Lecture 28

Monday, 25 October 2021 3:00 PM

Simple IIR Filters

IIR LPF

$$H(z) = \left(\frac{1-\alpha}{2}\right)\frac{1+z^{-1}}{1-\alpha z^{-1}}$$

It also ensures that the max^m value of $|H(e^{j\omega})| = 1$.

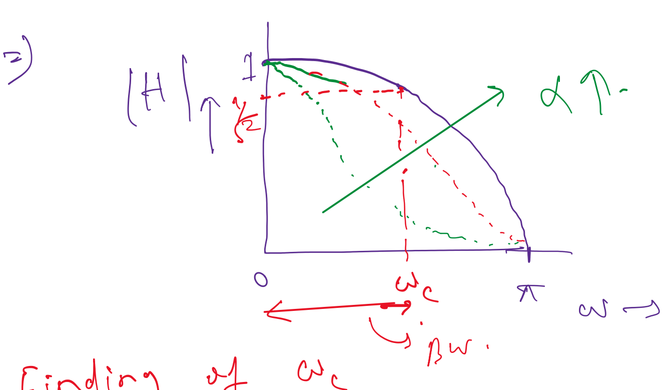
$$\boxed{|\alpha| < 1}$$

$z = \alpha \rightarrow$ pole.

$$|H(e^{j\omega})|^2 = \frac{(1-\alpha)^2}{2}\frac{1+\cos\omega}{1-2\alpha\cos\omega+\alpha^2}$$

$$\frac{d|H(e^{j\omega})|^2}{d\omega} < 0 \rightarrow 0 < \omega < \pi$$

$\Rightarrow$



Finding of ω_c

$$|H(e^{j\omega_c})|^2 = \frac{1}{2}$$

$$\Rightarrow \frac{(1-\alpha)^2}{2}\frac{1+\cos\omega_c}{1+2\alpha\cos\omega_c+\alpha^2} = \frac{1}{2}$$

$$\Rightarrow \boxed{\cos\omega_c = \frac{2\alpha}{1+\alpha^2}}$$

$\rightarrow$ ω_c is controlled by α

For a given ω_c

$$\alpha = \frac{1 \pm \sin\omega_c}{\cos\omega_c}$$

$$\boxed{\text{Acceptable } \alpha = \frac{1-\sin\omega_c}{\cos\omega_c}}$$

$\rightarrow |\alpha| < 1$.

Ex

$\omega_c = \pi/2$

$\Rightarrow \alpha = 0$.

$H(z) = \frac{1}{2}(1+z^{-1})$ $\rightarrow$ FIR LPF.

$\rightarrow$ BW of LPF $= \omega_c$

IIR HPF

$$H_{HP}(z) = H_{LP}(-z)$$

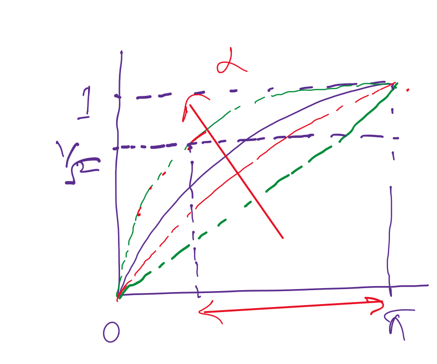
$$H_{HP}(z) = \frac{1-\alpha}{2}\frac{1-z^{-1}}{1+\alpha z^{-1}}$$

or

$$\left(\frac{1+\alpha}{2}\right)\frac{1-z^{-1}}{1-\alpha z^{-1}}$$

$\rightarrow$ At $z = -1$: $\frac{2}{1+\alpha}$

$$H_{HP}(z) = \left(\frac{1+\alpha}{2}\right)\frac{1-z^{-1}}{1-\alpha z^{-1}}$$



$\alpha\uparrow$ the ch. approaches to ideal

$$\cos\omega_c = \frac{2\alpha}{1+\alpha^2}$$

$$\rightarrow \alpha = \frac{1-\sin\omega_c}{\cos\omega_c} \rightarrow |\alpha| < 1.$$

$\rightarrow$ BW of HPF $= \pi - \omega_c$.

IIR Band pass Filter

$$\rightarrow H(z) = \frac{1-z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}\left(\frac{1-\alpha}{2}\right)$$

$z = \pm 1 \rightarrow H(z) = 0$.

