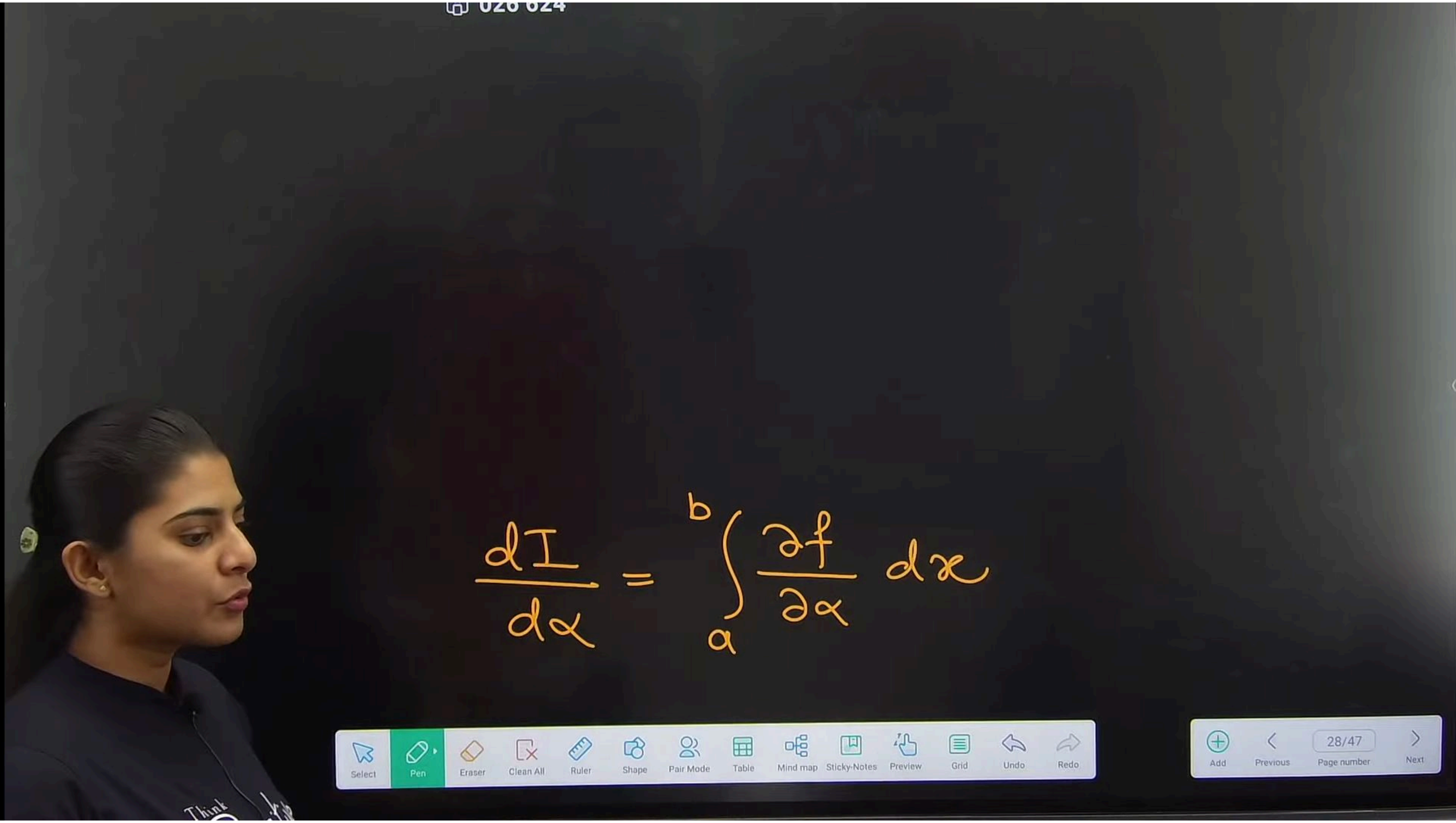
$\frac{dI}{d\alpha} = \int_a^b \frac{\partial f}{\partial \alpha} dx$
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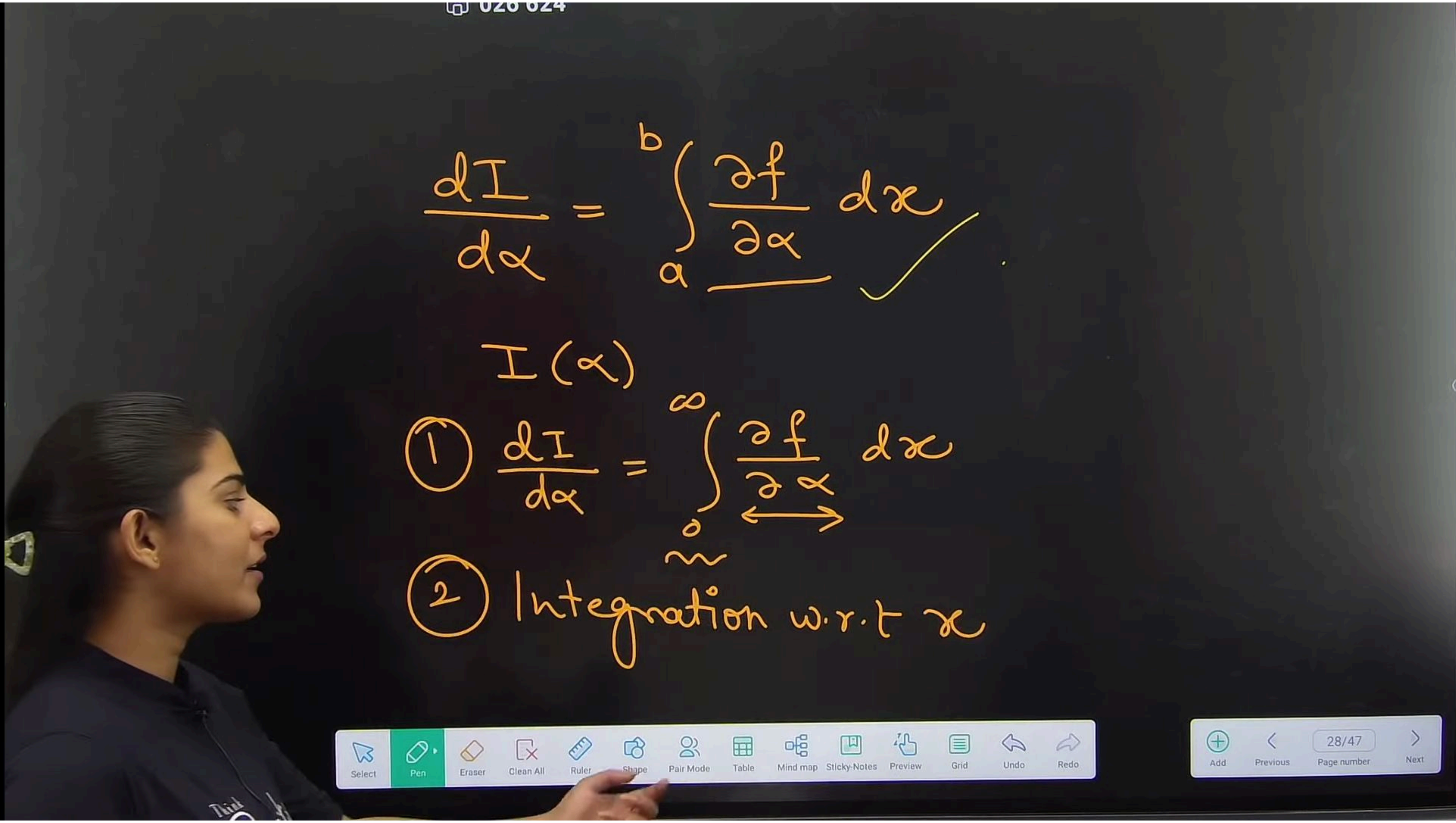
$\frac{dI}{d\alpha} = \int_a^b \frac{\partial f}{\partial \alpha} \, dx$
$I(\alpha)$
① $\frac{dI}{d\alpha} = \int_0^\infty \frac{\partial f}{\partial \alpha} \, dx$
② Integration w.r.t x
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(2) Integration w.r.t x

