⇒ construction of DFA from NFA with ϵ.

let $M = (Q, \Sigma, \delta, q_0, F)$ is NFA with ϵ-moves then $M' = (Q, \Sigma, \delta_D', q_0', F')$ is DFA.

Procedure :-

Step 1:- Find ϵ-closure for all state.

Step 2:- if ϵ-closure $(q_0) = \{P_1, P_2, P_3, ---, P_n\}$ then initial state of DFA $q_0' = [P_1 P_2 P_3 ---- P_n]$.

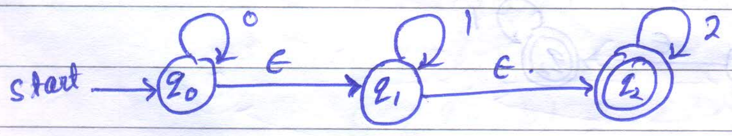
Step 3:- Determine the Transition function of Reachable state and start from initial state of DFA.

$$\delta_D'([P_1 P_2 P_3 --- P_n], a) = [\epsilon\text{-closure}(\delta(P_1, P_2, --- P_n), a)]$$

$$\delta_D'([P_1 P_2 P_3 --- P_n], a) = [\epsilon\text{-closure}(\delta[P_1, a) \cup \delta(P_2, a) \cup \delta(P_3, a) \cup ----- \delta(P_n, a))]$$

Step 4:- The states containing final state of NFA-with-ϵ is a Final state in DFA.

Q1) Convert the Following NFA with ϵ into DFA.



Solution :-

state/input	0	1	2	ϵ
$\rightarrow q_0$	$\{q_0\}$	–	–	$\{q_1\}$
q_1	–	$\{q_1\}$	–	$\{q_2\}$
q_2	–	–	$\{q_2\}$	–

step1:- Find ϵ-closure for all state:

$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$

$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$

$\epsilon\text{-closure}(q_2) = \{q_2\}$.

step2:- Here q_0 is the initial state of NFA with ϵ.
if $\epsilon\text{-closure}(q_0) = \{q_0 q_1, q_2\}$ then
initial state of DFA $= [q_0 q_1, q_2]$

step3:- Determine the Transaction for Reachable state and start from ~~initial~~ initial state of DFA.

$$\delta'_D([q_0 q_1, q_2], 0) = [\epsilon\text{-closure}(\delta([q_0 q_1, q_2], 0))]$$
$$= [\epsilon\text{-closure}(\delta[q_0, 0] \cup \delta(q_1, 0) \cup (q_2, 0))]$$
$$= [\epsilon\text{-closure}(\{q_0\} \cup \phi \cup \phi)$$
$$= [\epsilon\text{-closure}(q_0)]$$
$$\delta'_D([q_0 q_1, q_2], 0) = [q_0 q_1, q_2]$$

Similarly:- $\delta'_D([q_0 q_1, q_2], 1) = [q_1 q_2]$

$\delta'_D([q_0 q_1, q_2], 2) = [q_2]$

then New Rechable states are $[q_1 q_2]$ and $[q_2]$. ~~and~~ so find the Transition for new states.

$\delta'_D([q_1 q_2], 0) = \phi$

$\delta'_D([q_1 q_2], 1) = [q_1 q_2]$

$\delta'_D([q_1 q_2], 2) = \cancel{\phi}\ [q_2]$

$\delta'_D([q_2], 0) = \phi$

$\delta'_D([q_2, 1]) = \phi$

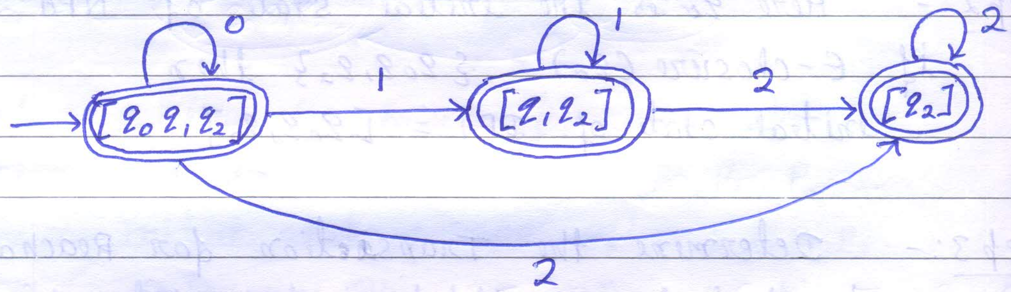
$\delta'_D([q_2, 2]) = [q_2]$

~~step~~ ~~8~~

NO new reachable states are found. so stop this process. and Total Reachable states ~~are~~ are $[q_0 q_1, q_2]$, $[q_1 q_2]$, $[q_2]$.

Step 4 :- set of Final state of NFA with ϵ $(F) = \{q_2\}$.
The states containing final states of NFA-with-ϵ is a final state in DFA.

So set of Reachable Final state of DFA $= \{[q_0q_1q_2], [q_1q_2], [q_2]\}$



Transition diagram

So DFA is represent by 5 Tupples $(Q', \Sigma, \delta', q_0', F')$ where,

(i) $Q' = 2^Q = \{\phi, [q_0], [q_1], [q_2], [q_0q_1], [q_0q_2], [q_1q_2], [q_0q_1q_2]\}$

(ii) $\Sigma = \{0, 1, 2\}$

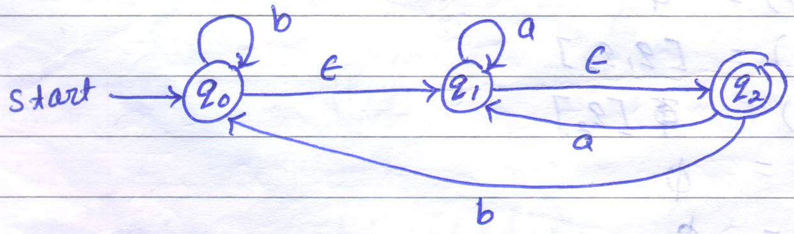
(iii) $q_0' = [q_0q_1q_2]$

(iv) $F' = \{[q_2], [q_0q_2], [q_1q_2], [q_0q_1q_2]\}$.

(v) δ' is defined in Transition diagram.

Notes :- only $[q_0q_1q_2]$, $[q_1q_2]$, $[q_2]$ are Reachable states and other states are Not Reachables.

Q2) Convert the following NFA with ϵ to equivalent DFA.



Solution :- let $M = (Q, \Sigma, \delta, q_0, F)$ is NFA with ϵ.
then $M' = (Q', \Sigma, \delta', q_0', F')$ is DFA.

state/input	a	b	ϵ
$\rightarrow q_0$	—	$\{q_0\}$	$\{q_1\}$
q_1	$\{q_1\}$	—	$\{q_2\}$
q_2	$\{q_1\}$	$\{q_0\}$	—

Step 1 :- Find ϵ-closure for all states.

$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$
$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$
$\epsilon\text{-closure}(q_2) = \{q_2\}$.

Step 2 :- initial state of DFA is $[q_0 q_1 q_2]$

Step 3 :- determine Transition for Reachable state and start from initial state.

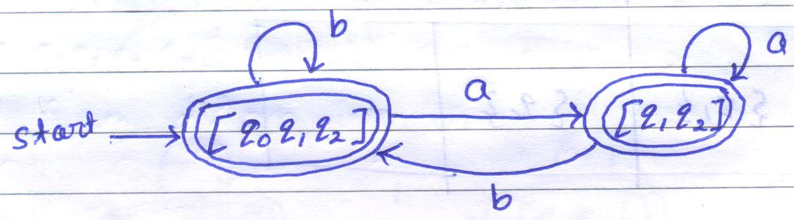
$\delta'_D([q_0 q_1 q_2], a) = \epsilon\text{-closure}(\delta([q_0 q_1 q_2], a))$
$\delta'_D([q_0 q_1 q_2], a) = \epsilon\text{-closure}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a))$
$= \epsilon\text{-closure}(\phi \cup \{q_1\} \cup \{q_1\})$
$= \epsilon\text{-closure}(q_1)$
$\delta'_D([q_0 q_1 q_2], a) = [q_1 q_2]$

Similarly :- $\delta'_D([q_0 q_1 q_2], b) = [q_0 q_1 q_2]$
$\delta'_D([q_1 q_2], a) = [q_1 q_2]$
$\delta'_D([q_1 q_2], b) = [q_0 q_1 q_2]$

So total reachable states are $[q_0 q_1 q_2]$ and $[q_1 q_2]$.

Step 4:- The states containing Final state of NFA-with ϵ is a Final state in DFA.

