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Date 29-11-19

PRACTICAL NO. 1

Topic:- Limits and Continuity

1] $\lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

2] $\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

3] $\lim_{x\to \pi/6}\left[\dfrac{\cos x-\sqrt{3}\sin x}{\pi-6x}\right]$

4] $\lim_{x\to \infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

5] Examine the continuity of the following function at given points

(i) $f(x) = \left.\begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & \text{for } 0<x\le \dfrac{\pi}{2} \\ \dfrac{\cos x}{\pi-2x}, & \text{for } \dfrac{\pi}{2}<x<\pi \end{cases}\right\}$ at $x=\dfrac{\pi}{2}$

(ii) $f(x) = \left.\begin{cases} \dfrac{x^2-9}{x-3} & 0<x<3 \\ x+3 & 3\le x<6 \\ \dfrac{x^2-9}{x+3} & 6\le x<9 \end{cases}\right\}$ at $x=3$ and $x=6$.

6] Find value of k, so that the function f(x) is continous at the indicated point.

(i) $f(x) = \left.\begin{cases} \dfrac{1-\cos 4x}{x^2}, & x<0 \\ k, & x=0 \end{cases}\right\}$ at $x=0$

(ii) $f(x) = \left.\begin{cases} (\sec^2 x)^{\cot^2 x}, & x\ne 0 \\ k, & x=0 \end{cases}\right\}$ at $x=0$

(iii) $f(x) = \left.\begin{cases} \dfrac{\sqrt{3}-\tan x}{\pi-3x}, & x\ne \dfrac{\pi}{3} \\ k, & x=\dfrac{\pi}{3} \end{cases}\right\}$ at $x=\dfrac{\pi}{3}$

7] Discuss the continuity of the following functions, which of these functions have a removable discontinuity? Redefine the function so as to remove the discontinuity.

(i) $f(x) = \left.\begin{cases} \dfrac{1-\cos 3x}{x\tan x}, & x\ne 0 \\ 9, & x=0 \end{cases}\right\}$ at $x=0$

(ii) $f(x) = \left.\begin{cases} \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}, & x\ne 0 \\ \dfrac{\pi}{60}, & x=0 \end{cases}\right\}$ at $x=0$.

8] If $f(x) = \dfrac{e^{x^2}-\cos x}{x^2}$ ~~for~~ $x\ne 0$ is continous at $x=0$, find $f(0)$.

9] If $f(x) = \dfrac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$ for $x\ne \dfrac{\pi}{2}$ is continous at $x=\dfrac{\pi}{2}$, find $f\left(\dfrac{\pi}{2}\right)$.

SOLUTIONS

1] $\lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$= \lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right] \times \dfrac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}$

$= \lim_{x\to a} \dfrac{(a+2x-3x)}{(3a+x-4x)} \dfrac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}$

$= \lim_{x\to a} \dfrac{(a-x)}{(3a-3x)} \dfrac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}$

$= \dfrac{1}{3} \lim_{x\to a} \dfrac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}$

$= \dfrac{1}{3} \lim_{x\to a} \dfrac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$

$= \dfrac{1}{3} \lim_{x\to a} \dfrac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$= \dfrac{1}{3} \lim_{x\to a} \dfrac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$= \dfrac{1}{3} \lim_{x\to a} \dfrac{4\sqrt{a}}{2\sqrt{3a}}$

$= \dfrac{1}{3} \cdot \dfrac{2\sqrt{a}}{\sqrt{3a}}$

$= \dfrac{2\sqrt{a}}{3\sqrt{3a}}$

2] $\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$= \lim_{y\to 0}\left(\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \dfrac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right)$

$= \lim_{y\to 0} \dfrac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$= \cancel{\lim \; y+y-a}$

$= \lim_{y\to 0} \dfrac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$= \dfrac{y}{y\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$

$= \cancel{\dfrac{1}{a(2a)}} \; \dfrac{1}{\sqrt{a}\,(2\sqrt{a})}$

$= \dfrac{1}{2a}$

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3.] $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3} \sin x}{\pi - 6x}$

By substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$

where $h \to 0$

$\lim_{h \to 0} \frac{\cos(h + \pi/6) - \sqrt{3}\sin(h + \pi/6)}{\pi - 6(h + \pi/6)}$

Using $\cos(A+B) = \cos A \cdot \cos A - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\lim_{h \to 0} \frac{\cos h \cdot \cos \pi/6 - \sin h \cdot \sin \pi/6 - \sqrt{3} \sin h \cos \pi/6 + \cos h \sin \pi/6}{\pi - 6\left(\frac{6h + \pi}{6}\right)}$

$\lim_{h \to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{\sqrt{2}} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cosh \frac{1}{\sqrt{2}}\right)}{\pi - 6h + \pi}$

$\lim_{h \to 0} \frac{\cos \frac{\sqrt{3}}{2} h - \sin \frac{h}{2} - \sin \frac{3h}{2} - \cos \frac{\sqrt{3}}{2} h}{-6h}$

$\lim_{h \to 0} \frac{-\sin 4h/2}{-6h}$

$\lim_{h \to 0} \frac{\sin 2h}{3 \times 2h}$

$\frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$

4.] $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$

By rationalizing numerator and denominator both.

$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$

$\lim_{x \to \infty} \left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$

$\lim_{x \to \infty} \frac{8(\sqrt{x^2+3} + \sqrt{x^2+1})}{2(\sqrt{x^2+5} + \sqrt{x^2-3})}$

$4 \lim_{x \to \infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$

After applying limit we get,

$= 4$.

5.](i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \leq \pi/2$

$= \frac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$

at $x = \frac{\pi}{2}$.

$f\left(\frac{\pi}{2}\right) = \frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2(\pi/2)}}$

$\therefore f(\pi/2) = 0$

f at $x = \frac{\pi}{2}$ define.

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$$\lim_{x\to\frac{\pi}{2}^+} f(x) = \lim_{x\to\frac{\pi}{2}^+} \frac{\cos x}{\pi-2x}$$

By Substituting method.

$$x-\frac{\pi}{2}=h$$

$$x=h+\frac{\pi}{2}$$

where $h\to 0$

$$=\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi-2\left(h+\frac{\pi}{2}\right)}$$

$$=\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi-2\left(\frac{2h+\pi}{2}\right)}$$

$$=\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{-2h}$$ Using $\cos(A+B)=\cos A\cos B-\sin A\sin B$

$$=\lim_{h\to 0} \frac{\cos h\cdot\cos\frac{\pi}{2}-\sin h\cdot\sin\frac{\pi}{2}}{-2h}$$

$$=\lim_{h\to 0} \frac{\cos h\cdot 0-\sin h}{-2h}$$

$$=\lim_{h\to 0} \frac{-\sin h}{-2h}$$

$$=\frac{1}{2}\lim_{h\to 0} \frac{\sin h}{h}$$

$$=\frac{1}{2}$$

(b) $$\lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$ Using $\sin 2x=2\sin x\cdot\cos x$.

$$\lim_{x\to\pi/2^-} \frac{2\sin x\cdot\cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x\to\pi/2^-} \frac{2\sin x\cdot\cos x}{\sqrt{2}\sin x}$$

$$\lim_{x\to\pi/2^-} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}}\lim_{x\to\pi/2^-} \cos x$$

$\because$ L.H.L. $\neq$ R.H.L.

$\therefore$ f is not continuous at $x=\frac{\pi}{2}$.

