

* Convert context free grammar into PDA.

If L is a context free language, then we construct a Pda A accepting L by empty store $L = N(A)$.

Let $L = L(\alpha)$, where $\alpha = (V_N, \Sigma, P, S)$ is a CFA. We construct a Pda A as

$$N(A) = (\{\epsilon\}, \Sigma, V_N \cup \Sigma, S, \emptyset, S, \emptyset)$$

where S is defined by the following rules:-

$$R_1: S(\emptyset, \lambda, A) = \{(\emptyset, \lambda)\} \quad | \quad A \rightarrow \alpha \text{ is in } P.$$

$$R_2: S(\emptyset, a, a) = \{(\emptyset, \lambda)\} \quad \text{for every } a \text{ in } \Sigma.$$

(Q1) Construct a Pda A equivalent to the following CFA

$$S \rightarrow 0BB, B \rightarrow 0S/1S/0$$

Test whether 010^4 is in $N(A)$.

Solution: Define Pda A as follows:-

$$A = (\{\epsilon\}, \{0, 1\}, \{S, B, 0, 1\}, S, \emptyset, S, \emptyset)$$

S is defined by the following rules:-

$$R_1: S(\emptyset, \lambda, S) = \{(\emptyset, 0BB)\}$$

$$R_2: S(\emptyset, \lambda, B) = \{(\emptyset, 0B), (\emptyset, 1S), (\emptyset, 0)\}$$

$$R_3: S(\emptyset, 0, 0) = \{(\emptyset, \lambda)\}$$

$$R_4: S(\emptyset, 1, 1) = \{(\emptyset, \lambda)\}$$



Test whether 010^4 is in $N(G)$.

- $(\emptyset, 010^4, S) \rightarrow$ by Rule R₁
- $\vdash (\emptyset, 010^4, OBB)$ by Rule R₃
 - $\vdash (\emptyset, 10^4, BB)$ by Rule R₂
 - $\vdash (\emptyset, 10^4, ISB)$ by Rule R₄
 - $\vdash (\emptyset, 0^4, SB)$ by Rule R₁
 - $\vdash (\emptyset, 0^4, OBBB)$ by Rule R₃
 - $\vdash (\emptyset, 0^3, BBB)$ by Rule R₂
 - $\vdash (\emptyset, 0^3, OBB)$ by Rule R₃
 - $\vdash (\emptyset, 0^2, BB)$ by Rule R₂
 - $\vdash (\emptyset, 0^2, OB)$ by Rule R₃
 - $\vdash (\emptyset, 0, B)$ by Rule R₂
 - $\vdash (\emptyset, 0, O)$ by Rule R₃
 - $\vdash (\emptyset, \Lambda, \Lambda)$ Accept by null store.

Q2) convert the following CFN into PDA:-

$$S \rightarrow QBA / bA$$

$$A \rightarrow a / aS / bAA$$

$$B \rightarrow b / bS / aBB$$

Also parse the string baab.

Solution:-

The required PDA A can be defined as follows:

$$A = (\{q\}, \{a, b\}, \{S, A, B, a, b\}, q, S, \phi)$$

where S is defined by the following rules:-

$$R_1 : S(q, \Lambda, S) = \{(q, qB), (q, bA)\}$$

$$R_2 : S(q, \Lambda, A) = \{(q, a), (q, aS), (q, bAA)\}$$

$$R_3 : S(q, \Lambda, B) = \{(q, b), (q, bS), (q, aBB)\}$$

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R₄: $S(2, a, a) = (2, \lambda)$ so it is 00

R₅: $S(2, b, b) = (2, \lambda)$

R₆: S

Parse the string baab.

$(2, baab, S)$ by Rule R₁

$\vdash (2, baab, bA)$ by Rule R₅

$\vdash (2, aab, A)$ by Rule R₂

$\vdash (2, aab, aS)$ by Rule R₄

$\vdash (2, ab, S)$ by Rule R₁

$\vdash (2, ab, aB)$ by Rule R₄

$\vdash (2, b, B)$ by Rule R₃

$\vdash (2, b, b)$ by Rule R₅

$\vdash (2, \lambda, \lambda)$ accepted by Null store.

