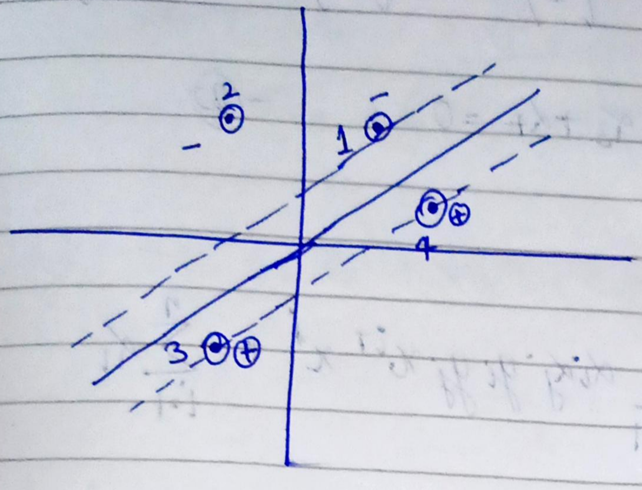
Question :



$x^1\begin{pmatrix}1\\2\end{pmatrix}$

$x^2\begin{pmatrix}-1\\2\end{pmatrix}$

$x^3\begin{pmatrix}-1\\-2\end{pmatrix}$

$x^4\begin{pmatrix}3\\1\end{pmatrix}$

SVM lagrangian:

$$\mathcal{L}(w,b,\alpha) = \frac{1}{2}\|w\|^2 - \sum_{i=1}^{n}\alpha_i\left(y_i\left(x^{iT}w+b\right)-1\right)$$

$$\mathcal{L}(w,b,\alpha) = \frac{1}{2}w^t w - \sum_{i=1}^{n}\alpha_i y_i x^{iT} w - b\left(\sum_{i=1}^{n}\alpha_i y_i\right) + \sum_{i=1}^{n}\alpha_i$$

$$\mathcal{L}(w,b,\alpha) = \frac{1}{2}\begin{bmatrix}w_1 & w_2\end{bmatrix}\begin{bmatrix}w_1\\w_2\end{bmatrix} - (-\alpha_1)\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}w_1\\w_2\end{bmatrix} - (-\alpha_2)\begin{bmatrix}-1 & 2\end{bmatrix}\begin{bmatrix}w_1\\w_2\end{bmatrix}$$
$$- (+\alpha_3)\begin{bmatrix}-1 & -2\end{bmatrix}\begin{bmatrix}w_1\\w_2\end{bmatrix} - (+\alpha_4)\begin{bmatrix}3 & 1\end{bmatrix}\begin{bmatrix}w_1\\w_2\end{bmatrix}$$
$$+ b\left(-\alpha_1-\alpha_2+\alpha_3+\alpha_4\right) + \alpha_1+\alpha_2+\alpha_3+\alpha_4$$

$$\mathcal{L}(w,b,\alpha) = \frac{1}{2}\left(w_1^2+w_2^2\right) + \alpha_1\left(w_1+2w_2\right) + \alpha_2\left(-w_1+2w_2\right)$$
$$- \alpha_3\left(-w_1-2w_2\right) - \alpha_4\left(3w_1+w_2\right)$$
$$+ b\left(-\alpha_1-\alpha_2+\alpha_3+\alpha_4\right) + \alpha_1+\alpha_2+\alpha_3+\alpha_4$$

$$w^* = \sum_{i=1}^{3}\alpha_i y_i x^i$$

$$w^* = -\alpha_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \alpha_2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \alpha_3 \begin{bmatrix} -1 \\ -2 \end{bmatrix} + \alpha_4 \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad -(1)$$

$$-\alpha_1 - \alpha_2 + \alpha_3 + \alpha_4 = 0 \quad -(2)$$

dual optimization problem:

$$\max \; -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j x^{iT} x^j + \sum_{i=1}^{n} \alpha_i$$

Training data points

(1 2)	$y_1 = -1$
(-1 2)	$y_2 = -1$
(-1 -2)	$y_3 = +1$
(3, 1)	$y_4 = +1$

$x^{1T}x^1 = 5$ $\quad x^{1T}x^2 = 3$ $\quad x^{1T}x^3 = -5$ $\quad x^{1T}x^4 = 5$

