Data605 Assignment 2

Problem set 1

① Show that $A^TA \neq AA^T$ in general (proof and demo)

Solution: →

$$A = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} x_1^2+y_1^2+z_1^2 & x_1x_2+y_1y_2+z_1z_2 & x_1x_3+y_{13}+z_1z_3 \\ x_2x_1+y_2y_1+z_2z_1 & x_2^2+y_2^2+z_2^2 & x_2x_3+y_2y_3+z_2z_3 \\ x_3x_1+y_3y_1+z_3z_1 & x_3x_2+y_3y_2+z_3z_2 & x_3^2+y_3^2+z_3^2 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} x_1^2+x_2^2+x_3^2 & x_1y_1+x_2y_2+x_3y_3 & x_1z_1+x_2z_2+x_3z_3 \\ y_1x_1+y_2x_2+y_3x_3 & y_1^2+y_2^2+y_3^2 & y_1z_1+y_2z_2+y_3z_3 \\ z_1x_1+z_2x_2+z_3x_3 & z_1y_1+z_2y_2+z_3y_3 & z_1^2+z_2^2+z_3^2 \end{bmatrix}$$

We can see here $A^TA \neq AAT$

Example :⇒

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 2 & 4 \\ 1 & 0 & 1 \end{bmatrix} \quad \therefore A^T = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 0 \\ 5 & 4 & 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 3 & 5 & 10 \\ 5 & 13 & 23 \\ 10 & 23 & 42 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 35 & 27 & 6 \\ 27 & 21 & 5 \\ 6 & 5 & 2 \end{bmatrix}$$

$\therefore A^TA \neq AA^T$

(2) For a special type of square matrix A, we get $A^TA = AA^T$! under what conditions could this be true.

Solution :⇒ $A^TA = AA^T$ only when $A^T = A$ i.e. the transpose of the matrix A is the same as matrix

Example

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore A^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore A^TA = AA^T$