

## CHAPTER 1—PROBLEM SOLUTIONS

1.1 (a)  $I = \frac{V}{R} = \frac{10 \text{ V}}{1 \text{ k}\Omega} = 10 \text{ mA}$

(b)  $R = \frac{V}{I} = \frac{10 \text{ V}}{1 \text{ mA}} = 10 \text{ k}\Omega$

(c)  $V = IR = 10 \text{ mA} \times 10 \text{ k}\Omega = 100 \text{ V}$

(d)  $I = \frac{V}{R} = \frac{10 \text{ V}}{100 \text{ }\Omega} = 0.1 \text{ A}$

*Note:* Volts, millamps, and kilo-ohms constitute a consistent set of units.

1.2 (a)  $P = I^2R = (30 \times 10^{-3})^2 \times 1 \times 10^3 = 0.9 \text{ W}$

Thus,  $R$  should have a 1-W rating.

(b)  $P = I^2R = (40 \times 10^{-3})^2 \times 1 \times 10^3 = 1.6 \text{ W}$

Thus, the resistor should have a 2-W rating.

(c)  $P = I^2R = (3 \times 10^{-3})^2 \times 10 \times 10^3 = 0.09 \text{ W}$

Thus, the resistor should have a  $\frac{1}{8}$ -W rating.

(d)  $P = I^2R = (4 \times 10^{-3})^2 \times 10 \times 10^3 = 0.16 \text{ W}$

Thus, the resistor should have a  $\frac{1}{4}$ -W rating.

(e)  $P = V^2/R = 20^2/(1 \times 10^3) = 0.4 \text{ W}$

Thus, the resistor should have a  $\frac{1}{2}$ -W rating.

(f)  $P = V^2/R = 11^2/(1 \times 10^3) = 0.121 \text{ W}$

Thus, a rating of  $\frac{1}{8}$  W should theoretically suffice though  $\frac{1}{4}$  W would be prudent to allow for consistent tolerances and measurement errors.

1.3 (a)  $V = IR = 10 \text{ mA} \times 1 \text{ k}\Omega = 10 \text{ V}$   
 $P = I^2 R = (10 \text{ mA})^2 \times 1 \text{ k}\Omega = 100 \text{ mW}$

(b)  $R = V/I = 10 \text{ V}/1 \text{ mA} = 10 \text{ k}\Omega$   
 $P = VI = 10 \text{ V} \times 1 \text{ mA} = 10 \text{ mW}$

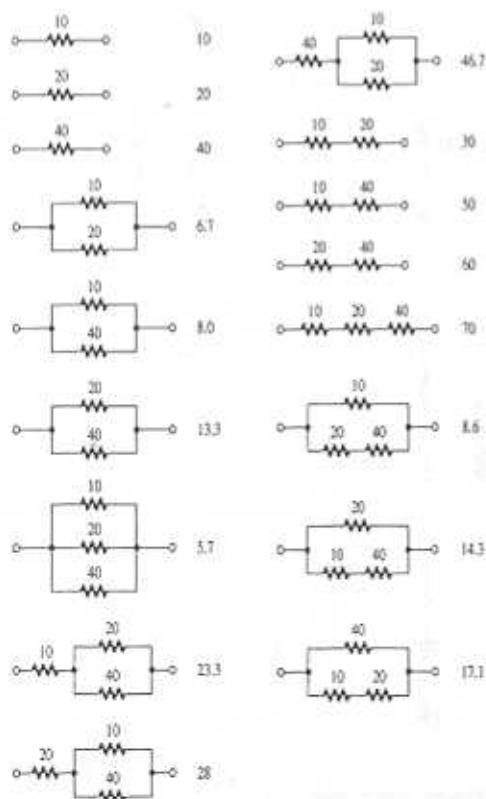
(c)  $I = P/V = 1 \text{ W}/10 \text{ V} = 0.1 \text{ A}$   
 $R = V/I = 10 \text{ V}/0.1 \text{ A} = 100 \Omega$

(d)  $V = P/I = 0.1 \text{ W}/10 \text{ mA} = 100 \text{ mW}/10 \text{ mA} = 10 \text{ V}$   
 $R = V/I = 10 \text{ V}/10 \text{ mA} = 1 \text{ k}\Omega$

(e)  $P = I^2 R \Rightarrow I = \sqrt{P/R}$   
 $I = \sqrt{1000 \text{ mW}/1 \text{ k}\Omega} = 31.6 \text{ mA}$   
 $V = IR = 31.6 \text{ mA} \times 1 \text{ k}\Omega = 31.6 \text{ V}$

Note: V, mA, kΩ, and mW constitute a consistent set of units.

1.4



Thus, there are 17 possible resistance values.

1.5 Shunting the 10 kΩ by a resistor of value  $R$  results in the combination having a resistance  $R_{eq}$ :

$$R_{eq} = \frac{10R}{R+10}$$

Thus, for a 1% reduction,

$$\frac{R}{R+10} = 0.99 \Rightarrow R = 990 \text{ k}\Omega$$

For a 5% reduction,

$$\frac{R}{R+10} = 0.95 \Rightarrow R = 190 \text{ k}\Omega$$

For a 10% reduction,

$$\frac{R}{R+10} = 0.90 \Rightarrow R = 90 \text{ k}\Omega$$

For a 50% reduction,

$$\frac{R}{R+10} = 0.50 \Rightarrow R = 10 \text{ k}\Omega$$

Shunting the 10 kΩ by

(a) 1 MΩ result in

$$R_{eq} = \frac{10 \times 1000}{1000 + 10} = \frac{10}{1.01} = 9.9 \text{ k}\Omega, \text{ a } 1\% \text{ reduction;}$$

(b) 100 kΩ results in

$$R_{eq} = \frac{10 \times 100}{100 + 10} = \frac{10}{1.1} = 9.09 \text{ k}\Omega, \text{ a } 9.1\% \text{ reduction;}$$

(c) 10 kΩ results in

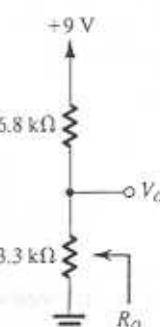
$$R_{eq} = \frac{10}{10 + 10} = 5 \text{ k}\Omega, \text{ a } 50\% \text{ reduction;}$$

1.6  $V_O = V_{DD} \frac{R_2}{R_1 + R_2}$

To find  $R_O$ , we short circuit  $V_{DD}$  and look back into node X<sub>1</sub>,

$$R_O = R_2 \parallel R_1 = \frac{R_1 R_2}{R_1 + R_2}$$

1.7



$$V_O = 9 \frac{3.3}{3.3 + 6.8} \\ = 2.94 \text{ V}$$

$$R_O = 2.22 \text{ k}\Omega$$

For  $\pm 5\%$  resistor tolerance the extreme values of  $V_O$  are

$$V_{O,\text{low}} = 9 \frac{3.3(1 - 0.05)}{3.3(1 - 0.05) + 6.8(1 + 0.05)} \\ = 2.75 \text{ V}$$

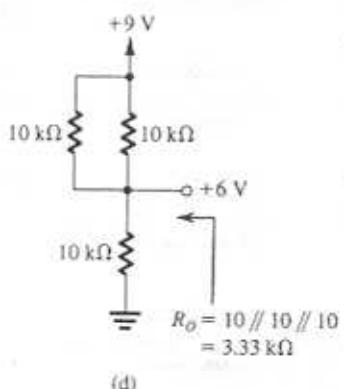
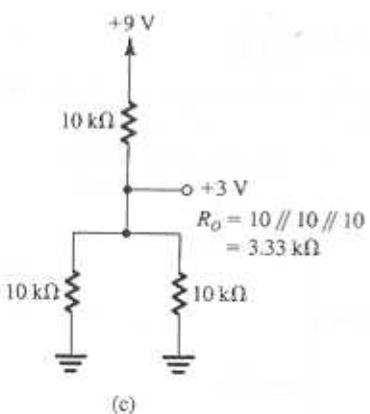
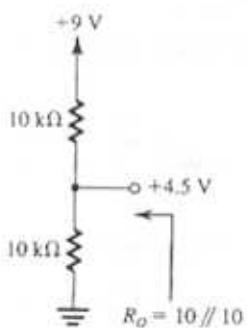
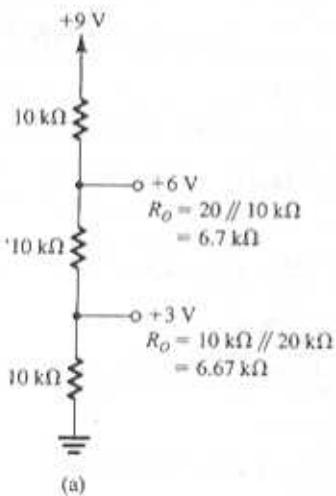
$$V_{O,\text{high}} = 9 \frac{3.3(1 + 0.05)}{3(1 + 0.05) + 6.8(1 - 0.05)} \\ = 3.14 \text{ V}$$

The extreme values of  $R_O$  are

$$R_{O,\text{low}} = \frac{3.3(1 - 0.05) \times 6.8(1 - 0.05)}{3.3(1 - 0.05) + 6.8(1 - 0.05)} \\ = 2.22(1 - 0.05) = 2.11 \text{ k}\Omega$$

$$R_{O,\text{high}} = 2.22(1 + 0.05) = 2.33 \text{ k}\Omega$$

1.8

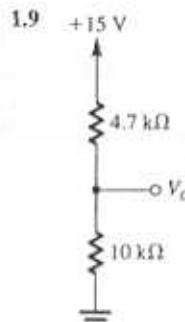


Voltages generated:

$+3 \text{ V}$  (two ways: (a) and (c) with (c) having lower output resistance)

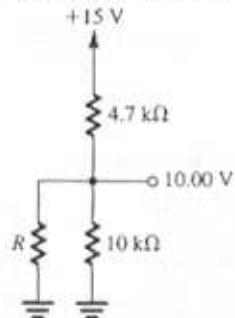
$+4.5 \text{ V}$  (b)

$+6 \text{ V}$  (two ways: (a) and (d) with (d) having a lower output resistance)



$$V_O = 15 \frac{10}{10 + 4.7} = 10.2 \text{ V}$$

To reduce  $V_O$  to 10.00 V we shunt the 10-kΩ resistor by a resistor  $R$  whose value is such that  $10 \parallel R = 2 \times 4.7$ .



Thus

$$\frac{1}{10} + \frac{1}{R} = \frac{1}{9.4}$$

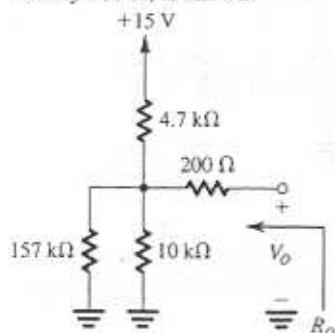
$$\Rightarrow R = 156.7 \approx 157 \text{ k}\Omega$$

Now,

$$R_O = 10 \text{ k}\Omega \parallel R \parallel 4.7 \text{ k}\Omega$$

$$= 9.4 \parallel 4.7 = \frac{9.4}{3} = 3.133 \text{ k}\Omega$$

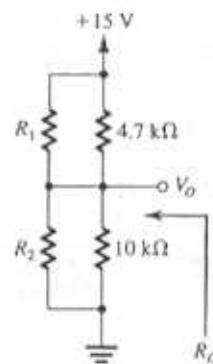
To make  $R_O = 3.33$  we add a series resistance of approximately 200 Ω, as shown.



To obtain  $V_O = 10.00 \text{ V}$  and  $R_O = 3 \text{ k}\Omega$  we have to shunt both the 4.7-kΩ and the 10-kΩ resistors as shown. To yield an output voltage  $V_O = 10.00 \text{ V}$  we must have

$$\frac{(R_2 \parallel 10)}{R'_2} = \frac{2(R_1 \parallel 4.7)}{R'_1}$$

$$R'_2 = 2R'_1 \quad (1)$$



For  $R_O = 3 \text{ k}\Omega$  we must have

$$R'_1 \parallel R'_2 = 3 \quad (2)$$

Solving (1) and (2) yields

$$R'_1 = 4.5 \text{ k}\Omega$$

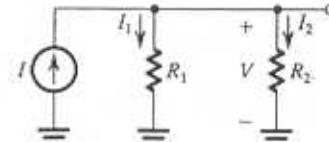
$$R'_2 = 9.0 \text{ k}\Omega$$

which can be used to find  $R_1$  and  $R_2$  respectively,

$$R_1 = 157 \text{ k}\Omega$$

$$R_2 = 90 \text{ k}\Omega$$

1.10



$$V = I(R_1 \parallel R_2)$$

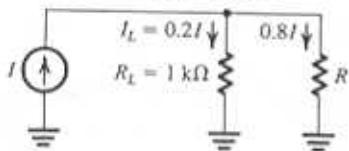
$$= I \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V}{R_1} = I \frac{R_2}{R_1 + R_2}$$

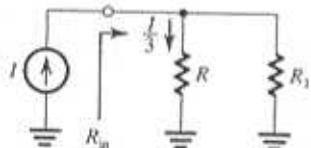
$$I_2 = \frac{V}{R_2} = I \frac{R_1}{R_1 + R_2}$$

**1.11** Connect a resistor  $R$  in parallel with  $R_L$ . To make  $I_L = 0.2I$  (and thus the current through  $R$ ,  $0.8I$ ),  $R$  should be such

$$0.2I \times 1 \text{ k}\Omega = 0.8I R \\ \Rightarrow R = 250 \text{ }\Omega$$



**1.12**



To make the current through  $R$  equal to  $I/3$  we shunt  $R$  by a resistance  $R_1$  of value such that the current through it will be  $2I/3$ ; thus

$$\frac{1}{3}R = \frac{2I}{3}R_1 \Rightarrow R_1 = \frac{R}{2}$$

The input resistance of the divider,  $R_{in}$ , is

$$R_{in} = R \parallel R_1 = R \parallel \frac{R}{2} = \frac{1}{3}R$$

Now if  $R_1$  is 10% too high, i.e.,

$$R_1 = 1.1 \frac{R}{2}$$

the problem can be solved in two ways:

(a) Connect a resistor  $R_2$  across  $R_1$  of value such that  $R_1 \parallel R_2 = R/2$ , thus

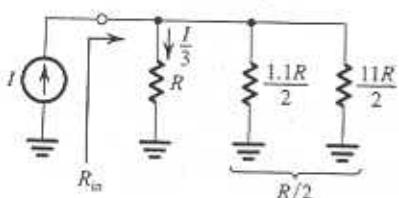
$$\frac{R_2(1.1R/2)}{R_2 + (1.1R/2)} = \frac{R}{2}$$

$$1.1R_2 = R_2 + \frac{1.1R}{2}$$

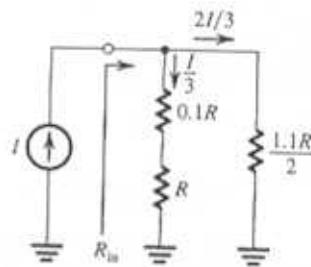
$$\Rightarrow R_2 = \frac{11R}{2} = 5.5R$$

$$R_{in} = R \parallel \frac{1.1R}{2} \parallel \frac{11R}{2}$$

$$= R \parallel \frac{R}{2} = \frac{R}{3}$$



(b) Connect a resistor in series with the load resistor  $R$  so as to raise the resistance of the load branch by 10%, thereby restoring the current division ratio to its desired value. The added series resistance must be 10% of  $R$  i.e.,  $0.1R$ .



$$R_{in} = 1.1R \parallel \frac{1.1R}{2} \\ = \frac{1.1R}{3}$$

i.e., 10% higher than in case (a).

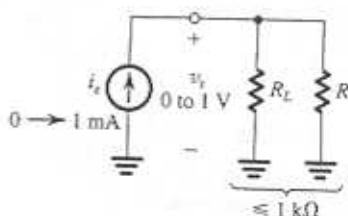
**1.13** If  $R_L = 10 \text{ k}\Omega$

then a voltage of 0 to 10 V may develop across the source. To limit the voltage to the specified maximum of 1 V, we have to shunt  $R_L$  with a resistor  $R$  whose value is such that the parallel combination of  $R_L$  and  $R$  is  $\leq 1 \text{ k}\Omega$ . Thus,

$$\frac{RR_L}{R + R_L} \leq 1$$

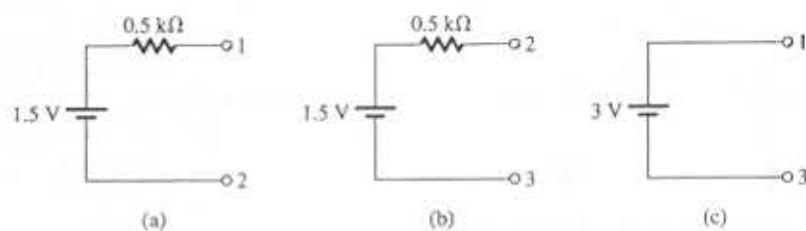
$$R \leq 1.111 \text{ k}\Omega$$

$$\Rightarrow R = 1.1 \text{ k}\Omega$$

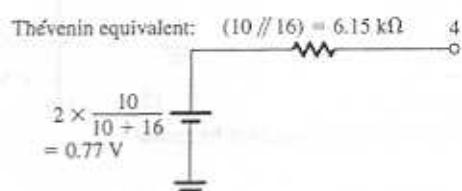
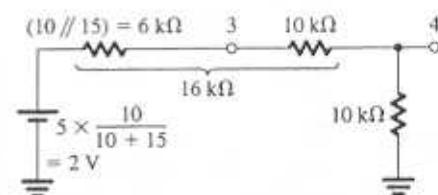
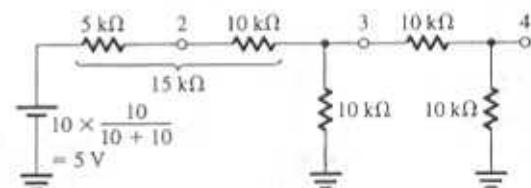
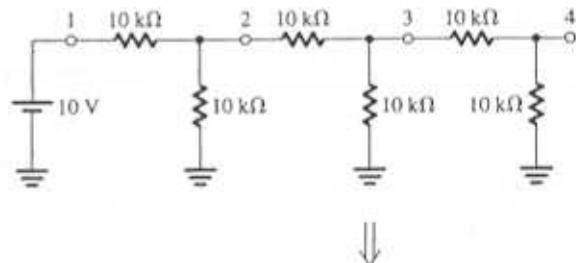


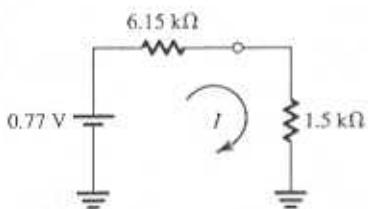
The resulting circuit, utilizing only one additional resistor of value  $1.1 \text{ k}\Omega$  creates a current divider across the source.

1.14



1.15





Now, when a resistance of  $1.5 \text{ k}\Omega$  is connected between 4 and ground,

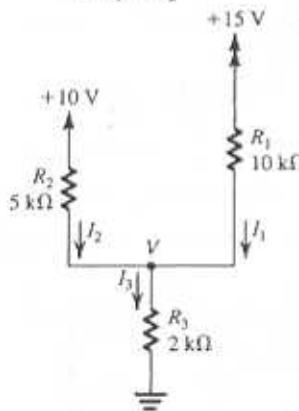
$$I = \frac{0.77}{6.15 + 1.5} \\ = 0.1 \text{ mA}$$

**1.16 (a)** Node equation at the common node yields

$$I_3 = I_1 + I_2$$

Using the fact that the sum of the voltage drops across  $R_1$  and  $R_3$  equals 15 V, we write

$$15 = I_1 R_1 + I_3 R_3 \\ = 10I_1 + (I_1 + I_2) \times 2 \\ = 12I_1 + 2I_2$$



That is,

$$12I_1 + 2I_2 = 15 \quad (1)$$

Similarly, the voltage drops across  $R_2$  and  $R_3$  add up to 10 V, thus

$$10 = I_2 R_2 + I_3 R_3 \\ = 5I_2 + (I_1 + I_2) \times 2$$

which yields

$$2I_1 + 7I_2 = 10 \quad (2)$$

Equations (1) and (2) can be solved together by multiplying (2) by 6.

$$12I_1 + 42I_2 = 60 \quad (3)$$

Now, subtracting (1) from (3) yields

$$40I_2 = 45 \\ \Rightarrow I_2 = 1.125 \text{ mA}$$

Substituting in (2) gives

$$2I_1 = 10 - 7 \times 1.125 \text{ mA} \\ \Rightarrow I_1 = 1.0625 \text{ mA} \\ I_3 = I_1 + I_2 \\ = 1.0625 + 1.125 \\ = 1.1875 \text{ mA}$$

$$V = I_3 R_3 \\ = 1.1875 \times 2 = 2.3750 \text{ V}$$

To summarize:

$$I_1 = 1.06 \text{ mA} \quad I_2 = 1.13 \text{ mA} \\ I_3 = 1.19 \text{ mA} \quad V = 2.38 \text{ V}$$

**(b)** A node equation at the common node can be written in terms of  $V$  as

$$\frac{15 - V}{R_1} + \frac{10 - V}{R_2} = \frac{V}{R_3}$$

Thus,

$$\frac{15 - V}{10} + \frac{10 - V}{5} = \frac{V}{2} \\ \Rightarrow 0.8V = 3.5 \\ \Rightarrow V = 2.375 \text{ V}$$

Now,  $I_1$ ,  $I_2$ , and  $I_3$  can be easily found as

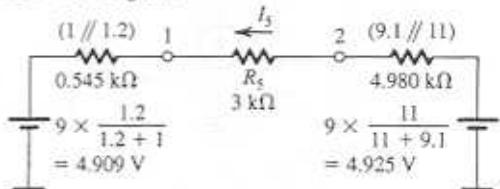
$$I_1 = \frac{15 - V}{10} = \frac{15 - 2.375}{10} = 1.0625 \text{ mA} = 1.06 \text{ mA}$$

$$I_2 = \frac{10 - V}{5} = \frac{10 - 2.375}{5} = 1.125 \text{ mA} = 1.13 \text{ mA}$$

$$I_3 = \frac{V}{R_3} = \frac{2.375}{2} = 1.1875 \text{ mA} = 1.19 \text{ mA}$$

Method (b) is much preferred; faster, more insightful and less prone to errors. In general, one attempts to identify the least possible number of variables and write the corresponding minimum number of equations.

**1.17** See diagram

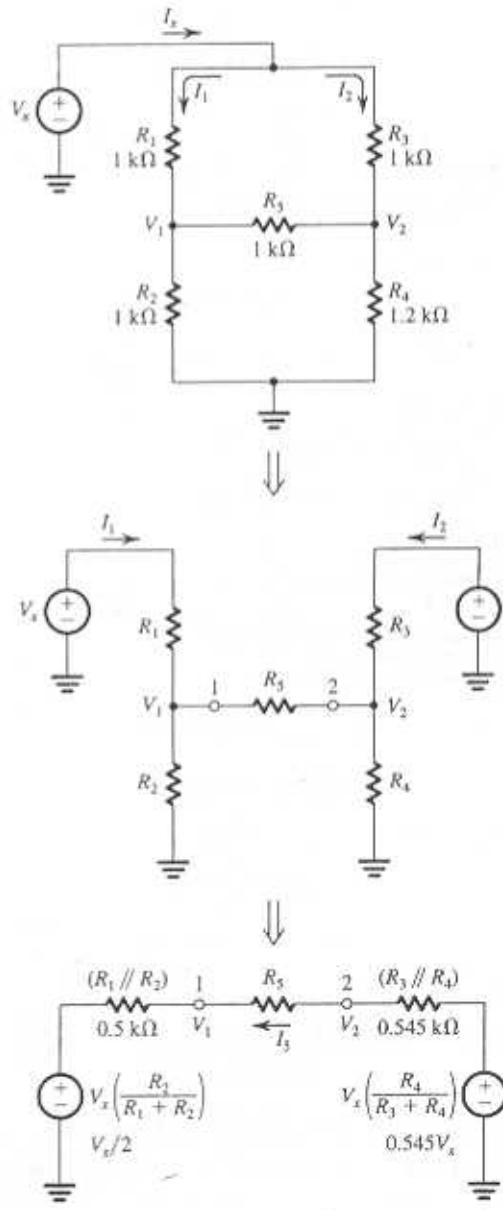
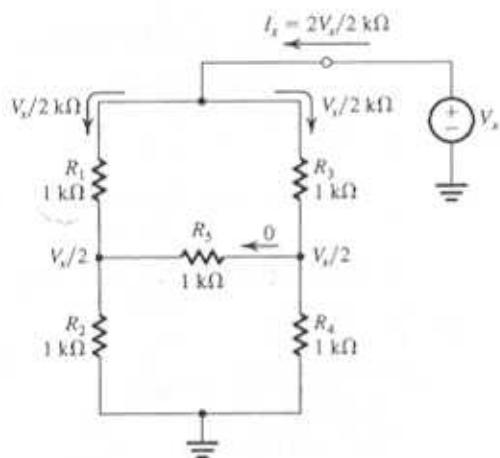


$$I_5 = \frac{4.925 - 4.909}{4.98 + 3 + 0.545} = 1.88 \mu\text{A}$$

$$V_5 = 1.88 \mu\text{A} \times 3 \text{ k}\Omega = 5.64 \text{ mV}$$

L.18 From the symmetry of the circuit, there will be no current in  $R_5$ . (Otherwise the symmetry would be violated.) Thus each branch will carry a current  $V_x/2 \text{ k}\Omega$  and  $I_x$  will be the sum of the two currents.

$$I_x = \frac{2V_x}{2 \text{ k}\Omega} = \frac{V_x}{1 \text{ k}\Omega}$$



Thus,

$$R_{eq} \equiv \frac{V_x}{I_x} = 1 \text{ k}\Omega$$

Now, if  $R_4$  is raised to  $1.2 \text{ k}\Omega$  the symmetry will be broken. To find  $I_x$  we use Thévenin's theorem as follows:

$$I_5 = \frac{0.545V_x - 0.5V_x}{0.5 + 1 + 0.545} = 0.022V_x$$

$$V_T = \frac{V_x}{2} + 0.022V_x \times 0.5 \\ = 0.5V_x \times 1.022 = 0.511V_x$$

$$V_2 = V_1 + I_5 R_5 = 0.533V_x$$

$$I_1 = \frac{V_x - V_1}{1 \text{ k}\Omega} = 0.489V_x$$

$$I_2 = \frac{V_x - V_2}{1 \text{ k}\Omega} = 0.467V_x$$

$$I_x = I_1 + I_2 = 0.956V_x$$

$$\Rightarrow R_{eq} \equiv \frac{V_x}{I_x} = 1.05 \text{ k}\Omega$$

1.19 (a)  $T = 10^{-4} \text{ ms} = 10^{-7} \text{ s}$

$$f = \frac{1}{T} = 10^7 \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^7 \text{ rad/s}$$

(b)  $f = 1 \text{ GHz} = 10^9 \text{ Hz}$

$$T = \frac{1}{f} = 10^{-9} \text{ s}$$

$$\omega = 2\pi f = 6.28 \times 10^9 \text{ rad/s}$$

(c)  $\omega = 6.28 \times 10^2 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 10^{-2} \text{ s}$$

(d)  $T = 10 \text{ s}$

$$f = \frac{1}{T} = 10^{-1} \text{ Hz}$$

$$\omega = 2\pi f = 6.28 \times 10^{-1} \text{ rad/s}$$

(e)  $f = 60 \text{ Hz}$

$$T = \frac{1}{f} = 1.67 \times 10^{-2} \text{ s}$$

$$\omega = 2\pi f = 3.77 \times 10^2 \text{ rad/s}$$

(f)  $\omega = 1 \text{ krad/s} = 10^3 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = 1.59 \times 10^2 \text{ Hz}$$

$$T = \frac{1}{f} = 6.28 \times 10^{-3} \text{ s}$$

(g)  $f = 1900 \text{ MHz} = 1.9 \times 10^9 \text{ Hz}$

$$T = \frac{1}{f} = 0.526 \times 10^{-9} \text{ s}$$

$$\omega = 2\pi f = 1.194 \times 10^9 \text{ rad/s}$$

1.20 (a)  $Z = 1 \text{ k}\Omega$  at all frequencies

(b)  $Z = 1/j\omega C = -j \frac{1}{2\pi f \times 10 \times 10^{-9}}$

At  $f = 60 \text{ Hz}$ ,  $Z = -j265 \text{ k}\Omega$

At  $f = 100 \text{ kHz}$ ,  $Z = -j159 \text{ }\Omega$

At  $f = 1 \text{ GHz}$ ,  $Z = -j0.016 \text{ }\Omega$

(c)  $Z = 1/j\omega C = -j \frac{1}{2\pi f \times 2 \times 10^{-12}}$

At  $f = 60 \text{ Hz}$ ,  $Z = -j1.33 \text{ G}\Omega$

At  $f = 100 \text{ kHz}$ ,  $Z = -j0.8 \text{ M}\Omega$

At  $f = 1 \text{ GHz}$ ,  $Z = -j79.6 \text{ }\Omega$

(d)  $Z = j\omega L = j2\pi f L = j2\pi f \times 10 \times 10^{-3}$

At  $f = 60 \text{ Hz}$ ,  $Z = j3.77 \text{ }\Omega$

At  $f = 100 \text{ kHz}$ ,  $Z = j6.28 \text{ k}\Omega$

At  $f = 1 \text{ GHz}$ ,  $Z = j62.8 \text{ M}\Omega$

1.21 (a)  $Z = R + \frac{1}{j\omega C}$

$$= 10^3 + \frac{1}{j2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}} \\ = (1 - j1.59) \text{ k}\Omega$$

(b)  $Y = \frac{1}{R} + j\omega C$

$$= \frac{1}{10^3} + j2\pi \times 10 \times 10^3 \times 0.01 \times 10^{-6} \\ = 10^{-3}(1 + j0.628) \text{ S}$$

$$Z = \frac{1}{Y} = \frac{1000}{1 + j0.628}$$

$$= \frac{1000(1 - j0.628)}{1 + 0.628^2} \\ = (717.2 - j450.4) \text{ }\Omega$$

(c)  $Y = \frac{1}{R} + j\omega C$

$$= \frac{1}{100 \times 10^3} + j2\pi \times 10 \times 10^3 \times 100 \times 10^{-12} \\ = 10^{-5}(1 + j0.628)$$

$$Z = \frac{10^5}{1 + j0.628}$$

$$= (71.72 - j45.04) \text{ k}\Omega$$

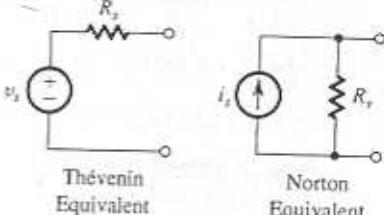
(d)  $Z = R + j\omega L$

$$= 100 + j2\pi \times 10 \times 10^3 \times 10 \times 10^{-3}$$

$$= 100 + j6.28 \times 100$$

$$= (100 + j628) \text{ }\Omega$$

1.22



$$v_{oc} = v_i$$

$$i_{sc} = i_s$$

$$v_i = i_s R_t$$

Thus,

$$R_t = \frac{v_{oc}}{i_{sc}}$$

(a)  $v_i = v_{oc} = 10 \text{ V}$

$$i_s = i_{sc} = 100 \mu\text{A}$$

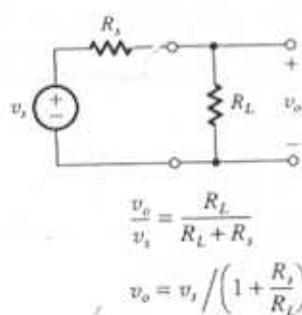
$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{10 \text{ V}}{100 \mu\text{A}} = 0.1 \text{ M}\Omega = 100 \text{ k}\Omega$$

(b)  $v_i = v_{oc} = 0.1 \text{ V}$

$$i_s = i_{sc} = 10 \mu\text{A}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{0.1 \text{ V}}{10 \mu\text{A}} = 0.01 \text{ M}\Omega = 10 \text{ k}\Omega$$

1.23



Thus,

$$\frac{v_i}{1 + \frac{R_s}{100}} = 30$$

and

$$\frac{v_i}{1 + \frac{R_s}{10}} = 10$$

Dividing (1) by (2) gives

$$\frac{1 + (R_s/10)}{1 + (R_s/100)} = 3$$

$$\Rightarrow R_s = 28.6 \text{ k}\Omega$$

Substituting in (2) gives

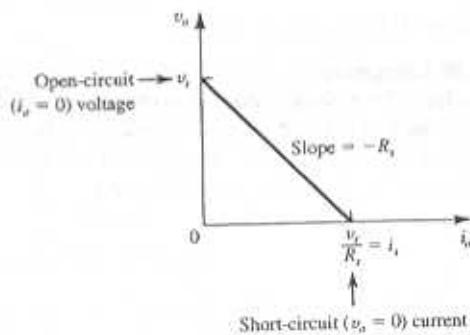
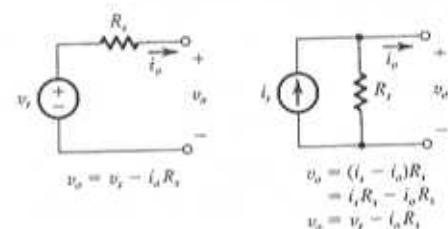
$$v_i = 38.6 \text{ mV}$$

The Norton current  $i_s$  can be found as

$$i_s = \frac{v_i}{R_t} = \frac{38.6 \text{ mV}}{28.6 \text{ k}\Omega} = 1.35 \mu\text{A}$$

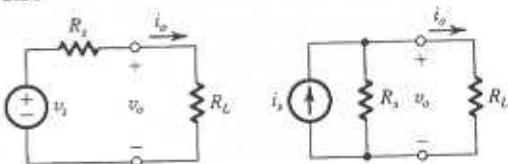
1.24 The observed output voltage is 1 mV/°C which is one half the voltage specified by the sensor, presumably under open-circuit conditions that is without a load connected. It follows that that sensor internal resistance must be equal to  $R_L$ , i.e., 10 kΩ.

1.25



(1) 1.26

(2)



$R_t$  represents the input resistance of the processor

For  $v_o = 0.9 v_i$ ,

$$0.9 = \frac{R_L}{R_L + R_t} \Rightarrow R_L = 9R_t$$

For  $i_o = 0.9 i_s$ ,

$$0.9 = \frac{R_t}{R_t + R_L} \Rightarrow R_L = R_t/9$$

1.27

Case	$\omega$ [rad/s]	f (Hz)	T (s)
a	$6.28 \times 10^9$	$1 \times 10^9$	$1 \times 10^{-9}$
b	$1 \times 10^9$	$1.59 \times 10^8$	$6.28 \times 10^{-9}$
c	$6.28 \times 10^{10}$	$1 \times 10^{10}$	$1 \times 10^{-10}$
d	$3.77 \times 10^2$	60	$1.67 \times 10^{-2}$
e	$6.28 \times 10^3$	$1 \times 10^3$	$1 \times 10^{-3}$
f	$6.28 \times 10^6$	$1 \times 10^6$	$1 \times 10^{-6}$

1.28 (a)  $V_{\text{peak}} = 117 \times \sqrt{2} = 165$  V

(b)  $V_{\text{rms}} = 33.9/\sqrt{2} = 24$  V

(c)  $V_{\text{peak}} = 220 \times \sqrt{2} = 311$  V

(d)  $V_{\text{peak}} = 220 \times \sqrt{2} = 311$  kV

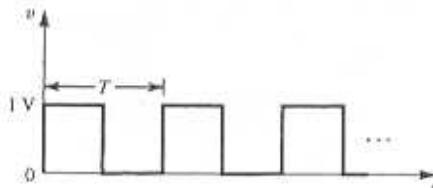
1.29 (a)  $v = 10 \sin(2\pi \times 10^4 t)$ , V

(b)  $v = 120\sqrt{2} \sin(2\pi \times 60)$ , V

(c)  $v = 0.1 \sin(1000t)$ , V

(d)  $v = 0.1 \sin(2\pi \times 10^4 t)$ , V

1.30 Comparing the given waveform to that described by Eq. 1.2 we observe that the given waveform has an amplitude of 0.5 V (1 V peak-to-peak) and its level is shifted up by 0.5 V (the first term in the equation). Thus the waveform look as follows.



Average value = 0.5 V

Peak-to-peak value = 1 V

Lowest value = 0 V

Highest value = 1 V

Period  $T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 10^{-3}$  s

1.31 The two harmonics have the ratio  $126/98 = 9/7$ . Thus, these are the 7th and 9th harmonics. From Eq. 1.2 we note that the amplitudes of these two harmonics will have the ratio 7 to 9, which is confirmed by the measurement reported. Thus the fundamental will have a frequency of  $98/7$  or 14 kHz and peak amplitude of  $63 \times 7 = 441$  mV. The rms value of the fundamental will be  $441/\sqrt{2} = 312$  mV. To find the peak-to-peak amplitude of the square wave we note

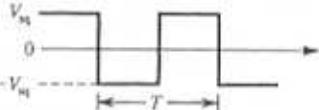
that  $4V/\pi = 441$  mV. Thus,

$$\text{Peak-to-peak amplitude} = 2V = 441 \times \frac{\pi}{2} = 693 \text{ mV}$$

$$\text{Period } T = \frac{1}{f} = \frac{1}{14 \times 10^3} = 71.4 \mu\text{s}$$

1.32 To be barely audible by a relatively young listener, the 5th harmonic must be limited to 20 kHz; thus the fundamental will be 4 kHz. At the low end, hearing extends down to about 20 Hz. For the fifth and higher to be audible the fifth must be no lower than 20 Hz. Correspondingly, the fundamental will be at 4 Hz.

1.33 If the amplitude of the square wave is  $V_{\text{sq}}$  then the power delivered by the square wave to



a resistance  $R$  will be  $V_{\text{sq}}^2/R$ . If this power is to equal that delivered by a sine wave of peak amplitude  $\hat{V}$  then

$$\frac{V_{\text{sq}}^2}{R} = \frac{(\hat{V}/\sqrt{2})^2}{R}$$

Thus,  $V_{\text{sq}} = \hat{V}/\sqrt{2}$ . This result is independent of frequency.

1.34 Decimal      Binary

0	0
5	101
8	1000
25	11001
57	111001

1.35  $b_3 \ b_2 \ b_1 \ b_0$       Value Represented

0 0 0 0	+0
0 0 0 1	+1
0 0 1 0	+2
0 0 1 1	+3
0 1 0 0	+4
0 1 0 1	+5
0 1 1 0	+6
0 1 1 1	+7
1 0 0 0	-0
1 0 0 1	-1
1 0 1 0	-2
1 0 1 1	-3
1 1 0 0	-4
1 1 0 1	-5
1 1 1 0	-6
1 1 1 1	-7

Note that there are two possible representation of zero: 0000 and 1000. For a 0.5-V step size, analog signals in the range  $\pm 3.5$  V can be represented

Input	Steps	Code
+2.5 V	+5	0101
-3.0 V	-6	1110
+2.7	+5	0101
-2.8	-6	1110

1.36 (a) For  $N$  bits there will be  $2^N$  possible levels, from 0 to  $V_{FS}$ . Thus there will be  $(2^N - 1)$  discrete steps from 0 to  $V_{FS}$  with the step size given by

$$\text{Step size} = \frac{V_{FS}}{2^N - 1}$$

This is the analog change corresponding to a change in the LSB. It is the value of the resolution of the ADC.

(b) The maximum error in conversion occurs when the analog signal value is at the middle of a step. Thus the maximum error is

$$\frac{1}{2} \times \text{step size} = \frac{1}{2} \frac{V_{FS}}{2^N - 1}$$

This is known as the quantization error.

$$(c) \frac{10 \text{ V}}{2^N - 1} \leq 5 \text{ mV}$$

$$2^N - 1 \geq 2000$$

$$2^N \geq 2001 \Rightarrow N = 11$$

For  $N = 11$ ,

$$\text{Resolution} = \frac{10 \text{ V}}{2^{11} - 1} = 4.9 \text{ mV}$$

$$\text{Quantization error} = \frac{4.9}{2} = 2.4 \text{ mV}$$

1.37 When  $b_i = 1$ , the  $i$ th switch is in position 1 and a current ( $V_{ref}/2^i R$ ) flows to the output. Thus  $i_o$  will be the sum of all the currents corresponding to "1" bits, i.e.,

$$i_o = \frac{V_{ref}}{R} \left( \frac{b_1}{2^1} + \frac{b_2}{2^2} + \cdots + \frac{b_N}{2^N} \right)$$

(b)  $b_N$  is the LSB  
 $b_1$  is the MSB

$$(c) i_{O\max} = \frac{10 \text{ V}}{5 \text{ k}\Omega} \left( \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6} \right)$$

$$= 1.96875 \text{ mA}$$

Corresponding to the LSB changing from 0 to 1 the output changes by  $10/5 \times 1/2^6 = 0.03125 \text{ mA}$ .

1.38 There will be 44,100 samples per second with each sample represented by 16 bits. Thus the throughput or speed will be  $44,100 \times 16 = 7.056 \times 10^5$  bits per second.

$$1.39 (a) A_v = \frac{v_o}{v_i} = \frac{10 \text{ V}}{100 \text{ mV}} = 100 \text{ V/V}$$

or,  $20 \log 100 = 40 \text{ dB}$

$$A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{v_i / i_i} = \frac{10 \text{ V} / 100 \Omega}{100 \mu\text{A}} = \frac{0.1 \text{ A}}{100 \mu\text{A}}$$

$$= 1000 \text{ A/A}$$

or,  $20 \log 1000 = 60 \text{ dB}$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i} = 100 \times 1000$$

$$= 10^5 \text{ W/W}$$

or  $10 \log 10^5 = 50 \text{ dB}$

$$(b) A_v = \frac{v_o}{v_i} = \frac{2 \text{ V}}{10 \mu\text{V}} = 2 \times 10^5 \text{ V/V}$$

or,  $20 \log 2 \times 10^5 = 106 \text{ dB}$

$$A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{v_i / i_i} = \frac{2 \text{ V} / 10 \text{ k}\Omega}{100 \text{ nA}} = \frac{0.2 \text{ mA}}{100 \text{ nA}} = \frac{0.2 \times 10^{-3}}{100 \times 10^{-9}} = 2000 \text{ A/A}$$

or  $20 \log A_i = 66 \text{ dB}$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i} = 2 \times 10^5 \times 2000$$

$$= 4 \times 10^8 \text{ W/W}$$

or  $10 \log A_p = 86 \text{ dB}$

$$(c) A_v = \frac{v_o}{v_i} = \frac{10 \text{ V}}{1 \text{ V}} = 10 \text{ V/V}$$

or,  $20 \log 10 = 20 \text{ dB}$

$$A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{v_i / i_i} = \frac{10 \text{ V} / 10 \Omega}{1 \text{ mA}} = \frac{1 \text{ A}}{1 \text{ mA}} = 1000 \text{ A/A}$$

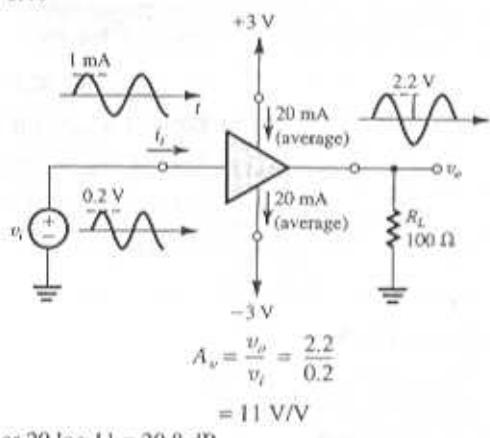
or,  $20 \log 1000 = 60 \text{ dB}$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i} \times \frac{i_o}{i_i}$$

$$= 10 \times 1000 = 10^4 \text{ W/W}$$

or  $10 \log_{10} A_p = 40 \text{ dB}$

1.40



or  $20 \log 11 = 20.8 \text{ dB}$

$$A_i = \frac{i_o}{i_i} = \frac{2.2 \text{ V}/100 \Omega}{1 \text{ mA}}$$

$$= \frac{22 \text{ mA}}{1 \text{ mA}} = 22 \text{ A/A}$$

or,  $20 \log A_i = 26.8 \text{ dB}$

$$A_p = \frac{P_o}{P_i} = \frac{(2.2/\sqrt{2})^2 / 100}{0.2 \times 10^{-3}} / \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 242 \text{ W/W}$$

or,  $10 \log A_p = 23.8 \text{ dB}$

Supply power =  $2 \times 3 \text{ V} \times 20 \text{ mA} = 120 \text{ mW}$

$$\text{Output power} = \frac{v_{o,\text{rms}}^2}{R_L} = \frac{(2.2/\sqrt{2})^2}{100 \Omega} = 24.2 \text{ mW}$$

$$\text{Input power} = \frac{24.2}{242} = 0.1 \text{ mW} \quad (\text{negligible})$$

Amplifier dissipation = Supply power - Output power  
 $= 120 - 24.2 = 95.8 \text{ mW}$

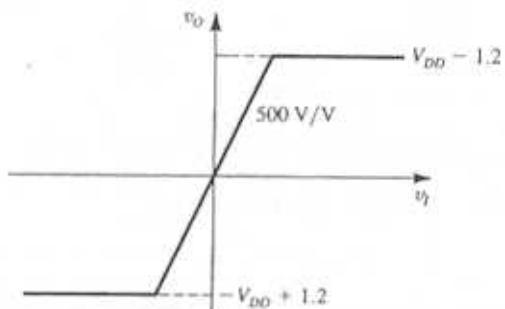
$$\text{Amplifier efficiency} = \frac{\text{Output power}}{\text{Supply power}} \times 100$$

$$= \frac{24.2}{120} \times 100 = 20.2\%$$

1.41 For  $V_{DD} = 5 \text{ V}$ :

The largest undistorted sine-wave output is of 3.8-V peak amplitude or  $3.8/\sqrt{2} = 2.7 \text{ V}_{\text{rms}}$ . Input needed is  $5.4 \text{ mV}_{\text{rms}}$ .

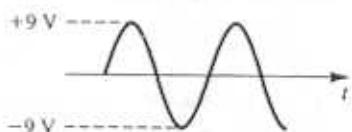
Supplies are  $V_{DD}$  and  $-V_{DD}$



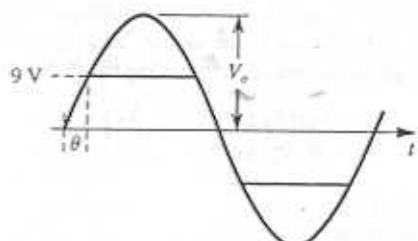
For  $V_{DD} = 10 \text{ V}$ , the largest undistorted sine-wave output is of 8.8-V peak amplitude or  $6.2 \text{ V}_{\text{rms}}$ . Input needed is  $12.4 \text{ mV}_{\text{rms}}$ .

For  $V_{DD} = 15 \text{ V}$ , the largest undistorted sine-wave output is of 13.8-V peak amplitude or  $9.8 \text{ V}_{\text{rms}}$ . The input needed is  $9.8 \text{ V}/500 = 19.6 \text{ mV}_{\text{rms}}$ .

1.42 (a) For an output whose extremes are just at the edge of clipping, i.e., an output of  $9 \text{ V}_{\text{peak}}$ , the input must have  $9 \text{ V}/1000 = 9 \text{ mV}_{\text{peak}}$ .



(b) For an output that is clipping 90% of the time,  $\theta = 0.1 \times 90^\circ = 9^\circ$  and  $V_p \sin 9^\circ = 9 \text{ V} \Rightarrow V_p = 57.5 \text{ V}$  which of course does not occur as the output saturates at  $\pm 9 \text{ V}$ . To produce this result, the input peak must be  $57.5/1000 = 57.5 \text{ mV}$ .



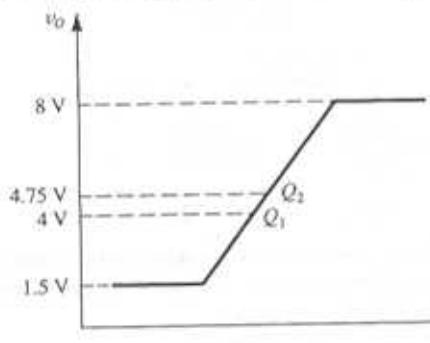
(c) For an output that is clipping 99% of the time,  $\theta = 0.01 \times 90^\circ = 0.9^\circ$

$$V_p \sin 0.9^\circ = 9 \text{ V}$$

$$\Rightarrow V_p = 573 \text{ V}$$

and the input must be  $573 \text{ V} / 1000$  or  $0.573 \text{ V}_{\text{peak}}$ .

**1.43** When the amplifier is biased at  $4 \text{ V}$  (i.e., at point  $Q_1$ ), the maximum possible amplitude of a sine-wave output without clipping is  $(4 - 1.5) = 2.5 \text{ V}_{\text{peak}}$ .

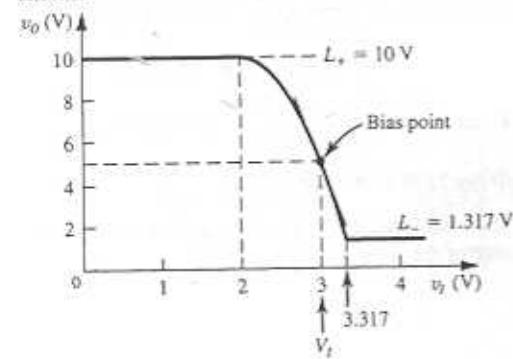


To obtain the largest undistorted sine wave output possible with this amplifier, it must be biased halfway between the saturation levels, i.e., at  $v_o = (8 + 1.5)/2 = 4.75 \text{ V}$  (point  $Q_2$ ) and the resulting output will have a peak value of  $8 - 4.75 = 4.75 - 1.50 = 3.25 \text{ V}$ .

**1.44**  $v_o = 10 - 5(v_i - 2)^2, \quad 2 \leq v_i \leq v_o + 2, \quad v_o \geq 0$

(a) For  $v_i \leq 2 \text{ V}$ ,  $v_o = 10 \text{ V}$ .

The upper limit on  $v_i$  is found by substituting  $v_i = v_o + 2$ , that is,  $v_o = v_i - 2$  in the transfer characteristic. The result is  $v_o = 10 - 5v_o^2$ , whose solution is  $v_o = 1.317 \text{ V}$  and the corresponding  $v_i = 3.317 \text{ V}$ . To obtain a sketch of  $v_o$  versus  $v_i$ , we evaluate  $v_o$  for values of  $v_i$  in the range  $2 \text{ V}$  to  $3.317 \text{ V}$ . The result is the following sketch:



(b) To obtain  $v_o = 5 \text{ V}$  we bias at  $v_i = 3 \text{ V}$ .

$$(c) \text{ Small-signal gain at bias point} = \left. \frac{\partial v_o}{\partial v_i} \right|_{v_i=3 \text{ V}} = -5 \times 2(v_i - 2) = -10 \text{ V/V}$$

(d)  $v_i = 3 + V_i \cos \omega t$

$$v_o = 10 - 5(3 + V_i \cos \omega t - 2)^2$$

$$= 10 - 5(1 + 2V_i \cos \omega t + V_i^2 \cos^2 \omega t)$$

$$= 5 - 10V_i \cos \omega t - 5V_i^2 \left( \frac{1}{2} + \frac{1}{2} \cos 2\omega t \right)$$

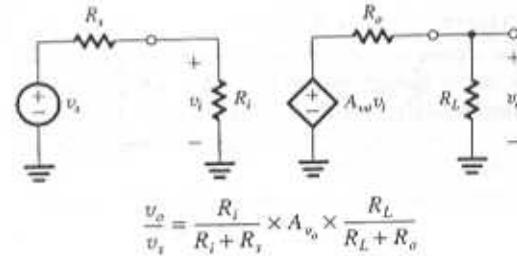
$$= \underbrace{(5 - 2.5V_i^2)}_{\text{dc}} - \underbrace{10V_i \cos \omega t}_{\text{Fundamental}} - \underbrace{2.5V_i^2 \cos 2\omega t}_{\text{2nd harmonic}}$$

For 1% second-harmonic distortion:  $2.5V_i^2/10V_i = 0.01$

Thus,

$$V_i = \frac{10 \times 0.01}{2.5} \text{ V} = 40 \text{ mV}$$

**1.45**



$$\frac{v_o}{v_i} = \frac{R_j}{R_i + R_j} \times A_{v_o} \times \frac{R_L}{R_L + R_o}$$

$$(a) \frac{v_o}{v_i} = \frac{10R_i}{10R_i + R_j} \times A_{v_o} \times \frac{10R_o}{10R_o + R_o}$$

$$= \frac{10}{11} \times 10 \times \frac{10}{11} = 8.26 \text{ V/V}$$

or,  $20 \log 8.26 = 18.3 \text{ dB}$

$$(b) \frac{v_o}{v_i} = \frac{R_j}{R_i + R_j} \times A_{v_o} \times \frac{R_o}{R_o + R_o}$$

$$= 0.5 \times 10 \times 0.5 = 2.5 \text{ V/V}$$

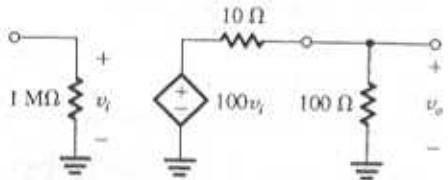
or,  $20 \log 2.5 = 8 \text{ dB}$

$$(c) \frac{v_o}{v_i} = \frac{R_i/10}{(R_i/10) + R_j} \times A_{v_o} \times \frac{R_o/10}{(R_o/10) + R_o}$$

$$= \frac{1}{11} \times 10 \times \frac{1}{11} = 0.083 \text{ V/V}$$

or  $20 \log 0.083 = -21.6 \text{ dB}$

1.46  $20 \log A_{v_o} = 40 \text{ dB} \Rightarrow A_{v_o} = 100 \text{ V/V}$



$$\begin{aligned} A_v &= \frac{v_o}{v_i} \\ &= 100 \times \frac{100}{100 + 10} \\ &= 90.9 \text{ V/V} \end{aligned}$$

or,  $20 \log 90.9 = 39.1 \text{ dB}$

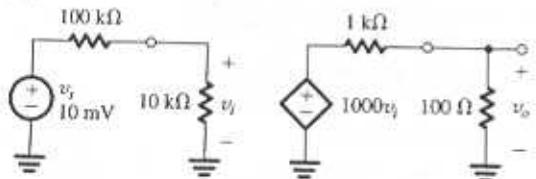
$$A_p = \frac{v_o^2 / 100 \Omega}{v_i^2 / 1 \text{ M}\Omega} = A_v^2 \times 10^4 = 8.3 \times 10^7 \text{ W/W}$$

or  $10 \log (8.3 \times 10^7) = 79.1 \text{ dB}$ .

For a peak output sine-wave current of  $100 \Omega$ , the peak output voltage will be  $100 \text{ mA} \times 100 \Omega = 10 \text{ V}$ . Correspondingly  $v_i$  will be a sine wave with a peak value of  $10 \text{ V}/A_v = 10/90.9$  or an rms value of  $10/(90.9 \times \sqrt{2}) = 0.08 \text{ V}$ .

$$\begin{aligned} \text{Corresponding output power} &= (10/\sqrt{2})^2 / 100 \Omega \\ &= 0.5 \text{ W} \end{aligned}$$

1.47

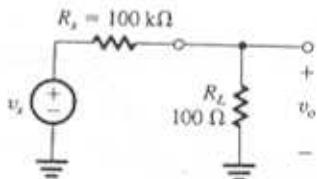


$$\begin{aligned} \frac{v_o}{v_i} &= \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 100 \text{ k}\Omega} \times 1000 \times \frac{100 \Omega}{100 \Omega + 1 \text{ k}\Omega} \\ &= \frac{10}{110} \times 1000 \times \frac{100}{1100} = 8.26 \text{ V/V} \end{aligned}$$

The signal loses about 90% of its strength when connected to the amplifier input (because  $R_i = R_s/10$ ). Also, the output signal of the amplifier loses approximately 90% of its strength when the load is connected

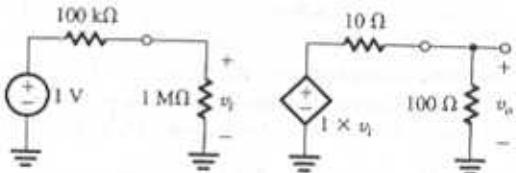
(because  $R_L = R_o/10$ ). Not a good design! Nevertheless, if the source were connected directly to the load,

$$\begin{aligned} \frac{v_o}{v_i} &= \frac{R_L}{R_L + R_s} \\ &= \frac{100 \Omega}{100 \Omega + 100 \text{ k}\Omega} \\ &= 0.001 \text{ V/V} \end{aligned}$$



which is clearly a much worse situation. Indeed inserting the amplifier increases the gain by a factor  $8.3/0.001 = 8300$ .

1.48



$$\begin{aligned} v_o &= 1 \text{ V} \times \frac{1 \text{ M}\Omega}{1 \text{ M}\Omega + 100 \text{ k}\Omega} \times 1 \times \frac{100 \Omega}{100 \Omega + 10 \Omega} \\ &= \frac{1}{1.1} \times \frac{100}{110} = 0.83 \text{ V} \end{aligned}$$

$$\text{Voltage gain} = \frac{v_o}{v_i} = 0.83 \text{ V/V} \quad \text{or} \quad -1.6 \text{ dB}$$

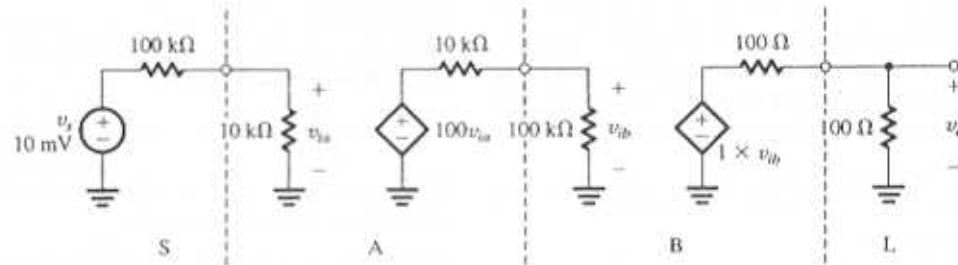
$$\begin{aligned} \text{Current gain} &= \frac{v_o/100 \Omega}{v_i/1.1 \text{ M}\Omega} = 0.83 \times 1.1 \times 10^4 \\ &= 9091 \text{ A/A} \quad \text{or} \quad 79.2 \text{ dB} \end{aligned}$$

$$\text{Power gain} = \frac{v_o^2/100 \Omega}{v_i^2/1.1 \text{ M}\Omega} = 7578 \text{ W/W}$$

or  $10 \log 7578 = 38.8 \text{ dB}$

(This takes into acct. the power dissipated in the internal resistance of the source.)

1.50 Case (a) S-A-B-L

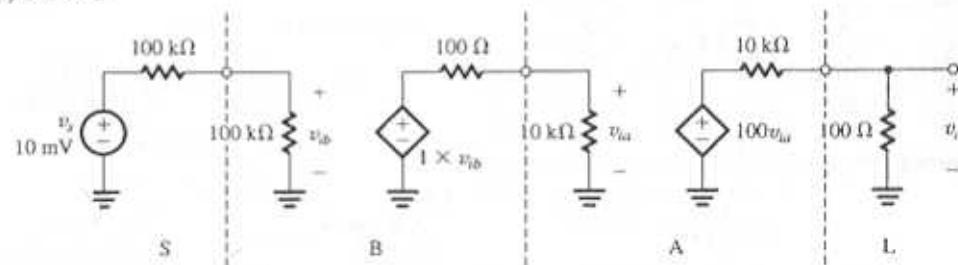


$$\frac{v_o}{v_s} = \frac{10}{10+100} \times 100 \times \frac{100}{100+10} \times 1 \times \frac{100}{100+100}$$

$$= 4.1 \text{ V/V}$$

or  $20 \log 4.1 = 12.3 \text{ dB}$

Case (b) S-B-A-L



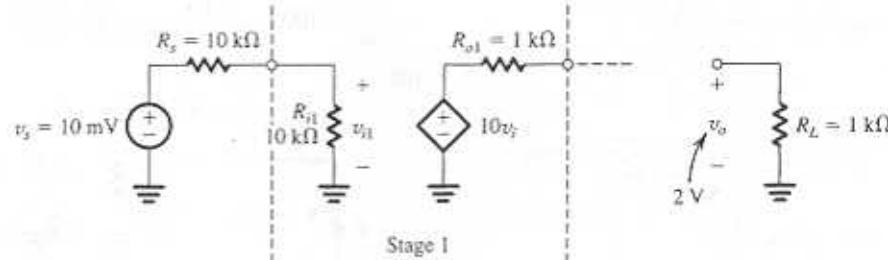
$$\frac{v_o}{v_s} = \frac{100}{100+100} \times 1 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 100 \text{ }\Omega} \times 100 \times \frac{100 \text{ }\Omega}{100 \text{ }\Omega + 10 \text{ k}\Omega}$$

$$= 0.5 \times \frac{10}{10.1} \times 100 \times \frac{0.1}{10.1}$$

$$= 0.5 \text{ V/V or } -6 \text{ dB}$$

Thus, obviously case (a) i.e., SABL is preferred.

1.51

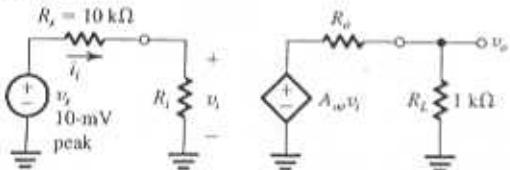


Required overall voltage gain =  $2 \text{ V} / 10 \text{ mV} = 200 \text{ V/V}$ . Each stage is capable of providing a maximum voltage gain of 10 (the open-circuit gain value). For  $n$  stages in cascade the maximum (unattainable) voltage gain is  $10^n$ . We thus see that we need at least 3 stages. For 3 stages, the overall voltage gain obtained is

$$\frac{v_o}{v_i} = \frac{10}{10+10} \times 10 \times \frac{10}{1+10} \times 10 \times \frac{10}{1+10} \times 10 \times \frac{1}{1+1} = 206.6 \text{ V/V}$$

Thus, three stages suffice and provide a gain slightly larger than required. The output voltage actually obtained is  $10 \text{ mV} \times 206.6 = 2.07 \text{ V}$ .

### 1.53



$$(a) \text{ Required voltage gain} = \frac{v_o}{v_i} = \frac{3 \text{ V}}{0.01 \text{ V}} = 300 \text{ V/V}$$

(b) The smallest  $R_i$  allowed is obtained from

$$0.1 \mu\text{A} = \frac{10 \text{ mV}}{R_s + R_i} \Rightarrow R_s + R_i = 100 \text{ k}\Omega$$

Thus  $R_i = 90 \text{ k}\Omega$ .

For  $R_i = 90 \text{ k}\Omega$ ,  $i_i = 0.1 \mu\text{A}$  peak, and

$$\begin{aligned} \text{Overall current gain} &= \frac{v_o}{i_i} \\ &= \frac{3 \text{ mA}}{0.1 \mu\text{A}} = 3 \times 10^4 \text{ A/A} \end{aligned}$$

$$\begin{aligned} \text{Overall power gain} &\equiv \frac{v_{o(\text{rms})}^2 / R_L}{v_{i(\text{rms})}^2 \times i_{i(\text{rms})}} \\ &= \frac{\left(\frac{3}{\sqrt{2}}\right)^2 / 1000}{\left(\frac{10 \times 10^{-3}}{\sqrt{2}}\right) \times \left(\frac{0.1 \times 10^{-6}}{\sqrt{2}}\right)} \\ &= 9 \times 10^6 \text{ W/W} \end{aligned}$$

(This takes into account the power dissipated in the internal resistance of the source.)

(c) If  $(A_{v_o} v_i)$  has its peak value limited to 5 V, the largest value of  $R_o$  is found from

$$5 \times \frac{R_L}{R_L + R_o} = 3 \Rightarrow R_o = \frac{2}{3} R_L = 667 \Omega$$

(If  $R_o$  were greater than this value, the output voltage across  $R_L$  would be less than 3 V.)

(d) For  $R_i = 90 \text{ k}\Omega$  and  $R_o = 667 \Omega$ , the required value of  $A_{v_o}$  can be found from

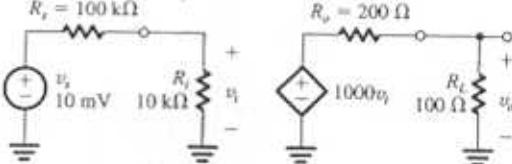
$$\begin{aligned} 300 \text{ V/V} &= \frac{90}{90+10} \times A_{v_o} \times \frac{1}{1+0.667} \\ \Rightarrow A_{v_o} &= 555.7 \text{ V/V} \end{aligned}$$

$$(e) R_i = 100 \text{ k}\Omega (1 \times 10^5 \Omega)$$

$$R_o = 100 \Omega (1 \times 10^2 \Omega)$$

$$\begin{aligned} 300 &= \frac{100}{100+10} \times A_{v_o} \times \frac{1000}{1000+100} \\ \Rightarrow A_{v_o} &= 363 \text{ V/V} \end{aligned}$$

### 1.54

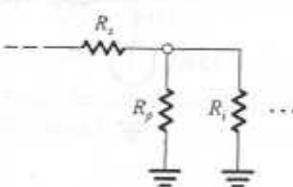


$$(a) v_o = 10 \text{ mV} \times \frac{10}{10+100} \times 1000 \times \frac{100}{100+200} = 303 \text{ mV}$$

$$(b) \frac{v_o}{v_i} = \frac{303 \text{ mV}}{10 \text{ mV}} = 30.3 \text{ V/V}$$

$$(c) \frac{v_o}{v_i} = 1000 \times \frac{100}{100+200} = 333.3 \text{ V/V}$$

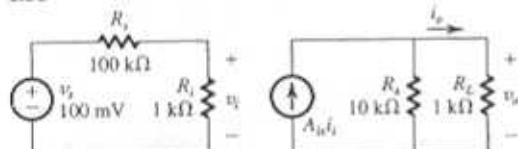
(d)



Connect a resistance  $R_p$  in parallel with the input and select its value from

$$\begin{aligned}\frac{(R_p \parallel R_i)}{(R_p \parallel R_i) + R_i} &= \frac{1}{2} \frac{R_i}{R_i + R_p} \\ \Rightarrow 1 + \frac{R_i}{R_p \parallel R_i} &= 22 \Rightarrow R_p \parallel R_i = \frac{R_i}{21} = \frac{100}{21} \\ \Rightarrow \frac{1}{R_p} + \frac{1}{R_i} &= \frac{21}{100} \\ R_p &= \frac{1}{0.21 - 0.1} = 9.1 \text{ k}\Omega\end{aligned}$$

1.55

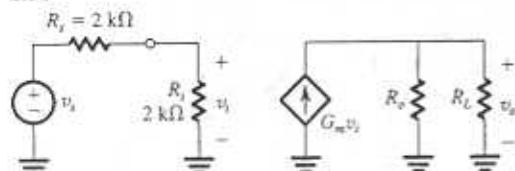


$$\begin{aligned}\text{(a) Current gain} &= \frac{i_o}{i_i} \\ &= A_{i_o} \frac{R_o}{R_o + R_L} \\ &= 100 \frac{10}{11} \\ &= 90.9 \frac{\text{A}}{\text{A}} = 39.2 \text{ dB}\end{aligned}$$

$$\begin{aligned}\text{(b) Voltage gain} &= \frac{v_o}{v_i} \\ &= \frac{i_o}{i_i} \frac{R_o}{R_o + R_i} \\ &= 90.9 \times \frac{1}{101} \\ &= 0.9 \text{ V/V} = -0.9 \text{ dB}\end{aligned}$$

$$\begin{aligned}\text{(c) Power gain} &= A_p = \frac{v_o i_o}{v_i i_i} \\ &= 0.9 \times 90.9 \\ &= 81.8 \text{ W/W} = 19.1 \text{ dB}\end{aligned}$$

1.56



$$G_m = 40 \text{ mA/V}$$

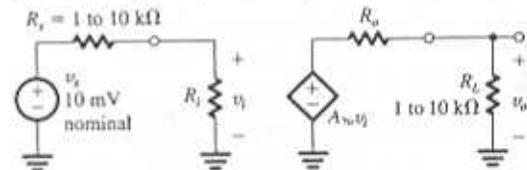
$$R_o = 20 \text{ k}\Omega$$

$$R_L = 1 \text{ k}\Omega$$

$$\begin{aligned}v_i &= v_t \frac{R_i}{R_i + R_f} \\ &= v_t \frac{2}{2+2} = \frac{v_t}{2} \\ v_o &= G_m v_i (R_L \parallel R_o) \\ &= 40 \frac{20 \times 1}{20+1} v_t \\ &= 40 \frac{20}{21} \frac{v_t}{2}\end{aligned}$$

$$\text{Overall voltage gain} = \frac{v_o}{v_t} = 19.05 \text{ V/V}$$

1.57 A voltage amplifier is required.



To limit the change in  $v_o$  to 10% as  $R_i$  varies from 1 to 10 kΩ we select  $R_i$  sufficiently large;

$$R_i \geq 10R_{i\max}$$

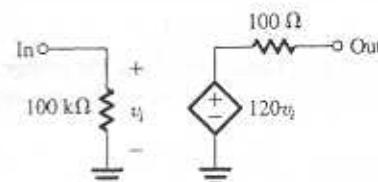
Thus  $R_i = 100 \text{ k}\Omega$ .

To limit the change in  $v_o$  corresponding to  $R_L$  varying in the range 1 to 10 kΩ, to 10%, we select  $R_o$  sufficiently small;

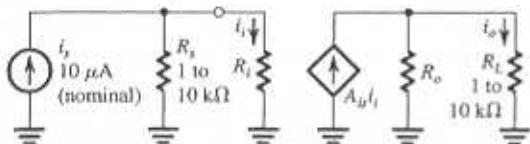
$$R_o \leq R_{L\min}/10$$

Thus,

$$\begin{aligned}R_o &= 100 \Omega \\ v_{o\min} &= v_t \frac{R_i}{R_i + R_{i\max}} A_{v_o} \frac{R_{L\min}}{R_{L\min} + R_o} \\ 1 &= 0.01 \frac{100}{100+10} A_{v_o} \frac{1000}{1000+100} \\ \Rightarrow A_{v_o} &= 121 \text{ V/V}\end{aligned}$$



**1.58 Current amplifier.**



To limit the change in  $i_o$  resulting from  $R_i$  varying over the range 1 to 10 kΩ, to 10% we select  $R_i$  sufficiently low so that,

$$R_i \leq R_{i,\min}/10$$

Thus,  $R_i = 100 \Omega$

To limit the change in  $i_o$  as  $R_L$  changes from 1 to 10 kΩ, to 10% we select  $R_o$  sufficiently large;

$$R_o \geq 10R_{L,\max}$$

Thus,  $R_o = 100 \text{ k}\Omega$

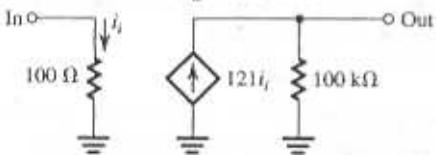
Now for  $i_s = 10 \mu\text{A}$ ,

$$i_{o,\min} = 10^{-5} \frac{R_{i,\min}}{R_{i,\min} + R_i} A_{i,i} \frac{R_o}{R_o + R_{L,\max}}$$

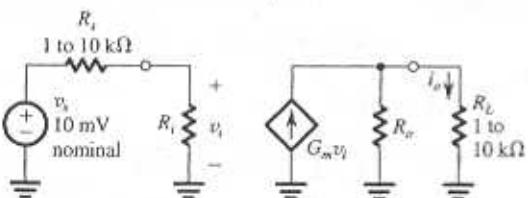
Thus,

$$10^{-3} = 10^{-5} \frac{1000}{1000 + 100} A_{i,i} \frac{100}{100 + 10}$$

$$\Rightarrow A_{i,i} = 121 \text{ A/A}$$



**1.59 Transconductance amplifier.**



For  $R_i$  varying in the range 1 to 10 kΩ, and  $\Delta i_o$  limited to 10% we have to select  $R_i$  sufficiently large;

$$R_i \geq 10R_{i,\max}$$

$$R_i = 100 \Omega$$

For  $R_L$  varying in the range 1 to 10 kΩ, the change in  $i_o$  can be kept to 10% if  $R_o$  is selected sufficiently large;

$$R_o \geq R_{L,\max}$$

Thus  $R_o = 100 \text{ k}\Omega$

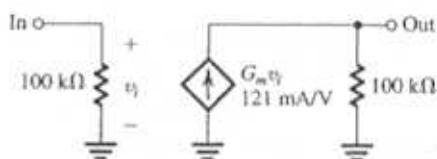
For  $v_t = 10 \text{ mV}$ ,

$$i_{o,\min} = 10^{-2} \frac{R_i}{R_i + R_{i,\max}} G_m \frac{R_o}{R_o + R_{L,\max}}$$

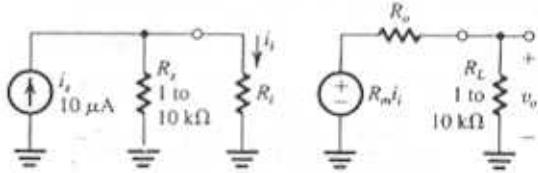
$$10^{-3} = 10^{-2} \frac{100}{100 + 10} G_m \frac{100}{100 + 10}$$

$$G_m = 1.21 \times 10^{-1} \text{ A/V}$$

$$= 121 \text{ mA/V}$$



**1.60 Transresistance amplifier**



To limit  $\Delta v_o$  to 10% corresponding to  $R_i$  varying in the range 1 to 10 kΩ, we select  $R_i$  sufficiently low;

$$R_i \leq \frac{R_{i,\min}}{10}$$

Thus,  $R_i = 100 \Omega$

To limit  $\Delta v_o$  to 10% while  $R_L$  varies over the range 1 to 10 kΩ, we select  $R_o$  sufficiently low;

$$R_o \leq R_{L,\min}/10$$

Thus,  $R_o = 100 \Omega$

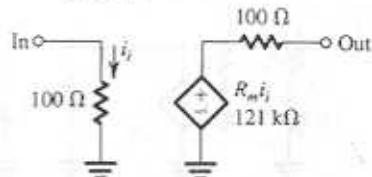
Now, for  $i_s = 10 \mu\text{A}$ ,

$$v_{o,\min} = 10^{-5} \frac{R_{i,\min}}{R_{i,\min} + R_i} R_m \frac{R_{L,\min}}{R_{L,\min} + R_o}$$

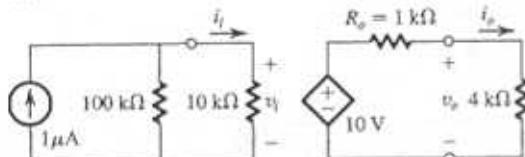
$$1 = 10^{-5} \frac{1000}{1000 + 100} R_m \frac{1000}{1000 + 100}$$

$$\Rightarrow R_m = 1.21 \times 10^5$$

$$= 121 \text{ k}\Omega$$



1.64



$$R_o = \frac{\text{Open-circuit output voltage}}{\text{Short-circuit output current}} = \frac{10 \text{ V}}{10 \text{ mA}} = 1 \text{ k}\Omega$$

$$v_o = 10 \times \frac{4}{1+4} = 8 \text{ V}$$

$$A_v = \frac{v_o}{v_i} = \frac{8}{1 \times 10^{-3} \times (100 // 10) \times 10^3} = 888 \text{ V/V or } 58.9 \text{ dB}$$

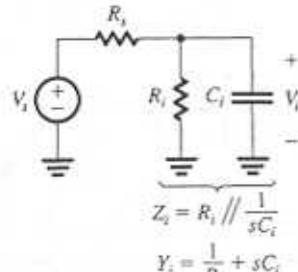
$$A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{10^{-3} \times \frac{100}{100+10}} = \frac{8/(4 \times 10^3)}{10^{-3} \times \frac{100}{110}} = 2200 \text{ A/A or } 66.8 \text{ dB}$$

$$A_t = \frac{v_o^2 / R_L}{i_i^2 R_i} = \frac{8^2 / (4 \times 10^3)}{\left(10^{-3} \times \frac{100}{100+10}\right)^2 10 \times 10^3} = 19.36 \times 10^5 \text{ W/W or } 62.9 \text{ dB}$$

$$\text{Overall current gain} = \frac{i_o}{1 \mu\text{A}}$$

$$= \frac{v_o / R_L}{1 \mu\text{A}} = \frac{8 / (4 \times 10^3)}{10^{-3}} = 2000 \text{ A/A or } 66 \text{ dB}$$

1.65 Using the voltage divider rule



$$\frac{V_j}{V_s} = \frac{Z_i}{Z_i + R_i}$$

$$= \frac{1}{1 + R_i Y_i}$$

$$= \frac{1}{1 + R_i \left( \frac{1}{R_i} + s C_i \right)} = \frac{1}{1 + R_i + s C_i R_i} = \frac{1}{1 + s C_i \frac{R_i}{1 + R_i}} = \frac{1}{1 + R_i}$$

$$= \frac{1}{1 + \frac{R_i}{R_i}} \frac{1}{1 + s C_i \left( \frac{R_i R_i}{R_i + R_i} \right)} = \frac{R_i}{R_i + R_i} \frac{1}{1 + s C_i (R_i // R_i)}$$

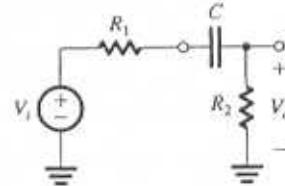
This transfer function is of the STC low-pass type with a dc gain  $K = R_i / (R_i + R_i)$  and a 3-dB frequency  $\omega_0 = 1 / C_i (R_i // R_i)$ .

For  $R_i = 20 \text{ k}\Omega$ ,  $R_o = 80 \text{ k}\Omega$ , and  $C_i = 5 \text{ pF}$ ,

$$\omega_0 = \frac{1}{5 \times 10^{-12} \times \frac{20 \times 80}{20+80} \times 10^3} = 1.25 \times 10^7 \text{ rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1.25 \times 10^7}{2\pi} = 2 \text{ MHz}$$

1.66 Using the voltage-divider rule,



$$T(s) = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 + \frac{1}{sC}}$$

$$= \frac{R_2}{R_1 + R_2} \frac{s}{s + \frac{1}{C(R_1 + R_2)}}$$

which from Table 1.2 is of the high-pass type with

$$K = \frac{R_2}{R_1 + R_2}$$

and

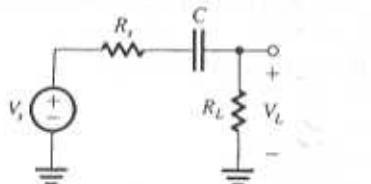
$$\omega_0 = \frac{1}{C(R_1 + R_2)}$$

As a further verification that this is a high-pass network and  $T(s)$  is a high-pass transfer function we observe as at  $s = 0$ ,  $T(s) = 0$ ; and that as  $s \rightarrow \infty$ ,  $T(s) = R_2/(R_1 + R_2)$ . Also, from the circuit observe as at  $s \rightarrow \infty$ ,  $(1/sC) \rightarrow 0$  and  $V_o/V_i = R_2/(R_1 + R_2)$ . Now, for  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 40 \text{ k}\Omega$ , and  $C = 0.1 \mu\text{F}$ ,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \times 0.1 \times 10^{-6} (10 + 40) \times 10^3} = 31.8 \text{ Hz}$$

$$|T(j\omega_0)| = \frac{K}{\sqrt{2}} = \frac{40}{\sqrt{10 + 40}} = 0.57 \text{ V/V}$$

1.67 Using the voltage divider rule,



$$\begin{aligned} \frac{V_o}{V_i} &= \frac{R_L}{R_L + R_i + \frac{1}{sC}} \\ &= \frac{R_L}{R_L + R_i} \frac{s}{s + \frac{1}{C(R_L + R_i)}} \end{aligned}$$

which is of the high-pass STC type (see Table 1.2) with

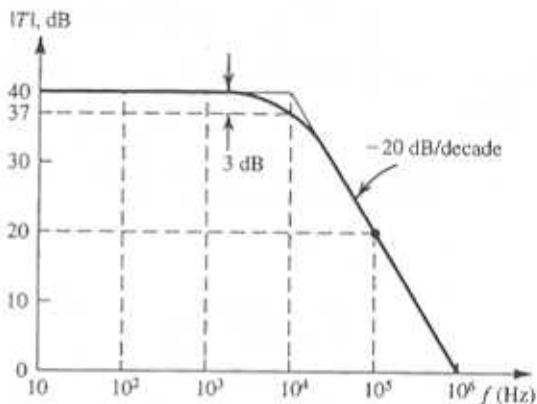
$$K = \frac{R_L}{R_L + R_i}, \quad \omega_0 = \frac{1}{C(R_L + R_i)}$$

For  $f_0 \leq 10 \text{ Hz}$

$$\begin{aligned} \frac{1}{2\pi C(R_L + R_i)} &\leq 10 \\ \Rightarrow C &\geq \frac{1}{2\pi \times 10(20 + 5) \times 10^3} \end{aligned}$$

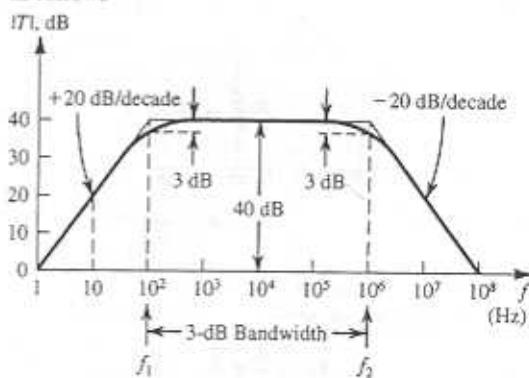
Thus, the smallest value of  $C$  that will do the job is  $C = 0.64 \mu\text{F}$ .

1.68 The given measured data indicate that this amplifier has a low-pass STC frequency response with a low-frequency gain of 40 dB, and a 3-dB frequency of  $10^4 \text{ Hz}$ . From our knowledge of the Bode plots for low-pass STC networks (Figure . . .) we can complete the Table entries and sketch the amplifier frequency response.



$f(\text{Hz})$	$ T (\text{dB})$	$\angle T(\text{degrees})$
1000	40	-5.7°
$10^4$	37	-45°
$10^5$	20	-84.3°
$10^6$	0	-90°

1.69 From our knowledge of the Bode plots of STC low-pass and high-pass networks we see that this amplifier has a mid-band gain of 40 dB, a low-frequency response of the high-pass STC type with  $f_{3\text{dB}} = 10^2 \text{ Hz}$ , and a high-frequency response of the low-pass STC type with  $f_{3\text{dB}} = 10^6 \text{ Hz}$ . We thus can sketch the amplifier frequency response and complete the table entries as follows



$f(\text{Hz})$	$10^1$	$10^2$	$10^3$	$10^4$	$10^5$	$10^6$	$10^7$	$10^8$
$ T (\text{dB})$	0	20	37	40	40	37	20	0

**1.72** Since the overall transfer function is that of three identical STC LP circuits in cascade (but with no loading effects since the buffer amplifiers have input and zero output resistances) the overall gain will drop by 3 dB below the value at dc at the frequency for which the gain of each STC circuit is 1 dB down. This frequency is found as follows: The transfer function of each STC circuit is

$$T(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

where

$$\omega_0 = 1/CR$$

Thus,

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{\omega_{1\text{dB}}}{\omega_0}\right)^2}} = -1$$

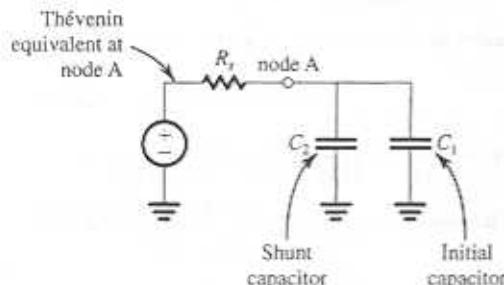
$$\Rightarrow 1 + \left(\frac{\omega_{1\text{dB}}}{\omega_0}\right)^2 = 10^{0.1}$$

$$\omega_{1\text{dB}} = 0.51\omega_0$$

$$\omega_{1\text{dB}} = 0.51/CR$$

**1.73**  $R_i = 100 \text{ k}\Omega$ , since the 3-dB frequency is reduced by a very high factor (from 6 MHz to 120 kHz)  $C_2$  must be much larger than  $C_1$ . Thus, neglecting  $C_1$  we find  $C_2$  from

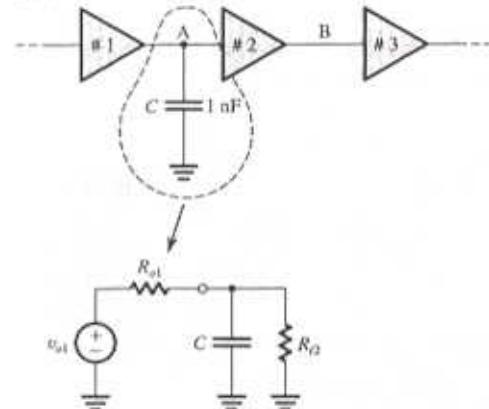
$$\begin{aligned} 120 \text{ kHz} &= \frac{1}{2\pi C_2 R_i} \\ &= \frac{1}{2\pi C_2 \times 10^5} \\ \Rightarrow C_2 &= 13.3 \text{ pF} \end{aligned}$$



If the original 3-dB frequency (6 MHz) is attributable to  $C_1$  then

$$\begin{aligned} 6 \text{ MHz} &= \frac{1}{2\pi C_1 R_i} \\ \Rightarrow C_1 &= \frac{1}{2\pi \times 6 \times 10^6 \times 10^5} \\ &= 0.26 \text{ pF} \end{aligned}$$

**1.74**



Since when  $C$  is connected the 3-dB frequency is reduced by a large factor, the value of  $C$  must be much larger than whatever parasitic capacitance originally existed at node A (i.e., between A and ground). Furthermore, it must be that  $C$  is now the dominant determinant of the amplifier 3-dB frequency (i.e., it is dominating over whatever may be happening at node B or anywhere else in the amplifier). Thus, we can write

$$\begin{aligned} 150 \text{ kHz} &= \frac{1}{2\pi C(R_{o1} \parallel R_{i2})} \\ \Rightarrow (R_{o1} \parallel R_{i2}) &= \frac{1}{2\pi \times 150 \times 10^3 \times 1 \times 10^{-9}} \\ &= 1.06 \text{ k}\Omega \end{aligned}$$

Now  $R_{i2} = 100 \text{ k}\Omega$ ,

Thus  $R_{o1} = 1.07 \text{ k}\Omega$

Similarly, for node B,

$$\begin{aligned} 15 \text{ kHz} &= \frac{1}{2\pi C(R_{o2} \parallel R_{i3})} \\ \Rightarrow R_{o2} \parallel R_{i3} &= \frac{1}{2\pi \times 15 \times 10^3 \times 1 \times 10^{-9}} \\ &= 10.6 \text{ k}\Omega \end{aligned}$$

$$R_{o2} = 11.9 \text{ k}\Omega$$

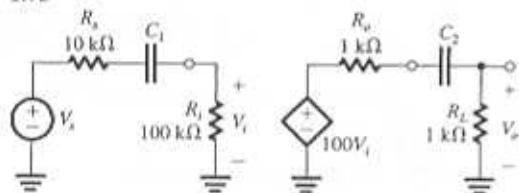
She should connect a capacitor of value  $C_p$  to node B where  $C_p$  can be found from,

$$10 \text{ kHz} = \frac{1}{2\pi C_p (R_{o2} // R_{i3})}$$

$$\Rightarrow C_p = \frac{1}{2\pi \times 10 \times 10^3 \times 10.6 \times 10^3} = 1.5 \text{ nF}$$

Note that if she chooses to use node A she would need to connect a capacitor 10 times larger!

1.75



For the input circuit, the corner frequency  $f_{01}$  is found from

$$f_{01} = \frac{1}{2\pi C_1 (R_i + R_t)}$$

For  $f_{01} \leq 100 \text{ Hz}$ ,

$$\frac{1}{2\pi C_1 (10 + 100) \times 10^3} \leq 100$$

$$\Rightarrow C_1 \geq \frac{1}{2\pi \times 110 \times 10^3 \times 10^2} = 4.4 \times 10^{-8}$$

Thus we select  $C_1 = 1 \times 10^{-7} \text{ F} = 0.1 \mu\text{F}$ . The actual corner frequency resulting from  $C_1$  will be

$$f_{01} = \frac{1}{2\pi \times 10^{-7} \times 110 \times 10^3} = 14.5 \text{ Hz}$$

For the output circuit,

$$f_{02} = \frac{1}{2\pi C_2 (R_o + R_L)}$$

For  $f_{02} \leq 100 \text{ Hz}$ ,

$$\frac{1}{2\pi C_2 (1 + 1) \times 10^3} \leq 100$$

$$\Rightarrow C_2 \geq \frac{1}{2\pi \times 2 \times 10^3 \times 10^2} = 0.8 \times 10^{-6}$$

Select  $C_2 = 1 \times 10^{-6} = 1 \mu\text{F}$

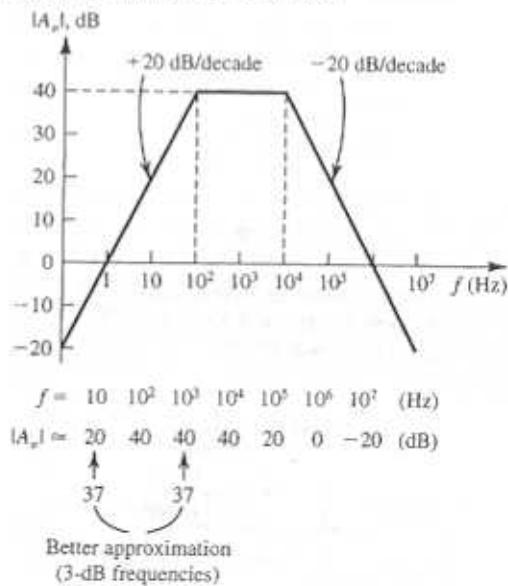
This will place the corner frequency at

$$f_{02} = \frac{1}{2\pi \times 10^{-6} \times 2 \times 10^3} = 80 \text{ Hz}$$

$$T(s) = 100 \frac{s}{\left(1 + \frac{s}{2\pi f_{01}}\right)\left(1 + \frac{s}{2\pi f_{02}}\right)}$$

1.76 The LP factor  $1/(1+jf/10^4)$  results in a Bode plot like that in Fig. 1.23(a) with the 3 dB frequency  $f_0 = 10^4 \text{ Hz}$ . The high-pass factor  $1/(1+10^4/f)$  results in a Bode plot like that in Fig. 1.24(a) with the 3 dB frequency  $f_0 = 10^4 \text{ Hz}$ .

The Bode plot for the overall transfer function can be obtained by summing the dB values of the two individual plots and then raising the resulting plot vertically by 40 dB (corresponding to the factor 100 in the numerator). The result is as follows:



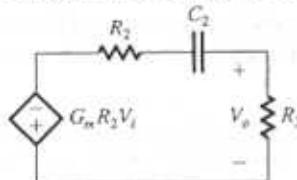
$$\text{Bandwidth} = 10^4 - 10^2 = 9900 \text{ Hz}$$

1.77

$$T_1(s) = \frac{V_i(s)}{V_t(s)} = \frac{1/s C_1}{1/s C_1 + R_1} = \frac{1}{s C_1 R_1 + 1} \quad \text{LP}$$

$$3 \text{ dB frequency} = \frac{1}{2\pi C_1 R_1} = \frac{1}{2\pi 10^{-11} 10^6} = 15.9 \text{ Hz}$$

For  $T_o(s)$ , the following equivalent circuit can be used:



$$T_o(s) = -G_m R_2 \frac{R_3}{R_2 + R_3 + 1/sC_2}$$

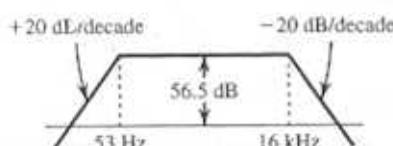
$$= -G_m (R_2 \parallel R_3) \frac{s}{s + \frac{1}{C_2(R_2 + R_3)}}$$

$$3 \text{ dB frequency} = \frac{1}{2\pi C_2(R_2 + R_3)}$$

$$= \frac{1}{2\pi 100 \times 10^{-9} \times 30 \times 10^3} = 53 \text{ Hz}$$

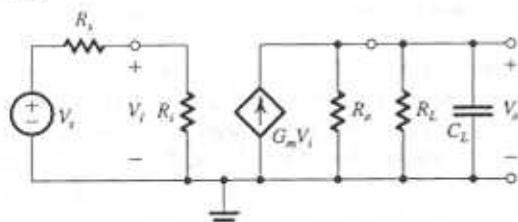
$$\therefore T(s) = T_i(s)T_o(s)$$

$$= \frac{1}{1 + \frac{s}{2\pi \times 15.9 \times 10^3}} \times -666.7 \times \frac{s}{s + (2\pi \times 53)}$$



$$\text{Bandwidth} = 16 \text{ kHz} - 53 \text{ Hz} \approx 16 \text{ kHz}$$

1.78



$$V_i = V_i \frac{R_i}{R_i + R_o} \quad (1)$$

To satisfy constraint (1), namely

$$V_i \geq \left(1 - \frac{x}{100}\right) V_i$$

We substitute in Eq.(1) to obtain

$$\frac{R_i}{R_i + R_o} \geq 1 - \frac{x}{100}$$

Thus

$$\frac{R_i + R_o}{R_i} \leq \frac{1}{1 - \frac{x}{100}}$$

$$\frac{R_o}{R_i} \leq \frac{1}{1 - \frac{x}{100}} - 1 = \frac{\frac{x}{100}}{1 - \frac{x}{100}}$$

which can be expressed as

$$\frac{R_o}{R_i} \geq \frac{1 - \frac{x}{100}}{\frac{x}{100}}$$

resulting in

$$R_o \geq R_i \left( \frac{100}{x} - 1 \right) \quad (1)$$

The 3-dB frequency is determined by the parallel  $RC$  circuit at the output,

$$f_0 = \frac{1}{2\pi} \omega_0 = \frac{1}{2\pi} \frac{1}{C_L(R_L \parallel R_o)}$$

Thus,

$$f_0 = \frac{1}{2\pi C_L} \left( \frac{1}{R_L} + \frac{1}{R_o} \right)$$

To obtain a value for  $f_0$  greater than a specified value  $f_{3dB}$  we select  $R_o$  so that

$$\frac{1}{2\pi C_L} \left( \frac{1}{R_L} + \frac{1}{R_o} \right) \geq f_{3dB}$$

$$\frac{1}{R_L} + \frac{1}{R_o} \geq 2\pi C_L f_{3dB}$$

$$\frac{1}{R_o} \geq 2\pi C_L f_{3dB} - \frac{1}{R_L}$$

$$R_o \leq \frac{1}{2\pi f_{3dB} C_L - \frac{1}{R_L}} \quad (2)$$

To satisfy constraint (3), we first determine the dc gain as

$$\text{dc gain} = \frac{R_o}{R_i + R_o} G_m (R_o \parallel R_L)$$

For the dc gain to be greater than a specified value  $A_0$ ,

$$\frac{R_o}{R_i + R_o} G_m (R_o \parallel R_L) \geq A_0$$

The first factor on the LHS is (from constraint (1))

greater or equal to  $(1 - x/100)$ . Thus

$$G_m \geq \frac{A_0}{\left(1 - \frac{x}{100}\right)(R_o \parallel R_L)} \quad (3)$$

Substituting  $R_o = 10 \text{ k}\Omega$  and  $x = 20\%$  in (1) results in

$$R_i \geq 10 \left( \frac{100}{20} - 1 \right) = 40 \text{ k}\Omega$$

Substituting  $f_{3dB} = 3 \text{ MHz}$ ,  $C_L = 10 \text{ pF}$  and  $R_L = 10 \text{ k}\Omega$  in Eq. (2) results in

$$R_o \leq \frac{1}{2\pi \times 3 \times 10^6 \times 10 \times 10^{-12} - \frac{1}{10^4}} = 11.3 \text{ k}\Omega$$

Substituting  $A_0 = 80$ ,  $x = 20\%$ ,  $R_L = 10 \text{ k}\Omega$ , and  $R_o = 11.3 \text{ k}\Omega$ , eq. (3) results in

$$G_m \geq \frac{80}{\left(1 - \frac{20}{100}\right)(10 \parallel 11.3) \times 10^3} = 18.85 \text{ mA/V}$$

If the more practical value of  $R_o = 10 \text{ k}\Omega$  is used then

$$G_m \geq \frac{80}{\left(1 - \frac{20}{100}\right)(10 \parallel 10) \times 10^3} = 20 \text{ mA/V}$$

**1.79** Using the voltage-divider rule we obtain

$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

where

$$Z_1 = R_1 \parallel \frac{1}{sC_1} \quad \text{and} \quad Z_2 = R_2 \parallel \frac{1}{sC_2}.$$

It is obviously more convenient to work in terms of admittances. Therefore we express  $V_o/V_i$  in the alternate form

$$\frac{V_o}{V_i} = \frac{Y_1}{Y_1 + Y_2}$$

and substitute  $Y_1 = (1/R_1) + sC_1$  and  $Y_2 = (1/R_2) + sC_2$  to obtain

$$\begin{aligned} \frac{V_o}{V_i} &= \frac{\frac{1}{R_1} + sC_1}{\frac{1}{R_1} + \frac{1}{R_2} + s(C_1 + C_2)} \\ &= \frac{C_1}{C_1 + C_2} \frac{s + \frac{1}{C_1 R_1}}{s + \frac{1}{(C_1 + C_2)(R_1 + R_2)}} \end{aligned}$$

This transfer function will be independent of frequency ( $s$ ) if the second factor reduces to unity. This in turn will happen if

$$\frac{1}{C_1 R_1} = \frac{1}{C_1 + C_2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

which can be simplified as follows

$$\frac{C_1 + C_2}{C_1} = R_1 \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \quad (1)$$

$$1 + \frac{C_2}{C_1} = 1 + \frac{R_1}{R_2}$$

or

$$C_1 R_1 = C_2 R_2$$

When this condition applies, the attenuator is said to be compensated, and its transfer function is given by

$$\frac{V_o}{V_i} = \frac{C_1}{C_1 + C_2}$$

which, using Eq. (1) above can be expressed in the alternate form

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{R_2}{R_1 + R_2}$$

Thus when the attenuator is compensated ( $C_1 R_1 = C_2 R_2$ ) its transmission can be determined either by its two resistors  $R_1$ ,  $R_2$  or by its two capacitors,  $C_1$ ,  $C_2$ , and the transmission is *not* a function of frequency.