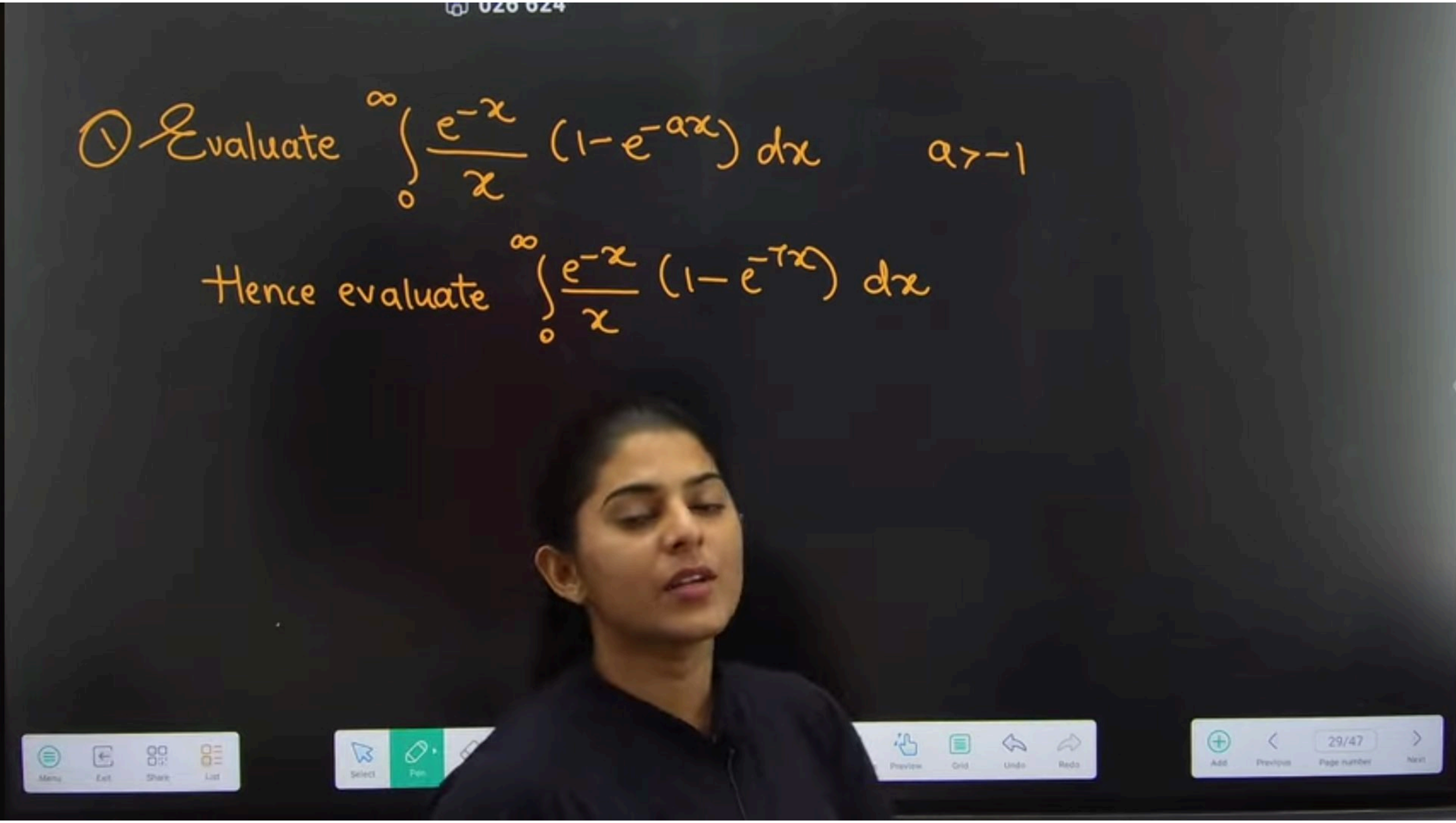
$$\frac{dI}{d\alpha} = \underline{\qquad\qquad}$$