(ii) $$\left.\begin{aligned} f(x) &= \frac{x^2-9}{x-3} && 0<x<3 \\ &= x+3 && 3\le x<6 \\ &= \frac{x^2-9}{x+3} && 6\le x<9 \end{aligned}\right\}$$ at $x=3$ and $x=6$.

at $x=3$

(a) $$f(3)=\frac{x^2-9}{x-3}=0$$

f at $x=3$ define.

(b) $$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$$

$f(3)=x+3=3+3=6$.

f is defined at $x=3$.

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6.$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)}$$

$\therefore$ L.H.L. = R.H.L.

f is continous at $x=3$.

Now, for $x=6$.

(a) $f(6) = \dfrac{x^2-9}{x+3} = \dfrac{36-9}{6+3} = 3.$

(b) $\displaystyle\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$$\lim_{x\to 6^+} \frac{(x-3)(x+3)}{(x+3)}$$

$$\lim_{x\to 6^+} (x-3) = 6-3 = 3.$$

$$\lim_{x\to 6^-} x+3 = 3+6 = 9.$$

$\therefore$ L.H.L. $\neq$ R.H.L.

f is not continous

6.(i) $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & x<0 \\ = k, & x=0 \end{cases}$ at $x=0$.

f is continous at $x=0$,

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k.$$

$$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k.$$

$$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k.$$

$$2(2)^2 = k$$

$$\boxed{\therefore k = 8}$$

(ii) $f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x}, & x\neq 0 \\ = k & x=0 \end{cases}$ at $x=0$.

$$f(x) = (\sec^2 x)^{\cot^2 x}$$

Using ~~sec~~ $\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$.

and $\cot^2 x = \dfrac{1}{\tan^2 x}$.

$$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$$

We know that,

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$$\boxed{\therefore k = e}$$

(xi) $f(x) = \left.\begin{array}{ll} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} & , x \neq \dfrac{\pi}{3} \\ = k & , x = \dfrac{\pi}{3} \end{array}\right\}$ at $x = \dfrac{\pi}{3}$.

$x - \dfrac{\pi}{3} = h$

$x = h + \dfrac{\pi}{3}$.

where $h \to 0$

$$f\left(\tfrac{\pi}{3} + h\right) = \frac{\sqrt{3} - \tan\left(\pi/3 + h\right)}{\pi - 3\left(\pi/3 + h\right)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan\left(\pi/3 + h\right)}{\pi - 3\left(\pi/3 + h\right)}$$

Using

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan \pi/3 \cdot \tan h\right) - \left(\tan \frac{\pi}{3} + \tan h\right)}{1 - \tan \frac{\pi}{3} \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\left(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h\right) - \left(\sqrt{3} + \tan h\right)}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{\dfrac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{-4\tan h}{-3h\left(1 - \sqrt{3}\tan h\right)}$$

$$\lim_{h \to 0} \frac{4 \tan h}{3h\left(1 - \sqrt{3}\tan h\right)}$$

$$\frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \lim_{h \to 0} \frac{1}{\left(1 - \sqrt{3}\tan h\right)} \quad , \frac{\tan h}{h} = 1.$$

$$= \frac{4}{3} = \frac{1}{1 - \sqrt{3}(0)}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right)$$

$$= \frac{4}{3}.$$

7.(i) $f(x) = \left.\begin{array}{ll} \dfrac{1 - \cos 3x}{x \tan x} & , x \neq 0 \\ = a & , x = 0 \end{array}\right\}$ at $x = 0$.

$$f(x) = \frac{1 - \cos 3x}{x \tan x}$$

$$\lim_{x \to 0} \frac{2\sin^2 3/2\, x}{x \tan x}$$

$$\lim_{x \to 0} \frac{\dfrac{2 \sin^2 \frac{3x}{2}}{x^2} \times x^2}{x \cdot \dfrac{\tan x}{x^2} \times x^2}$$

$$2\lim_{x\to 0}\frac{(3/2)^2}{1}$$

$$2\times\frac{9}{4} = \frac{9}{2}$$

$\lim_{x\to 0} f(x) = \frac{9}{2}$, $9 = f(0)$

$\therefore f$ is not continous at $x=0$.

Redefine function

$$f(x) = \begin{cases} \frac{1-\cos 3x}{x\tan x} & x\neq 0 \\ \frac{9}{2} & x=0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$.

(ii) $$f(x) = \left.\begin{cases} \frac{(e^{3x}-1)\sin x}{x^2} & x\neq 0 \\ = \frac{\pi}{6} & x=0 \end{cases}\right\}$$ at $x=0$.

$$\lim_{x\to 0}\frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0}\frac{e^{3x}-1}{x}\lim_{x\to 0}\frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3\lim_{x\to 0}\frac{e^{3x}-1}{3x}\lim_{x\to 0}\frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$3\cdot 1\cdot\frac{\pi}{180} = \frac{\pi}{60} = f(0)$

f is continous at $x=0$.

8) $f(x) = \frac{e^{x^2}-\cos x}{x^2}$ $x=0$

is continous at $x=0$

Given

f is continous at $x=0$.

$\lim_{x\to 0} f(x) = f(0)$

$$\lim_{x\to 0}\frac{e^{x^2}-\cos x -1+1}{x^2}$$

$$\lim_{x\to 0}\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x\to 0}\frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to 0}\left(\frac{\sin x/2}{x}\right)^2$$

Multiply ~~with 2~~ on Numerator and Denominator

$$= 1 + 2\times\frac{1}{4} = \frac{3}{2} = f(0)$$

110

9] $f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

f(0) is continous at $x=\pi/2$

$$\lim_{x\to\pi/2} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$\lim_{x\to\pi/2} \frac{2-1+\sin x}{\cos^2 x\,(\sqrt{2}+\sqrt{1+\sin x})}$$

$$\lim_{x\to\pi/2} \frac{1+\sin x}{1-\sin^2 x\,(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x\to\pi/2} \frac{1+\sin x}{(1-\sin x)(1+\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x\to\pi/2} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

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Date 06-12-19

PRACTICAL NO. 2

TOPIC :- Derivative.

Q.1] Show that the following function defined from R to R are differentiable.

(1) cot x

→ $f(x) = \cot x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\tan x - 1/\tan a}{x-a}$$

$$= \lim_{x\to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

put $x-a=h$

$x = a+h$

as $x\to a$, $h\to 0$.

$$D.f(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h\times \tan(a+h)\tan a}$$

Formula :- $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$\tan A - \tan B = \tan(A-B)(1+\tan A\tan B)$.

$$= \lim_{h\to 0} \frac{\tan(a-a-h)(1+\tan a\tan(a+h))}{h\times\tan(a+h)\tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a\tan(a+h)}{\tan(a+h)\tan a}$$

$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$

$= \frac{-\sec^2 a}{\tan^2 a}$

$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in R$.

(2) cosecx

$\rightarrow f(x) = \operatorname{cosec} x$

$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$

$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$

$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$

put $x-a=h$

$x = a+h$

as $x \to a$, $h \to 0$.

$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$

Formula:- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$

$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$

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$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$

$= \frac{-\cos a}{\sin^2 a}$

$= -\cot a \operatorname{cosec} a$

(3) secx

$\rightarrow f(x) = \sec x$

$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$

$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$

$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$

put $x-a=h$

$x = a+h$

as $x \to a$, $h \to 0$

$Df(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$

Formula:- $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$

$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\frac{-h}{2}}{\cos a \cos(a+h) \times -h/2} \times \frac{-1}{2}$.

