

Machine Learning Assignment 4

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1 Question 1: Convolution Operation

1. Convolution with stride of 1

$$\begin{bmatrix} 0 & 2 & 4 & 1 & 0 \\ 3 & 1 & 1 & 0 & 1 \\ 2 & 4 & 1 & 0 & 1 \\ 2 & 0 & 5 & 2 & 2 \\ 0 & 1 & 3 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$N = 5, F = 3, \text{stride} = 1$

$= (N-F)/\text{stride} + 1 = (5-3)/1 + 1 = 3$, so the output will be 3x3 matrix

$$\text{output} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$a = 0*1 + 2*0 + 4*-1 + 3*1 + 1*0 + 1*-1 + 2*1 + 4*0 + 1*-1 = -1$$

$$b = 2*1 + 4*0 + 1*-1 + 1*1 + 1*0 + 0*-1 + 4*1 + 1*0 + 0*-1 = 6$$

$$c = 4*1 + 1*0 + 0*-1 + 1*1 + 0*0 + 1*-1 + 1*1 + 0*0 + 1*-1 = 4$$

$$d = 3*1 + 1*0 + 1*-1 + 2*1 + 4*0 + 1*-1 + 2*1 + 0*0 + 5*-1 = 0$$

$$e = 1*1 + 1*0 + 0*-1 + 4*1 + 1*0 + 0*-1 + 0*1 + 5*0 + 2*-1 = 3$$

$$f = 1*1 + 0*0 + 1*-1 + 1*1 + 0*0 + 1*-1 + 5*1 + 2*0 + 2*-1 = 3$$

$$g = 2*1 + 4*0 + 1*-1 + 2*1 + 0*0 + 5*-1 + 0*1 + 1*0 + 3*-1 = -5$$

$$h = 4*1 + 1*0 + 0*-1 + 0*1 + 5*0 + 2*-1 + 1*1 + 3*0 + 2*-1 = 1$$

$$i = 1*1 + 0*0 + 1*-1 + 5*1 + 2*0 + 2*-1 + 3*1 + 2*0 + 1*-1 = 5$$

$$\text{output} = \begin{bmatrix} -1 & 6 & 4 \\ 0 & 3 & 3 \\ -5 & 1 & 5 \end{bmatrix}$$

2. Zero padding of 1 + convolution with stride of 1

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 5 & 2 & 2 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

N = 7, F = 3, stride = 1

= (N-F)/stride + 1 = (7-3)/1 + 1 = 5, so the output will be 5x5 matrix

$$output = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \\ p & q & r & s & t \\ u & v & w & x & y \end{bmatrix}$$

$$\begin{aligned} a &= 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 2*-1 + 0*1 + 3*0 + 1*-1 = -3 \\ b &= 0*1 + 0*0 + 0*-1 + 0*1 + 2*0 + 4*-1 + 3*1 + 1*0 + 1*-1 = -2 \\ c &= 0*1 + 0*0 + 0*-1 + 2*1 + 4*0 + 1*-1 + 1*1 + 1*0 + 0*-1 = 2 \\ d &= 0*1 + 0*0 + 0*-1 + 4*1 + 1*0 + 0*-1 + 1*1 + 0*0 + 1*-1 = 4 \\ e &= 0*1 + 0*0 + 0*-1 + 1*1 + 0*0 + 0*-1 + 0*1 + 1*0 + 0*-1 = 1 \\ f &= 0*1 + 0*0 + 2*-1 + 0*1 + 3*0 + 1*-1 + 0*1 + 2*0 + 4*-1 = -7 \\ g &= 0*1 + 2*0 + 4*-1 + 3*1 + 1*0 + 1*-1 + 2*1 + 4*0 + 1*-1 = -1 \\ h &= 2*1 + 4*0 + 1*-1 + 1*1 + 1*0 + 0*-1 + 4*1 + 1*0 + 0*-1 = 6 \\ i &= 4*1 + 1*0 + 0*-1 + 1*1 + 0*0 + 1*-1 + 1*1 + 0*0 + 1*-1 = 4 \\ j &= 1*1 + 0*0 + 0*-1 + 0*1 + 1*0 + 2*-1 + 0*1 + 1*0 + 1*-1 = 1 \\ k &= 0*1 + 3*0 + 1*-1 + 0*1 + 2*0 + 4*-1 + 0*1 + 2*0 + 0*-1 = -5 \\ l &= 3*1 + 1*0 + 1*-1 + 2*1 + 4*0 + 1*-1 + 2*1 + 0*0 + 5*-1 = 0 \\ m &= 1*1 + 1*0 + 0*-1 + 4*1 + 1*0 + 0*-1 + 0*1 + 5*0 + 2*-1 = 3 \\ n &= 1*1 + 0*0 + 1*-1 + 1*1 + 0*0 + 1*-1 + 5*1 + 2*0 + 2*-1 = 3 \\ o &= 0*1 + 1*0 + 0*-1 + 0*1 + 1*0 + 0*-1 + 2*1 + 2*0 + 0*-1 = 2 \\ p &= 0*1 + 2*0 + 4*-1 + 0*1 + 2*0 + 0*-1 + 0*1 + 0*0 + 1*-1 = -5 \\ q &= 2*1 + 4*0 + 1*-1 + 2*1 + 0*0 + 5*-1 + 0*1 + 1*0 + 3*-1 = -5 \\ r &= 4*1 + 1*0 + 0*-1 + 0*1 + 5*0 + 2*-1 + 1*1 + 3*0 + 2*-1 = 1 \\ s &= 1*1 + 0*0 + 1*-1 + 5*1 + 2*0 + 2*-1 + 3*1 + 2*0 + 1*-1 = 5 \\ t &= 0*1 + 1*0 + 0*-1 + 2*1 + 2*0 + 0*-1 + 2*1 + 1*0 + 0*-1 = 4 \\ u &= 0*1 + 2*0 + 0*-1 + 0*1 + 0*0 + 1*-1 + 0*1 + 0*0 + 0*-1 = -1 \\ v &= 2*1 + 0*0 + 5*-1 + 0*1 + 1*0 + 3*-1 + 0*1 + 0*0 + 0*-1 = -6 \\ w &= 0*1 + 5*0 + 2*-1 + 1*1 + 3*0 + 2*-1 + 0*1 + 0*0 + 0*-1 = -3 \\ x &= 5*1 + 2*0 + 2*-1 + 3*1 + 2*0 + 1*-1 + 0*1 + 0*0 + 0*-1 = 5 \\ y &= 2*1 + 2*0 + 0*-1 + 2*1 + 1*0 + 0*-1 + 0*1 + 0*0 + 0*-1 = 4 \end{aligned}$$

