Lecture 11

Friday, 10 September 2021 2:55 PM

FIR and IIR systems

Finite Impulse response (FIR)

→ If $h(n) \neq 0, \quad N_1 \leq n \leq N_2$
$= 0, \quad$ otherwise.

$h(n)$ nonzero between N_1 and N_2

→ If N_1 or N_2 or both $\to \infty$, then it will be infinite impulse response (IIR) system.

Ex

$$y(n) = \frac{1}{M}\sum_{k=0}^{M-1} x(n-k)$$

$$h(n) = \Big\{ \underset{\substack{\uparrow \\ n=0}}{\frac{1}{M}}, \frac{1}{M}, \cdots \underset{n=M-1}{\frac{1}{M}} \Big\}$$

↳ FIR system.

→ $x(n) \longrightarrow \boxed{\text{LTI DTS}}_{h(n)} \longrightarrow y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$

This is a non-recursive computation.

This computation is independent to FIR or IIR systems.

→ Both FIR & IIR LTI systems can be computed non-recursively.

→ FIR systems not necessarily non-recursive
IIR " " " recursive

Ex

$$y(n) = x(n) + x(n-1) \longrightarrow \text{Non-recursive}$$

↳ FIR system
$h(n) \to \{ \underset{\uparrow}{1}, 1 \}$

$\Rightarrow y(n-1) = x(n-1) + x(n-2)$

$\Rightarrow y(n) - y(n-1) = x(n) - x(n-2)$

$\Rightarrow y(n) = y(n-1) + x(n) - x(n-2)$

↳ recursive computation.

Frequency-domain representation of DT signal.

Discrete time Fourier Transform (DTFT)

$X(e^{j\omega}) = F\{x(n)\}$ $\quad \omega \to$ Normalized angular freq.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\,e^{-j\omega n}$$

↓ continous fun^n of ω.

→ $$X(e^{j(\omega+2\pi k)}) = \sum x(n)\,e^{-j(\omega+2\pi k)n}$$
$$= \sum x(n)\,e^{-j\omega n}$$
$$= X(e^{j\omega}).$$

→ $X(e^{j\omega})$ is a periodic function of ω with a period 2π.

$$\boxed{\underbrace{x(t)}_{\text{periodic}} = \sum_{k=-\infty}^{\infty} c_k\, e^{j2\pi f_0 k t} \quad \to \text{F.S.}}$$

$$c_k = \frac{1}{T}\int_0^T x(t)\,e^{-j2\pi f_0 t}$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,e^{j\omega n}\,d\omega$$
$$= \text{IDFT}\{X(e^{j\omega})\}$$
$$= F^{-1}\{X(e^{j\omega})\}.$$

$$x(n) \overset{F}{\longleftrightarrow} X(e^{j\omega}).$$

$X(e^{j\omega}) \to$ periodic function
↳ complex function of the real variable ω.

$$X(e^{j\omega}) = \underbrace{X_{re}(e^{j\omega})}_{} + j\,\underbrace{X_{im}(e^{j\omega})}_{}.$$

real function of ω.

$$X_{re}(e^{j\omega}) = \frac{1}{2}\{X(e^{j\omega}) + X^*(e^{j\omega})\}$$
$$X_{im}(e^{j\omega}) = \frac{1}{2j}\{X(e^{j\omega}) - X^*(e^{j\omega})\}$$

$$X(e^{j\omega}) = |X(e^{j\omega})|\,e^{j\theta(\omega)}.$$

$$\theta(\omega) = \arg\{X(e^{j\omega})\}.$$

$|X(e^{j\omega})| \to$ Magnitude spectrum. (even fun^n)

$\theta(\omega) \to$ Phase spectrum.

$$\tan(\theta(\omega)) = \frac{X_{im}(e^{j\omega})}{X_{re}(e^{j\omega})}$$

→ $\theta(\omega) \to \theta(\omega) + 2\pi k.$

$\Rightarrow X(e^{j\omega}) = |X(e^{j\omega})|\,e^{j(\theta(\omega)+2\pi k)}$
$= |X(e^{j\omega})|\,e^{j\theta(\omega)}.$

→ $\theta(\omega)$ can't be uniquely specified for any DTFT $\forall\, \omega$.

$\theta(\omega)$ is uncertain.

→ $-\pi \leq \theta(\omega) \leq \pi$

Symmetry Relations

$x(n) \longleftrightarrow X(e^{j\omega}) = X_{re}(e^{j\omega}) + j\,X_{im}(e^{j\omega}).$

$x(n) \longleftrightarrow X(e^{j\omega})$
↳ one-to-one mapping.

$x_{ev}(n) \longleftrightarrow X_{re}(e^{j\omega})$

$x_{od}(n) \longleftrightarrow j\,X_{im}(e^{j\omega}).$

$$X(e^{j\omega}) = X^*(e^{-j\omega}).$$

$$X_{re}(e^{j\omega}) = X_{re}(e^{-j\omega})$$
↳ Even function of ω.

$$X_{im}(e^{j\omega}) = -X_{im}(e^{-j\omega})$$
↳ odd function of ω.

$$\arg\{X(e^{j\omega})\} = -\arg\{X(e^{-j\omega})\}$$

$$\Rightarrow \theta(\omega) = -\theta(-\omega).$$
↳ odd function of ω.

If $x(n) \longleftrightarrow X(e^{j\omega})$

$x(-n) \longleftrightarrow X(e^{-j\omega}).$

$x^*(-n) \longleftrightarrow X^*(e^{j\omega}).$

$x_{re}(n) \longleftrightarrow \frac{1}{2}\{X(e^{j\omega}) + X^*(e^{-j\omega})\}$
$= X_{cs}(e^{j\omega})$

$j\,x_{im}(n) \longleftrightarrow \frac{1}{2}\{X(e^{j\omega}) - X^*(e^{-j\omega})\}$
$= X_{ca}(e^{j\omega})$

$x_{cs}(n) \longleftrightarrow X_{re}(e^{j\omega})$

$x_{ca}(n) \longleftrightarrow j\,X_{im}(e^{j\omega}).$

Ex

1) $x(n) = \delta(n)$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \delta(n)\,e^{-jn\omega} = 1.$$

$\delta(n) \longleftrightarrow 1.$
↳ one-for-one.

2) $x(n) = \alpha^n u(n), \quad |\alpha| < 1.$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} (\alpha)^n\,e^{-j\omega n}$$

$$= \frac{1}{1-\alpha e^{-j\omega}}, \quad |\alpha| < 1.$$

→ Here $|X(e^{j\omega})| < \infty$, so the F.T. of discrete time signal exists.