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Regularisation Techniques : Lasso (L1) & Ridge(L2) ¶

```
As we have observed in our Linear Regressiion - Problems Notebook. How Increased Features cause problems with Model results.

Lets quickly demonstrate here as well.
```

In [1]:

```python
#Importing libraries
import numpy as np
import pandas as pd
import random
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
%matplotlib inline
```

> we will make a dataframe which could mimic a Sine curve

In [2]:

```python
#Defining independent variable as angles from 60deg to 300deg converted to radians
x = np.array([i*np.pi/180 for i in range(10,360,3)])
```

In [3]:

```python
#Setting seed for reproducability
np.random.seed(10)
```

In [4]:

```python
#Defining the target/dependent variable as sine of the independent variable

# y = sin(x) + SOME NOISE BEING ADDED ON TOP OF IT
y_sin_noise = np.sin(x) + np.random.normal(0,0.15,len(x))
y_pure_sin = np.sin(x)

del_y = y_sin_noise - y_pure_sin
```

In [5]:

```python
#Creating the dataframe using independent and dependent variable
sin_df = pd.DataFrame(np.column_stack([x,y_sin_noise]),columns=['x','y'])
sin_df.head()
```

Out[5]:

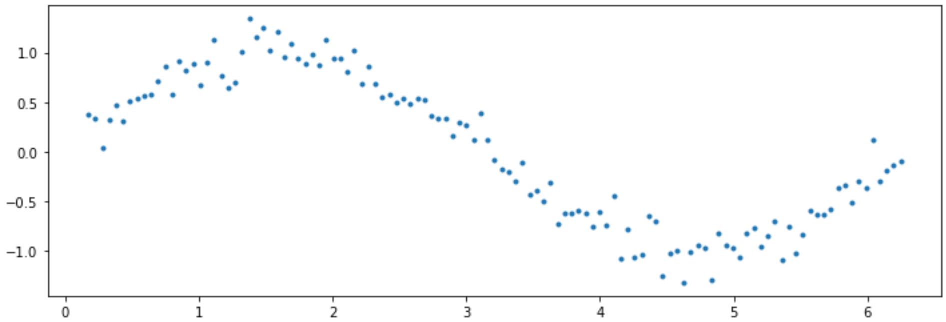
	x	y
0	0.174533	0.373386
1	0.226893	0.332243
2	0.279253	0.043827
3	0.331613	0.324311
4	0.383972	0.467807

In [6]:

```
# sine curve with noise added

#Plotting the dependent and independent variables
plt.figure(figsize=(12,4))
plt.plot(sin_df['x'],sin_df['y'],'.')
```

Out[6]:

```
[<matplotlib.lines.Line2D at 0x273a38238b0>]
```

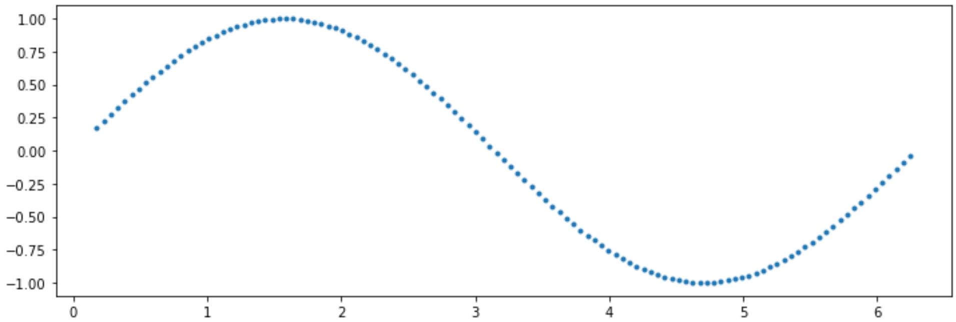


In [7]:

```
# this is how the pure sine column plot appears : without noise

#Plotting the dependent and independent variables
plt.figure(figsize=(12,4))
plt.plot(sin_df['x'],y_pure_sin,'.')
```

