



Financial Returns

Dakota Wixom

Quantitative Analyst | QuantCourse.com



Course Overview

Learn how to analyze investment return distributions, build portfolios and reduce risk, and identify key factors which are driving portfolio returns.

- Univariate Investment Risk
- Portfolio Investing
- Factor Investing
- Forecasting and Reducing Risk



Investment Risk

What is Risk?

- Risk in financial markets is a measure of uncertainty
- Dispersion or variance of financial returns

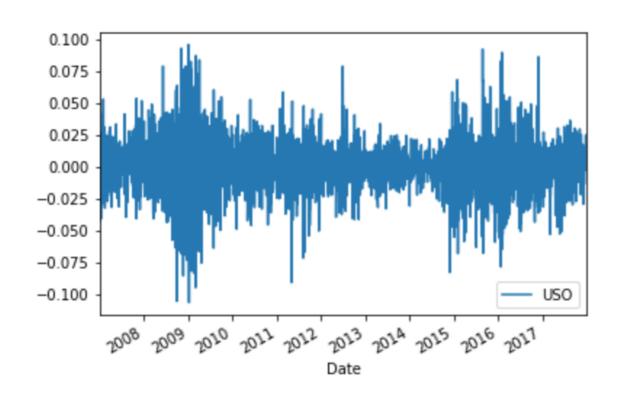
How do you typically measure risk?

- Standard deviation or variance of daily returns
- Kurtosis of the daily returns distribution
- Skewness of the daily returns distribution
- Historical drawdown

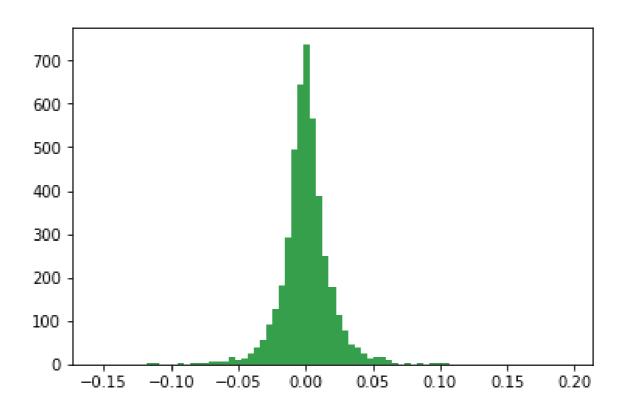


Financial Risk

RETURNS



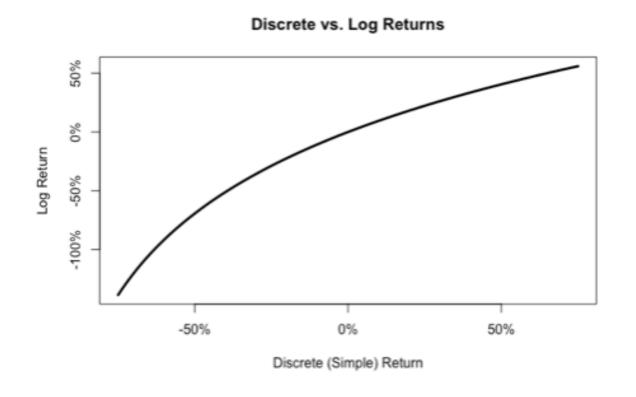
PROBABILITY





A Tale of Two Returns

- Returns are derived from stock prices
- Discrete returns (simple returns) are the most commonly used, and represent periodic (e.g. daily, weekly, monthly, etc.) price movements
- Log returns are often used in academic research and financial modeling. They assume continuous compounding.



