UNIT - 4

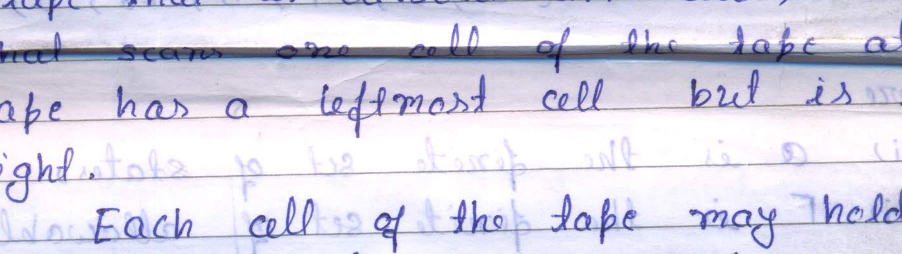
* Turing Machine (TM) *

The Turing Machine is a simple Mathematical Model of general Purpose computer. Turing Machine is a language acceptor as well as It is capable of performing any calculation which can be performed by any computing Machine.

* Model of TM *

The basic model has a finite control, an input tape that is divided into cells, and a tape head that scans one cell of the tape at a time. The tape has a leftmost cell but is infinite to the right. Each cell of the tape may hold exactly one of ~~the~~ a finite number of tape symbols.

Initially, the n leftmost cells, for some finite $n \geq 0$, hold the input. The remaining infinity of cells each hold the blank, which is a special tape symbol that is not an input symbol.



Basic Turing Machine.

In one move the Turing machine, depending upon the symbol scanned by the tape head and the state of the finite control,

1.) Changes state,
2.) Prints a symbol on the tape cell scanned, replacing what was written there,
3.) Move its head left or right one cell.

* Definition *

A Turing machine (TM) is denoted

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F),$$

where

(i) Q is the finite set of states,
(ii) Γ is the finite set of allowable tape symbols,
(iii) B, a symbol of Γ, is the blank,
(iv) Σ, a subset of Γ not including B, is the set of input symbols.
(v) δ is the next move function, a mapping from $Q \times \Gamma$ to $Q \times \Gamma \times \{L, R\}$.
(vi) q_0 in Q is the start state.

(vii) $F \subseteq Q$ is the set of final states.

* Instantaneous description (ID) of the Turing Machine M by $\alpha_1 q \alpha_2$. Here q, the current state of M, is in Q;
$\alpha_1 \alpha_2$ is the string in Γ^*

* **Moves in TM:-**

Let $X_1 X_2 \ldots X_{i-1}\, q\, X_i \ldots X_n$ be an ID.

① Suppose $\delta(q, X_i) = (P, Y, L)$. The change of ID is represented by

$$X_1 X_2 \ldots X_{i-1}\, q\, X_i \ldots X_n \vdash X_1 X_2 \ldots X_{i-2}\, P\, X_{i-1}\, Y\, X_{i+1} \ldots X_n$$

② Suppose $\delta(q, X_i) = (P, Y, R)$. The change of ID is represented by

$$X_1 X_2 \ldots X_{i-1}\, q\, X_i \ldots X_n \vdash X_1 X_2 \ldots X_{i-1}\, Y\, P\, X_{i+1} \ldots X_n.$$

* **Turing Machine (Language acceptor)**

Q1) The design of a TM M to accept the language $L = \{0^n 1^n \mid n \geq 1\}$

Solution:- Initially the tape of M contains $0^n 1^n$ followed by an infinity of blanks.
compare 0's and 1's. first compare first zero to first one. & so on.

Design:-

Step1:- q_0 is the initial state. The R/W head Replaces the leftmost 0 by X, ~~moves right to the leftmost 1.~~ ~~replace~~ and moves to the right. TM enters q_1.

Step2: Moves right to the leftmost 1, the R/W head replaces 1 by Y and moves to the left. TM enters q_2.

step3:- Moves left to find the rightmost X, then moves one cell right to the leftmost 0 and TM enters q_0 and repeats the cycle.

step4:- If however, when searching for a 1, M finds a blank instead, then M halts without accepting.

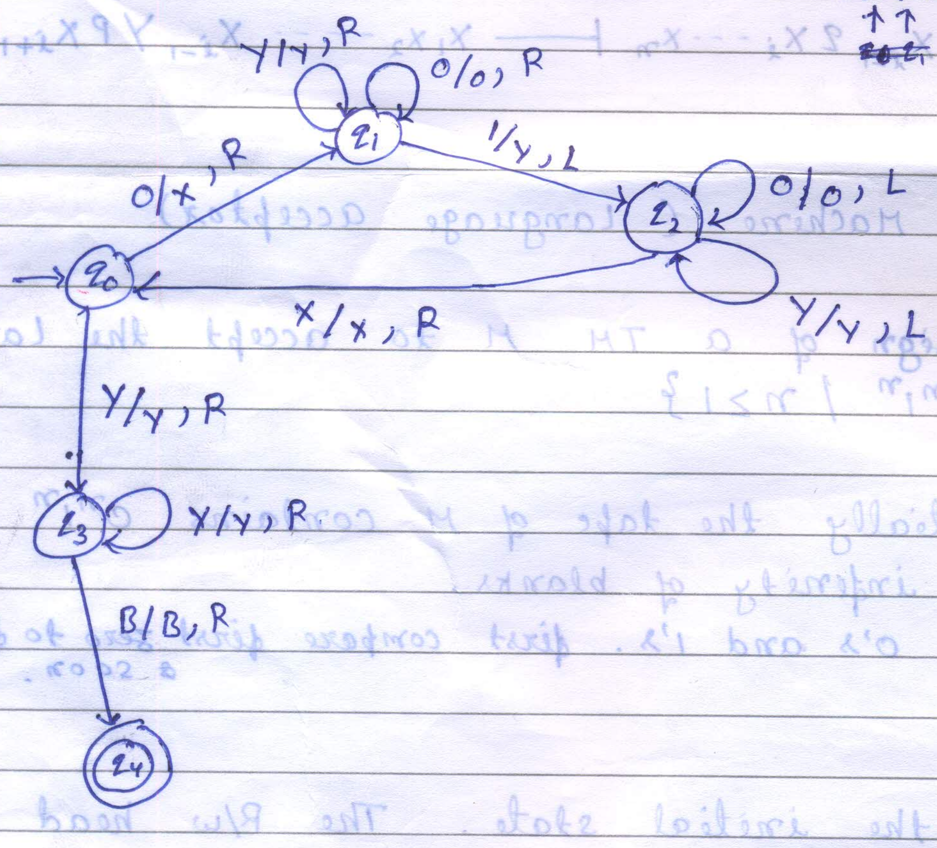
If M finds no more 0's then M checks that no more 1's remain, accepting if there are none.

Input String B00001111B

~~o/p~~ B X X X X Y Y Y Y B

↑↑ ↑↑

~~q_0 q_1~~ ~~q_1 q_2~~



- : Transition diagram : -

Transition Table :-

State	Symbol 0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)			(q_3, Y, R)	
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	
q_3				(q_3, Y, R)	(q_4, B, R)
q_4					

Here

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, X, Y, B\}$

$F = \{q_4\}$

So the Turing Machine M is defined as:-

$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$.

δ is defined in Transition Table.

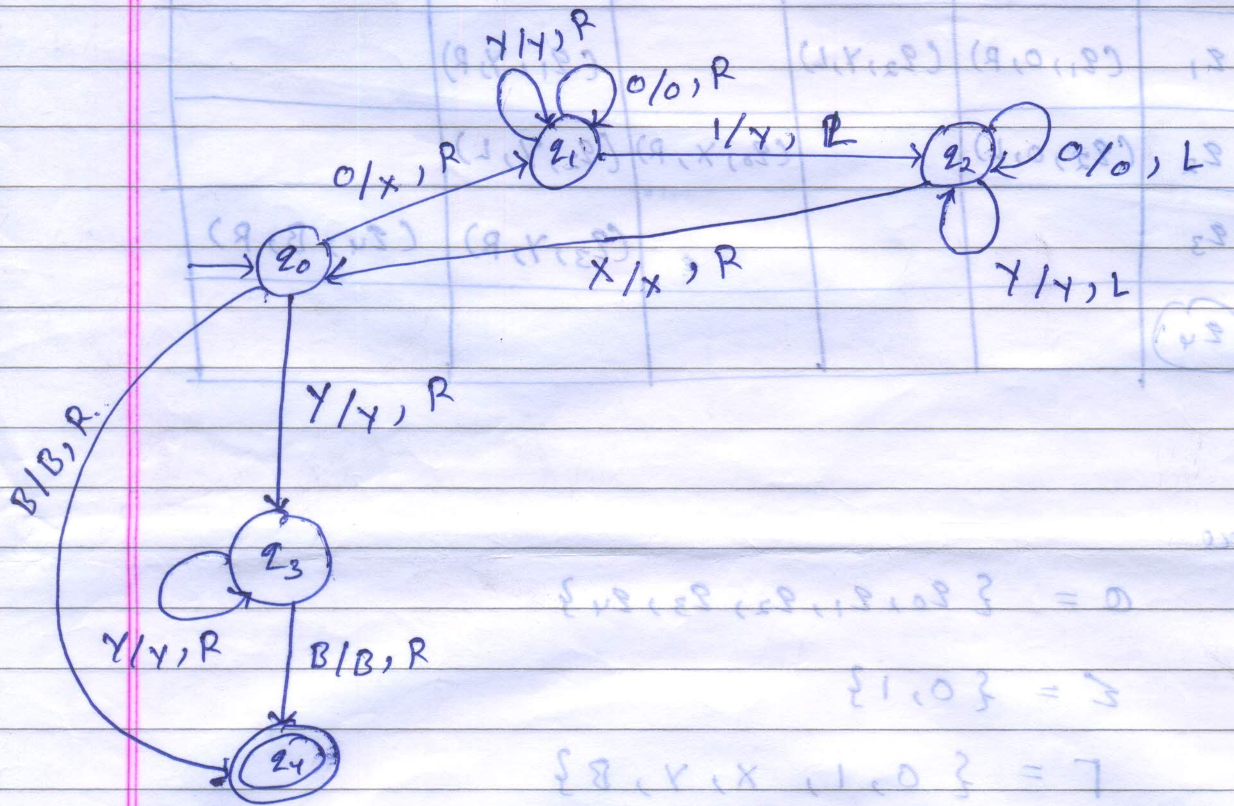
where δ is defined as:-

$\delta(q_0, 0) = (q_1, X, R)$
$\delta(q_0, Y) = (q_3, Y, R)$
$\delta(q_1, 0) = (q_1, 0, R)$
$\delta(q_1, 1) = (q_2, Y, L)$
$\delta(q_1, Y) = (q_1, Y, R)$
$\delta(q_2, 0) = (q_2, 0, L)$
$\delta(q_2, X) = (q_0, X, R)$
$\delta(q_2, Y) = (q_2, Y, L)$
$\delta(q_3, Y) = (q_3, Y, R)$
$\delta(q_3, B) = (q_4, B, R)$

Q2) Design Turing Machine to recognize the Language $\{0^n 1^n \mid n \geq 0\}$.

Solution:

B XXXXYYYY B



$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$, δ is defined in Transition Table:-

State/symbol	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)			(q_3, Y, R)	(q_4, B, R)
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	
q_3				(q_3, Y, R)	(q_4, B, R)
q_4	–		–		

Check the string 0011.

$q_0 0011 \vdash X q_1 011$
$\vdash X 0 q_1 11$
$\vdash X q_2 0 Y 1$
$\vdash q_2 X 0 Y 1$
$\vdash X q_0 0 Y 1$
$\vdash X X q_1 Y 1$
$\vdash X X Y q_1 1$
$\vdash X X q_2 Y Y$
$\vdash X q_2 X Y Y$
$\vdash X X q_0 Y Y$
$\vdash X X Y q_3 Y$
$\vdash X X Y Y q_3$
$\vdash X X Y Y B q_4$

Here Turing Machine has reached final state q_4, this input string 0011 is a accepted.

Q3) Design Turing Machine to recognize the language $\{1^n 2^n 3^n \mid n \geq 1\}$.

solution:- compare 1, 2 & 3. let us take 111222333B first compare first 1, first 2 and first 3. & so on.

Design:-

step1: q_0 is the initial state. The R/W head scans the leftmost 1, replaces 1 by X and moves to the right. TM enters q_1.

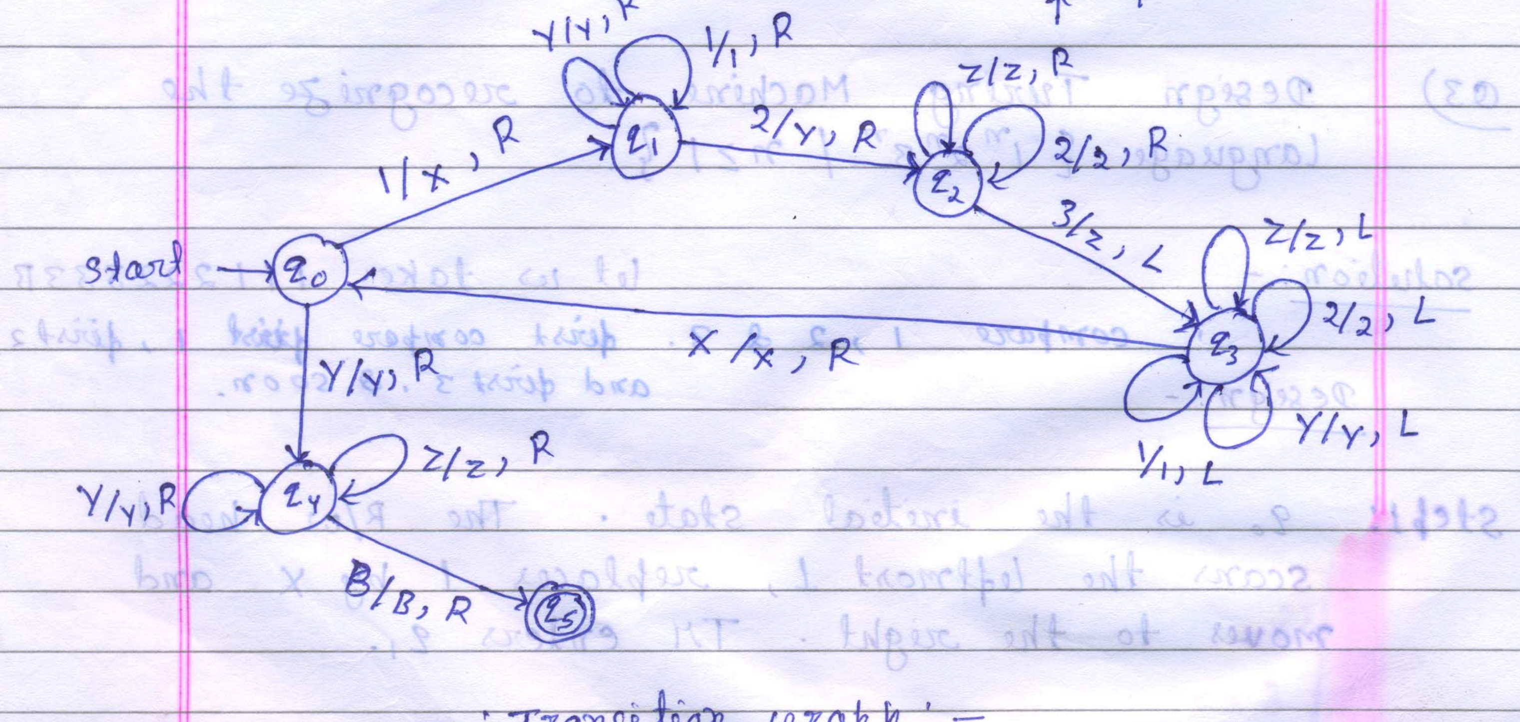
step2 : on scanning the leftmost 2, The R/w head replaces 2 by Y and moves to the right. TM enters q_2.

step3 : on scanning the leftmost 3, the R/w head replaces 3 by Z and moves to the left. M enters q_3.

step4 : It will moves left until it finds the leftmost X and then it will move right again and enters q_0.
and it again ~~scans the~~ Repeat step1 to 4.
and the Process goes on until all strings are replaced by X, Y and Z.

step5: After replacing every string it find blank, it will enter into final state.