Out[7]:

```
[<matplotlib.lines.Line2D at 0x273a5925580>]
```



In [8]:

```
# using polynomial regression from power 1 to 15
for i in range(2,16): #power of 1 is already there, hence starting with 2
    col_name = 'x_%d'%i # generating column name with the respective power
    sin_df[col_name] = sin_df['x']**i

sin_df.head()
```

Out[8]:

	x	y	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_10	x_11	x_12	x_13
0	0.174533	0.373386	0.030462	0.005317	0.000928	0.000162	0.000028	0.000005	8.610313e-07	1.502783e-07	2.622851e-08	4.577739e-09	7.989662e-10	1.394459e-10
1	0.226893	0.332243	0.051480	0.011681	0.002650	0.000601	0.000136	0.000031	7.023697e-06	1.593626e-06	3.615823e-07	8.204043e-08	1.861438e-08	4.223469e-09
2	0.279253	0.043827	0.077982	0.021777	0.006081	0.001698	0.000474	0.000132	3.698101e-05	1.032705e-05	2.883856e-06	8.053244e-07	2.248890e-07	6.280085e-08
3	0.331613	0.324311	0.109967	0.036466	0.012093	0.004010	0.001330	0.000441	1.462338e-04	4.849296e-05	1.608088e-05	5.332620e-06	1.768364e-06	5.864117e-07
4	0.383972	0.467807	0.147435	0.056611	0.021737	0.008346	0.003205	0.001231	4.724984e-04	1.814264e-04	6.966273e-05	2.674857e-05	1.027071e-05	3.943671e-06

Creating Train & Test set Randomly

In [9]:

```
sin_df['y_pure_sin'] = y_pure_sin

# allocating random int to each record and if it is <3 => train & >3 => test
# this is just a fancy way of doing train test split, nothing else
sin_df['randNumCol'] = np.random.randint(1, 6, sin_df.shape[0])
sin_df.head()
train=sin_df[sin_df['randNumCol']<=3]
test=sin_df[sin_df['randNumCol']>3]
train = train.drop('randNumCol', axis=1)
test = test.drop('randNumCol', axis=1)
```

Implementing Linear regression

In [10]:

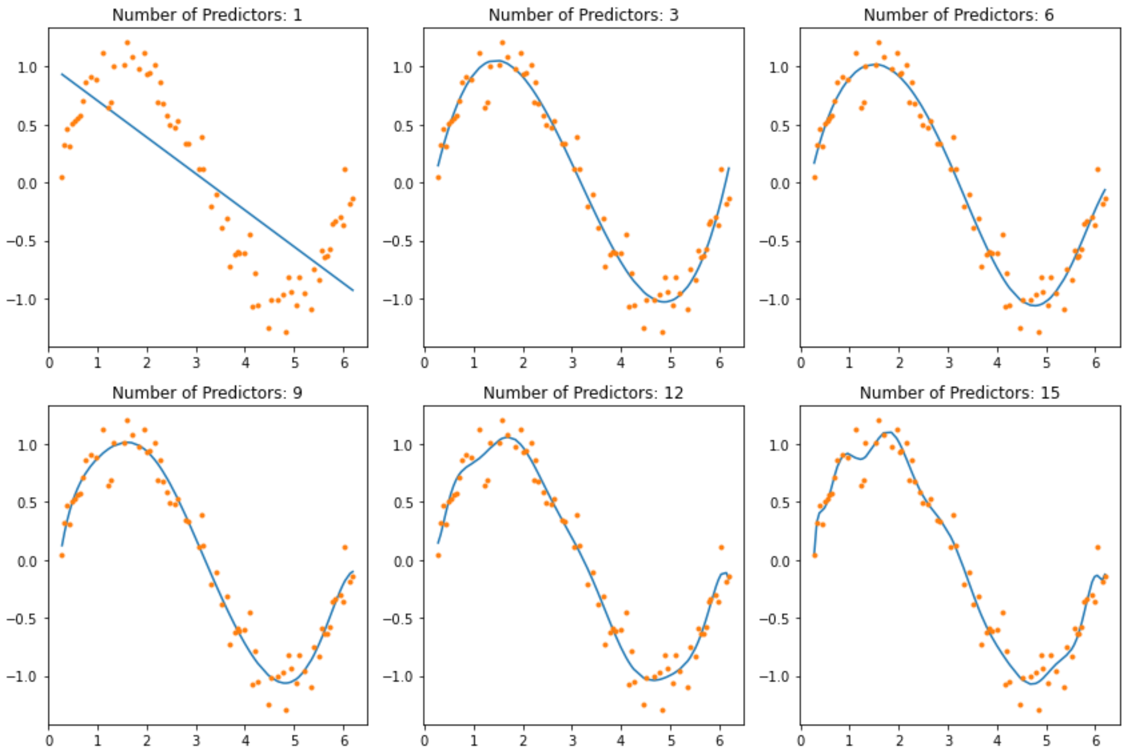
```
from sklearn.linear_model import LinearRegression

#Separating the independent and dependent variables
X_train = train.drop('y', axis=1).values
y_train = train['y'].values
y_sin_train = train['y_pure_sin'].values

X_test = test.drop('y', axis=1).values
y_test = test['y'].values
y_sin_test = test['y_pure_sin'].values
```

