Lecture 29

Wednesday, 27 October 2021 4:01 PM

BPF, BSF $\rightarrow$ α, β.

Band stop filter :-

$$Q = \frac{\omega_0}{BW}$$

$\rightarrow$ $|\alpha| < 1$, $|\beta| < 1$ $\Rightarrow$ ensures stability of BPF & BSF.

Ex

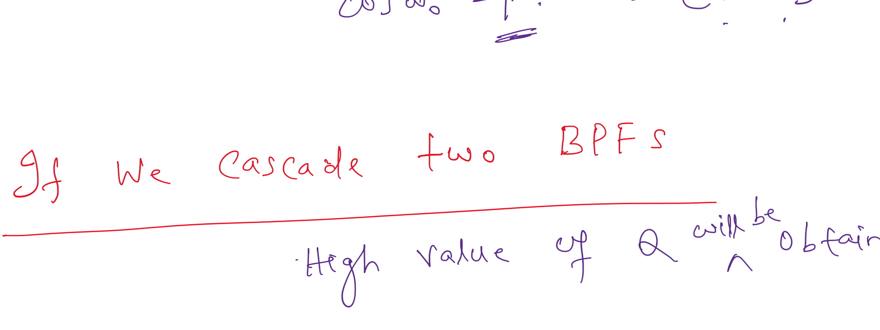
$\omega_0 = \pi/2$, $BW = \pi/4$.

$$H(z) = \frac{1 + z^{-2}}{1 + (\sqrt{2} - 1) z^{-2}} \frac{1}{\sqrt{2}}$$

$\cos \omega_0 = \beta$. $\rightarrow [-1, 1]$.

If we cascade two BPFs

High value of Q will be obtained



$\rightarrow$ If we cascade BSF, we will get worst filter.

IIR All pass Filter.

$\rightarrow$ $|A(e^{j\omega})|^2 = 1 \quad \forall \omega$.

$\rightarrow$ A(z) is APF.

$\angle A(e^{j\omega}) \rightarrow$ can be anything.

$$A(z) A(z^{-1}) = 1$$

$$A_N(z) = \frac{P_N(z)}{Q_N(z)}, \quad N \text{ is the order of TF.}$$

$$A_N(z) A_N(z^{-1}) = 1$$

$$\Rightarrow P_N(z) P_N(z^{-1}) = Q_N(z) Q_N(z^{-1})$$

$\rightarrow$ 1st let's consider a 1st order APF.

$$A(z) = K \frac{1 + a z^{-1}}{1 + b z^{-1}}, \quad |b| < 1.$$

K should be choosen that $A(z) A(z^{-1}) = 1$.

$$A(e^{j\omega}) = K \frac{1 + a \cos\omega - j a \sin\omega}{1 + b\cos\omega - j b \sin\omega}$$

$$|A(e^{j\omega})|^2 = K^2 \frac{1 + a^2 + 2a\cos\omega}{1 + b^2 + 2b\cos\omega}$$

$$= \frac{K^2 a}{b} \left(\frac{a + 1/a + 2\cos\omega}{b + 1/b + 2\cos\omega} \right)$$

$\rightarrow$ To get $|A(e^{j\omega})|^2 = 1$

$K = 1$ & $a = b$.

$\Rightarrow A(z) = 1 \rightarrow$ trivial APF

$\rightarrow$ phase $= 0$.

$\hookrightarrow$ It doesn't have any use.

Another way.

$a = 1/b$ and $K = b$.

$\Rightarrow |A(e^{j\omega})|^2 = 1$.

$$A(z) = b \left(\frac{1 + \frac{1}{b} z^{-1}}{1 + b z^{-1}} \right).$$

$$= \frac{b + z^{-1}}{1 + b z^{-1}}$$

$|A(e^{j\omega})|^2 = 1$ & $\phi(\omega) \neq 0$.

$$A(z) = \frac{b + z^{-1}}{1 + b z^{-1}} = \frac{N(z)}{D(z)}$$

$$= \frac{z^{-1} \underbrace{(1 + bz)}_{D(z^{-1})}}{1 + b z^{-1}}$$

$$= \frac{z^{-1} D(z^{-1})}{D(z)}$$

$$\boxed{A_N(z) = \frac{z^{-N} D_N(z^{-1})}{D_N(z)}}$$

$$A_N(z) = \frac{d_N + d_{N-1} z^{-1} + \cdots + d_1 z^{-(N-1)} + z^{-N}}{1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{N-1} z^{-(N-1)} + d_N z^{-N}}$$

