Lecture 10

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Continuation previous Example.

$$y(n) + y(n-1) - 6y(n-2) = \underbrace{2^n u(n)}_{x(n)}$$

$\lambda_1 = -3, \quad \lambda_2 = 2$

$y_c(n) = \alpha_1(-3)^n + \alpha_2(2)^n.$

$y_p(n) = \beta n 2^n.$

$$\beta n 2^n + \beta(n-1)2^{n-1} - 6\beta(n-2)2^{n-2} = 2^n u(n)$$

for $n \geq 0$

$$2^{n-2}\left(\beta n 2^2 + \beta(n-1)2 - 6\beta(n-2)\right) = 2^n.$$

$\Rightarrow \quad \beta = 0.4.$

$y(n) = y_c(n) + y_p(n)$

$= \alpha_1(-3)^n + \alpha_2(2)^n + 0.4n2^n, \quad n \geq 0$

$y(-2) = -1$
$y(-1) = 1$ $\Big\} \rightarrow$ initial cond^n (given).

Find the coefficients using these initial cond^{n}s.

$\alpha_1 = -5.04, \quad \alpha_2 = -0.96.$

$$y(n) = -5.04(-3)^n - 0.96(2)^n + 0.4n(2)^n.$$

Zero I/P and Zero-state response.

$\hookrightarrow$ Alternate approach to determine the total solution.

$$y(n) = \underbrace{y_{zi}(n)}_{\text{zero I/P response}} + \underbrace{y_{zs}(n)}_{\text{zero state response.}}$$

$y_{zi}(n) \neq y_c(n).$
$y_{zs}(n) \neq y_p(n)$

$y_{zi}(n) \rightarrow$ Has the same form as $y_c(n)$, but the constants will be determined using the initial cond^n, before adding $y_{zs}(n)$.

$y_{zs}(n) \rightarrow$ Find $y_c(n) + y_p(n)$, with constants determined from the zero initial cond^n.

Example.

$y(n) + y(n-1) - 6y(n-2) = x(n)$

$x(n) = 8u(n)$

$y(-1) = 1, \quad y(-2) = -1.$

Sol^n

Find $y_{zi}(n)$:

$y(n) = \lambda^n, \quad x(n) = 0.$

$\lambda^n + \lambda^{n-1} - 6\lambda^{n-2} = 0$

$\Rightarrow \lambda_1 = -3, \quad \lambda_2 = 2.$

$y_{zi}(n) = \alpha_1(-3)^n + \alpha_2(2)^n.$

$y(-1) = \alpha_1(-3)^{-1} + \alpha_2(2)^{-1} = 1$
$y(-2) = \alpha_1(-3)^{-2} + \alpha_2(2)^{-2} = -1$

$\alpha_2 = -1.6, \quad \alpha_1 = -5.4.$

$\Rightarrow y_{zi}(n) = -5.4(-3)^n - 1.6(2)^n.$

Find $y_{zs}(n)$:

$y_c(n) = c_1(-3)^n + c_2(2)^n$

$y_p(n) = \beta u(n)$

$\beta + \beta - 6\beta = 8, \quad n \geq 0$

$\Rightarrow \beta = -2. \quad \Rightarrow y_p(n) = -2u(n)$

$y_{zs}(n) = c_1(-3)^n + c_2(2)^n - 2$ for $n \geq 0$.

For $n=0$

$y(0) + \cancel{y(-1)}^{0} - 6\cancel{y(-2)}^{0} = 8.$

$y(0) = 8.$

For $n=1$

$y(1) + y(0) - \cancel{6y(-1)}^{0} = 8$

$\Rightarrow y(1) = 8 - y(0) = 0.$

$y(0) = c_1 + c_2 - 2 = 8.$
$y(1) = -3c_1 + 2c_2 - 2 = 0.$

$c_1 + c_2 = 10 \quad$ — ①
$-3c_1 + 2c_2 = 2 \quad$ — ②

$c_1 = 3.6, \quad c_2 = 6.4.$

$y_{zs}(n) = 3.6(-3)^n + 6.4(2)^n - 2, \quad n \geq 0$

$$y(n) = y_{zi}(n) + y_{zs}(n)$$
$$= \underbrace{-5.4(-3)^n - 1.6(2^n)}_{y_{zi}(n)} + \underbrace{3.6(-3)^n + 6.4(2)^n - 2}_{y_{zs}(n)}$$

$$\Rightarrow \boxed{y(n) = -1.8(-3)^n + 4.8(2)^n - 2, \quad n \geq 0}$$

Computation of Impulse response from Difference equation :

For LTI causal system

$\rightarrow$ o/p observed with $x(n) = \delta(n)$

$h(n) = 0, \quad n < 0.$

$\rightarrow$ Thus, it is simply $\underbrace{\text{zero state response}}_{\text{since all initial cond}^n \text{ are zero (causality).}}$ with $x(n) = \delta(n)$.

$\rightarrow$ For an I/P $x(n) = \delta(n)$, $x(n) = 0$, for $n > 0$, Thus, $y_p(n) = 0.$

$$y_{zs}(n) = \sum_{i=1}^{N} \alpha_i(\lambda_i)^n u(n) + \cancel{y_p(n)}^{0}$$

$$h(n) = \sum_{i=1}^{N} \alpha_i(\lambda_i)^n u(n)$$

$\lambda_i \rightarrow$ obtained from ch. eq^n.
$\alpha_i \rightarrow$ Obtained using zero initial cond^n.

Ex

Determine the impulse res. $h(n)$ for the following system

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

Find $y_c(n)$

$y(n) - 3y(n-1) - 4y(n-2) = 0$

$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$

$\Rightarrow \lambda_1 = -1, \quad \lambda_2 = 4.$

$y_c(n) = \alpha_1(-1)^n + \alpha_2(4)^n.$

$y_p(n) = 0$

$y_{zs}(n) = \alpha_1(-1)^n + \alpha_2(4)^n.$

$y(n) - 3y(n-1) - 4y(n-2) = \delta(n) + 2\delta(n-1)$

for $n=0$

$y(0) - 3\cancel{y(-1)}^{0} - 4\cancel{y(-2)}^{0} = 1 + \cancel{2\delta(-1)}^{0}$

$\Rightarrow y(0) = 1.$

for $n=1$

$y(1) - 3y(0) - 4\cancel{y(-1)}^{0} = 2.$

$\Rightarrow y(1) = 3y(0) + 2 = 5.$

$y_{zs}(0) = \alpha_1 + \alpha_2 = 1$
$y_{zs}(1) = -\alpha_1 + 4\alpha_2 = 5.$

$\alpha_1 = -\frac{1}{5}, \quad \alpha_2 = \frac{6}{5}.$

$$\Rightarrow \boxed{h(n) = \left(-\frac{1}{5}(-1)^n + \frac{6}{5}(4)^n\right)u(n)}$$