

**Birla Institute of Technology & Science, Pilani**  
**Work Integrated Learning Programmes Division**  
**First Semester 2022-2023**

**Comprehensive Examination**  
**(EC-3 Regular)**

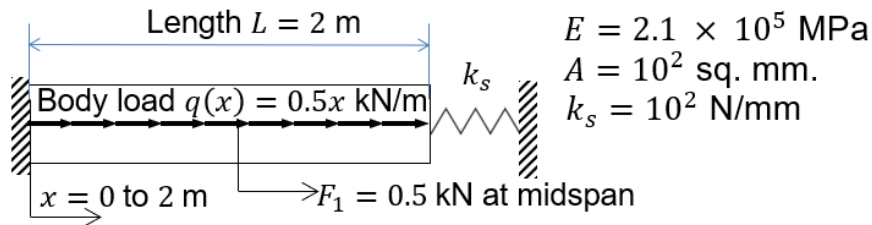
Course No. : DE ZG513  
Course Title : Finite Element Method  
Nature of Exam : Open Book  
Weightage : 40%  
Duration : 2 ½ Hours  
Date of Exam : 26/11/2022 (FN)

No. of Pages	= 2
No. of Questions	= 4

Note to Students:

1. Please follow all the *Instructions to Candidates* given on the cover page of the answer book.
2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
3. Assumptions made if any, should be stated clearly at the beginning of your answer.

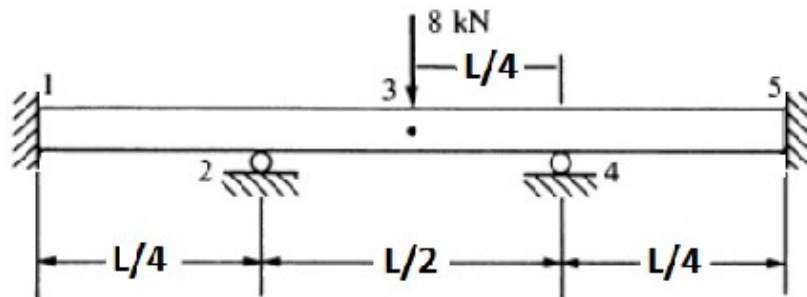
Q.1. Using two equal linear bar elements for a bar fixed at one end and supported by a spring of stiffness  $k_s$  at the other end shown in Figure 1, answer the following questions



**Figure 1**

- a) Derive the element level force vector for a generic element **[4 marks]**
- b) Write the element level equations in matrix form **[2 marks]**
- c) Write global matrix equations, apply BCs and show partitioning **[3 marks]**
- d) find the primary and secondary unknowns **[3 marks]**
- e) find the stress and strain in each element **[2 marks]**

Q.2. A beam of length  $L$  ( $=1\text{m}$ ) and square cross section (of side  $10 \text{ mm}$ ) is shown in Figure 2. Calculate the primary and secondary variables by using four beam elements. Young's modulus  $E = 200 \text{ GPa}$  and Poisson's ratio  $= 0.3$ . The nodes must be numbered as shown in Figure 2. Use Matlab software for computations. **[10 marks]**



**Figure 2**

- Q.3. Solve the Poisson equation  $-\nabla^2 u = 2$  in domain  $\Omega$  shown in Figure 3 with two boundary conditions, one,  $u=0$  on  $\Gamma_1$ , and the other,  $\partial u / \partial n = 0$  on  $\Gamma_2$ .  $\Omega$  is in the first quadrant bounded by the parabola, whose equation is  $y=1-x^2$ , and the coordinate axes.  $\Gamma_1$  and  $\Gamma_2$  are the boundaries of the 2D domain. Compute the primary and secondary unknowns using Matlab software. **[10 marks]**

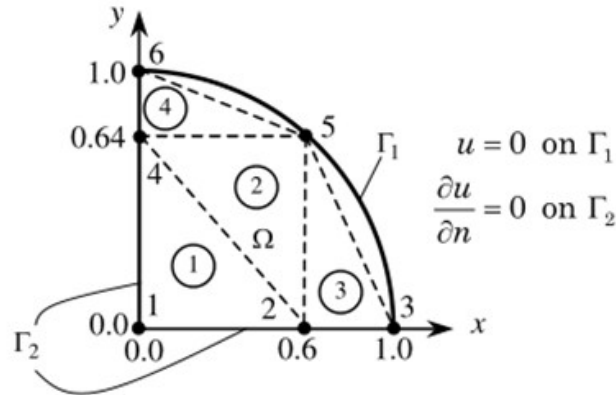
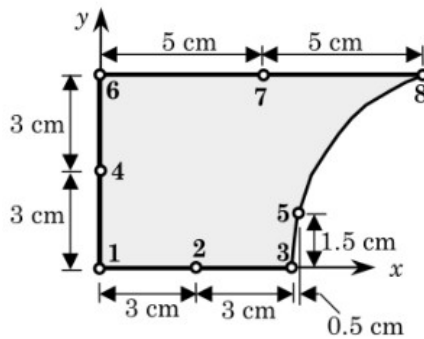
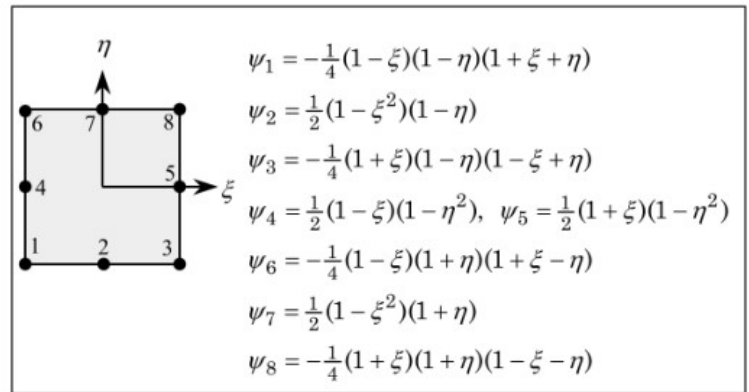


Figure 3

- Q.4. For quadratic quadrilateral element shown in Figure 4a, the corresponding master element and the respective shape functions are shown in Figure 4b.
- What are the geometry transformations equations for the coordinates  $x$  and  $y$ . **[4 marks]**
  - What is the Jacobian matrix for the transformation from the global to natural coordinate system **[2 marks]**



(a)



(b)

Figure 4

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