Convert context free Grammar into PDA.

If L is a context free Language, then we construct a Pda A accepting L by empty store $L = N(A)$.

let $L = L(G)$, where $G = (V_N, \Sigma, P, S)$ is a CFG.
we construct a Pda A as:-

$$N(A) = (\{q\}, \Sigma, V_N \cup \Sigma, \delta, q, S, \phi)$$

where δ is defined by the following rules:-

R_1: $\delta(q, \Lambda, A) = \{(q, \alpha)\} \mid A \to \alpha$ is in P.

R_2: $\delta(q, a, a) = \{(q, \Lambda)\}$ for every a in Σ.

Q1) Construct a Pda A equivalent to the following CFG

$$S \to 0BB \;,\; B \to 0S/1S/0$$

Test whether 010^4 is in $N(A)$.

Solution: Define Pda A as follows:-

$$A = (\{q\}, \{0,1\}, \{S, B, 0, 1\}, \delta, q, S, \phi)$$

δ is defined by the following rules:-

R_1: $\delta(q, \Lambda, S) = \{(q, 0BB)\}$

R_2: $\delta(q, \Lambda, B) = \{(q, 0S), (q, 1S), (q, 0)\}$

R_3: $\delta(q, 0, 0) = \{(q, \Lambda)\}$

R_4: $\delta(q, 1, 1) = \{(q, \Lambda)\}$

Test whether 010^4 is in $N(A)$.

$(q, 010^4, S)$ by Rule R_1

$\vdash (q, 010^4, 0BB)$ by Rule R_3

$\vdash (q, 10^4, BB)$ by Rule R_2

$\vdash (q, 10^4, 1SB)$ by Rule R_4

$\vdash (q, 0^4, SB)$ by Rule R_1

$\vdash (q, 0^4, 0BBB)$ by Rule R_3

$\vdash (q, 0^3, BBB)$ by Rule R_2

$\vdash (q, 0^3, 0BB)$ by Rule R_3

$\vdash (q, 0^2, BB)$ by Rule R_2

$\vdash (q, 0^2, 0B)$ by Rule R_3

$\vdash (q, 0, B)$ by Rule R_2

$\vdash (q, 0, 0)$ by Rule R_3

$\vdash (q, \Lambda, \Lambda)$ Accept by Null store.

Q2) Convert the following CFG into PDA:-

$S \rightarrow aB / bA$

$A \rightarrow a / aS / bAA$

$B \rightarrow b / bS / aBB$

Also parse the string baab.

Solution:- The required Pda A can be defined as follows:-

$A = (\{q\}, \{a, b\}, \{S, A, B, a, b\}, \delta, q, S, \phi)$

where δ is defined by the following rules:-

$R_1 : \delta(q, \Lambda, S) = \{(q, aB), (q, bA)\}$

$R_2 : \delta(q, \Lambda, A) = \{(q, a), (q, aS), (q, bAA)\}$

$R_3 : \delta(q, \Lambda, B) = \{(q, b), (q, bS), (q, aBB)\}$

$R_4: \delta(q, a, a) = (q, \Lambda)$
$R_5: \delta(q, b, b) = (q, \Lambda)$
~~Res~~
Parse the string baab.

$(q, baab, S)$ by Rule R_1
$\vdash (q, baab, bA)$ by Rule R_5
$\vdash (q, aab, A)$ by Rule R_2
$\vdash (q, aab, aS)$ by Rule R_4
$\vdash (q, ab, S)$ by Rule R_1
$\vdash (q, ab, aB)$ by Rule R_4
$\vdash (q, b, B)$ by Rule R_3
$\vdash (q, b, b)$ by Rule R_5
$\vdash (q, \Lambda, \Lambda)$ accept by Null store.