B X X X Y Y Y Z Z Z B



— : Transition graph : —

Transition Table :-

State	1	2	3	X	Y	Z	B
$\rightarrow q_0$	(q_1, X, R)	—	—	—	(q_4, Y, R)	—	—
q_1	$(q_1, 1, R)$	(q_2, Y, R)	—	—	(q_1, Y, R)	—	—
q_2	—	$(q_2, 2, R)$	(q_3, Z, L)	—	—	(q_2, Z, R)	—
q_3	$(q_3, 1, L)$	$(q_3, 2, L)$	~~$(q_3,$~~ —	(q_0, X, R)	(q_3, Y, L)	(q_3, Z, L)	—
q_4	—	—	—	—	(q_4, Y, R)	(q_4, Z, R)	(q_5, B, R)
q_5	—	—	—	—	—	—	—

(Symbol)

Transition Rules :-

$\delta(q_0, 1) = (q_1, X, R)$
$\delta(q_0, Y) = (q_4, Y, R)$
$\delta(q_1, 1) = (q_1, 1, R)$
$\delta(q_1, 2) = (q_2, Y, R)$
$\delta(q_1, Y) = (q_1, Y, R)$
$\delta(q_2, 2) = (q_2, 2, R)$
$\delta(q_2, 3) = (q_3, Z, L)$
$\delta(q_2, Z) = (q_2, Z, R)$.

$\delta(q_3, 1) = (q_3, 1, L)$
$\delta(q_3, 2) = (q_3, 2, L)$
$\delta(q_3, X) = (q_0, X, R)$
$\delta(q_3, Y) = (q_3, Y, L)$
$\delta(q_3, Z) = (q_3, Z, L)$
$\delta(q_4, Y) = (q_4, Y, R)$
$\delta(q_4, Z) = (q_4, Z, R)$
$\delta(q_4, B) = (q_5, B, R)$.

Turing Machine M is denoted as :-

$$M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{1, 2, 3\}, \{1, 2, 3, X, Y, Z, B\}, \delta, q_0, B, \{q_5\}).$$

δ is defined above.

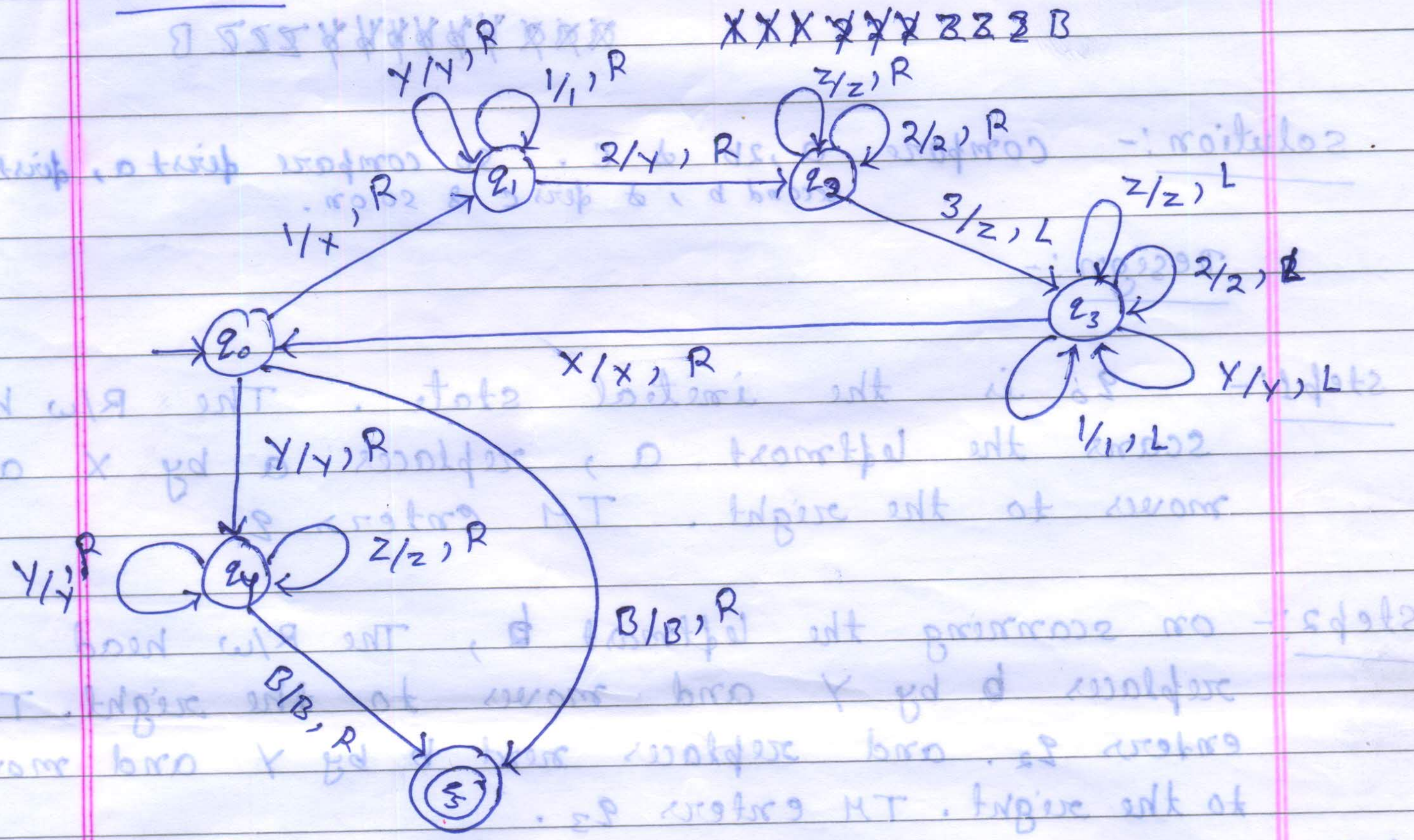
Check the string 112233.

$$q_0\, 112233 \vdash X\, q_1\, 12233$$
$$\vdash X\, 1\, q_1\, 2233$$
$$\vdash X\, 1\, Y\, q_2\, 233$$
$$\vdash X\, 1\, Y\, 2\, q_2\, 33$$
$$\vdash X\, 1\, Y\, \cancel{2}\, q_3\, 2\, Z\, 3$$
$$\vdash X\, 1\, q_3\, Y\, 2\, Z\, 3$$
$$\vdash X\, q_3\, 1\, Y\, 2\, Z\, 3$$
$$\vdash q_3\, X\, 1\, Y\, 2\, Z\, 3$$
$$\vdash X\, q_0\, 1\, Y\, 2\, Z\, 3$$
$$\vdash X\, X\, q_1\, Y\, 2\, Z\, 3$$
$$\vdash X\, X\, Y\, q_1\, 2\, Z\, 3$$
$$\vdash X\, X\, Y\, Y\, q_2\, Z\, 3$$
$$\vdash X\, X\, Y\, Y\, Z\, q_2\, 3$$
$$\vdash X\, X\, Y\, Y\, q_3\, Z\, Z$$
$$\vdash X\, X\, Y\, q_3\, Y\, Z\, Z$$
$$\vdash X\, X\, q_3\, Y\, Y\, Z\, Z$$
$$\vdash X\, q_3\, X\, Y\, Y\, Z\, Z$$
$$\vdash X\, X\, q_0\, Y\, Y\, Z\, Z$$
$$\vdash X\, X\, Y\, q_4\, Y\, Z\, Z$$
$$\vdash X\, X\, Y\, Y\, q_4\, Z\, Z$$
$$\vdash X\, X\, Y\, Y\, Z\, q_4\, Z$$
$$\vdash X\, X\, Y\, Y\, Z\, Z\, q_4$$
$$\vdash X\, X\, Y\, Y\, Z\, Z\, B\, q_5$$

q_5 is final state so it accept the string.

Q4) Design Turing machine to recognize the Language $\{ 1^n 2^n 3^n \mid n \geq 0 \}$.

solution:-



Turing Machine M is denoted as:-

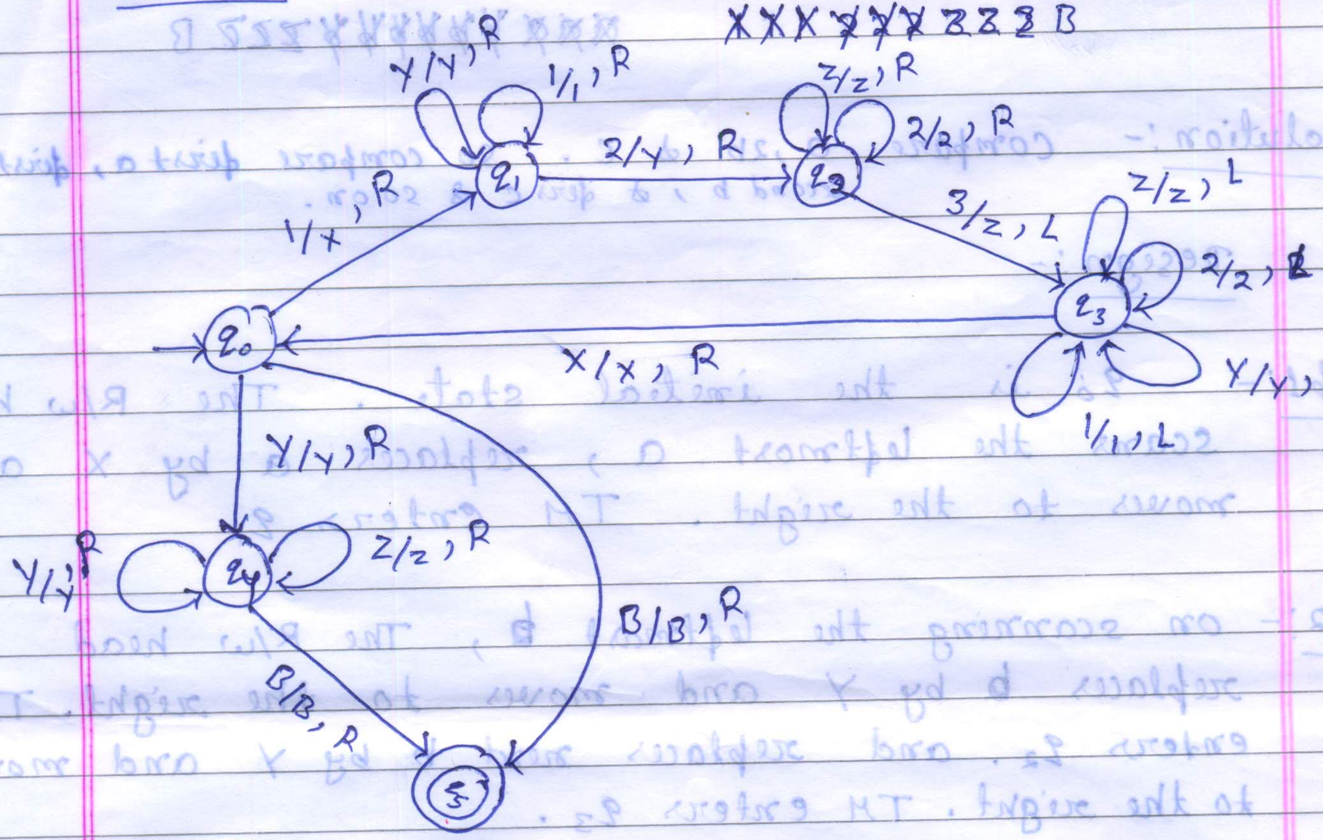
$M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{1, 2, 3\}, \{1, 2, 3, X, Y, Z, B\}, \delta, q_0, B, \{q_5\})$

δ is defined in transition Table.

State	1	2	3	X	Y	Z	B
$\rightarrow q_0$	(q_1, X, R)	–	–	–	(q_4, Y, R)	–	(q_5, B, R)
q_1	$(q_1, 1, R)$	(q_2, Y, R)	–	–	(q_1, Y, R)	–	–
q_2	–	$(q_2, 2, R)$	(q_3, Z, L)	–	–	(q_2, Z, R)	–
q_3	$(q_3, 1, L)$	$(q_3, 2, L)$	–	(q_0, X, R)	(q_3, Y, L)	(q_3, Z, L)	–
q_4	–	–	–	–	(q_4, Y, R)	(q_4, Z, R)	(q_5, B, R)
q_5	–	–	–	–	–	–	–

Q4) Design Turing machine to recognize the language $\{ 1^n 2^n 3^n \mid n \geq 0 \}$

solution:-



Turing Machine M is denoted as:-

$M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{1, 2, 3\}, \{1, 2, 3, X, Y, Z, B\}, \delta, q_0, B, \{q_5\})$

δ is defined in transition Table.

