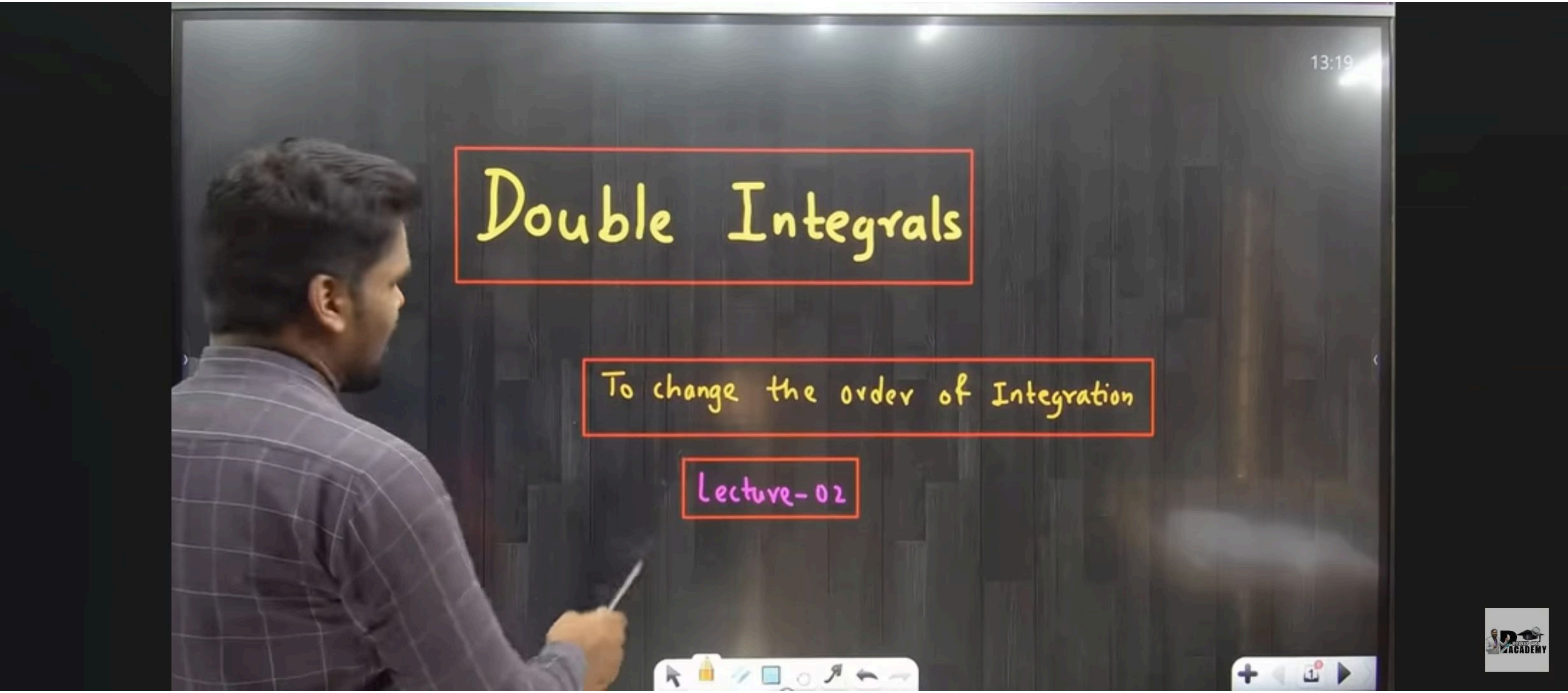
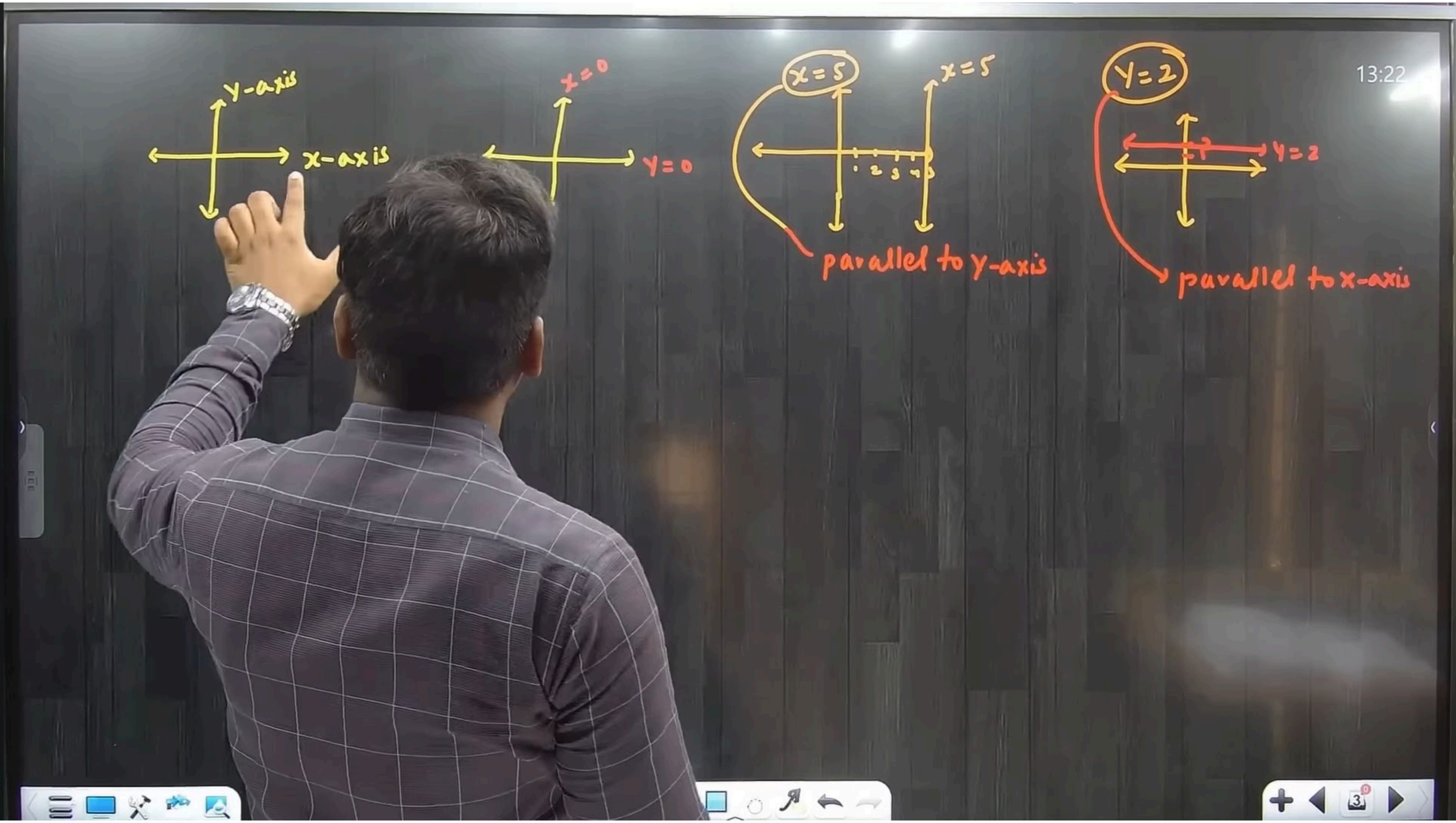
