

likelihood function.

$$P(t|x, w, \beta) = \prod_{n=1}^N \mathcal{N}(t_n | y(x_n, w), \beta^{-1})$$

apply log on both sides

$$\log P(t|x, w, \beta) = \log \left[\prod_{n=1}^N \mathcal{N}(t_n | y(x_n, w), \beta^{-1}) \right]$$

$$\log P(t|x, w, \beta) = \sum_{n=1}^N \left[-\frac{\beta}{2} [y(x_n, w) - t_n]^2 \right] - \sum_{n=1}^N \log (2\pi \beta^{-1})^{1/2}$$

substituted $\mathcal{N}(t_n | y(x_n, w), \beta^{-1}) = \frac{1}{(2\pi \beta^{-1})^{1/2}} e^{-\beta/2 [y(x_n, w) - t_n]^2}$

$$\Rightarrow \log P(t|x, w, \beta) = -\frac{\beta}{2} \sum_{n=1}^N (y(x_n, w) - t_n)^2 - \frac{n}{2} \log 2\pi + \frac{n}{2} \log \beta.$$

Given $(x_1, t_1) = (1, 1.2)$ $(x_2, t_2) = (2, 1.9)$ $(x_3, t_3) = (3, 3.2)$.
 $\beta = 1$

we know that

$$\frac{1}{\beta_{ML}} = \frac{1}{N} \sum_{n=1}^N \{y(x_n, w_{ML}) - t_n\}^2 \quad (\because \text{maximizing w.r.t } \beta)$$

$$1 = \frac{1}{3} \left[\sum_{n=1}^3 \{y(x_n, w_{ML}) - t_n\}^2 \right] \quad y(x, w) = w_1 x + w_0 = w^T x.$$

$$3 = (w_1(1) + w_0 - 1.2)^2 + (w_1(2) + w_0 - 1.9)^2 + (w_1(3) + w_0 - 3.2)^2 \rightarrow \textcircled{1}$$

Boo

maximizing w.r.t w

$$\Rightarrow -\frac{1}{2} \times 2 \sum_{n=1}^N \{y(x_n, w) - t_n\} \times \frac{\partial}{\partial w} y(x_n, w) = 0.$$

$$\Rightarrow \sum_{n=1}^N (y(x_n, w) - t_n) = 0 \quad \left[\frac{\partial}{\partial w} y(x_n, w) = 1 \text{ since } \text{linear function} \right]$$

$$w_1 + w_0 - 1.2 + w_1(2) + w_0 - 1.9 + w_1(3) + w_0 - 3.2 = 0$$

$$\Rightarrow 6w_1 + 3w_0 - 6.3 = 0$$

$$\Rightarrow \boxed{2w_1 + w_0 = 2.1} \rightarrow (2)$$

Substitute $w_0 = 2.1 - 2w_1$ in (1).

$$\Rightarrow 3 = (w_1 + 2.1 - 2w_1 - 1.2)^2 + (2w_1 + 2.1 - 2w_1 - 1.9)^2 + (3w_1 + 2.1 - 2w_1 - 3.2)^2$$

$$3 = (0.9 - w_1)^2 + (0.2)^2 + (w_1 - 1.1)^2$$

$$3 = w_1^2 + 0.81 - 1.8w_1 + 0.04 + w_1^2 + 1.21 - 2.2w_1$$

$$3 = 0.81 + w_1^2 - 1.8w_1 + 0.04 + w_1^2 + 1.21 - 2.2w_1$$

$$3 = 2w_1^2 - 4w_1 + 2.06$$

$$\Rightarrow 2w_1^2 - 4w_1 - 0.94 = 0$$

$$\Rightarrow w_1^2 - 2w_1 - 0.47 = 0$$

$$\Rightarrow w_1 = \frac{2 \pm \sqrt{4 - 4(1)(-0.47)}}{2}$$

$$\Rightarrow w_1 = \frac{2 \pm 2.42}{2}$$

$$w_1 = 2.21, -0.21$$

$$\text{for } w_1 = 2.21$$

$$w_0 = 2.1 - 2w_1$$

$$\Rightarrow w_0 = 2.1 - 4.42$$

$$w_0 = -2.32$$

$$\Rightarrow y = w_1 x + w_0$$

$$\Rightarrow y = 2.21x - 2.32 \quad \text{and} \quad y = -0.21x + 2.52$$

we have to check between ③ and ④ lines

$$\text{for } w_1 = -0.21$$

$$w_0 = 2.1 - 2w_1$$

$$= 2.1 - 2(-0.21)$$

$$= 2.1 + 0.42$$

$$= 2.52$$

\Rightarrow consider line ③ since it gives less error difference

$$y = 2.21x - 2.32 \text{ plot it}$$