In [11]:

```python
def check_features_vs_result(train_x, train_y, test_x, test_y, features, models_to_plot):

    '''
    Takes input train and test dataset, features and a dictionary with number of features to plot with respective plot location
    and returns train v/s test results plot to better understand the overfitting / underfitting results.

    Params :
        train_x : training data
        train_y : training target feature
        test_x : testing data
        test_y : testing target feature
        features : (int) number of features to consider while plotting
        models_to_plot : dictionary : key -> number of features & value -> Plot location in subplot

    Returns :
        Respective train v/s test plot
    '''


    # fitting the model
    lr = LinearRegression(normalize=True)
    lr.fit(train_x, train_y)
    train_y_pred = lr.predict(train_x)
    test_y_pred = lr.predict(test_x)

    # checking features for which plot is to be made:
    if features in models_to_plot :
        plt.subplot(models_to_plot[features])
        plt.tight_layout()
        plt.plot(train_x[:, 0:1], train_y_pred)
        plt.plot(train_x[:, 0:1], train_y, '.')
        plt.title('Number of Predictors: %d'%features)

    rss_train = sum((train_y_pred-train_y)**2)/train_x.shape[0]
    return_list = [rss_train]

    rss_test = sum((test_y_pred-test_y)**2)/test_x.shape[0]
    return_list.extend([rss_test])

    return_list.extend([lr.intercept_])
    return_list.extend(lr.coef_)

    return return_list

# Making DataFrame to store the results

col =  ['mrss_train','mrss_test','intercept'] + ['coef_Var_%d'%i for i in range(1,16)]
ind = ['Number_of_variable_%d'%i for i in range(1,16)]
coef_matrix_simple = pd.DataFrame(index=ind, columns=col)

# defining a dictionary to store subpolot locations for respective number of features
models_to_plot = {1:231,3:232,6:233,9:234,12:235,15:236}

```

In [12]:

```
# Iterating through all powers of polynomial reg and storing results in the dataframe made above
plt.figure(figsize=(12,8))

for i in range(1,16):
    train_x = X_train[:,0:i]
    train_y = y_train
    test_x = X_test[:,0:i]
    test_y = y_test

    # row = i-1 because we need to start from 0th location
    # column = i+3 because there are somdefault columns like x and y axis
    coef_matrix_simple.iloc[i-1, 0:i+3] = check_features_vs_result(
                                            train_x, train_y, test_x, test_y,
                                            features=i,
                                            models_to_plot=models_to_plot
    )
```



when the features were given more & more, the model got Overfitted and Learned the noise present in data as well.

