Lecture 33

Friday, 5 November 2021 3:00 PM

Power complementary Filter

$\{H_i(z)\}$ $0 \le i \le M-1$ power complementary.

iff $\sum_{i=0}^{M-1} |H_i(e^{j\omega})|^2 = K \quad \forall \omega$.

$$\downarrow$$

$$\sum_{i=0}^{M-1} |H_i(e^{j\omega})|^2 = 1$$

M = 2

$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 = 1$$

$$\Downarrow$$

$$H_0(z)\, H_0(z^{-1}) + H_1(z)\, H_1(z^{-1}) = 1$$

Consider
$$H_0(z) = \frac{1}{2}[A_0(z) + A_1(z)]$$
$$H_1(z) = \frac{1}{2}[A_0(z) - A_1(z)]$$

$$H_0(z)\, H_0(z^{-1}) = \frac{1}{4}[A_0(z) + A_1(z)][A_0(z^{-1}) + A_1(z^{-1})]$$

$$= \frac{1}{4}[A_0(z) A_0(z^{-1}) + A_1(z) A_1(z^{-1}) + A_0(z) A_1(z^{-1}) + A_1(z) A_0(z^{-1})]$$

$$H_1(z)\, H_1(z^{-1}) = \frac{1}{4}[A_0(z) A_0(z^{-1}) + A_1(z) A_1(z^{-1}) - A_0(z) A_1(z^{-1}) - A_1(z) A_0(z^{-1})]$$

$$H_0(z)\, H_0(z^{-1}) + H_1(z)\, H_1(z^{-1}) = \frac{1}{4}[2 \underbrace{A_0(z) A_0(z^{-1})}_{=1} + 2 \underbrace{A_1(z) A_1(z^{-1})}_{=1}]$$

$$= \frac{1}{4}[2 + 2] = 1.$$

$\Rightarrow$ $H_0(z)$ & $H_1(z)$ are also power complementary.

Theorem

If $H_0(z)$ & $H_1(z)$ are APC, then they are also power complementary.

(Not necessarily true for more than 2 TFs).

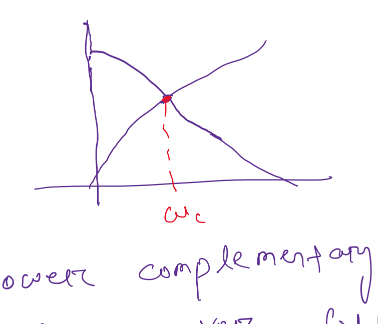
$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 = 1$$

At $\omega = \omega_c$ $\rightarrow$ $|H_0(e^{j\omega_c})|^2 = 1/2$

3-dB freq.

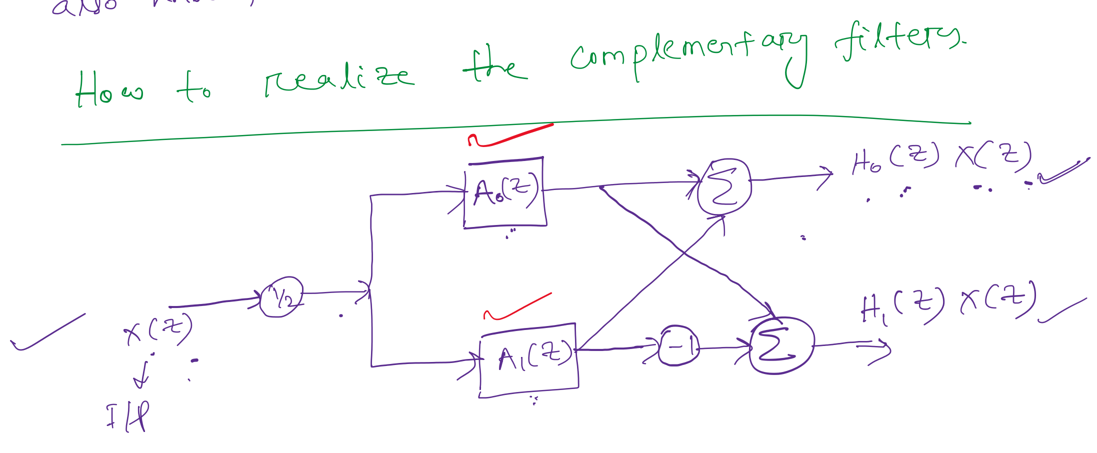
$$|H_1(e^{j\omega_c})|^2 = 1/2.$$

$\rightarrow$ Since $|H_0(e^{j\omega_c})| = |H_1(e^{j\omega_c})|$, there will be a cross-over at $\omega = \omega_c$. Therefore, ω_c is known as cross-over frequency.



