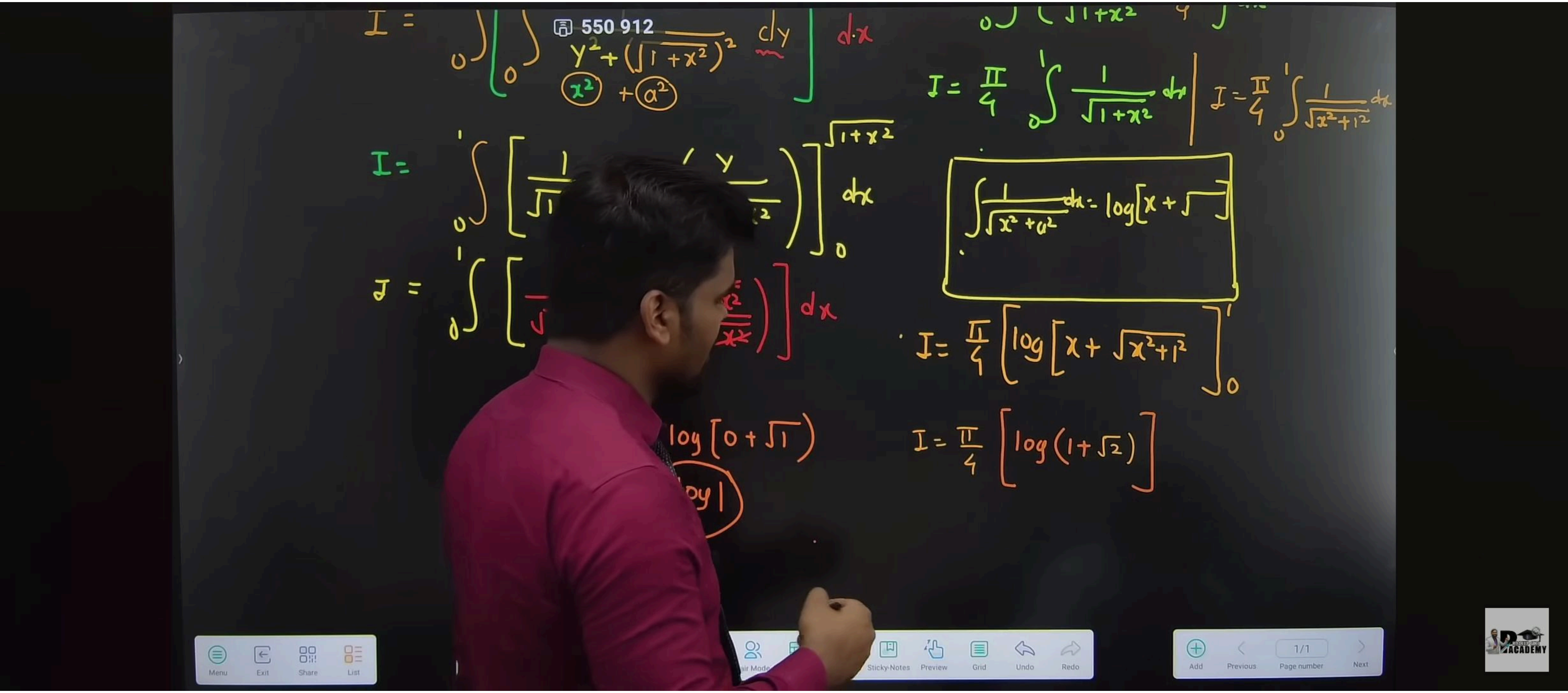
$I = \int_0^1 \left[\frac{1}{\sqrt{1+x^2}} \tan^{-1}(1) \right] dx$

$I = \int_0^1 \left[\frac{1}{\sqrt{1+x^2}} \times \frac{\pi}{4} \right] dx$

$I = \frac{\pi}{4} \int_0^1 \frac{1}{\sqrt{1+x^2}}\, dx \quad \Big| \quad I = \frac{\pi}{4} \int_0^1 \frac{1}{\sqrt{x^2+1^2}}\, dx$

$$\boxed{\int \frac{1}{\sqrt{x^2+a^2}}\, dx = \log\left[x + \sqrt{}\right]}$$

$I = \frac{\pi}{4} \left[\log\left[x + \sqrt{x^2+1^2}\right] \right]_0^1$


550 912
I = π/4 ∫₀¹ 1/√(1+x²) dx
I = π/4 ∫₀¹ 1/√(x²+1²) dx
∫ 1/√(x²+a²) dx = log[x + √]
I = π/4 [log[x + √(x²+1²)]]₀¹
I = π/4 [log(1+√2)]
log[0 + √1]
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ACADEMY

026 624

$$I = \int_0^{a\sqrt{3}} \int_0^{\sqrt{x^2+a^2}} \frac{x}{y^2+x^2+a^2}\, dy\, dx$$

$$I = \int_0^{a\sqrt{3}} \left[\int_0^{\sqrt{x^2+a^2}} \frac{x}{y^2+\left(\sqrt{x^2+a^2}\right)^2}\, dy \right] dx$$

$$\int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

$$I =$$

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$$\int_{0} \left[\int_{0} \ldots (\ldots + a^2)^2 \ldots \right]$$

$$\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

026 624

$$I = \int_{0}^{a\sqrt{3}} x \cdot \left[\frac{1}{\sqrt{x^2+a^2}} \tan^{-1}\left(\frac{y}{\sqrt{x^2+a^2}} \right) \right]_{0}^{\sqrt{x^2+a^2}} dx$$

$$I = \int_{0}^{a\sqrt{3}} x \cdot \left[\frac{1}{\sqrt{x^2+a^2}} \cdot \frac{\pi}{4} \right] dx$$

$$\int \frac{f'(x)}{\sqrt{f(x)}} = 2\sqrt{f(x)}$$

$$I = \frac{\pi}{4} \frac{1}{2} \int_{0}^{a\sqrt{3}} \frac{2x}{\sqrt{x^2+a^2}} dx.$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$I = \int_0^{a\sqrt{3}} x \cdot \left[\frac{1}{\sqrt{x^2+a^2}}\tan^{-1}\left(\frac{y}{\sqrt{x^2+a^2}}\right)\right]_0^{\sqrt{x^2+a^2}} dx$$