$= -\frac{1}{2} \times \frac{-2\sin(2a+0)}{\cos a \cos(a+0)}$

$= -\frac{1}{2} \times \frac{-2\sin a}{\cos a \times \cos a}$

$= \tan a \sec a$

Q.2] If $f(x) = 4x+1 \quad , x \le 2$

$\quad\quad = x^2+5 \quad , x>0$, at $x=2$ then find function is differentiable or not.

→ L.H.D :

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{(x-2)}$$

$\boxed{\therefore Df(2^-) = 4}$

R.H.D :-

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$$= 2+2$$

$\boxed{\therefore Df(2^+) = 4}$

$\because$ L.H.D. = R.H.D.

$\therefore f$ is differentiable at $x=2$

Q.3] If $f(x) = 4x+7 \quad , x<3$

$\quad\quad = x^2+3x+1 \quad , x\ge 3$ at $x=3$, then find f is differentiable or not?

→ R.H.D :-

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)(x-3)}{(x-3)}$$

$$= 3+6$$

$\boxed{\therefore Df(3^+) = 9}$

L.H.D :-

$$Df(3^-) = \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x\to 3} \frac{4(x-3)}{(x-3)}$$

$\boxed{\therefore Df(3^-) = 4}$

$\because$ L.H.D. $\neq$ R.H.D.

$\therefore$ f is not differentiable at $x=3$.

Q4] If $f(x) = 8x-5$, $x \le 2$
$= 3x^2-4x+7$, $x>2$ at $x=2$, then find f is differentiable or not.

→ $f(2) = 8\times2-5 = 16-5 = 11$

R.H.D :-

$Df(2^+) = \lim_{x\to2^+} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to2^+} \frac{3x^2-4x+7-11}{x-2}$

$= \lim_{x\to2^+} \frac{3x^2-4x-4}{x-2}$

$= \lim_{x\to2^+} \frac{3x^2-6x+2x-4}{x-2}$

$= \lim_{x\to2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3\times2+2$

$\boxed{Df(2^+) = 8}$

L.H.D :-

$Df(2^-) = \lim_{x\to2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to2^-} \frac{8x-5-11}{x-2}$

$= \lim_{x\to2^-} \frac{8x-16}{x-2}$

$= \lim_{x\to2^-} \frac{8(x-2)}{(x-2)}$

$\boxed{Df(2^-) = 8}$

$\because$ L.H.D. = R.H.D.

$\therefore$ f is differentiable at $x=3$.

A+
20/12/19

PRACTICAL No. 3

TOPIC : Application of Derivative .

Q.1] Find the intervals in which functions is increasing or decreasing.

(1) $f(x) = x^3 - 5x - 11$

$\rightarrow f'(x) = 3x^2 - 5$

f is increasing iff $f'(x) > 0$.

$\therefore 3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > \frac{5}{3}$

$x > \pm\sqrt{\frac{5}{3}}$

$+ \quad -\sqrt{\frac{5}{3}} \quad - \quad \sqrt{\frac{5}{3}} \quad +$

$\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < \frac{5}{3}$

$x < \pm\sqrt{\frac{5}{3}}$

$+ \quad -\sqrt{\frac{5}{3}} \quad - \quad \sqrt{\frac{5}{3}} \quad +$

$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right).$

(2) $f(x) = x^2 - 4x$

$\rightarrow$ $f'(x) = 2x - 4$

$\therefore$ f is increasing iff $f'(x) > 0$.

$2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x > 2$.

$\therefore x \in (2, \infty)$

$\therefore$ f is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$

$x < 2$.

$\therefore x \in (-\infty, 2)$

(3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\rightarrow$ $f'(x) = 6x^2 + 2x - 20$

f is increasing iff $f'(x) > 0$

$6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x - 10)(x+2) > 0$.

+ $\quad -2 \quad$ − $\quad \frac{10}{6} \quad$ +

$\therefore x \in (-\infty, -2) \cup (10/6, \infty)$

f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$.

$6x^2 + 12x - 10x - 20 < 0$.

$6x(x+2) - 10(x+2) < 0$

$(6x - 10)(x+2) < 0$

+ $\quad -2 \quad$ − $\quad \frac{10}{6} \quad$ +

$\therefore x \in \left(-2, \frac{10}{6}\right)$

(4) $f(x) = x^3 - 27x + 5$

$\rightarrow$ $f'(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

+ $\quad -3 \quad$ − $\quad 3 \quad$ +

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

f is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

+ $\quad -3 \quad$ − $\quad 3 \quad$ +

$\therefore x \in (-3, 3)$

(5) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$\rightarrow$ $f'(x) = -24 - 18x + 6x^2$

i.e. $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

f is increasing iff $f'(x) > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x^2-4x+x-4>0$

$x(x-4)+1(x-4)>0$

$(x+1)(x-4)>0$

$+ \quad -1 \quad - \quad 4 \quad +$

$\therefore x \in (-\infty,-1) \cup (4,\infty)$

f is decreasing iff $f'(x)<0$

$6(x^2-3x-4)<0$

$x^2-3x-4<0$

$x^2-4x+x-4<0$

$x(x-4)+1(x-4)<0$

$(x+1)(x-4)<0$

$+ \quad -1 \quad - \quad 4 \quad +$

$\therefore x \in (1,4)$

Q.2] Find the intervals in which function is concave upwards and concave downwards.

(1) $y=3x^2-2x^3$

$\rightarrow$ Let, $f(x)=y=3x^2-2x^3$

$\therefore f'(x)= 6x-6x^2$

$f''(x) = 6-12x$

$= 6(1-2x)$

$f''(x)$ is concave upwards iff,

$f''(x)>0$

$6(1-2x)>0$

$1-2x>0$

$-2x>-1$

$2x<1$

$x<\frac{1}{2}$.

$\therefore x \in \left(-\infty,\frac{1}{2}\right)$

$f''(x)$ is concave downwards iff,

$f''(x)<0$

$6(1-2x)<0$

$1-2x<0$

$-2x<-1$

$x>\frac{1}{2}$

$\therefore x \in \left(\frac{1}{2},\infty\right)$.

(2) $y=x^4-6x^3+12x^2+5x+7$

$\rightarrow$ Let, $f(x)=y=x^4-6x^3+12x^2+5x+7$

$f'(x)=4x^3-18x^2+24x+5$

$\therefore f''(x)=12x^2-36x+24$

$=12(x^2-3x+2)$

$f''(x)$ is concave upwards iff.

$f''(x)>0$

$12(x^2-3x+2)>0$

$x^2-3x+2>0$

$x^2-x-2x+2>0$

$x(x-1)-2(x-1)>0$

$(x-2)(x-1)>0$

$+ \quad 1 \quad - \quad 2 \quad +$

$\therefore x \in (-\infty,1) \cup (2,\infty)$

$f''(x)$ is concave downwards iff,

$f''(x)<0$.

$12(x^2-3x+2)<0$

$x^2-3x+2<0$

$x^2 - x - 2x + 2 < 0$
$x(x-1) - 2(x-1) < 0$
$(x-2)(x-1) < 0$.