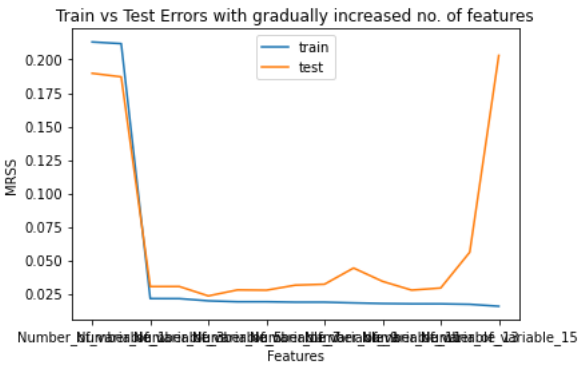
We can observe how the last plot is able to mimic the sine curve including noise perfectly

In [13]:

```
coef_matrix_simple[['mrss_train','mrss_test']].plot()
plt.xlabel('Features')
plt.ylabel('MRSS')
plt.title('Train vs Test Errors with gradually increased no. of features')
plt.legend(['train', 'test'])
```

Out[13]:

```
<matplotlib.legend.Legend at 0x273a6742430>
```



The solution to avoid this is Regularisation

Ridge Regularisation (L2)

in L2, We simply add `Square of Coefficients` to the regular `Cost Function`

In [14]:

```
# importing ridge from sklearn linear_model module
from sklearn.linear_model import Ridge
```

In [15]:

```
# list of various lambda/ Alpha i.e Learning Rates to try (to ensure the degree of control of Regularization)
learning_rate = [0, 1e-4, 1e-3, 1e-2, 1, 5]
```

In [16]:

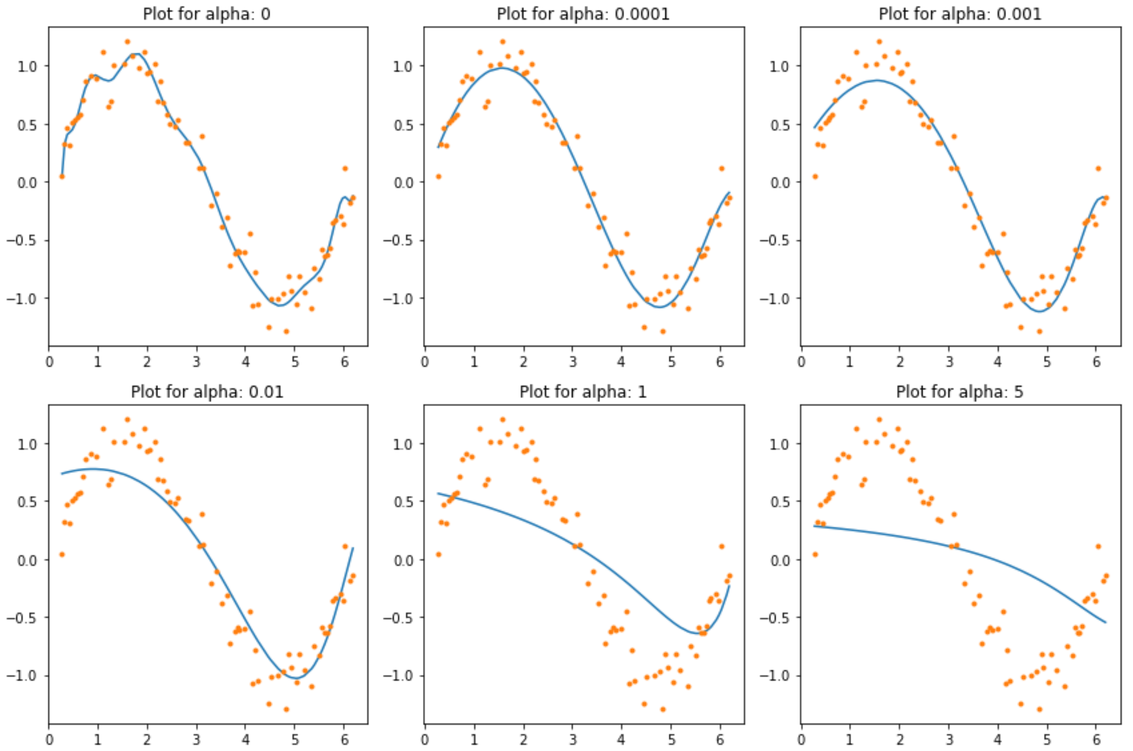
```
# defining a function which will fit ridge regression model, plot the results, and return the coefficients
def ridge_regression(train_x, train_y, test_x, test_y, alpha, models_to_plot={}):
    # Fit the model
    ridge = Ridge(alpha= alpha, normalize= True)
    ridge.fit(train_x, train_y)
    train_pred = ridge.predict(train_x)
    test_pred = ridge.predict(test_x)

    # plotting results
    if alpha in models_to_plot:
        plt.subplot(models_to_plot[alpha])
        plt.tight_layout()
        plt.plot(train_x[:, 0:1], train_pred) # predicted values
        plt.plot(train_x[:,0:1],train_y,'.') # actual values
        plt.title('Plot for alpha: %.3g'%alpha)

    #Return the result in pre-defined format
    mrss_train = sum((train_pred-train_y)**2)/train_x.shape[0]
    ret = [mrss_train]

    mrss_test = sum((test_pred-test_y)**2)/test_x.shape[0]
    ret.extend([mrss_test])

    ret.extend([ridge.intercept_])
    ret.extend(ridge.coef_)

    return ret


