Lecture 4

Wednesday, 25 August 2021 3:55 PM

Convolution Sum :

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$y(n) = x(n) \circledast h(n) = h(n) * x(n).$

Steps to compute

→ Find $x(k)$

$\{x(k)\} \to \{x(-2)\ x(-1)\ x(0)\ x(1)\ x(2)\}$

→ $h(k) \to \{h(-1)\ h(0)\ h(1)\}$

→ Find $h(-k)$ → Folding operation.

$y(1)$ → Find $h(1-k)$ → shifting of folded seq.

⋮

→ Multiply $x(k)$ with $h(-k)$ then sum it, $y(0)$ will be computed.

Example

$x(k) \to \{x(-1)\ x(0)\ x(1)\ x(2)\}$

$h(k) \to \{h(-2)\ h(-1)\ h(0)\ h(1)\}$

	-2	-1	0	1	2
$x(k)$ →		$x(-1)$	$x(0)$	$x(1)$	$x(2)$
$h(-k)$ →		$h(1)$	$h(0)$	$h(-1)$	$h(-2)$

$y(0) = x(-1)\,h(1) + x(0)\,h(0) + x(1)\,h(-1) + x(2)\,h(-2)$

$h(1-k) \to \quad h(1)\quad h(0)\quad h(-1)\quad h(-2)$

$y(1) = x(0)\,h(1) + x(1)\,h(0) + x(2)\,h(-1)$

⋮

NOTE :

→ $y(n) = x_1(n) * x_2(n).$

Length of $x_1(n)$ is N_1.

" " $x_2(n)$ " N_2.

Length of $y(n) = N = N_1 + N_2 - 1$

$x_1(n) \to [n_{1min}$ to $n_{1max}]$

$x_2(n) \to [n_{2min}$ to $n_{2max}]$.

$y(n) = x_1(n) \circledast x_2(n).$

$[n_{1min} + n_{2min},\ n_{1max} + n_{2max}]$

→ $N_1 + N_2 - 1$

→ For the above example.

$y(n) \to n \in [-3, 3]$ → 7 samples.

$\{y(-3) \cdots y(3)\}$ → 7 samples.

Simple trick

-2	-1	0	1	2	
	$x(-1)$	$x(0)$	$x(1)$	$x(2)$	→ $x(k)$
$h(-2)$	$h(-1)$	$h(0)$	$h(1)$		→ $h(k)$

			h_1x_{-1}	h_1x_0	h_1x_1	h_1x_2 → 1+2 = 3
		h_0x_{-1}	h_0x_0	h_0x_1	h_0x_2	
	$h_{-1}x_{-1}$	$h_{-1}x_0$	$h_{-1}x_1$	$h_{-1}x_2$		
$h_{-2}x_{-1}$	$h_{-2}x_0$	$h_{-2}x_1$	$h_{-2}x_2$			
$y(-3)$	$y(-2)$	$y(-1)$	$y(0)$	$y(1)$	$y(2)$	$y(3)$

$y(3) = h(1)\,x(2)$

⋮

$y(0) = h(1)\,x(-1) + h(0)\,x(0) + h(-1)\,x(1) + h(-2)\,x(2)$

⋮

$y(-3) = h(-2)\,x(-1).$

$x(n) \to 0 \le n \le N-1$

$x(n-2) \to$ range is $\notin [0, N-1]$ → $[2, N+1]$.

Circular shift of a sequence.

↳ In this operation the range of sequence remains same.

$\langle m \rangle_N = m$ modulo N.

$r = \langle m \rangle_N$

↳ integer with a value between 0 & N-1 $\in [0, N-1]$

$x(n) \to$ sample values $0 \le n \le N-1$

$x(n) = 0, \quad n < 0$ & $n > N-1$.

$r = \langle m \rangle_N = m + lN$

l is a positive or negative integer chosen to make r an integer between 0 & N-1.

$r = \langle 25 \rangle_7 = 25 + 7l = 25 - 7 \times 3 = 4.$ ($l = -3$)

$[0, 6]$

$\langle 25 \rangle_7 = 4.$

$\langle -15 \rangle_7 = -15 + 7l$
$= -15 + 7 \times 3 = 6.$

Circular time-reversal operation

$x(n) \to \{x(0),\ x(1),\ x(2),\ x(3),\ x(4)\}$ 5-1, N=5

$y(n) = x(\langle -n \rangle_N)$

$y(0) = x(\langle 0 \rangle_5) = x(0)$ → $[0, N-1]$

$y(1) = x(\langle -1 \rangle_5) = x(4)$

$y(2) = x(\langle -2 \rangle_5) = x(3)$

$y(3) = x(2)$

$y(4) = x(1)$

flipped.

$y(n) \to \{x(0)\ x(4)\ x(3)\ x(2)\ x(1)\}$