This is one form of APF.

$$A_1(z) = \frac{d_1 + z^{-1}}{1 + d_1 z^{-1}}$$

$$A_2(z) = \frac{d_2 + d_1 z^{-1} + z^{-2}}{1 + d_1 z^{-1} + d_2 z^{-2}}.$$

Bounded Real (BR) Transfer fn

$\Rightarrow$ A causal stable real-coefficient fncn $H(z)$ is defined as a BR transfer fun, if $|H(e^{j\omega})| \leq 1 \quad \forall \omega$.

Ex

$$H(z) = \frac{K}{1 - \alpha z^{-1}}, \quad 0 < |\alpha| < 1$$

$\rightarrow$ It will be BR fn for $K = \pm (1 - \alpha)$.

$$|H(e^{j\omega})| \leq 1$$

$$\Rightarrow |H(e^{j\omega})|^2 \leq 1$$

$$\Rightarrow |Y(e^{j\omega})|^2 \leq |X(e^{j\omega})|^2$$

$$\Rightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} |Y(e^{j\omega})|^2 d\omega \leq \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

$$\Rightarrow \sum y^2(n) \leq \sum x^2(n)$$

$\Rightarrow$ o/p energy $\leq$ I/p energy.

$\Rightarrow$ BR functions are passive.

$\Rightarrow$ If equality holds then it is loss less BR (LBR) TF.

Properties of APF

Prop-1

$\rightarrow$ A causal stable real coefficient APF is a LBR transfer function.

Prop-2

If z_0 is a pole of APF

$\Rightarrow D_N(z_0) = 0$.

Then we can get $D_N(z^{-1}) = 0$ when $z = 1/z_0$.

$\rightarrow$ poles and zeros of APFs occur in reciprocal pairs.

Prop-3

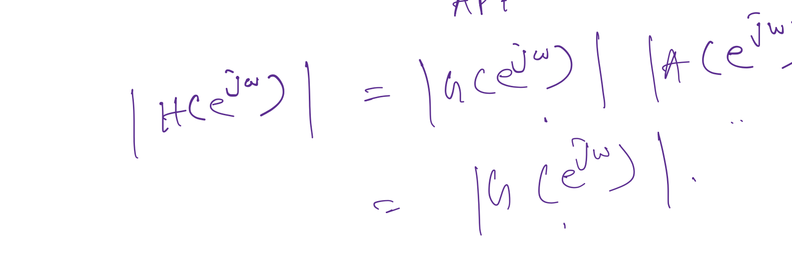
$$|A_N(z)| \begin{cases} < 1 & \text{for } |z| > 1. \\ = 1 & \text{for } |z| = 1 \\ > 1 & \text{for } |z| < 1 \end{cases}$$

Application of APF

Delay Equalizer.

Non linear phase response $\Rightarrow$ Delay distortion

$\rightarrow$ Group delay $\neq$ const.



$$|H(e^{j\omega})| = |G(e^{j\omega})| \, |A(e^{j\omega})|$$

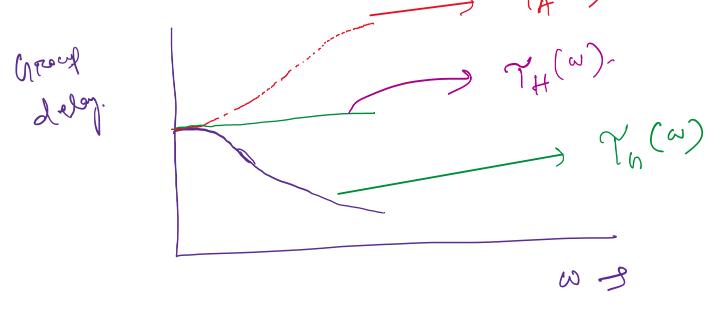
$$= |G(e^{j\omega})|.$$

$\theta_G(\omega) = \angle G(e^{j\omega})$, $\theta_A(\omega) = \angle A(e^{j\omega})$

$\theta_H(\omega) = \angle H(e^{j\omega})$

$$\theta_H(\omega) = \theta_G(\omega) + \theta_A(\omega)$$

$$\Rightarrow \tau_H(\omega) = \tau_G(\omega) + \tau_A(\omega).$$



Comb Filter. (FIR or IIR)