$\rightarrow$ APC & power complementary filters are also known as cross-over filters.

How to realize the complementary filters



$\rightarrow$ The realization of Allpass filters are required.

Doubly-Complementary Transfer function

$\{H_i(z)\}$, $0 \le i \le M-1$, satisfying both allpass & power complementary.

$\rightarrow$
$$H_0(z) = \frac{1}{2}[A_0(z) + A_1(z)]$$
$$H_1(z) = \frac{1}{2}[A_0(z) - A_1(z)]$$

$\rightarrow$ Doubly-complementary filters.

Ex

Let $H_0(z) = \frac{1+z^{-1}}{1-\alpha z^{-1}} \left(\frac{1-\alpha}{2}\right)$

$\downarrow$ 1st order IIR LPF.

$$\Rightarrow H_0(z) = \frac{1}{2}\left[\frac{1 + z^{-1} - \alpha - \alpha z^{-1}}{1 - \alpha z^{-1}}\right]$$

$$= \frac{1}{2}\left[\frac{1 - \alpha z^{-1} - \alpha + z^{-1}}{1 - \alpha z^{-1}}\right]$$

$$= \frac{1}{2}\left[\underbrace{1}_{\text{Trivial APF.}} + \underbrace{\frac{-\alpha + z^{-1}}{1 - \alpha z^{-1}}}_{\text{1st order IIR APF.}}\right]$$

$$= \frac{1}{2}[A_0(z) + A_1(z)]$$

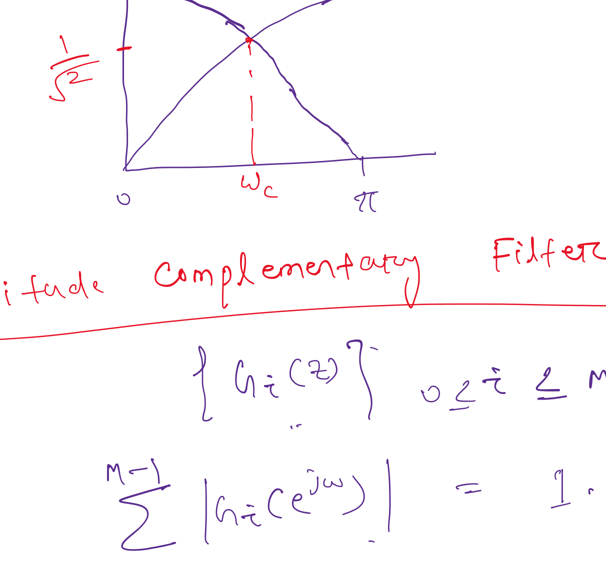
$$H_1(z) = \frac{1}{2}\left[1 - \frac{-\alpha + z^{-1}}{1 - \alpha z^{-1}}\right]$$

$$= \frac{1}{2}\left[\frac{1 - \alpha z^{-1} + \alpha - z^{-1}}{1 - \alpha z^{-1}}\right]$$

$$= \frac{1+\alpha}{2} \frac{1 - z^{-1}}{1 - \alpha z^{-1}}$$

$\rightarrow$ 1st order IIR HPF.

$$\boxed{\omega_c = \cos^{-1} \frac{2\alpha}{1+\alpha^2}} \rightarrow \text{cross-over freq.}$$



Magnitude Complementary Filter

$\{G_i(z)\}$ $0 \le i \le M-1$

$$\sum_{i=0}^{M-1} |G_i(e^{j\omega})| = 1.$$

$$|G_0(e^{j\omega})| + |G_1(e^{j\omega})| = 1.$$

$$H_0(z) = \frac{1}{2}[A_0(z) + A_1(z)]$$
$$H_1(z) = \frac{1}{2}[A_0(z) - A_1(z)]$$

$$G_0(z) = H_0^2(z).$$
$$G_1(z) = H_1^2(z).$$

magnitude complementary pairs.

$$|G_0(e^{j\omega})| \rightarrow |H_0^2(e^{j\omega})| = |H_0(e^{j\omega})|^2.$$

$$|G_p(e^{j\omega})| \rightarrow |H_1^2(e^{j\omega})| = |H_1(e^{j\omega})|^2$$

$$\Rightarrow |G_0(e^{j\omega})| + |G_1(e^{j\omega})|$$

$$= |H_0^2(e^{j\omega})| + |H_1^2(e^{j\omega})|$$

$$= |H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2$$

$$= 1.$$

$\rightarrow$ Here $G_0(z)$ & $G_1(z)$ are magnitude complementary filter pair.

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