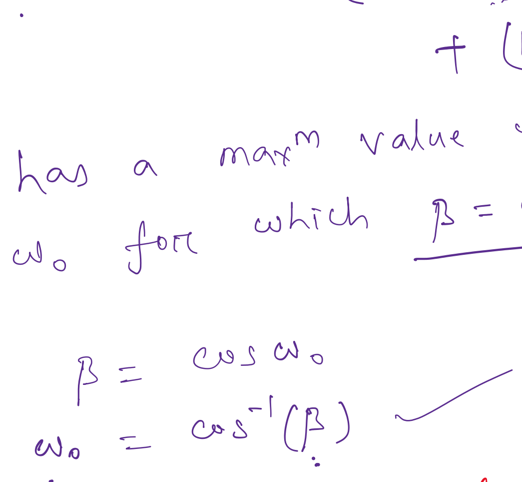
Find $\omega = \omega_0$ for which $|H(e^{j\omega})|^2$ is max^m

$$|H(e^{j\omega})|^2 = \frac{(1-\alpha)^2}{4}\frac{4\sin^2\omega}{(1+\alpha)^2(\beta-\cos\omega)^2+(1-\alpha)^2\sin^2\omega}$$

$\rightarrow$ It has a max^m value of 1 at $\omega = \omega_0$ for which $\beta = \cos\omega_0$.

$\beta = \cos\omega_0$

$\omega_0 = \cos^{-1}(\beta)$ $\rightarrow$ center freq. of the BPF.



$$\boxed{\omega_2 - \omega_1 = BW = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)}$$

$\rightarrow$ Quality factor $= Q = \frac{\omega_0}{BW}$.

$\beta \rightarrow$ determines the center freq.

$\alpha \rightarrow$ " " BW.

$\rightarrow$ Zeros: $z = \pm 1$.

Poles: $z = \frac{\beta(1+\alpha) \pm \sqrt{\beta^2(1+\alpha)^2 - 4\alpha}}{2}$.

$|\alpha| < 1$

Ex

$\omega_0 = \frac{\pi}{2}$, $BW = \frac{\pi}{4}$. Design IIR BPF

Sol^n

$$Q = \frac{\omega_0}{BW} = 2.$$

$$\beta = \cos\omega_0 = 0$$

$$\frac{2\alpha}{1+\alpha^2} = \cos(BW) = \frac{1}{\sqrt{2}}.$$

$$\Rightarrow \alpha = \sqrt{2} \pm 1.$$

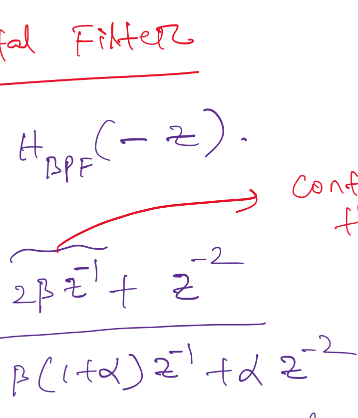
$$\alpha = \sqrt{2} - 1.$$

Since $\beta = 0$

$z = \pm\sqrt{-\alpha} = \pm j\sqrt{\alpha}$

If $\alpha = \sqrt{2} - 1$.

$\Rightarrow z = \pm j\sqrt{\sqrt{2}-1}$



$$\boxed{H(z) = \left(1 - \frac{1}{\sqrt{2}}\right)\frac{1-z^{-2}}{1+(\sqrt{2}-1)z^{-2}}}$$

Band-stop IIR Digital Filter

$$H_{BSF}(z) \neq H_{BPF}(-z).$$

$$H(z) = \frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}.$$

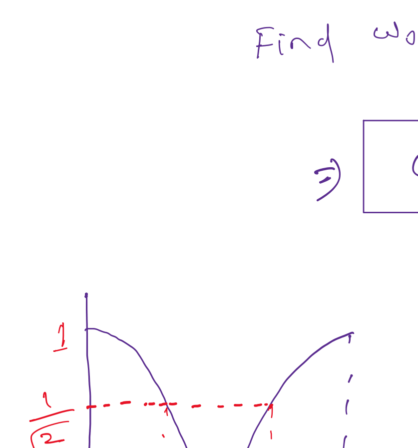
controls the Notch freq.

$\rightarrow$ This is a simple 2nd order IIR BSF.

How to find the Notch freq.

Find ω_0 s.t. $|H(e^{j\omega_0})|^2 = 0$.

$$\Rightarrow \boxed{\cos\omega_0 = \beta}$$



$$BW = \omega_2 - \omega_1 = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right).$$

$|\alpha| < 1$.