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In this notebook, We will see what are the problems of Linear Regression and Why do we need `Regularisation`

In [1]:

```python
#Importing libraries
import numpy as np
import pandas as pd
import random
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
%matplotlib inline
```

> we will make a dataframe which could mimic a Sine curve

In [2]:

```python
#Defining independent variable as angles from 60deg to 300deg converted to radians
x = np.array([i*np.pi/180 for i in range(10,360,3)])
```

In [3]:

```python
#Setting seed for reproducability
np.random.seed(10)
```

In [4]:

```python
#Defining the target/dependent variable as sine of the independent variable

# y = sin(x) + SOME NOISE BEING ADDED ON TOP OF IT
y_sin_noise = np.sin(x) + np.random.normal(0,0.15,len(x))
y_pure_sin = np.sin(x)

del_y = y_sin_noise - y_pure_sin
```

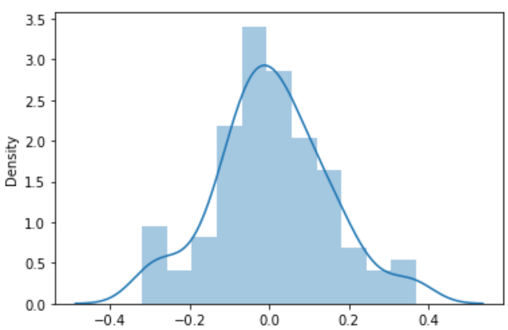
In [5]:

```python
import seaborn as sns
sns.distplot(del_y, hist=True)

# we can see the errors being generated from noise form a normal distribution
```

```
C:\Users\rkt7k\anaconda3\lib\site-packages\seaborn\distributions.py:2619: FutureWarning: `distplot` is a deprecated functio
n and will be removed in a future version. Please adapt your code to use either `displot` (a figure-level function with sim
ilar flexibility) or `histplot` (an axes-level function for histograms).
  warnings.warn(msg, FutureWarning)
```

Out[5]:

```
<AxesSubplot:ylabel='Density'>
```



In [6]:

```python
#Creating the dataframe using independent and dependent variable
sin_df = pd.DataFrame(np.column_stack([x,y_sin_noise]),columns=['x','y'])
```

In [7]:

```
sin_df.head()
```

Out[7]:

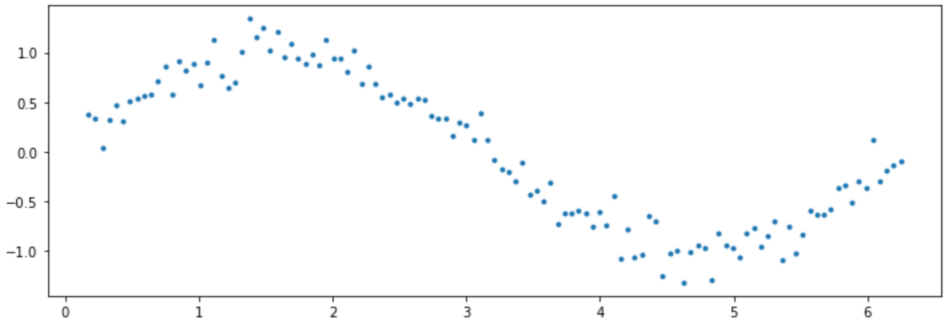
	x	y
0	0.174533	0.373386
1	0.226893	0.332243
2	0.279253	0.043827
3	0.331613	0.324311
4	0.383972	0.467807

In [8]:

```
# sine curve with noise added

#Plotting the dependent and independent variables
plt.figure(figsize=(12,4))
plt.plot(sin_df['x'],sin_df['y'],'.')
```

