11/03/2022

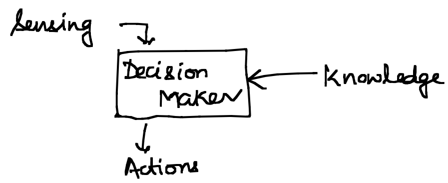
Artificial Intelligence

Module 3: Knowledge Representation and Logic

What Knowledge representation means?

Representing the knowledge about the world in a manner that facilitates inferencing (i.e. drawing conclusions) from knowledge.

Does Knowledge has any role in demonstrating intelligent behavior?



How we can represent knowledge?

→ language to represent the domain knowledge.

→ must have a method to use this knowledge.

→ Inference mechanism.

→ Syntax and Semantics of language.

e.g. The pen cuts the road. ← Syntactically correct ← Semantically incorrect.

Propositional Logic

Intelligent (Ram) : P

- Ram is intelligent.
- Ram is hardworking.

} Propositions

Compound proposition

- If Ram is intelligent and Ram is hardworking then Ram scores high grades.

Hardworking (Ram) : Q

(P and Q) $P \wedge Q$
(P or Q) $P \vee Q$
} Compound Prop.

Syntactic Elements of Propositional Logic

Vocabulary : A set of propositional symbols (e.g. P, Q ...) each of which can be either True or False.

- Set of Logical operators And ($\wedge$), or ($\vee$), Implies ($\rightarrow$) grouping ( ), not ($\neg$)

- Logical constants : True (T), False (F)

Propositional Sentence

- Each symbol (propositional or constant) is a sentence.

If P is a sentence and Q is a sentence then

* (P) is a sentence
* $P \wedge Q$ is a sentence
* $P \vee Q$ " "
* $\neg P$ " "
* $P \rightarrow Q$ " "
* Nothing else is a sentence.

Note: Sentences are also known as well formed formula (wff)

Examples wff :-

P ✓

True ✓

$(P \vee Q) \rightarrow R$ ✓

$(P \wedge Q) \vee R \rightarrow S$ ✓

$\neg(P \vee Q)$ ✓

$\neg(P \vee Q) \rightarrow R \wedge S$ ✓

Implication operator

$P \rightarrow Q$

Sufficient but not necessary

If P is true then Q is true.

If it rains then roads are wet.

$P \rightarrow Q \equiv \neg P \vee Q$

If the roads are wet then it rains ?

Equivalence ($\Leftrightarrow$)

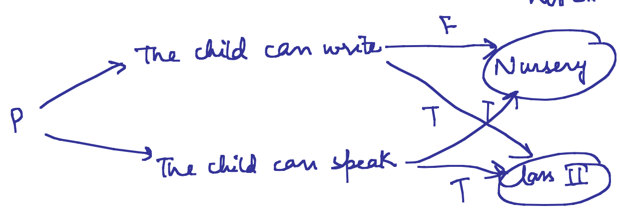
$P \Leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

If two sides of a Δ are equal then two base angles of Δ are equal.

Evaluating a compound proposition

1. Interpret each atomic proposition in same world.
2. Assign True to atomic propositions.
3. Compute value of compound proposition.

World.



- Interpretation is a world.
- When interpret a sentence in a world, we assign meaning to it & evaluate to T or F.

Validity of a sentence

If a propositional sentence is true under all the possible interpretations / world, it is valid.

Tautology $P \vee \neg P$ always true whether P is T or F.

Exercise

1. It rains in July.

   Rains(July) ✓

2. If it rains today and Tom does not carry umbrella then he will be drenched.

   Rains(Today) $\wedge$ $\neg$ carry(Tom, umbrella) $\rightarrow$ Drenched(Tom). ✓

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$ $(\neg P \vee Q)$
T	T	T
T	F	F
F	T	T
F	F	T

Draw the truth table of $(\neg P \vee Q) \rightarrow (P \wedge Q)$?

P	Q	$(\neg P \vee Q) \rightarrow (P \wedge Q)$
T	T	T
T	F	T
F	T	F
F	F	F

DeMorgan's Theorem

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$
$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

Verify from truth table

Reasoning

P: It is month of July

Q: It rains

R: $P \rightarrow Q$

R: If it is month of July then it rains

P: It is month of July

Conclude: It rains.