```

In [17]:

```
#Initialize the dataframe for storing coefficients.
col = ['mrss_train','mrss_test','intercept'] + ['coef_Var_%d'%i for i in range(1,16)]
ind = ['alpha_%.2g'%learning_rate[i] for i in range(0,6)]
coef_matrix_ridge = pd.DataFrame(index=ind, columns=col)

#Define the alpha value for which a plot is required:
models_to_plot = {0:231, 1e-4:232, 1e-3:233, 1e-2:234, 1:235, 5:236}
```

In [18]:

```
#Iterate over the 10 alpha values:
plt.figure(figsize=(12,8))
for i, alpha in enumerate(learning_rate):
    coef_matrix_ridge.iloc[i,] = ridge_regression(train_x, train_y, test_x, test_y, alpha, models_to_plot)
```



Ridge Regression Results

> with `learning rate = 0` : There is no change in model, its same overfitted model, capturing noises

> with even a slight change in `learning rate =  1e-4` : the model is no longer capturing the noises and has learned the underlying relation of a sine curve

> with `increased learning rate` : model is gradually underfitting and eventually depecting a straight line

In [19]:

```
#Set the display format to be scientific for ease of analysis
pd.options.display.float_format = '{:,.2g}'.format
coef_matrix_ridge
```

Out[19]:

	mrss_train	mrss_test	intercept	coef_Var_1	coef_Var_2	coef_Var_3	coef_Var_4	coef_Var_5	coef_Var_6	coef_Var_7	coef_Var_8	coef_Var_
alpha_0	0.016	0.2	-25	2.7e+02	-1.3e+03	3.3e+03	-5.4e+03	5.9e+03	-4.5e+03	2.5e+03	-1e+03	3e+C
alpha_0.0001	0.02	0.026	-0.023	1.2	-0.36	-0.023	0.0016	0.00058	9e-05	9.4e-06	5.5e-07	-4.1e-C
alpha_0.001	0.028	0.034	0.28	0.72	-0.19	-0.021	-0.00068	0.00019	5.2e-05	9e-06	1.2e-06	1.4e-C
alpha_0.01	0.059	0.058	0.7	0.17	-0.075	-0.011	-0.00085	-1.7e-06	1.6e-05	4e-06	7.4e-07	1.1e-C
alpha_1	0.19	0.2	0.59	-0.093	-0.013	-0.0016	-0.00019	-2.1e-05	-1.9e-06	-1e-07	1.1e-08	5.6e-C
alpha_5	0.35	0.37	0.29	-0.036	-0.0049	-0.00066	-8.7e-05	-1.1e-05	-1.3e-06	-1.5e-07	-1.6e-08	-1.4e-C

