

# Time Series

Date \_\_\_\_\_  
Page \_\_\_\_\_

weather forecasting  
finance  
weather  
medical  
economics

## → Non-time Series

- House Price Prediction  
↳ size, location, bedrooms, price.
- Regression problem.

## Time Series

- Sales Data  
↳ time dependent data.

	sales
Day 1	50 k
Day 2	60 k
Day 3	70 k

• effect of "previous" timestamp.

## → Interpolation vs Extrapolation

### Interpolation

- Find out value in range itself.
- It assumes there should be a linear relationship

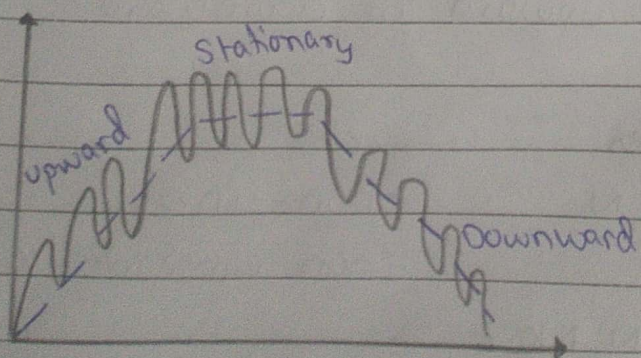
### Extrapolation

- Find out value out of the range.
- Based on previous data we try to forecast future data.
- Time series problem statement will be extrapolation

## → Components of Time Series

i. Trend , ii. Season , iii. Cycle , iv. Noise

### ↗ Trend



• overall direction of the series.

→ 3 parts

- Upward
- Downward
- Flat [horizontal]



- stock, people's sentiment
- Sales: forecasting.

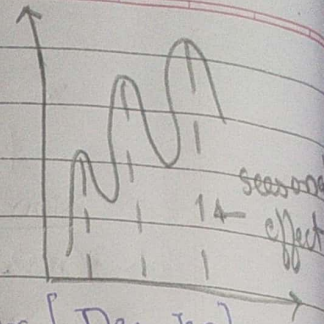
## 2) Season

- frequent repetition

- Example: i) Sales of ice cream in summer

ii) Traffic at 5pm in my area

iii) Tourist in Goa at end of year [Dec, Jan]



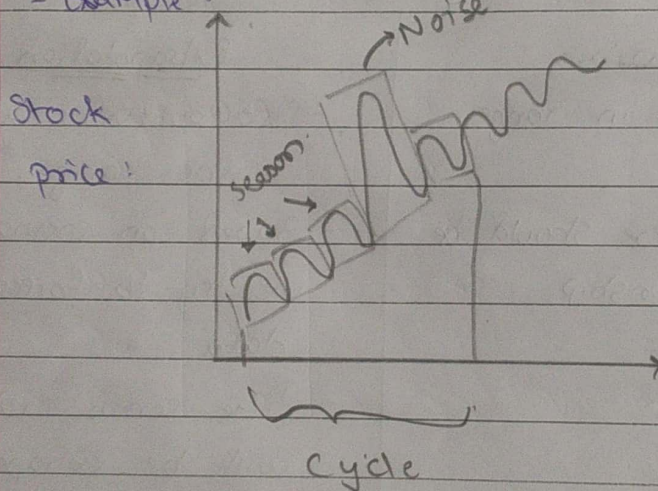
- Repetition on daily, monthly, hourly or yearly basis

## 3) Cycle

- Time series behaviour over long time/period

- Cycle = Season + Noise (fluctuation)

- Example:



Politics: Cycle for 5 years

## 4) Noise

Some uncertainty, randomness in my data because of unpredictable reason.

→ Pandemic, war, report, Current News

- Sudden fluctuation is known as noise.

- Fluctuation can be upward or downward

## • Multiplicative Time Series

$$Y_t = T \times S \times N$$

• non-linear

• non constant variance

T: Trend, S: Season, N: Noise



## • Additive Time Series

$$Y_t = T + S + N$$

- Linear over the time
- Constant Variance

\* Moving Average → Use for Smoothing of Time Series data.

→ Types of moving average.

• Move: Moving over time axis in specific window time.

### 1] SMA [Simple Moving Average]

Example:

Window = 3

1) Window Size

2) Average Calculation

Time	Sales	Avg
D <sub>1</sub>	10	→ NAN
D <sub>2</sub>	12	→ NAN
D <sub>3</sub>	15	→ 12.33 ← 1st avg
D <sub>4</sub>	13	→ 13.33 ← 2nd avg
D <sub>5</sub>	14	→ 14
D <sub>6</sub>	16	→ 14.33
D <sub>7</sub>	17	→ 15.66

### \* SMOOTHING

- To remove all effect from the data.
- We perform Smoothing with the help of MA

→ Benefits.

- 1) Pattern Recognition from data
- 2) Analyzing trend of data
- 3) Reduce effect of outlier
- 4) Enhancing the visualization

### 2] CMA [Cumulative Moving Average]

- Findout average of all datapoint upto given time stamp.