State	1	2	3	X	Y	Z	B
	symbol						
$\rightarrow q_0$	(q_1, X, R)	–	–	–	(q_4, Y, R)	–	(q_5, B, R)
q_1	$(q_1, 1, R)$	(q_2, Y, R)	–	–	(q_1, Y, R)	–	–
q_2	–	$(q_2, 2, R)$	(q_3, Z, L)	–	–	(q_2, Z, R)	–
q_3	$(q_3, 1, L)$	$(q_3, 2, L)$	–	(q_0, X, R)	(q_3, Y, L)	(q_3, Z, L)	–
q_4	–	–	–	–	(q_4, Y, R)	(q_4, Z, R)	(q_5, B, R)
q_5	–	–	–	–	–	–	–

Q5) Design Turing Machine to recognize the language $\{ a^n b^{2n} c^n \mid n \geq 0 \}$.

XXXYYYYYYZZZB

solution:- compare a, 2b & c. so compare first a, first & second b, & first c & so on.

Design:-

step1:- q_0 is the initial state. The R/W head scans the leftmost a, replaces a by X and moves to the right. TM enters q_1.

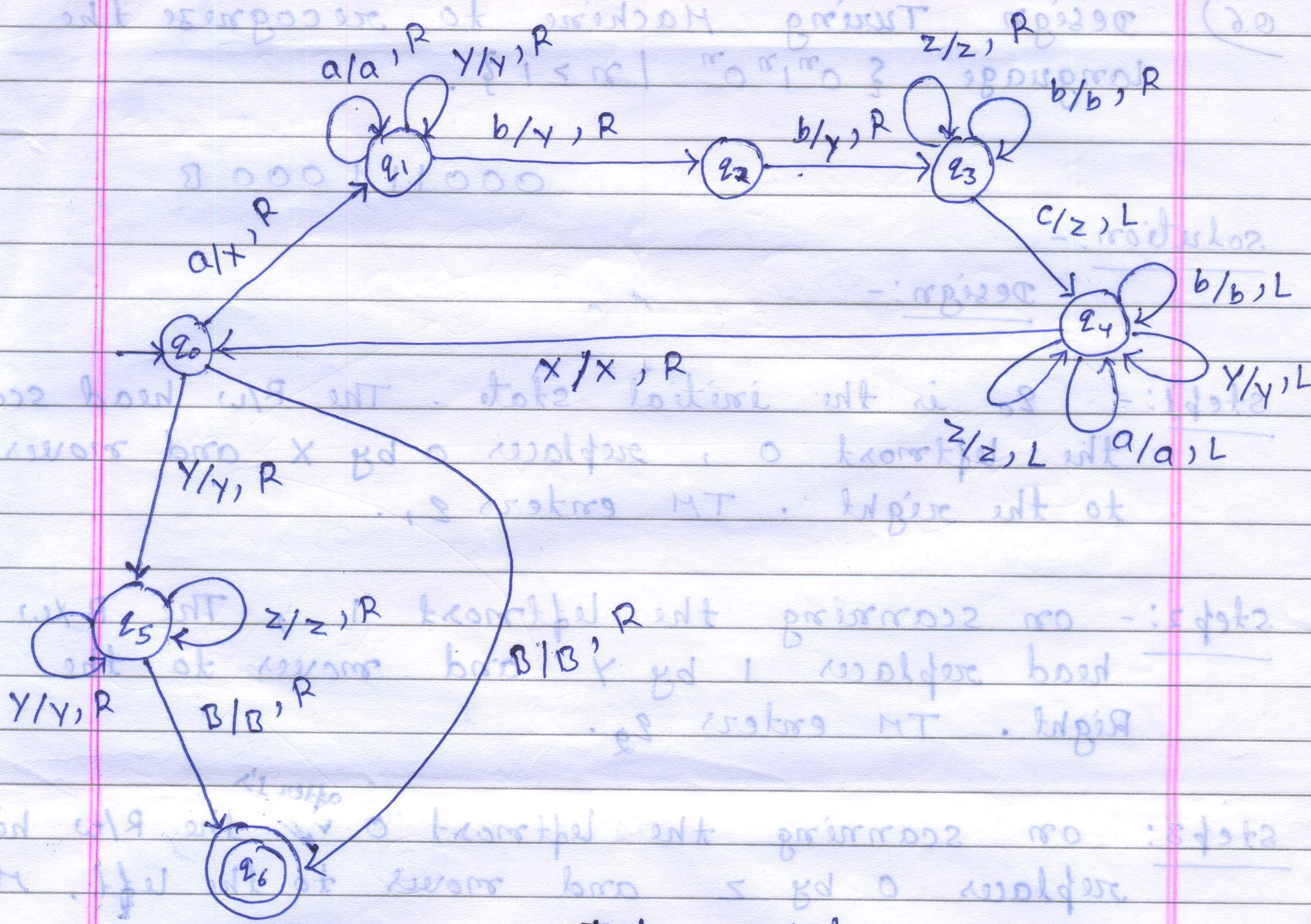
step2:- on scanning the leftmost b, The R/W head replaces b by Y and moves to the right. TM enters q_2. and replaces next b by Y and moves to the right. TM enters q_3.

step3:- on scanning the leftmost c, the R/W head replaces c by Z and moves to the left. M enters q_4.

step4:- It will moves left until it finds the X and then it will move right again and enters q_0.

and Repeat step 1 to 4. and the process goes on until all strings are replaced by X, Y & Z.

step5. After replacing every string it find blank, it will enters into final state.

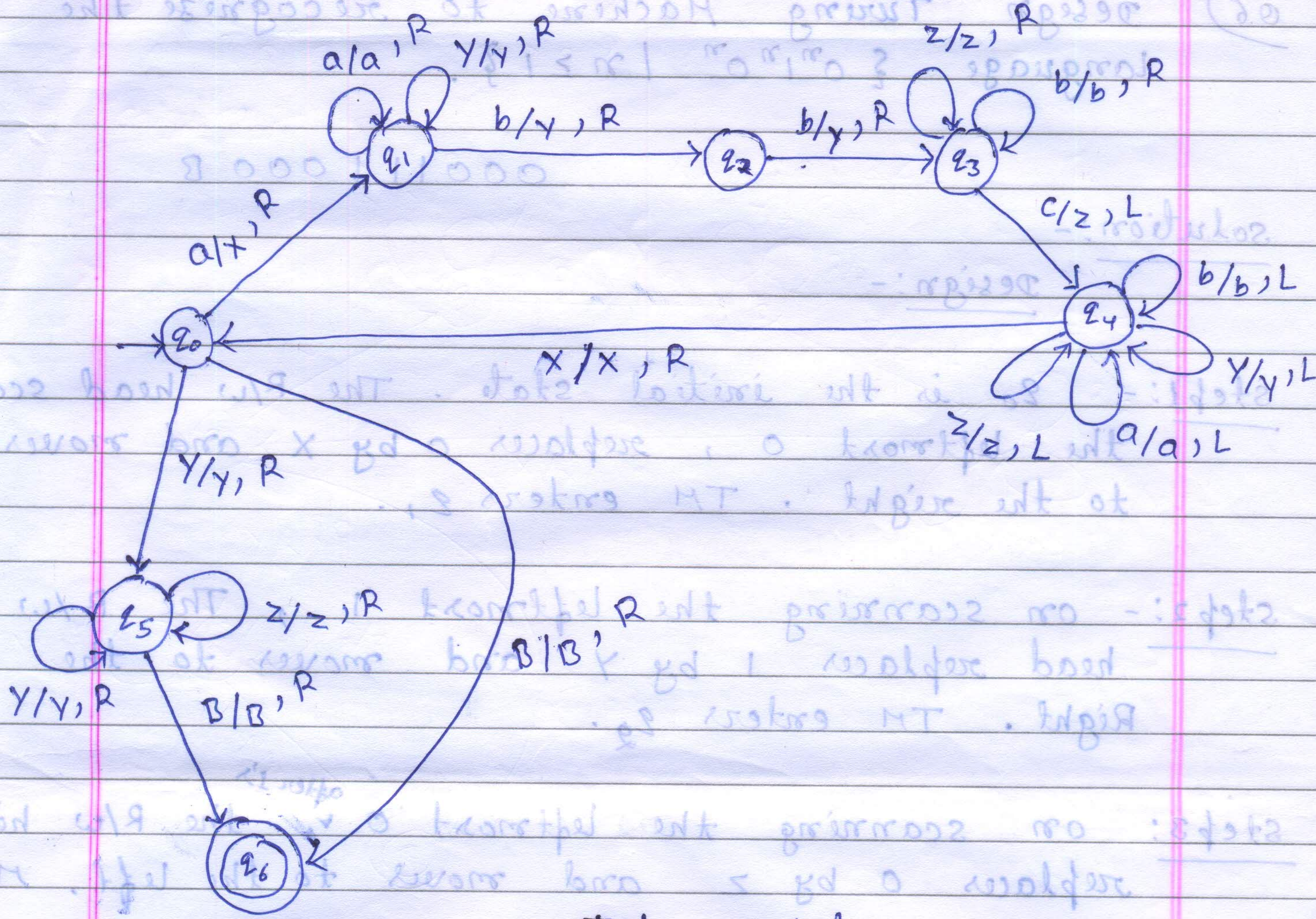


Tape symbol

Present state	a	b	c	X	Y	Z	B
$\rightarrow q_0$	(q_1, X, R)	—	—	—	(q_5, Y, R)	—	(q_6, B, R)
q_1	(q_1, a, R)	(q_2, Y, R)	—	—	(q_1, Y, R)	—	—
q_2	—	(q_3, Y, R)	—	—	—	—	
q_3	—	(q_3, b, R)	(q_4, Z, L)			(q_3, Z, R)	—
q_4	(q_4, a, L)	(q_4, b, L)	—	(q_0, X, R)	(q_4, Y, L)	(q_4, Z, L)	—
q_5	—	—	—	—	(q_5, Y, R)	(q_5, Z, R)	(q_6, B, R)
q_6	—	—	—	—	—	—	—

$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{a, b, c\}, \{a, b, c, X, Y, Z, B\}, \delta, q_0, B, \{q_6\})$

δ is defined in Transition Table



Tape symbol

Present state	a	b	c	X	Y	Z	B
$\rightarrow q_0$	(q_1, X, R)	—	—	—	(q_5, Y, R)	—	(q_6, B, R)
q_1	(q_1, a, R)	(q_2, Y, R)	—	—	(q_1, Y, R)		—
q_2	—	(q_3, Y, R)	—	—	—	—	
q_3	—	(q_3, b, R)	(q_4, Z, L)			(q_3, Z, R)	—
q_4	(q_4, a, L)	(q_4, b, L)	—	(q_0, X, R)	(q_4, Y, L)	(q_4, Z, L)	—
q_5	—	—	—	—	(q_5, Y, R)	(q_5, Z, R)	(q_6, B, R)
q_6	—	—	—	—	—	—	—

$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{a, b, c\}, \{a, b, c, X, Y, Z, B\}, \delta, q_0, B, \{q_6\})$

δ is defined in Transition Table

Q6) Design Turing Machine to recognize the language $\{ 0^n 1^n 0^n \mid n \geq 1 \}$.

000111000 B

solution:-

Design:-

step1:- q_0 is the initial state. The R/W head scans the leftmost 0, replaces 0 by X and moves to the right. TM enters q_1.

step2:- on scanning the leftmost 1, The R/W head replaces 1 by Y and moves to the Right. TM enters q_2.

step3: on scanning the leftmost 0 after 1's, the R/W head replaces 0 by Z and moves to the left. M enters q_2.

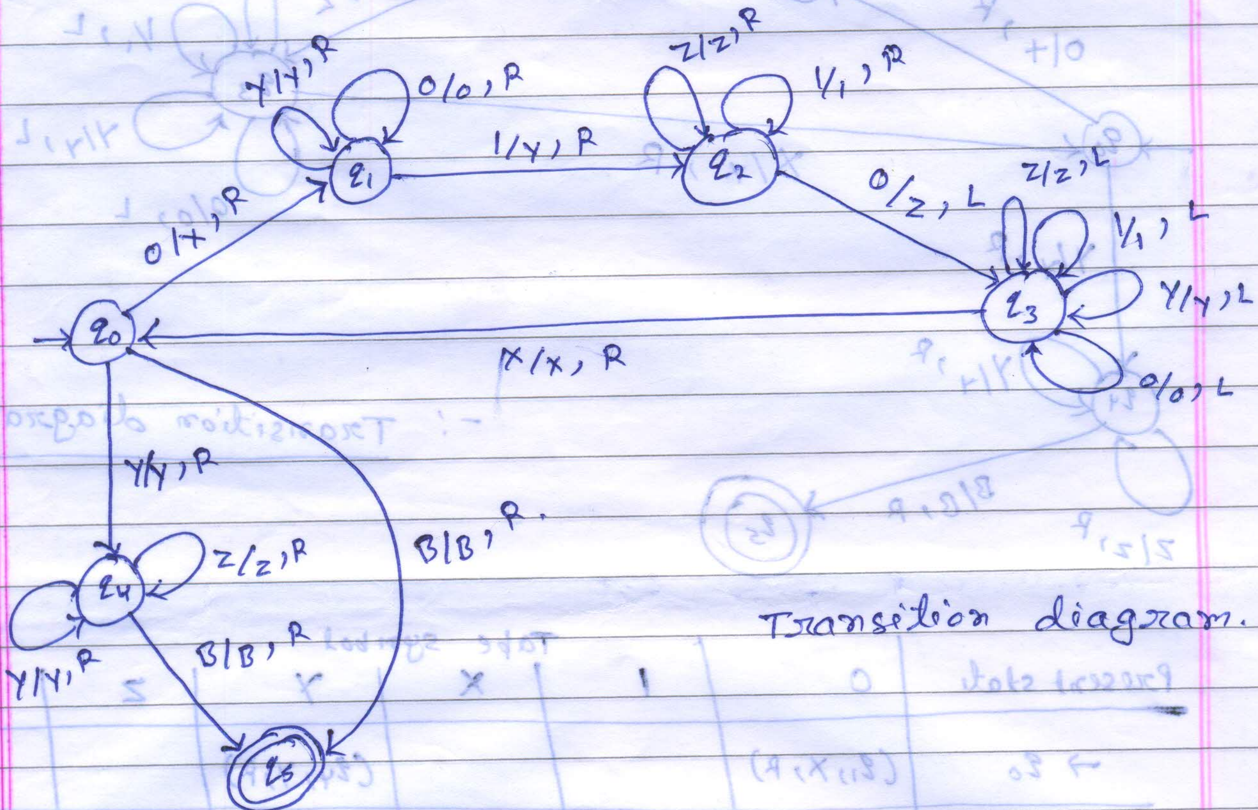
step4: It will moves left until it finds the X and then it will moves Right again and enters q_0.

and then Repeat step 1 to 4. and the Process goes on until all strings & replaced by X, Y, Z.

step5: After replacing every string it find blank, it will enter into final state.