Log returns are always smaller than discrete returns



Calculating Stock Returns

 Discrete returns are calculated as the change in price as a percentage of the previous period's price

Calculating Discrete Returns

$$R_{t_2} = \frac{(P_{t_2} - P_{t_1})}{P_{t_1}}$$

Calculating Log Returns

- Log returns are calculated as the difference between the log of two prices
- Log returns aggregate across time,
 while discrete returns aggregate
 across assets

Calculating Log Returns

$$Rl_{t_2} = \frac{ln(P_{t_2})}{ln(P_{t_1})} = ln(P_{t_2}) - ln(P_{t_1})$$



Calculating Stock Returns in Python

STEP 1:

Load in stock prices data and store it as a pandas DataFrame organized by date:

```
In [1]: import pandas as pd
In [2]: StockPrices = pd.read_csv('StockData.csv', parse_dates=['Date'])
In [3]: StockPrices = StockPrices.sort_values(by='Date')
In [4]: StockPrices.set_index('Date', inplace=True)
```



Calculating Stock Returns in Python

STEP 2:

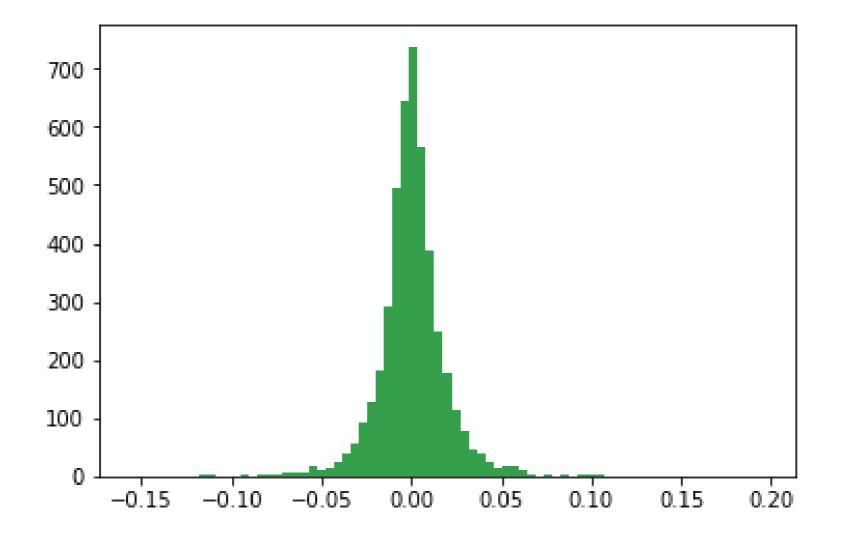
Calculate daily returns of the adjusted close prices and append the returns as a new column in the DataFrame:

```
In [1]: StockPrices["Returns"] = StockPrices["Adj Close"].pct_change()
In [2]: StockPrices["Returns"].head()
```



Visualizing Return Distributions

```
In [1]: import matplotlib.pyplot as plt
In [2]: plt.hist(StockPrices["Returns"].dropna(), bins=75, density=False)
In [3]: plt.show()
```







Let's practice!





Mean, Variance, and Normal Distributions

Dakota Wixom

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Moments of Distributions

Probability distributions have the following moments:

- 1) Mean (µ)
- 2) Variance (σ^2)
- 3) Skewness
- 4) Kurtosis

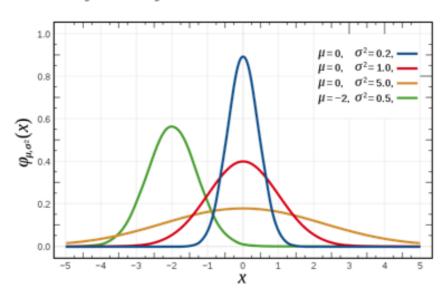
The Normal Distribution

There are many types of distributions. Some are normal and some are nonnormal. A random variable with a **Gaussian distribution** is said to be normally distributed.