Out[8]:

```
[<matplotlib.lines.Line2D at 0x1f01e2720d0>]
```

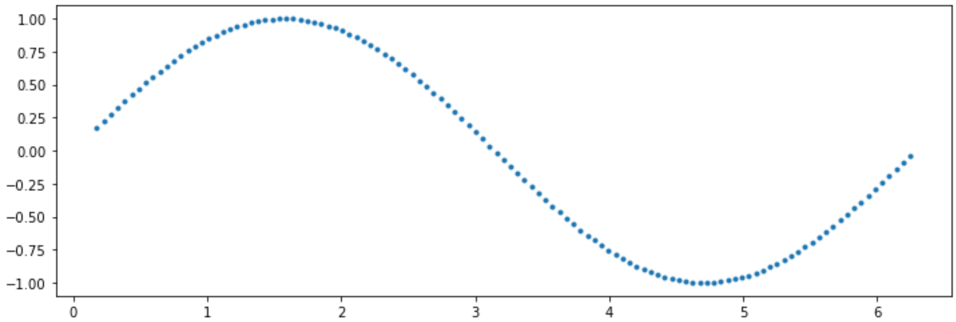


In [9]:

```
# this is how the pure sine column plot appears : without noise

#Plotting the dependent and independent variables
plt.figure(figsize=(12,4))
plt.plot(sin_df['x'],y_pure_sin,'.')
```

Out[9]:

```
[<matplotlib.lines.Line2D at 0x1f01e2d0f70>]
```



In [10]:

```
# using polynomial regression from power 1 to 15
for i in range(2,16): #power of 1 is already there, hence starting with 2
    col_name = 'x_%d'%i # generating column name with the respective power
    sin_df[col_name] = sin_df['x']**i

sin_df.head()
```

Out[10]:

	x	y	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_10	x_11	x_12	x_13
0	0.174533	0.373386	0.030462	0.005317	0.000928	0.000162	0.000028	0.000005	8.610313e-07	1.502783e-07	2.622851e-08	4.577739e-09	7.989662e-10	1.394459e-10
1	0.226893	0.332243	0.051480	0.011681	0.002650	0.000601	0.000136	0.000031	7.023697e-06	1.593626e-06	3.615823e-07	8.204043e-08	1.861438e-08	4.223469e-09
2	0.279253	0.043827	0.077982	0.021777	0.006081	0.001698	0.000474	0.000132	3.698101e-05	1.032705e-05	2.883856e-06	8.053244e-07	2.248890e-07	6.280085e-08
3	0.331613	0.324311	0.109967	0.036466	0.012093	0.004010	0.001330	0.000441	1.462338e-04	4.849296e-05	1.608088e-05	5.332620e-06	1.768364e-06	5.864117e-07
4	0.383972	0.467807	0.147435	0.056611	0.021737	0.008346	0.003205	0.001231	4.724984e-04	1.814264e-04	6.966273e-05	2.674857e-05	1.027071e-05	3.943671e-06

Creating Train & Test set Randomly

In [11]:

```
sin_df['y_pure_sin'] = y_pure_sin

# allocating random int to each record and if it is <3 => train & >3 => test
# this is just a fancy way of doing train test split, nothing else
sin_df['randNumCol'] = np.random.randint(1, 6, sin_df.shape[0])
sin_df.head()
train=sin_df[sin_df['randNumCol']<=3]
test=sin_df[sin_df['randNumCol']>3]
train = train.drop('randNumCol', axis=1)
test = test.drop('randNumCol', axis=1)
```

In [12]:

```
sin_df.randNumCol.value_counts()
# we can see the distribution is almost same
```

Out[12]:

```
1    28
3    26
5    22
4    22
2    19
Name: randNumCol, dtype: int64
```

Implementing Linear Regression

In [13]:

```
from sklearn.linear_model import LinearRegression
```

In [14]:

```
#Separating the independent and dependent variables
X_train = train.drop('y', axis=1).values
y_train = train['y'].values
y_sin_train = train['y_pure_sin'].values

X_test = test.drop('y', axis=1).values
y_test = test['y'].values
y_sin_test = test['y_pure_sin'].values
```