So set of Reachable Final states are $\{ [q_0 q_1 q_2], [q_1 q_2] \}$.



Transition diagram

DFA represents by 5 Tupples $(Q', \Sigma, \delta', q_0', F')$

where,

(i) $Q' = 2^Q = \{ \phi, [q_0], [q_1], [q_2], [q_0 q_1], [q_0 q_2], [q_1 q_2], [q_0 q_1 q_2] \}$

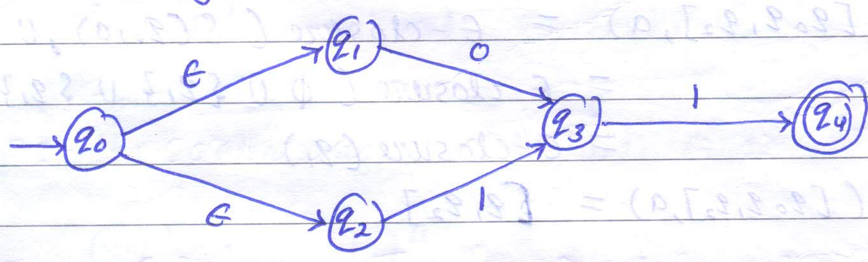
(ii) $\Sigma = \{a, b\}$

(iii) $q_0' = [q_0 q_1 q_2]$

(iv) $F = \{ [q_2], [q_0 q_2], [q_1 q_2], [q_0 q_1 q_2] \}$

(v) δ is defined in Transition diagram.

Q3) Convert the given NFA with ϵ to its equivalent DFA.



Solution:- Let $M = (Q, \Sigma, \delta, q_0, F)$ is NFA with ϵ.

then $M' = (Q', \Sigma, \delta', q_0', F')$ is DFA.

state/input	0	1	ϵ
$\rightarrow q_0$	-	-	$\{q_1, q_2\}$
q_1	$\{q_3\}$	-	-
q_2	-	$\{q_3\}$	-
q_3	-	$\{q_4\}$	-
q_4	-	-	-

Step 1:- Find the ϵ-closure for all state.

$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$

$\epsilon\text{-closure}(q_1) = \{q_1\}$

$\epsilon\text{-closure}(q_2) = \{q_2\}$

$\epsilon\text{-closure}(q_3) = \{q_3\}$

$\epsilon\text{-closure}(q_4) = \{q_4\}$

Step 2:- initial state of DFA is $[q_0 q_1 q_2]$.

Step 3:- Determine Transition for Reachable state and start from initial state.

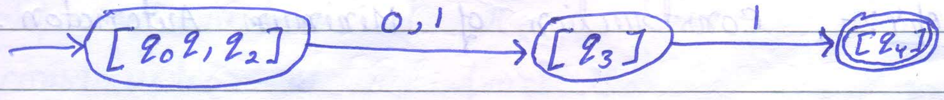
$\delta'_D([q_0 q_1 q_2], 0) = \epsilon\text{-closure}(\delta([q_0 q_1 q_2], 0))$
$\delta'_D([q_0 q_1 q_2], 0) = [q_3]$

Similarly :-

$\delta'_D([q_0 q_1 q_2], 1) = [q_3]$
$\delta'_D([q_3], 0) = \phi$
$\delta'_D([q_3], 1) = [q_4]$
$\delta'_D([q_4], 0) = \phi$
$\delta'_D([q_4], 1) = \phi$

Step 4:- The state containing Final state of NFA-with-ϵ is a Final state in DFA.

So set of Reachable Final states are $\{[q_0 q_1 q_2], [q_3], [q_4]\}$.



Transition diagram

DFA represent by 5 Tupples $(Q', \Sigma, \delta', q_0', F')$ where

$Q' = 2^Q = \{\phi, [q_0], [q_1], [q_2], [q_3], [q_4], [q_0q_1], [q_0q_2], [q_0q_3], [q_0q_4], [q_1q_2], [q_1q_3], [q_1q_4], [q_2q_3], [q_2q_4], [q_3q_4], [q_0q_1q_2], [q_0q_1q_3], [q_0q_1q_4], [q_0q_2q_3], [q_0q_2q_4], [q_0q_3q_4], [q_1q_2q_3], [q_1q_2q_4], [q_2q_3q_4], [q_0q_1q_2q_3],\}$

$[q_0 q_1 q_2 q_4], [q_0 q_1 q_3 q_4], [q_1 q_2 q_3 q_4], [q_0 q_1 q_2 q_3 q_4] \}$

(iii) $q_0' = [q_0 q_1 q_2]$

(iv) $F' = \{ [q_4], [q_0 q_4], [q_1 q_4], [q_2 q_4], [q_3 q_4], [q_0 q_1 q_4], [q_1 q_2 q_4], [q_2 q_3 q_4], [q_0 q_1 q_2 q_4], [q_0 q_2 q_3 q_4], [q_1 q_2 q_3 q_4], [q_0 q_1 q_2 q_3 q_4] \}$

(v) δ' is defined in Transition diagram.

⇒ **Construction of Minimum automaton.**

Procedure:-

Step 1:- Construction of π_0

Partition the states of given automaton as set of final states and set of non-final states.

By definition of 0-Equivalence $\pi_0 = \{Q_1^0, Q_2^0\}$, where

Q_1^0 = the set of all final states.

Q_2^0 = the set of non-final states. $Q_2^0 = Q - Q_1^0$.

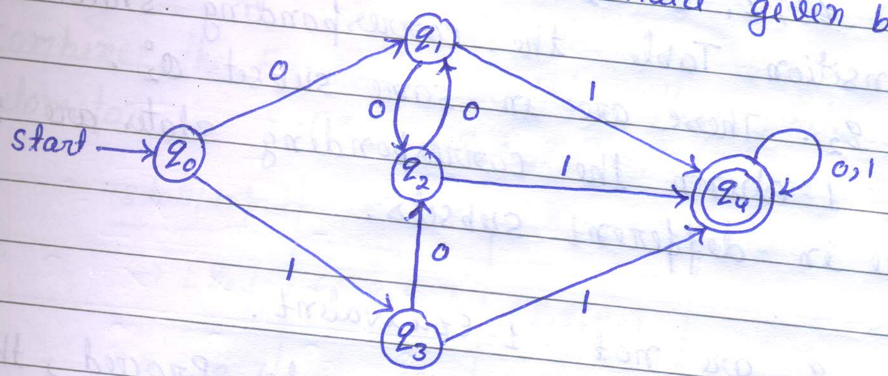
Step 2:- Construction of π_{K+1} from π_K.

Step 3:- Construct π_n for $n = 1, 2, \ldots$ until $\pi_n = \pi_{n+1}$.

Step 4:- Construction of Minimum Automaton.

Note:- Two state q_1 & q_2 are equivalent if both $\delta(q_1, x)$ and $\delta(q_2, x)$ are final states or both of them are non final states for all $x \in \Sigma^*$.

Q1) Minimization of finite Automata given below:-



Solution:- By Set Method

State/input	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_2	q_4
q_2	q_1	q_4
q_3	q_2	q_4
q_4	q_4	q_4

Transition Table

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

Step 1:- Construction of π_0. (Partition the states of given automaton as set of final state and set of Non-final states.

$\pi_0 = \{Q_1^0, Q_2^0\}$

$\pi_0 = \{\{q_4\}, \{q_0, q_1, q_2, q_3\}\}$

Step 2 & Step 3: Construction of π_{K+1} from π_K. So construct π_n where $n = 1, 2, 3 \ldots K$. until $\pi_n = \pi_{n+1}$.

(i) construction of π_1 from π_0.

We cant Partition of $Q_1^0 = \{q_4\}$

Partitioning of $Q_2^0 = \{q_0, q_1, q_2, q_3\}$.

⇒ consider $q_0, q_1 \in Q_2^\circ$.
In Transition Table the corresponding states in 0-column are q_1, q_2. These are in same subset Q_2°.
But in 1-column the corresponding states are q_3, q_4.
These are in different subsets.

so q_0, q_1 are not 1-equivalent.
In the same way, we have to Proceed, the result is q_0 is not 1-equivalent with q_2, q_3 also.

⇒ consider $q_1, q_2 \in Q_2^\circ$
In transition Table the corresponding states in 0-column are q_2, q_1. These belongs to same subset Q_2°. the corresponding states in 1-column are q_4, q_4 which is in subset Q_1°.
so q_1, q_2 are 1-equivalent. In the same way, we have to Proceed, the result is q_1 is 1-equivalent with q_3.
so q_1, q_2 and q_3 are 1-equivalent.

$$\pi_1 = \{ \{q_4\}, \{q_0\}, \{q_1, q_2, q_3\} \}$$

(ii) constructruction of π_2 from π_1.

using Procedure of (i), find that q_1, q_2 and q_3 are 2-equivalent.

$$\pi_2 = \{ \{q_4\}, \{q_0\}, \{q_1, q_2, q_3\} \}$$

Here $\pi_1 = \pi_2$

⇒ ~~Step 3.~~ Stop the construction of π_n because $\pi_n = \pi_{n+1}$ is True.