Normal Distributions have the following properties:

- Mean = μ
- Variance = σ^2
- Skewness = 0
- Kurtosis = 3

Probability Density Function of Normal Distributions



Probability Density Function Equation of a Standard Normal Distribution

$$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The Standard Normal Distribution

The **Standard Normal** is a special case of the Normal Distribution when:

- $\sigma = 1$
- $\mu = 0$



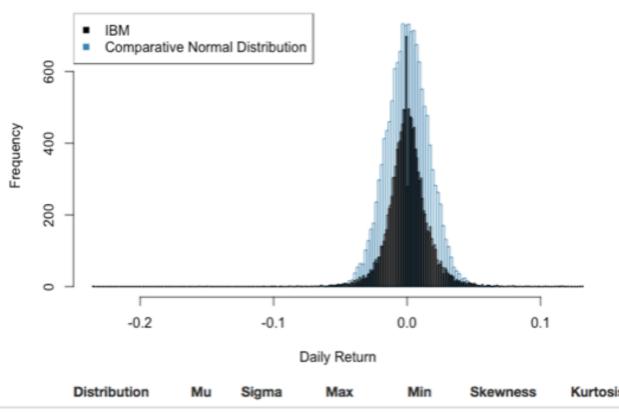
Comparing Against a Normal Distribution

- Normal distributions have a skewness near 0 and a kurtosis near 3.
- Financial returns tend not to be normally distributed
- Financial returns can have high kurtosis



Comparing Against a Normal Distribution





Distribution	Mu	Sigma	Max	Min	Skewness	Kurtosis
Normal Distribution	4e-04	0.0158	0.0530	-0.0610	-0.02	2.96
IBM	4e-04	0.0158	0.1316	-0.2352	0.02	13.20



Calculating Mean Returns in Python

To calculate the average daily return, use the np.mean() function:

```
In [1]: import numpy as np
In [2]: np.mean(StockPrices["Returns"])
Out [2]: 0.0003
```

To calculate the average annualized return assuming 252 trading days in a year:

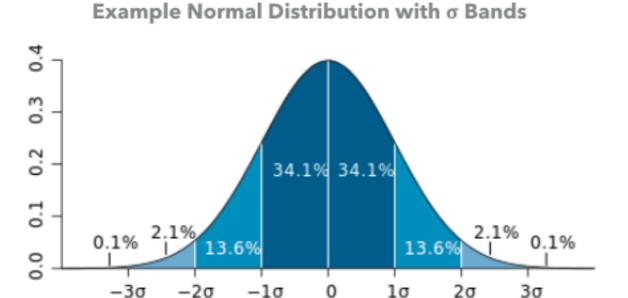
```
In [1]: import numpy as np
In [2]: ((1+np.mean(StockPrices["Returns"]))**252)-1
Out [2]: 0.0785
```



Standard Deviation and Variance

Standard Deviation (Volatility)

- Variance = σ^2
- Often represented in mathematical notation as σ , or referred to as volatility
- An investment with higher σ is viewed as a higher risk investment
- Measures the dispersion of returns



Note that 2 σ from the mean contains approximately 95.4% of all data in a normal distribution



Standard Deviation and Variance in Python

Assume you have pre-loaded stock returns data in the StockData object. To calculate the periodic standard deviation of returns:

```
In [1]: import numpy as np
In [2]: np.std(StockPrices["Returns"])
Out [2]: 0.0256
```

To calculate variance, simply square the standard deviation:

```
In [1]: np.std(StockPrices["Returns"])**2
Out [2]: 0.000655
```

Scaling Volatility

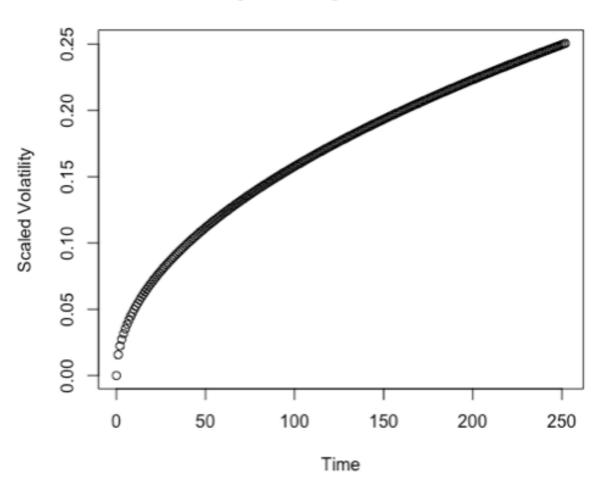
- Volatility scales with the square root of time
- You can normally assume 252 trading days in a given year, and 21 trading days in a given month

Example Volatility Scaling Equations

$$\sigma_{Annual} = \sigma_{Daily} * \sqrt(252)$$

$$\sigma_{Monthly} = \sigma_{Daily} * \sqrt(21)$$

IBM Daily Volatility Scaled Over Time





Scaling Volatility in Python

Assume you have pre-loaded stock returns data in the StockData object. To calculate the annualized volatility of returns:

```
In [1]: import numpy as np
In [2]: np.std(StockPrices["Returns"]) * np.sqrt(252)
Out [2]: 0.3071
```





Let's practice!