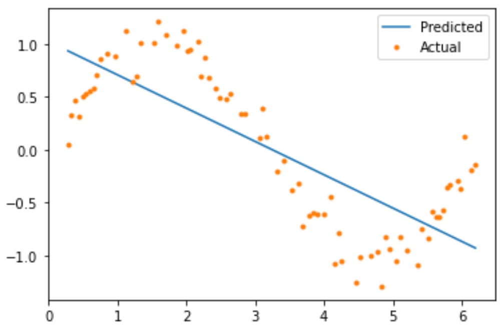
In [15]:

```
#Linear regression with one features

# on TRAINING SET
independent_variable_train = X_train[:,0:1] # this is array slicing => slicing only 1st feature from all components

lr = LinearRegression(normalize=True)
lr.fit(independent_variable_train, y_train)
y_train_pred = lr.predict(independent_variable_train)

from sklearn.metrics import mean_squared_error as mse
mse_train = mse(y_train_pred, y_train)

# ON TESTING SET
independent_variable_test = X_test[:,0:1]
y_test_pred = lr.predict(independent_variable_test)
mse_test = mse(y_test_pred, y_test)

# printing results
print("Train Error ", mse_train)
print("Test Error ", mse_test)

# plotting scores
plt.plot(X_train[:,0:1], y_train_pred, label = 'Predicted')
plt.plot(X_train[:,0:1], y_train, '.', label = 'Actual')
plt.legend()
```

```
Train Error  0.213144309581738 97
Test Error  0.18974033118165387
```

Out[15]:

```
<matplotlib.legend.Legend at 0x1f01ec54130>
```



As we are using only one feature to predict, the output is a stringht line

This demonstrates that our model is a `UNDERFITTED` model, as its not able to capture the relationship as seen above

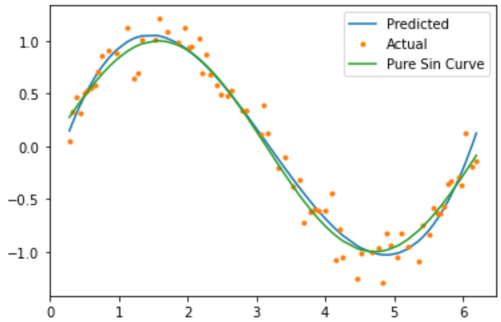
In [16]:

```
# performing the same task as above but with 3 features


# on TRAINING SET
independent_variable_train = X_train[:,0:3] # this is array slicing => slicing only 1st feature from all components

lr = LinearRegression(normalize=True)
lr.fit(independent_variable_train, y_train)
y_train_pred = lr.predict(independent_variable_train)

from sklearn.metrics import mean_squared_error as mse
mse_train = mse(y_train_pred, y_train)

# ON TESTING SET
independent_variable_test = X_test[:,0:3]
y_test_pred = lr.predict(independent_variable_test)
mse_test = mse(y_test_pred, y_test)

# printing results
print("Train Error ", mse_train)
print("Test Error ", mse_test)

# plotting scores
plt.plot(X_train[:,0:1], y_train_pred, label = 'Predicted')
plt.plot(X_train[:,0:1], y_train, '.', label = 'Actual')
plt.plot(X_train[:,0:1], y_sin_train, label = 'Pure Sin Curve')
plt.legend()
```

```
Train Error  0.02147248177096577
Test Error  0.03045187888196913
```

Out[16]:

```
<matplotlib.legend.Legend at 0x1f01eca6790>
```




```
we can observe that with the increased features, model is trying to predict better as its getting closer to the sine curve
```

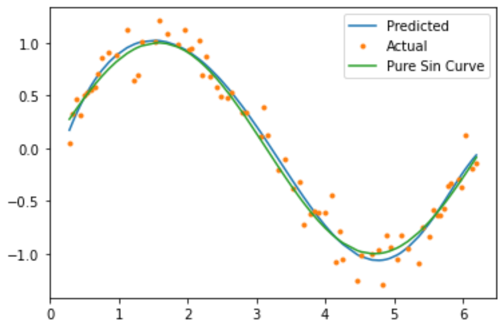
In [17]:

```
# performing the same task as above but with 7 features


# on TRAINING SET
independent_variable_train = X_train[:,0:7] # this is array slicing => slicing only 1st feature from all components

lr = LinearRegression(normalize=True)
lr.fit(independent_variable_train, y_train)
y_train_pred = lr.predict(independent_variable_train)

from sklearn.metrics import mean_squared_error as mse
mse_train = mse(y_train_pred, y_train)

# ON TESTING SET
independent_variable_test = X_test[:,0:7]
y_test_pred = lr.predict(independent_variable_test)
mse_test = mse(y_test_pred, y_test)

# printing results
print("Train Error ", mse_train)
print("Test Error ", mse_test)

# plotting scores
plt.plot(X_train[:,0:1], y_train_pred, label = 'Predicted')
plt.plot(X_train[:,0:1], y_train, '.', label = 'Actual')
plt.plot(X_train[:,0:1], y_sin_train, label = 'Pure Sin Curve')
plt.legend()
```

```
Train Error  0.019095054809245043
Test Error  0.027717292848481637
```