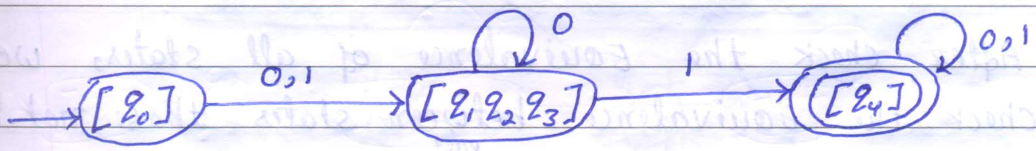
so $\pi_1 = \{ \{q_4\}, \{q_0\}, \{q_1, q_2, q_3\} \}$

Step 4:- Construction of Minimum Automaton.

Combine the Equivalent states thus states of Minimum Automaton is $\{[q_0], [q_1 q_2 q_3], [q_4]\}$.

stat/input	0	1
$\rightarrow [q_0]$	$[q_1 q_2 q_3]$	$[q_1 q_2 q_3]$
$[q_1 q_2 q_3]$	$[q_1 q_2 q_3]$	$[q_4]$
$[q_4]$	$[q_4]$	$[q_4]$

Transition Table of Minimize DFA



Transition diagram

DFA represents by 5 Tupples $(Q, \Sigma, \delta, q_0, F)$, where

(i) $Q = \{[q_0], [q_1 q_2 q_3], [q_4]\}$

(ii) $\Sigma = \{0, 1\}$

(iii) $[q_0]$ is the initial state.

(iv) $F = \{[q_4]\}$

(v) δ is defined in Transition diagram.

Solution:- by cross (X) method.

~~Construction~~ of π_0.

(i) initialy, X is Placed in each entry corresponding to one final stat and one non-final state.

(ii) Check the Equivalence of q_0 with q_1. In Transition Table the corresponding states in 1-column are q_3, q_4.

because X is exist between q_3 and q_4. So we Place X between q_0 and q_1.
In the same way, we have to Proceed then result is
Place X between q_0 and q_2.
Place X between q_0 and q_3.

q_1	X			
q_2	X	-		
q_3	X	-	-	
q_4	X	X	X	X
	q_0	q_1	q_2	q_3

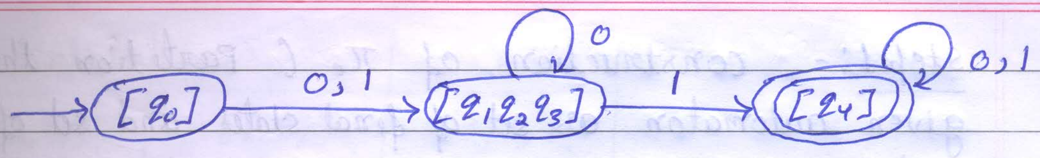
After check the Equivalence of all states, we again check the Equivalence between states those not have X sign.
check the Equivalence of pair $(q_1$ & $q_2)$, $(q_2$ & $q_3)$, $(q_1$ & $q_3)$.
If No change is met then stop this Procedure.
otherwise Repeat this Procedure.

So Equivalent states are (q_1, q_2), (q_2, q_3), (q_1, q_3).
<u>or</u> (q_1, q_2, q_3).

combine the Equivalent states. then states are
$\{[q_0], [q_1 q_2 q_3], [q_4]\}$.

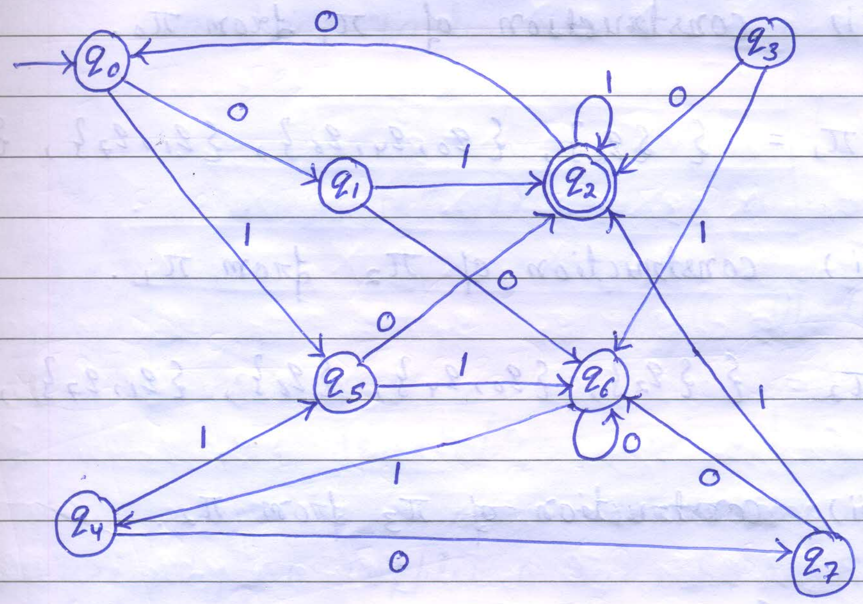
state/input	0	1
$\rightarrow [q_0]$	$[q_1 q_2 q_3]$	$[q_1 q_2 q_3]$
$[q_1 q_2 q_3]$	$[q_1 q_2 q_3]$	$[q_4]$
$[q_4]$	$[q_4]$	$[q_4]$

Transition Table.



Transition diagram

Q2) Construct a minimum state Automaton Equivalent to the given Finite Automaton.



Solution :-

state/input	0	1
$\rightarrow q_0$	q_1	q_5
q_1	q_6	q_2
q_2	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

by set Method

Transition Table.

Here $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$

Step 1:- construction of π_0 (Partition the states of given automaton as set of final states and set of non-final states.

$$\pi_0 = \{ Q_1^0 , Q_2^0 \}$$

$$\pi_0 = \{ \{q_2\}, \{q_0, q_1, q_3, q_4, q_5, q_6, q_7\} \}$$

Step 2 & step 3: construction of π_{k+1} from π_k. so construct π_n where $n = 1, 2, 3, \dots k$. until $\pi_n = \pi_{n+1}$.

(i) construction of π_1 from π_0.

$$\pi_1 = \{ \{q_2\}, \{q_0, q_4, q_6\}, \{q_1, q_7\}, \{q_3, q_5\} \}$$

(ii) construction of π_2 from π_1.

$$\pi_2 = \{ \{q_2\}, \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{q_3, q_5\} \}$$

(iii) construction of π_3 from π_2.

$$\pi_3 = \{ \{q_2\}, \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{q_3, q_5\} \}$$

Here $\pi_2 = \pi_3$

stop the construction of π_n because $\pi_n = \pi_{n+1}$ is True.

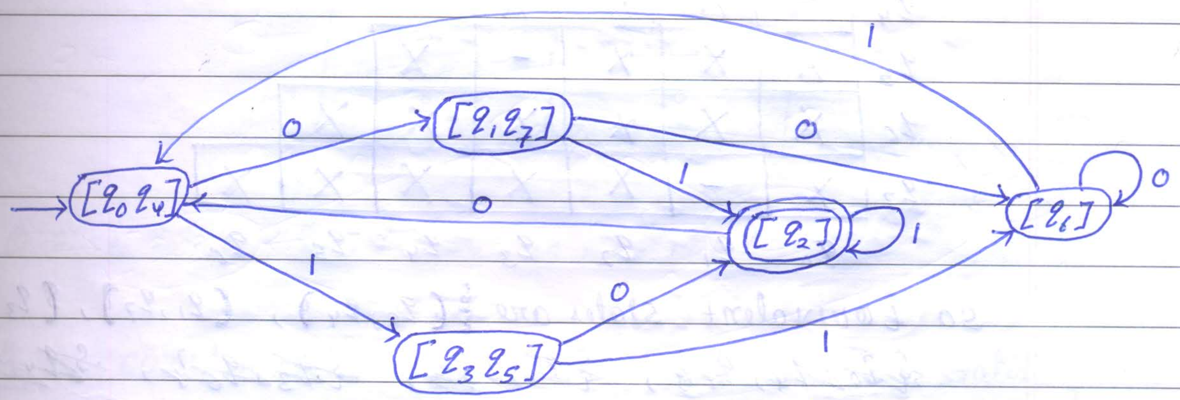
so $\pi_2 = \{ \{q_2\}, \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{q_3, q_5\} \}$

Step 4:- Construction of Minimum Automaton.