$+ \quad 1 \quad - \quad 2 \quad +$

$\therefore x \in (1, 2)$

(3) $y = x^3 - 27x + 5$
$\rightarrow$ Let, $f(x) = y = x^3 - 27x + 5$
$\therefore f'(x) = 3x^2 - 27$
$\therefore f''(x) = 6x$.
$f''(x)$ is concave ~~iff~~ upwards iff,
$f''(x) > 0$
$6x > 0$
$x > 0$
$\therefore x \in (0, \infty)$
$f''(x)$ is concave downwards iff,
$f''(x) < 0$
$6x < 0$
$x < 0$
$\therefore x \in (-\infty, 0)$

(4) $y = 69 - 24x - 9x^2 + 2x^3$.
$\rightarrow$ Let, $f(x) = y = 69 - 24x - 9x^2 + 2x^3$.
$f'(x) = -24 - 18x + 6x^2$.
$f''(x) = -18 + 12x$.
$f''(x)$ is concave upwards iff,
$f''(x) > 0$
$-18 + 12x > 0$

$12x > 18$
$x > \frac{18}{12}$
$\therefore x \in \left(\frac{3}{2}, \infty\right)$
$f''(x)$ is concave downwards iff,
$f''(x) < 0$
$-18 + 12x < 0$
$12x < 18$
$x < \frac{18}{12}$.
$\therefore x \in \left(-\infty, \frac{3}{2}\right)$

(5) $y = 2x^3 + x^2 - 20x + 4$
$\rightarrow$ Let, $f(x) = y = 2x^3 + x^2 - 20x + 4$
$f'(x) = 6x^2 + 2x - 20$
$f''(x) = 12x + 2$
$= 2(6x + 1)$
$\therefore f''(x)$ is concave upwards iff,
$f''(x) > 0$
$2(6x+1) > 0$
$6x + 1 > 0$
$6x > 1$
$x > \frac{1}{6}$.
$\therefore x \in \left(\frac{1}{6}, \infty\right)$
$f''(x)$ is concave downwards iff,
$f''(x) < 0$
$2(6x - 1) < 0$
$6x - 1 < 0$

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20/12/19

$6x < 1$

$x < \frac{1}{6}$

$\therefore x \in \left(-\infty, \frac{1}{6}\right)$

PRACTICAL NO. 4

TOPIC :- Application of Derivative and Newton's Method.

Q.1] Find maximum and minimum value of following functions:

(1) $f(x) = x^2 + \frac{16}{x^2}$

$\rightarrow f'(x) = 2x - \frac{32}{x^3}$

Now consider,

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$\therefore x^4 = 16$

$\boxed{\therefore x = \pm 2}$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$ and $x = -2$

(2) $f(x) = 3 - 5x^3 + 3x^5$.

$\rightarrow$ $\therefore f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$\boxed{x = \pm 1}$

$\therefore f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0$

$\therefore f$ has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum value at $x = -1$.

$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore f$ has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$.

(3) $f(x) = x^3 - 3x^2 + 1 \quad [-1/2, 4]$

$\rightarrow$ $f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore f$ has maximum value at $x = 0$.

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$f(2) = -3$

$\therefore f$ has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$.

(4) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider, $f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$
$= 18 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$
$= 2(8) - 3(4) - 24 + 1$
$= 16 - 12 - 24 + 1$
$= -19$

$\because f''(-1) = 12(-1) - 6$
$= -12 - 6$
$= -18 < 0$

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$
$= -2 - 3 + 12 + 1$
$= 8$

$\therefore f$ has maximum value at $x = -1$ and f has minimum value -19 at $x = 2$.

Q.2.] Find the root of the following equation by Newton's method (Take 4 iteration only) correct upto 4 decimal.

(1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

$\rightarrow f'(x) = 3x^2 - 6x - 55$

By newton's method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$
$= 0.0051 - 0.0895 - 9.4985 + 9.5$
$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$
$= 6.0895 - 1.0362 - 55$
$= -55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$
$= 0.1727 - 0.0829/55.9467$
$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$
$= 0.0050 - 0.0879 - 9.416 + 9.5$
$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= -55.9393$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$
$= 0.1712$

$\therefore$ The root of the equation is 0.1712.

(2) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

$\rightarrow f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$

$f(3) = 3^3 - 4(3) - 9$
$= 27 - 12 - 9$
$= 6$

Let $x_0 = 3$ be the initial approximation.

By Newton's Method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$
$= 3 - 6/23$
$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9$
$= 0.596$
$f'(x_1) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$= 18.5096$
$x_2 = x_1 - f(x_1)/f'(x_1)$
$= 2.7392 - 0.596/18.5096$
$= 2.7071$
$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$
$= 19.8386 - 10.8284 - 9$
$= 0.0102$
$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$
$x_3 = 2.7071 - 0.0102/17.9851$
$= 2.7071 - 0.0056$
$= 2.7015$
$\therefore f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= 19.7158 - 10.806 - 9$
$= -0.0901$
$f'(3) = 3(2.7015)^2 - 4$
$= 21.8943 - 4$
$= 17.8943$
$x_4 = 2.7015 + 0.0901/17.8943$
$= 2.7015 + 0.0050$
$= 2.7065$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1,2]$
$\rightarrow f'(x) = 3x^2 - 3.6x - 10$
$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2$
$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 + 20 + 17$
$= -2.2$

Let $x_0 = 2$ be initial approximation by Newton's method
$x_{n+1} = x_n - f(x_n)/f'(x_n)$
$x_1 = x_0 - f(x_0)/f'(x_0)$
$= 2 - 2.2/5.2$
$= 2 - 0.4230$
$= 1.577$
$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 0.6755$
$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$
$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$
$= 1.577 + 0.6755/8.2164$
$= 1.577 + 0.0822$
$= 1.6592$
$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$
$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - f(x_2) / f'(x_2)$
$= 1.6592 + 0.0204 / 7.7143$
$= 1.6592 + 0.0026$
$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$
$= 4.5892 - 4.9708 - 16.618 + 17$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= 8.2849 - 5.9824 - 10$
$= -7.6977$

$x_4 = x_3 - f(x_3) / f'(x_3)$
$= 1.8618 + 0.0004 / 7.6977$

$x_4 = 1.6618$.

PRACTICAL NO. 5

Topic :- Integration.

Q.1] Solve the following integration.

(1) $\dfrac{dx}{\sqrt{x^2+2x-3}}$

$\rightarrow \int \dfrac{1}{x^2+2x-3} \, dx$

$= \int \dfrac{1}{\sqrt{x^2+2x-3}} \, dx$

$= \int \dfrac{1}{\sqrt{x^2+2x+1-4}}$

$= \int \dfrac{1}{(x+1)^2-4} \, dx \quad \rightarrow [a^2+2ab+b^2 = (a+b)^2]$

Substitute $x-1=t$.

$dx = \dfrac{1}{t} \times dt$ where $t=1$, $t=x+1$

$\int \dfrac{1}{\sqrt{t^2-4}} \, dt$

Using, $\int \dfrac{1}{\sqrt{x^2-(a)^2}} \, dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+c$

(2) $\int (4e^{3x}+1)\,dx$

$\rightarrow = \int (4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}dx + \int 1\,dx \qquad \rightarrow \left[\int e^{ax}dx = \frac{1}{a} \times e^{ax}\right]$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

(3) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$\rightarrow = \int 2x^2 - 3\sin x + 5x^{1/2}\,dx \qquad \# \sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2 dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + c.$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

(4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

split the denominator.

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}dx + \int 3x^{1/2}dx + \int \frac{4}{x^{1/2}}\,dx.$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

(5) $\int t^7 \times \sin(2t^4)\,dt$

Put : $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du$

$= \frac{t^4 \times \sin(2t^4)}{8}\,du$

Substitute t^4 with $\frac{u}{2}$.