Out[17]:

```
<matplotlib.legend.Legend at 0x1f01ed2a580>
```




```
Increasing the features more, we can observe that dependence is followed more closely now
```

Defining a function to automate this process and iterate th. range of features and plot results

In [18]:

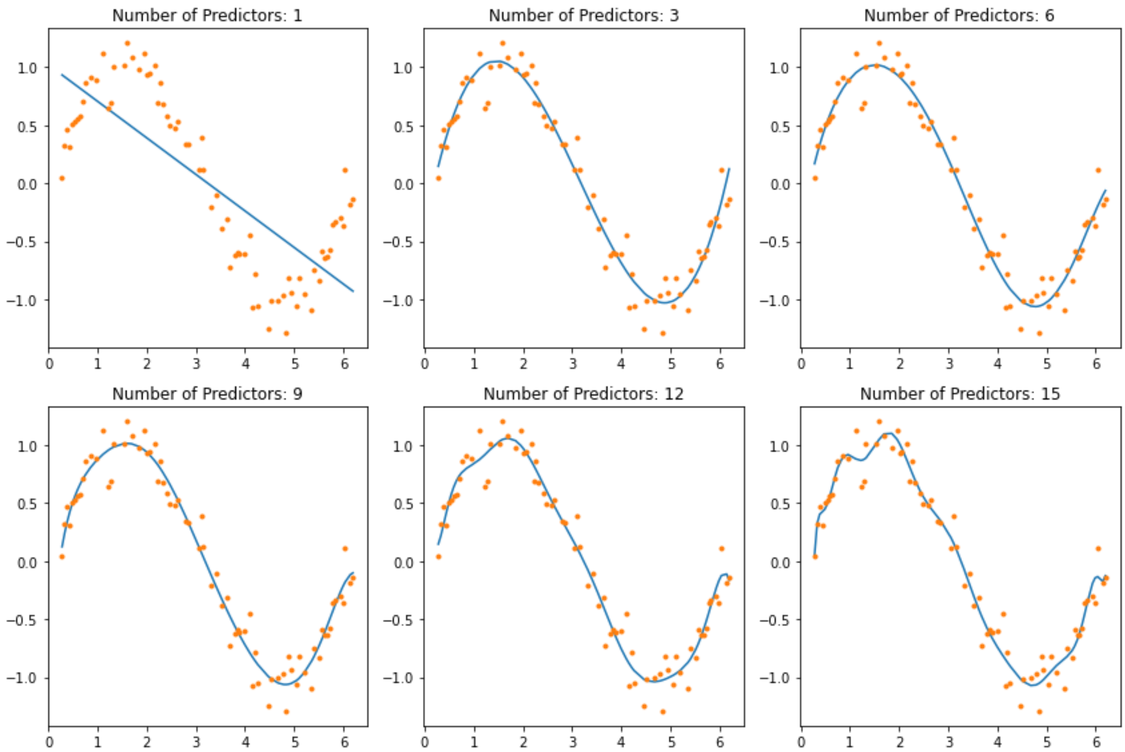
```
def check_features_vs_result(train_x, train_y, test_x, test_y, features, models_to_plot):

    '''
    Takes input train and test dataset, features and a dictionary with number of features to plot with respective plot location
    and returns train v/s test results plot to better understand the overfitting / underfitting results.

    Params :
        train_x : training data
        train_y : training target feature
        test_x : testing data
        test_y : testing target feature
        features : (int) number of features to consider while plotting
        models_to_plot : dictionary : key -> number of features & value -> Plot location in subplot

    Returns :
        Respective train v/s test plot
    '''


    # fitting the model
    lr = LinearRegression(normalize=True)
    lr.fit(train_x, train_y)
    train_y_pred = lr.predict(train_x)
    test_y_pred = lr.predict(test_x)

    # checking features for which plot is to be made:
    if features in models_to_plot :
        plt.subplot(models_to_plot[features])
        plt.tight_layout()
        plt.plot(train_x[:, 0:1], train_y_pred)
        plt.plot(train_x[:, 0:1], train_y, '.')
        plt.title('Number of Predictors: %d'%features)

    rss_train = sum((train_y_pred-train_y)**2)/train_x.shape[0]
    return_list = [rss_train]

    rss_test = sum((test_y_pred-test_y)**2)/test_x.shape[0]
    return_list.extend([rss_test])

    return_list.extend([lr.intercept_])
    return_list.extend(lr.coef_)

    return return_list