Combine the Equivalent states thus states of Minimum Automaton is $\{ [q_0 q_4], [q_1 q_7], [q_2], [q_3 q_5], [q_6] \}$.

state / input	0	1
$\rightarrow [q_0 q_4]$	$[q_1 q_7]$	$[q_3 q_5]$
$[q_1 q_7]$	$[q_6]$	$[q_2]$
$[q_2]$	$[q_0 q_4]$	$[q_2]$
$[q_3 q_5]$	$[q_2]$	$[q_6]$
$[q_6]$	$[q_6]$	$[q_0 q_4]$

Transition Table.



Transition diagram

DFA represent by 5 Tupples $(Q, \Sigma, \delta, q_0, F)$, where

(i) $Q = \{ [q_0 q_4], [q_1 q_7], [q_2], [q_3 q_5], [q_6] \}$.

(ii) $\Sigma = \{0, 1\}$

(iii) $q_0 = [q_0 q_4]$

(iv) $F = \{ [q_2] \}$

(v) δ is defined in Transition diagram.

Solution :- by X cross Method.

(i) initially, X is Placed in each entry corresponding to one final state and one non-final state.

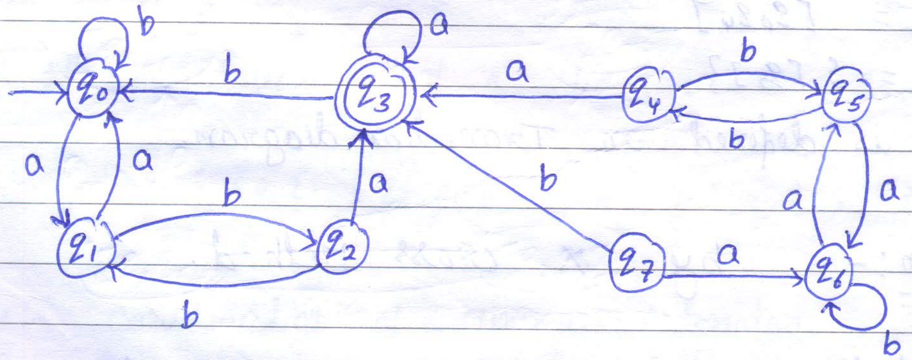
(ii) check the Equivalence of q_0 with q_1. In Transition Table the corresponding state in 1-column are q_5, q_2. because X is exist between q_5 & q_2 so we place X between q_0 & q_1. In the same way, check the Equivalence of all states.

After check the Equivalence of all states, we again check the Equivalence between states those not have X sign.
If No change is met then stop this Procedure otherwise Repeat this Procedure.

q_1	X						
q_2	X	X					
q_3	X	X	X				
q_4	–	X	X	X			
q_5	X	X	X	–	X		
q_6	X	X	X	X	X	X	
q_7	X	–	X	X	X	X	X
	q_0	q_1	q_2	q_3	q_4	q_5	q_6

so Equivalent states are $\{\{q_0, q_4\}, \{q_1, q_7\}, \{q_3, q_5\}\}$.
~~$\{q_0, q_4, q_4\}$~~, ~~$\{q_1, q_7\}$~~, ~~$\{q_3, q_5\}$~~, ~~$\{q_7, q_2\}$~~
so combine the Equivalent states. then states are
$\{ [q_0 q_4], [q_1 q_7], [q_2], [q_3 q_5], [q_6] \}$.

Q3) Minimize the following Automaton.



solution :-

by set Method

state/input	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
q_3	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3

$Q = \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7 \}$.

Step 1 :- construction of π_0. (Partition the states of given automata as set of final states and set of Non-final states.

$$\pi_0 = \{ Q_1^0 , Q_2^0 \}$$

$$\pi_0 = \{ \{ q_3 \}, \{ q_0, q_1, q_2, q_4, q_5, q_6, q_7 \} \}$$

Step2 and step3 :- construction of π_{k+1} from π_k.
so construct π_n where $n = 1, 2, \ldots$ until $\pi_n = \pi_{n+1}$.

(i) construction of π_1 from π_0.

$\pi_1 = \{ \{ q_3 \}, \{ q_0, q_1, q_5, q_6 \}, \{ q_2, q_4 \}, \{ q_7 \} \}$.

(ii) construction of π_2 from π_1.

$\pi_2 = \{ \{ q_3 \}, \{ q_0, q_6 \}, \{ q_1, q_5 \}, \{ q_2, q_4 \}, \{ q_7 \} \}$.

(iii) construction of π_3 from π_2.

$\pi_3 = \{ \{ q_3 \}, \{ q_0, q_6 \}, \{ q_1, q_5 \}, \{ q_2, q_4 \}, \{ q_7 \} \}$.

Here $\pi_2 = \pi_3$.

stop the construction of π_n because $\pi_n = \pi_{n+1}$ is True.

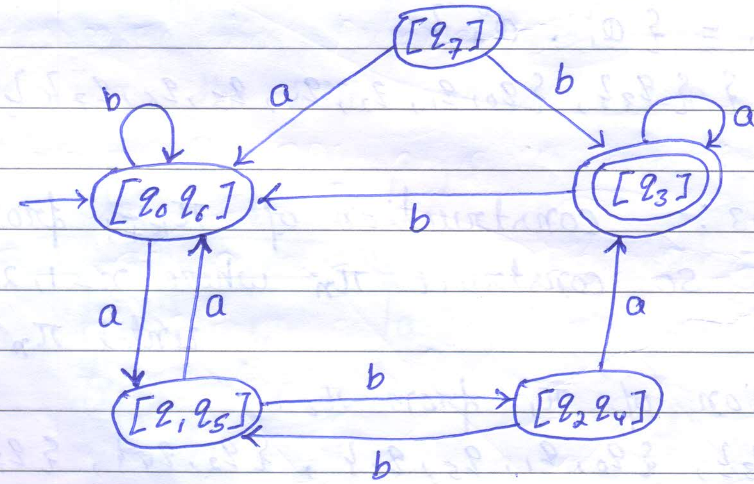
So $\pi_2 = \{ \{q_3\}, \{q_0, q_6\}, \{q_1, q_5\}, \{q_2, q_4\}, \{q_7\} \}$.

Step 4:- construction of Minimum Automaton.

combine the Equivalent states thus states of Minimum Automaton is $\{ [q_0 q_6], [q_1 q_5], [q_2 q_4], [q_3], [q_7] \}$.

state / input	a	b
$\rightarrow [q_0 q_6]$	$[q_1 q_5]$	$[q_0 q_6]$
$[q_1 q_5]$	$[q_0 q_6]$	$[q_2 q_4]$
$[q_2 q_4]$	$[q_3]$	$[q_1 q_5]$
$[q_3]$	$[q_3]$	$[q_0 q_6]$
$[q_7]$	$[q_0 q_6]$	$[q_3]$

Transition Table



Transition diagram.

DFA represents by 5 Tupples $(Q, \Sigma, \delta, q_0, F)$, where

(i) $Q = \{ [q_0 q_6], [q_1 q_5], [q_2 q_4], [q_3], [q_7] \}$

(ii) $\Sigma = \{a, b\}$

(iii) $q_0 = [q_0 q_6]$

(iv) $F = [q_3]$

(v) δ is defined in Transition diagram.

Solution :- by X cross method

q_1	X						
q_2	X	X					
q_3	X	X	X				
q_4	X	X	-	X			
q_5	X	-	X	X	X		
q_6	-	X	X	X	X	X	
q_7	X	X	X	X	X	X	X
	q_0	q_1	q_2	q_3	q_4	q_5	q_6

Equivalent ~~sate~~ states are (q_0, q_6), (q_1, q_5), (q_2, q_4).

Combine the Equivalent states so state of automaton are $\{ [q_0 q_6], [q_1 q_5], [q_2 q_4], [q_3], [q_7] \}$.

Q4) Minimize the Following automaton.

state / input	a	b
$\rightarrow q_0$	q_0	q_3
q_1	q_2	q_5
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_6
q_5	q_1	q_4
q_6	q_1	q_3

(BY set Method)

Solution :- $Q = \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \}$.

Step 1 :- construction of π_0. (Partition the states of given automata as set of final states and set of Non-final states.

$\pi_0 = \{ Q_1^0, Q_2^0 \}$.

$\pi_0 = \{ \{q_6\}, \{q_0, q_1, q_2, q_3, q_4, q_5\} \}$

Step 2 and step 3:- construction of π_{K+1} from π_K
So construct π_n where $n = 1, 2, \ldots$ until $\pi_n = \pi_{n+1}$.