Q7) Design Turing Machine to recognize the Language $\{0^n 1^n 0^n \mid n \ge 0\}$.

~~XXX~~YYYZZZ B



Transition diagram.

Present state	Tape symbol 0	1	X	Y	Z	B
$\rightarrow q_0$	(q_1, X, R)	–	–	(q_4, Y, R)	–	(q_5, B, R)
q_1	$(q_1, 0, R)$	(q_2, Y, R)	–	(q_1, Y, R)	–	–
q_2	(q_3, Z, L)	$(q_2, 1, R)$	–	–	(q_2, Z, R)	–
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, X, R)	(q_3, Y, L)	(q_3, Z, L)	–
q_4	–	–	–	(q_4, Y, R)	(q_4, Z, R)	(q_5, B, R).
q_5	–	–	–	–	–	–

$M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{0, 1\}, \{0, 1, X, Y, Z, B\}, \delta, q_0, B, \{q_5\})$ δ is defi[ned in] Transitio[n table]

Q8) Design Turing Machine to recognize the language $\{ a^n b^n c^n \mid n \geq 1$

Solution:-

XXXYYYZZZB
↑

Design:-

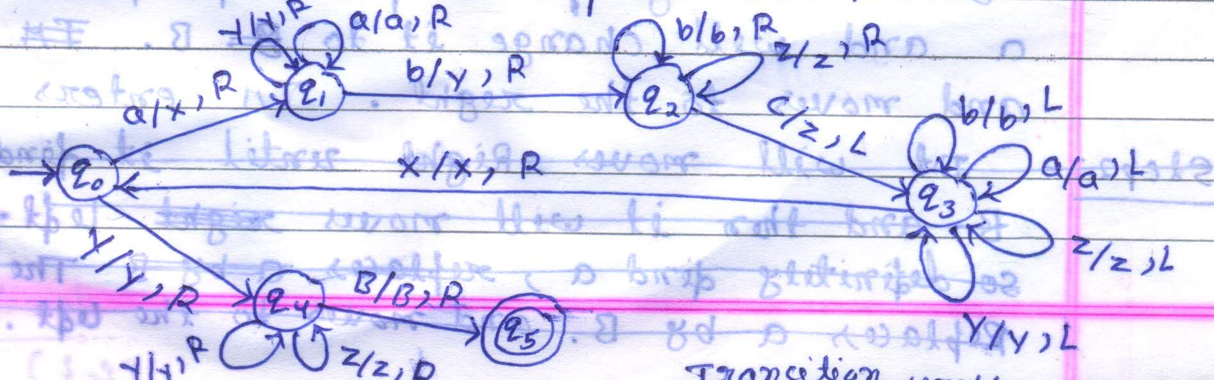
Step 1:- q_0 is the initial state. The R/w head scans the leftmost a, Replaces a by X and moves to the Right. TM enters q_1.

Step 2:- On scanning the leftmost b, The R/w head replaces b by Y and moves to the right. TM enters q_2.

Step 3:- On scanning the leftmost c, the R/w head replaces c by Z and moves to the left. M enters q_3.

Step 4:- It will moves left until it finds the X and then it will move right again and enters q_0.
and it again Repeat step 1 to 4.
and the Process goes on until all strings are replaced by X, Y, Z.

Step 5:- After replacing every string it find blank, it will enters into final state.



Transition graph.

Transition Table :-

State	a	b	c	X	Y	Z	B
$\rightarrow q_0$	(q_1, X, R)	-	-	-	(q_4, Y, R)	-	-
q_1	(q_1, a, R)	(q_2, Y, R)	-	-	(q_1, Y, R)	-	-
q_2	-	(q_2, b, R)	(q_3, Z, L)	-	-	(q_2, Z, R)	-
q_3	(q_3, a, L)	(q_3, b, L)	-	(q_0, X, R)	(q_3, Y, L)	(q_3, Z, L)	-
q_4	-	-	-		(q_4, Y, R)	(q_4, Z, R)	(q_5, B, R)
q_5	-	-	-	-	-	-	-

Turing Machine M is defined as :-

$$M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b, c\}, \{a, b, c, X, Y, Z, B\}, \delta, q_0, B, \{q_5\})$$

δ is defined in Transition Table.

Q9) Design Turing Machine to recognize the language $L = \{ \omega \omega^R , \text{ where } \omega \in \{a, b\}^* \}$

~~step~~ Solution :- $L = \omega\omega^R$

Design :- ω^R is the Reverse of ω. ~~so~~ ~~leftmost symb~~

Step 1 :- q_0 is the initial state. ~~The R/W head~~ ~~scans~~ Reading first from left, if TM will read a and will change it to ~~B.~~ B. ~~TM enters q_1.~~ and moves to the right. TM enters q_1.

~~Step 2 :- It will moves Right until it finds the B and then it will moves right~~ left ~~. and TM enters q_2.~~ ~~so definitely find a ,~~ ~~replaces a by B~~ ~~The R/W head Replaces a by B. and moves to the left. TM~~

step2: It will move right until it finds the B. and then it will moves left and TM enters q_2.

step3 so definitly find a, The R/w head Replaces a by B, and move to the left. TM enters q_3.

step3: It will move left until it finds the B, then it will move Right. TM enters q_0.

If again ~~a~~ reading a then Repeat step 1 to 3.
If reading b then Repeat step 4 to 6.

step4: q_0 is the initial state. if TM will read b and will change it to B. and moves to the right. TM enters q_4.

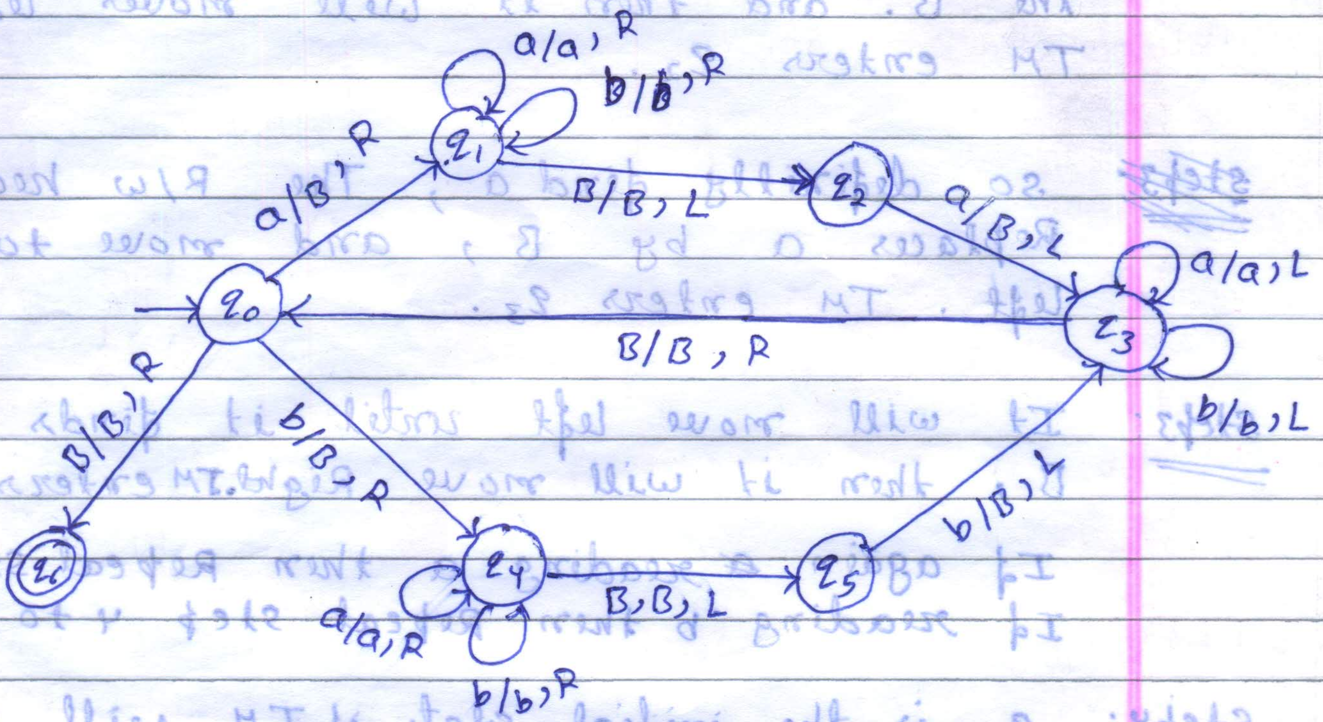
step5: It will move right until it finds the B. and then it will moves left and TM enters q_5.

So definitly find b, The R/w head Replaces b by B, and move to the left. TM enters q_5.

step6: It ~~move~~ will move left until it finds the B, then it will move Right. TM enters q_0.
If again reading b then repeat step 4 to 6.
If reading a then repeat step 1 to 3.

step7: If It find Blank on q_0. It enter into final state.

$\underbrace{B\,a\,b\,b}_{\omega}\underbrace{b\,b\,a\,a}_{\omega^R}\,B$



Transition Table:-

State	a	b	B
$\rightarrow q_0$	(q_1, B, R)	(q_4, B, R)	(q_6, B, R)
q_1	(q_1, a, R)	(q_1, b, R)	(q_2, B, L)
q_2	(q_3, B, L)	–	
q_3	(q_3, a, L)	(q_3, b, L)	(q_0, B, R)
q_4	(q_4, a, R)	(q_4, b, R)	(q_5, B, L)
q_5		(q_3, B, L)	
q_6	–	–	–

The Turing Machine M is defined as:-

$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_6\})$

δ is defined in Transition Table.

Q10) Design Turing Machine to recognize the language $L = \{wcw^R, \text{ where } w \in (a+b)^*\}$

Solution :-

Design :- w^R is the reverse of w.

Step 1: q_0 is the initial state. Reading first from left, if TM will read a and will change it to B and moves to the right. TM enters q_1.

Step 2: It will move right until it find c and then ~~it will~~ again it will move right. TM enters q_2.

Step 3: It will move right until it finds the B and then it will moves left and TM enters q_3.
So definetely find a, The R/w head Replaces a by B, and move to the left. TM enters q_4.

Step 4: It will move left until it finds the B, then it will move Right. TM enters q_0.
If again reading a then Repeat step 1 to 4.
If Reading b then Repeat step 5 to 8.

Step 5: q_0 is the initial state. Reading first from left, if TM will read b and will change it to B. and move to the Right. TM enters q_5.

Step 6: It will move right until it find c and then again it will move right. TM enters. q_6.

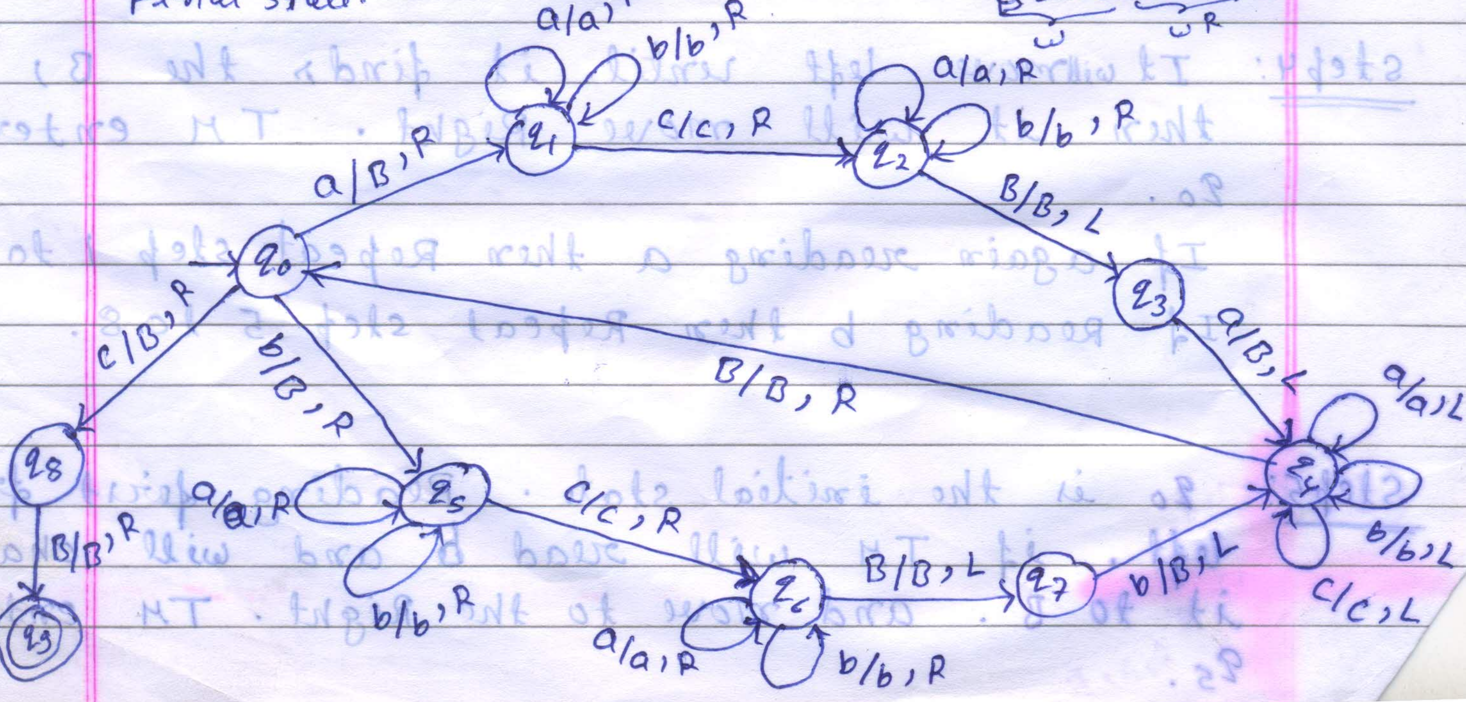
Step 7: It will move right until it finds the B and then it will move left and TM enters q_7.
So here definetely find b, The R/W head Replaces b by B. and move to the left. TM enters q_8.

Step 8. It will move left until it finds the B., then it will move right. TM enters q_0.

If again reading b then Repeat step 5 to 8.
If Reading a then Repeat step 1 to 4.

