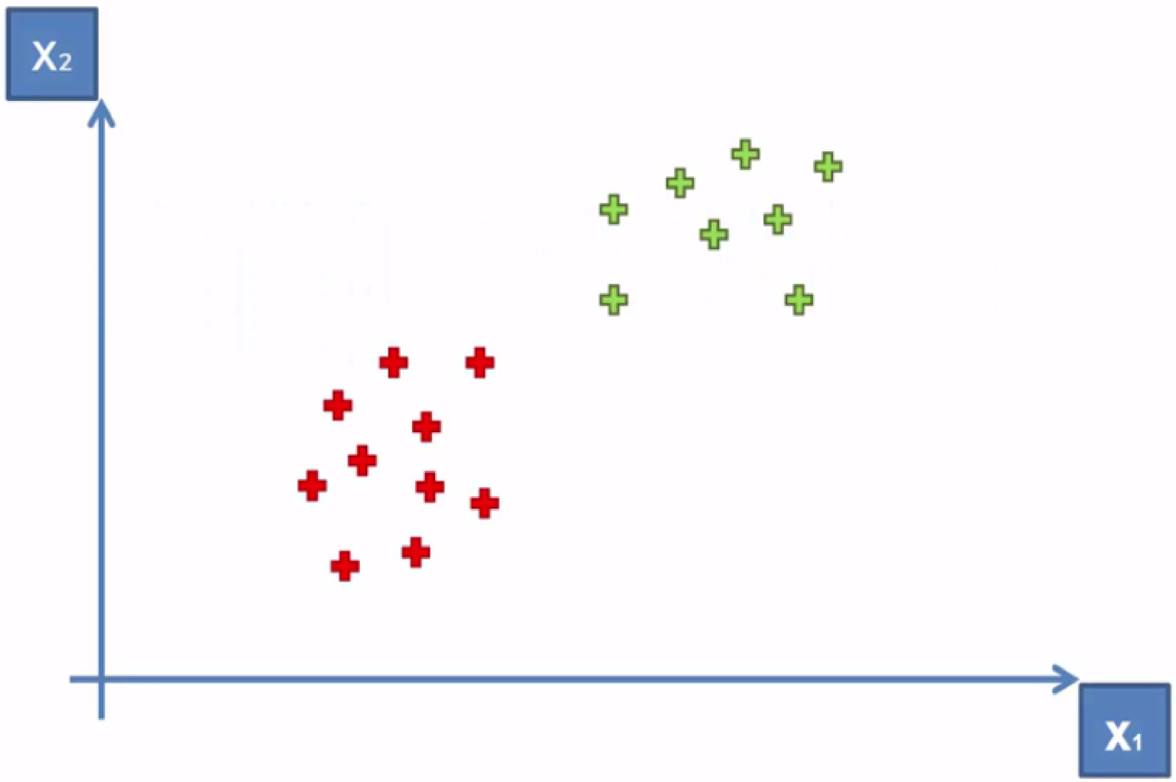
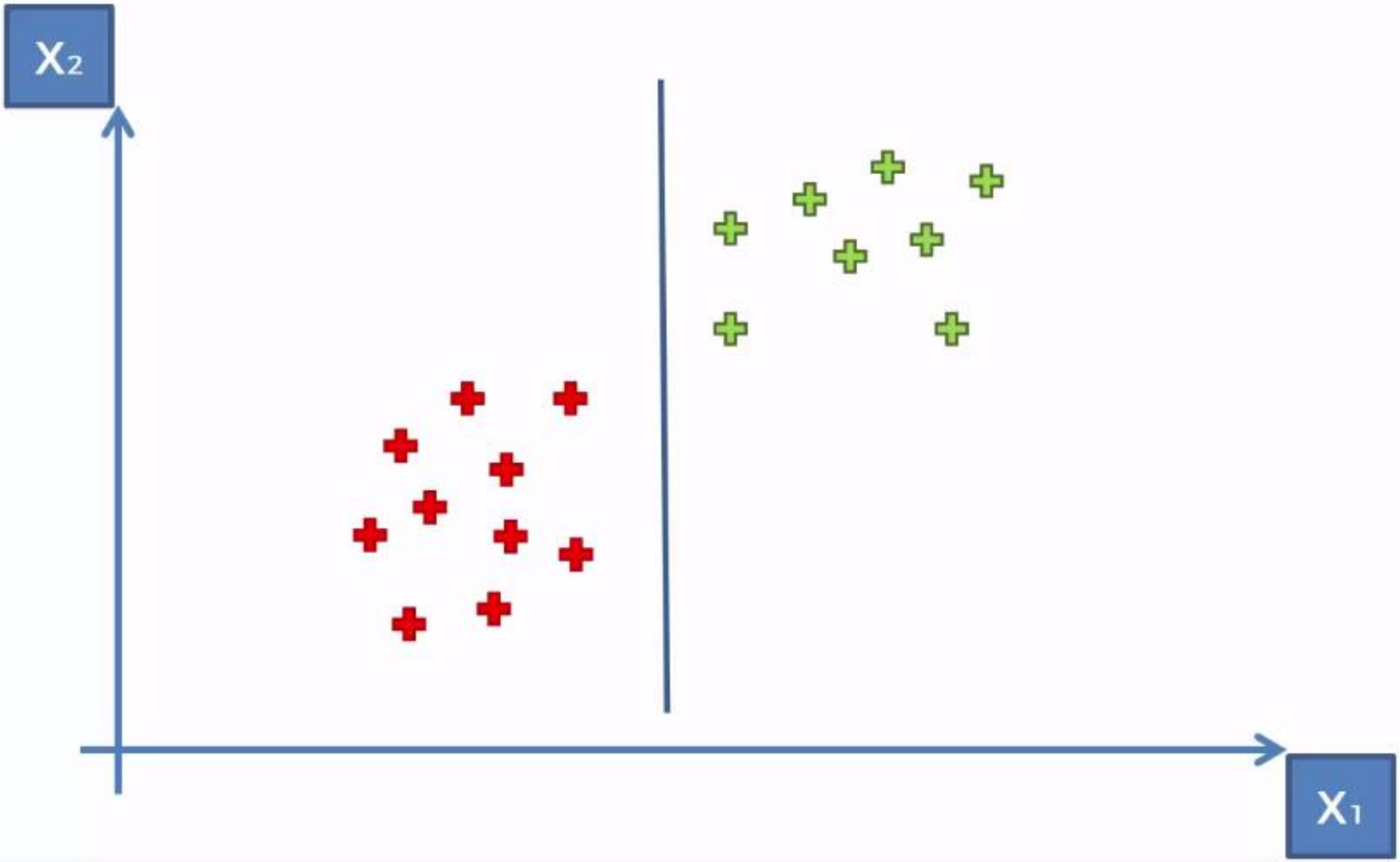
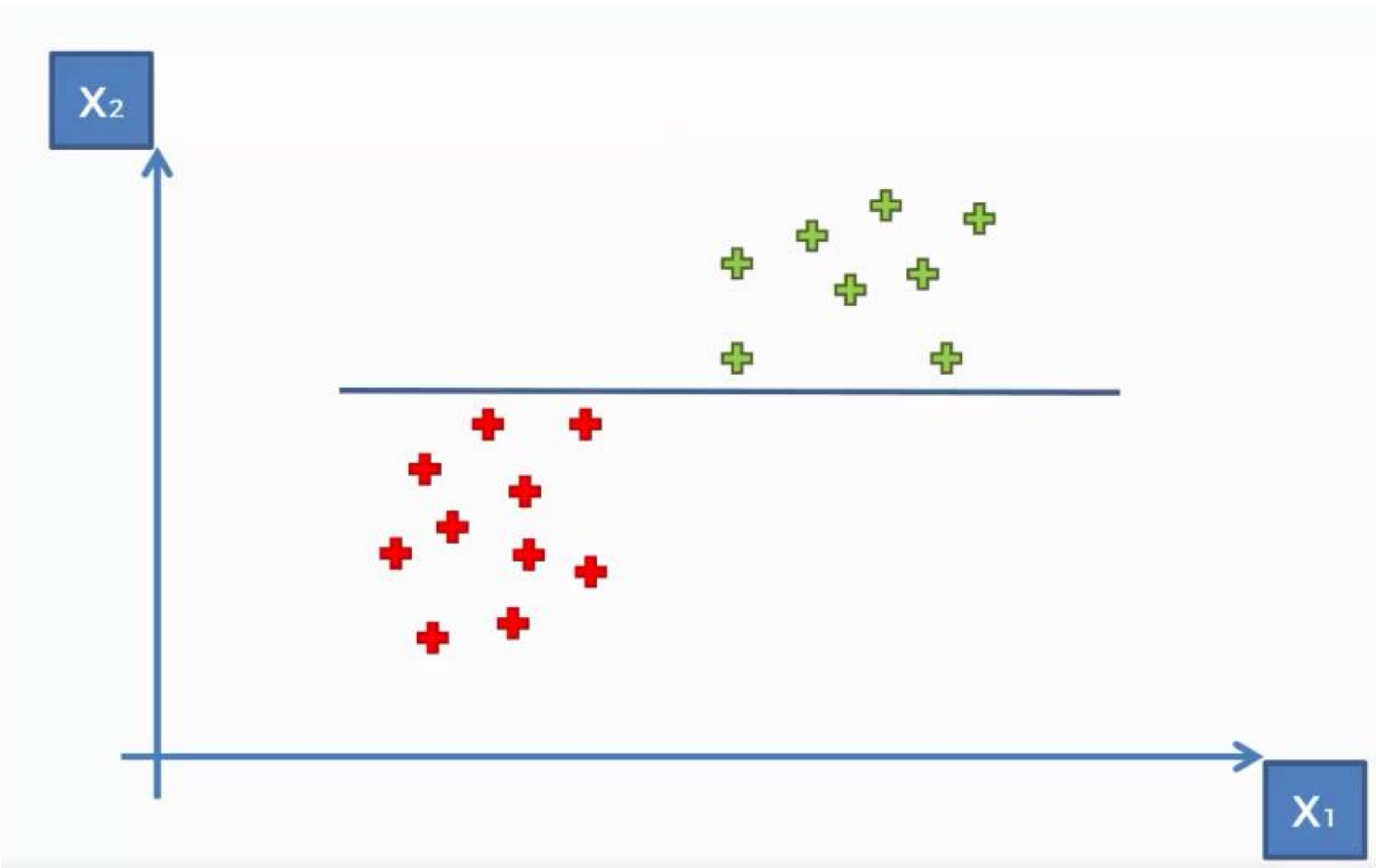
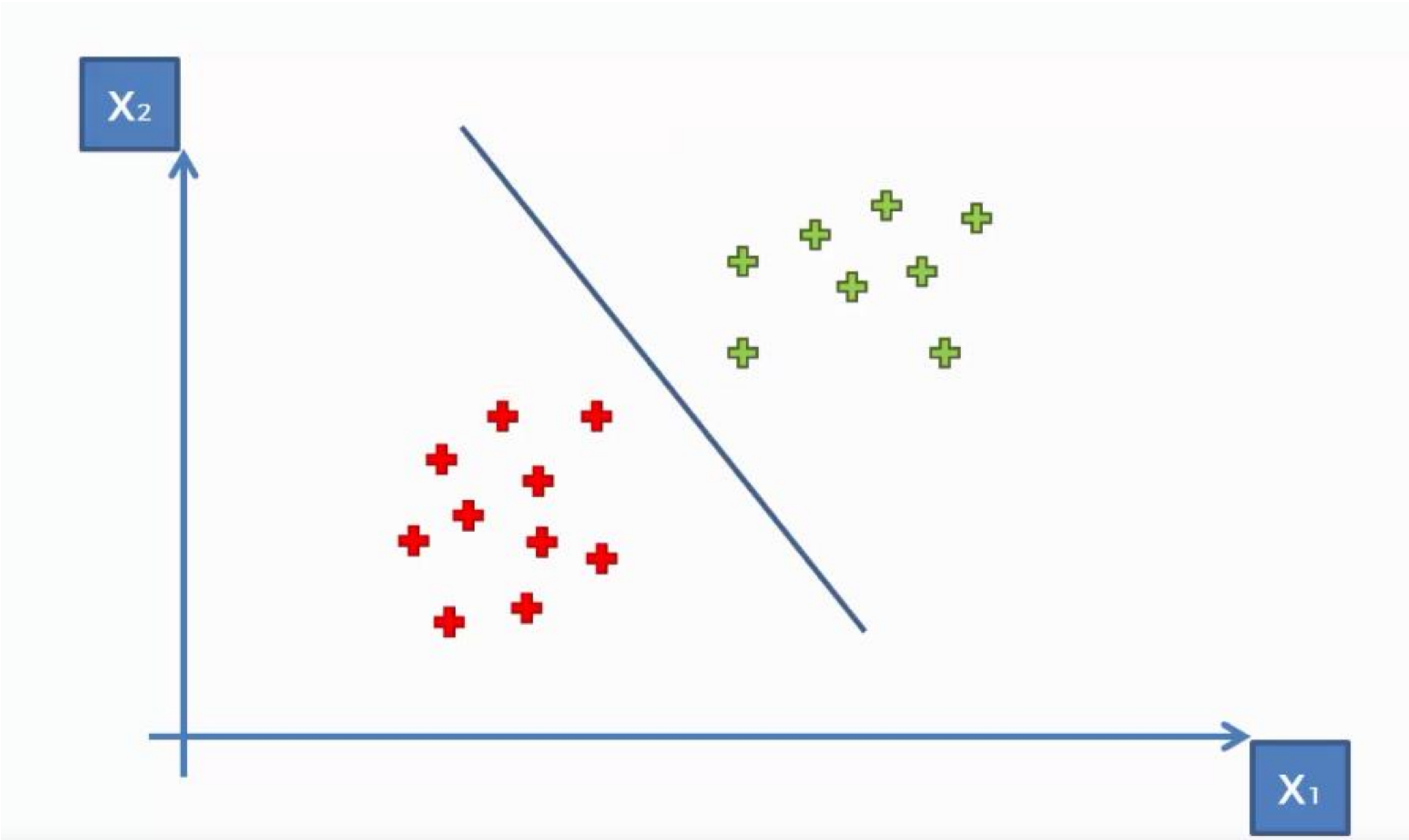
SVM Intuition

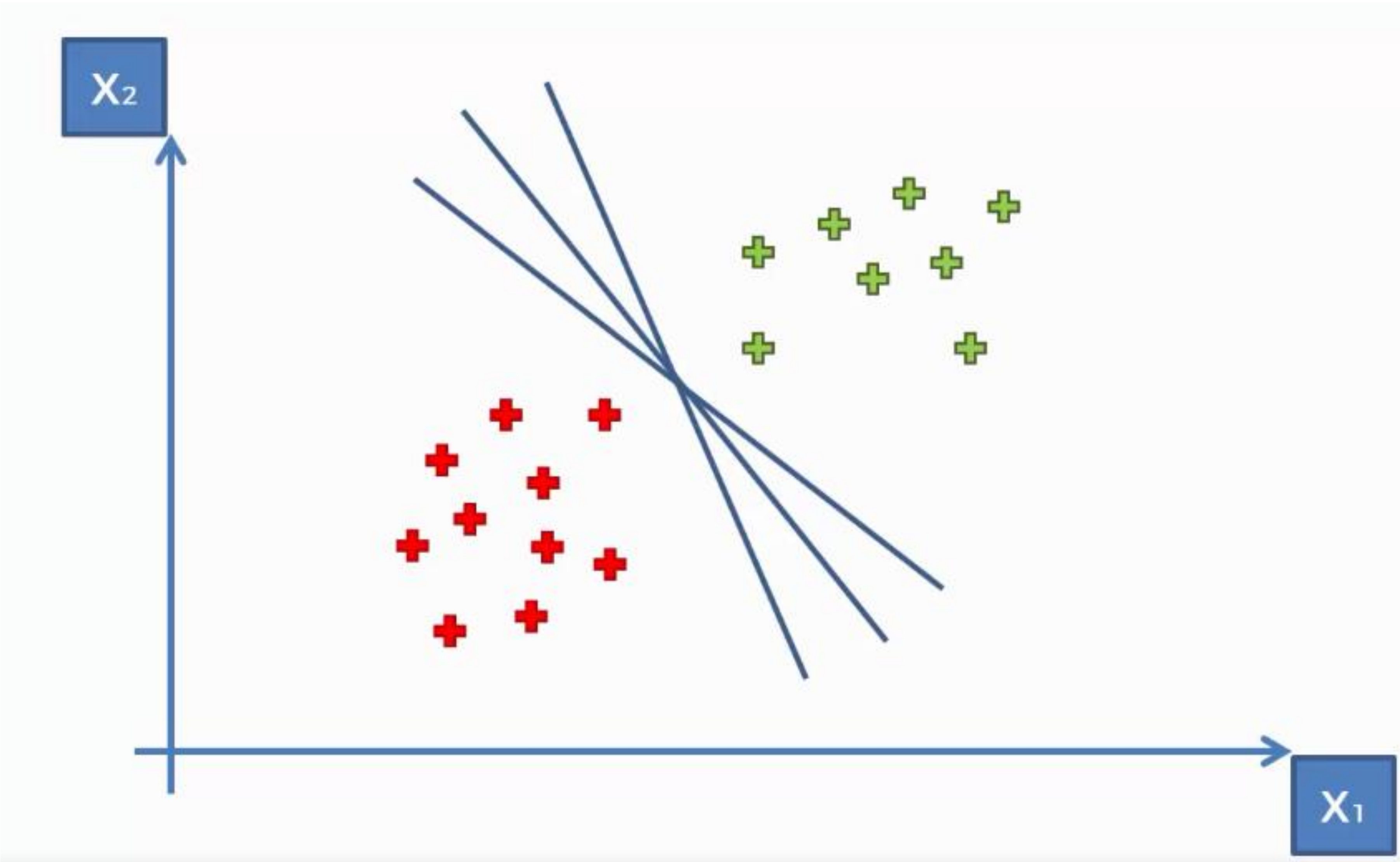
How to separate these points ?

