Lecture 21

Wednesday, 6 October 2021 3:58 PM

→ If $g(n) \overset{z}{\longleftrightarrow} G(z)$

$$G(z) = \frac{P(z)}{Q(z)} \rightarrow \text{ratio of polynomials}$$

↓

rational function of z^{-1}

$$G(z) = \frac{P(z)}{Q(z)} = K \frac{1 + p_1 z^{-1} + \cdots + p_M z^{-M}}{1 + q_1 z^{-1} + \cdots + q_N z^{-N}}$$

$$= K \frac{\prod_{l=1}^{M} (1 - \alpha_l z^{-1})}{\prod_{l=1}^{N} (1 - \lambda_l z^{-1})}$$

$$= K z^{N-M} \frac{\prod_{l=1}^{M} (z - \alpha_l) \cdots}{\prod_{l=1}^{N} (z - \lambda_l) \cdots}$$

At $z = \alpha_l$, $G(z) = 0$

$\Rightarrow \alpha_l \rightarrow$ zeros of $G(z)$

At $z = \lambda_l$, $G(z) = \infty$.

$\Rightarrow \lambda_l \rightarrow$ poles of $G(z)$.

$N > M \Rightarrow N-M$ zeros at $z = 0$.

$N < M \Rightarrow M-N$ poles at $z = 0$.

$N = M \Rightarrow$ No poles & zeros at $z = 0$ & $z = \infty$.

→ In general, for any rational function, the no. of poles = the no. of zeros including the points at 0 & ∞.

Finite sequence.

Ex $x(n) = \{1, \underset{\uparrow\, n=0}{2}, 1\}$ → both sided seq.

$X(z) = z + 2 + z^{-1}$

$z = 0 \rightarrow |X(z)| = \infty$

$z = 0$ is a pole

$z = \infty$ is also a pole.

ROC will be entire z-plane except $z = 0$ & $z = \infty$.

Note

ROC doesn't include poles.

→ For the right sided finite sequences all poles are at $z = 0$.

ROC → $|z| > 0$.

→ For the left sided finite sequences all poles at $z = \infty$.

ROC → $|z| < \infty$.

NOTE

If $x(n)$ is a finite seq., ROC is the entire z-plane except possibly $z = 0$ and/or $z = \infty$.

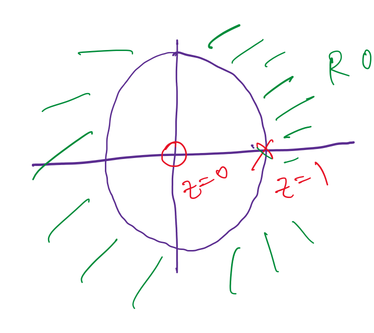
For Infinite Sequences

Here ROC is bounded by poles.

Ex $x(n) = u(n)$

$$u(n) \longleftrightarrow \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}$$

$|z| > 1$ → ROC.

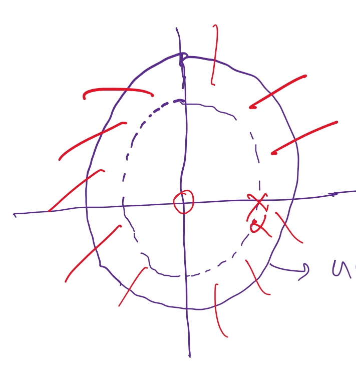


→ ROC doesn't include unit circle.

Ex $x(n) = \alpha^n u(n)$. → Right sided Exponential seq.

$$X(z) = \frac{1}{1 - \alpha z^{-1}} = \frac{z}{z - \alpha},$$

$|z| > |\alpha|$.



unit circle

If $|\alpha| < 1$ → F.T. exists.

Ex $h(n) = ((0.2)^n + (-0.6)^n)\, u(n)$

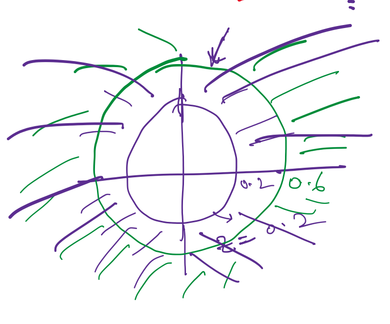
Imp. res.

