Data 605: Week 3 Assignment

Problem Set 2 [manual work]

1. Compute the eigenvalues and eigenvectors of matrix A.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

For Eigenvalue :→

$$A\vec{V} = \lambda\vec{V}$$

$$\lambda I_3 \vec{V} - A\vec{V} = \vec{0}$$

$$(\lambda I_3 - A)\vec{V} = \vec{0}$$

$$\det(\lambda I_3 - A) = 0$$

$$\det\left\langle \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \right\rangle = 0$$

$$\det \begin{bmatrix} \lambda - 1 & -2 & -3 \\ 0 & \lambda - 4 & -5 \\ 0 & 0 & \lambda - 6 \end{bmatrix} = 0$$

$$(\lambda - 1)\left[(\lambda - 4)(\lambda - 6) - (-5) \times 0\right] = 0$$

$$(\lambda - 1)(\lambda^2 - 10\lambda + 24) = 0$$

$$(\lambda^3 - 11\lambda^2 + 34\lambda - 24) = 0$$

The eqn can be factorized as

$$(\lambda-1)(\lambda-6)(\lambda-4) = 0$$

∴ The eigenvalues of A are

$$(1, 6, 4)$$

$$\Downarrow$$

Same as calculated by R

Substitute Eigenvalues into eqn

$$(\lambda I_3 - A)\vec{V} = \vec{0}$$

① $\lambda = 1$

$$\begin{bmatrix} 0 & -2 & -3 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_2 = 0 \quad v_3 = 0$

$$E_{\lambda=1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(2) $\lambda = 6$

$$\begin{bmatrix} 5 & -2 & -3 \\ 0 & 2 & -5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad v_1 = \frac{2}{5} v_2 + \frac{3}{5} v_3 = \frac{8}{5} v_3$$

$$v_2 = \frac{5}{2} v_3$$

$$E_{\lambda=6} = S\left(\begin{bmatrix} 8/5 \\ 5/2 \\ 1 \end{bmatrix} \right)$$

$$\text{length} = \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{5}{2}\right)^2 + 1^2} = 3.1321$$

$$E_{\lambda=6} = \begin{bmatrix} 0.5108 \\ 0.7982 \\ 0.3193 \end{bmatrix}$$

(3) $\lambda = 4$

$$\begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & -5 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v_1 = \frac{2}{3} v_3$$

$$v_3 = 0$$

$$\text{length} = \sqrt{\left(\frac{2}{3}\right)^2 + 1^2 + 0^2} = 1.2019$$

$$E_{\lambda=4} = \begin{bmatrix} 0.5547 \\ 0.8321 \\ 0 \end{bmatrix}$$

∴ Eigen-vectors for $\lambda = 1$, $\lambda = 6$ & $\lambda = 4$ are

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0.5108 \\ 0.7982 \\ 0.3193 \end{bmatrix} \quad \begin{bmatrix} 0.5547 \\ 0.8321 \\ 0 \end{bmatrix}$$