D <sub>1</sub>	10	D <sub>1</sub> = 10
D <sub>2</sub>	12	D <sub>1</sub> + D <sub>2</sub> / 2 = 10 + 12 / 2 = 11
D <sub>3</sub>	15	D <sub>1</sub> + D <sub>2</sub> + D <sub>3</sub> / 3 = 10 + 12 + 15 / 3
D <sub>4</sub>	14	10 + 12 + 15 + 14 / 4 =
D <sub>5</sub>	16	10 + 12 + 15 + 14 + 16 / 5 =
D <sub>6</sub>	17	10 + 12 + 15 + 14 + 16 + 17 / 6 =

• we use CMA for long time period.



### 3] EMA [Exponential Moving Average] or EWMA [Exponential weighted . . . .]

- In EMA we give more weightage to the recent datapoint or give more weightage to recent time stamp.

$$V_t = \beta V_{t-1} + (1-\beta) \theta_t$$

$\therefore V_t = \text{EMA at time } t$

$$\beta = 0 < \beta < 1$$

$V_{t-1} = \text{EMA at previous time stamp}$

$\theta_t = \text{Data at } t \text{ time stamp.}$

Ex:  $\beta = 0.9$

$D_1$	25	$V_0 = 0$	$V_1 = \beta \times V_0 + (1-\beta) \theta_1$
$D_2$	13	$V_1 = 1.3$	$V_1 = 0.9 \times 0 + (1-0.9) 13$
$D_3$	17	$V_2 = 2.87$	$V_1 = 0 + 0.1 \times 13 = 1.3$
$D_4$	31		
$D_5$	43		

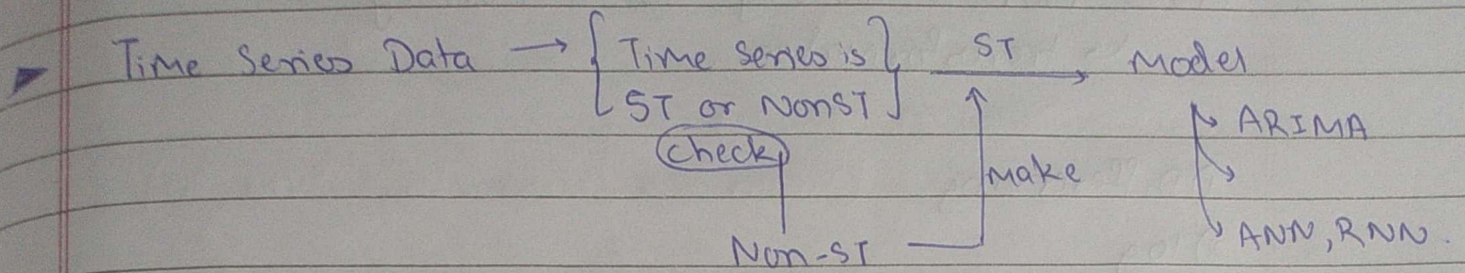
$$V_2 = \beta \times V_1 + (1-\beta) \theta_1$$
$$= 0.9 \times 1.3 + (0.1) 17$$
$$= 1.17 + 1.7$$
$$V_2 = 2.87$$

### \* Stationary & Non-Stationary Time Series

↓  
Mean & variance  
will be constant

↓  
Mean & variance will not be  
constant.

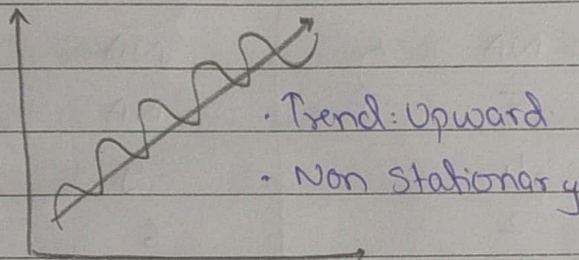




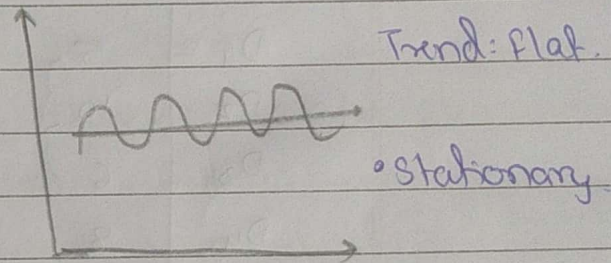
→ For checking whether Time Series is ST or Non-ST ??

1) Visualization.

↳ check trend, moving average.



• Moving average is increasing



• Moving average is constant.

2) Stats Based test.

ADF : Augmented dickey fuller test.

↳ 1) Static test

2) P-value

3) Critical value

$H_0$  = Null hypothesis

↳ My data is non stationary

$H_a$  : Alternative hypothesis

↳ data is stationary

• White Noise

- random process is white noise process.

- Errors are serially uncorrelated if they are independent & identically distributed (iid).

$\therefore P \leq 0.05$

↳ reject null hypothesis  
 ↳ Stationary

$\therefore P > 0.05$

↳ accept null hypothesis  
 ↳ non-stationary.



→ From Non-stationary To Stationary  
↳ transformation.