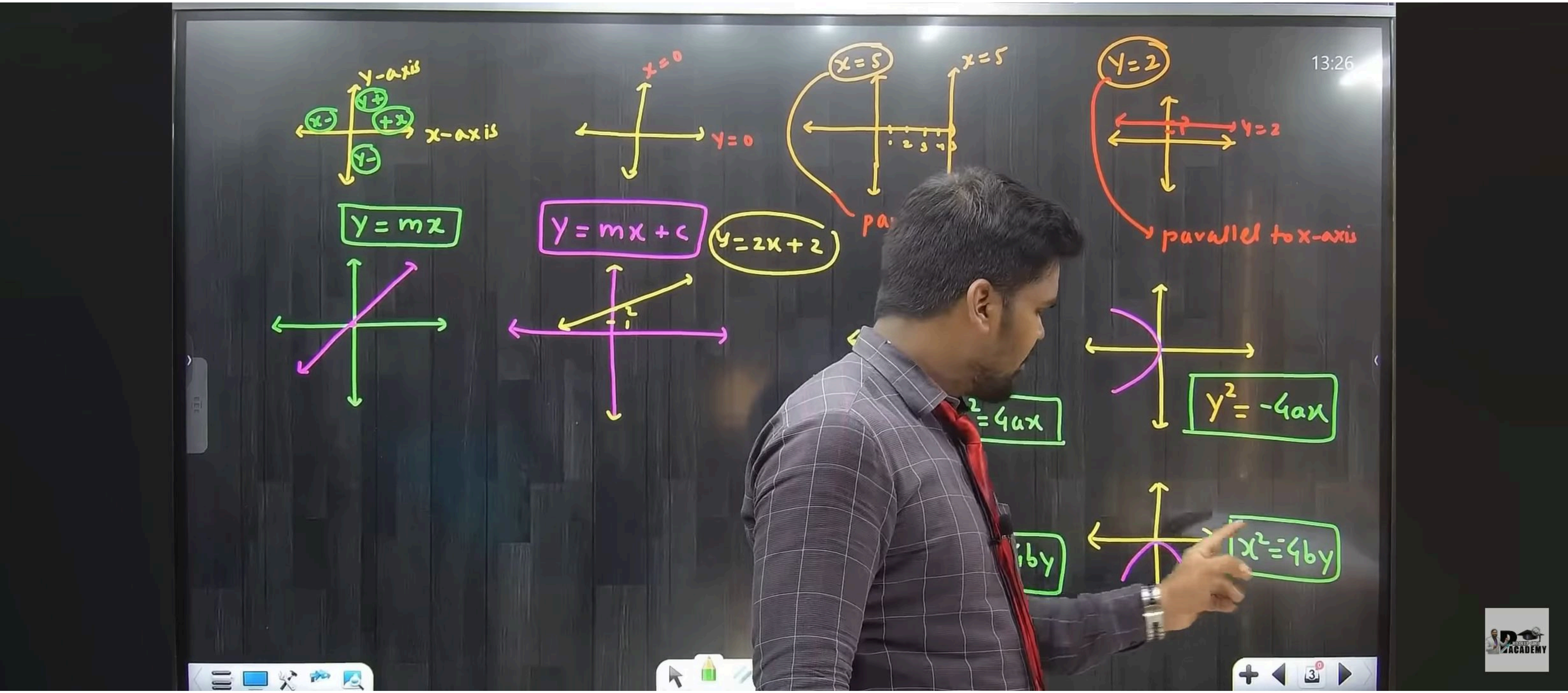
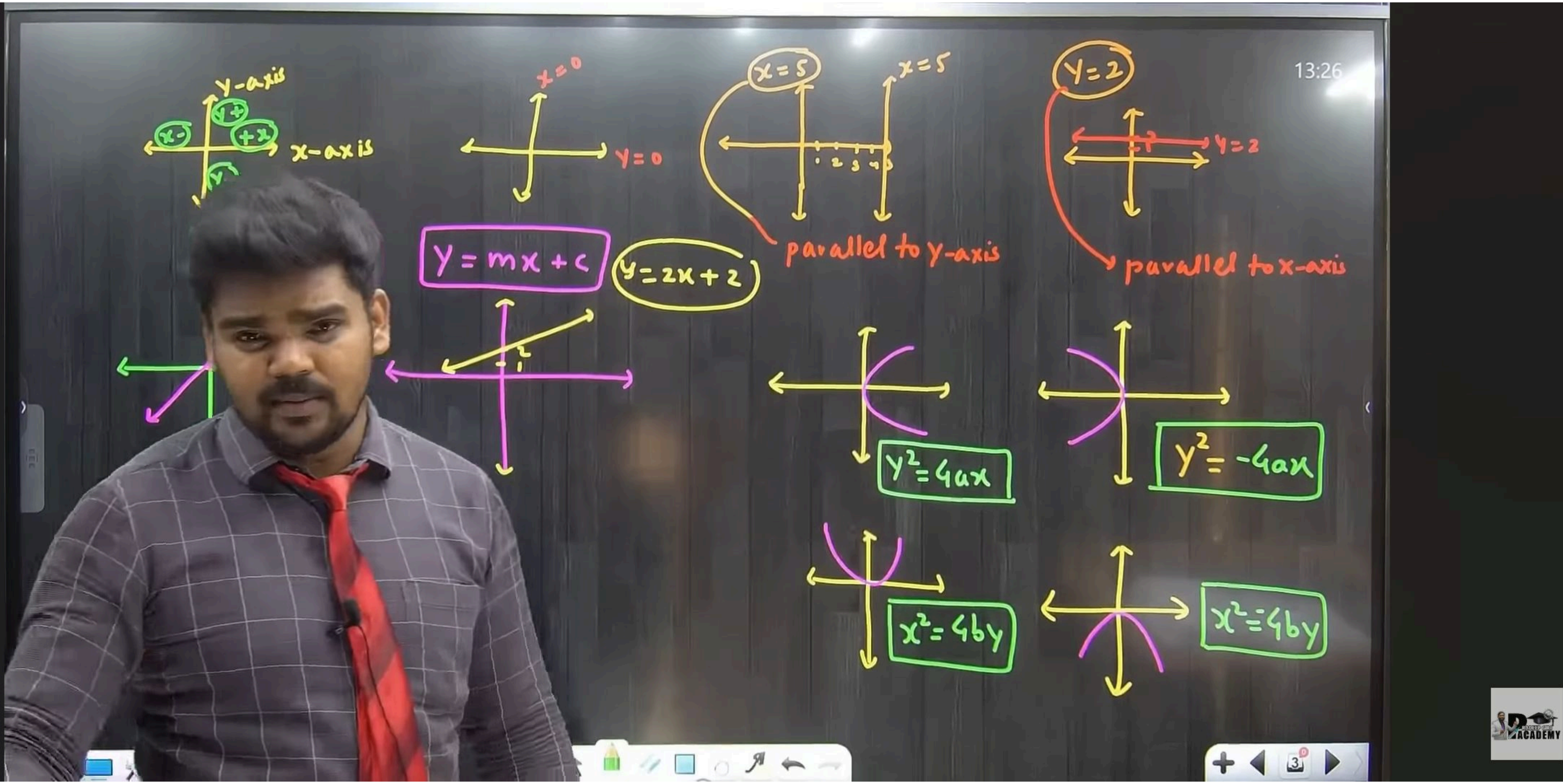

