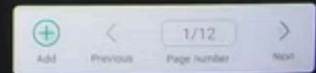
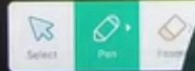
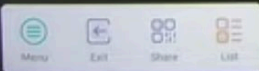


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Triple Integration



$$\int_{x=a}^{x=b} \int_{y=\phi_1(x)}^{y=\phi_2(x)} \int_{z=f_1(x,y)}^{z=f_2(x,y)} f(x,y,z) dz dy dx$$

① $\int_{y=0}^{y=1} \int_{x=y^2}^{x=1} \int_{z=0}^{z=1-x} x \, dz \, dx \, dy$

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$$\int_0^1 \int_{y^2}^1 [xz]_0^{1-x} dx dy$$

$$\int_0^1 \int_{y^2}^1 [x - x^2] dx dy$$

$$\int_{y_2}^1 \left[\frac{x^2}{2} - \frac{x^3}{3} \right] dy$$

$$(y^2)^3 \int_0^1 \left[\frac{x^2}{2} - \frac{x^3}{3} \right] dy$$

$$\int_0^1 \left[\frac{1}{2} - \frac{1}{3} - \left(\frac{y^4}{2} - \frac{y^6}{3} \right) \right] dy$$

$$= \int_0^1 \left[\frac{1}{6} - \frac{y^4}{2} + \frac{y^6}{3} \right] dy$$

$$\begin{aligned}
 & \left[\frac{y^5}{2} - \frac{y^6}{3} \right]_0^1 \\
 &= \int_0^1 \left(\frac{y^4}{2} - \frac{y^5}{3} \right) dy \\
 &= \left[\frac{y^5}{10} - \frac{y^6}{18} \right]_0^1 \\
 &= \left[\frac{1}{10} - \frac{1}{18} - 0 \right]
 \end{aligned}$$

$$= \left[\frac{4}{6} - \frac{4}{10} + \frac{4}{21} \right]_0^1$$

$$= \left[\frac{1}{6} - \frac{1}{10} + \frac{1}{21} \right]$$

$$= 4/35$$



$$\textcircled{2} \int_0^2 \int_1^2 \int_0^{yz} xyz \, dx dy dz$$

$$= \int_0^2 \int_1^2 \left(\left[\frac{x^2}{2} \right] yz \right)_0^{yz} dy dz$$

$$= \int_0^2 \int_1^2 \frac{(yz)^3}{2} dy dz$$

$$= \int_0^2 \int_1^2 \left(\left[\frac{x^2}{2} \right] yz \right) dy dz$$

$$= \int_0^2 \int_1^2 \frac{y^3 z^3}{2} dy dz$$

$$= \frac{1}{8} \int_0^2 \left(\left[y^4 \right] z^3 \right) dz$$

$$= \frac{1}{8} \int_0^2 \left[2^4 z^3 - z^3 \right]$$

$$= \frac{15}{8} \int_0^2 z^3 dz$$

$$= \frac{15}{8} \left[\frac{z^4}{4} \right]_0^2$$

$$= \frac{15}{8} \left[\frac{2^4}{4} \right]$$

$$= \frac{15}{8} \times 4$$

$$= \frac{15}{2}$$

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(3)
$$\int_0^{\log 2} \int_0^x \int_0^{x+y} \frac{e^{x+y+z}}{e^{x+y} \cdot e^z} dz dy dx$$

$$\int_0^{\log 2} \int_0^x \left(e^{x+y} e^z \right)_0^{x+y} dy dx$$

$$\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y} e^z \, dz \, dy \, dx$$

$$\int_0^{\log 2} \int_0^x \left(e^{x+y} e^z \right)_0^{x+y} dy \, dx$$

$$= \int_0^{\log 2} \int_0^x e^{x+y} e^{x+y} - e^{x+y} \, dy \, dx$$

$$\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y} \cdot e^z \, dz \, dy \, dx$$

$$\int_0^{\log 2} \int_0^x \left(e^{x+y} e^z \right)_0^{x+y} dy \, dx$$

$$= \int_0^{\log 2} \int_0^x e^{2x+2y} - e^{x+y} dy \, dx$$

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$$= \int_0^{\log 2} \int_0^x \frac{e^{2x+2y} - e^{x+y}}{e^{2x} \cdot e^{2y} - e^x \cdot e^y} dy dx$$

$$= \int_0^{\log 2} \left[e^{2x} \frac{e^{2y}}{2} - e^x e^y \right]_0^x dx$$

$$= \int_0^{\log 2} \left[e^{2x} \cdot \frac{e^{2x}}{2} \right]$$

$$e^{2x} \cdot e^{2y} - e^x \cdot e^y$$

$$= \int_0^{\log 2} \left[e^{2x} \frac{e^{2y}}{2} - e^x e^y \right]_0^x dx$$

$$= \int_0^{\log 2} \left[\frac{e^{4x}}{2} - e^{2x} - \frac{e^{2x}}{2} + e^x \right] dx$$

$$= \int_0^{\log 2} \left[\frac{e^{4x}}{2} - e^{2x} - \frac{e^{2x}}{2} + e^x \right] dx$$

$$= \left[\frac{e^{4x}}{8} - \frac{e^{2x}}{2} - \frac{e^{2x}}{4} + e^x \right]_0^{\log 2}$$

$$= \left[\frac{e^{4 \log 2}}{8} - \frac{e^{2 \log 2}}{2} - \frac{e^{2 \log 2}}{4} + e^{\log 2} \right] - \left[\frac{1}{8} - \frac{1}{2} - \frac{1}{4} + 1 \right]$$

