Lab assignment 1

Optimization in ML (CSL4010)

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Compute $\nabla f(x^0)$ and $\nabla^2 f(x^0)$ of the following function. Check whether f is convex or not.

1. $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x) = (x_1 - r)^2 + (x_2 - r)^2$, $-2r \leq x_1, x_2 \leq 2r$, $x^0 = (-0.5r, 1.5r)^T$ where r is the last digit of your roll no. Use $r = 1.5$ if last digit of your roll no is 0.

   For next problems chose x^0 uniformly distributed in (lb, ub). For question 2-9 solve for f_1 if last two digits of your roll no is odd and f_2 if last last two digits of your roll no is even.

2. $f_1, f_2 : \mathbb{R}^n \to \mathbb{R}$, $n = 5, 10, 30$

$$
\begin{aligned}
I_1 &= \{i \in \{2, 3, ..., n\} \mid i \mod 2 = 1\} \\
I_2 &= \{i \in \{2, 3, ..., n\} \mid i \mod 2 = 0\}; \\
f_1(x) &= x_1 + \frac{2}{|I_1|} \sum_{i \in I_1} (x_i - sin(6\pi x_1 + i\pi/n))^2 \\
f_2(x) &= 1 - \sqrt{x_1} + \frac{2}{|I_2|} \sum_{i \in I_2} (x_i - sin(6\pi x_1 + i\pi/n))^2 \\
0.001 &\leq x_1 \leq 1, \quad -1 \leq x_i \leq 1, \quad i = 2, 3, ..., n.
\end{aligned}
$$

3. $f_1, f_2 : \mathbb{R}^n \to \mathbb{R}$, $n = 5, 10, 30$

$$
\begin{aligned}
I_1 &= \{i \in \{2, 3, ..., n\} \mid i \mod 2 = 1\} \\
I_2 &= \{i \in \{2, 3, ..., n\} \mid i \mod 2 = 0\} \\
\text{if } i \in I_1 \text{ then } y_i &= x_i - (0.3 * x_1^2 \cos(24\pi x_1 + 4i\pi/n) + 0.6x_1) \cos(6\pi x_1 + i\pi/n) \\
\text{otherwise} \quad y_i &= x_i - (0.3 * x_1^2 \cos(24\pi x_1 + 4i\pi/n) + 0.6x_1) \sin(6\pi x_1 + i\pi/n) \\
f_1(x) &= x_1 + \frac{2}{|I_1|} \sum_{i \in I_1} y_i^2 \\
f_2(x) &= 1 - \sqrt{x_1} + \frac{2}{|I_2|} \sum_{i \in I_2} y_i^2 \\
0.001 &\leq x_1 \leq 1, \quad -1 \leq x_i \leq 1, \quad i = 2, 3, ..., n.
\end{aligned}
$$

4. $f_1, f_2 : \mathbb{R}^n \rightarrow \mathbb{R}, n = 5, 10, 30$

$$\begin{aligned}
I_1 &= \{i \in \{2,3,...,n\} \mid i \mod 2 = 1\} \\
I_2 &= \{i \in \{2,3,...,n\} \mid i \mod 2 = 0\} \\
y_i &= x_i - x_1^{0.5(1+3\frac{i-2}{n-2})}, \;\; i = 2,3,...,30 \\
f_1(x) &= x_1 + \frac{2}{|I_1|}\left(4\sum_{i\in I_1} y_i^2 - 2\prod_{i\in I_1}\cos(20y_i\pi/\sqrt{i}) + 2\right) \\
f_2(x) &= 1 - \sqrt{x_1} + \frac{2}{|I_2|}\left(4\sum_{i\in I_2} y_i^2 - 2\prod_{i\in I_2}\cos(20y_i\pi/\sqrt{i}) + 2\right) \\
0.001 &\leq x_1 \leq 1, \;\; -1 \leq x_i \leq 1, \;\; i = 2,3,...,n.
\end{aligned}$$

5. $f_1, f_2 : \mathbb{R}^n \rightarrow \mathbb{R}, n = 30$

$$\begin{aligned}
I_1 &= \{i \in \{2,3,...,n\} \mid i \mod 2 = 1\} \\
I_2 &= \{i \in \{2,3,...,n\} \mid i \mod 2 = 0\} \\
y_i &= x_i - sin(6\pi x_1 + i\pi/n), \;\; i = 2,3,...,n \\
f_1(x) &= x_1^{0.2} + \frac{2}{|I_1|}\sum_{i\in I_1} y_i^2 \\
f_2(x) &= 1 - x_1{}^{0.2} + \frac{2}{|I_2|}\sum_{i\in I_2} y_i^2 \\
0.001 &\leq x_1 \leq 1, \;\; -1 \leq x_i \leq 1, \;\; i = 2,3,...,n.
\end{aligned}$$

