Lecture 17

Friday, 24 September 2021 2:58 PM

Implementation of Linear Convolution using Circular conv.

Generalized.

$g(n) \to 0 \le n \le N-1$.

$h(n) \to 0 \le n \le M-1$

$y_L(n) = g(n) * h(n)$

$L = M+N-1$

$0 \le n \le M+N-2$.

$$\tilde{g}(n) = \begin{cases} g(n), & 0 \le n \le N-1 \\ 0, & N \le n \le L-1 \end{cases}$$

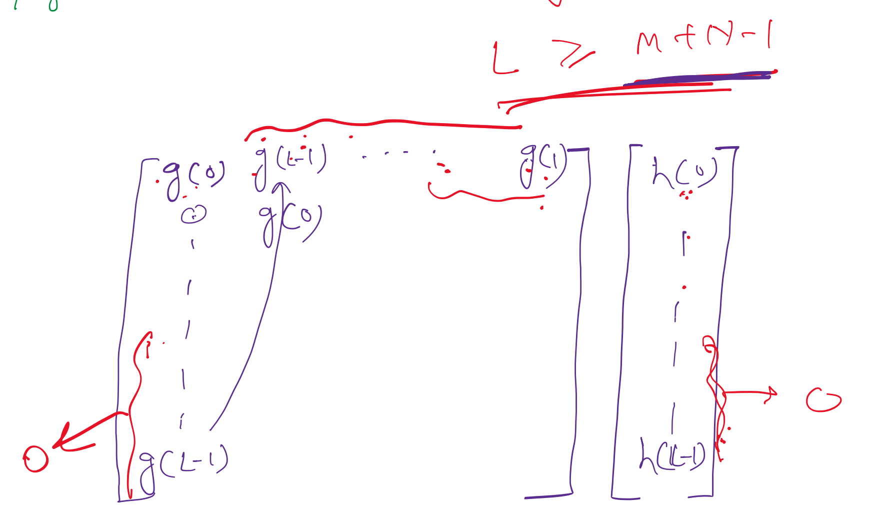
Appending $L-N = M-1$ zeros.

$$\tilde{h}(n) = \begin{cases} h(n), & 0 \le n \le M-1 \\ 0, & M \le n \le L-1 \end{cases}$$

Appending $L-M = N-1$ zeros.

$\underset{N\text{-length}}{g(n)} * \underset{M\text{-length}}{h(n)} = \tilde{g}(n) \, Ⓛ \, \tilde{h}(n)$

$L \ge M+N-1$

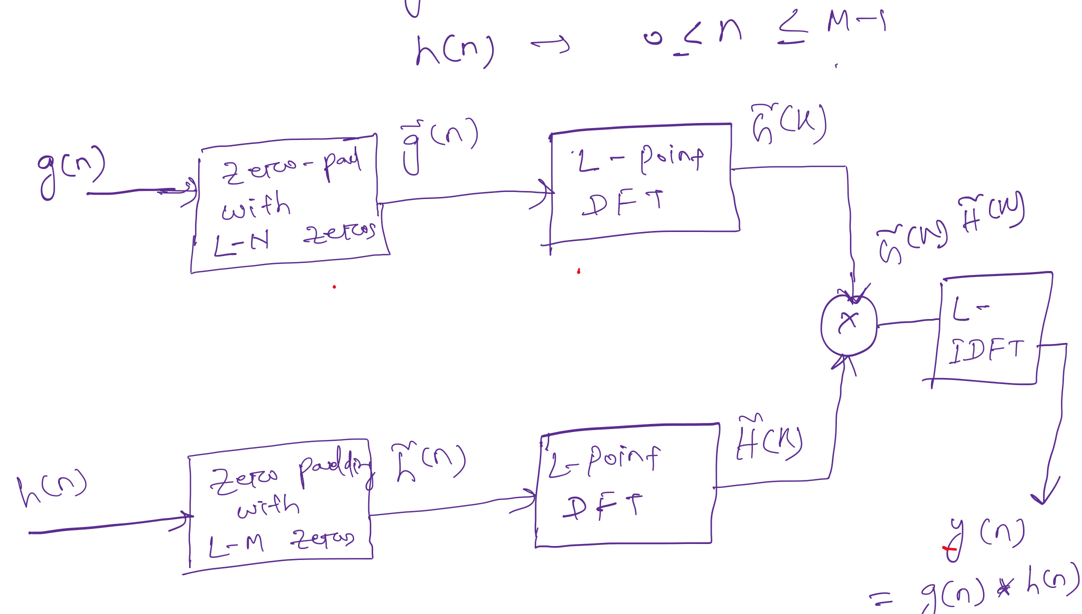


$\boxed{L \ge M+N-1}$ → $L = 2^n$.

