Lecture 3

Wednesday, 18 August 2021 4:01 PM

Sinusoidal signal.

$x(t) = A\cos(\Omega t + \phi)$

$\Omega = 2\pi f$

$\hookrightarrow$ Analog freq. (rad/sec)

$-\infty < \Omega < \infty$.

$x(n)$

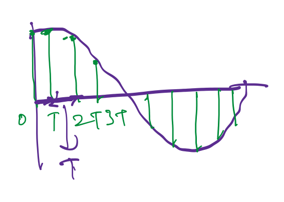
$A\cos(\Omega t + \phi) \xrightarrow[T]{f} A\cos(\Omega nT + \phi)$

$t_n = nT$.

$n = 0, 1 \qquad 0, T, 2T. \rightarrow$ sampling instants

$T \rightarrow$ sampling interval.

$T = \frac{1}{f_s}$



$\rightarrow$ sampling freq.

$\omega = \Omega T = \frac{\Omega}{f_s} = \frac{2\pi f}{f_s}$

$\hookrightarrow$ Normalized angular freq. (rad).

$\hookrightarrow$ Digital freq.

$x(n) = A\cos(\Omega nT + \phi) = A\cos(\omega n + \phi)$

$x(n) = A\cos(\Omega_0 nT + \phi) = A\cos(\omega_0 n + \phi)$

real sinusoidal sequence.

$\rightarrow x(n)$ is bounded.

$\rightarrow$ Periodic ? $\rightarrow$ It may or mayn't be periodic.

$x(n+N) = x(n) \rightarrow$ periodic.

$\cos(\omega_0(n+N) + \phi) = \cos(\omega_0 n + \phi)$

$\Rightarrow \boxed{\omega_0 N = 2\pi r} \rightarrow$ +ve integer.

$\hookrightarrow$ This ensures the periodicity of $x(n)$

$\Rightarrow \boxed{\frac{\omega_0}{2\pi} = \frac{r}{N}}$

Ex $\cos 3n \rightarrow$ Not periodic.

$A\cos(\Omega t + \phi) \xrightarrow{f} A\cos(\omega n + \phi)$

$\omega = \Omega T = \frac{\Omega}{f_s} = \frac{2\pi f}{f_s}$

$\Omega = 2\pi f$.

$\boxed{f_s \geq 2f} \rightarrow$ Nyquist Criteria.

$\Rightarrow f \leq f_s/2$

$\Rightarrow f_{max} = f_s/2$.

$\omega_{max} = \frac{2\pi f_{max}}{f_s} = \frac{2\pi f_s/2}{f_s} = \pi$

$-\pi \leq \omega \leq \pi \rightarrow$ ~~Actual~~ range of dig. freq.

or

$0 \leq \omega \leq 2\pi$

$\rightarrow$ For any $\Omega \in (-\infty, \infty)$, $\omega \in [-\pi, \pi]$.

Exponential sequence.

$x(n) = A\alpha^n$

A & $\alpha \rightarrow$ real or complex.

Let $\alpha = e^{(\sigma_0 + j\omega_0)}$

$A = |A| e^{j\phi}$.

$x(n) = |A| e^{j\phi} e^{(\sigma_0 n + j\omega_0 n)}$

$= |A| e^{\sigma_0 n} e^{j(\omega_0 n + \phi)}$

Complex exponential seq.

$= \underbrace{|A| e^{\sigma_0 n} \cos(\omega_0 n + \phi)}_{\text{real}} + j\underbrace{|A| e^{\sigma_0 n} \sin(\omega_0 n + \phi)}_{\text{real}}$

$= x_{re}(n) + j\, x_{im}(n)$

$x_{re}(n) = |A| e^{\sigma_0 n} \cos(\omega_0 n + \phi)$

$x_{im}(n) = |A| e^{\sigma_0 n} \sin(\omega_0 n + \phi)$

$\rightarrow$ real sinusoidal sequences.

If $\sigma_0 = 0$, $\Rightarrow$ const. amplitude.

$\sigma_0 > 0 \rightarrow$ Amplitude will grow.

$\sigma_0 < 0 \rightarrow$ " " decay.

Note

$\rightarrow$ The real sinusoidal sequence is periodic of period N if $\omega_0 N = 2\pi r$ $\rightarrow$ +ve integer.

$\rightarrow$ The complex exp. seq. with $\sigma_0 = 0$, is periodic of period N if $\omega_0 N = 2\pi r$

Ex $x_1(n) = e^{j\omega_1 n}$, $x_2(n) = e^{j\omega_2 n}$

$\omega_2 = \omega_1 + 2\pi k$.