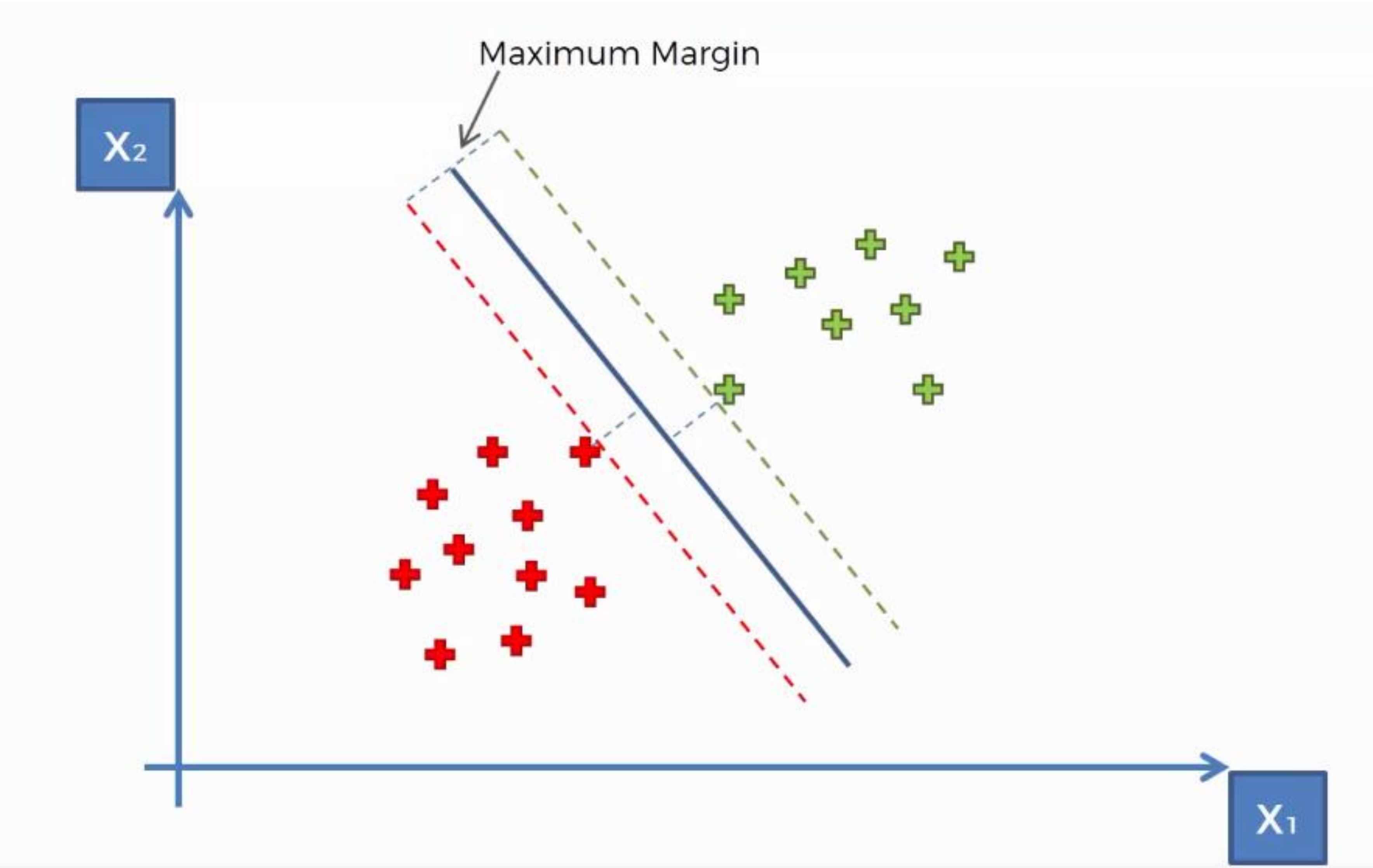


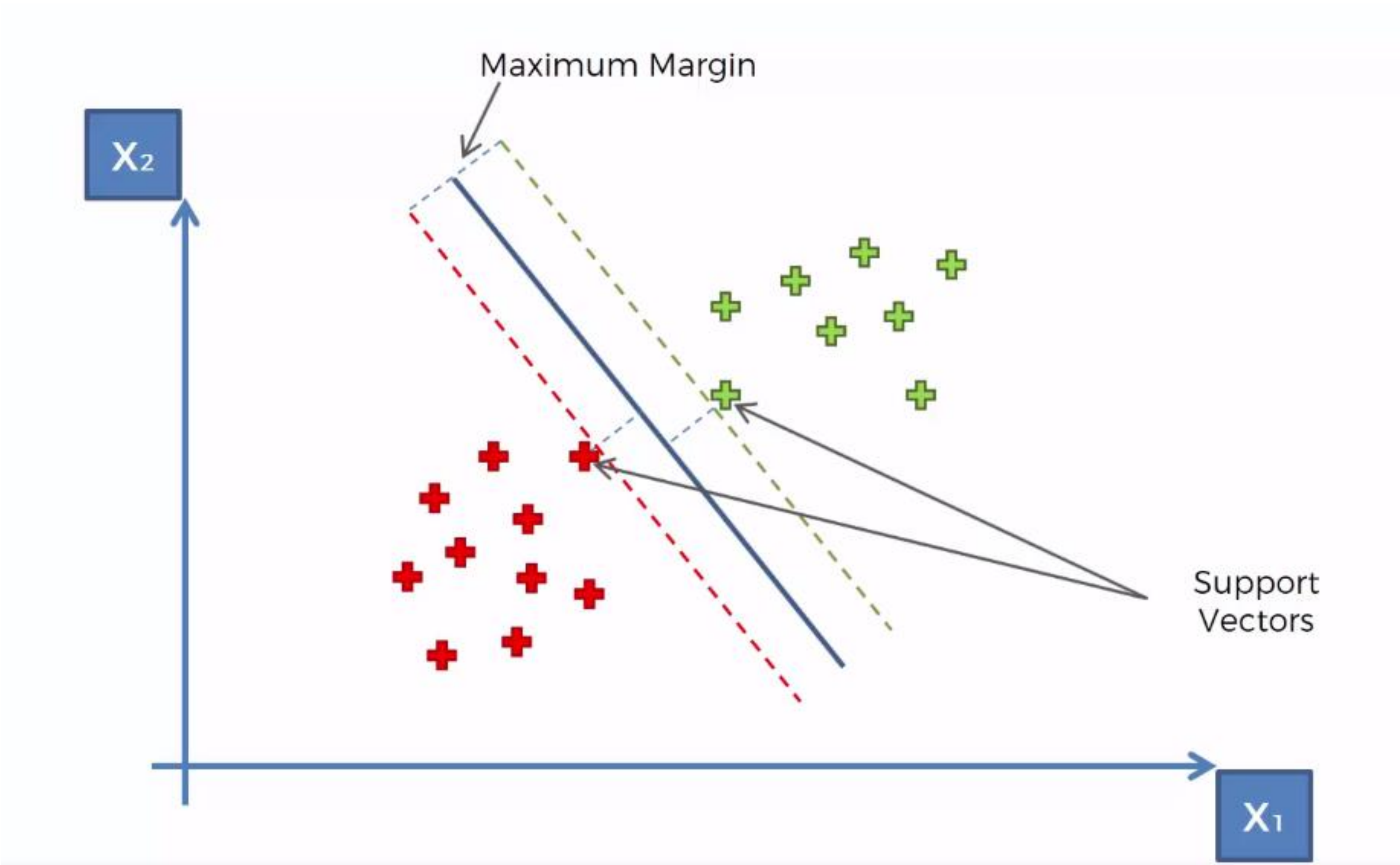

X2
X1

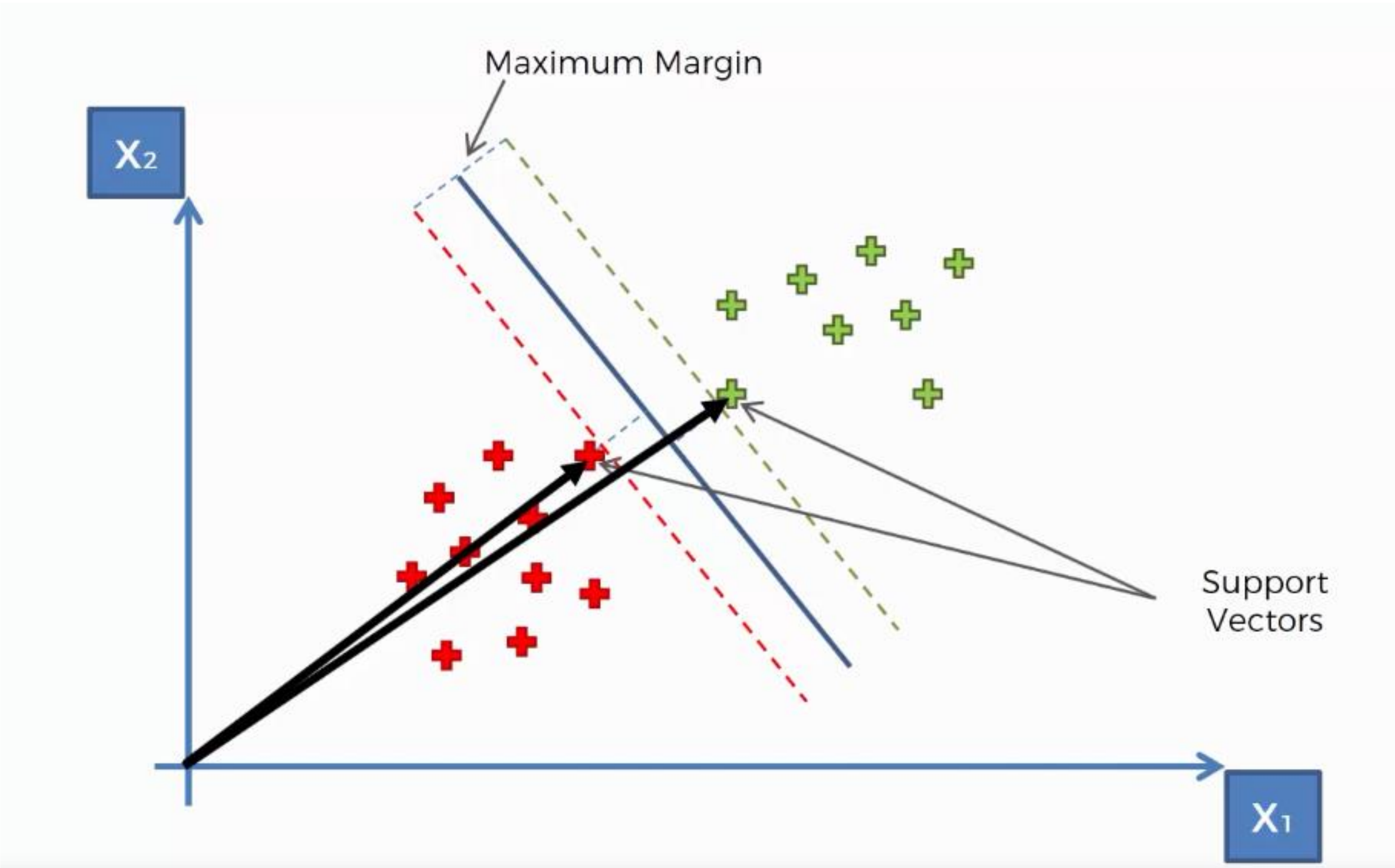

X2
X1

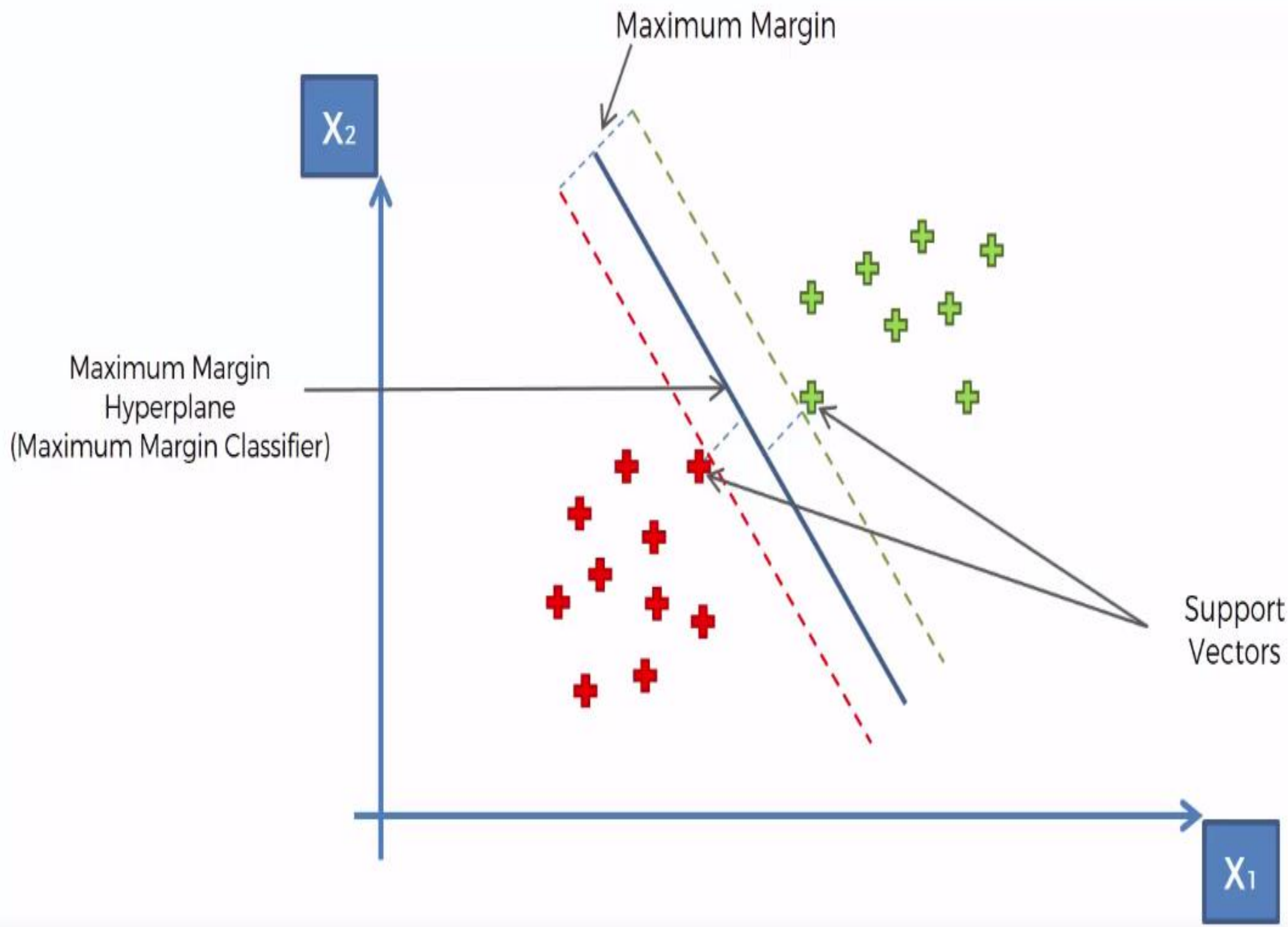

X2
X1


X2
X1

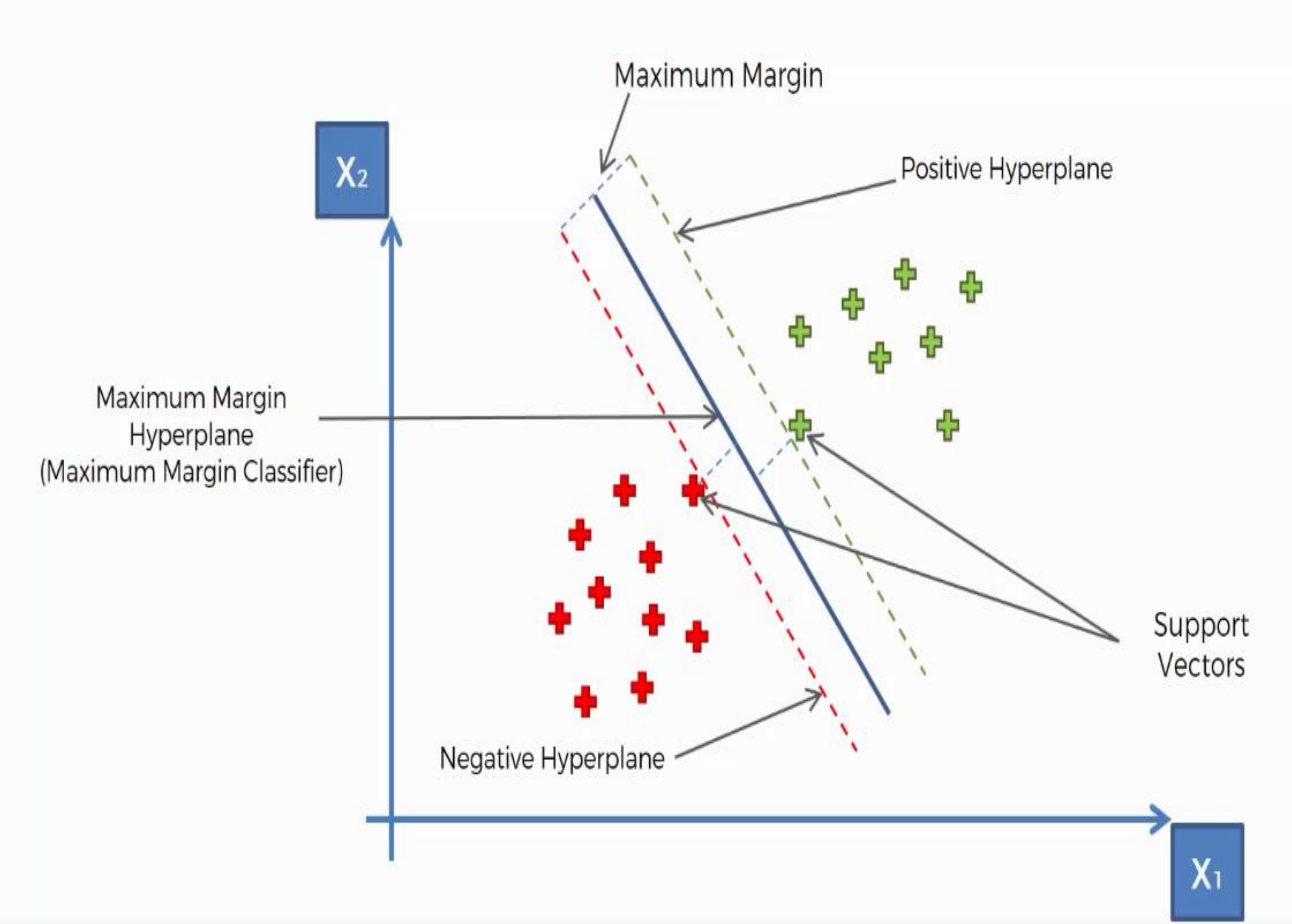

Maximum Margin
X2
X1


Maximum Margin
X2
Support Vectors
X1


Maximum Margin
X2
X1
Support
Vectors



Maximum Margin
X_2
Maximum Margin
Hyperplane
(Maximum Margin Classifier)
Support
Vectors
X_1


Maximum Margin
Positive Hyperplane
X_2
Maximum Margin
Hyperplane
(Maximum Margin Classifier)
Support
Vectors
Negative Hyperplane
X_1

What’s So Special About SVMs?

