Lecture 2

Wednesday, 18 August 2021 3:29 PM

General signal.

$x[n] = x_e[n] + x_o[n]$

↓

Neither even nor odd.

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

Periodic Signal.

If $x[n]$ satisfies

$x[n] = x[n + KN] \quad \forall n$ — (1)

then $x[n]$ is called a periodic seq. with period N.

$N \rightarrow$ +ve integer

$K \rightarrow$ Any integer

period of $x[n]$.

$N_f \rightarrow$ fundamental period

= Smallest value of N for which (1) satisfies.

Sum of two or more periodic signals

$x_1[n]$, $x_2[n]$, & $x_3[n]$

↓ fundament. peri. $= N_1$ ↓ N_2 ↓ N_3.

$x_4[n] = \alpha\, x_1[n] + \beta\, x_2[n] + \gamma\, x_3[n]$

↓

It is always periodic.

Fundamental period of $x_4[n]$

$= N_4 = LCM(N_1, N_2, N_3)$.

Ex

$N_1 = 3,\ N_2 = 5,\ N_3 = 12$

$N_4 = LCM(3, 5, 12) = 60$

⇒ At 60, $x_1[n]$ completed 20 cycles, $x_2[n]$ completed 12 cycles. $x_3[n]$ completed 5 cycles.

Product of two or more periodic signal.

$w[n] = x_a[n]\, x_b[n]$

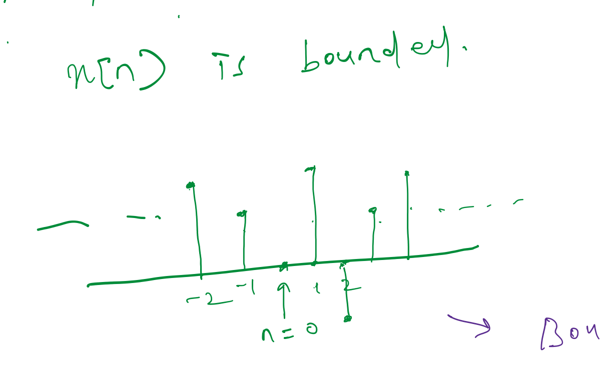
↓ period N. ↓ N_a ↓ N_b.

$N = LCM(N_a, N_b)$.

Bounded signal :-

If $|x[n]| \le B_x < \infty \quad \forall n$

then $x[n]$ is bounded.



→ Bounded signal

$|x(n)| \rightarrow$ finite.

Absoluty summable.

$$\sum_{n=-\infty}^{\infty} |x[n]| \le P < \infty.$$

then $x[n]$ is absoluty summable.

Square summable.

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 \le Q < \infty.$$

↳ Finite energy.

Ex

$$x[n] = \begin{cases} \dfrac{\sin \omega_c n}{\pi n} & n<0,\ n>0 \\ \dfrac{\omega_c}{\pi}, & n = 0. \end{cases}$$

↓

Square summable but not absoluty summable.

$x[n] = \sin \omega_c n \quad -\infty < n < \infty.$

$x(n) = K \quad -\infty < n < \infty$

↳ Neither absolute. summable nor sq. summable.

Energy of the signal.

$$\mathcal{E}_x = \sum_{n=-\infty}^{\infty} |x(n)|^2 \rightarrow \text{energy of the signal } x[n]$$

Ex

$$x(n) = \begin{cases} \frac{1}{n}, & n \ge 1 \\ 0, & n \le 0 \end{cases}$$

↙ Infite length seq. ↘ Finite energy $= \pi^2/6$.

→ The energy of ∞ length seq. may or may n't be finite

Ex

$$x(n) = \begin{cases} \frac{1}{\sqrt{n}}, & n \ge 1 \\ 0, & n \le 0. \end{cases}$$

↙ ∞ length seq. ↘ ∞ energy.

Average power of the signal.

power → Energy per time.

For aperiodic signal.

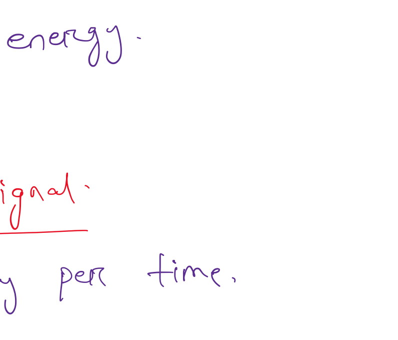
$$P_{avg} = \lim_{K \to \infty} \frac{1}{2K+1} \sum_{n=-K}^{K} |x(n)|^2$$

→ Energy over the length 2K+1

↓ Avg. power over 2K+1

For a periodic signal.

$$P_{avg} = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$



↓ periodic with fund. period = 3.