(3) Integrate w.rt α

$$I = \underline{\qquad\quad} + C$$

(4) Put $\alpha = 0$

$$C = \boxed{\qquad\quad}$$



① Evaluate $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})\,dx$ $a>-1$
Hence evaluate $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-7x})\,dx$
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$$I(a) = \int_0^\infty \frac{e^{-x}}{x}\left(1 - e^{-ax}\right) dx$$

$$\boxed{\frac{dI}{da} = \int_0^\infty \frac{\partial f}{\partial a}\, dx}$$

Diff.... w.r.t a

$$\frac{dI}{da} = \int_0^\infty$$

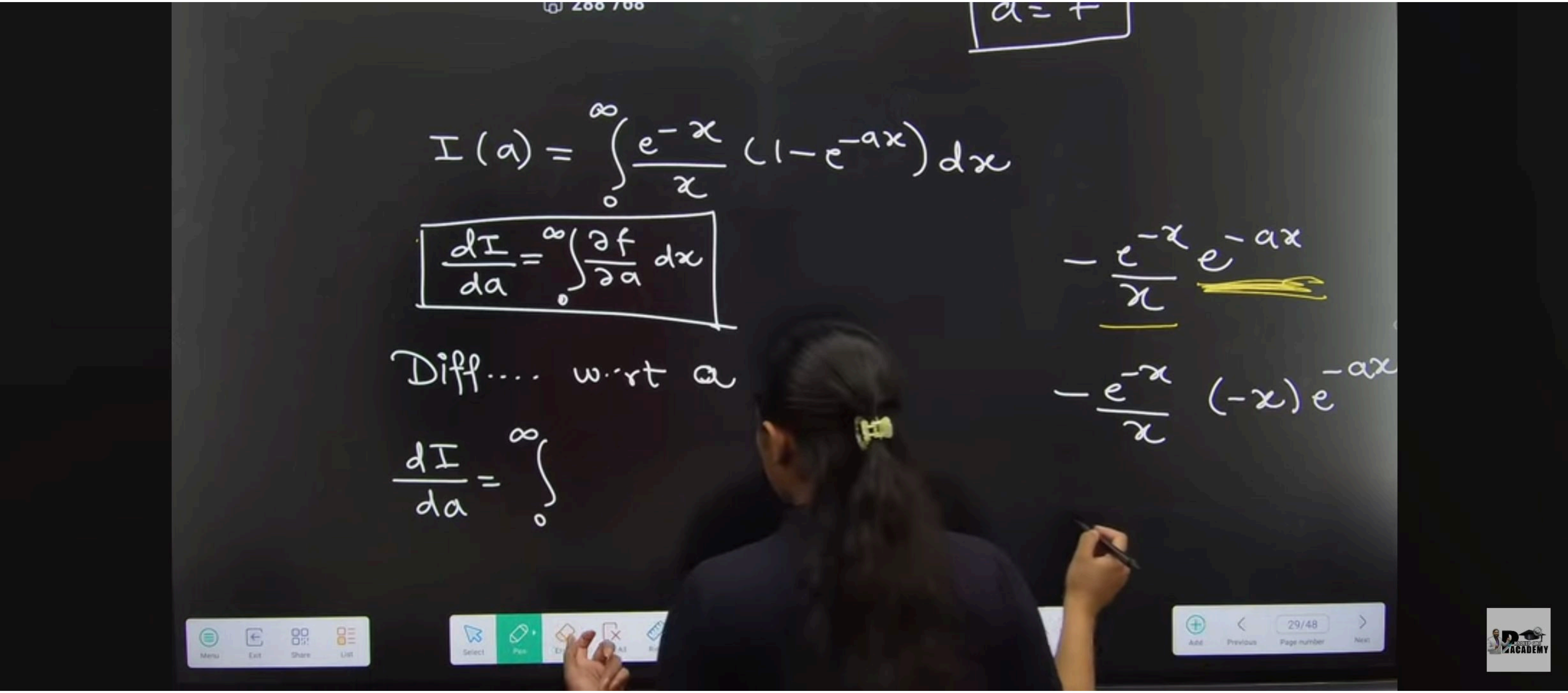
$$I(a) = \int_0^\infty \frac{e^{-x}}{x}\left(1-e^{-ax}\right)dx$$