Step 9: If It finds ~~Blank~~ C on q_0. It enters into ~~final~~ ~~state~~. q_8. then if find B ~~then~~ so enter into Final state.

$\underbrace{B\,b\,b\,a}_{w}\,c\,\underbrace{a\,b\,b}_{w^R}\,a\,B$



Transition Table:-

State	a	b	c	B
$\rightarrow q_0$	(q_1, B, R)	(q_5, B, R)	(q_8, B, R)	–
q_1	(q_1, a, R)	(q_1, b, R)	(q_2, c, R)	–
q_2	(q_2, a, R)	(q_2, b, R)	–	(q_3, B, L)
q_3	(q_4, B, L)	–	–	–
q_4	(q_4, a, L)	(q_4, b, L)	(q_4, c, L)	(q_0, B, R)
q_5	(q_5, a, R)	(q_5, b, R)	(q_6, c, R)	–
q_6	(q_6, a, R)	(q_6, b, R)		(q_7, B, L)
q_7	–	(q_4, B, L)	–	–
q_8	–	–	–	(q_9, B, R)
q_9	–	–		

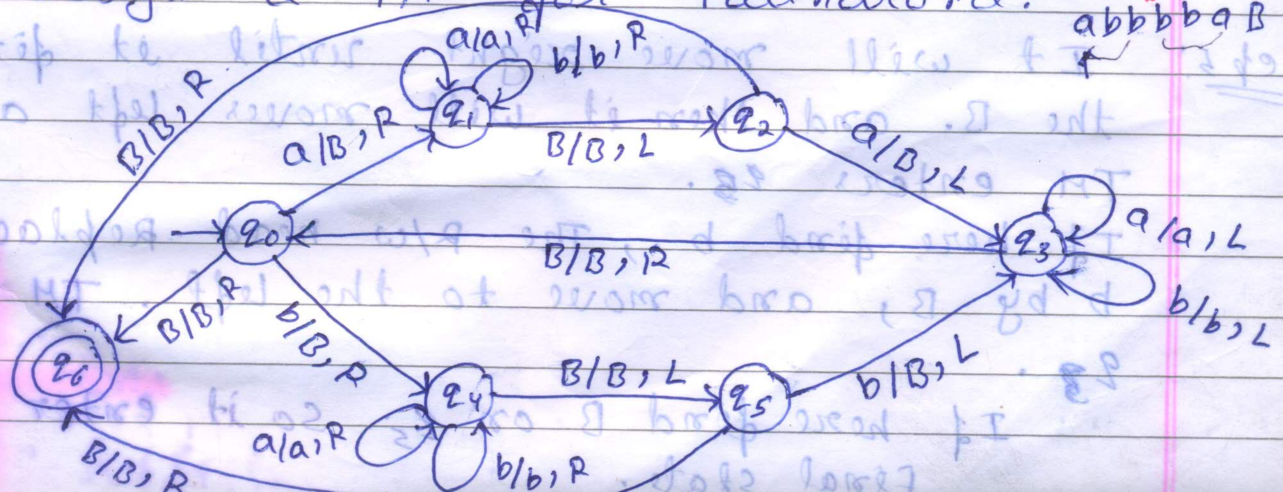
Turing Machine M is defined as:-

$$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9\}, \{a, b, c\}, \{a, b, c, B\}, \delta, q_0, B, \{q_9\})$$

δ is defined in Transition Table.

Q11) Design a TM for Palindrome.

abbbbaB



step1:- q_0 is the initial state. Reading first from left. if TM will read a and will change it to B. and moves to the right. TM enters q_1.

step2: It will move right until it finds the B. and then it will moves left and TM enters q_2.

If ~~so definetly~~ find a, The R/W head Replaces a by B, and move to the left. TM enters q_3.

~~on~~ q_2, If here find B on q_2 so it enter into final state.

step3: ~~It will~~ on q_3, It will move left until it finds the B, then it will move right. TM enters q_0.

If again reading a then repeat step 1 to 3.
If reading b then repeat step 4 to 6.

step4: q_0 is the initial state. if TM will read b and will change it to B. and moves to the right. TM enters q_4.

step5: It will move right until it finds the B. and then it will moves left and TM enters q_5.
If here find b, The R/W head Replaces b by B, and move to the left. TM enters q_3.
If here find B on q_5 so it enter into final state.

Step 6: on q_3, It will move left until it find the B, then it will move right. T.M. enters q_0.

on q_0, If again reading 'a' then repeat step 1 to 3.

on q_0, If reading 'b' then repeat step 4 to 6.

Step 7: If It find Blank on q_0. It enters into final state.

The Turing Machine M is defined as:-

$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, q_6)$

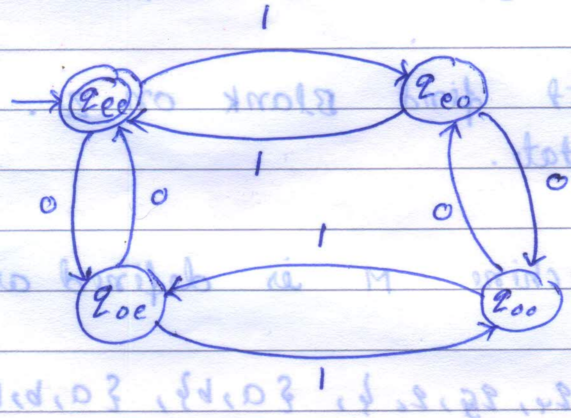
δ is defined as:-

$\delta(q_0, a) = (q_1, B, R)$
$\delta(q_0, b) = (q_4, B, R)$
$\delta(q_0, B) = (q_6, B, R)$
$\delta(q_1, a) = (q_1, a, R)$
$\delta(q_1, b) = (q_1, b, R)$
$\delta(q_1, B) = (q_2, B, L)$
$\delta(q_2, a) = (q_3, B, L)$
$\delta(q_2, B) = (q_6, B, R)$
$\delta(q_3, a) = (q_3, a, L)$
$\delta(q_3, b) = (q_3, b, L)$
$\delta(q_3, B) = (q_0, B, R)$
$\delta(q_4, a) = (q_4, a, R)$
$\delta(q_4, b) = (q_4, b, R)$
$\delta(q_4, B) = (q_5, B, L)$
$\delta(q_5, b) = (q_3, B, L)$
$\delta(q_5, B) = (q_6, B, R)$.

Q 12) construct a TM for the languages of even number of 1's and even number of 0's over $\Sigma = \{0, 1\}$.

or

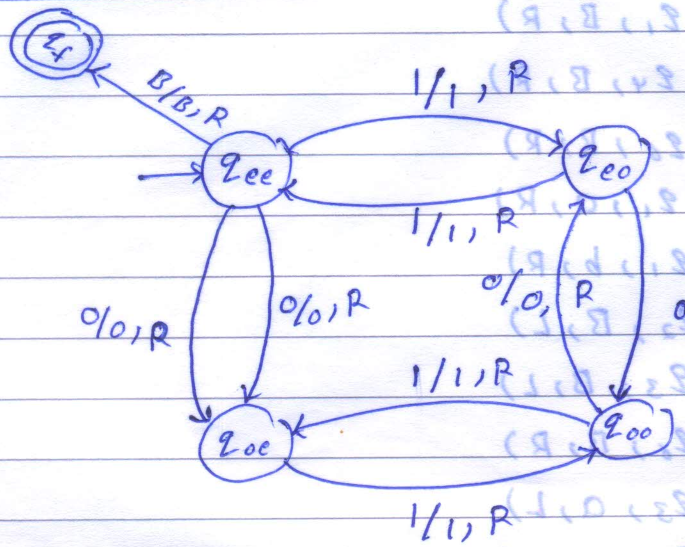
convert Following Finite automata into TM.



Finite automata.

Solution:-

convert into TM:-



Transition diagram

Turing Machine $M = (\{q_{ee}, q_{eo}, q_{oe}, q_{oo}, q_f\}, \{0,1\}, \{0,1,B\}, \delta, q_{ee}, B, q_f)$.

δ is defined :-

$\delta(q_{ee}, 0) = (q_{oe}, 0, R)$

$\delta(q_{ee}, 1) = (q_{eo}, 1, R)$

$\delta(q_{oe}, 0) = (q_{ee}, 0, R)$

$\delta(q_{oe}, 1) = (q_{oo}, 1, R)$.

$\delta(q_{eo}, 0) = (q_{oo}, 0, R)$

$\delta(q_{eo}, 1) = (q_{ee}, 1, R)$

$\delta(q_{oo}, 0) = (q_{eo}, 0, R)$

$\delta(q_{oo}, 1) = (q_{oe}, 1, R)$

$\delta(q_{ee}, B) = (q_f, B, R)$.

Q8) Construct a TM for a language having equal number of a's and b's in it over the input set $\Sigma = \{a, b\}$. (Two-way Infinite Tape)

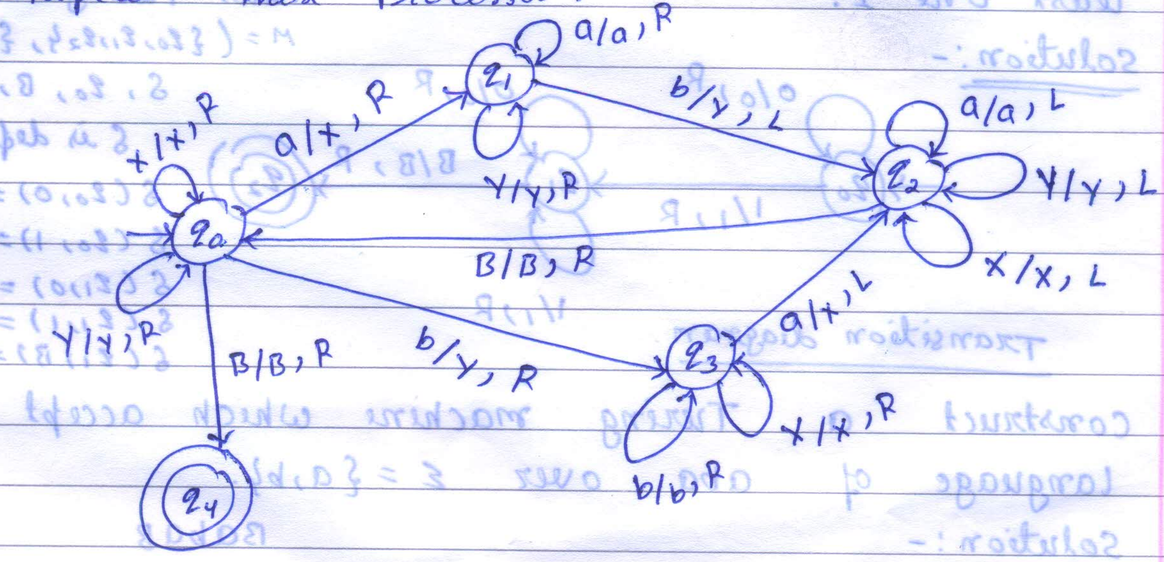
Solution: Here we will clearly take the advantage of infinite two ends of the input tape.

B b aa bbb aa B
↑

we will have a simple logic as if we get 'a'. we will replace it by X and move right in search of 'b', if we get 'b' we will replace it by Y and move left.

If we get 'b'. we will replace it by Y and move right in search of 'a', if we get 'a' we will replace it by X and move left.

Repeat these Processes.



Turing Machine M is defined :-

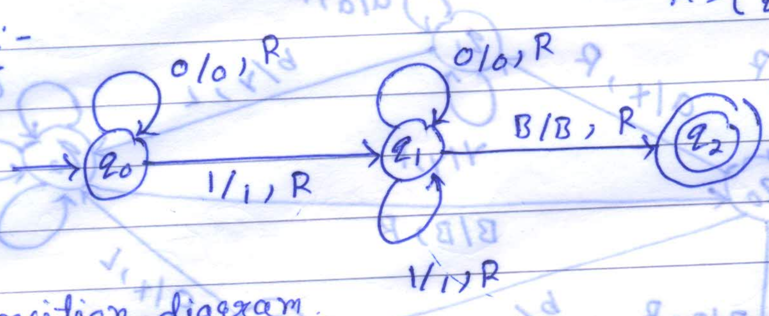
$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, X, Y, B\}, \delta, q_0, B, q_4)$.

δ is defined as

$\delta(q_0, a) = (q_1, X, R)$
$\delta(q_0, X) = (q_0, X, R)$
$\delta(q_0, b) = (q_3, Y, R)$
$\delta(q_0, Y) = (q_0, Y, R)$
$\delta(q_0, B) = (q_4, B, R)$
$\delta(q_1, a) = (q_1, a, R)$
$\delta(q_1, b) = (q_2, Y, L)$
$\delta(q_1, Y) = (q_1, Y, R)$
$\delta(q_2, a) = (q_2, a, L)$
$\delta(q_2, X) = (q_2, X, L)$
$\delta(q_2, Y) = (q_2, Y, L)$
$\delta(q_2, B) = (q_0, B, R)$
$\delta(q_3, a) = (q_2, X, L)$
$\delta(q_3, b) = (q_3, b, R)$
$\delta(q_3, X) = (q_3, X, R)$

Q(14) Design a Turing machine that recognizes the set of all strings of 0's and 1's containing at least one 1.

Solution:-



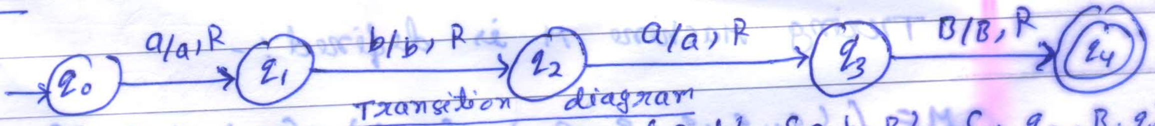
Transition diagram.

Thus Turing Machine
$M = (\{q_0, q_1, q_2\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, q_2)$
δ is defined by
$\delta(q_0, 0) = (q_0, 0, R)$
$\delta(q_0, 1) = (q_1, 1, R)$
$\delta(q_1, 0) = (q_1, 0, R)$
$\delta(q_1, 1) = (q_1, 1, R)$
$\delta(q_1, B) = (q_2, B, R)$.

Q15) Construct a Turing machine which accept the language of aba over $\Sigma = \{a, b\}$.

BabaB

Solution:-



Transition diagram

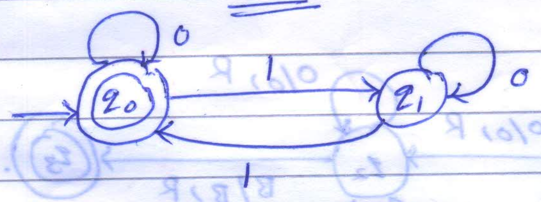
Turing Machine $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, q_4)$.