$= \int \frac{u/2 \times \sin(u)\,du}{8}$

$= \int \frac{u \times \sin(u)}{2} / 8\,du$

$= \int \frac{u \times \sin(u)}{16}\,du$

$= \frac{1}{16}\int u \times \sin(u)\,du$

$\# \int u\,dv = uv - \int v\,du$

where $u = u$

$dv = \sin(u) \times du$

$du = 1\,du$

$v = -\cos(u)$

$= \frac{1}{16}(u \times (-\cos(u))) - \int -\cos(u)\,du$

$= \frac{1}{16} \times (u \cdot t \cos u)) + \int \cos(u)\, du$

$\cos x\, dx = \sin x$

$= \frac{1}{16} \times (4 \times (-\cos(u)) + \sin(u))$

Return the substitution $u = 2 + t^4$

$= \frac{1}{16} \times (2t^4 \times (-\cos(2+t^4)) + \sin(2+t^4))$

$= -\frac{t^4 \times \cos(2+t^4)}{8} + \frac{\sin(2+t^4)}{16} + C$

(6) $\int \sqrt{x}\,(x^2 - 1)\, dx$

$\rightarrow = \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\, dx$

$= \int x^{5/2} - x^{1/2}\, dx$

$= \int x^{5/2}\, dx - \int x^{1/2}\, dx$

$= I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$

(7) $\int \frac{\cos x}{3\sqrt{\sin^2 x}}\, dx$

$\rightarrow = \int \frac{\cos x}{\sin x^{2/3}}\, dx$

Put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos x}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)\, dt}$

$= \frac{1}{\sin x^{3/2}}\, dt$

$= \frac{1}{t^{2/3}}\, dt$

$I = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3 - 1)t^{2/3} - 1}$

$= \frac{-1}{-1/3\, t^{2/3} - 1} = \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$

Return substitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + C$

(8) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dx$

Put $x^3 - 3x^2 + 1 = dt$

$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x}\, dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x}\, dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)}\, dt$

$= \int \frac{1}{x^3 - 3x^2 + 1} \times \frac{1}{3}\, dt$

$= \int \frac{1}{3(x^3 - 3x^2 + 1)}\, dt = \int \frac{1}{3t}\, dt$

$= \frac{1}{3}\int \frac{1}{t}\,dt$ # $\int \frac{1}{x}dx = \ln|x|$

$= \frac{1}{3} \times \ln|t| + c$

$= \frac{1}{3} \times \ln(|x^3 - 3x^2 + 1|) + c$

(9) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$\rightarrow I = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

Let,

$\frac{1}{x^2} = t$

$\therefore x^{-2} = t$

$\therefore \frac{-2}{x^3}dx = dt$

$I = \frac{1}{-2}\int \frac{-2}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$= \frac{-1}{2}\int \sin t\,dt$

$= \frac{-1}{2}[-\cos t] + c$

$= \frac{1}{2}\cos t + c$

Resubstituting $t = \frac{1}{x^2}$

$\therefore I = \frac{1}{2}\cos\left(\frac{1}{x^3}\right) + c$

(10) $\int e^{\cos^2 x}\sin^2 x\,dx$

$I = \int e^{\cos^2 x}\sin^2 x\,dx$

Let $\cos^2 x = t$

$-2\cos x\sin x\,dx = dt$

$-2\sin 2x\,dx = dt$

$I = -\int -\sin 2x\, e^{\cos^2 x}\,dx$

$= -\int e^t\,dt$

$= -e^t + c$

Resubstituting $t = \cos^2 x$

$\therefore I = -e^{\cos^2 x} + c$

AK

03/01/2020

PRACTICAL NO. 6

Topic :- Application of integration and Numerical integration.

Q.1] Find the length of the following curve.

(1) $x = t\sin t$, $y = 1-\cos t$, $t \in [0, 2\pi]$

(2) $y = \sqrt{4-x^2}$, $x \in [-2, 2]$

(3) $y = x^{3/2}$ in $[0, 4]$

(4) $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q.2] Using Simpson's rule, solve the following.

(1) $\int_0^2 e^{x^2} dx$ with $n = 4$.

(2) $\int_0^4 x^2 dx$ with $n = 4$.

(3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$.

ANSWERS

Q.1] (1) $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} + dt$

$x = t - \sin t$ $\quad \therefore \frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t$ $\quad \therefore \frac{dy}{dt} = 0 - (-\sin t) = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

$= \int_0^{2\pi} \sqrt{2(1 - \cos t)}$

$= \int_0^{2\pi} \sqrt{2 \times 2\sin^2 t/2}$

$= \int_0^{2\pi} \sqrt{4\sin^2 t/2}$

$= \int_0^{2\pi} 2\,|\sin t/2|\, dt$

$= \int_0^{2\pi} 2\sin t/2\, dt$

$= \left(-4\cos(t/2)\right)_0^{2\pi}$

$= (-4\cos\pi) - (-4\cos 0)$

$= 4 + 4$

$= 8$

(2) $y = \sqrt{4 - x^2}$ $\quad , x \in [-2, 2]$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$y = \sqrt{4 - x^2}$

$\therefore \frac{dy}{dx} = 2\int_0^2 \left(1 + \left(\frac{-x}{\sqrt{4 - x^2}}\right)^2\right) dx$

$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4 - x^2}}\, dx$

$= 4\int_0^2 \frac{1}{\sqrt{4 - x^2}}\, dx$

$= 4\left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

(3) $y = x^{3/2}$ in $[0, 4]$

$f'(x) = \frac{3}{2} x^{1/2}$

$[f'(x)]^2 = \frac{9}{4} x$

$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_0^4 \sqrt{1 + \frac{9}{4} x}\, dx$

$= \int_0^4 \sqrt{\frac{4 + 9x}{4}}\, dx$

$= \frac{1}{2} \int_0^4 \sqrt{4 + 9x}\, dx$

$= \frac{1}{2} \left[\frac{(4 + 9x)^{1/2 + 1}}{1/2 + 1}\right]_0^4 dx$

$= \frac{1}{27} \left[(4 + 9x)^{3/2}\right]_0^4$

$= \frac{1}{27} \left[(4 + 0)^{3/2} - (4 + 36)^{3/2}\right]$

$$= \frac{1}{27}\left[(4)^{3/2} - (40)^{3/2}\right] \quad \approx 30$$

(4) $x = 3\sin t \qquad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} (3\cos t)^2 + (-3\sin)^2 dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{9(1)}\, dt$$

$$= \int_0^{2\pi} 3\, dt$$

$$= 3\int_0^{2\pi} dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 6\pi$$

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y} \qquad y \in [1,2]$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4 - 1}{2y^2}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$$

$$= \frac{17}{12}.$$

Q3(1) $\int_0^2 e^{x^2} dx$ with $n = 4$

$a = 0,\ b = 2,\ n = 4$

$$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule,

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(1 + 54.5981) + 4(1.2840 + 9.4877) + 2(2.7182 + 54.5981)\right]$$

$$= \frac{0.5}{3}\left[55.5981 + 43.0868 + 114.6326\right]$$

$$= 1.1779$$

(2) $\int_0^4 x^2 dx$

$$L = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2\,dx = \frac{1}{3}\left[16+4(10)+8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2\,dx = 21.533$$

(3) $\int_0^{\pi/3} \sqrt{\sin x}\,dx \qquad n=6$

$$l = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$6\pi/18$	$8\pi/18$
y	0	0.4166	0.58	0.70	0.80087	0.8727	0.9904

$$\int_0^{\pi/3} \sqrt{\sin x} = \pi/54 \times 12.1163$$

$$= \int_0^{\pi/3} \sqrt{\sin x}\,dx = 0.7049$$

PRACTICAL NO. 7

Topic:- Differential Equation.

Q.1] Solve the following differential equations.