(i) construction of π_1 from π_0
$\pi_1 = \{ \{q_6\}, \{q_0, q_1, q_2, q_3, q_5\}, \{q_4\} \}$

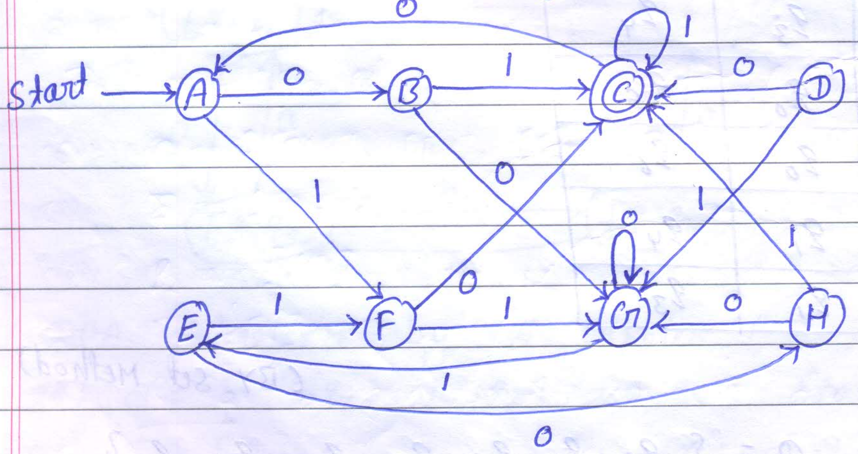
(ii) construction of π_2 from π_1.
$\pi_2 = \{ \{q_6\}, \{q_0, q_1, q_3\}, \{q_2, q_5\}, \{q_4\} \}$.

(iii) construction of π_3 from π_2.
$\pi_3 = \{ \{q_6\}, \{q_0\}, \{q_1\}, \{q_3\}, \{q_2, q_5\}, \{q_4\} \}$.

(iv) construction of π_4 from π_3.
$\pi_4 = \{ \{q_6\}, \{q_0\}, \{q_1\}, \{q_3\}, \{q_2\}, \{q_5\}, \{q_4\} \}$.

No Equivalent states are found so can not minimize the Automaton.

Q5) Minimize the DFA as given below:-



solution:- by set method

state/input	0	1
→A	B	F
B	G	C
C	A	C
D	C	G
E	H	F
F	C	G
G	G	E
H	G	C

Transition Table.

$Q = \{A, B, C, D, E, F, G, H\}$

Step 1:- construction of π_0. (Partition the states of Given automaton as set of final states and set of Non-final states.

$\pi_0 = \{Q_1^0, Q_2^0\}$

$\pi_0 = \{\{C\}, \{A, B, D, E, F, G, H\}\}$.

Step 2 and step 3:- construction of π_{K+1} from π_K.
So construct π_n where $n = 1, 2 \ldots$ until $\pi_n = \pi_{n+1}$

(i) construction of π_1 from π_0.

$\pi_1 = \{\{C\}, \{A, E, G\}, \{B, H\}, \{D, F\}\}$

(ii) construction of π_2 from π_1.

$\pi_2 = \{\{C\}, \{A, E\}, \{G\}, \{B, H\}, \{D, F\}\}$

(iii) construction of π_3 from π_2.

$\pi_3 = \{\{C\}, \{A, E\}, \{G\}, \{B, H\}, \{D, F\}\}$

Here $\pi_2 = \pi_3$

stop the construction of π_n because $\pi_n = \pi_{n+1}$ is True. so

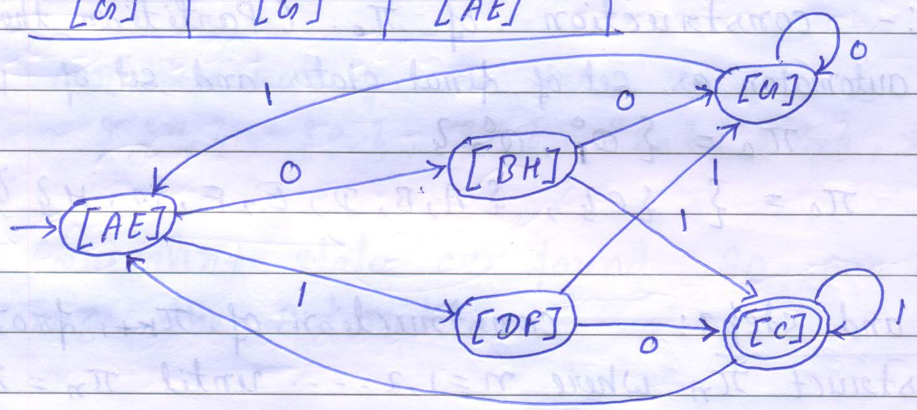
$\pi_2 = \{\{C\}, \{A, E\}, \{G\}, \{B, H\}, \{D, F\}\}$.

~~Thus we get the Pair of Equivalent state.~~
~~(A, E), (B, H), (D, F).~~

Step 4 :- Construction of Minimum Automaton.
Combine the Equivalent states thus states of Minimum Automaton is { [A, E], [BH], [C], [DF], [G] }

state/input	0	1
→ [AE]	[BH]	[DF]
[BH]	[G]	[C]
[C]	[AE]	[C]
[DF]	[C]	[G]
[G]	[G]	[AE]

Transition Table



Transition diagram

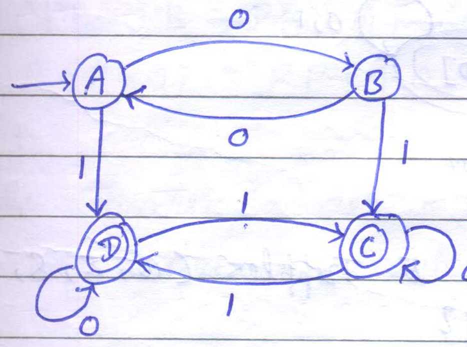
DFA represented by 5 Tupples $(Q, \Sigma, \delta, q_0, F)$, where
(i) Q = { [AE], [BH], [C], [DF], [G] }
(ii) Σ = { 0, 1 }
(iii) q_0 = [AE]
(iv) F = { [C] }
(v) δ is defined in Transition diagram.

Solution:-

	A	B	C	D	E	F	G
B	X						
C	X	X					
D	X	X	X				
E	-	X	X	X			
F	X	X	X	-	X		
G	X	X	X	X	X	X	
H	X	-	X	X	X	X	X

Thus we get Equivalent Pair (A, E), (B, H), (D, F).
Combine the Equivalent states so we get the states of Automaton { [AE], [BH], [C], [DF], [G] }.

Q6) Minimize the DFA as given below:-



State/input	0	1
A	B	D
B	A	C
C	C	D
D	D	C

Transition Table.

Solution:- By set Method.
$Q = \{A, B, C, D\}$

Step1:- construction of π_0. (Partition the states of given automaton as set of final states and set of Non-final states.

$\pi_0 = \{Q_1^0, Q_2^0\} = \{\{C, D\}, \{A, B\}\}$.

$\pi_0 = \{\{A, B\}, \{C, D\}\}$.

Step2 & Step3:- Construction of π_{K+1} from π_K. So construct the π_n ~~for~~ where $n = 1, 2 ----$ until $\pi_n = \pi_{n+1}$.

(i) construction of π_1 from π_0.

$\pi_1 = \{\{A, B\}, \{C, D\}\}$

construction of π_2 from π_1.

$\pi_2 = \{ \{A,B\}, \{C,D\} \}$.

Here $\pi_1 = \pi_2$

so stop the construction of π_n because $\pi_n = \pi_{n+1}$ is True.

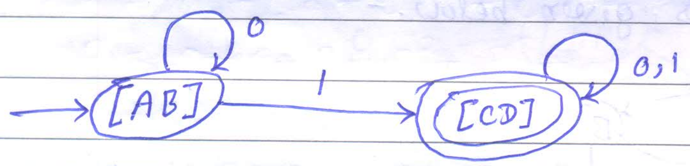
so $\pi_1 = \{ \{A,B\}, \{C,D\} \}$.

Step 4:- construction of Minimum Automaton.

combine the Equivalent states thus states of Minimum Automaton are $\{ [AB], [CD] \}$.

state/input	0	1
→ [AB]	[AB]	[CD]
[CD]	[CD]	[CD]

Transition Table



Transition diagram

DFA is represented by 5 Tupples $(Q, \Sigma, \delta, q_0, F)$, where

(i) $Q = \{ [AB], [CD] \}$

(ii) $\Sigma = \{0, 1\}$

(iii) $q_0 = [AB]$

(iv) $F = \{ [CD] \}$

(v) δ is defined in Transition diagram.