What's So Special About SVMs?

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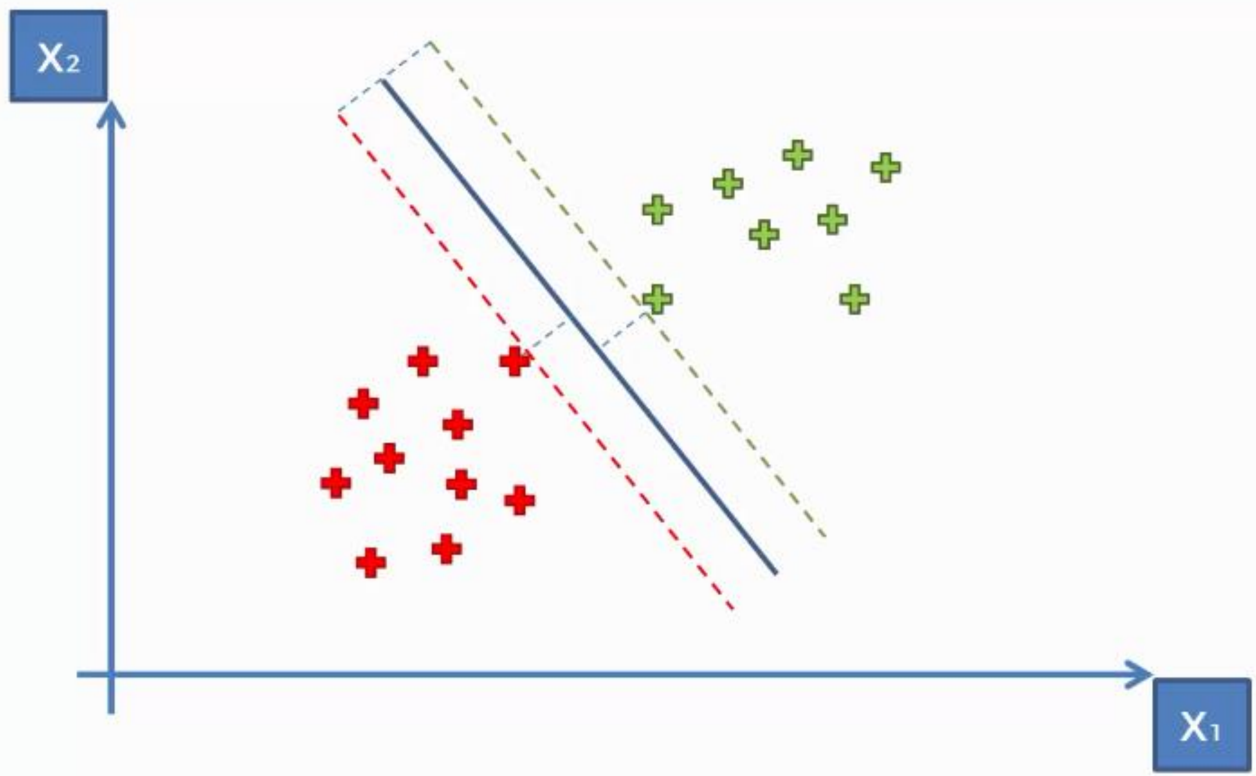
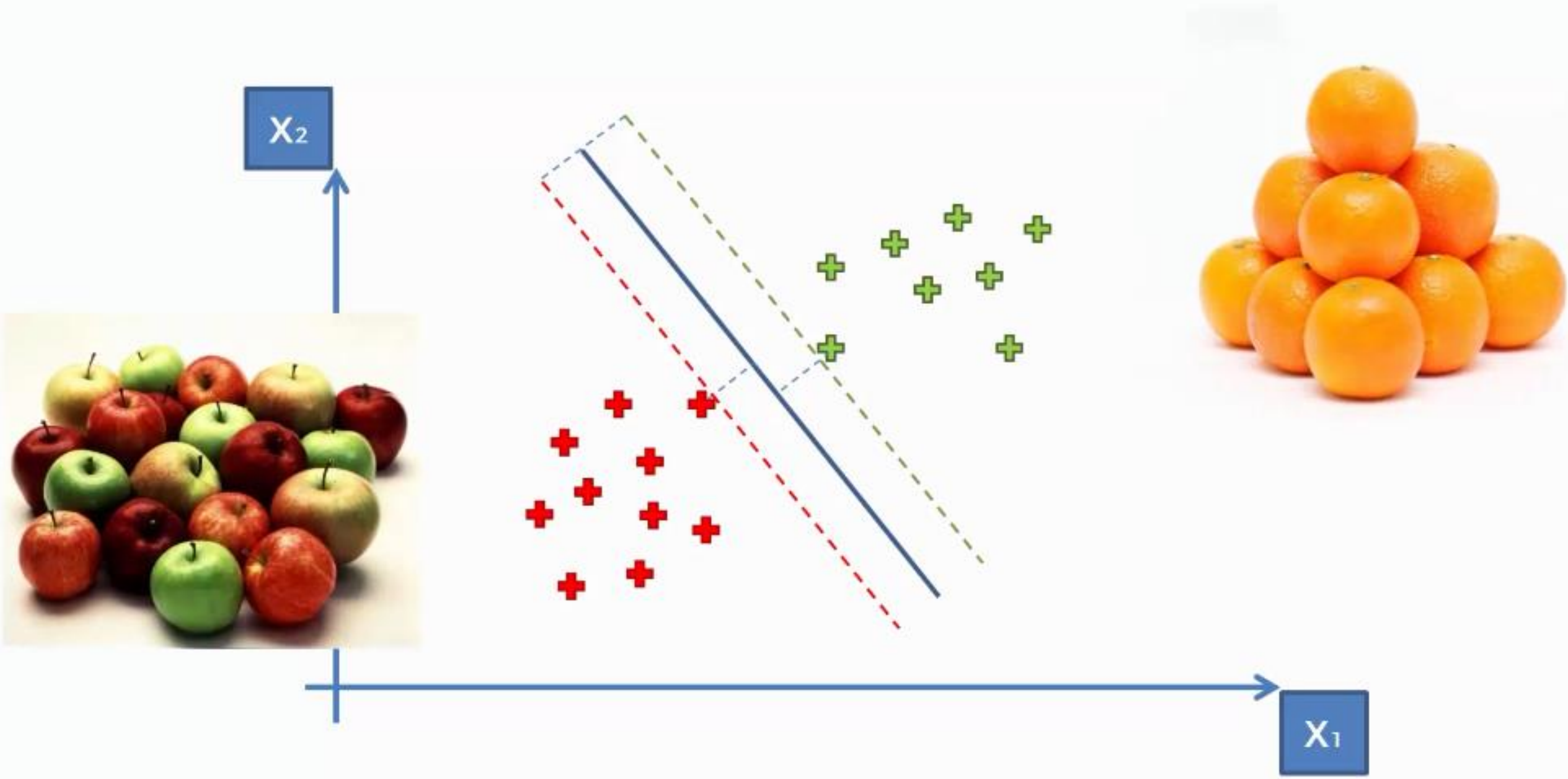
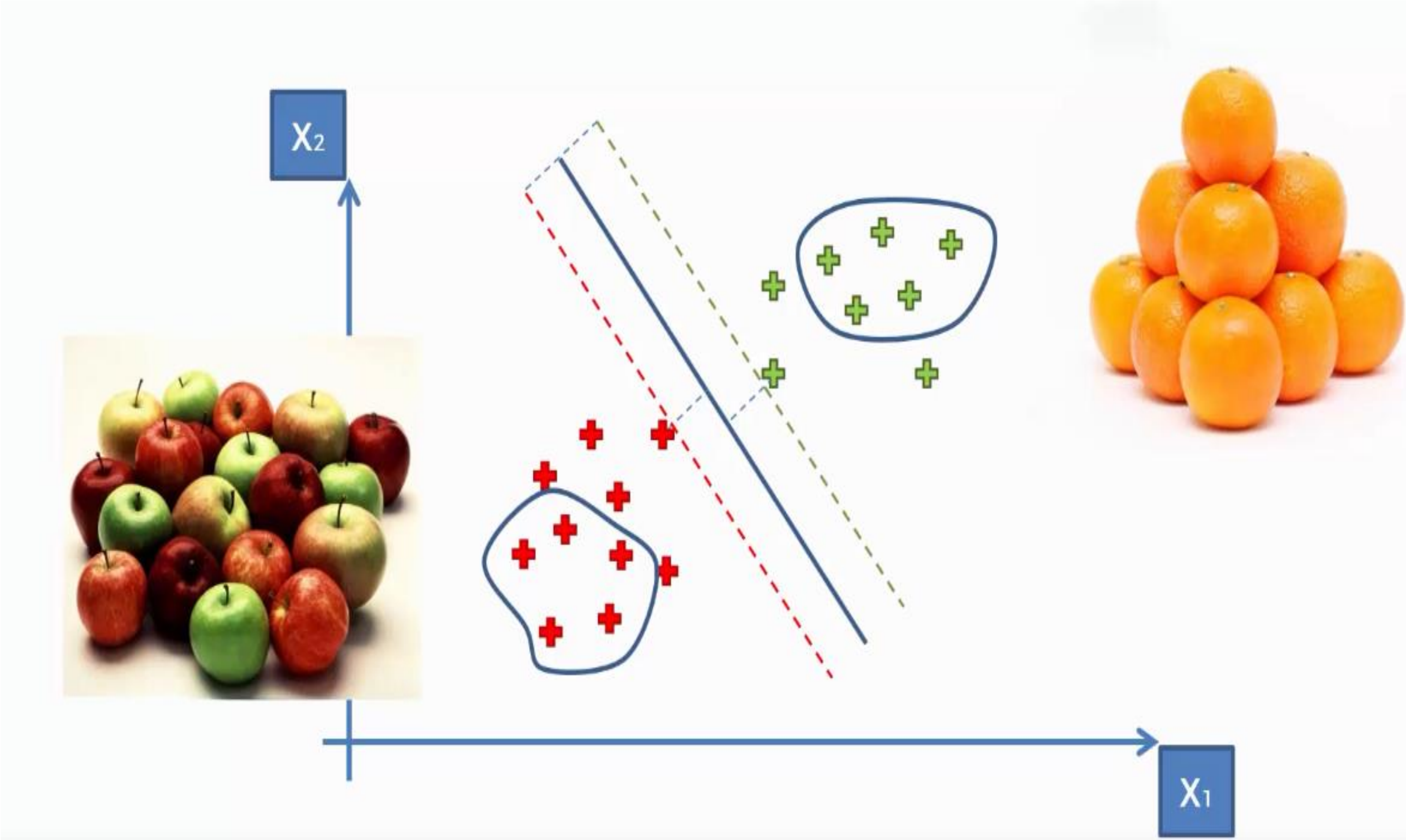
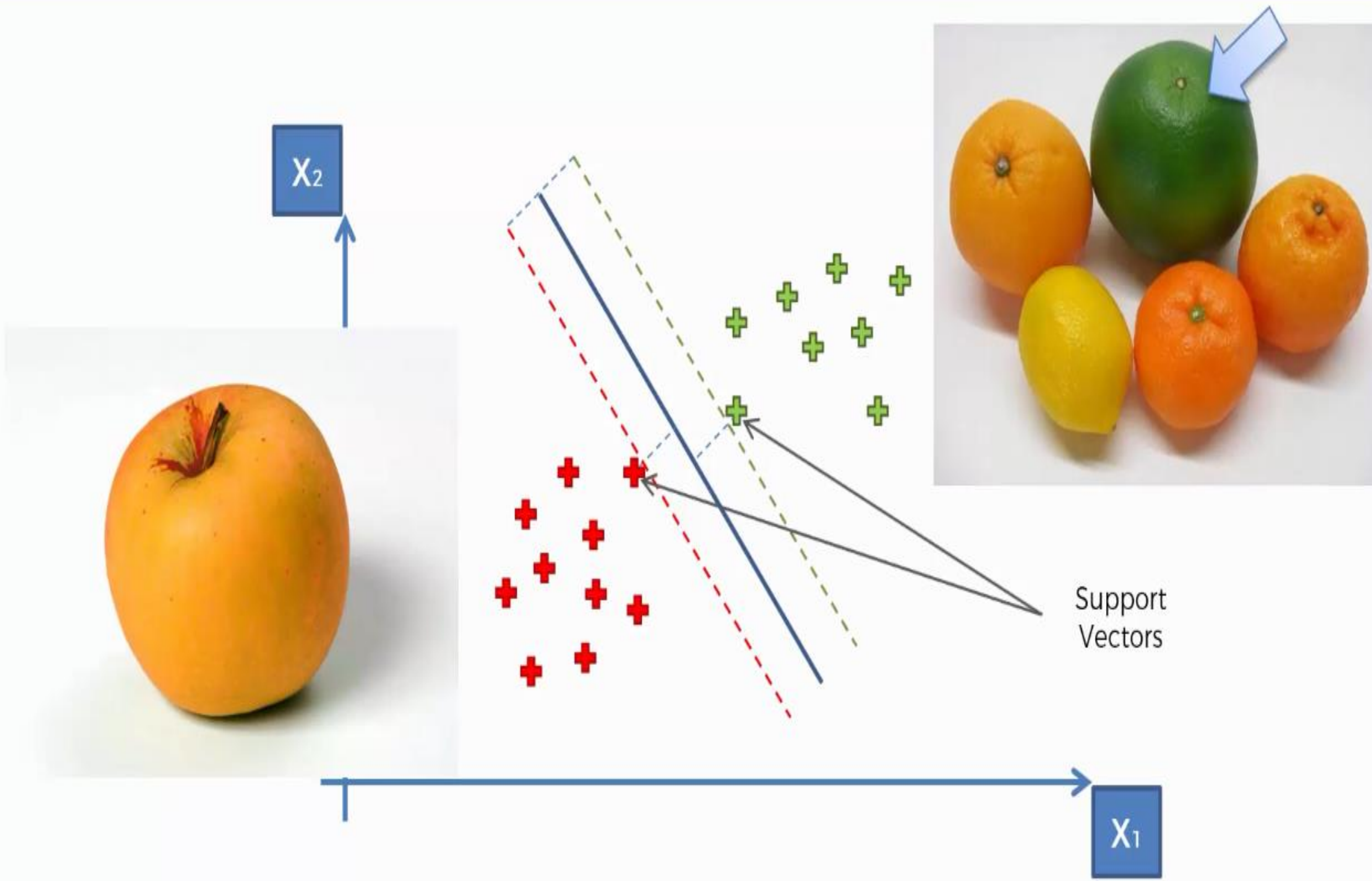


What's So Special About SVMs?



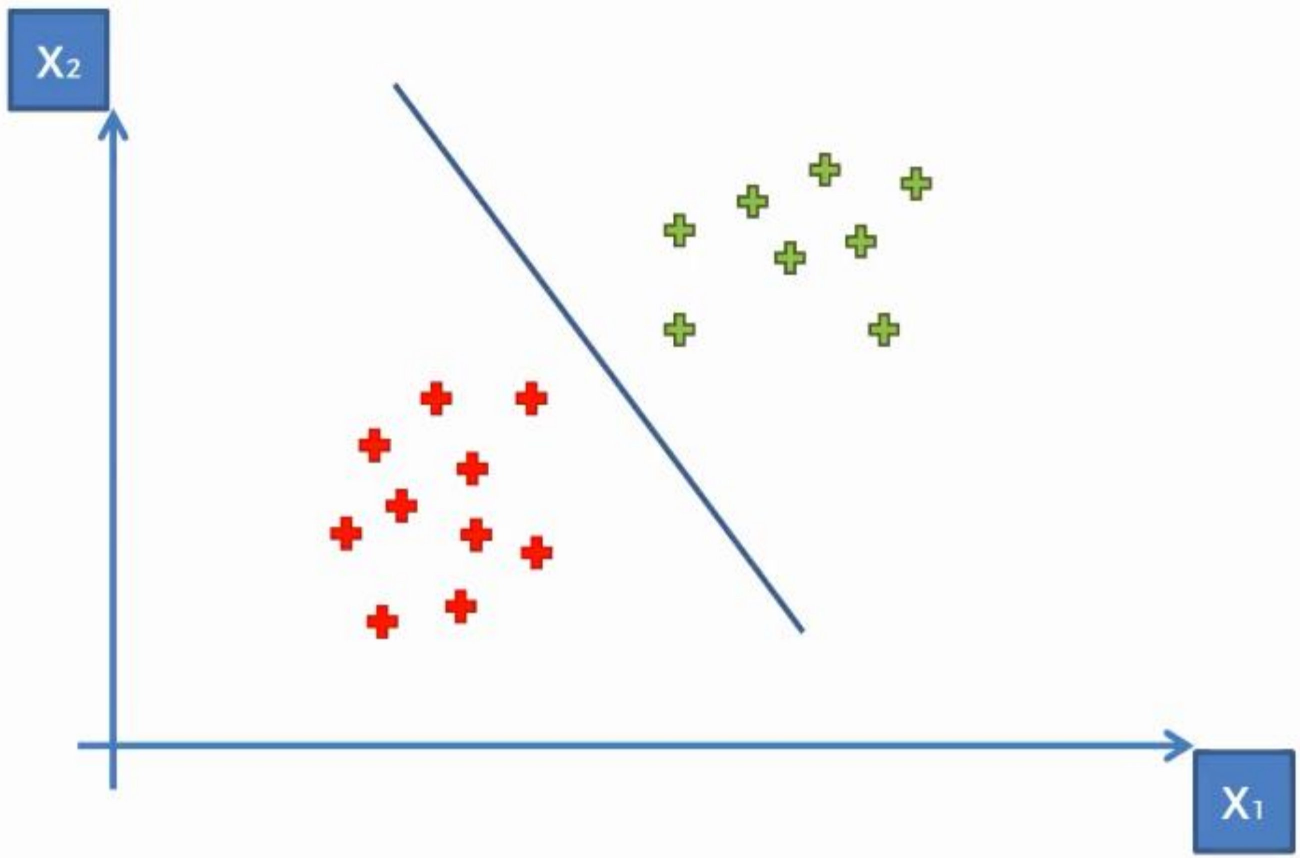

X2
X1

What's So Special About SVMs?