(Q3) construct the Pda equivalent to the following grammar

$S \rightarrow aAA$

$A \rightarrow aS / bS / a$

Also Parse the string aabaaa

Solution: The Pda A can be defined as follows:

$A = (\{a, b\}, \{S, A\}, \{a, b\}, S, \lambda, \emptyset)$

where S is defined by the following rules:-

R₁: $S(\lambda, \lambda, S) = (2, aAA)$

R₂: $S(\lambda, \lambda, A) = \{(2, aS), (2, bS), (2, a)\}$

R₃: $S(\lambda, a, a) = \{(2, \lambda)\}$

R₄: $S(\lambda, b, b) = \{(2, \lambda)\}$

$(2, aabaaa, S) \vdash (2, aabaaa, aAA) \vdash (2, abaaa, AA)$

$\vdash (2, abaaa, aA) \vdash (2, baaa, A) \vdash (2, baaa, bS)$

$\vdash (2, aaa, S) \vdash (2, aaa, aAA) \vdash (2, aa, AA)$

$\vdash (2, aa, aA) \vdash (2, a, A) \vdash (2, a, a) \vdash (2, \lambda, \lambda)$

accept by Null store.

Q4) Construct an NPDA corresponding to the grammar.

$$S \rightarrow aABBB / aAA$$

$$A \rightarrow aBB / a$$

$$B \rightarrow bBB / A$$

Solution:-

The required NPDA can be defined as follows:-

$$A = (\{ \epsilon \}, \{ a, b \}, \{ S, A, B \}, \{ a, b \}, S, \epsilon, S, \phi)$$

where S is defined by the following rules:-

$$R_1: S(\epsilon, \Lambda, S) = \{ (\epsilon, aABB), (\epsilon, aAA) \}$$

$$R_2: S(\epsilon, \Lambda, A) = \{ (\epsilon, aBB), (\epsilon, a) \}$$

$$R_3: S(\epsilon, \Lambda, B) = \{ (\epsilon, bBB), (\epsilon, A) \}$$

$$R_4: S(\epsilon, a, a) = \{ (\epsilon, \Lambda) \}$$

$$R_5: S(\epsilon, b, b) = \{ (\epsilon, \Lambda) \}$$

Q5) Construct PDA for the given grammar:-

$$S \rightarrow AA$$

$$A \rightarrow aABC / bB / a$$

$$B \rightarrow b$$

$$C \rightarrow c$$

Solution:-

$$L = (\{ \epsilon \}, \{ a, b, c \}, \{ a, b, c, S, A, B, C \}, S, \epsilon, S, \phi).$$

S is defined as:

$$(AAD, \epsilon) = (\epsilon, \Lambda, \epsilon)$$

$$R_1: S(\epsilon, \Lambda, S) = (\epsilon, AA)$$

$$R_2: S(\epsilon, \Lambda, A) = \{ (\epsilon, aABC), (\epsilon, bB), (\epsilon, a) \}$$

$$R_3: S(\epsilon, \Lambda, B) = \{ (\epsilon, b) \}$$

$$R_4: S(\epsilon, \Lambda, C) = \{ (\epsilon, c) \}$$

$$R_5: S(\epsilon, a, a) = (\epsilon, \Lambda)$$

$$R_6: S(\epsilon, b, b) = (\epsilon, \Lambda)$$

$$R_7: S(\epsilon, c, c) = (\epsilon, \Lambda)$$



Convert Push down Automata into CFG :-

If $A = (\Omega, \Sigma, \Gamma, S, \delta_0, z_0, F)$ is a Pda,
then there exists a context free grammar G
such that $L(G) = L(A)$.