**1.80** The HP STC circuit whose response determines the frequency response of the amplifier in the low-frequency range has a phase angle of  $11.4^\circ$  at  $f = 100 \text{ Hz}$ . Using the equation for  $\angle T(j\omega)$  from Table 1.2 we obtain

$$\tan^{-1} \frac{f_0}{100} = 11.4^\circ \Rightarrow f_0 = 20.16 \text{ Hz.}$$

The LP STC circuit whose response determines the amplifier response at the high-frequency end has a phase angle of  $-11.4^\circ$  at  $f = 1 \text{ kHz}$ . Using the relationship for  $\angle T(j\omega)$  given in Table 1.2 we obtain for the LP STC circuit

$$-\tan^{-1} \frac{10^3}{f_0} = -11.4^\circ \Rightarrow f_0 = 4959.4 \text{ Hz}$$

At  $f = 100$  Hz the drop in gain is due to the HP STC network, and thus its value is

$$20 \log \frac{1}{\sqrt{1 + \left(\frac{20.16}{100}\right)^2}} = -0.17 \text{ dB}$$

Similarly, at  $f = 1$  kHz the drop in gain is caused by the LP STC network. The drop in gain is

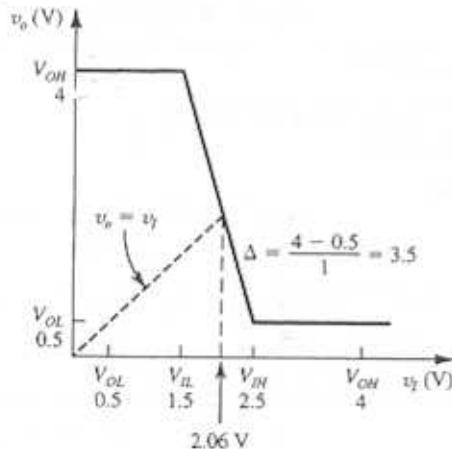
$$20 \log \frac{1}{\sqrt{1 + \left(\frac{1000}{4959.4}\right)^2}} = -0.17 \text{ dB}$$

The gain drops by 3 dB at the corner frequencies of the two STC networks, that is, at  $f = 20.16$  Hz and  $f = 4959.4$  Hz.

$$1.81 \quad NM_H = V_{OH} - V_{IH} = 3.3 - 1.7 = 1.6 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = 1.3 - 0 = 1.3 \text{ V}$$

1.82



$$(a) NM_H = V_{OH} - V_{IH} = 4 - 2.5 = 1.5 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = 1.3 - 0.5 = 1 \text{ V}$$

(b) In the transition region

$$\begin{aligned} V_O &= 4 - 3.5(V_I - 1.5) \\ &= 9.25 - 3.5V_I \end{aligned}$$

If

$$V_O = V_I \Rightarrow 4.5V_O = 9.25$$

$$V_O = V_I = 2.06 \text{ V}$$

(c) Slope =  $-3.5 \text{ V/V}$

1.83

$$NM_H = V_{OH} - V_{IH} = 0.8 V_{DD} - 0.6 V_{DD} = 0.2 V_{DD}$$

$$NM_L = V_{IL} - V_{OL} = (0.4 - 0.1)V_{DD} = 0.3 V_{DD}$$

$$\text{width of transition region} = V_{IH} - V_{IL} = 0.2 V_{DD}$$

for a minimum  $NM$  of 1 V  $\Rightarrow 0.2 V_{DD} = 1$

$$V_{DD} = 5 \text{ V}$$

1.84

$$(a) \text{Worse case } NM_H = V_{OH,\min} - V_{IH} = 2.4 - 2 = 0.4 \text{ V}$$

$$\text{Worse case } NM_L = V_{IL,\max} - V_{OL} = 0.8 - 0.4 = 0.4 \text{ V}$$

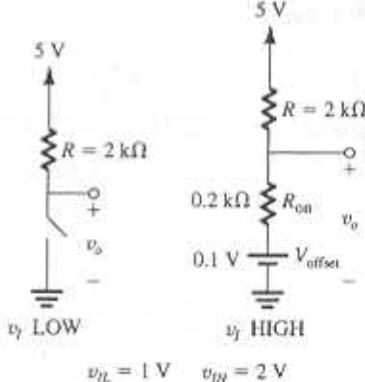
$$(b) P_{D_H} = \frac{1}{20} [5 \times 3 + 5 \times 1] = 10 \text{ mW}$$

$$(c) \text{Dynamic power dissipation} = f_C V_{DD}^2 = 10^6 \times 45 \times 10^{-12} \times 25 = 1.13 \text{ mW}$$

$$(d) t_P(\text{typical}) = \frac{1}{2}(t_{PHL} + t_{PLH}) = \frac{1}{2}(7 + 11) = 9 \text{ ns}$$

$$t_P(\text{maximum}) = \frac{1}{2}(15 + 22) = 18.5 \text{ ns}$$

1.85



$$v_{IL} = 1 \text{ V} \quad v_{IH} = 2 \text{ V}$$

$$(a) V_{OL} = \frac{5 - 0.1}{2.2} = 0.2 + 0.1 = 0.545 \text{ V}$$

$$V_{OH} = 5 \text{ V}$$

$$NM_H = V_{OH} - V_{IH} = 3 \text{ V}$$

$$NM_L = V_{IL} - V_{OL} = 0.455 \text{ V}$$

$$(b) V_{OH} = 5 - N(0.2 \times 10^{-3})R = 5 - 0.4N$$

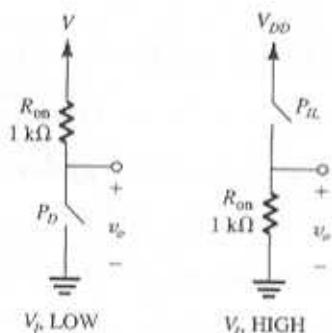
$$NM_H = 5 - 0.4N - 2 = 3 - 0.4N = 0.455 \quad \therefore N = 6$$

$$(c) (i) P_{D_{v_L} \text{LOW}} = (5 - 0.1)^2 / 2.2 \text{ k}\Omega = 10.9 \text{ mW}$$

$$(ii) P_{D_{v_L} \text{HIGH}} = 5 \times (0.2 \times 6) = 6 \text{ mW}$$

1.86 (a)  $V_{OL} = 0$     $V_{OH} = 5$     $NM_L = V_{IL} - V_{OL} = 2.5 - 0 = 2.5 \text{ V}$

$$NM_H = V_{OH} - V_{IH} = 5 - 2.5 = 2.5 \text{ V}$$



$$(b) V_O(t) = 0 - (0 - 5)e^{-t/R_{on}C} = 5e^{-t/R_{on}C}$$

$$\text{For } t_{PLH} \Rightarrow V_O(t) = 5e^{-t_{PLH}/R_{on}C} = \frac{1}{2}(5) = 2.5$$

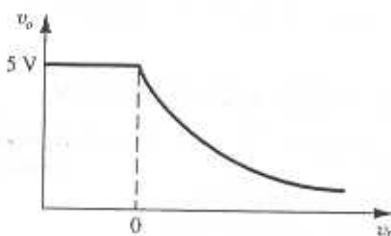
$$t_{PLH} = -(10^3)(10^{-9}) \ln \frac{2.5}{5} = 0.69 \text{ ns}$$

$$\text{For } t_{THL} \quad V_O(t) = 5e^{-t_{THL}/R_{on}C} = 4.5 \text{ V}$$

$$t_1 = 0.01 \text{ ns} \quad V_O(t) = 5e^{-t_1/R_{on}C} = 0.5 \text{ V}$$

$$t_2 = 2.3 \text{ ns}$$

$$\therefore t_{THL} = t_2 - t_1 = 2.2 \text{ ns}$$



$$(c) V_O(t) = 5 - (5 - 0)e^{-t/R_{on}C} = 5 - 5e^{-t/R_{on}C}$$

$$V_O = 5 - 5e^{-t_{PLH}/R_{on}C} = 2.5 \text{ ns}$$

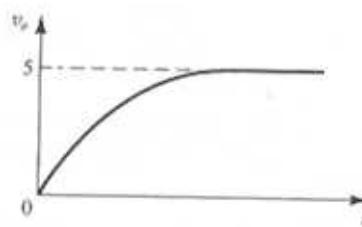
$$t_{PLH} = 0.69 \text{ ns}$$

For  $t_{TLH}$ ,

$$V_O(t) = 5 - 5e^{-t_1/R_{on}C} = 0.5 \Rightarrow t_1 = 0.10 \text{ ns.}$$

$$V_O(t) = 5 - 5e^{-t_2/R_{on}C} = 4.5 \Rightarrow t_2 = 2.3 \text{ ns.}$$

$$t_{TLH} = 2.3 - 0.1 = 2.2 \text{ ns}$$



1.87

$$V_{OH} = 5 \text{ V}$$

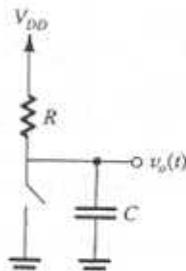
$$V_{OL} = 5 - 2 \times 1 = 3 \text{ V}$$

$$1.88 P_{\text{dynamic}} = f C V_{DD}^2 = 100 \times 10^6 \times 10 \times 10^{-12} \times 25 = 25 \text{ mW}$$

$$P = V_{DD} I_{avg} = 5 I_{avg} = 25 \text{ mW}$$

$$I_{avg} = 5 \text{ mA}$$

1.89



$v_O(t)$  begins at  $V_{OL}$  and rises toward  $V_{OH}$  (in this case  $V_{OH} = V_{DD}$ ) according to

$$\begin{aligned} v_O(t) &= v_{ss} - (v_{ss} - v_{oh}) e^{-t/CR} \\ &= V_{OH} - (V_{OH} - V_{OL}) e^{-t/CR} \end{aligned}$$

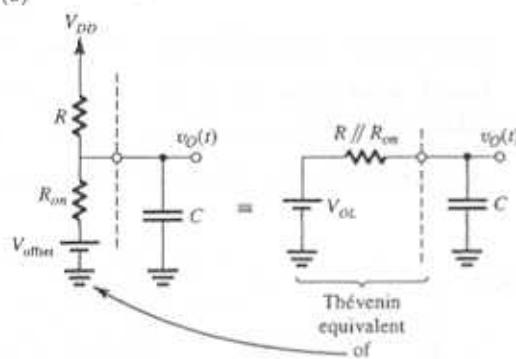
$$= V_{OH} - (V_{OH} - V_{OL}) e^{-t/\tau_1}, \quad \tau_1 = CR \quad \text{Q.E.D.}$$

$$v_O(t) \text{ reaches } \frac{1}{2}(V_{OH} + V_{OL}) \text{ at } t = t_{PLH},$$

$$\frac{1}{2}(V_{OH} + V_{OL}) = V_{OH} - (V_{OH} - V_{OL}) e^{-t_{PLH}/\tau_1}$$

$$\Rightarrow t_{PLH} = \tau_1 \ln 2 = 0.69 CR \quad \text{Q.E.D.}$$

(b)



$$V_{OL} = V_{DD} \frac{R_{on}}{R + R_{on}} + V_{offset} \frac{R}{R + R_{on}}$$

$$v_O(t) = v_m - (v_m - v_{ol}) e^{-t/\tau_2}, \quad \tau_2 = C(R // R_{on})$$

$$= V_{OL} - (V_{OL} - V_{OH}) e^{-t/\tau_2}$$

$$= V_{OL} + (V_{OH} - V_{OL}) e^{-t/\tau_2} \quad \text{Q.E.D.}$$

$$v_O(t_{PHL}) = \frac{1}{2}(V_{OL} + V_{OH})$$

Thus,

$$t_{PHL} = \tau_2 \ln 2 = 0.69C(R // R_{on})$$

$$= 0.69 CR_{on}, \quad R_{on} \ll R \quad \text{Q.E.D.}$$

$$(c) \quad \tau_p = \frac{1}{2}(t_{PLH} + t_{PHL}) = \frac{1}{2} \times 0.69(CR + CR_{on})$$

$$= 0.35CR, \quad \text{for } R_{on} \ll R \quad \text{Q.E.D.}$$

(d) Static power is dissipated only when the output is low, in which case.

$$P = \frac{V_{DD}^2}{R + R_{on}} \approx \frac{V_{DD}^2}{R}, \quad \text{for } R_{on} \ll R,$$

and assuming that  $V_{offset} \ll V_{DD}$ . Thus if the inverter spends only half the time in this low-output state,

$$P = \frac{1}{2} \frac{V_{DD}^2}{R}$$

(e) Large  $R$  results in low  $P$  but high  $\tau_p$  and vice-versa. For  $V_{DD} = 5$  V and  $C = 10$  pF,

- $\tau_p \leq 10$  ns  $\Rightarrow 0.35CR \leq 10$  ns
- $\Rightarrow R \leq 2875 \Omega$
- $P \leq 10$  mW  $\Rightarrow \frac{1}{2} \times \frac{25}{R} \leq 10 \times 10^{-3}$
- $\Rightarrow R \geq 1250 \Omega$

Thus,

$$1250 \leq R \leq 2875 \Omega$$

Selecting  $R = 2$  k $\Omega$ ,

$$\tau_p = 0.35 \times 10 \times 10^{-12} \times 2 \times 10^3 = 7 \text{ ns}$$

$$P = \frac{1}{2} \times \frac{25}{2} = 6.25 \text{ mW}$$

## Unnumbered 1.48

$$\frac{v_{i1}}{v_i} = 0.5 \quad \frac{v_{i2}}{v_{i1}} = 100 \quad \frac{1000}{1001} \approx 100$$

$$\frac{v_{i2}}{v_{i1}} = 10 \times \frac{10}{11} = 9.09$$

$$\frac{v_L}{v_{i3}} = \frac{10}{11} = 0.909$$

$$A_v = 826.3$$

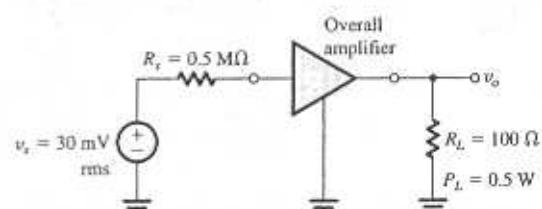
$$\frac{v_o}{v_i} = 413.1$$

$$0.5 \times 100 \times 10 \times \frac{10}{11} = 450$$

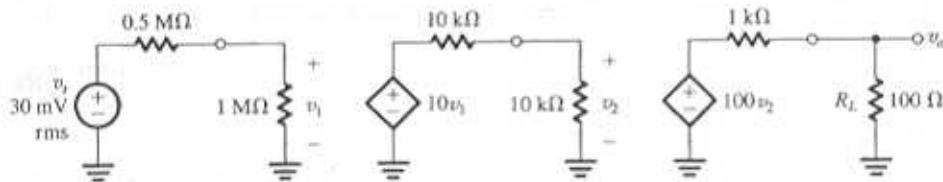
$$P_L = 0.5 \text{ W} = \frac{v_o^2}{R_L}$$

$$v_o = \sqrt{0.5 \times 100} = 7.07 \text{ V}$$

## Unnumbered 1.50

Required overall voltage gain =  $7.07/0.03 = 235.7 = 235$  V/V.

In order to avoid the loss of more than 2/3 of the signal strength in coupling the source to the first stage of the amplifier, we have used the type (1), high-input resistance amplifier as the input stage. Both type 2 and type 3 do not satisfy this requirement. The type (1) amplifier has an open-circuit voltage gain of 10 and thus connects by itself to satisfy the overall gain requirement.



We next consider cascading the type (1) input stage with the type (2), high-gain stages. The result would be

Overall voltage gain

$$= \frac{1}{1+0.5} \times 10 \times \frac{10}{10+10} \times 100 \times \frac{100}{100+1000}$$

$$= 30.3 \text{ V/V}$$

Thus, this cascade amplifier does not meet specs. The reason is obvious from the gain calculation: the output resistance,  $1 \text{ k}\Omega$ , is too high for feeding a load of  $100 \Omega$  and indeed results in a loss of gain by a factor of 11!

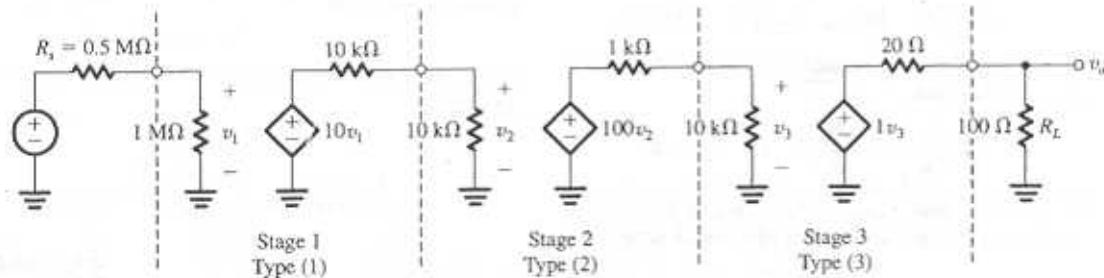
That's where the type (3) amplifier stage can be beneficial. While its open-circuit gain is only 1, its output resistance is  $20 \Omega$ , five times lower than the  $100 \Omega$  load. The overall amplifier would then look as follows:

$$\text{Overall voltage gain} = \frac{v_o}{v_s}$$

$$= \frac{1}{1+0.5} \times 10 \times \frac{10}{10+10} \times 100 \times \frac{10}{10+1} \times 1 \times \frac{100}{100+20}$$

$$= 252.5 \text{ V/V}$$

which meets the specified gain of 235 V/V.



## Chapter 2 - Problems

2.1

The minimum number of pins required by dual-op-amp is 8. Each op-amp has 2 input terminals (4 pins) and one output terminal (2 pins). Another 2 pins are required for power.

Similarly, the minimum number of pins required by quad-opamp is 14:

$$4 \times 2 + 4 \times 1 + 2 = 14$$

2.2

Refer to Fig. P2.2.  $V_o = V_1 \frac{1KR_2}{1M\Omega + 1K\Omega} = \frac{4}{1001} V$

$$V_o = A V_+ \Rightarrow A = \frac{4}{4/1001} = 1001$$

2.3

The voltage at the positive input has to be  $-3.000V$ .

$$V_+ = -3.020V, A = \frac{V_o}{(V_+ - V_-)} = \frac{-2}{-3.020 - (-3)} = 100$$

2.4

#	$V_1$	$V_2$	$\frac{V_{ds}}{V_2 - V_1}$	$V_o$	$V_o/V_d$
1	0.00	0.00	0.00	0.00	-
2	1.00	1.00	0.00	0.00	-
3	② 1.00	③ 1.00	1.00	-	
4	1.00	1.10	0.10	10.1	101
5	2.01	2.00	-0.01	-0.99	99
6	1.99	2.00	0.01	1.00	100
7	5.10	④ ⑤ -5.10	-	-	-

experiments 4, 5, 6 show that the gain is

approximately  $100 V/V$ . The missing entry for experiment #3 can be predicted as follows:

$$\textcircled{b} \quad V_d = \frac{V_2}{A} = \frac{1.00}{100} = 0.01 V.$$

$$\textcircled{a} \quad V_o = V_2 - V_d = 1.00 - 0.01 = 0.99 V$$

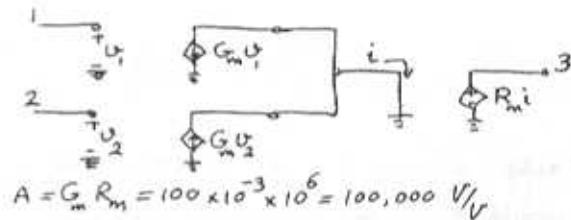
The missing entries for experiment #7:

$$\textcircled{d} \quad V_d = \frac{-5.10}{100} = -0.051 V$$

$$\textcircled{c} \quad V_o = V_1 + V_d = 5.10 - 0.051 = 5.049 V$$

All the results seem to be reasonable.

2.5



$$A = G_m R_m = 100 \times 10^{-3} \times 10^6 = 100,000 V/V$$

2.6

$$V_{CM} = 10V \sin(2\pi 60t) = \frac{1}{2} (V_1 + V_2)$$

$$V_d = 0.01 \sin(2\pi 1000t) = V_1 - V_2$$

$$V_1 = V_{CM} - V_d/2 = \sin(120\pi t) - 0.005 \sin 2000\pi t$$

$$V_2 = V_{CM} + V_d/2 = \sin 120\pi t + 0.005 \sin 2000\pi t$$

2.7

$$V_d = R(G_{m2}V_2 - G_{m1}V_1) \quad \text{Refer to Fig. 2.4.}$$

$$V_o = V_3 = \mu V_d = \mu R(G_{m2}V_2 - G_{m1}V_1)$$

$$V_o = \mu R(G_{m2}\frac{V_2}{2} + \frac{1}{2}\Delta G_m V_2 - G_{m1}\frac{V_1}{2} + \frac{1}{2}\Delta G_m V_1)$$

$$V_o = \mu R G_m \underbrace{(V_2 - V_1)}_{V_{Id}} + \frac{1}{2} \mu R \Delta G_m \underbrace{(V_1 + V_2)}_{2V_{ICM}}$$

$$\text{we have } V_o = A_d V_{Id} + A_{CM} V_{ICM}$$

$$\Rightarrow A_d = \mu R G_m, \quad A_{CM} = \mu R \Delta G_m$$

$$CMRR = 20 \log |A_d| / |A_{CM}| = 20 \log \frac{G_m}{\Delta G_m}$$

cont.

$$20 \log_{10} A_d = 80 \text{ dB} \Rightarrow A_d = 10^4$$

$$\frac{A_{CM}}{A_d} = \frac{\Delta G_m}{G_m} \Rightarrow A_{CM} = 10^4 \times \frac{0.1}{100} = 10$$

$$CMRR = 20 \log \frac{G_m}{\Delta G_m} = 20 \log \frac{1}{0.1/100} = 60$$

2.8

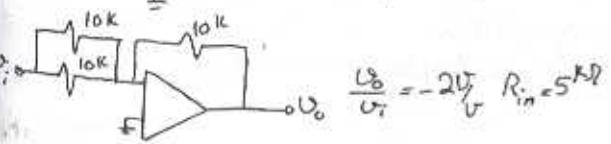
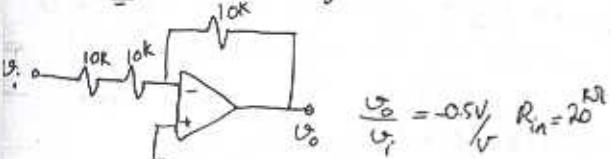
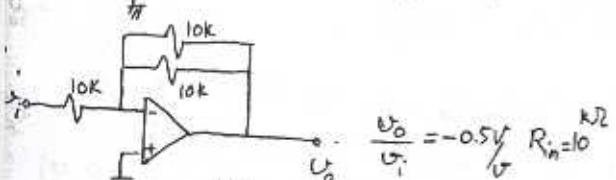
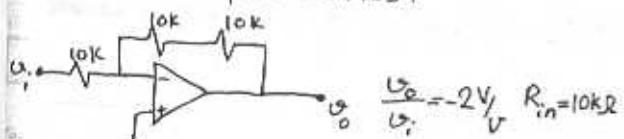
circuit	$\frac{V_o}{V_i}$ (V/V)	$R_{in}$ (kΩ)
a	$\frac{-100}{10} = -10$	10
b	-10	10
c	-10	10
d	-10	10 no current in 10kΩ

2.9

Closed loop gain =  $-1 \text{ V/V}$ . For  $V_i = 5 \text{ V} \Rightarrow V_o = -5 \text{ V}$   
 Gain would be in the range of  $\frac{-0.95}{1.05}$  to  $\frac{-1.05}{0.95} \Rightarrow -0.9 < G < -1.1$   
 For  $V_i = 5 \Rightarrow 4.5 < V_o < -5.5 \text{ V}$

2.10

There are four possibilities:



2.11

- a.  $G = 1 \text{ V/V}$   
 c.  $G = 0.1 \text{ V/V}$   
 e.  $G = 10 \text{ V/V}$

- b.  $G = 10 \text{ V/V}$   
 d.  $G = 100 \text{ V/V}$

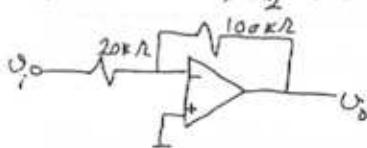
2.12

- a.  $G = -1 \text{ V/V} = -\frac{R_2}{R_1} \Rightarrow R_1 = R_2 = 10 \text{ k}\Omega$   
 b.  $G = -2 \text{ V/V} = -\frac{R_2}{R_1} \Rightarrow R_1 = 10 \text{ k}\Omega, R_2 = 20 \text{ k}\Omega$   
 c.  $G = -0.5 \text{ V/V} = -\frac{R_2}{R_1} \Rightarrow R_1 = 20 \text{ k}\Omega, R_2 = 10 \text{ k}\Omega$   
 d.  $G = -100 \text{ V/V} = -\frac{R_2}{R_1} \Rightarrow R_1 = 10 \text{ k}\Omega, R_2 = 1 \text{ M}\Omega$

2.13

$$\frac{U_o}{U_i} = -5 = -\frac{R_2}{R_1} \Rightarrow R_2 = 5R_1$$

$$R_1 + R_2 = 120 \text{ k}\Omega \Rightarrow 5R_1 + R_1 = 120 \text{ k}\Omega \Rightarrow R_1 = 20 \text{ k}\Omega \Rightarrow R_2 = 100 \text{ k}\Omega$$



2.14

$$20 \log |G| = 26 \text{ dB} \Rightarrow G = 19.95 \text{ V/V} = \frac{U_o}{U_i} = -\frac{R_2}{R_1}$$

$$\Rightarrow R_2 = 19.95 R_1 \leq 10 \text{ M}\Omega$$

For largest possible input resistance, select  
 $R_2 = 10 \text{ M}\Omega \Rightarrow R_1 \approx 500 \text{ k}\Omega$   
 $R_{in} = 500 \text{ k}\Omega$

2.15

$$G = \frac{U_o}{U_i} = \frac{-R_2}{R_1} = \frac{-100}{10} = -10$$

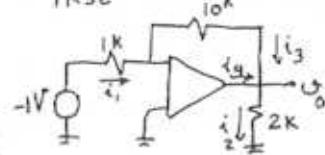
$$U_{low} = -10 \text{ V}, U_{high} = 0, U_{avg} = -5 \text{ V}$$

2.16

$$\frac{U_0}{U_i} = -\frac{R_2}{R_1} \Rightarrow U_0 = -1 \times \frac{-10k\Omega}{1k\Omega} = 10V$$

$$i_2 = \frac{U_0}{2k\Omega} = 5mA$$

$$i_1 = i_3 = \frac{U_0}{10k\Omega} = 1mA$$



$i_4 = i_2 - i_3 = 4mA$  This additional current comes from the output of the op-amp.

2.17

$$|Gain| = \frac{R_2}{R_1} = \frac{R_2(1+x/100)}{R_1(1+x/100)} \approx \frac{R_2}{R_1} (1 \pm \frac{2x}{100})$$

$\Rightarrow 2x\%$  is the tolerance on the closed loop gain ( $G$ ).

$$G = -100V/V, x=5 \Rightarrow -110 < G < -90$$

$$\text{or more precisely: } -100 \times \frac{105}{95} < G < -100 \times \frac{95}{105}$$

$$-110.5 < G < -90.5$$

2.18

$$G = \frac{U_0}{U_i} = -\frac{R_2}{R_1} \Rightarrow \frac{R_2}{R_1} = \frac{5}{15}$$

$$U_i = 0V, U_2 = U_0 = 5V$$

$$\text{For } \pm 1\% \text{ on } R_1, R_2: R_1 = 15 \pm 0.15 k\Omega$$

$$R_2 = 5 \pm 0.05 k\Omega$$

$$U_0 = U_i \frac{R_2}{R_1} = 15 \frac{R_2}{R_1} \Rightarrow 15 \times \frac{4.95}{15.15} < U_0 < 15 \times \frac{5.05}{14.85}$$

$$\Rightarrow 4.9V < U_0 < 5.1V$$

$$\text{For } U_i = -15 \pm 0.15 V \quad 14.85 \pm 4.95 < U_0 < 15.15 \pm 5.05$$

$$\Rightarrow 4.85V < U_0 < 5.15V$$

2.19

$$\frac{U_0}{U_i} = -\frac{R_2}{A} = -\frac{100}{200}$$

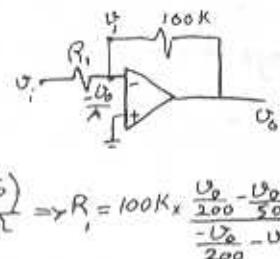
$$\frac{U_2}{U_i} = 50V/V$$

$$\frac{U_i - (-\frac{U_0}{A})}{R_1} = \frac{(-\frac{U_0}{A} - U_0)}{100k\Omega} \Rightarrow R_1 = 100k\Omega \times \frac{\frac{U_0}{200} - \frac{U_0}{50}}{\frac{-U_0}{200} - U_0}$$

$$\Rightarrow R_1 = 100k \times \frac{3}{201} = 1.49k\Omega$$

shunt Resistor  $R_a$ :  $R_a \parallel 2k\Omega = 1.49k\Omega$

$$\frac{R_a \times 2}{R_a + 2} = 1.49 \Rightarrow R_a = 5.84k\Omega$$



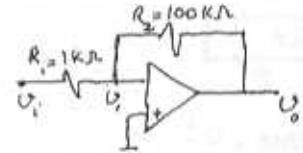
2.20

(a)

$$\frac{U_0}{U_i} = -\frac{R_2}{R_1} \Rightarrow -100V/V = -\frac{R_2}{1k\Omega} \Rightarrow R_2 = 100k\Omega$$

$$(b) A = 1000V/V$$

$$U_i = -\frac{U_0}{A}$$



$$\frac{U_i - U_1}{R_1} = \frac{U_i - U_0}{R_2}$$

$$\frac{U_0}{U_i} = \frac{-R_2/R_1}{1 + (1 + R_2/A)} = \frac{-100}{1 + \frac{101}{1000}} = -90.8V/V$$

$$\Rightarrow \frac{U_0}{U_i} = 90.8V/V$$

(b) Assume  $R'_1 = R_x \parallel R_1$  when  $R_1 = 1k\Omega$

$$\frac{U_0}{U_i} = 100V/V$$

$$\frac{U_i - U_1}{R'_1} = \frac{U_i - U_0}{R_2} \Rightarrow R'_1 = R_2 \times \left( \frac{U_0}{100} - \frac{U_0}{1000} \right) / \left( \frac{U_0}{100} - \frac{U_0}{1000} \right)$$

$$R'_1 = \frac{1 - 0.1}{1.001} = 0.899k\Omega = \frac{R_1 R_x}{R_1 + R_x} = \frac{R_x}{1 + R_x}$$

$$\Rightarrow R_x = 8.9k\Omega \approx 8.87k\Omega \pm 1\%$$

2.21

Voltage of the inverting input terminal

Cont.

will vary from  $\frac{-10V}{1000}$  to  $\frac{+10V}{1000}$ . Thus the virtual ground will depart from the ideal voltage of zero by a maximum of  $\pm 10mV$ .

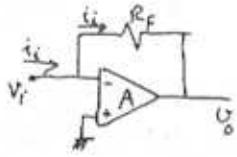
2.22

a) For  $A = \infty$ :  $V_i = 0$

$$V_o = -i_c R_F$$

$$R_m = \frac{V_o}{i_c} = -R_F$$

$$R_{ia} = \frac{V_o}{i_c} = 0$$



b) For  $A = \text{Finite}$ :  $V_i = -\frac{V_o}{A}$ ,  $V_o = V_i + i_c R_F$

$$\Rightarrow V_o = -\frac{V_o}{A} - i_c R_F \Rightarrow R_m = \frac{V_o}{i_c} = -\frac{R_F}{A} = \frac{1}{1+A}$$

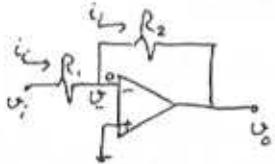
$$R_i = \frac{V_i}{i_c} = \frac{R_F}{1+A} \quad R_L > \frac{V_i}{i_c} = \frac{-V_o/A}{i_c} = \frac{-V_o}{A} + i_c R_F$$

2.23

$$V_o = -AV_+ = V_- - i_c R_2$$

$$i_c R_2 = (1+A) V_-$$

$$V_- = \frac{i_c R_2}{1+A}$$



$$\text{Now: } V_i = i_c R_1 + V_- = i_c R_1 + i_c \frac{R_2}{1+A}$$

$$R_{in} = \frac{V_i}{i_c} = R_1 + \frac{R_2}{1+A}$$

2.24

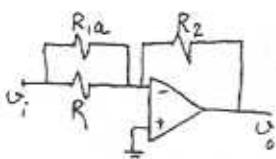
$$G = \frac{-R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}}$$

$$\text{Gain Error} \epsilon = \left(1 + \frac{R_2}{R_1}\right) / A \times 100$$

$$\epsilon \mid \begin{array}{ccc} 0.1\% & 1\% & 10\% \\ 1000(1 + \frac{R_2}{R_1}) & 100(1 + \frac{R_2}{R_1}) & 10(1 + \frac{R_2}{R_1}) \end{array}$$

Gain with  $R_{ia}$ :

$$G = \frac{R_2}{R_1} \left(1 + \frac{R_2}{R_{ia}}\right) \quad \frac{R_2}{1 + \frac{1 + R_2/R_1}{A}}$$



where we have neglected the effect of  $R_{ia}$  on

the error on the denominator. To restore the gain to its nominal value of  $R_2/R_1$ , we use:

$$\frac{R_i}{R_{ia}} = \frac{1 + R_2/R_1}{A} = \frac{\epsilon}{100} \rightarrow R_{ia} = \frac{100 R_1}{\epsilon}$$

$\frac{\epsilon}{100}$	$0.1\%$	$1\%$	$10\%$
	$1000R_1$	$100R_1$	$10R_1$

2.25

$$R'_i = R_i \parallel R_C \quad G' = \frac{-R_2/R'_i}{1 + \frac{1 + R_2/R'_i}{A}}$$

$$G = \frac{-R_2}{R_i}$$

$$\text{In order for } G' = G, \quad G = \frac{-R_2/R'_i}{1 + \frac{1 + R_2/R'_i}{A}} = \frac{-R_2}{R_i}$$

$$R'_i = \frac{R_i R_C}{R_i + R_C}$$

$$\Rightarrow \frac{R_i + R_C}{R_i R_C} = \frac{1}{R_i} \left(1 + \frac{1 + \frac{R_2(R_i + R_C)}{R_i R_C}}{A}\right)$$

$$(R_i + R_C) A = A R_C + R_C + \frac{R_2}{R_i} (R_i + R_C)$$

$$R_i A = R_C + G R_i + G R_C$$

$$\frac{R_C}{R_i} = \frac{A - G}{1 + G}$$

2.26

$$G = \frac{-R_2/R_1}{1 + \frac{1 + R_2/R_1}{A}} \quad G_{\text{Nominal}} = \frac{-R_2}{R_1}$$

$$\epsilon = \left| \frac{G - G_{\text{Nominal}}}{G_{\text{Nominal}}} \right| = \left| \frac{G}{G_{\text{Nominal}}} - 1 \right|$$

$$\epsilon = \left| \frac{1}{1 + \frac{1 + R_2/R_1}{A}} - 1 \right| = \left| \frac{-1 - R_2/R_1}{1 + 1 + R_2/R_1} \right| = \frac{1}{\frac{A}{1 + R_2/R_1} + 1}$$

which can be rearranged to yield:

$$\frac{A}{1 + \frac{R_2}{R_1}} + 1 = \frac{1}{\epsilon} \Rightarrow A = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{1}{\epsilon} - 1\right)$$

$$\text{or } A = \left(1 - \frac{G_{\text{Nominal}}}{\epsilon}\right) \left(\frac{1}{\epsilon} - 1\right)$$

For  $G_{\text{Nominal}} = -100 \frac{V_o}{V_i}$  and  $\epsilon = 0.1\% = 0.1$

$$A = \left(1 + 100\right) \left(\frac{1}{0.1} - 1\right) = 909 \frac{V_o}{V_i}$$

This is the minimum required value for  $A$ .

2.27

$$\begin{aligned} |G| &= \frac{R_2/R_1}{1 + \frac{R_2/R_1}{A}} & A \rightarrow A(1 - \frac{x}{100}) \\ |G'| &= \frac{R_2/R_1}{1 + \frac{R_2/R_1}{A(1 - \frac{x}{100})}} & G \rightarrow G' \\ \text{For } |G'| = |G| (1 - \frac{x}{100}) \end{aligned}$$

$$\begin{aligned} \frac{R_2/R_1}{1 + \frac{R_2/R_1}{A(1 - \frac{x}{100})}} &= \frac{R_2/R_1}{1 + \frac{R_2/R_1}{A}} (1 - \frac{x}{100}) \\ \frac{1 + \frac{R_2/R_1}{A(1 - \frac{x}{100})}}{A} &= (1 + \frac{1 + R_4/R_1}{A}) / (1 - \frac{x}{100}) \\ \frac{1 - \frac{x}{100K} + \frac{1 + R_4/R_1}{A}}{1 - \frac{x}{100}} &= 1 + \frac{1 + R_4/R_1}{A} \end{aligned}$$

$$\frac{1 + R_4/R_1}{A} \frac{1 - x/100K - 1 + x/100}{1 - x/100} = \frac{x}{100K}$$

$$A = \frac{1 + K}{1 - \frac{x}{100}} (1 + R_4/R_1) = (\frac{K+1}{1 - \frac{x}{100}}) (1 + \frac{R_4}{R_1})$$

$$\begin{aligned} \text{For } \frac{R_2}{R_1} = 100, x = 50, K = 100; A = \frac{99}{0.5} \times 101 = 19998 \\ A \approx 2 \times 10^4 V/V \end{aligned}$$

Thus for  $A = 2 \times 10^4 V/V$ , a reduction of 50% results in only 0.5% reduction of the closed loop gain whose nominal value is  $\frac{R_2}{R_1}$  (100).

2.28

From the results of example 2.2, the gain of the circuit in fig. 2.8 is given by:

$$\begin{aligned} \frac{V_o}{V_i} &= -\frac{R_2}{R_1} (1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}) \\ \text{For } R_1 = R_2 = R_4 = 1M\Omega \Rightarrow \frac{V_o}{V_i} &= -(1 + 1 + \frac{1}{R_3}) \end{aligned}$$

a)  $\frac{V_o}{V_i} = -10 V/V \Rightarrow 10 = 2 + \frac{1}{R_3} \Rightarrow R_3 = \frac{1}{12.5} M\Omega = 125 k\Omega$   
 b)  $\frac{V_o}{V_i} = -100 V/V \Rightarrow 100 = 2 + \frac{1}{R_3} \Rightarrow R_3 = \frac{1}{98} M\Omega = 10.2 k\Omega$   
 c)  $\frac{V_o}{V_i} = -2 V/V \Rightarrow 2 = 2 + \frac{1}{R_3} \Rightarrow R_3 = \infty$ ; eliminate  $R_3$ .

2.29

$$R_2/R_1 = 1000, R_2 = 100 k\Omega \Rightarrow R_1 = 100 \Omega$$

$$a) R_{in} = R_1 = 100 \Omega$$

$$\begin{aligned} b) \frac{V_o}{V_i} &= \frac{-R_2}{R_1} (1 + \frac{R_4}{R_2} + \frac{R_4}{R_3}) = -1000 \\ \text{If } R_2 = R_1 = R_4 = 100 k\Omega \Rightarrow R_3 = \frac{100 k\Omega}{1000 - 2} \approx 100 \Omega \\ R_{in} = R_1 = 100 \Omega \end{aligned}$$

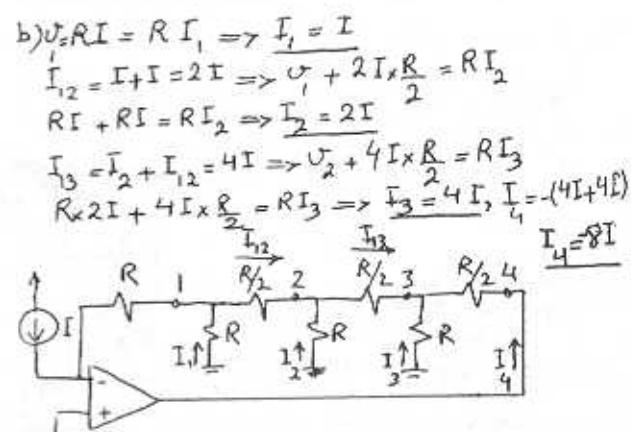
2.30

$$\begin{aligned} V_x &= v - i, R_2, i = \frac{V_i}{R_1} \Rightarrow V_x = -V_i \frac{R_2}{R_1} \\ \frac{V_x}{V_i} &= -\frac{R_2}{R_1} \\ V_x &= V_i \frac{R_2 || R_3}{R_2 || R_3 + R_4} = \frac{V_i R_2 R_3}{R_2 R_3 + R_4 R_2 + R_4 R_3} \\ \frac{V_o}{V_x} &= \frac{R_2 R_3 + R_2 R_4 + R_3 R_4}{R_2 R_3} = 1 + \frac{R_4}{R_3} + \frac{R_4}{R_2} \\ \frac{V_o}{V_x} &= \frac{V_o}{V_i} = \frac{(1 + R_4/R_3 + R_4/R_2)}{1 - R_1/R_2} \\ \frac{V_o}{V_i} &= -\frac{R_2}{R_1} (1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}) \end{aligned}$$

2.31

$$a) R_1 = R$$

$$\begin{aligned} R_2 &= R || R + \frac{R}{2} = \frac{R}{2} + \frac{R}{2} = R \\ R_3 &= R_2 || R + \frac{R}{2} = R || R + \frac{R}{2} = R \\ R_4 &= R_3 || R + \frac{R}{2} = R || R + \frac{R}{2} = R \end{aligned}$$



Cont.

2.34

$$\begin{aligned}
 a) V_1 &= I_1 R = IR \\
 V_2 &= I_2 R = 2IR \\
 V_3 &= -I_3 R = -4IR \\
 V_4 &= -\frac{I}{3} R + I_4 \frac{R}{2} = -4IR - 8I \frac{R}{2} = -8IR
 \end{aligned}$$

2.32

$$\begin{aligned}
 a) I_1 &= \frac{1V}{10k\Omega} = 0.1mA \\
 I_2 &= 0.1mA, I_2 \times 10k\Omega = I_3 \times 100\Omega \Rightarrow I_3 = 10mA \\
 V_x &= 10mA \times 100\Omega = 1V
 \end{aligned}$$

$$\begin{aligned}
 b) V_x &= R_L I_L + V_o, I_1 = I_2 + I_3 = 10.1mA \\
 1V &= R_L \times 10.1mA + V_o \\
 R_L &= \frac{1-V_o}{10.1} \Rightarrow R_{L\max} = \frac{1-V_{o\min}}{10.1} = \frac{14}{10.1} = 1.38k\Omega \\
 R_{L\min} &=
 \end{aligned}$$

$$c) 100\Omega < R_L < 1k\Omega$$

$I_L$  stays fixed at 10.1mA

$$V_o = V_x - R_L I_L = 1 - R_L \times 10.1 \Rightarrow -9.1 \leq V_o \leq -0.01$$

2.33

$$\begin{aligned}
 a) \frac{i_L}{i_I} &= 20 \Rightarrow i_L = 20i_I \\
 -10k\Omega \times i_I &= R(i_I - i_L) \\
 R &= \frac{10k\Omega \times i_I}{20i_I - i_I} = 0.53k\Omega
 \end{aligned}$$

$$\begin{aligned}
 b) R_L &= 1k\Omega, -12 \leq V_o \leq 12V \\
 V_o &= R_L i_L + 10k\Omega \times i_I = i_I (1k\Omega \times \frac{i_L}{i_I} + 10k\Omega) \\
 V_o &= i_I (1 \times 20 + 10) = 30i_I \\
 i_I &= \frac{V_o}{30} \Rightarrow -12 \leq i_I \leq 12 \Rightarrow -0.4 \leq i_I \leq 0.4
 \end{aligned}$$

$$\begin{aligned}
 c) R_L &= \frac{V_o}{i_I} = \frac{0}{i_I} = 0 \\
 V_o &= 0 \Rightarrow i_I = 0 \\
 \Rightarrow i_L &= 1mA
 \end{aligned}$$

From part a:  $i_L = 20 \times i_I = 20 \text{ mA}$

$$R_2 \gg R_3, \text{ if we ignore the current across } R_2 : V_A = \frac{V_o R_3}{R_3 + R_4}$$

$$\frac{V_T}{R_1} = \frac{0 - V_A}{R_2} \Rightarrow V_A = -\frac{R_2 V_T}{R_1}$$

$$\frac{V_o}{R_3 + R_4} = -\frac{R_2}{R_1} \times V_I \Rightarrow \frac{V_o}{V_I} = -\frac{R_2}{R_1} \left( 1 + \frac{R_4}{R_3} \right)$$

Now if we recalculate  $V_A$  considering that there is a voltage divider between  $R_4$  and  $R_3 \parallel R_2$ :

$$V_A = V_o \frac{R_3 \parallel R_2}{R_4 + R_3 \parallel R_2} = V_o \frac{R_3 R_2}{R_4 (R_3 R_2) + R_2^2}$$

$$V_A = V_o \frac{R_2 R_3}{R_3 R_4 + R_2 R_4 + R_2 R_3}$$

$$V_A = V_o \frac{1}{R_4/R_2 + R_4 + 1}$$

$$V_A = -\frac{R_2}{R_1} \cdot V_I \Rightarrow \frac{V_o}{V_I} = -\frac{R_2}{R_1} \left( \frac{R_4}{R_2} + \frac{R_4}{R_3} + 1 \right)$$

same as example 2.2.

2.35

$$R_L = 100k\Omega, -10 \leq \frac{V_o}{V_I} \leq -1 \frac{V}{V}$$

$$R_1 = R_2 = 100k\Omega$$

$$\frac{V_o}{V_I} = \frac{-R_2}{R_1} \left( \frac{R_4}{R_3} + \frac{R_4}{R_2} + 1 \right)$$

$$R_4 = 0 \Rightarrow \frac{V_o}{V_I} = \frac{-R_2}{R_1} = -1 \Rightarrow R_2 = 100k\Omega$$

$$R_4 = 10k\Omega \Rightarrow \frac{V_o}{V_I} = -10 = -1 \times \left( \frac{10k\Omega}{R_3} + \frac{10k\Omega}{100k\Omega} + 1 \right)$$

$$+10 = \left( \frac{10}{R_3} + 1.1 \right) \Rightarrow R_3 = 1.12k\Omega$$

Potentiometer in the middle:  $\frac{V_o}{V_I} = -1 \left( \frac{5}{5+R_3} + \frac{5}{100} + 1 \right)$

$$\frac{V_o}{V_I} = -1.87 \frac{V}{V}$$

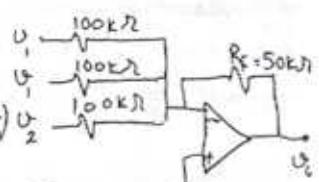
2.36

According to eq. 2.7:

$$U_o = -\left(\frac{R_E}{R_1} U_1 + \frac{R_E}{R_2} U_2 + \frac{R_E}{R_n} U_n\right)$$

$$U_o = -\left(\frac{50k}{100k} U_1 + \frac{50k}{100k} U_2 + \frac{50k}{100k} U_n\right)$$

$$U_o = -(U_1 + U_2) \quad U_1 = 3, U_2 = -3 \Rightarrow U_o = -1.5V$$

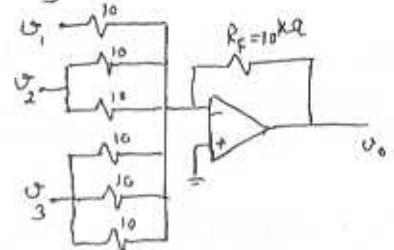


$$\frac{R_E}{R_3} = 3 \Rightarrow R_3 = \frac{10}{3} k\Omega$$

$$R_{I1} = 10k\Omega$$

$$R_{I2} = 5k\Omega$$

$$R_{I3} = 3.3k\Omega$$



2.37

We choose the weighted Summer configuration

$$U_o = -[4U_1 + \frac{U_2}{3}]$$

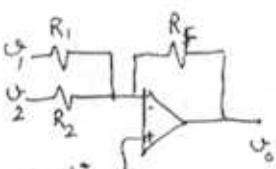
$$i_1 = \frac{U_1}{R_1}, \quad i_2 = \frac{U_2}{R_2}$$

$$i_1, i_2 \leq 0.1mA \text{ for } U_1, U_2 = 1V$$

$$R_1, R_2 \geq 10k\Omega$$

$$\frac{R_E}{R_1} = 4, \text{ if } R_1 = 10k\Omega \Rightarrow R_f = 40k\Omega$$

$$\frac{R_E}{R_2} = \frac{1}{3} \Rightarrow R_2 = 120k\Omega$$



$$b) U_o = -(U_1 + U_2 + 2U_3 + 2U_4)$$

$$\frac{R_E}{R_1} = 1 \Rightarrow R_1 = 10k\Omega$$

$$\frac{R_E}{R_2} = 1 \Rightarrow R_2 = 10k\Omega$$

$$\frac{R_E}{R_3} = 2 \Rightarrow R_3 = \frac{10}{2} k\Omega$$

$$\frac{R_E}{R_4} = 2 \Rightarrow R_4 = \frac{10}{2} k\Omega$$

$$R_{I1} = R_{I2} = 10k\Omega$$

$$R_{I3} = R_{I4} = 5k\Omega$$

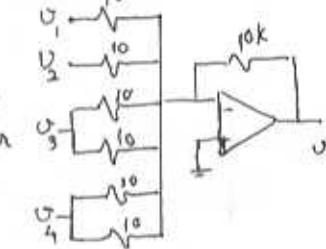
$$c) U_o = -(U_1 + 5U_2)$$

$$R_1 = 10k$$

$$R_2 = \frac{10k}{5}$$

$$R_{I1} = 10k\Omega$$

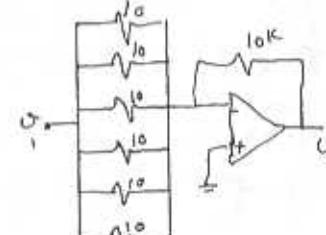
$$R_{I2} = 2k\Omega$$



$$d) U_o = -6U_1$$

$$R_1 = 10k$$

$$R_{I1} = 1.67k\Omega$$



Suggested configurations:

$$U_o = -(2U_1 + 2U_2 + 2U_3) \rightarrow$$

$$U_o = -(3U_1 + 3U_2)$$

$$U_o = -(U_1 + U_2 + U_3 + U_4)$$

$$U_o = -(U_1 + 5U_2)$$

$$U_o = -(U_1 + 2U_2 + 3U_3)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + U_5)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + 2U_5)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + 2U_5 + U_6)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + 2U_5 + 2U_6)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + 2U_5 + 2U_6 + U_7)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + 2U_5 + 2U_6 + 2U_7)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + 2U_5 + 2U_6 + 2U_7 + U_8)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + 2U_5 + 2U_6 + 2U_7 + 2U_8)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + 2U_5 + 2U_6 + 2U_7 + 2U_8 + U_9)$$

$$U_o = -(U_1 + 2U_2 + 2U_3 + 2U_4 + 2U_5 + 2U_6 + 2U_7 + 2U_8 + 2U_9)$$

2.39

$$a) U_o = -(U_1 + 2U_2 + 3U_3)$$

$$\frac{R_E}{R_1} = 1 \Rightarrow R_1 = 10k\Omega, \quad \frac{R_E}{R_2} = 2 \Rightarrow R_2 = 5k\Omega$$

$$\frac{R_E}{R_3} = 3 \Rightarrow R_3 = 3.3k\Omega$$

$$\frac{R_E}{R_4} = 4 \Rightarrow R_4 = 2.5k\Omega$$

$$\frac{R_E}{R_5} = 5 \Rightarrow R_5 = 2k\Omega$$

$$\frac{R_E}{R_6} = 6 \Rightarrow R_6 = 1.67k\Omega$$

$$\frac{R_E}{R_7} = 7 \Rightarrow R_7 = 1.43k\Omega$$

$$\frac{R_E}{R_8} = 8 \Rightarrow R_8 = 1.25k\Omega$$

$$\frac{R_E}{R_9} = 9 \Rightarrow R_9 = 1.11k\Omega$$

$$\frac{R_E}{R_{10}} = 10 \Rightarrow R_{10} = 1k\Omega$$

$$\frac{R_E}{R_{11}} = 11 \Rightarrow R_{11} = 909\Omega$$

$$\frac{R_E}{R_{12}} = 12 \Rightarrow R_{12} = 833\Omega$$

$$\frac{R_E}{R_{13}} = 13 \Rightarrow R_{13} = 769\Omega$$

$$\frac{R_E}{R_{14}} = 14 \Rightarrow R_{14} = 708\Omega$$

$$\frac{R_E}{R_{15}} = 15 \Rightarrow R_{15} = 647\Omega$$

$$\frac{R_E}{R_{16}} = 16 \Rightarrow R_{16} = 588\Omega$$

$$\frac{R_E}{R_{17}} = 17 \Rightarrow R_{17} = 531\Omega$$

$$\frac{R_E}{R_{18}} = 18 \Rightarrow R_{18} = 477\Omega$$

$$\frac{R_E}{R_{19}} = 19 \Rightarrow R_{19} = 424\Omega$$

$$\frac{R_E}{R_{20}} = 20 \Rightarrow R_{20} = 375\Omega$$

$$\frac{R_E}{R_{21}} = 21 \Rightarrow R_{21} = 326\Omega$$

$$\frac{R_E}{R_{22}} = 22 \Rightarrow R_{22} = 286\Omega$$

$$\frac{R_E}{R_{23}} = 23 \Rightarrow R_{23} = 250\Omega$$

$$\frac{R_E}{R_{24}} = 24 \Rightarrow R_{24} = 216\Omega$$

$$\frac{R_E}{R_{25}} = 25 \Rightarrow R_{25} = 188\Omega$$

$$\frac{R_E}{R_{26}} = 26 \Rightarrow R_{26} = 164\Omega$$

$$\frac{R_E}{R_{27}} = 27 \Rightarrow R_{27} = 143\Omega$$

$$\frac{R_E}{R_{28}} = 28 \Rightarrow R_{28} = 125\Omega$$

$$\frac{R_E}{R_{29}} = 29 \Rightarrow R_{29} = 111\Omega$$

$$\frac{R_E}{R_{30}} = 30 \Rightarrow R_{30} = 909\Omega$$

$$\frac{R_E}{R_{31}} = 31 \Rightarrow R_{31} = 769\Omega$$

$$\frac{R_E}{R_{32}} = 32 \Rightarrow R_{32} = 647\Omega$$

$$\frac{R_E}{R_{33}} = 33 \Rightarrow R_{33} = 531\Omega$$

$$\frac{R_E}{R_{34}} = 34 \Rightarrow R_{34} = 477\Omega$$

$$\frac{R_E}{R_{35}} = 35 \Rightarrow R_{35} = 424\Omega$$

$$\frac{R_E}{R_{36}} = 36 \Rightarrow R_{36} = 375\Omega$$

$$\frac{R_E}{R_{37}} = 37 \Rightarrow R_{37} = 326\Omega$$

$$\frac{R_E}{R_{38}} = 38 \Rightarrow R_{38} = 286\Omega$$

$$\frac{R_E}{R_{39}} = 39 \Rightarrow R_{39} = 250\Omega$$

$$\frac{R_E}{R_{40}} = 40 \Rightarrow R_{40} = 216\Omega$$

$$\frac{R_E}{R_{41}} = 41 \Rightarrow R_{41} = 188\Omega$$

$$\frac{R_E}{R_{42}} = 42 \Rightarrow R_{42} = 164\Omega$$

$$\frac{R_E}{R_{43}} = 43 \Rightarrow R_{43} = 143\Omega$$

$$\frac{R_E}{R_{44}} = 44 \Rightarrow R_{44} = 125\Omega$$

$$\frac{R_E}{R_{45}} = 45 \Rightarrow R_{45} = 909\Omega$$

$$\frac{R_E}{R_{46}} = 46 \Rightarrow R_{46} = 769\Omega$$

$$\frac{R_E}{R_{47}} = 47 \Rightarrow R_{47} = 647\Omega$$

$$\frac{R_E}{R_{48}} = 48 \Rightarrow R_{48} = 531\Omega$$

$$\frac{R_E}{R_{49}} = 49 \Rightarrow R_{49} = 477\Omega$$

$$\frac{R_E}{R_{50}} = 50 \Rightarrow R_{50} = 424\Omega$$

$$\frac{R_E}{R_{51}} = 51 \Rightarrow R_{51} = 375\Omega$$

$$\frac{R_E}{R_{52}} = 52 \Rightarrow R_{52} = 326\Omega$$

$$\frac{R_E}{R_{53}} = 53 \Rightarrow R_{53} = 250\Omega$$

$$\frac{R_E}{R_{54}} = 54 \Rightarrow R_{54} = 216\Omega$$

$$\frac{R_E}{R_{55}} = 55 \Rightarrow R_{55} = 188\Omega$$

$$\frac{R_E}{R_{56}} = 56 \Rightarrow R_{56} = 164\Omega$$

$$\frac{R_E}{R_{57}} = 57 \Rightarrow R_{57} = 143\Omega$$

$$\frac{R_E}{R_{58}} = 58 \Rightarrow R_{58} = 125\Omega$$

$$\frac{R_E}{R_{59}} = 59 \Rightarrow R_{59} = 909\Omega$$

$$\frac{R_E}{R_{60}} = 60 \Rightarrow R_{60} = 769\Omega$$

$$\frac{R_E}{R_{61}} = 61 \Rightarrow R_{61} = 647\Omega$$

$$\frac{R_E}{R_{62}} = 62 \Rightarrow R_{62} = 531\Omega$$

$$\frac{R_E}{R_{63}} = 63 \Rightarrow R_{63} = 477\Omega$$

$$\frac{R_E}{R_{64}} = 64 \Rightarrow R_{64} = 424\Omega$$

$$\frac{R_E}{R_{65}} = 65 \Rightarrow R_{65} = 375\Omega$$

$$\frac{R_E}{R_{66}} = 66 \Rightarrow R_{66} = 326\Omega$$

$$\frac{R_E}{R_{67}} = 67 \Rightarrow R_{67} = 250\Omega$$

$$\frac{R_E}{R_{68}} = 68 \Rightarrow R_{68} = 216\Omega$$

$$\frac{R_E}{R_{69}} = 69 \Rightarrow R_{69} = 188\Omega$$

$$\frac{R_E}{R_{70}} = 70 \Rightarrow R_{70} = 164\Omega$$

$$\frac{R_E}{R_{71}} = 71 \Rightarrow R_{71} = 143\Omega$$

$$\frac{R_E}{R_{72}} = 72 \Rightarrow R_{72} = 125\Omega$$

$$\frac{R_E}{R_{73}} = 73 \Rightarrow R_{73} = 909\Omega$$

$$\frac{R_E}{R_{74}} = 74 \Rightarrow R_{74} = 769\Omega$$

$$\frac{R_E}{R_{75}} = 75 \Rightarrow R_{75} = 647\Omega$$

$$\frac{R_E}{R_{76}} = 76 \Rightarrow R_{76} = 531\Omega$$

$$\frac{R_E}{R_{77}} = 77 \Rightarrow R_{77} = 477\Omega$$

$$\frac{R_E}{R_{78}} = 78 \Rightarrow R_{78} = 424\Omega$$

$$\frac{R_E}{R_{79}} = 79 \Rightarrow R_{79} = 375\Omega$$

$$\frac{R_E}{R_{80}} = 80 \Rightarrow R_{80} = 326\Omega$$

$$\frac{R_E}{R_{81}} = 81 \Rightarrow R_{81} = 250\Omega$$

$$\frac{R_E}{R_{82}} = 82 \Rightarrow R_{82} = 216\Omega$$

$$\frac{R_E}{R_{83}} = 83 \Rightarrow R_{83} = 188\Omega$$

$$\frac{R_E}{R_{84}} = 84 \Rightarrow R_{84} = 164\Omega$$

$$\frac{R_E}{R_{85}} = 85 \Rightarrow R_{85} = 143\Omega$$

$$\frac{R_E}{R_{86}} = 86 \Rightarrow R_{86} = 125\Omega$$

$$\frac{R_E}{R_{87}} = 87 \Rightarrow R_{87} = 909\Omega$$

$$\frac{R_E}{R_{88}} = 88 \Rightarrow R_{88} = 769\Omega$$

$$\frac{R_E}{R_{89}} = 89 \Rightarrow R_{89} = 647\Omega$$

$$\frac{R_E}{R_{90}} = 90 \Rightarrow R_{90} = 531\Omega$$

$$\frac{R_E}{R_{91}} = 91 \Rightarrow R_{91} = 477\Omega$$

$$\frac{R_E}{R_{92}} = 92 \Rightarrow R_{92} = 424\Omega$$

$$\frac{R_E}{R_{93}} = 93 \Rightarrow R_{93} = 375\Omega$$

$$\frac{R_E}{R_{94}} = 94 \Rightarrow R_{94} = 326\Omega$$

$$\frac{R_E}{R_{95}} = 95 \Rightarrow R_{95} = 250\Omega$$

$$\frac{R_E}{R_{96}} = 96 \Rightarrow R_{96} = 216\Omega$$

$$\frac{R_E}{R_{97}} = 97 \Rightarrow R_{97} = 188\Omega$$

$$\frac{R_E}{R_{98}} = 98 \Rightarrow R_{98} = 164\Omega$$

$$\frac{R_E}{R_{99}} = 99 \Rightarrow R_{99} = 143\Omega$$

$$\frac{R_E}{R_{100}} = 100 \Rightarrow R_{100} = 125\Omega$$

$$\frac{R_E}{R_{101}} = 101 \Rightarrow R_{101} = 909\Omega$$

$$\frac{R_E}{R_{102}} = 102 \Rightarrow R_{102} = 769\Omega$$

$$\frac{R_E}{R_{103}} = 103 \Rightarrow R_{103} = 647\Omega$$

$$\frac{R_E}{R_{104}} = 104 \Rightarrow R_{104} = 531\Omega$$

$$\frac{R_E}{R_{105}} = 105 \Rightarrow R_{105} = 477\Omega$$

$$\frac{R_E}{R_{106}} = 106 \Rightarrow R_{106} = 424\Omega$$

2.40

The output signal should be:

$$U_o = -5 \sin \omega t - 5$$

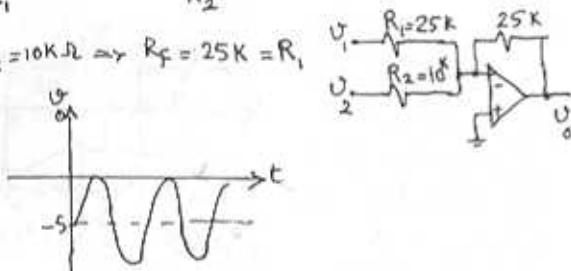
if we assume:  $U_1 = 5 \sin \omega t$

$$U_2 = 2V \quad \left\{ U_o = U_1 + 2.5U_2 \right.$$

In a weighted summer configuration:

$$\frac{R_f}{R_1} = +1 \quad \frac{R_f}{R_2} = 2.5$$

$$R_2 = 10k\Omega \Rightarrow R_f = 25k\Omega = R_1$$



2.41

$$U_o = U_1 + 2U_2 - 3U_3 - 4U_4 : \text{Consider Fig. 2.11.}$$

According to eq. 2.8 For a weighted summer circuit:

$$U_o = U_1 \frac{R_a}{R_1} \frac{R_c}{R_b} + U_2 \frac{R_a}{R_2} \frac{R_c}{R_b} - U_3 \frac{R_c}{R_3} - U_4 \frac{R_c}{R_4}$$

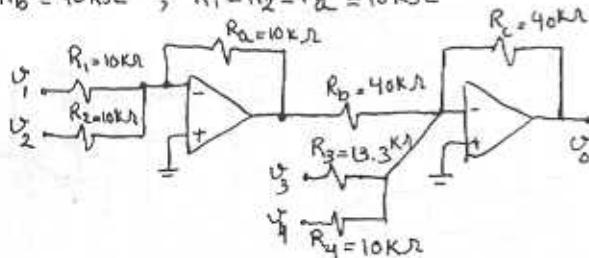
$$\frac{R_a}{R_1} \frac{R_c}{R_b} = 1, \quad \frac{R_a}{R_2} \frac{R_c}{R_b} = 1, \quad \frac{R_c}{R_3} = 3, \quad \frac{R_c}{R_4} = 4$$

assume:

$$R_4 = 10k\Omega \Rightarrow R_c = 40k\Omega \Rightarrow R_3 = \frac{40}{3} = 13.3k\Omega$$

$$\frac{R_a}{R_1} \frac{40}{R_b} = 1 \quad \frac{R_a}{R_2} \frac{40}{R_b} = 1$$

$$R_b = 40k\Omega, \quad R_1 = R_2 = R_a = 10k\Omega$$



2.42

$$U_1 = 3 \sin(2\pi \times 60t) + 0.01 \sin(2\pi \times 1000t)$$

$$U_2 = 3 \sin(2\pi \times 60t) - 0.01 \sin(2\pi \times 1000t)$$

we want to have:  $U_o = 10U_1 - 10U_2$

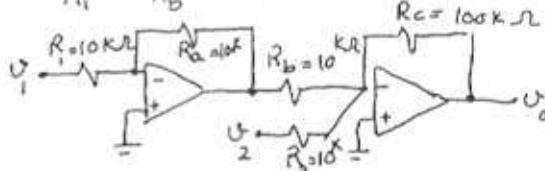
we use the circuit in Fig. 2.11.

According to Eq. 2.8:

$$U_o = U_1 \frac{R_a}{R_1} \frac{R_c}{R_b} - U_2 \frac{R_c}{R_3}$$

$$\frac{R_a}{R_1} \frac{R_c}{R_b} = 10, \quad \frac{R_c}{R_3} = 10, \text{ if } R_3 = 0 \Rightarrow R_c = 100k\Omega$$

$$\Rightarrow \frac{R_a}{R_1} \times \frac{100k\Omega}{R_3} = 10 \Rightarrow R_a = R_1 = R_b = 10k\Omega$$



$$U_o = 10U_1 - 10U_2 = 10 \times 0.02 \sin(2\pi \times 1000t)$$

$$U_o = 0.2 \sin(2\pi \times 1000t) \quad -0.2 \leq U_o \leq 0.2$$

2.43

This is a weighted summer circuit:

$$U_o = -\left(\frac{R_F}{R_0} U_0 + \frac{R_F}{R_1} U_1 + \frac{R_F}{R_2} U_2 + \frac{R_F}{R_3} U_3\right)$$

$$\text{we may write: } U_0 = 5^0 \times a_0, \quad U_1 = 5^1 \times a_1, \quad U_2 = 5^2 \times a_2$$

$$U_3 = 5^3 \times a_3, \quad U_o = -R_F \left( \frac{5a_0}{80k} + \frac{5}{40k} a_1 + \frac{5}{20k} a_2 + \frac{5}{10k} a_3 \right)$$

$$U_o = -R_F \left( \frac{a_0}{16} + \frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{2} \right)$$

$$U_o = -\frac{R_F}{16} (2^0 a_0 + 2^1 a_1 + 2^2 a_2 + 2^3 a_3)$$

$$-12 \leq U_o \leq 0 \Rightarrow \frac{R_F}{16} (2^0 + 2^1 + 2^2 + 2^3) =$$

$$= \frac{15}{16} R_F = 12 \quad \text{when } a_0 = a_1 = a_2 = a_3 = 1 \text{ we have the peak value at } U_o.$$

$$\Rightarrow R_F = 12.8k\Omega$$

2.44

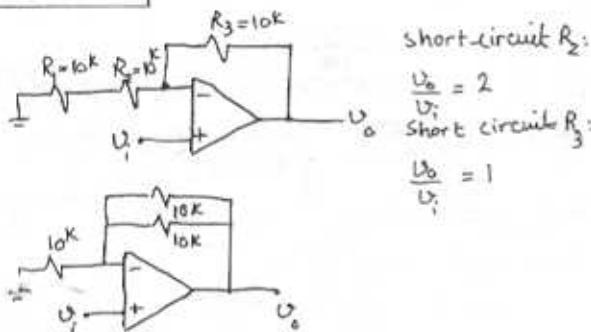
$$a) \frac{U_o}{U_1} = 1 = 1 + \frac{R_2}{R_1} \Rightarrow R_2 = 0, R_1 = 10k\Omega$$

$$b) \frac{U_o}{U_1} = 2 = 1 + \frac{R_2}{R_1} \Rightarrow R_1 = R_2 = 10k\Omega$$

Cont.

c)  $\frac{U_o}{U_i} = 101 \frac{V}{V} = 1 + \frac{R_2}{R} \Rightarrow \text{if } R_1 = 10k\Omega \Rightarrow R_2 = 100k\Omega$   
d)  $\frac{U_o}{U_i} = 100 \frac{V}{V} = 1 + \frac{R_2}{R} \Rightarrow \text{if } R_1 = 10k\Omega \Rightarrow R_2 = 990k\Omega$

2.45



2.46

$$U_+ = U_- = V = R \times i, i = 100 \mu A \text{ when } V = 10^5$$

$$\Rightarrow R = \frac{10}{0.1mA} = 100k\Omega$$

As indicated,  $i$  only depends on  $R$  and  $V$  and the meter resistance does not affect  $i$ .

2.47

Refer to the circuit in P2.47:

a) using superposition, we first set  $U_{P1}=U_{P2}=\dots=0$ . The output voltage that results in response to  $U_{N1}, U_{N2}, \dots, U_{Nn}$  is:

$$U_{ON} = -\left[ \frac{R_E}{R_{N1}} U_{N1} + \frac{R_F}{R_{N2}} U_{N2} + \dots + \frac{R_F}{R_{Nn}} U_{Nn} \right]$$

Then we set  $U_{N1}=U_{N2}=\dots=0$ , then:

$$R_N = R_{N1} \parallel R_{N2} \parallel R_{N3} \parallel \dots \parallel R_{Nn}$$

The circuit simplifies to:

$$U_{Op} = \left(1 + \frac{R_F}{R_N}\right) \times$$

$$\left( U_{P1} \frac{1}{R_{P1}} + U_{P2} \frac{1}{R_{P2}} + \dots + U_{Pn} \frac{1}{R_{Pn}} \right)$$

$$U_{Op} = \left(1 + \frac{R_E}{R_N}\right) \left( U_{P1} \frac{R_P}{R_{P1}} + U_{P2} \frac{R_P}{R_{P2}} + \dots + U_{Pn} \frac{R_P}{R_{Pn}} \right)$$

where:  
 $R_P = R_{P1} \parallel R_{P2} \parallel \dots \parallel R_{Pn}$

when all inputs are present:

$$U_o = U_{ON} + U_{op} = -\left( \frac{R_E}{R_N} U_{N1} + \frac{R_F}{R_{N2}} U_{N2} + \dots \right) + \left( 1 + \frac{R_E}{R_N} \right) \left( \frac{R_P}{R_{P1}} U_{P1} + \frac{R_P}{R_{P2}} U_{P2} + \dots \right)$$

$$\text{b) } U_o = -2U_{N1} + U_{P1} + 2U_{P2}$$

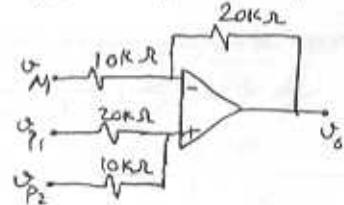
$$\frac{R_E}{R_N} = 2 \quad R_{N1} = 10k\Omega \Rightarrow R_F = 20k\Omega$$

$$\left(1 + \frac{R_E}{R_N}\right) \left( \frac{R_P}{R_{P1}} \right) = 1 \Rightarrow 3 \frac{R_P}{R_{P1}} = 1 \Rightarrow R_{P2} = \frac{R_{P1}}{2}$$

$$\left(1 + \frac{R_E}{R_N}\right) \left( \frac{R_P}{R_{P2}} \right) = 2 \Rightarrow 3 \frac{R_P}{R_{P2}} = 2 \Rightarrow R_{P2} = \frac{R_{P1}}{2}$$

where  $R_P = \frac{R_{P1} \times R_{P2}}{R_{P1} + R_{P2}}$  (ignoring  $R_{P3}$ )

Note that if the results from the last 2 constraints differ, we would use an additional resistor connected from the positive input to ground. ( $R_{P3}$ )



2.48

$$U_o = U_{I1} + 3U_{I2} - 2(U_{I3} + 3U_{I4})$$

Refer to P2.47.

$$\frac{R_E}{R_{N3}} = 2 \quad \text{if } R_{N3} = 10k\Omega \Rightarrow R_F = 20k\Omega$$

$$\frac{R_E}{R_{N4}} = 6 \Rightarrow R_{N4} = \frac{20}{6} = 3.3k\Omega$$

$$R_N = R_{N3} \parallel R_{N4} = 10k \parallel 3.3k = 2.48k\Omega$$

$$\left(1 + \frac{R_E}{R_N}\right) \frac{R_P}{R_{P1}} = 1 \Rightarrow \left(1 + \frac{20}{2.48}\right) \frac{R_P}{R_{P1}} = 1 \Rightarrow 9.06 \frac{R_P}{R_{P1}} = R_{P1}$$

$$R_P = R_{P1} \parallel R_{P2} \parallel R_{P3} \Rightarrow R_P = \frac{1}{\frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P3}}}$$

$$\left(1 + \frac{R_E}{R_N}\right) \frac{R_P}{R_{P2}} = 3 \Rightarrow 9.06 \frac{R_P}{R_{P2}} = 3 \Rightarrow R_{P2} \approx 3R_{P1}$$

$$R_{P1} \parallel R_{P2} = \frac{9 \times 3R_{P1}}{9+3} = 2.25R_{P1}, R_P = 2.25R_{P1} \parallel R_{P3}$$

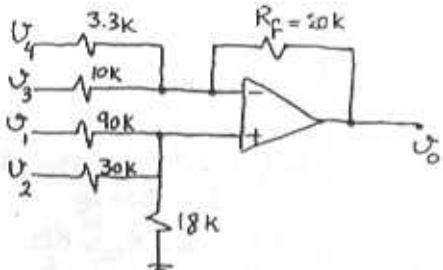
Cont.

$$2.25 R_p + R_{P_0} = 2.25 R_{P_0} \Rightarrow R_{P_0} = 1.8 R_p$$

if  $R_p = 10\text{ k}\Omega \Rightarrow R_{P_0} = 18\text{ k}\Omega$

$$R_{P_1} = 9 \times 10\text{ k} = 90\text{ k}\Omega$$

$$R_{P_2} = 3 \times 10\text{ k} = 30\text{ k}\Omega$$



2.49

$$U_+ = U_I \frac{R_4}{R_3 + R_4} = U$$

$$\frac{U_o}{R_1} = \frac{U_+ - U_-}{R_2} \rightarrow U_o = U_- (1 + \frac{R_2}{R_1})$$

from the two above equations :

$$\frac{U_o}{U_I} = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4}{R_3 + R_4}\right) = \frac{1 + R_2/R_1}{1 + R_3/R_4}$$

2.50

Refer to Fig. 2.50. Setting  $U_2 = 0$ , we obtain the output component due to  $U_1$  as:

$$U_{o1} = -20U_1$$

Setting  $U_1 = 0$ , we obtain the output component due to  $U_2$  as:

$$U_{o2} = U_2 \left(1 + \frac{20R}{R}\right) \left(\frac{20R}{20R + R}\right) = 20U_2$$

The total output voltage is :

$$U_o = U_{o1} + U_{o2} = 20(U_2 - U_1)$$

$$\text{For } U_1 = 10\sin 2\pi \times 60t - 0.1\sin(2\pi \times 1000t)$$

$$U_2 = 10\sin 2\pi \times 60t + 0.1\sin(2\pi \times 1000t)$$

$$U_o = 4\sin(2\pi \times 1000t)$$

2.51

$$\frac{U_o}{U_i} = 1 + \frac{R_2}{R_1} = 1 + \frac{(1-x)}{x} = 1 + \frac{1}{x} - 1 = \frac{1}{x}$$

$$0 < x \leq 1 \Rightarrow 1 + \frac{U_o}{U_i} \leq \infty$$

if we add a resistor on the ground path:

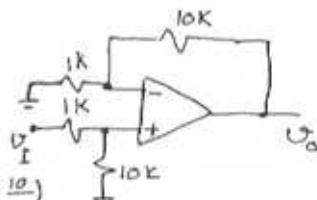
$$\frac{U_o}{U_i} = 1 + \frac{(1-x) \times 10\text{k}}{x \times 10\text{k} + R}$$

$$\text{Gain}_{\max} = 21 \text{ when}$$

$$x=0 \Rightarrow 21 = 1 + \frac{10\text{k}}{R} \Rightarrow R = \frac{10\text{k}}{20} = 0.5\text{k}\Omega$$



2.52



$$U_o = U_I \frac{10}{1+10} \left(1 + \frac{10}{1}\right), \quad U_o = 10U_I$$

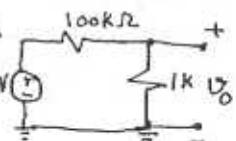
2.53

a) Source is connected directly,

$$U_o = 10 \times \frac{1}{1+10} = 0.99V$$

$$i_L = \frac{U_o}{1\text{k}} = \frac{0.99}{1\text{k}} = 0.99\text{mA}$$

current supplied by the 10V source is 0.99mA.



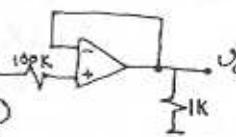
b) inserting a buffer

$$U_o = 10V$$

$$i_L = \frac{10V}{1\text{k}} = 10\text{mA}$$

current supplied by the 10V source is 0.

The load current  $i_L$  comes from the power supply of the op-amp.



2.54

$$U_o = U_i - \frac{U_o}{A}$$

$$\frac{U_o}{U_i} = \frac{1}{1 + \frac{1}{A}}$$

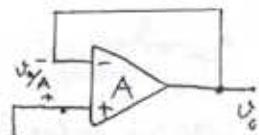
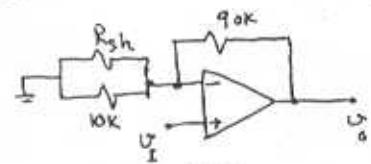
error of Gain magnitude

$$\left| \frac{U_o}{U_i} - 1 \right| = - \frac{1}{A+1}$$

 $A(V/V)$  $\frac{U_o}{U_i}(V/V)$ 

Gain Error

	1000	100	10
0.999	0.990	0.909	
-0.1%	-1%	-9.1%	

we can shunt a resistor with  $R_s$ .

$$R_{sh} : 10 = \frac{1 + \frac{90}{10} + \frac{90}{R_{sh}}}{1 + 1 + \frac{90}{10} + \frac{90}{R_{sh}}} \Rightarrow$$

$$10 \times (510 R_{sh} + 90 R_{sh} + 900) = 50 \times (10 R_{sh} + 90 R_{sh} + 900)$$

$$100 R_{sh} = 3600 \Rightarrow R_{sh} = 36 \text{ k}\Omega$$

if  $A = 100$  then :

$$\text{Uncompensated} = \frac{1 + \frac{90}{10}}{1 + 1 + \frac{90}{10}} = \frac{10}{1.1} = 9.09 V/V$$

$$\text{Compensated} = \frac{1 + \frac{90}{10} + \frac{90}{36}}{1 + 1 + \frac{90}{10} + \frac{90}{36}} = \frac{12.5}{1.125} = 11.1 V/V$$

2.55

for an inverting amplifier:

$$R_i = R_s, G = -\frac{R_2}{R_s}$$

for a non-inverting amplifier:

$$R_i = \infty, G = 1 + \frac{R_2}{R_s}$$

Case	Gain %	$R_{in}$	$R_s$	$R_2$
a	-10	10k	10k	100k
b	-1	100k	100k	100k
c	-2	50k	50k	100k
d	+1	$\infty$	10k	10k
e	+2	$\infty$	10k	10k
f	+11	$\infty$	10k	100k
g	-0.5	10k	10k	5k

2.56

$$A = 50 V/V, 1 + \frac{R_2}{R_s} = 10 V/V$$

$$\text{if } R_s = 10 \text{ k}\Omega \Rightarrow R_2 = 90 \text{ k}\Omega$$

$$\text{According to Eq. 2.11: } G = \frac{U_o}{U_i} = \frac{1 + \frac{R_2}{R_s}}{1 + \frac{1 + R_2/R_s}{A}}$$

$$G = \frac{1 + \frac{90}{10}}{1 + \frac{1 + 90/10}{50}} = \frac{10}{1.2} = 8.33 V/V$$

In order to compensate the gain drop,

2.57

$$G = \frac{G_o}{1 + \frac{G_o}{A}}, \frac{G_o - G}{G_o} \times 100 = \frac{G_o/A \times 100}{1 + \frac{G_o}{A}} \times \chi$$

$$\text{or } \frac{1 + \frac{G_o}{A}}{\frac{G_o}{A}} \geq \frac{100}{\chi} \Rightarrow \frac{A}{G_o} \geq \underbrace{\frac{(100)}{\chi} - 1}_{F}$$

$$\Rightarrow A \geq G_o F \text{ where } F = \frac{100}{\chi} - 1 \leq \frac{100}{\chi} F$$

$$\frac{\chi}{F} \mid \begin{array}{ccccc} 0.01 & 0.1 & 1 & 10 & 100 \end{array}$$

$$\chi = 0.01 : G_o(V/V) \quad 1 \quad 10 \quad 10^2 \quad 10^3 \quad 10^4$$

$$A(V/V) \quad 10^{-4} \quad 10^{-5} \quad 10^{-6} \quad 10^{-7} \quad 10^{-8}$$

too high to be practical

$$\chi = 0.1 : G_o(V/V) \quad 1 \quad 10 \quad 10^2 \quad 10^3 \quad 10^4$$

$$A(V/V) \quad 10^{-3} \quad 10^{-4} \quad 10^{-5} \quad 10^{-6} \quad 10^{-7}$$

$$\chi = 1 : G_o(V/V) \quad 1 \quad 10 \quad 10^2 \quad 10^3 \quad 10^4$$

$$A(V/V) \quad 10^{-2} \quad 10^{-3} \quad 10^{-4} \quad 10^{-5} \quad 10^{-6}$$

$$\chi = 10 : G_o(V/V) \quad 1 \quad 10 \quad 10^2 \quad 10^3 \quad 10^4$$

$$A(V/V) \quad 10 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5$$

2.58

for non-inverting amplifier, Eq. 2.11:

$$G = \frac{G_0}{1 + \frac{G_0}{A}}, E = \frac{G_0 - G}{G_0} \times 100$$

for inverting amplifier, Eq. 2.5:

$$G = \frac{G_0}{1 + \frac{1 - G_0}{A}}, E = \frac{G_0 - G}{G_0} \times 100$$

Case	$G_0(V_{in})$	$A(V/V)$	$G(V_{in})$	$E\%$
a	-1	10	-0.83	16
b	1	10	0.91	9
c	-1	100	-0.98	2
d	10	10	5	50
e	-10	100	9	10
f	-10	1000	-9.89	1.1
g	+1	2	0.67	33

2.59

Refer to Fig. P2.59, when potentiometer is set to the bottom:

$$V_o = V_+ = -15 + \frac{30 \times 20}{20 + 100 + 20} = -10.714V$$

$$\text{when set to the top: } V_o = -15 + \frac{30 \times 120}{20 + 100 + 20} = 10.714V$$

$$\Rightarrow -10.714 \leq V_o \leq +10.714$$

pot has 20 turns, each turn:  $\Delta V = \frac{2 \times 10.714}{20} = 1.07V$

2.60

Refer to Fig. 2.16. Notice that similar to eq. 2.15 we have:  $\frac{R_4}{R_3} = \frac{R_2}{R_1} = \frac{100}{10}$ . therefore according to 2.16:

$$V_o = \frac{R_2}{R_1} V_{id} \Rightarrow A = \frac{R_2}{R_1} = 10 \frac{V}{V}$$

According to 2.20:  $R_{id} = 2R_1 = 20k\Omega$

If  $\frac{R_2}{R_1}, \frac{R_4}{R_3}$  were different by  $\frac{1}{10}$ :

$$\frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3}$$

$$\text{Refer to eq. 2.19: } A_{cm} = \frac{R_4}{R_4 + R_3} \left( 1 - \frac{R_2}{R_1} \cdot \frac{R_3}{R_4} \right)$$

$$A_{cm} = \frac{100}{100 + 10} (1 - 0.99) = 0.009$$

$$\text{CMRR} = 20 \log \frac{|A_d|}{|A_{cm}|}, \text{ so let's calculate } A_d$$

$$A_d = \frac{V_o}{V_{id}} \text{ if we apply superposition:}$$

$$V_{o1} = -\frac{R_2}{R_1} V_+, \quad V_{o2} = V_+ \frac{R_4}{R_3 + R_4} \left( 1 + \frac{R_2}{R_1} \right)$$

$$V_o = V_{o2} + V_{o1} = V_+ \frac{R_4/R_3}{1 + \frac{R_4}{R_3}} \left( 1 + \frac{R_2}{R_1} \right) - \frac{R_2}{R_1} V_+$$

$$\text{Replace } \frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3} \Rightarrow \frac{R_2}{R_1} = 0.99 \times \frac{100}{10} = 9.9$$

$$V_o = V_+ \frac{10}{1 + 10} (1 + 9.9) - V_+ 9.9 = 9.9 (V_+ - V_+)$$

$$\frac{V_o}{V_{id}} = 9.9 = A_d \Rightarrow \text{CMRR} = 20 \log \frac{9.9}{0.009} = 60.8$$

$$\text{CMRR} = \underline{60.8}$$

2.61

If we assume  $R_3 = R_1, R_4 = R_2$ , then

$$\text{eq. 2.20: } R_{id} = 2R_1 \Rightarrow R_1 = \frac{20}{2} = 10k\Omega$$

(Refer to Fig. 2.16)

$$\text{a) } A_d = \frac{R_2}{R_1} = 1 \frac{V}{V} \Rightarrow R_2 = 10k\Omega$$

$$R_1 = R_2 = R_3 = R_4 = 10k\Omega$$

$$\text{b) } A_d = \frac{R_2}{R_1} = 2 \frac{V}{V} \Rightarrow R_2 = 20k\Omega = R_4$$

$$R_1 = R_3 = 10k\Omega$$

$$\text{c) } A_d = \frac{R_2}{R_1} = 100 \frac{V}{V} \Rightarrow R_2 = 1M\Omega = R_4$$

$$R_1 = R_3 = 10k\Omega$$

$$\text{d) } A_d = \frac{R_2}{R_1} = 0.5 \frac{V}{V} \Rightarrow R_2 = 5k\Omega = R_4$$

$$R_1 = R_3 = 10k\Omega$$

2.62

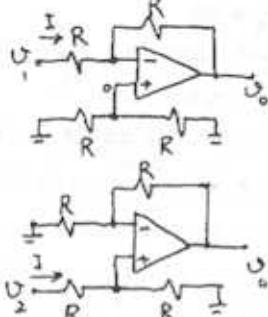
Refer to Fig P2.62 :

Cont.

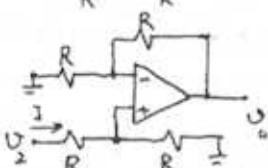
Considering that  $V_+ = V_+$ :

$$V_i + \frac{V_o - V_i}{2} = \frac{V_o}{2} \Rightarrow V_o = V_2 - V_i$$

$$V_i \text{ only: } R_I = \frac{V_i}{I} = R$$



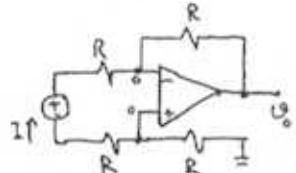
$$V_2 \text{ only: } R_I = \frac{V_2}{I} = 2R$$



$V_3$  between 2 terminals:

$$R_I = \frac{V}{I} = 2R$$

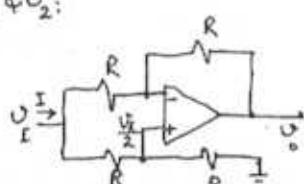
$$V_+ = V_- = 0$$



$V_3$  connected to both  $V_+$  &  $V_-$ :

$$R_I = \frac{V}{I} = R$$

$$V_+ = V_- = \frac{V_3}{2}$$



2.63

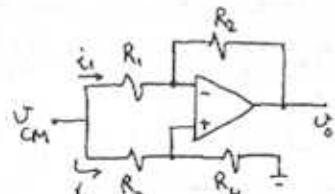
$$V_+ = V_{CM} \frac{R_4}{R_3 + R_4}$$

$$V_+ = V_-$$

$$i_2 = \frac{V_{CM}}{R_3 + R_4}$$

$$i_1 = \frac{V_{CM}}{R_1} - \frac{V_{CM} R_4}{R_3 + R_4} \times \frac{1}{R_1} = \frac{V_{CM}}{R_1} \frac{R_3}{R_3 + R_4}$$

$$i = i_1 + i_2 = \frac{V_{CM}}{R_1} \frac{R_3}{R_3 + R_4} + \frac{V_{CM}}{R_3 + R_4}$$



$$\frac{1}{R_I} = \frac{i}{V_{CM}} = \frac{1}{R_1} \frac{1}{\frac{R_4}{R_3} + 1} + \frac{1}{R_3 + R_4}$$

$$\text{if we replace } \frac{R_4}{R_3} \text{ with } \frac{R_2}{R_1}: \left( \frac{R_4}{R_3} + \frac{R_2}{R_1} \right)$$

$$\frac{1}{R_I} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} \Rightarrow R_I = (R_1 + R_2) \parallel (R_3 + R_4)$$

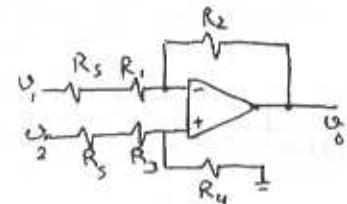
2.64

In order to have an ideal differential amp:

$$\frac{R_S + R_1}{R_2} = \frac{R_S + R_3}{R_4}$$

$$\frac{R_S/R_1 + 1}{R_2/R_1} = \frac{R_S/R_3 + 1}{R_4/R_3}$$

$$\text{Since } \frac{R_2}{R_1} = \frac{R_4}{R_3}:$$



$$\frac{R_S}{R_1} + 1 = \frac{R_S}{R_3} + 1 \Rightarrow R_1 = R_3 \Rightarrow R_2 = R_4$$

2.65

Refers to eq. 2.19 and Fig. P2.62:

$$A_{CM} = \frac{V_o}{V_{CM}} = \frac{R_4}{R_3 + R_4} \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

The worst case is when  $A_{CM}$  has its maximum value.

$$A_{CM} = \frac{1}{\frac{R_3}{R_4} + 1} \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right)$$

Max  $A_{CM} \Rightarrow \frac{R_3}{R_4}$  has to be at its minimum value and also  $\frac{R_4}{R_2}$  has to be minimum.

$$\frac{100-x}{100+x} \leq \frac{R_3}{R_4} \leq \frac{100+x}{100-x}, \quad \frac{100-x}{100+x} \leq \frac{R_2}{R_1} \leq \frac{100+x}{100-x}$$

$$\text{so if } \frac{R_3}{R_4} = \frac{100-x}{100+x} \text{ & } \frac{R_2}{R_1} = \frac{100-x}{100+x}$$

$$A_{CM Max} = \frac{1}{\frac{100-x}{100+x} + 1} \left( 1 - \frac{100-x}{100+x} \frac{100-x}{100+x} \right)$$

$$A_{CM Max} = \frac{1}{200} \frac{(100+x)^2 - (100-x)^2}{100+x} = \frac{2x}{100+x} \approx \frac{x}{50}$$

$$\begin{array}{c|ccc} x & 0.1 & 1 & 5 \\ \hline A_{CM Max} & 0.002 & 0.02 & 0.1 \end{array}$$

CHRR =  $20 \log \left| \frac{A_{CM}}{A_{CM}} \right|$ . Now we have to calculate  $A_{CM}$  based on values we chose for  $R_1 - R_4$  that gave us  $A_{CM Max}$ .

$$R_2 = R_3 = 100-x \quad R_1 = R_4 = 100+x$$

$V_o = V_{O1} + V_{O2}$  by applying superposition

$$V_o = -\frac{R_2}{R_1} V_1 + V_2 \frac{R_4}{R_3 + R_4} \left( 1 + \frac{R_2}{R_1} \right)$$

$$V_o = -\frac{100-x}{100+x} V_1 + V_2 \frac{100+x}{200} \left( 1 + \frac{100-x}{100+x} \right)$$

$$V_o = -\frac{100-x}{100+x} V_1 + V_2$$

$$\text{if we consider } \frac{100-x}{100+x} \ll 1 \Rightarrow \frac{V_o}{V_{id}} \approx 1$$

Cont.

$$CMRR = 20 \log \frac{A_d}{A_{cm}} = 20 \log \frac{1}{X_{SD}} = 20 \log \frac{50}{X}$$

X	0.1	1	5
CMRR	54db	34db	20db

2.67

$$A_d = 100$$

$$\text{we assume } \frac{R_2}{R_1} = \frac{R_4}{R_3} \text{ then } A_d = \frac{R_2}{R_1} = K$$

$$K \leq 100$$

$$R_{id} = 2R_1 = 20k\Omega \Rightarrow R_1 = 10k\Omega$$

$$CMRR = 80 \text{ db} = 20 \log \frac{A_d}{A_{cm}} \Rightarrow \frac{A_d}{A_{cm}} = 10^4$$

$$\Rightarrow A_{cm} = 0.01$$

$$A_d = 100 = \frac{R_2}{R_1} \Rightarrow R_2 = 1M\Omega$$

$$\text{Refer to P2.66: } CMRR = 20 \log \frac{K+1}{4E}$$

$$CMRR = 10^4$$

$$\Rightarrow E = 10^{-2} \times 0.25$$

$$\text{we assumed earlier } \frac{R_2}{R_1} = \frac{R_4}{R_3} \text{ then}$$

$$\frac{R_4}{R_3} \leq 100 \Rightarrow \text{if } R_3 = 10k \pm \epsilon$$

$$\Rightarrow R_4 = 1M\Omega \pm \epsilon$$

$$R_2 = 1M\Omega \pm \epsilon$$

$$\epsilon = 0.25\%$$

$$R_1 = 10k \pm \epsilon$$

2.68

Refer to Fig.P2.68 and Eq. 2.19.

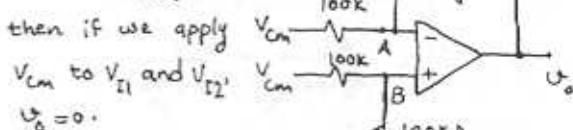
$$A_{cm} = \frac{R_1}{R_3 + R_4} \left(1 - \frac{R_2}{R_1} \frac{R_3}{R_4}\right) = \frac{100}{100+100} \left(1 - \frac{100 \cdot 100}{100 \cdot 100}\right)$$

$$A_{cm} = 0$$

$$\text{Refer to 2.17: } \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$\Rightarrow A_d = \frac{R_2}{R_1} = 1$$

b) Since  $A_{cm} = 0$ ,



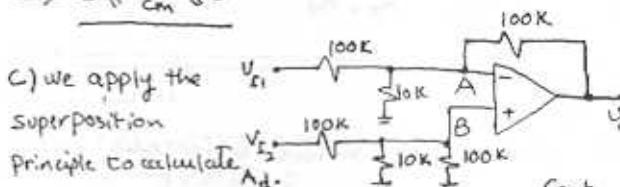
$$\text{Therefore, } V_A = V_{cm} \frac{100}{100+100} =$$

$$V_A = \frac{V_{cm}}{2}$$

$$\text{Similarly, } V_B = \frac{V_{cm}}{2}$$

$$\text{We know } V_A = V_B \text{ and } -2.5 \leq V_A \leq 2.5$$

$$\Rightarrow -5 \leq V_{cm} \leq 5$$



c) we apply the superposition principle to calculate  $A_d$ .

Superposition

principle to calculate  $A_d$ .

Cont.

$U_{O1}$  is the output voltage when  $V_{I2}=0$   
 $U_{O2}$  is the output voltage when  $V_{I1}=0$

$$U_O = U_{O1} + U_{O2}$$

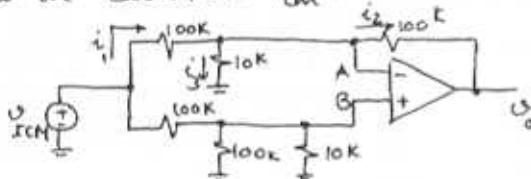
$$U_O = -\frac{R_2}{R_1} I_I \quad I_I = -U_I$$

$$U_{O2} = \frac{V_{I2}}{100K//10K} \left(1 + \frac{100K}{100K//10K}\right)$$

$$U_{O2} = \frac{V_{I2}}{100K//10K}$$

$$\Rightarrow U_O = U_{O1} + U_{O2} = -U_I + U_{I2} \Rightarrow A_d = 1$$

Now we calculate  $A_{cm}$ :



$$U_B = U_{ICM} \frac{100K//10K}{100K//10K+100K}, \quad U_A = U_B$$

$$i_1 = \frac{U_{ICM} - U_A}{100K}$$

$$U_O = U_A - 100K \cdot i_2 \quad \text{and} \quad i_2 = i_1 - i_3 = i_1 - \frac{U_A}{10K}$$

$$U_O = U_A - 100K \cdot i_1 + 10 \cdot U_A$$

$$U_O = U_A - U_{ICM} + U_A + 10 \cdot U_A$$

$$U_A = U_B \Rightarrow U_O = U_{ICM} \left(-1 + 12 \frac{100K//10K}{100K//10K+100K}\right)$$

$$\frac{U_O}{U_{ICM}} = A_{cm} = 0$$

Now we calculate  $U_{ICM}$  range:

$$-2.5 \leq U_B \leq 2.5 \Rightarrow -2.5 \cdot U_{ICM} \cdot \frac{100K//10K}{100K//10K+100K} \leq 2.5$$

$$-30 \leq U_{ICM} \leq 30$$

2.69

Refer to Fig. P2.69; we use superposition:

$$U_O = U_{O1} + U_{O2}$$

$$\text{calculate } U_{O1}: \quad U_+ = \frac{\beta U_{O1}}{2} = U_-$$

$$\frac{U_+ - \frac{\beta U_{O1}}{2}}{R} = \frac{\beta U_{O1} - U_0}{2R} \Rightarrow U_{O1} = \frac{U_0}{\beta - 1}$$

calculate  $U_{O2}$ :

$$U_2 = \frac{U_{O2}}{2} = U_+ \Rightarrow U_2 - \frac{U_{O2}}{2} = \frac{U_{O2}}{2} - \beta U_{O2} \\ \Rightarrow U_{O2} = \frac{U_2}{1-\beta}$$

$$U_O = U_{O1} + U_{O2} = \frac{U_1}{\beta-1} + \frac{U_2}{1-\beta} = \frac{1}{1-\beta} (U_2 - U_1)$$

$$A_d = \frac{U_O}{U_2 - U_1} = \frac{1}{1-\beta}$$

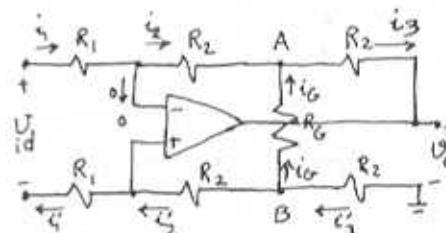
$$A_d = 10 \Rightarrow \beta = 0.9 = \frac{R_S}{R_S + R_6}$$

$$R_{id} = 2R = 2M\Omega \Rightarrow R = 1M\Omega$$

$$R_S + R_6 \leq \frac{R}{100} \Rightarrow R_S + R_6 \leq 10K\Omega$$

$$R_S = 6.8K\Omega \quad R_6 = 680\Omega \quad \Rightarrow \beta = \frac{6.8}{6.8+0.68} \approx 0.9$$

2.70



$U_+ = U_-$  so we can consider  $U_+, U_-$  a virtual

$$\text{short: } i_1 = U_{id}/2R_1 \Rightarrow i_2 = \frac{U_{id}}{2R_1}$$

$$i'_1 = i'_2 = \frac{U_{id}}{2R_1}$$

$$\text{then: } i'_2 R_2 + U_{AB} + i'_2 R_2 = 0 \Rightarrow U_{AB} = -\frac{U_{id}}{R_1} R_2$$

$$i'_G = \frac{U_{id}}{R_G} \times \frac{R_2}{R_1}$$

$$i_3 = i_2 + i'_G = \frac{U_{id}}{2R_1} + \frac{U_{id}}{R_G} \frac{R_2}{R_1}$$

$$i'_3 = i_3 + i'_2 = i_3$$

$$\Rightarrow U_O = -[i'_3 R_2 + U_{BA} + i_3 R_2]$$

$$U_O = -[2i_3 R_2 + U_{BA}]$$

$$U_O = -[\frac{2U_{id}R_2}{2R_1} + 2\frac{U_{id}R_2}{R_1} \frac{R_2}{R_G} + \frac{U_{id}R_2}{R_1}]$$

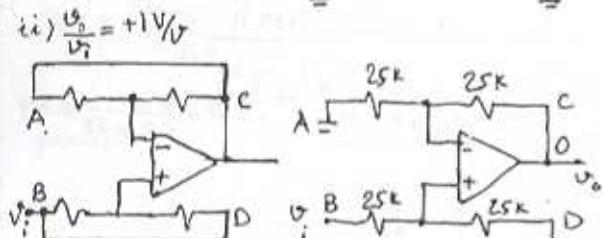
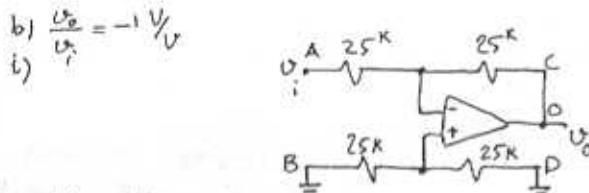
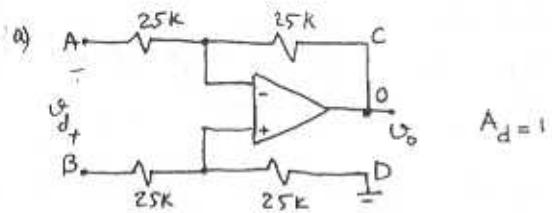
$$\frac{U_O}{U_{id}} = A_d = -2 \frac{R_2}{R_1} \left[1 + \frac{R_2}{R_G}\right]$$

2.71

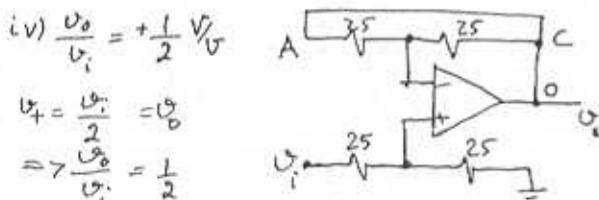
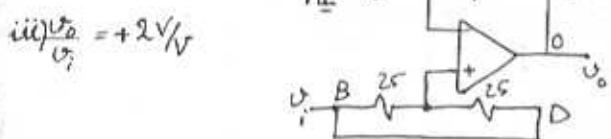
a)

Refer to Eq. 2.17:  $A_d = \frac{R_2}{R_1} = 1$ . Connect C and D together.

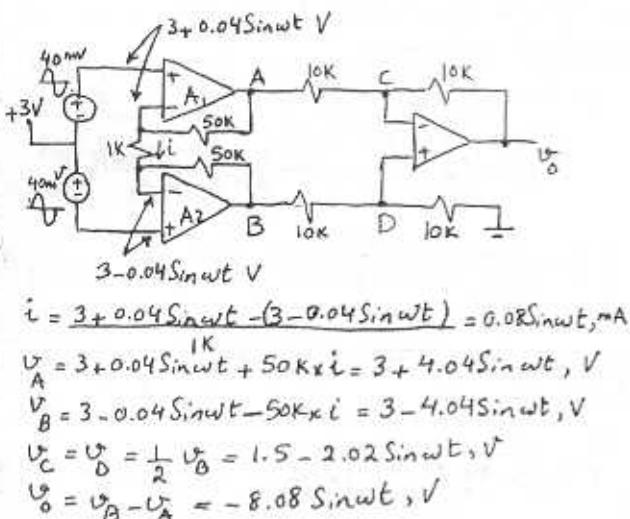
Cont.



The circuit on the left ideally has infinite input resistance.



2.72



2.73

Refer to Fig. 2.20.a.

The gain of the first stage is:  $(1 + \frac{R_2}{R_1}) = 101$

If the opamps of the first stage saturate at  $\pm 14 \text{ V}$ :  $-14 \leq U_1 \leq +14 \text{ V} \Rightarrow -14 \leq 101 U_{icm} \leq +14$   
 $\Rightarrow -0.14 \leq U_{icm} \leq 0.14$

As explained in the text, the disadvantage of circuit in Fig. 2.20.a is that  $U_{icm}$  is amplified by a gain equal to  $v_{id}(1 + \frac{R_2}{R_1})$  in the first stage and therefore a very small  $U_{icm}$  range is acceptable to avoid saturation.

b) In Fig. 2.20.b, when  $U_{icm}$  is applied, for both  $A_1$  &  $A_2$  is the same and therefore no current flows through  $ZR_1$ . This means voltage at the output of  $A_1$  and  $A_2$  is the same as  $U_{icm}$ .

$$-14 \leq U_o \leq 14 \Rightarrow -14 \leq U_{icm} \leq 14$$

This circuit allows for bigger range of  $U_{icm}$ .

2.74

$$U_{i1} = U_{cm} - U_d/2$$

$$U_{i2} = U_{cm} + U_d/2$$

Refer to Fig. 2.20.a.

output of the first stage:  $(1 + \frac{R_2}{R_1})(U_{cm} - U_d/2)$

$$U_{o1} = (1 + \frac{R_2}{R_1})(U_{cm} - U_d/2)$$

$$U_{o2} = (1 + \frac{R_2}{R_1})(U_{cm} + U_d/2)$$

$$U_{o2} - U_{o1} = (1 + \frac{R_2}{R_1}) U_d \Rightarrow A_{d(1)} = 1 + \frac{R_2}{R_1}$$

$$\frac{U_{o2} + U_{o1}}{2} = (1 + \frac{R_2}{R_1}) U_{cm} \Rightarrow A_{cm(1)} = 1 + \frac{R_2}{R_1}$$

$$\text{CMRR} = 20 \log |\frac{A_d}{A_{cm}}| = 0 \quad (\text{First stage})$$

Now Consider Fig. 2.20.b

$$U_{o1} = U_{i1} + R_2 \times \frac{(U_{i1} - U_{i2})}{2R_1}$$

$$U_{o1} = U_{cm} - \frac{U_d}{2} + \frac{R_2}{2R_1} (-U_d)$$

Cont.

$$V_{O1} = V_{CM} - \frac{V_d}{2} \left( 1 + \frac{R_2}{R_1} \right)$$

$$V_{O2} = V_{i2} - R_2 \times \frac{V_{i1} - V_{i2}}{2R_1} = V_{CM} + \frac{V_d}{2} + R_2 \cdot \frac{V_d}{2R_1}$$

$$V_{O2} = V_{CM} + \frac{V_d}{2} \left( 1 + \frac{R_2}{R_1} \right)$$

$$V_{O2} - V_{O1} = V_d \left( 1 + \frac{R_2}{R_1} \right) \Rightarrow A_{d(1)} = 1 + \frac{R_2}{R_1}$$

$$\frac{V_{O2} + V_{O1}}{2} = V_{CM} \Rightarrow A_{CM(1)} = 1$$

$$CMRR = 20 \log \left| \frac{A_d}{A_{CM}} \right| = 20 \log \left( 1 + \frac{R_2}{R_1} \right)$$

In 2.20.b, the common mode voltage is not amplified and it is only propagated to the outputs of the first stage.

2.75

Refer to eq. 2.2.2:

$$A_d = \frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_1} \right) = \frac{100K}{100K} \left( 1 + \frac{100K}{5K} \right) = 21 \text{ V/V}$$

$$A_{CM} = 0$$

$$CMRR = 20 \log \left| \frac{A_d}{A_{CM}} \right| = \infty$$

If all resistors are  $\pm 1\%$ :

$$A_d \approx 21$$

In order to calculate  $A_{CM}$ , apply  $V_{CM}$  to both inputs and note that  $V_{CM}$  will appear at both output terminals of the first stage.

Now we can evaluate  $V_o$  by analyzing the second stage as was done in problem 2.65.

In P2.65 we showed that if each 100k resistor has  $\pm x\%$  tolerance,  $A_{CM}$  of the differential amplifier is:  $A_{CM} = \frac{V_o}{V_{CM}} = \frac{x}{50}$ . Therefore the overall  $A_{CM}$  is also  $\frac{x}{50}$ .

$$x = 1 \Rightarrow A_{CM} = \frac{1}{50} = 0.02$$

$$CMRR = 20 \log \frac{21}{0.02} = 60 \text{ db}$$

$$\text{If } 2R_1 = 1K\Omega : A_d = \frac{R_4}{R_3} \left( 1 + \frac{R_2}{R_1} \right) = 201 \text{ V/V}$$

$$A_{CM} = 0.02 \text{ unchanged}$$

$$CMRR = 20 \log \frac{201}{0.02} = 80 \text{ db}$$

Conclusion: Large CMRR can be achieved by

having relatively large  $A_d$  in the first stage.

2.76

$$A_d \text{ of the second stage is } \frac{R_4}{R_3} = 0.5$$

$$R_4 = 100K\Omega, R_3 = 200K\Omega$$

We use a series configuration of  $R_F$  and  $R_1$  (pot):  $R_1 = 100K \text{ pot}$

$$\text{Minimum gain} = 0.5 \left( 1 + \frac{R_2}{R_1} \right) = 0.5 \left( 1 + \frac{R_2}{100K+R_1} \right)$$

$$1 \leq A_d \leq 100 \Rightarrow 1 = 0.5 \left( 1 + \frac{2R_2}{R_F+100K} \right)$$

$$\Rightarrow R_F + 100 = 2R_2 \quad \textcircled{1}$$

$$\text{Maximum gain} = 100 = 0.5 \left( 1 + \frac{R_2}{R_F/2} \right) \Rightarrow$$

$$2R_2 = 199R_F \quad \textcircled{2}$$

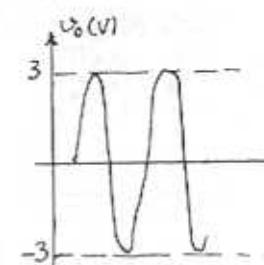
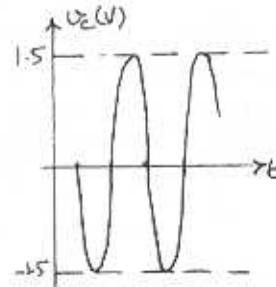
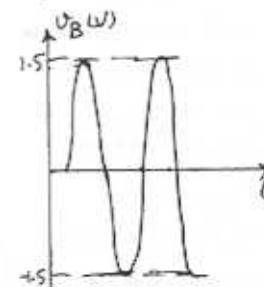
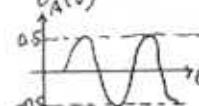
$$\textcircled{1}, \textcircled{2} \Rightarrow R_F = 0.505K\Omega \approx 0.5K\Omega$$

$$R_2 = 50.25 K\Omega \approx 50K\Omega$$

2.77

$$\text{a) } \frac{V_B}{V_A} = 1 + \frac{20}{10} = 3 \text{ V/V}, \frac{V_C}{V_A} = -\frac{30}{10} = -3 \text{ V/V}$$

$$\text{b) } V_o = V_B - V_C = 6 \text{ V} \Rightarrow \frac{V_o}{V_A} = 6 \text{ V/V}$$



c)  $V_B$  and  $V_C$  can be  $\pm 1V$  or  $28V$  p-p.

$$-28 \leq V_o \leq 28 \text{ or } \frac{V_o}{\sqrt{2}}$$

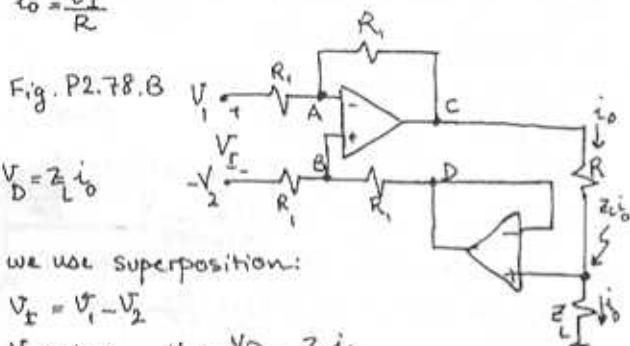
$$V_{o,\text{rms}} = \frac{19.8 V}{\sqrt{2}} = \frac{28}{\sqrt{2}}$$

2.78

Refer to Fig. P.2.78.a:

Since the inputs of the op-amp don't draw any current,  $V_i$  appears across  $R_1$ .

$$i_{o1} = \frac{V_i}{R_1}$$



We use superposition:

$$V_B = V_1 - V_2$$

$$V_1 \text{ only: } V_B = \frac{V_D}{2} = \frac{Z_L i_{o1}}{2}$$

$$\frac{V_1 - \frac{Z_L i_{o1}}{2}}{R_1} = \frac{\frac{Z_L i_{o1}}{2} - i_{o1}(Z_L + R)}{R_1}$$

$$\Rightarrow V_1 = i_{o1} R \Rightarrow i_{o1} = \frac{V_1}{R}$$

Now if only  $(-V_2)$  is applied:

$$V_B = \frac{-V_2 + Z_L i_{o2}}{2}, \quad V_A = \frac{i_{o2} \times (R + Z_L)}{2}$$

$$V_A = V_B \Rightarrow -V_2 + Z_L i_{o2} = i_{o2} R + i_{o2} Z_L$$

$$-V_2 = i_{o2} R \Rightarrow i_{o2} = \frac{-V_2}{R}$$

The total current due to both sources is:

$$i_o = i_{o1} + i_{o2} = \frac{V_1}{R} - \frac{V_2}{R} = \frac{V_1 - V_2}{R}$$

The circuit in Figure P.2.78(a) has ideally infinite input resistance, and it requires that both terminals of  $Z_L$  be available, while the other circuit has finite input resistance with one side of  $Z_L$  grounded.