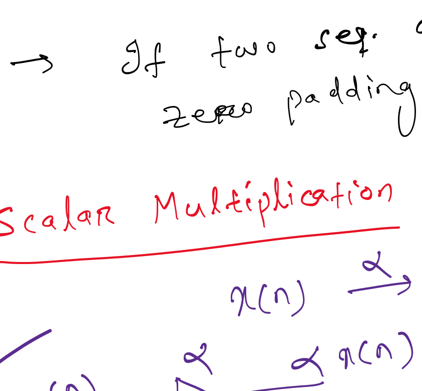
$x_2(n) = e^{j(\omega_1 + 2\pi k)n} = e^{j\omega_1 n} = x_1(n)$

Operations on sequence

Multiplication

$x(n), y(n)$

$w(n) = x(n)\, y(n)$



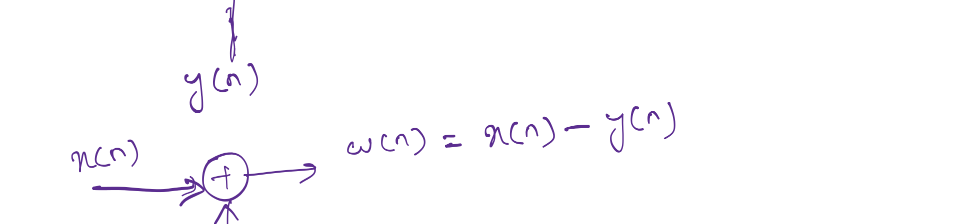
(For some particular application, this is also known as modulation operation.)

$\rightarrow$ It is non linear operation.

$\rightarrow$ If two seq. are not of same length, zero padding need to be done.

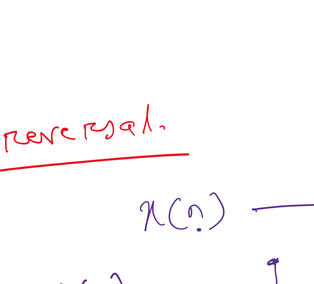
Scalar Multiplication

$x(n) \xrightarrow{\alpha} \alpha x(n)$

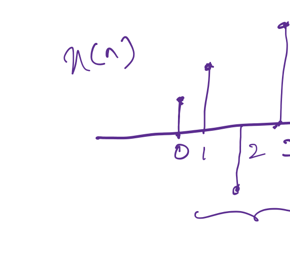


Scalar multiplier.

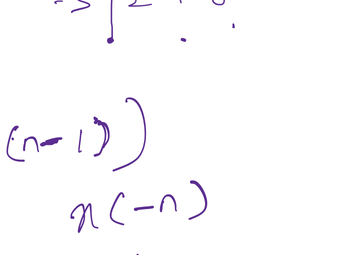
Addition:



$w(n) = x(n) + y(n)$



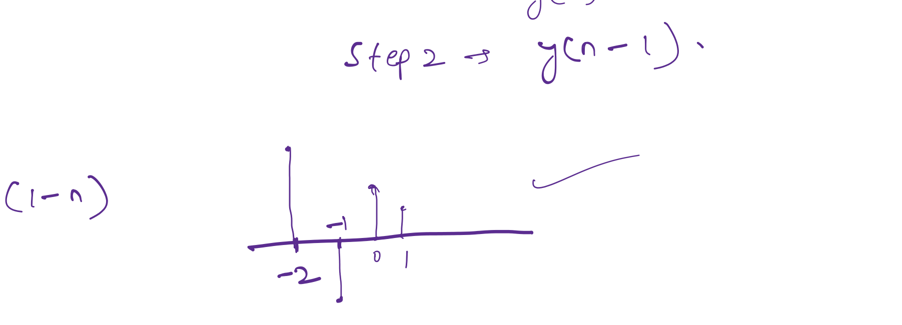
$w(n) = x(n) - y(n)$



Time reversal.

$x(n) \rightarrow x(-n)$.

Ex



$x(1-n)$, $\rightarrow x(-(n-1))$

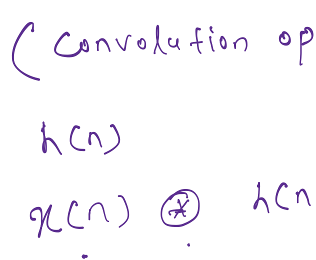
$\rightarrow$ step-1 $x(-n)$

$\downarrow$

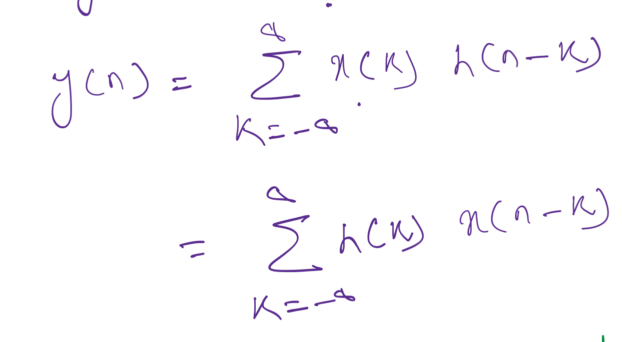
$y(n)$

Step 2 $\rightarrow y(n-1)$.

$x(1-n)$



Delay.



Convolution Sum. (Convolution operation)

$x(n), h(n)$

$y(n) = x(n) \circledast h(n) = h(n) * x(n)$

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$= \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

Steps to compute the convolution sum.

$x(n) \rightarrow \{\underset{\uparrow}{1}\ 2\ 3\}$

$y(n) \rightarrow \{\underset{\uparrow}{2}\ 3\}$

$y(n) \rightarrow \{\underset{\uparrow}{2}\ 3\ 0\}$ $\rightarrow$ Zeros padding.