Lecture 30

Friday, 29 October 2021 5:00 PM

→ Poles & zeros of APFs occur in reciprocal pairs.

→ All zeros are outside the unit circle
↳ maximum phase functions

→ All zeros are inside the unit circle
↳ minimum phase function

→ Zeros are at both inside outside of the unit circle
↳ mixed phase transfer function.

Comb Filter

→ Simple filters will be used to design the comb filter.

Comb filter → multiple pass bands & multiple stop bands.

→ If $H(z)$ → mag. response monotonically increasing or decreasing in the interval $[0, \pi]$.

then the TF of the comb filter.

$$G(z) = H(z^L)$$

$H(z)$ → simple digital filter.

Note

If the mag. response $|H(e^{j\omega})|$ has a peak or notch at ω_0, then $|G(e^{j\omega})|$ will exhibit L peaks or notches at

$$\frac{\omega_0 + 2\pi k}{L}, \quad 0 \le k \le L-1$$

over the interval $0 \le \omega < 2\pi$.

Design comb filter using prototype LPF.

$$H(z) = \frac{1}{2}(1 + z^{-1}) \rightarrow \text{LPF}.$$

Peak = $\omega_0 = 0$
Notch → $\omega_0 = \pi$

$$G(z) = H(z^L) = \frac{1}{2}(1 + z^{-L})$$

$$G(e^{j\omega}) = e^{-j\omega L/2} \cos\frac{\omega L}{2}$$

$|G(e^{j\omega})| = 1$, where $\left|\cos\frac{\omega L}{2}\right| = 1$

$$\Rightarrow \frac{\omega L}{2} = k\pi$$

$$\Rightarrow \omega = \frac{2\pi k}{L}$$

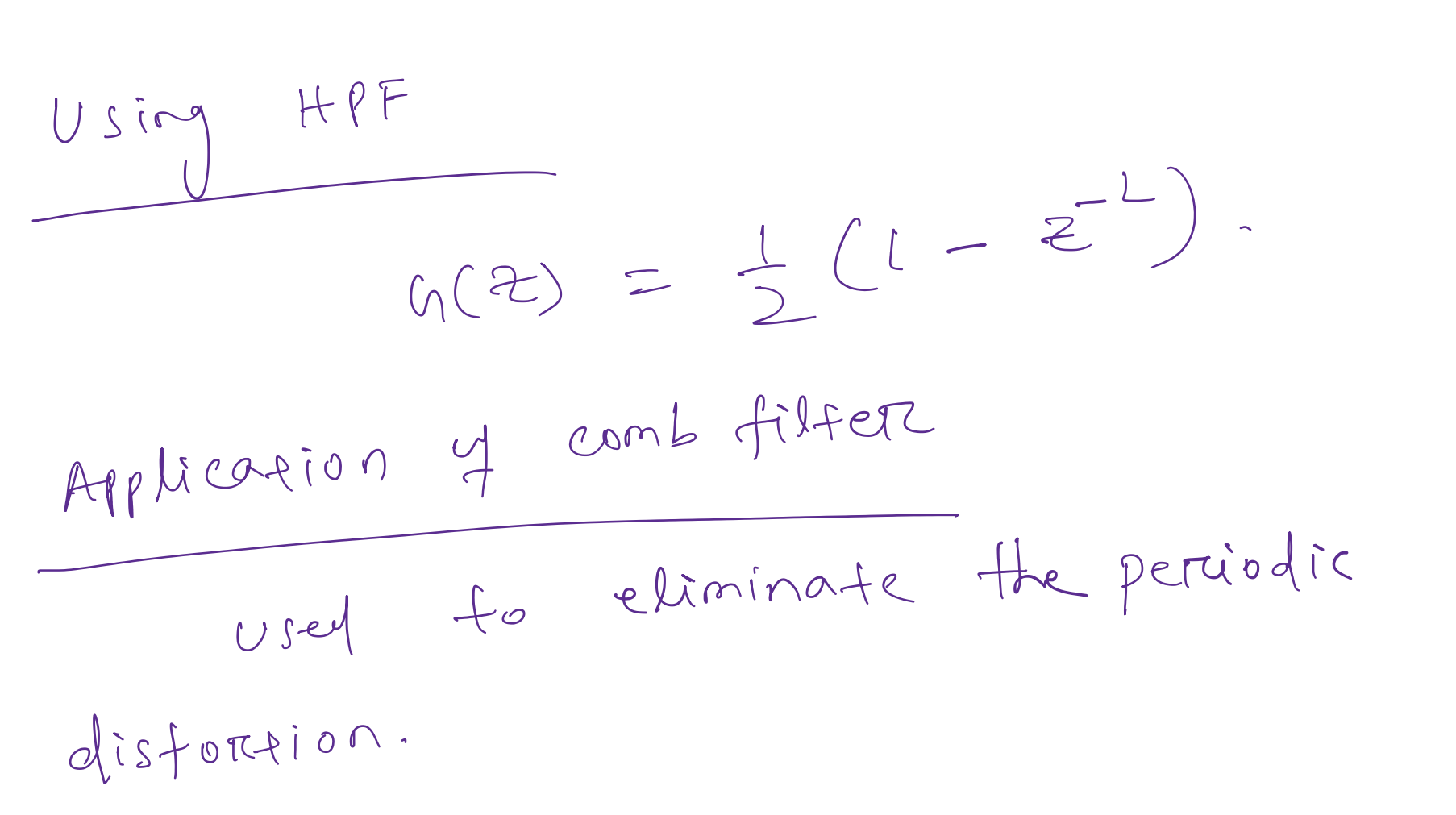
$k \rightarrow 0$ to $L-1$.

