Lecture 26

Friday, 22 October 2021 2:59 PM

If ROC of $H(z)$ includes the unit circle, $H(e^{j\omega}) = H(z)\big|_{z=e^{j\omega}}$.

LTI IIR Digital filters :-

Any LTI system can be represented by the diff. eqn

$$\sum_{k=0}^{N} d_k\, y(n-k) = \sum_{k=0}^{M} p_k\, x(n-k)$$

$$\Rightarrow \sum_{k=0}^{N} d_k z^{-k}\, Y(z) = \sum_{k=0}^{M} p_k z^{-k}\, X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{p_0 + p_1 z^{-1} + \cdots + p_M z^{-M}}{d_0 + d_1 z^{-1} + \cdots + d_N z^{-N}}$$

$$= z^{N-M} \cdot \frac{p_0}{d_0} \frac{\prod_{l=1}^{M}(z-\alpha_l)}{\prod_{l=1}^{N}(z-\lambda_l)}$$

$\alpha_1, \ldots \alpha_M \rightarrow$ finite zeros.

$\lambda_1, \ldots \lambda_N \rightarrow$ finite poles.

ROC for a causal IIR filter

$$|z| > \max_{k \rightarrow \{1, \ldots N\}} |\lambda_k|$$

$$H(e^{j\omega}) = H(z)\big|_{z=e^{j\omega}}$$

$$= \frac{p_0 + p_1 e^{-j\omega} + \cdots + p_M e^{-j\omega M}}{d_0 + d_1 e^{-j\omega} + \cdots + d_N e^{-j\omega N}}$$

Ex

$$y(n) = x(n-1) - 1.2\, x(n-2) + x(n-3) + 1.3\, y(n-1) - 1.04\, y(n-2) + 0.222\, y(n-3)$$

Soln

$$H(z) = \frac{z^{-1} - 1.2 z^{-2} + z^{-3}}{1 - 1.3 z^{-1} + 1.04 z - 0.222}$$

Zeros : $z = 0.6 + 0.8i$

$z = 0.6 - 0.8i$.

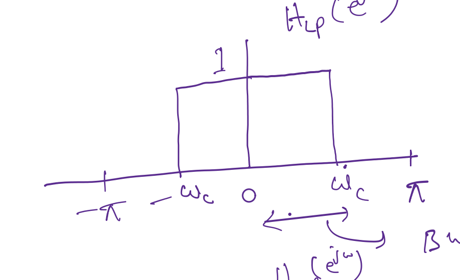
Poles: $z = 0.5 + 0.7i$

$z = 0.5 - 0.7i$

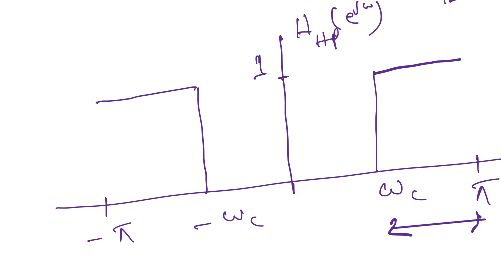
$z = 0.3$.

For causal $\Rightarrow |z| > \sqrt{0.74}$

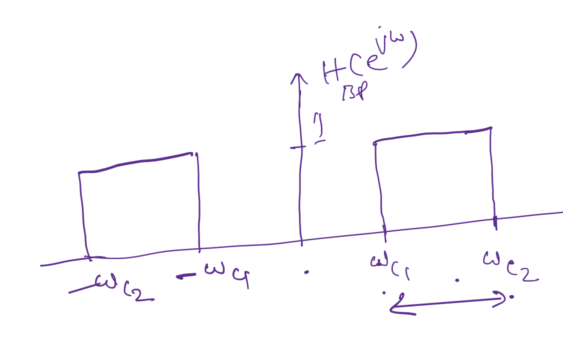
Simple Digital filters.



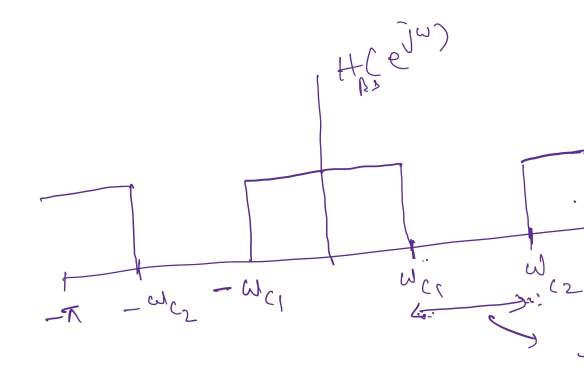
$\rightarrow$ Ideal LPF.



$\rightarrow$ Ideal HPF



$\rightarrow$ Ideal Band pass filter.



$\rightarrow$ Ideal Band stop filter.

stop band.