Solution: by X (cross) Method.

	A	B	C
B	-		
C	X	X	
D	X	X	-

Thus Equivalent states are (A,B) & (C,D).

combine the Equivalent states. then states of Automata are $\{ [AB], [CD] \}$.

⇒ Conversion of Finite Automaton to Regular Expression :-

Identities for Regular Expression :- These are useful for simplifying regular Expression.

1. $\phi + R = R$
2. $\phi R = R\phi = \phi$
3. $\Lambda R = R\Lambda = R$
4. $\Lambda^* = \Lambda$ and $\phi^* = \Lambda$
5. $R + R = R$
6. $R^* R^* = R^*$
7. $RR^* = R^* R$
8. $(R^*)^* = R^*$
9. $\Lambda + RR^* = R^* = \Lambda + R^* R$
10. $(PQ)^* P = P(QP)^*$
11. $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$
12. $(P+Q)R = PR + QR$ and
    $R(P+Q) = RP + RQ$.

⇒ **Arden's Theorem :-** let P and Q be two regular Expressions over Σ. If P does not contain Λ, then the following Equation in R,

$R = Q + RP$ has a unique solution given by $R = QP^*$.

PROOF :- L.H.S.

$$= R$$
$$= QP^* \qquad \{\because R = QP^*\}$$

R.H.S.

$$= Q + RP$$
$$= Q + (QP^*)P \qquad \{\because R = QP^*\}$$
$$= Q(\epsilon + P^* P)$$
$$= QP^*$$

L.H.S. = R.H.S.

This means $R = QP^*$ is a solution of Equation $R = Q + RP$.

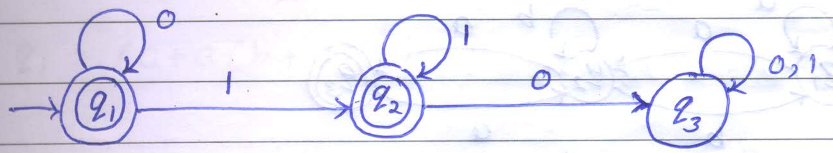
To Prove uniqueness, Here Replacing R by $Q + RP$ on the R.H.S.

$$R = Q + RP$$
$$R = Q + (Q + RP)P$$
$$R = Q + QP + RP^2$$
$$R = Q + QP + (Q + RP)P^2$$
$$R = Q + QP + QP^2 + RP^3$$
$$R = Q + QP + QP^2 + (Q + RP)P^3$$
$$R = Q + QP + QP^2 + QP^3 + RP^4$$
$$\vdots$$
$$R = Q + QP + QP^2 + QP^3 + \cdots + QP^i + RP^{i+1}$$
$$R = Q(\epsilon + P + P^2 + P^3 + \cdots + P^i) + RP^{i+1}$$

As P does not contain ϵ, RP^{i+1} has no string of length less than $i+1$, and so w is not in the set RP^{i+1}. This means w belongs to the set $Q(\epsilon + P + P^2 + \cdots + P^i)$, and hence to QP^*.

$$\boxed{R = QP^*}$$

Q1.) Find regular expression that denotes the set accepted by FA whose transition diagram is given below.



Solution:- Find Equation for Each state.

$q_1 = \epsilon + q_1 0$ ——— (i)

$q_2 = q_1 1 + q_2 1$ ——— (ii)

$q_3 = q_2 0 + q_3 (0+1)$ ——— (iii)

Here final states are q_1 & q_2 so find the regular Expression for q_1 & q_2.

Applying Arden's theorem on EQ-(i)

$$\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{0}_{P}$$

so we get $\quad q_1 = \epsilon 0^*$ $\quad \{ \because R = QP^* \}$

$q_1 = 0^*$

Put the value of q_1 into EQ-(ii)

$q_2 = q_1 1 + q_2 1$ ——— (ii)

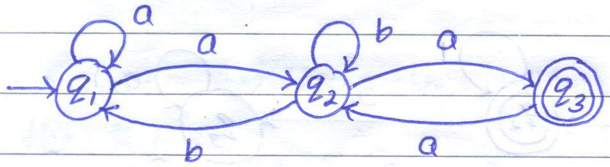
$$\underbrace{q_2}_{R} = \underbrace{0^* 1}_{Q} + \underbrace{q_2}_{R} \underbrace{1}_{P}$$

Apply Arden's theorem :-

$q_2 = 0^* 1 1^*$

Hence $q_1 + q_2 = 0^* + 0^* 1 1^* = 0^* (\epsilon + 1 1^*) = \underline{\underline{0^* 1^*}}$ Ans

Q2) consider the transition system given in fig. Prove that the string recognised are $(a + a(b+aa)^* b)^* a (b+aa)^* a$.



<u>Solution</u>:- Find Equation for each state.

$$q_1 = \epsilon + q_1 a + q_2 b \quad \text{——} \quad (i)$$
$$q_2 = q_1 a + q_2 b + q_3 a \quad \text{——} \quad (ii)$$
$$q_3 = q_2 a \quad \text{——} \quad (iii)$$

Here final state is q_3. so find the regular Expression for q_3.

By substituting the value of q_3 in EQ-(ii)

$$q_2 = q_1 a + q_2 b + q_3 a \quad \text{——} \quad (ii)$$
$$q_2 = q_1 a + q_2 b + q_2 a a$$
$$\underbrace{q_2}_{R} = \underbrace{q_1 a}_{Q} + \underbrace{q_2}_{R} \underbrace{(b + aa)}_{P}$$

Apply Arden's Theorem:-

$$q_2 = q_1 a (b+aa)^*$$

By substituting the value of q_2 in EQ-(i)

$$q_1 = \epsilon + q_1 a + q_2 b \quad \text{——} \quad (i)$$
$$q_1 = \epsilon + q_1 a + q_1 a (b+aa)^* b$$
$$\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{(a + a(b+aa)^* b)}_{P}$$

Apply Arden's theorem:

$$q_1 = \epsilon\,(a+a(b+aa)^* b)^*$$

$$q_1 = (a+a(b+aa)^* b)^*$$

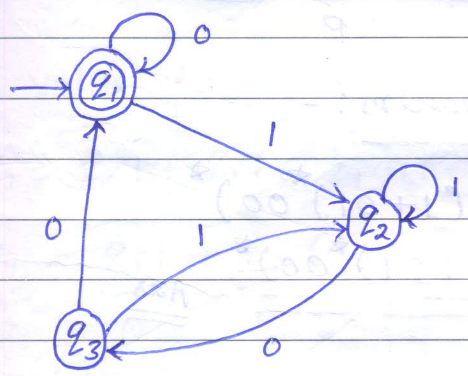
then $q_2 = (a+a(b+aa)^* b)^* a(b+aa)^*$

then $q_3 = (a+a(b+aa)^* b)^* a(b+aa)^* a$

since q_3 is a final state, the set of strings recognized by the graph is given by

$$(a+a(b+aa)^* b)^* a(b+aa)^* a.$$

Q3) find the Regular Expression for following Automaton.



Solution:- Find Equations for each state.

$$q_1 = \epsilon + q_1 0 + q_3 0 \quad \text{— (i)}$$

$$q_2 = q_1 1 + q_2 1 + q_3 1 \quad \text{— (ii)}$$

$$q_3 = q_2 0 \quad \text{— (iii)}$$

Here final state is q_1 so find the regular Expression for q_1.
By substituting the value of q_3 in EQ (ii)

$$q_2 = q_1 1 + q_2 1 + q_3 1 \quad \text{— (ii)}$$

$$q_2 = q_1 1 + q_2 1 + q_2 0 1$$

$$\underbrace{q_2}_{R} = \underbrace{q_1 1}_{Q} + \underbrace{q_2}_{R} \underbrace{(1+01)}_{P}$$

Apply Arden's theorem:-

$$q_2 = q_1 1 (1+01)^*$$

then $q_3 = q_1 1 (1+01)^* 0$

Put the value of q_3 in EQ-(i)

$$q_1 = \epsilon + q_1 0 + q_3 0 \quad \text{---} (i)$$

$$q_1 = \epsilon + q_1 0 + q_1 1 (1+01)^* 00$$

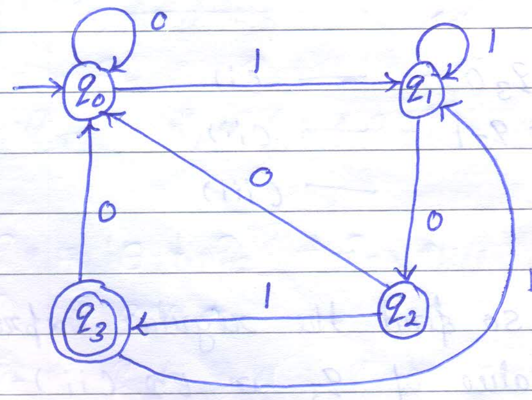
$$\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{(0 + 1(1+01)^* 00)}_{P}$$