(1) $x\dfrac{dy}{dx} + y = e^x$

(2) $e^x\dfrac{dy}{dx} + 2e^x y = 1$

(3) $x\dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

(4) $x\dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^2}$

(5) $e^{2x}\dfrac{dy}{dx} + 2e^{2x}y = 2x$

(6) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

(7) $\dfrac{dy}{dx} = \sin^2(x-y+1)$

(8) $\dfrac{dy}{dx} = \dfrac{2x+3y-1}{6x+9y+6}$

ANSWERS:-

(1) $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^x}{x}$.

$I.F. = e^{\int 1(x)\,dx}$

$= e^{\int 1/x\,dx}$

$= e^{\ln x}$

$I.F. = x$

$y(I.F.) = \int Q(x)(I.F.)\,dx + c$

$= \int \frac{e^x}{\cancel{x}} \cancel{x}\,dx + c$

$= \int e^x dx + 1$

$xy = e^x + c$

(2) $e^x\frac{dy}{dx} + 2e^xy = 1$

$\frac{dy}{dx} + 2\frac{e^x}{e^x}y = \frac{1}{e^x}$ (Dividing by e^x)

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $\quad Q(x) = e^{-x}$

$\int P(x)\,dx$

$I.F. = e^{\int 2\,dx}$

$= e^{2x}$

$y(I.F.) = \int Q(x)(I.F.)\,dx + c$

$y \cdot e^{2x} \int e^{-x} + 2x\,dx + c$

$= \int e^x dx + c$

$y \cdot e^{2x} = e^x + c$

(3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x)$ $\quad Q(x) = \frac{\cos x}{x^2}$

$IF = e^{\int P(x)\,dx}$

$= e^{\int 2/x\,dx}$

$= e^{2/x\,dx}$

$= \ln x^2$

$IF = x^2$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{\cos x}{x^2} - x^2\,dx + c$

$= \int \cos x + c$

$x^2y = \sin x + c$

(4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ (Dividing by x on both the sides)

$P(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e^{\int P(x)dx}$

$= e^{\int 3/x\, dx}$

$= e^{3/x}\, x$

$= e^{\ln x^3}$

$IF = x^3$

$y(IF) = \int Q(x)(IF)dx + C$

$= \int \frac{\sin x}{x^3} \cdot x^3 dx + C$

$= \int \sin x\, dx + C$

$x^3 y = -\cos x + C$

(5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$IF = e^{\int P(x)dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int 2xe^{-2x}e^{2x}\,dx + c$

$= \int 2x\,dx + c$

$ye^{2x} = x^2 + c$

(6) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\sec^2 x \tan y\, dx = -\sec^2 y \cdot \tan x\, dy$

$\frac{\sec^2 x\, dx}{\tan x} = \frac{-\sec^2 y\, dy}{\tan y}$

$\int \frac{\sec^2 x\, dx}{\tan x} = -\int \frac{\sec^2 y\, dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x - \tan y| = c$

$\tan x \cdot \tan y = e^c$

(7) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

Differentiating on both sides,

$x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\cancel{\frac{dv}{dx}} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v\, dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x + c$

(8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x+3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{v+2}$

$\int \left(\frac{v+2}{v+1}\right) dv = 3dx$

$= \int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = 3x$

$v + \log|x| = 3x + c$

$2x + 3y + \log|2x+3y+1| = 3x + c$

$3y = x - \log|2x+3y+1| + c$

10/01/2020

PRACTICAL NO. 8

TOPIC :- Euler's Method

1] $\frac{dy}{dx} = y + e^x - 2$, $y(0)=2$, $h=0.5$, find $y(2)$

2] $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$ find $y(1)$

3] $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0)=1$, $h = 0.2$ find $y(1)$

4] $\frac{dy}{dx} = 3x^2+1$, $y(1) = 2$, find $y(2)$
for $h=0.5$, $h=0.25$

5] $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$ find $y(1.2)$ with $h=0.2$

ANSWERS

1] $\frac{dy}{dx} = y + e^x - 2$

$f(x,y) = y + e^x - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_n+1
				2.5
0	0	2	1	
1	0.5	2.5	2.487	3.57435
2	1	3.5743	4.2925	5.3615

Using Euler's iteration formula

$\boxed{y_n + 1 = y_n + h\, f(x_n, y_n)}$

n	x_n	y_n	$f(x_n, y_n)$	y_n+1
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

∴ By Euler's formula;

$y(2) = 9.2831$

2.] $\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Euler's ~~rest~~ iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

∴ By Euler's formula,

$y(1) = 1.2942$

3] $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$ $x_0 = 0$, $h = 0.2$

Using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

4] $\frac{dy}{dx} = 3x^2 + 1$ $y_0 = 2$, $x_0 = 1$, $h = 0$

for $h = 0.5$

Using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

By Euler's formula,

$y(2) = 28.5$

for $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.185	8.9048
4	2	8.9048		

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By Euler's formula,

$y(2) = 8.9048$

5] $\frac{dy}{dx} = \sqrt{xy} + 2$ $\quad y_0 = 1, x_0 = 1, h = 0.2$

Using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

By Euler's formula

$y(1.2) = 1.6$

12/01/202-

PRACTICAL No. 9

TOPIC :- Limits and Partial Order Derivatives.

Q.1] Evaluate the following limits.

① $\lim_{(x,y) \to (-4,-1)} \frac{x^3 + 3y + y^2 - 1}{xy + 5}$

② $\lim_{(x,y) \to (2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

③ $\lim_{(x,y,z) \to (1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 yz}$

Q.2] Find f_x, f_y for each of the following f.

① $f(x,y) = xy e^{x^2+y^2}$

② $f(x,y) = e^x \cos y$

③ $f(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

Q.3] Using definition find values of f_x, f_y at $(0,0)$ for

$f(x,y) = \frac{2x}{1+y^2}$

Q.4] Find all second order partial derivative of f. Also verify whether $f_{xy} = f_{yx}$

① $f(x,y) = \frac{y^2 - xy}{x^2}$ $\quad$ ② $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

③ $f(x,y) = \sin(xy) + e^{x+y}$

[Q.5] Find the linearization of $f(x,y)$ at given point.

① $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

② $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

③ $f(x,y) = \log x + \log y$ at $(1,1)$

SOLUTIONS

Q.1] ① $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

at $(-4,-1)$, denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$

$= \frac{-64+3+1-1}{4+5}$

$= \frac{-61}{9}$

② $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(2,0)$, denominator $\neq 0$

By applying limit,

$= \frac{(0+1)((2)^2+0-4(2))}{2+0}$

$= \frac{(1)(4+0-8)}{2}$

$= -\frac{4}{2}$

$= -2$

③ $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

At $(1,1,1)$, denominator $= 0$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x-yz)(x+yz)}{x^2(x-yz)}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

On applying limit,

$= \frac{1+1(1)}{(1)^2}$

$= 2$

Q.2]

① $f(x,y) = xye^{x^2+y^2}$

$\therefore f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\boxed{\therefore f_x = 2xye^{x^2+y^2}}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\boxed{\therefore f_y = 2yxe^{x^2+y^2}}$

② $f(x,y) = e^x \cos y$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^x \cos y)$

$\boxed{\therefore f_x = e^x \cos y}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$\boxed{\therefore f_y = -e^x \sin y}$

③ $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\boxed{\therefore f_x = 3x^2y^2 - 6xy}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\boxed{f_y = 2x^3y - 3x^2 + 3y^2}$

Q-3]

