Module1

Complex function: Cartesian form: $w = f(z) = u(x,\ y) + \boldsymbol{i}\, v(x,\ y)$.

Polar form: $w = f(z) = u(r,\ \theta) + \boldsymbol{i}\, v(r,\ \theta)$.

Cartesian form	Polar form
If $f(z) = u(x,\ y) + \boldsymbol{i}\, v(x,\ y)$ is analytic then 1. $u_x = v_y,\ u_y = -v_x$ (C-R equations) 2. $u_{xx} + u_{yy} = 0$, and $v_{xx} + v_{yy} = 0$. 3. $f'(z) = u_x + iv_x = v_y - iu_y$ $= u_x - iu_y = v_y + iv_x$ If u or v are given then find $f'(z)$ by substituting $x = z$ and $y = 0$ in $f'(z)$ and find $f(z)$ by integrating $f'(z)$.	If $f(z) = u(r,\ \theta) + \boldsymbol{i}\, v(r,\ \theta)$ is analytic then 1. $ru_r = v_\theta,\ rv_r = -u_\theta$ (C-R equations) 2. $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$, and $v_{rr} + \frac{1}{r}v_r + \frac{1}{r^2}v_{\theta\theta} = 0$ 3. $f'(z) = \frac{ru_r + \boldsymbol{i}\, rv_r}{z} = \frac{v_\theta - \boldsymbol{i}\, u_\theta}{z}$ $= \frac{ru_r - \boldsymbol{i}\, u_\theta}{z} = \frac{v_\theta + \boldsymbol{i}\, rv_r}{z}$. If u or v are given then find $f'(z)$ by substituting $r = z$ and $\theta = 0$ in $f'(z)$ and find $f(z)$ by integrating $f'(z)$.

Module2:

1. Transformation $\boldsymbol{of\ w = e^z}$

Here, $w = u + iv = e^{x+iy} = e^x e^{iy} = \rho e^{i\varphi}$

$\Longrightarrow \rho = e^x$ & $\varphi = y$.

2. Transformation $\boldsymbol{w = z^2}$.

Here, w = u + iv $= (x + iy)^2 = (x^2 - y^2) + i\, 2xy$

$\Longrightarrow u = x^2 - y^2$ & $v = 2xy$

3. $\boldsymbol{Transformation\ of\ w = Z + \frac{1}{Z}}$, $z \neq 0$

Here $w = \left(r + \frac{1}{r}\right) cos\theta + i\left(r - \frac{1}{r}\right) sin\theta$, $\therefore u = \left(r + \frac{1}{r}\right) cos\theta$ & $v = \left(r - \frac{1}{r}\right) sin\theta$

$\frac{u^2}{\left(r+\frac{1}{r}\right)^2} + \frac{v^2}{\left(r-\frac{1}{r}\right)^2} = 1$, $r \neq 1$, and $\frac{u^2}{(2cos\theta)^2} - \frac{v^2}{(2sin\theta)^2} = 1$

Bilinear Transformation (BLT) :

The transformation $w = \frac{az+b}{cz+d}$ where a, b, c, d are real or complex constants such that $ad - bc \neq 0$ is called a BLT. If $w = \frac{az+b}{cz+d}$, then $z = \frac{dw-b}{-cw+a}$.

Cauchy's Theorem:

Statement: If $f(z)$ is an analytic function and $f'(z)$ is continuous at each point within and on a closed curve C, then $\int_C f(z)dz = 0$.

Cauchy's Integral Formula:

Statement: If $f(z)$ is analytic inside and on a simple closed curve C and if 'a' is any point within C then $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a} dz$.

Note: i) $\int_C \frac{f(z)}{z-a} dz = \begin{cases} 0 & if\ z = a\ lies\ outside\ C \quad (by\ Cauchy's\ Theorem) \\ 2\pi i f(a) & if\ z = a\ lies\ inside\ C \quad (by\ Cauchy\ Integral\ formula) \end{cases}$

ii) $\int_C \frac{f(z)}{(z-a)^{n+1}} dz = \begin{cases} 0 & if\ z = a\ lies\ outside\ C \quad (by\ Cauchy's\ Theorem) \\ \frac{2\pi i}{n!} f^n(a) & if\ z = a\ lies\ inside\ C \quad (by\ Cauchy\ Integral\ formula) \end{cases}$

Module3:

