Quiz 2 (ECC302: DSP Theory)

Name:

Date: 12 Nov 2022

Admission Number:

Time: 6 to 6.15 PM

Total Marks: 10

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1. A causal LTI system has the transfer function
$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{(1 + 0.5z^{-1})(1 - z^{-1})}$$
Find the impulse response of the system, $h[n]$. **[02]**
Answer:

$$h[n] = -2\delta[n] + \frac{1}{3}\left(-\frac{1}{2}\right)^n u[n] + \frac{8}{3} u[n].$$

2. Determine the Z.T and ROC of $x(n) = a^n u[n] + b^n u[n] + c^n u[-n-1]$, for $|a| < |b| < |c|$. **[02]**
Answer:

$$X(z) = \frac{1}{1-az^{-1}} + \frac{1}{1-bz^{-1}} - \frac{1}{1-cz^{-1}}$$

R.O.C. = $|b| < |z| < |c|$.

=

3. Find the ZT and the associated ROC of $x[n] = (0.1)^n u[-n-1] + (2)^n u[n]$ **[02]**
Answer:

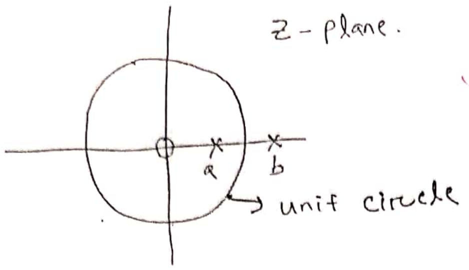
Z.T. doesn't exist.

No common ROC.

$$\frac{1}{1-2z^{-1}} - \frac{1}{1-(0.1)z^{-1}}$$

P.T.O

4. In the following figure, the pole-zero plots of $H(Z)$ is given. According to this plot, write the impossible nature(s) of $h(n)$ in terms of stability and causality. **[02]**



Answer:

Anti - causal & stable.

5. Considering the following pair of input and output z-transforms $X(z)$ and $Y(z)$, determine the ROC for the transfer function $H(z)$. **[01]**

$$X(z) = \frac{1}{1 - \frac{3}{4}z^{-1}}, \qquad |z| > \frac{3}{4}$$

$$Y(z) = \frac{1}{1 + \frac{2}{3}z^{-1}}, \qquad |z| > \frac{2}{3}$$

Answer:

$|z| > \frac{2}{3}$.

6. What will be the ROC of Z-transform of a finite causal sequence? **[01]**

Answer:

Comple z - plane except at $z = 0$.

Sol^n to Quiz 2

Q1

$$(1+\tfrac{1}{2}z^{-1})(1-z^{-1})$$

$$= 1 - z^{-1} + \tfrac{1}{2}z^{-1} - \tfrac{1}{2}z^{-2}$$

$$= 1 - \tfrac{1}{2}z^{-1} - \tfrac{1}{2}z^{-2}$$

$$
\begin{array}{r|l}
 & \quad -2 \\
-\tfrac{1}{2}z^{-2} - \tfrac{1}{2}z^{-1} + 1 & \; z^{-2} + 2z^{-1} + 1 \\
 & \; z^{-2} + z^{-1} - 2 \\
 & \; - \quad\; - \quad\; + \\
\hline
 & \quad z^{-1} + 3
\end{array}
$$

$$\Rightarrow H(z) = -2 + \frac{3+z^{-1}}{(1+\tfrac{1}{2}z^{-1})(1-z^{-1})}$$

$$= -2 + \frac{A}{1+\tfrac{1}{2}z^{-1}} + \frac{B}{1-z^{-1}}$$

$$A = \left.\frac{3+z^{-1}}{1-z^{-1}}\right|_{z^{-1} = -2}$$

$$= \frac{3-2}{1+2} \qquad \cancel{\tfrac{5}{2} \times \tfrac{2}{3} = 5/3}$$

$$= \frac{1}{3}$$

$$B = \left.\frac{3+z^{-1}}{1+\tfrac{1}{2}z^{-1}}\right|_{z^{-1}=1} = \frac{4}{1+\tfrac{1}{2}} = \frac{8}{3}$$

i) $H(z) = -2 + \frac{\frac{1}{3}}{1+\frac{1}{2}z^{-1}} + \frac{8/3}{1-z^{-1}}$

$h[n] = -2\delta[n] + \frac{1}{3}\left(-\frac{1}{2}\right)^n u[n]$

$+ \frac{8}{3} u[n]$

Q 2

$x[n] = a^n u[n] + b^n u[n] + c^n u[-n-1]$

$X(z) = \frac{1}{1-az^{-1}} + \frac{1}{1-bz^{-1}} - \frac{1}{1-cz^{-1}}$

$|b| < |z| < |c|.$

$= \frac{1-2cz^{-1} + (bc+ac-ab)z^{-2}}{(1-az^{-1})(1-bz^{-1})(1-cz^{-1})}$

Q 3

$x[n] = (0.1)^n u[-n-1] + (2)^n u[n]$

$X(z) = \frac{1}{1-2z^{-1}} - \frac{1}{1-(0.1)z^{-1}}$

$|z| > 2$ $\quad$ $|z| < 0.1$

No common ROC.

$\Rightarrow$ ZT. doesn't exist.

<u>Q4</u>

Impossible nature

✓ $h[n] \rightarrow$ anti-causal & stable.

<u>Q5</u>

ROC: $|z| > \cancel{3/4}\ 2/3 \rightarrow$ (it is common)

$$H(z) = \cancel{\text{for}}\ \frac{Y(z)}{X(z)}$$

$$= \frac{1 - \frac{3}{4}z^{-1}}{1 + \frac{2}{3}z^{-1}} \qquad |z| > 2/3.$$

<u>Q6</u>

"Complete z-plane except at $z = 0$".