Linear Convolution using DFT.

$g(n) \to 0 \le n \le N-1$

$h(n) \to 0 \le n \le M-1$



$\boxed{L \ge M+N-1}$

$0 \le n \le M+N-2$

Classification of complex sequences (Infinite length)

If $x(n) = x^*(-n)$

→ conjugate symmetric seq.

→ For real, it is even seq.

If $x(n) = -x^*(-n)$

→ conjugate anti-symmetric seq.

→ For real, it is odd seq.

→ Any complex seq. $x(n)$

$x(n) = x_{cs}(n) + x_{ca}(n)$

$x_{cs}(n)$: Conjugate symmetric part of $x(n)$; $x_{ca}(n)$: conjugate anti-symmetric part of $x(n)$

$x_{cs}(n) = \frac{1}{2}\left(x(n) + x^*(-n)\right)$

$x_{ca}(n) = \frac{1}{2}\left(x(n) - x^*(-n)\right)$.

Classification for finite complex seq.

→ Not same as that for infinite length seq.

$x(n) \to 0 \le n \le N-1$

$x(n) = x_{cs}(n) + x_{ca}(n)$

$x_{cs}(n) = \frac{1}{2}\left(x(n) + x^*(\langle -n \rangle_N)\right)$ → Circular-conjugate symmetric part of $x(n)$

$x_{ca}(n) = \frac{1}{2}\left(x(n) - x^*(\langle -n \rangle_N)\right)$. → Circular conjugate antisymmetric part of $x(n)$.

If $x(n)$ is real

$x_{cs}(n) = x_{ev}(n)$ → circular even part

$x_{ca}(n) = x_{odd}(n)$ → circular odd part.

→ If $x(n) = x^*(\langle -n \rangle_N) = x^*(\langle N-n \rangle_N)$.

then $x(n)$ is circular conjugate symmetric seq.

$x(n) \to 0 \le n \le N-1$

→ For real, circular even seq.

→ If $x(n) = -x^*(\langle -n \rangle_N) = -x^*(\langle N-n \rangle_N)$.

then $x(n)$ is circular conjugate antisymmetric seq.

for real $x(n)$, circular odd seq.

→ A complex DFT $X(k)$.

$X(k) = X_{cs}(k) + X_{ca}(k)$, $0 \le k \le N-1$

where $X_{cs}(k) = \frac{1}{2}\left(X(k) + X^*(\langle -k \rangle_N)\right)$

$X_{ca}(k) = \frac{1}{2}\left(X(k) - X^*(\langle -k \rangle_N)\right)$.

$0 \le k \le N-1$

Classification based on Geometric Symmetry.

Symmetric

If $x(n) = x(N-1-n)$

$x(n) \to 0 \le n \le \underset{\text{even or odd.}}{N-1}$

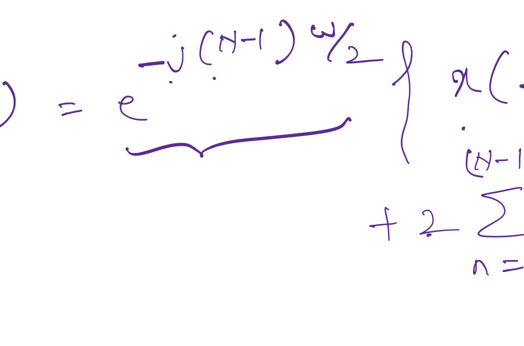
Antisymmetric

If $x(n) = -x(N-1-n)$

$x(n) \to 0 \le n \le N-1$

Symmetric seq with odd length.

Ex $N = 9$



Center of symmetric $\frac{N-1}{2}$.

$x(n) = x(N-1-n)$

$= x(8-n)$

DTFT

$$X(e^{j\omega}) = e^{-j(N-1)\omega/2} \left\{ x\left(\frac{N-1}{2}\right) + 2\sum_{n=1}^{(N-1)/2} x\left(\frac{N-1}{2} - n\right) \cos(\omega n) \right\}$$

Phase $= \alpha(\omega) = \frac{-(N-1)\omega}{2} + \beta$. ($\beta$: 0 or π)

$0 \le |\omega| \le \pi$

Linear phase.

DFT

$\omega = \frac{2\pi k}{N}$, $0 \le k \le N-1$

$$X(k) = e^{-j(N-1)\pi k/N} \left\{ x(N-1) + 2\sum_{n=1}^{(N-1)/2} x\left(\frac{N-1}{2} - n\right) \cos\left(\frac{2\pi k n}{N}\right) \right\}$$