FIR Digital filters

Low pass FIR Digital filter

$$H(z) = \frac{1}{2}(1 + z^{-1}) \rightarrow \text{Transfer fun}$$

If we consider $M=2$ in M-point MA filter

$$H(e^{j\omega}) = \frac{1}{2}(1 + e^{-j\omega})$$

$$|H(e^{j\omega})| = \begin{cases} 1, & \omega = 0 \\ 0, & \omega = \pi \end{cases}$$

$$\boxed{\begin{matrix} \omega = 0 \rightarrow z = 1 \\ \omega = \pi \rightarrow z = -1 \end{matrix}}$$

$$\left.\begin{matrix} z = 1, & H(z) = 1 \\ z = -1, & H(z) = 0 \end{matrix}\right\} \rightarrow \text{LPF.}$$

$$H(e^{j\omega}) = \frac{1}{2}(1 + e^{-j\omega})$$

$$= e^{-j\omega/2} \cos\left(\frac{\omega}{2}\right).$$

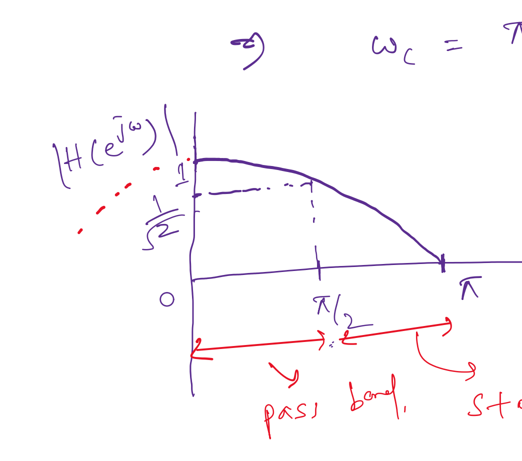
$\omega_c \rightarrow$ 3-dB cutoff freq.

$$|H(e^{j\omega_c})| = \frac{1}{\sqrt{2}} \max |H(e^{j\omega})|$$

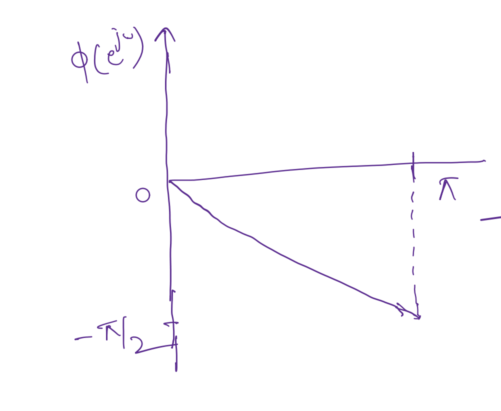
$$\Rightarrow |H(e^{j\omega_c})| = 1/\sqrt{2}$$

$$\Rightarrow \cos\frac{\omega_c}{2} = 1/\sqrt{2}$$

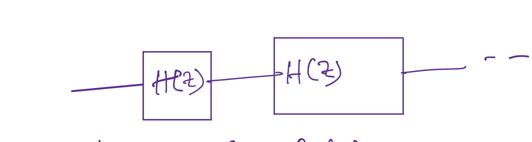
$$\Rightarrow \omega_c = \pi/2.$$



$$\phi(\omega) = -\frac{\omega}{2} + \angle \cos \omega/2$$



$\rightarrow$ phase response.



$\rightarrow$ A cascade of simple FIR filters, results an improved LP frequency.

$\rightarrow$ If M $H(z)$ are cascaded.

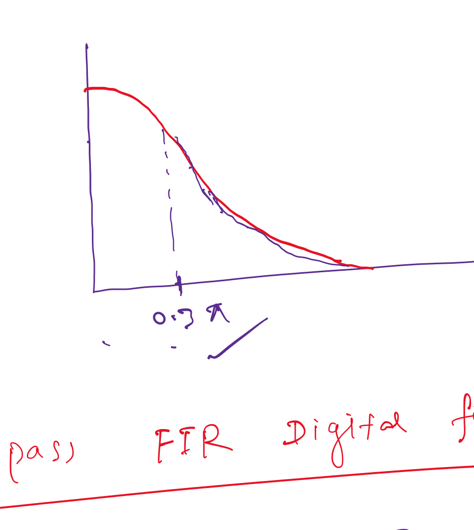
$$|H(e^{j\omega_c})| = 1/\sqrt{2}$$

$$\Rightarrow \left(\cos\frac{\omega_c}{2}\right)^M = 1/\sqrt{2}$$

$$\Rightarrow \omega_c = 2\cos^{-1}\left(2^{-1/2M}\right)$$

If $M = 3$,

$\omega_c = 0.3\pi$ rad.



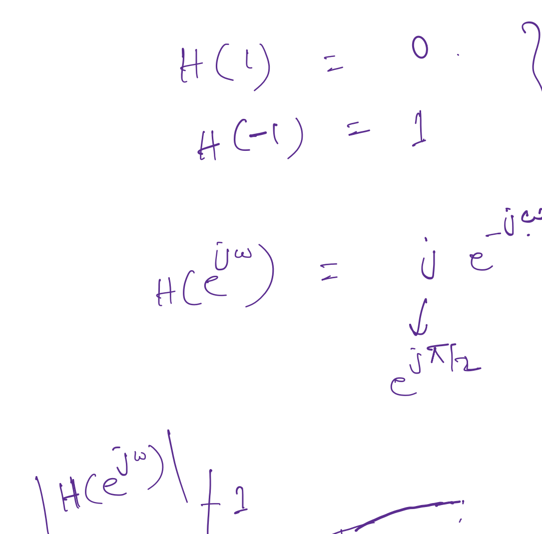
High pass FIR Digital filter.

$\rightarrow$ Just replace z with $-z$, the LPF will become HPF.

$$H(z) = \frac{1}{2}(1 - z^{-1})$$

$$\left.\begin{matrix} H(1) = 0 \\ H(-1) = 1 \end{matrix}\right\} \rightarrow \text{HPF.}$$

$$H(e^{j\omega}) = \underset{\downarrow \atop e^{j\pi/2}}{j}\, e^{-j\omega/2} \sin(\omega/2)$$



$\omega_c = \pi/2$

$$\phi(\omega) = -\frac{\omega}{2} + \frac{\pi}{2} + \angle \sin \omega/2$$