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Assignment - 1

Divide and Conquer   2K19/CO/319

ADA

Strassen's Matrix Multiplication

Divide and Conquer

1) Divide matrices A and B in 4 sub-matrices of size $N/2 \times N/2$
2) Calculate following values recursively.

$$ae + bg,\ af + bh,\ ce + dg \ \& \ cf + dh$$

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \times \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{C}{\begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}}$$

A, B and C are square metrices of size $N \times N$
a, b, c and d are submatrices of A, of size $N/2 \times N/2$
e, f, g and h are submatrices of B, of size $N/2 \times N/2$.

In the above method, we do 8 multiplications for matrices of size $N/2 \times N/2$ and 4 additions. Addition of two matrices taken $O(N^2)$ time. So the time complexity can be written as

$$T(N) = 8T(N/2) + O(N^2)$$

In the above divide and conquer method, the main component for high time complexity is 8 recursive calls. The idea of Strassen's method is to reduce the

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number of recursive calls to 7. Strassen's method is similar to above simple divide and conquer method in the sense that this method also divide matrices to sub-matrices of size $N/2 \times N/2$, but in Strassen's method the four sub-matrices of result are calculated using formulae.

$$p1 = a(f-h) \qquad p5 = (a+d)(e+h)$$
$$p2 = (a+b)h \qquad p6 = (b-d)(g+h)$$
$$p3 = (c+d)e \qquad p7 = (a-c)(e+f)$$
$$p4 = d(g-e)$$

The A x B can be calculated using above seven multiplications. Following are values of four sub-matrices of result C.

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \times \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{C}{\begin{bmatrix} p5+p4-p2+p6 & p1+p2 \\ p3+p4 & p1+p5-p3-p7 \end{bmatrix}}$$

p1, p2, p3, p4, p5, p6 and p7 are submitted sub matrices of size $N/2 \times N/2$

Time complexity of Strassen's Method

Addition and subtraction of two Matrices takes $O(N^2)$ time. So time complexity can be written as

$$T(N) = 7T(N/2) + O(N^2)$$

Generally Strassen's Method is not for practical applications for following reasons:

1) The constants used in strassen's method are high and for a typical application Naive method works better.

2) For sparse matrices, there are better methods especially designed for them.

3) The submatrices in recursion take extra space.

4) Because of the limited precision of computer arithmetic on noninteger, larger errors accumulate in Strassen's algorithm than in Naive Method.

$$Z = \begin{bmatrix} I & J \\ K & L \end{bmatrix} \quad X = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad Y = \begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

Using Strassen's Algorithm compute the following:

$$M_1 : (A+C) \times (E+F)$$

$$M_2 : (B+D) \times (G+H)$$

$$M_3 : (A-D) \times (E+H)$$

$$M_4 : A \times (F-H)$$

$$M_5 : (C+D) \times (E)$$

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$M_6 : (A+B) \times (H)$

$M_7 : D \times (G-E)$

Then,

$I = M_2 + M_3 - M_6 - M_7$

$J = M_4 + M_6$

$K = M_5 + M_7$

$L = M_1 - M_3 - M_4 - M_5$

Hence, the complexity of Strassen's matrix Multiplication algorithm is $\underline{O(n^{\log 7})}$.