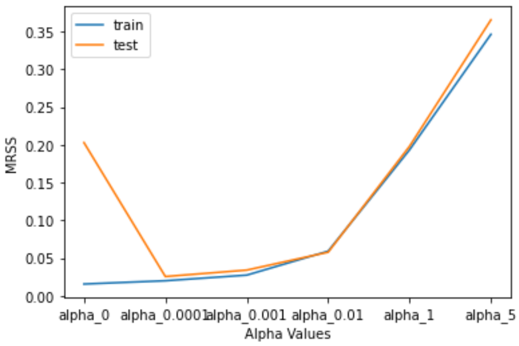
```
we can note that with increased value of aplhas, the coefficients are decreasing
BUT NOTE : EVEN WITH ALPHA =5, THE VALUES ARE NOT ABSOLUTE ZERO => SOME NON ZERO ELEMENT IS PRESENT
this thing is improved with Lasso Regression and hence that is used for Feature selection while Ridge is frequently used to
AVOID OVERFITTING
```

In [20]:

```
coef_matrix_ridge[['mrss_train','mrss_test']].plot()
plt.xlabel('Alpha Values')
plt.ylabel('MRSS')
plt.legend(['train', 'test'])
```

Out[20]:

```
<matplotlib.legend.Legend at 0x273a69afc70>
```




```
in this plot, in left most area we can observe huge gap in train and test errors => overfitting
in middle region, the model is performing best => best fit model
in right most region, train and test both errors are too high => model is underfitted
```

Lasso Regularisation (L1)

In [21]:

```
#Importing Lasso model from sklearn's linear_model module
from sklearn.linear_model import Lasso
```

In [22]:

```
#Define the alpha values to test
alpha_lasso = [0, 1e-10, 1e-8, 1e-5,1e-4, 1e-3,1e-2, 1, 5, 10]
```

