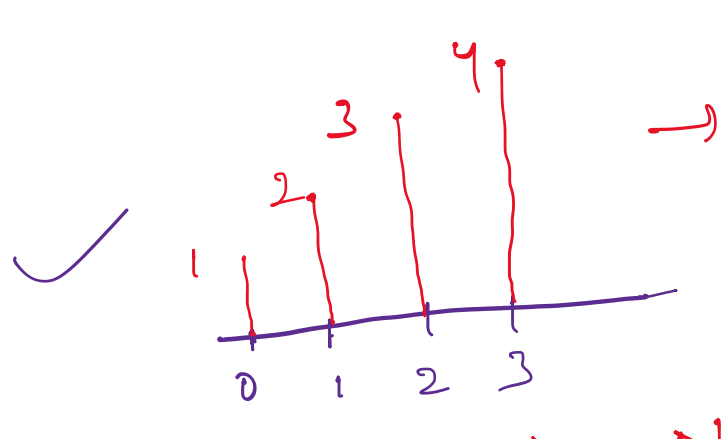
Lecture 5

Friday, 27 August 2021 3:02 PM

Circular shift :-

$y(n) = x(\langle n-n_0\rangle_N)$ → right circular shift by n_0

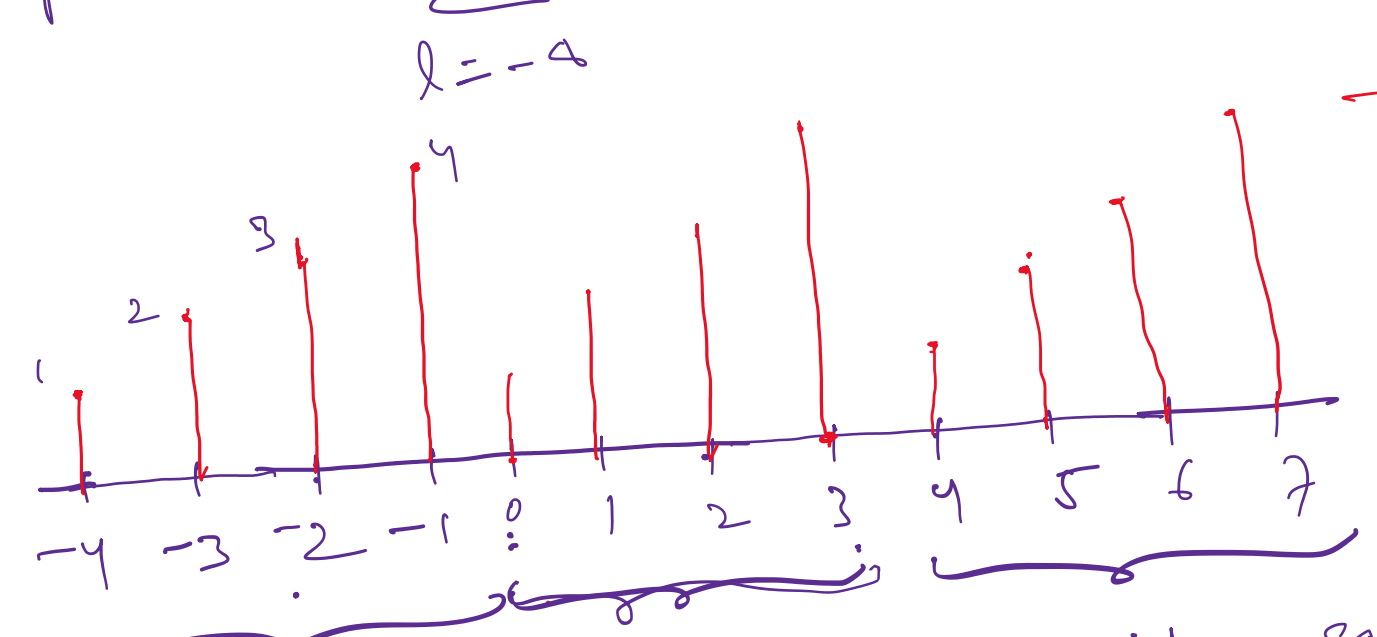
$y(n) = x(\langle n+n_0\rangle_N)$ → Left circular shift by n_0.