13:19
Double Integrals
To change the order of Integration
Lecture-02
ACADEMY

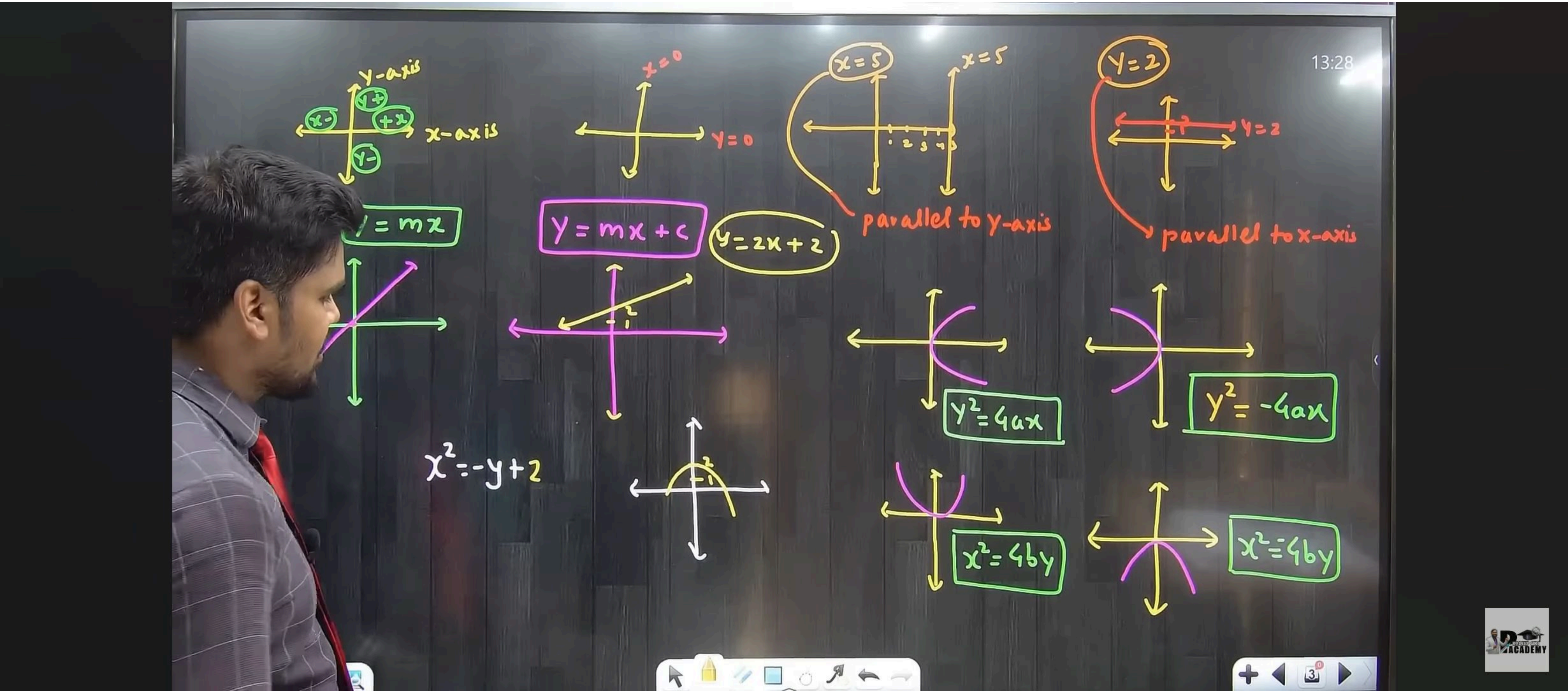

y-axis
x-axis
x=0
y=0
x=5
x=5
parallel to y-axis
y=2
y=2
parallel to x-axis
13:22
ACADEMY

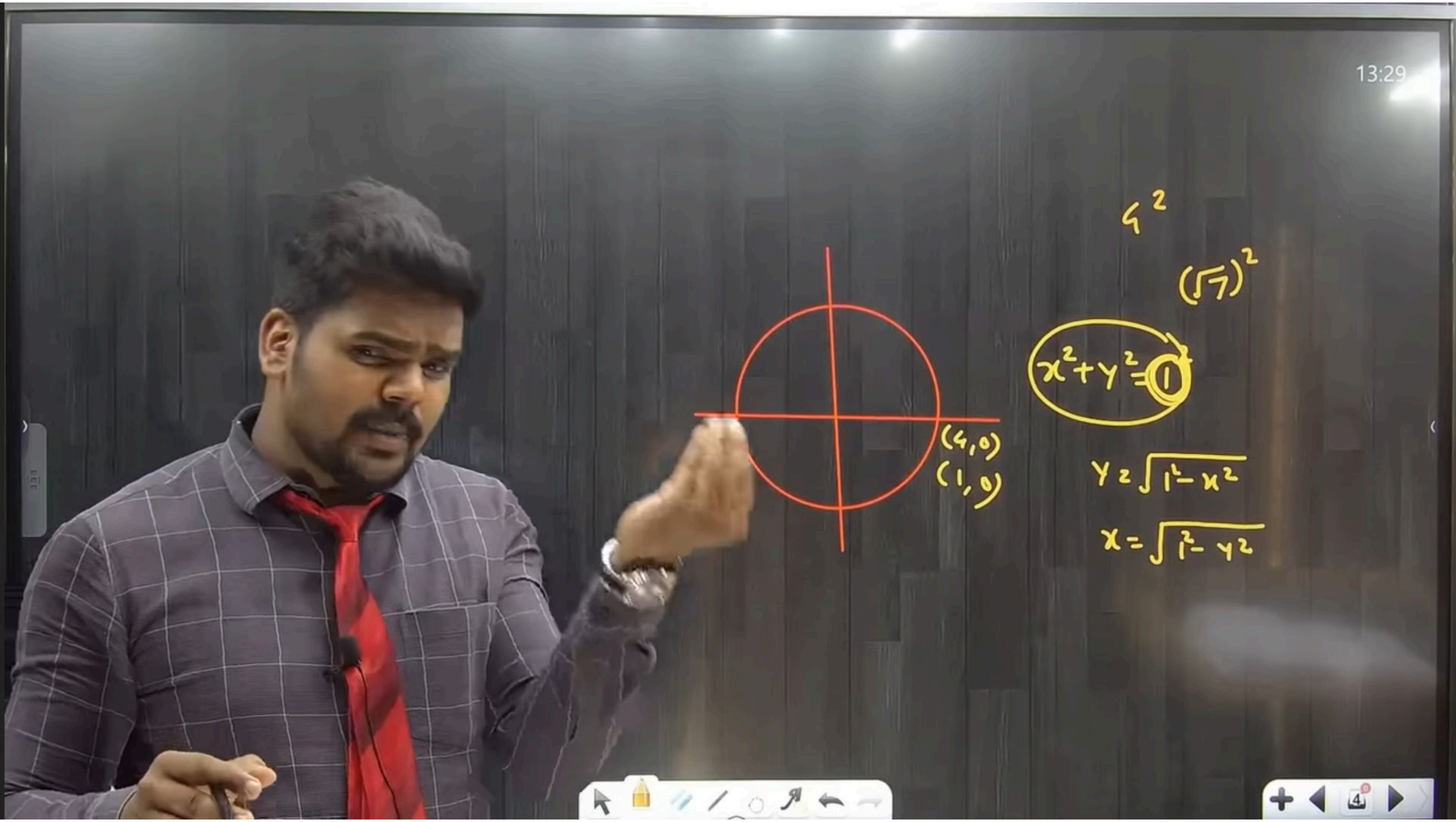


13:26
y-axis
x-axis
x=0
y=0
x=5
x=5
y=2
y=2
parallel to x-axis
y=mx
y=mx+c
y=2x+2
y²=-4ax
x²=4by
ACADEMY



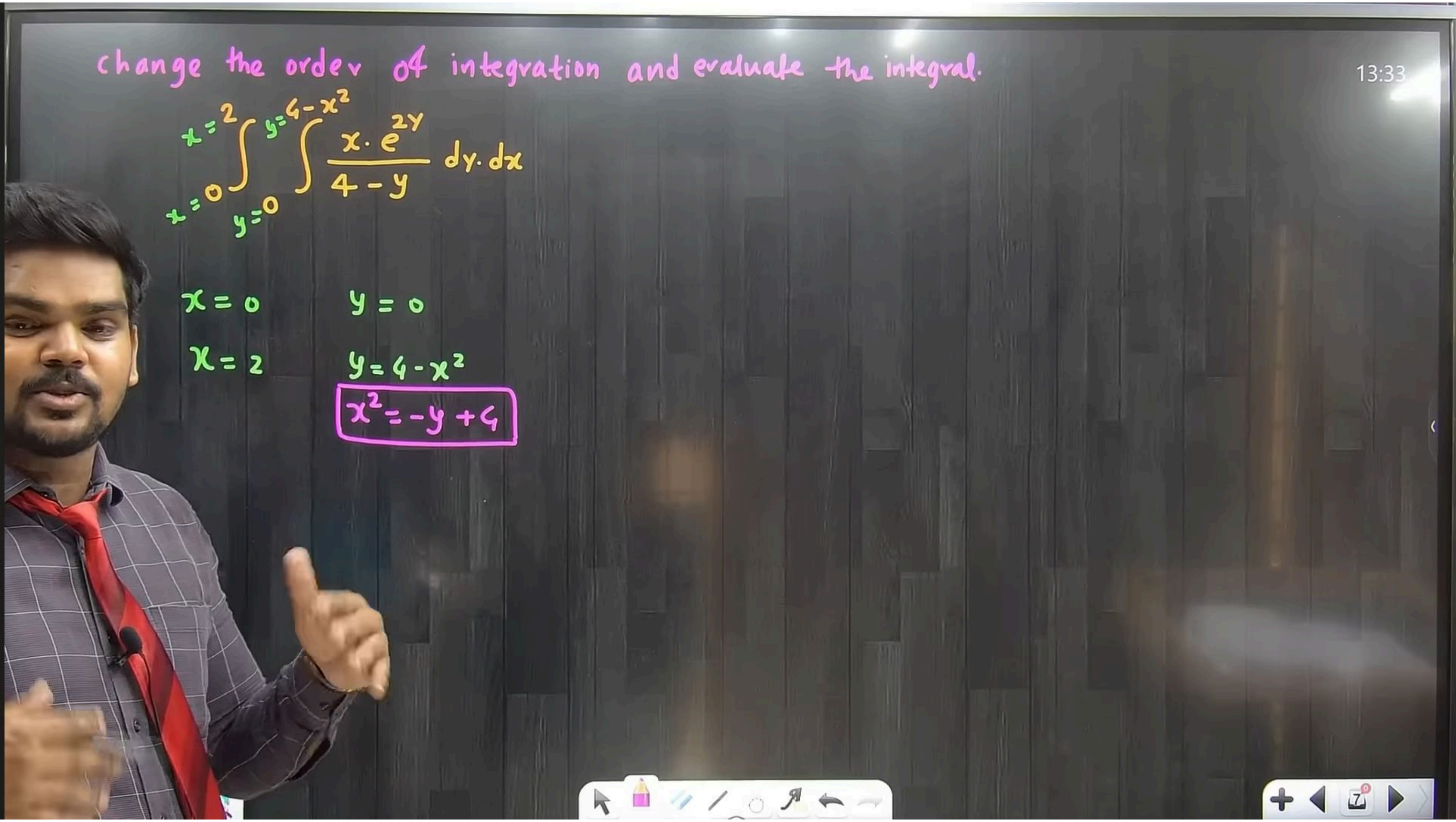
13:26
y-axis
x-axis
x=0
y=0
x=5
x=5
y=2
y=2
parallel to y-axis
parallel to x-axis
y = mx + c
y=2x+2
y²=4ax
y²= -4ax
x²=4by
x²=-4by
ACADEMY