> construction of G :- we define $G = (V_N, \Sigma, P, S)$,

$$\text{where } V_N = \{S\} \cup \{[q, z, z']\} / q, z' \in \Omega, z \in \Gamma$$

The productions in P are induced by moves of Pda
as follows :-

R_1 : S productions are given by $S \rightarrow [q_0, z_0, z]$

R_2 : Each move erasing a pushdown symbol given
by

$$\delta(q, a, z) = (q', \lambda) \quad [q, z, z'] \xrightarrow{a} [q', z, z']$$

R_3 : Each move not erasing a pushdown symbol
given by

$$\delta(q, a, z) = (q_1, z_1, z_2, \dots, z_m) \quad [q, z, z'] \xrightarrow{a} [q_1, z_1, z_2] [q_2, z_2, z_3] \dots [q_m, z_m, z']$$

where each of the states q_1, q_2, \dots, q_m can be any state in Ω .

Note: - no. of production rules for each transition rule = $C m^n$

where $m = \text{no. of states in } Q$

$n = \text{no. of stack variable in transition rule}$, $(A)N = (0)$ for respective

(Q1) Construct a PDA accepting the set of all strings over $\{a, b\}$, having equal number of $a's$ & $b's$.

$$N(A) = (\{\epsilon\}, \{a, b\}, \{z_0, a, b\}, S, \{z_*, z_0\}, \emptyset)$$

	NO. of Rules (m^n)
① $S(z, a, z_0) = (z, az_0)$	$(1)^2 = 1$
② $S(z, b, z_0) = (z, bz_0)$	$(1)^2 = 1$
③ $S(z, a, a) = (z, aa)$	$(1)^2 = 1$
④ $S(z, b, a) = (z, \lambda)$	$(1)^0 = 1$
⑤ $S(z, a, b) = (z, \lambda)$	$(1)^0 = 1$
⑥ $S(z, b, b) = (z, bb)$	$(1)^2 = 1$
⑦ $S(z, \lambda, z_0) = (z, \lambda)$	$(1)^0 = 1$

Solution:

$$\text{we define } G = (V_N, \Sigma, P, S)$$

where set of Variable V_N :

$$V_N = \{S\} \cup \{z, z_0, z'\} \mid z, z' \in Q, z \neq \epsilon$$

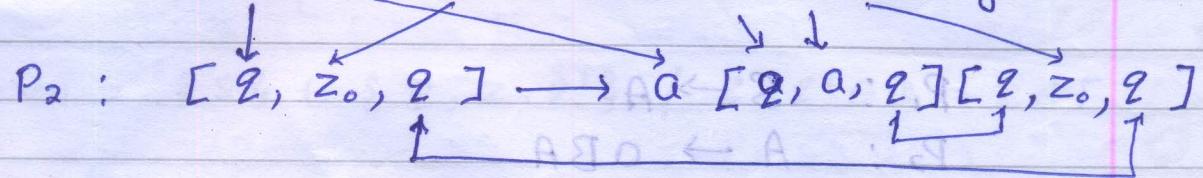
$$\text{so } V_N = \{S, [z, z_0, z'], [z, a, z], [z, b, z]\}$$

» Set of Production Rules:-

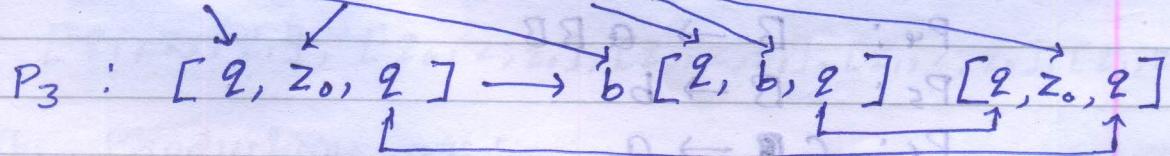
$$P_1: S \rightarrow [z_0, z_0, z] \text{ for every } z \in \Sigma.$$