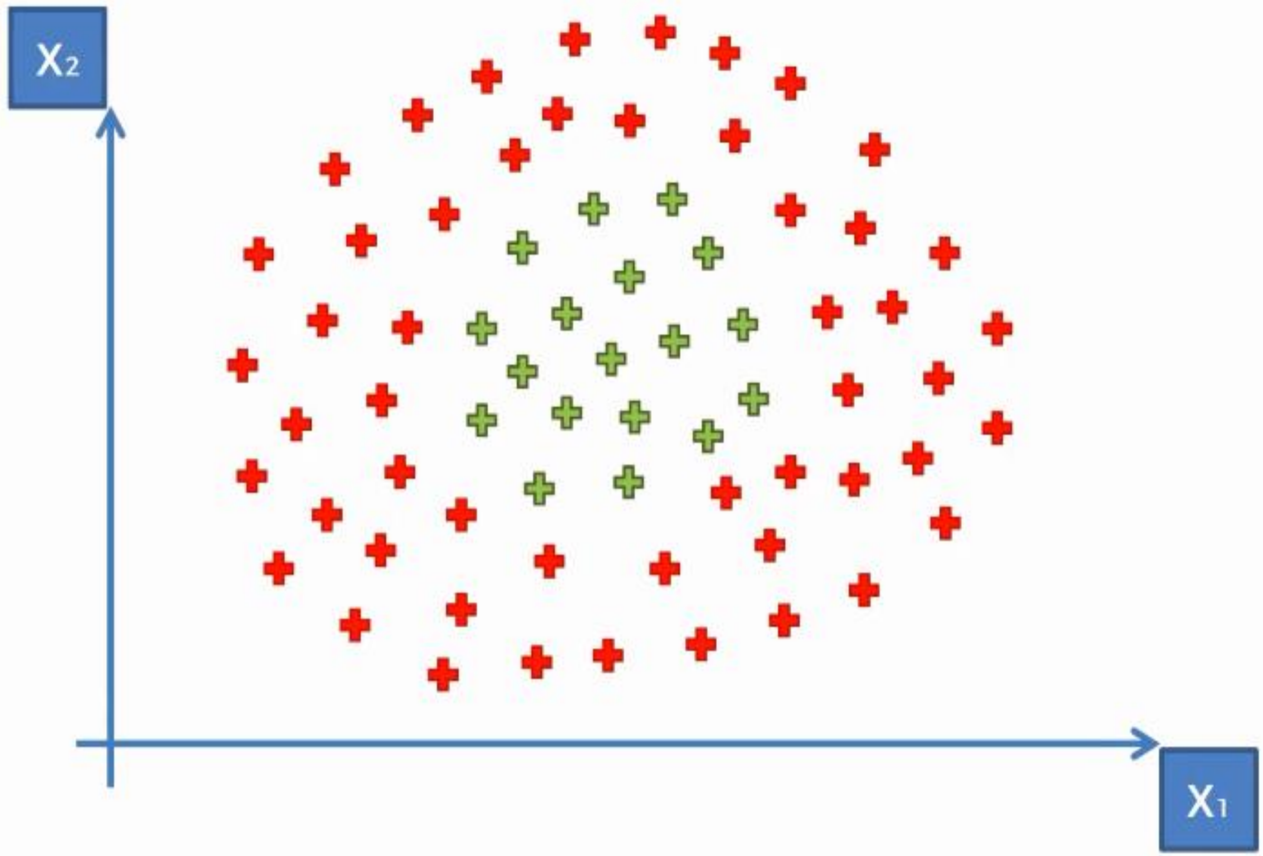


Kernel SVM Intuition

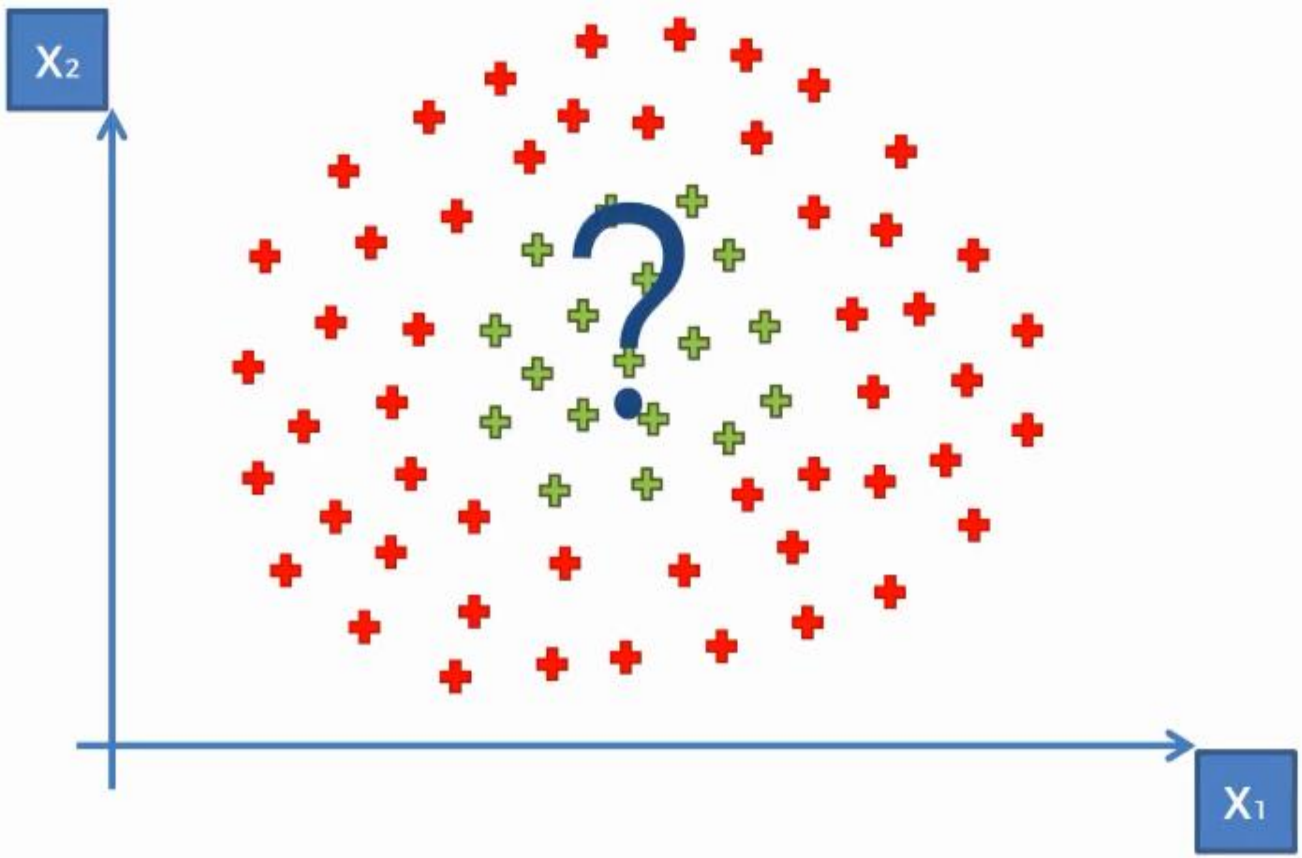
SVM separates well these points



What about these points ?



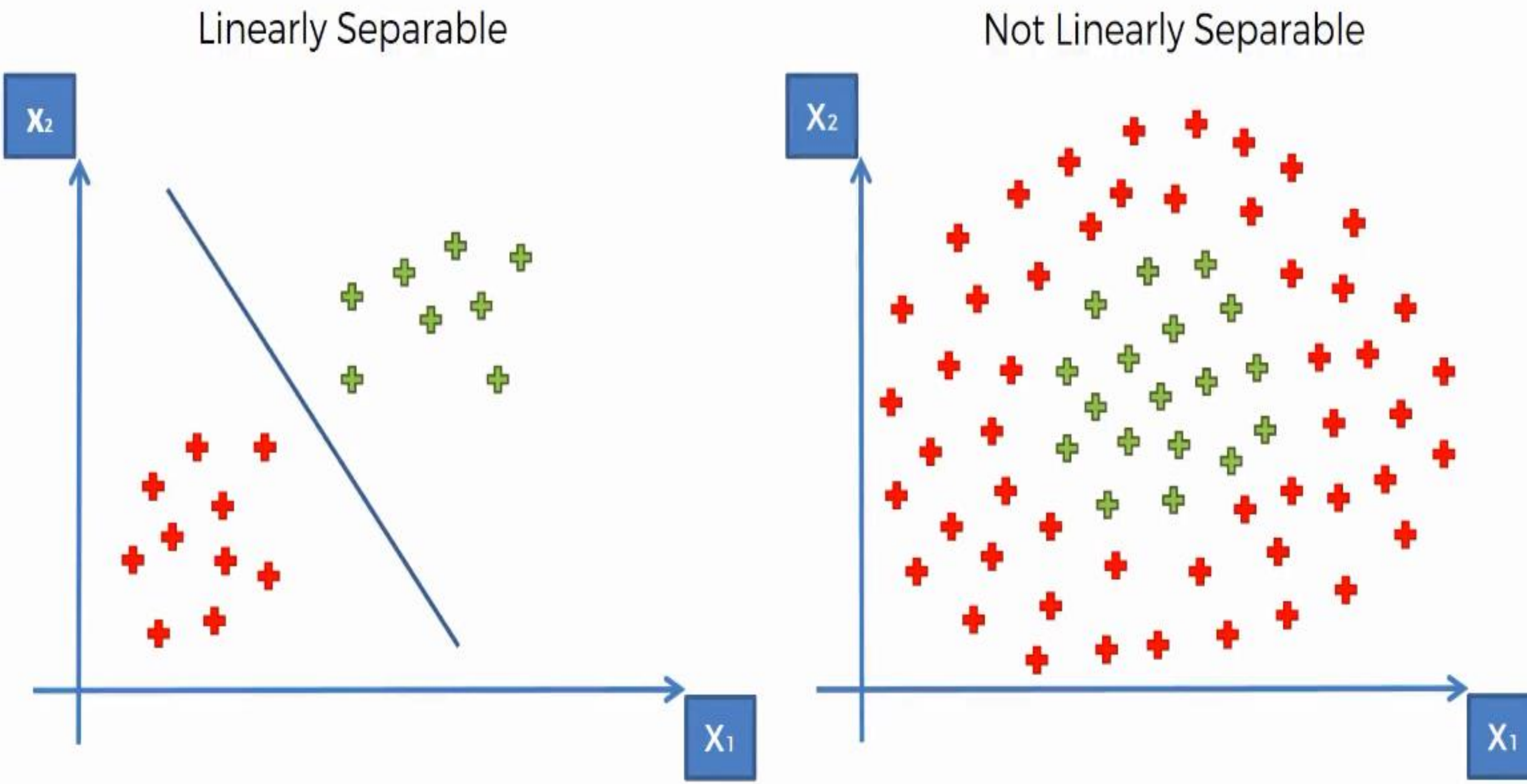
What about these points ?



Why ?

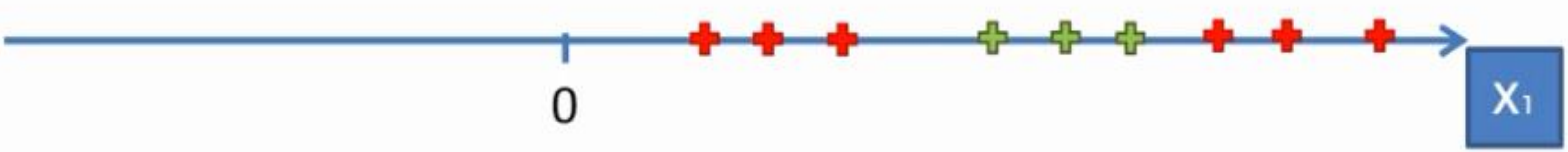
Because the data points are not LINEARLY SEPARABLE