If $L = 5$

peaks → $0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}$, and $\frac{8\pi}{5}$

$0 \le \omega < 2\pi$.

Notches → $\frac{\pi}{5}, \frac{3\pi}{5}, \frac{5\pi}{5}, \frac{7\pi}{5}$ and $9\pi/5$.



Using HPF

$$G(z) = \frac{1}{2}(1 - z^{-L}).$$

Application of comb filter

used to eliminate the periodic distortion.

Zero-phase Filters.

→ The frequency response should be real.

Ex $h(n) = \{\alpha, \beta, \alpha\}$ (↑ $n=0$ at β)

$$H(e^{j\omega}) = \beta + 2\alpha\cos\omega.$$

$\phi = 0$ or ↳ multiple of π.

→ Zero phase filters are non-causal.

→ It can be made causal by multiplying z^{-n}.

If $H(z) = \alpha z + \beta + \alpha z^{-1}$

$$z^{-1}H(z) = z^{-1}(\alpha z + \beta + \alpha z^{-1})$$
$$= \alpha + \beta z^{-1} + \alpha z^{-2}.$$

Linear phase Filter.

$$y(n) = x(n-d)$$
$$Y(e^{j\omega}) = e^{-j\omega d} X(e^{j\omega}).$$
$$e^{j\omega d} \cdot Y(e^{j\omega}) = X(e^{j\omega})$$

→ Linear phase filter can only be designed using FIR filter

→ Approximate linear phase response
↳ can be obtained using IIR.

→ Exact linear phase → FIR.

→ To achieve linear phase, impulse response $h(n)$ must be symmetrical or antisymmetrical.

$$h(n) = \pm h(N-n)$$

(+ → symmetry, − → anti-symmetry.)

$h(n) \rightarrow 0 \le n \le N$

Length of filter = $N+1$.

$h(n)$ →	symmetry	anti symmetry.
length →	odd	even.

↓ 4-types of FIR filters with linear phase.

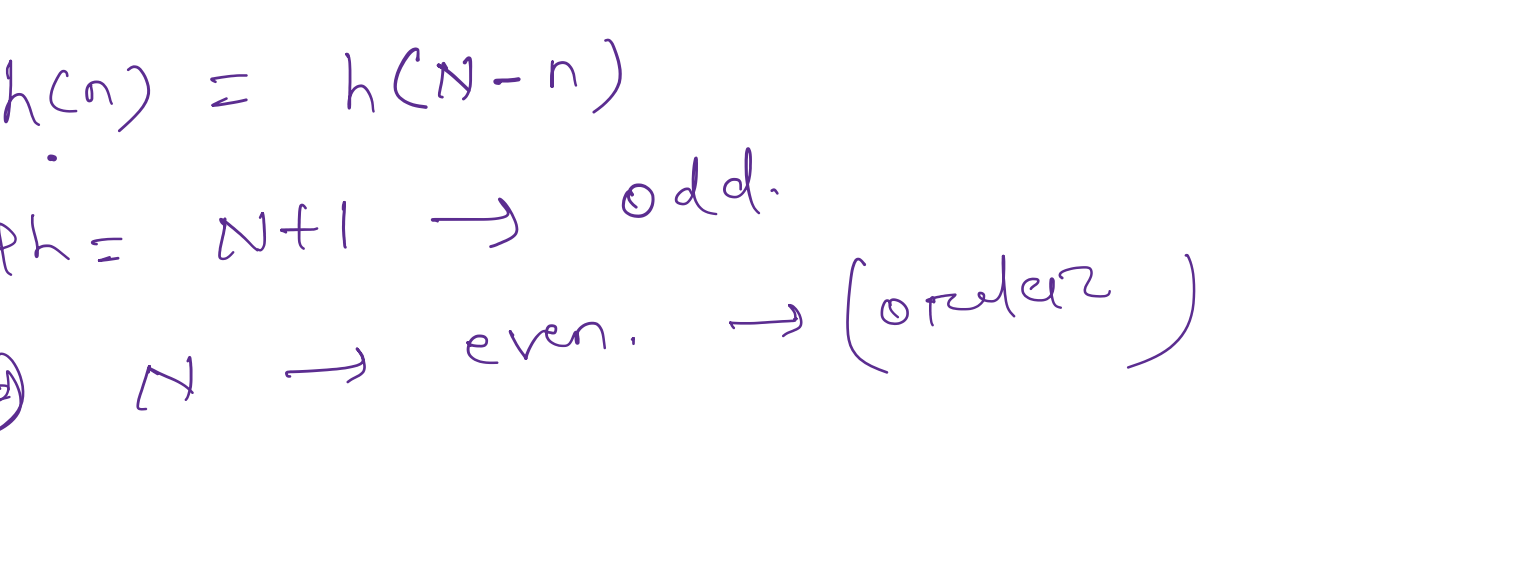
Type-I FIR linear phase filter (symmetric, odd length)

$$h(n) = h(N-n)$$

length $= N+1$ → odd.

$\Rightarrow N$ → even. → (order)

Let $N = 8$



$h(n) = h_n$

symmetric around $\frac{N}{2}$ th sample.

$$H(z) = h_0 + h_0 z^{-8} + h_1 z^{-1} + h_1 z^{-7} + h_2 z^{-2} + h_2 z^{-6} + h_3 z^{-3} + h_3 z^{-5} + h_4 z^{-4}$$

$$= h_0(1 + z^{-8}) + h_1(z^{-1} + z^{-7}) + h_2(z^{-2} + z^{-6}) + h_3(z^{-3} + z^{-5}) + h_4 z^{-4}.$$