$\star \star P_1 : S \rightarrow [q, z_0, q]$

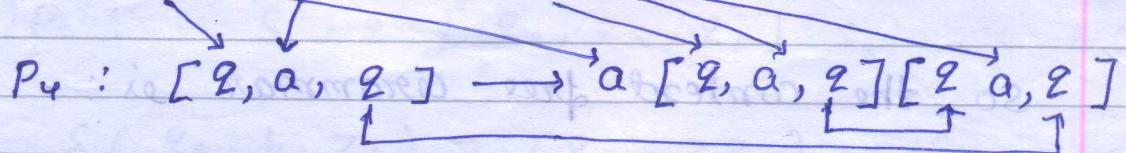
$$\star \star S(q, a, z_0) = (q, az_0) \text{ gives:-}$$



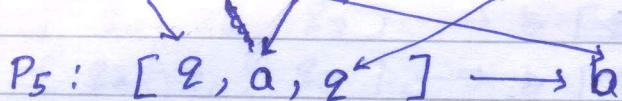
$$\star \star S(q, b, z_0) = (q, bz_0) \text{ gives:-}$$



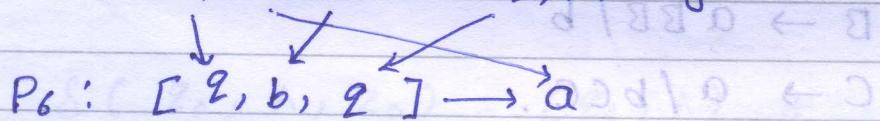
$$\star \star S(q, a, a) = (q, aa) \text{ gives:-}$$



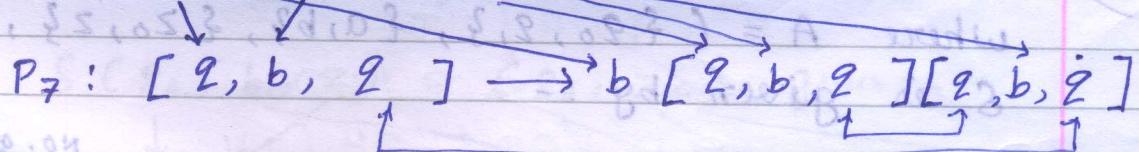
$$\star \star S(q, b, a) = (q, \lambda) \text{ gives:-}$$



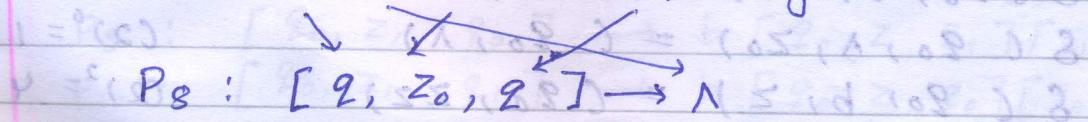
$$\star \star S(q, a, b) = (q, \lambda) \text{ gives:-}$$



$$\star \star S(q, b, b) = (q, bb) \text{ gives:-}$$



$$\star \star S(q, \lambda, z_0) = (q, \lambda) \text{ gives:-}$$



$$S(q, \lambda, z_0) = (q, \lambda)$$

$$S(q, \lambda, z_0) = (q, \lambda)$$

$$S(q, \lambda, z_0) = (q, \lambda)$$

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ANSWER $[z_0, z_0, z]$, $[z, a, z]$, $[z, b, z]$ is

A, B, C respectively.