Linear Separability

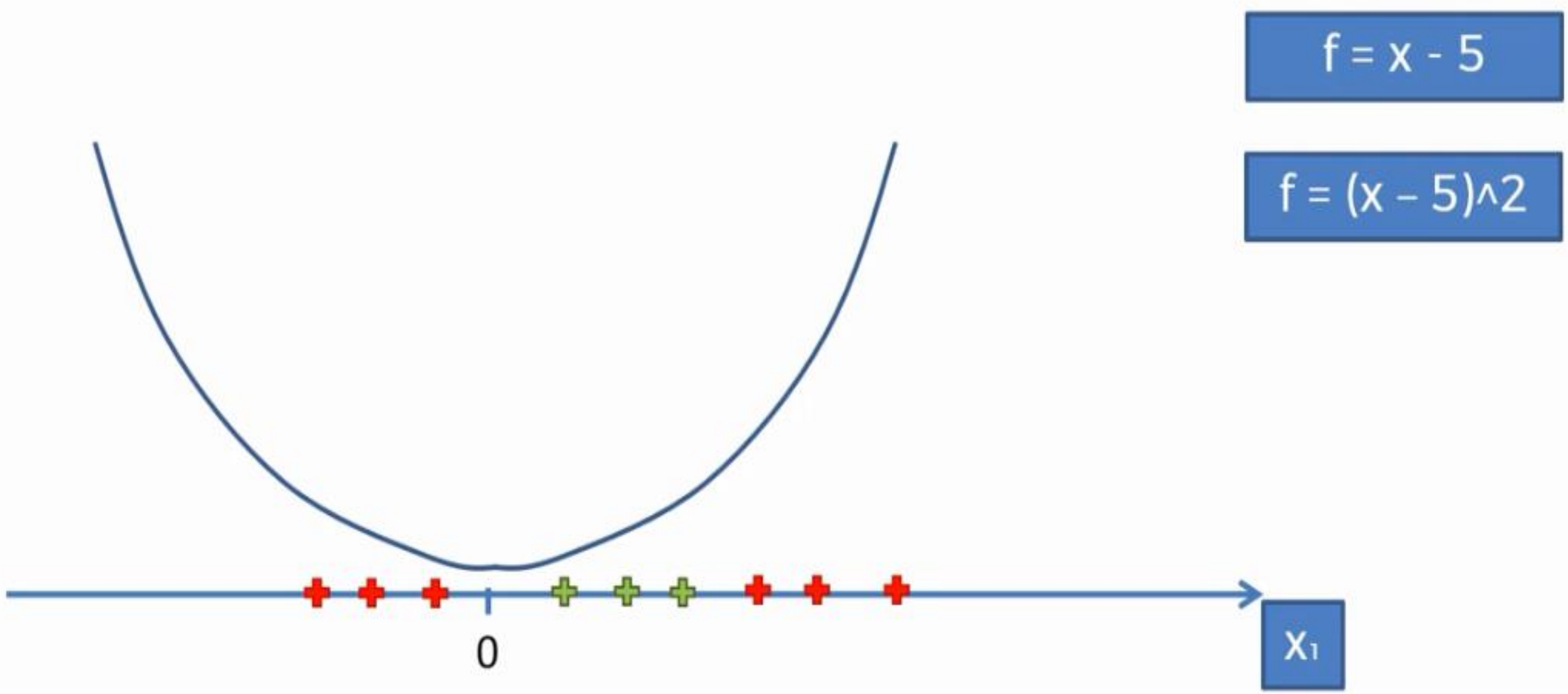


A Higher-Dimensional Space

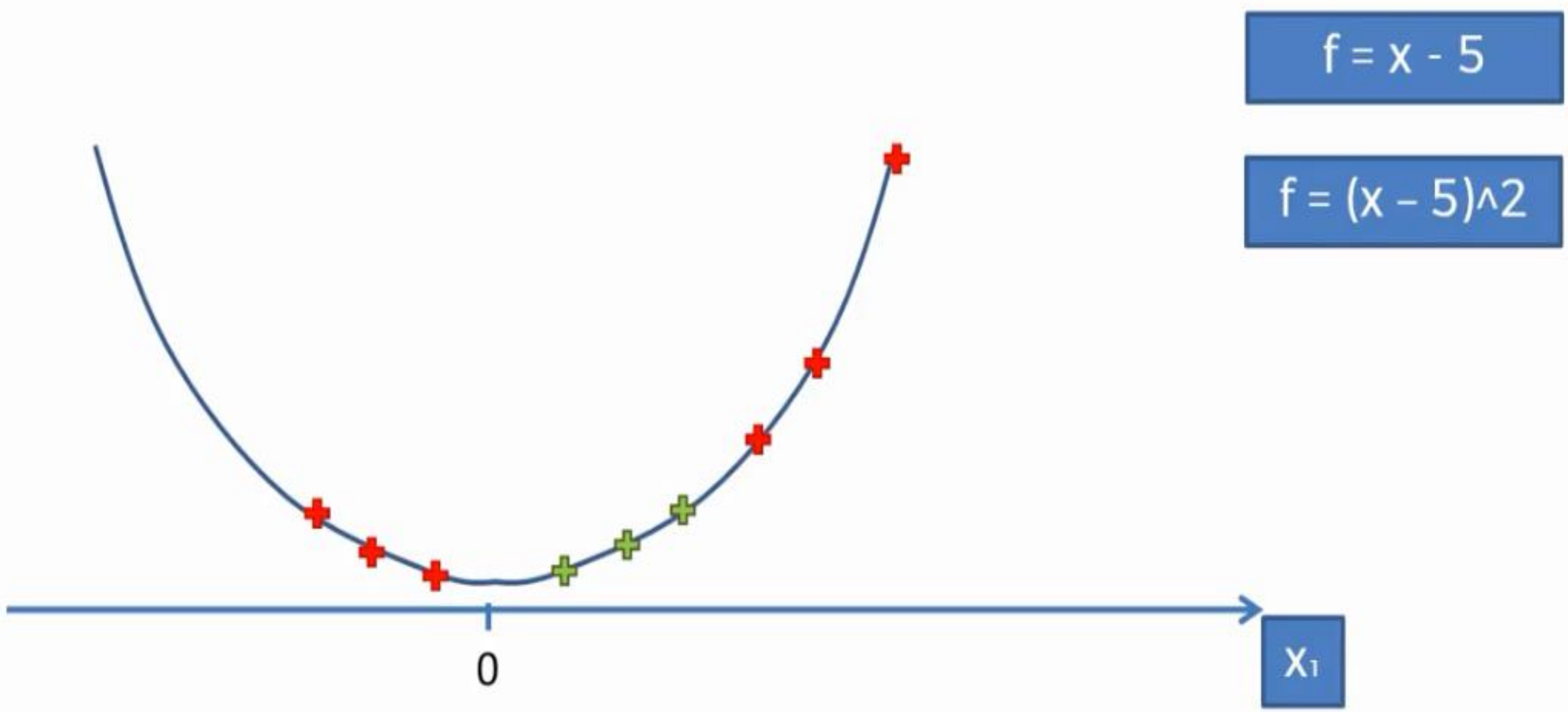
Mapping to a Higher Dimension

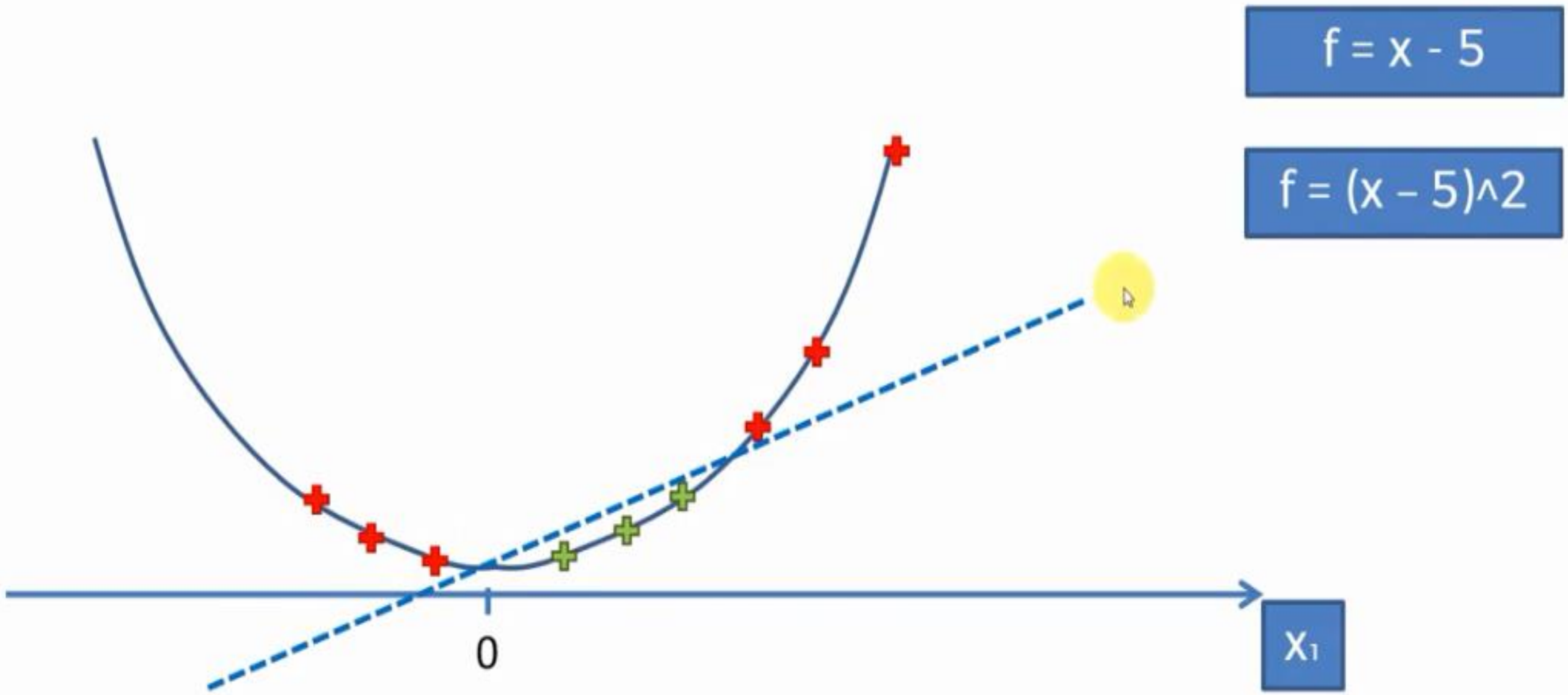


Mapping to a Higher Dimension

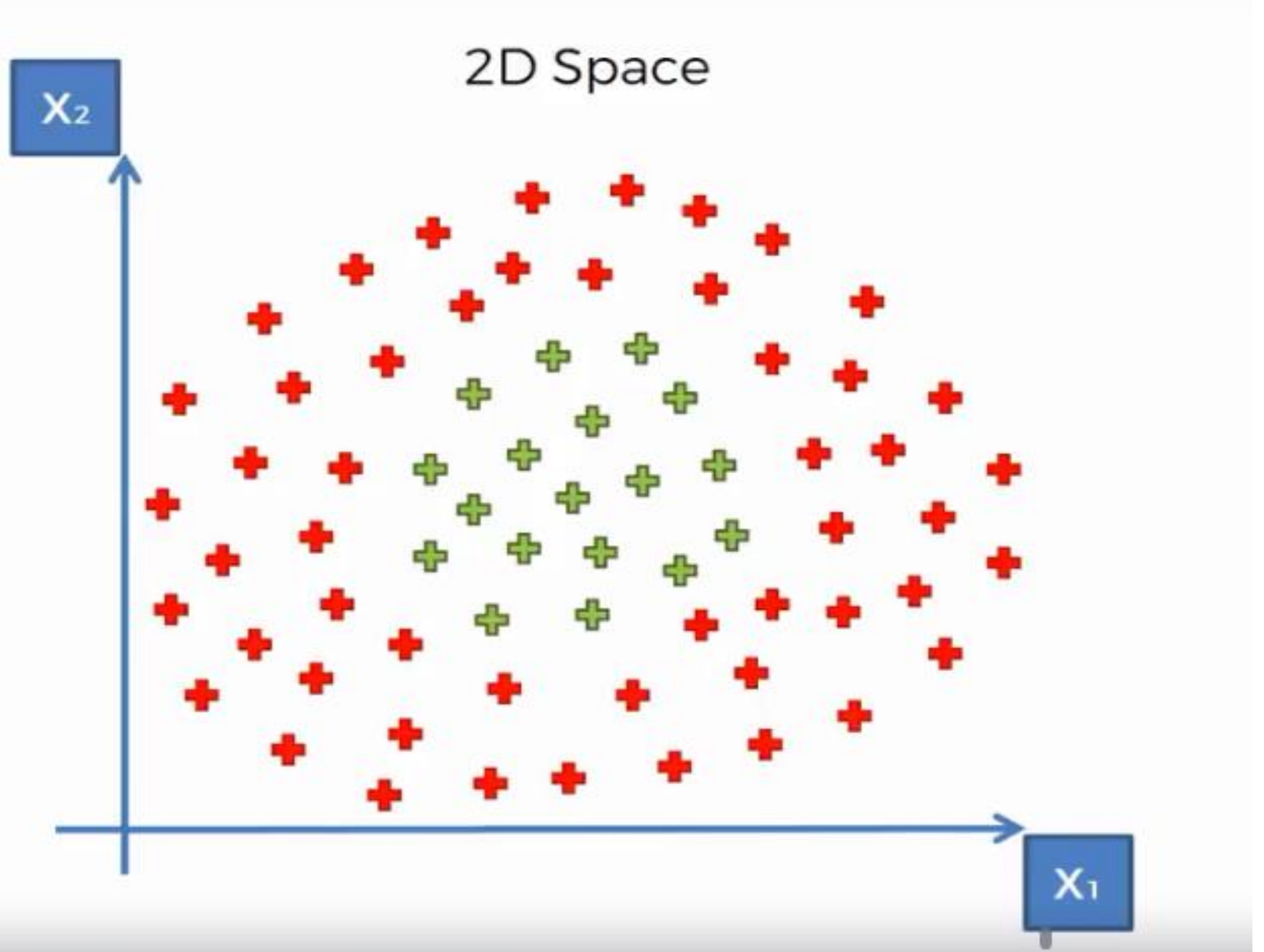


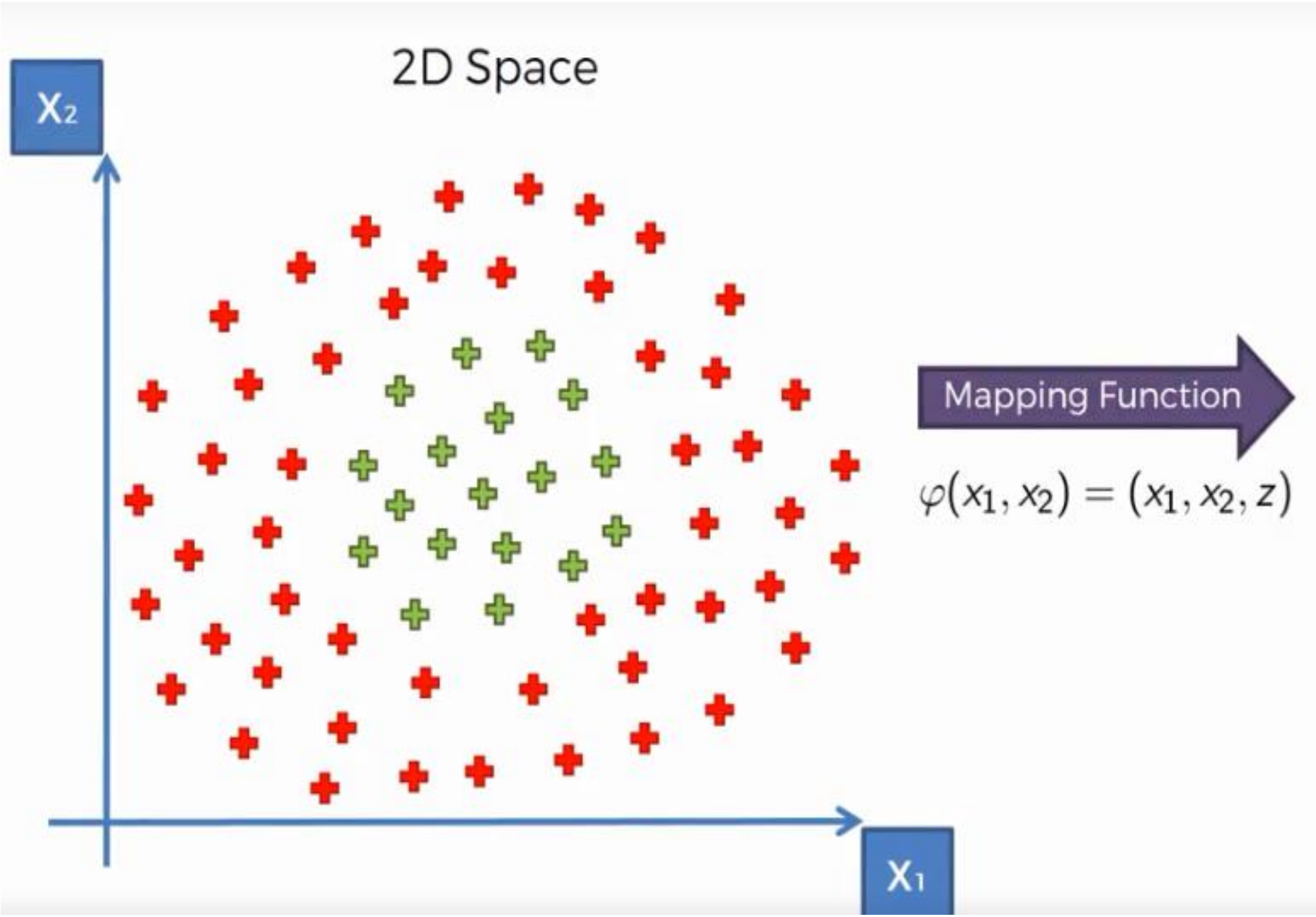
Mapping to a Higher Dimension



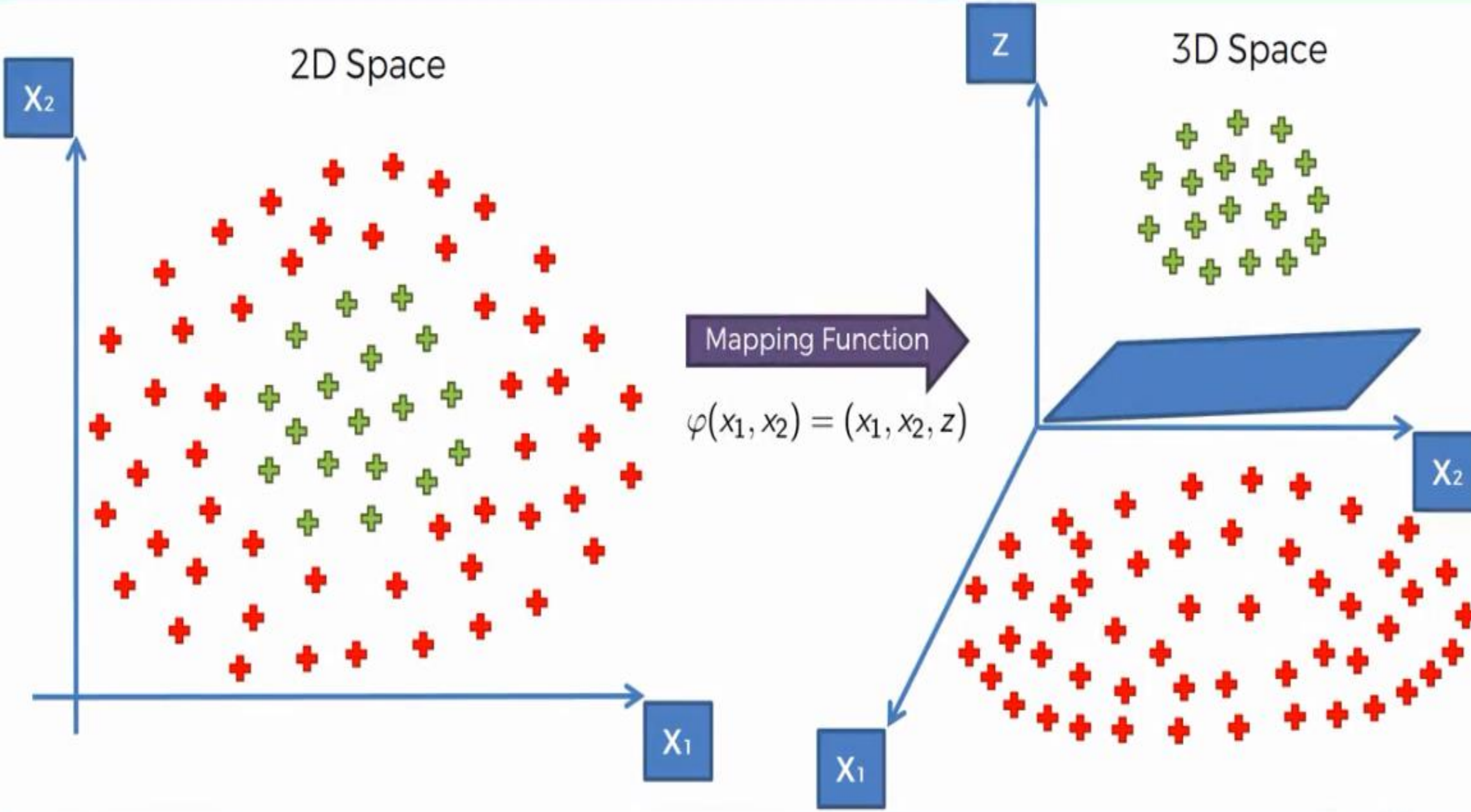


f = x - 5
f = (x – 5)^2
0
x1

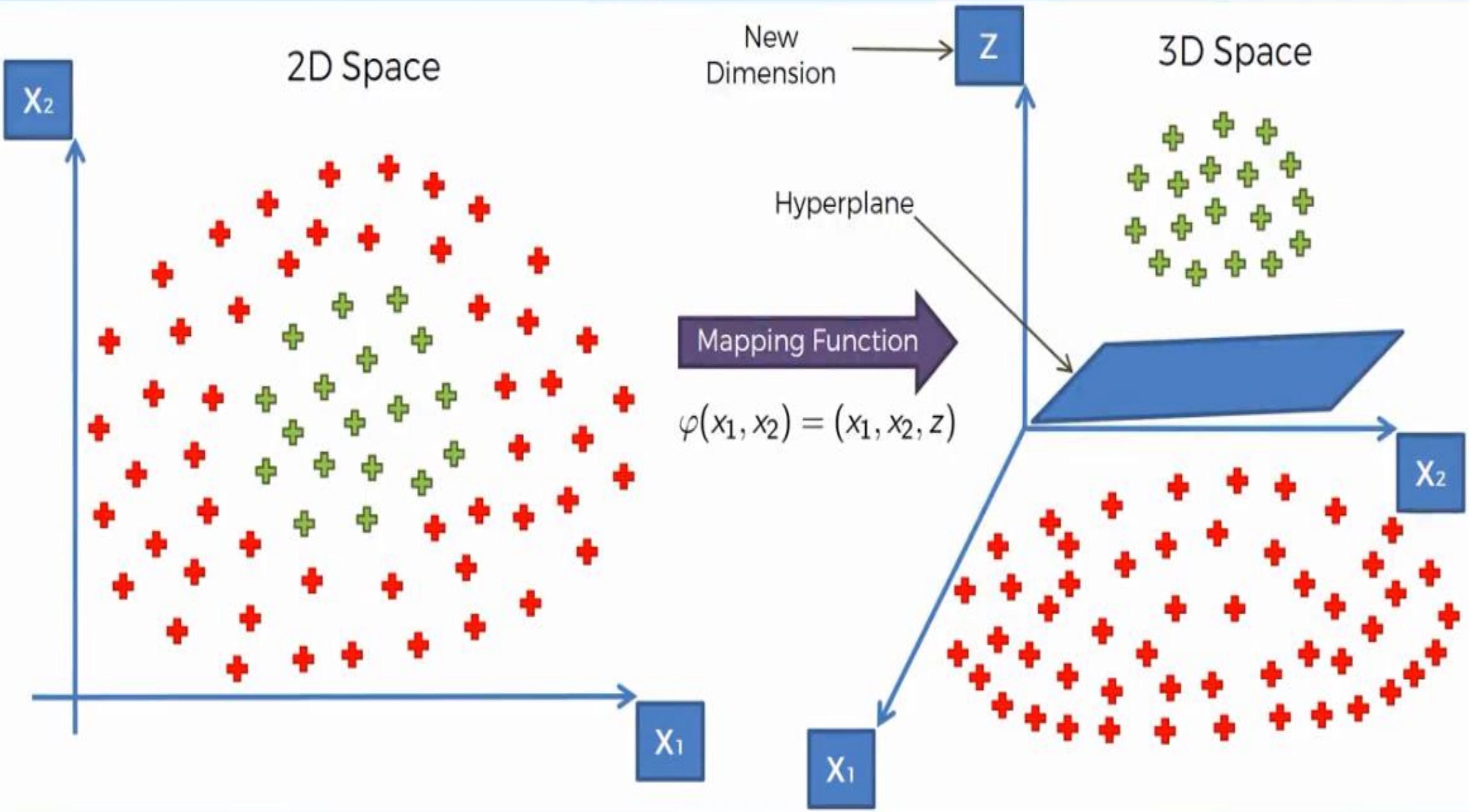

2D Space
X2
X1


2D Space
X2
X1
Mapping Function
$\varphi(x_1, x_2) = (x_1, x_2, z)$

