**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer: B = 0.26

Using python calculation, Probability that car will be ready within 1 hr = P

P= stats.norm.cdf(50,loc=45,scale=8)

P = 0.7340144709512995

So, Probability that car will not be ready within 1 hr = 1-P = 1- 0.7340144709512995 = 0.26

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:

1. Statement is true.

Using python, probability of employees with age 44 is P\_44=stats.norm.cdf(44,loc=38,scale=6)

P\_44 = .8413

So, probability of employees >44 age is 1 – P\_44 = 1 - .8413 or 84.13%

As data follows normal distribution, so

Probability of employees between age 38 and 44 = P\_44 – 0.5

= 0.8413 -0.5 = 0.3413 or 34.13%

1. Statement is true.

Probaility of employees with age 30 is P\_30 = stats.norm.cdf(30, loc=38,scale=6)

P\_30 = 0.09121

No. of employees with probability 0.09121 is = 0.09121\*400 = 36.48

36 Employees.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

X = N(µ1, σ1^2 ), and Y = N(µ2, σ2^2 ) are two independent random variables then

X + Y = N(µ1 + µ2, σ1^2 + σ2^2 ).

If Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z = N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore in the question

2X1= N(2 u,4 σ^2) and

X1+X2 = N(µ + µ, σ^2 + σ^2 ) = N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer: 48.5, 151.5

Z value by python formula is 2.57 using code stats.norm.ppf(0.995)

a and b are symmetric about the mean, so one z will have values 2.57 and -2.57

Z=(x-mean)/std. deviation

X = z\*std. deviation + mean

X = 2.57\*20+100 = 151.5

X = (-2.57)\*20+100 = 48.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

1. Total profit = P1 + P2

Mean = 45\*(profit 1+profit 2) = 45\*(5+7) =540 rupees

SD = SD (profit 1 + profit 2) = 45\*√(9+16)= 225 rupees.

Therefore, From the normal distribution plot rule, Approximately 95% of the data falls within two standard deviation of the mean.

μ ± 2σ = 540 + 2\*225 = 540 + 450 = 990 and 540 – 450 = 90

90 and 990

1. From the above normal distribution we can say that to find 5th percentile from the left side formula to be used = μ - 1.5σ

540 - (1.5\*225) = 202.5 rupees.

1. Division 2 has larger probability of making a loss.

P1=stats.norm.cdf(0,loc=5,scale=3) = 0.0477

P2=stats.norm.cdf(0,loc=7,scale=4) = 0.0400