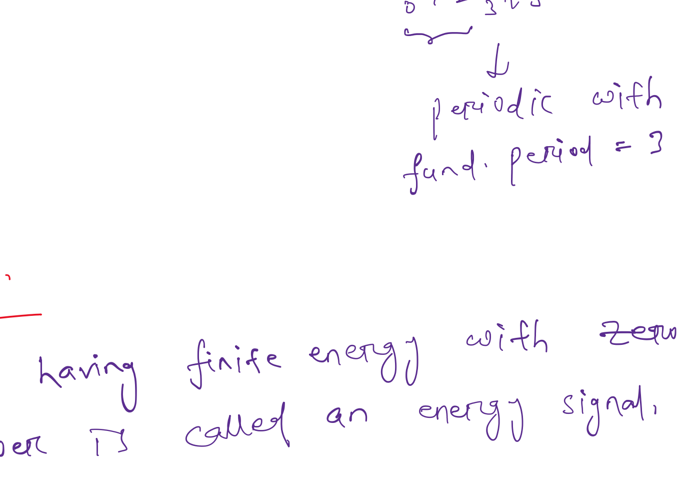
Energy signal.

A signal having finite energy with zero. avg. power is called an energy signal.

Power signal.

Signal having ∞ energy with finite Avg. power, is known as power signal.

Some typical Elementary Digital signals.

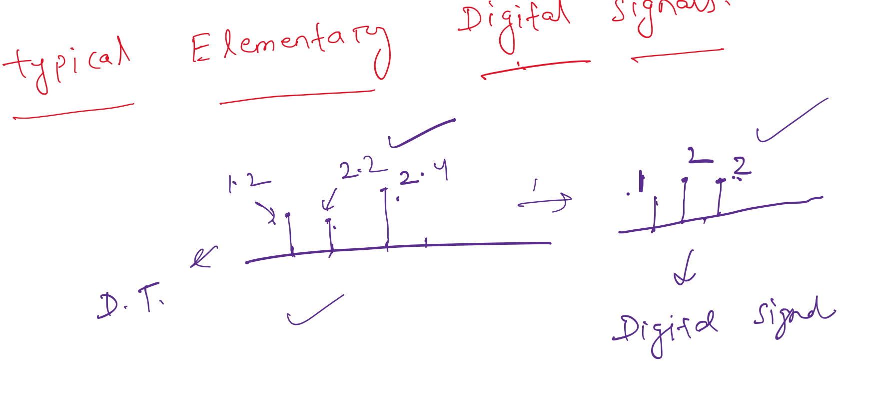


1) Unit impulse signal.

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \ne 0. \end{cases}$$

→ discrete-time-impulse.

↓ Kronecker delta

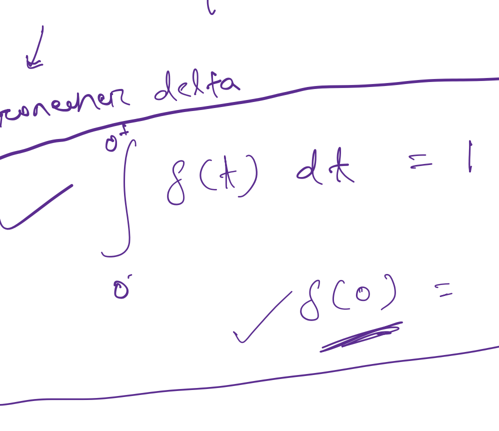


$$\int_{0^-}^{0^+} \delta(t)\, dt = 1$$

$\delta(0) = \infty$

2) Unit step. seq.

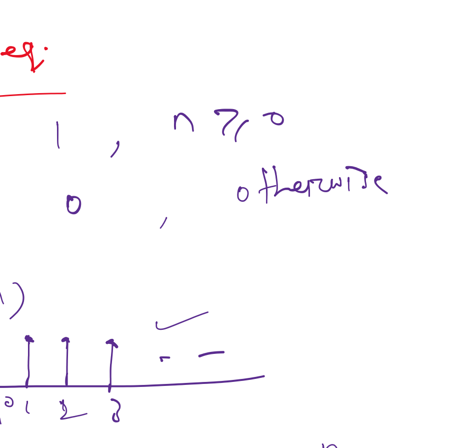
$$u[n] = \begin{cases} 1, & n \ge 0 \\ 0, & \text{otherwise} \end{cases}$$



$x(n)$

$$u[n] = \sum_{m=0}^{\infty} \delta(n-m) = \sum_{k=-\infty}^{n} \delta[k]$$

$\delta[n] = u[n] - u[n-1]$



→ Any arbitrary sequence can be expressed in terms of $\delta[n]$ and $u[n]$.

$x[n] = \{ \ldots x(-1)\ x(0)\ x(1) \ldots \}$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$$

$x[n] = \{1, 2, 1, 1, 0, 3\}$ (↑ $n=0$ at 2)

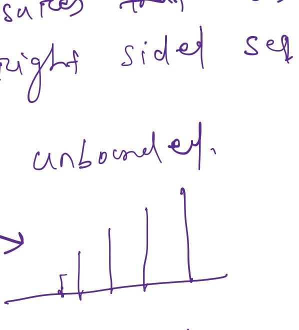
$= \delta(n+1) + 2\delta(n) + 1\cdot\delta(n-1) + \delta(n-2) + 0\cdot\delta(n-3) + 3\cdot\delta(n-4).$

Exponential Sequence

$x[n] = A\alpha^n u[n]$

↓ It ensures that $x[n]$ is a right sided seq.

$|\alpha| > 1, \quad x(n) \rightarrow$ unbounded.



$|\alpha| < 1, \quad x[n] \rightarrow$ bounded.

$x[n] = A\alpha^n \rightarrow$ unbounded, irrespective of $|\alpha|$.

$A\,1^n$