y-axis
x-axis
y+
x-
+x
y-
x=0
y=0
x=5
x=5
y=2
y=2
parallel to y-axis
parallel to x-axis
y = mx
y = mx + c
y = 2x + 2
y² = 4ax
y² = -4ax
x² = -y + 2
x² = 4by
x² = 4by
13:28
ACADEMY


13:29
4²
(√7)²
x²+y²=1
(4,0)
(1,0)
y = √(1²−x²)
x = √(1²−y²)


13:31
x = 5
y = √(1 - x²)
x = 0
y = 0
dy dx
y=
x=
y=
x=
dx dy
ACADEMY


13:31
x = 5
y = √1 − x
x = 0
y = 0
dy dx
y =
y =
dy
ACADEMY


change the order of integration and evaluate the integral.
13:33
$\int_{x=0}^{x=2}\int_{y=0}^{y=4-x^2} \frac{x \cdot e^{2y}}{4-y}\, dy \cdot dx$
$x = 0$
$y = 0$
$x = 2$
$y = 4 - x^2$
$\boxed{x^2 = -y + 4}$

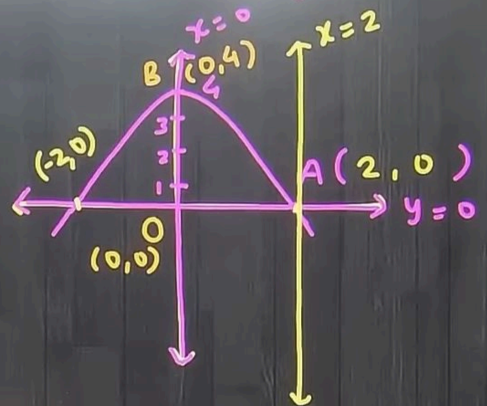
change the order of integration and evaluate the integral.

$$\int_{x=0}^{x=2}\int_{y=0}^{y=4-x^2} \frac{x \cdot e^{2y}}{4-y}\, dy \cdot dx$$

$\boxed{x = 0}$ ✓ $\boxed{y = 0}$ ✓

$\boxed{x = 2}$ $y = 4 - x^2$

$\boxed{x^2 = -y + 4}$ ✓



Find co-ordinate of A ≡ (   )

To solve

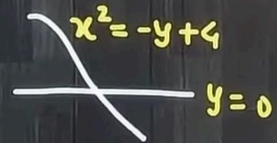
$x^2 = -y + 4$

$Y = 0$

$x^2 = -0 + 4$

$x^2 = 4$ $x = \sqrt{4}$

$x = \pm 2$



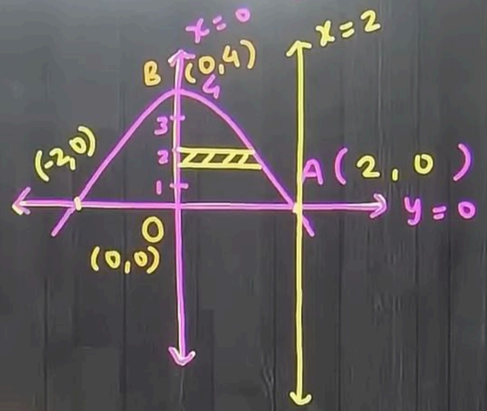
13:40

ACADEMY

13:41

change the order of integration and evaluate the integral.

