Applied DSP II: Processing PPG Signals

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Outline

Continuing Where we Left..

Here we try to understand the end-to-end pipeline for PPG processing as given in Figure 1.

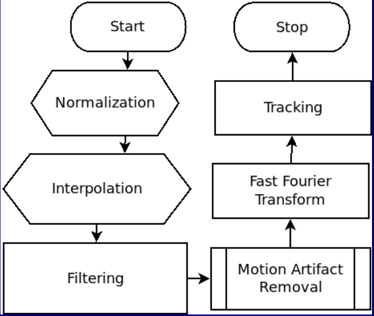


Figure 1: Algorithm Pipeline

Recap

We have already seen the motion artifact removal block in some detail, now we examine more of Singular Value Decomposition and the rest of the blocks.

Algorithm for Computing Singular Values

- For a matrix A, compute AA^T this will be a square matrix 'P'
- Find a scalar value λ such that $|A - \lambda I| = 0$, where 'I' is the identity matrix of same order.
- Singular values are positive square roots all possible values of λ. If λ values are 0, they are ignored, negative values indicate non-decomposing matrix.

Note

Positive values determine the rank of singular values of a matrix, which may reduce the dimension, further smaller values can be ignored.

Estimating the Eigenvectors

Computing the Left Eigenvector

- The left eigenvector is a *row vector* L_R
- For matrix A, it follows $AL_R = \lambda L_R$
- so for any of the eigenvalues, we have to find such a row vector.
- There is no direct solution, it is typically determined "iteratively".

Computing the Right Eigenvector

- The right eigenvector is a *column vector* R_C
- For matrix A, it follows $AR_C = \lambda R_C$
- so for any of the eigenvalues, we have to find such a column vector.

The vectors are then converted to their unit vector form

An example

Let, $A = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$

$=> A^T A = \begin{bmatrix} 25 & 15 \\ -15 & 25 \end{bmatrix}$

$=> \lambda = \{10, 40\}$

For $\lambda = 10$,

$=> \begin{bmatrix} 15 & -15 \\ -15 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$=> x_1 = x_2$ For $x_1 = 1$, $x_2 = 1$

$=> R_C = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Converting to unit vector, we get: $R_C = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

Points to Ponder...

There is *no unique* solution and also *no direct equation* to solve the above. Hence *iterative* methods are used.

Windowing

- Windowing converts a continuous time series into manageable chunks
- It helps us process the data online
- It reduces storage requirements of the algorithm

PPG Windowing Scheme

In our approach we have used rectangular windows with 50% overlap.

Normalization

- Normalization allows us to scale signals to a common range
- One of the common ways employed here is to subtract mean (μ) divide by standard deviation (σ).
- The formula for normalization is given in equation 1.

Formula for normalization

$$X_{norm}(t) = \frac{X(t) - \mu}{\sigma} \tag{1}$$

Interpolation

- Interpolation ensures *uniform sampling*.
- Sensor data has *jitter* which makes filters ineffective
- Interpolation is needed before filtering.
- Also makes the signal *smooth*.

Our Approach: Cubic Spline Interpolation

1. We assume p_t and p_{t+1} are joined by a equation as in equation 2.
2. values of a,b,c,d are derived by *empirical curve fitting*.

$$p_\tau = a\tau^3 + b\tau^2 + c\tau + d; t \leq \tau \leq t+1 \tag{2}$$

Filtering

- Filtering allows selective amplification of a frequency range.
- It can be used to select/reject or subdue certain frequencies within the signal.
- There are multiple kinds of filters
  1. band-pass
  2. Band-stop
  3. Loss-pass
  4. High-pass
- By means of technique there are *Infinite Impulse Response (IIR)* and *Finite Impulse Response (FIR)* filter

PPG Filter

- We have used a IIR band-pass filter.
- The frequency range is based on normal heart-rate range of 30-200 BPM. So we have designed a 0.5-3Hz band-pass filter.

