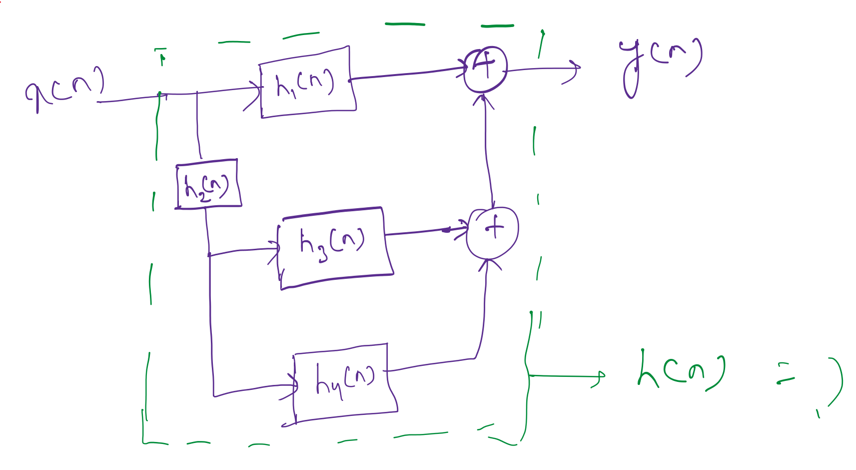
Lecture 9

Monday, 6 September 2021 2:58 PM

Ex



$\to h(n) =)$

$h_1(n) = \delta(n) + \frac{1}{2}\delta(n-1)$

$h_2(n) = \frac{1}{2}\delta(n) - \frac{1}{4}\delta(n-1)$

$h_3(n) = 2\delta(n)$

$h_4(n) = -2\left(\frac{1}{2}\right)^n u(n)$.

$h(n) = ?$

Ans

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \delta(n) - \frac{1}{2}\delta(n-1) - \delta(n) = \delta(n).$$

Difference eq^n

Ex 1 $\quad y(n) - \alpha y(n-1) = x(n)$

↳ recursive diff. eq^n.

Ex 2 $\quad y(n) = x(n) + x(n-1) + x(n-2)$

↳ Non recursive diff. eq^n.

→ In general an LTI system is expressed by a difference eq^n.

$$y(n) + b_1 y(n-1) + \cdots + b_N y(n-N) = a_0 x(n) + a_1 x(n-1) + \cdots + a_M x(n-M).$$

$b_0 = 1$

$$\sum_{k=0}^{N} b_k y(n-k) = \sum_{k=0}^{M} a_k x(n-k)$$

order of the system $= \max(N, M)$.

Assume system is causal.

$$y(n) = -\sum_{k=1}^{N} \frac{b_k}{b_0} y(n-k) + \sum_{k=0}^{M} \frac{a_k}{b_0} x(n-k)$$

$y(n)$, $n \geq n_0$. Initial cond^n.

Calculation of the total sol^n of diff. eq^n.

Sol^n to diff. eq^n consist of two parts

i) $y_c(n) \to$ Complementary solution / Homogeneous solution.

↳ Solution to the given eq^n with $x(n) = 0$.

ii) $y_p(n) \to$ Perticular solution

→ Sol^n to the given eq^n with $x(n) \neq 0$.

Method of computing $y_c(n)$:-

Assume that the form of $y_c(n)$ as

$y_c(n) = \lambda^n$

$$y(n) + b_1 y(n-1) + \cdots + b_N y(n-N) = a_0 x(n) + \cdots + a_M x(n-M) \quad (1)$$

$$y(n) + b_1 y(n-1) + \cdots + b_N y(n-N) = 0.$$

↳ Homegeneous eq^n.

Sol^n to this eq^n is $y_c(n)$.

$$\lambda^n + b_1 \lambda^{n-1} + \cdots + b_N \lambda^{n-N} = 0.$$

$$\Rightarrow \lambda^{n-N}\left(\lambda^N + b_1 \lambda^{N-1} + \cdots + b_N\right) = 0.$$

$$\Rightarrow \lambda^N + b_1 \lambda^{N-1} + \cdots + b_N = 0.$$

↳ Characteristic polynomial of the DTS given in (1).