① $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2-0}{(1+y^2)^2}$

$= \frac{2(1+y^2)}{(1+y^2)(1+y^2)}$

$= \frac{2}{(1+y^2)}$

At $(0,0)$

$f_x = \frac{2}{1+0}$

$\boxed{f_x = 2}$

$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

At $(0,0)$,

$f_y = \frac{-4(0)(0)}{(1+0)^2} = 0.$

Q.4]

① $f(x,y) = \frac{y^2 - xy}{x^2}$

$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$

$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$

$\boxed{f_x = \frac{-x^2 y - 2x(y^2 - xy)}{x^4}}$

$f_y = \frac{2y - x}{x^2}$

$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$

$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2 y - 2xy^2 + 2x^2 y)\right) - (-x^2 y - 2xy + 2x^2 y)\frac{\partial}{\partial x}(x^4)}{(x^4)^2}$

$f_{xx} = \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2 y - 2xy + 2x^2 y)}{x^6} \rightarrow (1)$

$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right)$

$= \frac{2 - 0}{x^2}$

$f_{yy} = \frac{2}{x^2} \rightarrow (2)$

$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}\right)$

$= \frac{-x^2 - 4xy + 2x^2}{x^4} \rightarrow (3)$

$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right)$

$= \frac{x^2 \frac{\partial}{\partial x}(2y - x) - (2y - x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$

$= \frac{-x^2 - 4xy + 2x^2}{x^4} \rightarrow (4)$

from (3) and (4),

$\boxed{f_{xy} = f_{yx}}$

③ $f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(x,y) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$\therefore f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot y + e^{x+y}(1) \rightarrow (1)$

$= -y^2\sin(xy) + e^{x+y}$

$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y} \rightarrow (2)$

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy) + \cos(xy) + e^{x+y} \rightarrow (3)$

$f_{yx} = \frac{\partial}{\partial x}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y} \rightarrow (4)$

from ② and ③,

$\boxed{f_{xy} \neq f_{yx}}$

Q.5

① $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2}$

$= \sqrt{2}$

$f_x = \dfrac{1}{2\sqrt{x^2+y^2}}(2x)$

$= \dfrac{x}{\sqrt{x^2+y^2}}$

$f_y = \dfrac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \dfrac{y}{\sqrt{x^2+y^2}}$

f_y at $(1,1) = \dfrac{1}{\sqrt{2}}$

f_x at $(1,1) = \dfrac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \dfrac{1}{\sqrt{2}}(x-1) + \dfrac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \dfrac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \dfrac{1}{\sqrt{2}}(x+y-2)$

$\cancel{= \sqrt{2} + \dfrac{1}{\sqrt{2}}(x+y-}$

$= \cancel{\sqrt{2} + \dfrac{1}{\sqrt{2}}x} + \dfrac{1}{\sqrt{2}}y - \dfrac{2}{\sqrt{2}}$

$= \dfrac{x+y}{\sqrt{2}}$

② $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \dfrac{\pi}{2} + 0 = 1 - \dfrac{\pi}{2}$

$f_x = 0 - 1 + y\cos x$

f_x at $(\pi/2, 0) = -1 + 0$

$= -1.$

$f_y = 0 - 0 + \sin x$

f_y at $(\pi/2, 0) = \sin\dfrac{\pi}{2} = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \dfrac{\pi}{2} + (-1)(x - \pi/2) + 1(y-0)$

$= 1 - \dfrac{\pi}{2} - x + \dfrac{\pi}{2} + y$

$= 1 - x + y.$

③ $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0.$

$f_x = \dfrac{1}{x} + 0$

$f_y = 0 + \dfrac{1}{y}$

f_x at $(1,1) = 1$

f_y at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

AK
21/01/2022

Practical no. 10

Topic :- Directional derivative, Gradient vector and maxima, minima, tangent, normal vectors.

Q.1] Find the directional derivative of the following function at given points and in the direction of given vector.

(1) $f(x,y) = x+2y-3$, $a=(1,1)$, $u=3i-j$
(2) $f(x,y) = y^2-4x+1$, $a=(3,4)$; $u=i+5j$
(3) $f(x,y) = 2x+3y$, $a=(1,2)$, $u=3i-4j$

Q.2] Find gradient vector for the following function of given point.

(1) $f(x,y) = x^y+y^x$, $a=(1,1)$
(2) $f(x,y) = (\tan^{-1}x)\cdot y^2$, $a=(1,-1)$
(3) $f(x,y,z) = xyz-e^{x+y+z}$, $a=(1,-1,0)$

Q.3] Find the equation of tangent and normal to each of the following curves at given points.

(1) $x^2\cos y+e^{xy}=2$ at $(1,0)$
(2) $x^2+y^2-2x+3y+2=0$ at $(2,-1)$

Q.4] Find the equation of tangent and normal line to each of the following surfaces.

~~(1) $f(x,y)=3x^2+y^2-3xy+6x-4y$~~

(1) $x^2-2yz+3y+xz=7$ at $(2,1,0)$

(2) $3xyz-x-y+z=-4$ at $(1,-1,2)$

Q.5] Find the local maxima and minima for the following functions.

(1) $f(x,y)=3x^2+y^2-3xy+6x-4y$
(2) $f(x,y)=2x^4+3x^2y-y^2$

SOLUTIONS

:-TO

Q.1]

(1) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1),\ u = 3i - j$

→ Here,

$u = 3i - j$ is not a unit vector

$\bar{u} = 3\bar{i} - \bar{j}$

$|u| = \sqrt{10}$

∴ Unit vector along u is $\frac{\bar{u}}{|u|} = \frac{1}{\sqrt{10}}(3i - j)$

$$= \frac{1}{\sqrt{10}}(3,-1)$$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

Now,

$$f(a+hu) = f\left((1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$$

$$= 1 + 2(-1) - 3 + \left(\frac{3h}{\sqrt{10}}, \frac{-h}{\sqrt{10}}\right) f\left(1 + \frac{3h}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right)$$

$$= -4 + \frac{3h}{\sqrt{10}} - 4 + 1 + \frac{3h}{\sqrt{10}} + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$= 1 - 2 - 3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$$

$$= -4 + \frac{h}{\sqrt{10}}$$

$$\therefore D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{-4 + \frac{h}{\sqrt{10}} - (-4)}{h}$$

$$= \lim_{h\to 0} \frac{h}{h\sqrt{10}}$$

$$= \frac{1}{\sqrt{10}}$$

(2) $f(x,y) = y^2 - 4x + 1$, $\quad a = (3,4)$, $u = i + 5j$

→ Here,

$u = i + 5j$ is not a unit vector.