Discrete Random Variable	**Continuous Random Variable**
$P(x)$ is called probability function for a discrete random variable X, If (i) $P(x) \geq 0$ and (ii) $\sum_x P(x) = 1$.	$f(x)$ is called probability density function (p.d.f.) for a continuous random variable X, If (i) $f(x) \geq 0$ and (ii) $\int_{-\infty}^{\infty} f(x)dx = 1$.
Mean= $\mu = \sum_x xP(x)$.	Mean= $\mu = \int_{-\infty}^{\infty} xf(x)dx$
Variance = $V = \sum_x x^2P(x) - \mu^2$.	Variance = $V = \int_{-\infty}^{\infty} x^2f(x)dx - \mu^2$.
Standard deviation= $\sigma = \sqrt{V}$	Standard deviation= $\sigma = \sqrt{V}$
Cumulative distributive function: $F(x) = P(X \leq x)$ $\therefore \quad F(x_0) = \sum_x P(X \leq x_0)$. (c.d.f. is defined for all real values of x)	Cumulative distributive function (c.d.f.): $F(x) = \int_{-\infty}^{x} f(x)dx$. Therefore p.d.f. $f(x) = \frac{d}{dx}[F(x)]$. $P(a < X < b) = \int_a^b f(x)dx = F(b) - F(a)$.
Expectation of $h(X) = E[h(X)] = \sum_x h(x)P(x)$. Therefore $\mu = E[X]$ and $V = E[X^2] - \{E[X]\}^2$.	Expectation of $h(x) = E[h(x)] = \int_{-\infty}^{\infty} h(x)f(x)dx$.
Binomial Distribution: If a Bernoulli's trail is conducted n times, Probability of x successes in n trail is given by $P(x) = {}^{n}c_x p^x q^{n-x}$. $p + q = 1$. **Mean**= $\mu = np$. **Variance** = $V = npq$. **Standard deviation** = $\sigma = \sqrt{V} = \sqrt{npq}$.	**Exponential distribution**: Probability density function is $f(x) = \begin{cases} \propto e^{-\propto x}, & for\ x \geq 0 \\ 0, & for\ x < 0 \end{cases}$. Mean= $\mu = \frac{\mathbf{1}}{\propto}$. Variance = $V = \frac{\mathbf{1}}{\propto^{\mathbf{2}}}$. Standard deviation= $\sigma = \sqrt{V} = \frac{\mathbf{1}}{\propto}$. Mean and standard deviation of an exponential distribution are same.
Poisson distribution: The limiting case of the binomial distribution by making n very large and p very small, and keeping np fixed ($np = m$) is Poisson distribution. Probability function is $P(x) = \frac{m^x e^{-m}}{x!}$. **Mean**= $\mu = m$. **Variance** = $V = m$. **Standard deviation** = $\sigma = \sqrt{V} = \sqrt{m}$. Mean and variance of a Poisson distribution are same.	**Normal distribution:** A continuous random variable X from $-\infty$ to ∞ is said to have normal distribution with parameter μ and σ^2 if its p.d.f. is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$, mean= $E(X) = \mu,\ \ and\ \ variance = V(X) = \sigma^2$. If $\mu = 0\ and\ \sigma = 1$ then the normal distribution is called standard normal distribution. Let x be a normal variable with mean μ and standard deviation σ , then $z = \frac{x-\mu}{\sigma}$.

Module4: Co-efficient of correlation: The numerical measure of correlation is called co-efficient of correlation r. Let $X = x - \bar{x}$, $Y = y - \bar{y}$ where $\bar{x}$ and $\bar{y}$ are means of x and y respectively.

Standard deviations of x is $\sigma_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$

Then the co-efficient of correlation, $r = \frac{\sum[(x-\bar{x})(y-\bar{y})]}{n\sigma_x\sigma_y} = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}}$.

Line of regression of y on x is, $y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$.

Line of regression of x on y is, $x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$.

$r\frac{\sigma_y}{\sigma_x}$ is called regression coefficient of y on x, and $r\frac{\sigma_x}{\sigma_y}$ is called regression coefficient of x on y.

If θ is the angle between the regression lines, then $\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$.

Note: 1. $r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$

2. Line of regression of y on x is $y - \bar{y} = \frac{n\sum xy - \sum x \sum y}{[n\sum x^2 - (\sum x)^2]}(x - \bar{x})$.

Similarly the line of regression of x on y is,

$$x - \bar{x} = \frac{n\sum xy - \sum x \sum y}{[n\sum y^2 - (\sum y)^2]}(y - \bar{y}).$$

3. $\sigma_x = \frac{\sqrt{n\sum x^2 - (\sum x)^2}}{n}$

Rank correlation between x and y is $\rho = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}} = 1 - \frac{6\sum {d_i}^2}{(n^3-n)}$ where $d_i = x_i - y_i$.