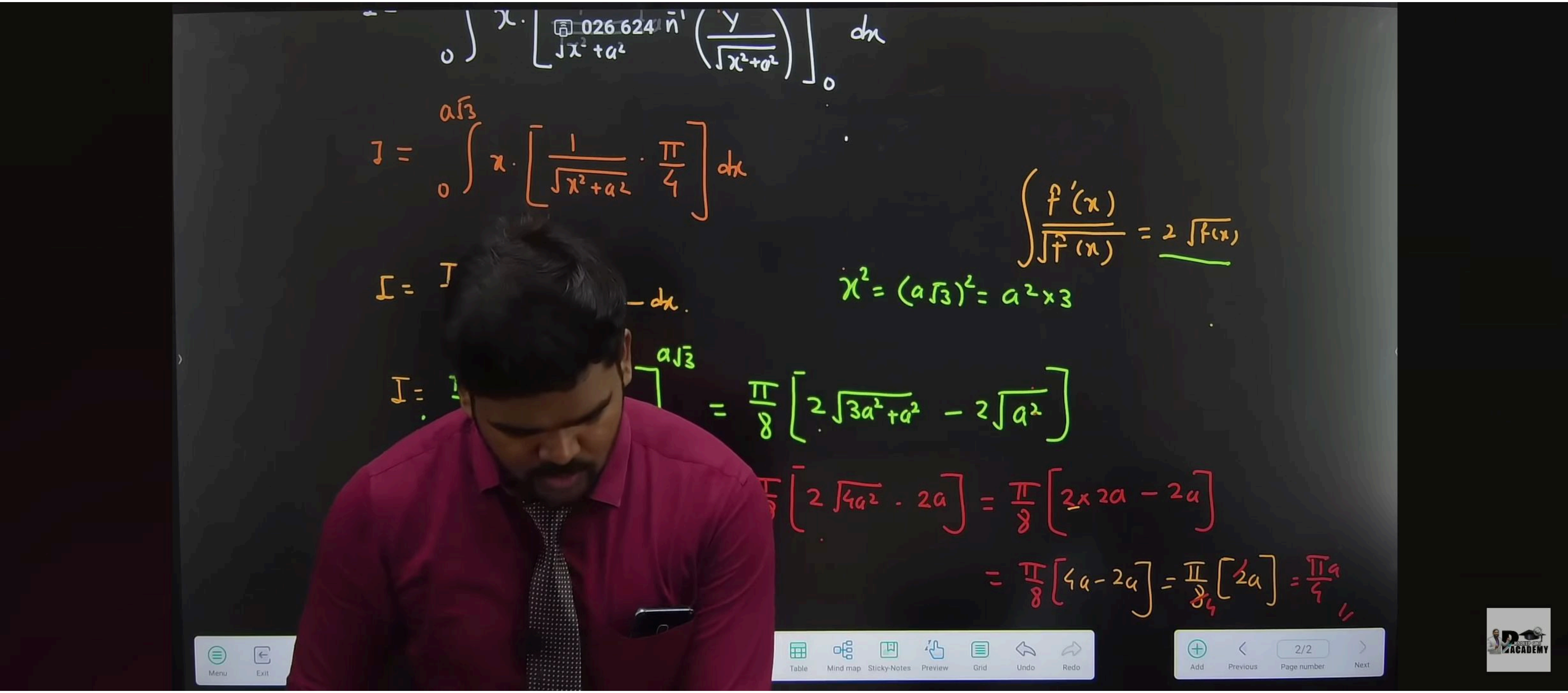
$$\int^{a\sqrt{3}} x \cdot \left[\frac{1}{\sqrt{x^2+a^2}} \cdot \frac{\pi}{4}\right] dx$$

$$\frac{\pi}{4}\,\frac{1}{2}\int_0^{a\sqrt{3}} \frac{2x}{\sqrt{x^2+a^2}}\,dx.$$

$$\int \frac{f'(x)}{\sqrt{f(x)}} = 2\sqrt{f(x)}$$

$$x^2 = (a\sqrt{3})^2 = a^2 \times 3$$

$$I = \frac{\pi}{8}\left[2\sqrt{x^2+a^2}\right]_0^{a\sqrt{3}} = \frac{\pi}{8}\left[2\sqrt{3a^2+a^2} - \right.$$


026 624
I = ∫₀^(a√3) x · [1/√(x²+a²) · π/4] dx
∫ f′(x)/√f(x) = 2√f(x)
x² = (a√3)² = a²×3
= π/8 [2√(3a²+a²) − 2√a²]
π/8 [2√4a² − 2a] = π/8 [2×2a − 2a]
= π/8 [4a − 2a] = π/8 [2a] = πa/4
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