$$\int_{x=0}^{x=2}\int_{y=0}^{y=4-x^2} \frac{x\cdot e^{2y}}{4-y}\,dy\cdot dx$$



$\boxed{x = 0}$ $\boxed{y}$

$\boxed{x = 2}$ y

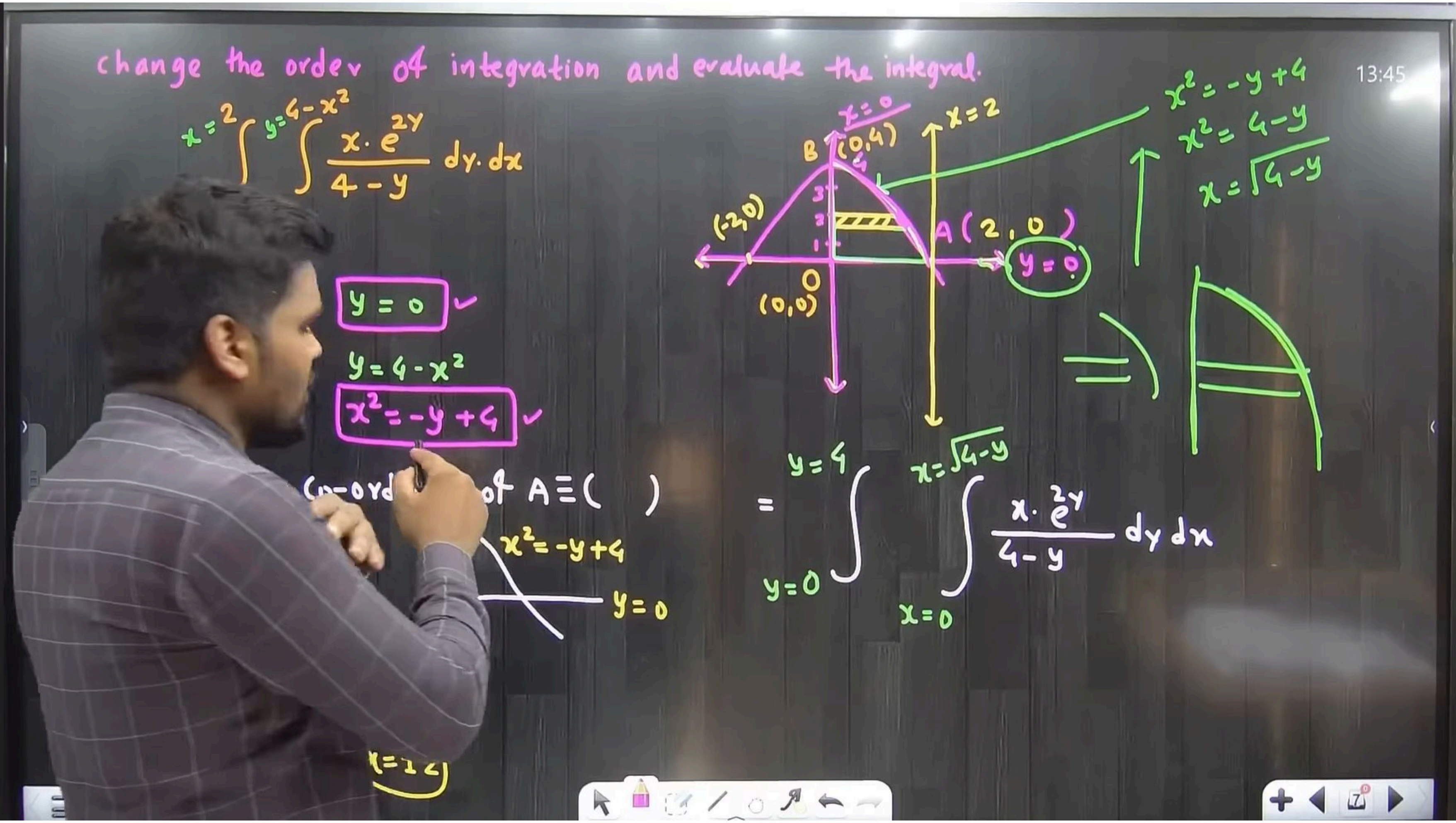
TOSN

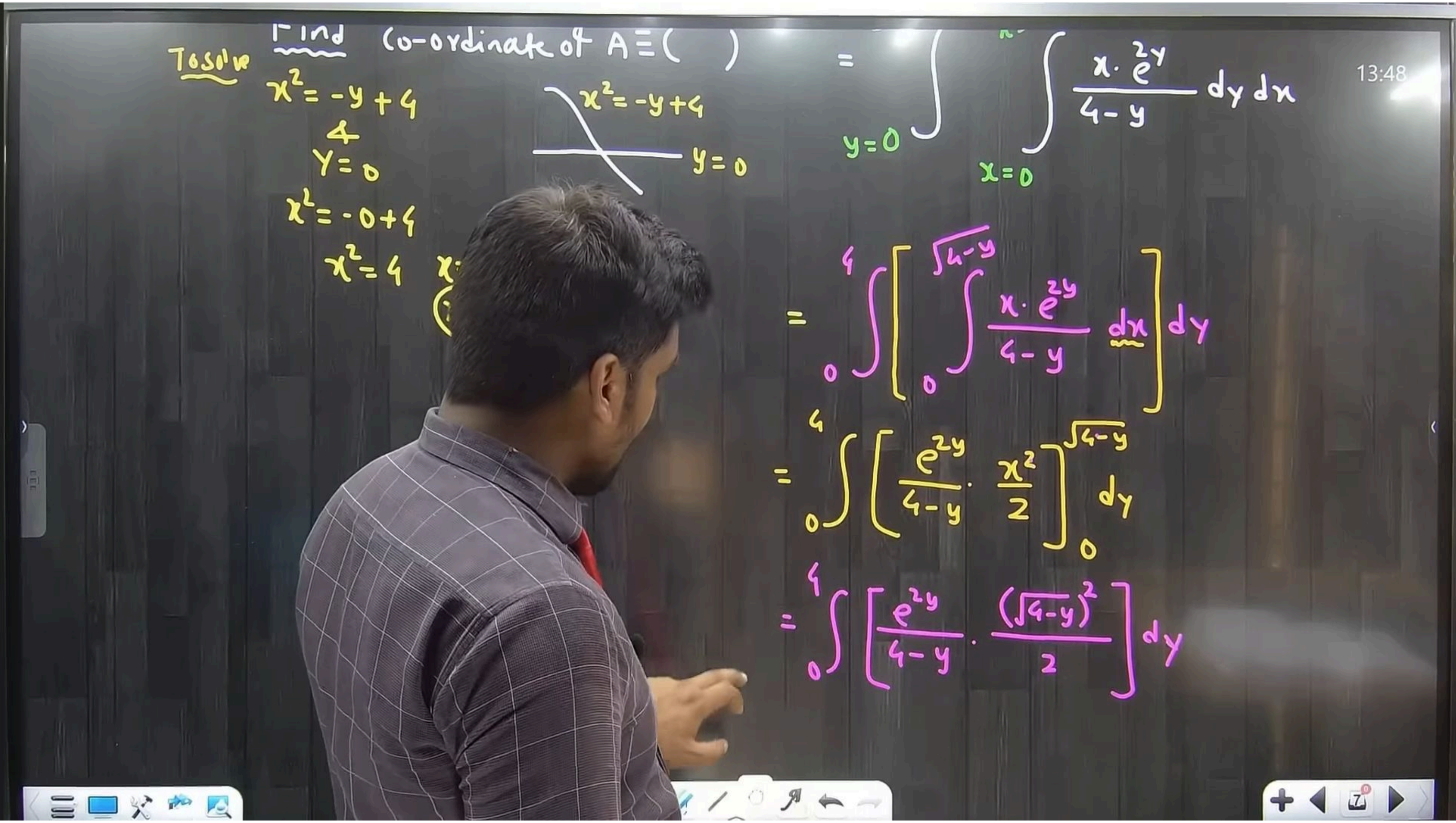
A ≡ ( )

=4

=0

Because limit me $4-x^2$
hai isliye strip will be
parallel to x
Agar y ka term aata toh
parallel to y


change the order of integration and evaluate the integral.
x=2 ∫ y=4-x² ∫ x·e^2y / 4-y dy·dx
y = 0
y = 4 - x²
x² = -y + 4
co-ord of A ≡ ( )
x² = -y + 4
y = 0
B (0,4)
x = 0
x = 2
(-2,0)
A (2, 0)
y = 0
O (0,0)
x² = -y + 4
x² = 4 - y
x = √4-y
= y=4 ∫ y=0 x=√4-y ∫ x=0 x·e^2y / 4-y dy dx
13:45


To solve
Find co-ordinate of A ≡ ( )
$x^2 = -y + 4$
4
$y = 0$
$x^2 = -0 + 4$
$x^2 = 4$
$x^2 = -y + 4$
$y = 0$
$= \int_{y=0} \int_{x=0} \frac{x \cdot e^{2y}}{4-y} \, dy \, dx$
13:48
$= \int_0^4 \left[\int_0^{\sqrt{4-y}} \frac{x \cdot e^{2y}}{4-y} \, dx \right] dy$
$= \int_0^4 \left[\frac{e^{2y}}{4-y} \cdot \frac{x^2}{2} \right]_0^{\sqrt{4-y}} dy$
$= \int_0^4 \left[\frac{e^{2y}}{4-y} \cdot \frac{(\sqrt{4-y})^2}{2} \right] dy$

$$= \int_0^4 \left[\frac{e^{2y}}{4-y} \cdot \frac{x^2}{2} \right]_0^{\sqrt{4-y}} dy$$

$$= \int_0^4 \left[\frac{e^{2y}}{4-y} \cdot \frac{(\sqrt{4-y})^2}{2} \right] dy$$

$$= \int_0^4 \left[\frac{e^{2y}}{\cancel{(4-y)}} \times \frac{\cancel{(4-y)}}{2} \right] dy$$

$$= \frac{1}{2} \int_0^4 e^{2y}\, dy$$

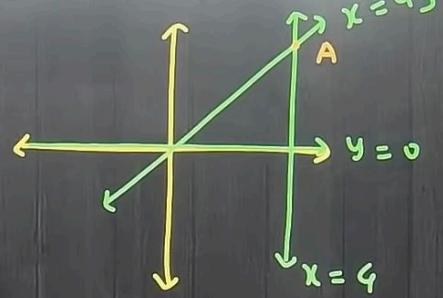
$$= \frac{1}{2} \left[\frac{e^{2y}}{2} \right]_0^4$$