δ is defined $\delta(q_0, a) = (q_1, a, R)$
$\delta(q_1, b) = (q_2, b, R)$
$\delta(q_2, a) = (q_3, a, R)$
$\delta(q_3, B) = (q_4, B, R)$.

☆ FA and Turing Machine :-

Q16) construct TM for languages consisting of strings having any number of 0's and only even number of 1's over the input set $\Sigma = \{0,1\}$.

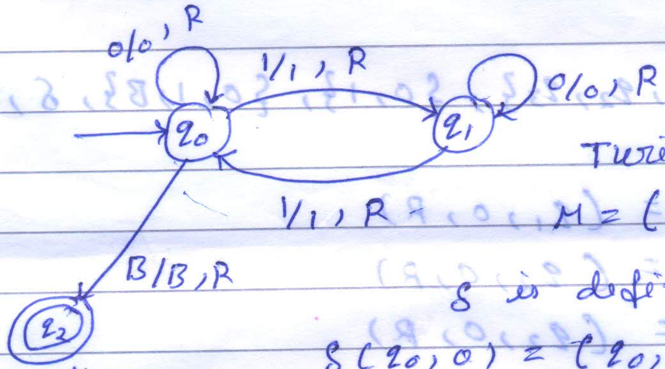
or convert following FA to TM.



Finite Automata

solution:

convert into Turing Machine:-



Transition diagram

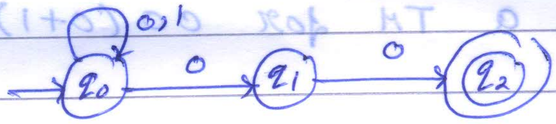
Turing Machine is defined:

$M = (\{q_0, q_1, q_2\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, q_2)$.

δ is defined as:-

$\delta(q_0, 0) = (q_0, 0, R)$	$\delta(q_1, 0) = (q_1, 0, R)$
$\delta(q_0, 1) = (q_1, 1, R)$	$\delta(q_1, 1) = (q_0, 1, R)$
$\delta(q_0, B) = (q_2, B, R)$	

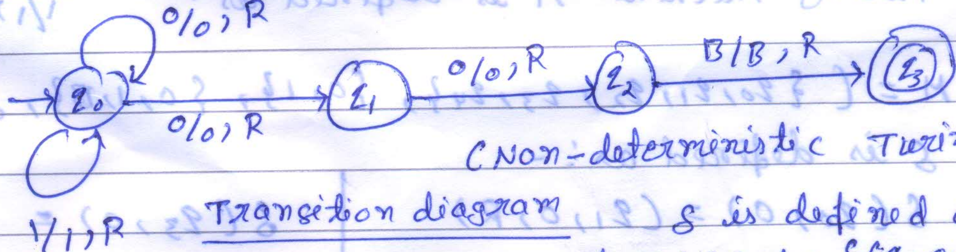
Q 17) construct a TM for the given FA.



Non-deterministic Finite Automata

solution:-

then Turing Machine:



(Non-deterministic Turing Machine

Transition diagram

So Turing Machine: ~~M =~~

$M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, q_3\}$

δ is defined as:-

$\delta(q_0, 0) = \{(q_0, 0, R), (q_1, 0, R$

$\delta(q_0, 1) = (q_0, 1, R)$

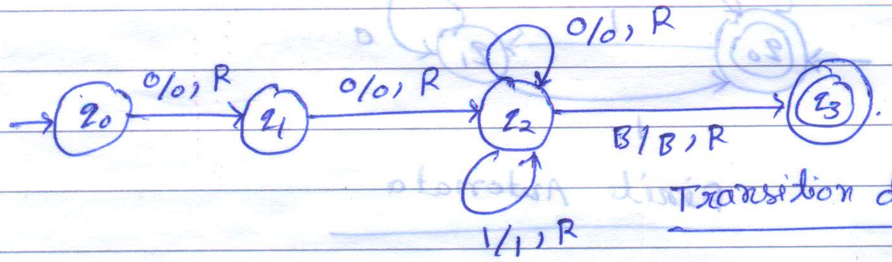
$\delta(q_1, 0) = (q_2, 0, R)$

$\delta(q_2, B) = (q_3, B, R)$.

Conversion RE to TM :-

Q1) Construct a TM for $00(0+1)^*$

Solution:-



Transition diagram

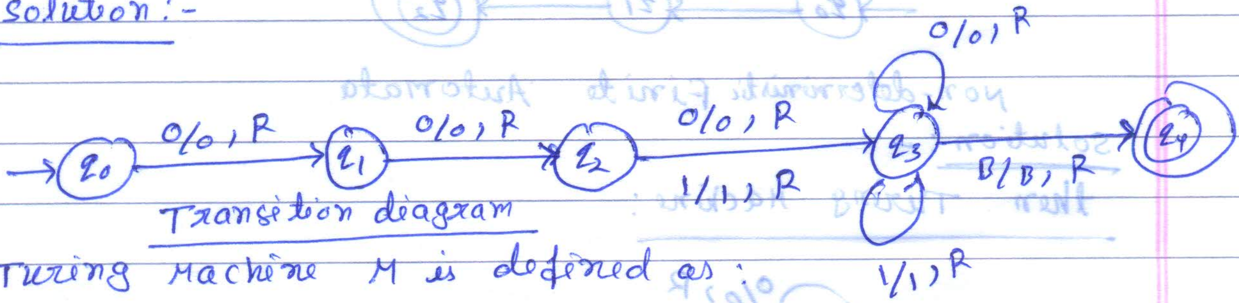
Turing Machine M is defined as:-

$$M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, q_3).$$

$\delta(q_0, 0) = (q_1, 0, R)$
$\delta(q_1, 0) = (q_2, 0, R)$
$\delta(q_2, 0) = (q_2, 0, R)$
$\delta(q_2, 1) = (q_2, 1, R)$
$\delta(q_2, B) = (q_3, B, R)$.

Q2) ~~00~~ Construct a TM for $00(0+1)^*$.

Solution:-



Transition diagram

Turing Machine M is defined as:

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, q_4).$$

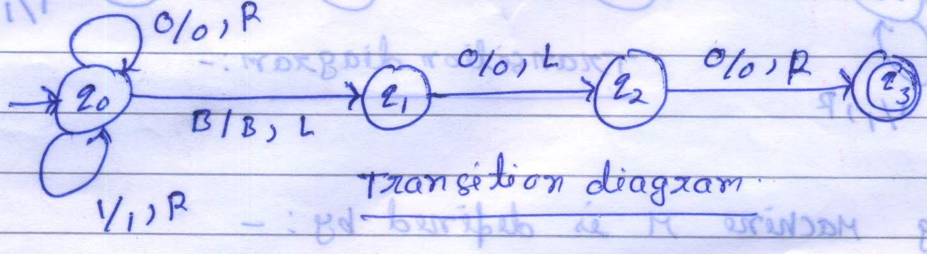
δ is defined as:-

$\delta(q_0, 0) = (q_1, 0, R)$	$\delta(q_3, 0) = (q_3, 0, R)$
$\delta(q_1, 0) = (q_2, 0, R)$	$\delta(q_3, 1) = (q_3, 1, R)$
$\delta(q_2, 0) = (q_3, 0, R)$	$\delta(q_3, B) = (q_4, B, R)$.
$\delta(q_2, 1) = (q_3, 1, R)$	

* Check from back side :-

Q1) Construct TM for $(0+1)^* 00$.

Solution:-



Transition diagram

So Turing Machine $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, q_3)$.

δ is defined :-

$\delta(q_0, 0) = (q_0, 0, R)$
$\delta(q_0, 1) = (q_0, 1, R)$
$\delta(q_0, B) = (q_1, B, L)$
$\delta(q_1, 0) = (q_2, 0, L)$
$\delta(q_2, 0) = (q_3, 0, R)$.

Procedure:

$(0+1)^* 0 0$ (Read all 0's & 1's on q_0)
$\uparrow$
q_0

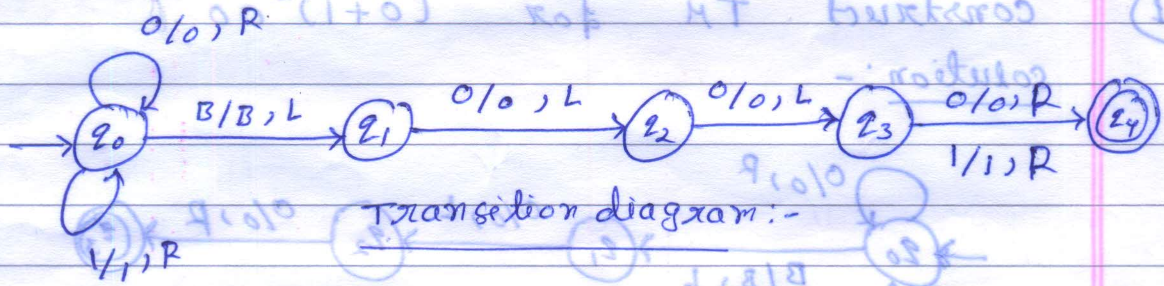
$(0+1)^* 0 0 B$ (then Back from Blank)
$\uparrow$
q_0

$(0+1)^* 0 0 B$ (Read one zero from back)
$\uparrow$
q_1

$(0+1)^* 0 0 B$ (Read second zero from back)
$\uparrow$
q_2

Q2). Construct Turing Machine for $(0+1)^{*}00$.

Solution:-



Transition diagram:-

Turing Machine M is defined by:-

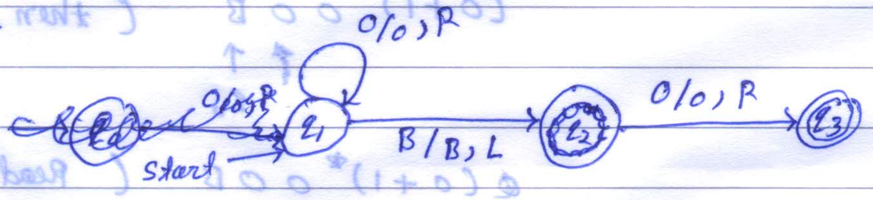
$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, q_4)$.

where δ is defined by:-

$\delta(q_0, 0) = (q_0, 0, R)$
$\delta(q_0, 1) = (q_0, 1, R)$
$\delta(q_0, B) = (q_1, B, L)$
$\delta(q_1, 0) = (q_2, 0, L)$
$\delta(q_2, 0) = (q_3, 0, L)$
$\delta(q_3, 0) = (q_4, 0, R)$
$\delta(q_3, 1) = (q_4, 1, R)$.

Q3) Construct a TM for 00^{*} where $\Sigma = \{0,1\}$.

Solution:-



Transition diagram

Turing Machine $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0,B\}, \delta, q_0, B, q_3)$.

δ is defined by:-

$\delta(q_0, 0) = (q_1, 0, R)$
$\delta(q_1, B) = (q_2, B, L)$
$\delta(q_2, 0) = (q_3, 0, R)$

Turing Machine as computable Functions :-

A Turing Machine is a language acceptor which checks whether a string x is accepted by a language L.

In addition to that it may be viewed as computer which performs computations of functions.

Q1) Design a Turing Machine that computes m+n for the given two positive integers m, n.

Solution :

let the number is unary. we know that unary number is made up of only one characters (Assume '0').

For Example 5 can be written in unary number system as '00000'.

So Assume m & n are two numbers in ~~every~~ unary notation seperated by single '1'.

$$\underbrace{00000}_{m}\ 1\ \underbrace{000}_{n}\ B$$

So construct a TM to perform addition of two unary numbers.

Design concept : (i) The seperating symbol 1 is found and replaced by 0.

(ii) The Right most 0 is found and replaced by Blank(B).

Design Procedure:-

Step 1: Initially the TM will be in its initial state q_0. It will scan & read '0' & move right until seperating symbol '1' is found.

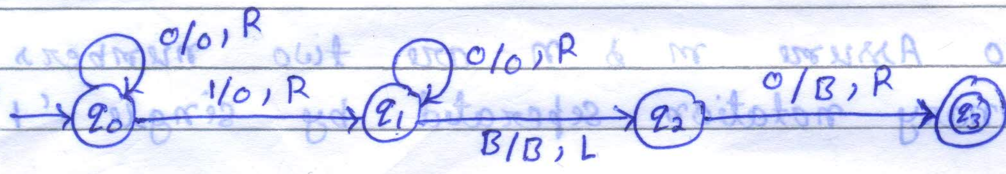
Step 2: when seperating symbol '1' is found, it should be replaced by '0'. and move right. and TM enters q_1.

Step 3: then It will scan & read 0, move right until B is found.

Step 4: when B is found, it will move left & enters q_2.

Step 5: Replace Rightmost 0 by B and enters into final state.

00000B000B



-: Transition Diagram :-

Transition Table:

State	0	1	B
$\rightarrow q_0$	$(q_0, 0, R)$	$(q_1, 0, R)$	–
q_1	$(q_1, 0, R)$	(–	(q_2, B, L)
q_2	(q_3, B, R)	–	–
q_3	–	–	–

So ~~Exansi~~ Turing Machine is defined by:-

$$M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, \{q_3\}).$$

δ is defined in Transition Table.

Q2) Design a Turing machine over $\{1, b\}$ which can compute a concatenation function over $\Sigma = \{1\}$. If a pair of words (w_1, w_2) is the input, the output has to be $w_1 w_2$.

Solution:

Let us assume that the two words w_1 and w_2 are written initially on the input tape separated by the symbol b.

for example, if $w_1 = 11$, $w_2 = 111$, then the input and output tapes.

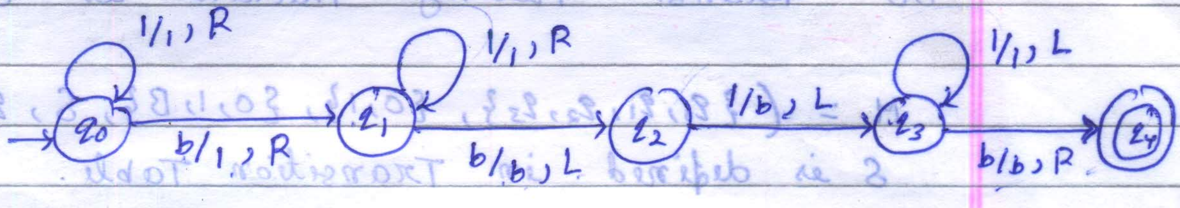
b	1	1	b	1	1	1	b
	w_1				w_2		

b	1	1	1	1	1	b	b
			$w_1 w_2$				

Input and output tapes.

we observe that the main task is to remove the symbol b. This can be done in the following manner:-

(i) The separating symbol b is found and replaced by 1.
(ii) The rightmost 1 is found and replaced by a blank b.
(iii) The R/W head returns to the starting position.