$[S, \{a, b\}, S \rightarrow A]$

$P_2: A \rightarrow aBA$

$P_3: B \rightarrow bCA$

$P_4: B \rightarrow aBB$

$P_5: B \rightarrow b$

$P_6: C \rightarrow a$

$P_7: C \rightarrow bCC$

$P_8: A \rightarrow \Lambda$

So the context free grammar is :-

$G = (\{S, A, B, C\}, \{a, b\}, P, S)$

P is defined as :-

$S \rightarrow A$

$A \rightarrow aBA / bCA / \Lambda$

$B \rightarrow aBB / b$

$C \rightarrow a / bCC$

Q2) construct a (CFG) G which accepts NCA,
where $A = (\{z_0, z_1\}, \{a, b\}, \{z_0, z_1\}, S, z_0, z_0, \emptyset)$.
 S is given by :-

NO. of rules:-

$S(z_0, b, z_0) = (z_0, zz_0)$ $(2)^2 = 4$

$S(z_0, \Lambda, z_0) = (z_0, \Lambda)$ $(2)^0 = 1$

$S(z_0, b, z) = (z_0, zz)$ $(2)^2 = 4$

$S(z_0, a, z) = (z_1, z)$ $(2)^1 = 2$

$S(z_1, b, z) = (z_1, \Lambda)$ $(2)^0 = 1$

$S(z_1, a, z_0) = (z_0, z_0)$ $(2)^1 = 2$

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Solution: Let $\alpha = (V_N, \{a, b\}, P, S)$.

where set of variable V_N :-

$$V_N = \{S\} \cup \{Z, Z_0, Z_1\} \mid Z, Z' \in Q, Z \in \Gamma$$

$$\text{So } V_N = \{S, [Z_0, Z_0, Z_0], [Z_0, Z_0, Z_1], [Z_1, Z_0, Z_0], \\ [Z_1, Z_0, Z_1], [Z_0, Z, Z_0], [Z_0, Z, Z_1], [Z_1, Z, Z_0], [Z_1, Z, Z_1]\}$$