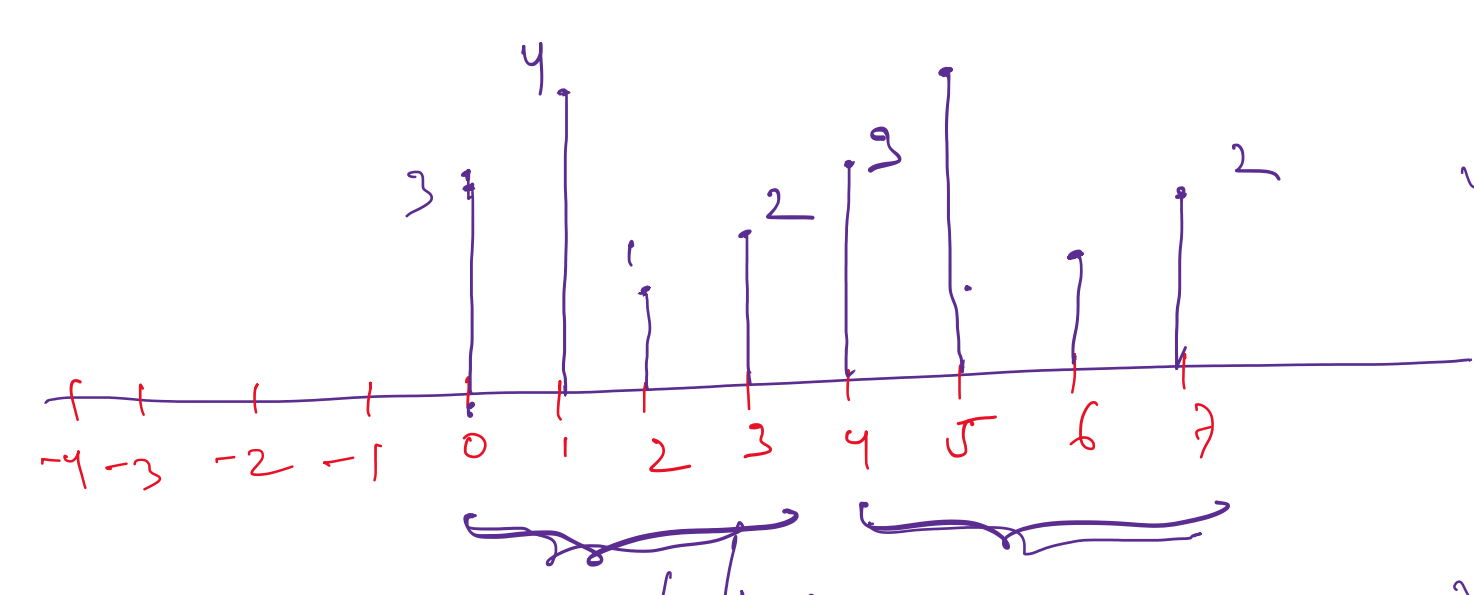
→ $N = 4$.

$y(n) = x(\langle n-2\rangle_4) = ?$

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n+lN)$$



Compute $x_p(n-2)$ then consider samples $0 \le n \le N-1$



→ $y(n) = x(\langle n-2\rangle_4)$.

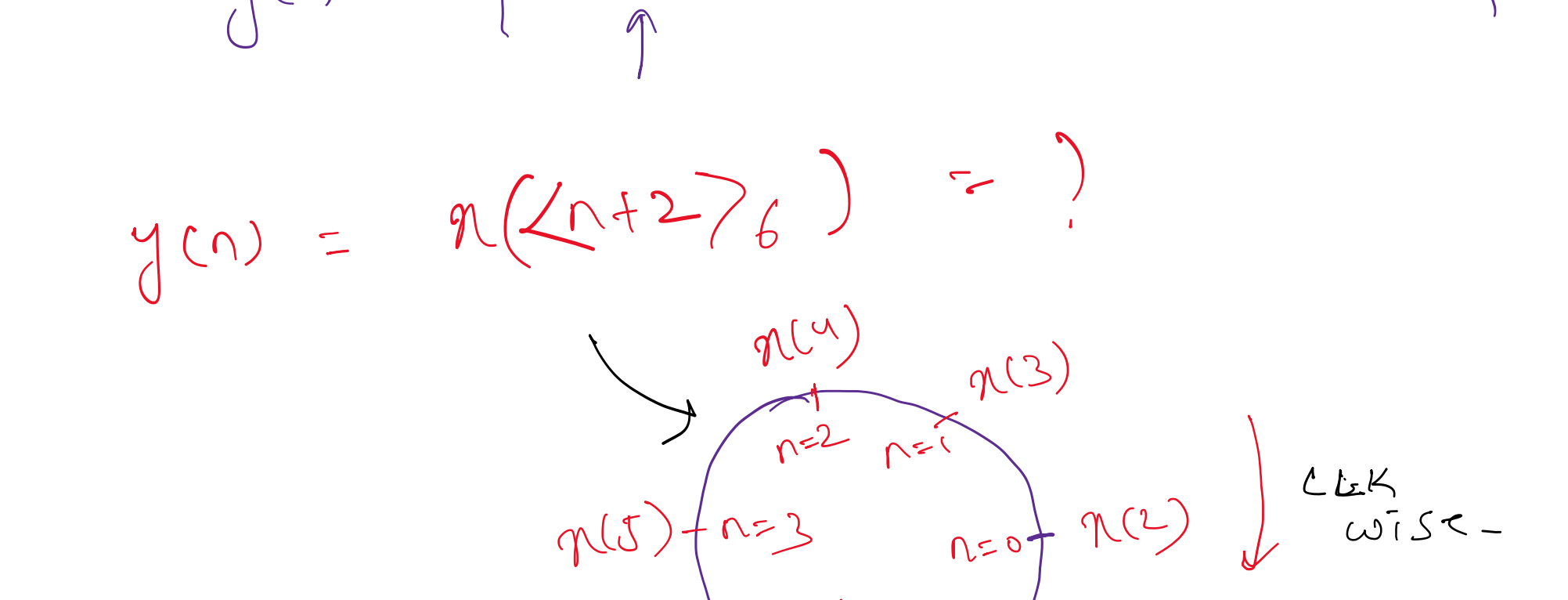
2nd method.

