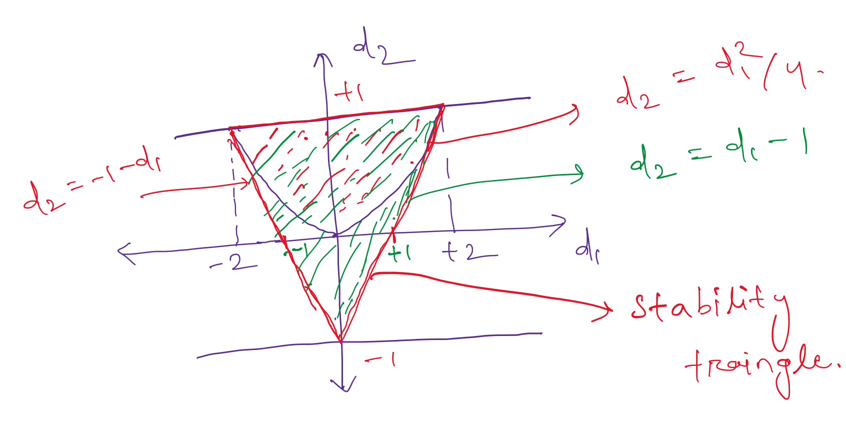
Lecture 35

Monday, 8 November 2021 3:01 PM

$|d_2| < 1$.



$$P_1 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} - d_2}$$

$$P_2 = -\frac{d_1}{2} - \sqrt{\frac{d_1^2}{4} - d_2}$$

If poles are complex:

$d_2 > d_1^2/4$.

$P_{1,2} = \alpha \pm j\beta$.

$|P_1|^2 = |P_2|^2 = P_1 P_2 = d_2 = \alpha^2 + \beta^2$.

$d_2 = \frac{d_1^2}{4} \rightarrow$ parabola.

If poles are real.

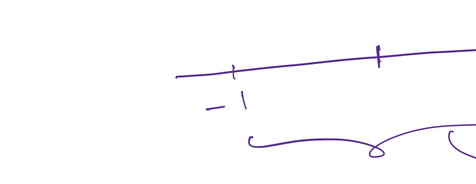
$d_2 < d_1^2/4$.

$$P_1 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} - d_2} \quad < 1.$$

$$P_2 = -\frac{d_1}{2} - \sqrt{\frac{d_1^2}{4} - d_2} \quad > -1$$

P_2 can be < -1

Two conditions ensures stability



P_1 & P_2 should belong to this region

$$-\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} - d_2} < 1$$

$$\Rightarrow \sqrt{\frac{d_1^2}{4} - d_2} < 1 + d_1/2$$

$$\Rightarrow \cancel{\frac{d_1^2}{4}} - d_2 < (1 + d_1/2)^2$$
$$= 1 + \cancel{\frac{d_1^2}{4}} + d_1$$

$$\Rightarrow -d_2 < 1 + d_1$$

$$\Rightarrow \boxed{d_2 > -1 - d_1} \quad — (1)$$

$$-\frac{d_1}{2} - \sqrt{\frac{d_1^2}{4} - d_2} > -1$$

$$\Rightarrow -\sqrt{\frac{d_1^2}{4} - d_2} > -1 + \frac{d_1}{2}$$

$$\Rightarrow \sqrt{\frac{d_1^2}{4} - d_2} < 1 - \frac{d_1}{2}.$$

$$\Rightarrow \cancel{\frac{d_1^2}{4}} - d_2 < 1 + \cancel{\frac{d_1^2}{4}} - d_1$$

$$\Rightarrow \boxed{d_2 > d_1 - 1} \quad — (2)$$

$d_2 = -1 - d_1$

d_1	d_2
0	-1
-1	0

→ How to test the stability for any TF $H_M(z)$, without finding the roots of the denominator $D_M(z)$.

→ We follow the stability test procedure using AP functions.