Q3) Construct the Pda equivalent to the following grammar

$S \to aAA$
$A \to aS / bS / a$

Also Parse the string aabaaa

<u>Solution</u>: The Pda A can be defined as follows:-
$A = (\{q\}, \{a, b\}, \{S, A, a, b\}, \delta, q, S, \phi)$
where δ is defined by the following rules:-

$R_1: \delta(q, \Lambda, S) = (q, aAA)$
$R_2: \delta(q, \Lambda, A) = \{(q, aS), (q, bS), (q, a)\}$
$R_3: \delta(q, a, a) = \{(q, \Lambda)\}$
$R_4: \delta(q, b, b) = \{(q, \Lambda)\}$

$(q, aabaaa, S) \vdash (q, aabaaa, aAA) \vdash (q, abaaa, AA)$
$\vdash (q, abaaa, aA) \vdash (q, baaa, A) \vdash (q, baaa, bS)$
$\vdash (q, aaa, S) \vdash (q, aaa, aAA) \vdash (q, aa, AA)$
$\vdash (q, aa, aA) \vdash (q, a, A) \vdash (q, a, a) \vdash (q, \Lambda, \Lambda)$
accept by Null store.

Q4) Construct an NPDA corresponding to the grammar

$S \to aABB / aAA$

$A \to aBB / a$

$B \to bBB / A$

Solution:-

The required NPDA A can be defined as follows:-

$A = (\{q\}, \{a,b\}, \{S,A,B,a,b\}, \delta, q, S, \phi)$

where δ is defined by the following rules:-

$R_1: \delta(q, \Lambda, S) = \{(q, aABB), (q, aAA)\}$

$R_2: \delta(q, \Lambda, A) = \{(q, aBB), (q, a)\}$

$R_3: \delta(q, \Lambda, B) = \{(q, bBB), (q, A)\}$

$R_4: \delta(q, a, a) = \{(q, \Lambda)\}$

$R_5: \delta(q, b, b) = \{(q, \Lambda)\}$

Q5) Construct Pda for the given grammar:-

$S \to aA$

$A \to aABC / bB / a$

$B \to b$

$C \to c$

Solution:-

$L = (\{q\}, \{a,b,c\}, \{a,b,c,S,A,B,C\}, \delta, q, S, \phi)$.

δ is defined as:

$R_1: \delta(q, \Lambda, S) = (q, aA)$

$R_2: \delta(q, \Lambda, A) = \{(q, aABC), (q, bB), (q, a)\}$

$R_3: \delta(q, \Lambda, B) = \{(q, b)\}$

$R_4: \delta(q, \Lambda, C) = \{(q, c)\}$

$R_5: \delta(q, a, a) = (q, \Lambda)$

$R_6: \delta(q, b, b) = (q, \Lambda)$

$R_7: \delta(q, c, c) = (q, \Lambda)$

Convert Push down Automata into CFG :-

If $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ is a Pda, then there exists a context free grammar G such that $L(G) = N(A)$.

> construction of G :- we define $G = (V_N, \Sigma, P, S)$,
where $V_N = \{S\} \cup \{[q, Z, q']\} / q, q' \in Q, Z \in \Gamma$

The productions in P are induced by moves of Pda as follows :-

R1 : S productions are given by $S \to [q_0, Z_0, q]$ for every q in Q.

R2 : Each move erasing a pushdown symbol given by

$$\delta(q, a, Z) = (q', \Lambda)$$

$$[q, Z, q'] \to a$$

R3 :- Each move not erasing a pushdown symbol given by

$$\delta(q, a, Z) = (q_1, Z_1 Z_2 \cdots Z_m)$$

$$[q, Z, q'] \to a[q_1, Z_1, q_2][q_2, Z_2, q_3] \cdots [q_m, Z_m, q']$$

where each of the states $q', q_2, \cdots q_m$ can be any state in Q.

Note:- no. of production rules for each transition rule $= (m)^n$
where m = no. of state in Q.
n = no. of stack variable in respective transition rule.