$$= \frac{1}{4} \left[e^{2y} \right]_0^4$$

$$= \frac{1}{4} \left[e^{2\times 4} - e^{2\times 0} \right]$$

$$= \frac{1}{4} \left[e^{8} - 1 \right]$$

$$\int_{y=0}^{y=1}\int_{x=4y}^{x=4} e^{x^2}\,dx\,dy$$



$y = 0$ $\quad \boxed{x = 4y}$

$y = 1$ $\quad x = 4$

Find $A \equiv (4, 1)$

$\boxed{x = 4}$ & $x = 4y$

$4 = 4y$

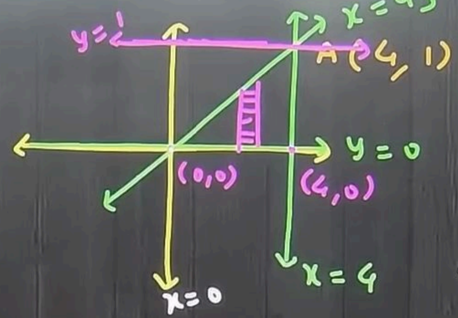
$\frac{4}{4} = y$

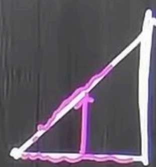
$\boxed{1 = y}$

$$\int_{y=0}^{y=1}\int_{x=4y}^{x=4} e^{x^2}\,dx\,dy$$

$y = 0$

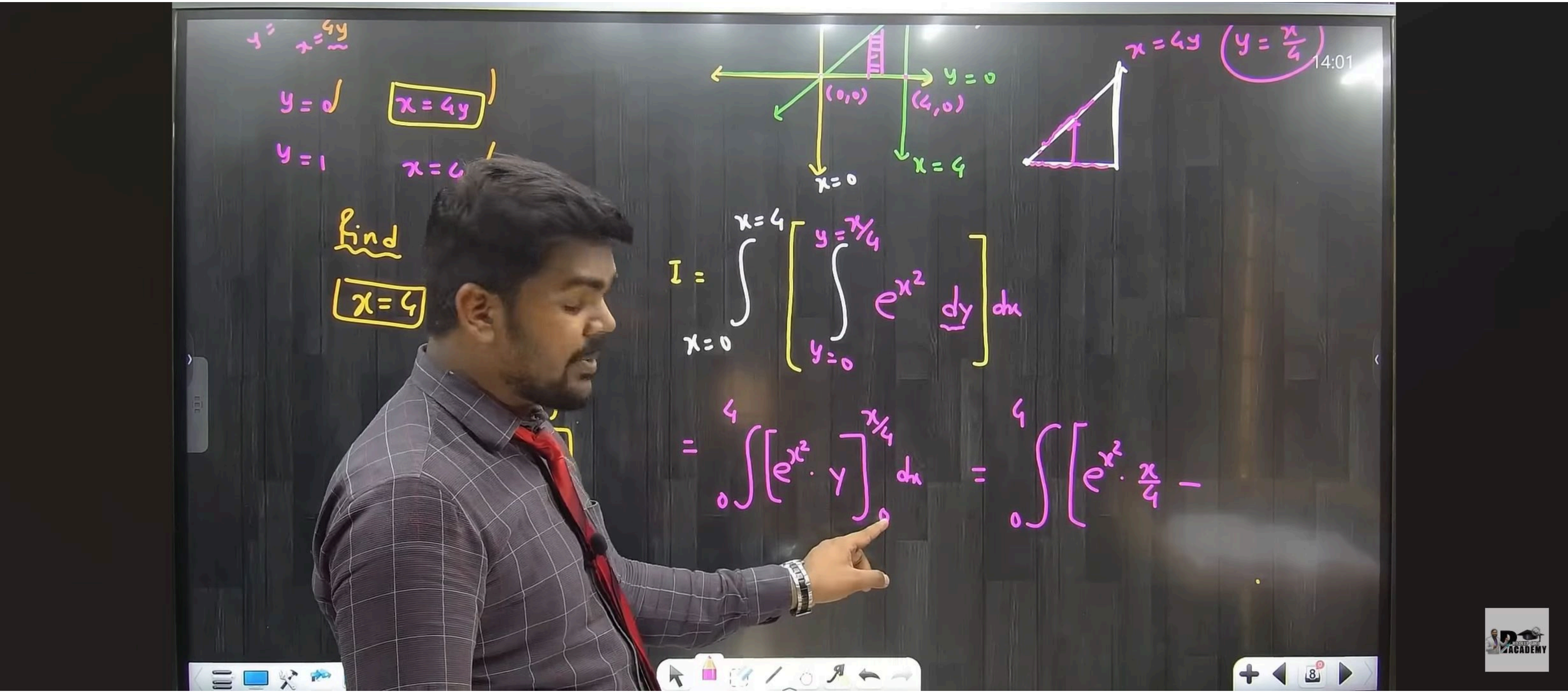
$y = 1$



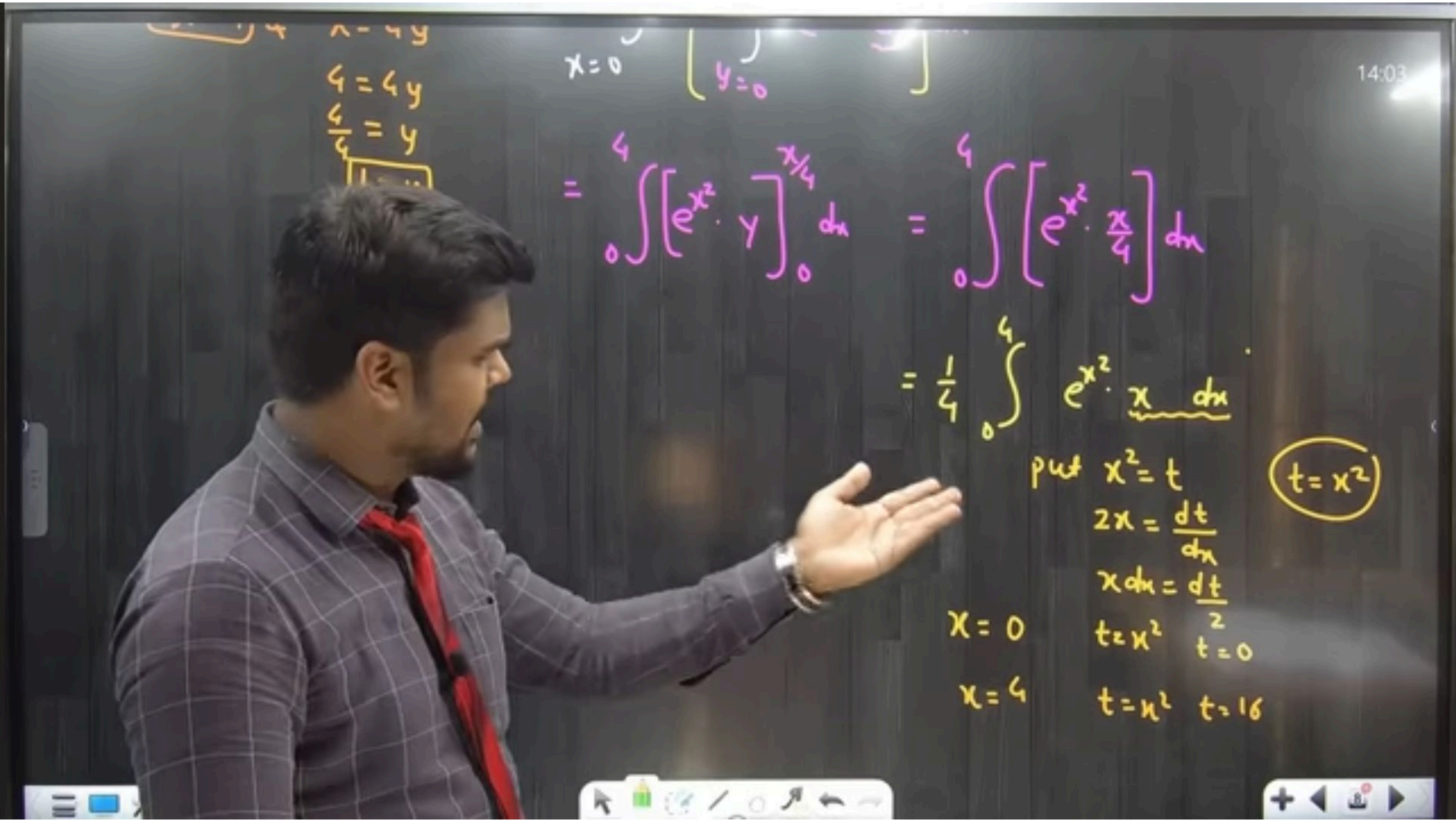


$y = \frac{x}{4}$

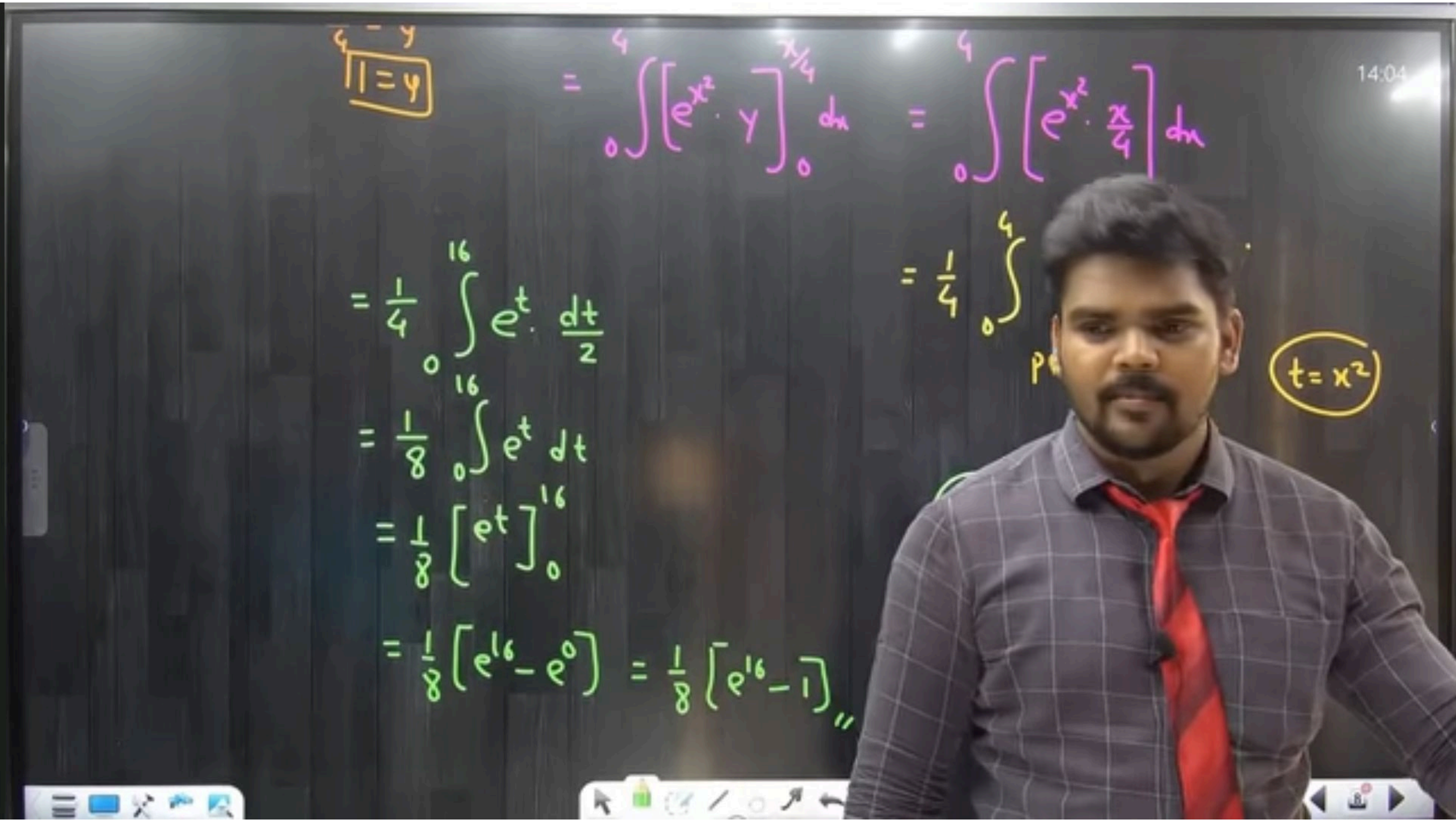
$$\int_{x=0}^{x=4}\int_{y=0}^{y=x/4}$$



x = 4y
y = 0
x = 4y
y = 1
Find
x = 4
y = 0
(0,0)
(4,0)
x = 0
x = 4
x = 4y
y = x/4
14:01
I = ∫_{x=0}^{x=4} [∫_{y=0}^{y=x/4} e^{x^2} dy] dx
= ∫_0^4 [e^{x^2} · y]_0^{x/4} dx
= ∫_0^4 [e^{x^2} · x/4 −
ACADEMY



14:03
4 = 4y
4/4 = y
x = 0
y = 0