2.79

$A_o$	$f_b$ (Hz)	$f_t$ (Hz)	eq. 2.28:
$10^5$	$10^2$	$10^7$	$\omega_t = A_o \omega_b$
$10^6$	1	$10^6$	$\Rightarrow f_t = A_o f_b$
$10^5$	$10^3$	$10^8$	
$10^7$	$10^1$	$10^6$	
$2 \times 10^5$	10	$2 \times 10^6$	

2.80

$$\text{Eq. 2.25: } A = \frac{A_o}{1 + j \frac{f}{f_b}} \Rightarrow |A| = \frac{|A_o|}{\sqrt{1 + (\frac{f}{f_b})^2}}$$

$$A_o = 86 \text{ dB}, A = 40 \text{ dB} @ f = 100 \text{ kHz}$$

$$20 \log \sqrt{1 + (\frac{f}{f_b})^2} = 20 \log \frac{|A|}{|A_o|} = 20 \log A_o - 20 \log A$$

$$= 86 - 40 = 46 \text{ dB}$$

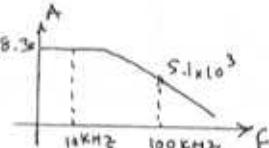
$$1 + \left(\frac{100 \text{ kHz}}{f_b}\right)^2 = (199.5)^2 \Rightarrow f_b = 0.501 \text{ kHz}$$

$$f_t = A_o f_b = \underbrace{1.995 \times 10^4}_{86 \text{ dB}} \times 501 = 9.998 \text{ MHz} \approx 10 \text{ MHz}$$

2.81

$$A_o = 8.3 \times 10^3 \text{ V/V}$$

$$\text{Eq. 2.25: } A = \frac{A_o}{1 + j \frac{f}{f_b}}$$



$$f_t = A_o f_b$$

$$5.1 \times 10^3 = \frac{8.3 \times 10^3}{\sqrt{1 + \left(\frac{100 \text{ kHz}}{f_b}\right)^2}} \Rightarrow 1 + \left(\frac{100 \text{ kHz}}{f_b}\right)^2 = 2.65$$

$$f_b = 60.7 \text{ kHz}$$

$$f_t = A_o f_b = 8.3 \times 10^3 \times 60.7 = 503 \text{ MHz}$$

2.82

we have:

$$A_o = 20 \text{ dB} + A_{(dB)} \quad 20 \text{ dB} = 20 \log 10 \Rightarrow A_o = 10 \text{ A}$$

$$\text{a) } A_o = 10 \times 3 \times 10^5 = 3 \times 10^6 \text{ Hz} \text{ V/V}$$

$$A = \frac{A_o}{1 + j \frac{f}{f_b}} \Rightarrow |1 + j \frac{f}{f_b}| = \frac{A_o}{A} = 10 \Rightarrow \frac{10^{12}}{f_b} = \sqrt{99} \Rightarrow f_b = 1 \text{ Hz}$$

$$\text{b) } A = 50 \times 10^5 \text{ V/V} \Rightarrow A_o = 10 \times 50 \times 10^5 = 50 \times 10^6 \text{ V/V}$$

$$|1 + j \frac{f}{f_b}| = \frac{A_o}{A} = 10 \Rightarrow \frac{10^{12}}{f_b} = \sqrt{99} \Rightarrow f_b = 1 \text{ Hz}$$

$$\text{c) } A = 1500 \text{ V/V} \Rightarrow A_o = 15000 \text{ V/V}$$

Cont.

$$|1 + \frac{jF}{f_b}| = 10 \Rightarrow \frac{0.1 \times 10^6}{f_b} = \sqrt{99} \Rightarrow f_b = 10 \text{ MHz}$$

$$f_t = 15000 \times 10^6 = 150 \text{ MHz}$$

$$b) 1 + \frac{R_2}{R_1} = 100 \text{ V/V} \quad f_{3db} = 100 \text{ kHz}$$

$$f_t = f_{3db} (1 + \frac{R_2}{R_1}) = 10 \text{ MHz}$$

$$d) A_o = 10 \times 100 = 1000 \text{ V/V}$$

$$|1 + \frac{jF}{f_b}| = 10 \Rightarrow \frac{0.1 \times 10^6}{f_b} = \sqrt{99} \Rightarrow f_b = 10 \text{ MHz}$$

$$f_t = 1000 \times 10 \text{ MHz} = 100 \text{ GHz}$$

$$c) 1 + \frac{R_2}{R_1} = 2 \text{ V/V} \quad f_{3db} = 10 \text{ kHz}$$

$$f_t = 10 \text{ MHz} \times 2 = 20 \text{ MHz}$$

$$e) A_o = 25 \text{ V/mV} \times 10 = 25 \times 10^4 \text{ V/V}$$

$$|1 + \frac{jF}{f_b}| = 10 \Rightarrow \frac{25 \text{ kHz}}{f_b} = \sqrt{99} \Rightarrow f_b = 2.51 \text{ kHz}$$

$$f_t = A_o f_b = 25 \times 10^4 \times 2.51 \times 10^3 = 62.75 \text{ MHz}$$

$$d) -\frac{R_2}{R_1} = -2 \text{ V/V} \quad f_{3db} = 10 \text{ kHz}$$

$$f_t = 10 \text{ MHz} (1+2) = 30 \text{ MHz}$$

$$e) -\frac{R_2}{R_1} = -1000 \text{ V/V} \quad f_{3db} = 20 \text{ kHz}$$

$$f_t = 20 \text{ kHz} (1+1000) = 20.02 \text{ MHz}$$

2.83

$$G_{num} = -\frac{R_2}{R_1} = -20 \quad A_o = 10^4 \text{ V/V} \quad f_b = 10^6 \text{ Hz}$$

$$\text{Eq. 2.35: } \omega_{3db} = \frac{\omega_b}{1 + R_2/R_1} = \frac{2\pi \times 10^6}{1 + 20} = 2\pi \times 47.6 \text{ kHz}$$

$$f_{3db} = 47.6 \text{ kHz}$$

$$\text{Eq. 2.34: } \frac{V_o}{V_i} \approx \frac{-R_2/R_1}{1 + \frac{s}{\omega_b/(1+R_2/R_1)}} = \frac{-20}{1 + \frac{215}{2\pi \times 10^6}}$$

$$F = 0.1 f_{3db} \Rightarrow \left| \frac{V_o}{V_i} \right| = \frac{-20}{\sqrt{1 + (0.1)^2}} = 19.9 \text{ V/V}$$

$$F = 10 f_{3db} \Rightarrow \left| \frac{V_o}{V_i} \right| = \frac{-20}{\sqrt{1 + 100}} = 1.99 \text{ V/V}$$

$$f) 1 + \frac{R_2}{R_1} = 1 \text{ V/V} \quad f_{3db} = 1 \text{ kHz}$$

$$f_t = 1 \text{ M} \times 1 = 1 \text{ MHz}$$

$$g) -\frac{R_2}{R_1} = -1 \text{ V/V} \quad f_{3db} = 1 \text{ kHz}$$

$$f_t = 1 \text{ M} (1+1) = 2 \text{ MHz}$$

2.86

$$1 + \frac{R_2}{R_1} = 100 \text{ V/V} \quad f_{3db} = 8 \text{ kHz}$$

$$f_t = 8 \times 100 = 800 \text{ kHz}$$

$$\text{for } f_{3db} = 20 \text{ kHz} : G_o = \frac{800}{20} = 40 \text{ V/V}$$

2.87

$$f_{3db} = f_t = 1 \text{ MHz}$$

$$|G| = \frac{1}{\sqrt{1 + (\frac{f}{f_{3db}})^2}} = \frac{1}{\sqrt{1 + f^2}} \quad f \text{ in MHz}$$

$$|G| = 0.99 \Rightarrow f = 0.142 \text{ MHz}$$

The follower behaves like a low-pass STC circuit with a time constant  $\tau = \frac{1}{\omega_{3db}}$

$$\text{Thus: } \tau = \frac{1}{2\pi \times 10^6} = \frac{1}{2\pi} \mu\text{s}$$

$$t_r = 2.2\tau = 0.35 \mu\text{s} \quad (\text{Refer to Appendix F})$$

2.85

$$a) -\frac{R_2}{R_1} = 100 \text{ V/V} \quad f_{3db} = 100 \text{ kHz}$$

$$\text{Eq. 2.35: } \omega_t = \omega_{3db} (1 + \frac{R_2}{R_1}) \Rightarrow f_t = 100 \times 101 = 10.1 \text{ MHz}$$

2.88

$$1 + \frac{R_2}{R_1} = 10 \text{ V/V} \quad R_1 = 1 \text{ k}\Omega \quad R_2 = 9 \text{ k}\Omega$$

If we consider 5% the time that it takes for the output voltage to reach 99% of its final value, then:  $5\% = 100 \text{ ns} \Rightarrow T = 20 \text{ ns}$

$$T = \frac{1}{\omega_{3\text{db}}} \Rightarrow \omega_{3\text{db}} = 50 \times 10^6 \Rightarrow f_{3\text{db}} = 7.96 \text{ MHz}$$

$$f_t = \left(1 + \frac{R_2}{R_1}\right) f_{3\text{db}} = 10 \times 7.96 = 79.6 \text{ MHz}$$

2.89

a) Assume two identical stages, each with a gain function:

$$G = \frac{G_o}{1 + j \frac{f}{f_i}} = \frac{G_o}{1 + j F/F_i}$$

$$G = \frac{G_o}{\sqrt{1 + (F/F_i)^2}}$$

Overall gain of the cascade is  $\frac{G_o}{1 + (F/F_i)^2}$   
 The gain will drop by 3dB when:  $1 + \left(\frac{f_{3\text{db}}}{F_i}\right)^2 = \sqrt{2}$ , Note  $3\text{dB} = 20 \log \sqrt{2}$   
 $f_{3\text{db}} = F_i \sqrt{\sqrt{2} - 1}$

b)  $40 \text{ dB} = 20 \log G_o \Rightarrow G_o = 100 = 1 + \frac{R_2}{R_1}$   
 $f_{3\text{db}} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{1 \text{ MHz}}{100} = 10 \text{ kHz}$

c) Each stage should have 20dB gain or  $1 + \frac{R_2}{R_1} = 10$  and therefore a 3dB frequency of:  $f = \frac{10^6}{10} = 10^5 \text{ Hz}$ .  
 The overall  $f_{3\text{db}} = 10^5 \sqrt{\sqrt{2} - 1} = 64.35 \text{ kHz}$  which is 6 times greater than the bandwidth achieved using single op-amp. (case b above)

2.90

$f_t = 100 \times 5 = 500 \text{ MHz}$ . if single op-amp is used.

With op-amp that has only  $f_t = 40 \text{ MHz}$ , the possible closed loop gain at 5MHz is:  
 $K = \frac{10}{5} = 8 \text{ V/V}$ .

To obtain an overall gain of 100, three such amplifiers cascaded, would be required. Now, if each of the 3 stages, has a low-frequency (d) closed loop gain  $K$ , then its 3dB frequency will be  $\frac{40}{K} \text{ MHz}$ . Thus for each stage the closed loop gain is:  $|G| = \frac{K}{\sqrt{1 + \left(\frac{F}{K}\right)^2}}$  which at  $F = 5 \text{ MHz}$  becomes:

$$|G_{5\text{MHz}}| = \frac{K}{\sqrt{1 + \left(\frac{5}{K}\right)^2}}$$

The overall gain of 100:  $100 = \left[ \frac{K}{\sqrt{1 + \left(\frac{5}{K}\right)^2}} \right]^3$   
 $K = 5.7$

Thus for each cascade stage:  $f_{3\text{db}} = \frac{40}{5.7}$   
 $f_{3\text{db}} = 7 \text{ MHz}$

The 3-dB frequency of the overall amplifier,  $F_i$ , can be calculated as:

$$\left[ \frac{5.7}{\sqrt{1 + \left(\frac{5}{5.7}\right)^2}} \right]^3 = \frac{(5.7)^3}{\sqrt{2}} \Rightarrow F_i = 3.6 \text{ MHz}$$

2.91

a)  $\frac{R_2}{R_1} = K \quad f_{3\text{db}} = \frac{f_t}{1 + \frac{R_2}{R_1}} = \frac{f_t}{1 + K}$   
 $\text{GBP} = \text{Gain} \times f_{3\text{db}}$   
 $\text{GBP} = K \frac{f_t}{1 + K}$

b)  $1 + \frac{R_2}{R_1} = K \quad f_{3\text{db}} = \frac{f_t}{K}$   
 $\text{GBP} = K \frac{f_t}{K} = f_t$

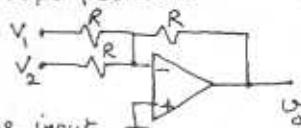
The non-inverting amplifier realizes a higher GBP and it's independent of  $K$ .

2.92

To find  $f_{3\text{db}}$  we use superposition:

set  $V_2 = 0$

Now Using Thevenin's  
 Theorem to Simplify the input circuit results in:



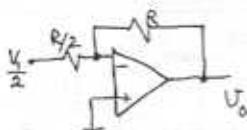
Cont.

$$\frac{V_o}{V_{1/2}} = \frac{-R/R/2}{1 + S \frac{1+R/R/2}{\omega_E}}$$

which gives:

$$\frac{V_o}{V_i} = \frac{-1}{1 + S(\omega_E/3)}$$

$f_{3dB} = \frac{f_E}{3}$ . Similar results can be obtained for  $\frac{V_o}{V_2}$ .



Thevenin's equivalent

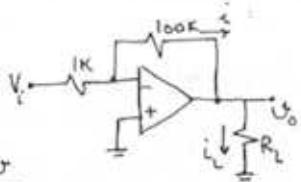
2.93

The peak value of the largest possible sine wave that can be applied at the input without output clipping is:  $\frac{\pm 12V}{100} = 0.12V = 120mV$   
rms value =  $\frac{120}{\sqrt{2}} = 85mV$

2.94

a)  $R_L = 1k\Omega$

for  $V_{o\max} = 10V$ :  $V_p = \frac{10}{100} = 0.1V$



when output is at its peak,  $i_L = \frac{10}{1k} = 10mA$   
 $i = \frac{10}{100k} = 0.1mA$ . therefore  $i_o = 10 + 0.1 = 10.1mA$   
is well under  $i_{omax} = 20mA$ .

b)  $R_L = 100\Omega$

If output is at its peak:  $i_L = \frac{10}{0.1} = 100mA$

which exceeds  $i_{omax} = 20mA$ . Therefore  $V_o$  cannot go as high as 10V. instead:

$$20mA = \frac{V_o}{100\Omega} + \frac{V_o}{100k} \Rightarrow V_o = \frac{20}{100} = 2V$$

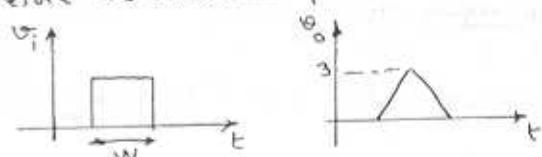
$$V_p = \frac{2}{100} = 0.02V = 20mV$$

c)  $R_L = ?$   $i_{omax} = 20mA = \frac{10V}{R_{min}} + \frac{10V}{100k}$   
 $20 - 0.1 = \frac{10}{R_{min}} \Rightarrow R_{min} = 502\Omega$

2.95

The output is triangular with the slew rate

of  $20V/\mu s$ . In order to reach 3V, it takes  $\frac{3}{20} \mu s = 0.15 \mu s = 150ns$ . Therefore the minimum pulse width is 150ns.



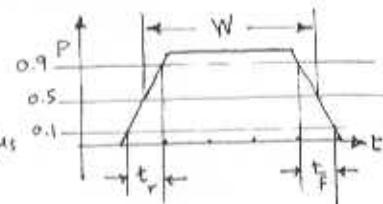
2.96

$W = 2\mu s$

$t_r + t_f = 0.2W = 0.4\mu s$

$t_r = t_f = 0.2\mu s$

$SR = \frac{(0.9 - 0.1)V}{t_r} = \frac{0.8 \times 10}{0.2} = 40V/\mu s$



2.97

Slope of the triangle wave =  $\frac{20V}{T/2} = SR$

thus  $\frac{20}{T} \times 2 = 10V/\mu s$

$\Rightarrow T = 4\mu s$  or  $f = \frac{1}{T} = 250kHz$

for a Sine wave  $V_o = A \sin(2\pi \times 250 \times 10^3 t)$

$$\left| \frac{dV_o}{dt} \right| = 2\pi \times 250 \times 10^3 A = SR$$

$$\Rightarrow A = \frac{10 \times 10^6}{2\pi \times 10^3 \times 250} = 6.37V$$

2.98

$$V_o = 10 \sin \omega t \Rightarrow \frac{dV_o}{dt} = 10\omega \cos \omega t \Rightarrow \left| \frac{dV_o}{dt} \right| = 10\omega$$

The highest frequency at which this wave output is possible is that for which:

$$\left| \frac{dV_o}{dt} \right|_{max} = SR \Rightarrow 10\omega_{max} = 60 \times 10^6 \Rightarrow \omega_{max} = 6 \times 10^6$$

$$\Rightarrow f_{max} = 45.5 kHz$$

2.99

a)  $V_i = 0.5 \Rightarrow V_o = 10 \times 0.5 = 5V$

Contd.

Output distortion will be due to slew rate limitation and will occur at the frequency for which  $\frac{dV_o}{dt}|_{\text{max}} = SR$

$$\omega_{\text{max}} \times 5 = \frac{1}{10^{-8}} = 2\pi \times 10^5 \text{ rad/s} \Rightarrow f_{\text{max}} = 31.8 \text{ kHz}$$

b) The output will distort at the value of  $V_i$  that results in  $\frac{dV_o}{dt}|_{\text{max}} = SR$ .

$$V_o = 10 V_i \sin 2\pi \times 20 \times 10^3$$

$$\frac{dV_o}{dt}|_{\text{max}} = 10 V_i \times 2\pi \times 20 \times 10^3$$

$$\text{Thus } V_i = \frac{1/10^6}{10 \times 2\pi \times 20 \times 10^3} = 0.795 \text{ V}$$

$$c) V_i = 50 \text{ mV} \quad V_o = 500 \text{ mV} = 0.5 \text{ V}$$

Slew rate begins at the frequency for which  $\omega \times 0.5 = SR$

$$\text{which gives } \omega = \frac{1/10^6}{0.5} = 2 \times 10^6 \text{ rad/s or } f = 31.8 \text{ kHz}$$

However the small signal 3db frequency is  $f_{3\text{db}} = \frac{f_0}{1 + \frac{R_2}{R_1}} = \frac{2 \times 10^6}{10} = 200 \text{ kHz}$

Thus the useful frequency range is limited at 200 kHz.

d) for  $f = 5 \text{ kHz}$ , the slew rate limitation occurs at the value of  $V_i$  given by

$$\omega \times 10 V_i = SR \Rightarrow V_i = \frac{1/10^6}{2\pi \times 5 \times 10^3 \times 10} = 3.18 \text{ V}$$

Such an input voltage, however would ideally result in an output of 31.8 V which exceeds  $V_{\text{max}}$ . Thus  $V_{i\text{max}} = \frac{V_{\text{max}}}{10} = 1 \text{ V peak}$ .

2.102

$$\text{Output DC offset, } V_{o5} = 3 \text{ mV} \times 1000 = 3 \text{ V}$$

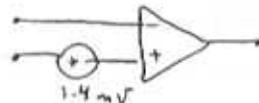
Therefore the maximum amplitude of an input sinusoid is the one that results in an output peak amplitude of  $13 - 3 = 10 \text{ V} \Rightarrow V_i = \frac{10}{1000} = 10 \text{ mV}$

If the amplifier is capacity coupled, then:

$$V_{i\text{max}} = \frac{13}{1000} = 13 \text{ mV}$$

2.103

$$V_{o5} = \frac{1.4}{100} = 1.4 \text{ mV}$$



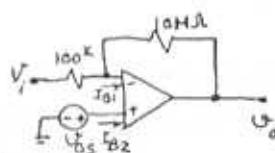
2.104

$$a) I_B = (I_{B1} + I_{B2})/2$$

open input:

$$V_o = V_+ + R_2 I_{B1} = V_{o5} + R_2 I_{B1}$$

$$9.31 = V_{o5} + 10000 I_{B1} \quad (1)$$



input connected to ground:

$$V_o = V_+ + R_2 (\bar{I}_{B1} + \frac{V_{o5}}{R_1}) = V_{o5} (1 + \frac{R_2}{R_1} + R_2 \bar{I}_{B1})$$

$$9.09 = V_{o5} \times 101 + 10000 \bar{I}_{B1} \quad (2)$$

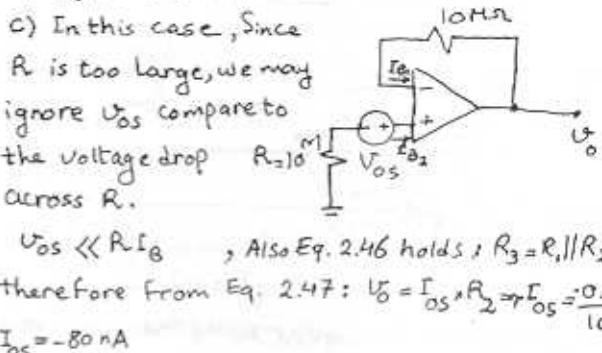
$$(1), (2) \Rightarrow 100 V_{o5} = -0.22 \Rightarrow V_{o5} = -2.2 \text{ mV}$$

$$\Rightarrow I_{B1} = 930 \text{ nA}$$

$$I_B \approx I_{B1} = 930 \text{ nA}$$

$$b) V_{o5} = -2.2 \text{ mV}$$

c) In this case, since  $R$  is too large, we may ignore  $V_{o5}$  compared to the voltage drop  $R_2 \times 10^3$  across  $R$ .



$V_{o5} \ll R_2 I_B$ , Also Eq. 2.46 holds:  $R_3 = R_1 // R_2$   
therefore from Eq. 2.47:  $V_o = I_{o5} R_2 \Rightarrow I_{o5} = \frac{0.8}{10^3}$   
 $I_{o5} = -80 \text{ nA}$

2.100

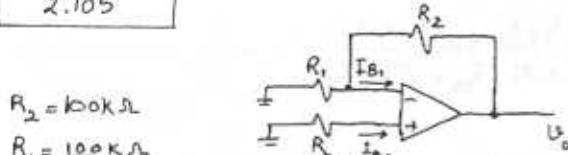
$$V_o = V_{o5} (1 + \frac{R_2}{R_1}) \Rightarrow -0.3 = V_{o5} (1 + \frac{100}{1}) = 3 \text{ mV}$$

2.101

$$V_{o5} = \pm 2 \text{ mV}$$

$$V_o = 0.01 \sin \omega t \times 200 + V_{o5} \times 200 = 2 \sin \omega t \pm 0.4 \text{ V}$$

2.105



$$R_2 = 100k\Omega$$

$$R_1 = 100k\Omega$$

$$R_3 = 5k\Omega$$

$$I_{B1} = 1 \pm 0.05 \mu A, V_{OS} = 0$$

$$I_{B2} = 1 \pm 0.05 \mu A$$

a) From Eq. 2.45:  $V_0 = -I_{B2}R_3 + R_2(I_{B1} - I_{B2}\frac{R_2}{R_1})$

For  $I_{B1} = 1.05 \mu A, I_{B2} = 0.95 \mu A$

$$V_0 = -0.95 \times 5 + 100(1.05 - 0.95 \times \frac{5}{100} \times 9) = 57.5 \text{ mV}$$

b) For  $I_{B1} = 0.95 \mu A, I_{B2} = 1.05 \mu A$

$$V_0 = -1.05 \times 5 + 100(0.95 - 1.05 \times \frac{5}{100} \times 9) = 42.5 \text{ mV}$$

$$\Rightarrow 42.5 \leq V_0 \leq 57.5 \text{ mV}$$

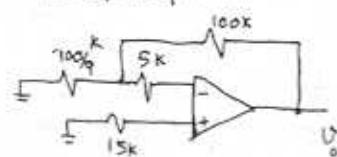
From the discussion in the text we know that to minimize the dc output voltage resulting from the input bias current, we should make the total DC resistance in the inputs of the op-amp equal. Currently, the negative input sees a resistance of  $R_1 || R_2 = \frac{100}{9} || 100 = 10k\Omega$  while the positive input terminal sees  $5k\Omega$  source resistance. Therefore we should add  $5k\Omega$  series resistor to the positive input terminal to make the effective resistance  $5k\Omega + 5k\Omega = 10k\Omega$ . The resulting  $V_0$  can be found as follows:

$$V_0 = -I_{B2} \times 10 + 100(I_{B1} - I_{B2} \frac{10}{100/9}) = (I_{B1} - I_{B2}) \times 100$$

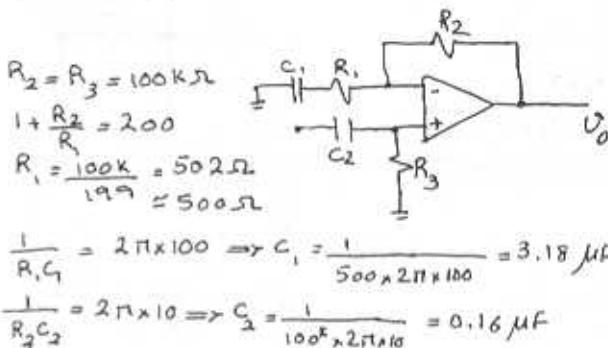
$$V_0 = I_{OS} \times 100 = \pm 0.1 \times 100 = \pm 10 \text{ mV}$$

$$V_0 = \pm 10 \text{ mV}$$

If the signal source resistance is  $15k\Omega$ , then the resistances can be equalized by adding a  $5k\Omega$  resistor in series with the negative input load of the op-amp.



2.106



$$R_2 = R_3 = 100k\Omega$$

$$1 + \frac{R_2}{R_1} = 200$$

$$R_1 = \frac{100k}{199} = 50.2 \Omega \approx 50k\Omega$$

$$\frac{1}{R_1 C_1} = 2\pi \times 100 \Rightarrow C_1 = \frac{1}{500 \times 2\pi \times 100} = 3.18 \mu F$$

$$\frac{1}{R_2 C_2} = 2\pi \times 10 \Rightarrow C_2 = \frac{1}{100 \times 2\pi \times 10} = 0.16 \mu F$$

2.107

The output component due to  $V_{OS}$  is:

$$V_{O1} = V_{OS} \left(1 + \frac{1M}{10k}\right)$$

$$V_{O1} = 4(1+100) = 404 \text{ mV}$$

The output component due to  $I_B$  or input bias current is:

$$I_{B1} = I_B + \frac{I_{OS}}{2}, I_{B2} = I_B - \frac{I_{OS}}{2}$$

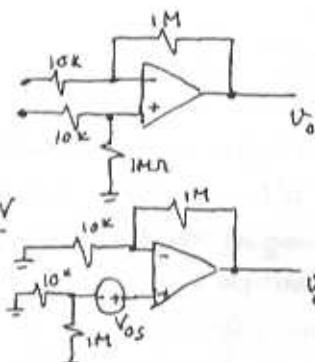
$$I_{B1} = 0.3 + \frac{0.05}{2} = 0.325 \mu A, I_{B2} = 0.275 \mu A$$

$$V_+ = -I_{B2} \times (10k || 1M)$$

$$V_+ = -2.72 \text{ mV}$$

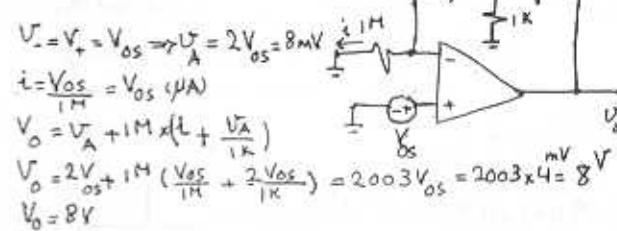
$$V_{O2} = V_+ + \left(1M \times \left(I_{B1} + \frac{V_+}{10k}\right)\right)$$

$$V_{O2} = 50 \text{ mV}$$



The worst case (largest) DC offset voltage at the output is  $404 + 50 = 454 \text{ mV}$

2.108



$$V_+ = V_0 = V_{OS} \Rightarrow V_A = 2V_{OS} = 8 \text{ mV}$$

$$i = \frac{V_{OS}}{1M} = V_{OS} (\mu A)$$

$$V_0 = V_A + 1M \times \left(1 + \frac{V_A}{1K}\right)$$

$$V_0 = 2V_{OS} + 1M \left(\frac{V_{OS}}{1M} + 2 \frac{V_{OS}}{1K}\right) = 2003V_{OS} = 2003 \times 4 = 8 \text{ V}$$

$$V_0 = 8 \text{ V}$$

Cont.

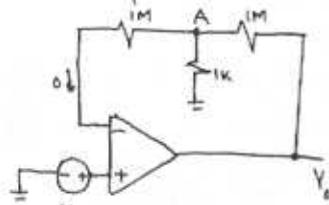
for capacitively coupled input:

$$V_f = V_+ = V_{OS}$$

$$V_A = V_{OS}$$

$$V_B = V_A + 1M \times \frac{V_{OS}}{1k}$$

$$V_B = 100I_{OS} \approx \pm 4.004V$$



for capacitively coupled 1k to ground:

$$V_f = V_+ = V_{OS}$$

$$V_A = 2V_{OS}$$

$$V_B = 3V_{OS} \approx \pm 12mV$$

This is much smaller than capacitively coupled input case.

2.109

At  $0^\circ C$ , we expect  $\pm 10 \times 25 \times 1000 \mu = \pm 250mV$

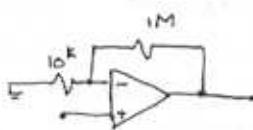
At  $75^\circ C$ , we expect  $\pm 10 \times 50 \times 1000 \mu = \pm 500mV$

We expect these quantities to have opposite polarities.

2.110

$$100 = 1 + \frac{R_2}{R_1} \Rightarrow R_1 = 10k\Omega$$

$$\text{a) } V_0 = 100 \times 10^{-9} \times 1 \times 10^6 = 0.1V$$



b) Largest output offset is:

$$V_D = 1mV \times 100 + 0.1V = 200mV = 0.2V$$

c) For bias current compensation we connect a resistor  $R_3$  in series with the positive input terminal of the op-amp, with:  $R_3 = R_1 \parallel R_2$

$$I_{OS} = \frac{100}{10} = 10nA$$

$$R_3 = 10^3 \parallel 1M = 10k\Omega$$

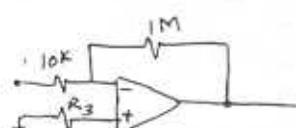
The offset current alone results in an output offset voltage of  $I_{OS} \times R_3 = 10 \times 10^{-9} \times 1 \times 10^6 = 10mV$

$$\text{d) } V_0 = 100mV + 10mV = \underline{\underline{110mV}}$$

2.111

$$R_3 = R_1 \parallel R_2 = 9.9k\Omega$$

(Refer to 2.46)

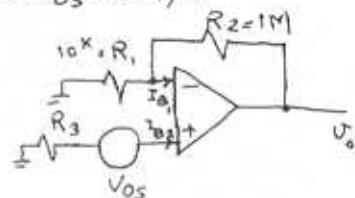


$$V_0 = I_{OS} R_2 \quad \text{Eq. 2.47}$$

$$V_0 = 0.21 = I_{OS} \times 1M \Rightarrow I_{OS} = 0.21 \mu A$$

$$\text{If } V_{OS} = 1mV$$

$$V_+ = -I_{OS} R_3 + V_{OS}$$



$$I_{B_1} = \frac{R_3 I_{B_2} + V_{OS}}{R_1} + \frac{0.21 + R_3 I_{B_2} + V_{OS}}{R_2}$$

$$I_{B_1} = R_3 I_{B_2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) + V_{OS} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow I_{B_1} - I_{B_2} = \pm V_{OS} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\Rightarrow I_{OS} = \pm \frac{1mV}{9.9k} = \pm 0.1 \mu A$$

If we apply the same current as  $I_{OS}$  to the other end of  $R_3$ , then it will cancel out the offset current effect on the output.  $\pm 0.1 \mu A$

Now if we use  $\pm 15V$  supplies:

2.112

$$\frac{V_o}{V_i} = -\frac{1}{SCR} = -\frac{1}{j\omega CR} = \frac{1}{-j\omega \times 10 \times 10^{-9} \times 100 \times 10^3}$$

$$\frac{V_o}{V_i} = -\frac{10^3}{j\omega}$$

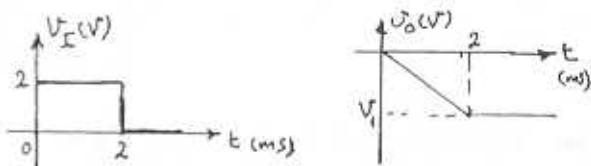
$$\text{a) } \frac{V_o}{V_i} = 1 \Rightarrow \omega = 1K \text{ rad/s} \quad f = 159 \text{ Hz}$$

b)  $\frac{1}{j}$  indicates  $90^\circ$  lag, but since its  $\frac{-1}{j}$ , it results in output leading the input by  $90^\circ$ .

c)  $\frac{V_o}{V_i} = -\frac{10^3}{j\omega}$  If frequency is lowered by a factor of 10, then the output would increase by a factor of 10.

Cont.

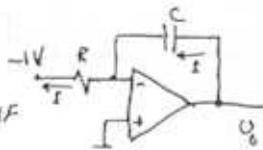
d) The phase does not change and the output still leads the input by  $90^\circ$



2.113

$$R_{in} = R = 100\text{k}\Omega$$

$$CR = 15 \Rightarrow C = \frac{1}{100 \times 10^3} = 10\text{nF}$$

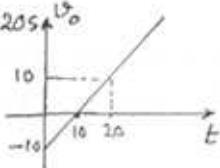


with a  $-1\text{V}$  DC input applied, the capacitor charges with a constant current:

$I = \frac{1\text{V}}{R}$  =  $0.01\text{mA}$  and its voltage rises linearly:

$$V_o(t) = -10 + \frac{1}{C} \int_0^t I dt = -10 + \frac{I}{C} t = -10 + \frac{t}{RC}$$

the voltage reaches  $0\text{V}$  at  $t = 10\text{RC} = 10\text{s}$  and it reaches  $10\text{V}$  at  $t = 20\text{s}$



2.114

$$|T| = \frac{1}{\omega RC} \quad \text{IF } |T| = 100\text{V/V} \text{ for } F = 1\text{kHz}, \text{ then}$$

$$\text{for } |T| = 1\text{V/V}, f \text{ has to be } 1\text{kHz} \times 100 = 100\text{kHz}.$$

$$\text{Also } RC = \frac{1}{\omega T} = \frac{1}{2\pi \times 1\text{kHz} \times 100} = 1.59\mu\text{s}$$

2.115

$R_{in} = R$ , Thus  $R = 100\text{k}\Omega$ .

$$|T| = \frac{1}{\omega RC} = 1 \text{ at } \omega = \frac{1}{RC}$$

$$\omega = 1000 = \frac{1}{RC} \Rightarrow C = \frac{1}{1000 \times 100} = 10\text{nF}$$

with a  $2\text{V}-2\text{ms}$  pulse at the input, the output falls linearly until  $t = 2\text{ms}$  at which

$$V_o = V_i, \quad V_o = \frac{-I}{C} t = \frac{-2}{RC} t = -2t \text{ Volts}$$

where  $t$  in ms

Thus  $V_o = -4\text{V}$

with  $V_i = 2\sin 1000t$  applied at the input,

$$V_o(t) = 2 \cdot \frac{1}{1000 \times 10^3} \sin(1000t + 90^\circ)$$

$$V_o(t) = 2 \sin(1000t + 90^\circ)$$

2.116

$$R_{in} = R = 20\text{k}\Omega$$

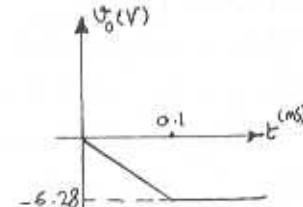
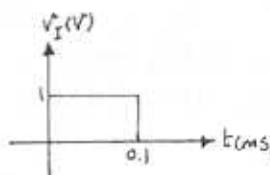
$$|T| = \frac{1}{\omega RC} = 1 \text{ at } \omega = 2\pi \times 10^3 \text{ rad/s} = \frac{1}{2\pi \times 10^3 \times 20} \text{ s}^{-1}$$

$$C = 0.796 \text{ nF}$$

Refer to discussion in page 110:

$\frac{V_o}{V_i} = \frac{R_f/R}{1 + sCR_f}$  and the finite DC gain is  $\frac{-R_f}{R}$ . Therefore for 40dB gain or equivalently  $100\text{V/V}$  we have:  $\frac{-R_f}{R} = 100 \text{V/V}$   
 $\Rightarrow R_f = 100 \times 20\text{k} = 2\text{M}\Omega$

The corner frequency  $\frac{1}{CR_f}$  is:  $\frac{1}{0.796 \times 2\text{M}} = 628$  Hz



a) when no  $R_f$

$$V_o(t) = -\frac{1}{RC} \int_0^t i dt = -62.8t \quad \text{if } t \leq 0.1\text{ms}$$

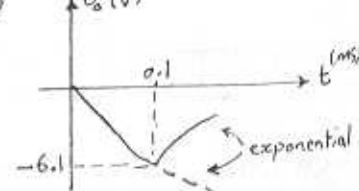
$$V_o(0.1) = -6.28\text{V}$$

$$\text{b) with } R_f: \quad V_o(t) = V_o(\infty) (1 - e^{-t/CR_f})$$

(Refer to pg. 112)

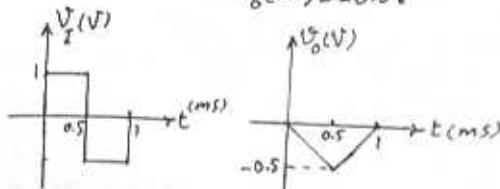
$$V_o(\infty) = -I \times R_f = -\frac{1\text{V}}{20\text{k}} \times 2\text{M} = -100\text{V}$$

$$V_o(t) = -100(1 - e^{-t/0.1})$$



2.117

For  $0 \leq t \leq 0.5 \text{ ms}$ :  $V_o(t) = V_o(0) - \frac{1}{RC} \int_0^t V_i dt$   
 $V_o(t) = 0 - \frac{t}{RC} = -\frac{t}{1 \text{ ms}}$   
 $V_o(0.5) = -0.5 \text{ V}$



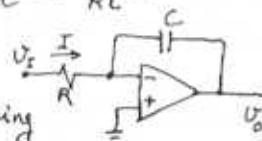
For  $0.5 \leq t \leq 1 \text{ ms}$ :  $V_o(t) = V_o(0.5) - \frac{1}{RC} \int_{0.5}^t 1 \text{ dt}$   
 $V_o(t) = -0.5 + \frac{1}{RC} (t - 0.5)$   
 $V_o(1 \text{ ms}) = -0.5 + \frac{0.5}{1} = 0 \text{ V}$

Another way of thinking about this circuit is as follows:

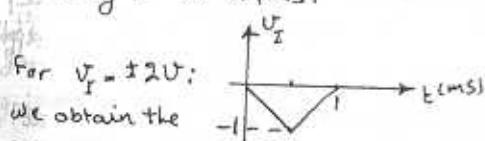
for  $0 \leq t \leq 0.5 \text{ ms}$  a current  $I = \frac{V_i}{R}$  flows through R and C in the direction indicated on the diagram. At time  $t$  we write:

$$I \cdot t = -C(V_o(t)) \Rightarrow V_o(t) = -\frac{1}{C} t = -\frac{1}{RC} t$$

which indicates that the output voltage is linearly decreased, reaching  $-0.5 \text{ V}$  at  $t = 0.5 \text{ ms}$ .



Then for  $0.5 \leq t \leq 1 \text{ ms}$ , the current flows in the opposite direction and  $V_o$  rises linearly reaching  $0 \text{ V}$  at  $t = 1 \text{ ms}$ .



Following waveform: (assuming time constant is the same)

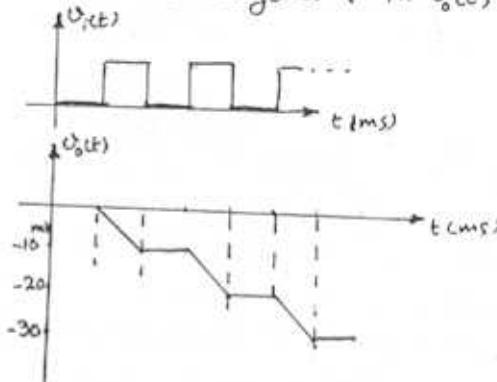
If AC is also doubled, then the waveform becomes the same as the first case where  $V_i = \pm 1 \text{ V}$  and  $RC = 1 \text{ ms}$ .

2.118

Each pulse lowers the output voltage by:

$$\Delta V_o = \frac{1}{RC} \int_0^{10 \text{ ms}} 1 \cdot dt = \frac{10 \mu\text{s}}{RC} = \frac{10 \mu\text{s}}{1 \text{ ms}} = 10 \text{ mV}$$

Therefore a total of 100 pulses are required to cause a change of 1V in  $V_o(t)$ .



2.119

Refer to Fig. P2.119.

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{Y_1}{Y_2 + SC} = -\frac{R_2/R_1}{1 + SCR_2}$$

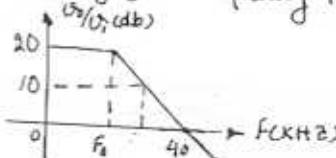
which is an STC LP circuit with a dc gain of  $-\frac{R_2}{R_1}$  and a 3-dB frequency  $\omega_0 = \frac{1}{CR_2}$ . The input resistance equal to  $R_1$ . So for:

$$R_1 = 1 \text{ k} \Omega \Rightarrow R_1 = 1 \text{ k} \Omega \text{ and for dc gain of } 20 \text{ dB}$$

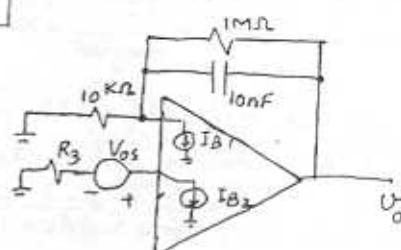
$$10 : \frac{R_2}{R_1} = 10 \Rightarrow R_2 = 10 \text{ k} \Omega$$

$$\text{For } 3 \text{ dB Frequency of } 4 \text{ kHz: } \omega_0 = 2\pi \times 4 \times 10^3 = \frac{1}{CR_2} \\ \Rightarrow C = \frac{1}{4\pi \times 10^3 \times 10^3} \text{ F}$$

the unity gain frequency is (dB) is 40 kHz

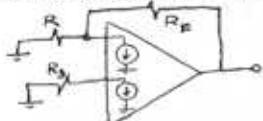


2.120



Cont.

a) To compensate for the effect of dc bias current  $I_B$ , we can consider the following model



Similar to the discussion leading to equation (2.46) we have:  $R_3 = R \parallel R_F = 10\text{k}\Omega \parallel 1\text{M}\Omega \Rightarrow R_3 = 9.9\text{k}\Omega$

(b) As discussed in Section 2.8.2 the dc output voltage of the integrator when the input is grounded is:  $V_o = V_{os} \left(1 + \frac{R_F}{R}\right) + I_{os} R_F$   
 $V_o = 3\text{mV} \left(1 + \frac{1\text{M}\Omega}{10\text{k}\Omega}\right) + 10\text{nA} \times 1\text{M}\Omega = 0.303\text{V} + 0.01\text{V}$   
 $V_o = 0.313\text{V}$

2.121

$$\frac{V_{o(1s)}}{V_i} = -SRC = -5 \times 0.01 \times 10^{-6} \times 10 \times 10^3 = -10^{-5}$$

$$\frac{V_o}{V_i} (j\omega) = -j\omega \times 10^{-4} \Rightarrow \left| \frac{V_o}{V_i} \right| = \omega \times 10^{-4} \Rightarrow$$

$$\left| \frac{V_o}{V_i} \right| = 1 \text{ when } \omega = 10^4 \text{ Rad/s or } f = 1.59\text{kHz}$$

For an input 10 times this frequency, the output will be 10 times as large as the input: 10V peak-to-peak. The (-j) indicates that the output lags the input by 90°. Thus  $V_o(t) = -5 \sin(10^5 t + 90^\circ)$  Volts

2.122

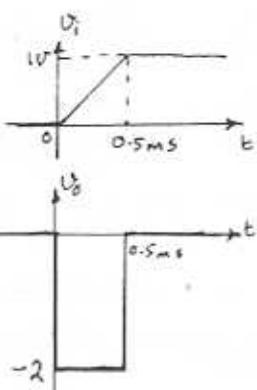
$$V_o = -CR \frac{dV_i}{dt}$$

therefore:

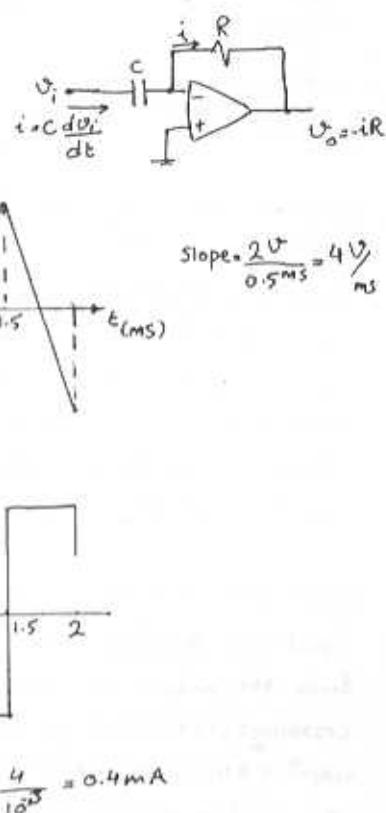
for  $0 \leq t \leq 0.5$ :

$$V_o = -1\text{ms} \times \frac{1\text{V}}{0.5\text{ms}} = -2\text{V}$$

and  $V_o = 0$  otherwise



2.123



Thus the peak value of the output square wave is  $0.4\text{mA} \times 10^6 \Omega = 4\text{V}$ . The frequency of the output is the same as the input (1kHz).

The average value of the output is 0. To increase the value of the output to 10V, R has to be increased to  $\frac{10}{0.4} = 25\text{k}\Omega$ , i.e. 2.5.

When a 1-kHz, 1V peak input sine wave is applied  $V_i = \sin(2\pi \times 1000t)$

a sinusoidal signal appears at the output.

It can be determined by one of the following methods:

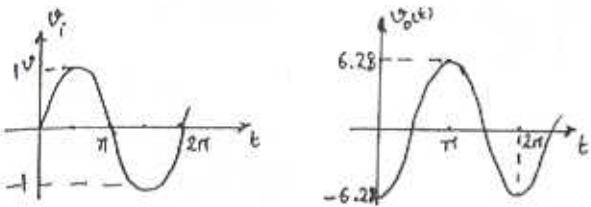
$$a) V_o(t) = -RC \frac{dV_i}{dt} = -0.1 \times 10^{-6} \times 10 \times 10^3 \frac{dV_i}{dt} = -10 \frac{dV_i}{dt}$$

$$V_o(t) = -10 \times 2\pi \times 1000 \times C_00 (2\pi \times 1000t)$$

$$V_o(t) = -2\pi \cos(2\pi \times 1000t)$$

Thus the peak amplitude is 6.28V and the negative peaks occur at  $t = 0, \frac{2\pi}{2\pi \times 1000}, \dots$

Cont.



2.125

Refer to Fig. P2.125:

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = \frac{-R_2}{R_1 + \frac{1}{jC}} = \frac{-(R_2/C)}{s + \frac{1}{R_1 C}}$$

which is the transfer function of an STC HP filter with

a high frequency gain  $K = -\frac{R_2}{R_1}$  and a 3-dB frequency  $\omega_0 = \frac{1}{R_1 C}$

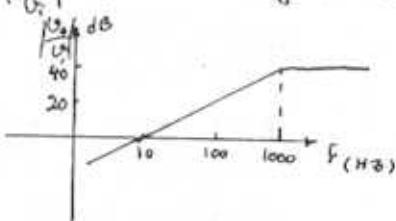
The high-frequency input impedance approaches  $R_1$ . (as  $\frac{1}{j\omega C}$  becomes negligibly small) So we can select  $R_1 = 10k\Omega$

To obtain a high-frequency gain of 40dB (i.e. 100) :  $\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 1M\Omega$ .

For a 3-dB frequency of 1000 Hz:

$$\frac{1}{R_1 C} = 2\pi \times 1000 \Rightarrow C = 15.9nF$$

From the Bode-diagram below, we see that  $\left|\frac{V_o}{V_i}\right|$  reduces to unity at  $f = 0.01 f_0 = 10\text{Hz}$



2.124

$$RC = 10^{-3} \text{ when } C = 10^{-6}F \Rightarrow R = 100k\Omega$$

$$\frac{V_o}{V_i} = -SRC \quad \frac{V_o}{V_i}(j\omega) = -j\omega RC \quad \phi = -90^\circ$$

always

$$\left|\frac{V_o}{V_i}\right| = 1 \Rightarrow \omega = \frac{1}{\text{unity } RC} = 10\text{ rad/s}$$

Gain is 10 times the unity gain, when the frequency is 10 times the unity gain frequency. Similarly for  $\omega = \frac{1}{10}$  rad/s, gain is  $0.1V_o/V_i$ . (for  $\omega = 10\text{ rad/s}$ , gain =  $10V_o/V_i$ )

For high frequencies C is short-circuited,  $R_1$  is short-circuited,  $R$  is open-circuited.

$$\frac{V_o}{V_i} = \frac{R}{R_1} = 100 \Rightarrow R_1 = 1k\Omega$$

$$\frac{V_o}{V_i} = \frac{-RCS}{R_1 CS + 1} = \frac{-10^{-3}}{10^{-5}S + 1} \Rightarrow \omega_{3\text{dB}} = 100\text{ rad/s or } f = 15.9\text{ kHz}$$

For unity gain:  $|10^{-3}| = |10^{-5} + 1| \Rightarrow \omega_H = 1.01\text{ rad/s}$

$$\text{if } \omega = 10.1\text{ rad/s} : \left|\frac{V_o}{V_i}\right| = \frac{10.1}{1.01} = 10, \quad \phi = -95.77^\circ$$

2.126

Refer to the circuit in Fig. P2.126:

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{1}{Z_1 Y_2} = -\frac{1}{(R_1 + \frac{1}{j\omega C_1})(\frac{1}{R_2} + j\omega C_2)}$$

$$\frac{V_o}{V_i} = -\frac{R_2/R_1}{(1 + \frac{1}{R_1 C_1})(1 + j\omega R_2 C_2)}$$

$$\frac{V_o}{V_i}(j\omega) = \frac{-R_2/R_1}{(1 + \frac{1}{j\omega R_1 C_1})(1 + j\omega R_2 C_2)} = \frac{-R_2/R_1}{(1 + \frac{\omega_1}{j\omega})(1 + \frac{\omega_2}{j\omega})}$$

$$\text{where } \omega_1 = \frac{1}{R_1 C_1}, \quad \omega_2 = \frac{1}{R_2 C_2}$$

a) For  $\omega \ll \omega_1 \ll \omega_2$ 

$$\frac{V_o}{V_i}(j\omega) \approx \frac{-R_2/R_1}{(1 + \frac{\omega_1}{j\omega})} \approx \frac{-R_2/R_1}{\omega_1/j\omega} = -j \frac{R_2}{R_1} \frac{\omega}{\omega_1}$$

Cont.

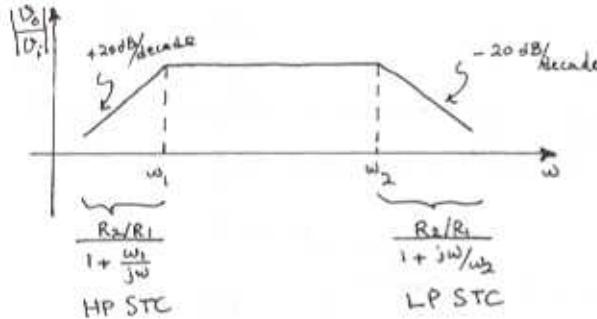
b) for  $\omega_1 \ll \omega \ll \omega_2$

$$\frac{V_o}{V_i} (j\omega) \approx -\frac{R_2}{R_1}$$

c) for  $\omega \gg \omega_2$  and  $\omega_2 \gg \omega_1$ :

$$\frac{V_o}{V_i} (j\omega) \approx \frac{-R_2/R_1}{1 + j\omega/\omega_2} \approx \frac{-R_2/R_1}{j\omega/\omega_2} = j \left( \frac{R_2}{R_1} \right) \left( \frac{\omega_2}{\omega} \right)$$

from the results of a), b) and c) we can draw  
the Bode-plot:



Design:  $\frac{R_2}{R_1} = 1000$  (60dB gain in the mid-frequency range)

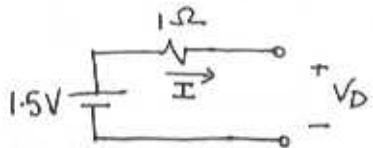
Rin for  $\omega \gg \omega_1$  =  $R_1 = 1\text{ k}\Omega$   $\Rightarrow$   $R_2 = 1\text{ M}\Omega$

$$f_1 = 100\text{ Hz} \Rightarrow \omega_1 = 2\pi \times 100 = \frac{1}{R_1 C_1} \Rightarrow C_1 = 1.59\text{ }\mu\text{F}$$

$$f_2 = 10\text{ kHz} \Rightarrow \omega_2 = 2\pi \times 10 \times 10^3 = \frac{1}{R_2 C_2} \Rightarrow C_2 = 15.9\text{ pF}$$

## CHAPTER 3 - PROBLEMS

**3.1**



The diode can be reverse-biased and thus no current would flow, or forward-biased where current would flow.

(a) Reverse biased  $I = 0\text{A}$   $V_D = 1.5\text{V}$

(b) Forward biased  $I = 1.5\text{A}$   $V_D = 0\text{V}$

**3.2**

(a) Diode is conducting and thus has a 0V drop across it. Consequently

$$V = \underline{\underline{-3\text{V}}}$$

$$I = \frac{3 - (-3)}{10k\Omega} = \underline{\underline{0.6\text{mA}}}$$

(b) Diode is cut off.

$$V = \underline{\underline{3\text{V}}} \quad I = \underline{\underline{0\text{A}}}$$

(c) Diode is conducting

$$V = \underline{\underline{3\text{V}}}$$

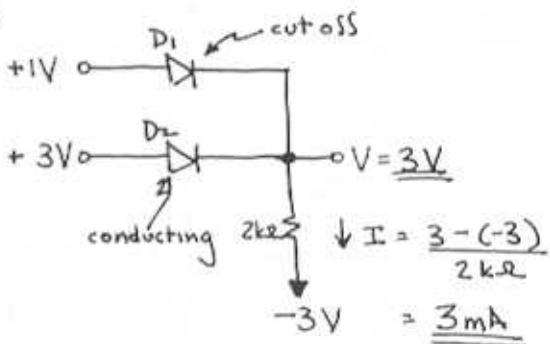
$$I = \frac{3 - (-3)}{10k\Omega} = \underline{\underline{0.6\text{mA}}}$$

(d) Diode is cut off.

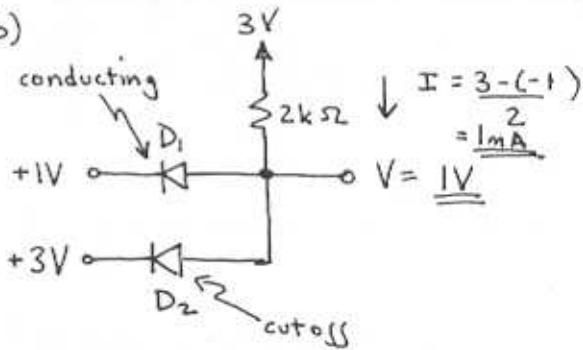
$$V = \underline{\underline{-3\text{V}}} \quad I = \underline{\underline{0\text{A}}}$$

**3.3**

(a)

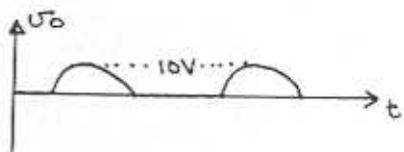


(b)



**3.4**

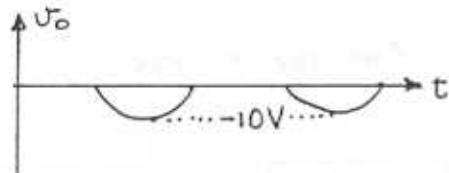
(a)



$$V_{p+} = \underline{\underline{10\text{V}}} \quad V_{p-} = \underline{\underline{0\text{V}}} \\ f = 1\text{kHz}$$

### CHAPTER 3 PROBLEMS

(b)



$$V_{p+} = \underline{0V} \quad V_{p-} = \underline{-10V}$$

$$f = 1\text{kHz}$$

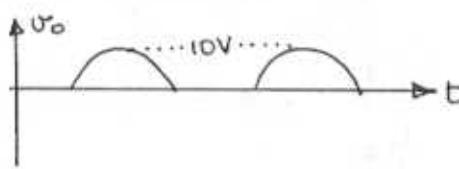
(c)



$$V_o = \underline{0V}$$

Neither  $D_1$  nor  $D_2$  conducts so there is no output.

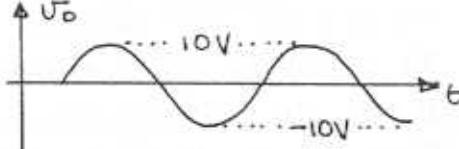
(d)



$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{0V} \quad f = 1\text{kHz}$$

Both  $D_1$  and  $D_2$  conduct when  $U_I > 0$

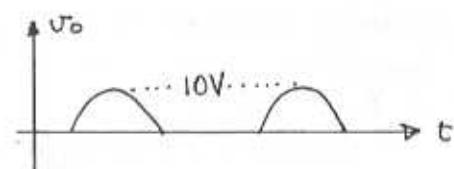
(e)



$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{-10V} \quad f = 1\text{kHz}$$

$D_1$  conducts when  $U_I > 0$  and  $D_2$  conducts when  $U_I < 0$ . Thus the output follows the input.

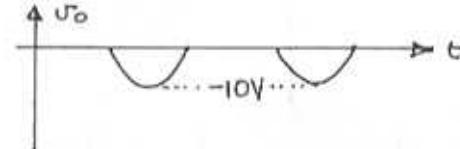
(f)



$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{0V} \quad f = 1\text{kHz}$$

$-D_1$  is cut off when  $U_I < 0$

(g)



$$V_{p+} = \underline{0V} \quad V_{p-} = \underline{-10V} \quad f = 1\text{kHz}$$

$D_1$  shorts to ground when  $U_I > 0$  and is cut off when  $U_I < 0$  whereby the output follows  $U_I$ .

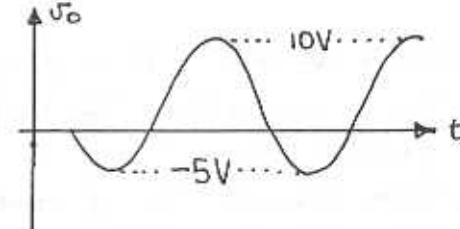
(h)



$$V_o = \underline{0V}$$

The output is always shorted to ground as  $D_1$  conducts when  $U_I > 0$  and  $D_2$  conducts when  $U_I < 0$ .

(i)

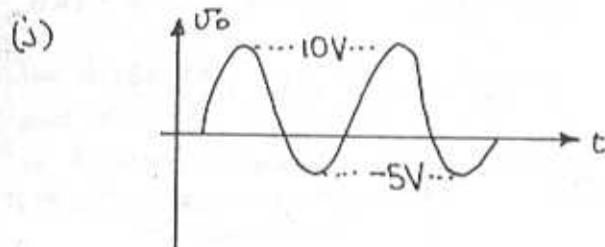


$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{-5V} \quad f = 1\text{kHz}$$

When  $U_I > 0$ ,  $D_1$  is cut off and  $V_o$  follows  $U_I$ .

-When  $U_I < 0$ ,  $D_1$  is conducting and the circuit becomes a voltage divider where the negative peak is

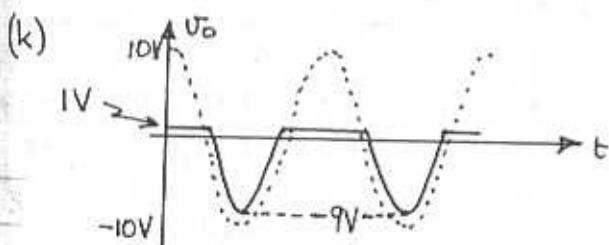
$$\frac{1k\Omega}{1k\Omega + 1k\Omega} \cdot -10V = -5V$$



$$V_{p+} = \underline{10V} \quad V_{p-} = \underline{-5V} \quad f = 1kHz$$

-When  $U_I > 0$ , the output follows the input as  $D_1$  is conducting.

-When  $U_I < 0$ ,  $D_1$  is cut off and the circuit becomes a voltage divider.



$$V_{p+} = \underline{1V} \quad V_{p-} = \underline{-9V} \quad f = 1kHz$$

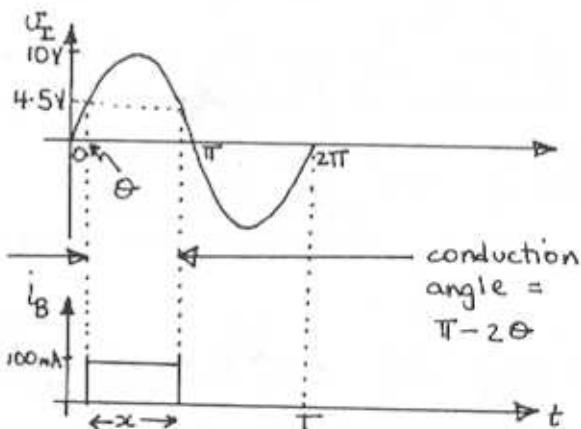
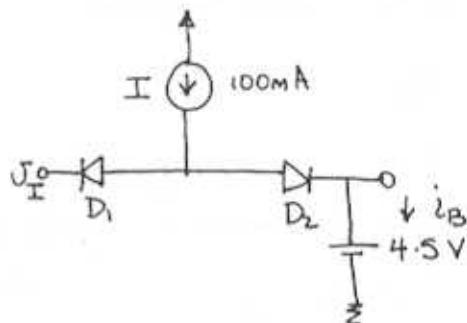
-When  $U_I > 0$ ,  $D_1$  is cut off and  $D_2$  is conducting. The output becomes 1V.

-When  $U_I < 0$ ,  $D_1$  is conducting and  $D_2$  is cut off. The output becomes :-

$$U_o = U_I + 1V .$$

### CHAPTER 3 PROBLEMS

3.5



-When  $U_I < 4.5V$   $D_1$  conducts and  $D_2$  is cut off so  $i_B = 0A$ . For  $U_I > 4.5V$   $D_2$  conducts and  $D_1$  is cut off thus disconnecting the input  $U_I$ . All of the current then flows through the battery.

$$10 \sin \theta = 4.5V$$

$$\theta = \sin^{-1}(4.5/10)$$

$$\text{conduction angle} = \pi - 2\theta$$

Fraction of cycle that  $i_B = \underline{100mA}$  is given by :-

$$x = \frac{\pi - 2\theta}{2\pi} = 0.35$$

### CHAPTER 3 PROBLEMS

$$i_{B\text{avg}} = \frac{1}{T} \int_T i_B dt$$

$$= \frac{1}{T} \left[ 100 + 0.35T \right]$$

$$= \underline{\underline{35 \text{ mA}}}$$

If  $V_T$  is reduced by 10% the peak value of  $i_B$  remains the same

$$i_{B\text{peak}} = \underline{\underline{100 \text{ mA}}}$$

but the fraction of the cycle for conduction changes

$$x = \frac{\pi - 2\theta}{2\pi} = \frac{\pi - 2 \sin^{-1}(4.5/9)}{2\pi}$$

$$= \frac{1}{3}$$

Thus:

$$i_{B\text{avg}} = \frac{1}{T} \left[ 100 \cdot \frac{1}{3} \right]$$

$$= \underline{\underline{33.3 \text{ mA}}}$$

$-x$  and  $y$  are the same for  $A = B$

$-x$  and  $y$  are opposite if  $A \neq B$

3.7

$$\frac{5-O}{R} \leq 0.1 \text{ mA}$$

$$R \geq \frac{5}{0.1} = \underline{\underline{50 \text{ k}\Omega}}$$

3.8

The maximum input current occurs when one input is low and the other two are high.

$$\frac{5-O}{R} \leq 0.1 \text{ mA}$$

$$R \geq \underline{\underline{50 \text{ k}\Omega}}$$

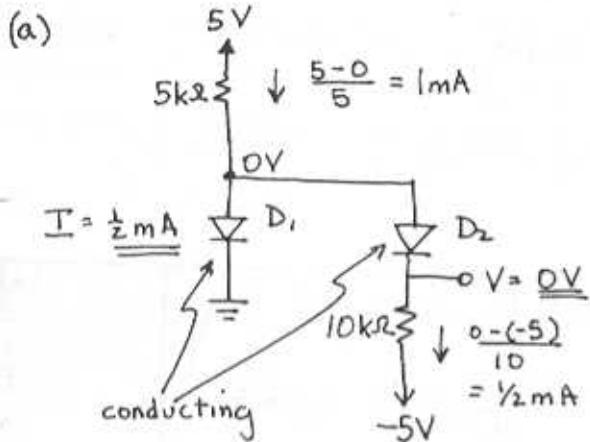
3.6

A	B	x	y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

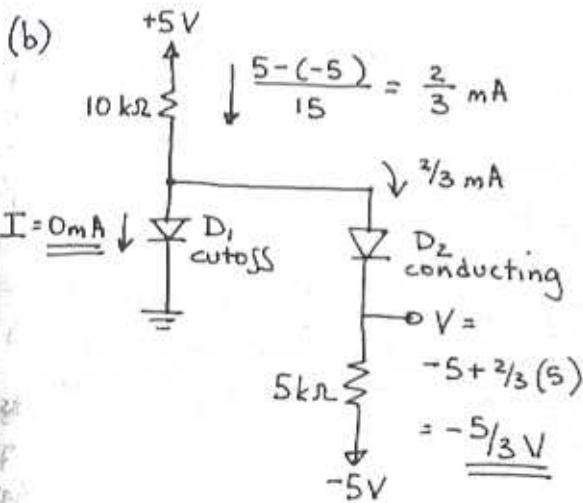
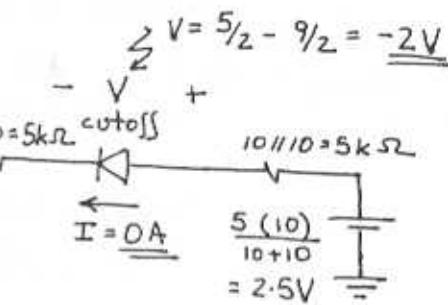
$$x = \underline{\underline{A \cdot B}}$$

$$y = \underline{\underline{A + B}}$$

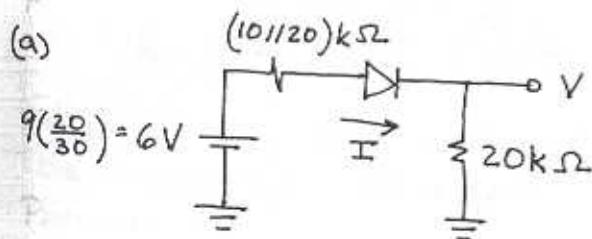
3.9



(b)



3.10



$$I = \frac{6}{(10//20)+20} = 0.225\text{mA}$$

$$V = \frac{20}{(10//20)+20} \times 6 = 4.5\text{V}$$

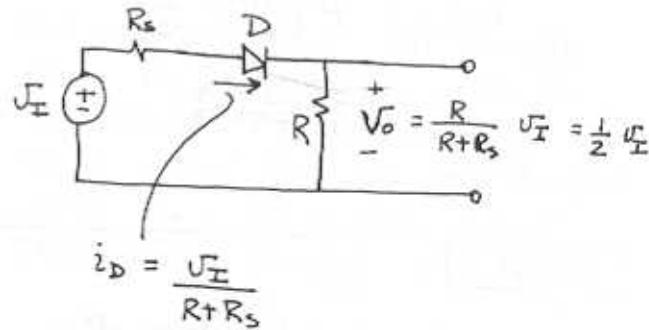
3.11

$$R \geq \frac{120\sqrt{2}}{50} \geq 3.4\text{k}\Omega$$

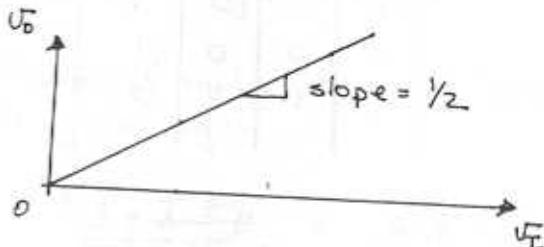
The largest reverse voltage appearing across the diode is equal to the peak input voltage

$$120\sqrt{2} = 169.7\text{V}$$

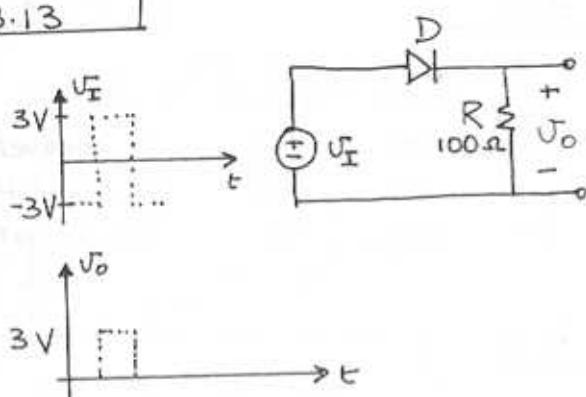
3.12



D starts to conduct when  $V_I > 0$



3.13



$$U_{O, \text{peak}} = 3V$$

$$\begin{aligned} U_{O, \text{avg}} &= \frac{1}{T} \int U_O dt \\ &= \frac{1}{T} \left[ 3 \cdot \frac{T}{2} \right] = \underline{\underline{1.5V}} \end{aligned}$$

$$i_{D, \text{peak}} = \frac{3}{100} = \underline{\underline{30mA}}$$

$$i_{D, \text{avg}} = \frac{1.5}{100} = \underline{\underline{15mA}}$$

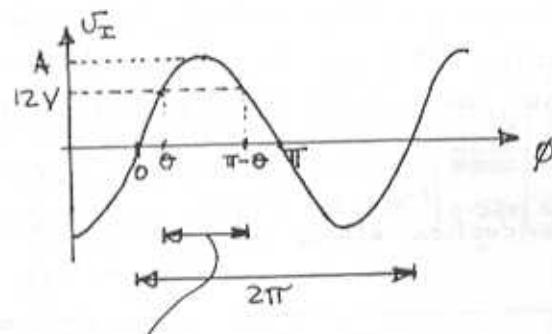
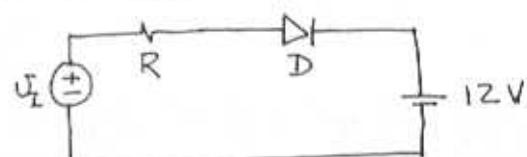
The maximum reverse diode voltage is 3V

$$i_{D, \text{peak}} = \frac{U_{O, \text{peak}}}{100} = \underline{\underline{50mA}}$$

$$i_{D, \text{avg}} = i_{D, \text{peak}}/2 = \underline{\underline{25mA}}$$

maximum reverse voltage = 1V

3.15



conduction occurs

$$U_I = A \sin \theta = 12 \sim \text{conduction across } D \text{ occurs}$$

For a conduction angle ( $\pi - 2\theta$ ) that is 20% of a cycle

$$\frac{\pi - 2\theta}{2\pi} = \frac{1}{5}$$

$$\theta = 0.3\pi$$

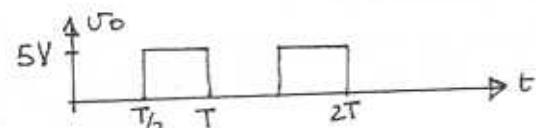
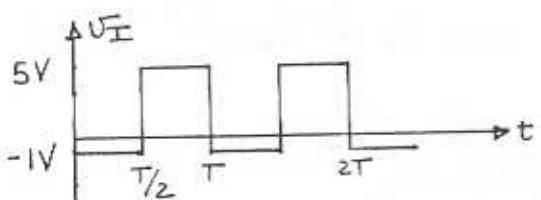
$$A = 12 / \sin \theta = 14.83V$$

$$\begin{aligned} \textcircled{o} \text{ Peak-to-peak sine wave voltage} \\ &= 2A = \underline{\underline{29.67V}} \end{aligned}$$

Given the average diode current to be

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{A \sin \phi - 12}{R} d\phi = 100mA$$

3.14



$$U_{O, \text{peak}} = \underline{\underline{5V}}$$

$$U_{O, \text{avg}} = \underline{\underline{2.5V}}$$

$$\frac{1}{2\pi} \left[ \frac{-14.83 \cos \phi - 12\phi}{R} \right]_{\phi=0.3\pi}^{\phi=0.7\pi} = 0.1$$

$$R = \underline{3.75 \Omega}$$

$$\text{Peak diode current} = \frac{A-12}{R} = \underline{0.75A}$$

$$\text{Peak reverse voltage} = A+12 = \underline{26.83V}$$

For resistors specified to only one significant digit and peak-to-peak voltage to the nearest volt then choose  $A = 15$  so the peak-to-peak sine wave voltage = 30V and  $R = \underline{3\Omega}$

$$\begin{aligned} \text{Conduction starts at } U_E &= A \sin \theta = 12 \\ 15 \sin \theta &= 12 \\ \theta &= 0.93 \text{ rad} \end{aligned}$$

Conduction stops at  $\pi - \theta$

$$\therefore \text{Fraction of cycle that current flows is } \frac{\pi - 2\theta}{2\pi} \times 100 = 20.5 \approx \underline{20\%}$$

Average diode current =

$$\frac{1}{2\pi} \left[ \frac{-15 \cos \phi - 12\phi}{3} \right]_{\phi=0.93}^{2.21} = \underline{136mA}$$

Peak diode current

$$= \frac{15-12}{3} = \underline{1A}$$

Peak reverse voltage =

$$A+12 = \underline{27V}$$

3.16

V	RED	GREEN	
3V	ON	OFF	- $D_1$ conducts
0	OFF	OFF	- No current flows
-3V	OFF	ON	- $D_2$ conducts

3.17

$$V_T = \frac{kT}{q} \quad \text{where } k = 1.38 \times 10^{-23} \text{ J/K}$$

$$T = 273 + x^\circ \text{C}$$

$$q = 1.60 \times 10^{-19} \text{ C}$$

x [°C]	V <sub>T</sub> [mV]
-40	20
0	23.5
40	27
150	36.5

$$\begin{aligned} \text{for } V_T &= 25 \text{ mV} \\ T &= \underline{16.8^\circ \text{C}} \end{aligned}$$

3.18

$$i = I_s e^{\frac{U}{2x0.025}}$$

$$\therefore 1000 I_s = I_s e^{\frac{U}{0.05}}$$

$$U = \underline{0.345V}$$

$$\text{at } U = 0.7V$$

$$i = I_s e^{\frac{0.7}{0.05}} = \underline{1.2 \times 10^6 I_s}$$

3.19

$$i_1 = I_s e^{-V_1/nV_T} = 10^{-3}$$

$$i_2 = I_s e^{-V_2/nV_T}$$

$$\frac{i_2}{i_1} = \frac{i_2}{10^{-3}} = e^{\frac{0.5 - 0.7}{0.025}}$$

$$i_2 = \underline{\underline{0.335 \mu A}}$$

3.20

$$i = I_s e^{V_1/nV_T} = I_s e^{0.7/0.025} = 5(10^{-3})$$

$$I_s = 5(10^{-3}) e^{-0.7/0.025} = \underline{\underline{3.46 \times 10^{-5} A}}$$

$V$	$i$
0.71 V	7.46 mA
0.8 V	273.21 mA
0.69 V	3.35 mA
0.6 V	91.65 $\mu A$

$$\text{Let } i_1 = I_s e^{V_1/0.025}$$

$$i_2 = 10i_1 = I_s e^{V_2/0.025}$$

$$\frac{i_2}{i_1} = 10 = e^{\frac{V_2 - V_1}{0.025}}$$

$$\therefore \Delta V = V_2 - V_1 = \underline{\underline{57.56 mV}}$$

3.21

To calculate  $I_s$  use

$$I_s = I e^{-V_1/nV_T} = I e^{-V_1/n \times 0.025}$$

To calculate the voltage at 1% of the measured current use

$$i_2 = 0.01 i_1 \quad \text{so,}$$

$$\frac{i_2}{i_1} = 0.01 = e^{\frac{V_2 - V_1}{nV_T}}$$

$$\begin{aligned} V_2 &= V_1 + nV_T \ln 0.01 \\ &= V + n(0.025) \ln(0.01) \end{aligned}$$

$V$ [V]	$I$ [A]	$n=1$	$I_s$ [A]	$V$ [V]	$V$ [V]
		[A]	$n=2$		
0.7	1 A	$6.91 \times 10^{-13}$	$8.32 \times 10^{-7}$	0.585	0.470
0.650	1 mA	$5.11 \times 10^{-15}$	$2.26 \times 10^{-9}$	0.535	0.420
0.650	10 $\mu A$	$5.11 \times 10^{-17}$	$2.26 \times 10^{-11}$	0.535	0.420
0.7	10 $\mu A$	$6.91 \times 10^{-15}$	$8.32 \times 10^{-9}$	0.584	0.470

3.22

Let  $I_1 = I_s e^{V_1/nV_T}$  and

$$I_2 = I_s e^{V_2/nV_T} = I_1/10$$

Calculate  $n$  by :-

$$\frac{I_2}{I_1} = e^{\frac{V_2 - V_1}{nV_T}}$$

$$n = \frac{1}{V_T} \left[ \frac{V_2 - V_1}{\ln \frac{I_2}{I_1}} \right] = \frac{1}{0.025} \left[ \frac{V_2 - V_1}{\ln 0.1} \right]$$

Calculate  $I_s$  by :-

$$I_s = I_1 e^{-V_1/nV_T}$$

Calculate the diode voltage at 10I<sub>s</sub> by :-  $V_3 = nV_T \ln \frac{10I_1}{I_s}$

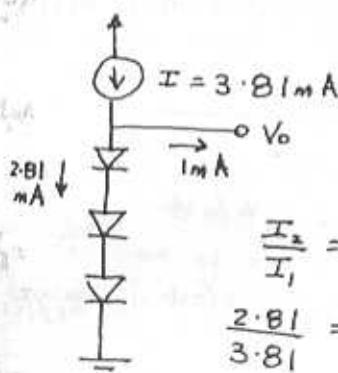
$I$	$V_1$ [V]	$V_2$ [V]	$n$	$I_s$ [A]	$V_3$ [V]
10mA	0.7	0.6	1.737	$10^{-9}$	0.8
1mA	0.7	0.6	1.737	$10^{-10}$	0.8
10A	0.8	0.7	1.737	$10^{-7}$	0.9
1mA	0.7	0.58	2.085	$1.47 \times 10^{-9}$	0.82
10μA	0.7	0.64	1.042	$2.15 \times 10^{-17}$	0.7

3.23

∴ The voltage across each diode is  $V_0/3$

$$I = I_s e^{\frac{V_0/3}{nV_T}} = 10^{-14} e^{\frac{2/3}{0.025}}$$

$$\Rightarrow \underline{3.81 \text{ mA}}$$



$$\Delta V = V_2 - 2 = \underline{-22.8 \text{ mV}}$$

through it is

$$I = I_s e^{\frac{V_1}{nV_T}}$$

With two diodes in parallel, the current splits between each diode so that the diodes each has half the current

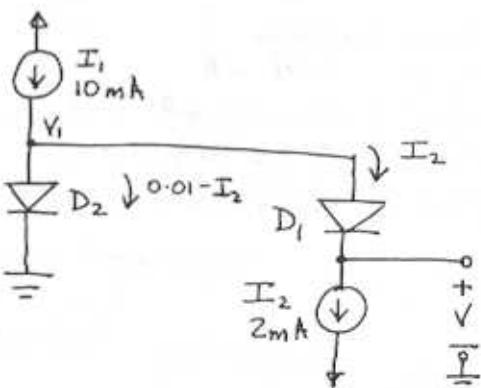
$$\frac{I}{2} = I_s e^{\frac{V_2}{nV_T}}$$

$$\therefore \frac{I/2}{I} = e^{\frac{V_2 - V_1}{nV_T}}$$

The change in voltage is

$$\Delta V = V_2 - V_1 = nV_T \ln\left(\frac{1}{2}\right) = \underline{-17.3 \text{ mV}}$$

3.25



The current through  $D_1$  is

$$10 I_s e^{\frac{V_1 - V}{nV_T}} = I_2 \quad (A)$$

The current through  $D_2$  is

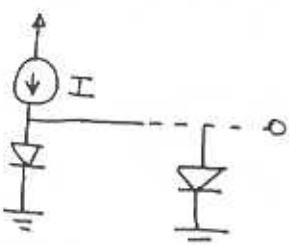
$$I_s e^{\frac{V_1}{nV_T}} = 0.01 - I_2$$

$$I_s = (0.01 - I_2) e^{\frac{V_1}{nV_T}} \quad (B)$$

(B)  $\rightarrow$  (A)

$$10 (0.01 - I_2) e^{\frac{-V}{nV_T}} = I_2$$

3.24



With one diode the current

$$V = -V_T \ln\left(\frac{I_2}{10(0.01 - I_2)}\right)$$

$$= 0.025 \ln\left(\frac{2}{10(8)}\right) = \underline{\underline{92.2 \text{ mV}}}$$

For  $V = 50 \text{ mV}$

$$-V_T \ln\left(\frac{I_2}{10(10 - I_2)}\right) = 50 \times 10^{-3}$$

$$I_2 = 10(10 - I_2) e^{-2}$$

$$I_2 (1 + 10e^{-2}) = 100e^{-2}$$

$$I_2 = \underline{\underline{5.75 \text{ mA}}}$$

3.27

AT A CONSTANT TEMPERATURE, THE DIODE VOLTAGE DROP CHANGES WITH CURRENT ACCORDING TO

$$\Delta V = V_T \ln\left(\frac{I_2}{I_1}\right)$$

WHERE

$$V_T = \frac{kT}{q} = \frac{1.38 \times 10^{-23} (273 + \text{Temp. } ^\circ\text{C})}{1.6 \times 10^{-19}}$$

THUS:

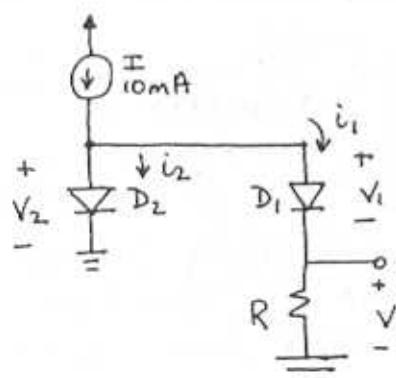
TEMP (°C)	0	50	75	100	-50
V <sub>T</sub> (mV)	23.5	27.9	30	32.2	19.2

AT A CONSTANT CURRENT, THE DIODE VOLTAGE DROP CHANGES WITH TEMPERATURE ACCORDING TO

$$\Delta V = -2 \text{ (mV)} \times \text{TEMPERATURE CHANGE (°C)}$$

THUS:

- (a) 620 mV at  $10 \mu\text{A}$  AND  $0^\circ\text{C}$
- 728 mV at  $1 \text{ mA}$  AND  $0^\circ\text{C}$
- 678 mV at  $1 \text{ mA}$  AND  $25^\circ\text{C}$



Given for each diode  
 $i = I_s e^{\frac{V}{nV_T}} \Rightarrow 10 \times 10^{-3} = I_s e^{0.7/n \times 0.025} \quad \text{①}$   
 $100 \times 10^{-3} = I_s e^{0.8/n \times 0.025} \quad \text{②}$

$$\text{②/①} \quad 10 = e^{0.1/n(0.025)}$$

$$n = 1.737$$

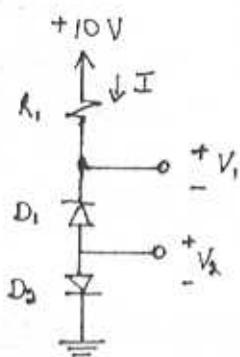
$$V = V_2 - V_1 = nV_T \ln\left(\frac{i_2}{i_1}\right) = 80 \text{ mV}$$

$$1.737 (25 \times 10^{-3}) \ln\left(\frac{0.01 - i_1}{i_1}\right) = 80$$

$$i_1 = 1.4 \text{ mA}$$

$$R = \frac{80}{i_1} = \frac{80}{1.4} = \underline{\underline{57.1 \Omega}}$$

3.28

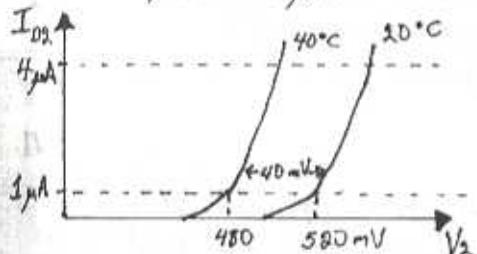


At 20°C :

$$V_{R1} = V_d = 520 \text{ mV}$$

$$R_1 = 520 \text{ k}\Omega$$

$$I = \frac{520 \text{ mV}}{520 \text{ k}\Omega} = 1 \mu\text{A}$$

At 40°C,  $I = 4 \mu\text{A}$ 

$$V_d = 480 + 2.3 \times 1 \times 25 \log 4 \\ = 514.6 \text{ mV}$$

$$V_{R1} = 4 \mu\text{A} \times 520 \text{ k}\Omega = \underline{2.08 \text{ V}}$$

$$\text{At } 0^\circ\text{C, } I = \frac{1}{4} \mu\text{A}$$

$$V_d = 560 - 2.3 \times 1 \times 25 \log 4 \\ = \underline{525.4 \text{ mV}}$$

$$V_{R1} = \frac{1}{4} \times 520 = \underline{0.13 \text{ V}}$$

3.29

The voltage drop =  $700 - 580 = 120 \text{ mV}$   
 Since the diode voltage decreases by approximately 2mV for every  $1^\circ\text{C}$  increase in temperature, the junction temperature must have increased by

$$\frac{120}{2} = \underline{60^\circ\text{C}}$$

Power being dissipated =

$$580 \times 10^{-3} \times 15 = \underline{8.7 \text{ W}}$$

$$\begin{aligned} \text{Thermal Resistance} &= \text{temperature rise}/\text{watt} \\ &= 60/8.7 = \underline{6.9^\circ\text{C/W}} \end{aligned}$$

3.30

$$i = I_s e^{V/nkT}$$

$$10 = I_s e^{0.8/2(0.025)}$$

$$I_s = 1.12 \times 10^{-6} \text{ A}$$

For current varying between  $i_1 = 0.5 \text{ mA}$  to  $i_2 = 1.5 \text{ mA}$ , the voltage varies from

$$V_1 = 2(0.025) \ln \left( \frac{0.5 \times 10^{-3}}{1.12 \times 10^{-6}} \right) = \underline{0.305 \text{ V}}$$

to:

$$V_2 = 2(0.025) \ln \left( \frac{1.5 \times 10^{-3}}{1.12 \times 10^{-6}} \right) = \underline{0.360 \text{ V}}$$

∴ the voltage decreases by approximately 2mV for every  $1^\circ\text{C}$  increase in temperature, the voltage may vary by  $\pm 50 \text{ mV}$  for the  $\pm 25^\circ\text{C}$  temperature variation.

3.31

$$i = I_s e^{V/nV_T}$$

$$\frac{I_{s2}}{I_{s1}} = \frac{1}{0.1 \times 10^{-3}} = 10^4$$

For identical currents

$$I_{s1} e^{V_1/nV_T} = I_{s2} e^{V_2/nV_T}$$

$$e^{\frac{V_1 - V_2}{nV_T}} = 10^4$$

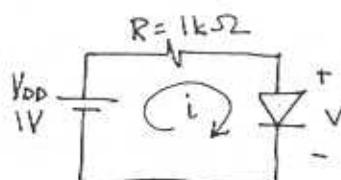
$$V_1 - V_2 = nV_T \ln 10^4$$

$$= 26 \times 10^{-3} \ln 10^4$$

$$= +0.23 \text{ V}$$

I.E. THE VOLTAGE DIFFERENCE BETWEEN THE TWO DIODES IS +0.23 V INDEPENDENT OF THE CURRENT. HOWEVER, SINCE THE TWO CURRENTS CAN VARY BY A FACTOR OF 3 (0.5 mA TO 1.5 mA) THE DIFFERENCE VOLTAGE WILL BE:  
 $0.23 \text{ V} \pm nV_T \ln 3 = 0.23 \text{ V} \pm 2.75 \text{ mV}$   
 SINCE TEMPERATURE CHANGE AFFECTS BOTH DIODES SIMILARLY THE DIFFERENCE VOLTAGE REMAINS CONSTANT.

3.32



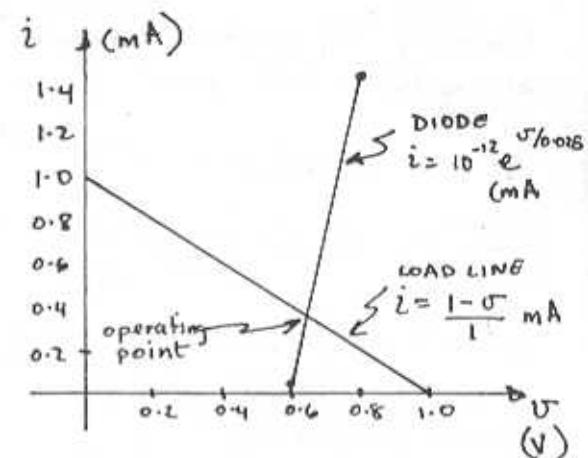
$$i = 10^{-5} e^{V/10^{-5}}$$

where  $n=1$

$$V = 0.7 \text{ V} \quad i = 1.45 \text{ mA}$$

$$V = 0.6 \text{ V} \quad i = 0.026 \text{ mA}$$

A sketch of the graphical construction to determine the operating point is shown below.



From the above sketch we see that the operating point must lie between  $V = 0.6$  and  $0.7 \text{ V}$  and  $i \times 0.3$  to  $0.4 \text{ mA}$ . To find the point more accurately an enlarged graph is plotted.

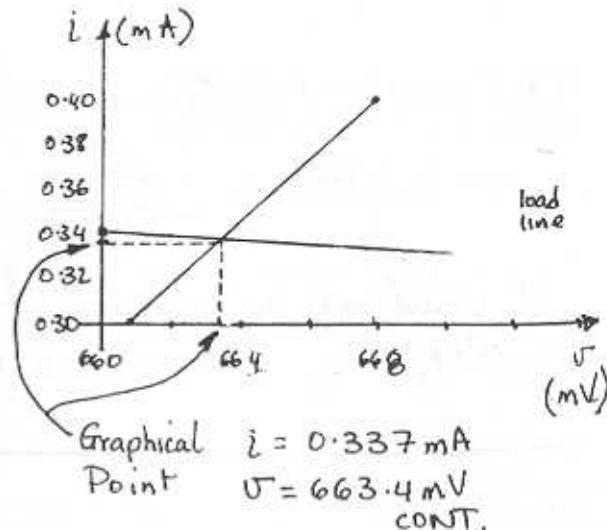
$$\text{For } i = 0.3 \text{ mA} = 10^{-12} e^{V/0.025} \\ \Rightarrow V = 660.7 \text{ mV}$$

$$\text{For } i = 0.4 \text{ mA} = 10^{-12} e^{V/0.025} \\ \Rightarrow V = 667.9 \text{ mV}$$

For the load line:

$$V = 660 \text{ mV} \Rightarrow i = 0.34 \text{ mA}$$

$$V = 670 \text{ mV} \Rightarrow i = 0.33 \text{ mA}$$



Comparing the graphical results to the exponential model gives:

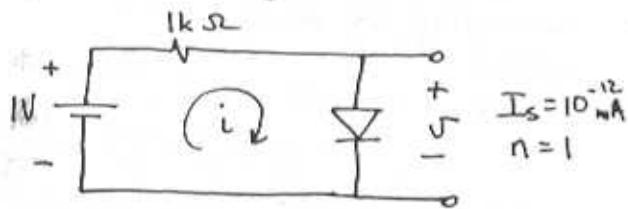
$$\text{At } i = 0.337 \text{ mA} = 10^{-12} e^{\frac{V}{0.025}}$$

$$\Rightarrow V = 663.6 \text{ mV}$$

which is only  $(663.6 - 663.4) = \underline{0.2 \text{ mV}}$   
greater than the value  
found graphically!

### 3.33

Iterative Analysis:



$$\#1 \quad V = 0.7 \text{ V} \quad i = \frac{1 - 0.7}{1} = 0.3 \text{ mA}$$

$$\#2 \quad V = 0.25 \ln\left(\frac{0.3}{10^{-12}}\right) = 0.6607 \text{ V}$$

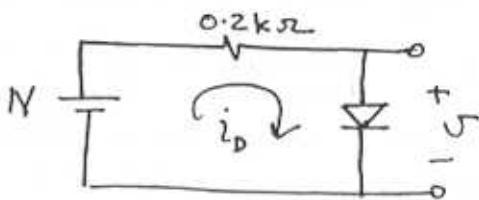
$$i = \frac{1 - 0.6607}{1} = 0.3393 \text{ mA}$$

$$\#3 \quad V = 0.25 \ln\left(\frac{0.3393}{10^{-12}}\right) = \underline{0.6638 \text{ V}}$$

$$i = \frac{1 - 0.6638}{1} = \underline{0.3362}$$

∴ i did not change by much  
stop here.

### 3.34



$$(a) \quad i_D = \frac{1 - 0.7}{0.2} = \underline{1.5 \text{ mA}}$$

(b) Iterative Analysis given  $V_D = 0.7 \text{ V}$   
at  $i_D = 1 \text{ mA}$

$$\#1 \quad V = 0.7 \text{ V} \quad i_D = \frac{1 - 0.7}{0.2} = 1.5 \text{ mA}$$

$$\#2 \quad \text{so } i = I_s e^{\frac{V}{nV_T}} \quad n=2 \\ \frac{i_2}{i_1} = e^{\frac{V_2 - V_1}{0.05}}$$

$$\text{thus } V_2 = V_1 + 0.05 \ln \frac{i_2}{i_1}$$

$$\text{so for } i = 1.5 \text{ mA}$$

$$V = 0.7 + 0.05 \ln \frac{1.5}{1} \quad \frac{1}{1} \quad i_D = \frac{1 - 0.720}{0.2} \\ = 0.720 \text{ V} \quad = 1.4 \text{ mA}$$

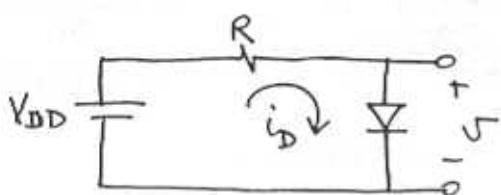
### #3

$$V = 0.720 + 0.05 \ln\left(\frac{1.4}{1.5}\right) \quad \frac{1}{1} \quad i_D = \frac{1 - 0.716}{0.2} \\ = 0.716 \text{ V} \quad = 1.42 \text{ mA}$$

### #4

$$V = 0.716 + 0.05 \ln\left(\frac{1.42}{1.4}\right) \quad \frac{1}{1} \quad i_D = \underline{1.42 \text{ mA}} \\ = 0.716 \text{ V}$$

3.35



Derivation of iterative equation

$$i_D = I_s e^{\frac{V}{V_T}}$$

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{V_2 - V_1}{nV_T}}$$

$$\begin{aligned} V_2 &= V_1 + nV_T \ln\left(\frac{i_{D2}}{i_{D1}}\right) \\ &= V_1 + \Delta V \log\left(\frac{i_{D2}}{i_{D1}}\right) \end{aligned}$$

$$(a) \quad V = 0.7 \text{ V} \quad i_D = \frac{10 - 0.7}{9.3} = 1 \text{ mA}$$

$$(b) \quad V = 0.7 \text{ V} \quad i_D = \frac{3 - 0.7}{2.3} = 1 \text{ mA}$$

~ for both these cases the diode is rated at 1mA for 0.7V so stop.

(c)  $V_{DD} = 2 \text{ V} \quad R = 2k\Omega$ 

$$\#1 \quad V = 0.7 \text{ V} \quad i_D = \frac{2 - 0.7}{2} = 0.65 \text{ mA}$$

$$\#2 \quad V = 0.7 + 0.1 \log\left(\frac{0.650}{i_D}\right) = 0.581 \text{ V}$$

$$i_D = \frac{2 - 0.581}{2} = 0.709 \text{ mA}$$

$$\#3 \quad V = 0.581 + 0.1 \log\left(\frac{0.709}{0.650}\right)$$

$$= \underline{0.584 \text{ V}}$$

$$i_D = \frac{2 - 0.584}{2} = \underline{0.708 \text{ mA}}$$

$$\begin{aligned} (d) \quad V_{DD} &= 2 \text{ V} \quad R = 2k\Omega \\ \#1 \quad V &= 0.7 \text{ V} \quad i_D = \frac{2 - 0.7}{2} \\ &= 0.650 \text{ mA} \end{aligned}$$

#2.

$$\begin{aligned} V &= 0.7 + 0.1 \log\left(\frac{0.650}{1}\right) \quad i_D = \frac{2 - 0.681}{2} \\ &= 0.681 \text{ V} \quad = 0.659 \text{ mA} \end{aligned}$$

#3

$$\begin{aligned} V &= 0.681 + 0.1 \log\left(\frac{0.659}{0.650}\right) \quad i_D = \frac{2 - 0.682}{2} \\ &= \underline{0.682 \text{ V}} \quad = \underline{0.659 \text{ mA}} \end{aligned}$$

(e)  $V_{DD} = 1 \text{ V} \quad R = 0.3k\Omega$ 

$$\#1 \quad V = 0.7 \text{ V} \quad i_D = \frac{1 - 0.7}{0.3} = 1 \text{ mA}$$

#2

$$\begin{aligned} V &= 0.7 + 0.1 \log \frac{1}{10} \quad i_D = \frac{1 - 0.6}{0.3} = 1.333 \text{ mA} \\ &= 0.6 \text{ V} \end{aligned}$$

#3

$$\begin{aligned} V &= 0.6 + 0.1 \log \frac{1.333}{1} \quad i_D = \frac{1 - 0.612}{0.3} = 1.293 \text{ mA} \\ &= 0.612 \text{ V} \end{aligned}$$

#4

$$\begin{aligned} V &= 0.612 + 0.1 \log \frac{1.293}{1.333} \quad i_D = \frac{1 - 0.611}{0.3} = 1.297 \text{ mA} \\ &= 0.611 \text{ V} \end{aligned}$$

#5

$$\begin{aligned} V &= 0.611 + 0.1 \log \frac{1.297}{1.293} \quad i_D = \underline{1.297 \text{ mA}} \\ &= \underline{0.611 \text{ V}} \end{aligned}$$

(f)  $V_{DD} = 1 \text{ V} \quad R = 0.3k\Omega$ 

$$\#1 \quad V = 0.7 \text{ V} \quad i_D = \frac{1 - 0.7}{0.3} = 1 \text{ mA}$$

CONT.

$$\#2 \quad i = 0.7 + 0.06 \log \frac{1}{10} \quad i_D = \frac{1 - 0.640}{0.3} \\ = 0.640 V \quad = 1.2 MA$$

$$\#2$$

$$V = 0.7 + 0.1 \log \frac{3.333 \times 10^{-3}}{10} \quad i_D = \frac{0.5 - 0.352}{30}$$

$$= 0.352 \text{ V} \quad = 4.933 \mu\text{A}$$

$$\#3 \\ V = +0.06 \log \frac{I_2}{I} \quad i_D = \frac{I - 0.645}{0.3} \\ = 0.645V \quad = 1.183mA$$

$$I_p = \frac{0.5 - 0.369}{30} = 4.367 \mu A$$

$$V = +0.06 \log \frac{1.183}{1.2} \\ = 0.645V \quad i_D = 1.183mA$$

$$\begin{aligned} \text{#4} \\ V &= 0.369 + 0.1 \log \frac{4.367}{4.933} & i_D &= \frac{0.5 - 0.364}{30} \\ &= 0.364V & &= 4.533 \mu A \end{aligned}$$

$$\#1 \quad V = 0.7 \quad i_D = \frac{1-0.7}{0.3} = 1 \text{ mA}$$

$$\text{#5} \quad U = 0.364 + 0.1 \log \left( \frac{4.533}{4.367} \right) \quad C_D = \frac{0.5 - 0.364}{30}$$

$$= 0.366V \quad = 4.467\mu A$$

$$U = 0.7 + 0.12 \log \frac{I}{10} \quad i_D = \frac{1 - 0.580}{0.3} \\ = 0.580V \quad = 1.381mA$$

$$\#6 \quad U = 0.366 + 0.1 \log \frac{4.467}{4.533} \quad i_D = \frac{0.5 - 0.363}{30} \\ = 0.365V \quad = 4.5mA$$

$$\#3 \quad V = 0.680 + 0.12 \log \frac{1.381}{1} \quad I_D = \frac{1 - 0.697}{0.3} \\ = 0.697V \quad = 1.343mA$$

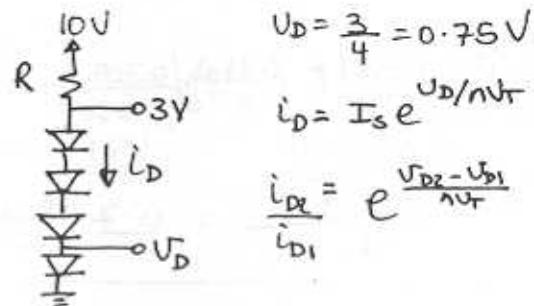
$$U = 0.365 + 0.1 \log \frac{4.5}{4.467}$$

$$\begin{aligned} \#5 \\ U &= 0.596 + 0.121 \log \frac{1.347}{1.343} \\ &= \underline{0.596 \text{ V}} \quad I_D = \underline{1.347 \text{ mA}} \end{aligned}$$

3.36

(h)  $V_{DP} = 0.5V$        $R = 30k\Omega$

$$\#1 \text{ let } V = 0.4V \quad i_D = \frac{0.5 - 0.4}{30} \\ = 3.333 \mu A$$



CONT.

$$\therefore i_D = i_{D2} = i_{D1} e^{\frac{U_{D2}-U_{D1}}{nV_T}}$$

$$= 1 \times e^{\frac{0.76-0.7}{1 \times 0.025}}$$

$$= 7.389 \text{ mA}$$

$$\therefore R = \frac{10-3}{i_D} = \frac{10-3}{7.389} = \underline{0.947 \text{ k}\Omega}$$

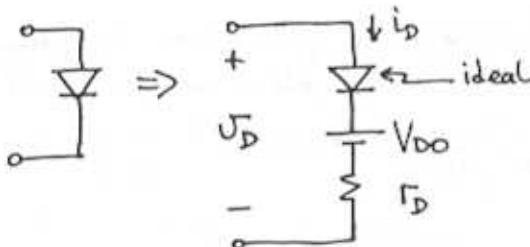
$$i_D = \frac{0.815 - V_{DD}}{R_D} = 10 \text{ mA} \quad \textcircled{2}$$

$$\textcircled{2}/\textcircled{1} \Rightarrow \frac{0.815 - V_{DD}}{0.7 - V_{DD}} = 10$$

$$V_{DD} = \frac{0.687 \text{ V}}{R_D} = \frac{0.7 - V_{DD}}{1} = \underline{12.8 \text{ }\Omega}$$

3.37

Piecewise linear model:



Given  $n=2$ ,  $V_D=0.7 \text{ V}$   $i_D=1 \text{ mA}$

The current through the diode is given by:

$$i_D = \frac{V_D - V_{DD}}{R_D} \quad \text{need to find the parameters } V_{DD} \text{ and } R_D$$

Using the exponential model to find the diode voltage at 10mA

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{U_{D2}-U_{D1}}{nV_T}}$$

$$U_{D2} = nV_T \ln \left( \frac{i_{D2}}{i_{D1}} \right) = 0.05 \ln \left( \frac{10}{1} \right)$$

$$= 0.815 \text{ V}$$

FINDING  $V_{DD}$  &  $R_D$  using the given facts:

$$i_D = \frac{0.7 - V_{DD}}{R_D} = 1 \text{ mA} \quad \textcircled{1}$$

Using the piecewise linear model

$$i_D = \frac{V_D - 0.687}{12.8} \Rightarrow V_D = 0.687 + 12.8 i_D$$

Using the exponential model

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{V_D - 0.7}{0.05}} \Rightarrow V_D = 0.7 + 0.05 \ln \left( \frac{i_{D2}}{i_{D1}} \right)$$

$i_D$ (mA)	PIECEWISE LINEAR $U_D$ (V)	EXPONENTIAL $V_D$ (V)	ERROR (mV)
0.5	0.693	0.655	28
5	0.751	0.780	-29.5
14	0.866	0.832	34

3.38

Looking at the copy of Fig 3.12 below, we see at

$$i_D = 1 \text{ mA} \rightarrow U_D = 0.7 \text{ V}$$

$$i_D = 10 \text{ mA} \rightarrow U_D = 0.8 \text{ V}$$

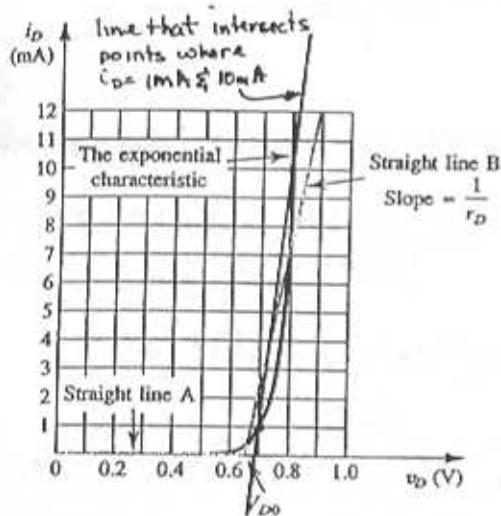
$$\therefore \text{slope} = \frac{1}{R_D} = \frac{0.8 - 0.7}{0.8 - 0.7} = 90 \frac{\text{mA}}{\text{V}}$$

$$\therefore R_D = \frac{1}{90 \times 10^{-3}} = \underline{11.1 \Omega}$$

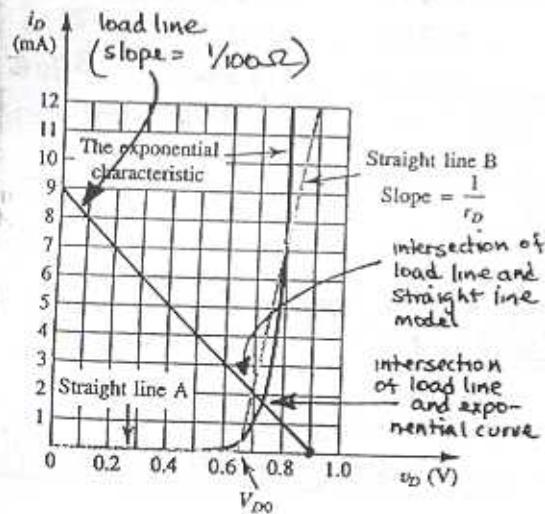
To find  $V_{DD}$ :

$$i_D = \frac{U_D - V_{DD}}{r_D}$$

$$10^{-3} = \frac{0.7 - V_{DD}}{11.1} \Rightarrow V_{DD} = 0.689V$$



3.39



(a) The load line intersects the exponential model at:

$$U_D = 0.7V \quad i_D = 1.7mA$$

(b) The load line intersects the straight-line model at

$$U_D = 0.7V \quad i_D = 2mA$$

3.40

Calculating the parameters  $r_D$  &  $V_{DD}$  for the battery plus resistor model

$$i_D = I_s e^{\frac{V_D}{nV_T}} \quad n=1$$

For  $i_D = 0.1 I_s$

$$U_{D2} = 0.7 + 0.025 \ln(0.1) = 0.642V$$

For  $i_D = 10 I_s$

$$U_{D3} = 0.7 + 0.025 \ln(10) = 0.758V$$

Note that since the specifications for all of the diodes are given for 0.7V, the end voltages are the same as the voltage change for relative currents are independent to  $I_D$  &  $I_s$ .

$$\therefore U_{D2} = V_{DD} + i_{D2} r_D \quad ①$$

$$U_{D3} = V_{DD} + i_{D3} r_D \quad ②$$

② → ①

$$U_{D3} - U_{D2} = (i_{D3} - i_{D2}) r_D$$

CONT.

$$0.758 - 0.642 = (0.1I_D - 0.1I_D) R_D$$

$$0.166 = 0.1 I_D R_D$$

$$R_D = \frac{0.0117}{I_D} \quad (3)$$

for (a)  $I_D = 1\text{mA}$        $R_D = \frac{0.0117}{1} = 11.7\Omega$

(b)  $I_D = 1\text{A}$        $R_D = \frac{0.0117}{1} = 0.0117\Omega$

(c)  $I_D = 10\mu\text{A}$        $R_D = \frac{0.0117}{10\mu\text{A}} = 1.17\text{k}\Omega$

(3)  $\rightarrow$  (1)

$$0.642 = V_{D0} + 0.1 I_D \times \frac{0.0117}{I_D}$$

$$= V_{D0} + 0.00117$$

$$V_{D0} = \underline{0.641\text{V}} \quad \leftarrow \text{same for all diodes}$$

3.41

Since for a current of  $10\text{mA}$ , the diode voltage is  $0.8\text{V}$ , this would be a suitable choice for the constant-voltage-drop model.

