Lecture 6

Wednesday, 1 September 2021 3:53 PM

Examples of DTS.

Ex 1 Accumulator

$$y(n) = \sum_{l=-\infty}^{n} x(l)$$ → Non recursive.

$$y(n) = \sum_{l=-\infty}^{n-1} x(l) + x(n)$$

(where $\sum_{l=-\infty}^{n-1} x(l) \to y(n-1)$)

$$= y(n-1) + x(n)$$

↳ recursive.

↳ Avoid the ∞ summation

↳ It needs feedback.

$$y(n) = \sum_{l=-\infty}^{-1} x(l) + \sum_{l=0}^{n} x(l)$$

(where $\sum_{l=-\infty}^{-1} x(l) \to y(-1)$)

$$y(n) = y(-1) + \sum_{l=0}^{n} x(l)$$

$y(-1)$ ↓ Initial condn. ↓ Is used for a causal I/P signal.

Ex-2 M-point Moving Average DTS.

$$y(n) = \frac{1}{M} \sum_{K=0}^{M-1} x(n-K)$$

↓ Average.

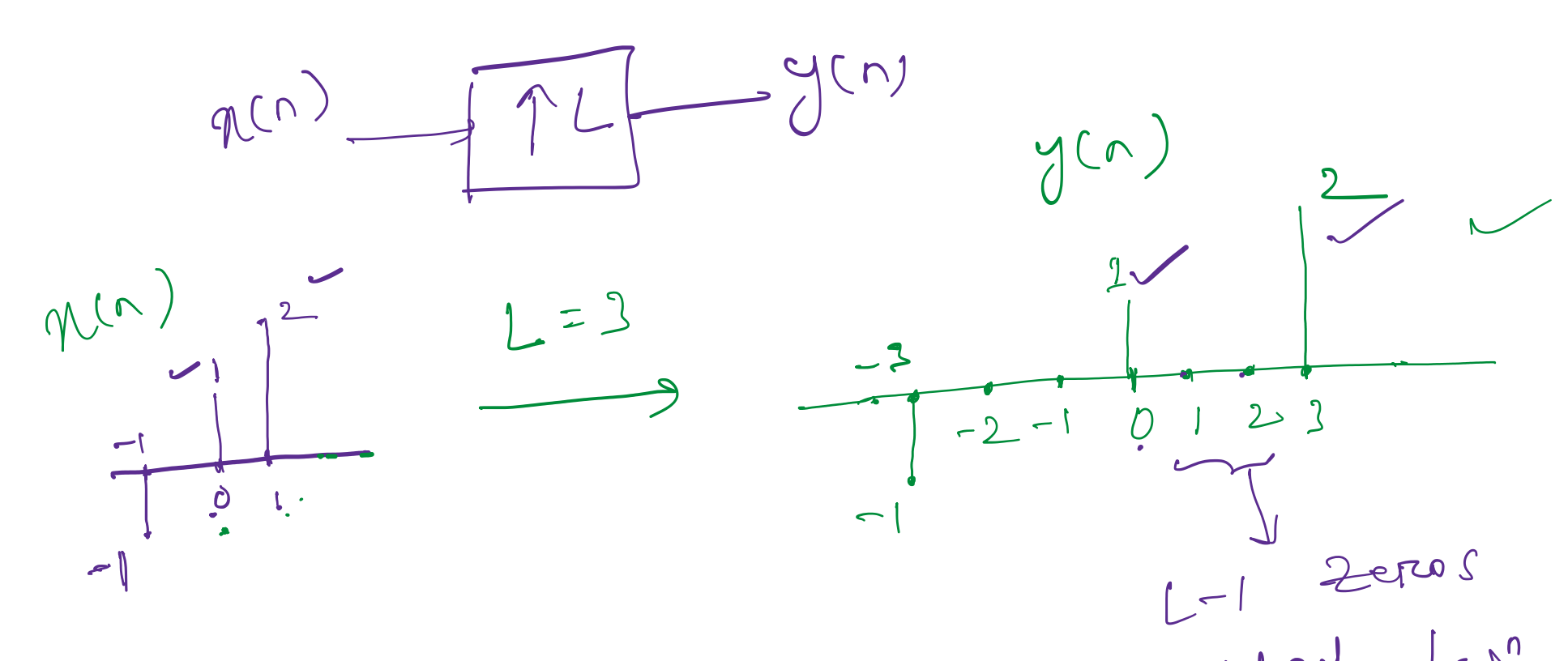
↓ This Avg. changes with 'n' ⇒ moving Avg. system.