Q1) Construct a PDA accepting the set of all strings over $\{a,b\}$, having equal number of a's & b's.

$$N(A) = (\{q\}, \{a,b\}, \{z_0, a, b\}, \delta, q_0, z_0, \phi)$$

		No. of Rules (m^n)
①	$\delta(q, a, z_0) = (q, az_0)$	$(1)^2 = 1$
②	$\delta(q, b, z_0) = (q, bz_0)$	$(1)^2 = 1$
③	$\delta(q, a, a) = (q, aa)$	$(1)^2 = 1$
④	$\delta(q, b, a) = (q, \Lambda)$	$(1)^0 = 1$
⑤	$\delta(q, a, b) = (q, \Lambda)$	$(1)^0 = 1$
⑥	$\delta(q, b, b) = (q, bb)$	$(1)^2 = 1$
⑦	$\delta(q, \Lambda, z_0) = (q, \Lambda)$	$(1)^0 = 1$

Solution:

we define $G = (V_N, \Sigma, P, S)$

where set of variable V_N:-

$$V_N = \{S\} \cup \{q, z, q'\} \,/\, q, q' \in Q, z \in \Gamma$$

so $V_N = \{S, [q, z_0, q], [q, a, q], [q, b, q]\}$

» Set of Production Rules:-

$R_1 : S \rightarrow [q_0, z_0, q]$ for every q in Q.

$P_1 : S \rightarrow [q, z_0, q]$

★ $\delta(q, a, z_0) = (q, az_0)$ gives :-

$P_2 : [q, z_0, q] \longrightarrow a\,[q, a, q]\,[q, z_0, q]$

★ $\delta(q, b, z_0) = (q, bz_0)$ gives :-

$P_3 : [q, z_0, q] \longrightarrow b\,[q, b, q]\,[q, z_0, q]$

★ $\delta(q, a, a) = (q, aa)$ gives.

$P_4 : [q, a, q] \longrightarrow a\,[q, a, q]\,[q, a, q]$

★ $\delta(q, b, a) = (q, \wedge)$ gives

$P_5 : [q, a, q] \longrightarrow b$

★ $\delta(q, a, b) = (q, \wedge)$ gives

$P_6 : [q, b, q] \longrightarrow a$

★ $\delta(q, b, b) = (q, bb)$ gives

$P_7 : [q, b, q] \longrightarrow b\,[q, b, q]\,[q, b, q]$

★ $\delta(q, \wedge, z_0) = (q, \wedge)$ gives

$P_8 : [q, z_0, q] \longrightarrow \wedge$

Assume $[q, z_0, q]$, $[q, a, q]$, $[q, b, q]$ is A, B, C respectively.

$P_1 :\ S \rightarrow A$
$P_2 :\ A \rightarrow aBA$
$P_3 :\ A \rightarrow bCA$
$P_4 :\ B \rightarrow aBB$
$P_5 :\ B \rightarrow b$
$P_6 :\ C \rightarrow a$
$P_7 :\ C \rightarrow bCC$
$P_8 :\ A \rightarrow \Lambda$

So the context free grammar is :-

$G = (\{S, A, B, C\}, \{a, b\}, P, S)$

P is defined as :

$S \rightarrow A$

$A \rightarrow aBA / bCA / \Lambda$

$B \rightarrow aBB / b$

$C \rightarrow a / bCC$.

Q2) Construct a CFG G which accepts N(A), where $A = (\{q_0, q_1\}, \{a, b\}, \{z_0, z\}, \delta, q_0, z_0, \phi)$. δ is given by :-

	No. of rules :
$\delta(q_0, b, z_0) = (q_0, zz_0)$	$(2)^2 = 4$
$\delta(q_0, \Lambda, z_0) = (q_0, \Lambda)$	$(2)^0 = 1$
$\delta(q_0, b, z) = (q_0, zz)$	$(2)^2 = 4$
$\delta(q_0, a, z) = (q_1, z)$	$(2)^1 = 2$
$\delta(q_1, b, z) = (q_1, \Lambda)$	$(2)^0 = 1$
$\delta(q_1, a, z_0) = (q_0, z_0)$	$(2)^1 = 2$

<u>Solution:</u> let $G = (V_N, \{a,b\}, P, S)$.