Its roots are the eigenvalues.

(i) If all roots are distinct.

$\lambda_1, \lambda_2 \cdots \lambda_N \to N$ distinct roots.

$$y_c(n) = \alpha_1 \lambda_1^n + \alpha_2 \lambda_2^n + \cdots + \alpha_N \lambda_N^n$$

Constant.

→ Constants will be computed using the initial cond^n on the total sol^n.

(ii) If the roots are not distinct.

$\lambda_1 \to$ repeated L time.

remaining N-L roots are distinct.

$$\underbrace{\lambda_1, \lambda_1 \cdots \lambda_1}_{\text{L repeated}}, \underbrace{\lambda_{L+1}, \lambda_{L+2} \cdots \lambda_N}_{\text{distinct}}.$$

$$y_c(n) = \lambda_1^n\left(\alpha_1 + n\alpha_2 + n^2\alpha_3 + \cdots + n^{L-1}\alpha_L\right) + \alpha_{L+1}\lambda_{L+1}^n + \alpha_{L+2}\lambda_{L+2}^n + \cdots + \alpha_N \lambda_N^n.$$

Computation of the perticular sol^n $y_p(n)$

→ Assume that $y_p(n)$ is also of the same form as $x(n)$.

If $x(n) = K$, $\Rightarrow y_p(n) = K_1$

$x(n) = A\cos(n\omega_0 + \phi) \Rightarrow y_p(n) = B\cos(n\omega_0 + \phi)$

$x(n) = \alpha^n \Rightarrow y_p(n) = c\,\alpha^n$.

Ex

$$y(n) + y(n-1) - 6y(n-2) = x(n)$$

For this DTS, determine the total solution $y(n)$ for $n \geq 0$, for the I/P $x(n) = 8u(n)$, with the initial cond^{n}s $y(-1) = 1$, & $y(-2) = -1$.

Determine $y_c(n)$

Set $x(n) = 0$, $y(n) = \lambda^n$.

$\Rightarrow y(n) + y(n-1) - 6y(n-2) = 0$

$\Rightarrow \lambda^n + \lambda^{n-1} - 6\lambda^{n-2} = 0$

$\Rightarrow \lambda^{n-2}(\lambda^2 + \lambda - 6) = 0$

$\lambda^2 + \lambda - 6 = 0 \Rightarrow (\lambda+3)(\lambda-2) = 0$

$\Rightarrow \lambda_1 = -3, \quad \lambda_2 = 2. \to$ Distinct

$y_c(n) = \alpha_1(-3)^n + \alpha_2 2^n$

For the particular sol^n

Assume $y_p(n) = \beta$.

substitute it on the diff. eq^n.

$\beta + \beta - 6\beta = 8u(n)$

for $n \geq 0$,

$\beta + \beta - 6\beta = 8$

$\Rightarrow \beta = -2$.

Total solution is of the form

$y(n) = y_c(n) + y_p(n)$

$= \alpha_1(-3)^n + \alpha_2(2)^n - 2$.

$y(-2) = \alpha_1(-3)^{-2} + \alpha_2(2)^{-2} - 2 = -1$.

$y(-1) = \alpha_1(-3)^{-1} + \alpha_2(2)^{-1} - 2 = 1$

$\Rightarrow \alpha_1 = -1.8, \quad \alpha_2 = 4.8$.

The total sol^n is

$$y(n) = -1.8(-3)^n + 4.8(2)^n - 2, \quad n \geq 0.$$

Example

Same question as the previous example except the I/P $x(n) = 2^n u(n)$.

Sol^n

$y(n) + y(n-1) - 6y(n-2) = x(n)$.

$\lambda_1 = -3, \quad \lambda_2 = 2$

$y_c(n) = \alpha_1(-3)^n + \alpha_2(2)^n$.

It is of same form as that of I/P.

$\boxed{y_p(n) \text{ \& } y_c(n) \text{ should n't have the same form.}}$

$y_p(n) = \beta n 2^n$