Modus Ponens Rule (Inference Rule)

$$\frac{\begin{array}{c} P \rightarrow Q \\ P \end{array}}{Q}$$

Given $P \rightarrow Q$ P

$\neg P \vee Q$ P

$(\neg P \vee Q) \wedge P$

$\Downarrow$

$\underbrace{(\neg P \wedge P)}_{} \vee (Q \wedge P)$

$F \vee (Q \wedge P)$

$Q \wedge P$

$Q \wedge T$

Q

Conjunctive Normal Form (CNF)

↳ Conjunction of Disjunctions

$(X_1 \vee X_2) \wedge (X_3 \vee X_4) \wedge (\ldots$

Other rules

* If P and Q then P
* If P then P or Q
* If (not(not(P))) then P
* Chain Rule: If P $\underbrace{\text{then Q} \quad \text{If Q}}$ then R

 If P then R.

Satisfiability

A sentence is satisfiable by an interpretation of under that interpretation the sentence evaluates to True.

↳ S is satisfiable if it is true in some world.

S is unsatisfiable if it is false in all the worlds.

S is valid if it is true in all worlds.

e.g. $P \rightarrow Q$ satisfiable

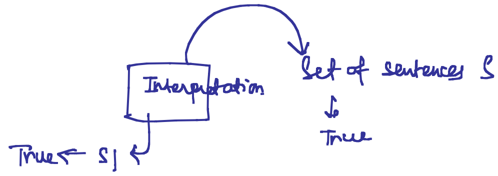
$R \rightarrow \neg R$ satisfiable

$S \wedge (W \wedge \neg S) \equiv (S \wedge W) \wedge \underbrace{(S \wedge \neg S)}_{F} \equiv F$ unsatisfiable

T: True $T \vee \neg T$ = valid

$X \rightarrow X$ = valid

Entailment



$S \vdash S_1 \Rightarrow S_1$ is logically follows form S.
S logically entails S_1

e.g. $S: x > 10$ $\quad S \vdash S_1$
$S_1: x \geq 4$

Literal : A single proposition or its negation. $P, \neg P$

Clause : Disjunction of literals $P \vee Q \vee \neg R$

Converting a compound proposition to the clausal form.

Given : $\neg(A \rightarrow B) \vee (C \rightarrow A)$

1. Eliminate $\rightarrow$: $\neg(\neg A \vee B) \vee (\neg C \vee A)$
2. DeMorgan's law : $(A \wedge \neg B) \vee (\neg C \vee A)$
3. Distributive law : $(A \vee \neg C \vee A) \wedge (\neg B \vee \neg C \vee A)$
4. $\underbrace{(A \vee \neg C)} \wedge \underbrace{(\neg B \vee \neg C \vee A)}$

Clauses : $(A \vee \neg C)$
$(\neg B \vee \neg C \vee A)$

Resolution

→ A sound inference mechanism.

→ Suppose x is a literal and S1 and S2 are two sets of propositional sentences represented in clausal form.

If we have $(x \vee S1)$ and $(\neg x \vee S2)$
then we get $S1 \vee S2$

$S1 \vee S2$: Resolvant
x : Resolved upon

Steps

Proof by Refutation.

1. Convert given proposition to clausal form.
2. Convert negation of sentence to be proved in clausal form.
3. Combine clauses in a set
4. Iteratively apply resolution to clauses set and add resolvant to set
5. Continue until no further resolvants can be obtained or null clause is obtained.

Example :- Given :- If a Δ is equilateral then it is isosceles.
If a Δ is isosceles then two sides AB, AC are equal.
If AB, AC are equal then angles B, C are equal.
ABC is equilateral Δ.

To prove :- B and C angles are equal.

P1: Equi(ABC) → Iso(ABC)
P2: Iso(ABC) → Equal(AB, AC)
P3: Equal(AB, AC) → Equal(B, C)
P4: Equi(ABC)

Given.

To prove:- Equal(B, C)

Resolution

Step 1:- Convert P1, P2, P3 in clausal form.

P1: ¬Equi(ABC) ∨ Iso(ABC)
P2: ¬ Iso(ABC) ∨ Equal(AB, AC)
P3: ¬ Equal(AB, AC) ∨ Equal(B, C)

Equi(ABC)

Step 2:- ¬Equal(B, C)

Step 3. Combine in a set.

Step 4:- $\underbrace{T}_{S2} \vee \underbrace{\neg \text{Equal}(B,C)}_{\neg x}$ $\underbrace{\neg \text{Equal}(AB,AC)}_{S1} \vee \underbrace{\text{Equal}(B,C)}_{x}$

¬Equal(AB, AC) ¬Iso(ABC) ∨ Equal(AB,AC)

¬ Equi(ABC) ∨ Iso(ABC) ¬Iso(ABC)

¬Equi(ABC) Equi(ABC)

Null clause

∴ Equal(B, C) is true.

Exercise :- Given:- 1. Mammals drink milk.
2. Man is mortal
3. Man is mammal.
4. Tom is man.

To prove:- 1. Tom drinks milk.
2. Tom is mortal.

Using Modus Ponens and Resolution.