$= \int_0^4 \left[e^{x^2} \cdot y\right]_0^{x/4} dx = \int_0^4 \left[e^{x^2} \cdot \frac{x}{4}\right] dx$
$= \frac{1}{4} \int_0^4 e^{x^2} \cdot x \, dx$
put $x^2 = t$
$2x = \frac{dt}{dx}$
$x\,dx = \frac{dt}{2}$
$x = 0$ $t = x^2$ $t = 0$
$x = 4$ $t = x^2$ $t = 16$
$t = x^2$



$\boxed{1 = y}$
$= \int_0^4 \left[e^{x^2}\cdot y\right]_0^{x/4} dx = \int_0^4 \left[e^{x^2}\cdot \frac{x}{4}\right] dx$
$= \frac{1}{4}\int_0^4$
$t = x^2$
$= \frac{1}{4}\int_0^{16} e^t \cdot \frac{dt}{2}$
$= \frac{1}{8}\int_0^{16} e^t \, dt$
$= \frac{1}{8}\left[e^t\right]_0^{16}$
$= \frac{1}{8}\left[e^{16} - e^0\right] = \frac{1}{8}\left[e^{16} - 1\right]$
14:04
ACADEMY

$$\int_{x=0}^{x=1}\int_{y=0}^{y=\sqrt{1-x^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dy\,dx$$

$x=0 \qquad y=0$

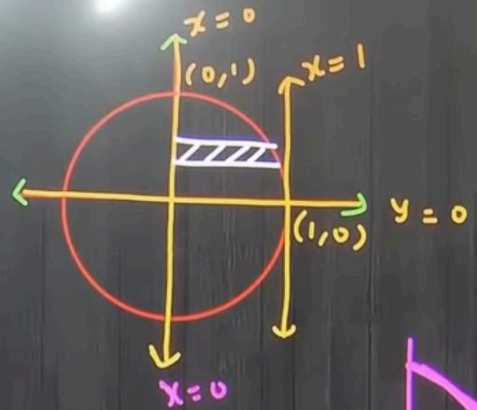
$x=1 \qquad y=\sqrt{1-x^2}$

$$y^2=1-x^2$$

$$\boxed{x^2+y^2=1}$$

14:07

$$\int_{x=0}^{x=1}\int_{y=0}^{y=\sqrt{1-x^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dy\,dx$$



