

Applied DSP: Motion Artifact Correction in PPG Signals

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Outline

Introduction

- Background

- Problem Overview and Solution Approach Taken

Singular Spectral Analysis

- Singular Value Decomposition

- Hankel Matrix

- SSA Algorithm

Application Context

Conclusion

Problem Definition

- ▶ Design a wearable system that is able to get robust heart-rate (HR) measurement from ambulatory Photoplethysmogram (PPG)
- ▶ System should work for lone-workers who are constantly in motion
- ▶ The wearable system is of watch form-factor having PPG and accelerometer sensors

The PPG Sensor and Acquisition System

- ▶ The PPG sensor consists of the monochromatic light source and a photo detector
- ▶ green or red light is impinged on skin and the reflected/transmitted intensity is observed using a photo-detector.
- ▶ in wearable devices, typically reflective PPG is used as illustrated in Figure 1

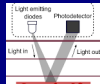


Figure 1: PPG Sensor Schematic

PPG Principle

Intensity of light reflected from the skin is proportional to the volume of blood, the time-series of this intensity gives the pulse signal.

PPG Motion Artifact

- ▶ PPG under motion is corrupted due to turbulent blood flow
- ▶ In Figure 2, the top plot shows PPG under motion and bottom one in rest. As is clearly visible, the motion PPG is unusable for heart-rate measurement.

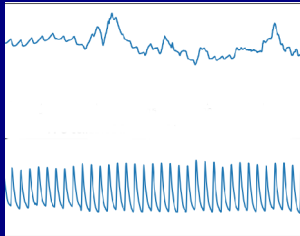


Figure 2: PPG Signal During Motion

Nature of Noise

Using system identification methods, it has been established that motion noise is an additive noise in the *frequency domain*.

Motivation and Challenges

- ▶ *Background:* Motion artifacts are a major issue in getting stable heart-rate measurements from smartwatch during activity
- ▶ *Challenges:* Motion noise is both *out-of-band* and *in-band*. It is *additive* in the frequency (spectral) domain

Solution Approach

We propose to use frequency domain separation methods viz. *Singular Spectrum Analysis (SSA)* to remove motion artifacts from PPG signal.

Introduction to Singular Value Decomposition (SVD)

- ▶ *Non parametric Decomposition:* SVD is a non-parametric matrix decomposition technique.
- ▶ *Linear Transformation:* SVD decomposes a matrix into three metrics as shown in equation 1.

Mathematical Representation of SVD

Mathematically, SVD is represented as equation 1. The diagonal values of Σ are defined uniquely by A and are called its “Singular Values”

$$\underbrace{A}_{W \times D} = \underbrace{U}_{W \times W} \times \underbrace{\Sigma}_{W \times D} \times \underbrace{V^T}_{D \times D} = \begin{pmatrix} \sigma_1 & & & \\ & \ddots & & 0 \\ & & \ddots & \\ 0 & & & \sigma_r & \\ & & & & 0 \end{pmatrix} \quad (1)$$

Significance of “Singular Values”

- ▶ SVD generalizes the eigendecomposition of a square matrix. to any $M \times N$ matrix.
- ▶ Singular Values and also called “Eigenvalues”
- ▶ Matrices U and V represent the eigenvectors of matrix A as in equation 1.

More about Eigenvectors

1. They are basis factors of the linear transformation done by the decomposition technique
2. It is another form of representing a matrix in a different vector space which ensures orthogonality
3. Each component is essentially ‘independent’ as with Fourier transform, where each frequency is independent.

Representing Signal as a Matrix: The Hankel Matrix

- ▶ Hankel matrix is a square matrix where each ascending skew diagonal from left to right is constant.
- ▶ For a signal it is constructed as in equation 2, and it represents the state-space of the signal.
- ▶ The rank of the matrix depends on the window of observation needed for the event

Representation of Signal in Hankel Form

The Hankel matrix of a signal $x(t)$ is represented as a rank n Square Matrix $G(n)$ as shown in equation 2

$$G(n) = \begin{bmatrix} x(t-n) & \dots & \dots & \dots & x(t) \\ \dots & \dots & \dots & x(t) & x(t+1) \\ \dots & \dots & \dots & \dots & \dots \\ x(t-1) & x(t) & \dots & \dots & \dots \\ x(t) & \dots & \dots & \dots & x(t+n) \end{bmatrix} \quad (2)$$

Singular Spectrum Analysis (SSA)

- ▶ It is a non-parametric spectral estimation method for signal processing
- ▶ It decomposes a signal into orthogonal components, ensuring each are independent of one another
- ▶ It also lets us know how many such components can represent a major part (say 97%) of the signal, which helps us concentrate on these components

Algorithm for SSA

1. Compute Hankel Matrix of Signal $x(t)$
2. Perform SVD of the Hankel Matrix

Each of the singular vectors (Eigenvectors) are independent spectral components of the signal. Their Eigenvalues is a measure of how much “importance” is there of that component in the signal.

Application of SSA to the Problem

1. Smartwatch has a PPG sensor and tri-axial accelerometer
2. We use SSA to decompose PPG (p) and Acceleration signals (a_x, a_y, a_z)
3. We find “cosine similarity” between components of p and $\{a_x, a_y, a_z\}$
4. Similar vectors in p are rejected as they represent motion
5. p is reconstructed from the rest of the major eigenvectors.

Selection Criteria

$$\text{cosineSimilarity}(p_i, a_{x|y|z_j}) > \tau \Rightarrow \text{Selected}$$

where τ is an application specific threshold

About “Cosine Similarity”

- ▶ Cosine similarity between two vectors is their dot (.) product
- ▶ It is a measure of “alignment” between the two vectors
- ▶ In context of the application, it shows how well an accelerometer and PPG signal is aligned
- ▶ In turn it means how much current component of the signal represents motion

Expression for Cosine Similarity




cosineSimilarity = $A \cdot B = \|A\| \times \|B\| \times \cos(\theta)$ where θ is the projection angle of A over B. Numerically, it is given by equation 3.

$$\text{cosineSimilarity} = \frac{\sum_{i=0}^n A_i B_i}{\sqrt{\sum_{i=0}^n A_i^2} \sqrt{\sum_{i=0}^n B_i^2}} \quad (3)$$

In The Next Lecture...

In the next lecture, we will learn about pre-processing (interpolation, filtering) and post-processing (tracking) algorithm used in this application

Further Reading

-  Ahmed, Nasimuddin, Shalini Mukhopadhyay, Varsha Sharma, and Avik Ghose. "Heart rate estimation algorithm from wrist-based photoplethysmogram using subspace learning method." In 2019 IEEE International Conference on Pervasive Computing and Communications Workshops (PerCom Workshops), pp. 145-150. IEEE, 2019.
-  Mukhopadhyay, Shalini, Nasimuddin Ahmed, Dibyanshu Jaiswal, and Avik Ghose. "A Robust and Customizable Tracking Algorithm for Accurate Heart Rate Estimation." In Proceedings of the 17th Annual International Conference on Mobile Systems, Applications, and Services, pp. 665-666. 2019.
-  Ahmed, Nasimuddin, Varsha Sharma, Arijit Chowdhury, Shalini Mukhopadhyay, and Avik Ghose. "A weiner filter based robust algorithm for estimation of heart rate from wrist based photoplethysmogram." In Adjunct Proceedings of the 2019 ACM International Joint Conference on Pervasive and Ubiquitous Computing and Proceedings of the 2019 ACM

Thank You!