3.42

Constant Voltage drop Model:

$$\text{Using } V_D = 0.7\text{V} \Rightarrow I_{D1} = \frac{V - 0.7}{R}$$

$$\text{Using } V_D = 0.6\text{V} \Rightarrow I_{D2} = \frac{V - 0.6}{R}$$

For the difference in currents to vary by only  $1\%$   $\Rightarrow$

$$\frac{I_{D2}}{I_{D1}} = 1.01 \quad I_{D1} = 1.01 (V - 0.7)$$

$$V = \underline{10\text{V}}$$

$$\text{For } V = 2\text{V} \quad \frac{1}{R} = 1\text{k}\Omega$$

$$\text{At } V_0 = 0.7\text{V} \quad I_{D1} = \frac{2 - 0.7}{1} = 1.3\text{mA}$$

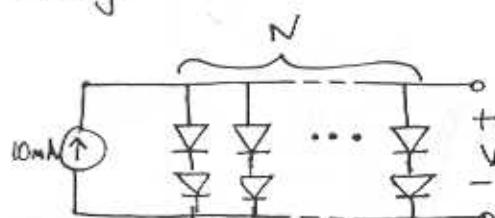
$$V_D = 0.6\text{V} \quad I_{D2} = \frac{2 - 0.6}{1} = 1.4\text{mA}$$

$$\frac{I_{D2}}{I_{D1}} = \frac{1.4}{1.3} = 1.08$$

Thus the percentage difference is  
8%

3.43

Since  $2V_D = 1.4\text{V}$  is close to the required  $1.25\text{V}$ , use  $N$  parallel pairs of diodes to split the current evenly.



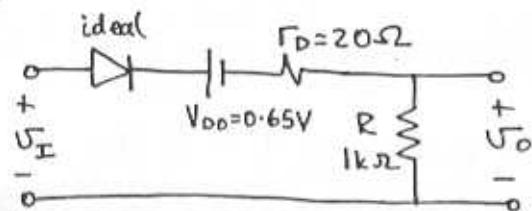
$$V = 2 \left[ 0.7 + 0.1 \log \frac{10/N}{20} \right] = 1.25\text{V}$$

$$N = 2.8 \Rightarrow \text{Use } \underline{3 \text{ sets of diodes}}$$

$$V = 2 \left( 0.7 + 0.1 \log \frac{10/3}{20} \right) = \underline{1.244\text{V}}$$

3.44

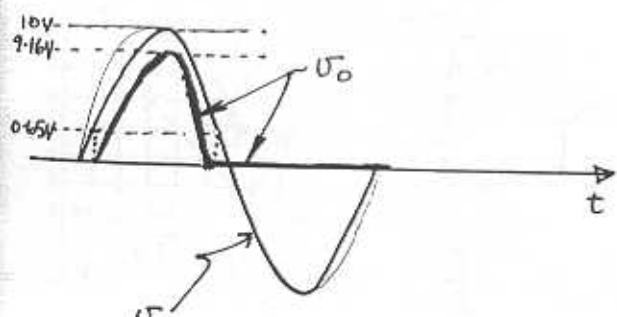
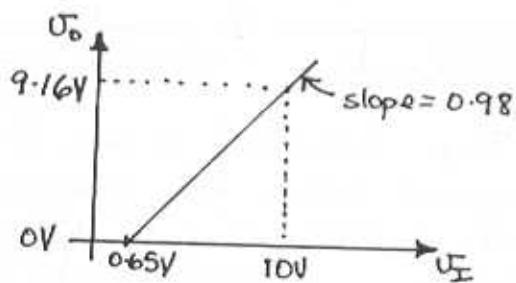
Piecewise linear model in a half-wave rectifier.



$$U_O = \frac{U_I - V_{D0}}{R + r_D} R, \text{ for } U_I \geq 0.65V$$

$$U_O = 0 \quad \text{for } U_I < 0.65V$$

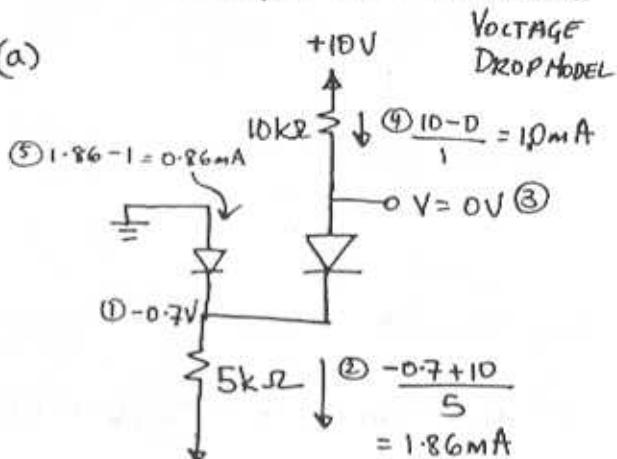
Sketch of transfer characteristic :-



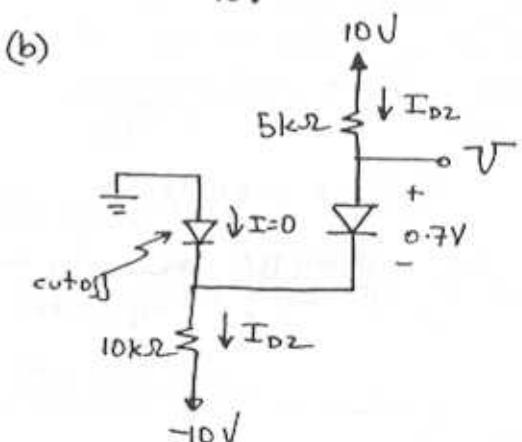
3.45

Refer to example 3-2 ~ CONSTANT VOLTAGE DROP MODEL

(a)



(b)

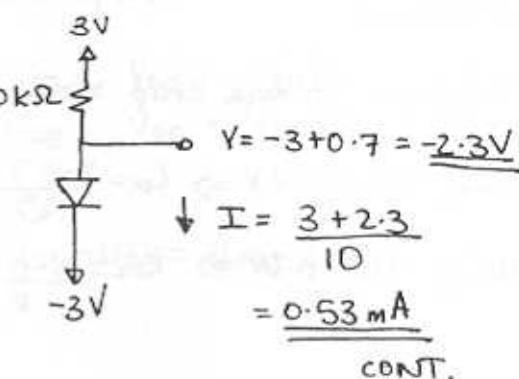


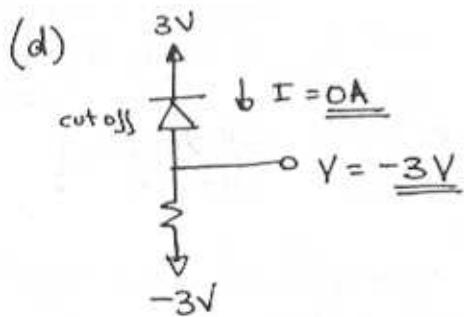
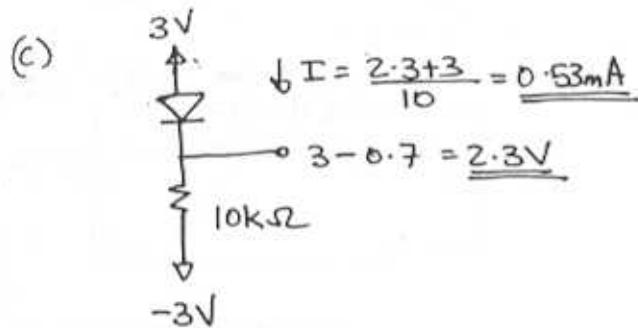
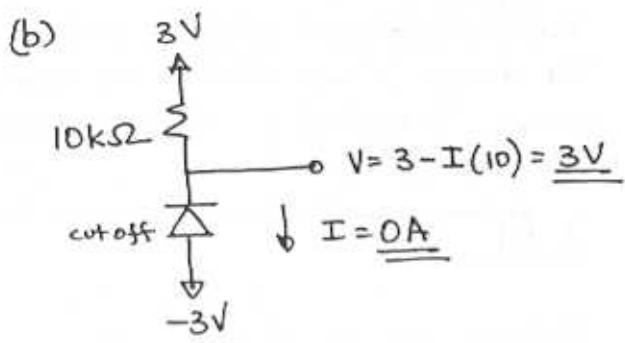
$$I_{D2} = \frac{10 - (-10) - 0.7}{15} = 1.29mA$$

$$U_O = -10 + 1.29(10) + 0.7 = \underline{\underline{3.6V}}$$

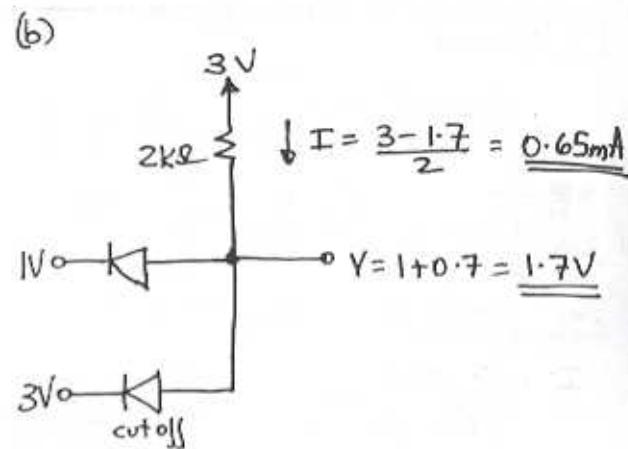
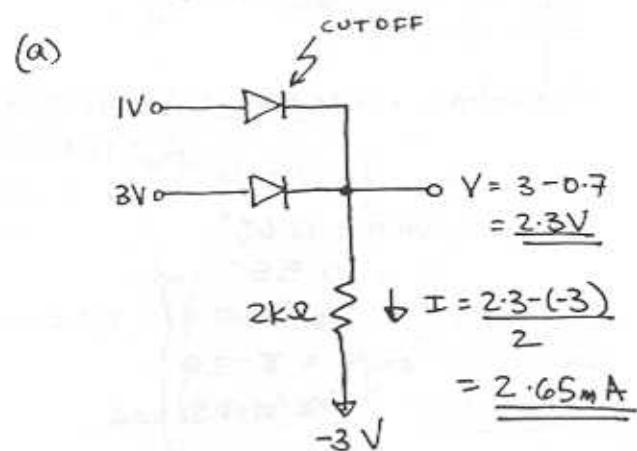
3.46

(a)

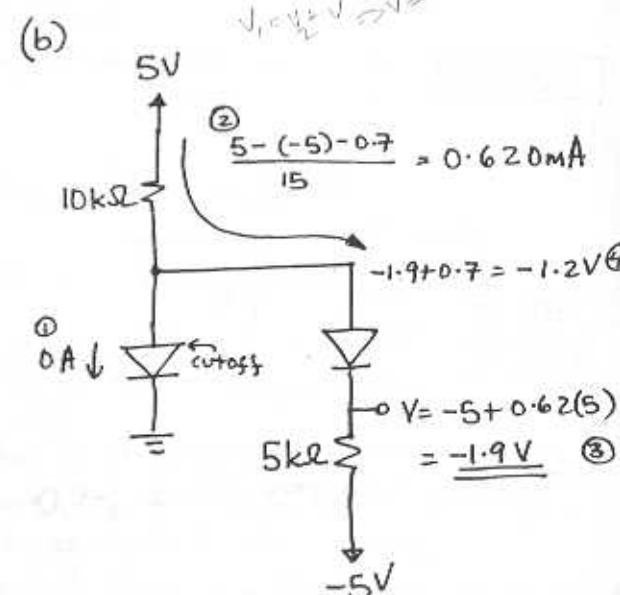
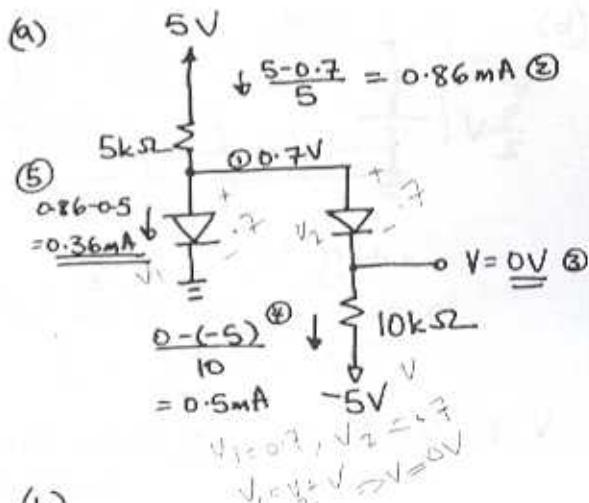




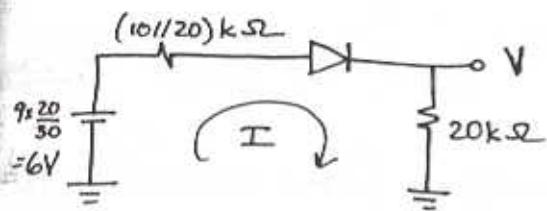
3.47



3.48



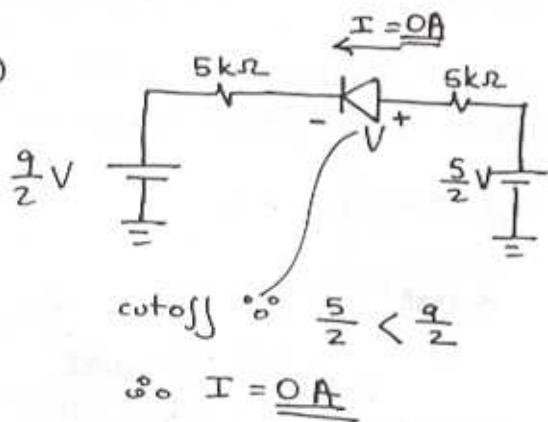
3.49



$$I = \frac{6 - 0.7}{(10/120) + 20} = 0.199 \text{ mA}$$

$$V = 20I = 3.98 \text{ V}$$

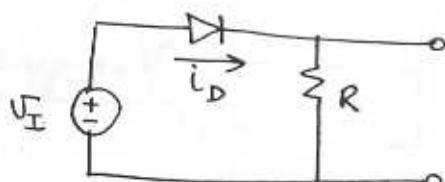
(b)



$$\therefore I = 0 \text{ A}$$

$$V = 9/2 - 9/2 = -2 \text{ V}$$

3.50



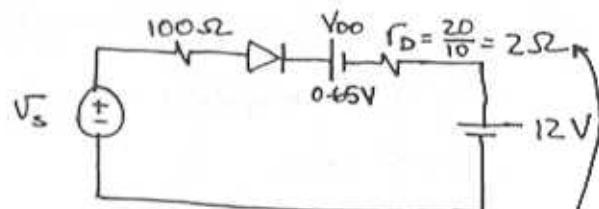
$$i_{D, \text{peak}} = \frac{Vi, \text{peak} - 0.7}{R} \ll 50$$

$$R \geq \frac{120\sqrt{2} - 0.7}{50} = 3.38 \text{ k}\Omega$$

Reverse voltage =  $120\sqrt{2} = 169.7 \text{ V}$ .  
The design is essentially the same  
since the supply voltage  $\gg 0.7 \text{ V}$

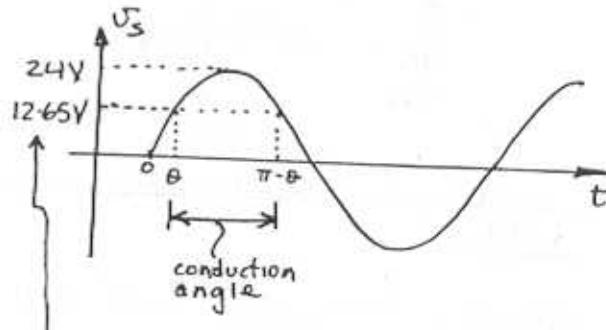
3.51

Battery plus resistance model



since the diode has 10x the area  $r_D$  is  $1/10$  as big.

$$i_D = I_s e^{\frac{V}{nV_F}} = \frac{V_s - V_{DD} - 12}{100 + 2}$$



Conduction starts when  $V_s > 12 + V_{DD}$   
 $V_s > 12.65 \text{ V}$

$$\therefore 24 \sin \theta = 12.65 \text{ V}$$

$$\theta = 0.555 \text{ rad}$$

$$\text{Conduction angle} = \pi - 2\theta$$

$$= 2.031 \text{ rad.}$$

$$\text{Fraction of cycle for conduction} = \frac{2.031}{2\pi} = 0.323$$

CONT.

$$i_{D, \text{peak}} = \frac{24 - 12.65}{100 + 2} = \underline{\underline{0.111 \text{ A}}}$$

Maximum reverse voltage occurs across the diode when  $V_S$  is at its negative peak and is equal to:

$$24 + 12 = \underline{\underline{36 \text{ V}}}$$

3.52

Using the exponential model

$$i_D = I_S e^{\frac{DV}{nV_T}}$$

FOR A +10mV CHANGE

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{0.01}{n(0.025)}} = e^{0.01/n}$$

$$= \begin{cases} 1.492 & \sim n=1 \\ 1.221 & \sim n=2 \end{cases}$$

$$\% \text{ CHANGE} = \frac{i_{D2} - i_{D1}}{i_{D1}} \times 100$$

$$= \begin{cases} (1.492 - 1) \times 100 = +49.2\% & n=1 \\ (1.221 - 1) \times 100 = 22.1\% & n=2 \end{cases}$$

FOR A -10mV CHANGE

$$\frac{i_{D2}}{i_{D1}} = 10^{-0.01/n(0.025)} = \begin{cases} 0.670 & n=1 \\ 0.819 & n=2 \end{cases}$$

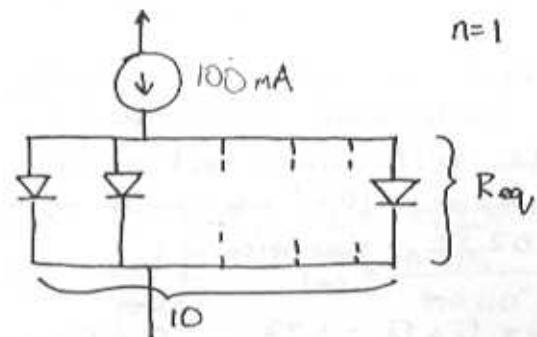
$$\% \text{ CHANGE} = \begin{cases} (0.670 - 1) \times 100 = -33\% & n=1 \\ (0.819 - 1) \times 100 = -18\% & n=2 \end{cases}$$

For a current change limited to  $\pm 10\%$

$$\frac{i_{D2}}{i_{D1}} = e^{\frac{DV}{n \times 0.025}} = 0.9 \text{ to } 1.1$$

$$DV = \begin{cases} -2.634 \text{ mV to } 2.383 \text{ mV} & n=1 \\ -5.268 \text{ mV to } 4.766 \text{ mV} & n=2 \end{cases}$$

3.53



Each diode has the current

$$i_D = \frac{0.1}{10} = 0.01 \text{ A}$$

Each diode has a small-signal resistance

$$r_d = \frac{nV_T}{I_D} = \frac{0.025}{0.01} = \underline{\underline{2.5 \Omega}}$$

$$R_{eq} = r_d / 10 = \underline{\underline{0.25 \Omega}}$$

For one diode conducting 0.1 A

$$r_d = nV_T / 0.1 = \frac{0.025}{0.1} = \underline{\underline{0.25 \Omega}}$$

This is the same as  $R_{eq}$ . We can think of the parallel connection as equivalent to a single diode having 10 times the junction area of each diode. This large diode

is fed with  $10\times$  the current (or  $0.1A$ ) and this exhibits the same small-signal resistance as 10 parallel smaller diodes.

Now consider the series resistance of  $0.2\Omega$  to connect a diode. For the parallel combination above:

$$R_{eq} = \frac{1}{10} (0.2 + 2.5) = 0.27\Omega$$

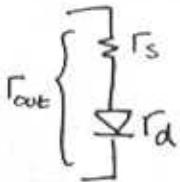
To have an equivalent resistance, the single diode conducting all of the  $0.1A$  would need a series resistance  $10\times$  as small or  $0.025\Omega$ . Specifically:

$$r_{out} = r_s + r_d = 0.27$$

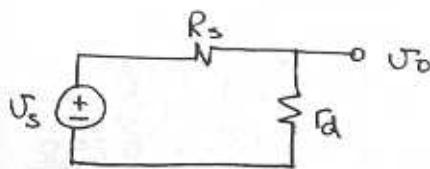
$$= r_s + \frac{nV_T}{I_D} = 0.27$$

$$= r_s + 0.25 = 0.27$$

$$r_s = 0.27 - 0.25 = \underline{\underline{0.025\Omega}}$$



3.54



SMALL SIGNAL EQUIVALENT CIRCUIT

To find the small-signal response,  $v_o$ , open the dc current source  $I$ , and short the capacitors  $C_1$  and  $C_2$ . Also replace the diode with its small signal resistance:

$$r_d = \frac{nV_T}{I} \quad n=2$$

Now:

$$v_o = v_s \frac{r_d}{r_d + r_s}$$

$$= v_s \frac{nV_T/I}{nV_T/I + R_s} = v_s \frac{nV_T}{nV_T + IR_s}$$

Q.E.D.

$$v_o = 10mV \frac{0.05}{0.05 + 10^3 I}$$

$$= \begin{cases} 0.476 \text{ mV} & \sim I = 1mA \\ 3.333 \text{ mV} & \sim I = 0.1mA \\ 9.804 \text{ mV} & \sim I = 1\mu A \end{cases}$$

$$\text{For } v_o = \frac{1}{2} v_s = v_s \times \frac{0.05}{0.05 + 10^3 I}$$

$$I = \underline{\underline{50\mu A}}$$

3.55

$r_s = 10k\Omega$ ,  $n=1$ , a  $1mA$  diode

$$v_o/v_i = \frac{0.025}{0.025 + r_s I}$$

$$= \frac{0.025}{0.025 + 10^4 I} \quad ①$$

For the current change limited to  $\pm 10\%$  of  $I$  & using the exponential model we get

$$\frac{i_{D2}}{i_{D1}} = e^{\Delta V/nV_T} = 0.9 \text{ to } 1.1$$

CONT.

$$\Delta U = -2.63 \text{ mV} \text{ to } +2.38 \text{ mV}$$

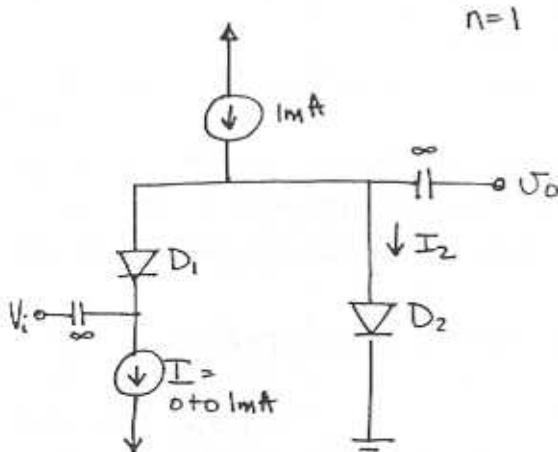
This is the amount the output will vary for a 10% change in diode current. Divide this by the specific gains given in the problem to find the limit on the input signal.

$$\Delta U_s = \frac{\Delta U_o}{U_o/U_I}$$

$$= -2.63 \text{ mV} \text{ to } +2.38 \text{ mV} \quad (2)$$

$U_o/U_I$	$I_{\text{use}} \quad (1) \quad (\mu\text{A})$	$U_s \text{ (using } 2\text{)} \quad (\text{mV})$
0.5	0.0025	5.26 to 4.76
0.1	0.0225	26.3 to 23.8
0.01	0.25	263 to 238
0.001	2.5	2630 to 2380

3.54



small signal model when  $D_1$  &  $D_2$  are conducting



$$(a) I = 0 \mu\text{A}$$

$$\text{Di-cutoff} \Rightarrow \frac{U_o}{U_I} = 0 \text{ V/V}$$

$$I_2 = 1 \text{ mA}$$

$$(b) I = 1 \mu\text{A} \quad I_2 = 999 \mu\text{A}$$

$$r_{d1} = \frac{nV_t}{I} \quad r_{d2} = \frac{0.025}{999 \times 10^{-6}}$$

$$= \frac{0.025}{I} \quad = 25.025 \Omega$$

$$= 25 \Omega$$

$$\frac{U_o}{U_I} = \frac{r_{d2}}{r_{d1} + r_{d2}} = \frac{0.001 \text{ V}}{\text{V}}$$

$$(c) I = 10 \mu\text{A} \quad I_2 = 990 \mu\text{A}$$

$$r_{d1} = \frac{0.025}{10 \times 10^{-6}} \quad r_{d2} = \frac{0.025}{990 \times 10^{-6}}$$

$$= 2.5 \text{ k}\Omega \quad = 25.25 \Omega$$

$$\frac{U_o}{U_I} = 0.01 \text{ V/V}$$

$$(d) I = 100 \mu\text{A} \quad I_2 = 900 \mu\text{A}$$

$$r_{d1} = \frac{0.025}{100 \times 10^{-6}} \quad r_{d2} = \frac{0.025}{900 \times 10^{-6}}$$

$$= 250 \Omega \quad = 27.78 \Omega$$

$$\frac{U_o}{U_I} = 0.1 \text{ V/V}$$

$$(e) I = 500 \mu\text{A} \quad I_2 = 500 \mu\text{A}$$

$$r_{d1} = r_{d2} = \frac{0.025}{500 \times 10^{-6}} = 50 \Omega$$

$$\frac{U_o}{U_I} = \frac{1}{2} \text{ V/V}$$

$$(f) I = 600 \mu\text{A} \quad I_2 = 400 \mu\text{A}$$

$$r_{d1} = \frac{0.025}{600 \times 10^{-6}} \quad r_{d2} = \frac{0.025}{400 \times 10^{-6}}$$

$$= 41.67 \Omega \quad = 62.5 \Omega$$

56 CONT.

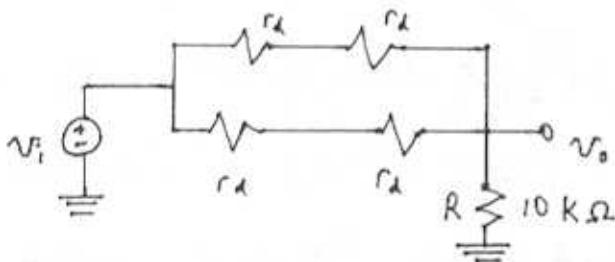
EN THE BIAS CURRENT IN EACH  
DE IS  $3.10 \mu\text{A}$ , THE DIODE RESISTANCE  
BE  $\leq 2.5 \text{k}\Omega$ . TO LIMIT THE  
CURRENT SIGNAL TO A MAXIMUM OF 10%  
BIAS, THE CURRENT SIGNAL MUST BE  
 $0.10 \mu\text{A}$ . THUS, THE SIGNAL VOLTAGE ACROSS  
"STARVED" DIODE WILL BE  
mV WHICH IS APPROXIMATELY THE  
VOLTAGE TO WHICH THE INPUT SIGNAL  
SHOULD BE LIMITED.

3.57

$$(a) \frac{V_o}{V_i} = \frac{R}{R + (2r_d // 2r_d)}$$

$$= \frac{R}{R + r_d}$$

WHERE  $r_d = \frac{V_T}{I/2} = \frac{2V_T}{I}$   
 $= \frac{0.05V}{I}$



I (mA)	$V_o/V_i$ (V/V)
0	0
$10^{-3}$	0.167
0.01	0.667
0.1	0.952
1.0	0.995
10	0.9995

(b) IF THE SIGNAL CURRENT IS TO BE LIMITED TO  $\pm 10I$ , THE CHANGE IN DIODE VOLTAGE  $\Delta V_D$  CAN BE FOUND FROM

$$\frac{i_D}{I} = e^{\Delta V_D / nV_T} = 0.9 \text{ TO } 1.1$$

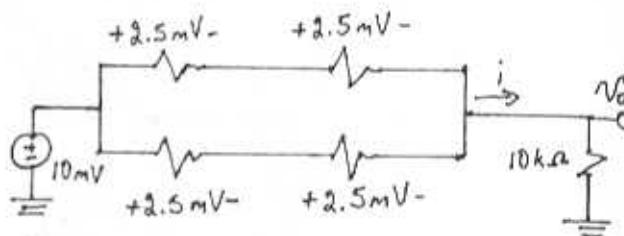
THUS, FOR  $n=1$

$$\Delta V_D = -2.63 \text{ mV TO } +2.38 \text{ mV}$$

OR APPROXIMATELY  $\pm 2.5 \text{ mV}$

3.57 cont.

(b cont.) FOR THE DIODE CURRENT TO REMAIN WITHIN  $\pm 10\%$  OF THEIR DC BIAS CURRENTS, THE SIGNAL VOLTAGE ACROSS EACH DIODE MUST BE LIMITED TO  $2.5 \text{ mV}$ . NOW, IF  $V_{i\text{PEAK}} = 10 \text{ mV}$  WE CAN OBTAIN THE FOLLOWING SITUATION



WE SEE THAT  $V_o = 5 \text{ mV}$  AND  $i = \frac{5 \text{ mV}}{10 \text{ k}\Omega} = 0.5 \mu\text{A}$ .

THUS, EACH DIODE IS CARRYING A CURRENT SIGNAL OF  $0.25 \text{ mA}$ . FOR THIS TO BE AT MOST  $10\%$  OF THE DC CURRENT, THE DC CURRENT IN EACH DIODE MUST BE AT LEAST  $2.5 \mu\text{A}$ . IT FOLLOWS THAT THE MINIMUM VALUE OF I MUST BE  $5 \mu\text{A}$ .

(c) FOR  $I = 1 \text{ mA}$ ,  $I_D = 0.5 \text{ mA}$ , AND FOR MAXIMUM SIGNAL OF  $10\%$ ,  $I_D = 0.05 \text{ mA}$ . THUS  $i_D = 2i_d = 0.1 \text{ mA}$  AND THE CORRESPONDING MAXIMUM  $V_o$  IS  $0.1 \text{ mA} \times 10 \text{ k}\Omega = 1 \text{ V}$ .

THE CORRESPONDING PEAK INPUT CAN BE FOUND BY DIVIDING  $V_o$  BY THE TRANSMISSION FACTOR OF 0.995, THUS

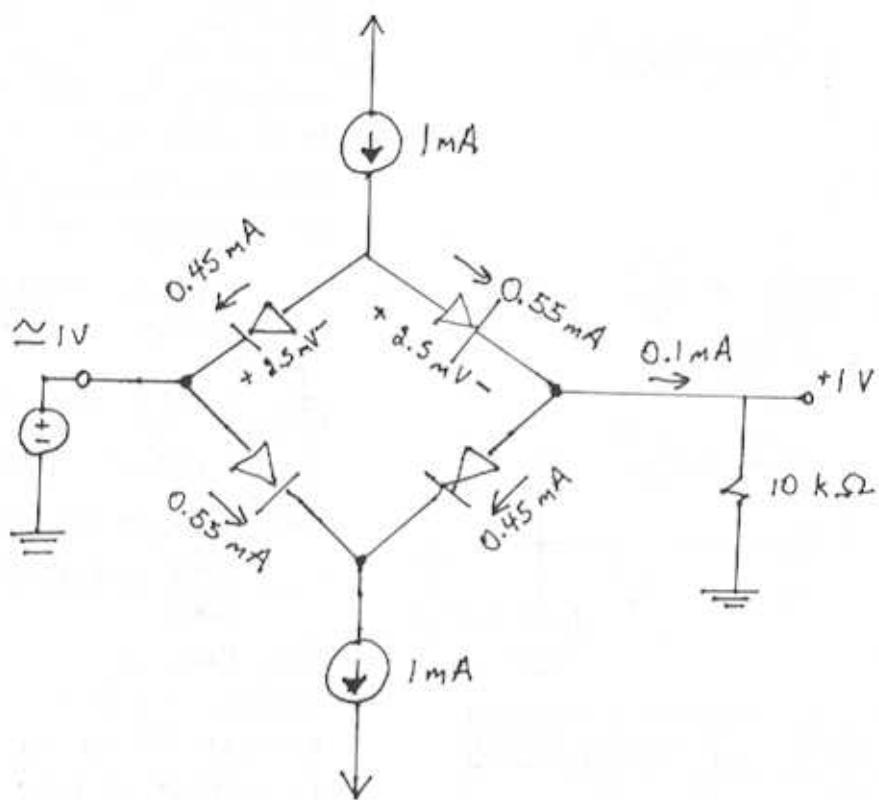
$$V_{i\text{MAX}} = \frac{1 \text{ V}}{0.995 \text{ V}} = \underline{\underline{1.005 \text{ V}}}$$

CONT.

SEE FIGURE.

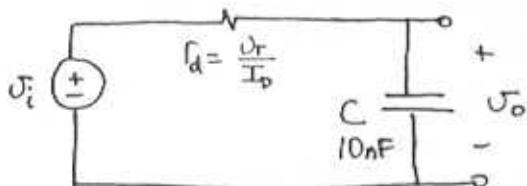
EACH DIODE HAS  $r_d = 50\Omega$

3.57 CONT.



3.58

Opening the current source we get  
the following small-signal circuit :  
(n=1)



$$\frac{V_o}{V_i} = \frac{\gamma_{sc}}{\gamma_{sc} + r_d} = \frac{1}{1 + sCr_d}$$

$$\begin{aligned}\text{Phase Shift} &= -\tan^{-1}\left(\frac{\omega Cr_d}{1}\right) \\ &= -\tan^{-1}\left(2\pi 10^5 \times 10 \times 10^{-9} \times 0.025/\pi\right)\end{aligned}$$

For a phase shift of  $-45^\circ$  we have

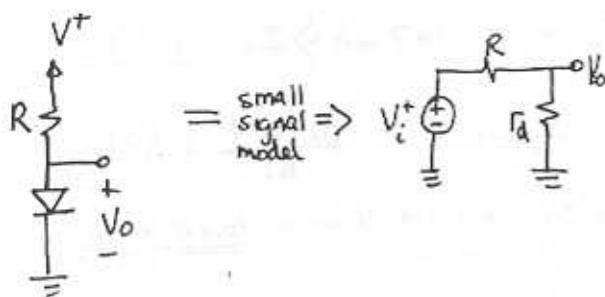
$$2\pi 10^5 \times 10(10^{-9}) \times \frac{0.025}{\pi} = 1$$

$$I = \underline{157 \mu A}$$

Range of phase shift for  $I = 15.7 \mu A$   
to  $157 \mu A$  is :

$$\underline{-84.3^\circ \text{ to } -5.71^\circ}$$

3.59



CONT.

$$(a) \frac{\Delta V_o}{\Delta V^+} = \frac{r_d}{r_d + R} = \frac{nV_T/I}{nV_T/I + R}$$

$$= \frac{nV_T}{nV_T + IR} \quad \text{where at no load}$$

$$I = \frac{V^+ - 0.7}{R}$$

$$= \frac{nV_T}{nV_T + V^+ - 0.7} \quad \text{Q.E.D.}$$

$$\Delta V_o = - I_L (R \parallel r_d)$$

$$\frac{\Delta V_o}{I_L} = \frac{-(R \parallel r_d)}{R} \quad \text{Q.E.D.}$$

$$(b) \text{ Given at DC } I_D = \frac{V^+ - 0.7}{R}$$

$$\text{Also } r_d = \frac{nV_T}{I_D}$$

(b) For  $m$  diodes in series use

$$I = \frac{V^+ - m \times 0.7}{R}$$

Thus:

$$\frac{\Delta V_o}{\Delta V^+} = \frac{m r_d}{m r_d + R} = \frac{m(nV_T)}{m(nV_T) + IR}$$

$$= \frac{m(nV_T)}{m(nV_T) + V^+ - 0.7m}$$

$$\frac{\Delta V_o}{I_L} = - \frac{1}{\frac{1}{R} + \frac{1}{r_d}}$$

$$= - \frac{1}{\frac{I_D}{V^+ - 0.7} + \frac{I_D}{nV_T}}$$

$$= - \frac{nV_T}{I_D} \frac{1}{1 + \frac{nV_T}{V^+ - 0.7}}$$

$$= - \frac{nV_T}{I_D} \frac{V^+ - 0.7}{V^+ - 0.7 + nV_T} \quad \text{Q.E.D.}$$

(c) Line Regulation for  $V^+ = 10V$ ,  $n=2$

$$i) m=1 \quad \frac{\Delta V_o}{\Delta V^+} = \underline{\underline{5.35 \text{ mV/V}}}$$

$$\text{for } \frac{\Delta V_o}{I_L} \leq 5 \frac{\text{mV}}{\text{mA}}$$

$$ii) m=3 \quad \frac{\Delta V_o}{\Delta V^+} = \underline{\underline{18.63 \text{ mV/V}}}$$

$$-\frac{2 \times 0.015}{I_D} \times \frac{10 - 0.7}{10 - 0.7 + 0.05} \leq \frac{5 \times 10^{-3}}{10^3}$$

$$I_D > 9.947 \text{ mA} \Rightarrow I_D = \underline{\underline{10 \text{ mA}}}$$

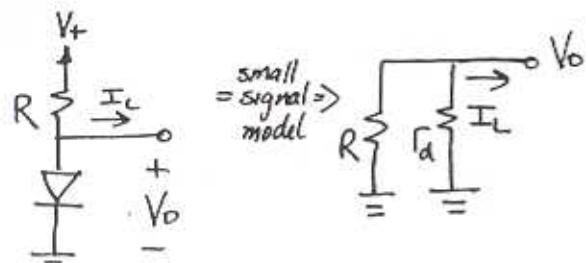
$$R = \frac{V^+ - 0.7}{I_D} = \frac{10 - 0.7}{10} = \underline{\underline{930 \Omega}}$$

Thus the diode should be a 10mA diode.

(c) For  $m$  diodes

$$I_D = \frac{V^+ - 0.7m}{R} \quad \text{and} \quad r_d = \frac{m(nV_T)}{I_D}$$

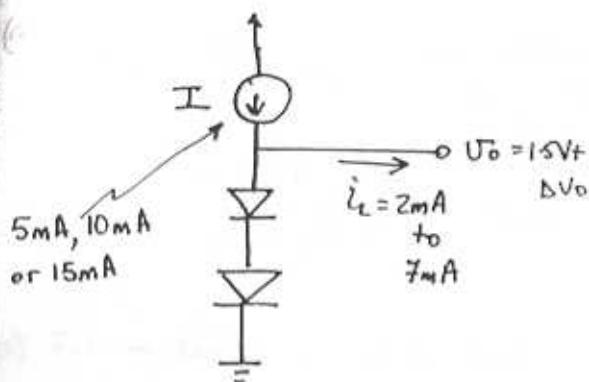
CONT.



$$\begin{aligned}
 \frac{\Delta V_o}{I_L} &= \frac{-1}{\frac{1}{R} + \frac{1}{G}} \\
 &= \frac{-1}{\frac{I_D}{V^+ - 0.7m} + \frac{I_D}{mnV_T}} \\
 &= -\frac{I_D}{mnV_T} \cdot \frac{1}{\frac{I_D}{V^+ - 0.7m} + 1} \\
 &= -\frac{mnV_T}{I_D} \cdot \frac{V^+ - 0.7m}{V^+ - 0.7m + mnV_T}
 \end{aligned}$$

3.61

3.62



For a load current of 2 to 7 mA,  $I$  must be greater than 7 mA. Thus the 5 mA source would not do.

We are left to choose between the 10 and 15 mA sources. The 15 mA source provides lower load regulation because the diodes will have more current flowing through them at all times. This is shown below:

Load Regulation if  $I = 10 \text{ mA}$

$$\text{use } \frac{I_{D2}}{I_D} = e^{\frac{\Delta V}{2nV_T}}$$

↑  
2 diodes

$$\therefore e^{\frac{\Delta V}{0.05 \times 2}} = \frac{3}{10} \text{ to } \frac{8}{10}$$

$$\Delta U_o = -120 \text{ mV to } -22.3 \text{ mV}$$

∴ The peak to peak ripple is  
 $-120 - (-22.3) \approx -100 \text{ mV}$

$$\text{Load Regulation} = \frac{\Delta U_o}{I_L} = \frac{-100}{5}$$

$$= -20 \frac{\text{mV}}{\text{mA}}$$

Load Regulation for  $I = 15 \text{ mA}$ .  
 Here the current through the diodes change from 8 to 13 mA corresponding to

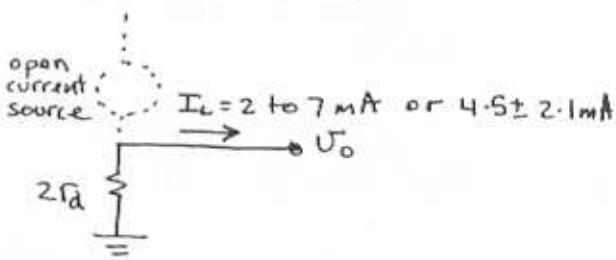
$$\Delta U_o = 0.1 \ln \left( \frac{8}{13} \right)$$

$$= -49 \text{ mV}$$

$$\text{Load Regulation} = \frac{-49}{5} \approx -10 \frac{\text{mV}}{\text{mA}}$$

The obvious disadvantage of using the 15 mA supply is the requirement of higher current and higher power dissipation.

Alternate solution of Line Regulation using the small signal model



$$\text{Load Regulation} = \frac{\Delta U_o}{I_L} = -2R_d = -\frac{2nV_T}{I_D}$$

Where the bias current  $I_D = 10 - 4.5$  for the 10 mA source.

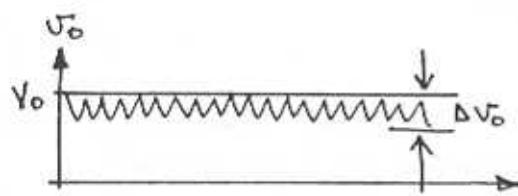
$$\Rightarrow \frac{\Delta U_o}{I_L} = -\frac{2 \times 2 \times 0.025}{10 - 4.5} = -18.2 \frac{\text{mV}}{\text{mA}}$$

For 15 mA source  $I_D = 15 - 4.5$

$$\frac{\Delta U_o}{I_L} = \frac{-0.1}{15 - 4.5} = -9.5 \frac{\text{mV}}{\text{mA}}$$

CONT.

Sketch of output:-



3.63

3.64

$$(a) V_Z = V_{Z0} + r_Z I_{ZT}$$

$$10 = 9.6 + r_Z \times 50 \times 10^{-3}$$

$$r_Z = \underline{8\Omega}$$

Power rating:

$$V_Z = V_{Z0} + r_Z \times 2I_{ZT}$$

$$= 9.6 + 8 \times 100 \times 10^{-3}$$

$$= 10.4V$$

$$P = 10.4 \times 100 \times 10^{-3} = \underline{1.04W}$$

$$(b) V_Z = V_{Z0} + r_Z I_{ZT}$$

$$9.1 = V_{Z0} + 30 \times 10 \times 10^{-3}$$

$$V_{Z0} = \underline{8.8V}$$

$$V_Z = 8.8 + 30 \times 20 \times 10^{-3} = 9.4V$$

$$P = 9.4 \times 20 \times 10^{-3} = \underline{188mW}$$

$$(c) 6.8 = 6.6 + 2 \times I_{ZT}$$

$$I_{ZT} = \underline{100mA}$$

$$V_Z = 6.6 + 2 \times 200 \times 10^{-3} = 7V$$

$$P = 7 \times 200 \times 10^{-3} = \underline{1.4W}$$

$$(d) 18 = 17.2 + r_Z \times 5 \times 10^{-3}$$

$$r_Z = \underline{160\Omega}$$

$$V_Z = 17.2 + 160 \times 10 \times 10^{-3} = 18.8V$$

$$P = 18.8 \times 10 \times 10^{-3} = \underline{188mW}$$

$$(e) 7.6 = V_{Z0} + 1.5 \times 200 \times 10^{-3}$$

$$V_{Z0} = \underline{7.2V}$$

$$V_Z = 7.2 + 1.5 \times 400 \times 10^{-3} = 7.8V$$

$$P = 7.8 \times 400 \times 10^{-3} = \underline{3.12W}$$

3.65

(a) Three 6.8V zeners provide  $3 \times 6.8 = 20.4V$  with  $3 \times 10 = 30\Omega$  resistance. Neglecting  $R$ , we have

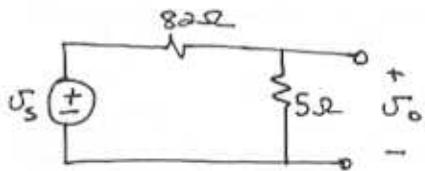
$$\text{Load Regulation} = -30 \text{ mV/mA}$$

(b) For 5.1V zeners we use 4 diodes to provide 20.4V with  $4 \times 30 = 120\Omega$  resistance.

$$\text{Load regulation} = -120 \text{ mV/mA}$$

3.66

Small signal model for line regulation:



$$\frac{\Delta V_o}{\Delta V_s} = \frac{5}{5+82}$$

$$\Delta V_o = \frac{5}{87} \times \Delta V_s$$

$$= \frac{5}{87} \times 1.3$$

$$= \underline{74.7 \text{ mV}}$$

3.67

$$V_Z = V_{Z0} + r_Z I_{ZT}$$

$$9.1 = V_{Z0} + 5 \times 28 \times 10^{-3}$$

$$V_{Z0} = 8.96V$$

$$V_Z = V_{Z0} + 5I_Z = 8.96 + 5I_Z$$

$$\text{FOR } I_Z = 10mA \quad V_Z = \underline{\underline{9.01V}}$$

$$\text{FOR } I_Z = 100mA \quad V_Z = \underline{\underline{9.46V}}$$

3.68

$$r_e = 30 \Omega$$

$$I_{ZK} = 0.5 \text{ mA}$$

$$V_Z = 7.5 \text{ V}$$

$$I_Z = 12 \text{ mA}$$

$$7.5 = V_{Z0} + 12 \times 30 \times 10^{-3}$$

$$\Rightarrow V_{Z0} = 7.14 \text{ V}$$

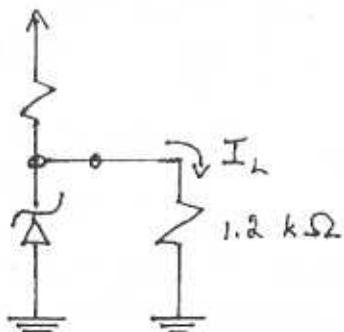
$$I_Z = \frac{7.5}{1.2} = 6.25 \text{ mA}$$

SELECT  $I = 10 \text{ mA}$

SO THAT  $I_Z = 3.7 \text{ mA}$

WHICH IS  $> I_{ZK}$

$$R = \frac{10 - 7.5}{10} = \underline{\underline{250 \Omega}}$$

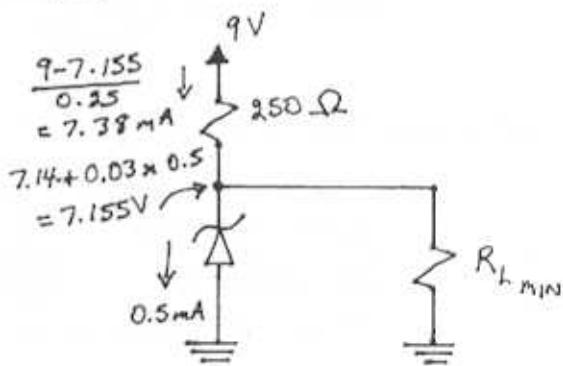
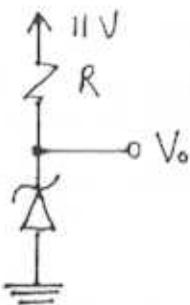


FOR  $\Delta V^+ = \pm 1 \text{ V}$

$$\begin{aligned}\Delta V_o &= \pm 1 \times \frac{1.2 // 0.03}{0.250 + (1.2 // 0.03)} \\ &= \pm 0.1 \text{ V}\end{aligned}$$

THUS  $V_o = +7.4 \text{ V}$  TO  $+7.6 \text{ V}$   
WITH  $V^+ = 11 \text{ V}$  AND  $I_L = 0$

$$\begin{aligned}V_o &= V_{Z0} + \frac{11 - V_o}{0.25} \times 0.03 \\ \Rightarrow V_o &= \underline{\underline{7.55 \text{ V}}}\end{aligned}$$



$$\begin{aligned}R_{L_{\min}} &= \frac{7.155}{7.38 - 0.5} \\ &= \underline{\underline{1.04 \text{ k} \Omega}}\end{aligned}$$

$$\therefore R = \frac{9 - 0.68}{20} = \underline{\underline{110.52}}$$

$$\begin{aligned}\text{Line Regulation} &= \frac{\Delta V_o}{\Delta V_s} = \frac{r_2}{r_2 + R} \\ &= \frac{5}{5 + 110} \\ &= \underline{\underline{43.5 \frac{mV}{V}}}\end{aligned}$$

SECOND DESIGN ~ limited current from 9V supply

$$I_{zK} = 0.25 \text{ mA}$$

$$V_z = V_{zK} \approx V_{z0} - \text{calculate } V_{z0} \text{ from}$$

$$V_z = V_{z0} + r_2 I_{zT}$$

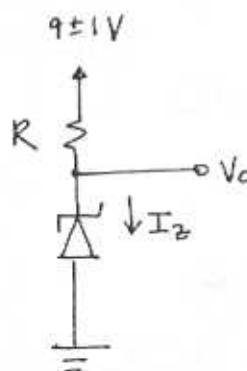
$$6.8 = V_{z0} + 5 \times 0.02$$

$$V_{z0} = 6.7 \text{ V}$$

$$\therefore R = \frac{9 - 6.7}{0.25} = \underline{\underline{9.2 \text{ k}\Omega}}$$

$$\begin{aligned}\text{LINE REGULATION} &= \frac{\Delta V_o}{\Delta V_s} = \frac{7.50}{750 + 9200} \\ &= \underline{\underline{75.4 \frac{mV}{V}}}\end{aligned}$$

3.69



GIVEN PARAMETERS

$$V_z = 6.8 \text{ V}, r_2 = 5 \Omega, I_{zK} = 20 \text{ mA}$$

By knee

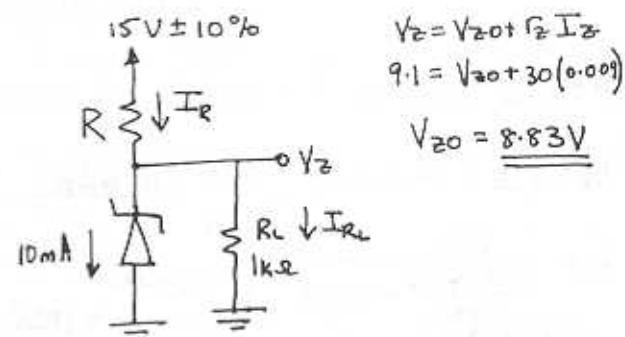
$$I_{zK} = 0.25 \text{ mA}$$

$$r_2 = 7.50 \Omega$$

FIRST DESIGN ~ 9V supply can easily supply current

Let  $I_z = 20 \text{ mA}$  ~ well above knee

3.70



$$V_z = V_{z0} + r_2 I_{zT}$$

$$9.1 = V_{z0} + 30(0.009)$$

$$V_{z0} = \underline{\underline{8.83 \text{ V}}}$$

CONT.

$$V_Z = 8.83 + 30(0.01) = 9.13 \text{ V}$$

$$I_{RL} = 9.13/1k\Omega = 9.13 \text{ mA}$$

$$I_R = 10 + 9.13 = \underline{19.13 \text{ mA}}$$

$$\therefore R = \frac{15 - 9.13}{19.13} = 306.8 \Omega \approx \underline{\underline{300 \Omega}}$$

$$V_Z = 8.83 + 30 \left( \frac{15 - V_Z}{300} - \frac{V_Z}{1000} \right)$$

$$= 10.33 - V_Z/10 - 3/100 V_Z$$

$$V_Z = 9.14 \text{ V}$$

$$V_Z = 8.83 + 30 \left( \frac{15 \pm 1.5 - V_Z}{300} - \frac{V_Z}{1000} \right) \\ = \frac{1}{1.13} \left[ 8.83 + 15 \pm 0.15 \right] = 9.14 \pm 0.13 \text{ V}$$

$\therefore \pm 0.13 \text{ V}$  variation in output voltage  
Halving the load current  $\equiv R_L$  doubling

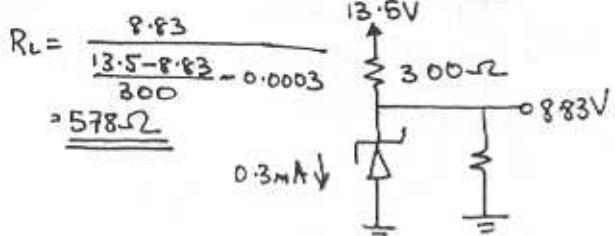
$$V_Z = 8.83 + 30 \left( \frac{15 - V_Z}{300} - \frac{V_Z}{2000} \right)$$

$$= \frac{10.33}{1.115} = 9.26 \text{ V}$$

$\therefore 9.26 - 9.14 = 0.12 \text{ V}$  increase in output voltage.

At the edge of the breakdown region

$$V_Z \approx V_{Z0} = 8.83 \text{ V} \quad I_{ZK} = 0.3 \text{ mA}$$



lowest output voltage = 8.83 V

$$\text{Line Regulation} = \frac{V_Z}{R + R_2} = \frac{30}{300 + 30} \\ = 90 \text{ mV/V}$$

$$\text{Load Regulation} = -(R_2 || R) = -29.1 \text{ mV/mA}$$

3.71

$$(a) V_{ZT} = V_{Z0} + r_z I_{ZT} \\ 10 = V_{Z0} + 7(0.025) \\ \Rightarrow V_{Z0} = \underline{\underline{9.825 \text{ V}}}$$

(b) The minimum zener current of 5mA occurs when  $I_L = 20 \text{ mA}$  and  $V_S$  is at its minimum of  $20(1 - 0.25) = 15 \text{ V}$ . See the circuit below:

$$R \leq \frac{15 - (V_{Z0} + r_z I_Z)}{20 + 5} \\ \leq \frac{15 - (9.825 + 7(0.005))}{25} \\ \leq 205.6 \Omega.$$

$\therefore$  Use  $R = \underline{\underline{205 \Omega}}$

$$(c) \text{Line Regulation} = \frac{7}{205 + 7} = 33 \text{ mV/V}$$

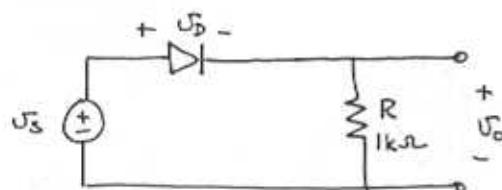
$\pm 25\%$  change in  $V_S \equiv \pm 5 \text{ V}$

$V_0$  changes by  $\pm 5 \times 33 = \pm \underline{\underline{165 \text{ mV}}}$

corresponding to  $\frac{\pm 165}{10} \times 100 = \pm \underline{\underline{1.65\%}}$

CONT.

3.75



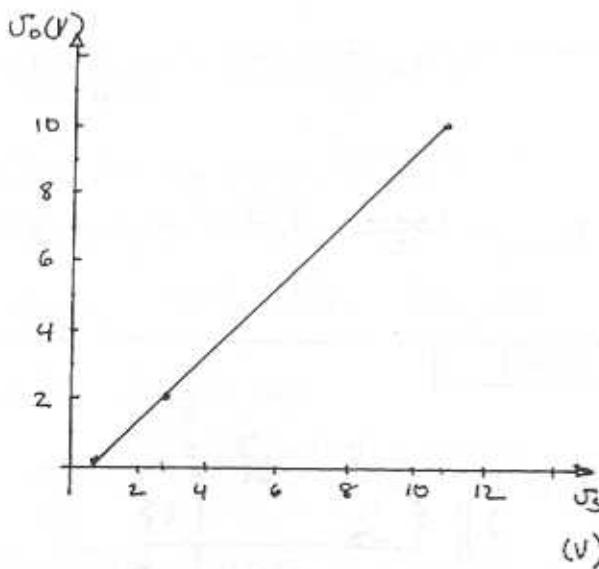
$$i_D = I_s e^{\frac{U_D}{nV_T}}$$

$$\frac{i_D}{1mA} = e^{\frac{U_D - 0.7}{nV_T}}$$

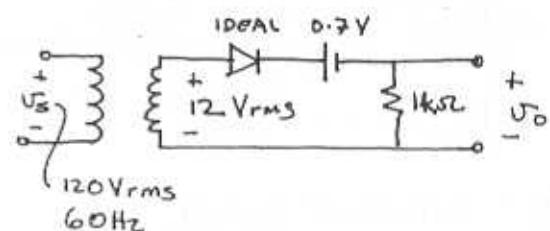
$$U_D - 0.7 = nV_T \ln\left(\frac{i_D}{10^{-3}}\right) = 0.1 \log\left(\frac{i_D}{10^{-3}}\right)$$

$$\begin{aligned} U_D &= 0.7 + 0.1 \log\left(\frac{U_o}{R}\right) \quad R = 1k\Omega \\ &= 0.7 + 0.1 \log\left(\frac{U_o}{1}\right) \end{aligned}$$

$U_o$ (V)	$U_D$ (V)	$U_s = U_D + U_o$ (V)
0.10	0.6	0.7
0.5	0.67	1.17
1	0.7	1.7
2	0.73	2.73
5	0.77	5.77
10	0.8	10.8



3.76



$$V_o = 12\sqrt{2} - 0.7 = 16.27V$$

Conduction begins at

$$\begin{aligned} U_s &= 12\sqrt{2} \sin\theta = 0.7 \\ \theta &= \sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right) \\ &= 0.0412 \text{ rad} \end{aligned}$$

Conduction ends at  $\pi - \theta$

$$\therefore \text{Conduction angle} = \pi - 2\theta = 3.06 \text{ rad}$$

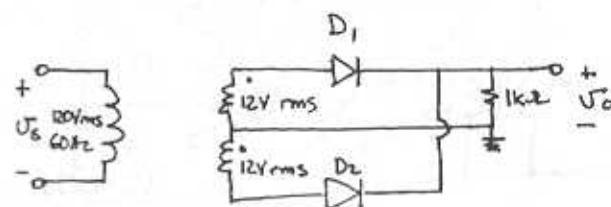
The diode conducts for

$$\frac{3.06}{2\pi} \times 100 = 48.7\% \text{ of the cycle}$$

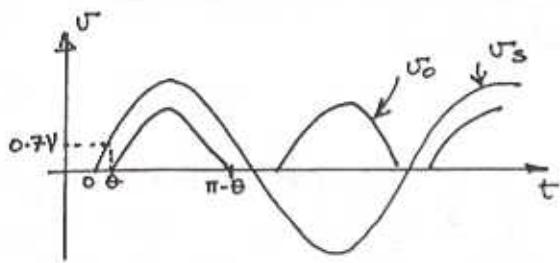
$$\begin{aligned} V_{o, \text{avg}} &= \frac{1}{2\pi} \int_{\theta}^{\pi-\theta} 12\sqrt{2} \sin\phi - 0.7 d\phi \\ &= 5.06V \end{aligned}$$

$$i_{D, \text{avg}} = \frac{V_{o, \text{avg}}}{R} = 5.06mA$$

3.77



CONT.



$$\hat{U}_o = 12\sqrt{2} - V_{D0} = \underline{16.27V}$$

$$\text{Conduction starts at } \theta = \sin^{-1} \frac{0.7}{12\sqrt{2}} \\ = 0.0412 \text{ rad}$$

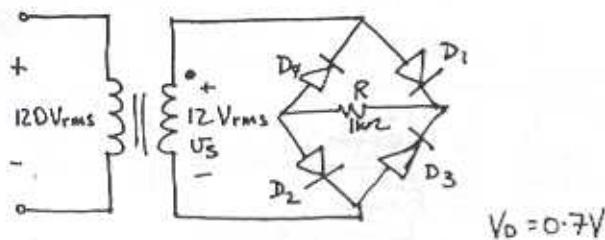
and ends at  $\pi - \theta$ . Conduction angle  $= \pi - 2\theta = 3.06 \text{ rad}$  in each half cycle. Thus the fraction of a cycle for which one of the two diodes conduct  $= \frac{2(3.06)}{2\pi} \times 100$   
 $= \underline{97.4\%}$

Note that during 97.4% of the cycle there will be conduction. However each of the two diodes conducts for only half the time, i.e. for 48.7% of the cycle.

$$U_{o,\text{avg}} = \frac{1}{\pi} \int_0^{\pi-\theta} 12\sqrt{2} \sin \phi - 0.7 \, d\phi \\ = \underline{10.12V}$$

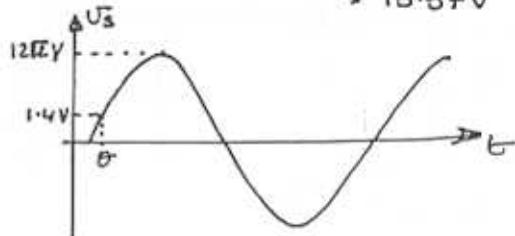
$$i_{D,\text{avg}} = \frac{10.12}{1k\Omega} = \underline{10.12mA}$$

3.78



$$\text{Peak voltage across } R = 12\sqrt{2} - 2V_D$$

$$= 12\sqrt{2} - 1.4 \\ > 16.57V$$



$$\theta = \sin^{-1} \frac{1.4}{12\sqrt{2}} = 0.0826 \text{ rad}$$

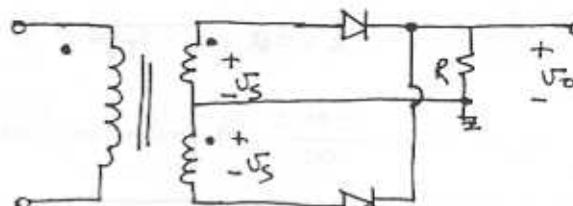
Fraction of cycle that D<sub>1</sub> & D<sub>2</sub> conduct is  $\frac{\pi - 2\theta}{2\pi} \times 100 = \underline{47.4\%}$

Note D<sub>3</sub> & D<sub>4</sub> conduct in the other half cycle  $\Rightarrow$  that there is 2(47.4) = 94.8% conduction interval.

$$U_{o,\text{avg}} = \frac{2}{2\pi} \int_0^{\pi-\theta} 12\sqrt{2} \sin \phi - 2V_D \, d\phi \\ = \frac{1}{\pi} \left[ -12\sqrt{2} \cos \phi - 1.4 \phi \right]_0^{\pi-\theta} \\ = \frac{2(12\sqrt{2} \cos \theta)}{\pi} - \frac{1.4(\pi - 2\theta)}{\pi} \\ = \underline{9.44V}$$

$$i_{R,\text{avg}} = \frac{U_{o,\text{avg}}}{R} = \frac{9.44}{1} = 9.44 \text{ mA}$$

3.79



CONT.

For  $V_{DD} \ll V_s$ ,

$$V_{o,\text{avg}} \approx \frac{2}{\pi} V_s - V_{DD}$$

(a) For  $V_{o,\text{avg}} = 10V$

$$10 = \frac{2}{\pi} V_s - 0.7$$

$$\Rightarrow V_s = 16.81V$$

$$\text{Line peak} = 120\sqrt{2}$$

Thus,

$$\text{turns ratio} = \frac{120\sqrt{2}}{16.81} = \underline{\underline{10.1:1}}$$

to each half  
of the secondary

OR 5.05:1 centre tapped

(b) For  $V_{o,\text{avg}} = 100V$

$$V_s = \frac{\pi}{2} (100.7) = \underline{\underline{158.2V}}$$

$$\text{Turns Ratio} = \frac{120\sqrt{2}}{158.2} = \underline{\underline{1.07:1}} \text{ to each half}$$

OR 0.535:1 centre tapped

3.80

Refer to Fig 3.27.

For  $2V_{DD} \ll V_s$ ,

$$V_{o,\text{avg}} = \frac{2}{\pi} V_s - 2V_{DD} = \frac{2}{\pi} V_s - 1.4$$

(a) For  $V_{o,\text{avg}} = 10V$

$$V_s = \frac{\pi}{2} \times 11.4 = 17.91V$$

$$\text{Turns Ratio} = \frac{120\sqrt{2}}{17.91} = \underline{\underline{9.477 \text{ to } 1}}$$

(b) For  $V_{o,\text{avg}} = 100V$

$$V_s = \frac{\pi}{2} \times 101.4 = 159.3V$$

$$\text{Turns Ratio} = \frac{120\sqrt{2}}{159.3} = \underline{\underline{1.065 \text{ to } 1}}$$

3.81

$$120\sqrt{2} \pm 10\% : 24\sqrt{2} \pm 10\%$$

$$\Rightarrow \text{turns Ratio} = 5 : 1$$

$$V_s = \frac{24\sqrt{2}}{2} \pm 10\%$$

$$\text{PIV} = 2V_{s_{\max}} | V_{DD} |$$

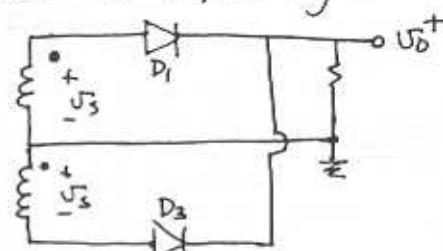
$$= 2 \times \frac{24\sqrt{2}}{2} \times 1.1 - 0.7$$

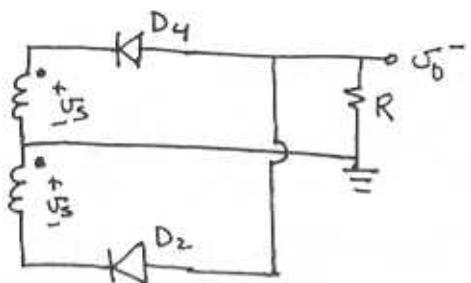
$$= \underline{\underline{36.6V}}$$

Using a factor of 1.5 for safety we select a diode having a PIV rating of 55V.

3.82

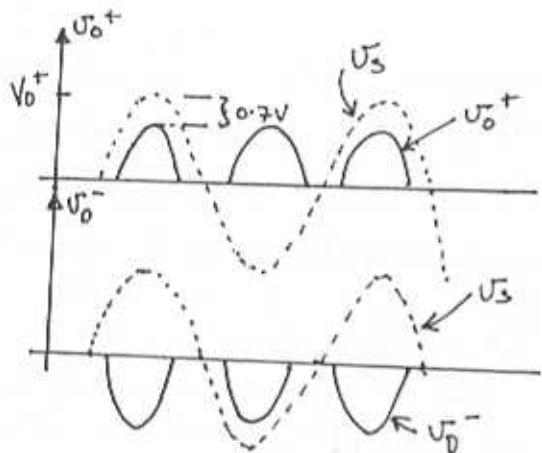
The circuit is a full wave rectifier with centre tapped secondary winding. The circuit can be analyzed by looking at  $V_o^+$  and  $V_o^-$  separately.





If choosing a diode, allow a safety margin of  $1.5 \text{ PIV} = \underline{\underline{73V}}$

3.83

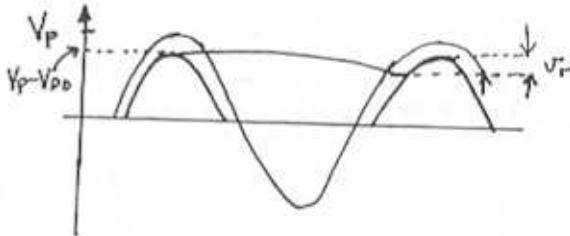
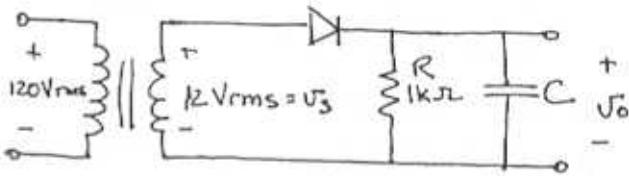


$$\begin{aligned} V_{D,\text{avg}} &= \frac{1}{2\pi} \int V_s \sin \phi - 0.7 \, d\phi = 15 \\ &= \frac{2V_s}{\pi} - 0.7 = 15 \quad \text{assumed } V_s > 0.7V \\ V_s &= \frac{15 + 0.7}{2} \pi = 24.66V \end{aligned}$$

Thus voltage across secondary winding  
=  $2V_s = \underline{\underline{49.32V}}$

Looking at  $D_4$

$$\begin{aligned} \text{PIV} &= V_s - V_o^- \\ &= V_s + (V_s - 0.7) \\ &= 2V_s - 0.7 \\ &= \underline{\underline{48.6V}} \end{aligned}$$



$$(i) \bar{V}_r \approx (V_p - V_{DD}) \frac{T}{CR} \quad \text{Eq. (3.28)}$$

$$0.1(V_p - V_{DD}) = (V_p - V_{DD}) \frac{T}{CR}$$

$$C = \frac{1}{0.1 \times 60 \times 10^3} = \underline{\underline{166.7 \mu F}}$$

$$(ii) \text{ For } \bar{V}_r = 0.01(V_p - V_{DD}) = (V_p - V_{DD}) \frac{T}{CR}$$

$$C = \underline{\underline{1667 \mu F}}$$

(a)

$$\begin{aligned} (i) \bar{V}_o, \text{avg} &= V_p - V_{DD} - \frac{1}{2} V_r \\ &= 12\sqrt{2} - 0.7 - \frac{1}{2}(2\sqrt{2} - 0.7) 0.1 \\ &= (12\sqrt{2} - 0.7)(1 - \frac{0.1}{2}) \\ &= \underline{\underline{15.5V}} \end{aligned}$$

$$\begin{aligned} (ii) \bar{V}_o, \text{avg} &= (12\sqrt{2} - 0.7)(1 - \frac{0.01}{2}) \\ &= \underline{\underline{16.19V}} \end{aligned}$$

CONT.

(b)

i) Using eq (3.30) we have the conduction angle =

$$\begin{aligned} \omega \Delta t &\approx \sqrt{\frac{2V_r}{(V_p - V_{DD})}} \\ &= \sqrt{\frac{2 \times 0.1 (V_p - 0.7)}{(V_p - 0.7)}} \\ &= \sqrt{0.2} \\ &= 0.447 \text{ rad} \end{aligned}$$

∴ Fraction of cycle for conduction =  $\frac{0.447}{2\pi} \times 100$   
 $= \underline{\underline{7.1\%}}$

ii)  $\omega \Delta t \approx \sqrt{\frac{2\pi 0.01 (V_p - 0.7)}{V_p - 0.7}} = 0.141 \text{ rad}$

Fraction of cycle =  $\frac{0.141}{2\pi} \times 100 = \underline{\underline{2.25\%}}$

(c) (i) Use eq (3.31)

$$\begin{aligned} i_{D,\text{avg}} &= I_L \left( 1 + \pi \sqrt{\frac{2(V_p - V_{DD})}{V_r}} \right) \\ &= \frac{V_{o,\text{avg}}}{R} \left( 1 + \pi \sqrt{\frac{2(V_p - V_{DD})}{0.1(V_p - V_{DD})}} \right) \\ &= \frac{15.5}{10^3} \left( 1 + \pi \sqrt{\frac{2}{0.1}} \right) \\ &= \underline{\underline{2.33 \text{ mA}}} \end{aligned}$$

ii)  $i_{D,\text{avg}} = \frac{16.19}{10^3} \left( 1 + \pi \sqrt{200} \right)$   
 $= \underline{\underline{7.35 \text{ mA}}}$

NB Text uses  $I_L \approx V_p/R = \frac{V_p - V_{DD}}{R}$   
 but here we used  $i_{D,\text{avg}} = \frac{V_p - V_{DD} - \frac{1}{2}V_C}{R}$

which is more accurate.

(d) i)  $i_{D,\text{peak}} = I_L \left( 1 + 2\pi \sqrt{\frac{2(V_p - V_{DD})}{V_r}} \right)$

$$\begin{aligned} &= \frac{15.42}{10^3} \left( 1 + 2\pi \sqrt{\frac{2}{0.1}} \right) \\ &= \underline{\underline{4.49 \text{ mA}}} \end{aligned}$$

ii)  $i_{D,\text{peak}} = \frac{16.19}{10^3} \left( 1 + 2\pi \sqrt{\frac{2}{0.01}} \right)$   
 $= \underline{\underline{14.55 \text{ mA}}}$

3.84

i)  $V_r = 0.1(V_p - V_{DD}) = \frac{(V_p - V_{DD})}{2fCR}$

the factor of 2 accounts for discharge occurring only half of the period  $\frac{1}{2} = \frac{1}{2f}$

$$C = \frac{1}{(2fR)0.1} = \frac{1}{2(60)10^3 \times 0.1} = \underline{\underline{83.3 \mu F}}$$

ii)  $C = \frac{1}{2(60)10^3 0.01} = \underline{\underline{833 \mu F}}$

(a) i)  $V_o = V_p - V_{DD} - \frac{1}{2}V_r$   
 $= (V_p - V_{DD})(1 - \frac{0.1}{2})$   
 $= (16.27)(1 - \frac{0.1}{2})$   
 $= \underline{\underline{15.5 \text{ V}}}$

ii)  $V_o = (16.27)(1 - \frac{0.01}{2}) = \underline{\underline{16.19 \text{ V}}}$

(b)

(i) Fraction of cycle =  $\frac{2\omega \Delta t}{2\pi} \times 100$   
 $= \frac{\sqrt{2V_r/(V_p - V_{DD})}}{\pi} \times 100$   
 $= \frac{1}{\pi} \sqrt{\frac{2(0.1)}{0.1}} \times 100 = \underline{\underline{14.2\%}}$

CONT.

$$\text{i)} \text{ Fraction of Cycle} = \frac{2\sqrt{2(0.01)}}{2\pi} \times 100 \\ = \underline{\underline{4.5\%}}$$

(c) Use eq (3.34)

$$\text{i)} \hat{I}_{D,\text{avg}} = I_L \left( 1 + \pi \sqrt{\frac{V_p - V_{DD}}{2V_r}} \right) \\ = \frac{15.5}{1} \left( 1 + \pi \sqrt{\frac{1}{2(0.1)}} \right) = \underline{\underline{124.4 \text{mA}}} \\ \text{ii)} \hat{I}_{D,\text{avg}} = \frac{16.19}{1} \left( 1 + \pi \frac{1}{\sqrt{2(0.01)}} \right) = \underline{\underline{376 \text{mA}}}$$

(d) Use eq (3.35)

$$\text{i)} \hat{I}_D = I_L \left( 1 + 2\pi \frac{1}{\sqrt{2(0.1)}} \right) = \underline{\underline{233 \text{mA}}} \\ \text{ii)} \hat{I}_D = I_L \left( 1 + 2\pi \frac{1}{\sqrt{0.02}} \right) = \underline{\underline{735 \text{mA}}}$$

3.85

$$\text{i)} V_r = 0.1 (V_p - V_{DD} \times 2) = \frac{V_p - 2V_{DD}}{2fCR}$$

discharge occurs only over  $T = \frac{1}{2f}$

$$C = \frac{(V_p - 2V_{DD})}{(V_p - 2V_{DD}) \cdot \frac{1}{2(0.1)fR}} = \underline{\underline{83.3 \mu F}}$$

$$\text{ii)} C = \frac{1}{2(0.01)fR} = \underline{\underline{833 \mu F}}$$

$$\text{(b)} \text{ Fraction of cycle} = \frac{2\omega dt}{2\pi} \times 100$$

$$= \frac{\sqrt{2(0.1)}}{\pi} \times 100 = \underline{\underline{14.2\%}}$$

$$\text{iii)} \text{ Fraction of cycle} = \frac{\sqrt{2(0.01)}}{\pi} \times 100 = \underline{\underline{4.5\%}}$$

(c) i)

$$\hat{I}_{D,\text{avg}} = \frac{14.79}{1} \left( 1 + \pi \sqrt{\frac{1}{0.2}} \right) = \underline{\underline{119 \text{mA}}}$$

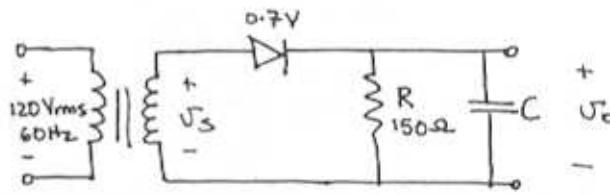
$$\text{ii)} \hat{I}_{D,\text{avg}} = \frac{15.49}{1} \left( 1 + \pi / \sqrt{0.02} \right) = \underline{\underline{356 \text{mA}}}$$

(d)

$$\text{i)} \hat{I}_D = \frac{14.79}{1} \left( 1 + 2\pi \sqrt{\frac{1}{0.2}} \right) = \underline{\underline{223 \text{mA}}}$$

$$\text{ii)} \hat{I}_D = \frac{15.49}{1} \left( 1 + 2\pi \sqrt{\frac{1}{0.02}} \right) = \underline{\underline{704 \text{mA}}}$$

3.86



$$V_{O,\text{peak}} = V_p - V_{DD} = 16$$

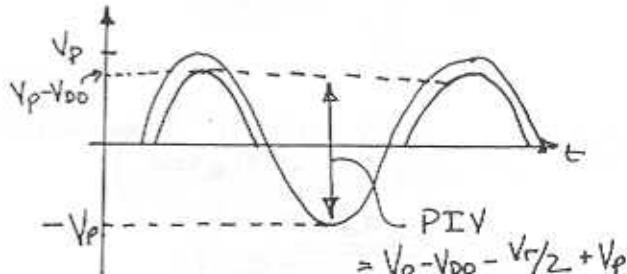
$$V_p = 16.7 \text{ V}$$

$$V_{rms} = \frac{16.7}{\sqrt{2}} = 11.8$$

$$\text{(b)} \text{ } V_r = (V_p - V_{DD}) \frac{1}{CR} \quad \text{Eq (3.28)}$$

$$2 = \frac{16}{60 \times C \times 150}$$

$$C = 889 \mu F$$



For a 50% safety margin  $\text{PIV} = 1.5 \times 31.7 \\ = \underline{\underline{47.6 \text{V}}}$

CONT.

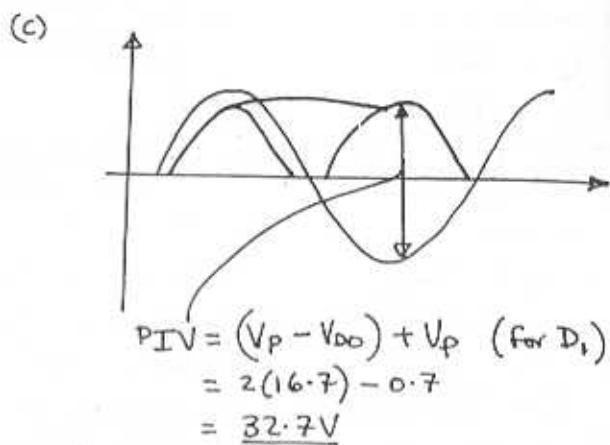
$$(d) i_{D, \text{avg}} = I_L \left( 1 + \pi \sqrt{\frac{2(V_p - V_{DD})}{V_r}} \right)$$

using  $I_L = \frac{U_{D, \text{avg}}}{R} = \frac{15}{R}$  we have

$$i_{D, \text{avg}} = \frac{15}{150} \left( 1 + \pi \sqrt{\frac{2(16)}{2}} \right) \\ = \underline{1.36 \text{ A}}$$

$$(e) i_{D, \text{peak}} = I_L \left( 1 + 2\pi \sqrt{\frac{2(V_p - V_{DD})}{V_r}} \right)$$

$$= \frac{15}{150} \left( 1 + 2\pi \sqrt{\frac{2(16)}{2}} \right) \\ = \underline{2.61 \text{ A}}$$



so Using a 50% margin

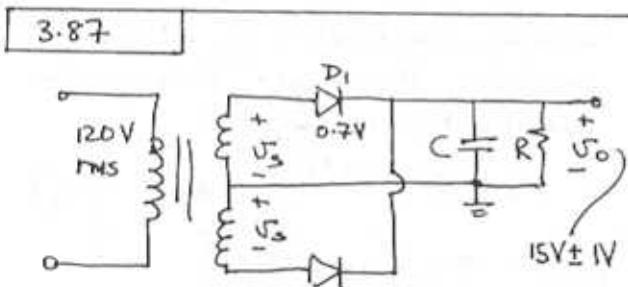
$$\text{PIV} = 1.5(32.7) = \underline{49 \text{ V}}$$

(d) Using Eq (3.34)

$$i_{D, \text{avg}} = I_L \left( 1 + \pi \sqrt{\frac{V_p}{2V_r}} \right) \\ = \frac{15}{150} \left( 1 + \pi \sqrt{\frac{16}{2 \times 2}} \right) \\ = \underline{0.73 \text{ A}}$$

(e) Using Eq. (3.35)

$$i_{D, \text{max}} = I_L \left( 1 + 2\pi \sqrt{\frac{V_p}{2V_r}} \right) \\ = \frac{15}{150} \left( 1 + 2\pi \sqrt{\frac{16}{2 \times 2}} \right) \\ = \underline{1.36 \text{ A}}$$



$$(a) \hat{U}_{D_1} = 16 \text{ V}$$

$$\therefore \hat{U}_S = 16 + U_{DD} = 16.7 \text{ V}$$

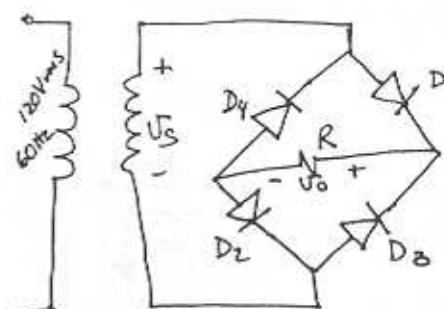
RMS Voltage across secondary  
=  $\frac{2 \times 16.7}{\sqrt{2}}$   
= 23.6 \text{ V}

(b) Using Eq. (3.29)

$$V_r = \frac{V_p}{2fCR} = \frac{16}{2 \times 60 \times 10^{-6} \times 150} = 2$$

$$C = \underline{444.4 \mu\text{F}}$$

3.88



$$U_o = 15 \pm 1 \text{ V}, R = 150 \Omega$$

CONT.

$$\hat{V}_o = 16V$$

$$V_s = 16 + 2V_{D0} = 17.4V$$

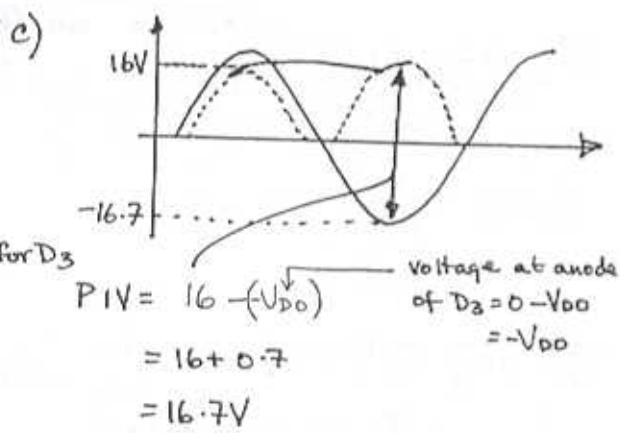
$$\text{RMS Secondary Voltage} = \frac{17.4}{\sqrt{2}} = 12.3V$$

$$(b) V_r = \frac{V_p}{2FCR}$$

$$Z = \frac{16}{2 \times 60 \times C \times 150}$$

$$C = 444.4 \mu F$$

note: we got the same value for  $C$  as the full wave rectifier as discharge is over the same amount of time  $T/2$  and the peak is the same - 16V.



$$\text{Allowing a } 50\% \text{ margin} = 16.7 \times 1.5$$

$$= 25V$$

(d) Using 3.34

$$i_{D,\text{avg}} = I_L \left( 1 + \pi \sqrt{V_p / 2V_r} \right)$$

$$= \frac{15}{150} \left( 1 + \pi \sqrt{16 / 2 \times 2} \right)$$

$$= 0.73A$$

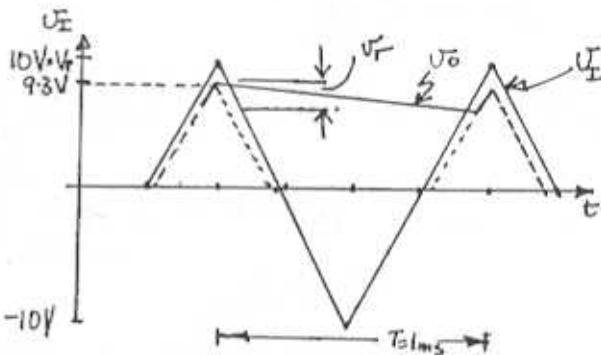
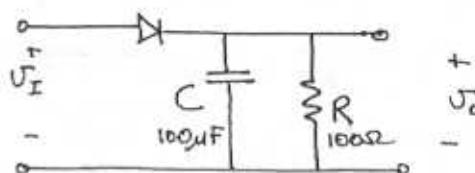
(e) Using (3.35)

$$i_{D,\text{max}} = I_L \left( 1 + 2\pi \sqrt{V_p / 2V_r} \right)$$

$$= \frac{15}{150} \left( 1 + 2\pi \sqrt{16 / 2 \times 2} \right)$$

$$= 1.36A$$

3.89



During the diode's off interval, the capacitor discharges through the resistor  $R$  according to :

$$U_o = 9.3 e^{-t/RC} \approx 9.3 (1 - t/CR)$$

$$\overset{\circ}{U_r} = 9.3 - 9.3 (1 - t/CR)$$

$$= 9.3 t / CR$$

$$= \frac{9.3}{FCR}$$

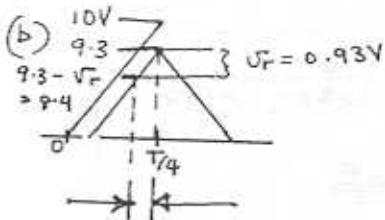
$$= 0.93V$$

NB this is Eq(3.38)

$$U_{o,\text{avg}} = V_o - V_{D0} - \frac{1}{2} U_r$$

$$= 9.3 - \frac{1}{2} 0.93$$

$$= 8.84V$$



$$\Delta t \Rightarrow \frac{16}{T/4} = \frac{0.93}{\Delta t}$$

$$\Delta t = 0.02325 T$$

$$= 0.02325 \text{ ms}$$

CONT.

$$(c) \text{ Charge gained during conduction} = \text{Charge lost during discharge}$$

$$i_{C,\text{avg}} \Delta t = C V_r$$

$$i_{C,\text{avg}} = \frac{C V_r}{\Delta t} = \frac{100 \times 10^{-6} \times 0.93}{0.02325 \times 10^{-3}}$$

$$= 4.0 \text{ A}$$

$$i_{D,\text{avg}} \approx i_{C,\text{avg}} + i_{L,\text{avg}} \quad \leftarrow \frac{V_{o,\text{avg}}}{R}$$

$$\approx 4.0 + \frac{8.84}{100} = \underline{\underline{4.09 \text{ A}}}$$

$$(d) i_{C,\text{max}} = C \frac{\delta V_o}{\Delta t} \Big|_{\text{at onset of conduction}}$$

$$= C \frac{\delta V_r}{\Delta t}$$

$$= 100 \times 10^{-6} \times 40 \times 10^3$$

$$= \underline{\underline{4 \text{ A}}}$$

$$i_{D,\text{max}} = i_{C,\text{max}} + i_{L,\text{max}}$$

$$= 4 + \frac{V_{o,\text{max}}}{100}$$

$$= 4 + \frac{9.3}{100}$$

$$= \underline{\underline{4.09 \text{ A}}}.$$

Note that in this case  $i_{D,\text{avg}} = i_{D,\text{max}}$  owing to the linear input ( $i_C$  is constant and  $i_L$  is approximately constant).

3.90

Refer to Fig P3.8.2 and let capacitor  $C$  be connected across each of the load resistors  $R$ . The two supplies,  $V_0^+$  and  $V_0^-$  are identical. Each is a full-wave rectifier similar to that based on the centre-tapped-transformer circuit. For each supply, the dc output is 15V and the ripple is 1V peak-to-peak. Thus  $V_0 = 15 \pm \frac{1}{2} \text{ V}$ . It follows that the peak value of  $V_S$  must be  $15.5 \pm 0.7 = 16.2 \text{ V}$ .  
 $\therefore$  Voltage across secondary =  $2(16.2) = 32.4 \text{ V}$

$$\text{RMS across secondary} = \frac{32.4}{\sqrt{2}} = \underline{\underline{22.9 \text{ V rms}}}$$

$$\text{Turns Ratio} = \frac{120}{22.9} = 5.24 : 1$$

Use Eq. (3.35) to find

$$i_{D,\text{max}} = I_L \left( 1 + 2\pi \sqrt{\frac{V_p}{2V_r}} \right)$$

$$= 0.2 \left( 1 + 2\pi \sqrt{\frac{15.5}{2}} \right)$$

$$= \underline{\underline{3.70 \text{ A}}}$$

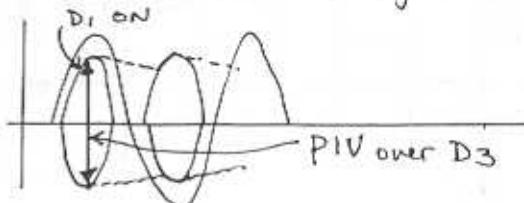
$$V_r = \frac{V_p}{2fCR} = 1 \quad \text{Eq. (3.28)}$$

$\downarrow$  DISCHARGE OCCURS OVER  $T/2 = \frac{1}{2f}$

$$\Rightarrow C = \frac{15.5}{2 \times 60 \times 75} \quad \text{where } 200 \text{ mA} = \frac{15}{R}$$

$$= \underline{\underline{1722 \mu\text{F}}} \quad R = \frac{15}{0.2} = 75 \Omega$$

Consider  $D_3$  when looking at PIV



CONT.

$$PIV = \hat{U}_o + \hat{U}_s \\ = 15.5 + 16.2 = 31.7 \text{ V.}$$

Allowing for 50% safety margin

$$PIV = 1.5 \times 31.7 = 47.6 \text{ V}$$

Use Eq (3.34) to find

$$\hat{i}_{D, \text{avg}} = I_L \left( 1 + \pi \sqrt{V_F / 2V_T} \right) \\ = 0.2 \left( 1 + \pi \sqrt{15.5 / 2} \right) \\ = 1.95 \text{ A}$$

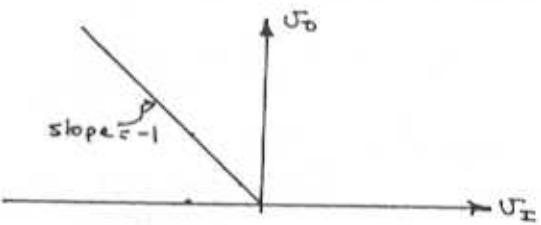
$$U_o, \text{avg} = 5 \text{ V}$$

3.92

$U_I > 0$   $D_1$  conducts  $D_2$  cutoff

$U_I < 0$   $D_1$  cut off

$$D_2 \text{ conducts } \sim \frac{U_o}{U_x} = -1$$



3.91

$$U_o = U_I \left( 1 + R/R \right)$$

$= 2U_I$  when the diode is conducting.

$$(a) U_I = +1 \text{ V} \quad U_o = 2 \text{ V} \quad U_A = 1.7 \text{ V} \quad U_- = U_I = 1 \text{ V}$$

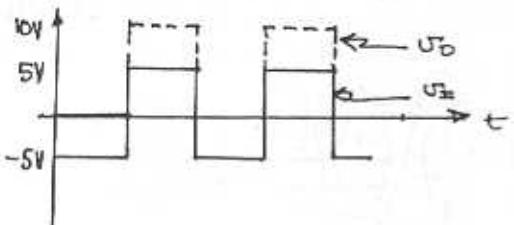
$$(b) U_I = 2 \text{ V} \quad U_o = 4 \text{ V} \quad U_A = 1.7 \text{ V} \quad U_- = 2 \text{ V}$$

$$(c) U_I = -1 \text{ V} \quad U_A = -1.2 \text{ V} \sim \text{diode is cut off}$$

$$U_o = 0 \text{ V} \\ U_- = 0 \text{ V}$$

d)

$$U_I = -2 \text{ V} \quad U_A = -1.2 \text{ V} \quad U_o = 0 \text{ V} \quad U_- = 0 \text{ V}$$



$$(a) U_I = 1 \text{ V} \quad U_o = 0 \text{ V}$$

$U_A = -0.7 \text{ V}$  - keeps  $D_2$  off so no current flows through  $R$

$$\Rightarrow U_- = 0 \text{ V} - \text{virtual ground as feedback is closed through } D_1$$

$$(b) U_I = 2 \text{ V}$$

$$U_o = 0 \text{ V}$$

$$U_A = -0.7 \text{ V}$$

$$U_- = 0 \text{ V}$$

$$(c) U_I = -1 \text{ V}$$

$$U_o = 1 \text{ V}$$

$$U_A = 1.7 \text{ V}$$

$$U_- = 0 \text{ V}$$

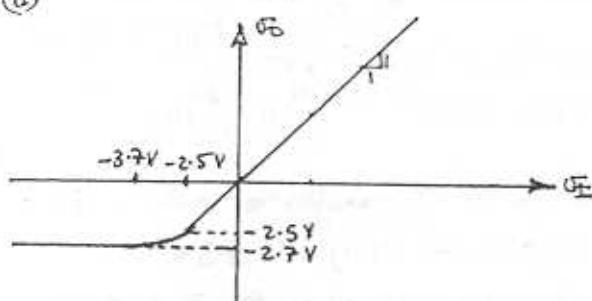
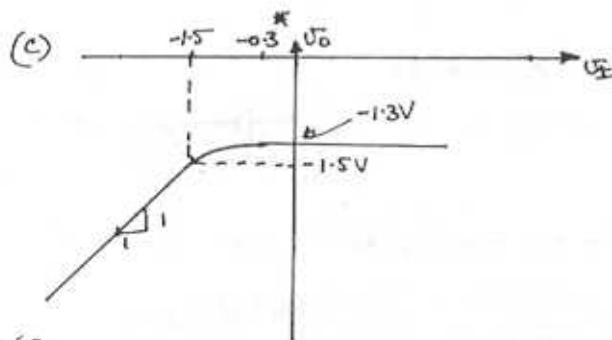
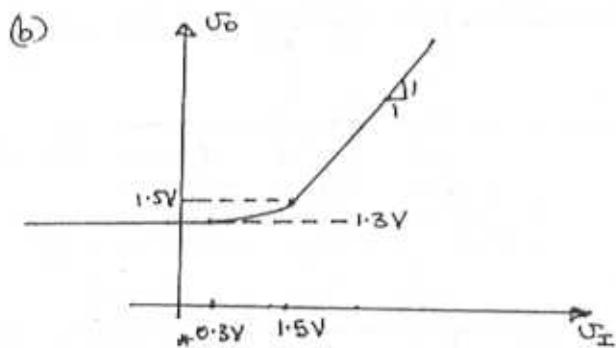
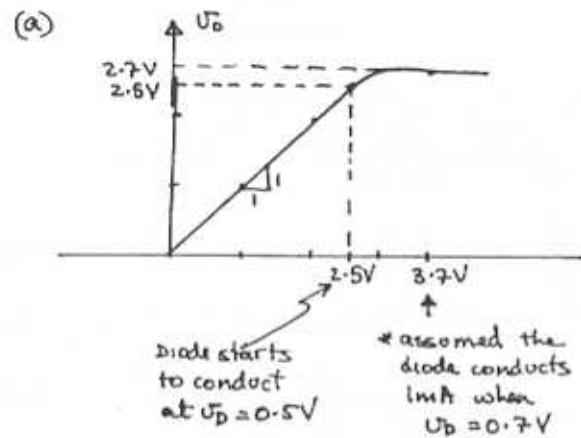
~ virtual ground as negative feedback is closed through  $R$ .

$$(d) U_I = -2 \text{ V} \Rightarrow U_o = 2 \text{ V}$$

$$U_A = 2.7 \text{ V}$$

$$U_- = 0 \text{ V}$$

3.93

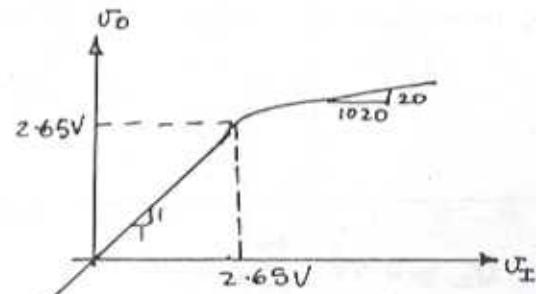
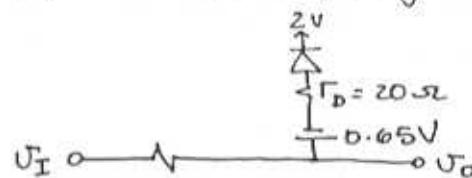


3.94

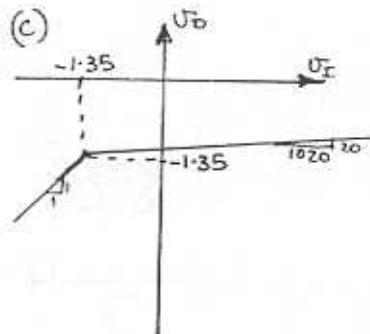
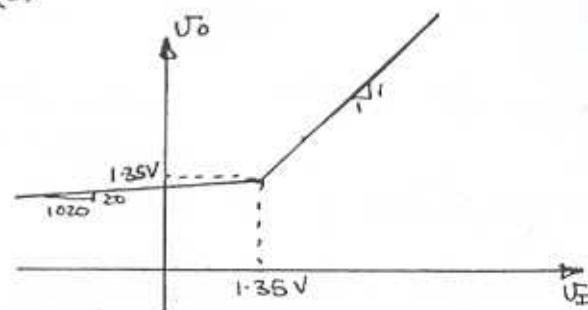
(a)  $U_x < 2.65V \rightarrow$  the diode is off and the circuit reduces to:



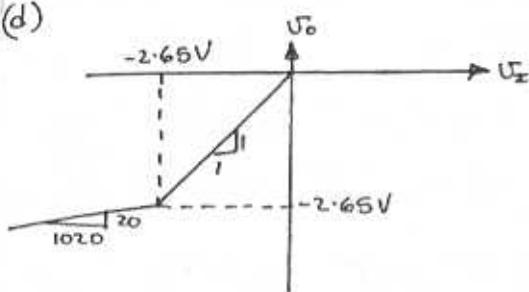
With the diode conducting we have:



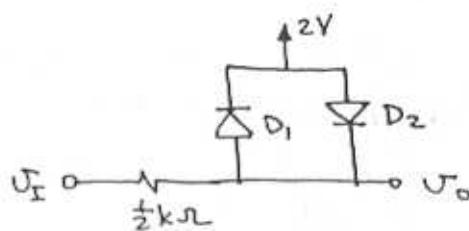
Similarly for (b), (c), (d)



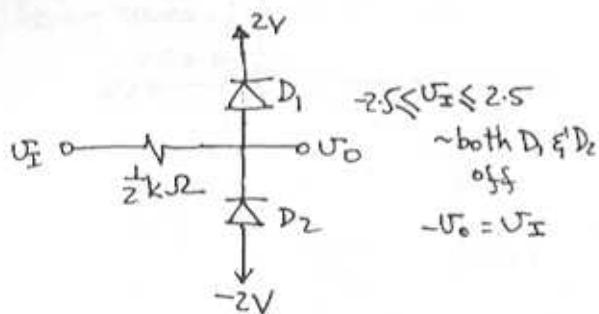
(d)



3.96

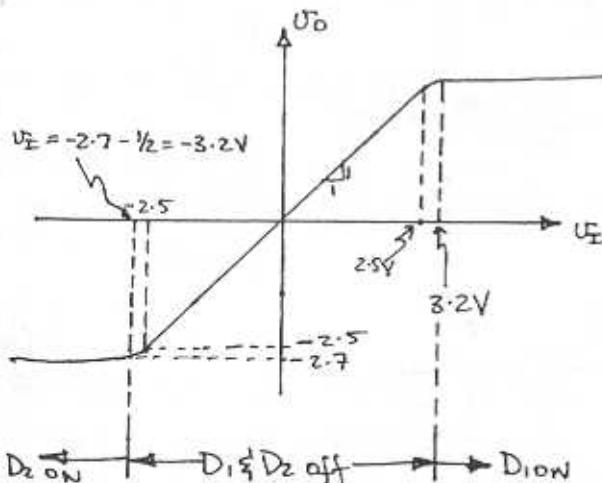
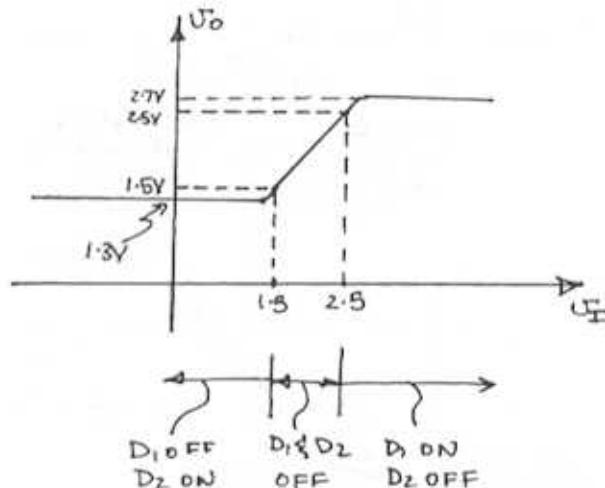


3.95

For  $U_I \geq 2.5V \sim D_1$  on

$$U_{D1} = 0.7 \text{ at } i_{D1} \geq 1 \text{ mA}$$

$$U_o = 2.7V \text{ at } U_I = 2.7 + \frac{1}{2} \times 1 \\ = \underline{\underline{3.2V}}$$

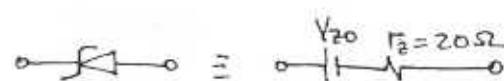


3.97

For each diode



For the zener-diode



$$g \cdot z = V_{z0} + 10 \times 10^{-3} \times 20$$

$$V_{z0} = 8.0V$$

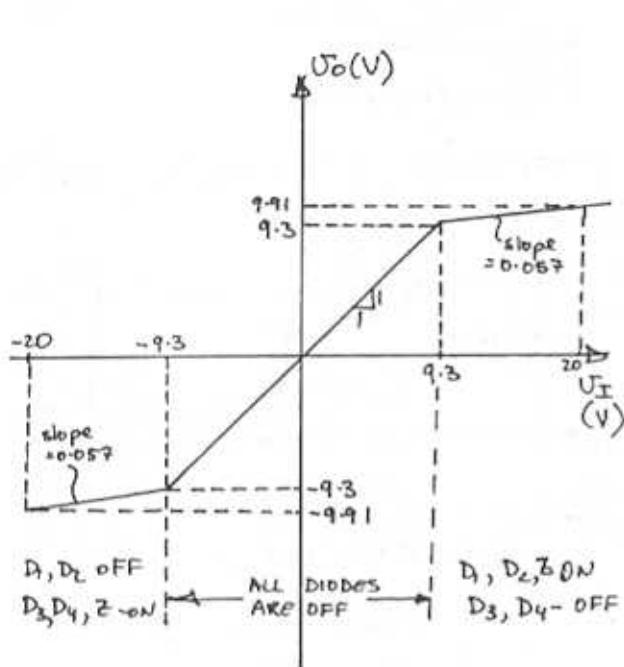
The limiter thresholds are

$$\pm (2 \times 0.65 + 8.0) = \pm 9.3V$$

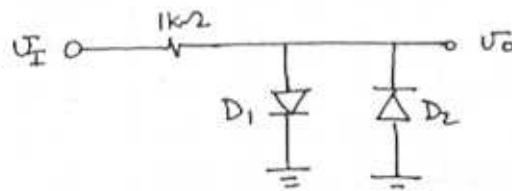
For  $U_I > 9.3V$  (as well as for  $U_I < -9.3V$ )

$$\frac{\partial U_o}{\partial U_I} = \frac{r_{D1} + r_2 + r_{D2}}{1k\Omega + r_{D1} + r_2 + r_{D2}} = \frac{3(20)}{1k\Omega + 3(20)} = 0.057 \frac{V}{V}$$

CONT.