$\bar{u} = \bar{i} + 5\bar{j}$

$|\bar{u}| = \sqrt{26}$

∴ Unit vector along u is $\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{26}}(\bar{i} + 5\bar{j})$

$$= \frac{1}{\sqrt{26}}(1,5)$$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

Now,

$$f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

077

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$

$= \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}}}{h}$

$= \lim_{h\to 0} \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$

$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$

$= \frac{36}{\sqrt{26}}$

(3) $f(x,y) = 2x + 3y$; $a = (1,2)$, $u = 3i + 4j$

$\rightarrow$ Here,

$u = 3i + 4j$ is not a unit vector

$\bar{u} = 3\bar{i} + 4\bar{j}$

$|\bar{u}| = \sqrt{25} = 5$

$\therefore$ Unit vector along $u = \frac{\bar{u}}{|\bar{u}|} = \frac{1}{5}(3\bar{i} + 4\bar{j})$

$= \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

078

Now $f(a+hu) = f\left((1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4h}{5}\right)$

$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= 8 + \frac{18h}{5}$

$\therefore D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{8 + \frac{18h}{5} - 8}{h}$

$= \lim_{h\to 0} \frac{18h}{5h}$

$= \frac{18}{5}$

Q.2

(1) $f(x,y) = x^y + y^x$, $a = (1,1)$

$f_x = y(x^{y-1}) + y^x \log y$

$f_y = x(y^{x-1}) + x^y \log x$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y, xy^{x-1} + x^y \log x)$

$\nabla f(x,y)$ at $(1,1)$

$= (1(1)^0 + 1^1 \log(1), 1(1)^{(1-1)} + \log 1)$

$= (1,1)$

850

(2) $f(x,y) = (\tan^{-1}x)\cdot y^2$, $a = (1,-1)$

$f_x = y^2\left(\frac{1}{1+x^2}\right) = \frac{y^2}{1+x^2}$

$f_y = 2y\tan^{-1}x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y\tan^{-1}x\right)$

$\nabla f(x,y)$ at $(1,-1)$

$= \left(\frac{(-1)^2}{1+(1)^2}, 2(-1)\tan^{-1}(1)\right)$

$= \left(\frac{1}{2}, -\frac{2\pi}{4}\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

(3) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$\nabla f(x,y,z)$ at $(1,-1,0)$

$= (-1(0) - e^{1-1+0}, 1(0) - e^{1-1+0}, 1(-1) - e^{1-1+0})$

$= (0-1, 0-1, -1-1)$

$= (-1,-1,-2)$

Q3]

(1) $x^2\cos y + e^{xy} = 2$ at $(1,0)$

$f(x,y) = x^2\cos y + e^{xy} - 2$

$f_x = 2x\cos y + ye^{xy}$

$f_y = -x^2\sin y + xe^{xy}$

$(x_0, y_0) = (1,0)$

f_x at $(1,0) = 2(1)\cos 0 + 0$

$= 2$

f_y at $(1,0) = -(1)^2\sin(0) + 1(e)^{1(0)}$

$= 1$

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ → Equation of the Tangent.

Now,

For equation of normal:

$bx + ay + d = 0$

$x + 2y + d = 0$

$\therefore (1) + 2(0) + d = 0$ at $(1,0)$

$1 + a = 0$

$d = -1$

$\therefore x + 2y - 1 = 0$ → Equation of Tangent.

(2) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

f_x at $(2,-2) = 2(2) - 2$

$= 2$

f_y at $(2,-2) = 2(-2) + 3$

$= -1$

$\therefore$ Equation of tangent,

$f_x(x-x_0)+f_y(y-y_0)=0$

$2(x-2)+(-1)(y+2)=0$

$2x-4-y-2=0$

$2x-y-6=0$ → Equation of Tangent.

For equation of Normal,

$bx+ay+d=0$

$-x+2y+d=0$

$-2+2(-2)+d=0$ at $(2,-2)$

$-2-4+d=0$

$d=6$

$\therefore -x+2y+6=0$ → Equation of Normal.

Q.4]

(1) $x^2-2yz+3y+xz=7$ at $(2,1,0)$

$f(x,y,z)=x^2-2yz+3y+xz-7$

$f_x=2x-0+0+z-0$ $\therefore f_x$ at $(2,1,0)=2(2)+0$

$=2x+z$ $=4$

$f_y=-2z+3+0-0$ $\therefore f_y$ at $(2,1,0)=2(0)+3$

$=-2z+3$ $=3$

$f_z=0-2y+0+x-0$ $\therefore f_z$ at $(2,1,0)=-2(1)+2$

$=-2y+x$ $=0$

Equation of Tangent,

$f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$

$4(x-2)+3(y-1)+0(z-0)=0$

$4x-8+3y-3=0$

$\therefore 4x+3y-11=0$ → Equation of tangent.

Equation of Normal,

$$\frac{x-x_0}{f_x}=\frac{y-y_0}{f_y}=\frac{z-z_0}{f_z}$$

$$\frac{x-2}{4}=\frac{y-1}{3}=\frac{z-0}{0}$$ → Equation of Normal.

(2) $3xyz-x-y+z=-4$ at $(1,-1,2)$

$f(x,y,z)=3xyz-x-y+z+4$

$f_x=3yz-1-0+0+0$ $\therefore f_x$ at $(1,-1,2)=3(-1)(2)-1$

$=3yz-1$ $=-7$

$f_y=3xz-0-1+0+0$ $\therefore f_y$ at $(1,-1,2)=3(1)(2)-1$

$=3xz-1$ $=5$

$f_z=3xy-0-0+1-0$ $\therefore f_z$ at $(1,-1,2)=3(1)(-1)+1$

$=3xy+1$ $=-2$

Equation of tangent,

$f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$

$-7(x-1)+5(y+1)+(-2)(z-2)=0$

$-7x+7+5y+5-2z+4=0$

$-7x+5y-2z+16=0$ → Equation of tangent.

Equation of Normal.

$$\frac{x-x_0}{f_x}=\frac{y-y_0}{f_y}=\frac{z-z_0}{f_z}$$

$$\frac{x-1}{-7}=\frac{y+1}{5}=\frac{z-2}{-2}$$ → Equation of normal.

Q-5]

(a) $f(x,y) = 3x^2 - y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6 \longrightarrow 1$

$f_y = 2y - 3x + 0 - 4$

$= 2y - 3x - 4 \longrightarrow 2$

$f_x = 0$ | $f_y = 0$

$6x - 3y + 6 = 0$ | $2y - 3x - 4 = 0$

$3(2x - y + 2) = 0$ | $2y - 3x = 4 \longrightarrow 4$

$2x - y + 2 = 0$

$2x - y = -2 \longrightarrow 5$

Multiplying 5 by 2 and subtracting 4 from 5.

$4x - 2y = -4$

$-\ 2y - 3x = 4$

$7x = 0$

$x = 0$

Substituting value of x in 5

$2(0) - y = -2$

$-y = -2$

$y = 2$

$\therefore$ Critical points are $(0,2)$

Now,

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

$rt - s^2 = 12 - 9$

$= 3 > 0$

Here $r > 0$ and $rt - s^2 > 0$

$\therefore f$ has minimum at $(0,2)$

$\therefore f(0,2) = 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

(b) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy - 0$

$= 8x^3 - 6xy$

$f_y = 0 + 3x^2 - 2y$

$= 3x^2 - 2y$

Now,

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 6xy) = 0$

$4x^2 + 6xy = 0 \longrightarrow 1$

$f_y = 0$

$3x^2 - 2y = 0$

$3x^2 - 2y = 0$

$3x^2 - 2y = 0 \longrightarrow 2$

Multiply in 1 by 3 and 2 by 4 and subtracting 2 from 1

$12x^2 + 18y = 0$

$-12x^2 - 8y = 0$

$2y = 0$

$\boxed{y = 0} \longrightarrow 3$

Substituting 3 in 2

$3x^2 - 2(0) = 0$

$3x^2 = 0$

$x^2 = 0$

$\boxed{x = 0} \longrightarrow 4$

Critical points are $(0,0)$

Now,

$x = f_{xx} = 24x^2 + 6y$

$t = f_{yy} = 6 - 2$

$s = f_{xy} = 6x$

$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$
$= -48x^2 - 12y - 36x^2$
$= -84x^2 - 12y$

At $(0,0)$
$r = 24(0)^2 + 6(0)$
$= 0$
$s = 6(0) = 0$
$rt - s^2 = -84(0)^2 - 12(0) = 0$
$r = 0$ and $rt - s^2 = 0$
$\therefore$ Nothing can be stated.

AK
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