$x(n) \to \{x(0), x(1), x(2), x(3), x(4), x(5)\}$

↓

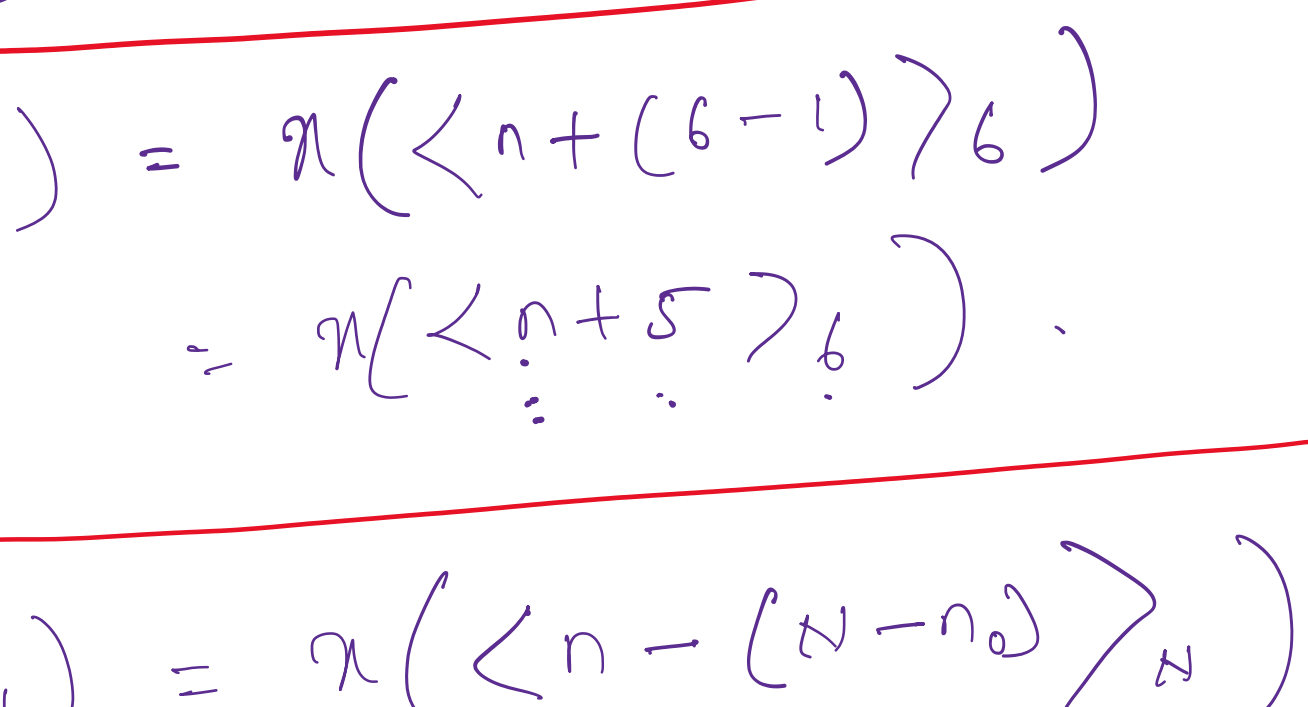
$N = 6$.