$x = 0$ ✓ $\quad y = 0$ ✓

$x = 1$ ✓ $\quad y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

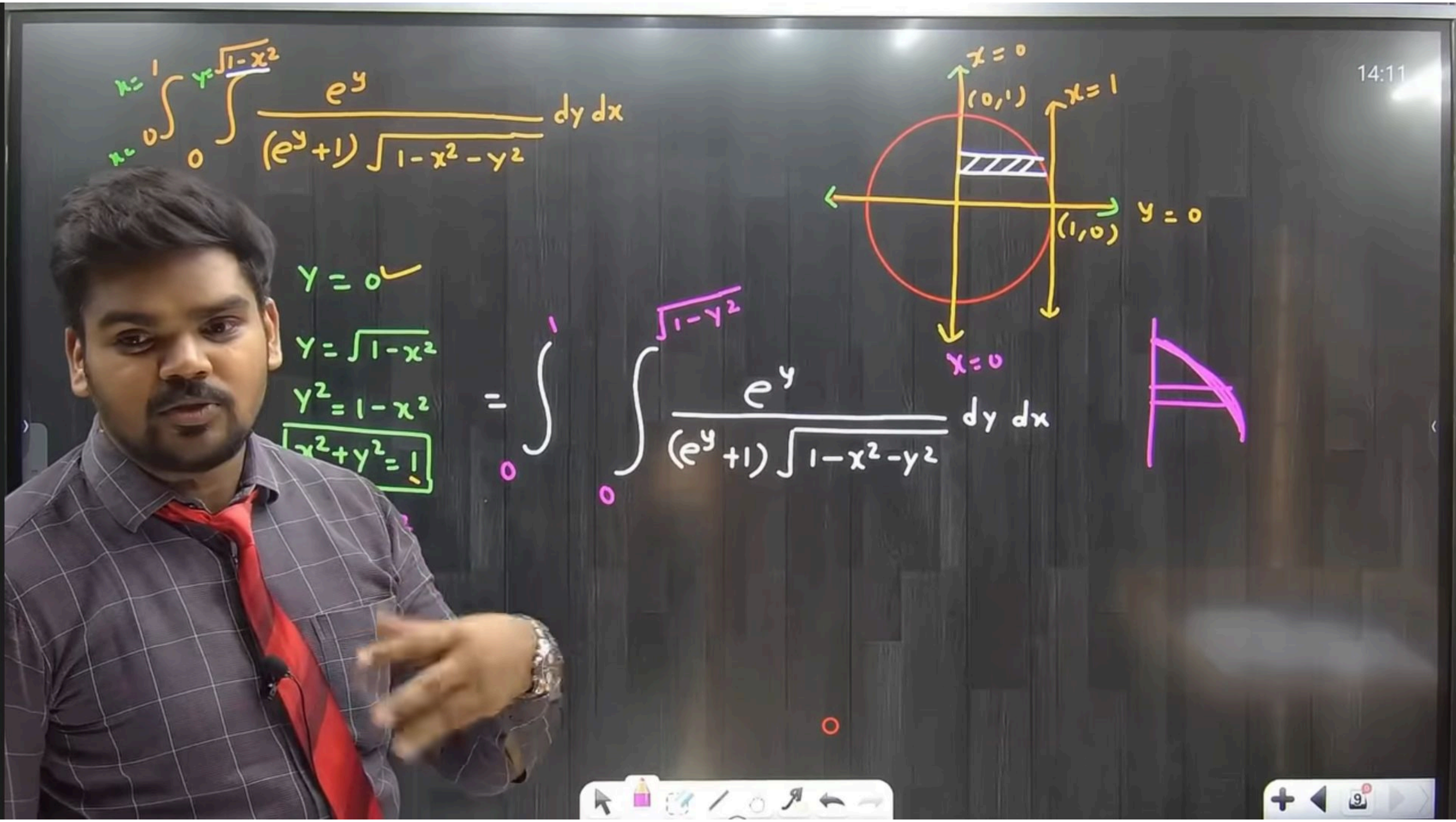
$\boxed{x^2 + y^2 = 1}$

$x^2 = 1 - y^2$

$x = \sqrt{1-y^2}$

$$= \int_0^1 \int_0 \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dy\,dx$$

14:10



$\int_{x=0}^{x=1}\int_{0}^{y=\sqrt{1-x^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dy\,dx$
$y=0$
$y=\sqrt{1-x^2}$
$y^2=1-x^2$
$\boxed{x^2+y^2=1}$
$=\int_0^1\int_0^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dy\,dx$
$x=0$
$(0,1)$
$x=1$
$y=0$
$(1,0)$
$x=0$
14:11
ACADEMY

$x = 0$ $\quad y = 0$

$x = 1$ $\quad y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

$\boxed{x^2 \ldots}$

x^2

$x = 0$

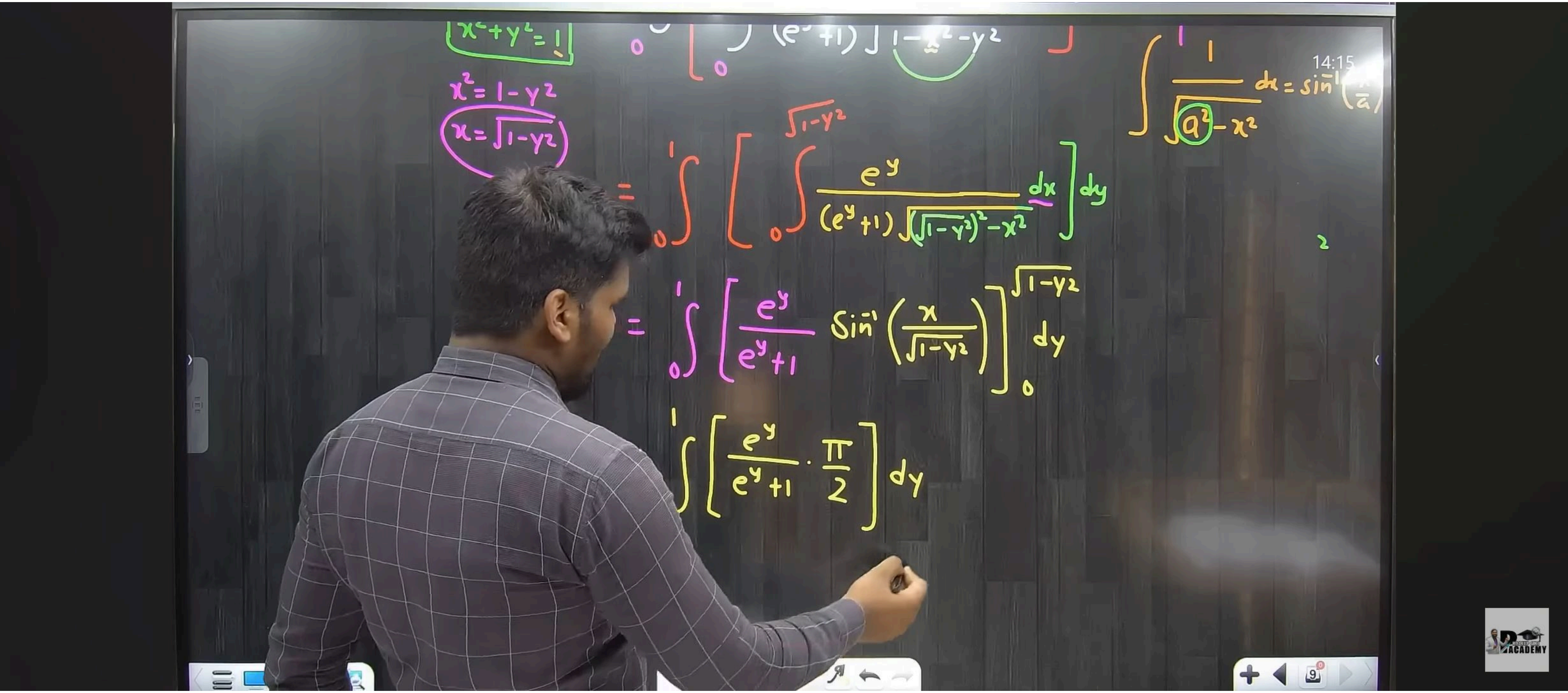
$$= \int_0^1 \left[\int_0^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\, dx \right] dy$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\int_0^1 \left[\int_0^{\sqrt{1-y^2}} \frac{e^y}{(e^y+1)\sqrt{\left(\sqrt{1-y^2}\right)^2 - x^2}}\, dx \right] dy$$

$$\left[\frac{e^y}{e^y+1} \sin^{-1}\left(\frac{x}{\sqrt{1-y^2}}\right) \right]_0^{\sqrt{1-y^2}} dy$$

2


$x^2+y^2=1$
$x^2=1-y^2$
$x=\sqrt{1-y^2}$
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\left(\frac{x}{a}\right)$
14:15
$=\int_0^1\left[\int_0^{\sqrt{1-y^2}}\frac{e^y}{(e^y+1)\sqrt{(\sqrt{1-y^2})^2-x^2}}\,dx\right]dy$
2
$=\int_0^1\left[\frac{e^y}{e^y+1}\sin^{-1}\left(\frac{x}{\sqrt{1-y^2}}\right)\right]_0^{\sqrt{1-y^2}}dy$
$\int_0^1\left[\frac{e^y}{e^y+1}\cdot\frac{\pi}{2}\right]dy$
ACADEMY

14:17

$$\int \frac{f'(x)}{f(x)}\,dx = \log f(x)$$

$$= \frac{\pi}{2} \int_0^1 \frac{e^y}{e^y+1}\,dy$$

$$= \frac{\pi}{2}\left[\log\left(e^y+1\right)\right]_0^1$$

$$= \frac{\pi}{2}\left[\log\left(e^1+1\right) - \log\left(e^0+1\right)\right]$$

$$= \frac{\pi}{2}\left[\log(\underset{m}{e+1}) - \log \underset{n}{2}\right]$$

$$\log m - \log n = \log\left(\frac{m}{n}\right)$$

$$= \frac{\pi}{2}\log\left(\frac{e+1}{2}\right)$$

//