```

In [19]:

```
# Making DataFrame to store the results

col =  ['mrss_train','mrss_test','intercept'] + ['coef_Var_%d'%i for i in range(1,16)]
ind = ['Number_of_variable_%d'%i for i in range(1,16)]
coef_matrix_simple = pd.DataFrame(index=ind, columns=col)

# defining a dictionary to store subpolot locations for respective number of features
models_to_plot = {1:231,3:232,6:233,9:234,12:235,15:236}
```

In [20]:

```
# Iterating through all powers of polynomial reg and storing results in the dataframe made above
plt.figure(figsize=(12,8))

for i in range(1,16):
    train_x = X_train[:,0:i]
    train_y = y_train
    test_x = X_test[:,0:i]
    test_y = y_test

    # row = i-1 because we need to start from 0th location
    # column = i+3 because there are somdefault columns like x and y axis
    coef_matrix_simple.iloc[i-1, 0:i+3] = check_features_vs_result(
                                            train_x, train_y, test_x, test_y,
                                            features=i,
                                            models_to_plot=models_to_plot
    )
```



Note : It is easily understood from above that initially the model was `Underfitted` as the number of features were less

It starting understaing the sine pattery as number of features increased

But when the features were given more & more, the model got `Overfitted` and Learned the noise present in data as well

In [21]:

```
coef_matrix_simple
```

Out[21]:

	mrss_train	mrss_test	intercept	coef_Var_1	coef_Var_2	coef_Var_3	coef_Var_4	coef_Var_5	coef_Var_6	coef_Var_7	
Number_of_variable_1	0.213144	0.18974	1.022026	-0.314825	NaN	NaN	NaN	NaN	NaN	NaN	
Number_of_variable_2	0.211909	0.187095	1.108613	-0.394472	0.012378	NaN	NaN	NaN	NaN	NaN	
Number_of_variable_3	0.021472	0.030452	-0.395777	2.211256	-0.985564	0.103652	NaN	NaN	NaN	NaN	
Number_of_variable_4	0.021433	0.030495	-0.429469	2.297581	-1.042236	0.117041	-0.001036	NaN	NaN	NaN	
Number_of_variable_5	0.01977	0.023449	-0.096495	1.196542	0.008412	-0.292937	0.068681	-0.00429	NaN	NaN	
Number_of_variable_6	0.019096	0.027915	-0.402994	2.460917	-1.644525	0.655694	-0.197059	0.031391	-0.001837	NaN	
Number_of_variable_7	0.019095	0.027717	-0.383432	2.364513	-1.484218	0.532799	-0.147927	0.020809	-0.000674	-0.000051	
Number_of_variable_8	0.01877	0.031513	-0.848928	5.059678	-7.017272	5.985346	-3.076467	0.923889	-0.160126	0.01492	
Number_of_variable_9	0.018764	0.032176	-0.938864	5.654966	-8.466056	7.732466	-4.26288	1.406429	-0.279989	0.032718	
Number_of_variable_10	0.018237	0.044286	-2.379595	16.259649	-38.022791	49.70171	-38.754405	18.930563	-5.952893	1.204084	
Number_of_variable_11	0.017761	0.034306	-0.337851	-0.367154	14.686782	-37.637253	46.978086	-34.37047	15.790354	-4.688212	
Number_of_variable_12	0.017555	0.027785	1.635685	-18.04956	77.796076	-157.789059	184.977871	-136.636942	66.609825	-21.947205	
Number_of_variable_13	0.01754	0.02935	0.84298	-10.307477	47.123532	-91.902126	98.300673	-61.92383	22.668915	-3.904411	
Number_of_variable_14	0.01718	0.056087	-5.499372	56.503431	-242.75995	599.968571	-927.089989	947.704446	-665.789454	329.548704	
Number_of_variable_15	0.015731	0.203073	-24.703978	273.274842	-1264.577986	3285.223124	-5365.479472	5882.772284	-4515.085155	2492.578419	-1

