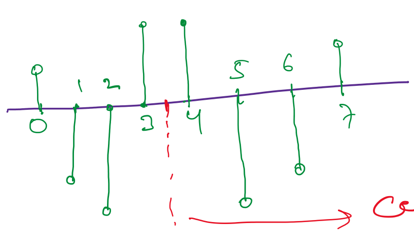
Lecture 18

Saturday, 25 September 2021 3:00 PM

Symmetric with even length.

$N = 8$



center of symmetry

$x(n) = x(N-1-n) = x(7-n)$

w.r.t. the half sample point $n = \frac{N-1}{2}$

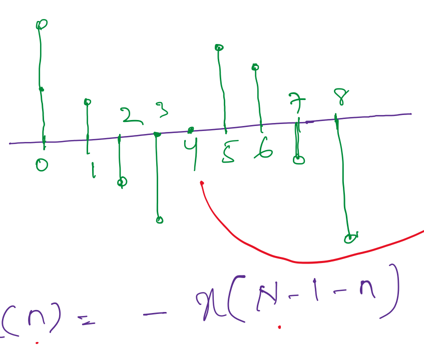
$$X(e^{j\omega}) = e^{-j(N-1)\omega/2} \left\{ 2 \sum_{n=1}^{N/2} x\left(\frac{N}{2}-n\right) \cos\left(\omega\left(n-\frac{1}{2}\right)\right) \right\}$$

$$\theta(\omega) = -\left(\frac{N-1}{2}\right)\omega + \underbrace{\beta}_{0 \text{ or } \pi}$$

$\hookrightarrow$ Linear phase.

Antisymmetric with odd length.

$N = 9.$



center of symmetric $\frac{N-1}{2}$.

$x(n) = -x(N-1-n) = -x(8-n)$

$$\boxed{x\left(\frac{N-1}{2}\right) = 0}$$

$$X(e^{j\omega}) = e^{-j(N-1)\omega/2} \cdot e^{j\pi/2} \times \left\{ 2 \sum_{n=1}^{(N-1)/2} x\left(\frac{N-1}{2}-n\right) \sin(\omega n) \right\}$$

$$\theta(\omega) = -\frac{(N-1)\omega}{2} + \frac{\pi}{2} + \underbrace{\beta}_{0 \text{ or } \pi}$$

Linear phase.

$$X(k) = e^{-j(N-1)\pi k/N} \times \left\{ 2 \sum_{n=1}^{(N-1)/2} x\left(\frac{N-1}{2}-n\right) \sin\left(\frac{2\pi k n}{N}\right) \right\}$$

Antisymmetric with Even length.

$$X(e^{j\omega}) = j e^{-j(N-1)\omega/2} \times \left\{ 2 \sum_{n=1}^{N/2} x\left(\frac{N}{2}-n\right) \sin\left(\omega\left(n-\frac{1}{2}\right)\right) \right\}$$

$e^{j\pi/2}$

$$\theta(\omega) = -\frac{(N-1)\omega}{2} + \frac{\pi}{2} + \underbrace{\beta}_{0 \text{ or } \pi}$$

$$X(k) = j e^{-j(N-1)\pi k/N} \times \left\{ 2 \sum_{n=1}^{N/2} x\left(\frac{N}{2}-n\right) \sin\left(\frac{\pi k(2n-1)}{N}\right) \right\}$$

DFT symmetry Relations

Finite length complex sequence.

$x(n) \longrightarrow$ N-point complex seq. $0 \le n \le N-1$

$x(n) = x_{re}(n) + j\, x_{im}(n)$

$x(n) \xleftrightarrow{DFT} X(k) = X_{re}(k) + j\, X_{im}(k)$. $\quad 0 \le k \le N-1$

$X_{re}(k) = \frac{1}{2}\left(X(k) + X^*(k)\right)$

real $\{X(k)\}$ = Re $\{X(k)\}$.

$X_{im}(k) = \frac{1}{2}\left(X(k) - X^*(k)\right)$

im $\{X(k)\}$.

Properties

$x^*(n) \xleftrightarrow{DFT.} X^*(\langle -k \rangle_N)$.

$x^*(\langle -n \rangle_N) \longleftrightarrow X^*(k)$.

$x_{re}(n) \xleftrightarrow{DFT} X_{cs}(k) = \frac{1}{2}\left\{X(k) + X^*(\langle -k \rangle_N)\right\}$

$j\, x_{im}(n) \longleftrightarrow X_{ca}(k) = \frac{1}{2}\left\{X(k) - X^*(\langle -k \rangle_N)\right\}$

$x_{cs}(n) \xleftrightarrow{DFT} X_{re}(k)$

$x_{ca}(n) \longleftrightarrow j\, X_{im}(k)$.

Proof of $x^*(n) \xleftrightarrow{DFT} X^*(\langle -k \rangle_N)$.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j 2\pi n k/N}$$

$0 \le k \le N-1$

$$X^*(k) = \sum_{n=0}^{N-1} x^*(n)\, e^{j 2\pi n k/N}.$$

$\Rightarrow X^*(\langle -k \rangle_N) = X^*(N-k)$

$$\boxed{x(\langle -n \rangle_N) = x(N-n)}$$

$$\Rightarrow X^*(\langle -k \rangle_N) = X^*(N-k) = \sum_{n=0}^{N-1} x^*(n)\, e^{j 2\pi n (N-k)/N}.$$

$$= \sum_{n=0}^{N-1} x^*(n)\, \underbrace{e^{j2\pi n}}_{=1} e^{-j 2\pi k n/N}.$$

$$\Rightarrow X^*(\langle -k \rangle_N) = \sum_{n=0}^{N-1} x^*(n)\, e^{-j 2\pi k n/N}.$$

$= \text{DFT}\{x^*(n)\}$.

$\Rightarrow x^*(n) \longleftrightarrow X^*(\langle -k \rangle_N)$.

Finite length real sequences.

$x(n) \longrightarrow$ real seq. $0 \le n \le N-1$

$x(n) \longleftrightarrow X(k) = X_{re}(k) + j\, X_{im}(k)$.

$x(n) = x_{ev}(n) + x_{od}(n)$.

$x_{ev}(n) \longleftrightarrow X_{re}(k)$.

$x_{od}(n) \longleftrightarrow j\, X_{im}(k)$.

$X(k) = X^*(\langle -k \rangle_N)$.

$X_{re}(k) = X_{re}(\langle -k \rangle_N)$

$X_{im}(k) = -X_{im}(\langle -k \rangle_N)$

$|X(k)| = |X(\langle -k \rangle_N)|$

$\arg X(k) = -\arg X(\langle -k \rangle_N)$.

symmetry properties, holds only when $x(n)$ is real finite seq.