Transition diagram :- Here we want to return R/W at start.

Transition Table :

State	1	b
$\rightarrow q_0$	$(q_0, 1, R)$	$(q_1, 1, R)$
q_1	$(q_1, 1, R)$	(q_2, b, L)
q_2	(q_3, b, L)	-
q_3	$(q_3, 1, L)$	(q_4, b, R)
q_4	-	-

So Turing Machine defined by :-

$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{1, b\}, \{1, b\}, \delta, q_0, b, \{q_4\})$.

Q3) Proper substraction $m-n$ is defined to be $m-n$ for $m \geq n$, and zero for $m < n$.

Solution : started with $0^m 1 0^n$ on its tape, ~~halts~~ halts with 0^{m-n} on its tape.

Design concept :

M repeatedly replaces its leading 0 by blank, then searches right for a 1 followed by a 0 and changes the 0 to 1.

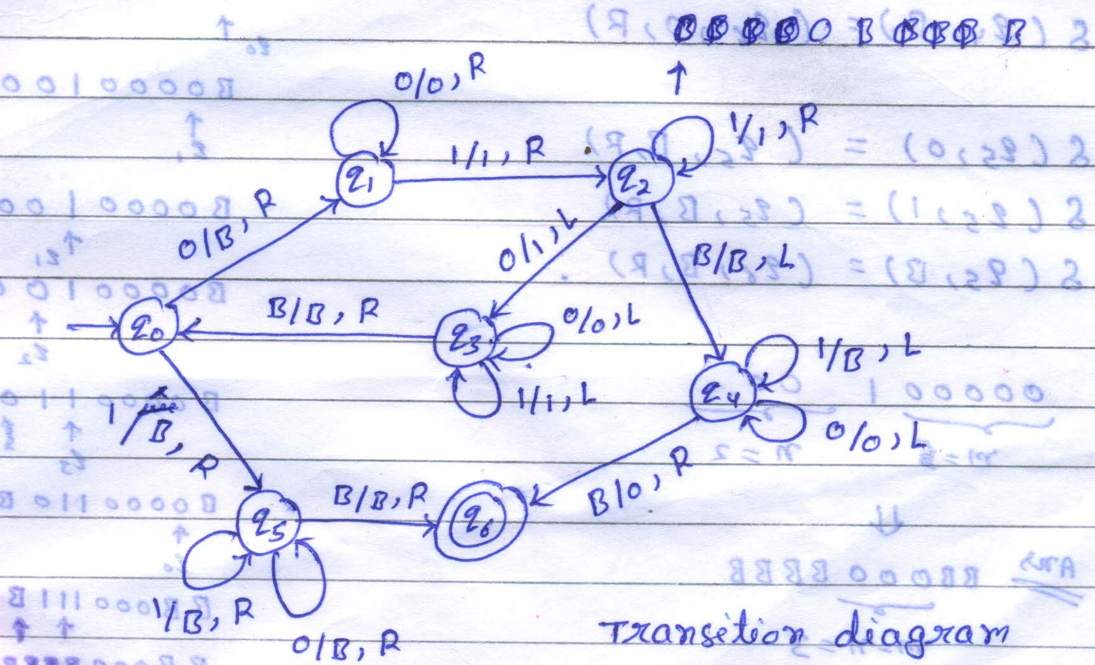
Next, M moves left until it encounters a blank and then repeats the cycle.

The repetition ends if.

(i) Searching right for a 0, M encounters a blank. Then, the n 0's in $0^m 1 0^n$ have all been changed to 1's and n+1 of the m 0's ~~have~~ have been changed to blank.

So when M encounters a blank. we will move left & replaces all 1 to Blank until Blank is found. then replace one blank to 0 and enter into final state.

(ii) Begining the cycle, M cannot find a '0' to change to a blank, because the first m 0's already have been changed. Then $n \geq m$, so $m-n = 0$. M replaces all remaining 1's and 0's by Blank.



Transition diagram

So M is defined by

$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, \{q_6\})$

δ is defined by :-

$\delta(q_0, 0) = (q_1, B, R)$
$\delta(q_0, 1) = (q_5, B, R)$

$\delta(q_1, 0) = (q_1, 0, R)$
$\delta(q_1, 1) = (q_2, 1, R)$

$\delta(q_2, 0) = (q_3, 1, L)$
$\delta(q_2, 1) = (q_2, 1, R)$
$\delta(q_2, B) = (q_4, B, L)$

$\delta(q_3, 0) = (q_3, 0, L)$
$\delta(q_3, 1) = (q_3, 1, L)$
$\delta(q_3, B) = (q_0, B, R)$

$\delta(q_4, 0) = (q_4, 0, L)$
$\delta(q_4, 1) = (q_4, B, L)$
$\delta(q_4, B) = (q_6, 0, R)$

$\delta(q_5, 0) = (q_5, B, R)$
$\delta(q_5, 1) = (q_5, B, R)$
$\delta(q_5, B) = (q_6, B, R)$.

B 00001 00 B
B BB 00 111 B
B B B 0 0 B B B B

$\underbrace{00000}_{m=5} \; 1 \; \underbrace{00}_{n=2}$

$\Downarrow$

Ans BB $\underbrace{000}_{m-n=3}$ BBBB

<u>Procedure</u> :-

00000 1 00 B
$\uparrow q_0$

B 0000 1 00 B
$\uparrow q_1$

B 0000 1 00 B
$\uparrow q_1$

B 0000 1 00 B
$\uparrow q_2$

B 0000 11 0 B
$\uparrow q_3$

B 0000 110 B
$\uparrow q_0$

BB 000 111 B

B B 000 111 B

BBB 00 111 B

BBB 00 BBB B

Q4) Construct TM for the function $f(x) = x+3$. if $x \geq 0$.

Solution:

Since the TM has to be constructed for unary number system. we Assume input set $\Sigma = \{1\}$. The logic is very simple. we just go to moving towards rightmost end of input string. Then replace Three Blank's to 1's. and enter into final state.

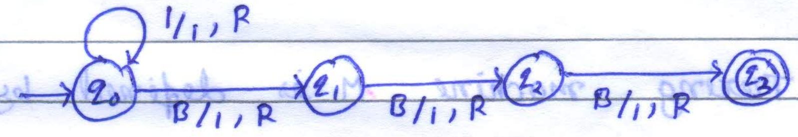
If ~~x~~ $\underbrace{11111}_{x=5}$ BBBB then ~~f(x)~~ $\underbrace{11111111}_{f(x)=8}$ B.

Procedure:

11111 BBBB
↑
q_0

11111 B BBB
↑

11111111 B
↑



Transition diagram.

Then Turing machine M is defined by.

$M = (\{q_0, q_1, q_2, q_3\}, \{1\}, \{1, B\}, \delta, q_0, B, q_3)$

where δ is defined by :-

$\delta(q_0, 1) = (q_0, 1, R)$

$\delta(q_0, B) = (q_1, 1, R)$

$\delta(q_1, B) = (q_2, 1, R)$

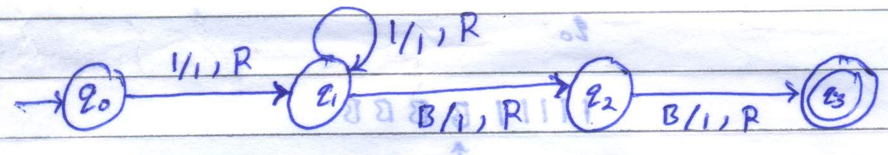
$\delta(q_2, B) = (q_3, 1, R)$.

Q5) construct Turing machine for the function $f(n) = n+2$ if $n > 0$.

Solution :- since the Turing machine has to be constructed for unary number system. we Assume input set $\Sigma = \{1\}$.

The logic is very much ~~sip~~ simple. we just going to moving towards rightmost end of input strings. Then replace two blank's to 1's and enter into final state.

If $\underbrace{1111}_{n=4}BBB$ then $\underbrace{111111}_{f(n)=6}B$



Transition diagram

Then Turing machine M is defined by.

$M = (\{q_0, q_1, q_2, q_3\}, \{1\}, \{1, B\}, \delta, q_0, B, q_3)$.

where δ is defined by

$\delta(q_0, 1) = (q_1, 1, R)$.

$\delta(q_1, 1) = (q_1, 1, R)$

$\delta(q_1, B) = (q_2, 1, R)$

$\delta(q_2, B) = (q_3, 1, R)$.

Q6) construct TM for the function:-

$$f(m) = \begin{cases} m-2 & , \text{if } m > 2. \\ 1 & , \text{if } m \leq 2 \end{cases}$$

Solution: since the Turing machine has to be constructed for unary number system. we Assume input set $\Sigma = \{1\}$.

The logic is very much simple.

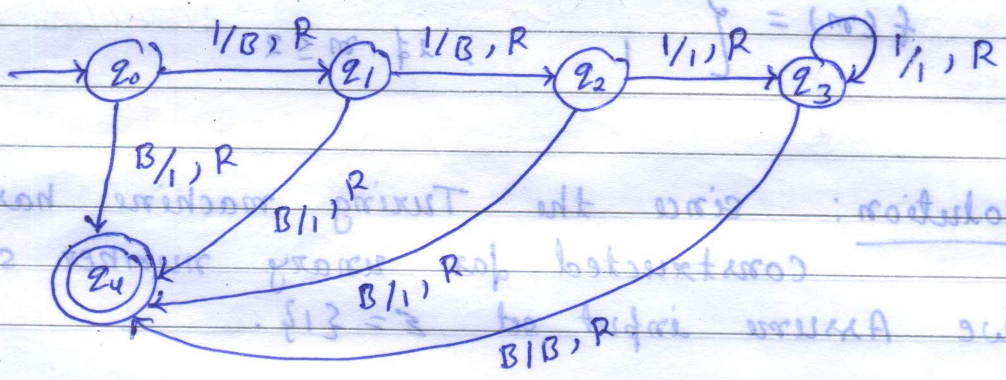
(i) when $m=0$, then input is BBB.
(ii) when $m=1$, then input is B1B.
(iii) when $m=2$, then input is B11B.
(iv) when $m=3$, then input is B111B.
(v) when $m=4$, then input is B1111B.

(1) when $m \leq 2$.

(i) If read Blank at start then replace Blank to '1' & Enter into final state.
(ii) Replace the first '1' to Blank. and scan Blank at second position then replace Blank to 1. & Enter into final state.
(iii) Replace the first & second '1' to Blank. and scan Blank at Third position then replace Blank to '1' & enter into final state.

(2) when $m > 2$.
Replace first & second '1' to Blank. then scan the 1's & move towards right until the Blank is found. then TM enters into final state.

so



Transition diagram:-

so Turing machine is defined by :-

$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{1\}, \{1, B\}, \delta, q_0, B, q_4)$

δ is defined by :-

$\delta(q_0, 1) = (q_1, B, R)$

$\delta(q_0, B) = (q_4, 1, R)$

$\delta(q_1, 1) = (q_2, B, R)$

$\delta(q_1, B) = (q_4, 1, R)$

$\delta(q_2, 1) = (q_3, 1, R)$

$\delta(q_2, B) = (q_4, 1, R)$

$\delta(q_3, 1) = (q_3, 1, R)$

$\delta(q_3, B) = (q_4, B, R)$

Q7) Construct TM for copying the input unary string on the tape.

Solution: The Turing machine is able to Perform copy operation also. In this TM will read each Input symbol and mark it as x and copy it at the right end after Blank.
Finally after copying the complete string we will move left and reconvert 'x' to 1 again.

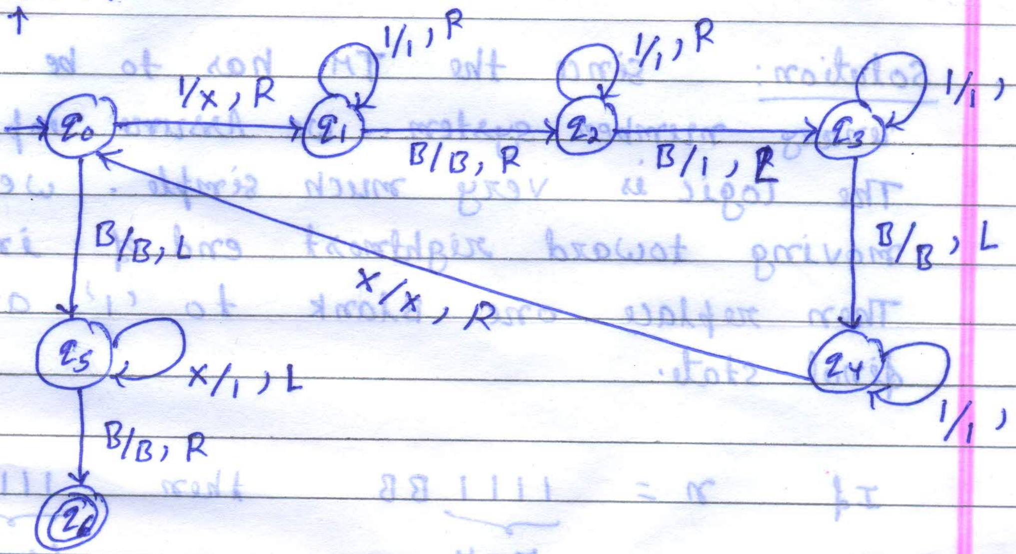
The TM will be as follows.