3.98



For D<sub>1</sub>

$$\text{Given } \frac{i_D}{i_{mA}} = e^{\frac{U_o - 0.7}{nV_T}}$$

$$(U_o - 0.7) = nV_T \ln\left(\frac{i_D}{i_{mA}}\right)$$

$$= 0.1 \log\left(\frac{i_D}{10^{-3}}\right) \leftarrow \text{can find } U_o \text{ from } i_D$$

$$\therefore i_D = 10^{-3} \times 10^{\frac{U_o - 0.7}{0.1}}$$

$$= 10^{-3} \times 10^{10(U_o - 0.7)}$$

$$\therefore U_I = U_o + \frac{i_D \times 10^3}{10(U_o - 0.7)}$$

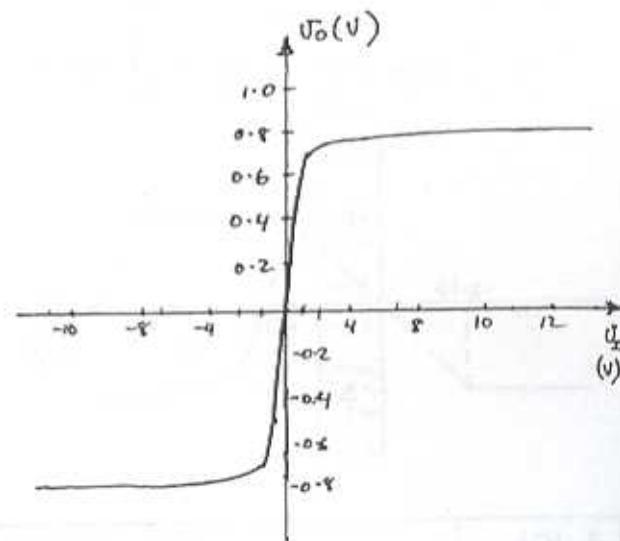
$$= U_o + 10^{-10}(U_o - 0.7)$$

$$\text{for } D_2: U_I = U_o - 10^{-10}(U_o - 0.7)$$

$U_o$ (V)	$U_I$ (V)
0.5	0.510
0.4	0.7
0.7	1.7
0.8	10.7
0	0
-0.5	-0.51
-0.6	-0.7
-0.7	-1.7
-0.8	-10.7

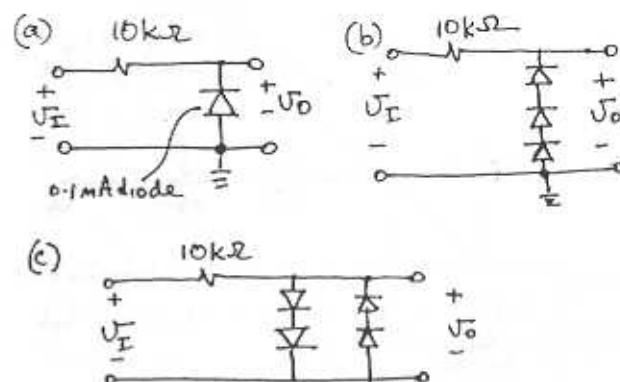
$D_1$  ON

$D_2$  ON

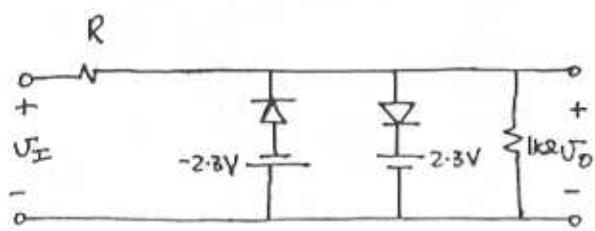


The limiter is fairly hard with a gain  
 $K \approx 1$   
 $L+ \approx 0.8V$ ,  $L- \approx -0.8V$

3.99



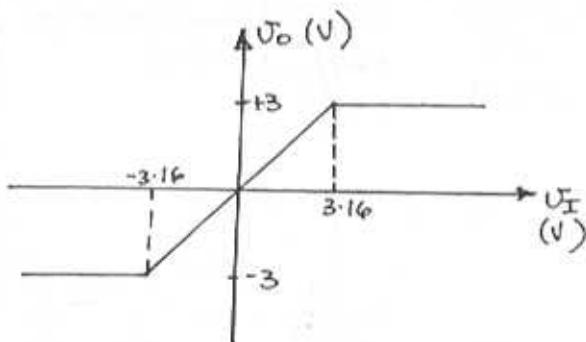
3.100



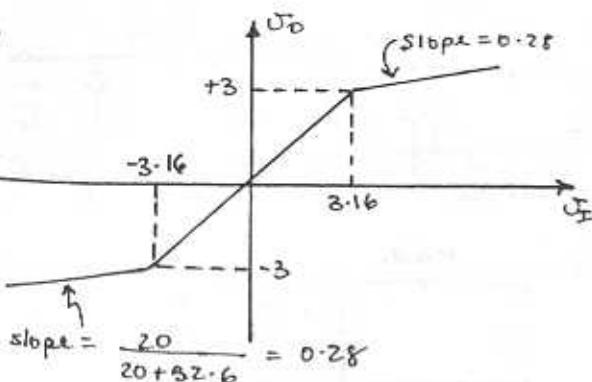
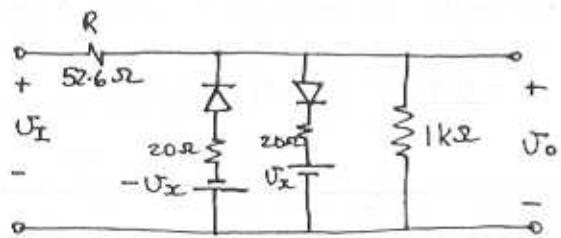
In the limiting region

$$\frac{U_o}{U_I} = \frac{1000}{1000 + R} \geq 0.95$$

$$R \leq \underline{52.6\Omega}$$

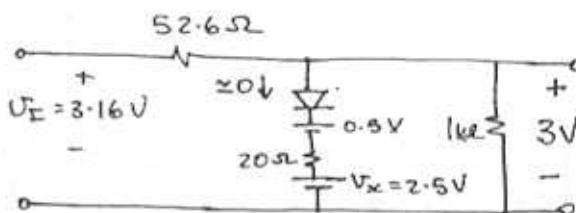


3.101



$$\frac{0.2}{10 \times 10^3} = 20\Omega \sim$$

At the verge of limiting in the positive direction we have :-



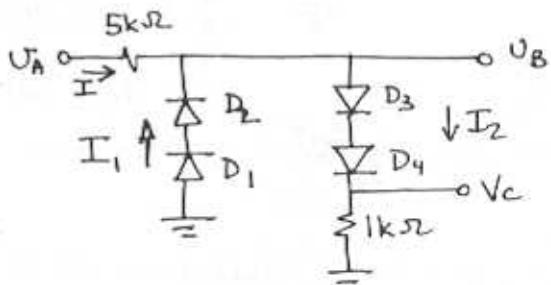
For  $U_I = 10V$

$$U_o \approx 3 + 0.28(10 - 3.16) \\ = \underline{4.9V}$$

For  $U_I = -10V$

$$U_o = \underline{-4.9V}$$

3.102



$$U_B = 0.7 + 0.1 \log \left( \frac{I_D}{0.1} \right)$$

(a) For  $U_A > 0$   $D_1, D_2$  off  $\Rightarrow I_1 = 0$

$$I = I_2 = U_C / 1k\Omega \quad U_A = V_B + I_2 5$$

(b) For  $U_A < 0$   $D_3, D_4$  off  $\Rightarrow U_C = 0$

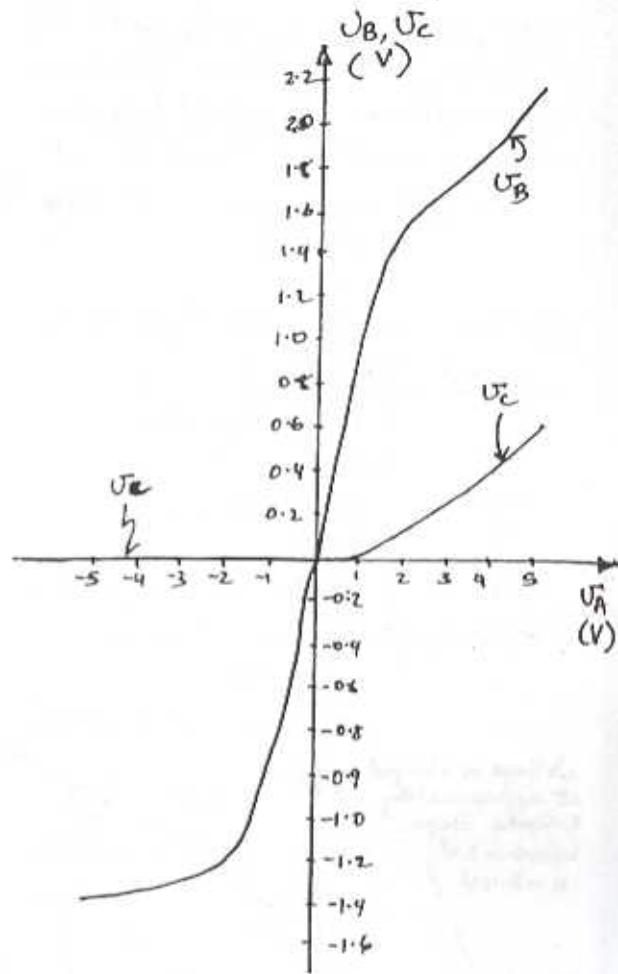
$$I = -I_1 \quad U_B = -(U_{D1} + U_{D2})$$

$$U_A = -(V_B + 5I_1)$$

CONT.

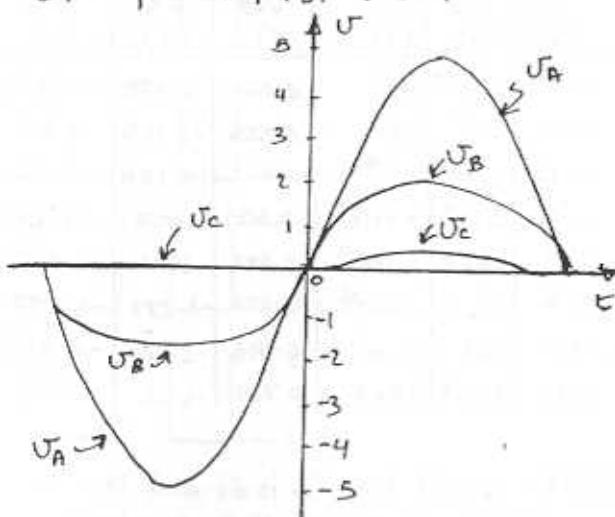
(a) List of points for  $U_A > 0$

$U_C$ (V)	$I_2$ (mA)	$U_{D3}, U_{D4}$ (V)	$U_B = U_C + U_{D3} + U_{D4}$ (V)	$U_A$ (V)
0.0001	0.0001	0.4	0.8	0.8
0.001	0.001	0.5	1.00	1.01
0.01	0.1	0.6	1.21	1.24
0.1	0.1	0.7	1.50	1.90
0.2	0.2	0.73	1.66	2.66
0.3	0.3	0.75	1.80	3.30
0.4	0.4	0.76	1.92	3.92
0.5	0.5	0.77	2.04	4.54
0.6	0.6	0.78	2.16	5.16

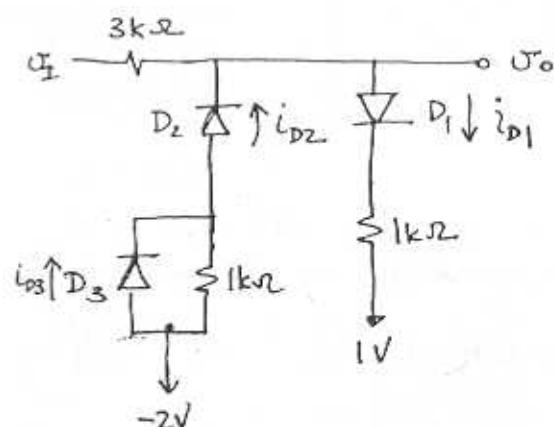


(b) List of Points for  $U_A < 0$

$I_1$ (mA)	$U_D, U_{DC}$ (V)	$U_B$ (V)	$U_A$ (V)
0.0001	0.4	-0.80	-0.80
0.001	0.5	-1.00	-1.01
0.01	0.6	-1.20	-1.25
0.10	0.7	-1.40	-1.90
0.20	0.73	-1.46	-2.46
0.30	0.75	-1.50	-3.00
0.40	0.76	-1.52	-3.52
0.50	0.77	-1.54	-4.04
0.60	0.78	-1.56	-4.56
0.70	0.785	-1.57	-5.07



3.103



At currents  $i_{D1} > 1mA$ ,  $U_{D1} \approx 0.7V$   
Let  $U_{D1} = 0.71V$      $U_A > 5.7V$

CONT.

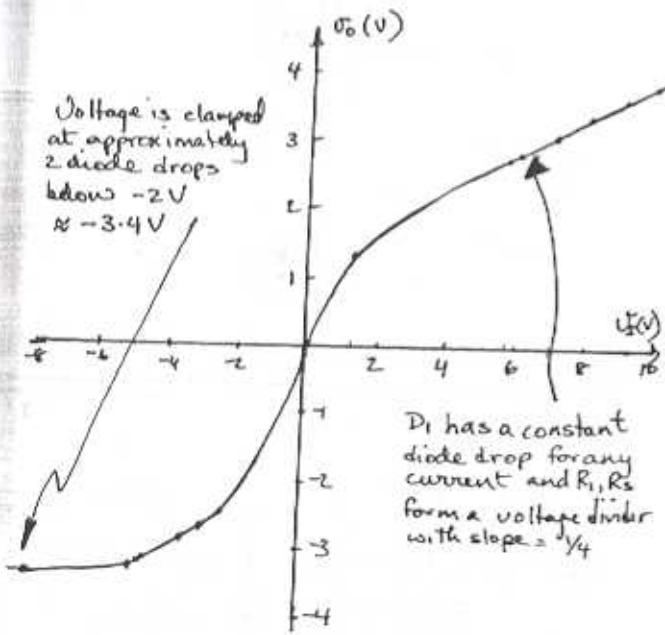
$$U_o = 1.71 + i_{D1} \times 1k\Omega$$

$$= 1.71 + \left( \frac{U_I - 1.71}{4} \right) \times 1$$

$$= \frac{U_I}{4} + 1.2825 \quad \text{NB slope} = \frac{1}{4}$$

For  $U_I > 5V$  slope  $\frac{U_o}{U_I} \approx \frac{1}{4}$

$U_I(V)$	$U_o(V)$
5.8	2.7325
6.0	2.7825
7.0	3.0325
8.0	3.2825
9.0	3.5325
10.0	3.7825



where points for  $-8 \leq U_I \leq 6 V$  are calculated as shown below:

$$i_D = 1mA \text{ at } U_D = 0.7V \quad n=1$$

$$i_D = I_S e^{\frac{U_D - 0.7}{0.025}} = 10^{-3}$$

$$I_S = 6.914 \times 10^{-16} A$$

For Diodes use  $i_D = 6.914 \times 10^{-16} e^{\frac{U_D - 0.7}{0.025}}$

D<sub>1</sub> conducting  $i_{D2} = 0$

$i_{D1}$ (A)	$U_{D1}$ (V)	$U_o$ (V)	$U_I = (4k)i_{D1} + U_{D1}$ (V)
$10^{-6}$	0.297	1.297	1.297 even at small $i_{D1}$
$10^{-5}$	0.527	1.528	$1.5313 \leftarrow U_o > IV, U_o \propto U_I$
$10^{-4}$	0.584	1.595	$1.625 \leftarrow$ since $i_{D1} \ll 0$
$10^{-3}$	0.64	1.742	2.042
$10^{-2}$	0.70	2.7	5.7
$0.2 \times 10^{-2}$	0.74	6.74	12.74
$10^{-1}$	0.758	11.75	41.75

For the D<sub>2</sub>, D<sub>3</sub> arm conducting use the following equations:  
Note  $U_I < -2.5V$

Starting with a value for  $U_A$  we have

$$U_{D3} = U_A + 2 \quad \text{①}$$

$$i_{D3} = I_S e^{\frac{U_{D3} - 0.7}{0.025}} \quad \text{②}$$

$$i_{D2} = i_{D3} + \frac{U_A + 2}{1} \quad \text{③}$$

$$U_{D2} = 0.025 \ln \left( \frac{i_{D2}}{6.914 \times 10^{-16}} \right) \quad \text{④}$$

$$U_o = U_A - U_{D2} \quad \text{⑤}$$

$$U_I = U_o - i_{D2} \times 3k\Omega \quad \text{⑥}$$

① $U_A$ (V)	② $i_{D3}$ (A)	③ $i_{D2}$ (A)	④ $U_{D2}$ (V)	⑤ $U_o$ (V)	⑥ $U_I$ (V)
-2.001	$7 \times 10^{-16}$	$10^{-6}$	0.527	-2.528	-2.531(A)
-2.01	$10^{-15}$	$10^{-5}$	0.585	-2.595	-2.625
-2.10	$3.8 \times 10^{-15}$	$10^{-4}$	0.642	-2.724	-3.024
-2.20	$2 \times 10^{-14}$	$0.2 \times 10^{-3}$	0.659	-2.859	-3.459
-2.5	$33.4 \mu A$	$0.5 \times 10^{-3}$	0.682	-3.128	-4.628(B)
-2.6	$18 \mu A$	$0.6 \times 10^{-3}$	0.687	-3.287	-5.087
-2.7	1mA	$1.7 \times 10^{-3}$	0.713	-3.413	-8.516(C)
-2.71	1.5mA	2.2mA	0.720	-3.43	-10

(A) for small  $i_{D2}$ , D<sub>3</sub> is off and D<sub>2</sub> is on  
 $\therefore i_2$  flows through  $1k\Omega$  resistor

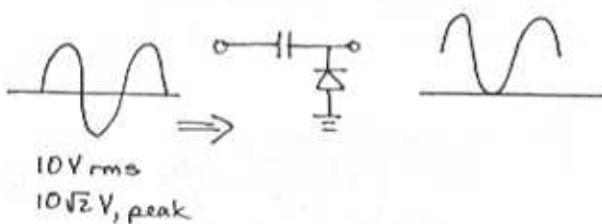
CONT.

(B) 0.5 V drop across  $D_3$  causes  $D_3$  to start to conduct

(C)  $U_A = -2.7V$

The 0.7 voltage across  $D_3$  clamps the voltage across  $R_3$  so that  $D_3$  controls the current  $i_{D2}$

3.104



Average (dc) value of output =  $10\sqrt{2}$   
 $= \underline{\underline{14.14V}}$

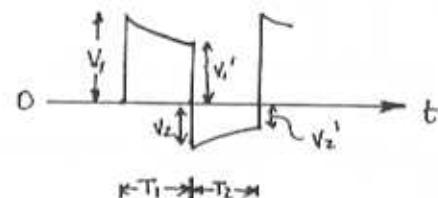
3.105

- (a)
- (b)
- (c)
- (d)
- (e)

(f) Here there are two different time constants involved. To calculate the output levels we shall exaggerate the discharge and charge waveforms.

During  $T_1$ ,  $V_0 = V_1 e^{-t/RC}$

$$\text{At } t = T_1 = T, V_0 = V_1' \\ = V_1 e^{-T/RC}$$



where for  $T \ll CR$

$$V_1' \approx V_1 (1 - T/CR) = V_1 (1 - \alpha) \quad \text{where } \alpha \ll 1$$

During the period  $T_2$

$$|V_0| = |V_2| e^{-t/CR_2}$$

at the end of  $T_2$ ,  $t = T$ ,  $|V_0| = V_2'$

where  $V_2' = |V_2| e^{-T/CR_2}$

$$\approx |V_2| (1 - \frac{T}{CR_2}) = |V_2| (1 - 2\alpha)$$

Now  $|V_1'| + |V_2| = 20 \Rightarrow V_1 + |V_2| - \alpha V_1 = 20 \quad \text{(1)}$

and  $|V_2'| + V_1 = 20 \Rightarrow V_1 + |V_2| - 2\alpha |V_2| = 20 \quad \text{(2)}$

from (1) & (2) we find that

$$V_1 = 2V_2$$

Then using (1) and neglecting  $\alpha V_1$  yields

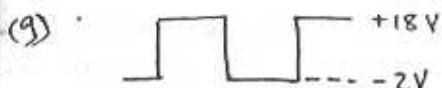
$$3|V_2| = 20 \Rightarrow |V_2| = \underline{\underline{6.67V}}$$

$$V_1 = \underline{\underline{13.33V}}$$

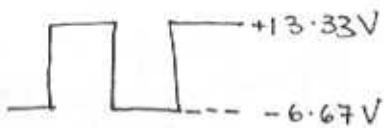
The result is:



CONT.



(h) Using a method similar to that employed for case (f) above we obtain.



3.106

$$n_i^2 = BT^3 e^{-E_g/kT}$$

$$B = 5.4 \times 10^{31}$$

$$E_g = 1.12 \text{ eV for silicon}$$

$$k = 8.62 \times 10^{-5} \frac{\text{eV}}{\text{K}}$$

$$\text{Fraction of ionized atoms} = n_i / 5 \times 10^{22}; T = 273 + x^\circ\text{C}$$

T (K)	$n_i$ (carriers/cm <sup>3</sup> )	Fraction of ionized atoms.
203	$2.68 \times 10^5$	$5.37 \times 10^{-4}$
273	$1.53 \times 10^9$	$3.07 \times 10^{-14}$
293	$8.65 \times 10^9$	$1.73 \times 10^{-13}$
373	$1.044 \times 10^{12}$	$2.89 \times 10^{-11}$
398	$4.75 \times 10^{12}$	$9.51 \times 10^{-11}$

3.107

(a) intrinsic silicon

$$\rho = \frac{1}{q(P_{n0}\mu_p + n_{p0}\mu_n)}$$

$$= \frac{1}{1.6 \times 10^{-9} (1.5 \times 10^{10} \times 480 + 1.5 \times 10^{10} + 1350)}$$

$$= 2.28 \times 10^5 \Omega \cdot \text{cm}$$

$$R = \rho L/A = 2.28 \times 10^5 \frac{10 \times 10^{-4}}{\frac{3.4 \times 10^{-4}}{10^4/3}} = 7.6 \times 10^9 \Omega$$

(b)

$$\rho = \frac{1}{q(P_{n0}\mu_p + n_{p0}\mu_n)}$$

$$= \frac{1}{(1.6 \times 10^{-9}) \left( \frac{(1.5 \times 10^{10})^2}{10^{16}} \times \frac{1200}{2.5} + 10^{16} \times 1200 \right)}$$

$$= 0.521 \Omega \cdot \text{cm}$$

$$R = \rho \times \frac{10^4}{3} = 1.74 \text{ k}\Omega$$

(c) n doped  $N_D = n_{n0} = 10^{18}$

$$n_{p0} = n_i^2 / n_{n0}$$

$$\rho = \frac{1}{1.6 \times 10^{-9} \left( \frac{(1.5 \times 10^{10})^2}{10^{18}} \times \frac{1200}{2.5} + 10^{18} \times 1200 \right)}$$

$$= 5.21 \times 10^{-3} \Omega \cdot \text{cm}$$

$$R = \rho \frac{10^4}{3} = 17.4 \Omega$$

(d) P doped  $N_A = p_{p0} = 10^{10}/\text{cm}^3$

$$\rho = \frac{1}{q(P_{n0}\mu_p + n_{p0}\mu_n)}$$

$$= \frac{1}{1.6 \times 10^{-9} \left( \frac{10^{10} \times 1200}{2.5} + \frac{(1.5 \times 10^{10})^2}{10^{10}} \times 1200 \right)}$$

$$= 2.05 \times 10^4 \Omega \cdot \text{cm}$$

$$R = \rho \frac{10^4}{3} = 68.35 \text{ k}\Omega$$

(e)  $R = 2.8 \times 10^4/3 = 0.933 \Omega$

3.108

$$P_{n0} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4/\text{cm}^3$$

CONT.

$$\frac{d\rho}{dx} = \frac{1000 \rho_{n0} - \rho_{n0}}{W}$$

$$\cong \frac{2.26 \times 10^{-1}}{5 \times 10^{-4}}$$

$$J_p = -qD_p \frac{d\rho}{dx}$$

$$= -1.6 \times 10^{-19} \times 12 \times \frac{-2.25 \times 10^{-1}}{5 \times 10^{-4}}$$

$$= \underline{\underline{8.64 \times 10^{-8} \text{ A/cm}^2}}$$

3.111

$$E = \frac{1V}{10 \times 10^{-4}} = \frac{1}{10^{-3}} \frac{V}{\text{cm}}$$

$$n_{n0} = N_D \quad \rho_{n0} = \frac{n_i^2}{N_D} \quad \text{assume } \rho_{n0} \ll n_{n0}$$

$$\therefore J_{\text{drift}} = q(\mu_n n + \mu_p p) E = \frac{10^{-3}}{10^{-8}} \frac{\text{A}}{\text{cm}^2}$$

$$= 1.6 \times 10^{-9} (1350 N_D) \frac{1}{10^{-3}} = 10^5$$

$$N_D = 4.6 \times 10^{17} / \text{cm}^3$$

3.109

$$V_{\text{drift},p} = \mu_p E$$

$$= 480 \frac{\text{cm}^2}{\text{V.s}} \times \frac{1}{10^3} \frac{\text{V}}{\text{cm}}$$

$$= \underline{\underline{4.8 \times 10^5 \text{ cm/s}}}$$

$$V_{\text{drift},n} = \mu_n E$$

$$= 1350 \frac{\text{cm}^2}{\text{V.s}} \times \frac{1}{10^3} \frac{\text{V}}{\text{cm}}$$

$$= \underline{\underline{1.35 \times 10^6 \text{ cm/s}}}$$

3.112

$$\rho_{p0} = N_A = 10^{16} / \text{cm}^3 \quad \text{at all temperatures}$$

$$\text{At } 25^\circ\text{C} \cong 300K$$

$$n_i^2 = BT^3 e^{-E_F/kT}$$

$$= 5.4 \times 10^{31} (300)^3 e^{-1.12/8.62 \times 10^{-5}(300)}$$

$$= 2.26 \times 10^{20}$$

$$\rho_{p0} = \frac{n_i^2}{N_A} = \underline{\underline{2.26 \times 10^{-4} / \text{cm}^3}}$$

At  $125^\circ\text{C} \cong 400\text{K}$

$$n_i^2 = 5.4 \times 10^{31} (400)^3 e^{-1.12/8.62 \times 10^{-5}(400)}$$

$$= 2.7 \times 10^{25}$$

$$\rho_{p0} = \frac{n_i^2}{N_A} = \underline{\underline{2.7 \times 10^{-9} / \text{cm}^3}}$$

3.110

$$J_{\text{drift}} = q(n\mu_n + p\mu_p) E \quad \text{A/cm}^2$$

$$I_{\text{drift}} = q(n\mu_n + p\mu_p) E \cdot A$$

$$= 1.6 \times 10^{-9} (10^5 \cdot 1350 + 10^{15} \cdot 480) \times$$

$$\frac{1}{10^{-3}} \times (5 \times 10^{-8})$$

$$= \underline{\underline{15.36 \mu\text{A}}}$$

3.113

DOPING CONCENTRATION	$\mu_n$ $\text{cm}^2/\text{V.s}$	$\mu_p$ $\text{cm}^2/\text{V.s}$	$D_n$ $\text{cm}^2/\text{s}$	$D_p$ $\text{cm}^2/\text{s}$
INTRINSIC	1350	480	34	12
$10^{16}$	1100	400	28	10
$10^{17}$	700	260	18	6
$10^{18}$	360	150	9	4

$$\text{where } D_n = V_T \mu_n = \underline{\underline{0.025 \mu_A}}$$

$$D_p = V_T \mu_p = \underline{\underline{0.025 \mu_P}}$$

3.114

$$V_0 = V_T \ln \left( \frac{N_A N_D}{n^2} \right) = 0.025 \ln \left( \frac{10^{16} 10^{15}}{10^{20}} \right) \\ = 1.27 \text{ V}$$

$$W_{dep} = x_n + x_p = \sqrt{\frac{2E_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0} \\ = \sqrt{\frac{2(11.7)(8.85 \times 10^{-14})}{1.6 \times 10^{-19}} \left( \frac{1}{10^{16}} + \frac{1}{10^{15}} \right) 1.27} \\ = 57 \times 10^{-6} \text{ cm} \\ = \underline{\underline{0.57 \mu\text{m}}}$$

$$\frac{x_n}{x_p} = \frac{N_A}{N_D} \Rightarrow x_n = x_p = \underline{\underline{0.28 \mu\text{m}}}$$

$$q_J = q_N = q_P = q \frac{N_A N_D}{N_A + N_D} A W_{dep} \\ = \underline{\underline{45.6 \times 10^{-15} \text{ C}}}$$

$$C_J = \frac{E_s A}{W_{dep}} = \frac{11.7 (8.85 \times 10^{-14})}{0.57 \times 10^{-9}} \times 100 \times 10^{-8} \\ = \underline{\underline{18.2 \text{ fF}}}$$

3.115

$$V_0 = V_T \ln \left( \frac{N_A N_D}{n_i^2} \right) = 0.025 \ln \left( \frac{10^{16} 10^{15}}{10^{20}} \right) \\ = 0.633 \text{ V}$$

$$W_{dep} = x_n + x_p = \sqrt{\frac{2E_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)}$$

$$W_{dep} = \sqrt{\frac{2(11.7)(8.85 \times 10^{-14})}{1.6 \times 10^{-19}} \left( \frac{1}{10^{16}} + \frac{1}{10^{15}} \right) (0.633 + 5)} \\ = 2.83 \times 10^{-4} \text{ cm} = \underline{\underline{2.83 \mu\text{m}}}$$

$$x_n = \frac{N_A}{N_D} x_p = 10 x_p$$

$$\therefore x_p = \frac{2.83}{11} = \underline{\underline{0.26 \mu\text{m}}}$$

$$x_n = \underline{\underline{2.57 \mu\text{m}}}$$

$$q_J = q_N = q_P = q \frac{N_A N_D}{N_A + N_D} A W_{dep} \\ = 1.6 \times 10^{-19} \left( \frac{10^{16} 10^{15}}{10^{16} + 10^{15}} \right) 400 \times 10^{-8} \times 2.83 \times 10^{-9} \\ = \underline{\underline{1.65 \times 10^{-13} \text{ C}}}$$

$$C_J = \frac{E_s A}{W_{dep}} = \frac{11.7 (8.85 \times 10^{-14})}{2.83 \times 10^{-9}} \times 400 \times 10^{-8} \\ = 14.6 \times 10^{-15} = \underline{\underline{14.6 \text{ fF}}}$$

3.116

$$q_J = q_N \times A = 1.6 \times 10^{-19} \times 10^{16} \times 0.1 \times 10^{-4} \times \\ 100 \times 10^{-8} = \underline{\underline{16 \text{ fC}}}$$

3.117

$$q_J = q \frac{N_A N_D}{N_A + N_D} A W_{dep} \\ = q \left( \frac{N_A N_D}{N_A + N_D} A \right) \sqrt{\frac{2E_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)}$$

CONT.

$$\frac{dQ_S}{dV_R} = \mu \frac{N_A N_D}{N_A + N_D} A \times \frac{1}{\sqrt{\frac{2\varepsilon_0}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)}} \\ \times \frac{2\varepsilon_0}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right)$$

$$= \frac{e_s A}{W_{dep}}$$

3.118

$$C_J = \frac{C_{in}}{(1 + V_R N_D)^m}$$

$$V_R = 1V \quad C_J = \frac{0.4 \text{ pF}}{(1 + 10/0.75)^{1/3}} = \underline{0.45 \text{ pF}}$$

$$V_R = 10V \quad C_J = \frac{0.6 \text{ pF}}{(1 + 10/0.75)^{1/3}} = \underline{0.25 \text{ pF}}$$

3.119

$$V_Z = 10$$

$$I_D = \frac{1}{2} \left( \frac{0.25}{10} \right) = \underline{12.5 \text{ mA}}$$

Since breakdown occurs only half the time, the average breakdown current can be twice the continuous value i.e. it can be 25 mA if the dissipation is limited to half the rated value or 50 mA if the dissipation is allowed to rise to the rated value.

3.120

$$I_P = A q n_i^2 \left( \frac{D_p}{L_p N_D} \right) \left( e^{V_R} - 1 \right)$$

$$I_n = A q n_i^2 \left( \frac{D_n}{L_n N_A} \right) \left( e^{V_R} - 1 \right)$$

$$\therefore \frac{I_P}{I_n} = \frac{D_p L_n N_A}{D_n L_p N_D} = \frac{10 \times 10 \times 10^{-16}}{20 \times 5 \times 10^{-16}} = \underline{\underline{100}}$$

$$I = I_P + I_n = 100 I_n + I_n = 100 I_n$$

$$I_n = 1/101 = \underline{9.9 \mu A}$$

$$I_P = 100 (9.9) = \underline{990 \mu A}$$

3.121

$$I_P = A q n_i^2 \frac{D_p}{L_p N_D} \left( e^{V_R} - 1 \right)$$

$$I_n = A q n_i^2 \frac{D_n}{L_n N_A} \left( e^{V_R} - 1 \right)$$

For  $P^+ - n \quad N_A \gg N_D$

$$\therefore I \approx I_P = A q n_i^2 \frac{D_p}{L_p N_D} \left( e^{V_R} - 1 \right)$$

$$I_S \approx A q n_i^2 \frac{D_p}{L_p N_D} = A q n_i^2 \left( \frac{D_p}{\sqrt{D_p T_p} N_D} \right)$$

$$= 10^4 (10^{-8}) 1.6 \times 10^{-19} \frac{(1.5 \times 10^{10})^2}{\sqrt{10 \times 0.1 \times 10^{-16}}} \times 5 \times 10^6$$

$$= \underline{0.72 \times 10^{-15} A}$$

CONT.

$$I = I_s (e^{V/V_T} - 1) = 0.2 \times 10^{-3}$$

$$0.72 \times 10^{-5} (e^{V/0.025} - 1) = 0.2 \times 10^{-3}$$

$$V = \underline{0.684 V}$$

Excess minority charge

$$= Q_p + Q_n$$

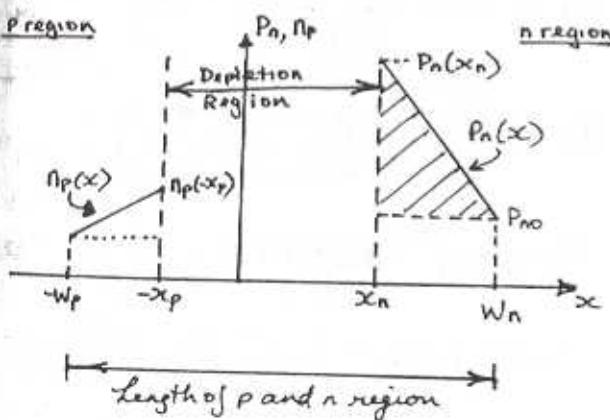
$$= \tau_p I_p + \tau_n I_n \cong \tau_p I_p$$

$$= (0.1 \times 10^{-6}) (0.2 \times 10^{-3}) = \underline{2 \times 10^{-11} C}$$

$$C_d = \frac{\tau_p I}{V_T} \cong \frac{\tau_p I}{V_T}$$

$$= \frac{0.1 \times 10^{-6} \times 0.2 \times 10^{-3}}{0.025} = \underline{800 \text{ pF}}$$

3.122



(a) First consider  $I_p$ :

$$P_n(x_n) = P_{n0} e^{V/V_T}$$

$$I_p = A J_p, \text{ and}$$

$$P_{n0} = \frac{n_i^2}{N_D}$$

$$\text{Thus, } I_p = A q n_i^2 \frac{D_p}{(W_n - x_n) N_D} (e^{V/V_T} - 1)$$

$$\& I_n = A q n_i^2 \frac{D_n}{(W_p - x_p) N_A} (e^{V/V_T} - 1)$$

$$\text{Thus: } I = I_p + I_n$$

$$= A q n_i^2 \left[ \frac{D_p}{(W_n - x_n) N_D} + \frac{D_n}{(W_p - x_p) N_A} \right] \times (e^{V/V_T} - 1) \quad \underline{\text{Q.E.D}}$$

The excess charge,  $Q_p$ , can be found by multiplying the area of the shaded triangle of the  $P_n(x)$  distribution graph by  $Aq$ .

$$Q_p = A q n \frac{1}{2} [P_n(x_n) - P_{n0}] [W_n - x_n]$$

$$= \frac{1}{2} A q P_{n0} (e^{V/V_T} - 1) (W_n - x_n)$$

$$= \frac{1}{2} A q \frac{A l^2}{N_D} (W_n - x_n) (e^{V/V_T} - 1)$$

$$= \frac{1}{2} \frac{(W_n - x_n)^2}{D_p} I_p$$

$$\approx \frac{1}{2} \frac{W_n^2}{D_p} I_p \quad \text{for } W_n \gg x_n$$

Q.E.D. //

$$(c) C_d = \frac{dQ}{dV} = \tau_T \frac{dI}{dV} \quad \text{But } I = I_s (e^{V/V_T} - 1)$$

$$\therefore C_d \approx \frac{\tau_T I}{V_T} \quad \frac{dI}{dV} = \frac{I_s e^{V/V_T}}{V_T} \approx I/V_T$$

$$(d) C_d = \frac{1}{2} \frac{W_n^2}{l^2} \times \frac{10^{-3}}{0.025} = 8 \times 10^{-12}$$

$$W_n = \underline{63.2 \mu M}$$

## Chapter 4 - Problems

4.1

The capacitance per unit area is:  $C_{ox} = \frac{E_{ox}}{t_{ox}}$

$$E_{ox} = 3.45 \times 10^{11} \text{ F/m}^2$$

$$t_{ox} = 5 \text{ nm} \Rightarrow C_{ox} = \frac{3.45 \times 10^{11}}{5 \times 10^{-9}} = 6.9 \text{ fF}/\mu\text{m}^2$$

$$t_{ox} = 20 \text{ nm} \Rightarrow C_{ox} = 0.86 \text{ fF}/\mu\text{m}^2$$

For 1PF capacitance, we require an area A:

$$A = \frac{10^{-12}}{6.9 \times 10^{-15}} = 145 \mu\text{m}^2 \text{ For } t_{ox} = 5 \text{ nm}$$

$$A = \frac{10^{-12}}{0.86 \times 10^{-15}} = 116.3 \mu\text{m}^2 \text{ For } t_{ox} = 20 \text{ nm}$$

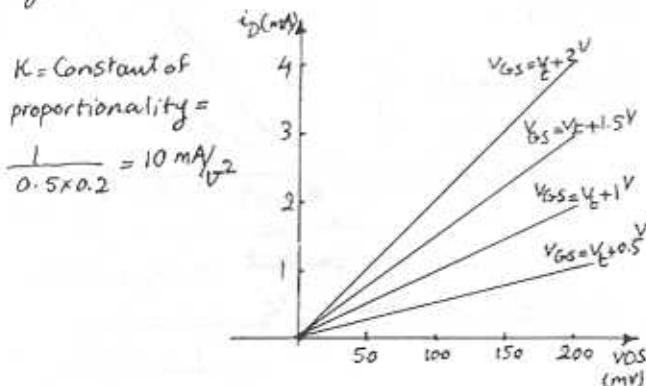
For a square plate capacitor of 10PF:

$$A = 10 \times 145 = 1450 \mu\text{m}^2 \text{ or } 38 \times 38 \mu\text{m}^2 \text{ square for } t_{ox} = 5 \text{ nm}$$

$$A = 10 \times 116.3 = 11630 \mu\text{m}^2 \text{ or } 108 \times 108 \mu\text{m}^2 \text{ square for } t_{ox} = 20 \text{ nm}$$

4.2

Drain current is directly proportional to the width of the channel. Therefore if width is 10 times greater, then  $i_D$  would be 10 times greater as well.



$$K = \text{Constant of proportionality} = \frac{1}{0.5 \times 0.2} = 10 \text{ mA/V}^2$$

$$r_{DS} = \frac{i_D}{V_{DS}} = \frac{1}{0.2} = 5 \text{ k}\Omega \text{ for } V_{OV} = 0.5 \text{ V}$$

$$r_{DS} = 10 \text{ k}\Omega \text{ for } V_{OV} = 1 \text{ V}$$

$$r_{DS} = 15 \text{ k}\Omega \text{ for } V_{OV} = 1.5 \text{ V}$$

$$r_{DS} = 20 \text{ k}\Omega \text{ for } V_{OV} = 2 \text{ V}$$

$$5 \text{ k}\Omega \leq r_{DS} \leq 20 \text{ k}\Omega \text{ for } 0.5 \leq V_{OV} \leq 2 \text{ V}$$

4.3

$$\text{eq. 4.6a : } i_D = \frac{1}{2} K_n \frac{W}{L} (V_{GS} - V_t)^2 \quad K_n = \mu_n C_{ox}$$

for equal drain currents:

$$\mu_n C_{ox} \frac{W_n}{L} = \mu_p C_{ox} \frac{W_p}{L} \Rightarrow \frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = \frac{1}{0.4} = 2.5$$

4.4

$$\text{for small } V_{DS}: \quad i_D = K_n \frac{W}{L} (V_{GS} - V_t) V_{DS}$$

$$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{K_n \frac{W}{L} (V_{GS} - V_t)} = \frac{1}{50 \times 10^{-6} \times 20 \times (5 - 0.8)}$$

$$r_{DS} = 238 \Omega \quad V_{DS} = r_{DS} \times i_D = 238 \text{ mV}$$

for the same performance of a p-channel device:

$$\frac{W_p}{W_n} = \frac{\mu_n}{\mu_p} = 2.5 \Rightarrow \frac{W_p}{L} = \frac{W_n \times 2.5}{L} = 20 \times 2.5$$

$$\Rightarrow \frac{W_p}{L} = 50$$

4.5

$$\text{Eq. 4.5a : } i_D = K_n \frac{W}{L} \left[ (V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \text{ triode region}$$

for small  $V_{DS}$ :

$$i_D = K_n \frac{W}{L} (V_{GS} - V_t) V_{DS}$$

$$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{K_n \frac{W}{L} (V_{GS} - V_t)}$$

for  $r_{DS} = 1 \text{ k}\Omega$ :

$$K_n = 100 \text{ mA/V}^2 : \quad 1000 = \frac{1}{100 \times 10^{-6} \frac{W}{L} \times (5 - 0.8)}$$

$$\Rightarrow W = 2.4 \mu\text{m}$$

4.6

$$\text{a) } C_{ox} = \frac{E_{ox}}{t_{ox}} = \frac{3.45 \times 10^{11}}{15 \times 10^{-9}} = 2.3 \text{ fF}/\mu\text{m}^2$$

$$K_n = \mu_n C_{ox} = 550 \times 10^{-4} \times 2.3 \times 10^{-3} = 126.5 \text{ mA/V}^2$$

$$\text{b) } i_D = \frac{1}{2} K_n \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow 100 = \frac{1}{2} \times 126.5 \times \frac{16}{0.8} (V_{GS} - 0.7)^2$$

$$V_{GS} - 0.7 = 0.28 \Rightarrow V_{OV} = 0.28 \text{ V}$$

$$V_{GS} = 0.98 \text{ V}$$

$$V_{DS\min} = V_{GS} - V_t = 0.28 \text{ V}$$

$$\text{c) For small } V_{DS} : \text{ (triode region)} \quad i_D = K_n \frac{W}{L} V_{OV} \cdot V_{DS}$$

Cont.

$$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{K_n \frac{W}{L} V_{OV}} = \frac{1}{126.5 \times 10^6 \times \frac{16}{0.8} \text{V}} = 1000$$

$$\Rightarrow V_{OV} = 0.4 \text{V}$$

$$V_{GS} = V_{OV} + V_t = 0.4 + 0.7 = 1.1 \text{V}$$

4.9

4.7

$$K'_n = K_n C_{ox} = K_n \frac{\rho_{ox}}{t_{ox}} = 650 \times 10^{-4} \times \frac{3.45 \times 10^{-11}}{20 \times 10^{-9}} = 112.1 \text{mA/V}^2$$

a) triode region:  $V_{DS} < V_{GS} - V_t$

$$i_D = K'_n \frac{W}{L} [(V_{GS} - V_t)V_{DS} - \frac{1}{2} V_{DS}^2]$$

$$i_D = 112.1 \times 10^6 \times 10 \left[ (5 - 0.8) \times 1 - \frac{1}{2} \times 1^2 \right] = 4.15 \text{mA}$$

b) edge of saturation region:  $V_{DS} = V_{GS} - V_t$

$$i_D = \frac{1}{2} K'_n \frac{W}{L} (V_{GS} - V_t)^2 = \frac{1}{2} \times 112.1 \times 10^6 \times 10 \times (1.2)^2 = 0.8 \text{mA}$$

c) triode region:  $V_{DS} < V_{GS} - V_t$

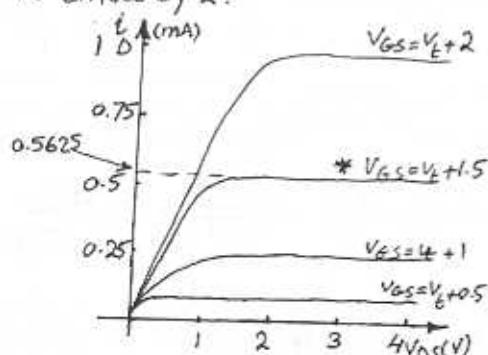
$$i_D = 112.1 \times 10^6 \times 10 \left[ (5 - 0.8) \times 0.2 - \frac{1}{2} \times 0.2^2 \right] = 0.92 \text{mA}$$

d) Saturation region:  $V_{DS} > V_{GS} - V_t$

$$i_D = \frac{1}{2} \times 112.1 \times 10^6 \times 10 \times (5 - 0.8)^2 = 9.9 \text{mA}$$

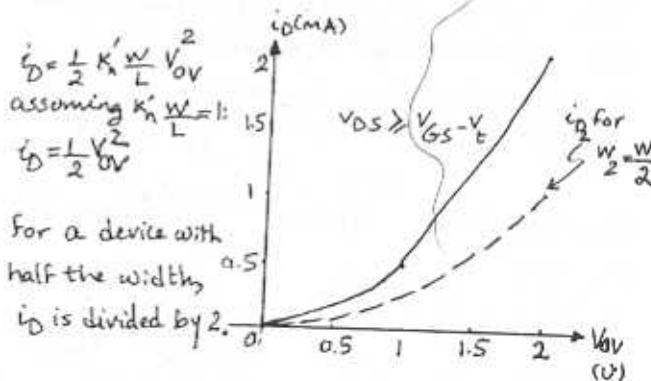
4.8

Refer to Fig. 4.11b,  $i_D \propto \frac{W}{L}$  so if  $W$  is halved, then  $i_D$  would also be halved. The vertical axis,  $i_D$ , has to be divided by 2:



For  $V_{OV} = 1.5 \text{V}$ , by looking at the corresponding curve,  $V_{GS} = V_t + 1.5$ , we observe that:  
 $i_D = 0.5625 \text{ mA}$

4.10



This graph is not dependent on  $V_t$ , while Fig. 4.12 is dependent on  $V_t$ .

4.11

$$\text{Eq. 4.13: } r_{DS} = [K_n \frac{W}{L} (V_{GS} - V_t)]^{-1} \text{ therefore:}$$

$$\frac{r_{DS1}}{r_{DS2}} = \frac{V_{GS2} - V_t}{V_{GS1} - V_t} \Rightarrow \frac{1000}{200} = \frac{V_{GS2} - 1}{1.5 - 1} \Rightarrow V_{GS2} = 3.5 \text{V}$$

Now for a device with twice the width:

Cont.

$$\frac{r_{DS1}}{r_{DS2}} = \frac{W_2(V_{GS2}-V_t)}{W_1(V_{GS2}-V_t)}$$

$$\text{for } V_{GS} = 1.5V \quad r_{DS1} = 2 = r_{DS2} = \frac{1000}{2} = 500\Omega$$

$$\text{for } V_{GS} = 3.5V \quad r_{DS1} = \frac{200}{2} = 100\Omega$$

4.12

$$i_D = \frac{1}{2} K_n \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow 0.2 \times 10 = \frac{1}{2} \times 0.1 \times 10 (V_{GS} - 1)^2$$

$$V_{GS} - 1 = 2 \Rightarrow V_{GS} = 3V$$

$$V_{DSmin} = V_{GS} - V_t = 3 - 1 = 2V$$

$$\text{for } i_D = 0.8mA: 0.8 = \frac{1}{2} \times 0.1 (V_{GS} - 1)^2$$

$$V_{GS} - 1 = 4 \Rightarrow V_{GS} = 5V$$

$$V_{DSmin} = V_{GS} - V_t = 5 - 1 = 4V$$

4.13

$V_{GS} = V_{DS}$  indicates operation in saturation mode:

$$i_D = \frac{1}{2} K_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$4 = \frac{1}{2} K_n \frac{W}{L} (5 - V_t)^2$$

$$1 = \frac{1}{2} K_n \frac{W}{L} (3 - V_t)^2 \Rightarrow 4 = \frac{(5 - V_t)^2}{(3 - V_t)^2} \Rightarrow$$

$$(5 - V_t) = 2(3 - V_t) \Rightarrow V_t = 1V \Rightarrow K_n \frac{W}{L} = 0.5mA/V^2$$

4.14

$$i_D = \frac{1}{2} K_n \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow 0.8 = \frac{1}{2} \times 50 \times 10 \frac{W}{L} (5 - 1)^2$$

$$\frac{W}{L} = 2 \Rightarrow W = 2 \times 2 = 4\mu m$$

4.15

In triode region:  $i_D = K_n \frac{W}{L} [V_{GS} - V_t] \frac{V_{DS} - \frac{V_{DS}}{2}^2]}{2}$

$$\frac{i_{D1}}{i_{D2}} = \frac{60mA}{160mA} = \frac{(2-V_t)0.1 - 0.01/2}{(4-V_t)0.1 - 0.01/2} \Rightarrow V_t = 0.75V$$

if  $K_n' = 50\mu A/V^2$ :  $60 = 50 \frac{W}{L} [(2 - 0.75)0.1 - \frac{0.01}{2}]$

$$\frac{W}{L} = 10$$

If  $V_{GS} = 3V$ ,  $V_{DS} = 0.15V$  then  $i_D = 50 \times 10 \left[ 2.25 \times 0.15 - \frac{0.01}{2} \right]$

$$i_D = 163.125\mu A$$

IF  $V_{GS} = 3V$ , the channel reach pinchoff at

$$V_{DS} = V_{GS} - V_t = 3 - 0.75 = 2.25V \text{ for which:}$$

$$i_D = \frac{1}{2} \times K_n' \frac{W}{L} (V_{GS} - V_t)^2 = \frac{1}{2} \times 50 \times 10 \times 2.25^2 = 1.3mA$$

$$i_D = 1.3mA$$

4.16

For the channel to remain continuous:

$$V_{DS} \leq V_{GS} - V_t \Rightarrow V_{DSmax} = 1.5 - 0.8 = 0.7V$$

4.17

$$\text{Eq. 4.15: } r_{DS} = \left[ K_n' \frac{W}{L} V_{ov} \right]^{-1} = \frac{1}{50 \times 10 \frac{W}{L} (V_{GS} - 1)} M\Omega$$

$$r_{DS} = \frac{1}{V_{GS} - 1} K_n' \Omega$$

$$V_{GS} = 1.1V \Rightarrow r_{DS} = 10K\Omega \Rightarrow 100\Omega \leq r_{DS} \leq 10K\Omega$$

$$V_{GS} = 11V \Rightarrow r_{DS} = 100\Omega$$

- a)  $r_{DS} \propto \frac{1}{W}$  so if  $W$  is halved,  $r_{DS}$  is doubled:  
 $200\Omega \leq r_{DS} \leq 20K\Omega$
- b)  $r_{DS} \propto L$  so if  $L$  is halved,  $r_{DS}$  is also halved:  $50\Omega \leq r_{DS} \leq 5K\Omega$
- c)  $r_{DS} \propto \frac{1}{W}$  so if both  $W$  and  $L$  are halved,  $r_{DS}$  stays unchanged and so does  $r_{DS}$ .  
 $100\Omega \leq r_{DS} \leq 10K\Omega$

4.18

According to eq. 4.17:  $V_{GD} \leq V_t$  for saturation region. Since  $V_{GD}$  is zero in Fig. P4.18, then the device is always in saturation region:

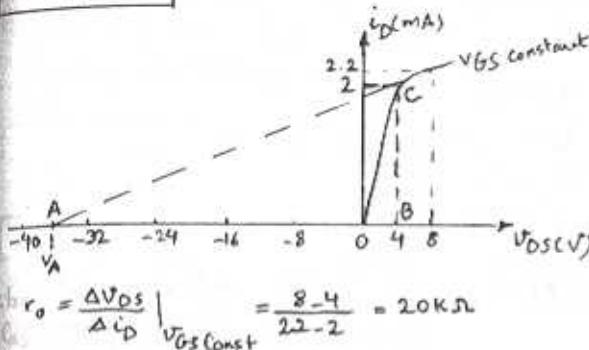
$$i_D = i = \frac{1}{2} K_n' \frac{W}{L} (V_{GS} - V_t)^2$$

if we replace  $V_{GS}$  by  $V$  and  $K' = \frac{1}{2} K_n'$

$$i = K' \frac{W}{L} (V - V_t)^2$$

$$r = \left( \frac{\partial i}{\partial V} \right)^{-1} = \frac{1}{2K' \frac{W}{L} (V - V_t)} = \frac{1}{K' \frac{W}{L} V_{ov}} \text{ where } V = |V_t| + V_{ov}$$

4.19



To calculate  $V_A$ , consider the ABC triangle:

$$V_A + 4 = 2 \text{ mA} \times r_0 = 2 \times 20 = 40 \text{ V} \Rightarrow V_A = 36 \text{ V}$$

$$\lambda = \frac{1}{V_A} = 0.028 \text{ V}^{-1}$$

For  $L=3 \mu\text{m}$  :  $V_A = 50 \times 3 = 150 \text{ V}$

$$r_0 = \frac{V_A}{I_D} = \frac{150}{0.08} = 1875 \text{ k}\Omega$$

$$r_0 = \frac{\Delta V_{DS}}{\Delta i_D} \Rightarrow \Delta i_D = \frac{\Delta V_{DS}}{r_0} = \frac{5-1}{1875} = 2.13 \mu\text{A}$$

For  $V_{DS}$  raised from 1 V to 5 V,  $i_D$  increases from  $80 \mu\text{A}$  to  $82.13 \mu\text{A}$ .

$$\frac{\Delta i_D}{i_D} = 2.7\% \text{ Change in } i_D.$$

In order to reduce  $\frac{\Delta i_D}{i_D}$  by a factor of 2,  $i_D$  has to be halved, or equivalently  $r_0$  has to be doubled. In order to double  $r_0$ ,  $V_A$  has to be doubled and this can be done by doubling the length.  $L = 2 \times 3 = 6 \mu\text{m}$

4.20

Eq. 4.26 :  $r_0 = \frac{V_A}{i_D} = \frac{50}{0.1} = 500 \text{ k}\Omega$

$$r_0 = \frac{\Delta V_{DS}}{\Delta i_D} \Rightarrow \Delta i_D = \frac{\Delta V_{DS}}{r_0} = \frac{1}{500}$$

$i_D = 0.1 \text{ mA}$     $\Delta i_D = 2 \mu\text{A}$     $\frac{\Delta i_D}{i_D} = 2\%$

$i_D = 1 \text{ mA}$     $\Delta i_D = 20 \mu\text{A}$     $\frac{\Delta i_D}{i_D} = 2\%$

4.23

$$V_A = V_A L = 20 \times 1.6 = 32 \text{ V} , \lambda = \frac{1}{V_A} = 0.031 \text{ V}^{-1}$$

$V_{DS} = 2 \text{ V} > (V_{GS} - V_E) = 0.5 \text{ V} \Rightarrow$  device in saturation region

$$i_D = \frac{L}{2} K_n \frac{W}{L} \frac{V^2}{0 \text{ V}} = \frac{1}{2} \times 130 \times \frac{16}{1.6} \times 0.5^2 = 162.5 \mu\text{A}$$

$$r_0 = \frac{V_A}{i_D} = \frac{32}{162.5 \mu\text{A}} = 197 \text{ k}\Omega$$

$$r_0 = \frac{\Delta V_{DS}}{\Delta i_D} \Rightarrow 197 \text{ k}\Omega = \frac{1}{\Delta i_D} \Rightarrow \Delta i_D = 5.1 \mu\text{A}$$

4.21

$V_A = V_A' L$  where  $V_A'$  is completely process dependent. Also,  $r_0 = \frac{V_A}{i_D}$ . therefore to achieve desired  $r_0$  (which is 4 times larger), we should increase  $L$ .  $L = 4 \times 2 = 8 \mu\text{m}$

In order to keep  $I_D$  unchanged,  $\frac{W}{L}$  ratio has to stay unchanged. Therefore:

$$W = 4 \times 10 = 40 \mu\text{m} . \quad (\text{So } \frac{W}{L} \text{ is kept at 5})$$

$$V_A = r_0 i_D = 0.5 \text{ M} \times 100 \mu\text{A} = 50 \text{ V} \quad (\text{For standard})$$

$$V_A = 4 \times 0.5 \text{ M} \times 100 = 200 \text{ V} \quad (\text{For new device})$$

4.24

MOS	1	2	3	4
$\lambda (\text{V}^{-1})$	0.02	0.01	0.1	0.005
$V_A (\text{V})$	50	100	10	200
$I_D (\text{mA})$	5	3.33	0.1	0.2
$r_0 (\text{k}\Omega)$	10	30	100	1000
$r_0 = \frac{V_A}{I_D}$	$, \lambda = \frac{1}{V_A}$			

4.25

$$V_A = \frac{1}{\lambda} = 100 \text{ V} \quad V_A = V_A' L \quad \text{if } L \text{ is doubled, so is } V_A'$$

$$V_A = 2 \times 100 = 200 \text{ V} , \lambda = \frac{1}{200} = 0.005 \text{ V}^{-1}$$

a) if  $V_{GS}$  is fixed, then  $i_D$  is halved, as a result of doubling  $L$ .  $I_D \propto \frac{1}{L} \Rightarrow I_D = \frac{1}{2} = 0.5 \text{ mA}$

(cont.)

4.22

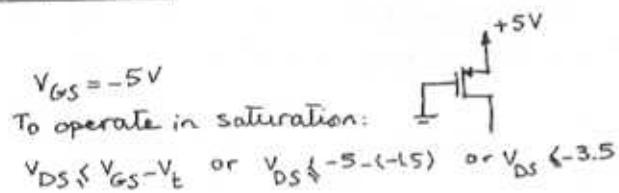
$$\lambda = 0.02 \text{ V}^{-1} \Rightarrow V_A = 50 \text{ V} \text{ for } L = 1 \mu\text{m}$$

$$V_A = V_A' L \Rightarrow V_A' = 50 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{200}{0.5} = 400 \text{ k}\Omega$$

b) If  $I_D$  is fixed:  $r_o = \frac{V_A}{I_D} = \frac{200}{1} = 200 \text{ k}\Omega$

4.26



a)  $V_D = 4V \Rightarrow V_{DS} = -1V > -3.5V \Rightarrow +\text{node region}$

$$i_D = K_p \frac{W}{L} [ (V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 ] \quad \text{eq. 4.29}$$

$$i_D = 80 \left[ (-5 - (-1.5)) \times (-1) - \frac{1}{2} \times 1 \right] = 0.24 \text{ mA}$$

b)  $V_D = 1.5V \Rightarrow V_{DS} = -3.5V = V_{GS} - V_t \Rightarrow \text{edge of saturation}$

$$i_D = \frac{L}{2} K_p \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS}) \quad \text{eq. 4.32}$$

$$i_D = \frac{80}{2} (3.5)^2 (1 + 0.02 \times 1) = 0.52 \text{ mA}$$

c)  $V_D = 0 \Rightarrow V_{DS} = -5V < -3.5 \Rightarrow \text{saturation}$

$$i_D = \frac{1}{2} \times 80 \times (3.5)^2 (1 + 0.02 \times 5) = 0.54 \text{ mA.}$$

d)  $V_D = -5V \Rightarrow V_{DS} = -10V < -3.5 \Rightarrow \text{saturation}$

$$i_D = \frac{1}{2} \times 80 \times (3.5)^2 (1 + 0.02 \times 10) = 0.59 \text{ mA.}$$

4.27

$$\begin{aligned} V_{GS} &= -3V & V_{SG} &= 3V & V_E &= -1V \\ V_{DS} &= -4V & V_{SD} &= 4V & V_A &= -50V \quad \lambda = -0.02 \end{aligned}$$

$$i_D = \frac{1}{2} K_p \frac{W}{L} (V_{GS} - V_E)^2 (1 + \lambda V_{DS})$$

$$3 = \frac{1}{2} K_p \frac{W}{L} (-3 + 1)^2 (1 + 0.02 \times 4) = 2.16 K_p \frac{W}{L}$$

$$K_p \frac{W}{L} = 1.39 \text{ mA}/V^2$$

4.28

Eq. 4.34 :  $\gamma = \sqrt{29N_A \epsilon_s} \Rightarrow N_A = \frac{\gamma^2 C_{ox}}{29 \epsilon_s}$

$$t_{ox} = 20 \text{ nm} \quad C_{ox} = \frac{C_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}}{20 \times 10^{-9}} = 1.73 \text{ F/F} / \mu\text{m}^2$$

$$N_A = \frac{2 \times 1.73 \times 10^{-6}}{2 \times 1.6 \times 10^{19} \times 11.7 \times 8.854 \times 10^{-14} \times 10^2} = 2.3 \times 10^{22} / \text{m}^3$$

$$N_A = 2.3 \times 10^{16} / \text{cm}^3$$

If  $t_{ox} = 100 \text{ nm}$   $\frac{\gamma_1}{\gamma_2} = \frac{C_{ox2}}{C_{ox1}} = \frac{t_{ox1}}{t_{ox2}} \Rightarrow \frac{0.5}{\gamma_2} = \frac{20}{100}$   
 $\Rightarrow \gamma_2 = 2.5 \sqrt{2}$

If  $\gamma$  is kept at  $0.5\sqrt{2}$ , then when  $t_{ox}$  is changed:

$$\frac{\sqrt{N_{A1}}}{C_{ox1}} = \frac{\sqrt{N_{A2}}}{C_{ox2}} \Rightarrow \frac{\sqrt{N_{A1}}}{\sqrt{N_{A2}}} = \frac{C_{ox1}}{C_{ox2}} = \frac{t_{ox2}}{t_{ox1}} = \frac{100}{20} = 5$$

$$\sqrt{N_{A2}} = \frac{\sqrt{N_{A1}}}{5} \Rightarrow N_{A2} = \frac{N_{A1}}{25} = 9.2 \times 10^{14} / \text{cm}^3$$

4.29

$$\text{Eq. 4.33} \quad V_E = V_{Eo} + \gamma \left[ \sqrt{2C_F + V_{SB}} - \sqrt{2C_F} \right]$$

$$V_{SB} = 0V \Rightarrow V_E = V_{Eo} = 1V$$

$$V_{SB} = 4V \Rightarrow V_E = 1 + 0.5 \left[ \sqrt{0.6+4} - \sqrt{0.6} \right] = 1.69V$$

$$1 \leq V_E \leq 1.69V$$

If  $t_{ox}$  is increased by a factor of 4:

from Eq. 4.34  $\gamma \propto \frac{1}{C_{ox}}$  or  $\gamma \propto t_{ox}$  or  $\frac{\gamma_1}{\gamma_2} = \frac{t_{ox1}}{t_{ox2}}$   
 $\Rightarrow \gamma_2 = 4 \times 0.5 = 2\sqrt{2}$

$$V_E = 1 + 2 \left( \sqrt{0.6+4} - \sqrt{0.6} \right) = 3.74V \text{ for } V_{SB} = 2V$$

$$V_E = 1V \text{ for } V_{SB} = 0$$

Therefore for  $\gamma = 2$ :  $1 \leq V_E \leq 3.74V$

4.30

$$|V_{SB}| = 3V \quad \text{Eq. 4.33} \quad V_E = V_{Eo} + \gamma \left[ \sqrt{2C_F + V_{SB}} - \sqrt{2C_F} \right]$$

$$V_E = -1 - 0.5 \left[ \sqrt{0.6+3} - \sqrt{0.6} \right] = -1.56V$$

4.31

a)  $i_D = \frac{1}{2} K_n \frac{W}{L} (V_{GS} - V_t)^2$

$$\frac{\partial i_D}{\partial T} = \frac{1}{2} \frac{W}{L} \frac{\partial K_n}{\partial T} (V_{GS} - V_t)^2 - K_n \frac{W}{L} (V_{GS} - V_t) \frac{\partial V_t}{\partial T}$$

$$\frac{\partial i_D / i_D}{\partial T} = \frac{\partial K_n / K_n}{\partial T} - 2 \frac{\partial V_t}{\partial T} \frac{1}{V_{GS} - V_t}$$

Cont.

b)  $\frac{\partial V_t}{\partial T} = -2 \text{ mV}/^\circ\text{C}$  For  $\frac{\partial i_D}{\partial T} = -0.2/\text{mV}/^\circ\text{C} = -0.002/\text{C}$

$$-0.002 = \frac{\partial K_n/K_n}{\partial T} - 2 \times (-2)^{nV} \frac{1}{5-1} \Rightarrow$$

$$\frac{\partial K_n/K_n}{\partial T} = -0.003/\text{C} = -0.3\%/\text{C}$$

4.32

Case	Transistor	$V_s$	$V_g$	$V_d$	$I_D$	Type	mode	$\mu_n C_{ox} \frac{W}{L}$	$V_t$
a	1	0	2	5	100	N	Sat.	200	1
	1	0	3	5	400	N	Sat.	200	1
b	2	5	3	-4.5	50	P	Sat.	400	-1.5
	2	5	2	-0.5	450	P	Sat.	400	-1.5
c	3	5	3	4	200	P	Sat.	400	-1
	3	5	2	0	800	P	Sat.	400	-1
d	4	-2	0	0	T2	N	Sat.	100	0.8
	4	-4	0	-3	270	N	Triode	100	0.8

Case a) Transistor 1:  $V_{GS} = 2\text{V}$   $V_{DS} = 5\text{V}$   $I_D = 100\text{mA}$

This must be an NMOS operating in saturation.

$$I_D = 100 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2 - V_t)^2$$

when  $V_{GS} = 3\text{V}$ :  $400 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (3 - V_t)^2 \Rightarrow 4 = \frac{(3 - V_t)^2}{(2 - V_t)^2}$

$$\Rightarrow 2(2 - V_t) = 3 - V_t \Rightarrow V_t = 1\text{V}, \mu_n C_{ox} \frac{W}{L} = 200 \frac{\text{mA}}{\text{V}^2}$$

Case b) Transistor 2:  $V_{GS} = 3.5 = -2\text{V}$   $V_{DS} = -9.5\text{V}$

Therefore  $V_{DS} < V_{GS} - V_t$  regardless of value of  $V_t$ , and the device operates in saturation (PMOS)

$$I_D = 50 = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (-2 - V_t)^2$$

$$450 = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (-3 - V_t)^2 \Rightarrow 9 = \frac{(3 + V_t)^2}{(2 + V_t)^2}$$

$$\Rightarrow 3(2 + V_t) = 3 + V_t \Rightarrow V_t = -1.5\text{V}, \mu_p C_{ox} \frac{W}{L} = 400 \frac{\text{mA}}{\text{V}^2}$$

Case c) Transistor 3:  $V_s = -2\text{V}$   $V_{DS} = -1\text{V}$   $\Rightarrow$  PMOS

This device can be either in saturation or triode region. First, we assume saturation region:

$$I_D = 200 = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (-2 - V_t)^2$$

$$800 = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (-3 - V_t)^2 \Rightarrow 4 = \frac{(3 + V_t)^2}{(2 + V_t)^2}$$

$$\Rightarrow 2(2 + V_t) = 3 + V_t \Rightarrow V_t = -1\text{V}, \mu_p C_{ox} \frac{W}{L} = 400 \frac{\text{mA}}{\text{V}^2}$$

So our assumption was right:

$$(V_{DS} = -1\text{V}) \& (V_{GS} - V_t) = -2 + 1 = -1\text{V}$$

edge of saturation

Case d) Transistor 4:  $V_{GS} = 2\text{V}$   $V_{DS} = 2\text{V} \Rightarrow$  NMOS

So  $V_{DS} > V_{GS} - V_t \Rightarrow$  saturation region

$$\textcircled{1} I_D = 72 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2 - V_t)^2$$

But for  $V_{GS} = 4\text{V}$ ,  $V_{DS} = 1\text{V}$  and considering that  $V_t < 2\text{V}$ , then the device is in triode region:

$$\textcircled{2} 270 = \mu_n C_{ox} \frac{W}{L} [(4 - V_t) \times 1 - \frac{1}{2} \times 1]$$

$$\textcircled{1,2} V_t = 0.8\text{V} \quad \mu_n C_{ox} \frac{W}{L} = 100 \frac{\text{mA}}{\text{V}^2}$$

4.33

$$\textcircled{a} I_D = \frac{1}{2} \mu_n \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow 2 = \frac{1}{2} \mu_n \frac{W}{L} (3 - 1)^2$$

$$\Rightarrow \mu_n \frac{W}{L} = 1 \frac{\text{mA}}{\text{V}^2}$$

$$V_t = V_{DS} = 3\text{V}$$

$$\textcircled{b} V_2 = V_s = V_D - V_{DS} = 1 - 3 = -2\text{V}$$

$$\textcircled{c} V_3 = V_s = V_D - V_{DS} = 0 - (-3) = 3\text{V}$$

$$\textcircled{d} V_4 = V_D = V_s + V_{DS} = 5 + (-3) = 2\text{V}$$

In order to calculate  $R_{max}$  that can be inserted in series with the drain,  $V_{DS}$  has to be equal to  $V_{GS} - V_t$ , so that the device is operating on the edge of saturation:

$|V_{DS}| = 3 - 1 = 2\text{V}$ . Note that since  $i_D$  is the same,  $V_{GS}$  stays the same.

$$\textcircled{a} R_{Dmax} = \frac{3 - 2}{2 \text{ mA}} = 0.5 \text{ k}\Omega$$

$$\textcircled{b} V_2 = -2\text{V} \Rightarrow V_D = -2 + 2 = 0 \Rightarrow R_{Dmax} = \frac{1}{2} = 0.5 \text{ k}\Omega$$

Note that  $V_2$  is fixed through  $V_{GS} = 3\text{V}$ .

$$\textcircled{c} V_{GS} = -3\text{V} \Rightarrow V_s = V_3 = 3\text{V}$$
. Now for  $V_{DS}$  to be  $-2\text{V}$ ,  $V_D$  has to be  $1\text{V}$ .

$$R_{Dmax} = \frac{1}{2 \text{ mA}} = 0.5 \text{ k}\Omega$$

$$\textcircled{d} V_{GS} = -3\text{V} \Rightarrow V_s = V_4 = 2\text{V}$$
. Adding the resistor between  $V_s$  and drain means that  $V_D$  has to be  $5 - 2 = 3\text{V}$  and this leaves  $1\text{V}$  voltage drop on the resistor:  $R_{Dmax} = \frac{1}{2} = 0.5 \text{ k}\Omega$

In order to calculate the largest resistor added to the gates, note that since the gate doesn't draw any current, the value of the resistor is immaterial.

Cont.

Now we calculate  $R_{\text{max}}$ , assuming that the voltage drop across the current source is at least 2V:

a)  $V_1 = 8V$  then  $V_{GS} = 3V \Rightarrow V_S = 8 - 3 = 5V$

$$R_{\text{max}} = \frac{5}{2} = 2.5 \text{ k}\Omega$$

b)  $V_2 = -9 + 2 = -7V$ ,  $V_S = 1 - |V_D| = -2V$

$$R_{\text{max}} = \frac{-2 - (-7)}{2} = 2.5 \text{ k}\Omega$$

c)  $V_3 = 10 - 2 = 8V$ ,  $V_S = 0 + |V_D| = 3V$

$$R_{\text{max}} = \frac{8 - 3}{2} = 2.5 \text{ k}\Omega$$

d)  $V_4 = -5 + 2 = -3V$ ,  $V_S = -3 + |V_D| = 0V$

$$R_{\text{max}} = \frac{5}{2} = 2.5 \text{ k}\Omega$$

4.34

$$I_D = \frac{V_{DD} - V_D}{R_D} \Rightarrow \frac{5 - 0}{R_D} = 1 \text{ mA} \Rightarrow R_D = 5 \text{ k}\Omega$$

$V_D = V_G$   $\Rightarrow$  Saturation

therefore:  $i_D = \frac{1}{2} K_n \frac{W}{L} (V_{GS} - V_t)^2$   
 $= \frac{1}{2} \times 60 \times 10^3 \times \frac{100}{3} (V_{GS} - 1)^2$

$$\Rightarrow V_{GS} = 2V \Rightarrow V_S = -2V$$

$$R_S = \frac{-2 - (-5)}{1} = 3 \text{ k}\Omega$$

4.35

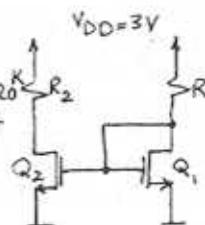
a)  $i_D = \frac{1}{2} K_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$

$$i_D = 0.2 = \frac{1}{2} \times 200 \times 10^3 \times \frac{8}{0.8} (V_{GS} - 0.6)^2$$

$$V_{GS} - 0.6 = \sqrt{0.2}$$

$$V_{GS} = 1.05V$$

$$R = \frac{3 - 1.05}{0.2} = 9.75 \text{ k}\Omega$$



b) For  $Q_2$  to conduct 0.5mA:

$$R_2 = \frac{3 - 1}{0.5} = 4 \text{ k}\Omega$$

Since the gates are connected together, both transistors have the same  $V_{GS}$  and hence same  $i_D$ . In order to conduct 0.5mA or multiply  $i_D$  by 5:  $\frac{W_2}{W_1} = 5 \Rightarrow V_2 = 5 \times 8$

$$W_2 = 40 \mu\text{m}$$

4.36

Refer to Fig. P4.36.

$$R = \frac{3.5}{0.115} = 3.04 \text{ k}\Omega$$

$$0.115 = \frac{1}{2} \times 60 \times 10^3 \frac{W}{0.8} (-1.5 - (-0.7)) \Rightarrow W = 4.8 \mu\text{m}$$

4.37

Refer to Fig. P4.37

$$V_{GS1} = 1.5V, 120 \mu\text{A} = \frac{1}{2} \times 120 \times \frac{W_1}{1} (1.5 - 1)^2$$

$$\Rightarrow W_1 = 8 \mu\text{m}$$

$$V_{GS2} = 3.5 - 1.5 = 2V, 120 = \frac{1}{2} \times 120 \frac{W_2}{1} (2 - 1)^2$$

$$\Rightarrow W_2 = 2 \mu\text{m}$$

$$R = \frac{5 - 3.5}{0.120} = 12.5 \text{ k}\Omega$$

4.38

Refer to Fig. P4.38.

$$V_{GS1} = 1.5V \quad 120 \mu\text{A} = \frac{1}{2} \times 120 \times \frac{W_1}{1} (1.5 - 1)^2 \Rightarrow W_1 = 8 \mu\text{m}$$

$$V_{GS2} = 2V \quad 120 \mu\text{A} = \frac{1}{2} \times 120 \frac{W_2}{1} (2 - 1)^2 \Rightarrow W_2 = 2 \mu\text{m}$$

$$V_{GS3} = 1.5V \quad W_3 = 8 \mu\text{m}$$

4.39

Refer to Fig. 4.23a,  $V_{GS} = 5 - 6I_D \Rightarrow$

$$I_D = \frac{1}{2} \times 2 \times (5 - 6I_D - 2)^2 \Rightarrow 36I_D^2 - 37I_D + 9 = 0$$

$$\Rightarrow I_D = 0.395 \text{ mA or } 0.633 \text{ mA}$$

$I_D = 0.633 \text{ mA}$  is not acceptable, because it results in  $V_{GS} = 1.2V$  which is lower than  $V_t = 2V$ .

Therefore  $I_D = 0.395 \text{ mA} \leq 0.4 \text{ mA}$

$$V_D = 10 - 6I_D = 7.6V$$

These results show that although  $V_t$  and  $K_n \frac{W}{L}$  are doubled,  $I_D$  is only decreased by 20% and  $V_D$  is only increased by 8.5%. Therefore the conclusion is

Contd.

that the circuit is tolerant to changes in device parameters.

4.40

$$V_G = 6V \Rightarrow R_{G_1} = 4M\Omega, R_{G_2} = 6M\Omega$$

$$I_D = 0.5mA \Rightarrow R_D = \frac{5}{0.5} = 10k\Omega$$

$$I_D = 0.5 = \frac{1}{2} \times 1 \times (V_{GS} + 1.5)^2$$

$$V_{GS} = -2.5V \text{ or } -0.5V$$

$$(V_{GS} = -0.5V \text{ is rejected, because it is less than } V_b)$$

$$V_{GS} = -2.5V \Rightarrow V_s = 8.5V \Rightarrow R_s = \frac{10 - 8.5}{0.5} = 3k\Omega$$

In circuit d:  $I_D = 2mA \Rightarrow V_{GS} = -4V \Rightarrow V_b = 6V$   
 $\Rightarrow V_2 = V_b - 4 = 2V$

If we replace the current source with a resistor in each of those circuits:

in circuit a:

$$R = \frac{-4 - (-10)}{2} \approx 3.01k\Omega \quad (\text{by looking at the table for } i_r \text{ resistors})$$

Now recalculate  $I_D$ :  $I_D = \frac{1}{2} \times 1 \times (V_{GS} - V_t)^2$   
 $V_{GS} - V_t = 0 - (-10 + 3.01I_D) - 2 = 8 - 3.01I_D \quad \Rightarrow$

$$2I_D = (8 - 3.01I_D)^2 \Rightarrow I_D = 1.99mA \Rightarrow V_2 = 2.04V$$

$$V_1 = -4.01V$$

4.41

Refer to Fig. P4.41.

$$V_I = V_{GS} = 5V \Rightarrow V_o = V_{DS} = 0.05V$$

$$r_{DS} = 50\Omega = \frac{V_{DS}}{I_D} \Rightarrow I_D = \frac{0.05}{50} = 0.001A = 1mA$$

$$R = \frac{V_{DD} - V_o}{I_D} = \frac{5 - 0.05}{1} = 4.95k\Omega$$

$$V_{DS} < V_{GS} - V_t \Rightarrow \text{triode region}$$

$$I_D = K_n \frac{W}{L} \left[ (V_{GS} - V_t) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$1 = 100 \times 10^3 \frac{W}{L} \left[ (5 - 1) \times 0.05 - \frac{0.05^2}{2} \right] \Rightarrow \frac{W}{L} = 50$$

in circuit b:

$$R = \frac{10 - 3.41}{2} = 6.59 \approx 6.65k\Omega$$

Then  $V_{GS} = 10 - 6.65I$

$$I = \frac{1}{2} \times 1 \times (10 - 6.65I - 2)^2 \Rightarrow I = 0.99mA$$

$$V_3 = 10 - 6.65 \times 0.99 = 3.41V$$

in circuit c:

$$R = \frac{10 - 4}{2} \approx 3.01k\Omega, V_{GS} = -(10 - 3.01I)$$

$$I = \frac{1}{2} \times 1 \times (10 + 3.01I + 2)^2 \Rightarrow I_D = 1.99mA$$

$$V_4 = 10 - 3.01 \times 1.99 = 4.01V$$

$$V_5 = -10 + 2.5 \times 1.99 = -5.03V$$

in circuit d:

$$R = \frac{1}{2} = 1k \text{ so } V_2 \text{ is still } 2V$$

4.43

Refer to Fig. P4.42.

in circuit a:  $V_2 = 10 - 4 \times 2 = 2V$

assume saturation:  $I_D = 2 = \frac{1}{2} \times 1 \times (V_{GS} - 2)^2 \Rightarrow V_{GS} = 4V$

$$\Rightarrow V_1 = -4V, V_{DS} = 6V > V_{GS} - V_t \text{ so our assumption was correct.}$$

circuit b:  $I_D = 1 = \frac{1}{2} \times 1 \times (V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.41V$   
 $V_3 = 3.41V$

circuit c:  $I_D = 2mA \Rightarrow V_{GS} = -4V \Rightarrow V_5 = 4 = V_4$   
 $V_5 = -10 \times 2.5 \times 2 = -5V$

- $V_{GS} = -V_1 \cdot 10 \mu A = \frac{1}{2} \times 0.4 \times 10^3 (V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.22V \Rightarrow V_1 = -1.22V$
- $100 \mu A = \frac{1}{2} \times 0.4 \times 10^3 (V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.71V, V_2 = -1.71V$
- $I = \frac{1}{2} \times 0.4 (V_{GS} - 1)^2 \Rightarrow V_{GS} = 3.23V \Rightarrow V_3 = -3.23V$
- $10 = \frac{1}{2} \times 0.4 \times 10^3 (V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.22V \Rightarrow V_4 = 1.22V$
- $I = \frac{1}{2} \times 0.4 (V_{GS} - V_t)^2 \Rightarrow V_{GS} = 3.24V \Rightarrow V_5 = 3.24V$
- $I = \frac{1}{2} \times 0.4 (5 - 100I - 1)^2 \Rightarrow I = 0.045mA, 0.036mA$   
 $V_6 = 5 - 100 \times 0.036 = 1.4V$

$$g) I = \frac{1}{2} \times 0.4 \times (5 - 1 \times I - 1)^2 \Rightarrow I = 1.38 \text{ mA}$$

$$V_g = 5 - 1.38 \times 1 = 3.62 \text{ V}$$

$$h) I = \frac{1}{2} \times 0.4 \times (5 - 100 - I)^2 \Rightarrow I = 0.045 \text{ mA}, 0.036 \text{ mA}$$

$$V_g = -5 + 100 \times 0.036 = -1.4 \text{ V}$$

Note that  $I = 0.045 \text{ mA}$  in circuits h and f is not acceptable, because it results in  $V_{GS} < V_t$  that is not physically possible.

4.44

Refer to Fig. P4.44.

$$a) V_{GS2} = -V_2, I = \frac{V_2 - (-5)}{1k} = \frac{1}{2} \times 2 \times (-V_2 - 1)^2$$

$$\Rightarrow V_2 + 5 = V_2^2 + 2V_2 + 1 \Rightarrow V_2^2 + V_2 - 4 = 0 \Rightarrow V_2 = 1.55 \text{ V}$$

$$V_2 = -2.56 \text{ V}$$

$V_2 = 1.55 \text{ V}$  is not acceptable because it results in  $V_{GS} < 0$  that is not possible for an NMOS.

Therefore  $V_2 = -2.56 \text{ V}$

$$i_{D1} = i_{D2} \Rightarrow \frac{V_2 - (-5)}{1k} = \frac{1}{2} \times 2(5 - V_1 - 1)^2 \Rightarrow$$

$$2.44 = (4 - V_1)^2 \Rightarrow 4 - V_1 = \pm 1.56 \text{ V} \Rightarrow V_1 = 2.44 \text{ V}$$

$$V_1 = 5.56 \text{ V} X$$

The second answer results in  $V_{GS} = 5 - 5.56 < 0$  which is not acceptable. Therefore  $V_1 = 2.44 \text{ V}$

$$b) \frac{10 - V_3}{1k} = \frac{V_5}{1k} = i_D \Rightarrow 10 - V_3 = V_5 \quad ①$$

$$i_{D1} = \frac{V_5}{1k} = \frac{1}{2} \times 2 \times (V_3 - V_4 - 1)^2 \Rightarrow V_5 = (V_3 - V_4 - 1)^2 \quad ②$$

$$i_{D2} = \frac{V_5}{1k} = \frac{1}{2} \times 2 \times (V_4 - V_5 - 1)^2 \Rightarrow V_5 = (V_4 - V_5 - 1)^2 \quad ③$$

$$②, ③ \Rightarrow V_3 - V_4 - 1 = V_4 - V_5 - 1 \Rightarrow V_5 = 2V_4 - V_3 \quad ④$$

$$①, ④ \Rightarrow 2V_4 - V_3 = 10 - V_3 \Rightarrow V_4 = 5 \text{ V}$$

$$③ \Rightarrow V_5 = (4 - V_5)^2 \Rightarrow V_5^2 - 9V_5 + 16 = 0 \Rightarrow V_5 = 6.55 \text{ V}$$

$$V_5 = 2.45 \text{ V}$$

$V_5 = 6.55$  results in  $i_D = 6.55 \mu\text{A}$ ,  $V_3 = 4.45 \text{ V}$  and this is not physically possible. So  $V_3 = 2.45 \text{ V}$

$$V_3 = 10 - 2.45 = 7.55 \text{ V}$$

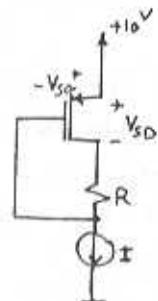
4.45

The PMOS transistor operates in saturation

region if  $V_{SD} \geq V_{SG} - |V_t|$

or  $V_{SD} \geq V_{SG} - 1$

Also,  $V_{SD} + IR = V_{SG} \Rightarrow V_{SD} = V_{SG} - IR$   
 $\Rightarrow IR \leq |V_t|$  For PMOS to be in saturation.



$$a) R = 0 \Rightarrow IR = 0 < |V_t|$$

Saturation:

$$I = 100 = \frac{1}{2} \times 8 \times 25 \times (V_{SG} - |V_t|)^2$$

$$V_{SG} - 1 = \pm 1 \Rightarrow V_{SG} = 2 \text{ V} = V_{SD}$$

$$b) R = 10 \text{ k}\Omega \Rightarrow IR = 10 \times 0.1 = 1 \text{ V} \Rightarrow \text{saturation}$$

$$V_{SG} = 2 \text{ V} \Rightarrow V_{SD} = 2 - 1 = 1 \text{ V}$$

$$c) R = 30 \text{ k}\Omega \Rightarrow IR = 30 \times 0.1 = 3 \text{ V} \Rightarrow \text{triode region}$$

$$100 = 8 \times 25 \left[ (V_{SG} - |V_t|) V_{SD} - \frac{1}{2} V_{SD}^2 \right]$$

$$0.5 = [(V_{SG} - 1)(V_{SG} - 3) - \frac{1}{2} (V_{SG} - 3)^2]$$

$$0.5 = 0.5 V_{SG}^2 - V_{SG} - 1.5 \Rightarrow V_{SG}^2 - 2V_{SG} - 4 = 0$$

$$V_{SG} = 3.24 \text{ V}, -1.2 \text{ V} X$$

$$V_{SD} = 3.24 - 3 = 0.24 \text{ V}$$

$$d) R = 100 \text{ k}\Omega \Rightarrow IR = 100 \times 0.1 = 10 \text{ V} \Rightarrow \text{triode region}$$

$$100 = 8 \times 25 \left[ (V_{SG} - 1)(V_{SG} - 10) - \frac{1}{2} (V_{SG} - 10)^2 \right]$$

$$0.5 = 0.5 V_{SG}^2 - V_{SG} - 40 \Rightarrow V_{SG}^2 - 2V_{SG} - 81 = 0$$

$$V_{SG} = 10.1 \text{ V} \Rightarrow V_{SD} = 0.1 \text{ V}$$

$$e) V_{SD} = V_{SG} \text{ when } R \rightarrow 0$$

f)  $V_{SD} = V_{SG}/2$  For the device in saturation,  $V_{SG} = 2 \text{ V}$  (for  $I = 100 \mu\text{A}$ ), therefore  $V_{SD} = \frac{V_{SG}}{2} = 1 \text{ V}$  implies:  $IR = 1 \text{ V} \Rightarrow R = 10 \text{ k}\Omega$

$$g) V_{SD} = \frac{V_{SG}}{10} \text{ In saturation: } V_{SG} = 2 \text{ V}, V_{SD} = 0.2 \text{ V}$$

$\Rightarrow V_{SD} < V_{SG} - |V_t| \Rightarrow$  therefore

the device can't be in saturation and it is in triode region:  $100 = \frac{1}{2} \times 8 \times 25 \left[ (V_{SG} - 1)(V_{SG}) - \left( \frac{V_{SG}}{10} \right)^2 \right]$

$$0.5 = \frac{V_{SG}^2}{10} - \frac{V_{SG}}{200} - 0.1 V_{SG} \Rightarrow 19 V_{SG}^2 - 20 V_{SG} - 100 = 0$$

$$\Rightarrow V_{SG} = 2.88 \text{ V}, -1.82 \text{ V} X$$

$$V_{SD} = \frac{2.88}{10} = 0.29 \text{ V} \Rightarrow IR = 2.88 - 0.29 = 2.59 \text{ V}$$

$$\Rightarrow R = 25.9 \text{ k}\Omega$$

4.46

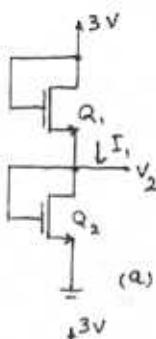
a)  $Q_1, Q_2$  operating in saturation:  $i_{D1} = i_{D2}$

$$\Rightarrow V_{GS1} = V_{GS2}$$

$$3V = V_{GS1} + V_{GS2} \Rightarrow V_{GS1} = V_{GS2} = 1.5V$$

$$V_2 = 1.5V$$

$$I_1 = \frac{1}{2} \times 20 \times \frac{30}{10} (1.5 - 1)^2 = 7.5\mu A$$



$$I_2 = \frac{1}{2} \times 50 \times \frac{100}{10} (2.5 - 1)^2 = 5.625\mu A \quad \text{or}$$

$$\frac{i_{D2}}{i_{D1}} = \frac{W_2}{W_1} = \frac{100}{10} \Rightarrow I_{D2} = 10 \times 562.5\mu A = 5.625\mu A$$

4.48

$$a) \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 20$$

$$V_{GS1} = V_{GS2} = -V_3$$

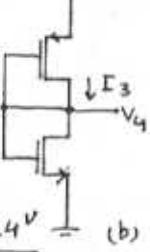
Assuming that the transistors are in saturation, since  $k_n'$  and  $W$  are the same for both  $Q_2$  and  $Q_3$ , we

$$can write: i_{D1} = i_{D2} = \frac{200}{2} = 100\mu A$$

$$100\mu A = \frac{1}{2} \times 100 \times 20 \times (V_{GS} - 1)^2 \Rightarrow 0.1 = (V_{GS} - 1)^2$$

$$V_{GS} = 1.32V = -V_3 \Rightarrow V_3 = -1.32V$$

$$V_1 = 5 - 40 \times 100 \times \frac{1}{2} = 1V$$



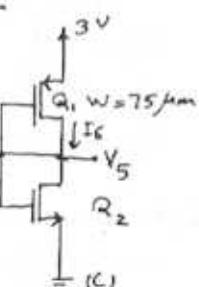
b) Both transistors have  $V_D = V_G$  and therefore they are operating in saturation:  $i_{D1} = i_{D2}$

$$\frac{1}{2} \mu_n C_o x \frac{W}{L} (V_4 - 1)^2 = \frac{1}{2} \mu_p C_o x \frac{W}{L} (3 - V_4)^2$$

$$2.5(V_4 - 1)^2 = (2 - V_4)$$

$$1.58(V_4 - 1) = (2 - V_4) \Rightarrow V_4 = 1.39V = 1.4V$$

$$I_3 = \frac{1}{2} \times 20 \times \frac{30}{10} (1.39 - 1)^2 = 4.6\mu A$$



$$c) \frac{W_1}{L_1} = \frac{75}{10} = 7.5$$

$$\frac{W_1}{L_2} = \frac{30}{10} = 3 \quad \frac{W_1}{L_2} = 2.5$$

$$i_{D1} = i_{D2}$$

$$\text{Since } \mu_n C_o x \frac{W_2}{L_2} = \mu_p C_o x \frac{W_1}{L_1}$$

$$\Rightarrow V_{GS1} = V_{GS2} = \frac{3}{2} = 1.5V = V_5$$

$$I_6 = \frac{1}{2} \times 20 \times \frac{30}{10} (1.5 - 1)^2 = 7.5\mu A$$

4.47

Refer to Fig. P4.47.

Since  $V_G = V_D$ , then  $Q_1$  is in saturation. We assume that  $Q_2$  is also in saturation, then because  $i_{D1} = i_{D2}$ ,  $V_{GS1}$  would be equal to  $V_{GS2}$ .

$$V_{GS1} = V_{GS2} = \frac{5}{2} = 2.5V$$

$$I_1 = \frac{1}{2} \times 50 \times \frac{100}{10} (2.5 - 1)^2 = 562.5\mu A$$

$V_{GS3} = V_{GS1} = 2.5V$ . Since  $Q_3$  and  $Q_4$  have the same drain current, then  $V_{GS3} = V_{GS4} = 2.5V$

This is based on the assumption that  $Q_3$  &  $Q_4$  are saturated:

$$V_{GS3} = V_{GS1} \Rightarrow I_2 = I_{GS3} = I_{GS1} = 562.5\mu A$$

$$V_2 = 5 - 2.5 = 2.5V$$

Now if  $Q_3$  and  $Q_4$  have  $W = 100\mu m$  then:

4.49

a)

$$\text{Point A : } V_I = V_E = 1V, V_{GS} = V_{DD} = 5V$$

For  $V_I < V_E$ , the transistor is not on &  $V_{GS} < V_E$ .

Point A is when  $V_{GS} = V_E$  and the transistor turns on. As  $V_I$  increases, the  $i_D$  increases and  $V_g$  decreases.  $V_g$  decreases to the point that it is below  $V_I$  by  $V_E$  Volts. At this point, B, the MOSFET enters the triode region:  $V_{DS} = V_{IB} - V_E$

$$\text{or } V_{DS} = V_{GS} - V_E$$

$$I = \frac{V_{DD} - (V_{GS} - V_E)}{R} = \frac{1}{2} \times k_n' \frac{W}{L} (V_{GS} - V_E)^2$$

$$\frac{5 - V_{GS} + 1}{24} = \frac{1}{2} \times 1 \times (V_{GS} - 1)^2 \Rightarrow 12V_{GS}^2 - 23V_{GS} + 6 = 0$$

Cont.

$$V_{GS} = 1.61V \Rightarrow V_I = 1.61V \quad V_O = 1.61 - 1 = 0.61V$$

$$\text{Point B: } V_{OB} = 0.61V \quad V_{EB} = 1.61V$$

$$\text{b) } I_Q = \frac{1}{2} \times 1 \times 0.5^2 = 0.125mA$$

$$V_{OQ} = 5 - 24 \times 0.125 = 2V$$

$$V_{EQ} = V_{GS} = V_{OV} + V_b = 0.5 + 1 = 1.5V$$

Now to calculate the incremental gain

$A_V$  at this bias point, From equation 4.41,

$$\text{we have: } A_V = -2 V_{RD} / V_{OV} = -2(V_{DD} - V_{OQ}) / V_{OV}$$

$$A_V = -\frac{2(5-2)}{0.5} = -12V/V$$

c)  $V_{SQ} = 1.5V$ ,  $V_t = 1V$ ,  $V_{IB} = 1.61V$ . Thus the largest amplitude of a sine wave that can be applied to the input while the transistor remains in saturation is:  $1.61 - 1.5 = 0.11V$

The amplitude of the output voltage signal that results is approximately equal to  $V_{OQ} - V_{OB} = 2 - 0.61 = 1.39V$ . The gain implied by this amplitudes is:  
gain =  $\frac{-1.39}{0.11} = 12.64V/V$

This gain is 5.3% different from the incremental gain calculated in part(b). This difference is due to the fact that the segment of the voltage transfer curve considered here is not perfectly linear.

4.50

$$V_{DS} = 1V = V_{OQ} \Rightarrow I_D = \frac{V_{DD} - V_{OQ}}{R} = \frac{10 - 1}{1k} = 0.5mA$$

$$I_D = 0.5 = \frac{1}{2} \times 1 \times V_{OV}^2 \Rightarrow V_{OV} = \sqrt{2 \times 0.5} = 1V$$

$$V_{OV} = V_{GS} - V_t \Rightarrow V_{GS} = 2V$$

$$A_V = \frac{-2 V_{RD}}{V_{OV}} = \frac{-2 \times 18 \times 0.5}{1} = -18V/V$$

$$V_O^+ = V_{DD} - V_{OQ} = 10 - 1 = 9V$$

$$V_O^- = V_{OQ} - V_{OB}$$

In order to find  $V_{OB}$  where  $V_{DS} = V_{GS} - V_t$  or

$$V_{OB} = V_{IB} - V_t : V_O = V_{DD} - R_D I_D$$

$$V_{OB} = 10 - 18 \times \frac{1}{2} \times 1 \times V_{OB}^2 \Rightarrow 9V_{OB}^2 + V_{OB} - 10 = 0 \Rightarrow V_{OB} = 1V$$

$$V_O^- = V_{OQ} - V_{OB} = 1 - 1 = 0$$

Using the same formulas we can calculate the

rest of the values for  $V_{DS} = 2, 3, \dots, 10V$

$V_{DS}$ (V)	$I_D$ (mA)	$V_{OV}$ (V)	$V_{GS}$ (V)	$A_V$	$V_O^+$ (V)	$V_O^-$ (V)
1	0.5	1	2	18	9	0
2	0.44	0.94	1.94	17.02	8	1
3	0.39	0.88	1.88	15.9	7	2
4	0.33	0.81	1.81	14.8	6	3
5	0.28	0.75	1.75	13.3	5	4
6	0.22	0.66	1.66	12.1	4	5
7	0.17	0.58	1.58	10.3	3	6
8	0.11	0.47	1.47	8.5	2	7
9	0.06	0.35	1.35	5.7	1	8
10	0	0	<1V	0	0	0

4.51

$$R_D = 20k\Omega, V_{RD} = 2V \Rightarrow I_D = 0.1mA$$

$$A_V = -\frac{2 V_{RD}}{V_{OV}} \Rightarrow -10 = -\frac{2 \times 2}{V_{OV}} \Rightarrow V_{OV} = 0.4V$$

$$V_{GS} = 1.2V \Rightarrow V_E = 1.2 - 0.4 = 0.8V$$

$$I_D = \frac{L K' W}{2} \frac{V_{OV}^2}{L} \Rightarrow 0.1 = \frac{1}{2} \times 50 \times 10^{-3} \frac{W}{L} \frac{0.4^2}{L}$$

$$\Rightarrow W = 25$$

4.52

$$\text{Eq. 4.41 : } A_V = -2(V_{DD} - V_{OQ})$$

$$\text{a) } A_{V_{max}} = -\frac{2(V_{DD} - 0)}{V_{OV_{min}}} = -\frac{2 \times 5}{0.2} = 50V/V$$

b)

c)

### 4.53

From Figure 4.26(c) we have:  $\hat{V}_o = V_{DS} - V_{OB}$

Assuming linear operation around the bias point:

$$A_v = -\frac{\hat{V}_o}{V_{IB} - V_{IQ}} = -\frac{\hat{V}_o}{V_{OB} + V_t - V_{IQ}} = -\frac{\hat{V}_o}{V_{OB} + V_t - (V_{ov} + V_t)}$$

$$A_v = -\frac{\hat{V}_o}{V_{OB} - V_{ov}} = -\frac{\hat{V}_o}{V_{OB} - V_{DS} + V_{DS} - V_{ov}} = -\frac{\hat{V}_o}{-\hat{V}_o + V_{DS} - V_{ov}}$$

$$-A_v \hat{V}_o + A_v (V_{DS} - V_{ov}) = -\hat{V}_o \Rightarrow \hat{V}_o (1 - A_v) = -A_v (V_{DS} - V_{ov})$$

$$\hat{V}_o = \frac{A_v (V_{DS} - V_{ov})}{A_v - 1} \Rightarrow \hat{V}_o = \frac{V_{DS} - V_{ov}}{1 - \frac{1}{A_v}}$$

$V_{DS} (V)$      $A_v (\frac{V_o}{V_D})$      $\hat{V}_o (V)$      $V_g (V)$

1      -16      0.47      0.029

1.5     -14      0.93      0.066

2      -12      1.38      0.12

2.5     -10      1.81      0.18

$$I_D = \frac{1}{2} \times 1 \times (0.5)^2 = 0.125 \text{ mA}$$

$$R_D = \frac{V_{DD} - V_{DS}}{I_D} = \frac{5 - 1}{0.125} = 32 \text{ k}\Omega$$

### 4.54

$$i_{D_1} = i_{D_2} \Rightarrow \frac{1}{2} K_n \left( \frac{W}{L} \right)_1 (V_{GS1} - V_t)^2 = L \frac{1}{2} K_n \left( \frac{W}{L} \right)_2 (V_{GS2} - V_t)^2$$

Note that  $V_{GS1} = V_I$

$$V_{GS2} = V_{DD} - V_o$$

$$\Rightarrow \left( \frac{W}{L} \right)_1 (V_I - V_t)^2 = \left( \frac{W}{L} \right)_2 (V_{DD} - V_o - V_t)^2$$

$$\sqrt{\left( \frac{W}{L} \right)_1 / \left( \frac{W}{L} \right)_2} (V_I - V_t) = V_{DD} - V_o - V_t$$

$$V_o = V_{DD} - V_t + \sqrt{\frac{\left( \frac{W}{L} \right)_1}{\left( \frac{W}{L} \right)_2}} V_t - \sqrt{\frac{\left( \frac{W}{L} \right)_1}{\left( \frac{W}{L} \right)_2}} V_I$$

$$\text{If } \left( \frac{W}{L} \right)_1 = \frac{50}{0.5} = 100 \quad \left( \frac{W}{L} \right)_2 = \frac{5}{0.5} = 10$$

then:

$$A_v = \frac{\partial V_o}{\partial V_I} = -\sqrt{\frac{100}{10}} = -3.16 \text{ V/V}$$

### 4.55

$$I_D = 2 \text{ mA} = \frac{1}{2} \times 80 \times 10^{-3} \times \frac{240}{6} \times (V_{GS} - 1.2)^2 \Rightarrow$$

$$V_{GS} = 2.32 \text{ V} \quad \text{Refer to Fig. 4.30c}$$

$$R_D I_D = \frac{15}{3} = 5 \text{ V} \Rightarrow R_D = \frac{5}{2 \text{ mA}} = 2.5 \text{ k}\Omega$$

$$R_S I_D = 5 \text{ V} \Rightarrow R_S = \frac{5}{2} = 2.5 \text{ k}\Omega$$

$$V_G = 5 + V_{GS} = 7.32 \text{ V}$$

$$\frac{15}{R_{G1} + R_{G2}} \times R_{G2} = 7.32 \quad R_{G1} = 2.2 \text{ M}\Omega \Rightarrow R_{G2} = 20.97 \text{ M}\Omega$$

$$V_{DS} = 5 \text{ V}$$

at the edge of saturation  $V_{DS} = V_{GS} - V_t$  or

$$V_{DS} = 2.32 - 1.2 = 1.12 \text{ V}. \text{ So } V_{DS} \text{ is } 5 - 1.12 = 3.88 \text{ V}$$

away from the edge of saturation.