$$\boxed{\frac{dI}{da} = \int_0^\infty \frac{\partial f}{\partial a}\,dx}$$

$$\left(\frac{e^{-x}}{x} - \frac{e^{-x}}{x}e^{-ax}\right)$$

Diff....

$$\frac{dI}{da} = \int^\infty$$



$I(a) = \int_0^\infty \frac{e^{-x}}{x}(1 - e^{-ax})\,dx$
$\boxed{\frac{dI}{da} = \int_0^\infty \frac{\partial f}{\partial a}\,dx}$
Diff.... w.r.t a
$\frac{dI}{da} = \int_0^\infty$
$-\frac{e^{-x}}{x} e^{-ax}$
$-\frac{e^{-x}}{x}(-x) e^{-ax}$
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$\boxed{a = 1}$

$$I(a) = \int_0^\infty \frac{e^{-x}}{x}\left(1-e^{-ax}\right)dx$$

$$\boxed{\frac{dI}{da} = \int_0^\infty \frac{\partial f}{\partial a}\,dx}$$

Diff.... w.r.t a

$$\frac{dI}{da} = \int_0^\infty$$

$$-\frac{e^{-x}}{x}\,e^{-ax}$$

$$e^{-x} \cdot e^{-ax}$$

$$e^{-x} \cdot e^{-ax}$$

$$e^{-x-ax}$$

$$e^{-x(1+a)}$$

$$\frac{dI}{da} = \int_0^\infty e^{-x(1+a)}\,dx$$

(2) Int. w.r.t x

$$\frac{dI}{da} = \left[\frac{e^{-x(1+a)}}{-(1+a)}\right]_0^\infty$$

$e^{-x} \cdot e^{-ax}$

e^{-x-ax}

$e^{-x(1+a)}$

d x

i d

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$$\frac{dI}{da} = \int_0^\infty e^{-x(1+a)}\,dx$$