In [22]:

```
#Set the display format to be scientific for ease of analysis
pd.options.display.float_format = '{:,.2g}'.format
coef_matrix_simple
```

Out[22]:

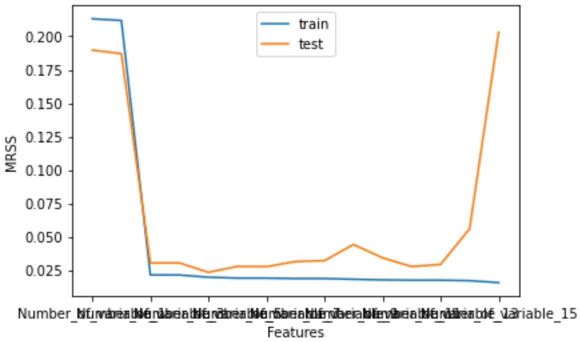
	mrss_train	mrss_test	intercept	coef_Var_1	coef_Var_2	coef_Var_3	coef_Var_4	coef_Var_5	coef_Var_6	coef_Var_7	coef_Var_8
Number_of_variable_1	0.21	0.19	1	-0.31	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Number_of_variable_2	0.21	0.19	1.1	-0.39	0.012	NaN	NaN	NaN	NaN	NaN	NaN
Number_of_variable_3	0.021	0.03	-0.4	2.2	-0.99	0.1	NaN	NaN	NaN	NaN	NaN
Number_of_variable_4	0.021	0.03	-0.43	2.3	-1	0.12	-0.001	NaN	NaN	NaN	NaN
Number_of_variable_5	0.02	0.023	-0.096	1.2	0.0084	-0.29	0.069	-0.0043	NaN	NaN	NaN
Number_of_variable_6	0.019	0.028	-0.4	2.5	-1.6	0.66	-0.2	0.031	-0.0018	NaN	NaN
Number_of_variable_7	0.019	0.028	-0.38	2.4	-1.5	0.53	-0.15	0.021	-0.00067	-5.1e-05	NaN
Number_of_variable_8	0.019	0.032	-0.85	5.1	-7	6	-3.1	0.92	-0.16	0.015	-0.00058
Number_of_variable_9	0.019	0.032	-0.94	5.7	-8.5	7.7	-4.3	1.4	-0.28	0.033	-0.002
Number_of_variable_10	0.018	0.044	-2.4	16	-38	50	-39	19	-6	1.2	-0.15
Number_of_variable_11	0.018	0.034	-0.34	-0.37	15	-38	47	-34	16	-4.7	0.9
Number_of_variable_12	0.018	0.028	1.6	-18	78	-1.6e+02	1.8e+02	-1.4e+02	67	-22	4.9
Number_of_variable_13	0.018	0.029	0.84	-10	47	-92	98	-62	23	-3.9	-0.31
Number_of_variable_14	0.017	0.056	-5.5	57	-2.4e+02	6e+02	-9.3e+02	9.5e+02	-6.7e+02	3.3e+02	-1.2e+02
Number_of_variable_15	0.016	0.2	-25	2.7e+02	-1.3e+03	3.3e+03	-5.4e+03	5.9e+03	-4.5e+03	2.5e+03	-1e+03

In [23]:

```python
coef_matrix_simple[['mrss_train','mrss_test']].plot()
plt.xlabel('Features')
plt.ylabel('MRSS')
plt.legend(['train', 'test'])
```

Out[23]:

```
<matplotlib.legend.Legend at 0x1f01f1d5250>
```




```
Same concept is being proved by this plot as well
```

Solution : Regularization Techniques