$$output = \begin{bmatrix} -3 & -2 & 2 & 4 & 1 \\ -7 & -1 & 6 & 4 & 1 \\ -5 & 0 & 3 & 3 & 2 \\ -5 & -5 & 1 & 5 & 4 \\ -1 & -6 & -3 & 5 & 4 \end{bmatrix}$$

3. Zero padding of 2 + convolution with stride of 2

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 5 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$N = 9, F = 3, \text{stride} = 2$$

$$= (N-F)/\text{stride} + 1 = (9-3)/2 + 1 = 4, \text{ so the output will be } 4 \times 4 \text{ matrix}$$

$$output = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

This question also require same calculation just that in this we will have a stride of 2

$$\begin{aligned} a &= 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 = 0 \\ b &= 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 + 0*1 + 2*0 + 4*-1 = -4 \\ c &= 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 + 4*1 + 1*0 + 0*-1 = 4 \\ d &= 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 = 0 \\ e &= 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 3*-1 + 0*1 + 0*0 + 2*-1 = -5 \\ f &= 0*1 + 2*0 + 4*-1 + 3*1 + 1*0 + 1*-1 + 2*1 + 4*0 + 1*-1 = -1 \\ g &= 4*1 + 1*0 + 0*-1 + 1*1 + 0*0 + 1*-1 + 1*1 + 0*0 + 1*-1 = 4 \\ h &= 0*1 + 0*0 + 0*-1 + 1*1 + 0*0 + 0*-1 + 1*1 + 0*0 + 0*-1 = 2 \\ i &= 0*1 + 0*0 + 2*-1 + 0*1 + 0*0 + 2*-1 + 0*1 + 0*0 + 0*-1 = -4 \\ j &= 2*1 + 4*0 + 1*-1 + 2*1 + 0*0 + 5*-1 + 0*1 + 1*0 + 3*-1 = -5 \\ k &= 1*1 + 0*0 + 1*-1 + 5*1 + 2*0 + 2*-1 + 3*1 + 2*0 + 1*-1 = 5 \\ l &= 1*1 + 0*0 + 0*-1 + 2*1 + 0*0 + 0*-1 + 1*1 + 0*0 + 0*-1 = 4 \\ m &= 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 = 0 \\ n &= 0*1 + 1*0 + 3*-1 + 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 = -3 \\ o &= 3*1 + 2*0 + 1*-1 + 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 = 2 \end{aligned}$$

$$p = 1*1 + 1*0 + 3*-1 + 0*1 + 0*0 + 0*-1 + 0*1 + 0*0 + 0*-1 = 1$$

$$output = \begin{bmatrix} 0 & -4 & 4 & 0 \\ -5 & -1 & 4 & 2 \\ -4 & -5 & 5 & 4 \\ 0 & -3 & 2 & 1 \end{bmatrix}$$

4. Convolution with stride of 1 + max pooling of 3 with stride of 1

So the output for the Convolution with stride of 1 from part 1 is:

$$output = \begin{bmatrix} -1 & 6 & 4 \\ 0 & 3 & 3 \\ -5 & 1 & 5 \end{bmatrix}$$

For max pooling with a pool size of 3x3 and a stride of 1 on this convolved feature, since the pool size is the same as the size of the convolved feature, the max pooling operation would consider the entire matrix at once.

So the max pooling would just be finding the maximum value in this entire 3x3 convolved matrix:

$$\max(output) = 6$$

5. Zero padding of 2 + convolution with stride of 1 + max pooling of 3 with stride of 1

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 5 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$N = 9, F = 3, \text{stride} = 1$$

$$= (N-F)/\text{stride} + 1 = (9-3)/1 + 1 = 7, \text{ so the output will be } 7 \times 7 \text{ matrix}$$

So similar to the calculations in 1,2,3; I will calculate the output matrix with stride 1

$$output = \begin{bmatrix} 0 & -2 & -4 & 1 & 4 & 1 & 0 \\ -3 & -3 & -2 & 2 & 4 & 1 & 1 \\ -5 & -7 & -1 & 6 & 4 & 1 & 2 \\ -7 & -5 & 0 & 3 & 3 & 2 & 4 \\ -4 & -5 & -5 & 1 & 5 & 4 & 4 \\ -2 & -1 & -6 & -3 & 5 & 4 & 3 \\ 0 & -1 & -3 & -1 & 2 & 2 & 1 \end{bmatrix}$$

By applying max pooling on this convolved feature with pooling size of 3x3 with a stride of 1:

$$result = \begin{bmatrix} 0 & 6 & 6 & 6 & 4 \\ 0 & 6 & 6 & 6 & 4 \\ 0 & 6 & 6 & 6 & 5 \\ 0 & 3 & 5 & 5 & 5 \\ 0 & 1 & 5 & 5 & 5 \end{bmatrix}$$