② Int. w.r.t x

$$\frac{dI}{da} = \left[\frac{e^{-x(1+a)}}{-(1+a)}\right]_0^\infty$$

$$= \cancel{-}\,\frac{1}{\cancel{-}(1+a)}$$

$$\frac{dI}{da} = \frac{1}{1+a}$$

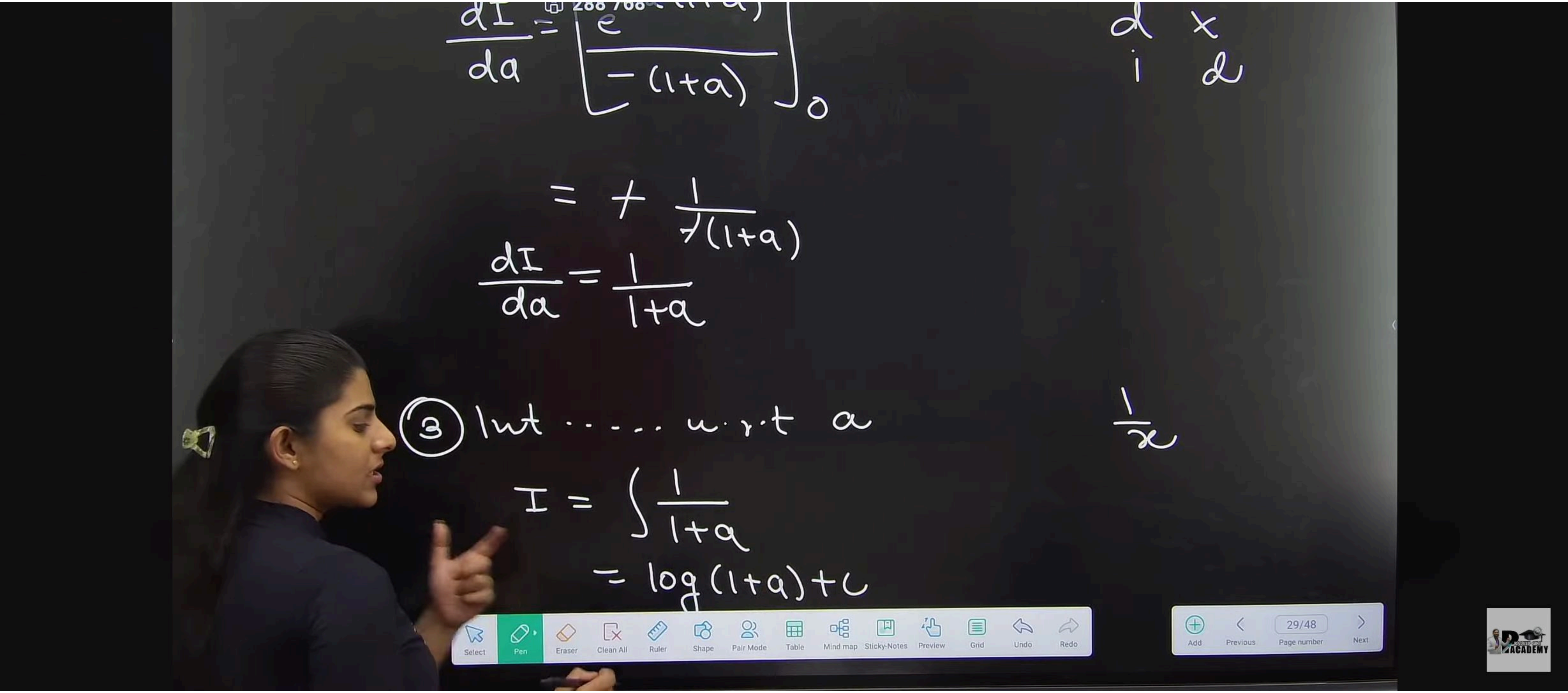
$e^{-x}\cdot e^{-ax}$

e^{-x-ax}

$e^{-x(1+a)}$

d x

i d



$\frac{dI}{da} = \left[\frac{e^{-x(1+a)}}{-(1+a)}\right]_0$
$= + \frac{1}{+(1+a)}$
$\frac{dI}{da} = \frac{1}{1+a}$
③ Int ..... w.r.t a
$I = \int \frac{1}{1+a}$
$= \log(1+a) + c$
d x
i d
$\frac{1}{x}$
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(4) Put $a = 0$