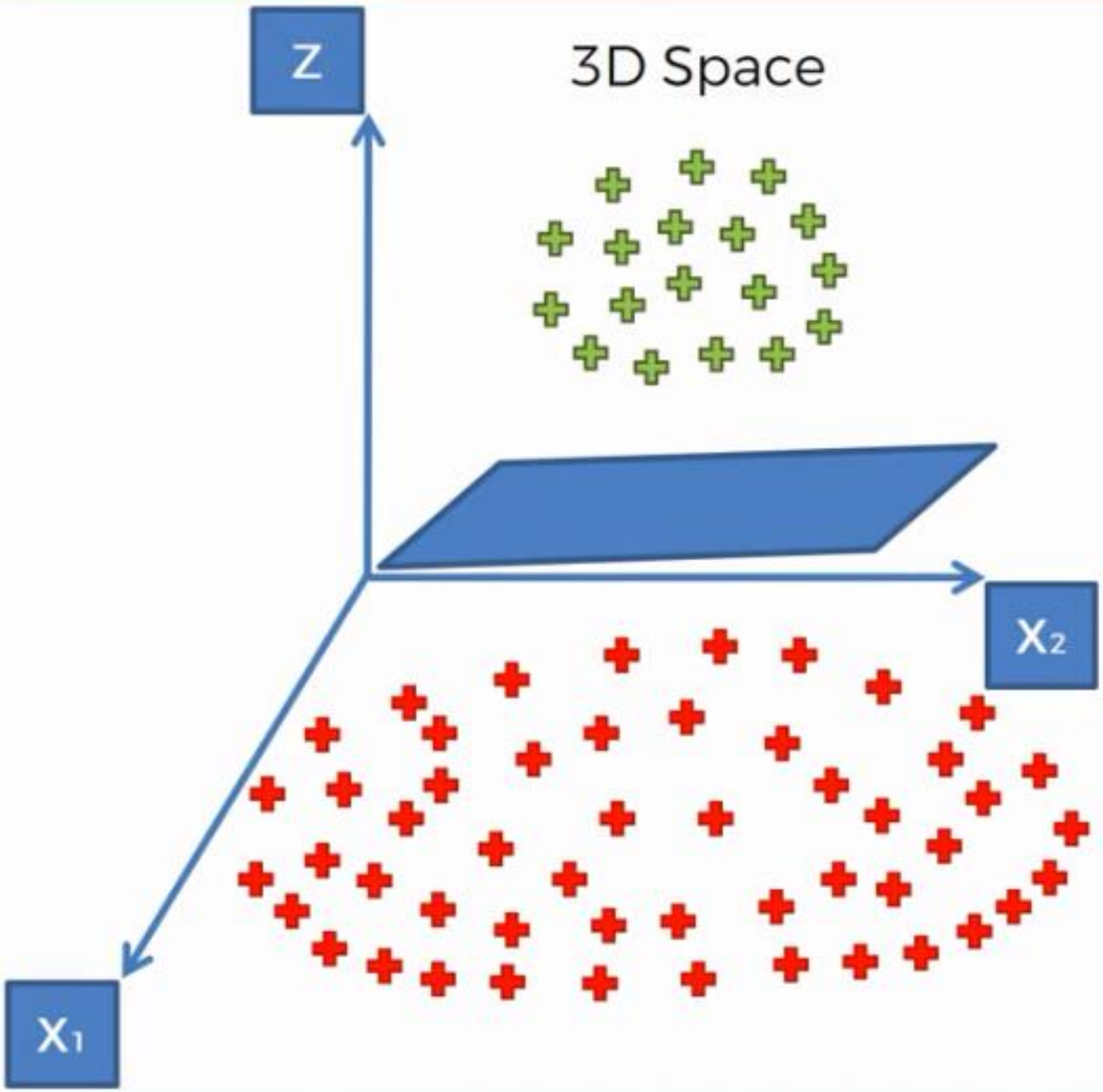
Mapping to a Higher Dimension



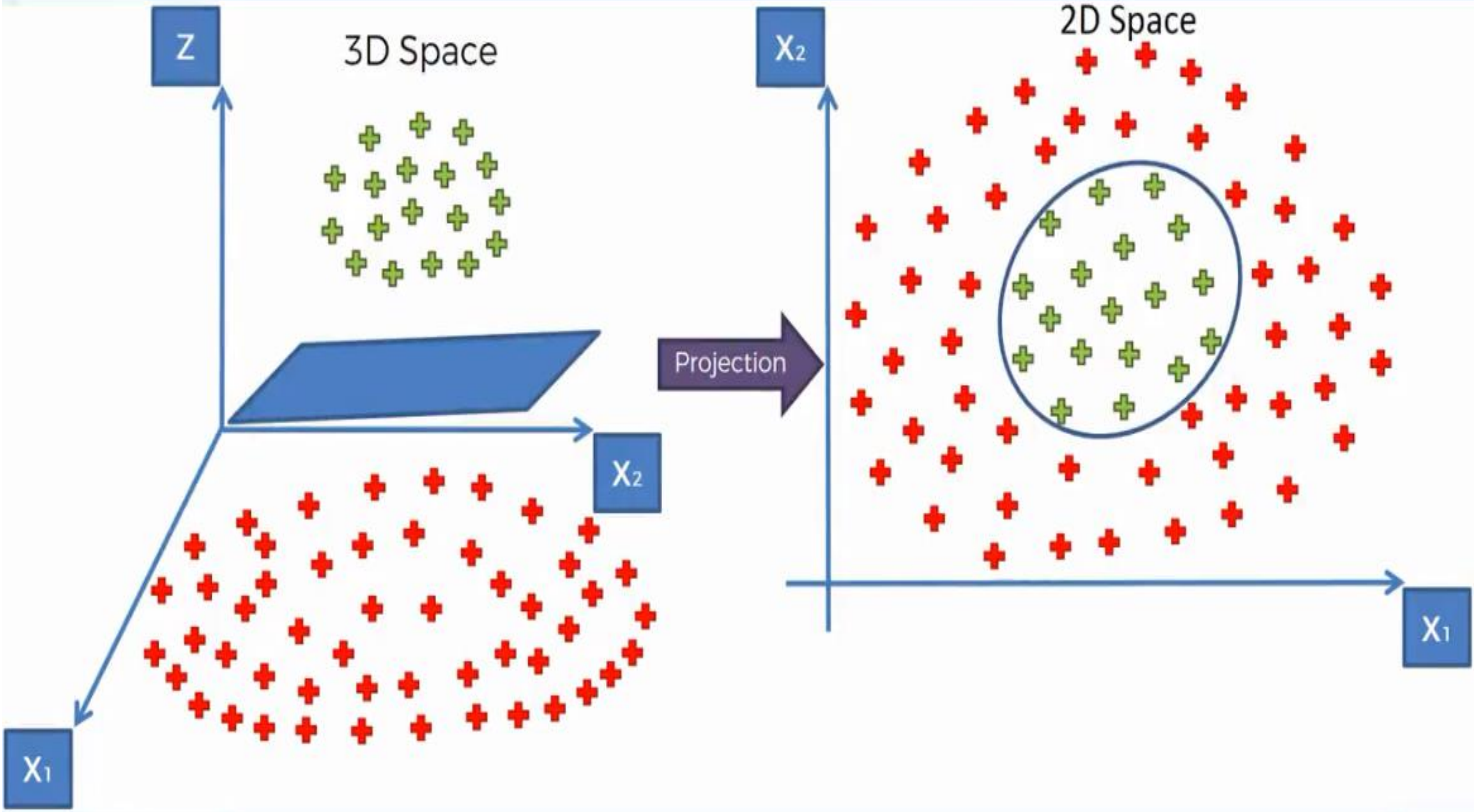
Mapping to a Higher Dimension



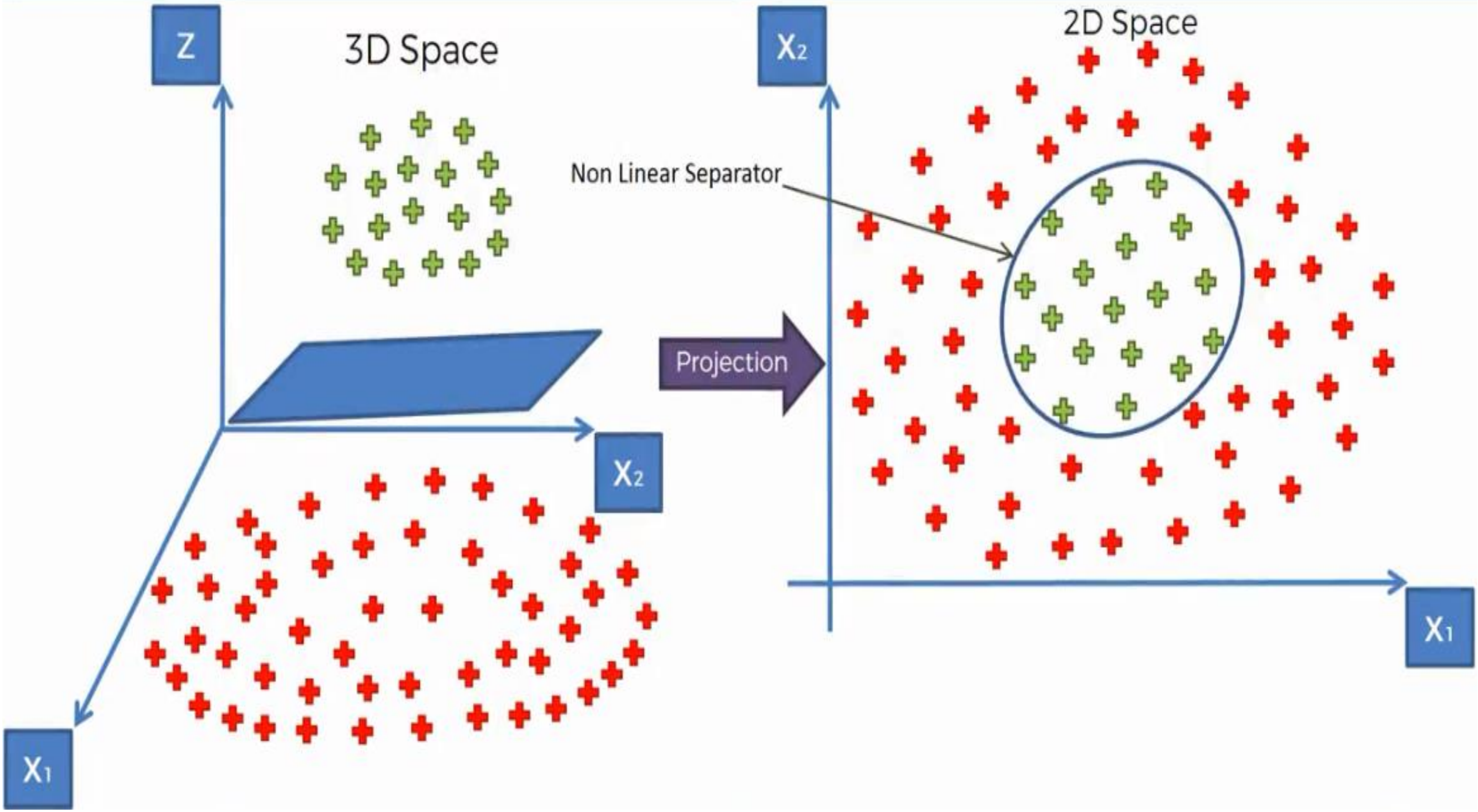
Projecting back to 2D Space



Projecting back to 2D Space



Projecting back to 2D Space



But there is a catch...

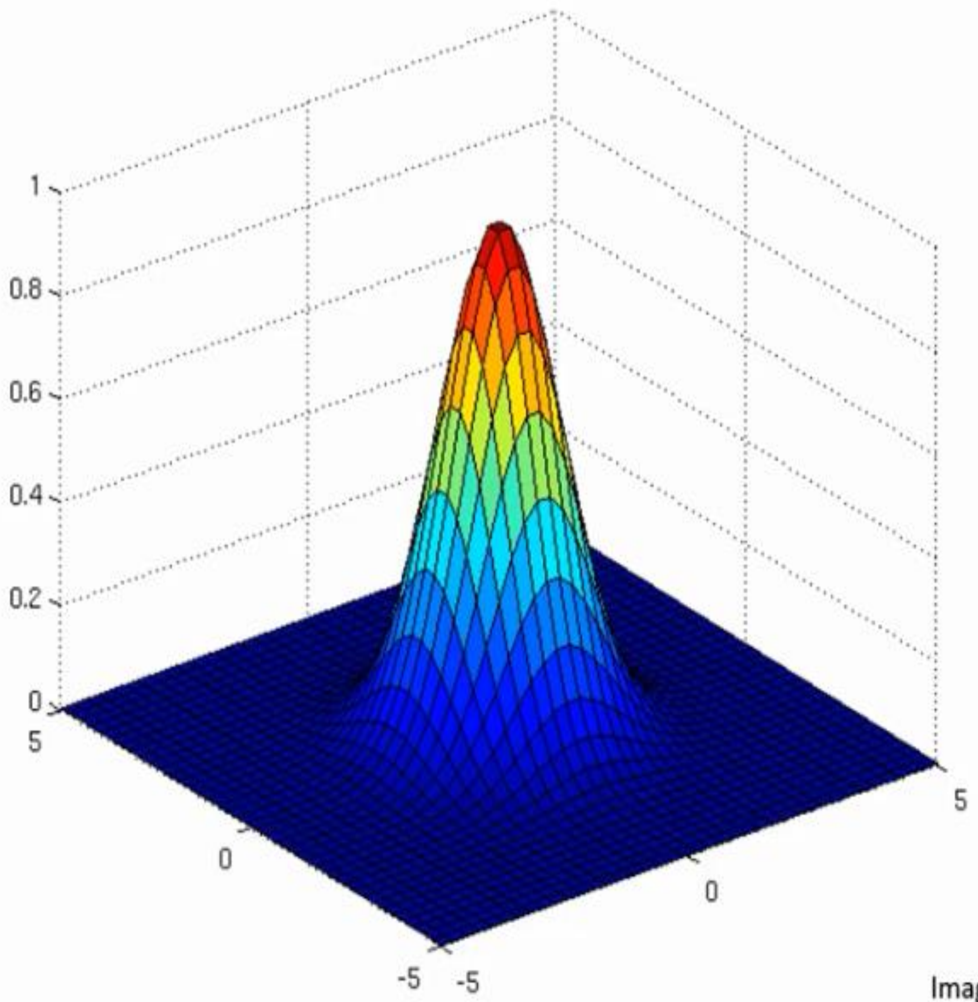
Mapping to a Higher Dimensional Space can be highly compute-intensive

The Kernel Trick

The Gaussian RBF Kernel

$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\left\|\vec{x} - \vec{l}^i\right\|^2}{2\sigma^2}}$$