$$(0.2)^n u(n) \overset{z}{\longleftrightarrow} \frac{1}{1 - 0.2 z^{-1}}, \quad |z| > 0.2$$

$$(-0.6)^n u(n) \overset{z}{\longleftrightarrow} \frac{1}{1 + 0.6 z^{-1}}, \quad |z| > 0.6$$

$$H(z) = \frac{1}{1 - 0.2 z^{-1}} + \frac{1}{1 + 0.6 z^{-1}}$$

$|z| > 0.2$ ↓ $|z| > 0.6$ ↓

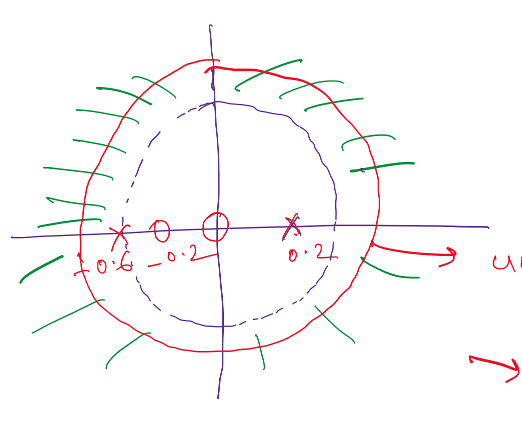


→ $|z| > 0.6$ → ROC of $H(z)$

$$H(z) = \frac{z(2z + 0.4)}{(z - 0.2)(z + 0.6)}$$

poles → $z = 0.2$, $z = -0.6$.

Zeros → $z = 0$, $z = -0.2$.



→ $|z| > 0.6$ ROC.

unit circle.

→ It includes unit circle.

$\Rightarrow$ It is a stable system (∵ $h(n)$ is absolutely summ.)

NOTE

If $x(n)$ is a Right sided sequence, the ROC extends outward from the outermost finite pole $X(z)$ to $z = \infty$.

Ex $x(n) = -a^n u(-n-1)$: → Left sided seq.

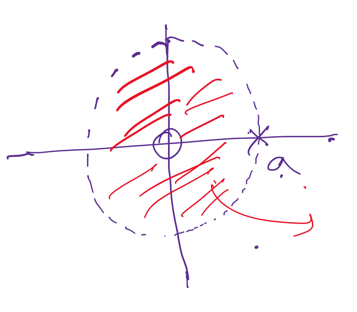
$$X(z) = \frac{1}{1 - a z^{-1}} = \frac{z}{z - a},$$

$|z| < |a|$.

↓

R.O.C.

pole → $z = a$



→ If $|a| < 1$, ROC doesn't include unit circle.

NOTE

If $x(n)$ is a left-sided sequence, the ROC extends inward from the innermost (smallest magnitude) non zero pole in $X(z)$ to $z = 0$.

Ex Two sided exponential seq.

$$h(n) = (0.2)^n u(n) - (-0.6)^n u(-n-1)$$

$$(0.2)^n u(n) \longleftrightarrow \frac{1}{1 - 0.2 z^{-1}}, \quad |z| > 0.2$$

$$-(-0.6)^n u(-n-1) \longleftrightarrow \frac{1}{1 + 0.6 z^{-1}}, \quad |z| < 0.6.$$

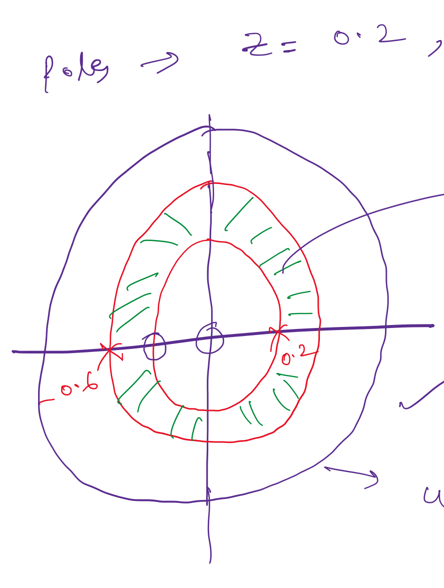
$$H(z) = \frac{1}{1 - 0.2 z^{-1}} + \frac{1}{1 + 0.6 z^{-1}}$$

$$= \frac{z(2z + 0.4)}{(z - 0.2)(z + 0.6)}$$

R.O.C → $0.2 < |z| < 0.6$.

zeros → $z = 0$, $z = -0.2$

poles → $z = 0.2$, $z = -0.6$.



ROC doesn't include unit circle.

unit circle.

Ex $x(n) = \alpha^n$

$$X(z) = \sum_{n=0}^{\infty} \alpha^n z^{-n} + \sum_{n=-\infty}^{-1} \alpha^n z^{-n}$$

↓ $|z| > |\alpha|$ ↓ $|z| < |\alpha|$

$z = \alpha$ → pole.

→ There is no common region.

$\Rightarrow$ No ROC.

$\Rightarrow$ No z-T.