$x^{2T}x^2 = 5$ $\quad x^{2T}x^3 = -3$ $\quad x^{2T}x^4 = -1$

$x^{3T}x^3 = 5$ $\quad x^{3T}x^4 = -5$

$x^{4T}x^4 = 10$

Expanding $\mathcal{L}(w^*, \alpha, b^*)$

(i, j)

$(1,1) + (1,2) + (1,3) + (1,4) + (2,1) + (2,2) + (2,3) + (2,4)$
$+ (3,1) + (3,2) + (3,3) + (3,4) + (4,1) + (4,2) + (4,3) + (4,4)$

$$-\frac{1}{2}\Big[\alpha_1^2(-1)^2(5) + \big(\alpha_1\alpha_2(3) + (-\alpha_1\alpha_3)(-5) + (-\alpha_1\alpha_4)5$$
$$+ (-\alpha_2\alpha_3)(-3) + (-\alpha_2\alpha_4)(-1) + (+\alpha_3\alpha_4)(-5)\big) \times 2$$
$$+ \alpha_2^2(5) + \alpha_3^2(5) + \alpha_4^2 10\Big]$$

~~$-\frac{1}{2}\left[5\alpha_1^2 + 5\alpha_2^2 + 5\alpha_3^3 + 10\alpha_4^2 + 6\alpha_1\alpha_2 + 10\alpha_1\alpha_3 - 10\alpha_1\alpha_4 + 6\alpha_2\alpha_3 + 2\alpha_2\alpha_4 +\right.$~~

$$-\frac{1}{2}\left[5\alpha_1^2 + 5\alpha_2^2 + 5\alpha_3^2 + 10\alpha_4^2 + 6\alpha_1\alpha_2 + 10\alpha_1\alpha_3 - 10\alpha_1\alpha_4 + 6\alpha_2\alpha_3 + 2\alpha_2\alpha_4 - 10\alpha_3\alpha_4\right]$$

dual optimization :

$$\max \; -\frac{1}{2}\left[5\alpha_1^2 + 5\alpha_2^2 + 5\alpha_3^2 + 10\alpha_4^2 + 6\alpha_1\alpha_2 + 10\alpha_1\alpha_3 - 10\alpha_1\alpha_4 + 6\alpha_2\alpha_3 + 2\alpha_2\alpha_4 - 10\alpha_3\alpha_4\right]$$

$$-\alpha_1 - \alpha_2 + \alpha_3 + \alpha_4$$

Computing partial derivative :

$$\alpha_4 = \alpha_1 + \alpha_2 - \alpha_3$$

~~only~~ on substituting

$$\max \; -\frac{1}{2}\left[5\alpha_1^2 + 17\alpha_2^2 + 25\alpha_3^2 + 18\alpha_1\alpha_2 - 20\alpha_1\alpha_3 - 26\alpha_2\alpha_3\right]$$

$$+ 2(\alpha_1 + \alpha_2 - \alpha_3)$$

$$\frac{\partial L}{\partial \alpha_1} = -5\alpha_1 - 9\alpha_2 + 10\alpha_3 + 2 \quad -①$$

$$\frac{\partial L}{\partial \alpha_2} = -17\alpha_2 - 9\alpha_1 + 13\alpha_3 + 2 \quad -②$$

$$\frac{\partial L}{\partial \alpha_3} = -25\alpha_3 + 10\alpha_1 + 13\alpha_2 - 2 \quad -③$$

only solving these eqns ①, ② & ③

$$-5\alpha_1 - 9\alpha_2 + 10\alpha_3 + 2 = 0$$

$$-17\alpha_2 - 9\alpha_1 + 13\alpha_3 + 2 = 0$$

$$-25\alpha_3 + 10\alpha_1 + 13\alpha_2 - 2 = 0$$

$$\alpha_1 = \frac{3}{5}, \quad \alpha_2 = 0, \quad \alpha_3 = 0, \quad \alpha_4 = \frac{2}{5}$$

so, $$w = -\frac{3}{5}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \frac{2}{5}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$w = \begin{bmatrix} -\frac{3}{5} + \frac{6}{5} \\ -\frac{6}{5} + \frac{2}{5} \end{bmatrix} = \begin{bmatrix} 3/5 \\ -4/5 \end{bmatrix}$$

Margin width $= \frac{2}{\|w\|} = \frac{2}{1}$

bais, $b = 1 + x^{2T} w = 1 + \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 3/5 \\ -4/5 \end{bmatrix} = 1 + \left(\frac{-3}{5} + \left(\frac{-6}{5}\right)\right) = \frac{-4}{5}$