$y(n) = x(\langle n-1\rangle_6)$



$y(n) = \{x(5)\ x(0)\ x(1)\ x(2)\ x(3)\ x(4)\}$

$y(n) = x(\langle n+2\rangle_6) = ?$



$x(n) \to 0 \le n \le N-1$

$x(\langle n-n_0\rangle_N) = x(\langle n+(N-n_0)\rangle_N)$

$x(\langle n-1\rangle_6) = x(\langle n+(6-1)\rangle_6)$
$= x(\langle n+5\rangle_6)$.

$x(\langle n+n_0\rangle_N) = x(\langle n-(N-n_0)\rangle_N)$

$x(\langle n+5\rangle_6) = x(\langle n-(6-5)\rangle_6)$
$= x(\langle n-1\rangle_6)$.

$x(n) \to 0 \le n \le N-1$

$x(\langle -n\rangle_N) = x(\langle N-n\rangle_N)$.

For $1 \le n \le N-1$, $x(\langle -n\rangle_N) = x(N-n)$

$x(\langle -3\rangle_6) = x(3)$
$x(\langle -2\rangle_6) = x(4)$

The sampling process (Introduction)

$\cos(\Omega_0 t + \phi)$ —/— (T) → $\cos(\Omega_0 nT + \phi) = \cos(\omega_0 n + \phi)$.

$t_n \to nT \to$ Sampling instances.

$\omega_0 = \Omega_0 T$.

$f_s = \frac{1}{T}$.

Example

Let's consider 3 signals. & $T = 0.1$ sec.

$T = 0.1$ sec. $\Rightarrow f_s = \frac{1}{T} = 10$ Hz.

$\cos 6\pi t$	$\cos 14\pi t$	$\cos 26\pi t$.
↓	↓	↓
$f_1 = 3$ Hz.	$f_2 = 7$ Hz	$f_3 = 13$ Hz.
↓	↓	↓
$f_s > 2f_1$	$f_s < 2f_2$	$f_s < 2f_3$.
↓ sampling.	↓	↓
$\cos(0.6\pi n)$	$\cos(1.4\pi n)$	$\cos(2.6\pi n)$
	$= \cos((2\pi-0.6\pi)n)$	$= \cos((2\pi+0.6\pi)n)$
	$= \cos(0.6\pi n)$	$= \cos(0.6\pi n)$
	↓ $f = 3$ Hz.	↓ 3 Hz.

→ All signals poses to 3 Hz signal.

→ Due to inadequate sampling process, the high freq. poses to low freq. → Aliasing effect.

$\cos(1.4\pi n)$ →
$\cos(2.6\pi n)$ → } → $\omega_0 > \pi$.

↓ This leads to the distortion.

$x(t) = \cos(6\pi t) + \cos(14\pi t) + \cos(26\pi t)$

$f_s \ge 2f_{max} = 2 \times 13 = 26$ Hz.

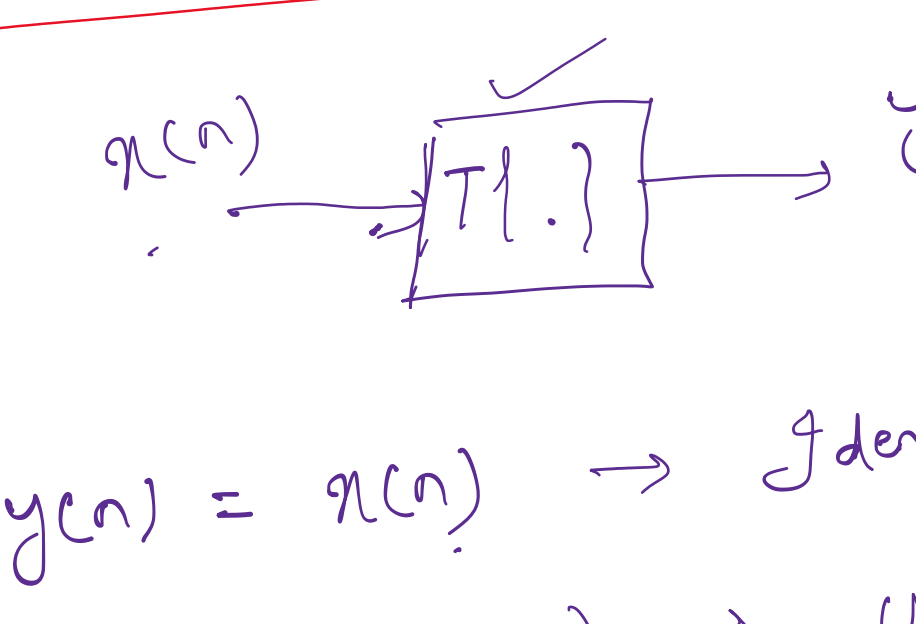
$f_{s_{min}} = 26$ Hz.