$$y(n) = y(n-1) + \frac{1}{M}\left(x(n) - x(n-M)\right)$$

Ex-3 Up sampler.

$$y(n) = \begin{cases} x(n/L), & n = 0, \pm L, \pm 2L \dots \\ 0, & \text{otherwise.} \end{cases}$$

$x(n) \to$ [↑L] $\to y(n)$

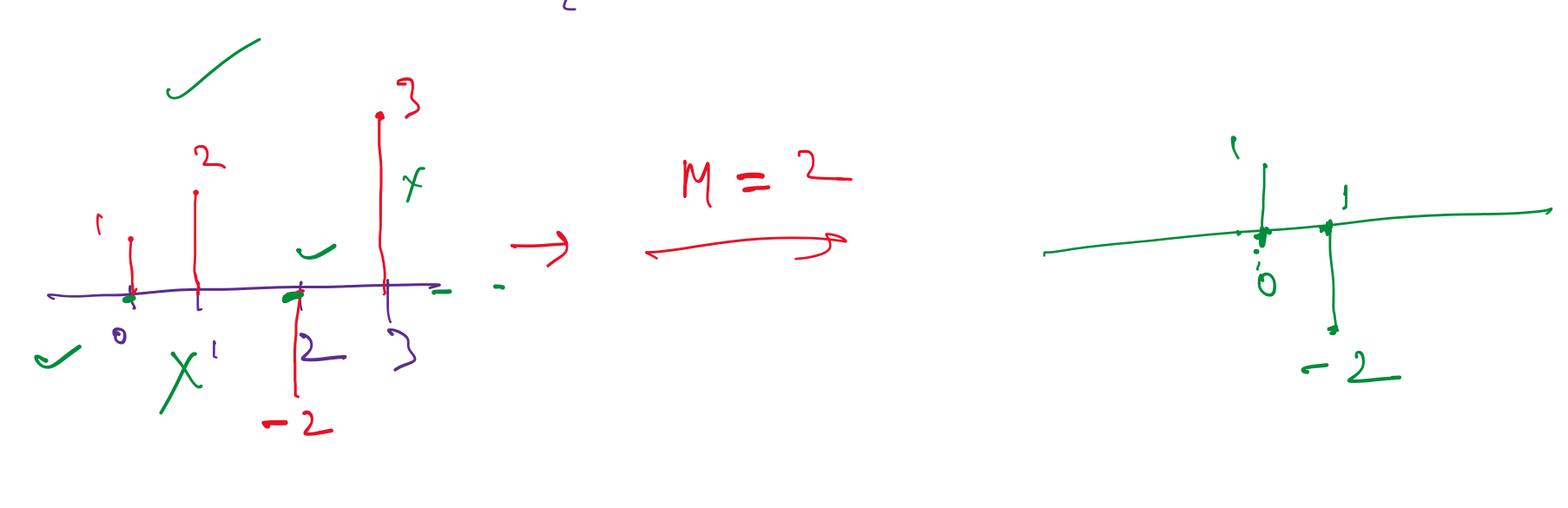


L−1 zeros padded betn the consequative samples.

Down sampler.

$x(n) \to$ [↓M] $\to y(n)$

$$y(n) = \begin{cases} x(Mn) \\ 0, & \text{otherwise.} \end{cases}$$



→ It ignores M−1 samples betn 0 & Mth sample.

