Lecture 27

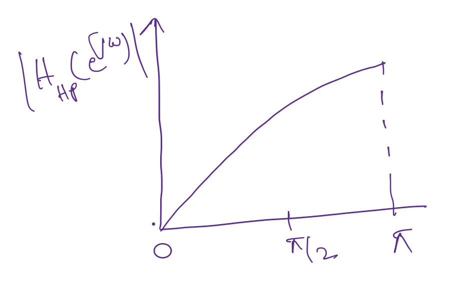
Saturday, 23 October 2021 2:57 PM

High pass FIR digital filter

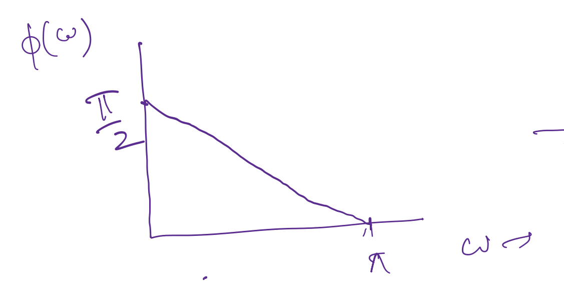
$$H(z) = \frac{1}{2}(1 - z^{-1})$$

$$H_{HP}(e^{j\omega}) = j\, e^{-j\omega/2} \sin(\omega/2)$$

$$\omega_c = \pi/2$$



$$\phi(\omega) = -\frac{\omega}{2} + \frac{\pi}{2} + \angle \sin \omega/2$$



→ Linear phase response.

→ Give $H_{LP}(z)$, we can always derive $H_{HP}(z)$ by $H_{LP}(-z) = H_{HP}(z)$

$$z \to e^{j\omega}$$

$$-z \to e^{-j\pi} e^{j\omega} = e^{j(\omega - \pi)}$$

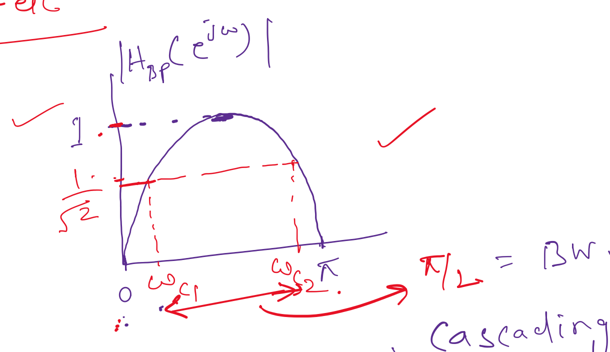
Ex

$$H_{LP}(z) = a_0 + a_1 z^{-1} + \dots + a_N z^{-N}.$$

$$H_{HP}(z) = H_{LP}(-z)$$

→ cut-off freq^n for HPF is not necessary same with LPF.

Band pass Filter



$\pi/2$ = BW.

→ Cascading LPF & HPF.