where set of variable V_N:-

$V_N = \{S\} \cup \{q, z, q'\} \;/\; q, q' \in Q \;,\; z \in \Gamma$

So $V_N = \{S, [q_0, z_0, q_0], [q_0, z_0, q_1], [q_1, z_0, q_0], [q_1, z_0, q_1], [q_0, z, q_0], [q_0, z, q_1], [q_1, z, q_0], [q_1, z, q_1]\}$

The Productions are:-

P_1 :- $S \longrightarrow [q_0, z_0, q_0]$

P_2 : $S \longrightarrow [q_0, z_0, q_1]$

* $\delta(q_0, b, z_0) = (q_0, zz_0)$ gives

P_3 : $[q_0, z_0, q_0] \longrightarrow b\,[q_0, z, q_0]\,[q_0, z_0, q_0]$

P_4 : $[q_0, z_0, q_0] \longrightarrow b\,[q_0, z, q_1]\,[q_1, z_0, q_0]$

P_5 : $[q_0, z_0, q_1] \longrightarrow b\,[q_0, z, q_0]\,[q_0, z_0, q_1]$

P_6 : $[q_0, z_0, q_1] \longrightarrow b\,[q_0, z, q_1]\,[q_1, z_0, q_1]$

* $\delta(q_0, \Lambda, z_0) = (q_0, \Lambda)$ gives:-

P_7 : $[q_0, q_0, q_0] \longrightarrow \Lambda$

* $\delta(q_0, b, z) = (q_0, zz)$ gives:-

P_8 : $[q_0, z, q_0] \longrightarrow b\,[q_0, z, q_0]\,[q_0, z, q_0]$

P_9 : $[q_0, z, q_0] \longrightarrow b\,[q_0, z, q_1]\,[q_1, z, q_0]$

P_{10} : $[q_0, z, q_1] \longrightarrow b\,[q_0, z, q_0]\,[q_0, z, q_1]$

P_{11} : $[q_0, z, q_1] \longrightarrow b\,[q_0, z, q_1]\,[q_1, z, q_1]$

☆ $\delta(q_0, a, Z) = (q_1, Z)$ gives:-

P_{12} : $[q_0, Z, q_0] \to a\,[q_1, Z, q_0]$

P_{13} : $[q_0, Z, q_1] \to a\,[q_1, Z, q_1]$

☆ $\delta(q_1, b, Z) = (q_1, \Lambda)$ gives:-

P_{14} : $[q_1, Z, q_1] \to b\,[\quad]$

☆ $\delta(q_1, a, Z_0) = (q_0, Z_0)$ gives:-

P_{15} : $[q_1, Z_0, q_0] \to a\,[q_0, Z_0, q_0]$

P_{16} : $[q_1, Z_0, q_1] \to a\,[q_0, Z_0, q_1]$

~~Asso~~ Let

$[q_0, Z_0, q_0]$ is A	$[q_1, Z_0, q_0]$ is E
$[q_0, Z_0, q_1]$ is B	$[q_1, Z_0, q_1]$ is F
$[q_0, Z, q_0]$ is C	$[q_1, Z, q_0]$ is G
$[q_0, Z, q_1]$ is D	$[q_1, Z, q_1]$ is H

So

$S \to A \mid B$

~~[illegible]~~

$A \to bCA \mid bDE$

~~[illegible]~~

$B \to bCB \mid bDF$

~~[illegible]~~ $C \to bCC \mid bDG$

$D \to bCD \mid bDH \mid aH$

$H \to b$

$E \to A$

$F \to B$

Grammar $G = (\{S, A, B, C, D, E, F, G\}, \{a, b\}, P, S\}$

P is defined above.