Skewness and Kurtosis

Dakota Wixom

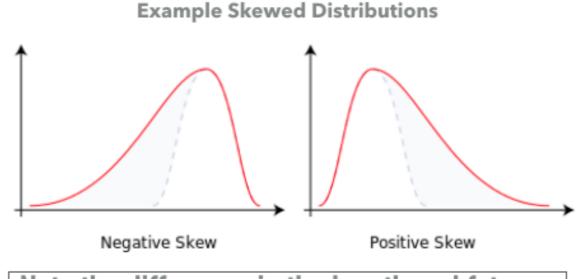
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Skewness

Skewness is the third moment of a distribution.

- Negative Skew: The mass of the distribution is concentrated on the right. Usually a right-leaning curve
- Positive Skew: The mass of the distribution is concentrated on the left.
 Usually a left-leaning curve
- In finance, you would tend to want positive skewness



Note the difference in the length and fatness of the tails depending on the skewness



Skewness in Python

Assume you have pre-loaded stock returns data in the StockData object.

To calculate the skewness of returns:

```
In [1]: from scipy.stats import skew
In [2]: skew(StockData["Returns"].dropna())
Out [2]: 0.225
```

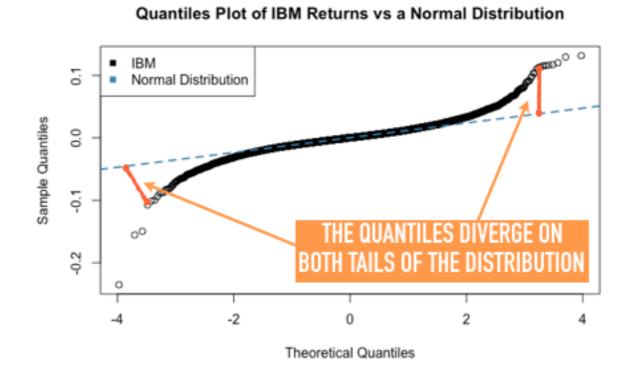
Note that the skewness is higher than 0 in this example, suggesting non-normality.



Kurtosis

Kurtosis is a measure of the thickness of the tails of a distribution

- Most financial returns are leptokurtic
- Leptokurtic: When a distribution has positive excess kurtosis (kurtosis greater than 3)
- Excess Kurtosis: Subtract 3 from the sample kurtosis to calculate "Excess Kurtosis"



Note the divergence near the tails?
That's an example of kurtosis



Excess Kurtosis in Python

Assume you have pre-loaded stock returns data in the StockData object. To calculate the excess kurtosis of returns:

```
In [1]: from scipy.stats import kurtosis
In [2]: kurtosis(StockData["Returns"].dropna())
Out [2]: 2.44
```

Note the excess kurtosis greater than 0 in this example, suggesting non-normality.



Testing for Normality in Python

How do you perform a statistical test for normality?

The null hypothesis of the **Shapiro-Wilk test** is that the data are normally distributed.

To run the Shapiro-Wilk normality test in Python:

```
In [1]: from scipy import stats
In [2]: p_value = stats.shapiro(StockData["Returns"].dropna())[1]
In [3]: if p_value <= 0.05:
In [4]:    print("Null hypothesis of normality is rejected.")
In [5]: else:
In [6]:    print("Null hypothesis of normality is accepted.")</pre>
```

The p-value is the second variable returned in the list. If the p-value is less than 0.05, the null hypothesis is rejected because the data are most likely non-normal.





Let's practice!