B 1111 B $\Rightarrow$ B X X X X B 1 1 1 1 B

after copying :-

$B\underbrace{1111}_{\text{original}}B\underbrace{1111}_{\text{copy}}B.$

B XXXX B 1111 BB



— Transition diagram :-

So, the Turing Machine M is defined by :-
$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{1\}, \{1, x, B\}, \delta, q_0, B, q_6).$

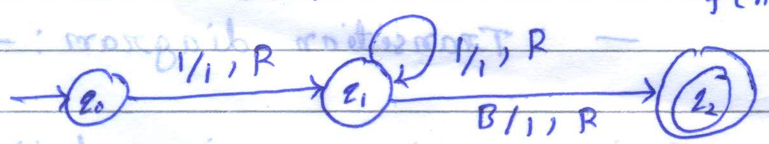
δ is defined as :-

$\delta(q_0, 1) = (q_1, X, R)$
$\delta(q_0, B) = (q_5, B, L)$
$\delta(q_1, 1) = (q_1, 1, R)$
$\delta(q_1, B) = (q_2, B, R)$
$\delta(q_2, 1) = (q_2, 1, R)$
$\delta(q_2, B) = (q_3, 1, L)$
$\delta(q_3, 1) = (q_3, 1, L)$
$\delta(q_3, B) = (q_4, B, L)$
$\delta(q_4, 1) = (q_4, 1, L)$
$\delta(q_4, X) = (q_0, X, R)$
$\delta(q_5, X) = (q_5, 1, L)$
$\delta(q_5, B) = (q_6, B, L)$

Q8) Construct a TM for a successor function for a given unary number i.e. $f(n) = n+1$, $n > 0$.

Solution: Since the TM has to be constructed for unary number system. We Assume input set $\Sigma = \{1\}$. The logic is very much simple. We just go to moving toward rightmost end of input string. Then replace one Blank to '1' and enter into final state.

If $n = \underbrace{1111}_{n=4}BB$ then $\underbrace{11111}_{f(n)=5}B$.



1111BB
↑

Turing Machine $M = (\{q_0, q_1, q_2\}, \{1\}, \{1, B\}, \delta, q_0, B, q_2)$
δ is defined $\delta(q_0, 1) = (q_1, 1, R)$, $\delta(q_1, 1) = (q_1, 1, R)$
$\delta(q_1, B) = (q_2, 1, R)$.

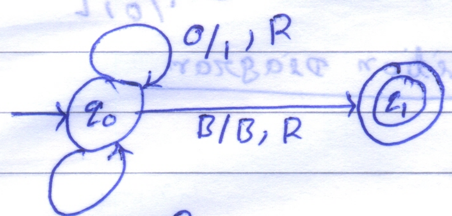
Q9) Construct TM for obtaining one's complement of a given binary number.

OR

Proove that function $f(w) = \bar{w}$ is computable where $w \in \{0,1\}^*$ and $\bar{w}$ the one's complement of w.

Solution: To make the 1's complement while the head is reading the input symbol from left to right, the 0's are replaced by 1's and 1's are replaced by 0's.

If the string is 10110 B then 1's complement is 01001 B.



The Turing Machine is :-

$M = (\{q_0, q_1\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, q_1)$.
where δ is defined by :-

$\delta(q_0, 0) = (q_0, 1, R)$

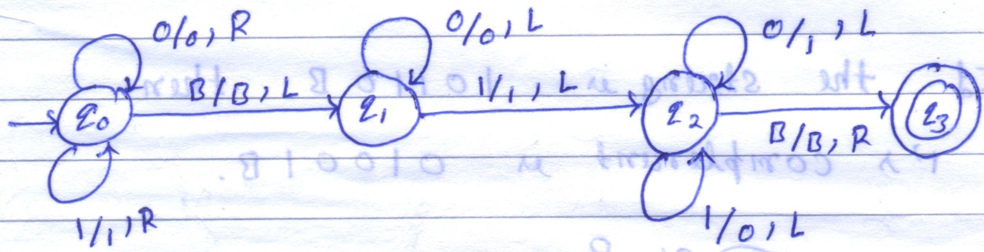
$\delta(q_0, 1) = (q_0, 0, R)$

$\delta(q_0, B) = (q_1, B, R)$

Example of Two-way Infinite Tape:-

Q10) construct TM for obtaining two's complement of a given binary number. (using Two way infinite Tape)

Solution: The logic for computing two's complement is, we read the binary string from LSB to MSB. From LSB, we keep all the zero's as it is. and move left till we do not get 1. After reading first 1 from LSB we move left and thereafter we convert 0 to 1 and 1 to 0. and go on moving towards left. The process continues upto leftmost Blank.



Transition Diagram

Example:-

B 100111000 B is a string.

After two's compliment B 011001000 B

Turing Machine M is defined as (Two-way Infinite Tape)

$M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, q_3)$

δ is defined by :-

$\delta(q_0, 0) = (q_0, 0, R)$

$\delta(q_0, 1) = (q_0, 1, R)$

$\delta(q_0, B) = (q_1, B, L)$

$\delta(q_1, 0) = (q_1, 0, L)$

$\delta(q_1, 1) = (q_2, 1, L)$

$\delta(q_2, 0) = (q_2, 1, L)$

$\delta(q_2, 1) = (q_2, 0, L)$

$\delta(q_2, B) = (q_3, B, R)$

Q11) construct a Turing Machine for the subroutine $f(a,b) = a * b$ where a and b are unary number.

or

construct a Turing machine for multiplication of two number.

<u>solution:</u> Here we represent unary number by 1's. so input $\Sigma = \{1\}$.

let two number is a & b stored into Tape seperated by 0. (Two-way Infinite Tape).

If $a = 4$ and $b = 3$ so $a * b = 12$.

$B\ \underbrace{1111}_{a=4}\ 0\ \underbrace{111}_{b=3}\ B$ then $B\ \underbrace{111111111111}_{a*b=12}\ B$

B 1111 0 111 B
↑

(i) Make first one to X and moving to right upto seperator 0.

B X 1 1 1 0 1 1 1 B
↑

(ii) Move Right after seperator 0 and Replace 1 to Y and copy this 1 after Blank.

B X 1 1 1 0 Y 1 1 B 1
↑

(iii) So Mark all 1's to Y after seperator 0, and copy these 1's after Blank.

B X 1 1 1 0 Y Y Y B 1 1 1 B

(iv) when copy of all 1's after seperator 0 is done then move left and reconverted all Y's to 1's. and Move left until X is found. then Move right.

B X 1 1 1 0 1 1 1 B 1 1 1 B

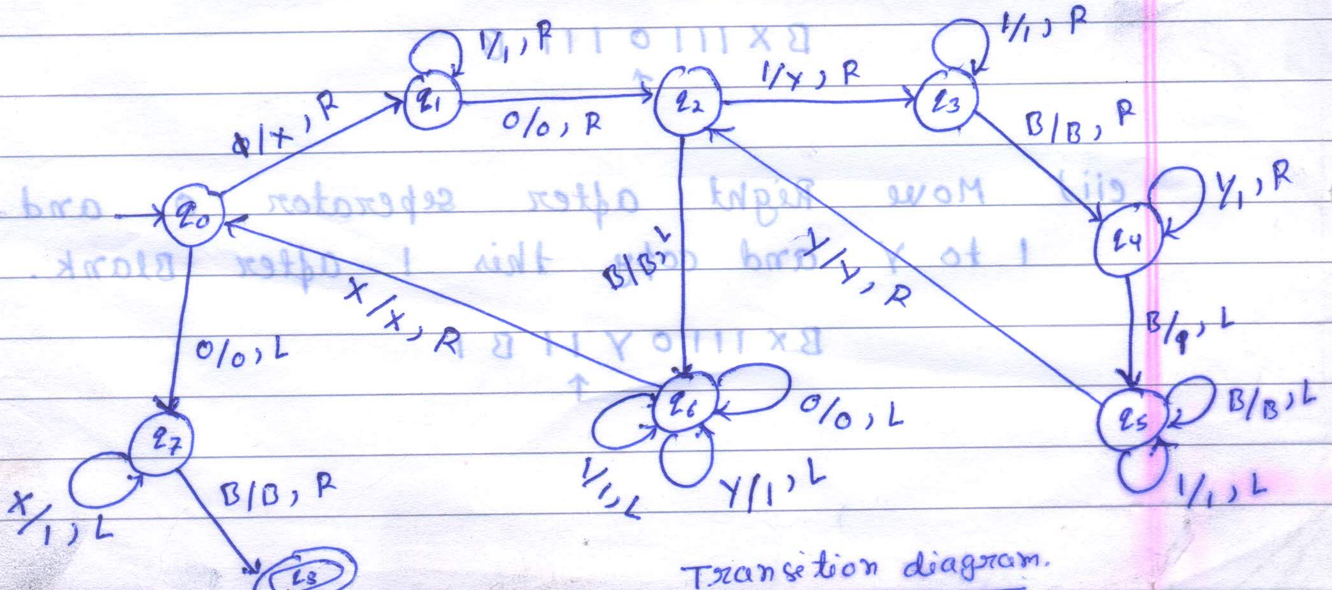
(v) Repeat the above steps until all 1's replaced by X before seperator 0.

B X X X X 0 1 1 1 B 1 1 1 1 1 1 1 1 1 1 1 1 B

(vi) Now we will move left changes all X's to 1's. and when Blank is found. Enter into final state.

$$B\ \underbrace{1111}_{a=4}\ 0\ \underbrace{111}_{b=3}\ B\ \underbrace{111111111111}_{a*b=12}\ B$$

B X X X X 0 Y Y Y B 1 1 1 1

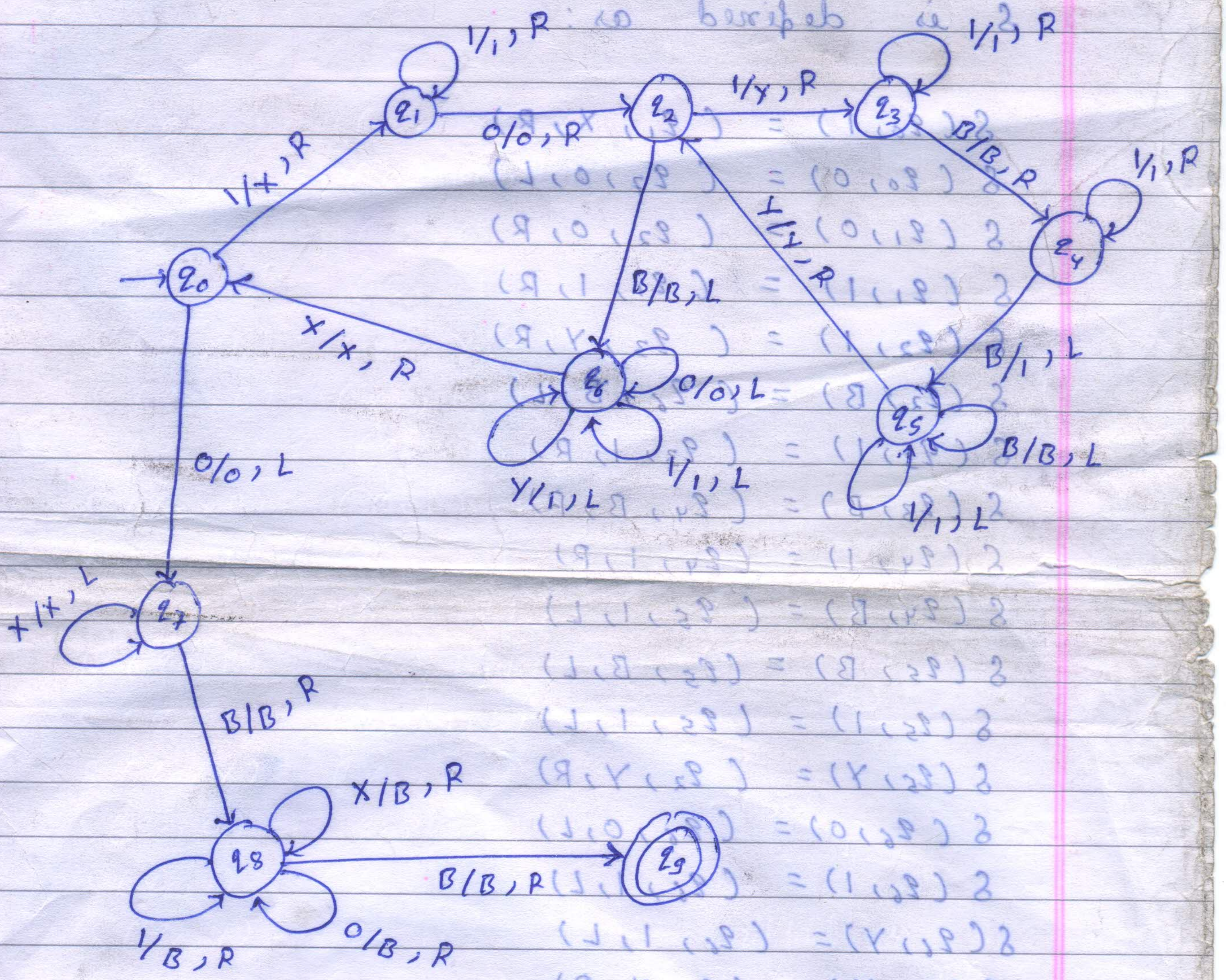


Transition diagram.

Q 12) Construct a Turing Machine which perform multiplication.

$1^m 0 1^n \rightarrow 1^{mn}$ (In this form)

Solution :-



B X X X X 0 1 1 1 B 1 1 1 1 1 1 1 1 1 1 1 1 B
↑

Now we will move left until Blank is found.

B X X X X 0 1 1 1 B 1 1 1 1 1 1 1 1 1 1 1 1 B
↑

Move right & Replace all X, 0 & 1 to Blank until next Blank is found.

When next Blank is found. enter into final state

B 1 1 1 1 1 1 1 1 1 1 1 1 B

Turing Machine M is defined by

$M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8\}, \{0, 1\}, \{0, 1, B\},$

$\delta, q_0, B, q_8)$.

δ is defined as:

$\delta(q_0, 1) = (q_1, X, R)$

$\delta(q_0, 0) = (q_7, 0, L)$

$\delta(q_1, 0) = (q_2, 0, R)$

$\delta(q_1, 1) = (q_1, 1, R)$

$\delta(q_2, 1) = (q_3, Y, R)$

$\delta(q_2, B) = (q_6, B, L)$

$\delta(q_3, 1) = (q_3, 1, R)$

$\delta(q_3, B) = (q_4, B, R)$

$\delta(q_4, 1) = (q_4, 1, R)$

$\delta(q_4, B) = (q_5, 1, L)$

$\delta(q_5, B) = (q_5, B, L)$

$\delta(q_5, 1) = (q_5, 1, L)$

$\delta(q_5, Y) = (q_2, Y, R)$

$\delta(q_6, 0) = (q_6, 0, L)$

$\delta(q_6, 1) = (q_6, 1, L)$

$\delta(q_6, Y) = (q_6, 1, L)$

$\delta(q_6, X) = (q_0, X, R)$

$\delta(q_7, X) = (q_7, 1, L)$

$\delta(q_7, B) = (q_8, B, R)$.

Input is $1^m 0 1^n$ then output is $1^m 0 1^n B 1^{mn}$