Apply Arden's theorem:-

$$q_1 = \epsilon (0 + 1(1+01)^* 00)^*$$

$$q_1 = (0 + 1(1+01)^* 00)^* \quad \text{Ans}$$

Q4) find the Regular Expression for following Finite Automaton



$$\underbrace{q_2}_{R} = \underbrace{q_1 1}_{Q} + \underbrace{q_2}_{R} \underbrace{(1+01)}_{P}$$

Apply Arden's theorem:-

$$q_2 = q_1 1 (1+01)^*$$

then $q_3 = q_1 1 (1+01)^* 0$

Put the value of q_3 in EQ-(i)

$$q_1 = \epsilon + q_1 0 + q_3 0 \quad \text{---} (i)$$

$$q_1 = \epsilon + q_1 0 + q_1 1 (1+01)^* 00$$

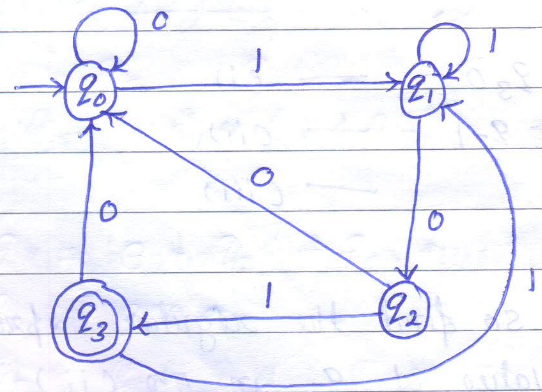
$$\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{(0 + 1(1+01)^* 00)}_{P}$$

Apply Arden's theorem:-

$$q_1 = \epsilon (0 + 1(1+01)^* 00)^*$$

$$q_1 = (0 + 1(1+01)^* 00)^*$$ Ans

Q4) find the Regular Expression for following Finite Automaton



solution:- Find Equation for each state.

$$q_0 = \epsilon + q_0 0 + q_2 0 + q_3 0 \quad \text{— (i)}$$
$$q_1 = q_0 1 + q_1 1 + q_3 1 \quad \text{— (ii)}$$
$$q_2 = q_1 0 \quad \text{— (iii)}$$
$$q_3 = q_2 1 \quad \text{— (iv)}$$

Here final state is q_3 so find the regular Expression for q_3.

Put the value of q_2 in EQ-(iv)

$$q_3 = q_2 1 \quad \text{— (iv)}$$
$$q_3 = q_1 01$$

Put the value of q_3 in EQ-(ii)

$$q_1 = q_0 1 + q_1 1 + q_1 011$$

$$\underbrace{q_1}_{R} = \underbrace{q_0 1}_{Q} + \underbrace{q_1}_{R} \underbrace{(1+011)}_{P}$$

Apply Arden's theorem:-

$$q_1 = q_0 1 (1+011)^*$$

find the q_2 & q_3 by putting the value of q_1.

then $q_2 = q_0 1 (1+011)^* 0$

then $q_3 = q_0 1 (1+011)^* 01$

substitute the value of q_2 and q_3 in EQ-(i)

$$q_0 = \epsilon + q_0 0 + q_2 0 + q_3 0 \quad \text{— (i)}$$
$$q_0 = \epsilon + q_0 0 + q_0 1 (1+011)^* 00 + q_0 1 (1+011)^* 010$$

$$\underbrace{q_0}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_0}_{R}\underbrace{(0+1(1+011)^*00+1(1+011)^*010)}_{P}$$

Apply Arden's theorem:-

$$q_0 = \epsilon(0+1(1+011)^*00+1(1+011)^*010)^*$$

$$q_0 = (0+1(1+011)^*(00+010))^*$$

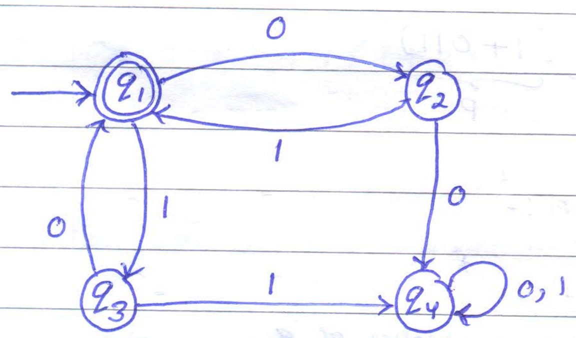
Put the value of q_0 and find the q_3.

$$q_3 = q_0 1(1+011)^* 01$$

$$q_3 = \underline{(0+1(1+011)^*(00+010))^* 1(1+011)^* 01}$$

Ans

Q5) Find Regular Expression for the DFA given in fig.



solution:- Find Equation for each state.

$q_1 = \epsilon + q_2 1 + q_3 0$ —— (i)
$q_2 = q_1 0$ —— (ii)
$q_3 = q_1 1$ —— (iii)
$q_4 = q_2 0 + q_3 1 + q_4(0+1)$ — (iv)

Here final state is q_1. So find the Regular Expression for q_1.

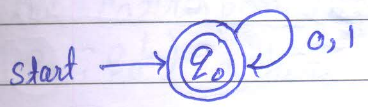
Put the value of q_2 and q_3 in EQ — (i)

$$q_1 = \epsilon + q_2 1 + q_3 0 \quad \text{— (i)}$$
$$q_1 = \epsilon + q_1 01 + q_1 10$$
$$\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R}\underbrace{(01+10)}_{P}$$

Apply Arden's theorem:-

$$q_1 = \epsilon\,(01+10)^*$$
$$q_1 = \underline{(01+10)^*} \quad \text{Ans}$$

Q6) Find the Regular Expression for following Finite Automaton.



Solution:- Find the Equation for q_0.

$$\underbrace{q_0}_{R} = \epsilon + q_0 (0+1) \quad \text{— (i)}$$

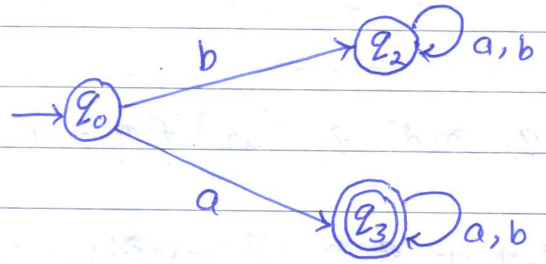
Here final state is q_0 so ~~solve~~ find the regular Expression for q_0.

$$\underbrace{q_0}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_0}_{R}\underbrace{(0+1)}_{P}$$

Apply Arden's theorem:-

$$q_0 = \epsilon\,(0+1)^*$$
$$q_0 = \underline{(0+1)^*} \quad \text{Ans}$$

Q7) Find the Regular Expression for following Finite Automaton.



Solution :- Find Equation for each state.

$$q_0 = \epsilon \quad \text{— (i)}$$
$$q_2 = q_0 b + q_2 (a+b) \quad \text{— (ii)}$$
$$q_3 = q_0 a + q_3 (a+b) \quad \text{— (iii)}$$

Here q_3 is final state so find the Regular Expression for q_3.

$$\underbrace{q_3}_{R} = \underbrace{q_0 a}_{Q} + \underbrace{q_3}_{R} \underbrace{(a+b)}_{P}$$

Apply Arden's theorem:-

$$q_3 = q_0 a (a+b)^*$$
$$q_3 = \epsilon a (a+b)^*$$
$$q_3 = \underline{\underline{a (a+b)^*}}$$

Ans

⇒ Equivalence of Two Finite State Machines.

Two Finite Automata over S are equivalent if they accept the same set of strings over S. Two Finite Automata over S are not equivalent if for some string w over S one automaton reaches a final state and the other automaton reaches a non-final state.

⇒ Procedure to test the equivalence of two FSM over S.

let M and M' be two FSM over Σ.

step 1:- construct comparision table consisting of $n+1$ columns, where n is the number of input symbols.
The first column consists of Pairs of vertices of the form (q, q'), where $q \in M$ and $q' \in M'$.
If (q, q') appears in some row of the first column, then the corresponding entry in the a-column $(a \in \Sigma)$ is (q_a, q_a') where $\delta(q, a) = q_a$, $\delta'(q', a) = q_a'$.

step 2:- comparision Table is constructed by starting with the Pair of initial vertices q_0, q_0' of M and M' in the first column.
The first elements in the subsequent columns are (q_a, q_a'), where $\delta(q_0, a) = q_a$, $\delta'(q_0', a) = q_a'$.