$$H_{BP}(z) = A(1 + z^{-1})(1 - z^{-1})$$

$$= A(1 - z^{-2})$$

$$H_{BP}(e^{j\omega}) = 2Aj\, e^{-j\omega} \sin\omega.$$

$$A = 1/2$$

$$\boxed{H_{BP}(z) = \frac{1}{2}(1 - z^{-2})}$$

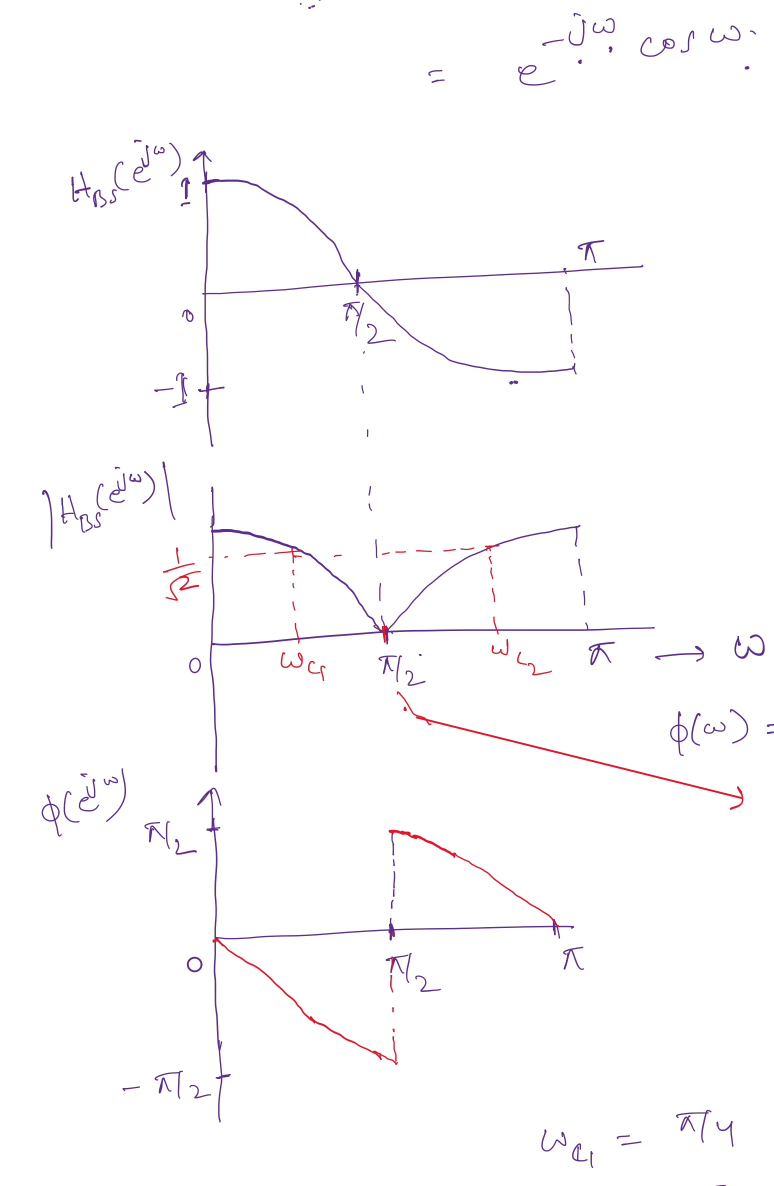
$$|H_{BP}(e^{j\omega_c})|^2 = 1/2.$$

Band-stop Filter

$$H_{BS}(z) = \frac{1}{2}(1 + z^{-2})$$

$$H_{BS}(e^{j\omega}) = \frac{1}{2} e^{-j\omega}\, 2\cos\omega.$$

$$= e^{-j\omega} \cos\omega.$$



$$\phi(\omega) = -\omega + \angle \cos\omega.$$

Notch.

$$\omega_{c1} = \pi/4$$

$$\omega_{c2} = \frac{\pi}{4} + \pi/2$$

$$BW = \omega_{c2} - \omega_{c1} = \pi/2.$$

→ Band-stop filter is also known as Notch filter.

Notch freq = $\pi/2$.

All pass Filter. (FIR Filter)

$$|H(e^{j\omega})| = 1 \quad \forall\ \omega.$$

→ It's angle will be different at diff. freq.

$$H(z) = z^{-N}.$$

$$|H(z)|_{z = e^{j\omega}} = 1.$$

$$\phi(\omega) = \angle H(z)|_{z = e^{j\omega}} = -N\omega.$$

↓ Linear phase response.

Group delay: $\tau_g(\omega) = -\dfrac{d\phi}{d\omega} = N.$

Phase delay: $\tau_p(\omega_0) = \dfrac{\phi(\omega_0)}{\omega_0}$

If there is a group of frequencies transmitted over a channel, whose TF is z^{-N}, then all of these frequencies shall arrive at the o/p N sample later.

→ If $\tau_g(\omega)$ is not constant, then distortion occurs at o/p waveform.

(Linear phase → const. group delay.)

gives a distortionless filter.

Simple IIR Digital filter.

IIR LPF

$$H(z) = \left(\frac{1-\alpha}{2}\right) \frac{1 + z^{-1}}{1 - \alpha z^{-1}} \to \text{LPF IIR.}$$

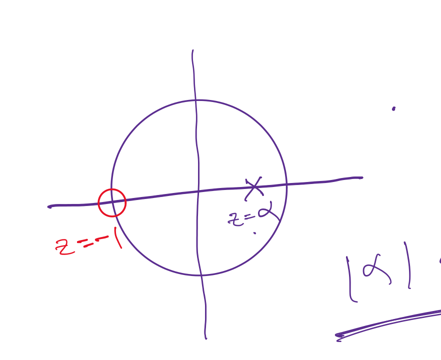
$$\boxed{|\alpha| < 1}$$

Multiplied to normalize the dc response to 1

$$\omega = 0 \to z = 1$$

$$\Rightarrow H(1) = \frac{2}{1-\alpha}.$$

DC response



$|\alpha| < 1$

→ This is a simple 1st order digital IIR LPF.

$$|H(e^{j\omega})|^2 = \left(\frac{1-\alpha}{2}\right)^2 \frac{1 + \cos\omega}{1 - 2\alpha\cos\omega + \alpha^2}$$

$$|\alpha| < 1.$$

$$\frac{d|H(e^{j\omega})|^2}{d\omega} < 0, \quad 0 < \omega < \pi$$

↓ Magnitude response is monotonic