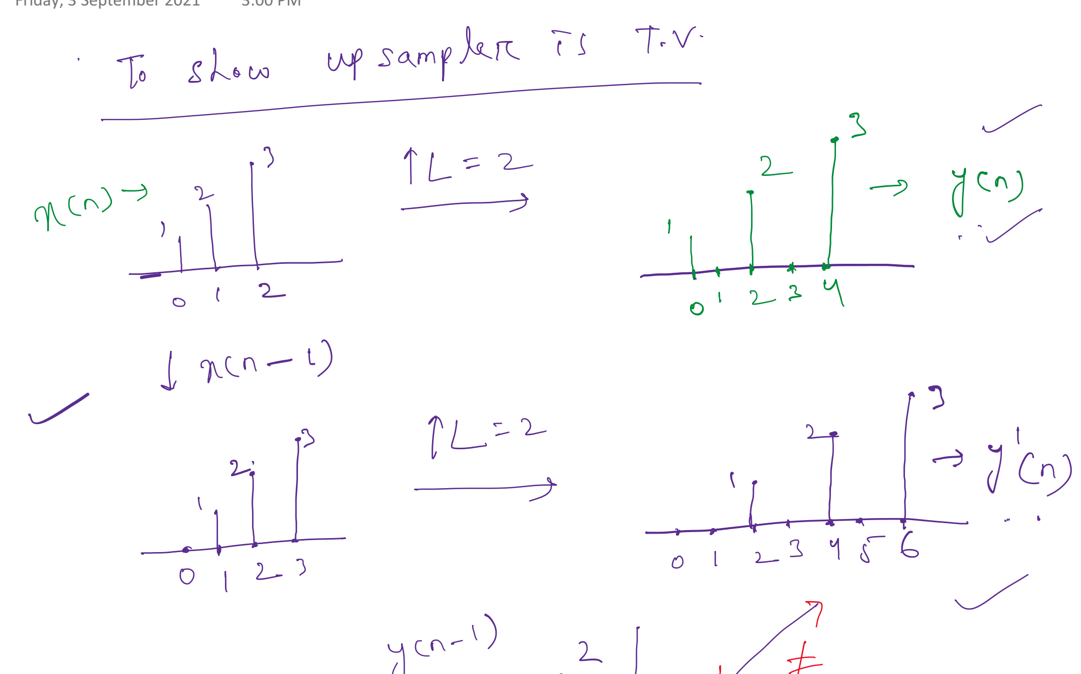
Lecture 7

Friday, 3 September 2021 3:00 PM

To show up sampler is T.V.



$y'(n) \neq y(n-1) \Rightarrow$ T.V. system.

Causal system

→ It can't predict the future.

→ $y(n)$ depends on $x(n)$, $x(n-i)$ & $y(n-j)$

$y(n)$ ↓ present o/p; $x(n)$ ↓ present I/p; $x(n-i)$: past I/p; $y(n-j)$: past o/p; i, j: +ve integer.

→ If $x_1(n) \rightarrow y_1(n)$ &

$x_2(n) \rightarrow y_2(n)$, then

$x_1(n) = x_2(n)$ for $n < N$, implies

$y_1(n) = y_2(n)$ for $n < N$.

Ex

1) $y(n) = x(n^2) \longrightarrow$ Non-causal.

2) $y(n) = \dfrac{x(n) + x(n-1) + x(n+1)}{3}$

↓ Non-causal.

3) $y(n) = \sum_{l=-\infty}^{n} x(l)$

↳ Causal

↳ LTI.

Stability.

(Bounded-Input-Bounded-o/p (BIBO) Stability.)

→ A DTS is stable iff.

$|x(n)| < B_x < \infty \quad \forall n.$ (Bounded I/p)

$\Rightarrow |y(n)| < B_y < \infty, \quad \forall n.$

↓ Bounded o/p.

Ex $y(n) = \alpha y(n-1) + x(n)$,

$x(n) = u(n)$.

↳ Bounded I/p.

$y(0) = \alpha y(-1) + x(0)$
$= \alpha y(-1) + 1$

$y(1) = \alpha y(0) + x(1)$
$= \alpha y(0) + 1$
$= \alpha^2 y(-1) + \alpha + 1$

$y(2) = \alpha^3 y(-1) + \alpha^2 + \alpha + 1$

$$\boxed{y(n) = \alpha^{n+1} y(-1) + \sum_{i=0}^{n} \alpha^i}$$

↳ stable system or not.

$0 < \alpha < 1$

$\lim_{n \to \infty} y(n) = \lim_{n \to \infty} \sum_{i=0}^{n} \alpha^i$

$= \dfrac{1}{1-\alpha} < \infty.$

$\Rightarrow$ It is a stable system.