$$I(0) = \log(1+0) + c$$

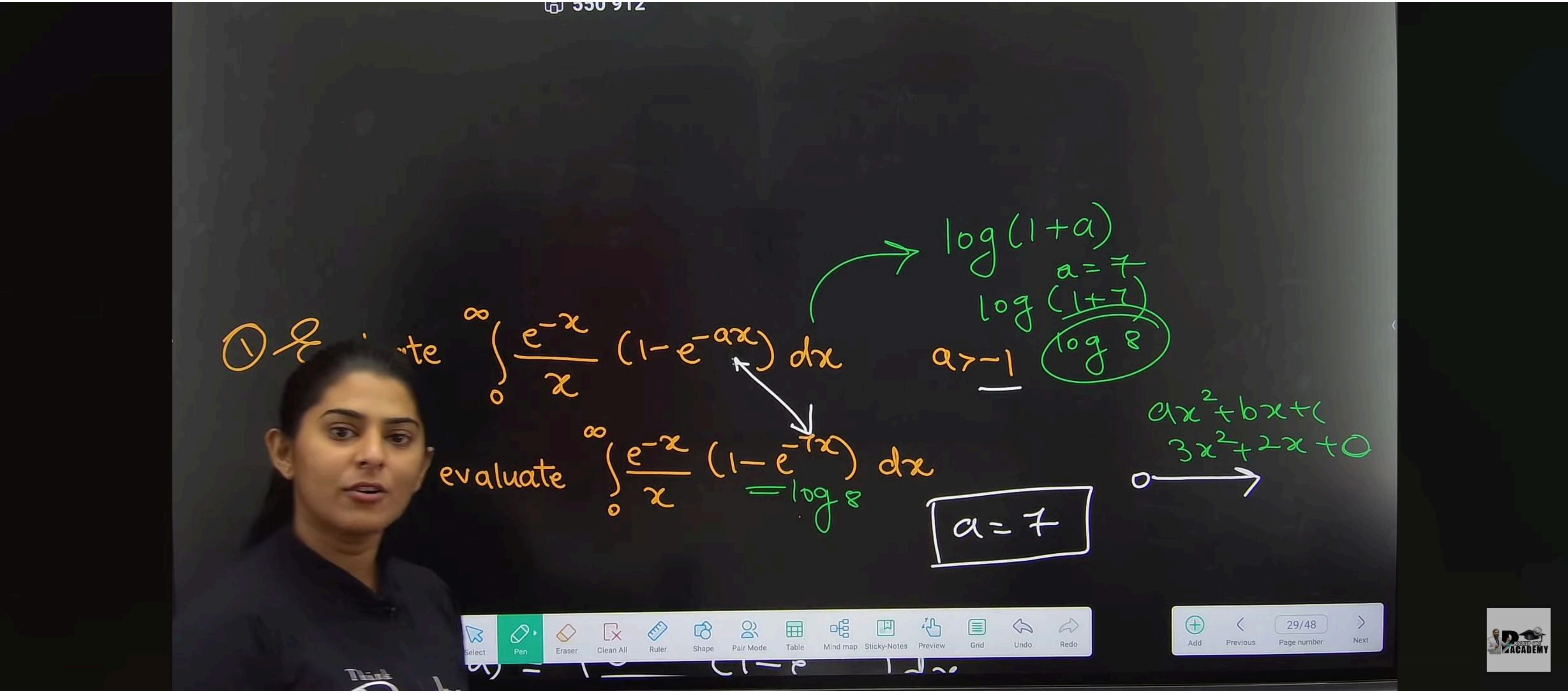
$$= \log 1 + c$$

$$I(0) = c$$

$$\int_0^\infty \frac{e^{-x}}{x}\Big(1-\underbrace{e^{0}}_{\substack{1-1 \\ 0}}\Big)dx = c$$

$$\int_0^\infty 0 \, dx = c$$

$$c = 0$$


550 912
$\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})\,dx$
log(1+a)
a=7
log(1+7)
log 8
a>-1
evaluate $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-7x})\,dx$
= log 8
ax^2+bx+c
$3x^2+2x+0$
a = 7
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Prove that $\int_0^{\frac{\pi}{2}} \frac{\log(1+a\sin^2 x)}{\sin^2 x}\, dx = \pi\left[\sqrt{a+1}-1\right], \; a>-1$

$0 \longrightarrow \infty$

$$I(a) = \int_0^{\frac{\pi}{2}} \frac{\log(1+a\sin^2 x)}{\sin^2 x}\, dx$$

$$\boxed{\frac{dI}{da} = \int_0^{\frac{\pi}{2}} \frac{\partial f}{\partial a}\, dx}$$

$$\boxed{\frac{dI}{da} = \int_0^{\frac{\pi}{2}} \frac{\partial f}{\partial a}\, dx}$$

$$\frac{dI}{da} = \int_0^{\frac{\pi}{2}} \frac{1}{\cancel{\sin^2 x}} \cdot \frac{1}{1 + a\sin^2 x}\, \cancel{\sin^2 x}\; dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{1 + a\sin^2 x}\, dx$$

$$\boxed{\frac{dI}{da} = \int_0^{\frac{\pi}{2}} \frac{\partial f}{\partial a}\, dx}$$

$$\frac{dI}{da} = \int_0^{\frac{\pi}{2}} \frac{1}{\cancel{\sin^2 x}} \cdot \frac{1}{1 + a\sin^2 x}\, \cancel{\sin^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{1 + a\sin^2 x}\, dx$$

multiply & Divide by $\cos^2 x$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{\sec^2 x + a\tan^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{1+a\sin^2 x}\,dx$$

multiply & Divide by $\cos^2 x$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{1+\tan^2 x + a\tan^2 x}\,dx$$

$$\sec^2 x = 1 + \tan^2 x$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{1+\tan^2 x(1+a)}$$

multiply & Divide by $\cos^2 x$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{1+\underline{\tan^2 x + a\tan^2 x}}\, dx$$

$$\sec^2 x = 1 + \tan^2 x$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{1+\tan^2 x(1+a)}\, dx$$

Put $t = \tan x$

$$\int_0 \frac{\cdots}{1+\tan x + a\tan^2 x}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{1+\tan^2 x(1+a)}\, dx$$

Put $t = \tan x$

Diff ..... t

$$1 = \sec^2 x \frac{dx}{dt}$$

$$dt = \sec^2 x\, dx$$

Put $t = \tan x$

Diff..... t

$$1 = \sec^2 x \frac{dx}{dt}$$

$$dt = \sec^2 x \, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{dt}{1^2 + (1+a)t^2}$$

$a = 1$

$(\sqrt{a})^2$

a

$$\boxed{\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(x/a\right) + c}$$

$$= \int_0^{\frac{\pi}{2}} \frac{dt}{1^2 + \left((\sqrt{1+a})t\right)^2}$$

026 624

$$dt = \sec^2 x \, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{dt}{1^2 + (1+a)t^2}$$

$a = 1$

$(\sqrt{a})^2$

a

$$\boxed{\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(x/a\right) + c}$$

$$= \int_0^{\frac{\pi}{2}} \frac{dt}{1^2 + \left((\sqrt{1+a})\,t\right)^2}$$

$$= \frac{1}{1}\tan^{-1}\left(\frac{(\sqrt{1+a})\,t}{1}\right) \times \frac{1}{\sqrt{1+a}}$$