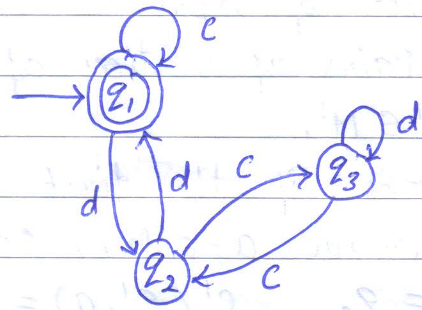
step 3:- Repeat the construction by considering the Pairs in the second and subsequent columns which are not in the first column.
The Row-wise construction is repeated. There are two cases:-

Case 1:- If we reach a Pair (q, q') such that q is a final state of M, and q' is a non-final state of M' or vice versa, we terminate the construction and conclude that M and M' are not equivalent.

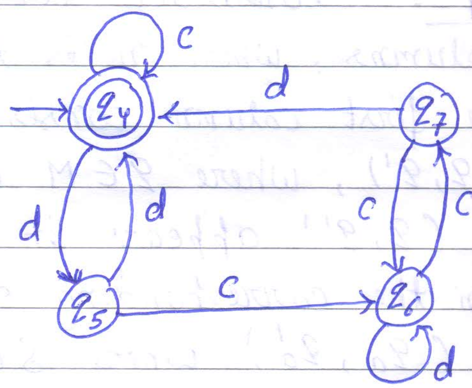
Case 2:- Here the construction of table is terminated when no new element appears in the second and subsequent columns which are not in the first column.

In this case we conclude that M, M' are equivalent.

Q1) Consider the following two DFA M and M' over $\{c, d\}$.



(a)



(b)

find that they are equivalent or not.

Solution :-

The initial states in M and M' are q_1 and q_4 respectively. Hence the first element of the first column in the comparision table must be (q_1, q_4).

The first element in second column is (q_1, q_4). ~~The first element in second column is (q_1, q_4)~~ since both q_1, q_4 are c-reachable from respective initial states.

The first element in third column in (q_2, q_5) since $\delta(q_1, d) = q_2$
$\delta(q_4, d) = q_5$

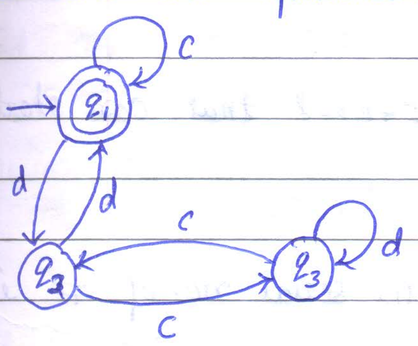
Repeating the construction by taking new pair (q_2, q_5) in first row and so on, gives the following table.

(q, q')	(q_c, q_c')	(q_d, q_d')
(q_1, q_4)	(q_1, q_4)	(q_2, q_5)
(q_2, q_5)	(q_3, q_6)	(q_1, q_4)
(q_3, q_6)	(q_2, q_7)	(q_3, q_6)
(q_2, q_7)	(q_3, q_6)	(q_1, q_4)

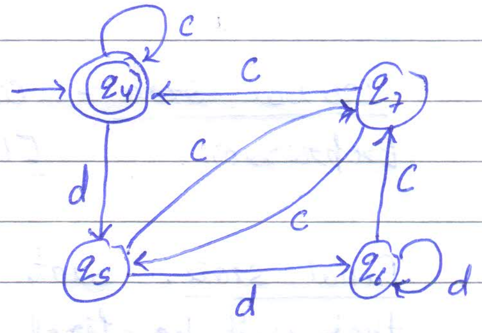
Comparision Table

M and M' are equivalent.

Q2) Show that the following automata M_1 & M_2 are not equivalent.



(a)



(b)

(q, q')	(q_c, q_c')	(q_d, q_d')
(q_1, q_4)	(q_1, q_4)	(q_2, q_5)
(q_2, q_5)	(q_3, q_7)	(q_1, q_6)

Comparision Table

~~As~~ ~~q_1~~ From table, q_1 and q_6 are d-reachable from q_2 and q_5 respectively. As q_1 is a final state in M_1 and q_6 is non-final state in M_2, we can say that M_1 and M_2 are not equivalent.

⇒ Regular Expression to DFA conversion:-

* **follow set :-** follow set of each input symbol of regular expression consists of symbols that come directly after that symbol.

Example:- $(a+b)\,a\,b\,\#$
with positions 1 2 3 4 5

followset of each input symbol:-

follow set (1) = {3}
follow set (2) = {3}
follow set (3) = {4}
follow set (4) = {5}
follow set (5) = ϕ

$\boxed{\begin{array}{l} a = 1, 3 \\ b = 2, 4 \end{array}}$

* **Initial state:-** group of symbol that can start the Expression. [12]

* **Final state:** state in which the serial no. of # is appeared that will be final state.

Q1) Design a DFA for following regular Expression:-
$(0+1)^*110\#$

Solution:- $(0+1)^*\ 1\ 1\ 0\ \#$
with positions 1 2 3 4 5 6

follow set

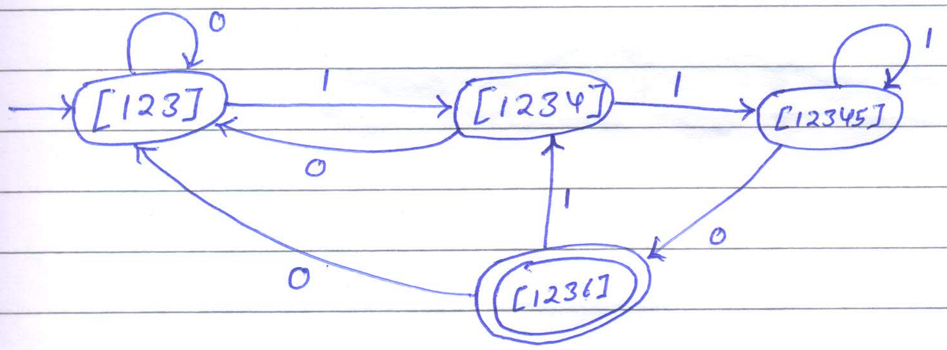
1. {1,2,3}
2. {1,2,3}
3. {4}
4. {5}
5. {6}
6. ϕ

Input 0 = 1,5

Input 1 = 2,3,4.

$\boxed{\begin{array}{l} \text{Start from [123] state.} \\ \text{first on Input 0 (1,5).} \\ \text{common between [123] \& (1,5) is 1.} \\ \text{then follow set of 1 is [123].} \end{array}}$

Then apply Input 1 on state [123]. find common between state [123] and Input 1 (=234) which is (2,3) so find union of follow set of 2 and 3. that is $\{1,2,3\} \cup \{4\} = \{1,2,3,4\}$.
so New state is [1234].



Transition diagram

Q2) find DFA for Regular Expression $a^*bb + (a+b)^+b$.

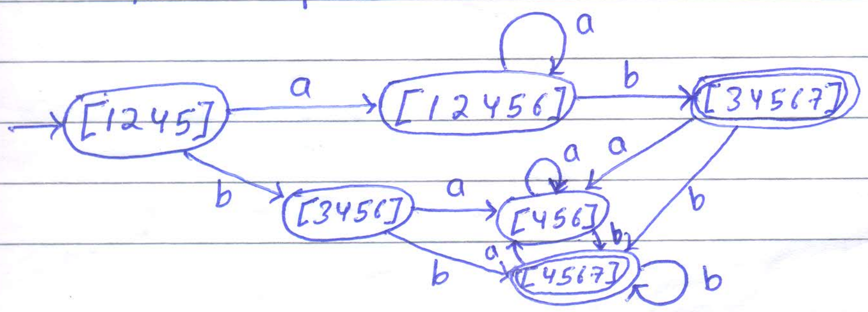
Solution:-

$$\underset{1}{[\,a}^*\underset{2}{b}\underset{3}{b} + (\underset{4}{a}+\underset{5}{b})^+\underset{6}{b}\,]\,\underset{7}{\#}$$

follow set :-

1. $\{1, 2\}$
2. $\{3\}$
3. $\{7\}$
4. $\{4, 5, 6\}$
5. $\{4, 5, 6\}$
6. $\{7\}$
7. ϕ

Input a = $\{1, 4\}$
Input b = $\{2, 3, 5, 6\}$.



Transition diagram