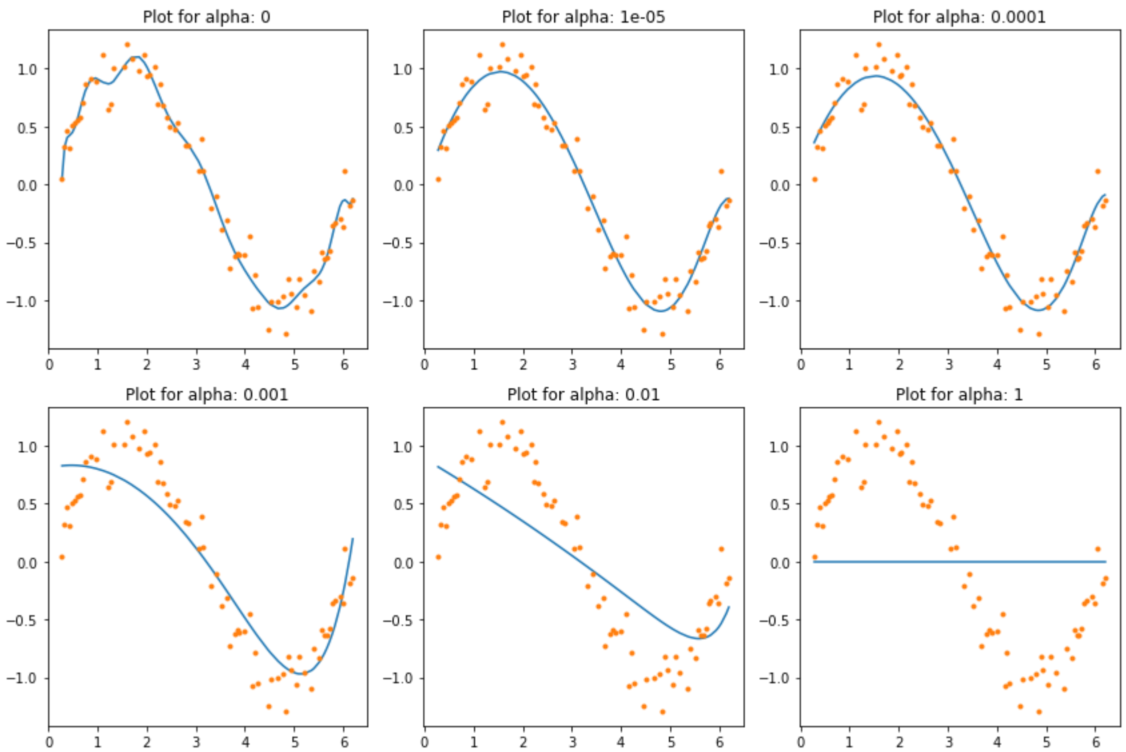
In [23]:

```
# defining a function which will fit lasso regression model, plot the results, and return the coefficients
def lasso_regression(train_x, train_y, test_x, test_y, alpha, models_to_plot={}):
    #Fit the model

    # lasso model by default does NOT allows to train a model with 0 Learning rate, hence we use Linear Regression for that.
    if alpha == 0:
        lassoreg = LinearRegression(normalize=True)
        lassoreg.fit(train_x, train_y)
        train_y_pred = lassoreg.predict(train_x)
        test_y_pred = lassoreg.predict(test_x)

    else:
        lassoreg = Lasso(alpha=alpha,normalize=True)
        lassoreg.fit(train_x,train_y)
        train_y_pred = lassoreg.predict(train_x)
        test_y_pred = lassoreg.predict(test_x)

    #Check if a plot is to be made for the entered alpha
    if alpha in models_to_plot:
        plt.subplot(models_to_plot[alpha])
        plt.tight_layout()
        plt.plot(train_x[:,0:1],train_y_pred)
        plt.plot(train_x[:,0:1],train_y,'.')
        plt.title('Plot for alpha: %.3g'%alpha)

    #Return the result in pre-defined format
    mrss_train = sum((train_y_pred-train_y)**2)/train_x.shape[0]
    ret = [mrss_train]

    mrss_test = sum((test_y_pred-test_y)**2)/test_x.shape[0]
    ret.extend([mrss_test])

    ret.extend([lassoreg.intercept_])
    ret.extend(lassoreg.coef_)

    return ret
```

In [24]:

```python
#Initialize the dataframe to store coefficients
col = ['mrss_train','mrss_test','intercept'] + ['coef_Var_%d'%i for i in range(1,16)]
ind = ['alpha_%.2g'%alpha_lasso[i] for i in range(0,10)]
coef_matrix_lasso = pd.DataFrame(index=ind, columns=col)

#Define the models to plot
models_to_plot = {0:231, 1e-5:232,1e-4:233, 1e-3:234, 1e-2:235, 1:236}

#Iterate over the 10 alpha values:
plt.figure(figsize=(12,8))
for i in range(10):
    coef_matrix_lasso.iloc[i,] = lasso_regression(train_x, train_y, test_x, test_y, alpha_lasso[i], models_to_plot)
```

```
C:\Users\rkt7k\anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_descent.py:530: ConvergenceWarning: Objective d
id not converge. You might want to increase the number of iterations. Duality gap: 0.7335671573239089, tolerance: 0.0039830
65126185541
  model = cd_fast.enet_coordinate_descent(
C:\Users\rkt7k\anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_descent.py:530: ConvergenceWarning: Objective d
id not converge. You might want to increase the number of iterations. Duality gap: 0.7333330322072866, tolerance: 0.0039830
65126185541
  model = cd_fast.enet_coordinate_descent(
C:\Users\rkt7k\anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_descent.py:530: ConvergenceWarning: Objective d
id not converge. You might want to increase the number of iterations. Duality gap: 0.5377192887698238, tolerance: 0.0039830
65126185541
  model = cd_fast.enet_coordinate_descent(
C:\Users\rkt7k\anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_descent.py:530: ConvergenceWarning: Objective d
id not converge. You might want to increase the number of iterations. Duality gap: 0.1355348910790386, tolerance: 0.0039830
65126185541
  model = cd_fast.enet_coordinate_descent(
```