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Q3) construct a CFG G which accepts N(A), where $A = (\{q_0, q_1\}, \{a, b, c\}, \{a, b, z_0\}, \delta, q_0, z_0, \phi)$. is a pda, where δ is defined as :-

		No. of Rules.
1	$\delta(q_0, a, z_0) = (q_0, az_0)$	$(2)^2 = 4$
2	$\delta(q_0, b, z_0) = (q_0, bz_0)$	$(2)^2 = 4$
3	$\delta(q_0, a, a) = (q_0, aa)$	$(2)^2 = 4$
4	$\delta(q_0, b, a) = (q_0, ba)$	$(2)^2 = 4$
5	$\delta(q_0, a, b) = (q_0, ab)$	$(2)^2 = 4$
6	$\delta(q_0, b, b) = (q_0, bb)$	$(2)^2 = 4$
7	$\delta(q_0, c, a) = (q_1, a)$	$(2)^1 = 2$
8	$\delta(q_0, c, b) = (q_1, b)$	$(2)^1 = 2$
9	$\delta(q_0, c, z_0) = (q_1, z_0)$	$(2)^1 = 2$
10	$\delta(q_1, a, a) = (q_1, \Lambda)$	$(2)^0 = 1$
11	$\delta(q_1, b, b) = (q_1, \Lambda)$	$(2)^0 = 1$
12	$\delta(q_1, \Lambda, z_0) = (q_1, \Lambda)$	$(2)^0 = 1$

Solution: set of variables $V_N := \{S\} \cup \{q, z, q'\}$
$q, q' \in Q, z \in \Gamma$

$V_N = \{ S, [q_0, z_0, q_0], [q_0, z_0, q_1], [q_0, a, q_0],$
$[q_0, a, q_1], [q_0, b, q_0], [q_0, b, q_1], [q_1, z_0, q_0],$
$[q_1, z_0, q_1], [q_1, a, q_0], [q_1, a, q_1], [q_1, b, q_0], [q_1, b, q_1]$

set of Production Rules :-

$R_1 :$ $S \rightarrow [q_0, z_0, q]$ for every q in Q.

$P_1 :$ $S \rightarrow [q_0, z_0, q_0]$
$P_2 :$ $S \rightarrow [q_0, z_0, q_1]$

* $\delta(q_0, a, z_0) = (q_0, az_0)$ gives :-

$P_3 :$ $[q_0, z_0, q_0] \longrightarrow a\, [q_0, a, q_0][q_0, z_0, q_0]$
$P_4 :$ $[q_0, z_0, q_0] \longrightarrow a\, [q_0, a, q_1][q_1, z_0, q_0]$
$P_5 :$ $[q_0, z_0, q_1] \longrightarrow a\, [q_0, a, q_0][q_0, z_0, q_1]$
$P_6 :$ $[q_0, z_0, q_1] \longrightarrow a\, [q_0, a, q_1][q_1, z_0, q_1]$

* $\delta(q_0, b, z_0) = (q_0, bz_0)$ gives:-

P_7 : $[q_0, z_0, q_0] \longrightarrow b\,[q_0, b, q_0]\,[q_0, z_0, q_0]$
P_8 : $[q_0, z_0, q_0] \longrightarrow b\,[q_0, b, q_1]\,[q_1, z_0, q_0]$
P_9 : $[q_0, z_0, q_1] \longrightarrow b\,[q_0, b, q_0]\,[q_0, z_0, q_1]$
P_{10} : $[q_0, z_0, q_1] \longrightarrow b\,[q_0, b, q_1]\,[q_1, z_0, q_1]$

* $\delta(q_0, a, a) = (q_0, aa)$ gives:-

P_{11} : $[q_0, a, q_0] \longrightarrow a\,[q_0, a, q_0]\,[q_0, a, q_0]$
P_{12} : $[q_0, a, q_0] \longrightarrow a\,[q_0, a, q_1]\,[q_1, a, q_0]$
P_{13} : $[q_0, a, q_1] \longrightarrow a\,[q_0, a, q_0]\,[q_0, a, q_1]$
P_{14} : $[q_0, a, q_1] \longrightarrow a\,[q_0, a, q_1]\,[q_1, a, q_1]$