→ Let $T = 0.02$ sec $\Rightarrow f_s = 50$ Hz

$\cos(0.12\pi n)$ $\cos(0.28\pi n)$ $\cos(0.52\pi n)$

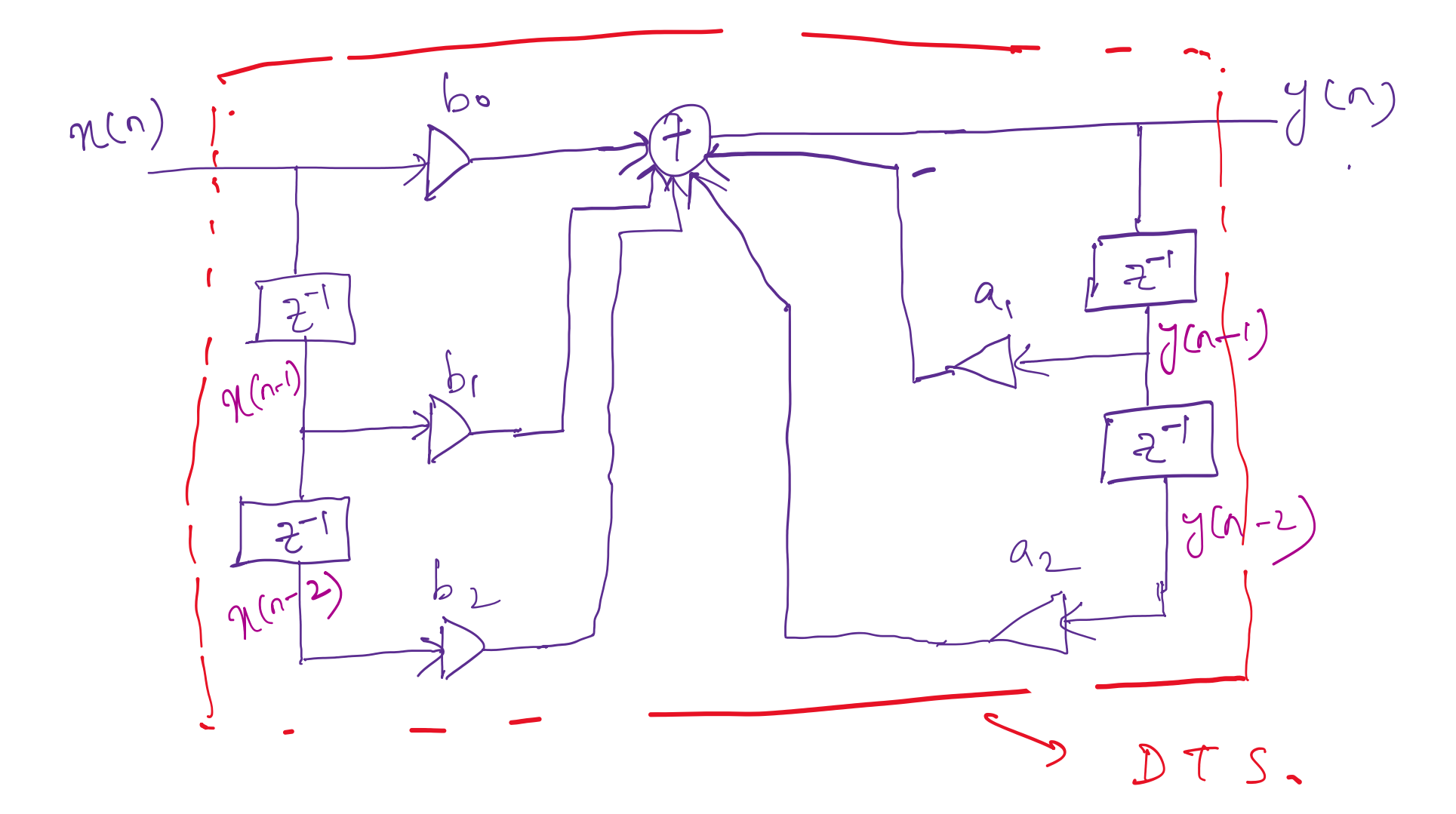
→ No aliasing

Discrete-Time systems:



$y(n) = T\{x(n)\}$.

$y(n) = x(n)$ → Identity system.
$y(n) = x(n-1)$ → Unit delay system.
$y(n) = x(n+1)$ → Unit advance system.



→ DTS.

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + a_1 y(n-1) + a_2 y(n-2)$$