Classification of DTS.

1) **Linear system.**

iff $x_1(n) \to y_1(n)$

$x_2(n) \to y_2(n)$

$\Rightarrow \alpha x_1(n) + \beta x_2(n) \to \alpha y_1(n) + \beta y_2(n).$

Then it is a linear system. → Superposition principle.

$\alpha x_1(n) \to \alpha y_1(n)$ → Homogeneity

$x_1(n) + x_2(n) \to y_1(n) + y_2(n)$ ↳ Additive

→ Superposition principle.

Ex $$y(n) = \sum_{l=-\infty}^{n} x(l)$$

↳ Linear.

$$x_1(n) \to y_1(n) = \sum_{l=-\infty}^{n} x_1(l)$$

$$x_2(n) \to y_2(n) = \sum_{l=-\infty}^{n} x_2(l)$$

$$\alpha x_1(n) + \beta x_2(n) \to \sum_{l=-\infty}^{n} \alpha x_1(l) + \beta x_2(l)$$

$$\alpha y_1(n) + \beta y_2(n) = \sum_{l=-\infty}^{n} \alpha x_1(l) + \sum_{l=-\infty}^{n} \beta x_2(l)$$

$$= \sum_{l=-\infty}^{n} (\alpha x_1(l) + \beta x_2(l))$$

=.

Ex $$y(n) = y(-1) + \sum_{l=0}^{n} x(l)$$

↳ Non linear system.

If $y(-1) = 0$, then this is a linear system.

Proof of Non linearity :-

$$x_1(n) \to y_1(n) = y_1(-1) + \sum_{l=0}^{n} x_1(l)$$

$$x_2(n) \to y_2(n) = y_2(-1) + \sum_{l=0}^{n} x_2(l)$$

$$\underbrace{\alpha x_1(n) + \beta x_2(n)}_{x_3(n)} \to y_3(-1) + \sum_{l=0}^{n} \alpha x_1(l) + \beta x_2(l)$$

$$\alpha y_1(n) + \beta y_2(n) = \alpha y_1(-1) + \beta y_2(-1) + \sum_{l=0}^{n} \alpha x_1(l) + \beta x_2(l)$$

($\alpha y_1(-1) + \beta y_2(-1)$) ↓ Not necessarily same as $y_3(-1)$

$\alpha x_1(n) + \beta x_2(n) \not\to \alpha y_1(n) + \beta y_2(n)$

⇒ Non linear system.

Time Invariance (Shift Invariance)

→ If $x(n) \to y(n) \Rightarrow x(n-n_0) \to y(n-n_0)$ then the system is time invariant.

↓ shape of o/p will remain constant

Ex Upsampler.

$$y(n) = \begin{cases} x(n/L), & n = 0, \pm L, \pm 2L. \\ 0, & \text{otherwise.} \end{cases}$$

↳ Linear system.

$$x(n) \to \begin{cases} x(n/L), & n = 0, \pm L, \pm 2L \\ 0, & \text{otherwise.} \end{cases}$$

$x_1(n) = x(n-n_0)$

$$x_1(n) \to \begin{cases} x_1(n/L), & n = 0, \pm L, \pm 2L \\ 0, & \text{other.} \end{cases}$$

$$= \begin{cases} x\left(\frac{n}{L} - n_0\right), & n = 0, \pm L, \pm 2L \dots \\ 0, & \text{otherwise} \end{cases}$$

↓ o/p for the I/P as $x(n-n_0)$

Not same.

$$y(n-n_0) = \begin{cases} x\left(\frac{n-n_0}{L}\right), & n-n_0 = 0, \pm L, \pm 2L \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} x\left(\frac{n-n_0}{L}\right), & n = n_0, n_0 \pm L, n_0 \pm 2L \\ 0, & \text{otherwise} \end{cases}$$

Here $x(n-n_0) \not\to y(n-n_0)$

⇒ Time variant system

Upsampler is LTV system.