The Productions are :-

$$P_1 : S \rightarrow [Z_0, Z_0, Z_0]$$

$$P_2 : S \rightarrow [Z_0, Z_0, Z_1]$$

$\star \quad S(Z_0, b, Z_0) = (Z_0, ZZ_0) \text{ gives}$

$$P_3 : [Z_0, Z_0, Z_0] \rightarrow b [Z_0, Z_1, Z_0] [Z_0, Z_0, Z_0]$$

$$P_4 : [Z_0, Z_0, Z_0] \rightarrow b [Z_0, Z, Z_1] [Z_1, Z_0, Z_0]$$

$$P_5 : [Z_0, Z_0, Z_1] \rightarrow b [Z_0, Z, Z_0] [Z_0, Z_0, Z_1]$$

$$P_6 : [Z_0, Z_0, Z_1] \rightarrow b [Z_0, Z, Z_1] [Z_1, Z_0, Z_1]$$

$\star \quad S(Z_0, \lambda, Z_0) = (Z_0, \lambda) \text{ gives:-}$

$$P_7 : [Z_0, Z_0, Z_0] \rightarrow \lambda$$

$\star \quad S(Z_0, b, Z) = (Z_0, ZZ) \text{ gives:-}$

$$P_8 : [Z_0, Z, Z_0] \rightarrow b [Z_0, Z, Z_0] [Z_0, Z_1, Z_0]$$

$$P_9 : [Z_0, Z, Z_0] \rightarrow b [Z_0, Z, Z_1] [Z_1, Z, Z_0]$$

$$P_{10} : [Z_0, Z, Z_1] \rightarrow b [Z_0, Z, Z_0] [Z_0, Z, Z_1]$$

$$P_{11} : [Z_0, Z, Z_1] \rightarrow b [Z_0, Z, Z_1] [Z_1, Z, Z_1]$$



$$\equiv S(z_0, a, z) = (z_1, z) \text{ gives:-}$$

$$P_{12} : [z_0, z, z_0] \rightarrow a [z_1, z, z_0]$$

$$P_{13} : [z_0, z, z_1] \rightarrow a [z_1, z, z_1]$$

$$\equiv S(z_1, b, z) = (z_1, z) \text{ gives:-}$$

$$P_{14} : [z_1, z, z_1] \rightarrow b E$$

$$\equiv S(z_1, a, z_0) = (z_0, z_0) \text{ gives:-}$$

$$P_{15} : [z_1, z_0, z_0] \rightarrow a [z_0, z_0, z_0]$$

$$P_{16} : [z_1, z_0, z_1] \rightarrow a [z_0, z_0, z_1]$$

Also let

$[z_0, z_0, z_0]$ is A

$[z_0, z_0, z_1]$ is B

$[z_0, z, z_0]$ is C

$[z_0, z, z_1]$ is D

$[z_1, z_0, z_0]$ is E

$[z_1, z_0, z_1]$ is F

$[z_1, z, z_0]$ is G

$[z_1, z, z_1]$ is H

So

$$S \rightarrow A / B$$

~~See~~ B

$$A \rightarrow bCA / bDE$$

~~See~~

$$B \rightarrow bCB / bDF$$

$$C \rightarrow bCC / bDG$$

$$D \rightarrow bCD / bDH / aH$$

$$H \rightarrow b$$

$$E \rightarrow A$$

$$F \rightarrow B$$

Grammar $G = (\{S, A, B, C, D, E, F, G\}, \{a, b\}, P, S)$
 P is defined above.

Q3) construct a CFN G which accepts N(A), where
 $A = (\{z_0, z_1\}, \{a, b, c\}, \{a, b, z_0\}, S, z_0, z_0, \emptyset)$.
 i) a PDA, where S is defined as :-

NO. of Rules.

- ① $S(z_0, a, z_0) = (z_0, az_0)$ $(2)^2 = 4$
- ② $S(z_0, b, z_0) = (z_0, bz_0)$ $(2)^2 = 4$
- ③ $S(z_0, a, a) = (z_0, aa)$ $(2)^2 = 4$
- ④ $S(z_0, b, a) = (z_0, ba)$ $(2)^2 = 4$
- ⑤ $S(z_0, a, b) = (z_0, ab)$ $(2)^2 = 4$
- ⑥ $S(z_0, b, b) = (z_0, bb)$ $(2)^2 = 4$
- ⑦ $S(z_0, c, a) = (z_1, a)$ $(2)^1 = 2$
- ⑧ $S(z_0, c, b) = (z_1, b)$ $(2)^1 = 2$
- ⑨ $S(z_0, c, z_0) = (z_1, z_0)$ $(2)^1 = 2$
- ⑩ $S(z_1, a, a) = (z_1, \lambda)$ $(2)^0 = 1$
- ⑪ $S(z_1, b, b) = (z_1, \lambda)$ $(2)^0 = 1$
- ⑫ $S(z_1, \lambda, z_0) = (z_1, \lambda)$ $(2)^0 = 1$

Solution: set of variables $V_N = \{S\} \cup \{z, z, z'\}$

$z, z' \in \emptyset, z \in T$

$V_N = \{S, [z_0, z_0, z_0], [z_0, z_0, z_1], [z_0, a, z_0],$
 $[z_0, a, z_1], [z_0, b, z_0], [z_0, b, z_1], [z_1, z_0, z_0],$
 $[z_1, z_0, z_1], [z_1, a, z_0], [z_1, a, z_1], [z_1, b, z_0], [z_1, b, z_1]$

Set of Production Rules:-

$R_1: S \rightarrow [z_0, z_0, z]$ for every $z \in \emptyset$.

$P_1: S \rightarrow [z_0, z_0, z_0]$

$P_2: S \rightarrow [z_0, z_0, z_1]$

* $S(z_0, a, z_0) = (z_0, az_0)$ gives:-

$P_3: [z_0, z_0, z_0] \xrightarrow{a} [z_0, a, z_0] \xrightarrow{a} [z_0, az_0, z_0]$

$P_4: [z_0, z_0, z_0] \xrightarrow{a} [z_0, a, z_1] \xrightarrow{a} [z_1, z_0, z_0]$