### 4.56

Refer to Fig. 4.30e

$$I_D = 2 \text{ mA} = \frac{1}{2} K_n \frac{W}{L} V_{GS}^2 \Rightarrow 2 = \frac{1}{2} \times 50 \times 10^{-3} \times \frac{200}{4} V_{GS}^2$$

$$V_{GS} = 1.26 \text{ V}$$

$$V_{DS} = V_{ov} \text{ edge of triode}$$

Midway of cutoff ( $V_{DS} = V_{DD}$ ) and beginning of triode operation ( $V_{DS} = V_{ov}$ ) is when  $V_{DS} = \frac{30 + 1.26}{2} = 15.63 \text{ V}$

$$V_{GS} = 2.32 \text{ V} \Rightarrow V_S = -2.32 \text{ V} \Rightarrow R_S = \frac{-2.32 + 15}{2}$$

$$R_S = 6.34 \text{ k}\Omega$$

$$V_D = V_S + V_{DS} = -2.32 + 15.63 = 13.31 \text{ V} \Rightarrow R_D = \frac{15 - 13.31}{2}$$

$$R_D = 0.85 \text{ k}\Omega$$

### 4.57

$$V_G = 12 \times \frac{2.2}{2.2 + 5.6} = 3.4 \text{ V}$$

$$K_n \frac{W}{L} = 220 \text{ to } 380 \text{ }\mu\text{A/V}^2$$

$$V_t = 1.3 \text{ to } 2.4 \text{ V}$$

$$I_D = \frac{1}{2} K_n \frac{W}{L} (3.4 - V_t)^2$$

$$I_{D\min} = \frac{1}{2} \times 220 (3.4 - 2.4)^2 = 110 \text{ }\mu\text{A}$$

$$I_{D\max} = \frac{1}{2} \times 380 (3.4 - 1.3)^2 = 838 \text{ }\mu\text{A}$$

to limit  $I_{D\max}$  to 150  $\mu\text{A}$ :

$$150 = \frac{1}{2} \times 380 (3.4 - 0.15 R_S - 1.3)^2$$

$$R_S = 8.1 \text{ k}\Omega$$

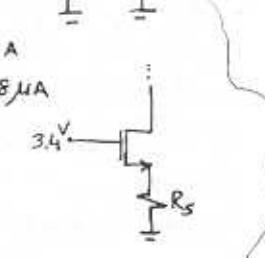
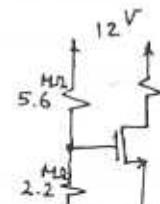
Select  $R_S = 8.2 \text{ k}\Omega$

$$I_{D\max} = \frac{1}{2} \times 380 \times (3.4 - I_{D\max} \times 8.2 - 1.3)^2$$

$$I_{D\max} = 0.15 \text{ mA} \text{ or } 0.4 \text{ mA}$$

The second answer results in negative  $V_{GS}$

Cont.



and therefore it is not acceptable.

$$I_{Dmin} = \frac{1}{2} \times 0.22 \times (3.4 - 8.2 I_{Dmin} - 2.4)^2$$

$$I_{Dmin} = 0.04 \text{ mA}$$

4.58

$$V_E = 2V, K' \frac{W}{L} = 2 \text{ mA/V}^2$$

$$I_D = \frac{1}{2} \times 2 \times (4 - I_D \times 1 - 2)^2$$

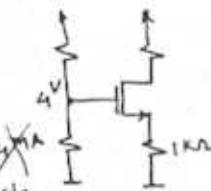
$$I_D = 4 + I_D^2 - 4I_D \Rightarrow I_D = 1 \text{ mA}, 4 \text{ mA}$$

$I_D = 4 \text{ mA}$  results in  $V_{GS} = 0$  which is not acceptable, therefore  $I_D = 1 \text{ mA}$ .

For  $K' \frac{W}{L}$  50% larger, i.e.  $K' \frac{W}{L} = 3 \text{ mA/V}^2$

$$I_D = \frac{1}{2} \times 3 \times (4 - I_D - 2)^2 \Rightarrow I_D = 1.13 \text{ mA}$$

$I_D$  increases by 13%.



4.59

Refer to Fig. 4.30.c

$$V_{GS} = 5 - 2 = 3V \Rightarrow I_D = \frac{V_S}{2R_S} = \frac{2}{1} = 2 \text{ mA}$$

$$I_D = 2 = \frac{1}{2} \times 2 \times (3 - V_E) \Rightarrow 1.41 = 3V_E \Rightarrow V_E = 1.59V$$

For a device with  $V_E = 1.59 - 0.5 = 1.09V$ :

$$I_D = \frac{1}{2} \times 2 \times (5 - I_D \times 1 - 1.09)^2 \Rightarrow I_D = 2.37 \text{ mA}$$

$$V_S = 2.37V$$

4.60

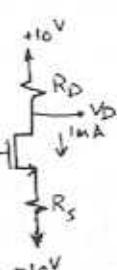
To maximize gain, we design for the lowest possible  $V_D$  consistent with allowing a 2V p-p signal swing.  $V_{Dmin} = V_D - 1$

$$V_{Dmin} = V_G - V_E = 0 - 2$$

$$V_D - 1 = -2 \Rightarrow V_D = -1V \Rightarrow R_D = \frac{10 - (-1)}{1 \text{ mA}} = 11 \text{ kΩ}$$

$$I_D = \frac{1}{2} \times 2 [0 - (-1 + 1 \times R_S) - 2]^2 \Rightarrow 1 = (8 - R_S)^2$$

$$R_S = 7 \text{ kΩ}$$



4.61

For p-channel MOSFET to be 1V from the edge of saturation:  $V_{DS} = V_{GS} - V_E - 1$ . Since  $V_D = 3V$  and  $I_D = 1 \text{ mA}$ :  $R_D = \frac{3}{1} = 3 \text{ kΩ}$ ,  $R_1 + R_2 = \frac{10V}{10 \text{ mA}} = 1 \text{ MΩ}$

$$\text{a) } |V_E| = 1V, K' \frac{W}{L} = 0.5 \text{ mA/V}^2$$

$$I_D = 0.5 \times \frac{1}{2} \times (V_{GS} + 1)^2 = 1 \Rightarrow V_{GS} = -3V$$

$$V_{DS} = -3 + 1 - 1 = -3V$$

$$V_S = 6V, V_G = 3V, R_S = \frac{10 - 6}{1} = 4 \text{ kΩ}$$

$$\frac{R_2}{R_1 + R_2} = \frac{3}{10} \Rightarrow R_1 = 0.7 \text{ MΩ}, R_2 = 0.3 \text{ MΩ}$$

$$\text{b) } |V_E| = 2V, K' \frac{W}{L} = 1.25 \text{ mA/V}^2$$

$$1 = \frac{1}{2} \times 1.25 (V_{GS} + 2)^2 \Rightarrow V_{GS} = -3.26V \text{ or } -0.74V$$

the second answer is not acceptable  $|V_{GS}| > |V_E|$

$$V_{GS} = -3.26V$$

$$V_{DS} = -3.26 + 2 - 1 = -2.26V$$

$$V_S = 3 + 2.26 = 5.26V$$

$$V_G = 2V$$

$$R_S = \frac{10 - 5.26}{1} = 4.74 \text{ kΩ}, R_D = 3 \text{ kΩ}$$

$$\frac{R_2}{R_1 + R_2} = \frac{2}{10} \Rightarrow R_1 = 0.2 \text{ MΩ}, R_2 = 0.8 \text{ MΩ}$$

4.62

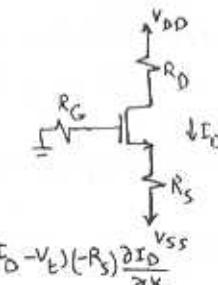
$$K = \frac{1}{2} K' \frac{W}{L}$$

$$\text{a) } I_D = \frac{1}{2} K' \frac{W}{L} (V_{GS} - V_E)^2$$

$$I_D = K (V_{GS} - R_S F_D - V_E)^2$$

$$\frac{\partial I_D}{\partial K} = (V_{SS} - R_S I_D - V_E)^2 +$$

$$+ 2K (V_{SS} - R_S I_D - V_E) (-R_S) \frac{\partial I_D}{\partial K}$$



$$\frac{\partial I_D}{\partial K} = \frac{I_D}{K} - 2R_S \sqrt{\frac{I_D}{K}} K \frac{\partial I_D}{\partial K}$$

$$\frac{\partial I_D}{\partial K} (1 + 2\sqrt{\frac{I_D}{K}} R_S) = \frac{I_D}{K} \Rightarrow S_K^I = \frac{\partial I_D}{\partial K} \frac{K}{I_D} = \frac{1}{1 + 2\sqrt{\frac{I_D}{K}} R_S}$$

$$\text{b) } K = 100 \mu A/V^2, \frac{\Delta K}{K} = \pm 10\%, V_E = 1V, I_D = 100 \mu A$$

$$\frac{\Delta I_D}{I_D} = \pm 1\%$$

$$S_K^I \approx \frac{\partial I_D / I_D}{\partial K / K} = \frac{1}{10} = 0.1 = \frac{1}{1 + 2\sqrt{100 \times 10^{-3} \times 100 \times 10^{-3}} R_S}$$

$$\Rightarrow R_S = 45 \text{ kΩ}$$

Cont.

$$100 = 100(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2V$$

$$V_{GS} = V_{SS} - I_D R_S \Rightarrow 2 = V_{SS} - 0.1 \times 45 \Rightarrow V_{SS} = 6.5V$$

c) For  $V_{SS} = 5V$ :

$$R_S = \frac{V_{GS} + V_{SS}}{I_D} = \frac{-2 + 5}{0.1} = 30k\Omega$$

$$S \frac{I_D}{K} = \frac{1}{1 + 2\sqrt{0.1 \times 0.1} \times 30} = \frac{1}{1 + 2 \times 0.1 \times 30} = 0.14$$

Therefore for  $\frac{\Delta K}{K} = \pm 10\%$ ,  $\frac{\Delta I_D}{I_D} = \pm 1.4\%$ .

4.63

Both cases are in saturation region, because

$$V_{DG} > V_E$$

$$V_D = 10 - 5 \times 1 = 5V$$

$$a) I = \frac{1}{2} \times 0.5 \times (V_{GS} - 1)^2 \Rightarrow V_{GS} = 3V, V_S = -3V$$

$$V_{DS} = 8V$$

$$b) I = \frac{1}{2} \times 1.25 \times (V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.3V, V_S = -3.3V$$

$$V_{DS} = 8.3V$$

4.64

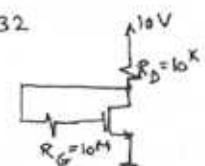
$$V_D = V_G = V_{GS} \quad \text{Refer to Fig. 4.32}$$

to operate in saturation:

$$V_{DS} \geq V_{GS} - V_E \Rightarrow V_{DG} \geq -V_E$$

$$V_{DG} = 0$$

$$a) \frac{10 - V_D}{10} = \frac{1}{2} \times 0.5 \times (V_D - 1)^2 \Rightarrow V_D = 2.7V$$



$$V_G = 2.7V$$

$$b) \frac{10 - V_D}{10} = \frac{1}{2} \times 1.25 \times (V_D - 2)^2 \Rightarrow V_D = 3.05V$$

$$V_G = 3.05V$$

4.65

For  $I_D = 0.2mA$ :

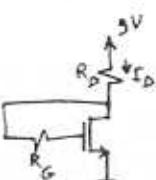
$$0.2 = \frac{1}{2} \times 0.4 \times (V_{GS} - 1)^2$$

$$V_{GS} = 2V, V_D = V_G = V_{GS} = 2V$$

$$R_D = \frac{9-2}{0.2} = 35k\Omega$$

$$\text{Select } R_D = 36k\Omega \Rightarrow \frac{9-V_D}{R_D} = \frac{1}{2} \times 0.4(V_D - 1)^2$$

$$\frac{9-V_D}{36} = 0.2(V_D - 1)^2 \Rightarrow V_D = 2V, I_D = 0.21mA.$$



4.66

$$I_D = 2 = \frac{1}{2} \times 3.2 \times (V_{GS} - 1.2)^2$$

$$V_{GS} - 1.2 = 1.12 \Rightarrow V_{GS} = 2.32V$$

$$V_G = 2.32V$$

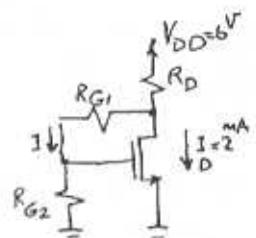
$$V_{DS\min} = V_{GS} - V_E = 1.12V$$

$$V_{DS} = V_{DS\min} + 2 = 3.12V$$

$$R_{G2} = 22M\Omega \Rightarrow I = \frac{2.32}{22} = 0.11mA$$

$$R_{G1} = \frac{3.12 - 2.32}{0.11} = 7.58M\Omega$$

$$R_D = \frac{6 - 3.12}{2 + 0.11 \times 10^3} = 1.44k\Omega$$



4.67

Eq. 4.57 implies:

$$i_D = I_D + K_n \frac{W}{L} (V_{GS} - V_E) V_{DS} \sin \omega t + \frac{1}{2} K_n' \frac{W}{L} V_{DS}^2 \sin 2\omega t$$

$$i_D = I_D + K_n \frac{W}{L} (V_{GS} - V_E) V_{DS} \sin \omega t + \frac{1}{2} K_n' \frac{W}{L} V_{DS}^2 \left( 1 - \frac{C_{as} 2\omega t}{2} \right)$$

$$i_D = I_D + K_n' \frac{W}{L} (V_{GS} - V_E) V_{DS} \sin \omega t + \frac{1}{4} K_n' \frac{W}{L} V_{DS}^2 - \frac{V_{DS}}{4} K_n' \frac{W}{L} C_{as} 2\omega t$$

$$\text{Second Harmonic Distortion} = \frac{\frac{1}{4} K_n' \frac{W}{L} V_{DS}^2 \times 100}{K_n' \frac{W}{L} (V_{GS} - V_E) V_{DS}}$$

$$= \frac{1}{4} \frac{V_{DS}}{V_{GS} - V_E} \times 100$$

$$\text{Second Harmonic Distortion} = \frac{1}{4} \frac{V_{DS}}{V_{GS}} \times 100$$

$$\text{For } V_{GS} = 10mV \quad \frac{1}{4} \times \frac{10 \times 10^{-3}}{V_{GS}} \times 100 \ll 1$$

$$\Rightarrow V_{GS} \gg 0.25V \Rightarrow V_{GS\min} = 0.25V$$

4.68

$$I_D = \frac{1}{2} K_n' \frac{W}{L} V_{DS}^2 \Rightarrow I_D = \frac{1}{2} \times 2 \times 1^2 = 1mA$$

$$I_D = \frac{1}{2} \times 2 \times (1+0.1)^2 = 1.21mA \quad (V_{GS} = 0.1V)$$

$$I_D = 1.21 - 1 = 0.21mA$$

$$\text{If } V_{GS} = -0.1V \Rightarrow I_D = \frac{1}{2} \times 2(1-0.1)^2 = 0.81mA$$

$$I_D = 0.81 - 1 = -0.19mA$$

$$\text{For positive increment: } g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{0.21}{0.1} = 2.1 \text{ mA/V}$$

$$\text{For negative increment: } g_m = \frac{0.19}{0.1} = 1.9 \text{ mA/V}$$

$$\text{An estimate of } g_m = \frac{2.1 + 1.9}{2} = 2 \text{ mA/V}$$

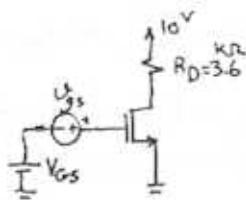
$$\text{Eq. 4.62 } g_m = K_n' \frac{W}{L} V_{DS} = 2 \times 1 = 2 \text{ mA/V. Same as estimated.}$$

4.69

$$a) I_D = \frac{1}{2} \times 1 \times (4-2)^2 = 2 \text{ mA}$$

$$V_D = V_{DD} - R_D I_D = 10 - 2 \times 3.6$$

$$\underline{V_D = 2.8 \text{ V}}$$



$$b) g_m = k_n' \frac{W}{L} V_{ov} = 1 \times (4-2) = 2 \text{ mA/V}$$

$$c) A_v = \frac{V_d}{V_{gs}} = -g_m R_D = -2 \times 3.6 = -7.2 \text{ V/V}$$

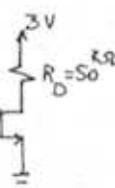
$$d) r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01 \times 2} = 50 \text{ kΩ}$$

$$A_v = \frac{V_d}{V_{gs}} = -g_m (R_D // r_o) = -2(3.6 // 50) = -6.7 \text{ V/V}$$

4.70

$$g_m R_D = 5 \Rightarrow g_m = \frac{5}{50} = 0.1 \text{ mA/V}$$

For 0.5V output signal and  
a gain of 5V/V,  $\frac{V_{ds}}{5} = 0.1$



$$\text{So we can write } V_{ds} - 0.5 \geq V_{gs} + 0.1 - V_t$$

$$\text{or } V_{ds} \geq V_{gs} + 0.6 - 0.8 \Rightarrow V_{ds} \geq V_{gs} - 0.2$$

$$\text{Also, from the other side: } V_{ds} + 0.5 \leq V_{DD}$$

$$\text{or } V_{ds} \leq 3 - 0.5 \Rightarrow V_{ds} \leq 2.5 \text{ V}$$

We design the circuit for lowest possible  $V_{DS}$  that guarantees the device operation in saturation:  $V_{DS} = V_{GS} - 0.2$

$$V_{DS} = V_{DD} - R_D I_D \Rightarrow V_{GS} - 0.2 = 3 - 50 \times I_D$$

$$\Rightarrow I_D = \frac{3 - V_{GS}}{50}$$

$$\text{Also, from eq. 4.71: } g_m = \frac{2 I_D}{V_{GS} - V_t} = 0.1$$

$$0.1 = \frac{2}{50} \times \frac{3 - V_{GS}}{V_{GS} - 0.8}$$

$$\Rightarrow V_{GS} = 1.49 \text{ V} \Rightarrow I_D = 0.034 \text{ mA}$$

$$V_{DS} = 1.49 - 0.2 = 1.29 \text{ V} \quad V_{ov} = 1.49 - 0.8 = 0.69 \text{ V}$$

$$\frac{W}{L} = \frac{I_D}{\frac{1}{2} k_n' V_{ov}^2} = \frac{0.034 \times 10^3}{\frac{1}{2} \times 100 \times 0.69^2} = 1.43$$

$$\underline{\frac{W}{L} = 1.43}$$

4.71

$$A_v = -g_m R_D \quad , \quad g_m = \frac{2 I_D}{V_{ov}} \quad \text{eq. 4.71} \quad \Rightarrow A_v = -\frac{2 R_D I_D}{V_{ov}} = -2 \frac{(V_{DD} - V_D)}{V_{ov}}$$

$$\textcircled{1}$$

Minimum  $V_{DS}$  for edge of saturation:

$$V_{DS} \geq V_{GS} - V_t \quad \text{or } V_{DSmin} = V_{GSmax} - V_t$$

$$V_{DS} - |A_v| \hat{V}_i = V_{GS} + \hat{V}_i - V_t$$

IF we replace  $A_v$  with  $\textcircled{1}$ :

$$V_D - \frac{2(V_{DD} - V_D)}{V_{ov}} \hat{V}_i = V + \hat{V}_i$$

$$\Rightarrow V_D (1 + \frac{2 \hat{V}_i}{V_{ov}}) = V_{ov} + \hat{V}_i + \frac{2 V_{DD} \hat{V}_i}{V_{ov}}$$

$$V_D = \frac{V_{ov} + \hat{V}_i + 2 V_{DD} (\hat{V}_i / V_{ov})}{1 + 2 (\hat{V}_i / V_{ov})}$$

$$V_{DD} = 3 \text{ V}, \hat{V}_i = 20 \text{ mV}, m = 10 = \frac{V_{ov}}{\hat{V}_i} \Rightarrow V_{ov} = 0.2 \text{ V}$$

$$V_D = \frac{0.2 + 0.02 + 2 \times 3 \times 10^{-3}}{1 + 2 \times 0.1} = 0.68 \text{ V}$$

$$A_v = \frac{2(3 - 0.68)}{0.2} = -23.2 \text{ V/V}$$

IF  $I_D = 100 \mu\text{A} = 0.1 \text{ mA}$ :

$$A_v = -\frac{2 R_D I_D}{V_{ov}} \Rightarrow 23.2 = \frac{2 \times R_D \times 0.1}{0.2} \Rightarrow$$

$$R_D = 23.2 \text{ kΩ}$$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{ov}^2 \Rightarrow 0.1 = \frac{1}{2} \times 100 \times 10 \frac{W}{L} 0.2^2$$

$$\Rightarrow \underline{\frac{W}{L} = 50}$$

4.72

$$k_n' = \mu_n C_{ox} = 500 \times 10 \times 0.4 \times 10^{-15} = 20 \text{ mA/V}^2, k_p' = 10 \mu\text{A/V}^2$$

Case	Type	$I_D$ (mA)	$V_{GS}$ (V)	$V_t$ (V)	$V_{ov}$ (V)	$W$ (μm)	$L$ (μm)	$\frac{W}{L}$	$g_m$ (mA/V)
a	N	1	3	2	1	100	1	100	2
b	N	1	1.2	0.7	0.5	50	$\frac{1}{8}$	400	8
c	N	10	?	?	2	250	1	250	5
d	N	0.5	?	?	0.5	?	?	200	4
e	N	0.1	?	?	1.41	10	2	5	0.1 0.14
f	N	0.1	1.8	0.8	1	40	4	10	0.2 0.2
g	P	1	?	?	2	?	?	25	0.25 1
h	P	1	3	1	2	?	?	50	0.5 1
i	P	10	?	?	1	4000	2	2000	20 20

cont.

Case	Type	$I_D$	$V_{GS}$	$V_t$	$V_{OV}$	$W$	$L$	$\frac{W}{L}$	$\frac{k'W}{L}$	$g_m$
j	P	10	?	?	4	?	?	125	1.25	5
K	P	0.05	?	?	1	30	3	10	0.1	0.1
L	P	0.1	?	?	5	?	?	0.8	0.000	0.04

$$V_{GS2} = 0.9 + \sqrt{2}(2 - 0.9) = 2.5V$$

$$\frac{g_{m1}}{g_{m2}} = \sqrt{\frac{I_{D1}}{I_{D2}}} \Rightarrow g_{m2} = \sqrt{2} g_{m1} = \underline{1.3 \text{ mA/V}}$$

$$\frac{r_{o1}}{r_{o2}} = \frac{I_{D2}}{I_{D1}} \Rightarrow r_{o2} = \frac{100}{2} = 50 \text{ k}\Omega$$

$$A_V = -1.3 \times (50 \text{ k} \parallel 10 \text{ k}) = -10.8 \text{ V/V}$$

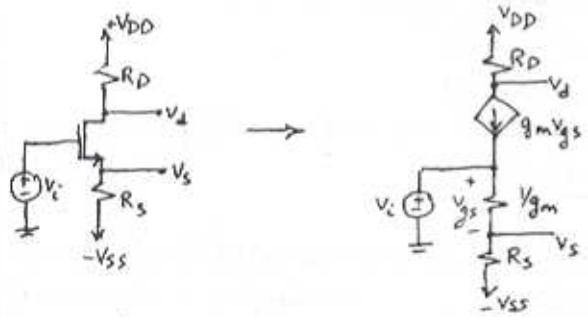
4.73

$$\text{Eq. 4.70: } g_m = \sqrt{2K_n \frac{W}{L} I_D} \Rightarrow \frac{W}{L} = \frac{g_m^2}{2K_n I_D}$$

$$\frac{W}{L} = \frac{1}{2 \times 50 \times 10^{-3} \times 0.5} = \underline{20 \text{ \mu m}}$$

$$g_m = \frac{2 I_D}{V_{OV}} \Rightarrow V_{OV} = \frac{2 \times 0.5}{1} = 1 \Rightarrow V_{GS} = 1 + V_t = 1.7V$$

4.74



$$\frac{V_d}{V_i} = \frac{R_s}{R_s + \frac{1}{g_m}} = \frac{R_s g_m}{R_s g_m + 1}$$

$$\frac{V_d}{V_i} = \frac{-g_m V_{GS} R_D}{V_i} = -g_m R_D \frac{1/g_m}{1/g_m + R_s} = \frac{-g_m R_D}{1 + g_m R_s}$$

4.75

$$r_o \approx \frac{V_A}{I_D} = \frac{50}{0.5} = 100 \text{ k}\Omega$$

$$g_m = \frac{2 I_D}{V_{GS} - V_t} \Rightarrow V_{GS} = V_{DS} = 2V$$

$$g_m = \frac{2 \times 0.5}{2 - 0.9} = 0.91 \text{ mA/V}$$

$$\frac{V_o}{V_i} = -g_m (r_o \parallel R_L) = -0.91 (100 \text{ k} \parallel 10 \text{ k}) = -8.3 \text{ V/V}$$

For  $I = 1 \text{ mA}$  or twice the current:

$$\frac{I_{D1}}{I_{D2}} = \frac{(V_{GS1} - V_t)^2}{(V_{GS2} - V_t)^2} \Rightarrow V_{GS2} = V_t + \sqrt{2(V_{GS1} - V_t)}$$

4.76

$$\text{NMOS: } g_m = \sqrt{2 K_n \frac{W}{L} I_D} = \sqrt{2 \times 9 \times 10^{-3} \times \frac{20}{2} \times 0.1} = 0.42 \text{ mA/V}$$

$$r_o = \frac{1/V_A}{I_D} = \frac{8 \times 2}{0.1} = \underline{160 \text{ k}\Omega}$$

$$X = \frac{Y}{2\sqrt{2g_f + Y_{SE}}} = \frac{0.5}{2\sqrt{2 \times 0.34 + 1}} = 0.2$$

$$g_{mb} = X g_m = 0.2 \times 0.42 = 0.084 \text{ mA/V}$$

$$g_m = \frac{2 I_D}{V_{OV}} \Rightarrow V_{OV} = \frac{2 \times 0.1}{0.42} = 0.48V$$

$$\text{PMOS: } g_m = \sqrt{2 \times 30 \times 10^{-3} \times \frac{20}{2} \times 0.1} = 0.24 \text{ mA/V}$$

$$r_o = \frac{1/V_A}{I_D} = \frac{12 \times 2}{0.1} = \underline{240 \text{ k}\Omega}$$

$$X = 0.2 \Rightarrow g_{mb} = 0.2 \times 0.24 = 0.048 \text{ mA/V}$$

$$V_{OV} = \frac{2 \times 0.1}{0.24} = 0.83V$$

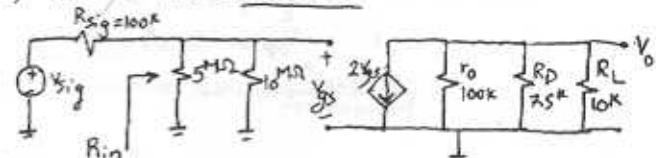
4.77

Refer to Fig. P4.77.

a)  $V_G = 15 \times \frac{5}{40+5} = 5V$      $V_S = 3 \times I_D \Rightarrow V_{GS} = 5 - 3I_D$   
 $I_D = \frac{1}{2} \times K \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow I_D = \frac{1}{2} \times 2 \times (5 - 3I_D - 1)^2$   
 $\Rightarrow 16 - 25I_D + 9I_D^2 = 0 \Rightarrow I_D = 1 \text{ mA}$   
 $V_{GS} = 2V$      $V_D = 15 - 7.5 = 7.5V$

b)  $g_m = \frac{2 I_D}{V_{OV}} = \frac{2 \times 1}{2 - 1} = 2 \text{ mA/V}$      $r_o = \frac{V_A}{I_D} = \underline{100 \text{ k}\Omega}$

c)  $R_{in} = 5M \parallel 10M = 3.33 M\Omega$



d)  $\frac{V_{GS}}{V_{S,ig}} = \frac{R_{in}}{R_{S,ig} + R_{in}} = \frac{3.33}{0.1 + 3.33} = 0.97 \text{ V/V}$

$$\frac{V_o}{V_{GS}} = 2 \times (S \parallel R_D \parallel R_L) = 2 \times 4.1K = 8.2 \text{ V/V}$$

$$\frac{V_o}{V_{S,ig}} = \frac{V_o}{8.2} = \underline{7.95 \text{ V/V}}$$

4.78

$i_D = \frac{1}{2} K_n \frac{W}{L} V_{DD}^2$   
 The equation for the tangent line passing from point A ( $V_{DD}$ ,  $I_D$ ) can be written as:

$$(i_D - I_D) = \frac{\partial i_D}{\partial V_{DD}} \Big|_{I_D=I_D} (V_{DD} - V_{DD}) + I_D$$

$$\frac{\partial i_D}{\partial V_{DD}} \Big|_{I_D=I_D} = K_n \frac{W}{L} V_{DD}$$

$$\Rightarrow i_D - I_D = K_n \frac{W}{L} V_{DD} (V_{DD} - V_{DD})$$

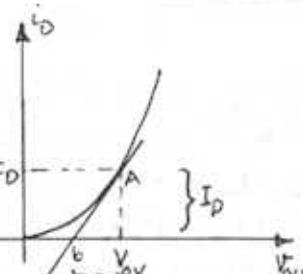
$$i_D - \frac{1}{2} K_n \frac{W}{L} V_{DD}^2 = K_n \frac{W}{L} V_{DD} (V_{DD} - V_{DD})$$

The tangent intersects the  $V_{DD}$  axis at  $i_D = 0$

$$0 - \frac{1}{2} K_n \frac{W}{L} V_{DD}^2 = K_n \frac{W}{L} V_{DD} (V_{DD} - V_{DD})$$

$$-\frac{V_{DD}}{2} = V_{DD} - V_{DD} \Rightarrow V_{DD} = \frac{V_{DD}}{2}$$

The slope of the tangent is  $g_m$  and it is equal to:  $g_m = \frac{\Delta i_D}{\Delta V_{DD}} = \frac{I_D}{V_{DD} - V_{DD}} = \frac{I_D}{\frac{V_{DD}}{2}} = \frac{2I_D}{V_{DD}}$



$$r_0 = \frac{V_A}{g_m} \Rightarrow r_0 = 4 \times r_{01} = 4 \times \frac{75}{0.5} = 600 \text{ k}\Omega$$

$$A_{V_{DD}} = -15 \times 2 = -30 \text{ V/V (without } r_0)$$

c) If we take  $r_0$  into account:

$$A_{V_{DD}} = -g_m (r_0 \parallel R_D) = -0.5 (600 \parallel 60) = -27.3 \text{ V/V}$$

$$R_{out} = R_D \parallel r_0 = 600 \parallel 60 = 54.5 \text{ k}\Omega$$

$$d) R_{in} = R_G = 4.7 \text{ M}\Omega$$

$$R_o = R_{out} = 54.5 \text{ k}\Omega$$

$$G_V = \frac{R_{in}}{R_{in} + R_{sig}} \quad A_{V_{DD}} \frac{R_L}{R_L + R_0} = \frac{4.7}{4.7+0.1} \frac{27.3 \times 15}{15+54.5}$$

$$G_V = 5.77 \text{ V/V}$$

e) As we can see by reducing  $V_{DD}$  to half of its value or equivalently multiplying drain current by 4,  $A_{V_{DD}}$  is almost doubled, while  $R_{out}$  is multiplied by 4.

As a result  $G_V$  which is proportional to both  $A_{V_{DD}}$  and  $\frac{1}{R_{out}}$  is only slightly reduced. ( $G_V$  was  $-7 \text{ V/V}$  before and it is  $5.8 \text{ V/V}$  now)

4.81

4.79

For this common-source amplifier we have:

$$g_m = \frac{2 \text{ mA}}{\text{V}} , r_0 = 50 \text{ k}\Omega , R_D = 10 \text{ k}\Omega$$

$$R_G = 10 \text{ M}\Omega , R_{sig} = 0.5 \text{ M}\Omega \text{ and } R_L = 20 \text{ k}\Omega$$

From equation 4.82 we have:

$$G_V = -\frac{R_L}{R_G + R_{sig}} g_m (r_0 \parallel R_D \parallel R_L) = -\frac{10 \times 2}{10 + 0.5} (50 \parallel 10 \parallel 20)$$

$$G_V = -11.2 \text{ V/V}$$

4.80

$$a) A_{V_{DD}} = -2 \frac{(V_{DD} - V_D)}{V_{DD}} = -2 \frac{(10 - 2.5)}{1} = -15 \text{ V/V}$$

b) if  $V_{DD}$  is halved ( $V_{DD} = 0.5$ ) then  $I_D$  is divided by 4, i.e.  $I_D = \frac{0.5}{4} = 0.125 \text{ mA}$

Since  $V_D$  is kept unchanged at  $2.5 \text{ V}$  then:

$$R_D = \frac{10 - 2.5}{0.125} = 60 \text{ k}\Omega , g_m = \frac{2I_D}{V_{DD}} = \frac{0.5 \text{ mA}}{0.5 \text{ V}}$$

For an NMOS common-gate amplifier with  $g_m = 5 \text{ mA/V}$ ,  $R_D = 5 \text{ k}\Omega$ ,  $R_L = 2 \text{ k}\Omega$ ,  $R_{sig} = 200 \text{ }\Omega$  we have:

$$R_{in} = \frac{1}{g_m} = \frac{1}{5} = 0.2 \text{ k}\Omega = 200 \text{ }\Omega$$

From Eq. 4.96b we know that the overall voltage gain of this amplifier is:

$$G_V = \frac{g_m (R_D \parallel R_L)}{1 + g_m R_{sig}} = \frac{5 (5 \parallel 2)}{1 + 5 \times 0.2} = 3.57 \text{ V/V}$$

If we increase the bias current by a factor of 4, while maintaining the other parameter constant (assuming linear operation), we have:

$$g_m = \sqrt{2 K_n \frac{W}{L} I_D} \Rightarrow \frac{g_m 2}{g_m} = \sqrt{\frac{I_{D2}}{I_{D1}}} = \sqrt{4} = 2$$

$$g_m = 2 \times 5 = 10 \text{ mA/V}$$

$$R_{in} = \frac{1}{g_m} = 0.1 \text{ k}\Omega = 100 \text{ }\Omega$$

$$G_V = \frac{g_m (R_D \parallel R_L)}{1 + g_m R_{sig}} = 10 \frac{(5 \parallel 2)}{1 + 10 \times 0.2} = 4.76 \text{ V/V}$$

4.82

Refer to Eq. 4.90:  $G_V = \frac{R_G}{R_G + R_{S1}g_m} \frac{g_m(R_D || R_L)}{1 + g_m R_S}$

 $G_{V1} = -16 V/V$  reduced by factor of 4:
 $\frac{G_{V2}}{G_{V1}} = \frac{1}{1 + g_m R_S} \Rightarrow \frac{1}{4} = \frac{1}{1 + 2R_S} \Rightarrow R_S = 1.5 k\Omega$

4.83

Eq. 4.90:  $G_V = \frac{R_G}{R_G + R_{S1}g_m} \frac{g_m(R_D || R_L)}{1 + g_m R_S}$

$G_{V1} = -10 V/V$  for  $R_S = 1 k\Omega$  and  $G_{V1} = -20 V/V$  for  $R_S = 2 k\Omega$

 $\frac{G_{V1}}{G_{V2}} = \frac{1 + g_m R_{S2}}{1 + g_m R_{S1}} \Rightarrow \frac{10}{20} = \frac{1 + g_m \times 2}{1 + g_m \times 1} \Rightarrow 1 + g_m = 2$

$g_m = 1 mA/V$

In order to have  $G_V = 8 V/V$ :

$$\frac{G_{V1}}{G_{V2}} = \frac{20}{8} = \frac{1 + g_m R_{S3}}{1 + 0} \Rightarrow 1 + R_{S3} = 2.5 \Rightarrow R_{S3} = 3.5 k\Omega$$

4.84

$A_{V0} = 0.98 V/V$  and  $A_{V0} = \frac{0.98}{2} = 0.49 V/V$  for  $R_L = 500 \Omega$

Eq. 4.102a:  $A_{V0} = \frac{R_o}{R_L + \frac{1}{g_m}} \Rightarrow 0.49 = \frac{0.5}{0.5 + \frac{1}{g_m}} \Rightarrow g_m = 1.92 mA/V$

Eq. 4.103:  $A_{V0} = \frac{r_o}{r_o + \frac{1}{g_m}} \Rightarrow 0.98 = \frac{r_o}{r_o + 0.52}$

 $\Rightarrow r_o = 25.5 k\Omega$

4.85

We have  $g_m = 5 mA/V$  and  $r_o = 20 k\Omega$ . From Eq. 4.103 we know:  $A_{V0} = \frac{r_o}{r_o + \frac{1}{g_m}} = \frac{20}{20 + \frac{1}{5}} = 0.99 V/V$

From Eq. 4.105:  $R_{out} = \frac{1}{g_m} || r_o = \frac{1}{5} || 20 = 19.8 \Omega$

 $R_{out} \approx 200 \Omega$

With  $R_L = 1 k\Omega$  we have:

$$A_{V0} = \frac{R_L || r_o}{(R_L || r_o) + \frac{1}{g_m}} = \frac{1 k || 20}{(1 k || 20) + \frac{1}{5}}$$
 $\Rightarrow A_{V0} = 0.83 V/V$

4.86

$R_{D2} = \frac{1}{g_{m2}} = 50 \Omega \Rightarrow g_{m2} = \frac{1}{50} = 20 mA/V$

If  $Q_1$  is biased the same as  $Q_2$ , then  $g_{m1} = g_{m2}$

 $i_{D1} = g_{m1}, V_{D1} = 20 \times 5 mV = 100 \mu A = 0.1 mA$ 
 $V_{D1} = i_{D1} \times 50 \Omega = 0.5 V$ 

In order to have  $V_{D2} = V_o = 1 V$ :

 $V_o = i_{D2} \times R_D \Rightarrow 1 = 0.1 \times R_D \Rightarrow R_D = 10 k\Omega$

4.87

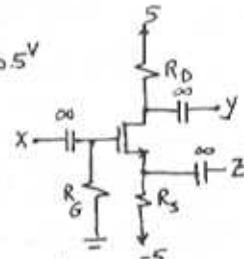
a)  $I_D = 0.1 = \frac{1}{2} \times 0.8 \times V_{DS}^2 \Rightarrow V_{DS} = 0.5 V$

 $\Rightarrow V_{GS} = 0.5 + 1 = 1.5 V$

$V_G = 0 \Rightarrow V_S = -1.5 V$ 
 $R_S = \frac{-1.5 - (-5)}{0.1} = 35 k\Omega$ 
 $V_{DS} = 5 - R_D \times 0.1$

Largest possible  $R_D$  is achieved for  $V_{DSmin}$

$V_{DS} \geq V_{GS} - V_t \Rightarrow V_{DSmin} = V_{GS} = 0.5 V \Rightarrow V_{DS} - 1 = V_{DS}$ 
 $\Rightarrow V_{DS} = 1 + 0.5 = 1.5 V \Rightarrow R_D = \frac{5 - 1.5}{0.1} = 35 k\Omega$ 
 $R_G = 10 M\Omega$



b)  $g_m = \frac{2 I_D}{V_{DS}} = \frac{2 \times 0.1}{0.5} = 0.4 mA/V$

 $r_o = \frac{V_{DS}}{I_D} = \frac{40}{0.1} = 400 k\Omega$

c) If Z is grounded then the circuit becomes a common-source configuration. The voltage gain according to Eq. 4.82:

$G_V = - \frac{R_G}{R_G + R_{S1}g_m} g_m (r_o || R_D || R_L)$

$G_V = \frac{10 M}{10 M + 10} \times 0.4 \times (400 k || 35 k || 40 k) = 6.5 V/V$ 
 $G_y = 6.5 V/V$

d) If y is grounded, then the circuit becomes a source follower configuration.

Eq. 4.103:  $A_{V0} = \frac{r_o}{r_o + \frac{1}{g_m}} = \frac{400}{400 + \frac{1}{0.4}} = 0.99 V/V$

 $R_{out} = \frac{1}{g_m} || r_o = \frac{1}{0.4} || 400 = 2.48 k\Omega$

(cont.)

e) If  $x$  is grounded, the circuit becomes a common-gate configuration.

$$R_{in} = \frac{1}{g_m} \parallel R_S = 35K \parallel \frac{1}{0.4} = 2.33K\Omega$$

$$\text{Eq. 4.98: } i_C = i_{S,ig} \frac{R_S g_m}{R_S g_m + R_{in}} = r$$

$$i_C = 10\mu A \frac{100K}{100K + 2.33K} = 9.77\mu A$$

$$V_y = R_D \times i_C = 35 \times 9.77\mu A = 0.34V$$

4.88

a) Fig. P4.88a is a source follower:

$$\text{Eq. 4.103: } A_{V_0} = \frac{r_o}{r_o + \frac{1}{g_m}}, \quad r_o \gg \frac{1}{g_m} \Rightarrow A = 1V/V$$

$$R_{out} = \frac{1}{g_m} = \frac{1}{5} = 0.2K\Omega$$

b) Fig. P4.88b is a common-gate configuration:

$$R_{in} = \frac{1}{g_m} = \frac{1}{5} = 0.2K\Omega$$

$$\text{Eq. 4.94: } A_V = g_m (R_D \parallel R_L) = 5 (5K \parallel 2K) = 7.1V/V$$

c) If we connect both stages together, then:

$$\text{for the first stage: } A_{V_1} = A_{V_0} \frac{R_L}{R_L + R_{out}}$$

where  $R_L$  is in fact  $R_{in}$  of the second stage

$$\text{Therefore: } A_{V_1} = 1 \times \frac{0.2K}{0.2 + 0.2} = 0.5V/V$$

$$\text{For the second stage: } A_{V_2} = 7.1V/V$$

$$\text{Overall gain } A_V = A_{V_1} A_{V_2} = 7.1 \times 0.5 = 3.55V/V$$

4.89

4.90

$$C_{ox} = \frac{E_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}}{8 \times 10^{-9}} = 4.3 \times 10^3 \text{ F/m}^2 = 4.3 \text{ pF/}\mu\text{m}^2$$

$$K_n' = K_n C_{ox} = 450 \times 10^{-4} \times 4.3 \times 10^3 = 193.5 \text{ MA/V}^2$$

$$I_D = 100 \mu\text{A} = \frac{1}{2} \times 193.5 \times \frac{20}{V_{ov}} \Rightarrow V_{ov} = 0.23 \text{ V}$$

$$V_{DS} = 1.5 \text{ V} > V_{ov} \Rightarrow \text{Saturation}$$

$$g_m = \frac{2 I_D}{V_{ov}} = \frac{880 \text{ mA/V}}{20 \text{ V}} \Rightarrow r_o = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 0.1} = 200 \text{ k}\Omega$$

$$X = \frac{Y}{2\sqrt{2g_f + g_s}} = \frac{0.5}{2\sqrt{0.65+1}} = 0.19$$

$$g_{mb} = X g_m = 167.2 \text{ }\mu\text{A/V}$$

$$C_{ov} = W L_{ov} C_{ox} = 20 \times 0.05 \times 4.3 = 4.3 \text{ pF}$$

$$C_{gs} = \frac{2}{3} W L C_{ox} + C_{ov} = \frac{2}{3} \times 20 \times 1 \times 4.3 + 4.3 = 61.6 \text{ pF}$$

$$C_{gd} = C_{ov} = 4.3 \text{ pF}$$

$$C_{sb} = \frac{C_{obs}}{\sqrt{1 + \frac{V_{SB}}{V_0}}} = \frac{15}{\sqrt{1 + \frac{1}{0.7}}} = 9.6 \text{ pF}$$

$$C_{db} = \frac{C_{obs}}{\sqrt{1 + \frac{V_{DB}}{V_0}}} = \frac{15}{\sqrt{1 + \frac{(1+1.5)}{0.7}}} = 7.5 \text{ pF}$$

4.91

$$g_m = \frac{2 I_D}{V_{ov}} = \frac{2 \times 0.1}{0.25} = 0.8 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{0.8 \times 10^{-3}}{2\pi(20+5) \times 10^{15}} = 5.1 \text{ GHz}$$

4.92

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \Rightarrow g_m = \sqrt{2 \mu C_{ox} \frac{W}{L} I_D}$$

Also  $C_{gs} \approx \frac{2}{3} W L C_{ox}$ , if  $C_{gs} \gg C_{gd}$  then we can ignore  $C_{gd}$ . If we replace for  $g_m$  and  $C_{gs}$  in the  $f_T$  formula, we have:

$$f_T = \frac{\sqrt{2 \mu C_{ox} W / L I_D}}{2\pi \times \frac{2}{3} W L C_{ox}} = \frac{1.5}{\pi L} \sqrt{\frac{\mu I_D}{2 C_{ox} W}}$$

Therefore we can see that the higher the current  $I_D$ , then the higher is the  $f_T$ . Also the frequency is reverse proportional to the size of the device, i.e. higher frequencies are achievable for smaller devices.

4.93

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \quad (1)$$

For  $C_{gs} \gg C_{gd}$  and the overlap capacitance of  $C_{gs}$  negligibly small:  $C_{gs} \approx \frac{2}{3} W L C_{ox}$   
Also  $g_m = \frac{2 I_D}{V_{ov}} = K_n \frac{W}{L} V_{ov}$

If we substitute  $g_m$  and  $C_{gs}$  in (1) from the above formulas:  $f_T = K_n \frac{W}{L} V_{ov} \frac{1}{2\pi \times \frac{2}{3} W L C_{ox}}$   
 $\Rightarrow f_T = \frac{3 K_n V_{ov}}{4\pi L^2}$

Therefore, for a given device  $f_T$  is proportional to  $V_{ov}$ .  $f_T \propto V_{ov}$

For  $L=1 \mu\text{m}$ ,  $V_{ov}=0.25$ :

$$f_T = \frac{3 \times 450 \times 10^{-4} \times 0.25}{4 \times \pi \times 10^{12}} = 2.7 \text{ GHz}$$

$$\text{For } V_{ov}=0.5 \text{ V} : \frac{f_{T1}}{f_{T2}} = \frac{V_{ov1}}{V_{ov2}} \Rightarrow \frac{f_{T2}}{f_T} = \frac{2.7 \times 0.5}{0.25}$$

$$f_T = 5.4 \text{ GHz}$$

4.94

$A_M = -27 \text{ V/V} \Rightarrow C_{gs} = 0.3 \text{ pF}, C_{gd} = 0.1 \text{ pF}$   
Common-Source configuration.

Eq. 4.127:  $C_{in} = C_{gs} + C_{gd} (1 + g_m R'_L)$ .

Also  $A_M = g_m R'_L$ , therefore:

$$C_{in} = 0.3 + 0.1 \times (1 + 27) = 3.1 \text{ pF}$$

Now to find the range of  $R_{sig}$  that results in 3-db frequencies over 10MHz, we use

$$\text{eq. 4.132: } f_H = \frac{1}{2\pi C_{in} R_{sig}}$$

If we neglect  $R_G$  effect then  $R'_{sig} \approx R_{sig}$ .

$$f_H \geq 10 \text{ MHz} \Rightarrow \frac{1}{2\pi \times 3.1 \times 10^{12} \times R_{sig}} \geq 10 \text{ MHz}$$

$$\Rightarrow R_{sig} \leq 5.1 \text{ k}\Omega$$

4.95

$$R_{sig} = 100 \text{ k}\Omega, R_{in} = 100 \text{ k}\Omega, C_{gs} = 1 \text{ pF}, C_{gd} = 0.2 \text{ pF}$$

Cont.

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m (r_0 \parallel R_D \parallel R_L) \quad (\text{Eq. 4.19})$$

Also  $R_{in} = 100k\Omega = R_G$

$$A_M = \frac{-100}{100+100} 3(50^k \parallel 8k \parallel 10^k) = -6.1 \text{ V/V}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} \quad (\text{Eq. 4.132})$$

$$R'_{sig} = R_{sig} \parallel R_G = 100 \parallel 100 = 50k\Omega$$

$$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$$

$$R'_L = r_0 \parallel R_D \parallel R_L = 4.1k\Omega$$

$$C_{in} = 1 + 0.2(1 + 3 \times 4.1) = 3.66 \mu F$$

Now we can calculate  $f_H$ :

$$f_H = \frac{1}{2\pi \times 3.66 \times 10^{-12} \times 50 \times 10^3} = 870 \text{ kHz}$$

In order to double  $f_H$ , we have to either decrease  $C_{in}$  (by reducing  $R_{out}$ ) or reduce  $R'_{sig}$  by reducing  $R_{in}$ .

If we reduce  $R_{out} = R_D \parallel r_0$ :

$$\frac{f_{H2}}{f_{H1}} = \frac{C_{in1}}{C_{in2}} \Rightarrow 2 = \frac{3.66 \mu F}{1 + 0.2(1 + 3 \times R'_L)}$$

$$\Rightarrow R'_L = 1.27k\Omega \quad R'_L = R_{out} \parallel R_L = R_{out} \parallel 10^k$$

$$\Rightarrow R_{out} = 1.45k\Omega$$

Therefore in order to double  $f_H$  to  $870 \times 2 = 1.74 \text{ MHz}$ , we have to reduce  $R_{out} = r_0 \parallel R_D$  to  $1.45k\Omega$  or equivalently reducing  $R_D$  to  $1.5k\Omega$ . The new midband gain would be:

$$\frac{A_{M2}}{A_{M1}} = \frac{R'_L}{R'_L} \Rightarrow A_{M2} = 6.1 \times \frac{1.27}{4.1} = 1.9 \text{ V/V}$$

Gain is almost reduced by a factor of 3.

If we reduce  $R_{in} = R_G$ :

$$\frac{f_{H2}}{f_{H1}} = \frac{R'_{sig1}}{R'_{sig2}} \Rightarrow 2 = \frac{50^k}{R'_{sig2}} \Rightarrow R'_{sig2} = 25k\Omega$$

$$\Rightarrow 25k\Omega = 100k \parallel R_G \Rightarrow R_G = 33k\Omega = R_{in}$$

Therefore in order to double  $f_H$ ,  $R_{in}$  is reduced by a factor of 3, from  $100k\Omega$  to  $33k\Omega$ .

The new midband gain would be:

$$\frac{A_{M2}}{A_{M1}} = \frac{R_G2}{R_G1} \frac{R_G1 + R_{sig}}{R_G2 + R_{sig}} \Rightarrow A_{M2} = 6.1 \times \frac{1}{3} \times \frac{100+100}{33+100}$$

$$A_{M2} = 3.06 \text{ V/V}$$

Gain is almost reduced by a factor of 2.

4.96

$$R_{in} = 2M\Omega, g_m = 4 \text{ mA/V}, r_0 = 100k\Omega, R_D = 10k\Omega$$

$$C_{gs} = 2 \text{ pF}, C_{gd} = 0.5 \text{ pF}, R_{sig} = 500k\Omega, R_L = 10k\Omega$$

a) using Eq. 4.19 and noting that  $R_G = R_{in}$ , we have:

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m (r_0 \parallel R_D \parallel R_L) = -\frac{2 \times 10^6}{2 + 0.5} (100 \parallel 10 \parallel 10)$$

$$A_M = -15.2 \text{ V/V}$$

b) Eq. 4.132:  $f_H = \frac{1}{2\pi C_{in} R'_{sig}}$  where

$$C_{in} = C_{gs} + C_{gd}(1 + g_m (r_0 \parallel R_D \parallel R_L)), R'_{sig} = R_{sig} \parallel R_L$$

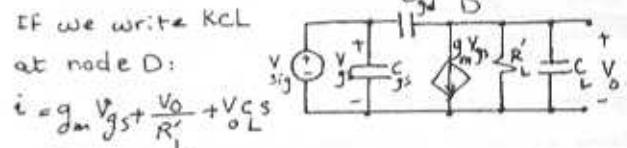
$$C_{in} = 2 + 0.5(1 + 4 \times (100 \parallel 10 \parallel 10)) = 12.02 \text{ pF}$$

$$R'_{sig} = 0.5M \parallel 2M\Omega = 0.4M\Omega = 400k\Omega$$

$$f_H = \frac{1}{2\pi \times 12.02 \times 10^{-12} \times 400 \times 10^3} = 33.1 \text{ kHz}$$

4.97

If we write KCL at node D:



$$\text{then: } V_{sig} = i \times \frac{1}{C_{gd}s} + V_o$$

$$V_{sig} = (g_m V_{gs} + \frac{V_o}{R'_L} + V_o C_L s) \frac{1}{C_{gd}s} + V_o, V_{sig} = V_{gs}$$

$$V_{sig} (1 - \frac{g_m}{C_{gd}s}) = V_o (1 + \frac{1}{R'_L C_{gd}s} + \frac{C_L s}{C_{gd}s})$$

$$\frac{V_o}{V_{sig}} = -g_m \frac{R'_L}{R'_L C_{gd}s + 1 + R'_L C_L s}$$

$$\frac{V_o}{V_{sig}} = -g_m \frac{\frac{C_{gd}}{g_m}}{1 + S(C_L + C_{gd})R'_L}$$

$$\text{If } (g_m/C_{gd}) \gg \omega \Rightarrow \frac{V_o}{V_{sig}} = \frac{-g_m R'_L}{1 + S(C_L + C_{gd})R'_L}$$

$$\text{For } C_{gd} = 0.5 \text{ pF}, C_L = 2 \text{ pF}, g_m = 4 \text{ mA/V}, R'_L = 5k\Omega$$

$$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + S_{WH}} \Rightarrow \left\{ \begin{array}{l} A_M = -g_m R'_L = -4 \times 5 = -20 \text{ V/V} \\ f_H = \frac{1}{2\pi(C_L + C_{gd})R'_L} \end{array} \right.$$

$$\Rightarrow f_H = \frac{10^12}{2\pi \times (2 + 0.5) \times 5 \times 10^3}$$

$$f_H = 12.7 \text{ MHz}$$

$$g_m/C_{gd} = \frac{4}{0.5} = 8 \text{ rad/s} \gg \omega_H$$

4.98

$$\text{Eq. 4.134} \quad \omega_{p_1} = \frac{1}{C_{L_1}(R_G + R_{S,ig})}$$

For  $f_p = 10 \text{ Hz}$ :

$$2\pi \times 10 = \frac{1}{C_{L_1} (1+1) \times 10^6} \Rightarrow C_{L_1} = 7.96 \text{nF}$$

To ensure that  $f_p$  is not exceeding 10 Hz,  $C_{L_1}$  has to be greater than 7.96 nF or:

$$C_{L_1} = 8 \text{nF}. \text{ If } C_{L_1} = 8 \text{nF} \Rightarrow f_p = 9.95 \text{ Hz}$$

To lower  $f_p$ ,  $R_G$  has to be increased. For the lowest possible  $f_p$ , the largest available  $R_G$  has to be used, which in this case is  $10 \times 1 = 10 \text{ M}\Omega$ .

To calculate the new  $f_p$  for  $R_G = 10 \text{ M}\Omega$ :

$$\frac{f_{p_2}}{f_{p_1}} = \frac{R_{G_1} + R_{S,ig}}{R_{G_2} + R_{S,ig}} \Rightarrow f_{p_2} = 9.95 \frac{1+1}{10+1} = 1.81 \text{ Hz}$$

$f_p$  is reduced by a factor of 5.5.

4.99

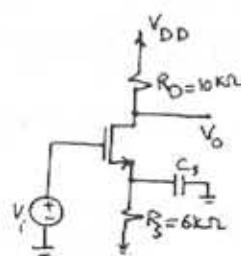
$$I_D = 1 \text{ mA}, g_m = 1 \text{ mA/V}$$

Using eq. 4.89 we have:

$$A_M = -\frac{g_m R_D}{1 + g_m R_S} = -\frac{1 \times 10}{1 + 1 \times 6} \\ A_M = 1.43 \frac{\text{V}_V}{\text{V}}$$

$$f_L = \frac{1}{2\pi \left( \frac{1}{g_m} \parallel R_S \right) C_S} = 10 \text{ Hz}$$

$$C_S = \frac{1}{2\pi \times 10 \left( 10^6 \parallel 10^6 \right)} = 18.57 \mu\text{F}$$



4.100

$$f_{C_{L_2}} = \frac{1}{2\pi C_{L_2} (R_L + R_D \parallel r_o)} \ll 10 \text{ Hz}$$

$$\Rightarrow C_{L_2} > \frac{1}{10 \times 2\pi \times (10^6 + 15^6 \parallel 150^6)} \Rightarrow C_{L_2} \geq 0.67 \text{nF}$$

$$\Rightarrow C_{L_2} = 0.7 \mu\text{F} \Rightarrow f_{C_{L_2}} = 9.62 \text{ Hz}$$

If  $I_D$  is doubled with both  $r_o$  and  $R_D$  halved:

$$f_{C_{L_2}} = \frac{1}{2\pi \times 0.7 \left( 10^6 + 15^6 \parallel \frac{150^6}{2} \right)} = 13.5 \text{ Hz}$$

For higher-power designs, where  $I_D$  is increased and consequently  $r_o$  and  $R_D$  are reduced.

For smallest  $r_o$  and  $R_D$  where  $r_o \parallel R_D \ll R_L$ ,  $R_L$  becomes dominant in determining the corner frequency:

$$f_{C_{L_2, \text{max}}} = \frac{1}{2\pi C_{L_2} (R_L)} = 22.75 \text{ Hz}$$

4.101

Refer to Fig. P4.101:  $\text{g}_m = 1 \text{ mA/V}$

$$A_M = -\frac{R_G}{R_G + R_{S,ig}} g_m (R_D \parallel R_L) \text{ where } R_G = 10^6 \parallel 10^6 \text{ M}\Omega \\ R_G = 8.25 \text{ M}\Omega$$

$$A_M = -\frac{8.25}{8.25 + 0.1} \times 1 \times (4.7 \parallel 10^6) = -3.16 \text{ V/V}$$

$$f_{p_1} = \frac{1}{2\pi C_{L_1} (R_G + R_{S,ig})} \quad (\text{Eq. 4.134})$$

$$f_{p_1} = \frac{1}{2\pi \times 0.01 \times 10^{-6} \times (8.25 + 0.1) \times 10^6} = 1.9 \text{ Hz}$$

$$f_{p_2} = \frac{1}{2\pi C_S (R_S \parallel \frac{1}{g_m})} = \frac{1}{2\pi \times 10 \times 10^{-6} \times (2^6 \parallel \frac{1}{1})} = 23.9 \text{ Hz}$$

$$f_{p_3} = \frac{1}{2\pi C_{L_2} (R_D + R_L)} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times (4.7 + 10) \times 10^6} = 108.3 \text{ Hz}$$

$$f_L \approx 108.3 \text{ Hz}$$

4.102

If  $\text{g}_m = 1 \text{ mA/V}$  and  $r_o = 100 \text{ k}\Omega$ :

$$A_M = -\frac{R_G}{R_G + R_{S,ig}} g_m (r_o \parallel R_D \parallel R_L) \text{ where } R_G = 10^6 \parallel 10^6 \text{ M}\Omega \\ R_G = 8.25 \text{ M}\Omega$$

$$A_M = -\frac{8.25}{8.25 + 0.1} \times (100^6 \parallel 4.7 \parallel 10^6) = -3.06 \text{ V/V}$$

$$f_H = \frac{1}{2\pi C_{in} R'_{S,ig}} \text{ where } R'_{S,ig} = R_{S,ig} \parallel R_G = 0.1 \parallel 8.25 \text{ M}\Omega \\ R'_{S,ig} = 0.1 \text{ M}\Omega$$

$$C_{in} = C_{GS} + C_{GD} (1 + g_m (r_o \parallel R_D \parallel R_L))$$

$$C_{in} = 1 + 0.2 (1 + 1 (100^6 \parallel 4.7 \parallel 10^6)) = 1.82 \text{ pF}$$

$$f_H = \frac{1}{2\pi \times 1.82 \times 10^{-12} \times 0.1 \times 10^6} = 875 \text{ kHz}$$

4.103

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m (r_0 || R_D || R_L) = -\frac{2 \times 3}{2+0.5} \times (20^6 || 10^5)$$

$$(r_0 = \infty) \\ A_M = -16 V/V$$

using equations 4.134, 4.136 and 4.138 we have:

$$f_{P1} = \frac{1}{2\pi C_{c1} (R_G + R_{sig})} \Rightarrow 3^{Hz} = \frac{1}{2\pi C_{c1} (2+0.5) \times 10^6}$$

$$\Rightarrow C_{c1} = 21.2 nF$$

$$f_{P2} = \frac{g_m}{2\pi C_S} \Rightarrow 50^{Hz} = \frac{3 \times 10^{-3}}{2\pi C_S} \Rightarrow C_S = 9.6 \mu F$$

$$f_{P3} = \frac{1}{2\pi C_{c2} (R_D + R_L)} \Rightarrow 10^{Hz} = \frac{1}{2\pi C_{c2} (10^5 + 20^5)}$$

$$\Rightarrow C_{c2} = 0.5 \mu F$$

$$f_L = 50^{Hz}$$

4.104

Refer to Fig. P4.104.

$$f_{P1} = \frac{1}{2\pi C_{c1} R_{in}} = 1^{Hz} \quad (\text{a decade lower than } 10^{Hz})$$

$$f_{P2} = \frac{1}{2\pi C_{c2} (R_L + R_D || R_O)} = 10^{Hz}$$

$$f_{P1} = 1^{Hz} = \frac{1}{2\pi C_{c1} \times 2.33 \times 10^6} \Rightarrow C_{c1} = 68.3 nF$$

$$f_{P2} = 10^{Hz} = \frac{1}{2\pi C_{c2} (10^K + 10^K || 47^K)} \Rightarrow C_{c2} = 873 nF$$

4.105

Note that  $K_n \frac{W_n}{L_n} = K_p \frac{W_p}{L}$  (matched)

a) when  $V_o$  is low: (eq. 4.140)

$$r_{DSN} = \frac{1}{K_n' \left( \frac{W}{L} \right)_n (V_{DD} - V_{th})} = \frac{1}{120 \times 10^6 \times \frac{1.2}{0.8} \times (3-0.7)}$$

$$r_{DSN} = 2.4 K\Omega$$

when  $V_o$  is high ( $V_{OH}$ ): (eq. 4.141)

$$r_{DSP} = \frac{1}{K_p' \left( \frac{W}{L} \right)_p (V_{DD} - |V_{tp}|)} = \frac{1}{60 \times 10^6 \times \frac{2.4}{0.8} \times (3-0.7)}$$

$$r_{DSP} = 2.4 K\Omega$$

b)  $I_{max} = k_n' \left( \frac{W}{L} \right)_n (V_{DD} - V_{th}) \times 0.1$

$$I_{max} = 120 \times 10^6 \times \frac{1.2}{0.8} \times (3-0.7) \times 0.1 = 41.4 mA$$

c) Eq. 4.148:  $V_{IH} = \frac{1}{3} (5V_{DD} - 2V_t)$

$$V_{IH} = \frac{1}{3} (5 \times 3 - 2 \times 0.7) = 1.7V$$

Eq. 4.149:  $V_{IL} = \frac{1}{3} (3V_{DD} + 2V_t) = 1.3V$

$$V_{MH} = V_{OH} - V_{IH} = 3 - 1.7 = 1.3V$$

$$V_{ML} = V_{IL} - V_{OL} = 1.3V - 0 = 1.3V$$

4.106

From the formula given in Exercise 4.44 we

$$\text{have: } V_{th} = \frac{r(V_{DD} - |V_{tp}|) + V_{en}}{1+r}$$

$$\text{where } r = \sqrt{\frac{K_p' (W/L)_p}{K_n (W/L)_n}}$$

We have  $V_{th} = |V_{tp}| = 0.7V$ ,  $V_{DD} = 3V$ ,  $K_n' = 120 \mu A/V^2$

$K_p' = 60 \mu A/V^2$ . Thus: For  $(\frac{W}{L})_p = (\frac{W}{L})_n$  we have:

$$r = \sqrt{\frac{60}{120}} = 0.71$$

$$V_{th} = \frac{0.71(3-0.7) + 0.7}{1+0.71} = 1.36V$$

For  $(\frac{W}{L})_p = 2 (\frac{W}{L})_n$  (the matched case)

$$\text{we have: } r = \sqrt{\frac{60 \times 2}{120}} = 1$$

$$V_{th} = \frac{1 \times (3-0.7) + 0.7}{1+1} = 1.5V$$

For  $(\frac{W}{L})_p = 4 (\frac{W}{L})_n$  we have:  $r = \sqrt{\frac{60 \times 4}{120}} = 1.41$

$$V_{th} = \frac{1.41 \times (3-0.7) + 0.7}{1+1.41} = 1.64V$$

The results are summarized in the following table:

$$(\frac{W}{L})_p = (\frac{W}{L})_n \quad V_{th} = 1.36V$$

$$(\frac{W}{L})_p = 2 (\frac{W}{L})_n \quad V_{th} = 1.5V$$

$$(\frac{W}{L})_p = 4 (\frac{W}{L})_n \quad V_{th} = 1.64V$$

4.107

equal sizes NMOS and PMOS, but  $k'_n = 2k'_p$   
 $V_t = 0.7V$

for  $V_{IH}$ :  $Q_N$  in triode and  $Q_p$  in saturation

$$k'_n \left( \frac{W}{L} \right)_n \left[ (V_I - V_t) V_o - \frac{1}{2} V_o^2 \right] = \frac{1}{2} k'_p \left( \frac{W}{L} \right)_p (V_{DD} - V_I - V_t)^2$$

$$4(V_I - V_t)V_o - 2V_o^2 = (V_{DD} - V_I - V_t)^2 \quad (1)$$

Differentiating both sides relative to  $V_I$   
 results in:

$$\frac{\partial}{\partial V_I} : 4(V_I - V_t) \frac{\partial V_o}{\partial V_I} + 4V_o - 4V_o \frac{\partial V_o}{\partial V_I} = 2(V_{DD} - V_I - V_t)(-1)$$

Substitute the values together with  $V_I = V_{IH}$ ,

$$\frac{\partial V_o}{\partial V_I} = -1 :$$

$$4(V_{IH} - 0.7)(-1) + 4V_o + 4V_o = 2(V_{IH} - 3 + 0.7)$$

$$V_{IH} = \frac{8V_o + 7.4}{6} = 1.33V_o + 1.23 \quad (2)$$

From (1) :  $4(V_{IH} - 0.7)V_o - 2V_o^2 = (3 - V_{IH} - 0.7)^2$

(3)  $4(V_{IH} - 0.7)V_o - 2V_o^2 = (2.3 - V_{IH})^2$

Solving (2) and (3) :  $1.55V_o^2 + 4.97V_o - 1.14 = 0$   
 $\Rightarrow V_o = 0.21V$

$$V_{IH} = 1.52V$$

For  $V_{IL}$ :  $Q_N$  is in saturation and  $Q_p$  in triode

$$\frac{1}{2} k'_n \left( \frac{W}{L} \right)_n (V_I - V_t)^2 = k'_p \left( \frac{W}{L} \right)_p [(V_{DD} - V_I - V_t)(V_{DD} - V_t) - \frac{1}{2} (V_{DD} - V_o)^2]$$

$$(V_I - 0.7)^2 = (3 - V_I - 0.7)(3 - V_o) - \frac{1}{2} (3 - V_o)^2 \quad (1)$$

$$(V_I - 0.7)^2 = (2.3 - V_I)(3 - V_o) - \frac{1}{2} (3 - V_o)^2$$

$$\frac{\partial}{\partial V_I} \Rightarrow 2(V_I - 0.7) = (2.3 - V_I) \left( -\frac{\partial V_o}{\partial V_I} \right) - (3 - V_o) + (3 - V_o) \frac{\partial V_o}{\partial V_I}$$

$$V_I = V_{IL} \text{ and } \frac{\partial V_o}{\partial V_I} = -1$$

$$2V_{IL} - 1.4 = 2.3 - V_{IL} - 3 + V_o + 3 - V_o$$

$$V_{IL} = \frac{2}{3}V_o - 1.15$$

From (1) :  $(V_{IL} - 0.7)^2 = (2.3 - V_{IL})(3 - V_o) - \frac{1}{2} (3 - V_o)^2$   
 $(0.66V_o - 1.85)^2 = (3.45 - 0.66V_o)(3 - V_o) - \frac{1}{2} (3 - V_o)^2$

$$V_o = 2.96V$$

$$V_{IL} = 0.81V$$

$$N_{MH} = 3 - 1.52 = 1.48V \quad , \quad N_{ML} = 0.81 - 0 = 0.81V$$

4.108

Matched MOSFETs with  $V_t = 1V$ ,  $V_{DD} = 10V$ .

$$V_{IL} = \frac{1}{8} (3V_{DD} + 2V_t) \quad \text{Eq. 4.149}$$

$$V_{IL} = \frac{1}{8} (3 \times 10 + 2 \times 1) = 4V$$

$$V_{IH} = \frac{1}{8} (5V_{DD} - 2V_t) \quad \text{Eq. 4.148}$$

$$V_{IH} = \frac{1}{8} (5 \times 10 - 2 \times 1) = 6V$$

$$N_{MH} = V_{OH} - V_{IH} = 10 - 6 = 4V$$

$$N_{ML} = V_{IL} - V_{OL} = 4 - 0 = 4V$$

For  $V_{DD} = 15V$

$$V_{IL} = \frac{1}{8} (3 \times 15 + 2 \times 1) = 5.875V$$

$$V_{IH} = \frac{1}{8} (5 \times 15 - 2 \times 1) = 9.125V$$

$$N_{MH} = 15 - 9.125 = 5.875V$$

$$N_{ML} = 5.875 - 0 = 5.875V$$

4.109

$$V_t = 0.5V$$

$$I_{max} = k'_n \left( \frac{W}{L} \right)_n \left[ (V_{DD} - V_{tn}) \times 0.5 - \frac{1}{2} \times 0.5^2 \right]$$

$$I_{max} = 20 \times 20 \left[ (10 - 0.5) \times 0.5 - 0.125 \right]$$

$$I_{max} = 185 \mu A$$

$$V_t = 1.5V \Rightarrow I_{max} = 16.5 \mu A$$

$$V_t = 2V \Rightarrow I_{max} = 155 \mu A$$

4.110

$$i_{Dn} = k'_n \left( \frac{W}{L} \right)_n \left[ (V_{DD} - V_t) V_o - \frac{1}{2} V_o^2 \right]$$

$$i_{Dnmax} = k'_n \left( \frac{W}{L} \right)_n \left[ (V_{DD} - 0.2V_{DD}) \times 0.1V_{DD} - \frac{1}{2} \times 0.1V_{DD}^2 \right]$$

$$i_{Dnmax} = k'_n \left( \frac{W}{L} \right)_n \times 0.075 V_{DD}^2$$

$$\text{IF } V_{DD} = 3V, k'_n = 120 \mu A/V^2, L_n = 0.8 \mu m, i_{Dmax} = 1mA$$

$$i_{Dnmax} = 1 = 120 \times 10^{-3} \times \frac{W_n}{0.8} \times 0.075 \times 9$$

$$W_n = 9.9 \leq 10 \mu m$$

4.111

we use the  $I_{\text{peak}}$  equation given in Table 4.6:

$$I_{\text{peak}} = \frac{1}{2} K_n \left( \frac{W}{L} \right)_n \left( \frac{V_{DD} - V_t}{2} \right)^2$$

$$I_{\text{peak}} = \frac{1}{2} \times 120 \times \frac{1.2}{0.8} \left( \frac{3}{2} - 0.7 \right)^2$$

$$I_{\text{peak}} = 57.6 \mu\text{A}$$

4.112

$$\text{Eq. 4.156: } t_{\text{PHL}} = \frac{2C}{K_n \left( \frac{W}{L} \right)_n (V_{DD} - V_t)} \left[ \frac{V_t}{V_{DD} - V_t} + \frac{1}{2} \ln \left( \frac{3(V_{DD} - V_t)}{3 - 0.7} \right) \right]$$

$$\Rightarrow t_{\text{PHL}} = \frac{2 \times 0.05 \times 10^{-12}}{120 \times 10^6 \times 1.2 \times 0.8} \left[ \frac{0.7}{3 - 0.7} + \frac{1}{2} \ln \left( \frac{3 \times 3 - 4 \times 0.7}{3} \right) \right]$$

$$t_{\text{PHL}} = 0.16 \text{ ns}$$

If we use the estimation formula of Eq. 4.157:

$$t_{\text{PHL}} = \frac{1.6C}{K_n \left( \frac{W}{L} \right)_n V_{DD}} = \frac{1.6 \times 10^{-12} \times 0.05}{120 \times 10^6 \times 1.2 \times 0.8} = 0.15 \text{ ns}$$

The value obtained using the estimation formula is 6% lower. Eq. 4.157 is for  $V_t \approx 0.2V_{DD}$ , but in our case  $V_t = 0.7 \approx 0.23V_{DD}$ .

4.113

We use the estimation formulas for  $t_{\text{PHL}}$  and  $t_{\text{PLH}}$  provided in Table 4.6:

$$t_{\text{PLH}} = \frac{1.6C}{K_p \left( \frac{W}{L} \right)_p V_{DD}} = \frac{1.6 \times 0.05 \times 10^{-12}}{60 \times 10^6 \times \left( \frac{W}{L} \right)_p \times 3} = \frac{444 \times 10^{-12}}{\left( \frac{W}{L} \right)_p} \text{ ps}$$

$$\Rightarrow \frac{444}{\left( \frac{W}{L} \right)_p} \ll 60 \Rightarrow \left( \frac{W}{L} \right)_p \gg 7.41 \Rightarrow W_p \gg 5.93 \mu\text{m}$$

$$\left( \frac{W}{L} \right)_p = 2 \left( \frac{W}{L} \right)_n \Rightarrow W_p = 2 W_n \Rightarrow W_n \geq 2.97 \mu\text{m}$$

We choose  $W_p = 6 \mu\text{m}$  and  $W_n = 3 \mu\text{m}$

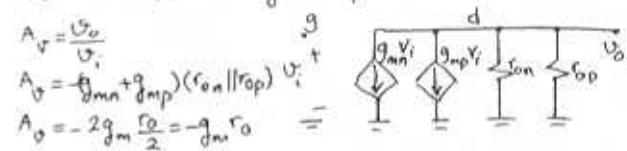
4.114

At  $V_i = V_o = \frac{V_{DD}}{2}$  both  $Q_N$  and  $Q_P$  are operating in saturation with  $I_D$  given by:

$$I_D = K_n \left( \frac{W}{L} \right)_n \left( \frac{V_{DD} - V_t}{2} \right)^2$$

Thus each device has  $g_m$  given by  $g_m = \frac{2ID}{V_{GS} - V_t}$   
 or  $g_m = \frac{2ID}{V_{DD}/2 - V_t}$  and  $r_0$  given by  $r_0 = \frac{V_{GS} - V_t}{ID}$  where  
 we have assumed matched devices:  $g_{mn} = g_{mp} = g_m$   
 $r_{op} = r_{on} = r_0$ .

From the small signal equivalent circuit:



Therefore:

$$A_V = -\frac{2NA_1}{V_{DD} - V_t}$$

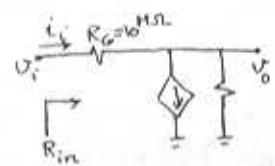
$$b) V_i = V_o = \frac{V_{DD}}{2} = 1.5 \text{ V}$$

$$A_V = -\frac{2 \times 50}{\frac{3}{2} - 0.7} = -\frac{100}{0.8} = -125 \text{ V/V}$$

$$R_{in} = \frac{V_i}{I_i} = \frac{V_i}{\frac{V_i - V_o}{R_G}} = \frac{R_G}{1 - A_V}$$

$$R_{in} = \frac{10 \times 10^3}{1 + 125} = 79.4 \text{ k}\Omega$$

$$R_{in} \approx 80 \text{ k}\Omega$$



4.115

we have  $K_n \frac{W}{L} = 2 \text{ mA/V}^2$ ,  $V_t = -3 \text{ V}$

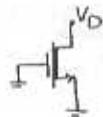
$V_{GS} = 0 > V_t \Rightarrow$  device is on

$$V_{GS} - V_t = 3 \text{ V}$$

a)  $V_D = 0.1 \text{ V}$ ,  $V_{DS} = 0.1 < V_{GS} - V_t \Rightarrow$  triode region

$$i_D = K_n \frac{W}{L} [(V_{GS} - V_t)V_{DS} - \frac{1}{2} V_{DS}^2]$$

$$i_D = 2 \times [3 \times 0.1 - \frac{1}{2} \times 0.1^2] = 0.59 \text{ mA}$$



b)  $V_D = 1 \text{ V} \Rightarrow V_{DS} = 1 \text{ V} < V_{GS} - V_t \Rightarrow$  triode region

$$i_D = 2 \times [3 \times 1 - \frac{1}{2} \times 1] = 5 \text{ mA}$$

c)  $V_D = 3 \text{ V} \Rightarrow V_{DS} = 3 \text{ V} = V_{GS} - V_t \Rightarrow$  edge of saturation

$$i_D = \frac{1}{2} \times n \frac{W}{L} (V_{GS} - V_t)^2 = \frac{1}{2} \times 2 \times 3^2 = 9 \text{ mA}$$

d)  $V_D = 5 \text{ V}$ ,  $V_{DS} = 5 \text{ V} > V_{GS} - V_t \Rightarrow$  saturation

$$i_D = \frac{1}{2} \times 2 \times 3^2 = 9 \text{ mA}$$

4.116

$$V_{GS} = 0 \quad V_t = -2V \quad V_{GS} - V_t = 2V$$

\*  $V_{DS} = 1V$  :  $V_{DS} < V_{GS} - V_t \Rightarrow$  triode

$$i_D = K_n \frac{W}{L} [(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2] = 200 [2 \times 1 - \frac{1}{2} \times 1]$$

$$i_D = 300 \mu A$$

If  $W$  is doubled, with  $L$  the same:  $i_D = \frac{i_D}{2} = \frac{W_2}{W_1} = 2$

$$\Rightarrow i_{D2} = 2 \times 300 = 600 \mu A$$

If  $W$  is doubled and  $L$  is also doubled, then  $\frac{W}{L}$  remains the same and therefore  $i_D = 300 \mu A$

\* If  $V_{DS} = 2V$ , then  $V_{DS} = V_{GS} - V_t \Rightarrow$  edge of saturation

$$i_D = \frac{1}{2} K_n \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS}) = \frac{1}{2} \times 200 \times 2^2 \times (1 + 0.02 \times 2)$$

$$i_D = 416 \mu A$$

If  $W$  is doubled and  $L$  is the same, then:

$$i_D = 2 \times 416 = 832 \mu A$$

If  $W$  is doubled and  $L$  is also doubled, then  $\frac{W}{L}$  stays the same but  $V_A = V_A L$  is doubled or equivalently  $\lambda$  is halved, thus:

$$i_D = \frac{1}{2} \times 200 \times 2^2 \times (1 + 0.01 \times 2) = 408 \mu A$$

\* If  $V_{DS} = 3V$ , then  $V_{DS} > V_{GS} - V_t \Rightarrow$  saturation

$$i_D = \frac{1}{2} \times 200 \times 2^2 \times (1 + 0.02 \times 3) = 424 \mu A$$

If  $W$  is doubled and  $L$  is the same:  $i_D = 2 \times 424 \Rightarrow$

$$i_D = 848 \mu A$$

If  $W$  is doubled and  $L$  is also doubled, then  $\frac{W}{L}$  stays unchanged,  $V_A$  is doubled,  $\lambda$  is halved:

$$\frac{i_{D2}}{i_{D1}} = \frac{(1 + \lambda_2 V_{DS})}{(1 + \lambda_1 V_{DS})} \Rightarrow i_{D2} = 424 \times \frac{(1 + 0.01 \times 3)}{(1 + 0.02 \times 3)} = 412 \mu A$$

\* If  $V_{DS} = 10V$ , then  $V_{DS} > V_{GS} - V_t \Rightarrow$  saturation

$$i_D = \frac{1}{2} \times 200 \times 2^2 \times (1 + 0.02 \times 10) = 480 \mu A$$

If  $W$  is doubled and  $L$  is the same:  $i_D = 960 \mu A$

If  $W$  is doubled and  $L$  is doubled:  $i_D = 480 \frac{(1 + 0.01 \times 10)}{1 + 0.02 \times 10} = 440 \mu A$

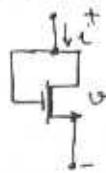
4.117

$V_{GS} = V_{DS} = V$ . For  $V > V_t$  the device will be conducting and since  $V_{DS} = V < V - V_t$ , the device

will be operating in the triode region and thus

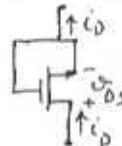
$$i_D = K_n \frac{W}{L} [(V - V_t)V_{DS} - \frac{1}{2}V_{DS}^2]$$

$$i_D = \frac{L}{2} K_n \frac{W}{L} (V^2 - 2V_t V) \quad V > V_t$$



For  $V \leq V_t$ , the source and drain exchange roles and the MOSFET operates with

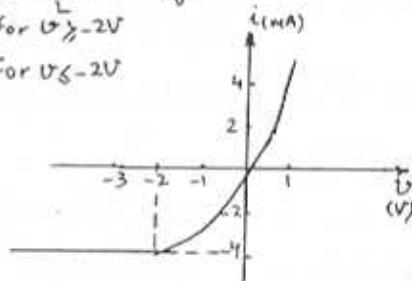
$V_{GS} = 0$  and  $V_{DS} = -V \geq -V_t$  which implies saturation region operation with  $i_D = \frac{1}{2} K_n \frac{W}{L} V_t^2$  (for  $V \leq V_t$ )



For  $V_b = -2V$  and  $K_n \frac{W}{L} = 2mA/V^2$

$$i_D = V^2 + 4V \quad \text{for } V \geq -2V$$

$$i_D = -4mA \quad \text{for } V \leq -2V$$



4.118

$$K_n \frac{W}{L} = 4mA/V^2, V_t = -2V$$

Since  $V_{DG} \leq |V_t|$ , the MOSFET will be operating in the triode region:

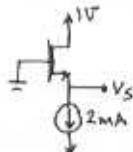
$$i_D = K_n \frac{W}{L} [(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2]$$

$$2 = 4 \left[ (-V_S + 2)(1 - V_S) - \frac{1}{2}(1 - V_S)^2 \right]$$

$$2 = +4V_S^2 + 8 - 12V_S - 2V_S^2 - 2 + 4V_S$$

$$2V_S^2 - 8V_S + 4 = 0 \Rightarrow V_S^2 - 4V_S + 2 = 0$$

$$V_S = 3.4V \text{ or } 0.59V$$



The first answer is not physically meaningful, for it results in  $V_{GS} = -3.4 < V_t$  and that implies cutoff. Therefore:  $V_S = 0.59V$

4.119

$$i_D = \frac{1}{2} K_n \frac{W}{L} [V_{GS} - V_t]^2, \lambda = 0$$

$$\begin{aligned} 1 &= \frac{K_n}{2} \frac{W}{L} (-1 - V_t)^2 \\ 9 &= \frac{K_n}{2} \frac{W}{L} (1 - V_t)^2 \end{aligned} \Rightarrow 9 = \frac{(1 - V_t)^2}{(-1 - V_t)^2} \Rightarrow 3 = \frac{1 - V_t}{1 + V_t}$$

$$\Rightarrow V_t = -0.5V, -2V$$

Cont.

$V_E = -0.5V$  is not acceptable, for it results in  $V_{GS} = -1V < V_E$ . Therefore  $V_E = -2V$ .

If we replace this in ①:

$$I_D = \frac{1}{2} K_n \frac{W}{L} (-1+2)^2 \Rightarrow K_n \frac{W}{L} = 2 \text{ mA/V}^2$$

$$I_{DSS} = \frac{1}{2} K_n \frac{W}{L} V_E^2 = \frac{1}{2} \times 2 \times 2^2 = 4 \text{ mA}$$

4.120

$$I_{DSS} = \frac{1}{2} K_n \frac{W}{L} V_E^2 \Rightarrow 4 = \frac{1}{2} K_n \frac{W}{L} \times 2^2 \Rightarrow K_n \frac{W}{L} = 2 \text{ mA/V}^2$$

Refer to Fig. P4.120

Consider  $Q_1$ :  $V_{GS1} = -I_D R_1$

$$I_D = \frac{1}{2} K_n \frac{W}{L} (V_{GS1} - V_E)^2 \Rightarrow 1 = \frac{1}{2} \times 2 \times (V_{GS1} + 2)^2 \Rightarrow$$

$$V_{GS1} = -1V$$

$$R_1 = \frac{V}{1 \text{ mA}} = 1 \text{ k}\Omega \quad , \quad R_2 = 1 \text{ k}\Omega$$

$$V_B = 6V \rightarrow R_3 = \frac{10-6}{1 \text{ mA}} = 4 \text{ k}\Omega$$

$$V_C = V_A - V_{GS2} - I_{D2} R_2 = 0 - V_{GS1} - I_{D2} R_2 = 0 - (-1) - 1 \times 1$$

$$V_C = 0V \quad (\text{for } V_A = 0)$$

$$\text{Now if } V_A = \pm 1V: \quad V_C = V_A - V_{GS1} - R_2 I_2 = V_A$$

$$\Rightarrow V_C = \pm 1V$$

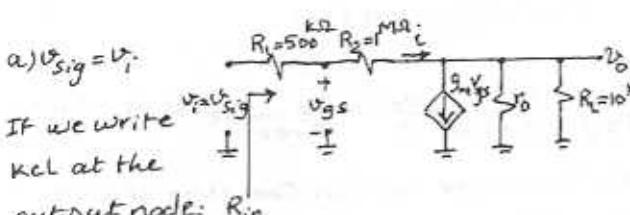
Since  $V_C = V_A$ , the source follower has zero offset.  $Q_2$  enters the triode region when:

$$V_E = V_A + |V_t| \Rightarrow V_A = 6 - 2 = 4V$$

$Q_1$  enters the triode region when  $V_C = -10 + 2$

$$V_C = -8V \quad \text{which corresponds to } V_A = -8V$$

4.121



$$\frac{V_i - V_o}{R_1 + R_2} = g_m V_{gs} + \frac{V_o}{R_2} + \frac{V_o}{R_L} \quad ①$$

$$\text{Also note that } V_{gs} = V_i + R_1 \frac{V_o - V_i}{R_1 + R_2}$$

$$V_{gs} = \frac{R_2}{R_1 + R_2} V_i + \frac{R_1}{R_1 + R_2} V_o$$

By substituting for  $V_{gs}$  in ①:

$$\frac{V_i - V_o}{R_1 + R_2} = g_m \frac{R_2}{R_1 + R_2} V_i + \frac{g_m R_1}{R_1 + R_2} V_o + \frac{V_o}{R_L} + \frac{V_o}{R_2}$$

$$\frac{V_i}{R_1 + R_2} \left( \frac{1 - g_m R_2}{R_1 + R_2} \right) = V_o \left( \frac{1 + g_m R_1}{R_1 + R_2} + \frac{1}{R_2} + \frac{1}{R_L} \right)$$

$$\frac{V_o}{V_i} = \frac{1 - g_m R_2}{1 + g_m R_1 + \frac{(R_1 + R_2)}{R_2 R_L}}$$

$$g_m = 1 \text{ mA/V}, \quad R_2 = 1 \text{ M}\Omega, \quad R_1 = 0.5 \text{ M}\Omega, \quad R_L = 10 \text{ k}\Omega$$

$$\frac{V_o}{V_i} = \frac{1 - 1 \times 1000}{1 + 1 \times 500 + \frac{1500}{10^4 / 100 \times 10^3}} = -1.5 \text{ V/V}$$

Now we calculate  $R_{in} = \frac{V_i}{i_c}$ :

$$V_i = (R_1 + R_2) i_c + (i_c - g_m V_{gs}) (R_2 || R_L)$$

$$V_{gs} = V_i - R_1 i_c$$

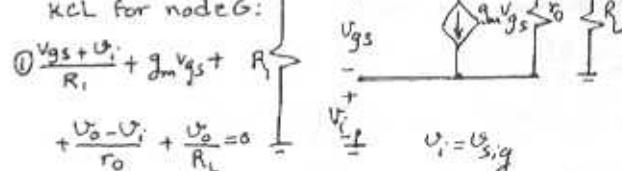
$$V_i = (R_1 + R_2) i_c + (i_c - g_m V_i + g_m R_1 i_c) (R_2 || R_L)$$

$$\frac{V_i}{i_c} = R_{in} = \frac{R_1 + R_2 + (1 + g_m R_1)(R_2 || R_L)}{1 + g_m (R_2 || R_L)}$$

$$R_{in} = \frac{500 + 1000 + (1 + 1 \times 500) \times (10^4 / 100 \times 10^3)}{1 + 1 \times 10^4 / 100 \times 10^3}$$

$$R_{in} = 600 \text{ k}\Omega$$

b) If we write a KCL for node G:



$$\text{Also: } V_{gs} = V_o \frac{R_1}{R_1 + R_2} - V_i$$

Substitute for  $V_{gs}$  in ①:

$$\frac{V_o}{R_1 + R_2} - \frac{V_i}{R_1} + \frac{V_i}{R_2} + g_m \frac{R_1}{R_1 + R_2} V_o - g_m i_c + \frac{V_o - V_i}{R_2} + \frac{V_o}{R_L} = 0$$

$$\frac{V_o}{R_1 + R_2} \left( 1 + g_m R_1 + \frac{1}{R_2} + \frac{1}{R_L} \right) = V_i \left( -g_m + \frac{1}{R_2} \right)$$

$$\frac{V_o}{V_i} = \frac{g_m + \frac{1}{R_2}}{\frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_2} + \frac{1}{R_L}}$$

$$\frac{V_o}{V_i} = \frac{1 + \frac{1}{100}}{\frac{1 \times 500 + 1}{500 + 100} + \frac{1}{100} + \frac{1}{10}} = 2.27 \text{ V/V}$$

Now we find  $R_i = \frac{V_i}{i_c}$ :

Cont.

$$i_c = \frac{V_o}{R_1 + R_2} + \frac{V_o}{R_L} \quad (\text{Note that } (R_1 + R_2) \text{ is parallel to } R_L)$$

$$i_c = V_o \left( \frac{1}{R_1 + R_2} + \frac{1}{R_L} \right)$$

$$\frac{i_c}{V_i} = \frac{V_o}{V_i} \left( \frac{1}{R_1 + R_2} + \frac{1}{R_L} \right)$$

$$R_i = \frac{V_i}{i_c} = \frac{1}{\frac{V_o}{V_i} \left( \frac{1}{R_1 + R_2} + \frac{1}{R_L} \right)}$$

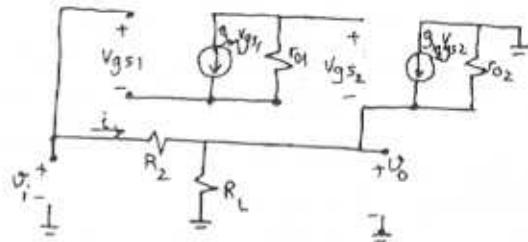
$$R_i = \left( \frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{r_{o1}} + \frac{1}{R_L} \right) / \left( g_m + \frac{1}{r_{o1}} \right) \left( \frac{1}{R_1 + R_2} + \frac{1}{R_L} \right)$$

$$R_i = \left( \frac{1 \times 500 + 1}{500 + 1000} + \frac{1}{100} + \frac{1}{10} \right) / \left( 1 + \frac{1}{100} \right) \left( \frac{1}{500 + 1000} + \frac{1}{10} \right)$$

$$R_i = 4.37 \text{ k}\Omega$$

$$\Rightarrow 0.8 = R_i \times 0.01 \Rightarrow R_i = 80 \text{ k}\Omega$$

$$(R_1 + R_2) \times 0.01 = 2 \Rightarrow R_2 = 120 \text{ k}\Omega$$



$$V_{GS1} = V_i$$

$$V_i = R_2 i + V_o \quad (1)$$

$$V_{GS2} = -g_{m1} V_{GS1} r_{o1} - V_o$$

$$i = \frac{V_o}{R_2} - g_{m2} V_{GS2} + \frac{V_o}{r_{o2}}$$

Substitute  $i$  in (1):

$$V_i = R_2 \frac{V_o}{R_2} + g_{m2} R_2 g_{m1} r_{o1} V_o + g_{m2} R_2 V_o + \frac{V_o}{r_{o2}} R_2$$

$$\frac{V_o}{V_i} = \frac{1 - g_{m2} R_2 g_{m1} r_{o1}}{R_2 + g_{m2} R_2 + \frac{R_2}{r_{o2}}}$$

We calculate the parameters:

$$g_{m1} = \frac{2 I_D}{V_{ov}} = \frac{2 \times 0.01}{0.8 - 0.6} = 0.1 \text{ mA/V}$$

$$g_{m2} = \frac{2 \times 1}{0.2} = 10 \text{ mA/V}$$

$$r_{o1} = \frac{V_A}{I_D} = \frac{20}{0.01} = 2000 \text{ k}\Omega = 2 \text{ M}\Omega$$

$$r_{o2} = \frac{20}{1} = 20 \text{ k}\Omega$$

$$\frac{V_o}{V_i} = \frac{1 - 10 \times 120 \times 0.1 \times 2000}{120 + 10 \times 120 + \frac{120}{20}} = -198.8 \text{ V/V}$$

$$\text{We can write: } V_i = V_o + R_2 i_c \Rightarrow i = \frac{V_o}{V_i} + R_2 \frac{i_c}{V_i}$$

$$\Rightarrow R_{in} = \frac{V_i}{i_c} = \frac{R_2}{1 - \frac{V_o}{V_i}}$$

$$R_{in} = \frac{120 \text{ k}}{1 + 198.8} = 0.6 \text{ k}\Omega$$

$$\frac{V_o}{V_{S,ig}} = \frac{R_{in}}{R_1 + R_{in}} \frac{V_o}{V_i} = \frac{0.6 \text{ k}}{80 + 0.6} \times (-198.8) = -1.48 \text{ V/V}$$

This two stage amplifier consisting of a CS stage followed by a source follower stage with a relatively high gain ( $\frac{V_o}{V_i}$ ) acts similar to an op-amp in an inverting configuration. Also note that  $\frac{V_o}{V_{S,ig}} \approx \frac{R_2}{R_1} = 1.5 \text{ V/V}$

Cont.

4.122

For DC-bias analysis purposes, both circuits are basically the same.

$$g_m = \frac{2 I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2 \times 0.1}{0.2} = 0.2 \text{ V}$$

$$I_D = \frac{1}{2} K_n' W L V_{ov}^2 \Rightarrow K_n' W L = \frac{2 \times 0.1}{0.2^2} = 5 \text{ mA/V}^2$$

$$r_o = \frac{V_A}{I_D} \Rightarrow V_A = 100 \times 0.1 = 10 \text{ V}$$

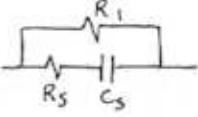
$$\begin{aligned} V_{DD} &= V_{DS} + (I_{R1} + I_D) R_L = 2.4 + (0.0016 + 0.1) \times 10 \text{ k} \\ V_{DD} &= 3.416 \text{ V} \end{aligned}$$

4.123

$$V_{R1} = V_{GS} = 0.8 \text{ V} = R_1 I_{R1} \Rightarrow I_{R1} = 0.01 \text{ mA} = 0.01 \text{ mA}$$

In order to increase the gain magnitude to  $+5\frac{1}{V}$ , we need to reduce the effective source resistance  $R_i$ . To do so, the following configuration can be used:

To calculate  $R_s$ :  $R'_i = R_i \parallel R_s$   
(midband)



$$\frac{V_o}{V_{sig}} = \frac{R_{in}}{R'_i + R_{in}} \frac{V_o}{V_i}$$

$$-5 = \frac{0.6}{R'_i + 0.6} \times (-19.8) \Rightarrow R'_i = 23.3 \text{ k}\Omega$$

$$R'_i = 23.3 \text{ k} = 80 \text{ k} \parallel R_s \Rightarrow R_s \approx 33 \text{ k}\Omega$$

4.124

$$I_i = 10 \mu\text{A} = I_{D1} = \frac{1}{2} \times 200 \times \left(\frac{W}{L}\right)_1 (0.8 - 0.6)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 2.5$$

$$I_{D2} = 1 \text{ mA} = \frac{1}{2} \times \frac{200}{1000} \times \left(\frac{W}{L}\right)_2 \times (0.8 - 0.6)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 250$$

## CHAPTER 5 PROBLEMS

5.1

Case	Mode
1	active
2	saturation
3	active
4	saturation
5	inverted active mode
6	active
7	cut-off
8	cut-off

5.2

Using eq. (5.4)

$$I_s = \frac{A_e q D_n n_i^2}{N_A W}$$

$$= \frac{10 \times 10 \times 10^{-4} \times 1.6 \times 10^{-19} \times 21.3 \times 1.5^2 \times 10^{20}}{10^{17} \times 1 \times 10^{-4}}$$

$$= \underline{\underline{7.7 \times 10^{-17} A}}$$

$$\beta = \frac{1}{\frac{D_p N_A W}{D_n N_A L_p} + \frac{1}{2} \frac{W^2}{D_n L_p}}$$

$$= \frac{1}{\frac{D_p N_A W}{D_n N_A L_p} + \frac{1}{2} \frac{W^2}{D_n L_p}} \quad \text{Eq (5.12)}$$

$$= \frac{1}{\frac{1.7 \times 10^{17} W}{21.3 \times 10^{19} \times 0.6 \times 10^{-14}} + \frac{1}{2} \frac{W^2}{19^2 \times 10^{-8}}}$$

- (a)  $W = 10^{-4} \text{ cm}$   $I_s = \underline{\underline{7.7 \times 10^{-17} A}}$   $\beta = \underline{\underline{368}}$   
 (b)  $W = 2 \times 10^{-4} \text{ cm}$   $I_s = \underline{\underline{3.8 \times 10^{-17} A}}$   $\beta = \underline{\underline{122}}$   
 (c)  $W = 5 \times 10^{-4} \text{ cm}$   $I_s = \underline{\underline{1.5 \times 10^{-17} A}}$   $\beta = \underline{\underline{24.2}}$

5.3

$$i_c = I_{s1} e^{\frac{V_{BE}}{V_T}}$$

For Device #1

$$0.2 \times 10^{-3} = I_{s1} e^{\frac{0.72}{0.025}}$$

$$I_{s1} = \underline{\underline{6.214 \times 10^{-17} A}}$$

For Device #2

$$12 \times 10^{-3} = I_{s2} e^{\frac{0.72}{0.025}}$$

$$I_{s2} = \underline{\underline{3.728 \times 10^{-15} A}}$$

Since  $I_s \approx A$ , the relative junction areas is :

$$\frac{A_2}{A_1} = \frac{I_{s2}}{I_{s1}} = \frac{i_{c2}}{i_{c1}} = \frac{12}{0.2} = \underline{\underline{60}}$$

5.4

$$i_c = \beta i_B$$

$$400 = \beta \times 7.5$$

$$\beta = \frac{400}{7.5} = \underline{\underline{53.3}}$$

$$\alpha = \frac{\beta}{\beta+1} = \frac{53.3}{54.3} = \underline{\underline{0.982}}$$

5.5

Use  $\beta = \alpha/(1-\alpha)$

$\alpha$	$\beta$	$\alpha$	$\beta$
0.5	1	0.99	99
0.8	4	0.995	199
0.9	9	0.999	999
0.95	19		

5.6

$$\text{Use } \alpha = \frac{\beta}{\beta+1}$$

$\beta$	$\alpha$	$\beta$	$\alpha$
1	0.5	100	0.9907
2	0.6667	200	0.9950
10	0.9091	1000	0.9990
20	0.9524	2000	0.9995

$$\alpha = \frac{\beta}{\beta+1} = \underline{0.9489}$$

$$I_c = \beta I_B = 18.571 \times 7 \times 10^{-6} = \underline{0.130 \text{ mA}}$$

$$I_c = I_s e^{\frac{V_{BE}}{N_T}}$$

$$I_s = 0.130 \times 10^{-3} e^{-0.58/0.025} = \underline{10.922 \times 10^{-15}}$$

$$(d) I_E = I_c + I_B = 10.1 + 0.120 = \underline{10.22 \text{ mA}}$$

$$I_c = \alpha I_E$$

$$\alpha = \frac{I_c}{I_E} = \frac{10.1}{10.22} = \underline{0.9883}$$

$$\beta = \frac{I_c}{I_B} = 10.1 / 0.120 = \underline{84.167}$$

$$(a) I_c = \beta I_B \\ 10^{-3} = \beta \times 50 \times 10^{-6} \\ \beta = \underline{20}$$

$$\alpha = \frac{\beta}{\beta+1} \\ = \frac{20}{21} = \underline{0.9524}$$

$$I_c = \alpha I_E \\ I_E = \frac{I_c}{\alpha} = \frac{10^{-3}}{0.9524} \\ = \underline{1.05 \text{ mA}}$$

$$I_c = I_s e^{\frac{V_{BE}}{N_T}} \\ I_s = I_c e^{-\frac{V_{BE}}{N_T}} \\ = 10^{-3} e^{-0.58/0.025} \\ = \underline{1.03 \times 10^{-15} \text{ A}}$$

$$I_c = 10.1 \times 10^{-3} = I_s e^{0.58/0.025} \\ I_s = \underline{284.66 \times 10^{-18} \text{ A}}$$

$$(e) I_E = I_c + I_B \\ I_c = I_E - I_B = 75 - 1.05 \\ = \underline{73.95 \text{ mA}}$$

$$\alpha = \frac{I_c}{I_E} = 73.95 / 75 = \underline{0.986}$$

$$\beta = \frac{I_c}{I_B} = 73.95 / 1.05 = \underline{70.429}$$

$$(b) I_c = \alpha I_E \\ 10^{-3} = \alpha \times 1.07 \times 10^{-3} \Rightarrow \alpha = \underline{0.9346}$$

$$\beta = \frac{\alpha}{1-\alpha} = \underline{14.286}$$

$$I_B = \frac{I_c}{\beta} = \frac{10^{-3}}{14.286} = \underline{7.0 \times 10^{-5} \mu\text{A}}$$

$$I_c = I_s e^{\frac{V_{BE}}{N_T}}$$

$$I_s = 10^{-3} e^{-0.58/0.025} = \underline{1.03 \times 10^{-15}}$$

$$(c) I_c = \alpha I_E = \beta I_B$$

$$\frac{\beta}{\beta+1} I_E = \beta I_B$$

$$I_E = (\beta+1) I_B$$

$$0.137 \times 10^{-3} = (\beta+1) 7 \times 10^{-6} \Rightarrow \beta = \underline{18.571}$$

5.8

$$i_c = I_s e^{\frac{V_{BE}}{N_T}} \\ 10 \times 10^{-3} = I_s e^{0.58/0.025} \Rightarrow I_s = \underline{6.273 \times 10^{-16}}$$

$$\text{FOR } V_{BE} = 0.7 \text{ V} \Rightarrow i_c = 6.273 \times 10^{-16} e^{0.7/0.025} \\ = \underline{0.907 \text{ mA}}$$

$$\text{For } i_c = 10 \mu\text{A} \Rightarrow 10 \times 10^{-3} = 6.273 \times 10^{-16} e^{\frac{V_{BE}}{0.025}} \\ \therefore V_{BE} = \underline{0.587 \text{ V}}$$

Alternate way - without calculating  $I_s$

For  $V_{BE} = 0.7V$

$$\frac{i_c}{10\text{mA}} = e^{\frac{0.7 - 0.76}{0.025}}$$

$$\therefore i_c = \underline{0.907\text{mA}}$$

For  $i_c = 10\text{mA}$

$$\frac{10 \times 10^{-6}}{10 \times 10^{-3}} = e^{\frac{V_{BE} - 0.76}{0.025}}$$

$$V_{BE} = \underline{0.587\text{V}}$$

5.9

$$\beta = \frac{\alpha}{1-\alpha}, \text{ for } \alpha \rightarrow \alpha + \Delta\alpha$$

$$\beta \rightarrow \beta + \Delta\beta$$

$$\beta + \Delta\beta = \frac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha} \quad \text{solve for } \Delta\beta$$

$$\Delta\beta = \frac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha} - \frac{\alpha}{1 - \alpha}$$

$$= \frac{\alpha}{1 - \alpha} \cdot \frac{1 + \frac{\Delta\alpha}{\alpha}}{1 - \frac{\Delta\alpha}{1 - \alpha}} - \frac{\alpha}{1 - \alpha}$$

$$= \beta \times \frac{\frac{\Delta\alpha}{\alpha} + \frac{\alpha}{1 - \alpha}}{1 - \frac{\Delta\alpha}{1 - \alpha}}$$

Thus,

$$\frac{\Delta\beta}{\beta} = \frac{\Delta\alpha}{\alpha} \cdot \frac{1 + \frac{\alpha}{1 - \alpha}}{1 - \frac{\Delta\alpha}{1 - \alpha}}$$

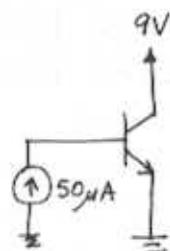
$$= \frac{\Delta\alpha}{\alpha} \cdot \frac{1 - \alpha + \alpha}{1 - \alpha - \Delta\alpha}$$

$$= \frac{\Delta\alpha}{\alpha} \cdot \frac{1}{1 - \alpha - \Delta\alpha}$$

for small  $\Delta\alpha$ ,  $\frac{1}{1 - \alpha - \Delta\alpha} \approx \beta$

$$\therefore \frac{\Delta\beta}{\beta} \approx \underline{\left( \frac{\Delta\alpha}{\alpha} \right) \beta} \quad \text{Q.E.D.}$$

5.10



$\beta = 60 \text{ to } 300$

$$I_c = \beta I_B \text{ ranges from}$$

$$= 60 \times 50\mu\text{A} \text{ to } 300 \times 50\mu\text{A}$$

$$= \underline{3\text{mA to } 15\text{mA}}$$

$$I_E = I_c + I_B \text{ ranges from}$$

$$= \underline{3.05\text{mA to } 15.05\text{mA}}$$

$$\text{Max Power} = 9 \times I_{c\max} = 9 \times 15$$

$$= \underline{135\text{mW}}$$

5.11

$$i_c = I_s e^{\frac{V_{BE}}{V_T}}$$

$$10 \times 10^{-3} = I_s e^{\frac{0.7}{0.025}}$$

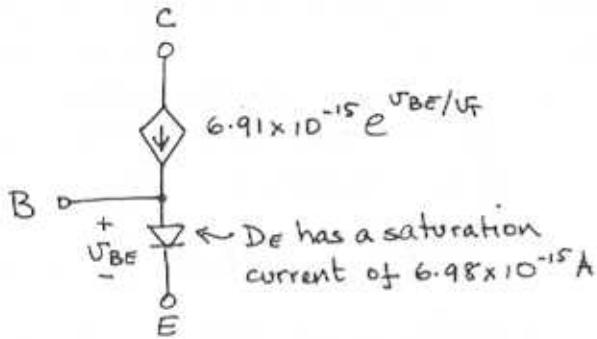
$$\Rightarrow I_s = 6.91 \times 10^{-15} \text{ A}$$

$$\alpha = \frac{i_c}{i_c + i_B} = \frac{10}{10 + 0.1} = 0.990$$

$$\beta = \frac{i_c}{i_B} = \frac{10}{0.1} = 100$$

$$\frac{I_s}{\alpha} = 6.98 \times 10^{-15} \text{ A}$$

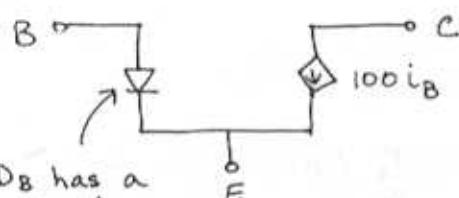
$$\frac{I_s}{\beta} = 6.91 \times 10^{-17} \text{ A}$$



$$i_c = \alpha i_e = 0.99 \times 3 \text{ mA}$$

$$V_c = 10 - 2i_c = 10 - 2 \times 0.99 \times 3 = 4.06 \text{ V}$$

5.13



$$\beta_p = 100 \quad \alpha_r = 0.01 \quad I_s = 10^{-15} \text{ A}$$

$$\Rightarrow \alpha_p = 0.99 \quad \beta_r = 0.111$$

(a) Forward Active Mode Operation  
 $I_B = 10 \mu\text{A}$     $V_{CB} = 1\text{V}$     $V_{CE}, I_C, I_E = ?$

Using the Ebers Moll model we have from eq. (5.28)

$$i_B = \frac{I_s}{\beta_p} \left( e^{\frac{V_{BE}/V_T}{-1}} - 1 \right) + \frac{I_s}{\beta_r} \left( e^{\frac{V_{CE}/V_T}{-1}} - 1 \right)$$

NB: NEGATIVE IN FORWARD ACTIVE MODE

$$10 \times 10^{-15} = \frac{10^{-15}}{100} \left( e^{\frac{V_{BE}/0.025}{-1}} - 1 \right) + \frac{10^{-15}}{0.111} \left( e^{\frac{1/0.025}{-1}} - 1 \right)$$

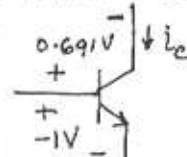
$$= 0.691 \text{ V}$$

$$I_C = I_s e^{\frac{V_{BE}/V_T}{-1}} = 10^{-15} e^{\frac{0.691}{0.025}}$$

$$= 1 \text{ mA} \quad \text{NB} \quad I_C = \beta I_B = 100 I_B$$

$$I_E = I_C + I_B = 1.01 \text{ mA}$$

(b) Reverse Active Mode Operation



$$i_E = I_{SE} e^{\frac{V_{BE}/V_T}{-1}}$$

$$3 \times 10^{-3} = \frac{10^{-15}}{0.99} e^{\frac{V_{BE}/0.025}{-1}}$$

$$V_{BE} = 0.718 \text{ V}$$

$$V_E = 0 - 0.718 = -0.718 \text{ V}$$

Eq (5.28)

$$i_B = \frac{I_s}{\alpha_F} (e^{\frac{V_{BE}/V_T}{\alpha_F}} - 1) + \underbrace{\frac{I_s}{\beta_R} (e^{\frac{V_{BC}/V_T}{\beta_R}} - 1)}_{\text{this term dominates in reverse active mode}}$$

$$= \underbrace{\frac{10^{-15}}{100} (e^{\frac{-1}{0.025}} - 1)}_{\approx 0} + \frac{10^{-15}}{0.111} (e^{\frac{0.691}{0.025}} - 1)$$

$$= -10^{-17} + 0.00909$$

$$= \underline{9.08 \text{ mA}}$$

Eq (5.27)

$$i_c = I_s (e^{\frac{V_{BE}/V_T}{\alpha_F}} - 1) - \frac{I_s}{\alpha_R} (e^{\frac{V_{BC}/V_T}{\alpha_R}} - 1)$$

$$= \underbrace{10^{-15} (e^{\frac{-1}{0.025}} - 1)}_{\approx 0} - \frac{10^{-15}}{0.1} (e^{\frac{0.691}{0.025}} - 1)$$

$$= -10^{-15} - 0.01009$$

$$= \underline{-10.09 \text{ mA}}$$

$$i_E = i_c + i_B = \underline{-1.01 \text{ mA}}$$

Check with Eq (5.26) - Ebers Moll.