A General stability test procedure.

$$H_M(z) = \frac{N_M(z)}{D_M(z)}$$

→ Form an Mth order allpass fn using $D_M(z)$.

$$A_M(z) = \frac{z^{-M} D_M(z^{-1})}{D_M(z)}$$

$$D_M(z) = 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_M z^{-M}.$$

$$A_M(z) = \frac{d_M + d_{M-1} z^{-1} + \cdots + z^{-M}}{1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_M z^{-M}}$$

$$D_M(z) = \prod_{i=1}^{M} (1 - P_i z^{-1})$$

$P_i \rightarrow$ roots of $D_M(z)$.

$$\boxed{d_M = (-1)^M \prod_{i=1}^{M} P_i}$$

For stability $|P_i| < 1$

$\Rightarrow |d_M| < 1$

$A_M(\infty) = d_M$.

Let's define $A_M(\infty) = d_M \triangleq K_M$

$$\boxed{K_M^2 < 1}$$

This a necessary cond^n for stability of $A_M(z)$.

→ Let's assume that $A_M(z)$ is stable with $K_M^2 < 1$.

Next, we form a new function $A_{M-1}(z)$

$$A_{M-1}(z) = \frac{z[A_M(z) - K_M]}{1 - K_M A_M(z)}$$
$$= \frac{z[A_M(z) - d_M]}{1 - d_M A_M(z)}$$

$$A_{M-1}(z) = \frac{N_{M-1}(z)}{D_{M-1}(z)}$$

Find $N_{M-1}(z)$

$$A_{M-1}(z) = \frac{z[A_M(z) - d_M]}{1 - d_M A_M(z)}$$
$$= \frac{z\left[\frac{N_M(z)}{D_M(z)} - d_M\right]}{1 - d_M \frac{N_M(z)}{D_M(z)}}$$
$$= \frac{z[N_M(z) - d_M D_M(z)]}{D_M(z) - d_M N_M(z)}$$
$$= \frac{N_{M-1}(z)}{D_{M-1}(z)}$$

$$N_{M-1}(z) = z[N_M(z) - d_M D_M(z)]$$
$$= (d_{M-1} - d_M d_1) + (d_{M-2} - d_M d_2) z^{-1} + \cdots + (1 - d_M^2) z^{-(M-1)}$$

$\downarrow$ $(M-1)$th order polynomial in z^{-1}

$$D_{M-1}(z) = (1 - d_M^2) + (d_1 - d_M d_{M-1}) z^{-1} + \cdots + (d_{M-1} - d_M d_1) z^{-(M-1)}$$

$\Rightarrow A_{M-1}(z)$ is an allpass fn.

$$A_{M-1}(z) = \frac{d'_{M-1} + d'_{M-2} z^{-1} + \cdots + z^{-(M-1)}}{1 + d'_1 z^{-1} + d'_2 z^{-2} + \cdots + d'_{M-1} z^{-(M-1)}}$$

where $\boxed{d'_i = \frac{d_i - d_M d_{M-i}}{1 - d_M^2}}$

→ Now we proove that $A_{M-1}(z)$ is stable.

$$A_{M-1}(z) = \frac{z[A_M(z) - K_M]}{1 - K_M A_M(z)}$$

Let λ_i be a pole of $A_{M-1}(z)$.

$\Rightarrow 1 - K_M A_M(\lambda_i) = 0$.

$\Rightarrow A_M(\lambda_i) = \frac{1}{K_M}$.

$\Rightarrow |A_M(\lambda_i)| = \frac{1}{|K_M|} > 1$.

We know that for a stable AP fn.

$$|A_M(z)| = \begin{cases} > 1 & \text{for } |z| < 1 \\ = 1 & \text{for } |z| = 1 \\ < 1 & \text{for } |z| > 1. \end{cases}$$

Since $|A_M(\lambda_i)| > 1$

$\Rightarrow \boxed{|\lambda_i| < 1}$

→ Therefore, $A_{M-1}(z)$ is stable AP fn,

Thus, if $A_M(z)$ is a stable AP fn & $K_M^2 < 1$, then $A_{M-1}(z)$ is a stable AP fn.