6. $f_1, f_2 : \mathbb{R}^4 \rightarrow \mathbb{R}$

$$\begin{aligned}
F &= 10, \;\; E = 2\times 10^5, \;\; L = 200, \;\; \sigma = 10 \\
f_1(x) &= L(2x_1 + \sqrt{2}x_2 + \sqrt{x_3 + x_4}) \\
f_2(x) &= FL/E(\frac{2}{x_1} + \frac{2\sqrt{2}}{x_2} - \frac{2\sqrt{2}}{x_3} + \frac{2}{x_4}) \\
\frac{F}{\sigma} &\leq x_1, x_4 \leq \frac{3F}{\sigma}, \;\; \frac{F}{\sqrt{2}\sigma} \leq x_2, x_3 \leq \frac{3F}{\sigma}
\end{aligned}$$

7. $f_1, f_2 : \mathbb{R}^2 \to \mathbb{R}$

$$\begin{aligned}
f_1(x) &= -2e^{15(-(x_1-0.1)^2-x_2^2)} - e^{20(-(x_1-0.6)^2-(x_2-0.6)^2)} + e^{20(-(x_1+0.6)^2-(x_2-0.6)^2)} \\
&\quad + e^{20(-(x_1-0.6)^2-(x_2+0.6)^2)} + e^{20(-(x_1+0.6)^2-(x_2+0.6)^2)} \\
f_2(x) &= 2e^{20(-x_1^2-x_2^2)} + e^{20(-(x_1-0.4)^2-(x_2-0.6)^2)} + -e^{20(-(x_1+0.5)^2-(x_2-0.7)^2)} \\
&\quad - e^{20(-(x_1-0.5)^2-(x_2+0.7)^2)} + e^{20(-(x_1+0.4)^2-(x_2+0.8)^2)} \\
-1 &\leq x_1, x_2 \leq 1
\end{aligned}$$

8. $f_1, f_2 : \mathbb{R}^2 \to \mathbb{R}$

$$\begin{aligned}
f_1(x) &= 1 - e^{-(x_1-1)^2-(x_2+1)^2} \\
f_2(x) &= 1 - e^{-(x_1+1)^2-(x_2-1)^2} \\
-4 &\leq x_1, x_2 \leq 4
\end{aligned}$$

9. $f_1, f_2 : \mathbb{R}^{10} \to \mathbb{R}$.

$$\begin{aligned}
f_1(x) &= 1 - e^{-\sum_{i=1}^{n}(x_i - \frac{1}{\sqrt{n}})^2} \\
f_2(x) &= 1 - e^{-\sum_{i=1}^{n}(x_i + \frac{1}{\sqrt{(n)}})^2} \\
-4 &\leq x_i \leq 4 \quad i = 1, 2, ...n.
\end{aligned}$$

For the rest problems solve f_1 if $rem(R, 3) = 0$, f_2 if $rem(R, 3) = 1$, and f_3 if $Rem(R, 3) = 2$, where R is the last two digits of your roll no

10. $f_1, f_2 : \mathbb{R}^n \to \mathbb{R}, n = 30$

$$\begin{aligned}
I_1 &= \{i \in \{3, 4, ..., n\} \mid i \mod 3 = 1\} \\
I_2 &= \{i \in \{3, 4, ..., n\} \mid i \mod 3 = 2\} \\
I_3 &= \{i \in \{3, 4, ..., n\} \mid i \mod 3 = 0\} \\
y_i &= x_i - 2x_2 \sin(2\pi x_1 + i\pi/n), \;\; i = 3, 4, ..., 30 \\
f_1(x) &= \cos(0.5x_1\pi)\cos(0.5x_2\pi) + \frac{2}{|I_1|}\sum_{i \in I_1} 4y_i^2 - \cos(8\pi y_i) + 1 \\
f_2(x) &= \cos(0.5x_1\pi)\sin(0.5x_2\pi) + \frac{2}{|I_2|}\sum_{i \in I_2} 4y_i^2 - \cos(8\pi y_i) + 1 \\
f_3(x) &= \sin(0.5x_1\pi) + \frac{2}{|I_3|}\sum_{i \in I_3} 4y_i^2 - \cos(8\pi y_i) + 1 \\
0 &\leq x_1, x_2 \leq 1, \;\; -2 \leq x_i \leq 2, \;\; i = 3, 4, ..., n.
\end{aligned}$$