If $\alpha > 1$

$\lim_{n \to \infty} y(n) = \infty.$

↳ unstable.

Ex $y(n) = \dfrac{1}{M} \sum_{K=0}^{M-1} x(n-K)$

↳ stable system.

For $|x(n)| < B_x < \infty$.

$|y(n)| = \left| \dfrac{1}{M} \sum_{K=0}^{M-1} x(n-K) \right|$

$\leq \dfrac{1}{M} \sum_{K=0}^{M-1} |x(n-K)|$

↳ $(|x+y| \leq |x| + |y|)$

$\leq \dfrac{1}{M} \times M \times B_x = B_x.$

$\Rightarrow |y(n)| \leq B_x < \infty.$

$\Rightarrow$ Stable system.

Passive & Lossless System.

$x(n) \rightarrow$ DTS $\rightarrow y(n)$.

$x(n)$: Finite energy sig.

$$\underbrace{\sum_{n=-\infty}^{\infty} |y(n)|^2}_{\text{o/p energy}} \leq \underbrace{\sum_{n=-\infty}^{\infty} |x(n)|^2}_{\text{I/p energy}}$$

↳ passive system,

→ If equality holds, it is a lossless system.

Ex $y(n) = \alpha x(n-N)$.

$\sum_{n=-\infty}^{\infty} |y(n)|^2 = |\alpha|^2 \sum_{n=-\infty}^{\infty} |x(n-N)|^2$

If $|\alpha| \leq 1$, $\Rightarrow$ passive system.

$|\alpha| = 1$, $\Rightarrow$ Lossless system.

Impulse and Step response.

Impulse response

$\delta(n) \rightarrow$ DTS $\rightarrow h(n)$

→ Impulse response.

Step-response.

$u(n) \rightarrow \boxed{} \rightarrow s(n)$

→ Step response.

Note

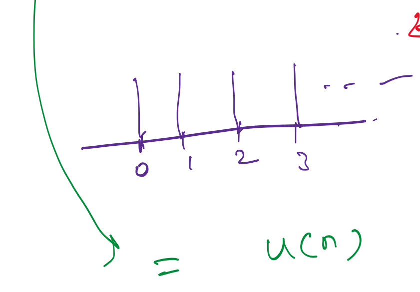
A LTI DTS is completly characterized in the time domain by its impulse response or step response.

Ex $y(n) = \sum_{l=-\infty}^{n} x(l)$

$h(n) = \sum_{l=-\infty}^{n} \delta(l)$

$= \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$

← $h(n)$.



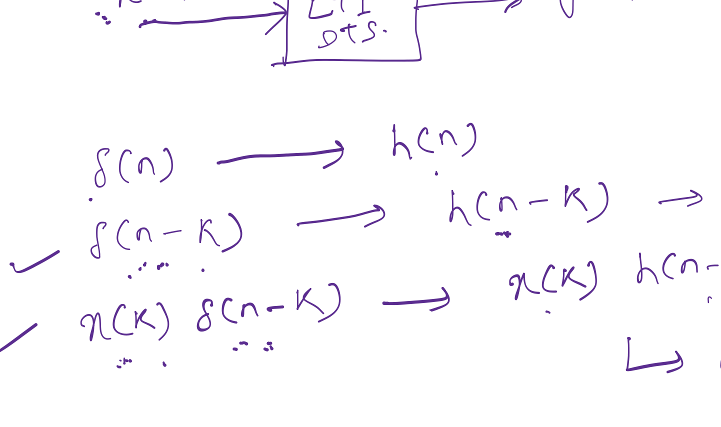
$= u(n)$

Ex Interpolator.

$y(n) = x(n) + \dfrac{1}{2}\left[x(n-1) + x(n+1)\right]$.

↳ Home Assignment.

Ans



Input-output relationship of LTI system.

$x(n) \rightarrow$ LTI DTS ($h(n)$) $\rightarrow y(n)$

$\delta(n) \longrightarrow h(n)$

$\delta(n-K) \longrightarrow h(n-K) \rightarrow$ due to TI

$x(K)\,\delta(n-K) \longrightarrow x(K)\,h(n-K)$

↳ Homogeneity.

$$\underbrace{\sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)}_{x(n).} \longrightarrow \sum_{K=-\infty}^{\infty} x(K)\,h(n-K)$$

↓ Superposition principle.

↓ $y(n)$

$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n) = \sum_{K=-\infty}^{\infty} x(K)\,h(n-K)$

$$\boxed{y(n) = \sum_{K=-\infty}^{\infty} x(K)\,h(n-K)}$$

↓ Convolution sum,

$y(n) = x(n) * h(n)$

$= h(n) * x(n)$