* $\delta(q_0, b, a) = (q_0, ba)$ gives:-

P_{15} : $[q_0, a, q_0] \longrightarrow b\,[q_0, b, q_0]\,[q_0, a, q_0]$
P_{16} : $[q_0, a, q_0] \longrightarrow b\,[q_0, b, q_1]\,[q_1, a, q_0]$
P_{17} : $[q_0, a, q_1] \longrightarrow b\,[q_0, b, q_0]\,[q_0, a, q_1]$
P_{18} : $[q_0, a, q_1] \longrightarrow b\,[q_0, b, q_1]\,[q_1, a, q_1]$

* $\delta(q_0, a, b) = (q_0, ab)$ gives:-

P_{19} : $[q_0, b, q_0] \longrightarrow a\,[q_0, a, q_0]\,[q_0, b, q_0]$
P_{20} : $[q_0, b, q_0] \longrightarrow a\,[q_0, a, q_1]\,[q_1, b, q_0]$
P_{21} : $[q_0, b, q_1] \longrightarrow a\,[q_0, a, q_0]\,[q_0, b, q_1]$
P_{22} : $[q_0, b, q_1] \longrightarrow a\,[q_0, a, q_1]\,[q_1, b, q_1]$

* $\delta(q_0, b, b) = (q_0, bb)$ gives:-

P_{23} : $[q_0, b, q_0] \longrightarrow b\,[q_0, b, q_0]\,[q_0, b, q_0]$
P_{24} : $[q_0, b, q_0] \longrightarrow b\,[q_0, b, q_1]\,[q_1, b, q_0]$
P_{25} : $[q_0, b, q_1] \longrightarrow b\,[q_0, b, q_0]\,[q_0, b, q_1]$
P_{26} : $[q_0, b, q_1] \longrightarrow b\,[q_0, b, q_1]\,[q_1, b, q_1]$

$\delta(q_0, c, a) = (q_1, a)$ gives:-

P_{27} : $[q_0, a, q_0] \rightarrow c\,[q_1, a, q_0]$
P_{28} : $[q_0, a, q_1] \rightarrow c\,[q_1, a, q_1]$

$\delta(q_0, c, b) = (q_1, b)$ gives:-

P_{29} : $[q_0, b, q_0] \rightarrow c\,[q_1, b, q_0]$
P_{30} : $[q_0, b, q_1] \rightarrow c\,[q_1, b, q_1]$

$\delta(q_0, c, z_0) = (q_1, z_0)$ gives:

P_{31} : $[q_0, z_0, q_0] \rightarrow c\,[q_1, z_0, q_0]$
P_{32} : $[q_0, z_0, q_1] \rightarrow c\,[q_1, z_0, q_1]$

$\delta(q_1, a, a) = (q_1, \Lambda)$ gives:-

P_{33} : $[q_1, a, q_1] \rightarrow a$

$\delta(q_1, b, b) = (q_1, \Lambda)$ gives:

P_{34} : $[q_1, b, q_1] \rightarrow b$

$\delta(q_1, \Lambda, z_0) = (q_1, \Lambda)$ gives:-

P_{35} : $[q_1, z_0, q_1] \rightarrow \Lambda$.

let $[q_0, z_0, q_0]$, $[q_0, z_0, q_1]$, $[q_0, a, q_0]$, $[q_0, a, q_1]$, $[q_0, b, q_0]$, $[q_0, b, q_1]$, $[q_1, z_0, q_0]$, $[q_1, z_0, q_1]$, $[q_1, a, q_0]$, $[q_1, a, q_1]$, $[q_1, b, q_0]$, $[q_1, b, q_1]$ is A, B, C, D, E, F, G, H, I, J, K, L respectively.

So $S \rightarrow A/B$, $A \rightarrow aCB/aDH/bEB/bFH/CH$
$C \rightarrow aCC/aDI/bEC/bFI/CI$, $D \rightarrow aCJ/aDJ/bEJ/bFJ/CJ$
$E \rightarrow aCE/aDK/bEE/bFK/CK$, $F \rightarrow aCF/aDL/bEF/bFL/CL$.
$J \rightarrow a$, $L \rightarrow b$, $H \rightarrow \Lambda$.