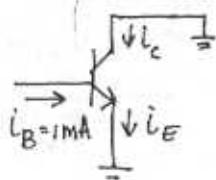
$$i_E = \underbrace{\frac{I_s}{\alpha_F} (e^{\frac{-1}{0.025}} - 1)}_{\approx 0} - I_s (e^{\frac{0.691}{0.025}} - 1)$$

$$= \underline{-1.01 \text{ mA}}$$

5.14

$$\alpha_F \approx 1$$

$$\therefore \beta_F = \infty$$



$$\alpha_F I_{SE} = \alpha_R I_{SC} = I_S$$

Since  $I_S \propto \text{Area}$

$$\frac{I_{SC}}{I_{SE}} \Rightarrow \frac{\alpha_F}{\alpha_R} = 10$$

$$\alpha_R = \frac{\alpha_F}{10} = \frac{1}{10}, \quad \beta_R = \frac{\alpha_R}{1-\alpha_R} = \frac{1}{9}$$

Using Eq (5.26)

$$i_E = \frac{I_s}{\alpha_F} (e^{\frac{V_{BE}/V_T}{\alpha_F}} - 1) - I_s (e^{\frac{V_{BC}/V_T}{\alpha_R}} - 1)$$

$$= \underline{0 \text{ A}} \quad - \text{ Since } V_{BE} = V_{BC}$$

Using Eq (5.28)

$$i_B = \frac{I_s}{\alpha_F} (e^{\frac{V_{BE}/V_T}{\alpha_F}} - 1) + \frac{I_s}{\beta_R} (e^{\frac{V_{BC}/V_T}{\beta_R}} - 1)$$

$$= I_s \left( \underbrace{\frac{1}{\alpha_F} + \frac{1}{\beta_R}}_{\approx 0} \right) (e^{\frac{V_{BC}/V_T}{\beta_R}} - 1)$$

$$\therefore I_s (e^{\frac{V_{BC}/V_T}{\beta_R}} - 1) = \beta_R i_B = \frac{10^{-3}}{9}$$

Using Eq (5.27)

$$i_c = I_s (e^{\frac{V_{BE}/V_T}{\alpha_F}} - 1) - \frac{I_s}{\alpha_R} (e^{\frac{V_{BC}/V_T}{\alpha_R}} - 1)$$

$$= I_s (1-10) (e^{\frac{V_{BC}/V_T}{\alpha_R}} - 1)$$

$$= -9 \times 10^{-3} / 9 = \underline{-1 \mu A}$$

5.15

Using the complete Ebers Moll equations we have:

From Eq (5.26)

$$i_E = \frac{I_s}{\alpha_p} \left( e^{\frac{V_{BE}}{V_T}} - 1 \right) - I_s \left( e^{\frac{V_{BC}}{V_T}} - 1 \right)$$

$$\Rightarrow I_s \left( e^{\frac{V_{BC}}{V_T}} - 1 \right) = \alpha_p (i_E + I_s \left( e^{\frac{V_{BC}}{V_T}} - 1 \right))$$

Substitute this expression in Eq (5.27):

$$i_C = I_s \left( e^{\frac{V_{BC}}{V_T}} - 1 \right) - \frac{I_s}{\alpha_R} \left( e^{\frac{V_{BC}}{V_T}} - 1 \right) \quad (A)$$

$$= \alpha_p i_E + I_s \left( \alpha_p e^{\frac{V_{BC}}{V_T}} - \alpha_p - \frac{I_s}{\alpha_R} e^{\frac{V_{BC}}{V_T}} + \frac{I_s}{\alpha_R} \right)$$

SEE FIG 5.9

$i_E = I_E$  since the npn transistor is fed with a constant current

$$= \alpha_p I_E - I_s \left( \frac{1}{\alpha_R} - \alpha_p \right) e^{\frac{V_{BC}}{V_T}} + I_s \left( \frac{1}{\alpha_R} - \alpha_p \right)$$

1

- neglect as  $I_s \ll 1$
- this is the same as neglecting terms not involving exponents

Thus,

$$i_C = I_E - I_s \left( \frac{1}{\alpha_R} - \alpha_p \right) e^{\frac{V_{BC}}{V_T}} \quad (B)$$

Q.E.D.

$$(b) I_s = 10^{-15} \quad \alpha_p \approx 1 \quad \alpha_R = 0.1 \quad \beta_F = \infty$$

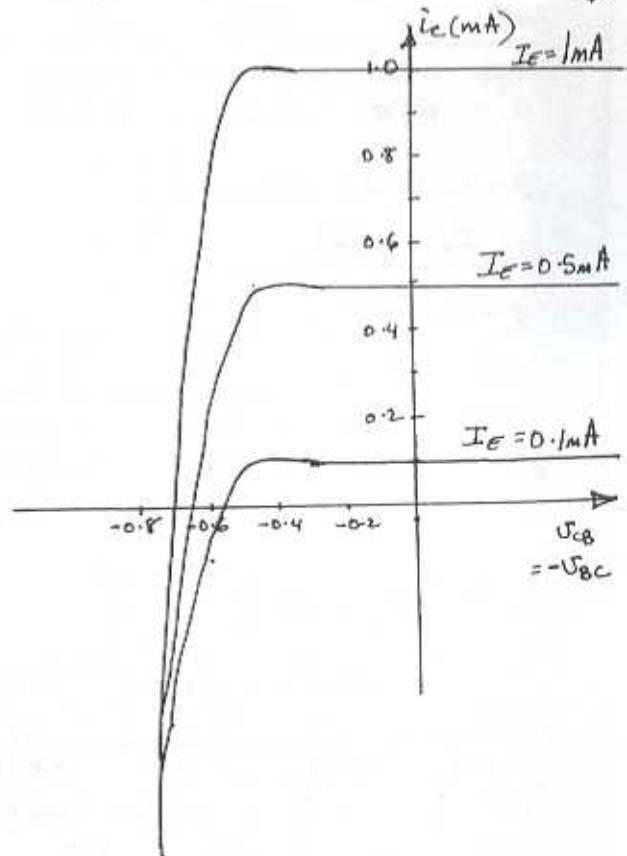
$$\text{from (B)} \quad i_C = I_E - 9 \times 10^{-15} e^{\frac{V_{BC}/0.025}{0.1}} \quad (C)$$

This expression is used to create the

following tables and hence the curves of  $i_C$  vs  $V_{BC}$ .

$V_{BC}$ (V)	$i_C$ (mA) $i_E = 0.1\text{mA}$	$i_C$ (mA) $i_E = 0.5\text{mA}$	$i_C$ (mA) $i_E = 1\text{mA}$
0.7	-12.92	-12.52	-12.02
0.6	-0.14	0.26	0.762
0.5	+0.1	0.5	1
0.45	+0.1	0.5	1
0.4	0.1	0.5	1
0.35	0.1	0.5	1
0.3	0.1	0.5	1
0	0.1	0.5	1
-1	0.1	0.5	1
-2	0.1	0.5	1

$V_{BC}$  is getting smaller so less current is subtracted from  $I_E$ .  $\therefore i_C \rightarrow I_E$



For a particular  $I_E$  and  $i_c$ ,  $V_{BC}$  can be calculated using (c)

$$i_c = I_E - 9 \times 10^{-15} e^{\frac{V_{BC}}{0.025}} \quad (1)$$

Having calculated  $V_{BC}$ ,  $V_{BE}$  can be calculated using (A)

$$i_c = 10^{-15} \left( e^{\frac{V_{BE}}{0.025}} - 1 \right) - 10^{-14} \left( e^{\frac{V_{BC}}{0.025}} - 1 \right) \quad (2)$$

Now  $V_{CE}$  can be calculated with  
 $V_{CE} = -V_{BC} + V_{BE}$  (3)

Thus,

for  $I_E = 0.1 \text{ mA}$

$i_c$ (mA)	$V_{BC}(1)$ (V)	$V_{BE}(2)$ (V)	$V_{CE}(3)$ (V)
$0.5I_E$	0.561	0.644	0.083
0	0.578	0.635	0.057

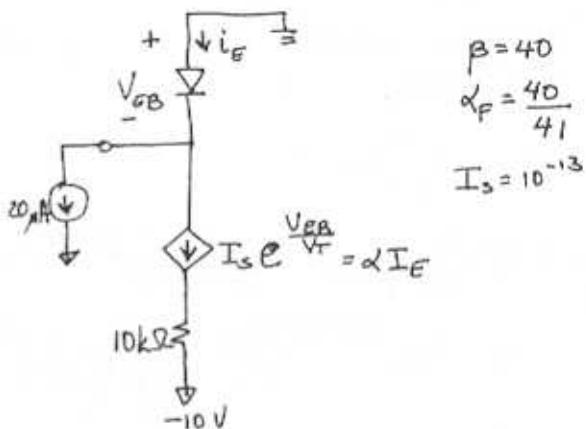
for  $I_E = 0.5 \text{ mA}$

$i_c$ (mA)	$V_{BC}(1)$ (V)	$V_{BE}(2)$ (V)	$V_{CE}(3)$ (V)
$0.5I_E$	0.601	0.674	0.073
0	0.618	0.675	0.057

for  $I_E = 1 \text{ mA}$

$i_c$ (mA)	$V_{BC}(1)$ (V)	$V_{BE}(2)$ (V)	$V_{CE}(3)$ (V)
$0.5I_E = \frac{1}{2}$	0.618	0.691	0.073
0	0.638	0.696	0.058

5.16



$$i_E = \frac{I_S}{\alpha} e^{\frac{V_{EB}/V_T}{1-\alpha}} = I_S e^{\frac{V_{EB}/V_T}{1-\alpha}} + 0.02 \times 10^{-3}$$

$$I_S e^{\frac{V_{EB}/V_T}{1-\alpha}} \left( \frac{1}{\alpha} - 1 \right) = 0.02 \times 10^{-3}$$

$$10^{-13} e^{\frac{V_{EB}/0.025}{1-\alpha}} \left( \frac{41}{40} - 1 \right) = 0.02 \times 10^{-3}$$

$$V_{EB} = 0.570 \text{ V} \Rightarrow V_B = -0.570 \text{ V}$$

$$i_E = \frac{I_S}{\alpha} e^{\frac{V_{EB}/V_T}{1-\alpha}} = \frac{10^{-13}}{40} e^{\frac{0.57}{0.025}} = 0.82 \text{ mA}$$

$$i_c = \alpha i_E \Rightarrow V_C = -10 + \alpha i_E \times 10 = -10 + \frac{40}{41} \times 0.82 \times 10 = -2 \text{ V}$$

5.17

$$\text{Given } i_c = I_S e^{\frac{V_{EB}/V_T}{1-\alpha}}$$

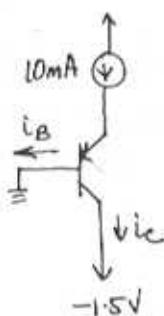
$$\text{Use } \frac{i_c}{1A} = e^{\frac{V_{EB}-0.8}{0.025}}$$

to calculate  $V_{EB}$  for a particular  $i_c$

CONT.

for  $i_c = 10 \text{ mA}$     $V_{EB} = 0.685 \text{ V}$   
 $i_c = 5 \text{ A}$     $V_{EB} = 0.840 \text{ V}$

5.18



$$\beta = 10$$

$$i_c = \alpha i_E = \frac{10}{11} \times 10 = \underline{\underline{9.09 \text{ mA}}}$$

$$i_B = i_E - i_c = \underline{\underline{0.91 \text{ nA}}}$$

$$i_c = I_s e^{\frac{V_{EB}}{V_T}}$$

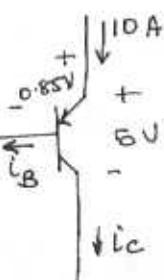
$$9.09 \times 10^{-3} = 10^{-14} e^{\frac{V_{EB}}{0.025}}$$

$$V_C = V_{EB} = \underline{\underline{0.803 \text{ V}}}$$

For  $\beta = 1000$

$$i_c = \frac{\beta}{\beta+1} i_E = \frac{1000}{1001} \times 10 = \underline{\underline{9.99 \text{ mA}}}$$

5.19



for  $\beta = 15$

$$i_E = (\beta+1) i_B$$

$$10 = (\beta+1) i_B$$

$$i_B = \frac{10}{16} = \underline{\underline{0.625 \text{ A}}}$$

Calculating  $I_{S1}$

$$i_c = \frac{\beta}{\beta+1} i_E = I_{S1} e^{\frac{V_{EB}}{V_T}}$$

$$\frac{15}{16} \times 10 = I_{S1} e^{0.85/0.025}$$

$$I_{S1} = 1.608 \times 10^{-14} \text{ A}$$

Compare this to  
 $I_{S2} = i_c e^{-\frac{V_{EB}}{V_T}}$

$$= 10^{-3} e^{-0.7/0.025}$$

$$= 6.914 \times 10^{-14}$$

$\therefore I_S \propto \text{Area}$

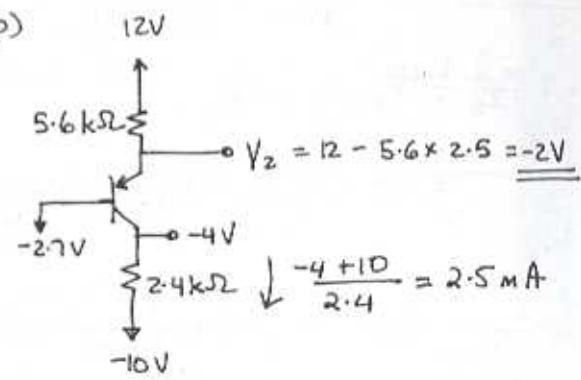
$$\frac{\text{Area 1}}{\text{Area 2}} = \frac{I_{S1}}{I_{S2}} = \frac{1.608 \times 10^{-14}}{6.914 \times 10^{-14}}$$

$$= \underline{\underline{23.3 \text{ times larger}}}$$

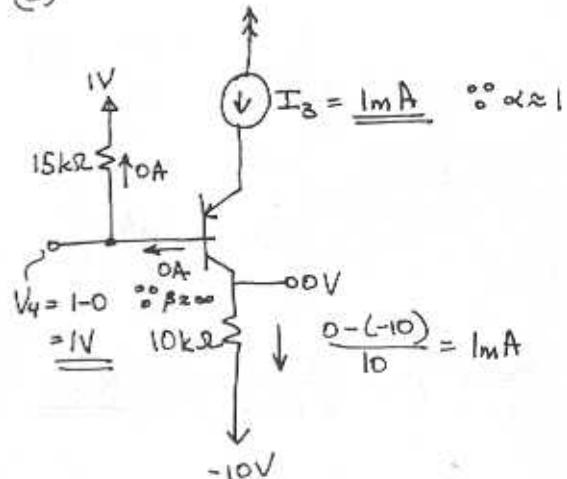
5.20

$$(a) I_1 = \frac{10.7 - 0.7}{10} = \underline{\underline{1 \text{ mA}}}$$

(b)

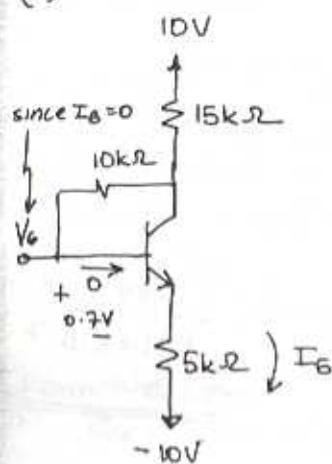


(c)



CONT

(d)



$$I_E = I_C$$

$$\frac{V_B - 0.7 + 10}{5} = \frac{10 - V_B}{15}$$

$$15V_B + 139.5 = 50 - 5V_B$$

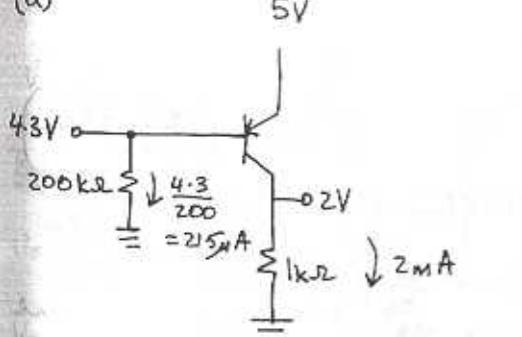
$$V_B = \underline{-4.475 \text{ V}}$$

$$I_E = \frac{V_B - 0.7 + 10}{5}$$

$$= \underline{-4.475 - 0.7 + 10} = \underline{0.965 \text{ mA}}$$

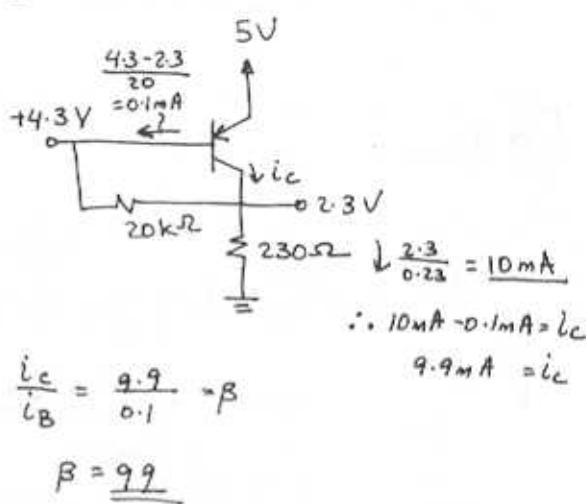
5.21

(a)



$$\frac{I_C}{I_B} = \beta = \frac{2}{0.0215} = \underline{93.0}$$

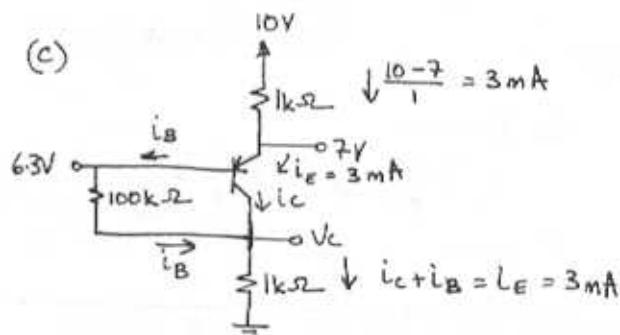
(b)



$$\frac{I_C}{I_B} = \frac{9.9}{0.1} = \beta$$

$$\beta = \underline{99}$$

(c)



$$V_C = 3 \times 1 = 3 \text{ V}$$

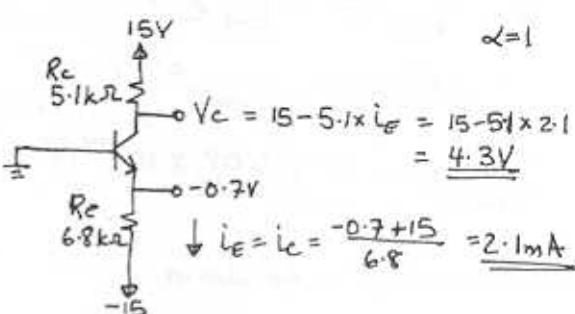
$$I_B = \frac{6.3 - 3}{100} = \frac{3.3}{100}$$

$$\beta + 1 = \frac{I_E}{I_B} = \frac{3 \times 100}{3.3}$$

$$\beta = \frac{300}{3.3} - 1 = \underline{89.9}$$

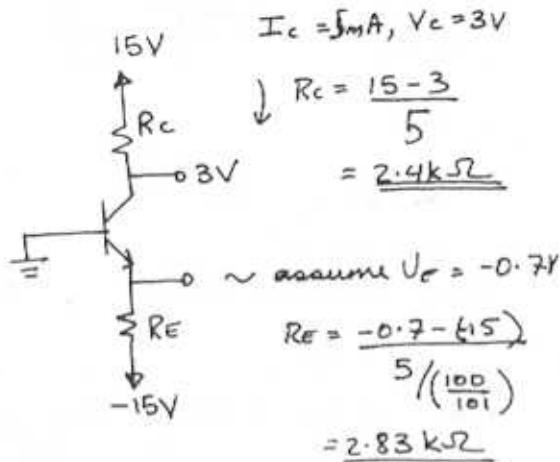
5.22

$$\alpha = 1$$



$$I_E = I_C = \frac{-0.7 + 15}{6.8} = \underline{2.1 \text{ mA}}$$

5.23



$$(c) V_B = 3 \text{ V}$$

$$V_E = V_B + 0.7 = 3.7 \text{ V}$$

$$I_E = \frac{9 - V_E}{1.1} = \frac{9 - 3.7}{1.1} = 4.82 \text{ mA}$$

$$I_c = \alpha I_e = \frac{30}{31} \times 4.82 = 4.66 \text{ mA}$$

$$V_c = I_c \times 0.56 = 2.62 \text{ V}$$

$$I_B = \frac{I_c}{\beta} = \frac{4.66}{30} = 0.155 \text{ mA}$$

(d)  $V_B = 3 \text{ V}$

$$V_E = 3 - 0.7 = 2.3 \text{ V}$$

$$I_E = V_E / 0.47 = 2.3 / 0.47 = 4.89 \text{ mA}$$

$$I_c = \alpha I_E = \frac{30}{31} \times 4.89 = 4.73 \text{ mA}$$

$$V_c = 9 - 1 \times I_c = 9 - 4.73 = 4.22 \text{ V}$$

$$I_B = I_c / \beta = \frac{4.73}{30} = 0.158 \text{ mA}$$

5.24

$$(a) V_B = 0 \text{ V}$$

$$V_E = V_B + 0.7 = 0.7 \text{ V}$$

$$I_E = \frac{-0.7 + 3}{2.2} = 1.05 \text{ mA}$$

$$I_c = \frac{30}{31} I_E = 1.02 \text{ mA}$$

$$V_c = 3 - 1.02 \times 2.2 = 0.756 \text{ V}$$

$$I_B = \frac{I_c}{\beta} = \frac{1.02}{30} = 0.034 \text{ mA}$$

$$(b) V_B = 0 \text{ V}$$

$$V_E = V_B + 0.7 = 0.7 \text{ V}$$

$$I_E = \frac{3 - V_E}{1} = \frac{3 - 0.7}{1} = 2.3 \text{ mA}$$

$$I_c = \alpha I_E = \frac{30}{31} \times 2.3 = 2.23 \text{ mA}$$

$$V_c = -3 + 1 \times I_c = -3 + 2.23$$

$$= -0.77 \text{ V}$$

$$I_B = \frac{I_c}{\beta} = \frac{2.23}{30} = 0.0743 \text{ mA}$$

5.25

Instead of  $V_{BE}$  being independent of the current level  $V_{BE} = 0.7 \text{ V}$  only at  $I_c = 1 \text{ mA}$ . If  $I_c$  is different  $V_{BE}$  will change.

$$(a) V_E = -V_{BE}$$

$$I_c = \alpha I_E = \frac{30}{31} \left( \frac{-V_{BE} + 3}{2.2} \right) = \frac{-30V_{BE} + 90}{31(2.2)}$$

$$\therefore \frac{I_c}{1 \text{ mA}} = e^{\frac{V_{BE} - 0.7}{0.025}} \text{ we have}$$

$$\frac{-30V_{BE} + 90}{31(2.2)} = e^{\frac{V_{BE} - 0.7}{0.025}}$$

CONT.

$$V_{BE} = 0.025 \ln \left( \frac{90 - 30V_{BE}}{31 \times 2.2} \right) + 0.7$$

Iteration #1

$$V_{BE} = 0.7 \rightarrow V_{BE} = 0.70029 \approx 0.7V$$

∴ All the values calculated in 5.24(a) remain the same. This makes sense as  $I_c$  in 5.24(a) was  $1.02mA$  which has a rated  $V_{BE}$  voltage of  $0.7V$ . Specifically we have:

$$\begin{aligned} V_B &= 0V \\ V_E &= -0.7V \\ V_C &= 0.756V \\ I_B &= 0.034mA \\ I_E &= 1.05mA \\ I_C &= 1.02mA \end{aligned}$$

(b) Continuing to use the iterative process on relative currents

$$\frac{I_c}{1mA} = e^{\frac{V_{BE}-0.7}{0.025}} \text{ we have:}$$

$$\frac{I_c}{1} = \frac{I_E}{1} = e^{\frac{V_{EB}-0.7}{0.025}}$$

$$\frac{30}{31} \left( \frac{3 - V_{EB}}{1} \right) = e^{\frac{V_{EB}-0.7}{0.025}}$$

$$V_{EB} = 0.7 + 0.025 \ln \left( \frac{90 - 30V_{EB}}{31} \right)$$

By Iteration

$$V_{EB} = 0.7V \rightarrow 0.72V \rightarrow 0.72V$$

$$\begin{aligned} V_E &= 0.72V \\ I_E &= \frac{3 - 0.72}{1} = 2.28mA \\ I_C &= \frac{30}{31} I_E = 2.21mA \end{aligned}$$

$$V_C = -3 + 2.21 = -0.79V$$

$$V_B = 0 \quad I_B = I_C/B = \frac{2.21}{30} = 0.074mA$$

$$(c) I_c = \alpha I_E = e^{\frac{V_{EB}-0.7}{0.025}}$$

$$\frac{30}{31} \left( \frac{3 - (3 + V_{EB})}{1} \right) = e^{\frac{V_{EB}-0.7}{0.025}}$$

$$V_{EB} = 0.7 + 0.025 \ln \left[ \frac{180 - 3V_{EB}}{31} \right]$$

By Iteration

$$V_{EB} = 0.7 \rightarrow 0.741 \rightarrow 0.741$$

$$\therefore V_E = 3.741V \quad I_E = \frac{9 - 3.741}{1.1} = 4.78mA$$

$$I_C = \frac{30}{31} I_E = 4.63mA \quad V_C = 0.56 I_C = 2.59V$$

$$I_B = \frac{I_C}{30} = 0.154mA \quad V_B = 3V$$

$$(d) I_c = \alpha I_E = e^{\frac{V_{BE}-0.7}{0.025}}$$

$$\frac{30}{31} \left( \frac{3 - V_{BE}}{0.47} \right) = e^{\frac{V_{BE}-0.7}{0.025}}$$

$$V_{EB} = 0.7 + \ln \left[ \frac{90 - 30V_{BE}}{31 \times 0.47} \right]$$

By Iteration:

$$V_{BE} = 0.7 \rightarrow 0.739 \rightarrow 0.738 \rightarrow 0.738$$

$$\therefore V_E = 3 - 0.738 = 2.26V$$

$$I_E = V_E / 0.47 = 4.81mA$$

CONT.

$$I_c = \frac{30}{31} I_E = \underline{4.65 \text{ mA}} \quad V_c = 9 - I_c \times 1 \\ = \underline{4.35 \text{ V}}$$

$$I_B = \frac{I_c}{30} = \underline{0.155 \text{ mA}} \quad V_B = \underline{3 \text{ V}}$$

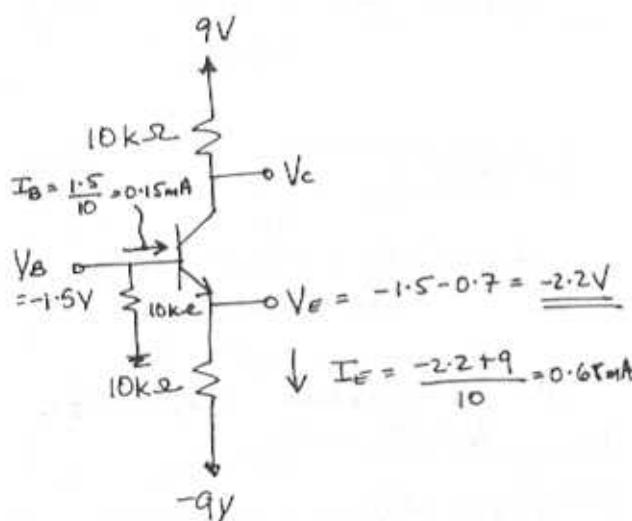
$$I_E = 0.68 \text{ mA}$$

$$I_B = 0 \quad V_B = \underline{0}$$

$$V_c = 9 - 10 I_E = 9 - 6.8 = \underline{2.2 \text{ V}}$$

5. 26

FIRST WITH FINITE  $\beta$



$$\frac{I_e}{I_b} = \beta + 1$$

$$\frac{0.68}{0.15} = \beta + 1 \Rightarrow \beta = \underline{3.63}$$

$$\therefore \alpha = \frac{\beta}{\beta+1} = \underline{0.779}$$

$$V_c = 9 - 10(\alpha I_e) \\ = 9 - 10(0.779 \times 0.68) \\ = \underline{3.7 \text{ V}}$$

SECOND WITH INFINITE  $\beta$

$$\beta = \infty \quad V_{BE} = 0.7 \text{ V} \Rightarrow V_E = \underline{-2.2 \text{ V}}$$

$$\alpha = 1$$

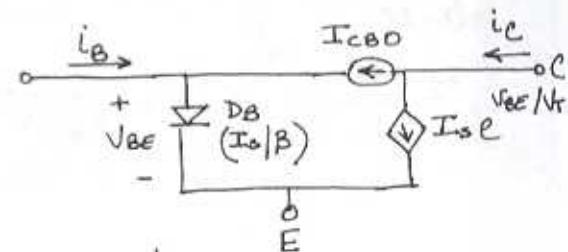
5. 27

$I_{CBO}$  doubles for every  $10^\circ\text{C}$  rise in temperature.

Thus if  $I_{CBO} = 20 \text{ nA}$  at  $25^\circ\text{C}$

$$\text{At } 85^\circ\text{C} \quad I_{CBO} = 2^{\frac{85-25}{10}} \times 20 \\ = \underline{1280 \text{ nA}}$$

5. 28



$$i_B = \frac{I_s}{\beta} e^{\frac{V_{BE}}{V_T}} - I_{CBO} \quad (1)$$

$$i_C = I_s e^{\frac{V_{BE}}{V_T}} + I_{CBO} \quad (2)$$

$$i_E = I_s (1 + \frac{1}{\beta}) e^{\frac{V_{BE}}{V_T}} \quad (3)$$

for  $\beta$  open circuited,  $i_B = 0$  and (1) gives:

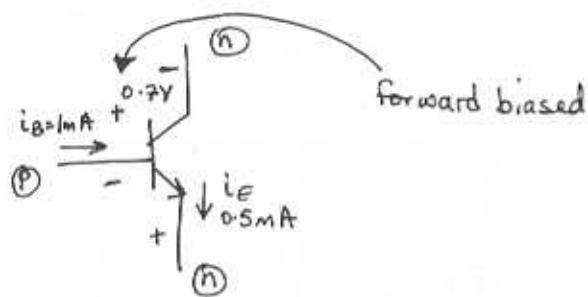
$$\frac{I_s}{\beta} e^{\frac{V_{BE}}{V_T}} = I_{CBO} \Rightarrow e^{\frac{V_{BE}}{V_T}} = \frac{\beta I_{CBO}}{I_s}$$

SUBSTITUTE INTO (2) & (3) =>

$$i_C = (\beta + 1) I_{CBO}$$

$$i_E = (\beta + 1) I_{CBO}$$

5.29



Reverse Active Mode - see Table 5.1

Use Ebers Moll Model:

$$\text{Eq (5.26)} \quad i_E = \frac{I_s}{\alpha_F} \left( e^{\frac{V_{BE}}{V_T}} - 1 \right) - I_s \left( e^{\frac{V_{CE}}{V_T}} - 1 \right)$$

NEGLECT

$I_s$  is small  
NEGATIVE AND  
THUS VERY  
SMALL

$$\text{Eq (5.28)} \quad i_E = \frac{I_s}{\beta_F} \left( e^{\frac{V_{EB}}{V_T}} - 1 \right) + \frac{I_s}{\beta_R} \left( e^{\frac{V_{EC}}{V_T}} - 1 \right)$$

NEGLECT  
THIS TERM

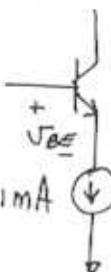
∴ Neglecting the first terms in the expressions for  $i_E$  and  $i_B$  we have

$$i_E = -I_s \left( e^{\frac{V_{EB}}{V_T}} - 1 \right) = 0.5 \times 10^{-3}$$

$$i_B = \frac{I_s}{\beta_R} \left( e^{\frac{V_{EC}}{V_T}} - 1 \right) = 10^{-3}$$

$$\left| \frac{i_E}{i_B} \right| = \underline{\beta_R = \frac{1}{2}} \quad \underline{\alpha_R = \frac{\beta_R}{\beta_R + 1} = \frac{1}{3}}$$

5.30



$V_{BE}$  changes by  $-2 \text{ mV}/^\circ\text{C}$

GIVEN  $V_{BE} = 0.69 \text{ V}$  at  $25^\circ\text{C}$

$$\begin{aligned} \text{Thus,} \\ \text{at } 0^\circ\text{C} \quad V_{BE} &= 0.69 - 2 \times 10^{-3}(-25) \\ &= \underline{\underline{0.74 \text{ V}}} \end{aligned}$$

$$\begin{aligned} \text{At } 100^\circ\text{C} \quad V_{BE} &= 0.69 - 2 \times 10^{-3}(75) \\ &= \underline{\underline{0.54 \text{ V}}} \end{aligned}$$

5.31

GIVEN  $i_E = 0.5 \text{ mA}$       } AT  $20^\circ\text{C}$   
 $V_{EB} = 0.692 \text{ V}$

(a) The junction temperature rises to  $50^\circ\text{C}$

$$\begin{aligned} V_{EB} &= 0.692 - 2 \times 10^{-3}(50 - 20) \\ &= \underline{\underline{0.632 \text{ V}}} \end{aligned}$$

(b) The Base-Emitter Voltage is fixed  
 $V_{EB} = 0.7 \text{ V}$  at ALL TEMPERATURES

At  $20^\circ\text{C}$   $\sim i_E = 0.5 \text{ mA}$  at  $V_{EB} = 0.692 \text{ V}$   
 Thus for  $V_{EB} = 0.7 \text{ V}$  we have

$$\frac{i_E}{0.5 \times 10^{-3}} = e^{\frac{0.7 - 0.692}{0.025}}$$

$$i_E = \underline{\underline{0.689 \text{ mA}}}$$

CONT.

Now if  $T = 50^\circ\text{C}$  &  $V_{EB} = 0.7\text{V}$

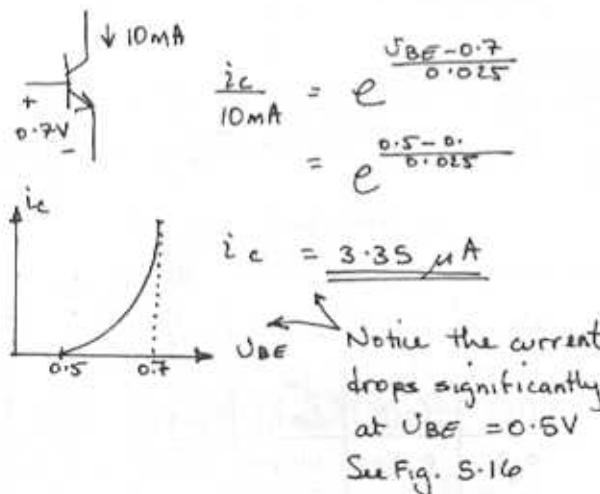
from (a) we see that at  $50^\circ\text{C}$ ,  
 $I_E = 0.5\text{mA}$ ,  $V_{EB} = 0.632\text{V}$

Therefore for  $V_{EB} = 0.7\text{V}$

$$\frac{i_E}{0.5 \times 10^{-3}} = e^{\frac{0.7 - 0.632}{0.025}}$$

$$i_E = \underline{7.59\text{mA}}$$

5.32



5.33

$V_{BE}$  changes by  $-2\text{mV}/^\circ\text{C}$  FOR A PARTICULAR CURRENT. Given that at  $25^\circ\text{C}$   $V_{BE} = 0.7\text{V}$  and  $i_C = 10\text{mA}$

Thus

$$\begin{aligned} @ -25^\circ\text{C} \quad V_{BE} &= 0.7 - 2 \times 10^{-3}(-50) \\ &= \underline{0.8\text{V}} \quad \text{and} \quad i_C = 10\text{mA} \end{aligned}$$

$$\begin{aligned} @ 125^\circ\text{C} \quad V_{BE} &= 0.7 - 2 \times 10^{-3}(100) \\ &= \underline{0.5\text{V}} \quad \text{and} \quad i_C = 10\text{mA} \end{aligned}$$

5.34

Using the complete Ebers Moll model in eqs. (5.26) and (5.27) and neglecting terms not containing exponentials the  $i_c$  dependence on  $V_{BC}$  with fixed  $i_e = I_E$  can be derived :-

$$\begin{aligned} i_c &= I_S \left( e^{\frac{V_{BE}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left( e^{\frac{V_{BC}}{V_T}} - 1 \right) \\ &\approx I_S e^{\frac{V_{BE}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_R}}} - \frac{I_S}{\alpha_R} e^{\frac{V_{BC}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_R}}} \quad (1) \end{aligned}$$

$$\begin{aligned} i_E &= \frac{I_S}{\alpha_F} \left( e^{\frac{V_{BC}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_F}}} - 1 \right) - I_S \left( e^{\frac{V_{BC}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_F}}} - 1 \right) \\ &\approx \frac{I_S}{\alpha_F} e^{\frac{V_{BC}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_F}}} - I_S e^{\frac{V_{BC}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_F}}} \end{aligned}$$

Noting the emitter current as being fixed at  $I_E$  we get

$$I_S e^{\frac{V_{BE}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_F}}} = \alpha_F I_E + \alpha_F I_S e^{\frac{V_{BC}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_F}}}$$

Substituting this into (1) we get:

$$\begin{aligned} i_c &= \alpha_F I_E + \alpha_F I_S e^{\frac{V_{BC}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_F}}} - \frac{I_S}{\alpha_R} e^{\frac{V_{BC}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_R}}} \\ &= \underline{\alpha_F I_E + I_S \left( \alpha_F - \frac{1}{\alpha_R} \right) e^{\frac{V_{BC}/V_T}{1 + \frac{V_{BC}/V_T}{\alpha_F}}}} \quad \equiv \text{Eq. (5.35)} \\ &\quad \text{Q.E.D.} \end{aligned}$$

5.35

See details in problem 5.15(b).

5.36

For the saturated transistor in Fig P536  
 $V_{BC} > 0 \Rightarrow e^{\frac{V_{BC}}{V_T}} \gg 1$

$$V_{BE} > 0 \Rightarrow e^{\frac{V_{BE}}{V_T}} \gg 1$$

Therefore only exponential terms in the complete Ebers Moll model are kept. Specifically in

Eg(5.27) we have

$$i_c = I_s e^{\frac{V_{BE}}{V_T}} - \frac{I_s}{\alpha_R} e^{\frac{V_{BC}}{V_T}}$$

and in Eq (5.26) we have

$$i_E = \frac{I_s}{\alpha_F} e^{\frac{V_{BE}}{V_T}} - I_s e^{\frac{V_{BC}}{V_T}}$$

$\approx 1$

Noting that  $V_{BE} = V_{BC} + V_{CE,sat}$  we get:

$$\begin{aligned} \frac{i_c}{i_E} &= \frac{I_s e^{\frac{V_{BC}+V_{CE,sat}}{V_T}} - \frac{I_s}{\alpha_R} e^{\frac{V_{BC}}{V_T}}}{I_s e^{\frac{V_{BC}+V_{CE,sat}}{V_T}} - I_s e^{\frac{V_{BC}}{V_T}}} \\ &= \frac{e^{\frac{V_{CE,sat}}{V_T}} - \frac{1}{\alpha_R}}{e^{\frac{V_{CE,sat}}{V_T}} - 1} \end{aligned}$$

$$(e^{\frac{V_{CE,sat}}{V_T}} - 1) \frac{I_{CE,sat}}{I_E} = e^{\frac{V_{CE,sat}}{V_T}} - \frac{1}{\alpha_R}$$

$$e^{\frac{V_{CE,sat}}{V_T}} \left[ 1 - \frac{I_{CE,sat}}{I_E} \right] = \frac{1}{\alpha_R} - \frac{I_{CE,sat}}{I_E}$$

$$V_{CE,sat} = V_T \ln \left[ \frac{\frac{1}{\alpha_R} - \frac{I_{CE,sat}}{I_E}}{1 - \frac{I_{CE,sat}}{I_E}} \right] \quad Q.E.D.$$

Given  $\alpha_R = 0.1$  then

$$V_{CE,sat} = 0.025 \ln \left[ \frac{10 - \frac{I_{CE,sat}}{I_E}}{1 - \frac{I_{CE,sat}}{I_E}} \right]$$

$I_{CE,sat}/I_E$	$V_{CE,sat} (V)$
0.9	0.113
0.5	0.0736
0.1	0.06
0	0.058

5.37

Eg(5.36) :

$$\begin{aligned} i_c &= I_s e^{\frac{V_{BE}}{V_T}} \left( 1 + \frac{V_{CE}}{V_A} \right) \leftarrow I_s = 10^{-15} A \quad V_A = 100V \\ &= 10^{-15} e^{\frac{V_{BE}/0.025}{V_T}} \left( 1 + \frac{V_{CE}}{100} \right) \end{aligned}$$

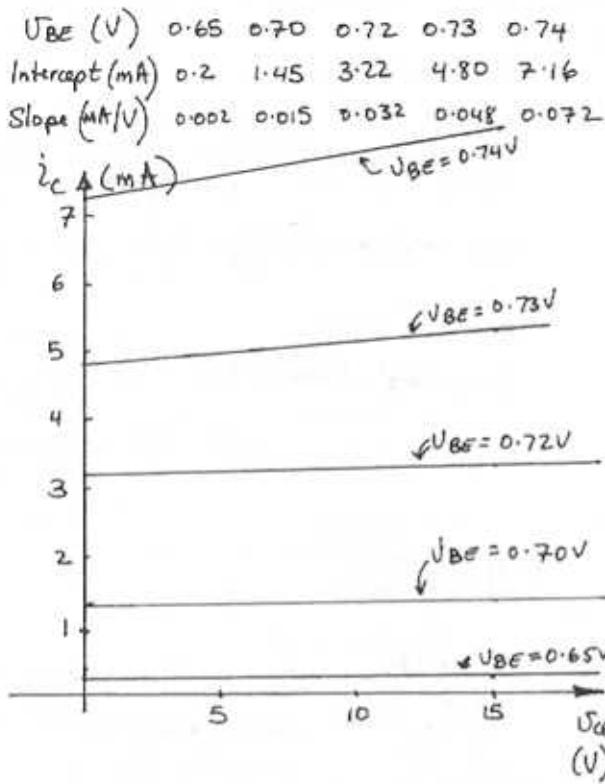
$V_{BE}$ (V)	$i_c$ (mA)	$i_c$ (mA)	$i_c$ (mA)	$i_c$ (mA)	$i_c$ (mA)
0.2	0.1961	1.4491	3.2251	4.8113	7.1777
0.8	0.1973	1.4578	3.2445	4.8402	7.2207
1.0	0.1977	1.4607	3.2509	4.8498	7.2350
4.0	0.2036	1.5041	3.3475	4.9938	7.4499
10	0.2153	1.5909	3.5406	5.2819	7.8797
12	0.2192	1.6198	3.6409	5.3780	8.0230
15	0.2436	1.6632	3.7015	5.5220	8.2379

To find the intercept of the straight-line characteristics on the  $i_c$  axis, we substitute  $V_{CE} = 0$  and evaluate

$$i_c = 10^{-15} e^{\frac{V_{BE}/0.025}{V_T}} A$$
 for the given

value of  $V_{BE}$ . The slope of each straight line is equal to this value divided by  $100V (V_A)$ . Thus we obtain

CONT.



5.38

$$r_o = \frac{1}{3 \times 10^{-5}} = 33.3 k\Omega$$

$$V_A = r_o I_c = 33.3 \times 10^3 \times 3 \times 10^{-3} = 100 V$$

$$r_o = \frac{V_A}{I_c} = \frac{100}{30} = 3.3 k\Omega$$

5.39

$$r_o = V_A / I_c = 200 / I_c$$

$$@ I_c = 1 mA, r_o = 200 k\Omega$$

$$@ I_c = 100 \mu A, r_o = \frac{200}{0.1} = 2.0 M\Omega$$

5.40

$$U_{BE} = 0.72V - i_c = 1.8 \text{ mA} \quad U_{CE} = 2V \\ i_c = 2.4 \text{ mA} \quad U_{CE} = 14V$$

$$r_o = \frac{\Delta U_{CE}}{\Delta i_c} = \frac{14 - 2}{2.4 - 1.8} = 20 k\Omega$$

Near saturation  $U_{CE} = 0.3V$

$$\therefore \frac{\Delta U_{CE}}{\Delta i_c} = \frac{0.3 - 2}{i_c - 1.8} = 20$$

$$i_c = 1.72 \text{ mA}$$

calculating  $U_{CE}$  for  $i_c = 2.0 \text{ mA}$

$$\frac{\Delta U_{CE}}{\Delta i_c} = r_o$$

$$\frac{U_{CE} - 2}{2 - 1.8} = 20 \Rightarrow U_{CE} = 6V$$

Take the ratio of currents to find the Early Voltage (with Eq 5.36)

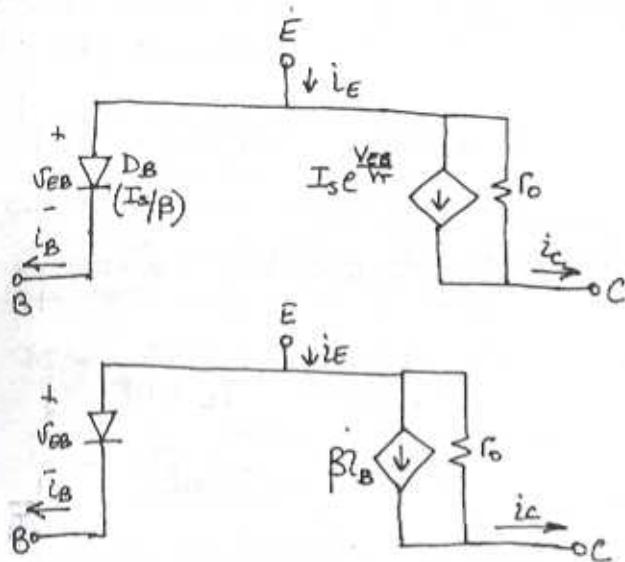
$$\frac{2.4}{1.8} = \underbrace{e^{\frac{U_{CE} - U_{BE}}{V_T}}}_{= 2} \left( \frac{1 + \frac{14}{V_A}}{1 + \frac{2}{V_A}} \right)$$

$$2.4 + \frac{4.8}{V_A} = 1.8 + \frac{2.52}{V_A}$$

$$V_A = 34 V$$

$$r_o = \frac{V_A}{I_c'} \quad \text{where } I_c' \text{ is the current near saturation} \Leftrightarrow \text{active boundary. As calculated above } I_c' = 1.72 \\ = \frac{34}{1.72} \\ = 19.8 k\Omega \quad \leftarrow \approx \text{to the above calculation of } 20 k\Omega$$

5.41



5.42

Large signal or DC  $\beta$ :

$$h_{FE} = \frac{i_c}{i_B} = \frac{1.2\text{mA}}{8\mu\text{A}} = 150$$

$$\text{Small signal } h_{fe} = \frac{0.1\text{mA}}{0.8\mu\text{A}} = 125$$

$$r_o = \frac{V_A}{I_c} = \frac{100V}{1.2\text{mA}} = 83.3\text{k}\Omega$$

$$\Delta i_c = h_{fe} \Delta i_B + \frac{\Delta V_{CE}}{r_o}$$

$$= 125 \times 3\mu\text{A} + \frac{2}{83.3\text{k}\Omega} = 0.274\text{mA}$$

$$\therefore i_c = 1.2\text{mA} + \Delta i_c = 1.474\text{mA}$$

5.43

	-55°C	25°C	125°C
$I_c = 100\mu\text{A}$	$\beta \geq 100$	187	300
$I_c = 10\text{mA}$	$\beta \geq 78$	178	322

For  $I_c = 100\mu\text{A}$

$$\text{Temp. coeff below } 25^\circ\text{C} = \frac{187 - 100}{25 + 55} = 1.09/\text{°C}$$

$$\text{Temp. coeff above } 25^\circ\text{C} = \frac{300 - 187}{125 - 25} = 1.13/\text{°C}$$

For  $I_c = 10\text{mA}$

$$\text{Temp. coeff below } 25^\circ\text{C} = \frac{178 - 78}{25 + 55} = 1.25/\text{°C}$$

$$\text{Temp. coeff above } 25^\circ\text{C} = \frac{322 - 178}{125 - 25} = 1.44/\text{°C}$$

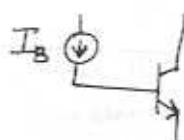
5.44

Use Eq. (5.27) & (5.28)  
but neglect  $(e^{V_{BC}/V_T} - 1) = 0$   
terms as  $V_{BC} = 0$ .  
 $V_{CB} = 0$   
 $\therefore$  active mode operation

$$\begin{aligned} i &= i_c + i_B \\ &= I_s (e^{\frac{V_{BE}}{V_T}} - 1) + \frac{I_s}{\beta_F} (e^{\frac{V_{BC}}{V_T}} - 1) \\ &= I_s (e^{\frac{V_{BE}}{V_T}} - 1) (1 + \frac{1}{\beta_F}) \approx \frac{\beta_F + 1}{\beta_F} = \frac{1}{\beta_F} \approx 1 \end{aligned}$$

Q.E.D.

5.45



$$V_{cesat} = V_{ce0} + I_{sat} R_{sat}$$

$0^\circ\text{C}$  collector is open

$$\begin{aligned} &= V_T \ln \left( \frac{1}{\alpha_R} \right) \\ &= 0.025 \ln \frac{1}{0.2} = 40.2\text{mV} \end{aligned}$$

5.46

$$0.1mA \text{ (V)} \quad \beta_{\text{forced}} = 20 \\ \beta_p = 50, \alpha_p = 0.98 \\ \beta_R = 0.2, \alpha_R = 0.167.$$

Eq (5.48)

$$R_{\text{cesat}} = \frac{1}{10\beta_p I_B} = \frac{1}{10 \times 50 \times 0.1} = 20\Omega$$

Eq (5.49)

$$V_{CE,\text{sat}} = V_T \ln \left[ \frac{1 + \frac{\beta_{\text{forced}} + 1}{\beta_R}}{1 - \beta_{\text{forced}}/\beta_p} \right] \\ = 0.025 \ln \left[ \frac{1 + 21/0.2}{1 - 20/50} \right] = 0.187V$$

5.47

Using Eq (5.47) which gives the collector current for a transistor in saturation and letting  $\alpha = \frac{i_c}{\beta_p I_B}$

we have:

$$i_c = \beta_p I_B \left( e^{\frac{V_{CE}}{V_T}} - e^{\frac{V_{RE}}{V_T}} \right)$$

solve for  $V_{CE}$  & take derivative

$$\frac{dV_{CE}}{di_c} \Big|_{i_B = I_B, i_c = I_{\text{sat}}} = R_{\text{cesat}}$$

$$\therefore \frac{i_c}{\beta_p I_B} \left( e^{\frac{V_{CE}}{V_T}} + \frac{\beta_p}{\beta_R} \right) = e^{\frac{V_{CE}}{V_T}} - \frac{1}{\alpha_R}$$

$$e^{\frac{V_{CE}}{V_T}} \left( \frac{i_c}{\beta_p I_B} - 1 \right) = - \frac{\beta_p}{\beta_R} \frac{i_c}{\beta_p I_B} - \frac{1}{\alpha_R}$$

$$\frac{V_{CE}}{V_T} (a - 1) = - \frac{\beta_p}{\beta_R} a - \frac{1}{\alpha_R}$$

$$V_{CE} = V_T \ln \left[ \frac{a \frac{\beta_p}{\beta_R} + \frac{1}{\alpha_R}}{1 - a} \right]$$

Now:

$$R_{\text{cesat}} = \frac{\partial V_{CE}}{\partial i_c} \Big|_{i_c = I_{\text{sat}}} \\ = \frac{\partial V_{CE}}{\partial a} \frac{\partial a}{\partial i_c} = \frac{\partial V_{CE}}{\partial a} \times \frac{1}{\beta_p I_B} \\ = V_T \left[ \frac{1-a}{\frac{\alpha \beta_p}{\beta_R} + \frac{1}{\alpha_R}} \times \left( \frac{\beta_p / \beta_R}{1-a} + \frac{\alpha \beta_p / \beta_R + \frac{1}{\alpha_R}}{(1-a)^2} \right) \right] \frac{1}{\beta_p I_B} \\ = \frac{V_T}{\beta_p I_B} \left[ \underbrace{\frac{\beta_p / \beta_R}{\alpha \beta_p / \beta_R + \frac{1}{\alpha_R}}}_{\approx \frac{1}{a}} + \frac{1}{1-a} \right] \\ \approx \frac{1}{a} \quad \because \frac{\beta_p}{\beta_R} a \gg \frac{1}{\alpha_R}$$

But at  $i_c = I_{\text{sat}}$   
 $a = \frac{I_{\text{sat}}}{\beta_p I_B} = \infty$

$$= \frac{V_T}{\beta_p I_B} \left[ \frac{1-a+a}{a(1-a)} \right]$$

$$= \frac{V_T}{\beta_p I_B} \times \frac{1}{a(1-a)}$$

$$= \frac{V_T}{\beta_p I_B} \times \frac{1}{x(1-x)}$$

Q.E.D.

Using this expression to find  $R_{\text{cesat}}$  at  $\beta_{\text{forced}} = \beta_p/2 \Rightarrow x = \frac{\beta_{\text{forced}}}{\beta_p} = \frac{1}{2}$

$$R_{\text{cesat}} = \frac{V_T}{\beta_p I_B} \times \frac{1}{\frac{1}{2}(1-\frac{1}{2})} \\ = \frac{4 \times 0.025}{\beta_p I_B} \\ = \frac{1}{10 \beta_p I_B} \quad \leftarrow \text{same as eq (5.48)}$$

5.48

$$\beta_F = 70 \rightarrow \alpha_F = 70/71$$

$$\beta_R = 0.7 \rightarrow \alpha_R = 0.7/0.7$$

$$I_B = 2 \text{ mA}$$

$$\text{FOR } i_c = 3 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{3}{2} = 1.5$$

Note the small  $\beta_{\text{forced}}$  means we are deeper in saturation.  
Hence Eq. (5.48) which approximates the slope at  $i_c = \frac{\beta_R I_B}{2}$  is not useful here!

Thus we have to use the expression in Problem 5.47

$$\begin{aligned} R_{CE,\text{sat}} &= \frac{V_T}{\beta_F I_B} \left[ \frac{1}{\frac{\beta_{\text{forced}}}{\beta_F} \left( 1 - \frac{\beta_{\text{forced}}}{\beta_F} \right)} \right] \\ &= \frac{0.025}{70 \times 2 \times 10^{-3}} \left[ \frac{1}{\frac{1.5}{70} \left( 1 - \frac{1.5}{70} \right)} \right] \\ &= \underline{8.516 \Omega} \end{aligned}$$

Using Eq (5.49) to calculate  $V_{CE,\text{sat}}$

$$\begin{aligned} V_{CE,\text{sat}} &= V_T / n \left[ \frac{1 + \frac{\beta_{\text{forced}} + 1}{\beta_R}}{1 - \frac{\beta_{\text{forced}}}{\beta_P}} \right] \\ &= 0.025 \ln \left[ \frac{1 + \frac{2.5}{0.7}}{1 - \frac{1.5}{70}} \right] \\ &= 38.5 \text{ mV} \end{aligned}$$

$$\begin{aligned} V_{CE,\text{sat}} &= V_{CE,\text{off}} + R_{CE,\text{sat}} I_{C,\text{sat}} \\ 38.5 &= V_{CE,\text{off}} + 8.516 \times 3 \times 10^{-3} \end{aligned}$$

$$V_{CE,\text{off}} = \underline{12.95 \text{ mV}}$$

Now for  $i_c = 0.3 \text{ mA}$

$$\beta_{\text{forced}} = \frac{0.3}{2} = 0.15$$

$$\begin{aligned} R_{CE,\text{sat}} &= \frac{V_T}{\beta_F I_B} \left[ \frac{1}{\frac{\beta_{\text{forced}}}{\beta_F} \left( 1 - \frac{\beta_{\text{forced}}}{\beta_F} \right)} \right] \\ &= \frac{0.025}{70 \times 2 \times 10^{-3}} \left[ \frac{1}{\frac{0.15}{70} \left( 1 - \frac{0.15}{70} \right)} \right] \\ &= 83.51 \Omega \end{aligned}$$

$$\begin{aligned} V_{CE,\text{sat}} &= 0.025 \ln \left[ \frac{1 + \frac{\beta_{\text{forced}} + 1}{\beta_R}}{1 - \frac{\beta_{\text{forced}}}{\beta_P}} \right] \\ &= 0.025 \ln \left[ \frac{1 + \frac{1.15}{0.7}}{1 - \frac{0.15}{70}} \right] \\ &= 24.35 \text{ mV} \end{aligned}$$

Thus  $V_{CE,\text{off}}$  can be calculated from

$$V_{CE,\text{sat}} = V_{CE,\text{off}} + R_{CE,\text{sat}} I_{C,\text{sat}}$$

$$\begin{aligned} V_{CE,\text{off}} &= V_{CE,\text{sat}} - R_{CE,\text{sat}} I_{C,\text{sat}} \\ &= 83.51 - 83.51 \times 0.3 \times 10^{-3} \\ &= \underline{-0.7 \text{ mV}} \end{aligned}$$

5.49

$$\beta_F = 150 \rightarrow \alpha_F = \frac{150}{151}$$

$$\alpha_F I_{SE} = \alpha_R I_{SC} = I_S$$

$$\frac{\text{ACBJ}}{\text{ACBJ}} \propto \frac{I_{SC}}{I_{SE}} = \frac{\alpha_F}{\alpha_R} = 10$$

$$\therefore \alpha_R = \frac{\alpha_F}{10} = 0.0993$$

$$\beta_R = \frac{\alpha_R}{1-\alpha_R} = 0.110$$

Calculate  $V_{CEsat}$  with

$$V_{CEsat} = V_T \ln \left[ \frac{1 + \frac{\beta_{forced} + 1}{\beta_R}}{1 - \frac{\beta_{forced}}{\beta_F}} \right]$$

$$= 0.025 \ln \left[ \frac{1 + \frac{\beta_{forced} + 1}{0.110}}{1 - \frac{\beta_{forced}}{\beta_F}} \right]$$

$\frac{\beta_{forced}}{\beta_F}$	$\beta_{forced}$	$V_{CEsat}$ (V)
0.99	148.5	0.294
0.95	142.5	0.254
0.9	135	0.236
0.5	76	0.180
0.1	15	0.127
0.01	1.5	0.079
0	0	0.058

As  $\beta_{forced}$  is decreased,  $V_{CEsat}$  decreases and the BJT is drawn further into saturation.

5.50

$$V_{BE} = 0.72V \text{ at } i_C = 0.6 \text{ mA}$$

(a)

$$\beta_F = 150 \rightarrow \alpha_F = \frac{150}{151} = \underline{\underline{0.993}}$$

$$\frac{\text{ACBJ}}{\text{ACBJ}} \propto \frac{I_{SC}}{I_{SE}} = \frac{\alpha_F}{\alpha_R} = 20$$

$$\Rightarrow \alpha_R = \frac{\alpha_F}{20} = \frac{0.993}{20} = \underline{\underline{0.0497}}$$

$$\therefore \beta_R = \underline{\underline{0.0523}}$$

(b)  $i_C = 5 \text{ mA}$  ~ assuming active mode

Using the Ebers Moll model in active mode - Eq (5.32)

$$i_C = I_S e^{\frac{V_{CE}/V_T}{1 + \frac{1}{\alpha_R} - 1}}$$

$$0.6 \times 10^{-3} = I_S \left( e^{0.72/0.025} + \frac{1}{0.0497} - 1 \right)$$

$$I_S = 1.86 \times 10^{-16}$$

Therefore at  $i_C = 5 \text{ mA}$  we have

$$5 \times 10^{-3} = 1.86 \times 10^{-16} \left( e^{\frac{V_{CE}/0.025}{0.0497}} + \frac{1}{0.0497} - 1 \right)$$

$$V_{BE} = \underline{\underline{0.773V}}$$

Use Eq (5.33) to calculate  $i_B$

$$i_B = \frac{I_S}{\beta_F} e^{\frac{V_{BE}/V_T}{1 + \frac{1}{\beta_F}}} - I_S \left( \frac{1}{\beta_F} + \frac{1}{\beta_R} \right)$$

$$= \frac{1.86 \times 10^{-16}}{150} e^{\frac{0.773}{0.025}} - 1.86 \times 10^{-16} \left( \frac{1}{150} + \frac{1}{0.0523} \right)$$

$$= \underline{\underline{33.2 \mu A}}$$

CONT.

This value for  $i_B$  makes sense as  $\frac{i_C}{\beta_F} = \frac{5}{150} = 33.3 \mu A$ .

It is a little less due to the reverse leakage currents.

(c) The base current is doubled

$$\left. \begin{array}{l} i_B = 66.4 \mu A \\ i_C = 5 \text{ mA} \end{array} \right\} \beta_{\text{forced}} = \frac{i_C}{i_B} = \frac{I_{C\text{sat}}}{i_B}$$

$$= \frac{5}{0.0664}$$

$$= \underline{\underline{75.3}}$$

Note  $\beta_{\text{forced}}$  is not small so we are not deep in saturation. Hence we can use Eq (5.48) to calculate  $R_{C\text{sat}}$ :

$$R_{C\text{sat}} = \frac{1}{10 \beta_F I_B} = \frac{1}{10 \times 150 \times 66.4 \mu A}$$

$$= \underline{\underline{10.04 \Omega}}$$

Checking this result with the expression from Prob. 5.47 we get the same result:

$$R_{C\text{sat}} = \frac{V_T}{\beta_F I_B} \left( \frac{1}{\beta_{\text{forced}} \left( 1 - \frac{\beta_{\text{forced}}}{\beta_F} \right)} \right)$$

$$= \frac{0.025}{150 \times 66.4 \times 10^{-6}} \left( \frac{1}{\frac{75.3}{150} \left( 1 - \frac{75.3}{150} \right)} \right)$$

$$= \underline{\underline{10.04 \Omega}}$$

From Eq (5.49)

$$|V_{C\text{sat}}| = V_T \ln \left[ \frac{1 + \frac{\beta_{\text{forced}} + 1}{\beta_R}}{1 - \frac{\beta_{\text{forced}}}{\beta_F}} \right]$$

$$|V_{C\text{sat}}| = 0.025 \ln \left[ \frac{1 + \frac{75.3}{0.0623}}{1 - \frac{75.3}{150}} \right]$$

$$= \underline{\underline{0.1996 V}}$$

Use the complete Ebers Moll equations to calculate  $V_{BE}$  and  $V_{BC}$ .

$$\begin{aligned} V_{CE} &= V_{CB} + V_{BE} \\ &= -V_{BC} + V_{BE} \end{aligned}$$

$$\therefore V_{BE} = V_{C\text{sat}} + V_{BC}$$

Eg (5.27) :

$$i_C = I_a \left( e^{\frac{V_{BE}}{V_T} - 1} \right) - I_s \frac{e^{\frac{V_{BC}}{V_T} - 1}}{\alpha R} \left( e^{\frac{V_{BC}}{V_T} - 1} \right)$$

$$\frac{i_C}{I_a} = e^{\frac{V_{BE}}{V_T} - 1} - \frac{e^{\frac{V_{BC}}{V_T} - 1}}{\alpha R} + \frac{1}{\alpha R}$$

$$\frac{5 \times 10^{-3}}{1.86 \times 10^{-16}} = e^{\frac{0.1996}{0.025}} e^{\frac{V_{BC}}{V_T} - 1} + \frac{1}{\alpha R}$$

$$V_{BC} = \frac{0.025 \ln \left[ \frac{5 \times 10^{-3}}{1.86 \times 10^{-16}} + 1 - \frac{1}{0.0497} \right]}{\frac{0.1996}{0.025} - \frac{1}{0.0497}}$$

$$= \underline{\underline{0.5736}}$$

Now

$$\begin{aligned} V_{BE} &= V_{C\text{sat}} + V_{BC} \\ &= 0.1996 + 0.5736 \\ &= \underline{\underline{0.7732 V}} \end{aligned}$$

5.51

Eq (5.49)

$$V_{CEsat} = V_T \ln \left[ \frac{1 + \frac{\beta_{forward}}{\beta_R}}{1 - \frac{\beta_{forward}}{\beta_F}} \right]$$

For E grounded and C open:  $\beta_{forward} = 0$

$$\therefore 60 = 25 \ln \left[ \frac{1 + \frac{0}{\beta_R}}{1 - 0} \right]$$

$$\ln \left( 1 + \frac{0}{\beta_R} \right) = \frac{60}{25} = 2.4$$

$$\beta_R = \underline{0.01}$$

For C grounded and E open

$$\beta_{forward} = 0 \quad \beta_R^+ = \beta_F^- \quad \beta_P = \beta_R$$

$$\therefore 1 = 25 \ln \left[ \frac{1 + \frac{0+1}{\beta_F}}{1 - 0} \right]$$

$$\ln \left( 1 + \frac{1}{\beta_F} \right) = \frac{1}{25}$$

$$\beta_F = \underline{24.5}$$

Check:

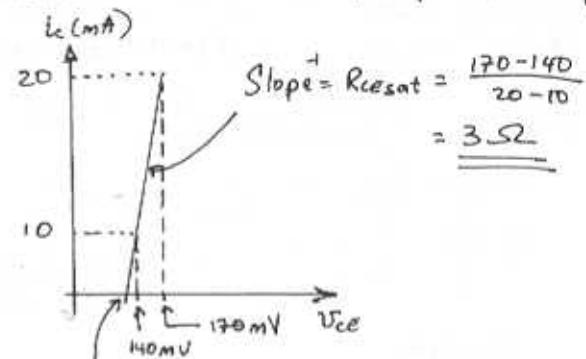
$$-(-1) = 25 \ln \left[ \frac{1 + \frac{1}{24.5}}{1 - 0} \right] = 1mV.$$

5.52

Information given (where  $\beta_{forward} = \frac{I_c}{I_B}$ )

$I_B(mA)$	$I_c(mA)$	$\beta_{forward}$	$V_{CEsat}(mV)$
0.5	10	20	140
0.5	20	40	170

Given information displayed pictorially



$$\begin{aligned} V_{ce,off} &= 140 - 10 R_{ce(sat)} \\ &= 140 - 10 \times 3 \\ &= \underline{110 \text{ mV}} \end{aligned}$$

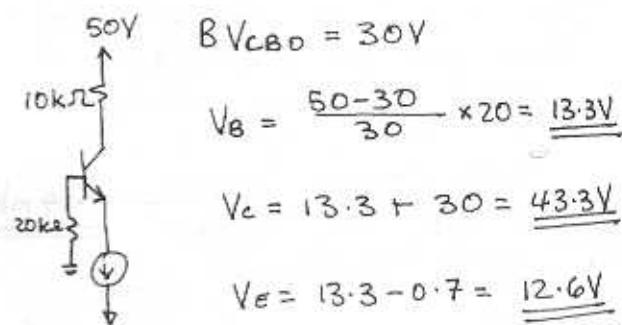
Inserting the given data into Eq (5.49) gives the following two equations

$$140 = 25 \ln \left[ \frac{1 + \frac{21}{\beta_R}}{1 - 20/\beta_F} \right] \quad (1)$$

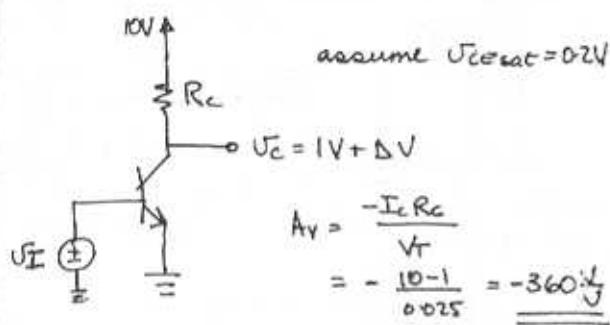
$$170 = 25 \ln \left[ \frac{1 + \frac{41}{\beta_R}}{1 - 40/\beta_F} \right] \quad (2)$$

which can be solved together to yield:  $\beta_F = \underline{68.2}$      $\beta_R = \underline{0.111}$

5.53



5.54



the range  
At saturation  $U_{CE(sat)} = 0.3V$   
 $\therefore U_C = 1 + \Delta V = 0.3$ .

$$\Delta V = -0.7V$$

$$\therefore U_O = 0.3V \quad i_c = \frac{10-0.3}{R_C}$$

$$\frac{i_{C2}}{i_{C1}} = \frac{9.7/R_C}{(10-0.3)/R_C} = e^{\frac{\Delta V}{V_T}}$$

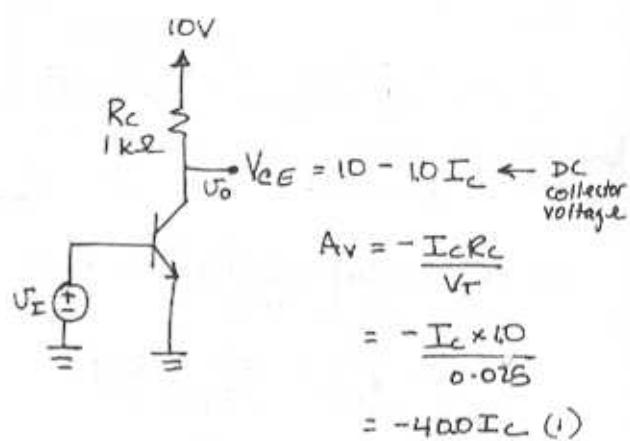
so maximum input signal

$$\Delta V = 0.025 \ln \frac{9.7}{9} = 1.87mV$$

If we assume linear operation right to saturation we can use the gain  $A_V$  to calculate the maximum input swing. Thus for an output swing  $\Delta U_O = 0.8$  we have

$$\Delta U_i = \frac{-\Delta U_O}{A_V} = \frac{-0.7}{-360} = 1.94mV$$

5.55



-Assuming the output voltage  $U_O = 0.3V$  is the lowest  $U_{CE}$  to stay out of saturation.  
 $\therefore U_O = 0.3 = 10 - i_c R_C$

$$= 10 - I_c R_C + \Delta U_O$$

$$\Delta U_O = -10 + 0.3 + I_c \times 1 \quad (2)$$

-Max output voltage before the transistor is cut off

$$V_{CE} + \Delta U_O = V_{CC}$$

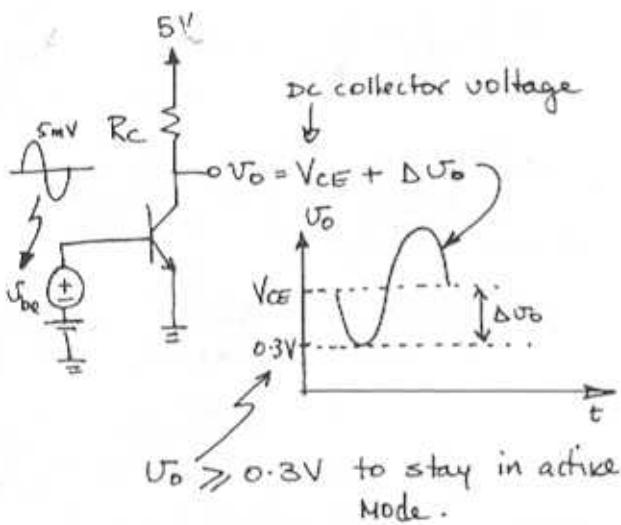
$$\begin{aligned} \Delta U_O &= V_{CC} - V_{CE} \\ &= 10 - 10 + 10I_c \\ &= 10 I_c \end{aligned} \quad (3)$$

Use (1) to calculate the gain and (2), (3) to calculate the output limits in order to stay in active mode for a particular bias current  $I_c$ .

$I_c$ (mA)	$A_V$ (V/V)	$\Delta U_O$ (V)
1	-40	-8 to 1
2	-80	-7 to 2
5	-200	-4.7 to 5
8	-320	-1.7 to 8
9	-360	-0.7 to 9

5.56

Since we are assuming linear operation we don't have to go to  $i_c = I_s e^{\frac{V_{BE}}{V_T}}$  equation.



$$A_v = -\frac{I_c R_C}{V_T} = -\frac{V_{CC} - V_{CE}}{V_T}$$

On the verge of saturation

$$V_{CE} - \Delta U_O = 0.3V \quad \text{for linear operation}$$

$$\Delta U_O = A_v U_{be}$$

$$V_{CE} - |A_v U_{be}| = 0.3$$

$$(5 - I_c R_C) - A_v \times 5 \times 10^{-3} = 0.3$$

$$5 - |A_v V_T| - |A_v \times 5 \times 10^{-3}| = 0.3$$

$$|A_v (0.025 + 0.005)| = 5 - 0.3$$

$|A_v| = 156.67$  Note  $A_v$  is negative.

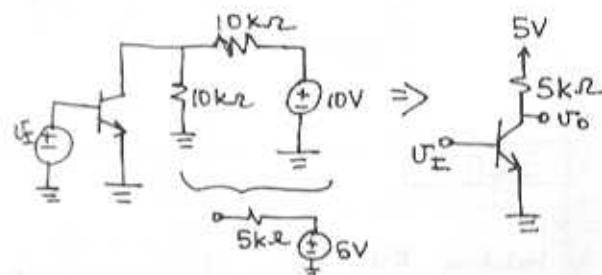
$$\therefore A_v = -156.67 \text{ V/V}$$

Now we can find the dc collector voltage. Refer to sketch of the output voltage, we see that  $|\Delta U_O| = |(A_v \times 0.005)|$

$$\therefore V_{CE} = 0.3 + |A_v| \times 0.005$$

$$= \underline{1.08V}$$

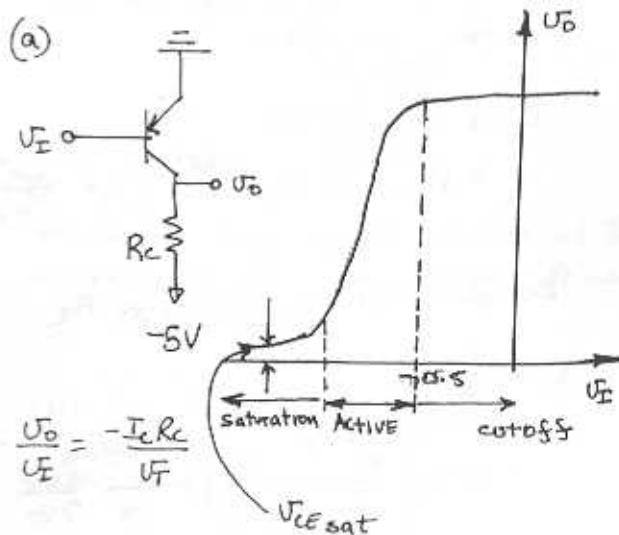
5.57



$$\frac{U_O}{U_I} = -\frac{I_c R_C}{V_T} = -\frac{0.5 \times 5}{0.025}$$

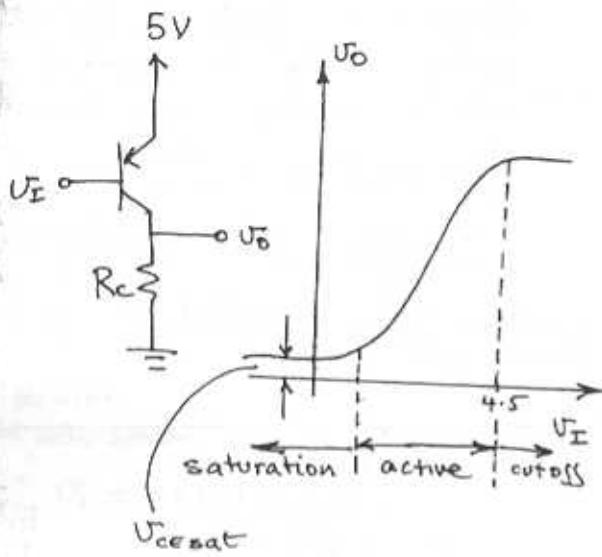
$$= \underline{-100 \text{ V/V}}$$

5.58



CONT.

(b)



5.69

Including the Early effect we note that:

$$i_c = I_s e^{\frac{U_{BE}}{V_T}} \left( 1 + \frac{U_{CE}}{V_A} \right)$$

Also, note  $I_c = I_s e^{\frac{U_{BE}}{V_T}}$  Eq (5.38b)  
is the value of the collector current  
with the Early voltage neglected.

Starting with the voltage at the  
collector we have:

$$\begin{aligned} U_O &= V_{CC} - i_c R_C \\ &= V_{CC} - R_C I_s e^{\frac{U_{BE}}{V_T}} \left( 1 + \frac{U_{CE}}{V_A} \right) \end{aligned}$$

Take derivative to get gain  $A_v$

$$\begin{aligned} A_v &= \frac{dU_O}{dU_I} \\ &= -R_C I_s \left[ \frac{e^{\frac{U_{BE}}{V_T}}}{V_T} \left( 1 + \frac{U_{CE}}{V_A} \right) + \frac{e^{\frac{U_{BE}}{V_T}}}{V_A} \frac{dU_{CE}}{dU_I} \right] \end{aligned}$$

$$A_v = -\frac{R_C I_s}{V_T} e^{\frac{U_{BE}}{V_T}} \left[ 1 + \frac{U_{CE}}{V_A} + \frac{V_T}{V_A} \frac{dU_{CE}}{dU_I} \right]$$

$$= -\frac{R_C I_c}{V_T} \left[ 1 + \frac{U_{CE}}{V_A} + \frac{V_T}{V_A} A_v \right]$$

$$-A_v \left[ \frac{1}{R_C I_c} + \frac{V_T}{V_A} \right] = 1 + \frac{U_{CE}}{V_A} = \frac{V_A + U_{CE}}{V_A}$$

$$-A_v \left[ \frac{V_A + R_C I_c}{R_C I_c V_A} \right] = \frac{V_A + U_{CE}}{V_A}$$

$$-A_v \frac{R_C I_c}{V_T} = \frac{V_A}{V_A + R_C I_c} \times \frac{V_A + U_{CE}}{V_A}$$

$$= \frac{V_A + U_{CE}}{V_A + R_C I_c} \stackrel{\text{divide by } V_A + U_{CE}}{\text{by } V_A + U_{CE}}$$

$$= \frac{1}{\frac{V_A}{V_A + U_{CE}} + \frac{R_C I_c}{V_A + U_{CE}}}$$

This term is  $\approx 1$   
 $\therefore V_A \gg U_{CE}$

$$\therefore A_v \approx \left[ -\frac{R_C I_c / V_T}{\left( 1 + \frac{R_C I_c}{V_A + U_{CE}} \right)} \right]$$

Q.E.D.

For  $V_{CC} = 6V$   $U_{CE} = 2.5V$   $V_A = 100V$

Ignoring the Early Voltage:

$$A_v = -\frac{I_c R_C}{V_T} = \frac{V_{CC} - U_{CE}}{V_T} = \frac{5 - 2.5}{0.025} = \underline{\underline{100V}}$$

With the Early Voltage

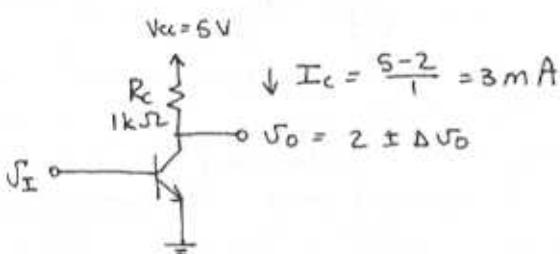
$$A_v \approx \frac{-I_c R_C / V_T}{1 + \frac{R_C I_c}{V_A + U_{CE}}}$$

CONT.

But  $V_{CE} = 0.5V$  &  $\frac{I_c R_c}{V_T} = 100$  as shown above.

$$\therefore A_v = \frac{-100}{1 + \frac{2.5}{100+2.5}} = -\underline{97.7 \text{ V/V}}$$

5.60



Small signal voltage gain

$$A_v = -\frac{I_c R_c}{V_T} = \frac{-3 \times 1}{0.025} = -\underline{120 \text{ V/V}}$$

If we assume linear operation around the bias point we can use  $A_v$ .

$$\Delta V_O = -120 \times 5 \times 10^{-3} = \underline{-0.6 \text{ V}} \quad \text{for } \Delta V_{BE} = 5 \text{ mV}$$

$$\Delta V_O = 120 \times 5 \times 10^{-3} = \underline{0.6 \text{ V}} \quad \text{for } \Delta V_{BE} = -5 \text{ mV}$$

Using the transistor exponential characteristics as in Example 5.2:  
for  $\Delta V_{BE} = 5 \text{ mV}$ :

$$\frac{i_c}{3 \text{ mA}} = e^{\frac{\Delta V_{BE}/V_T}{1}} = e^{5/25}$$

$$i_c = 3.66 \text{ mA}$$

$$V_O = 5 - 3.66 = 1.34 \text{ V}$$

$$\Delta V_O = 1.34 - 2 = -\underline{0.66 \text{ V}}$$

For  $\Delta V_{BE} = -5 \text{ mV}$

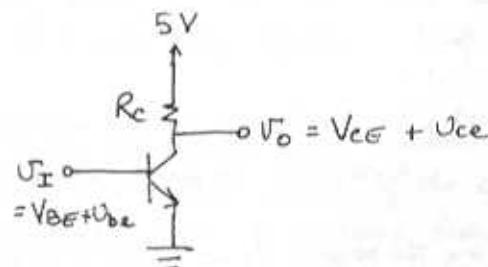
$$i_c = 3 \times e^{\frac{-5/25}{1}} = 2.46 \text{ mA}$$

$$V_O = 5 - 2.46 \times 1 = 2.54 \text{ V}$$

$$\Delta V_O = 2.54 - 2 = \underline{0.54 \text{ V}}$$

$\Delta V_{BE} (\text{mV})$	$\Delta V_O (\text{V})$ Exponential	$\Delta V_O (\text{V})$ Linear
+0.5	-0.64	-0.6
-0.5	+0.54	+0.6

5.61



(a) For maximum gain you would bias at the lowest current since  $A_v = -I_c R_c / V_T$ . This also means you would bias at the edge of saturation  $A_v = -V_{ce} - V_{cesat}$

$$= \frac{-5 - 0.3}{0.025} = \underline{-188 \text{ V/V}}$$

However any signal swing at the output would automatically drive it into saturation.

(b) for  $A_v = -100 \text{ V/V}$

$$A_v = \frac{V_{ce} - V_{cesat}}{V_T} = \frac{5 - V_{ce}}{V_T} = 100$$

$$V_{ce} = \underline{2.5 \text{ V}}$$

CONT.

(c) For a dc collector current of 0.5mA

$$R_c = \frac{5-2.5}{0.5} = 5 \text{ k}\Omega$$

$$(d) I_s = 10^{-15} \text{ A} \Rightarrow$$

$$I_c = I_s e^{\frac{V_{BE}}{V_T}}$$

$$0.5 \times 10^{-3} = 10^{-15} e^{\frac{V_{BE}}{0.025}}$$

$$V_{BE} = 0.673 \text{ V}$$

(e) If we assume linear operation we can use  $A_v$  to find the output change for  $V_{be} = 5 \text{ mV}$

$$V_{ce} = A_v V_{be} = -100 \times 0.005 \\ = -0.5 \text{ V} \sim \text{peak sine wave.}$$

∴ the output is a 0.5V p sine wave

(f) for  $V_{ce} = 0.5$

$$i_c = \frac{0.5}{5} = 0.1 \text{ mA peak}$$

This current is superimposed on  $I_c$ .

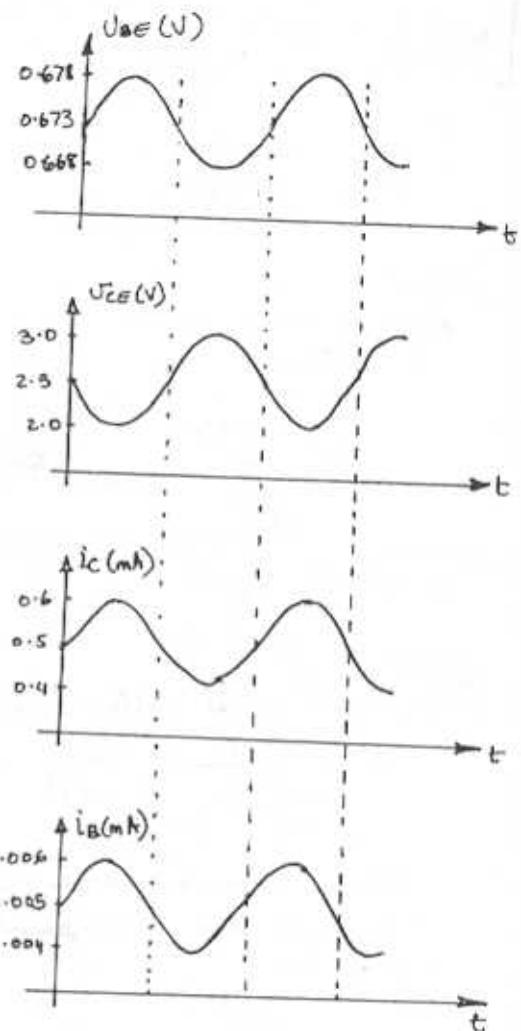
$$(g) I_b = I_c / \beta = \frac{0.5}{100} = 0.005 \text{ mA}$$

$$i_b = \frac{i_c}{\beta} = \frac{0.1}{100} = 0.001 \text{ mA}$$

$$(h) r_i = \frac{V_{be}}{i_b} = \frac{0.005}{0.001 \times 10^{-3}} = 5 \text{ k}\Omega$$

$$= 5 \text{ k}\Omega$$

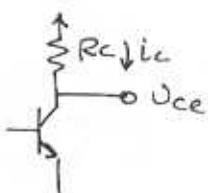
(i) See sketches that follow:



5.62

Eq (5.56)

$$A_v = \frac{V_{ce}}{V_{be}} = -\frac{I_c R_c}{V_T}$$



But  $V_{ce} = -i_c R_c$

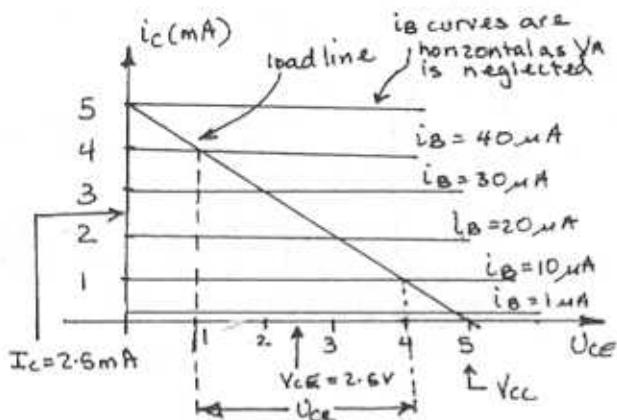
$$\therefore -\frac{i_c R_c}{V_{be}} = -\frac{I_c R_c}{V_T}$$

Now  $g_m = \frac{\text{output current}}{\text{input voltage}} = \frac{i_c}{V_{be}}$

$$\therefore g_m R_c = \frac{I_c R_c}{V_T}$$

$$g_m = \frac{I_c}{V_T}$$

5.63



For  $i_B$  varying from  $10 \mu A$  to  $40 \mu A$ ,  $i_c$  varies from  $1mA$  to  $4mA$  ( $\beta = 100$ ), and  $U_C = V_{CC} - R_C i_C$  varies from  $4$  to  $1V$ . Thus the peak-to-peak collector voltage swing is

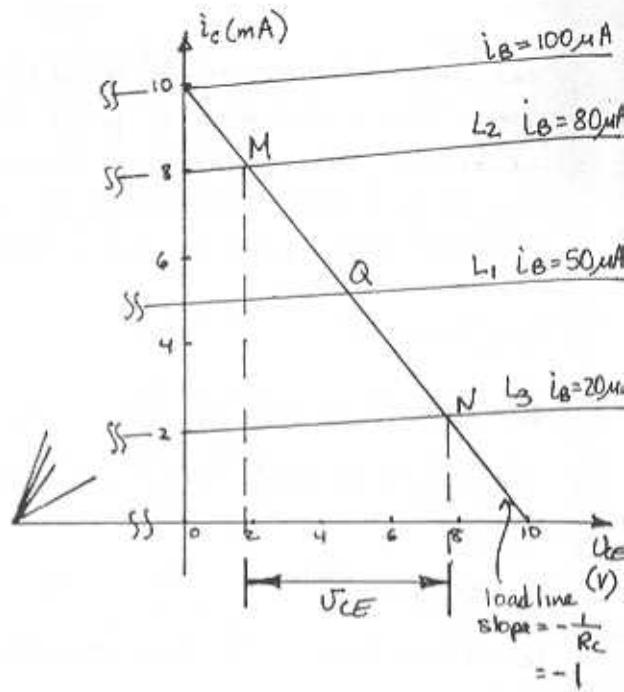
$$U_{CE} = 3V \text{ p-p}$$

$$\text{FOR } U_{CE} = \frac{1}{2} V_{CC} = 2.5V$$

$$I_C = 2.5mA$$

$$\therefore I_B = \frac{2.5}{100} = 25 \mu A$$

$$\begin{aligned} V_{BE} &= V_{BE} + I_B R_B \\ &= 0.7 + 0.025 \times 100 \\ &= 3.2V \end{aligned}$$



Given the base bias current of  $50 \mu A$ , the dc or bias point of the collector current  $I_C$  & voltage  $U_{CE}$  can be found from the intersection of the load line & the transistor line  $L_1$  of  $i_B = 50 \mu A$ . Specifically :

$$\begin{aligned} \text{Eq of } L_1 \Rightarrow i_c &= I_C \left( 1 + \frac{U_{CE}}{V_A} \right) \\ &= 5 \left( 1 + \frac{U_{CE}}{100} \right) \\ &= 5 + 0.05 U_{CE} \end{aligned}$$

$$\begin{aligned} \text{Eq of loadline} \Rightarrow i_c &= \frac{V_{CC} - U_{CE}}{R_C} \\ &= 10 - U_{CE} \end{aligned}$$

$$\therefore 10 - U_{CE} = 5 + 0.05 U_{CE}$$

$$U_{CE} = V_{CE} = 4.76V$$

$$I_C = i_c = 10 - U_{CE} = 5.24mA$$

Now for a signal of  $30 \mu A$  peak superimposed on  $I_B = 50 \mu A$ , the operating point moves along the

CONT.

5.64

See the graphical construction that follows. For this circuit :

$$\begin{aligned} V_{CC} &= 10V & \beta &= 100 \\ R_C &= 1k\Omega & V_A &= 100V \\ I_B &= 50 \mu A - \text{dc bias} & i_c &= \beta i_B \text{ at } U_{CE} = 0 \\ \therefore I_C &= 50 \times 100 & & \\ &= 5mA - \text{dc bias} & & \end{aligned}$$

load line between points N and M. To obtain the coordinates of point M, we solve the load line and line  $L_2$  to find the intersection M and the load line and line  $L_3$  to find N:

FOR POINT M:

$$i_c = 8 + \frac{8}{100} V_{CE} \quad i_c = 10 - V_{CE}$$

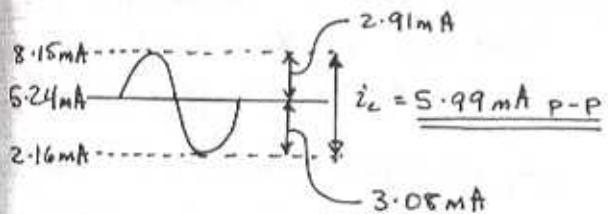
$$\therefore I_{cM} = 8.15 \text{ mA} \quad V_{CEM} = 1.85 \text{ V}$$

FOR POINT N:

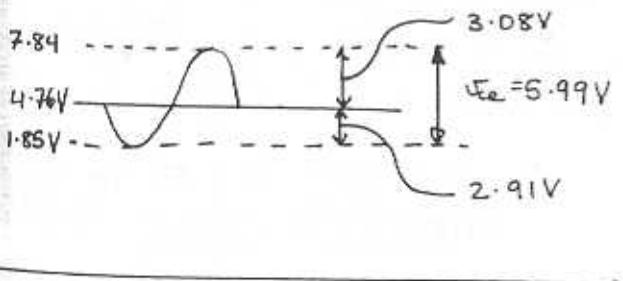
$$i_c = 2 + 0.02 V_{CE} \quad i_c = 10 - V_{CE}$$

$$V_{CEN} = 7.84 \text{ V} \quad i_{cN} = 2.16 \text{ mA}$$

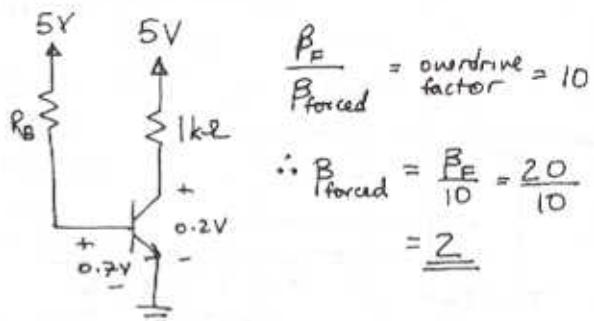
Thus the collector current varies as follows



And the collector voltage varies as:



5.65



$$\frac{\beta_e}{\beta_{\text{forced}}} = \text{overdrive factor} = 10$$

$$\therefore \beta_{\text{forced}} = \frac{\beta_e}{10} = \frac{2.0}{10} = \underline{\underline{2}}$$

$$\text{for } V_{CE\text{sat}} = 0.2 \text{ V}$$

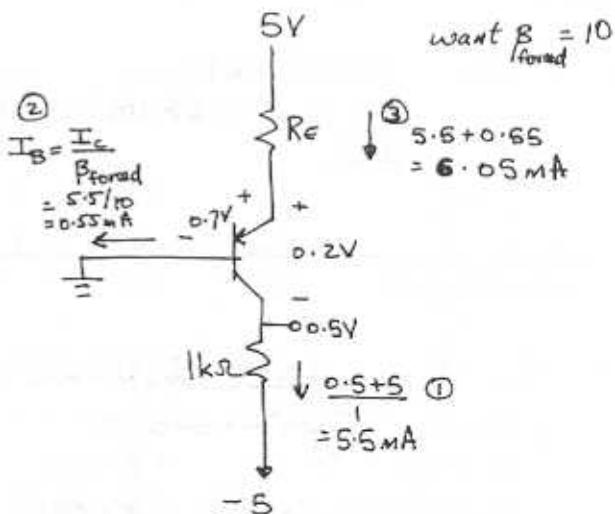
$$5 - I_{C\text{sat}} \times 1 = 0.2$$

$$I_{C\text{sat}} = 5 - 0.2 = 4.8 \text{ mA}$$

$$I_B = \frac{I_{C\text{sat}}}{\beta_{\text{forced}}} = \frac{4.8}{2} = 2.4 \mu\text{A}$$

$$\therefore R_B = \frac{5 - 0.7}{2.4} = \frac{4.3}{2.4} = \underline{\underline{1.8 \text{ k}\Omega}}$$

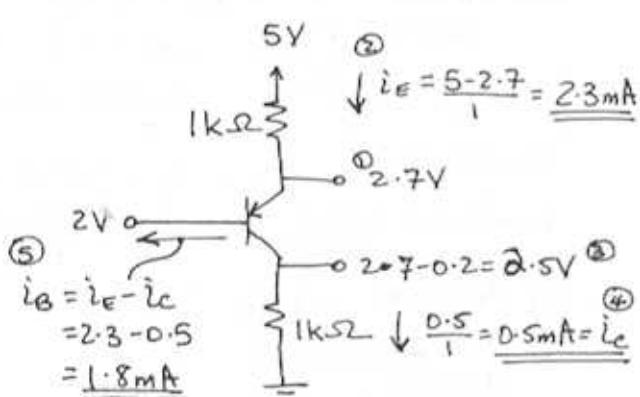
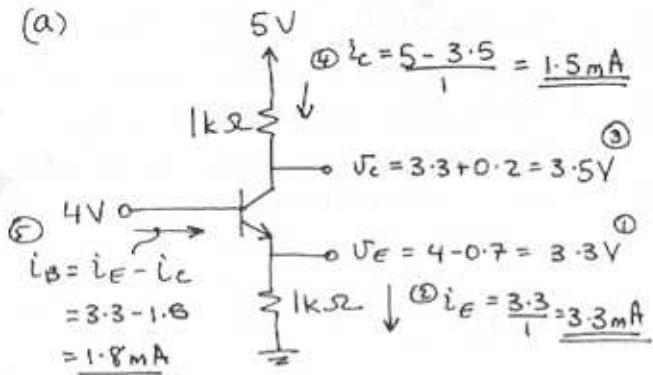
5.66



$$R_E = \frac{5 - 0.7}{6.05} = \underline{\underline{710 \Omega}}$$

5.67

(a)



FOR:

$$V_B = 1V$$

$$V_E = 1 - 0.3 = 0.7V$$

$$i_E = \frac{0.3}{1} = 0.3 \text{ mA}$$

$$i_C = \frac{\beta}{\beta + 1} i_E$$

$$= \frac{50}{51} \times 0.3$$

$$= 0.2941 \text{ mA}$$

$$V_C = 6 - 0.2941$$

$$= 5.71 \text{ V}$$

$$i_B = i_E - i_C \leftarrow \left( \equiv \frac{i_C}{\beta} \right) \rightarrow i_B = 2.3 - 2.25$$

$$= 0.0059 \text{ mA}$$

$$V_E = 6 - 2.25$$

$$= 3.75 \text{ V}$$

$$i_B = 2.3 - 2.25$$

$$= 0.0451 \text{ mA}$$

- let saturation begins at  $V_B = x$   
- at saturation starting  
 $V_{BE} = 0.4V$

$$V_C = x - 0.4 \Rightarrow i_C = \frac{6 - x + 0.4}{1}$$

$$V_E = x - 0.7 \Rightarrow i_E = \frac{x - 0.7}{1}$$

since we are still in active mode  $i_C = \frac{50}{51} i_E$

$$(6.4 - x) = \frac{50}{51} (x - 0.7)$$

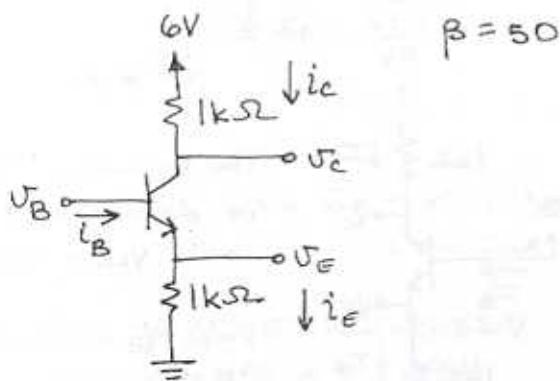
$$101x = 361.4$$

$$\therefore x = V_B = 3.58 \text{ V}$$

$$i_C = 6 - x + 0.4 = 2.82 \text{ mA}$$

$$i_B = \frac{i_C}{\beta} = \frac{2.82}{50} = 0.0564 \text{ mA}$$

5.68



Since conduction does not start until  $V_{BE} = 0.5V$ ,  $i_C$  &  $i_E$  are essentially 0 for  $V_B \leq 0.5V$

For  $V_B = 4V$  and  $6V$ , the transistor is in saturation. Assume  $V_{BE} = 0.7V$   
 $i_{cesat} = 0.7 - 0.6 = 0.1V$

CONT.