$P_5: [z_0, z_0, z_1] \xrightarrow{a} [z_0, a, z_0] \xrightarrow{a} [z_0, az_0, z_1]$

$P_6: [z_0, z_0, z_1] \xrightarrow{a} [z_0, a, z_1] \xrightarrow{a} [z_1, z_0, z_1]$

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$\star \quad S(2_0, b, z_0) = (2_0, bz_0)$ gives:-

$$\begin{array}{l} P_7 : [2_0, z_0, 2_0] \rightarrow b [2_0, b, 2_0] [2_0, z_0, 2_0] \\ P_8 : [2_0, z_0, 2_0] \rightarrow b [2_0, b, 2_1] [2_1, z_0, 2_0] \\ P_9 : [2_0, z_0, 2_1] \rightarrow b [2_0, b, 2_0] [2_0, z_0, 2_1] \\ P_{10} : [2_0, z_0, 2_1] \rightarrow b [2_0, b, 2_1] [2_1, z_0, 2_1] \end{array}$$

$\star \quad S(2_0, a, a) = (2_0, aa)$ gives:-

$$\begin{array}{l} P_{11} : [2_0, a, 2_0] \rightarrow a [2_0, a, 2_0] [2_0, a, 2_0] \\ P_{12} : [2_0, a, 2_0] \rightarrow a [2_0, a, 2_1] [2_1, a, 2_0] \\ P_{13} : [2_0, a, 2_1] \rightarrow a [2_0, a, 2_0] [2_0, a, 2_1] \\ P_{14} : [2_0, a, 2_1] \rightarrow a [2_0, a, 2_1] [2_1, a, 2_1] \end{array}$$

$\star \quad S(2_0, b, a) = (2_0, ba)$ gives:-

$$\begin{array}{l} P_{15} : [2_0, a, 2_0] \rightarrow b [2_0, b, 2_0] [2_0, a, 2_0] \\ P_{16} : [2_0, a, 2_0] \rightarrow b [2_0, b, 2_1] [2_1, a, 2_0] \\ P_{17} : [2_0, a, 2_1] \rightarrow b [2_0, b, 2_0] [2_0, a, 2_1] \\ P_{18} : [2_0, a, 2_1] \rightarrow b [2_0, b, 2_1] [2_1, a, 2_1] \end{array}$$

$\star \quad S(2_0, a, b) = (2_0, ab)$ gives:-

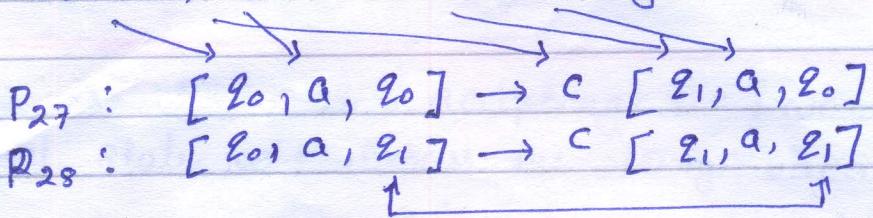
$$\begin{array}{l} P_{19} : [2_0, b, 2_0] \rightarrow a [2_0, a, 2_0] [2_0, b, 2_0] \\ P_{20} : [2_0, b, 2_0] \rightarrow a [2_0, a, 2_1] [2_1, b, 2_0] \\ P_{21} : [2_0, b, 2_1] \rightarrow a [2_0, a, 2_0] [2_0, b, 2_1] \\ P_{22} : [2_0, b, 2_1] \rightarrow a [2_0, a, 2_1] [2_1, b, 2_1] \end{array}$$