Curve fitting: For the curve $y = a + bx + cx^2$ normal equations are

$$na + b\sum x + c\sum x^2 = \sum y$$
$$a\sum x + b\sum x^2 + c\sum x^3 = \sum xy$$
$$a\sum x^2 + b\sum x^3 + c\sum x^4 = \sum x^2 y$$

For the curve $y = a + bx$, normal equations are $na + b\sum x = \sum y$ and $a\sum x + b\sum x^2 = \sum xy$.

For the curve $y = ax^b$, first taking log, i.e. $\log y = \log a + b\log x$

Or $Y = A + bX$. Where $Y = \log y$, $A = \log a$, $X = \log x$

Normal equations are, $nA + b\sum X = \sum Y$ and $A\sum X + b\sum X^2 = \sum xY$. And $a = e^A$.

Module5

Joint probability distribution: $P(x, y)$ is called joint Probability function for two discrete random variables X and Y If i) $P(x, y) \geq 0$ for all x, y ii) $\sum_x \sum_y P(x, y) = 1$.

Then $[(x, y), P(x, y)]$ is called joint probability distribution.

Marginal distribution of X : $[x, f(x)]$ where $f(x) = \sum_y P(x, y)$.

Marginal distribution of Y : $[y, g(y)]$ where $g(y) = \sum_x P(x, y)$.

$$E(X) = \mu_X = \sum x f(x), \qquad E(Y) = \mu_Y = \sum y g(y).$$
$$V(X) = \sum x^2 f(x) - (\mu_X)^2, \qquad V(Y) = \sum y^2 f(y) - (\mu_Y)^2.$$
$$\sigma_X = \sqrt{V(X)} \qquad \sigma_Y = \sqrt{V(Y)}$$
$$E(XY) = \sum_x \sum_y xyP(x, y)$$

$Cov\,(X,\ Y) = E(XY) - E(X)E(Y)$

Correlation $\rho(X,\ Y) = \frac{Cov\,(X,Y)}{\sigma_X \sigma_Y}$.

If $P(x,\ y) = f(x)g(y)$ for all $x,\ y$ then X and Y are independent.

Simple sampling of attributes: The expected value of success in a sample of size n is np, and standard deviation is $\sqrt{npq}$.

Mean proportion of successes $= \frac{np}{n} = p$.

Standard error of proportion of successes$= \sqrt{\frac{pq}{n}}$.

Precision of the proportion of successes$= \sqrt{\frac{n}{pq}}$.

Test of significance for large samples: If x be the observed number of successes in the large sample and z is the standard normal variate then $z = \frac{x-\mu}{\sigma}$.

1. If $|z| < 1.96$, difference between the observed and expected number of successes is not significant.
2. If $|z| > 1.96$, difference is significant at 5% level of significance.
3. If $|z| > 2.58$, difference is significant at 1% level of significance.

Sampling distribution of the mean: If a population is distributed normally with mean μ and standard deviation σ, then the means of all positive random samples of size n, are also distributed normally with mean μ and standard error $\frac{\sigma}{\sqrt{n}}$. $\therefore z = \frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$.

Student's t − Distribution: Consider a small sample of size n, drawn from a normal population with mean μ and S.D. σ . If $\overline{x}$ *and* σ_s be the sample mean and S.D. Then the statistic, t is defined as

$t = \frac{\overline{x}-\mu}{\sigma_s}\sqrt{n-1}$, where $v = n-1$ denotes the degree of freedom of t.

Significance test of a sample mean: Given a random sample $x_1, x_2, x_3, \cdots\cdots x_n$ from a normal population, we have to test the hypothesis that the mean of the population is μ . for this, we first calculate $t = \frac{\overline{x}-\mu}{\sigma_s}\sqrt{n}$

Where $\overline{x} = \frac{\sum_1^n x_i}{n}$, $\sigma_s{}^2 = \frac{1}{n-1} \sum_1^n (x_i - \overline{x})^2$.

Then find the value of P for the given d.f. from the table.

If the calculated $> t_{0.05}$, the difference between $\overline{x}$ and μ is said to be significant at 5% level of significance.

If $> t_{0.01}$, the difference between $\overline{x}$ and μ is said to be significant at 1% level of significance.

If $< t_{0.05}$, the data is said to be consistent with the hypothesis .

CHI-SQUARE (χ^2) TEST: If O_i and E_i are observed and expected frequencies for $i = 1,2 \cdots n$.

Then $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$ with $n-1$ degrees of freedom.

The equation of χ^2 curve is $y = y_0 e^{-\frac{\chi^2}{2}} (\chi^2)^{\frac{\gamma-1}{2}}$, where $\gamma = n-1$.

Goodness of fit: The value of χ^2 is used to test whether the deviations of the observed frequencies from theoretical frequencies are significant or not.