- 1) Differencing
- 2) log
- 3) root
- 4) Seasonal adjustment

### • Differencing

		First difference	2nd differency
D <sub>1</sub>	5	NA	NA
D <sub>2</sub>	10	5	NA
D <sub>3</sub>	6	-4	-9
D <sub>4</sub>	8	2	6
D <sub>5</sub>	15	7	5

↓  
if data is stationary stop, if not do 2nd differencing

↳ Again check until we get stationary data.

### \* ACF [Auto Correlation Function]

AUTO + Correlation

Correlation ↓  
itself in the feature

↳ It's a relationship b/w two features.

{ Pearson  
Spearman rank  
Kendall

- ACF is a func of time displacement of time series itself.
- ACF Measures the correlation between time series and its lag value.



	Timeseries		1st lag	2nd lag	3rd lag
D <sub>1</sub>	10		NA	NA	NA
D <sub>2</sub>	25	↔	D <sub>1</sub> → 10	NA	NA
D <sub>3</sub>	14	↔	D <sub>2</sub> → 25	D <sub>1</sub> → <del>NA</del> 10	NA
D <sub>4</sub>	16	↔	D <sub>3</sub> → 14	D <sub>2</sub> → 25	D <sub>1</sub> → 10
D <sub>5</sub>	20	↔	D <sub>4</sub> → 16	D <sub>3</sub> → 14	D <sub>2</sub> → 25
D <sub>6</sub>	32	↔	D <sub>5</sub> → 20	D <sub>4</sub> → 16	D <sub>3</sub> → 14
	$[Y_t]$		$[Y_{t-1}]$	$[Y_{t-2}]$	$[Y_{t-3}]$

$\text{Corr}(Y_t, Y_{t-1}), \text{Corr}(Y_t, Y_{t-2}), \text{Corr}(Y_t, Y_{t-3})$

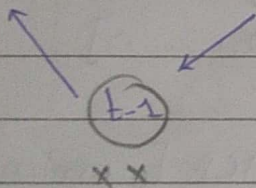
### \* PACF [Partial Auto Correlation Function]

- PACF is the conditional correlation between two variables under the assumptions that the effects of all previous lags on time series.

$t-2 \longleftrightarrow t$

Here,  $t$  is dependent on  $t-1$

$t-1$  is dependent on  $t-2$

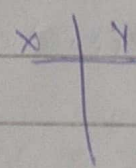


∴ Cancelling  $t-1$ ,

### \* Auto Regression [AR(p)]

Auto Regression  
↙  
itself in the variable.

Regression



$x \rightarrow$  Independent

$y \rightarrow$  Dependent

$$Y_t = \psi Y_{t-1} + C$$

$$y = mx + c$$

$$y = m_1 x_1 + m_2 x_2 + \dots + m_n x_n + c$$

$Y_t \rightarrow$  value at current timestamp

$\psi \rightarrow$  coefficient term

$C \rightarrow$  Constant

$\epsilon \rightarrow$  error

$$Y_t = \psi_1 Y_{t-1} + \psi_2 Y_{t-2} + \dots + \psi_n Y_{t-n} + C$$



## → Machine Learning

Data → Model

[Linear Reg, lasso, log, SVM, DT]

Time Series Data → Model

[ARIMA, SARIMA, SARIMAX, DL, RNN, Attention, transformer]

## \* ARIMA

AR	+	I	+	MA
[Auto Regression]		[Integration]		[Moving Average]
↓		↓		↓
p		d		q
[0, 1, 2, ..., n]		[0, 1, ..., n]		[0, 1, ..., n]

we decide AR  
value using  
PACF

Differencing  
[Non stationary/  
Stationary]

ACF [Corrlogram]

- Removing intermediate value & using partial value.

- Integration ⇒ Difference.

$$D_1 = (Y_t - Y_{t-1})$$

$$D_2 = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2})$$

$$D_2 = (Y_t - Y_{t-1} - Y_{t-1} + Y_{t-2})$$



- Moving Average

comes  
with error

$$Y_t = \epsilon_{t-1} \psi + c$$

$$Y_t = \epsilon_{t-1} \psi_{t-1} + \epsilon_{t-2} \psi_{t-2} + \dots + \epsilon_{t-n} \psi_{t-n} + c$$

ARIMA

AR(1)  $p=1$

I(2)  $d=2$

MA(1)  $q=1$

AR

I

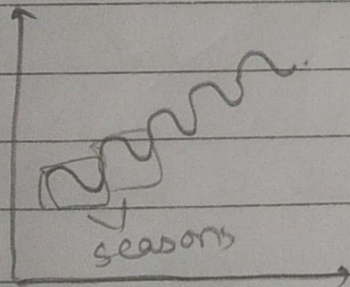
MA

$$Y_t = (\psi_{t-1} \psi_{t-1} + c) + (\psi_t - \psi_{t-1} - \psi_{t-2}) + (\epsilon_{t-1} \psi_{t-1} + c)$$

→ Variants of ARIMA

SARIMA

↳ seasonal



SARIMX

↳ exogenous [outlier]