Limits will change with substitution

Put $t = \tan x$

Diff ..... t

$$1 = \sec^2 x \, \frac{dx}{dt}$$

$$x \; \int_{0}^{\frac{\pi}{2}} \qquad \int_{0}^{\infty}$$

x

$$= \int_0^{\infty} \frac{dt}{1^2 + \left((\sqrt{1+a})t\right)^2}$$

$$= \frac{1}{1}\left[\tan^{-1}\left(\frac{(\sqrt{1+a})t}{1}\right)\right] \times \frac{1}{\sqrt{1+a}}$$

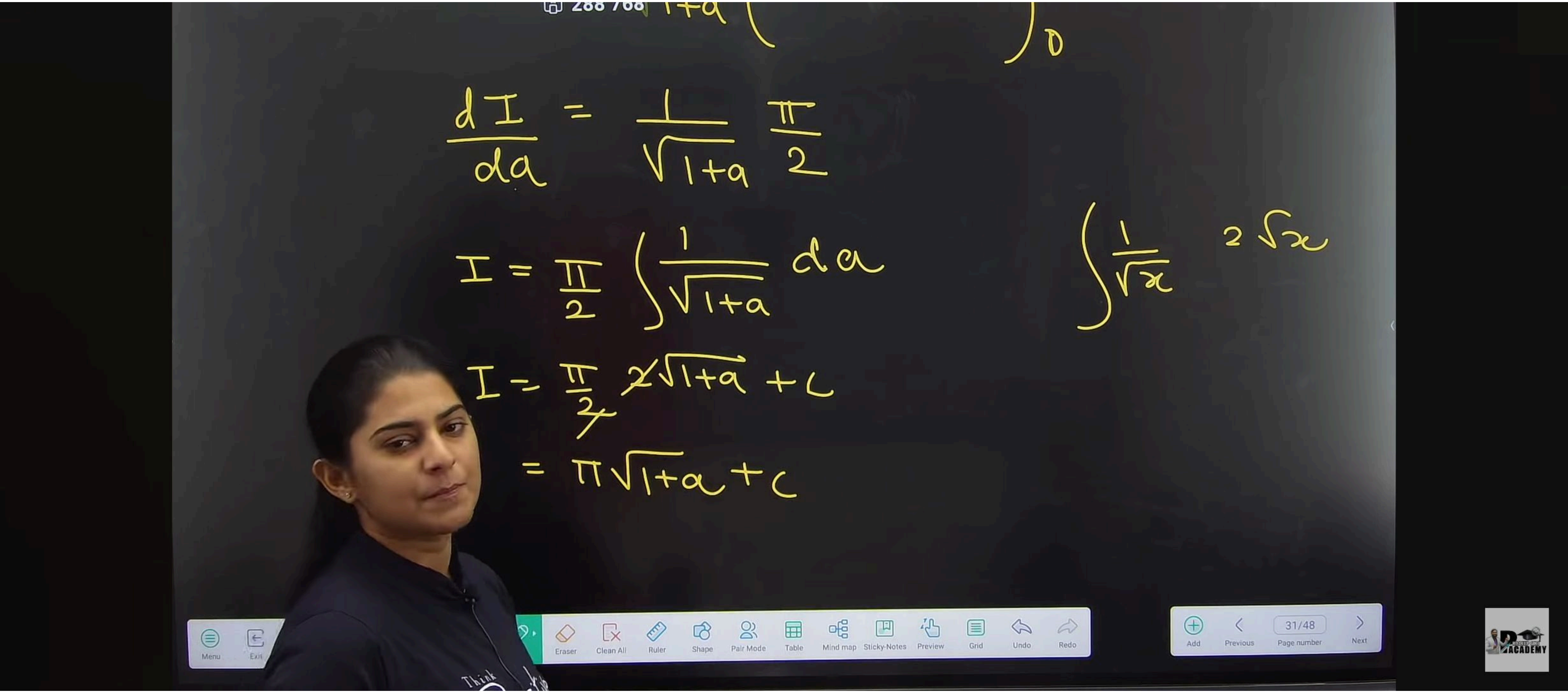
$$= \frac{1}{\sqrt{1+a}}\left(\tan^{-1}(\sqrt{1+a})t\right)_0^{\infty}$$

$$= \frac{1}{\sqrt{1+a}} \frac{\pi}{2}$$

$$= \frac{1}{1}\left[\tan^{-1}\left(\frac{\sqrt{1+a}\,t}{1}\right)\right] \times \frac{1}{\sqrt{1+a}}$$

$$= \frac{1}{\sqrt{1+a}}\left(\tan^{-1}(\sqrt{1+a})t\right)_0^\infty$$

$$\frac{dI}{da} = \frac{1}{\sqrt{1+a}}\,\frac{\pi}{2}$$


288 768
$\frac{dI}{da} = \frac{1}{\sqrt{1+a}} \frac{\pi}{2}$
$I = \frac{\pi}{2} \int \frac{1}{\sqrt{1+a}} \, da$
$I = \frac{\pi}{\cancel{2}} \cancel{2}\sqrt{1+a} + c$
$= \pi\sqrt{1+a} + c$
$\int \frac{1}{\sqrt{x}}$
$2\sqrt{x}$
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$$I(a) = \pi\sqrt{1+a} + c$$