Lasso Regression Results

> with `learning rate = 0` : There is no change in model, its same overfitted model, capturing noises

> with even a slight change in `learning rate = 1e-5` : the model is no longer capturing the noises and has learned the underlying relation of a sine curve

> with `increased learning rate` : model is gradually underfitting and eventually depecting a straight line.
>
> > NOTE : with increase in learning, after sometime, the plot is a straight line signifying ll variableshaven been reduced to absolute zero 0

In [25]:

```
#Set the display format to be scientific for ease of analysis
pd.options.display.float_format = '{:,.2g}'.format
coef_matrix_lasso
```

Out[25]:

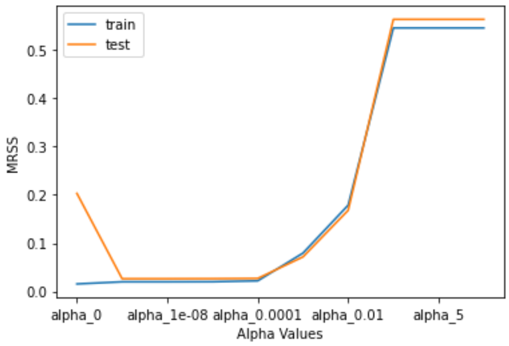
	mrss_train	mrss_test	intercept	coef_Var_1	coef_Var_2	coef_Var_3	coef_Var_4	coef_Var_5	coef_Var_6	coef_Var_7	coef_Var_8	coef_Var_
alpha_0	0.016	0.2	-25	2.7e+02	-1.3e+03	3.3e+03	-5.4e+03	5.9e+03	-4.5e+03	2.5e+03	-1e+03	3e+0
alpha_1e-10	0.02	0.027	-0.054	1.4	-0.44	-0.0036	0.0023	0.00039	4.7e-05	5e-06	4.5e-07	2.9e-0
alpha_1e-08	0.02	0.027	-0.054	1.4	-0.44	-0.0036	0.0023	0.00039	4.7e-05	5e-06	4.5e-07	2.9e-0
alpha_1e-05	0.02	0.027	-0.041	1.3	-0.44	-0.0032	0.002	0.0004	4.8e-05	5e-06	4.3e-07	2.4e-0
alpha_0.0001	0.022	0.027	0.071	1.1	-0.38	-0	0	0.00042	5.9e-05	5.4e-06	3.1e-07	
alpha_0.001	0.08	0.072	0.81	0.11	-0.11	-0	-0	0	0	0	9.6e-07	7.1e-0
alpha_0.01	0.18	0.17	0.89	-0.25	-0.0088	-0	-0	-0	0	0	0	
alpha_1	0.55	0.56	-0.003	-0	-0	-0	-0	-0	-0	-0	-0	-
alpha_5	0.55	0.56	-0.003	-0	-0	-0	-0	-0	-0	-0	-0	-
alpha_10	0.55	0.56	-0.003	-0	-0	-0	-0	-0	-0	-0	-0	-

In [26]:

```
coef_matrix_lasso[['mrss_train','mrss_test']].plot()
plt.xlabel('Alpha Values')
plt.ylabel('MRSS')
plt.legend(['train', 'test'])
```

Out[26]:

<matplotlib.legend.Legend at 0x273a6e08be0>




```
in this plot, in left most area we can observe huge gap in train and test errors => overfitting
in middle region, the model is performing best => best fit model
in right most region, train and test both errors are too high => model is underfitted
```

In [27]:

```
coef_matrix_lasso.apply(lambda x: sum(x.values==0),axis=1)
```

Out[27]:

```
alpha_0          0
alpha_1e-10      0
alpha_1e-08      0
alpha_1e-05      2
alpha_0.0001     7
alpha_0.001     11
alpha_0.01      12
alpha_1         15
alpha_5         15
alpha_10        15
dtype: int32
```

```
we could cross validate that how with increased alpha, the amount of features reduced to absolute zero are increasing.
```