Lecture 8

Saturday, 4 September 2021 2:08 PM

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)\, h(k)$$
$$= \sum h(n-k)\, x(k)$$

→ This convolution operation is only applicable for LTI system.

→ If the system is LTV

$$y(n) = \sum h(n,k)\, x(n-k)$$

may depend on time.

↳ Not a conv. operation.

→ An LTI system is completely characterized by its impulse response.

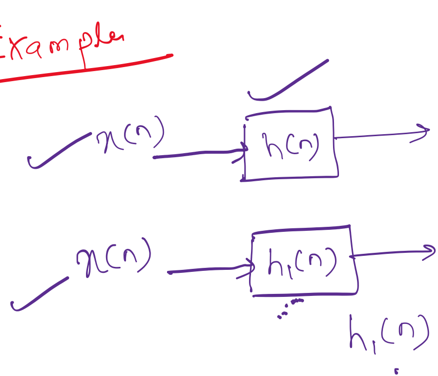
Relationship betn step response & IR.

$h(n) = s(n) - s(n-1)$

IR, S.R.

$$s(n) = u(n) * h(n)$$
$$= \sum_{k=0}^{\infty} \delta(n-k) * h(n)$$
$$= \sum_{k=0}^{\infty} h(n-k)$$

Example



$y(n) = x(n) * h(n)$

$y(n) = ?$

$h_1(n) = h(n-m)$

$$y(n) = x(n) * h_1(n)$$
$$= x(n) * h(n-m)$$
$$= \underbrace{x(n) * h(n)}_{y(n)} * \delta(n-m) \quad \to h(n-m)$$
$$= y(n) * \delta(n-m)$$
$$= y(n-m)$$

Stability condition in terms of the impulse response :

→ An LTI DTS is BIBO stable iff

$$S = \sum_{n=-\infty}^{\infty} |h(n)| < \infty.$$

Impulse Resp. should be absolutely summable.

Proof

part 1 If $S < \infty$, then $|y(n)| < \infty$.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k).$$

$|x(n)| \leq B_x < \infty.$ → Bounded I/P

$$\Rightarrow |y(n)| = \left| \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \right|$$
$$\leq \sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)|$$
$$\leq B_x \underbrace{\sum_{k=-\infty}^{\infty} |h(k)|}_{S}$$
$$= S B_x < \infty.$$

$\Rightarrow |y(n)| < \infty$ → Bounded o/p.

part 2 Given $|y(n)| < B_y < \infty$, then prove that $S < \infty$.

$$y(n) = \sum h(k)\, x(n-k)$$

Let's consider $x(n) = \mathrm{sgn}(h(n))$

$$\mathrm{sgn}(c) = \begin{cases} +1, & c \geq 0 \\ -1, & c < 0 \end{cases}$$

$$\Rightarrow x(n) = \begin{cases} +1, & h(n) \geq 0 \\ -1, & h(n) < 0. \end{cases}$$

$|x(n)| = 1 \quad \forall n \Rightarrow$ Bounded I/P.

$$|y(n)| = \sum_{k=-\infty}^{\infty} |h(k)| = S.$$

$\Rightarrow S < \infty.$ to make $y(n)$ to bounded.

Causality condn in terms of IR. :

→ An LTI DTS is causal iff its IR $h(n)$ is a causal sequence.

$h(n) = 0$ for $n < 0$.

Ex

$h(n) = \alpha^n u(n)$

↳ Causal system.

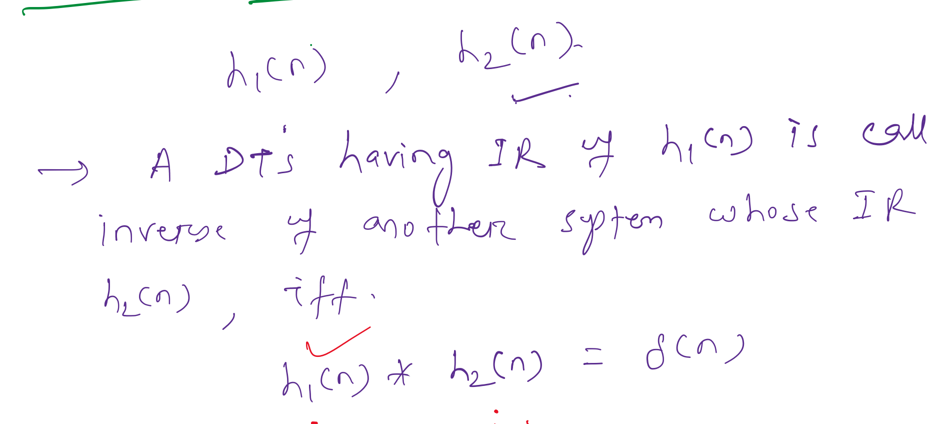
↳ stable ?

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0}^{\infty} |\alpha|^n$$
$$= \frac{1}{1-|\alpha|} \quad \text{for } |\alpha| < 1.$$

$\Rightarrow$ If $|\alpha| < 1$, it is BIBO stable system.

If $|\alpha| \geq 1$, unstable.

Interconnections betn LTI systems.



cascade connection.

Inverse system

$h_1(n), \quad h_2(n).$

→ A DTS having IR of $h_1(n)$ is called as inverse of another system whose IR is $h_2(n)$, iff.

$$h_1(n) * h_2(n) = \delta(n)$$

Ex

$$y(n) = \sum_{k=0}^{\infty} x(n-k) \quad \to \text{Accumulator.}$$

Find the inverse system.

$$h(n) = \sum_{k=0}^{\infty} \delta(n-k) = u(n).$$

Assume that $h_1(n)$ is IR of inverse system.

$$h(n) * h_1(n) = \delta(n).$$
$$\Rightarrow h_1(n) * \sum_{k=0}^{\infty} \delta(n-k) = \delta(n)$$
$$\Rightarrow \sum_{k=0}^{\infty} h_1(n-k) = \delta(n).$$
$$\Rightarrow h_1(n) + h_1(n-1) + h_1(n-2) + \cdots = \delta(n).$$

Assuming $h_1(n)$ is a causal system.

for $n = 0$

$h_1(0) + \cancel{h_1(-1)} + \cdots = \delta(0)$, with $h_1(-1) = 0$

$\Rightarrow h_1(0) = 1.$

for $n = 1$

$h_1(1) + h_1(0) = \delta(1) = 0$

$\Rightarrow h_1(1) = -h_1(0) = -1.$

for $n = 2$

$$h_1(2) + \underbrace{h_1(1) + h_1(0)}_{0} = \delta(2) = 0$$

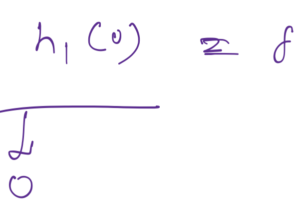
$\Rightarrow h_1(2) = 0.$

for $n = 3$

$h_1(3) = 0.$

$h_1(n) = 0, \quad n \geq 2.$

$h_1(n) \to$



$$\boxed{h_1(n) = \delta(n) - \delta(n-1)}$$

↳ Inverse system.

→ Inverse system will not exist if it is unstable.

Parallel connection