Put $a = 0$

$$I(0) = \pi\sqrt{1+0} + c$$

$$I(0) = \pi + c$$

$$\int_0^{\frac{\pi}{2}} \frac{\log(1+a\sin^2 x)}{\sin^2 x}\,dx = \pi + c$$

$$I(0) = \pi\sqrt{1+0} + C$$

$$I(0) = \pi + C$$

$$\int_0^{\frac{\pi}{2}} \frac{\log 1}{\sin^2 x}\,dx = \pi + C$$

$$\int_0^{\frac{\pi}{2}} 0\,dx = \pi + C$$

$$0 = \pi + C$$

$$\boxed{C = -\pi}$$

$$\int_0^{\frac{\pi}{2}} \frac{\log 1}{\sin^2 x}\, dx = \pi + c$$

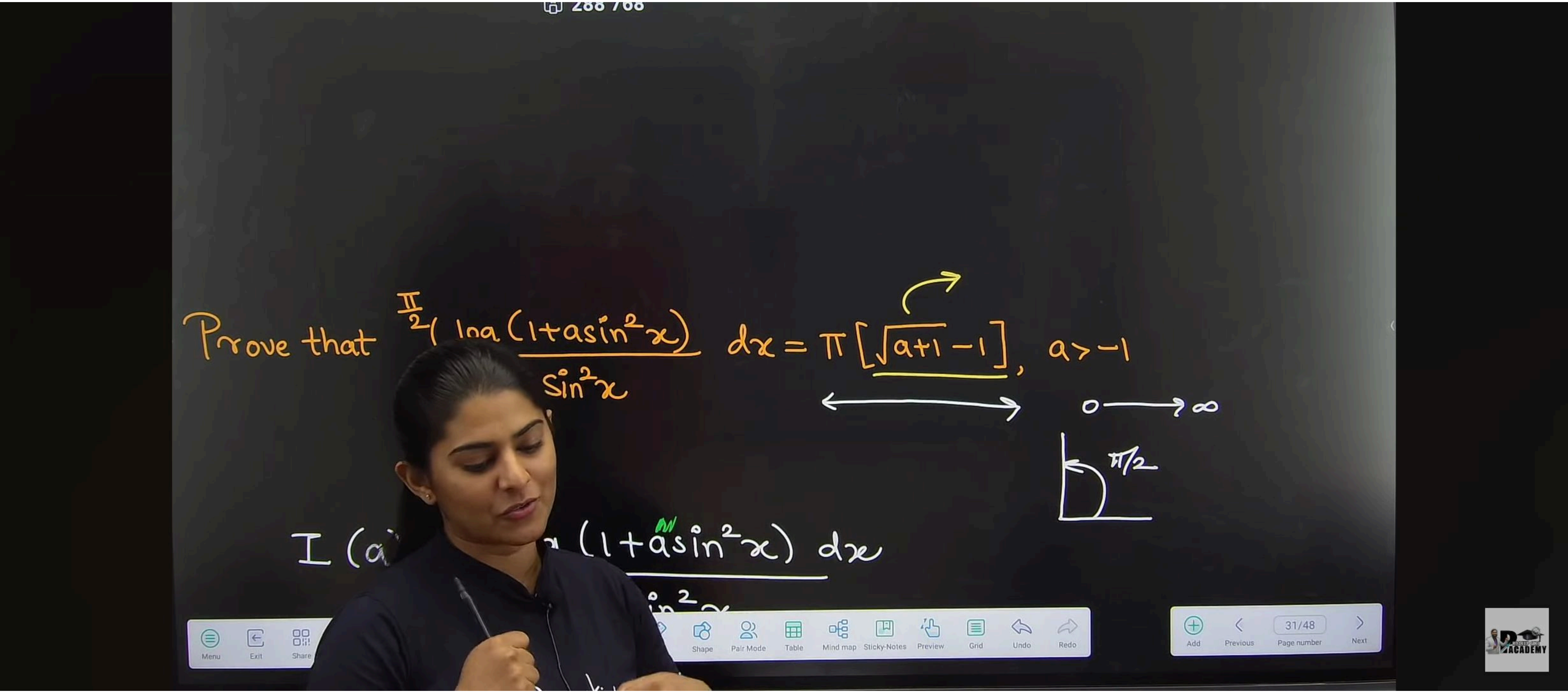
$$\int_0^{\frac{\pi}{2}} 0 \, dx = \pi + c$$

$$0 = \pi + c$$

$$\boxed{c = -\pi}$$

$$I = \pi\sqrt{1+a} - \pi$$

$$= \pi\left(\sqrt{1+a} - 1\right)$$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Prove that $\int^{\frac{\pi}{2}} \frac{\ln(1+a\sin^2 x)}{\sin^2 x}\, dx = \pi\left[\sqrt{a+1}-1\right]$, $a>-1$
$0 \longrightarrow \infty$
$\pi/2$
$I(a)$ … $(1+a\sin^2 x)\, dx$
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