$\star \quad S(2_0, b, b) = (2_0, bb)$ gives:-

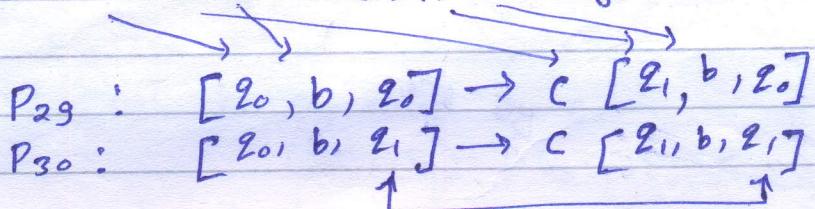
$$\begin{array}{l} P_{23} : [2_0, b, 2_0] \rightarrow b [2_0, b, 2_0] [2_0, b, 2_0] \\ P_{24} : [2_0, b, 2_0] \rightarrow b [2_0, b, 2_1] [2_1, b, 2_0] \\ P_{25} : [2_0, b, 2_1] \rightarrow b [2_0, b, 2_0] [2_0, b, 2_1] \\ P_{26} : [2_0, b, 2_1] \rightarrow b [2_0, b, 2_1] [2_1, b, 2_1] \end{array}$$

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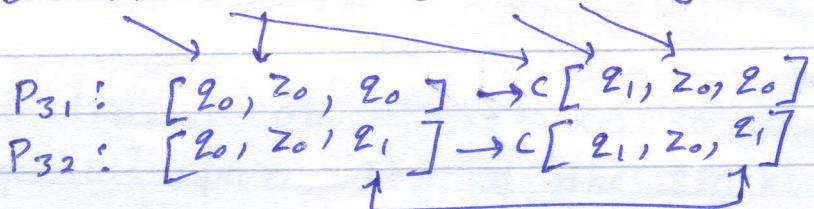
$$\Leftrightarrow S(z_0, c, a) = (z_1, a) \text{ gives:-}$$



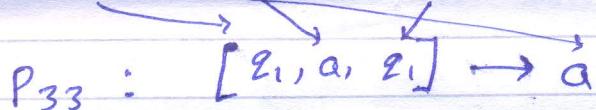
$$\Leftrightarrow S(z_0, c, b) = (z_1, b) \text{ gives:-}$$



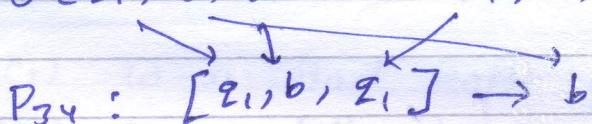
$$\Leftrightarrow S(z_0, c, z_0) = (z_1, z_0) \text{ gives:-}$$



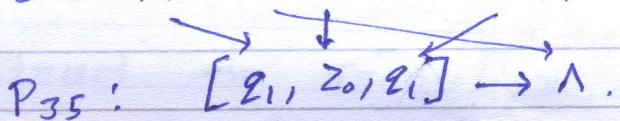
$$\Leftrightarrow S(z_1, a, a) = (z_1, \lambda) \text{ gives:-}$$



$$\star// S(z_1, b, b) = (z_1, \lambda) \text{ gives:-}$$



$$\Leftrightarrow S(z_1, \lambda, z_0) = (z_1, \lambda) \text{ gives:-}$$



Let $[z_0, z_0, z_0], [z_0, z_0, z_1], [z_0, a, z_0], [z_0, a, z_1]$
 $[z_0, b, z_0], [z_0, b, z_1], [z_1, z_0, z_0], [z_1, z_0, z_1], [z_1, a, z_0]$
 $[z_1, a, z_1], [z_1, b, z_0], [z_1, b, z_1]$ $\rightarrow A, B, C, D, E, F,$
 G, H, I, J, K, L respectively.

So $S \rightarrow A/B, A \rightarrow ACB/ADH/bEB/bFH/CH$
 $C \rightarrow ACC/ADI/bEC/bFI/CI, D \rightarrow ACD/ADJ/bED/bFJ/CJ$
 $E \rightarrow ACE/ADK/bEE/bFK/CK, F \rightarrow ACF/ADL/bEF/bFL/CL.$
 $J \rightarrow a, L \rightarrow b, H \rightarrow \lambda.$