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$$= \left[\frac{e^{4x}}{8} - \frac{e^{2x}}{2} - \frac{e^{2x}}{4} + e^x \right]_{\log^2 4}^{\log^2 0}$$

$$= \left[\frac{e^{4 \log^2 2}}{8} - \frac{e^{2 \log^2 2}}{2} - \frac{e^{2 \log^2 2}}{4} + e^{\log^2 2} \right] - \left[\frac{1}{8} - \frac{1}{2} - \frac{1}{4} + 1 \right]$$

$$= 1 - \frac{1}{8} + \frac{1}{2} + \frac{1}{4} - 1$$

$$= \frac{5}{8}$$

$$\textcircled{4} \int_{-1}^1 \int_z^2 \int_{x-z}^{x+z} (x+y+z) dy dx dz$$

$$\int_{-1}^1 \int_0^2 [(x+z) + y] dy dx dz$$

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$$= \int_{-1}^1 \int_0^z \int_{x-z}^{x+z} [(x+z) + y] dy dx dz$$

$$= \int_{-1}^1 \int_0^z \left[(x+z)y + \frac{y^2}{2} \right]_{x-z}^{x+z} dx dz$$

$$\int_0^z (x+z)$$

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$$= \int_{-1}^1 \int_0^z \left[(x+z)y + \frac{y^2}{2} \right]_{x-z}^{x+z} dx dz$$

$$= \int_{-1}^1 \int_0^z \left[(x+z)^2 + \frac{(x+z)^2}{2} - \left(x^2 - z^2 + \frac{(x-z)^2}{2} \right) \right] dx dz$$



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$$\int_{-1}^1 \int_0^2 \frac{(x+z)^2 + \frac{(x+z)^2}{2} - x^2 + z^2 - \frac{(x-z)^2}{2}}{2} dx dz$$

$$\int_{-1}^1 \int_0^2 \frac{3(x+z)^2}{2} - x^2 + z^2 - \frac{(x-z)^2}{2} dx dz$$

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$$\int_0^1 \int_0^2 \frac{3(x+z)^2}{2} - x^2 + z^2 - \frac{(x-z)^2}{2} dx dz$$

$$= \int_{-1}^1 \left[\frac{(x+z)^3}{2} - \frac{x^3}{3} + z^2 x - \frac{(x-z)^3}{6} \right] dz$$

$$\begin{aligned}
 & \int_0^1 \int_{-1}^1 \left[\frac{(x+z)^3}{2} - \frac{x^3}{3} + z^2x - \frac{(x-z)^3}{6} \right] dx dz \\
 &= \int_{-1}^1 \left[\frac{(2z)^3}{2} - \frac{z^3}{3} + z^3 - \left(\frac{z^3}{2} + \frac{z^3}{6} \right) \right] dz
 \end{aligned}$$

$$\int_0^1 \int_{-1}^1 \left[\frac{(x+z)^3}{2} - \frac{x^3}{3} + z^2x - \frac{(x-z)^3}{6} \right] dx dz$$

$$= \int_{-1}^1 \left[\frac{(x+z)^3}{2} - \frac{x^3}{3} + z^2x - \frac{(x-z)^3}{6} \right]_0^1 dz$$

$$= \int_{-1}^1 \left[4z^3 - \frac{z^3}{3} + z^3 - \frac{z^3}{2} - \frac{z^3}{6} \right] dz$$

$$= \int_{-1}^1 \left[\frac{z^4}{2} - \frac{z^3}{3} + z^2x - \frac{(x-z)^3}{6} \right] dz$$

$$= \int_{-1}^1 \left[4z^3 - \frac{z^3}{3} + z^2 - \frac{z^3}{2} - \frac{z^3}{6} \right] dz$$

$$= \left[\frac{4z^4}{4} - \frac{z^4}{12} + \frac{z^4}{4} - \frac{z^4}{8} - \frac{z^4}{24} \right]_{-1}^1$$

$$= \int_{-1}^1 \left(4z^5 - \frac{z^5}{3} + z^5 - \frac{z^5}{2} - \frac{z^5}{6} \right) dz$$

$$= \left[\frac{4z^6}{6} - \frac{z^6}{18} + \frac{z^6}{6} - \frac{z^6}{12} - \frac{z^6}{36} \right]_{-1}^1$$

$$= \left(1 - \frac{1}{18} + \frac{1}{6} - \frac{1}{12} - \frac{1}{36} \right) - \left(1 + \frac{1}{18} - \frac{1}{6} + \frac{1}{12} + \frac{1}{36} \right)$$

$$= \int_{-1}^1 4z^5 - \frac{z^5}{3} + z^5 - \frac{z^5}{2} - \frac{z^5}{6} dz$$

$$= \left[\frac{4z^6}{6} - \frac{z^6}{18} + \frac{z^6}{6} - \frac{z^6}{12} - \frac{z^6}{36} \right]_{-1}^1$$

$$= \left(\frac{4}{6} - \frac{1}{18} + \frac{1}{6} - \frac{1}{12} - \frac{1}{36} \right) - \left(\frac{4}{6} - \frac{1}{18} + \frac{1}{6} - \frac{1}{12} - \frac{1}{36} \right)$$

$$= 0$$