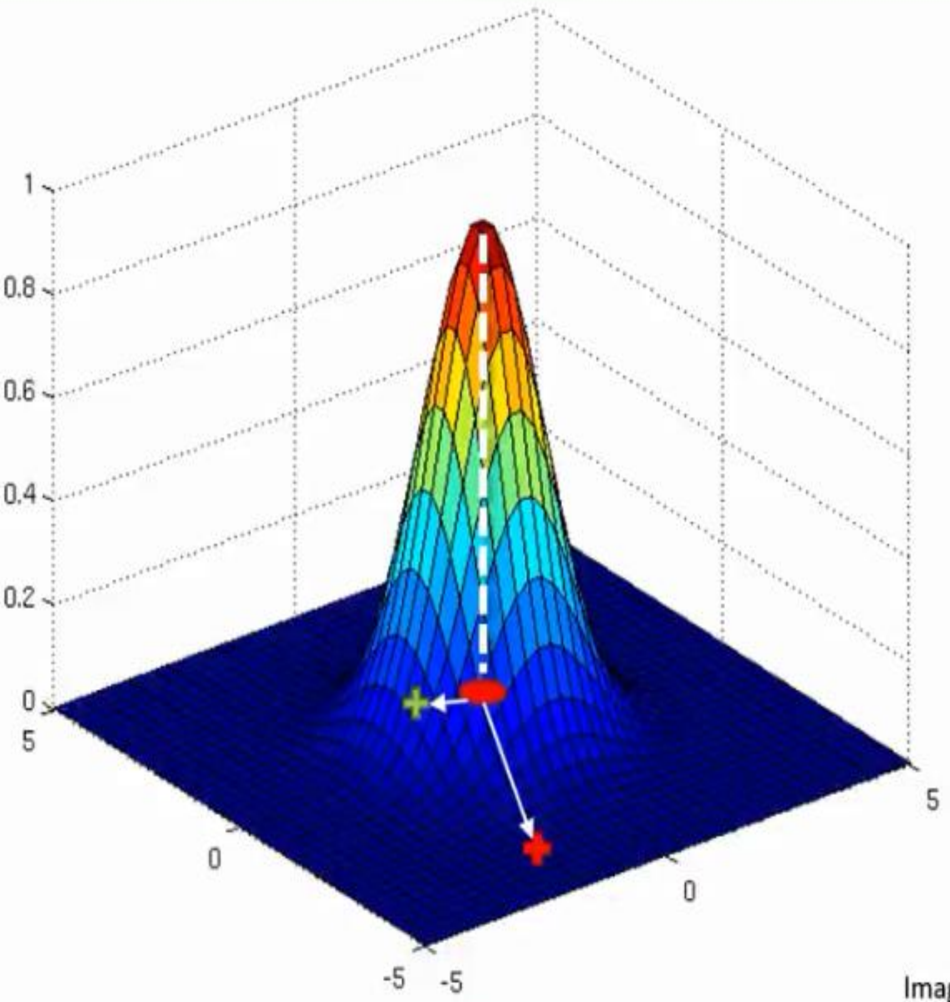
The Gaussian RBF Kernel



$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\|\vec{x}-\vec{l}^i\|^2}{2\sigma^2}}$$

Image source: http://www.cs.toronto.edu/~duvenaud/cookbook/inde

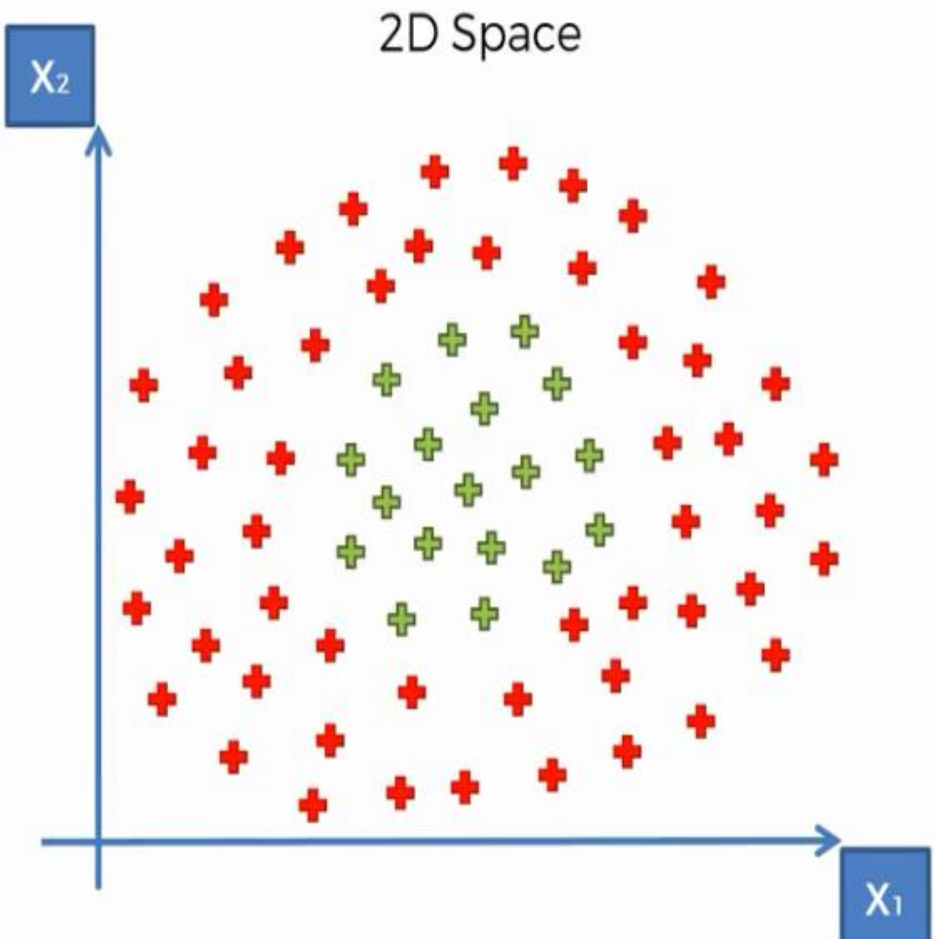
The Gaussian RBF Kernel

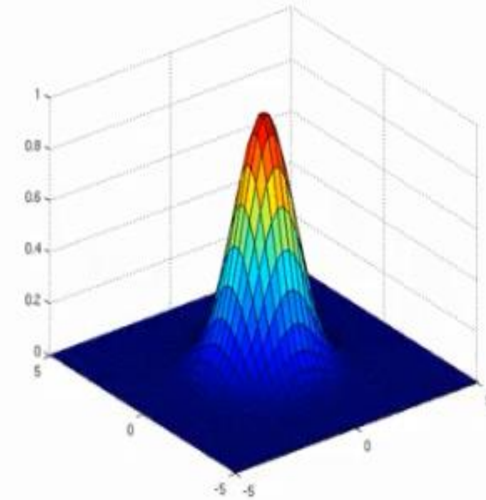


$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\|\vec{x}-\vec{l}^i\|^2}{2\sigma^2}}$$

Image source: http://www.cs.toronto.edu/~duvenaud/cookbook/index.html

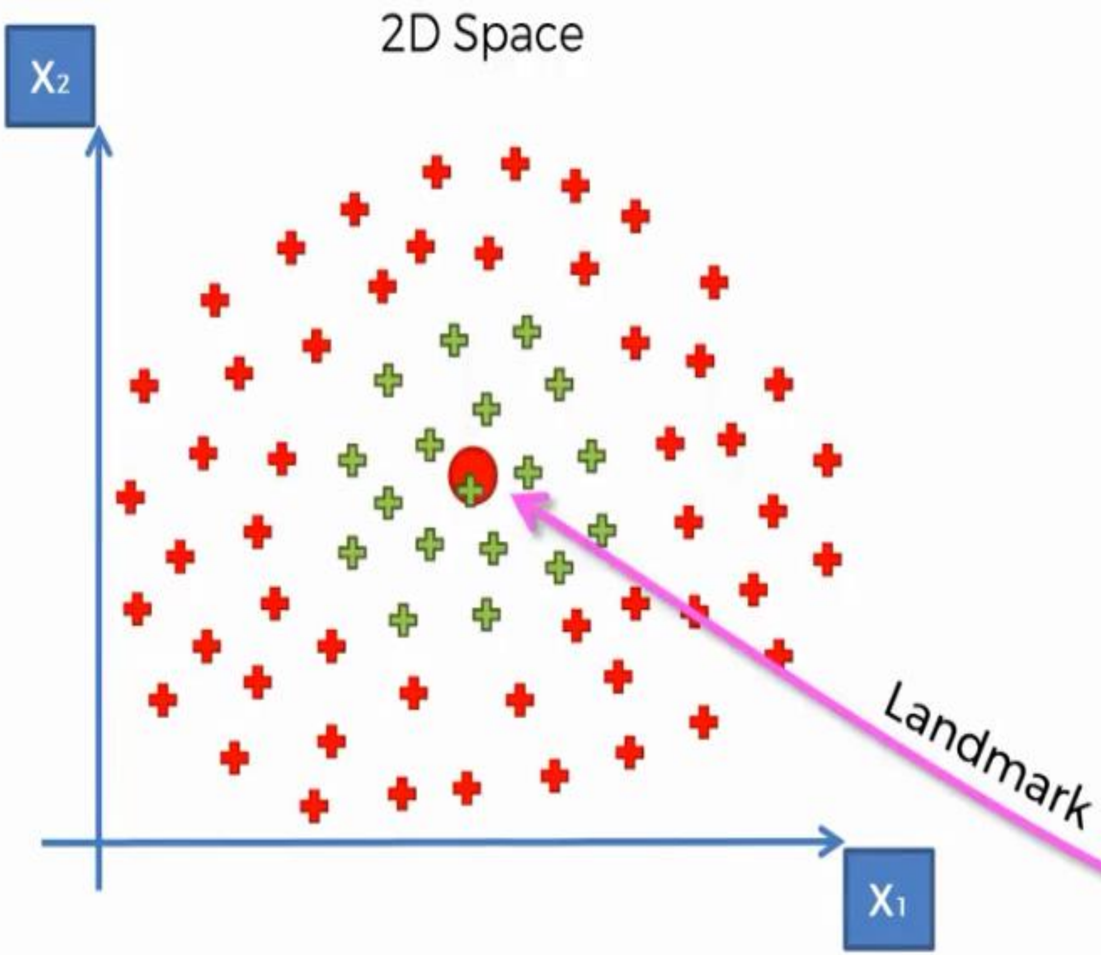
The Gaussian RBF Kernel

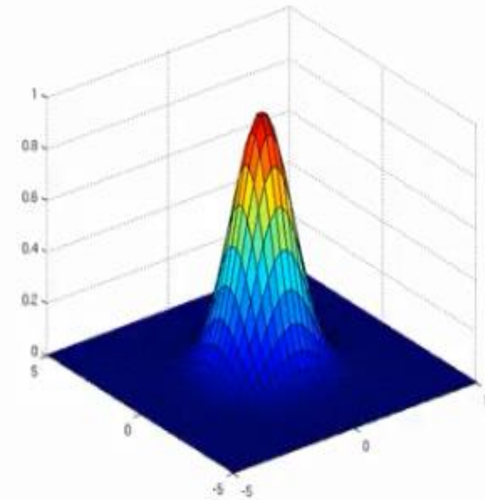




$$K\left(\vec{x}, \vec{l}^{i}\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}}$$

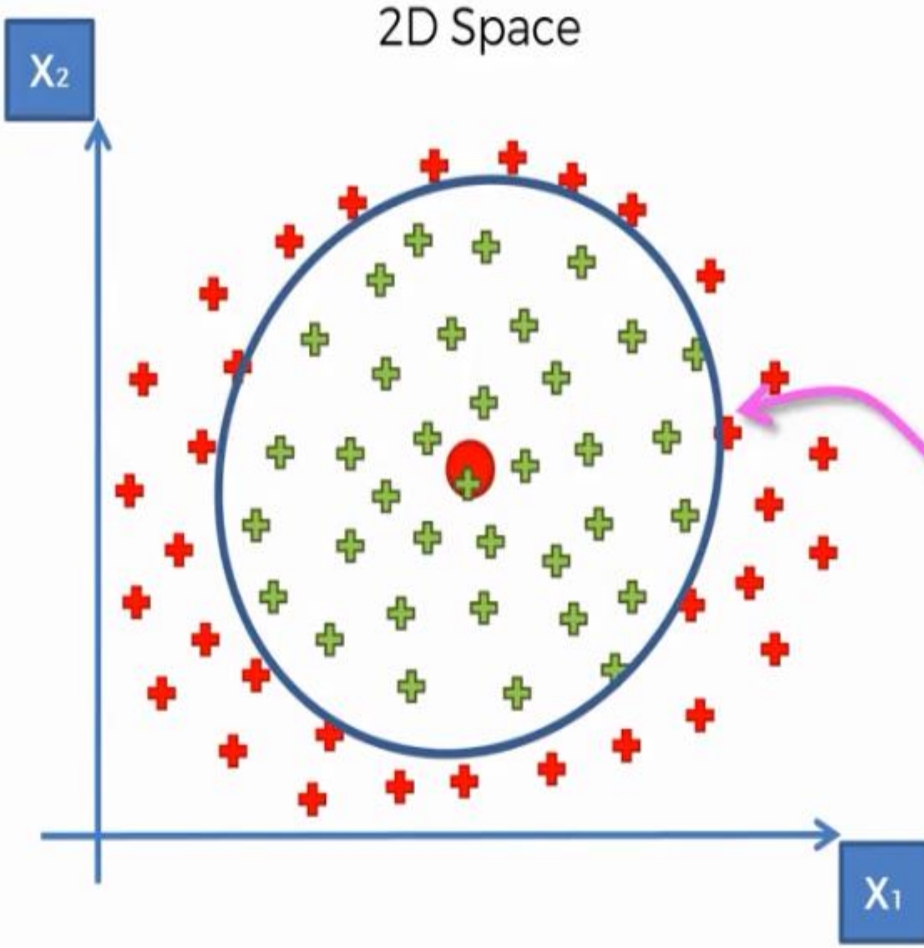
The Gaussian RBF Kernel



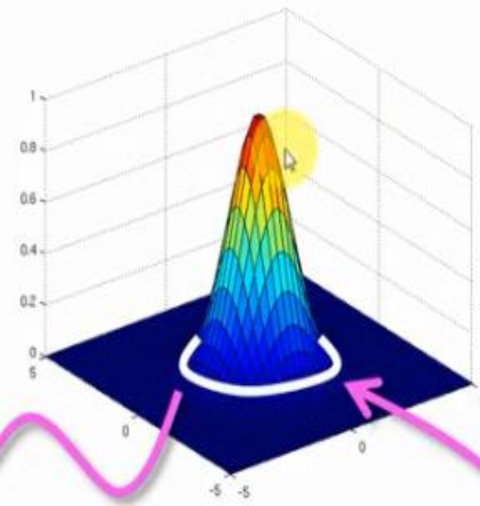


$$K\left(\vec{x}, \vec{l}^{i}\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}}$$