Zero Phase Filtering

- Biomedical signals are *quasi-periodic*, just like mechanical vibrations.
- Zero phase filtering ensures that phase is removed thereby giving better measures.

Zero-Phase IIR Butterwoth Filter

filtfilt algorithm works by applying the Butterwoth IIR filter on signal and then reverse of the signal in a window.

IIR Filter

- IIR filters are so called because an impulse theoretically is carried on the filtered signal till infinity.
- It is implemented via a method called *recursion* as shown in equation 3.

Mathematical Representation of IIR filter

$$\rho[t+1] = \frac{1}{a_0}(\Sigma_{i=0}^{P} b_i \times p[t-i] + \Sigma_{j=0}^{Q} a_j \times \rho[t-j]) \tag{3}$$

where, $\rho[t]$ is the output signal. $p[t]$ is the input signal. a_j are the feedback filter coefficients. b_i are the feed-forward filter coefficients. P is the feed-forward order and Q is the feedback filter order.

Fast Fourier Transform (FFT)

- Fourier transform transforms a time-series signal into temporal frequency domain from time-domain.
- The Fourier transform of a signal $X(t)$, denoted by $H(x)$ is given by equation 4.

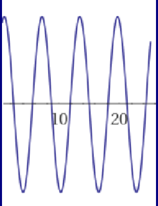
Fourier Transform Details

$$H(x) = \mathcal{R}\left\{ \frac{1}{2\pi} \int_0^\infty 2F(\omega)\mathrm{e}^{i\omega t}\, d\omega \right\} \tag{4}$$

also, $\mathrm{e}^{i\omega t} = Cos(\omega t) + i \times Sin(\omega t)$
and $\omega = 2\pi\nu$
where ν is a component frequency in the signal.

Significance of Fourier Transform

- Fourier transform decomposes a signal into independent *phase* and *amplitude* pairs each representing one oscillation in the signal
- In other words, it decomposes a signal into a sum of Sine and Cosine functions.
- Figure 2a and 2b shows how a signal and its Fourier transform look respectively.



(a) Signal

(b) FFT of signal

FFT and Heart-Rate

- In the FFT of a PPG signal filtered between 0.5-3Hz (30-180 bpm), the fundamental peak is considered to be the heart-rate.
- Standard peak detection methods are used as the frequency spectrum is "*clean*".

Computing Heart-Rate

Since heart-rate is expressed in beats per minute, it is advisable to convert the frequency as in equation 5, where F is the *dominant frequency* in Hz. The "round" function is used since heart-rate is expressed as a natural number.

$$HR = round(F * 60) \tag{5}$$

Tracking Algorithm

- *Why tracking?*: Because sometimes, even with much effort, we get a wrong HR measurement.
- Tracking is a rule-based post processing with simple physiological rules:
  1. If person is going from rest to motion then HR should increase.
  2. HR cannot change by more that 20 BPM (usually) within 10 seconds time.
  3. If abrupt HR is found, we replace it with last good HR measurement.

Schematic of Tracking

$$\mathrm{HR} = \mathrm{track}(\mathrm{lastHR}, \mathrm{activityStatus})$$

Results

- We present our accuracy in-terms of *Mean Absolute Deviation (MAD)*
- Result in presented for a scientific data-set [1] and our field collected data on over 13 workers during their daily duty over a span of 15 days.
- Summary of results in presented in Table 1.

Summary of Results

Errors are reported as Mean Absolute Deviation (MAD) in beats per minute (BPM). Formula for MAD is $MAD = \frac{\Sigma_{i=0}^{N} ||HR_{observed} - HR_{real}||}{N}$. The real HR was measured using a chest strap.

Data-set	MAD
Troika [1]	3
Field Collected	7

Table 1: Summary of Results

References

Zhang, Zhilin, Zhouyue Pi, and Benyuan Liu. "TROIKA: A general framework for heart rate monitoring using wrist-type photoplethysmographic signals during intensive physical exercise." IEEE Transactions on biomedical engineering 62, no. 2 (2014): 522-531.

Thank You!