11. $f_1, f_2, \ldots, f_m : \mathbb{R}^n \to \mathbb{R}$ $(m,n) = (3,3), (3,7)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= 100(k + \sum_{i=m}^{n} (x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5))) \\
f_1(x) &= 0.5(1 + gx) \prod_{i=1,2,\ldots,m-1} x_i \\
f_j(x) &= 0.5(1 + gx) \prod_{i=1,2,\ldots,m-j} x_i(1 - x_{m-j+1}), \quad j = 2, 3, ..., m \\
0 &\leq x_i \leq 1 \quad i = 1, 2, ..., n.
\end{aligned}$$

12. $f_1, f_2, \ldots, f_m : \mathbb{R}^n \to \mathbb{R}$ $(m,n) = (3,3), (3,10)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= \sum_{i=m}^{n} (x_i - 0.5)^2 \\
f_1(x) &= (1 + gx) \prod_{i=1,2,\ldots,m-1} \cos(0.5\pi x_i) \\
f_j(x) &= (1 + gx) \prod_{i=1,2,\ldots,m-j} \cos(0.5\pi x_i) \sin(0.5\pi x_{m-i+1}), \quad j = 2, ..., m \\
0 &\leq x_i \leq 1 \quad i = 1, ..., n.
\end{aligned}$$

13. $f_1, f_2, \ldots, f_m : \mathbb{R}^n \to \mathbb{R}$ $(m,n) = (3,3), (3,12)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= 100(k + \sum_{i=m}^{n} (x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5))) \\
f_1(x) &= (1 + gx) \prod_{i=1,2,\ldots,m-1} \cos(0.5\pi x_i) \\
f_j(x) &= (1 + gx) \prod_{i=1,2,\ldots,m-j} \cos(0.5\pi x_i) \sin(0.5\pi x_{m-i+1}), \quad j = 2, ..., m \\
0 &\leq x_i \leq 1 \quad i = 1, ..., n.
\end{aligned}$$

14. $f_1, f_2, \ldots, f_m : \mathbb{R}^n \to \mathbb{R}$ $(m, n) = (3, 3), (3, 10)$

$$\begin{aligned}
k &= n - m + 1, \ \ \alpha = 100 \\
y_i &= x_i^{\alpha} \quad i = 1, 2, ..., n \\
gx &= \sum_{i=m}^{n} (y_i - 0.5)^2 \\
f_1(x) &= (1 + gx) \prod_{i=1,2,\ldots,m-1} \cos(0.5\pi y_i) \\
f_j(x) &= (1 + gx) \prod_{i=1,2,\ldots,m-j} \cos(0.5\pi y_i) \sin(0.5\pi y_{m-i+1}), \ \ j = 2, ..., m \\
0 &\leq x_i \leq 1 \quad i = 1, ..., n.
\end{aligned}$$

15. $f_1, f_2, \ldots, f_m : \mathbb{R}^n \to \mathbb{R}$ $(m, n) = (3, 3), (3, 12)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= \sum_{i=m}^{n} x_i^{0.1} \\
\theta_i &= \frac{\pi(1 + 2 * gx * x_i)}{4(1 + gx)} \quad i = 1, 2, ..., n \\
f_1(x) &= (1 + gx) \cos(0.5\pi x_1) \prod_{i=1,2,\ldots,m-1} \cos(\theta_i) \\
f_j(x) &= (1 + gx) \cos(0.5\pi x_1) \prod_{i=1,2,\ldots,m-j} \cos(\theta_i) \sin(\theta_{m-i+1}), \ \ j = 2, ..., m \\
0.001 &\leq x_i \leq 1 \quad i = 1, ..., n.
\end{aligned}$$

16. $f_1, f_2, \ldots, f_m : \mathbb{R}^n \to \mathbb{R}$ $(m, n) = (3, 3), (3, 10)$

$$\begin{aligned}
k &= n - m + 1 \\
gx &= 1 + \frac{9}{k} \sum_{i=m}^{n} x_i \\
f_j(x) &= x_j, \ \ j = 1, ..., m - 1 \\
f_m(x) &= (1 + gx) * (m - \sum_{i=1}^{m-1} \frac{x_i}{1 + gx}(1 + sin(3\pi x_i)) \\
0 &\leq x_i \leq 1 \quad i = 1, ..., n.
\end{aligned}$$

17. $f_1, f_2, \ldots, f_m : \mathbb{R}^n \to \mathbb{R}$ $(m, n) = (3, 2)$.

$$
\begin{aligned}
f_1(x) &= (1 + x_3)(x_1^3 x_2^2 - 10x_1 - 4x_2) \\
f_2(x) &= (1 + x_3)(x_1^3 x_2^2 - 10x_1 + 4x_2) \\
f_3(x) &= 3(1 + x_3)x_1^2 \\
1 &\leq x_1 \leq 3.5, \;\; -2 \leq x_2 \leq 2, \;\; 0 \leq x_3 \leq 1
\end{aligned}
$$