The Gaussian RBF Kernel

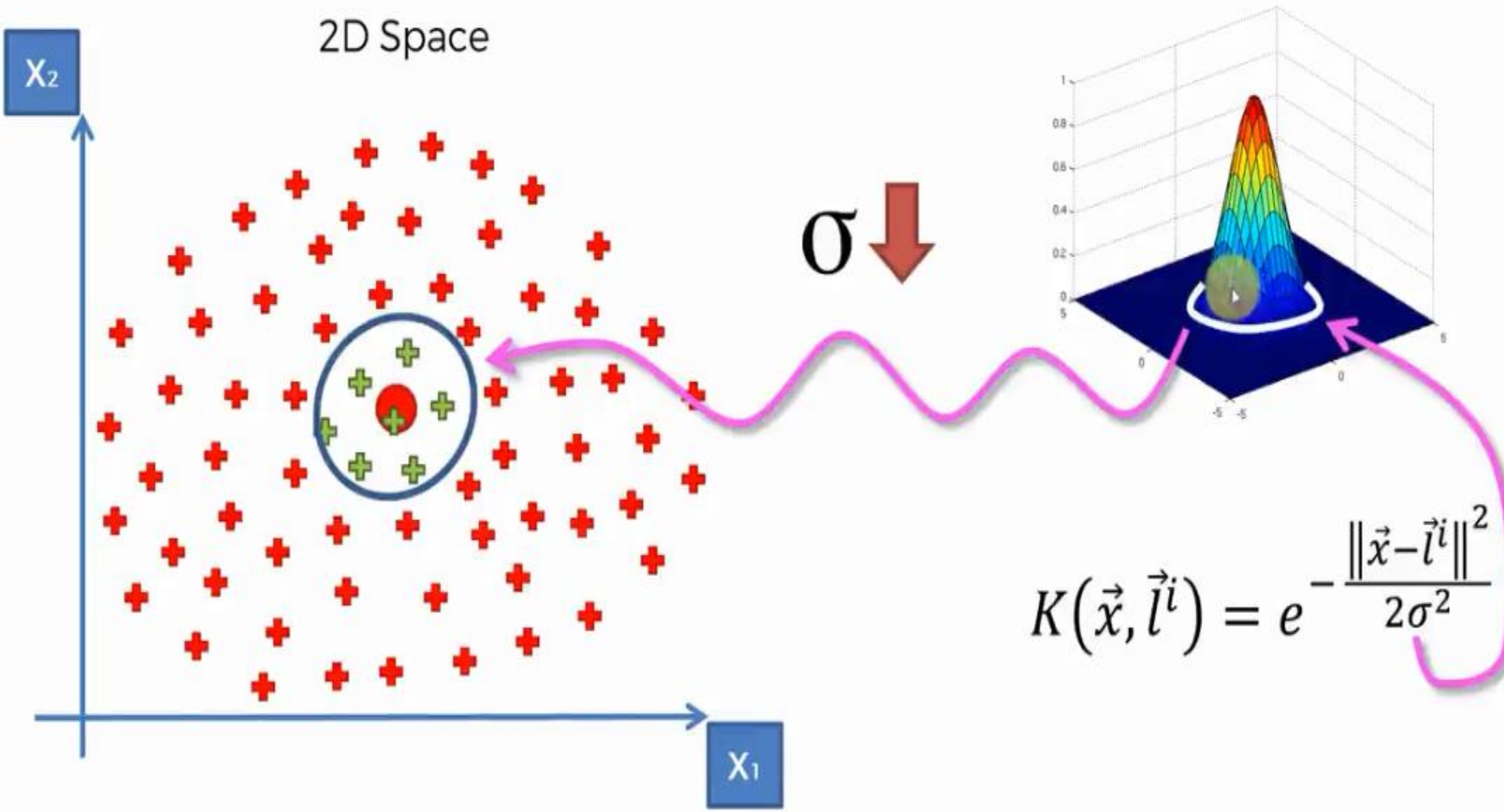


σ



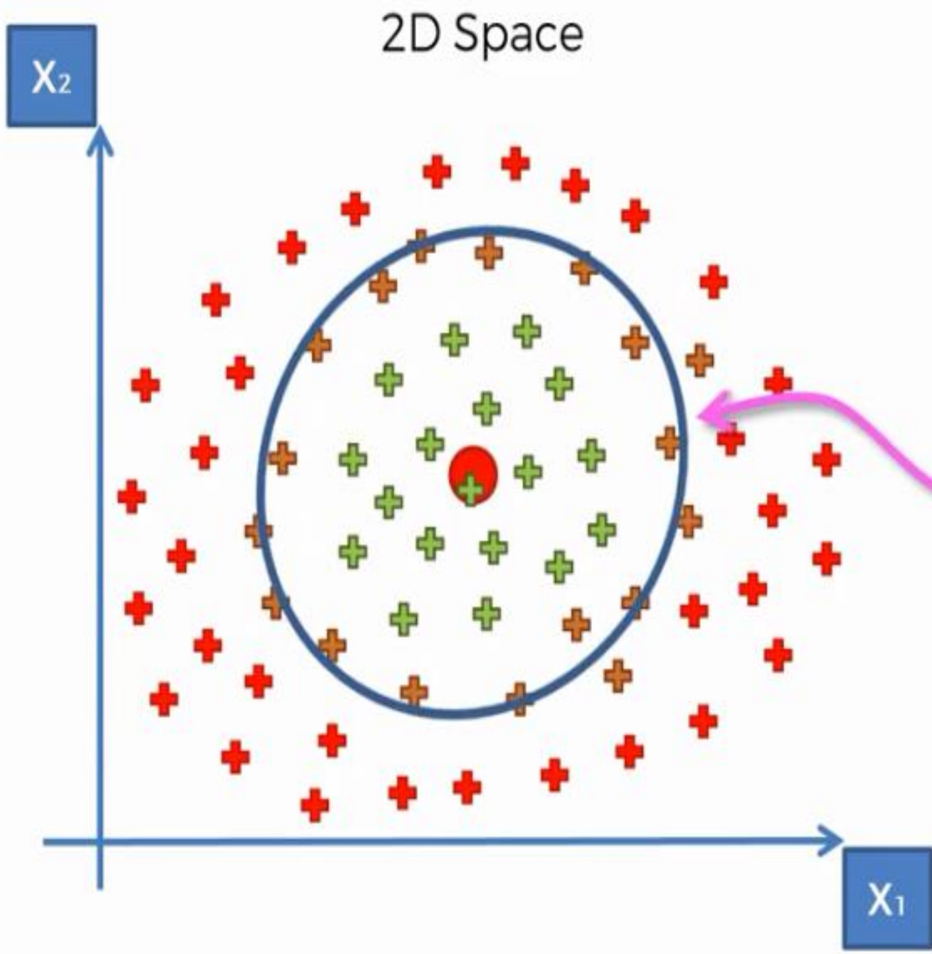
$$K\left(\vec{x}, \vec{l}^{i}\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}}$$

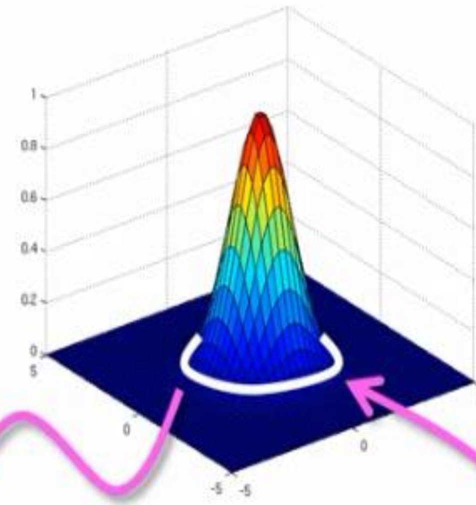
The Gaussian RBF Kernel



$$K\left(\vec{x}, \vec{l}^i\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^i\right\|^2}{2\sigma^2}}$$

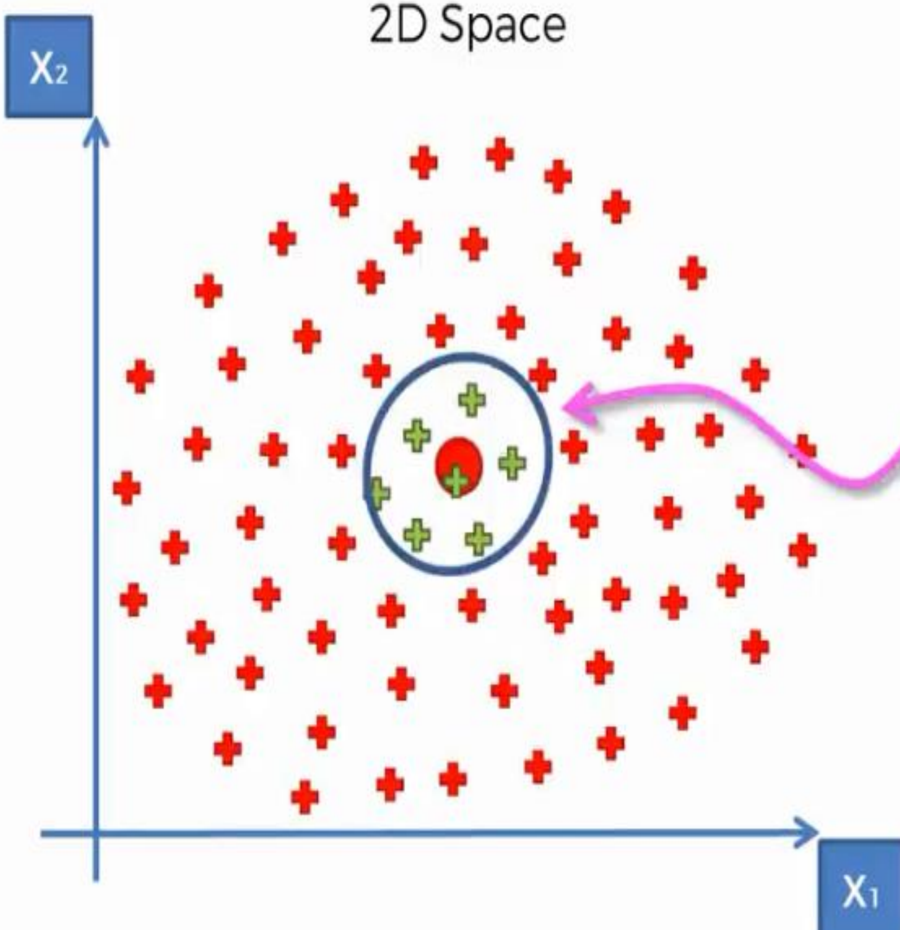
The Gaussian RBF Kernel



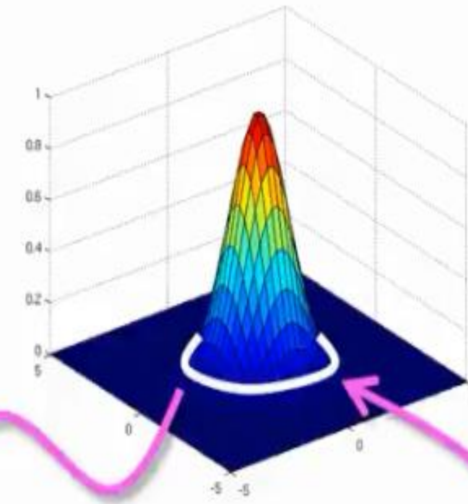


$$K\left(\vec{x},\vec{l}^{i}\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}}$$

The Gaussian RBF Kernel

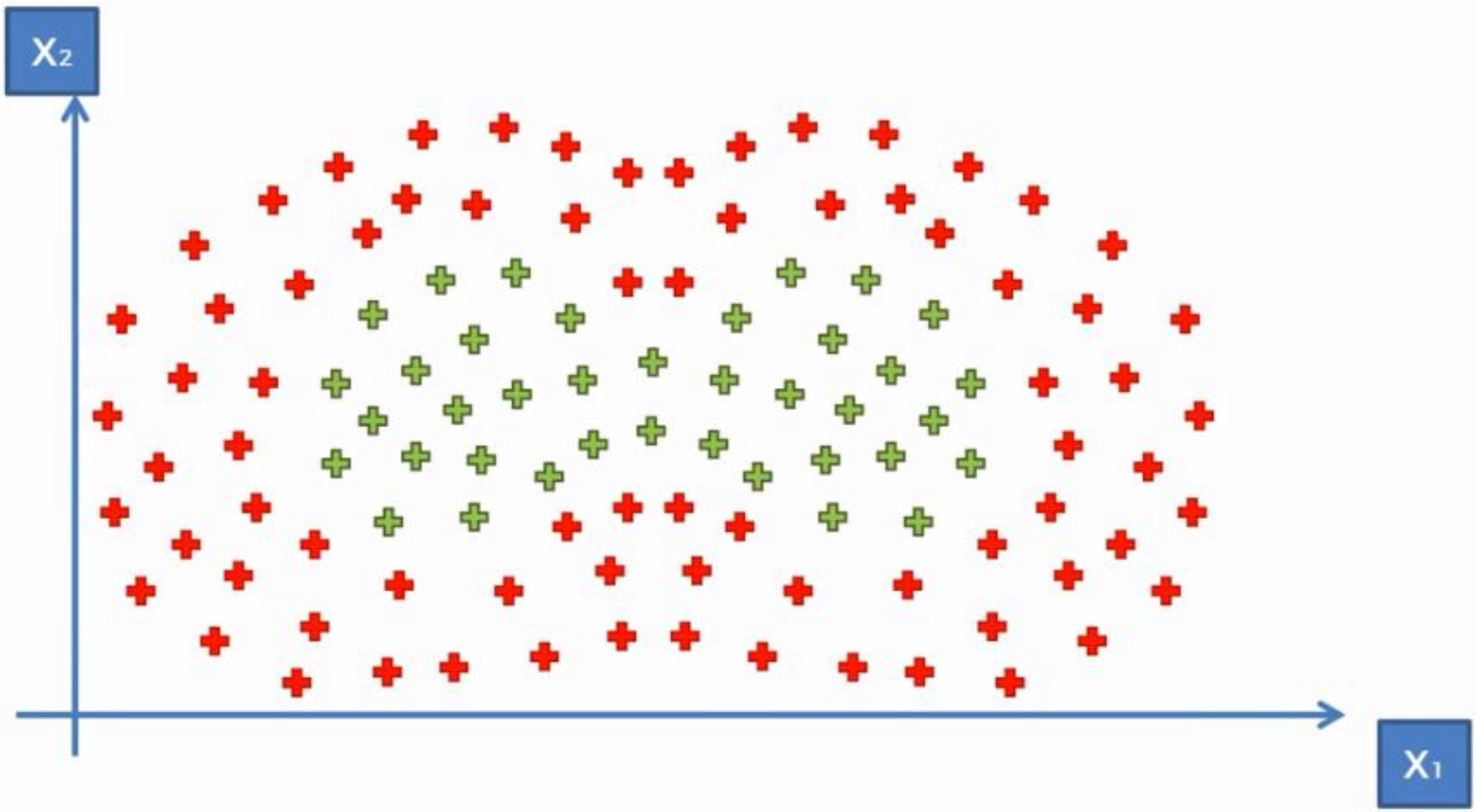


$\sigma \downarrow$

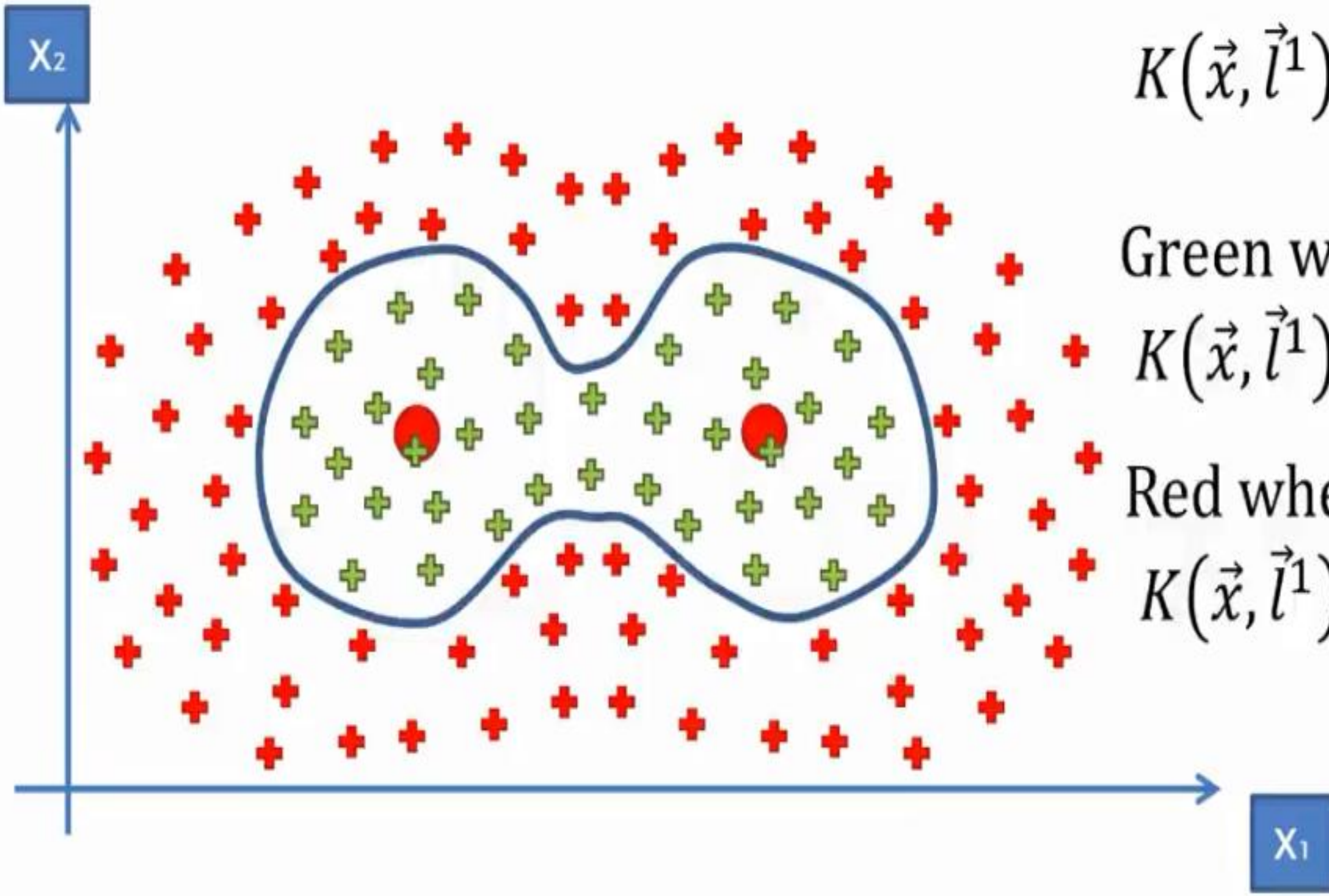


$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\|\vec{x}-\vec{l}^i\|^2}{2\sigma^2}}$$

The Gaussian RBF Kernel



The Gaussian RBF Kernel



$$K(\vec{x}, \vec{l}^1) + K(\vec{x}, \vec{l}^2)$$

(Simplified Formula)

Green when:

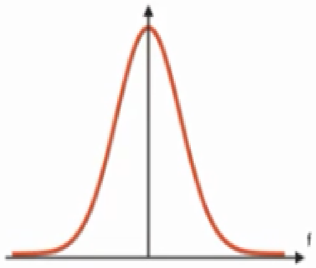
$$K(\vec{x}, \vec{l}^1) + K(\vec{x}, \vec{l}^2) > 0$$

Red when:

$$K(\vec{x}, \vec{l}^1) + K(\vec{x}, \vec{l}^2) = 0$$

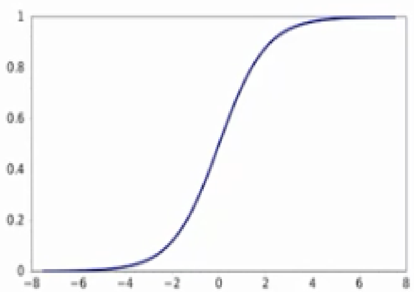
Types of Kernel Functions

Types of Kernel Functions



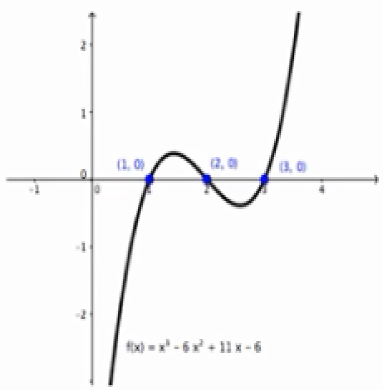
Gaussian RBF Kernel

$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\|\vec{x}-\vec{l}^i\|^2}{2\sigma^2}}$$



Sigmoid Kernel

$$K(X, Y) = \tanh(\gamma \cdot X^T Y + r)$$



Polynomial Kernel

$$K(X, Y) = (\gamma \cdot X^T Y + r)^d, \gamma > 0$$