For:

$$V_B = 4V$$

$$V_E = 4 - 0.7 = 3.3V$$

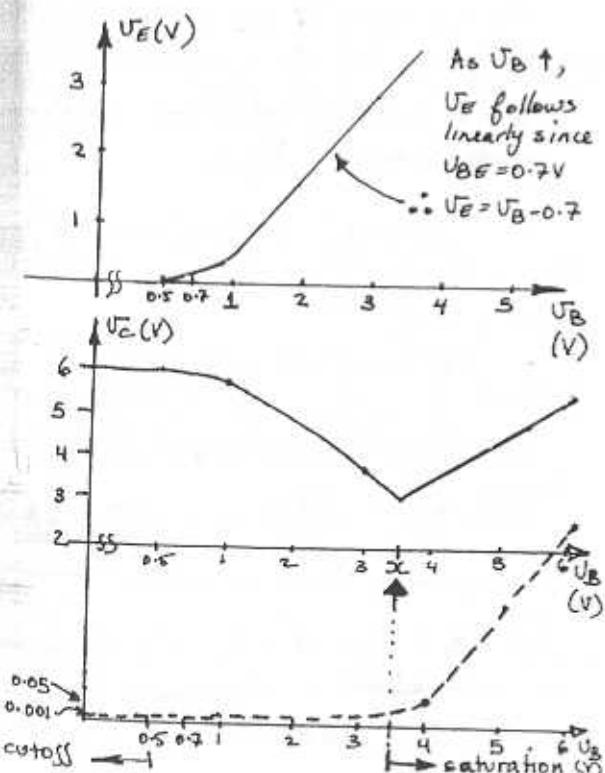
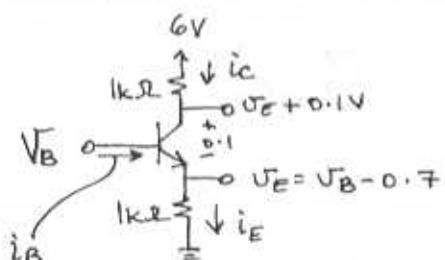
$$I_E = \frac{3.3}{1} = 3.3mA$$

$$V_C = 3.3 + 0.1 = 3.4V$$

$$I_C = \frac{6 - 3.4}{1} = 2.6mA$$

$$\begin{aligned} I_B &= I_E - I_C \\ &= 3.3 - 2.6 \\ &= 0.7mA \end{aligned}$$

CIRCUIT IN SATURATION:



$$V_B = 6V$$

$$V_E = 6 - 0.7 = 5.3V$$

$$I_E = \frac{5.3}{1} = 5.3mA$$

$$V_C = 5.3 + 0.1 = 5.4V$$

$$I_C = \frac{6 - 5.4}{1} = 0.6mA$$

$$\begin{aligned} I_B &= I_E - I_C \\ &= 5.3 - 0.6 \\ &= 4.7mA \end{aligned}$$

Notice for  $V_B \leq 0.7V$  the transistor is essentially cutoff. Then at saturation, as  $V_B$  increases, both  $V_E$  &  $V_C$  increases linearly. This occurs as  $V_{BE} = 0.7V$  &  $V_{CE} = 0V$  in saturation.

5.69

$$(a) V_B = 2V$$

$$V_E = 2 - 0.7 = 1.3V$$

$$I_E = \frac{1.3}{1} = 1.3mA$$

$$(b) V_B = 1V$$

$$V_E = 1 - 0.7 = 0.3V$$

$$I_E \approx I_c = 0.3mA$$

$$I_C \approx 1.3mA$$

$$V_C = 5 - 0.3$$

$$V_C = 5 - 1.3 = 3.7V$$

$$= 4.7V$$

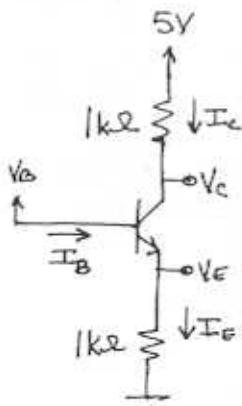
$$(c) V_B = 0V - \text{cutoff}$$

$$V_E = 0V$$

$$I_E = 0A$$

$$V_C = 5V$$

5.70



$\beta \sim \text{high}$

The transistor stays in active mode until  $V_{CB} = -0.4V$

$$\therefore V_C - V_B = -0.4V$$

$$\therefore I_C = I_E$$

$$\frac{5 - (V_B - 0.4)}{1} = \frac{V_B - 0.7}{1}$$

$$5.7 + 0.4 = 2V_B$$

$$V_B = 3.05V$$

CONT.

For operation with  $B_{\text{forced}} = 1 \Rightarrow$

$$B_{\text{forced}} = \frac{I_{C\text{sat}}}{I_B} = 1 \quad \left\{ \begin{array}{l} \text{assume} \\ V_{CE} = 0.2V \\ V_{CB} = -0.5V \end{array} \right.$$

$$\therefore I_{C\text{sat}} = I_B$$

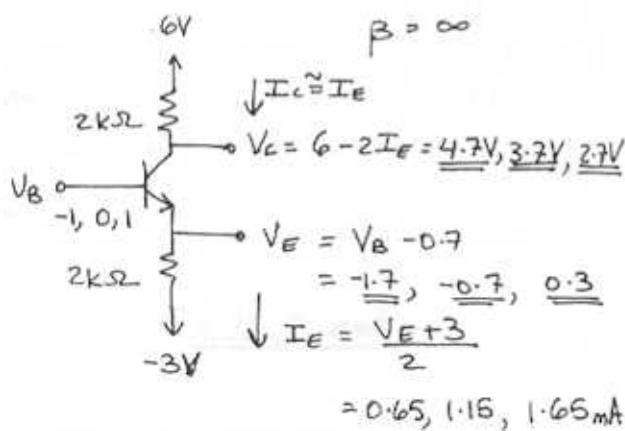
$$I_E = I_{C\text{sat}} + I_B = 2I_{C\text{sat}}$$

$$\frac{V_B - 0.7}{1} = 2 \times \frac{5 - (V_B - 0.5)}{1}$$

$$V_B - 0.7 = 2(5.5 - V_B)$$

$$V_B = \underline{\underline{3.9V}}$$

5.71



- Want  $V_B$  when  $I_E = \frac{1}{10} \times 1.15 \text{ mA} = 0.115 \text{ mA}$

$$V_E = -3 + 0.115 \times 2 = -2.77 \text{ V}$$

$$V_B = V_E + 0.7 = \underline{\underline{-2.07V}}$$

- Want  $V_B$  at the edge of conduction  
At the edge of conduction assume  $V_{BE} = 0.5 \text{ V}$

$$\therefore V_B - 0.5 - 2I_E + 3 = 0 \leftarrow I_E = 0$$

$$V_B = \underline{\underline{-2.6V}} \quad \text{at edge of conduction}$$

$$V_E = V_B - 0.5 = \underline{\underline{-3V}}$$

$I_c \approx 0 \text{ A} \therefore V_C = \underline{\underline{6V}}$   
At saturation assume  $V_{CE} = 0.2V$   
 $V_{CB} = -0.5V$

$$\therefore I_E = \frac{V_B - 0.7 + 3}{2} \approx I_c = \frac{6 - (V_B - 0.5)}{2}$$

$$\therefore V_B + 2.3 = 6.5 - V_B$$

$$V_B = \underline{\underline{2.1V}}$$

$$V_E = 2.1 - 0.7 = \underline{\underline{1.4V}} \quad V_C = V_B - 0.5 = \underline{\underline{1.6V}}$$

- Want  $V_B$  at  $B_{\text{forced}} = 2$ ,  $V_{CE} = 0.2V$   
 $V_{CB} = -0.5V$

$$B_{\text{forced}} = \frac{I_{C\text{sat}}}{I_B} = 2$$

$$I_E = I_B + I_{C\text{sat}} = \frac{I_{C\text{sat}}}{2} + I_{C\text{sat}} = \frac{3}{2} I_{C\text{sat}}$$

$$V_E = V_B - 0.7 = -3 + 2I_E = -3 + 3I_{C\text{sat}}$$

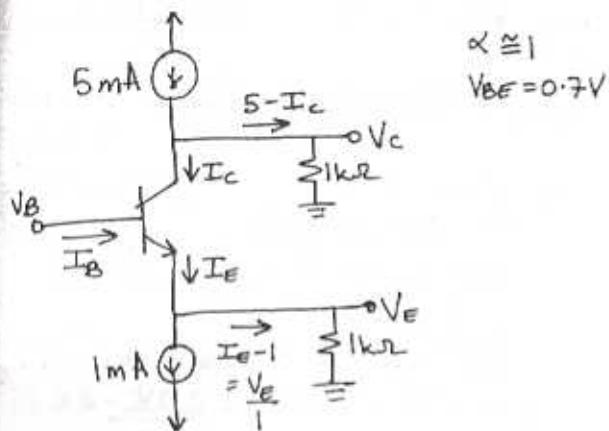
$$I_{C\text{sat}} = \frac{2.3 + V_B}{3}$$

$$I_C = \frac{V_C - (V_B - 0.5)}{2} = I_{C\text{sat}}$$

$$6.5 - V_B = \frac{2}{3}(2.3) + \frac{2}{3}V_B$$

$$V_B = \frac{6.5 - \frac{2(2.3)}{3}}{\frac{4}{3}} = \underline{\underline{2.98V}}$$

5.72



Under normal conduction if  $V_B = 0V$

$$V_E = 0 - 0.7 = -0.7V$$

$$I_E = 1 - \frac{0.7}{1} = 0.3mA \approx I_C$$

$$V_C = (5 - 0.3) 1 = 4.7V$$

At cutoff ~ all of the current in the 1mA source is supplied through  $R_E$   
 $\sim V_{BE} = 0.6V$

$$\therefore V_E = -1 \times 1 = -1V \text{ & } I_E = I_C = 0$$

$$\therefore V_B = V_E + 0.5 = -1 + 0.5$$

$$= -0.5V$$

- For Saturation  $V_{CE} = 0.2V$ ,  $V_{CB} = -0.5V$

$$V_C = V_E + 0.2$$

$$I_C = 5 - \frac{V_C}{1} = 5 - \left( \frac{V_E + 0.2}{1} \right)$$

$$= 5.2 - V_E$$

$$I_E = \frac{V_E + 1}{1}$$

At the edge of saturation  $I_E \approx I_C$  still!

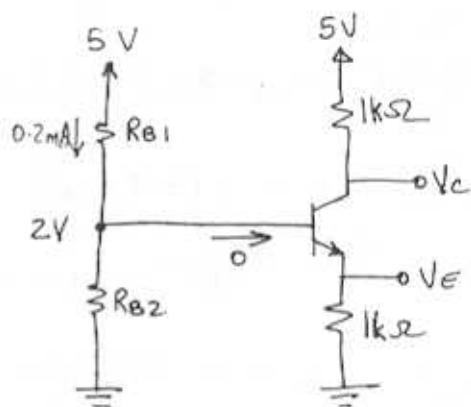
$$\therefore 5.2 - V_E = V_E + 1$$

$$V_E = 4.2/2 = 2.1V$$

$$V_B = V_E + 0.7 = 2.8V$$

$$V_C = V_E + 0.2 = 2.3V$$

5.73



for  $B = \infty$

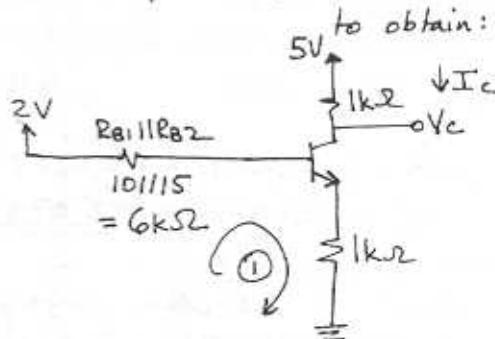
$$\frac{5}{R_{B1} + R_{B2}} = 0.2 \quad \& \quad \frac{R_{B2}}{R_{B1} + R_{B2}} 5 = 2$$

$$R_{B1} + R_{B2} = 25k\Omega \xrightarrow{\text{sub}}$$

$$\Rightarrow \frac{R_{B2}}{25} \times 5 = 2$$

$$R_{B2} = 10k\Omega \quad R_{B1} = 15k\Omega$$

Now for  $B = 100$ , use Thevenin's



$$\text{Loop ①} \quad 3 - 6 \left( \frac{I_E}{B_{eff}} \right) - 0.7 - I_E(1) = 0$$

$$I_E = 1.29mA$$

CONT.

$$I_C = \frac{100}{101} I_E = \frac{100}{101} \times 1.29 = \underline{1.28 \text{ mA}}$$

Using 5% resistor values

$$R_E = 3.9 \text{ k}\Omega \quad R_C = 22 \text{ k}\Omega$$

$$V_C = 5 - 1.28(1) = \underline{3.72 \text{ V}}$$

5.74

①  $I_E = \frac{5-1}{5} = 0.8 \text{ mA}$

②  $V_B = 1 - 0.7 = 0.3 \text{ V}$

③  $I_B = \frac{0.3}{20} = 0.015 \text{ mA}$

④  $V_C = 5 + 0.785(5) = \underline{8.925 \text{ V}}$   
 $V_C = -1.075 \text{ V}$

⑤  $I_C = I_E - I_B = 0.8 - 0.015 = \underline{0.785 \text{ mA}}$

⑥  $\beta = \frac{I_C}{I_B} = \frac{0.785}{0.015} = \underline{52.3}$

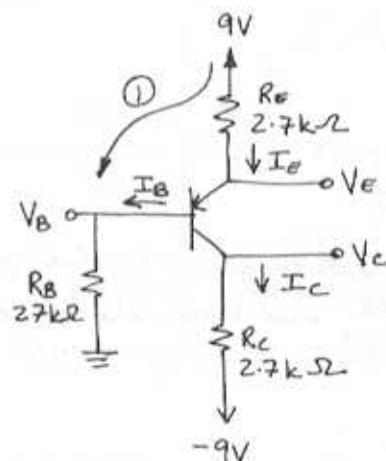
⑦  $\alpha = \frac{I_C}{I_E} = \frac{0.785}{0.8} = \underline{0.98}$

$$I_E = \frac{9 - 0.7}{3.9} = \underline{2.12 \text{ mA}}$$

$$V_C = -9 + 2.12 \times 2.2 = -4.3 \text{ V}$$

$$\therefore V_{BC} = \underline{4.3 \text{ V}}$$

5.76



5.75

①  $I_E = 2 \text{ mA}$

②  $9 - 2R_E - 0.7 - \frac{I_E}{R_B} R_B = 0$   
 $-2R_E + 9 = 0$   
 $6.4 - R_E - R_C = 0$

LET  $V_B = 0 \text{ V}$

$R_E = \frac{9 - 0.7}{2} = \underline{4.15 \text{ k}\Omega}$

$R_C = \frac{-4.5 + 9}{2} = \underline{2.25 \text{ k}\Omega}$

$$\text{Loop 1} \quad 9 - 2.7 I_E - 0.7 - \frac{I_E}{31} R_B = 0$$

$$I_E = 2.3243 \text{ mA}$$

$$V_B = R_B \times I_E / 31 = \underline{2.02 \text{ V}}$$

$$V_E = 9 - 2.7 I_E = \underline{2.72 \text{ V}}$$

$$V_C = -9 + \frac{30}{31} I_E (2.7) = \underline{-2.93 \text{ V}}$$

FOR  $R_B = 270 \text{ k}\Omega$

$$\text{Loop 1} \quad 9 - 2.7 I_E - 0.7 - \frac{R_B}{31} I_E = 0$$

$$I_E = 0.7274 \text{ mA}$$

CONT.

$$V_B = R_B \times \frac{I_E}{31} = \underline{\underline{6.34V}}$$

$$V_E = 9 - 2.7 I_E = \underline{\underline{7.04V}}$$

$$V_C = \frac{30}{31} I_E (2.7) - 9 = \underline{\underline{-7.10V}}$$

To return the voltages to the ones first calculated we have

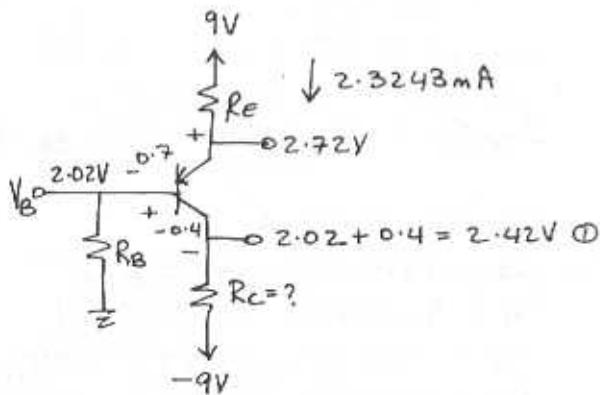
$$\text{Loop ① } \sim I_E = 2.3243 \text{ mA}$$

$$9 - 2.7 I_E - 0.7 - \frac{270}{\beta+1} I_E = 0$$

$$\beta = \underline{\underline{309}}$$

5.77

Using the values from the first part of P5.76 and for the edge of saturation  $V_{BC} > -0.4V$



CIRCUIT AT THE EDGE OF SATURATION

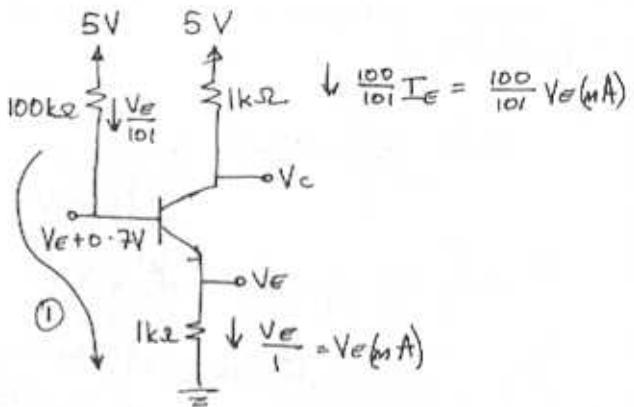
$$I_C = \frac{30}{31} I_E = \frac{30}{31} \times 2.3243$$

$$R_C = \frac{2.42 + 9}{\frac{30}{31} \times 2.3243} = \underline{\underline{5.08k\Omega}}$$

5.78

$$\beta = 100$$

(a)  $R_B = 100k\Omega$  -  $\circlearrowleft$   $R_B$  is large assume active mode.



Loop ①

$$5 - \frac{V_E}{101} \times 100 - 0.7 - V_E = 0$$

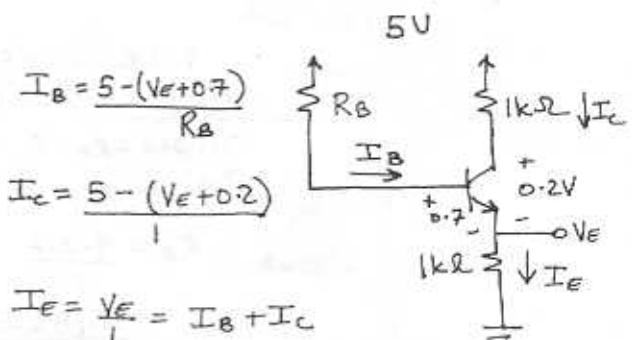
$$V_E = \underline{\underline{2.16V}}$$

$$V_B = V_E + 0.7 = \underline{\underline{2.86V}}$$

$$V_C = 5 - 1 \times \frac{100}{101} V_E = \underline{\underline{2.86V}}$$

Thus the BJT is in active mode as assumed.

(b)  $R_B = 10k\Omega$  - assume saturation



$$I_E = \frac{V_E}{1k\Omega} = I_B + I_C$$

$$\therefore V_E = \frac{4.3 - V_E}{10} + 4.8 - V_E$$

$$10V_E + V_E + 10V_E = 4.3 + 4.8$$

CONT

$$V_E = \underline{2.49V}$$

$$V_C = 2.49 + 0.2 = \underline{2.69V}$$

$$V_B = V_E + 0.7 = \underline{3.19V}$$

$$\text{Check: } I_C = \frac{5 - 2.69}{1} = 2.31mA$$

$$I_B = \frac{5 - 3.19}{10} = 0.181mA$$

$$\frac{I_C}{I_B} = \frac{2.31}{0.181} = 12.76 < 100$$

Hence we are in saturation as assumed!

(c)  $R_B = 1k\Omega$  - expect saturation  
- use circuit in (b)

$$I_B = \frac{5 - (V_E + 0.7)}{R_B} = \frac{4.3 - V_E}{1}$$

$$I_C = \frac{5 - (V_E + 0.2)}{1} = \frac{4.8 - V_E}{1}$$

$$I_E = I_B + I_C = V_E$$

$$4.3 - V_E + 4.8 - V_E = V_E$$

$$V_E = \underline{3V}$$

$$V_B = \underline{3.7V}$$

$$V_C = \underline{3.2V}$$

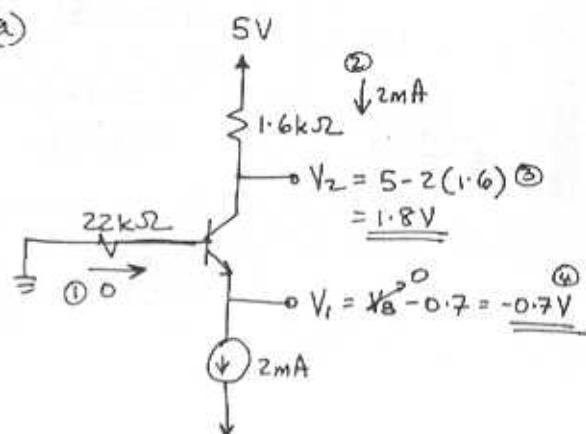
$$\text{Check } I_B = 4.3 - 3 = 1.3mA$$

$$I_C = 4.8 - 3 = 1.8mA$$

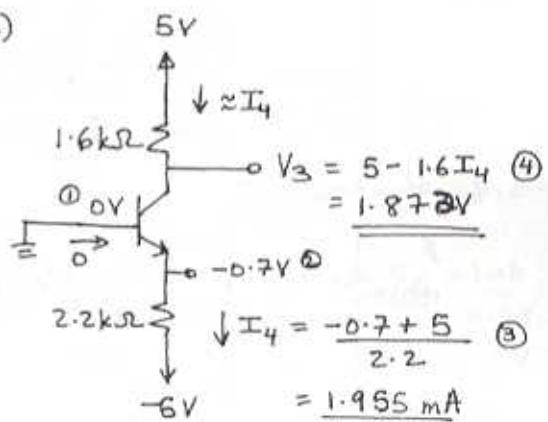
$$\frac{I_C}{I_B} = \frac{1.8}{1.3} = 1.4 < 100 \therefore \text{SATURATION AS ASSUMED}$$

5.79

(a)

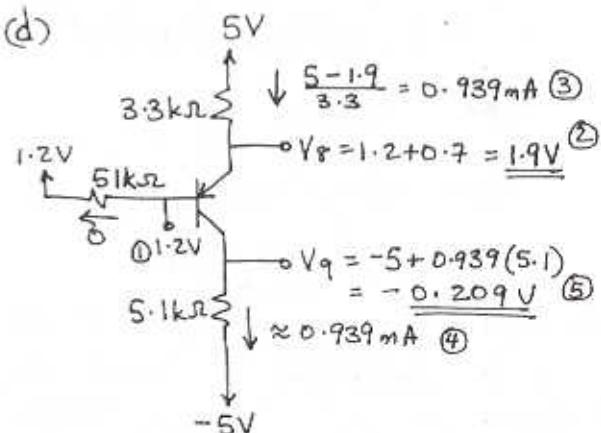


(b)



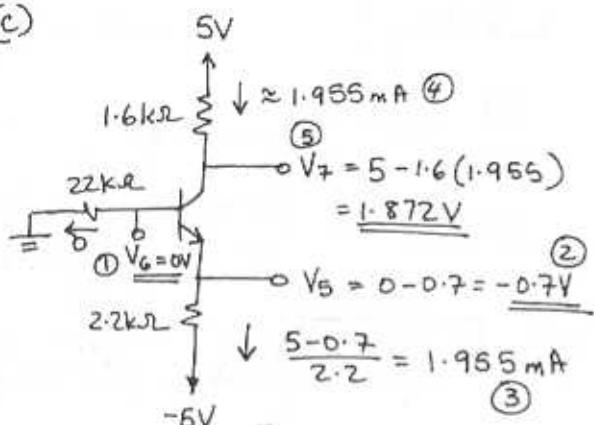
see below for part (c)

(d)



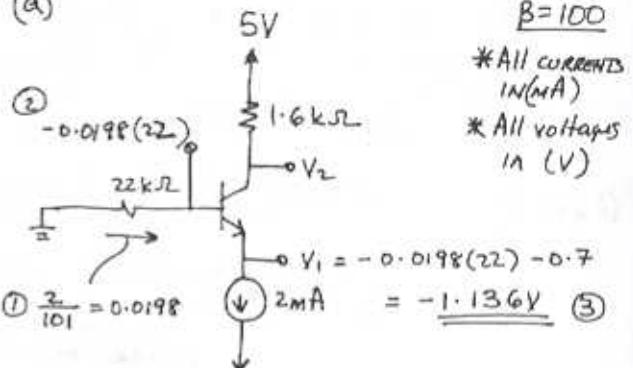
CONT.

(c)

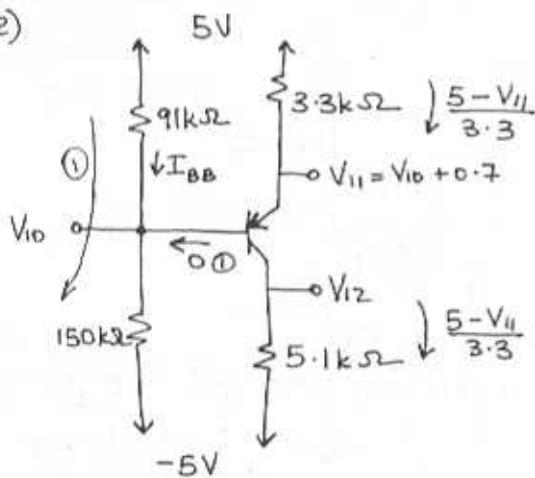


5.80

(a)

 $B=100$ \* All currents  
in (mA)  
\* All voltages  
in (V)

(e)



Loop ①

$$5 - 91I_{BB} - 150I_{BB} + 5 = 0$$

$$I_{BB} = \frac{10}{91+150}$$

$$V_{10} = -5 + 150I_{BB}$$

$$= -5 + \frac{150}{91+150} \times 10$$

$$= \underline{1.224V}$$

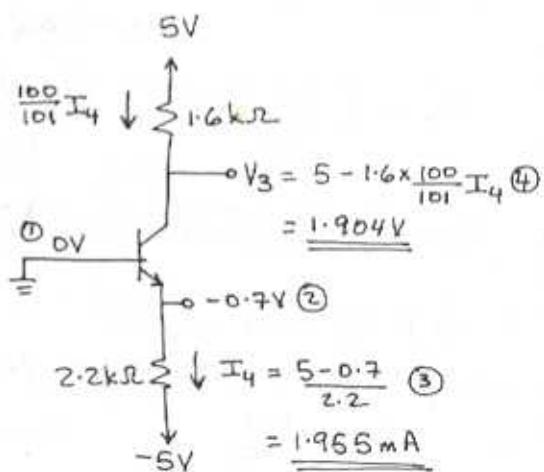
$$V_{11} = V_{10} + 0.7 = \underline{1.924V}$$

$$\therefore I_C \approx I_E = \frac{5 - V_{11}}{3.3}$$

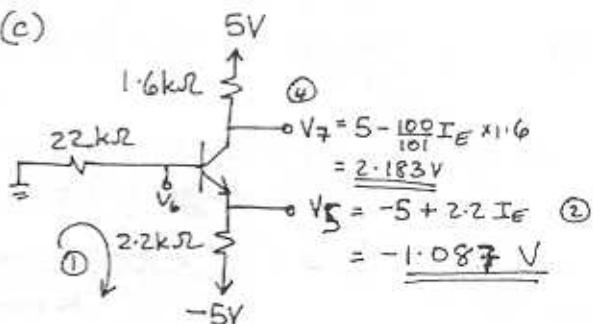
$$V_{12} = -5 + \left(\frac{5 - V_{11}}{3.3}\right) 5.1 = \underline{-0.246V}$$

$$④ V_2 = 5 - 2 \left(\frac{100}{101}\right) 1.6 = \underline{1.832V}$$

(b)



(c)



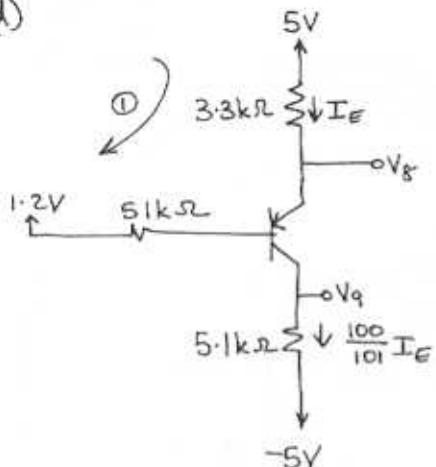
$$\text{Loop ① } 0 - \frac{I_E}{101} 22 - 0.7 - 2.2I_E + 5 = 0$$

$$I_E = 1.778 \text{ mA}$$

$$③ V_G = V_5 + 0.7 = \underline{-0.387V}$$

CONT

(d)



Loop ①

$$5 - 3 \cdot 3 I_E - 0.7 - \frac{I_E}{101} R_{BB} - 1.224 = 0$$

$$I_E = 0.7967 \text{ mA}$$

$$V_{11} = 5 - 3 \cdot 3 I_E = 2.371 \text{ V}$$

$$V_{12} = \frac{100}{101} I_E \times 5.1 - 5 = -0.977 \text{ V}$$

$$V_{10} = V_{11} - 0.7 = 1.67 \text{ V}$$

Loop ②

$$5 - 3 \cdot 3 I_E - 0.7 - \frac{I_E}{101} 51 - 1.2 = 0$$

$$I_E = 0.8147 \text{ mA}$$

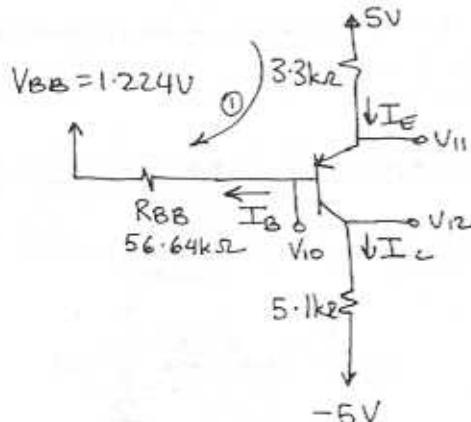
$$V_{11} = 5 - 3 \cdot 3 I_E = 2.3114 \text{ V}$$

$$V_9 = -5 + 5.1 \times \frac{100}{101} I_E = -0.8862 \text{ V}$$

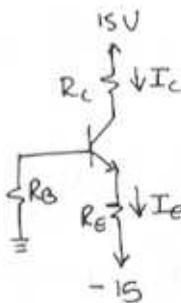
(e) Use Thévenin's theorem to simplify the bias network:

$$V_{BB} = -5 + \frac{150}{150+91} \times 10 = 1.224 \text{ V}$$

$$R_{BB} = 150 // 91 = 56.64 \text{ k}\Omega$$



5.8

Nominal  $\beta = 100$ .

$$\text{Thus, } \text{nominal } \alpha = \frac{100}{101} = 0.99$$

$$\text{nominal } I_E = 1 \text{ mA}$$

$$\text{nominal } I_C = 0.99 \text{ mA}$$

$$\text{nominal } V_C = 5 \text{ V}$$

$$\text{Thus, } R_C = \frac{15 - 5}{0.99} = 10.1 \text{ k}\Omega \xrightarrow{\text{use}} 10 \text{ k}\Omega$$

$$I_E = 1 = \frac{15 - 0.7}{R_E + \frac{R_B}{\beta + 1}}$$

$$= \frac{14.3}{R_E + \frac{R_B}{101}}$$

$$\Rightarrow R_E + \frac{R_B}{101} = 14.3 \quad (1)$$

As  $\beta$  varies from 50 to 150, need to limit the variation of  $I_E$  to  $\pm 10\%$  of 1mA. One can reason that the maximum variation in  $I_E$  occurs for  $\beta = 50$  (as opposed to  $\beta = 150$ ). To see this more that when  $\beta$  decreases from 100 to 50 the base current doubles while a change in  $\beta$  from

CONT.

100 to 150 causes the base current to decrease to  $\frac{2}{3}$  its nominal value. Thus our decision will be based on imposing the 10% limit for  $\beta = 50$ .

$$0.9 = \frac{14.3}{R_E + \frac{R_B}{\beta+1}} = \frac{14.3}{R_E + \frac{R_B}{51}}$$

$$R_E + \frac{R_B}{51} = 15.89 \quad (2)$$

$$(2) - (1) \Rightarrow R_B \left( \frac{1}{51} - \frac{1}{101} \right) = 1.59$$

$$\Rightarrow R_B = 163.8 \text{ k}\Omega \xrightarrow{\text{use}} \underline{\underline{164 \text{ k}\Omega}}$$

Sub into (1) gives

$$R_E = 12.7 \text{ k}\Omega \xrightarrow{\text{use}} \underline{\underline{13 \text{ k}\Omega}}$$

To find the expected range of  $I_C$  &  $V_c$  corresponding to  $\beta$  variation from 50 to 150 we use

$$I_C = \frac{14.3}{R_E + \frac{R_B}{\beta+1}}$$

$$\text{for } \beta = 50 \quad I_C = \frac{50}{51} \frac{14.3}{13 + \frac{164}{51}} = \underline{\underline{0.864 \text{ mA}}}$$

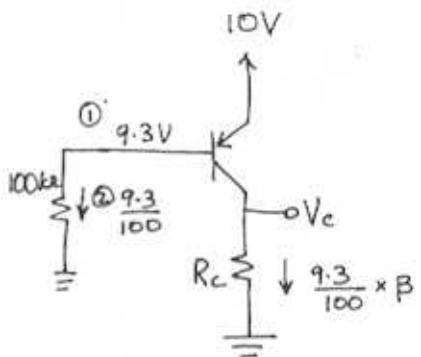
$$V_c = 15 - 0.864 \times 10 = \underline{\underline{6.36 \text{ V}}}$$

$$\text{for } \beta = 150 \quad I_C = \frac{150}{151} \times \frac{14.3}{13 + \frac{164}{151}}$$

$$= \underline{\underline{1.008 \text{ mA}}}$$

$$V_c = 15 - 1.008 \times 10 = \underline{\underline{4.92 \text{ V}}}$$

5.82



$$\text{For } V_c = 5 \text{ V} = \frac{9.3}{100} \times \beta \times R_C \quad \beta = 50$$

$$R_C = \frac{500}{9.3 \times 50} = \underline{\underline{1.08 \text{ k}\Omega}}$$

$$\text{For } \beta = 100$$

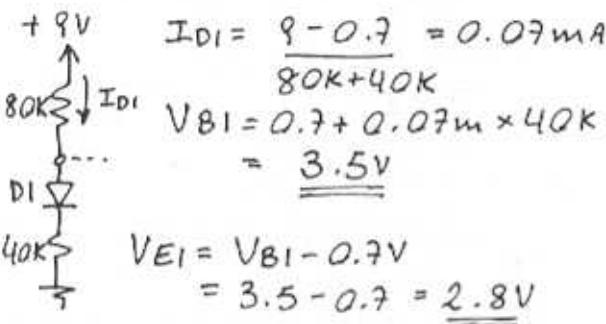
$$\begin{aligned} V_c &= \frac{9.3}{100} \times \beta \times R_C = \frac{9.3}{100} \times 100 \times 1.08 \\ &= \underline{\underline{10.04 \text{ V}}} \quad \leftarrow V_{BC} = 9.3 - 10.04 \\ &= -0.74 \end{aligned}$$

Since  $V_{BC} < -0.4 \text{ V}$  the transistor saturates!

5.83

For  $\beta = \infty$  and  $R_{open}$ :

$$I_{B1} = I_{B2} = 0, \alpha_1 = \alpha_2 = 1$$



$$I_{E1} = \frac{2.8V}{2k} = 1.4mA$$

$$I_{E1} = I_{C1} \text{ since } \alpha = 1$$

$$V_{C1} = 9V - 2k \times 1.4mA - 0.7 = 5.5V$$

$V_{CB} = 2V \rightarrow$  Transistor is in active mode.

$$V_{B2} = V_{C1} = 5.5V$$

$$V_{E2} = 5.5 + 0.7 = 6.2V$$

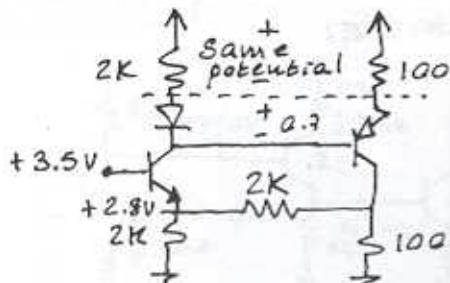
$$I_{E2} = \frac{9 - 6.2}{100} = 28mA$$

$$I_{E2} = I_{C2} \text{ since } \alpha = 1$$

$$\rightarrow V_{C2} = 28mA \times 100\Omega = 2.8V$$

For  $\beta = \infty$  and  $R$  connected:

$$\text{Still: } V_{B1} = 3.5V, V_{E1} = 2.8V$$

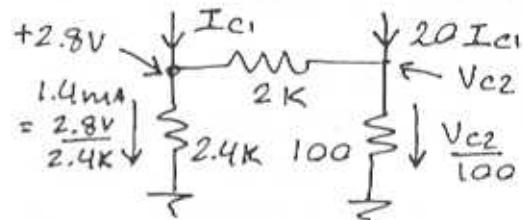


Since the voltage across the top two resistors is equal:

$$I_{C1} \times 2k = I_{E2} \times 100$$

$$I_{E2} = 20 I_{C1}$$

$$\text{also: } I_{C1} = I_{E1} \quad I_{C2} = I_{E2}$$



$$1.4mA - I_{C1} = 20 I_{C1} - \frac{V_{C2}}{100}$$

$$\rightarrow V_{C2} = 100 \times (21. I_{C1} - 1.4mA) \quad (1)$$

also:

$$\frac{V_{C2} - 2.8V}{2k\Omega} = 1.4mA - I_{C1}$$

$$\rightarrow V_{C2} = 5.6 - I_{C1} \times 2k \quad (2)$$

Solving for  $I_{C1}$  from (1) & (2)

$$I_{C1} = 1.4mA$$

Substituting in either (1) or (2)

$$V_{C2} = 2.8V$$

$$\text{and: } V_{E2} = 9 - 100 \times 1.4mA = 8.86V$$

$$V_{B2} = V_{C1} = 8.86 - 0.7 = 8.16V$$

For  $\beta = 100$  and  $R_{open}$ :

In the previous two cases

$$I_{D1} = 0.07mA, I_{E1} = 1.4mA$$

$$\text{if } \beta = 100 \rightarrow I_{B1} \approx 0.014mA$$

which is a significant amount compared to 0.07mA  
 → must be taken into account

The bottom two resistors have equal voltage drops thus,  
 CONT.

$$2K \times I_{E1} = 40K \times I_{D1}$$

$$\rightarrow I_{D1} = 0.05 \times I_{E1} \quad (3)$$

also:  $I_{D1} + I_{B1} = \frac{9 - V_{B1}}{80K}$

$$I_{E1} \left( \frac{1}{\beta+1} + 0.05 \right) \quad \text{FROM (3)}$$

for  $\beta = 100$ :

$$0.06 \times I_{E1} = \frac{9 - V_{B1}}{80K}$$

$$\rightarrow V_{B1} = 9 - 4800 \times I_{E1} \quad (4)$$

also:  $V_{B1} = 0.7 + I_{E1} \times 2K \quad (5)$

From (4) & (5)  $\begin{cases} V_{B1} = 3.11V \\ I_{E1} = 1.22mA \end{cases}$   
 $V_{E1} = 1.22mA \times 2K \rightarrow V_{E1} = \underline{\underline{2.44V}}$

$$I_{C1} = \alpha I_E = 0.99 \times 1.22mA$$

$$= 1.2mA$$

Again: voltage drop on top two resistors is equal

$$2K \cdot I_{D2} = 100 \cdot I_{E2}$$

$$I_{D2} = 0.05 I_{E2}$$

but  $I_{D2} = 1.2mA - \frac{I_{E2}}{\beta+1}$

$$\Rightarrow 1.2mA = \left( 0.05 + \frac{1}{\beta+1} \right) I_{E2}$$

$$I_{E2} = 20mA \quad \underline{0.05}$$

$$V_{E2} = 9 - 100 \times 20mA = \underline{\underline{7V}}$$

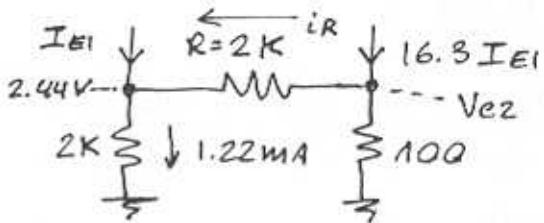
$$V_{B2} = V_{E1} = 7 - 0.7 = \underline{\underline{6.3V}}$$

$$I_{C2} = \alpha I_{E2} = 19.8mA$$

$$V_{C2} = 100 \times 19.8mA = \underline{\underline{1.98V}}$$

For  $\beta = 100$  and  $R$  connected:

To simplify the solution: assume  
 $i$  on  $R$  is  $\ll I_{E1} \rightarrow V_{E1} = 2.44V$



From top of circuit:

$$I_{E2} = I_{C1} / 0.06$$

$$I_{C2} = \frac{\alpha^2}{0.06} \cdot I_{E1}$$

$$I_{C2} = 16.3 \times I_{E1}$$

To obtain  $I_{E1}$ :

$$1.22mA - I_{E1} = 16.3 I_{E1} - \frac{V_{c2}}{100}$$

$$V_{c2} = 100 (17.2 I_{E1} - 1.22mA) \quad (6)$$

also:

$$\frac{V_{c2} - 2.44}{2K} = 1.22mA - I_{E1}$$

$$\rightarrow V_{c2} = 2.44 - 2000 I_{E1} + 2.44$$

$$= 4.88 - 2 \times 10^3 \cdot I_{E1} \quad (7)$$

From (7) & (8);  $I_{E1} = 1.34mA$

$$V_{c2} = \underline{\underline{2.18V}}$$

thus,

$$I_{C1} = 1.33mA$$

$$I_{E2} = 22.1mA$$

$$V_{E2} = 9 - 22.1mA \times 100 = 6.79V$$

$$V_{B2} = 6.79 - 0.7 = \underline{\underline{6.09V}}$$

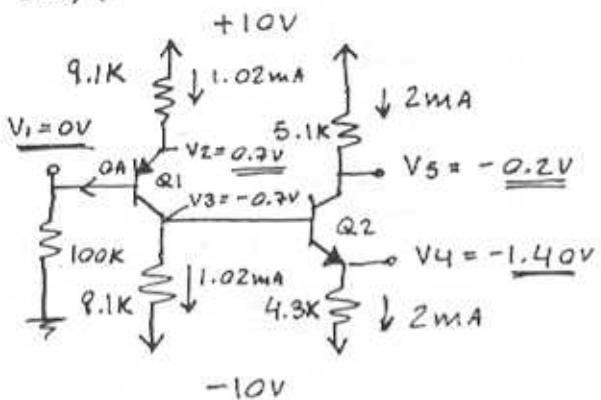
To confirm initial assumption  
on  $i$  of  $R$ :

$$\frac{2.44 - 2.18}{2K} = 0.13mA$$

which is 10 times smaller  
than  $I_{E1}$

5.84

(a)  $\beta = \infty$



$$R_1 = \frac{9.3}{2} = \underline{\underline{4.7\text{ k}\Omega}}$$

$$R_2 = \frac{10}{2} = 5 \rightarrow \underline{\underline{5.1\text{ k}\Omega}}$$

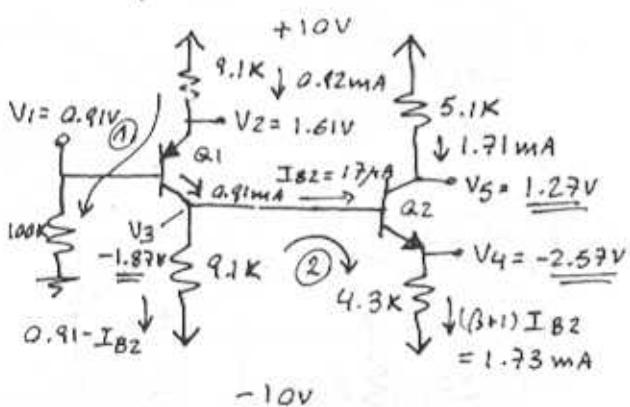
$$R_3 = \frac{9.3}{2} = \underline{\underline{4.7\text{ k}\Omega}}$$

$$R_4 = \frac{6}{2} = \underline{\underline{3\text{ k}\Omega}}$$

$$R_5 = \frac{8}{4} = \underline{\underline{2\text{ k}\Omega}}$$

$$R_6 = \frac{10 - 4.7}{4} = \underline{\underline{1.3\text{ k}\Omega}}$$

(b)  $\beta = 100$



$$\textcircled{1} \quad 10 - 9.1(\beta+1)I_{B1} - 0.7 - 100I_{B1} = 0$$

$$\rightarrow I_{B1} = 0.009\text{ mA}$$

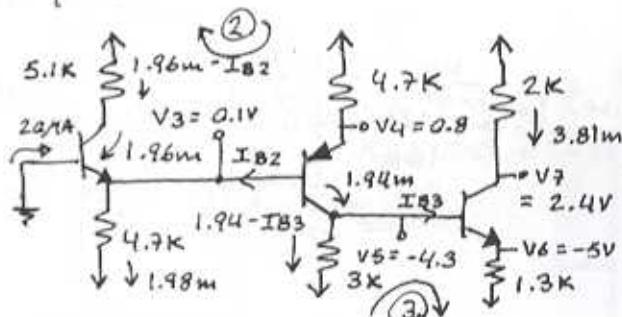
$$\textcircled{2} \quad (0.91 - I_{B2}) \times 9.1 =$$

$$0.7 + (\beta+1)I_{B2} \times 4.3$$

$$\rightarrow I_{B2} = 0.017\text{ mA}$$

$$\rightarrow I_{E2} = 1.73\text{ mA}$$

$$\beta = 100$$



$$\textcircled{2} \quad (1.96 - I_{B2}) \times 5.1 = (\beta+1)I_{B2} \times 4.7 + 0.7$$

$$I_{B2} = 0.0194\text{ mA}$$

$$I_{E2} = 1.96\text{ mA}$$

$$V3 = \underline{\underline{0.1V}} \quad V4 = \underline{\underline{0.8V}}$$

$$\textcircled{3} \quad (1.94 - I_{B3}) \times 3 = 0.7 + 1.3 \times (\beta+1) \cdot I_{B3}$$

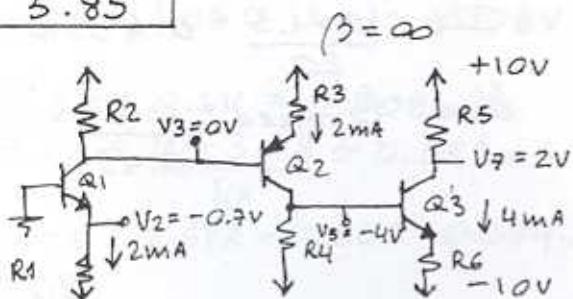
$$I_{B3} = 0.038\text{ mA}$$

$$I_{E3} = 3.85\text{ mA}$$

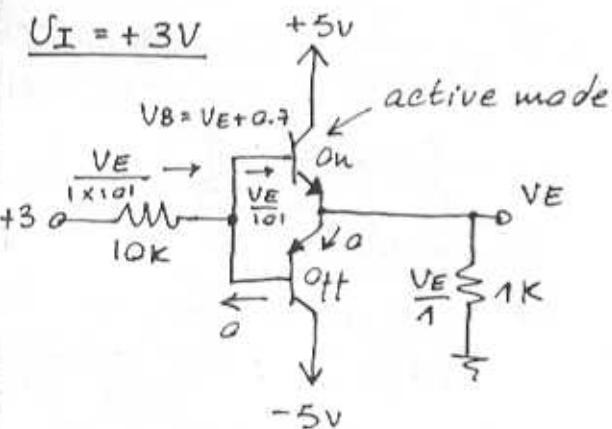
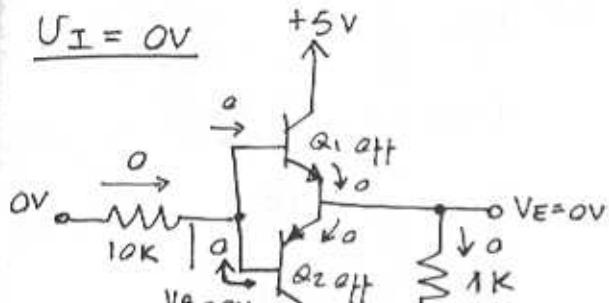
$$V5 = -4.3V \quad V6 = -5V$$

$$V7 = \underline{\underline{2.4V}}$$

5.85



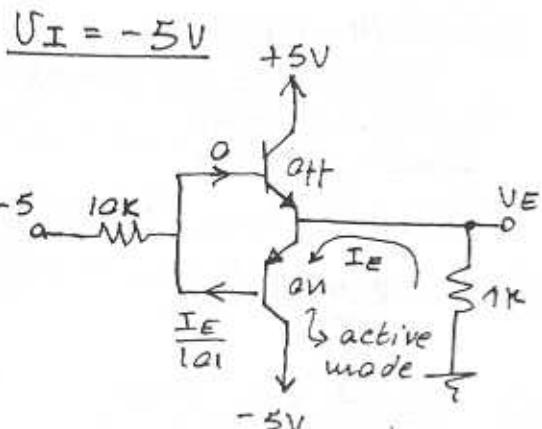
5.86



$$3 = \frac{V_E}{10} \times 10 + 0.7 + V_E$$

$$\Rightarrow V_E = \underline{\underline{2.09V}}$$

$$V_B = \underline{\underline{2.78V}}$$

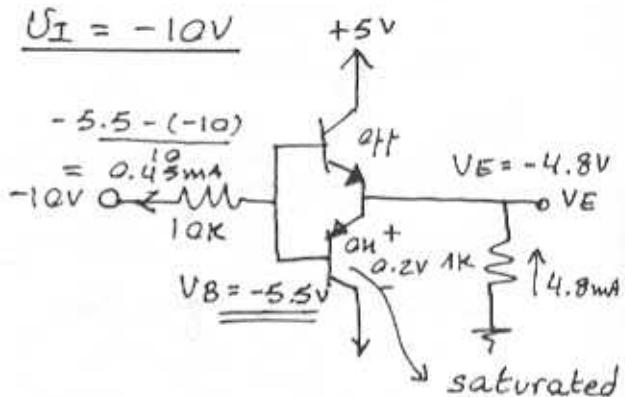


$$I_E = \frac{5 - 0.7}{1 + 10/10} = 3.91mA$$

$$V_E = -3.91V$$

$$V_B = \underline{\underline{-4.61V}}$$

$U_I = -10V$

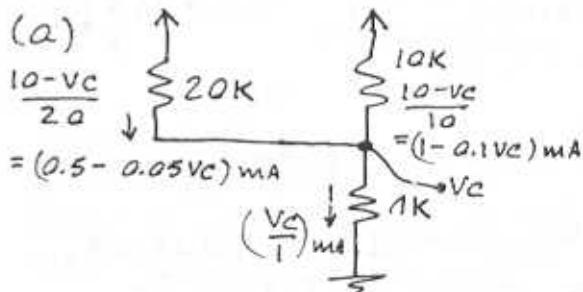


$$\frac{I_C}{I_B} = \frac{4.35}{0.45} = 9.7 < 100$$

thus, Q2 is saturated as assumed

$$V_E = \underline{\underline{-4.8V}} \quad V_B = \underline{\underline{-5.5V}}$$

5.87



$$(0.5 - 0.005V_c) + (1 - 0.1V_c) = V_c$$

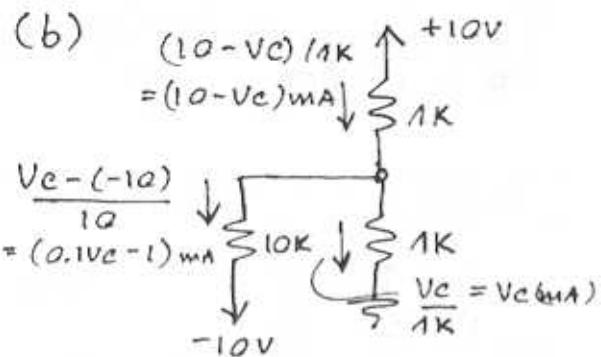
$$V_c = \underline{\underline{1.3V}}$$

$$I_C = \frac{10 - 1.3}{10} = 0.87mA$$

$$I_B = \frac{10 - 1.3}{20} = 0.435mA$$

$$\text{thus } \beta_{\text{forced}} = \frac{0.87}{0.435} = \underline{\underline{2}}$$

CONT.



$$10 - V_c = (0.1 V_c + 1) + (V_c)$$

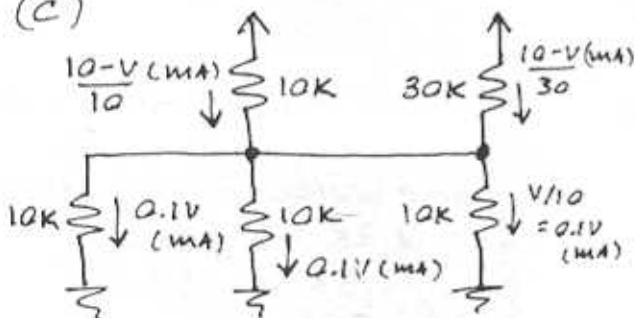
$$\Rightarrow V_c = \underline{\underline{+4.29V}}$$

$$I_c = 4.29 \text{ mA}$$

$$I_B = \frac{4.29 + 10}{10} = 1.43 \text{ mA}$$

$$\beta_{\text{forced}} = \frac{4.29}{1.43} = \underline{\underline{3}}$$

(c)



Node equation:

$$\frac{10-V}{10} + \frac{10-V}{30} = 0.1V + 0.1V + 0.1V$$

$$30 - 3V + 10 - V = 9V$$

$$40 = 13V$$

$$\Rightarrow V = \underline{\underline{3.08V}}$$

thus,  $V_{c3} \approx V_{c4} \approx 3.08V$

$$I_{B3} = 0.1V = 0.308 \text{ mA}$$

$$I_{E3} = \frac{10 - 3.08}{10} \approx 0.682 \text{ mA}$$

$$I_{c3} = 0.682 - 0.308 = 0.384 \text{ mA}$$

$$\beta_3 \text{ forced} = \frac{0.384}{0.308} = \underline{\underline{1.25}}$$

$$I_{c4} = \frac{10 - 3.08}{30} = 0.231 \text{ mA}$$

$$I_{E4} = 0.1V = 0.308 \text{ mA}$$

$$I_{B4} = 0.308 - 0.231 = 0.077 \text{ mA}$$

$$\beta_4 \text{ forced} = \frac{0.231}{0.077} = \underline{\underline{3}}$$

5.88

$$\frac{5 \times R_{B2}}{R_{B1} + R_{B2}} = 0.690$$

$$\rightarrow 5R_{B2} = 0.69R_{B1} + 0.69R_{B2}$$

$$\rightarrow 4.31R_{B2} = 0.69R_{B1}$$

$$\Rightarrow \frac{R_{B1}}{R_{B2}} = \underline{\underline{6.24}}$$

$$\textcircled{2} \text{ Since } V_{BE} = \frac{5 R_{B2}}{R_{B1} + R_{B2}}$$

If both  $R_{B2}$  &  $R_{B1}$  are at 0.99 or 1.01 of their nominal value  $\rightarrow V_{BE}$  will not be affected.

We must consider the cases when one resistor is at 0.99 and the other at 1.01 of their nominal value.

$$\text{If: } R_{B2}' = 1.01 R_{B2}$$

$$R_{B1}' = 0.99 R_{B1}$$

$$\Rightarrow V_{BE} = 0.702V$$

$$\text{If: } R_{B2}' = 0.99 R_{B2}$$

$$R_{B1}' = 1.01 R_{B1}$$

$$\Rightarrow V_{BE} = 0.678V$$

thus  $V_{BE}$  ranges from 0.678V to 0.702V

CONT.

For  $I_C$ :  $I_C = I_S e^{V_{BE}/V_T}$   
 for  $V_{BE} = 0.690 \rightarrow I_C = 1\text{mA}$   
 $\Rightarrow I_S = 1.032 \times 10^{-15}$

for  $V_{BE} = 0.678 \rightarrow I_C = 0.618\text{mA}$   
 $V_{BE} = 0.702 \rightarrow I_C = 1.62\text{mA}$

$I_C$  ranges from  $0.618\text{mA}$  to  $1.62\text{mA}$ .

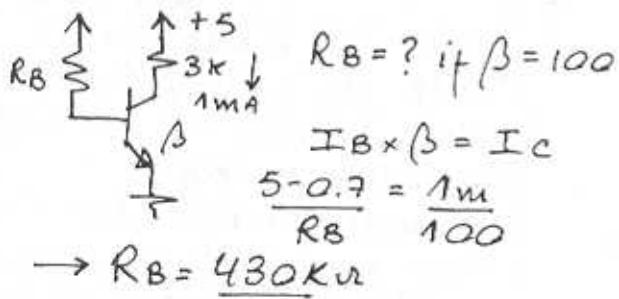
③ If  $R_C = 3\text{k}\Omega$

$$V_{CE} = 5 - 3\text{k} \times 0.62\text{mA} = 3.14\text{V}$$

$$V_{CE} = 5 - 3\text{k} \times 1.62\text{mA} = 0.14\text{V}$$

This circuit is too sensitive to parameter variations as shown here for a 1% resistor tolerance.

5.89



$$V_{CE} = 5\text{V} - 3\text{k} \cdot 1\text{mA} = 2\text{V}$$

If  $\beta = 50$ :  $I_C = \frac{5 - 0.7}{430\text{k}} \times 50$

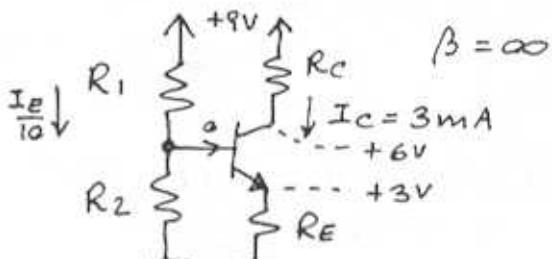
$$I_C = 0.50\text{mA}$$

$$\Rightarrow V_{CE} = 5 - 3\text{k} \times 0.5\text{mA} = +3.5\text{V}$$

If  $\beta = 150$ :  $I_C = 1.5\text{mA}$   
 $V_{CE} = 0.5\text{V}$

This design is too sensitive to variations of  $\beta$ .

5.90



$$R_C = \frac{3\text{V}}{3\text{mA}} = 1\text{k}\Omega$$

$$R_E = \frac{3\text{V}}{3\text{mA}} = 1\text{k}\Omega$$

$$V_b = 0.7 + 3 = 3.7\text{V}$$

$$R_1 = \frac{9 - 3.7}{I_E/10} = 17.7\text{k}\Omega$$

$$9\text{V} = (R_1 + R_2) \frac{I_E}{10} \rightarrow R_2 = 12.3\text{k}$$

Choose suitable 5% resistors

$$R_1 = 17.7\text{k} \rightarrow 18\text{k}\Omega$$

$$R_2 = 12.3\text{k} \rightarrow 13\text{k}\Omega$$

$$R_1 = R_2 = 1\text{k}$$

$$V_{BB} = \frac{9 \times 13}{18 + 13} = 3.77\text{V}$$

For these values of  $R$  and

$$\beta = 90: R_B = 18/13 = 7.55\text{k}\Omega$$

$$I_E = \frac{3.77 - 0.7}{1\text{k} + 7.55\text{k}} = 2.83\text{mA}$$

$$\alpha = 0.989 \Rightarrow I_C = 2.80\text{mA}$$

If  $R_E$  is reduced by  $\sim \frac{7.55\text{k}}{9\text{k}}$

$$\rightarrow R_E = 910\Omega$$

$$\Rightarrow I_E = 3.09\text{mA}$$

$$I_C = 3.05\text{mA}$$

5.91

For  $\beta = \infty$   $I_B = 0$ ,  $I_E = 3\text{mA}$

$$R_1 = \frac{9 - 3.7}{I_E/2} = 3.5\text{K}\Omega$$

$$9V = (R_1 + R_2) \frac{I_E}{2} \Rightarrow R_2 = 2.5\text{K}\Omega$$

Suitable 5% resistors:

$$R_1 = 3.5K \rightarrow 3.3\text{K}\Omega$$

$$R_2 = 2.5K \rightarrow 2.4\text{K}\Omega$$

$$R_E = R_C = 1\text{K}\Omega$$

$$V_{BB} = \frac{9 \times 2.4}{3.3 + 2.4} = 3.79V$$

For  $\beta = 50$ :

$$R_B = (3.3/2.4)K = 1.39\text{K}\Omega$$

$$I_E = \frac{3.79 - 0.7}{1K + \frac{1.39K}{91}} = 3.04\text{mA}$$

No need to adjust  $R_E$ .

As the current from the voltage divider increases the effect of  $I_B$  is reduced.

5.92

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta+1}}$$

(a) For  $\beta = 100$ , varying between 50 and 150 the maximum deviation in  $I_E$  (from the nominal value obtained for  $\beta = 100$ ) occurs at the low end of  $\beta$  values ( $\beta = 50$ ). Thus, to keep

$I_E$  within  $\pm 5\%$  of nominal we must impose the constraint

$$I_E(\beta=50) > 0.95 I_E(\beta=100)$$

$$\text{or, } \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{51}} > 0.95 \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{101}}$$

$$\text{or, } R_E + \frac{R_B}{51} > 0.95 \left( R_E + \frac{R_B}{101} \right)$$

$$0.05 R_E > R_B \left( \frac{0.95}{51} - \frac{1}{101} \right)$$

$$\Rightarrow \frac{R_B}{R_E} < 5.73$$

Thus, the largest ratio of  $R_B/R_E$  is 5.73

$$(b) I_E \cdot R_E = V_{CC}/3$$

$$\rightarrow \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta+1}} \cdot R_E = \frac{V_{CC}}{3}$$

$$\frac{V_{BB} - 0.7}{1 + \frac{R_B}{R_E} \cdot \frac{1}{\beta+1}} = \frac{V_{CC}}{3}$$

$$V_{BB} = \frac{1}{3} V_{CC} \left( 1 + \frac{5.73}{101} \right) + 0.7$$

$$\Rightarrow \underline{\underline{V_{BB} = 0.35 V_{CC} + 0.7}}$$

$$(c) V_{CC} = 10V$$

$$V_{BB} = 0.35 \times 10 + 0.7 = 4.2V$$

$$\rightarrow \frac{R_2}{R_1 + R_2} \times 10 = 4.2$$

$$\frac{R_2}{R_1 + R_2} = 0.42 \quad ①$$

$$I_E \cdot R_E = \frac{1}{3} V_{CC}$$

CONT.

$$2 \times R_E = \frac{1}{3} \times 10$$

$$\Rightarrow R_E = \underline{\underline{1.67 \text{ k}\Omega}}$$

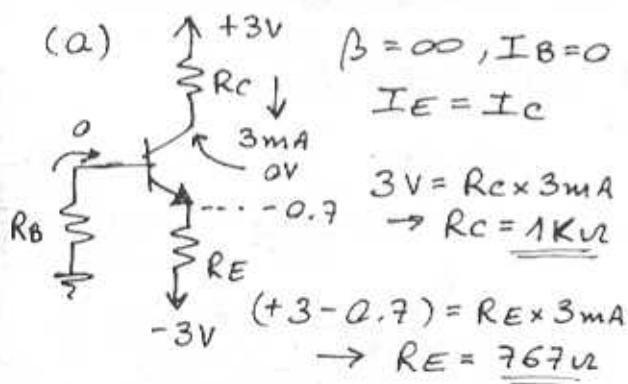
$$R_B = 5.73 \times 1.67 = 9.55 \text{ k}\Omega$$

$$\frac{R_1 \cdot R_2}{R_1 + R_2} = 9.55$$

Substituting from ① gives

$$R_1 = \frac{9.55}{0.42} = \underline{\underline{22.7 \text{ k}\Omega}}$$

5.93



(b)  $\beta = 90$   $\frac{V_{RE}}{10} = V_{RB}$

$$I_B \cdot R_B = \frac{I_E \cdot R_E}{10}$$

$$\rightarrow \frac{I_E}{(\beta+1)} \cdot R_B = \frac{I_E \cdot R_E}{10}$$

$$\rightarrow R_B = (\beta+1) R_E \quad \textcircled{1}$$

also,  $0 = V_{RB} + 0.7 + V_{RE} - 3$

$$2.3 = \frac{V_{RE}}{10} + V_{RE}$$

$$\rightarrow V_{RE} = \frac{2.3}{1.1} = 2.09V$$

$$2.09 = I_E \times R_E \quad \textcircled{2}$$

but:  $I_E = \frac{I_C}{\alpha} = \frac{3 \text{ mA}}{0.989} = 3.033 \text{ mA}$

Substituting in ②:

$$R_E = 689\Omega$$

from ①:

$$R_B = \underline{\underline{6269 \Omega}}$$

(c) Standard 5% values:

$$R_C = 1 \text{ k}\Omega$$

$$R_E = 689\Omega \rightarrow 680\Omega$$

$$R_B = 6269\Omega \rightarrow 6.2 \text{ k}\Omega$$

(d)  $\beta = \infty: I_B = 0$

$$I_C = I_E$$

$$V_B = 0$$

$$V_E = -0.7$$

$$I_E = \frac{3 - 0.7}{R_E} = \frac{3 - 0.7}{680} = \underline{\underline{3.38 \text{ mA}}}$$

$$V_C = 3 - 3.38 \text{ mA} \times 1 \text{ k} = \underline{\underline{-0.38V}}$$

For  $\beta = 90:$

$$I_E = \frac{2.3}{680 + 6.2 \text{ k}} = \frac{2.3}{91} = \underline{\underline{3.07 \text{ mA}}}$$

$$I_C = \alpha I_E = \underline{\underline{3.04 \text{ mA}}}$$

$$V_B = \frac{R_B \cdot I_E}{\beta + 1} = -0.209$$

$$V_E = -0.209 - 0.7 = \underline{\underline{0.909V}}$$

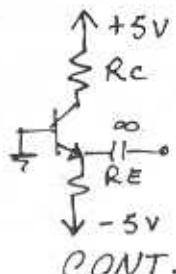
$$V_C = 3 - I_C \cdot R_C = 3 - 3.04 \times 1 = \underline{\underline{-0.04V}}$$

5.94

$$V_E = -0.7V$$

To obtain  $I_E = 1 \text{ mA}$

$$R_E = \frac{-0.7 - (-5)}{1} = 4.3 \text{ k}\Omega$$



CONT.

To maximize gain while allowing  $V_{\pm} \approx 1V$  signal at collector, design for a dc collector voltage of +1V.

Thus,

$$R_C = \frac{5-1}{I_C} \approx \frac{4}{1} = \underline{\underline{4 \text{ k}\Omega}} \quad (\alpha=1)$$

For 100°C rise in temperature,  $V_{BE}$  decreases by  $2 \times 100 = 200 \text{ mV}$  and thus  $I_E$  increases by  $\frac{0.2V}{R_E}$

$$= \frac{0.2V}{4.3 \text{ k}\Omega} = 0.047 \text{ mA}$$

i.e. an increase of 4.7%

The change in  $\beta$  from 50 to 150 causes  $\alpha$  to change from 0.980 to 0.993 which implies an increase in collector current of 1.3%. Thus the overall increase in  $I_C$  is 6%

### 5.95

To allow a collector voltage swing of  $\pm 1V$ , we design for:

$$V_C = V_B + 1 \\ = 0.7 + 1 = 1.7V$$

$$I_E = 0.5 \text{ mA}$$

$$\rightarrow R_C = \frac{5-1.7}{0.5} = \underline{\underline{6.6 \text{ k}\Omega}}$$

For  $\beta = 100$ :

$$I_B = \frac{I_E}{\beta+1} = \frac{0.5}{101} \approx 5 \mu\text{A}$$

$$I_B \cdot R_B = 1V$$

$$R_B = \frac{1V}{5 \mu\text{A}} = \frac{1}{5} \text{ M}\Omega = \underline{\underline{200 \text{ k}\Omega}}$$

Now, if the BJT used has  $\beta = 50$ , the emitter current resulting can be found from Eq (5.74)

$$I_E = \frac{V_{CC} - V_{BE}}{R_C + R_B} \quad (\beta+1) \\ = \frac{5 - 0.7}{6.6 + \frac{200}{51}} = \underline{\underline{0.41 \text{ mA}}}$$

$$\text{and } I_B = \frac{0.41}{51} \approx 8 \mu\text{A}$$

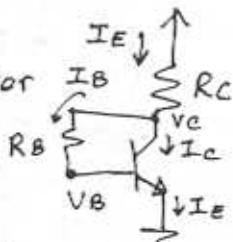
Thus the collector will be higher than the base by  $8 \times 0.2 = 1.6V$ , allowing for a  $\pm 1.6V$  signal swing at the collector.

For  $\beta = 150$ :

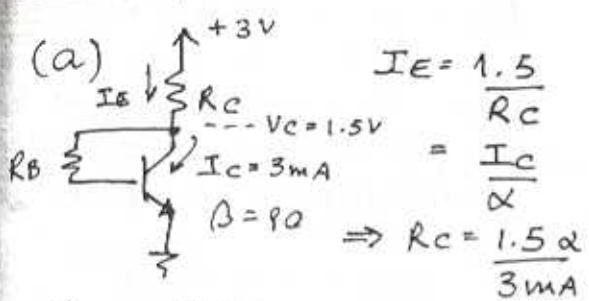
$$I_E = \frac{5 - 0.7}{6.6 + \frac{200}{151}} = 0.54 \text{ mA}$$

$$I_B = \frac{0.54}{151} = 36 \mu\text{A}$$

Thus the collector voltage will be higher than that of the base by  $3.6 \times 0.2 = 0.72V$  allowing for only  $\pm 0.72V$  signal swing.



5.96



(b) Standard 5% values

$$R_B = 24 \text{ k}\Omega$$

$$R_C = 495 \Omega \rightarrow 510 \Omega$$

$$\text{then, } I_E = \frac{3 - 0.7}{510 + 24 \text{ k}} = 2.97 \text{ mA}$$

$$I_C = I_E \cdot \alpha = 2.93 \text{ mA}$$

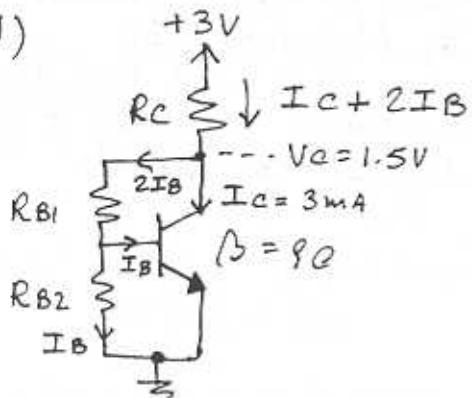
$$V_C = 3 - 2.97 \text{ mA} \times 510 = 1.48 \text{ V}$$

(c)  $\beta = \infty$   $I_B = 0, V_B = 0.7 \text{ V}$

$$I_C = I_E = \frac{3 - 0.7}{510} = 4.5 \text{ mA}$$

$$V_C = 3 - 4.5 \text{ mA} \times 510 = 0.7 \text{ V}$$

(d)



$$I_B = \frac{I_C}{\beta} = \frac{3 \text{ mA}}{90} = 0.033 \text{ mA}$$

$$1.5 \text{ V} = 2I_B \times R_B + 0.7$$

$$\rightarrow R_B = 12.1 \text{ k}\Omega$$

$$0.7 = I_B \times R_B$$

$$\rightarrow R_B = 21.2 \text{ k}\Omega$$

on  $R_C$ :

$$I_C + 2I_B = 3.066 \text{ mA}$$

$$R_C = \frac{1.5 \text{ V}}{3.066 \text{ mA}} = 489 \Omega$$

Standard 5% values:

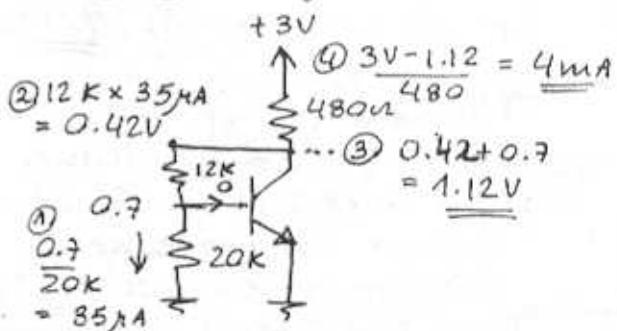
$$R_B = 12.1 \text{ k} \rightarrow 12 \text{ k}\Omega$$

$$R_B = 21.2 \text{ k} \rightarrow 20 \text{ k}\Omega$$

$$R_C = 489.2 \rightarrow 480 \Omega$$

Re-evaluate if  $\beta = \infty$ :

$$I_B = 0$$



5.97

$$I_B = I_C / \beta = 3 \text{ mA} / 90 = 0.033 \text{ mA}$$

$$V_C = R_B \cdot I_B + 0.7$$

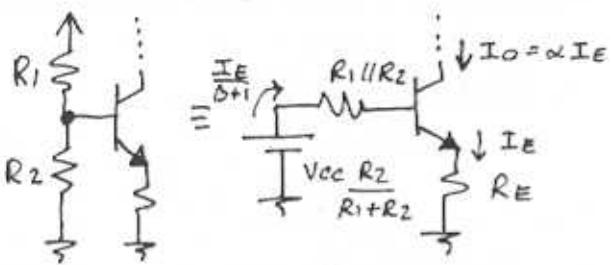
$$V_C = 1.5 \text{ V} \rightarrow R_B = 24.2 \text{ k}\Omega$$

$$I_E = \frac{I_C}{\alpha} = 3.03 \text{ mA}$$

$$I = I_C - I_B \equiv I_E$$

$$I = 3.03 \text{ mA}$$

5.98



$$\frac{V_{CC} \cdot R_2}{R_1 + R_2} = \frac{I_E}{\beta+1} (R_1 || R_2) + V_{BE} + I_E R_E$$

$$\Rightarrow I_E = \frac{V_{CC} \frac{R_2}{R_1 + R_2} - V_{BE}}{\frac{R_E + R_1 || R_2}{\beta+1}}$$

Thus,

$$I_o = \alpha I_E = \alpha \cdot \left( \frac{V_{CC} R_2}{R_1 + R_2} - V_{BE} \right)$$

$$\frac{R_E + R_1 || R_2}{\beta+1}$$

Q.E.D.

For  $\beta = 50$

$$\frac{5R_1}{R_1 + R_2}$$

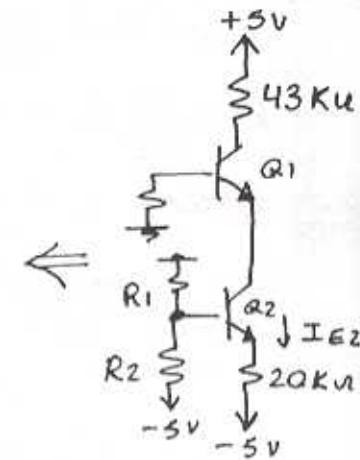
$$\frac{I_{E2}(\beta+1)}{R_1 || R_2}$$

$$Q_2$$

$$IE_2$$

$$20k$$

$$-5V$$



$$I_{E2} = -5 \cdot \frac{R_1}{R_1 + R_2} - 0.7 + 5$$

$$\frac{20 + R_1 || R_2}{\beta+1}$$

$$= 5 \cdot \frac{R_1}{R_1 + R_2} - 0.7$$

$$\frac{20 + \frac{1}{51} \cdot \frac{R_1 R_2}{R_1 + R_2}}{\beta+1}$$

Substitute  $R_1 = \frac{23}{27} R_2$

$$I_{E2} = \frac{5}{1 + 23/27} - 0.7$$

$$\frac{20 + \frac{1}{51} \cdot \frac{23/27 R_2}{1 + 23/27}}{\beta+1}$$

$$= \frac{2}{20 \left[ 1 + \frac{1}{20 \times 51} \times \frac{23 R_2}{50} \right]}$$

$$V_{RE2} = I_{E2} \times 20 = \frac{2}{1 + 0.46 \cdot \frac{R_2}{51 \times 20}}$$

For a reduction less than 5%

$$\frac{2}{1 + 0.46 R_2} > 0.95 \times 2$$

$$R_2 \leq \left( \frac{1}{0.95} - 1 \right) \frac{51 \times 20}{0.46} = 116.7 \text{ k}\Omega$$

CONT.

5.99

For  $\beta = \infty$ :

$$R_{E2} = \frac{2}{0.1} = 20 \text{ k}\Omega$$

$$R_C = \frac{5 - 0.8}{0.1} = 42 \text{ k}\Omega$$

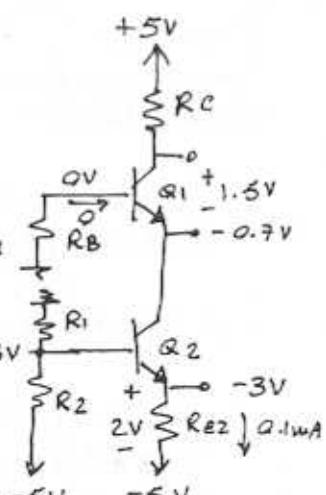
$$\text{Select } R_C = 43 \text{ k}\Omega$$

$$\text{Also } V_{R1} = 2.3 \text{ V}$$

$$V_{R2} = 2.7 \text{ V}$$

$$\rightarrow \frac{R_1}{R_2} = \frac{2.3}{2.7}$$

$$\Rightarrow R_1 = \frac{23}{27} \cdot R_2$$



Select  $R_2 = \underline{120\text{ k}\Omega}$

$$R_1 = \frac{23}{27} R_2 = 102.2\text{ k}\Omega$$

Select  $R_1 = \underline{100\text{ k}\Omega}$

For these values,

$$\beta = \infty : I_{E2} = -\frac{5 \times \frac{100}{220} - 0.7 + 5}{20} = \frac{2.027}{20} = 0.101\text{ mA}$$

$$V_{RE2} = 2.027\text{ V}$$

$$\beta = 50 : I_{E2} = \frac{2.027}{20 + \frac{120 \times 100}{220 \times 51}}$$

$$I_{E2} = 0.096\text{ mA}$$

$$I_{C2} = 0.98 \times 0.096 = 0.094\text{ mA}$$

$$I_{C1} = 0.98 \times 0.094 = 0.092\text{ mA}$$

To determine  $R_B$ :

$$V_{B1} = -I_{B1} \cdot R_B = -\frac{I_{C1}}{\beta} \cdot R_B = -\frac{0.094}{50} \cdot R_B$$

$$V_{E1} = V_{B1} - 0.7 = -\frac{0.094}{50} \cdot R_B - 0.7$$

$$V_{C1} = 5 - R_C I_{C1} = 5 - 48 \times 0.092 = 1.044\text{ V}$$

$$V_{CE1} = 1.044 + \frac{0.094}{50} \cdot R_B + 0.7 = 2.5\text{ V}$$

$$\Rightarrow R_B = 402\text{ k}\Omega$$

Select  $R_B = \underline{390\text{ k}\Omega}$

Now,

For  $\beta = 50$ :  $I_{C1} = \underline{0.092\text{ mA}}$

$$V_{B1} = -\frac{0.092}{50} \times 390 = -0.717\text{ V}$$

$$V_{E1} = -1.417\text{ V}$$

$$V_{CE1} = 1.044 + 1.417 = 2.46 \approx \underline{2.5\text{ V}}$$

For  $\beta = 100$ :

$$I_{E2} = \frac{2.027}{\frac{R_1 R_2}{R_1 + R_2} \cdot \frac{1}{100} + 20} = 0.099\text{ mA}$$

$$I_{C1} = 0.099 \times 0.99 \times 0.99 = 0.097\text{ mA}$$

$$V_{C1} = 0.829\text{ V}$$

$$V_{B1} = -0.378\text{ V}$$

$$V_{E1} = -1.078\text{ V}$$

$$V_{CE1} = \underline{1.91\text{ V}}$$

For  $\beta = 200$ :

$$I_{E2} = \frac{2.027}{\frac{R_1 R_2}{R_1 + R_2} \cdot \frac{1}{200} + 20} = 0.1\text{ mA}$$

$$I_{C1} = 0.1 \times 0.995 \times 0.995 = 0.099\text{ mA}$$

$$V_{C1} = 0.743\text{ V}$$

$$V_{B1} = -0.193\text{ V}$$

$$V_{E1} = -0.893\text{ V}$$

$$V_{CE1} = 1.636 \approx \underline{1.64\text{ V}}$$

5.100

Refer to Fig P5.100

Assuming  $\beta = \infty \rightarrow I_{B3} = 0$

$$V_{B3} = \frac{V_{CC} - V_{BE1} - V_{BE2} \times R_2}{R_1 + R_2} \times R_2 \dots + V_{BE1} + V_{BE2}$$

$$V_{E3} = V_{B3} - V_{BE3}$$

$$I_O = \frac{\alpha V_{E3}}{R_E}$$

$$I_O = \frac{\alpha}{R_E} \left[ \frac{(V_{CC} - V_{BE1} - V_{BE2}) R_2 + V_{BE1} + V_{BE2}}{R_1 + R_2} - V_{BE3} \right]$$

CONT.

Now, if the circuit is designed so that all three transistors conduct equal currents, then  $V_{BE1} = V_{BE2} = V_{BE3} = V_{BE}$

and  $I_o$  becomes:

$$I_o = \frac{\alpha}{R_E} \left[ (V_{CC} - 2V_{BE}) \frac{R_2}{R_1 + R_2} + V_{BE} \right]$$

To eliminate the  $V_{BE}$  terms in this equation we select  $R_1 = R_2$ , resulting in

$$I_o = \frac{\alpha V_{CC}}{2R_E} \quad Q.E.D.$$

Now, to make the current through  $Q_1$  and  $Q_2$  equal to that through  $Q_3$  (which is  $I_o/\alpha$ )

$$\frac{V_{CC} - 2V_{BE}}{2R_1} = \frac{V_{CC}}{2R_E}$$

$$\Rightarrow R_1 = R_2 = R_E \left( 1 - \frac{2V_{BE}}{V_{CC}} \right)$$

For  $V_{CC} = 10V$ ,  $\alpha = 1$

$$I_o = 0.5mA$$

$$0.5 = \frac{10}{2R_E} \Rightarrow R_E = \underline{\underline{10k\Omega}}$$

$$R_1 = R_2 = 10 \left( 1 - \frac{2 \times 0.7}{10} \right) \\ = \underline{\underline{8.6k\Omega}}$$

$$V_{C3/min} = I_o \cdot R_E + V_{BE} \\ = 0.5 \times 10 + 0.7 \\ = \underline{\underline{5.7V}}$$

5.101

Refer to Fig. P5.101

$$I_o = 2mA = \alpha \times \frac{5 - 0.7}{R} \approx \frac{4.3}{R}$$

$$\Rightarrow R = \underline{\underline{2.15k\Omega}}$$

$V_{C min} = \underline{\underline{0V}}$  (In actual practice,  $\underline{\underline{V_{C min}}} \approx 0.4V$ )

5.102

(a) Using the exponential characteristic:

$$I_c = I_C e^{V_{BE}/V_T}$$

thus,

$$I_c = I_C e^{\frac{V_{BE}}{V_T}} - I_C$$

$$\text{giving } \frac{I_c}{I_C} = e^{\frac{V_{BE}}{V_T}} - 1$$

(b) Using small-signal approximation:

$$I_c = g_m V_{BE} = \frac{I_C}{V_T} \cdot V_{BE}$$

$$\text{Thus, } \frac{I_c}{I_C} = \frac{V_{BE}}{V_T}$$

See Table below

For signals of  $\pm 5mV$ , the error introduced by the small-signal approximation is 10%.

The error increases to above 20% for signals of  $\pm 10mV$ .

CONT.

$U_{BE}$ (mV)	$i_C/i_C$ Expon.	$i_e/i_C$ small: equal:	% Error
+1	+0.041	+0.040	-2
-1	-0.039	-0.040	+2
+2	+0.083	+0.080	-4
-2	-0.077	-0.080	+4
+5	+0.221	+0.200	-9.5
-5	-0.181	-0.200	+10.3
+8	+0.377	+0.320	-15.2
-8	-0.274	-0.320	+16.8
+10	+0.492	+0.400	-18.7
-10	-0.330	-0.400	+21.3
+12	+0.616	+0.480	-22.1
-12	-0.381	-0.480	+25.9

### 5.104

$$g_m = \frac{I_C}{V_T} = \frac{1.2 \text{ mA}}{25 \text{ mV}} = 48 \text{ mA/V}$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{120}{48 \times 10^3} = 2.5 \text{ k}\Omega$$

$$r_e = \frac{r_{\pi}}{\beta + 1} = \frac{2500}{121} = 20.6 \text{ }\Omega$$

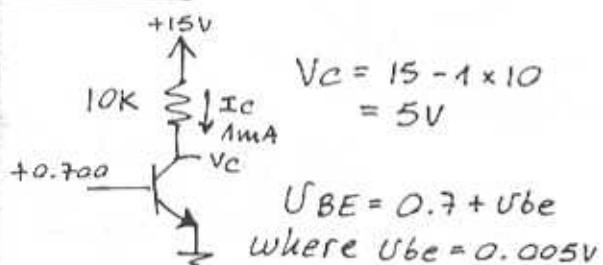
For a bias current of 120 mA  
i.e. 10 times lower:

$$g_m = \frac{48}{10} = 4.8 \text{ mA/V}$$

$$r_{\pi} = 10 \times 2.5 = 25 \text{ k}\Omega$$

$$r_e = 10 \times 20.6 = 206 \text{ }\Omega$$

### 5.103



$$I_C \approx I_C \left(1 + \frac{U_{BE}}{V_T}\right) \text{ Eq. (5.83)}$$

$$I_C \approx I_C + i_C \quad \text{where:}$$

$$i_C = \frac{1 \text{ mA} \times 0.005}{25 \text{ m}} = 0.2 \text{ mA}$$

$$I_C = 1 \text{ mA} + 0.2 \text{ mA}$$

$$V_C = V_{CC} - i_C R_C \quad \text{Eq. (5.101)}$$

$$\Rightarrow V_C = \overbrace{i_C R_C}^{0.2 \text{ mA} \times 10 \text{ k}}$$

$$V_C = 5 \text{ V} - 2 \text{ V}$$

$$\text{Gain} = \frac{-2 \text{ V}}{0.005 \text{ V}} = -400 \text{ V/V}$$

$$\text{while } -g_m \cdot R_C = -1 \text{ mA} \cdot 10 \text{ k} = -400 \text{ V/V}$$

### 5.105

$$I_C = 2 \text{ mA} \Rightarrow g_m = \frac{2 \text{ mA}}{25 \text{ mV}}$$

$$g_m = 80 \text{ mA/V}$$

$$r_e = \frac{V_T}{I_E}, \quad I_E = I_C \frac{(\beta + 1)}{\beta}$$

$$I_E = 2 \text{ mA} \times \frac{51}{50} = 2.04 \text{ mA}$$

$$r_e = \frac{25 \text{ m}}{2.04 \text{ mA}} = 12.25 \text{ }\Omega$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{50}{80 \times 10^{-3}} = 625 \text{ }\Omega$$

$$\text{gain: } -g_m \cdot R_C$$

For  $R_C = 5 \text{ k}\Omega$  and  $\hat{U}_{BE} = 5 \text{ mV}$

$$\hat{U}_o = -80 \text{ mA} \times 5 \text{ k} \times 5 \text{ mV} \\ = -2 \text{ V}$$

5.106

$$g_m = \frac{50 \text{ mA}}{V} = \frac{I_c}{V_T}$$

$$\Rightarrow I_c = g_m \times V_T = 50 \text{ m} \times 25 \text{ m} \\ = 1.25 \text{ mA}$$

$$r_{\pi} = 2k = \beta \Rightarrow \beta = 2k \times 50 \text{ m}$$

$$\beta = \frac{100}{g_m} \rightarrow \alpha = \frac{100}{101} = 0.99$$

$$I_E = \frac{I_c}{\alpha} = \frac{1.25 \text{ mA}}{0.99} = 1.26 \text{ mA}$$

$$i_{c(t)} = I_c + g_m v_{be}(t)$$

$$= \frac{1 + 40 \cdot 10^3 \times 0.005 \sin \omega t}{1 + 0.2 \sin \omega t, \text{ mA}}$$

$$v_{c(t)} = 5 - R_c i_{c(t)} \\ = \frac{2 - 0.6 \sin \omega t, \text{ V}}{100}$$

$$i_{b(t)} = i_{c(t)}/\beta \\ = \frac{1 + 0.2 \cdot 0.99 \sin \omega t, \text{ mA}}{100} \\ = \frac{10 + 2 \sin \omega t, \text{ mA}}{100}$$

$$\text{Voltage gain} = \frac{-0.6}{0.005} = -120 \text{ V}$$

5.107

$$g_m \text{ varies from: } 1.2 \times 60 = 72 \frac{\text{mA}}{V} \\ \text{to } 0.8 \times 60 = 48 \frac{\text{mA}}{V}$$

$$\beta \text{ varies from } 50 \text{ to } 200 \frac{V}{V}$$

$$r_{in/base} = r_{\pi} = \beta / g_m$$

$$\text{Largest value: } r_{\pi} = \frac{\beta_{\max}}{g_{m\min}} = \frac{200}{48 \text{ m}} \\ = 4.2 \text{ k}\Omega$$

$$\text{Smallest value: } r_{\pi} = \frac{\beta_{\min}}{g_{m\max}} = \frac{50}{72 \text{ m}} \\ = 694 \text{ }\Omega$$

5.108

Refer to Fig. 5.48.

$$V_C = 2 \text{ V} \Rightarrow I_c = \frac{V_{cc} - V_C}{R_c}$$

$$I_c = \frac{5 - 2}{3 \text{ k}} = 1 \text{ mA}$$

$$g_m = \frac{I_c}{V_T} = \frac{1 \text{ m}}{25 \text{ m}} = 40 \text{ mA/V}$$

5.109

$$i_c = I_c + g_m \hat{v}_{be} \sin \omega t$$

$$V_C = V_{cc} - I_c R_c \quad V_{BE} + \hat{v}_{be} \sin \omega t \\ - g_m \hat{v}_{be} R_c \sin \omega t$$

To maintain BJT in active region,  $V_C > V_{BE}$ , thus  $V_{cc} - I_c R_c - g_m R_c \hat{v}_{be} > V_{BE} + \hat{v}_{be}$

To obtain the largest possible output signal we design such that this constraint is satisfied with the equality sign, that is:

$$V_{cc} - R_c I_c - g_m R_c \hat{v}_{be} = V_{BE} + \hat{v}_{be}$$

substituting  $g_m = \frac{I_c}{V_T}$ , gives.

$$V_{cc} - R_c I_c - R_c I_c \frac{\hat{v}_{be}}{V_T} = V_{BE} + \hat{v}_{be}$$

$$\Rightarrow R_c I_c \left( 1 + \frac{\hat{v}_{be}}{V_T} \right) = V_{cc} - V_{BE} - \hat{v}_{be}$$

CONT.

$$R_c I_c = \frac{(V_{cc} - V_{BE} - \hat{V}_{be})}{(1 + \frac{\hat{V}_{be}}{V_T})} \quad Q.E.D.$$

$$\begin{aligned} \text{Voltage gain} &= -g_m \cdot R_c \\ &= -\frac{I_c}{V_T} \cdot R_c \\ &= -\frac{V_{cc} - V_{BE} - \hat{V}_{be}}{V_T + \hat{V}_{be}} \end{aligned}$$

For  $V_{cc} = 5V$ ,  $V_{BE} = 0.7V$  and  $\hat{V}_{be} = 5mV$

$$R_c I_c = \frac{5 - 0.7 - 0.005}{1 + \frac{0.005}{0.025}} = 3.6V$$

Thus,

$$V_C = 5 - 3.6 = +1.4V$$

$$\begin{aligned} \text{Amplitude of output signal is} \\ &= 1.4 - (V_{BE} + \hat{V}_{be}) \\ &= 1.4 - 0.7 - 0.005 \\ &= 0.695V \end{aligned}$$

$$\text{Voltage gain} = -\frac{0.695}{0.005} = -139V/V$$

Check

$$\begin{aligned} \text{Voltage gain} &= -\frac{(5 - 0.7 - 0.005)}{0.025 + 0.005} \\ &= -143V/V \end{aligned}$$

The difference is caused by decimal rounding-up of  $R_c I_c$ .

Otherwise:

$$\begin{aligned} \text{Voltage gain} &= -\frac{0.716}{0.005} \\ &= -143V/V \end{aligned}$$

5.110

	a	b	c	d	e	f	g
$\alpha$	1.000	0.990	0.98	1	0.990	0.90	0.941
$\beta$	$\infty$	100	50	$\infty$	100	9	16
$I_c(\text{mA})$	1.00	0.99	1.00	1.00	0.248	4.5	17.5
$I_E(\text{mA})$	1.00	1.00	1.02	1.00	0.25	5	18.6
$I_B(\text{mA})$	0	0.010	0.020	0	0.002	0.5	1.10
$g_m(\text{mA})$	40	39.6	40	40	0.01	180	700
$r_e(\text{V})$	25	25	24.5	25	100	5	1.34
$r_\pi(\text{V})$	$\infty$	2.5K	1.25K	$\infty$	10.1K	50	22A

5.111

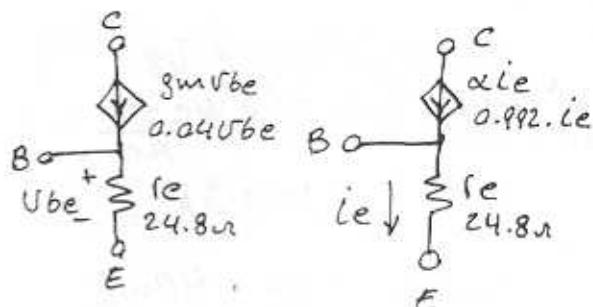
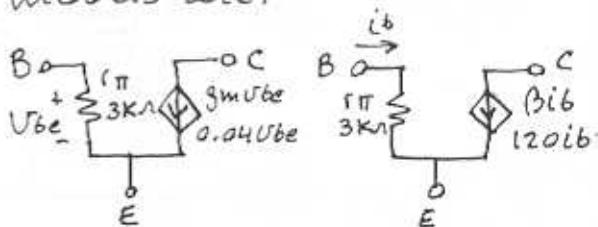
$$I_c = 1\text{mA}, \beta = 120, \alpha = 0.992$$

$$g_m = \frac{I_c}{V_T} = \frac{1}{25} = 40\text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40 \times 10^{-3}} = 3\text{K}\Omega$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{0.992}{40 \times 10^{-3}} = 24.8\Omega$$

The four equivalent circuit models are:



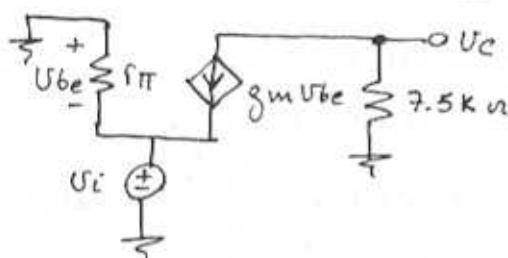
5.112

Refer to Fig P5.112  
 $\beta$  very high  $\rightarrow \alpha = 1$

$$I_C = I_E = 0.5 \text{ mA}$$

$$V_C = 5 - 7.5 \times 0.5 = + \underline{1.25} \text{ V}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{25 \mu\text{A}} = 20 \text{ mA/V}$$



Observe that  $U_{BE} = -U_i$   
 the output voltage  $V_C$  is found from:

$$V_C = -g_m U_{BE} \times 7.5 \text{ k}\Omega$$

Thus the voltage gain is

$$\begin{aligned} \frac{V_C}{U_i} &= g_m \times 7.5 \text{ k}\Omega \\ &= 20 \times 7.5 = \underline{150 \text{ V/V}} \end{aligned}$$

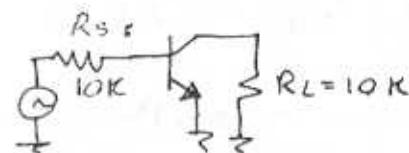
5.113

$$\begin{aligned} \frac{V_C}{U_{BE}} &= -g_m R_C \Rightarrow U_{BE} = \frac{1}{50 \times 2} \\ &= \underline{10 \mu\text{V p-p to p-p}} \end{aligned}$$

$$i_b = \frac{U_{BE}}{r_\pi} = \frac{10 \times 10^{-3}}{\beta/g_m} = \frac{0.01}{100/0.05}$$

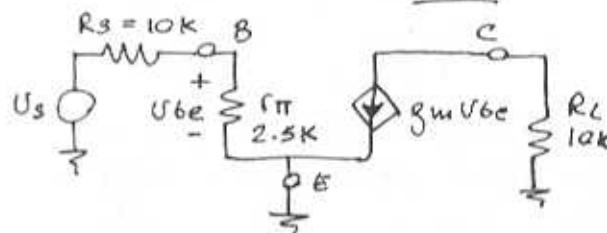
$$i_b = \underline{0.005 \text{ mA p-p to p-p}}$$

5.114



$$g_m = 40 \text{ mA/V}, r_\pi = 2.5 \text{ k}\Omega$$

$$r_\pi = \beta/g_m \Rightarrow \beta = 2.5 \text{ k}\Omega \times 40 \text{ mA} = \underline{100}$$



$$U_{BE} = \frac{U_s \cdot r_\pi}{R_B + r_\pi} = 0.2 \times U_s$$

$$U_C = -g_m U_{BE} \cdot R_L = -g_m (0.2 U_s) \cdot R_L$$

$$U_C = -80 U_s$$

$$\Rightarrow \text{gain } \frac{V_C}{U_s} = -\underline{80}$$

To double the gain:

If  $I_C$  is fixed  $\rightarrow g_m$  does not change.

$$\rightarrow \frac{U_{BE}}{U_s} = 0.4 \quad (2 \times 0.2)$$

$$\rightarrow \frac{r_\pi}{R_B + r_\pi} = 0.4 \quad \text{and } R_B = 10 \text{ k}\Omega$$

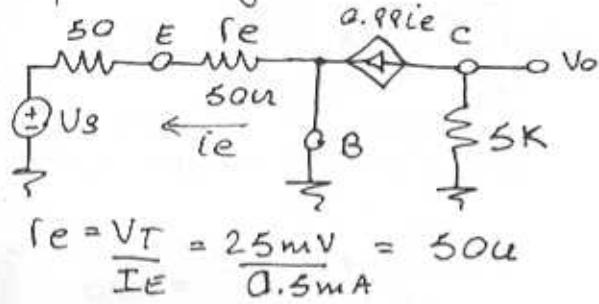
$$\rightarrow r_\pi = 6.6 \text{ k}\Omega$$

$$\text{Since } r_\pi = \frac{\beta}{g_m} \Rightarrow \beta = 6.6 \text{ k}\Omega \times 40 \text{ mA}$$

$$\beta = \underline{264}$$

5.115

Refer to Fig. P5.115



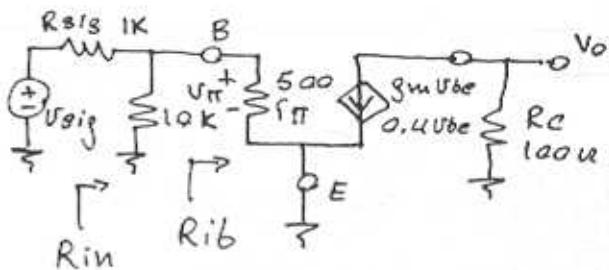
$$R_{in} = R_s + r_e = \underline{100\Omega}$$

$$V_o = -0.99ie \times 5K$$

but:  $ie = -\frac{Us}{R_{in}} = -\frac{Us}{100}$

$$\Rightarrow V_o = +0.99 \times 5K \cdot \frac{Us}{100}$$

$$\frac{V_o}{Us} = \underline{49.5 \text{ V/V}}$$



$$gm = \frac{I_c}{V_T} = \frac{9.95mA}{25m} = 0.4 A/V$$

$$r_\pi = \frac{\beta}{gm} = \frac{200}{0.4} = 500 \Omega$$

$$R_{ib} = r_\pi = \underline{500\Omega}$$

$$R_{in} = 10K \parallel r_\pi = \underline{476\Omega}$$

$$U_{be} = U_{sig} \times \frac{R_{in}}{R_{sig} + R_{in}} = U_{sig} \times 0.32$$

also:

$$\begin{aligned} U_o &= -gm U_{be} \cdot R_c \\ &= -gm R_c \times 0.32 U_{sig} \\ &= -0.4 \times 100 \times 0.32 U_{sig} \\ &= -12.8 U_{sig} \end{aligned}$$

$$\Rightarrow \text{gain } \frac{V_o}{Us} = -12.8 \approx -13 \text{ V/V}$$

If  $U_o = \pm 0.4V$ :

$$\hat{U}_s = \frac{\hat{U}_o}{13} = 30mV$$

$$U_{be} = 0.32 \times 30mV = 9.6mV$$

5.116

Refer to Fig. P5.116

$$\begin{aligned} \beta &= 200 \rightarrow \alpha = 0.995 \\ I_c &= \alpha I_E = 0.995 \times 10mA \\ &= 9.95mA \end{aligned}$$

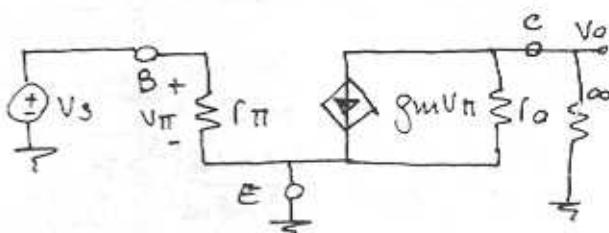
$$V_C = 9.95mA \times 100 = \underline{0.995V}$$

$$I_B \approx \frac{10mA}{200} = 0.05mA$$

$$V_B = 1.5 - 10K \times 0.05mA = 1V$$

$$\Rightarrow V_{BC} = +0.005 \rightarrow \text{Active region.}$$

5.117



$$V_S = V_\pi \Rightarrow \frac{V_o}{V_S} = -gm \times R_o$$

CONT.

$$\text{but: } r_o = \frac{V_A}{I_C} = \frac{V_A}{V_T \cdot g_m}$$

$$\Rightarrow \frac{V_O}{V_S} = -\frac{V_A}{V_T}$$

$$\text{if } V_A = 25V \Rightarrow \frac{V_O}{V_S} = -1000 \frac{V}{V}$$

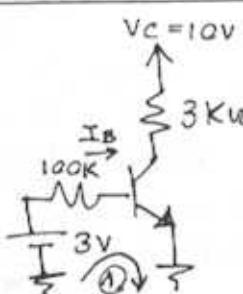
$$\text{if } V_A = 250V \Rightarrow \frac{V_O}{V_S} = -10,000 \frac{V}{V}$$

5.118

DC Analysis:

$$\textcircled{1} I_B = \frac{3 - 0.7}{100}$$

$$I_B = 0.023 \text{ mA}$$



Saturation begins to occur when  $V_c \leq 0.7V$

$$\therefore I_C \geq \frac{10 - 0.7}{3} = 3.1 \text{ mA}$$

$$I_C = \beta I_B \rightarrow \beta > \frac{3.1}{0.023} = 135$$

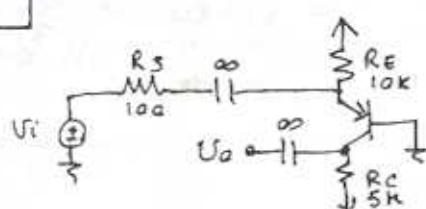
$\beta = 25$ :

$$r_e = \frac{V_T}{I_E} = \frac{V_T}{(\beta+1)I_B} = \frac{25 \times 10^{-3}}{26 \times 0.023 \times 10^3}$$

$$r_e = \underline{\underline{41.8 \mu V}}$$

$$g_m = \frac{\alpha}{r_e} = \frac{25/26}{41.8} = 23 \text{ mA/V}$$

5.119



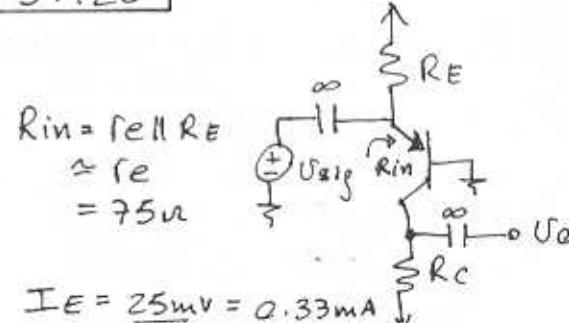
$$\begin{aligned} \frac{V_O}{V_i} &= \alpha \frac{r_e \parallel R_E}{R_E \parallel r_e + R_S} \cdot \frac{1}{r_e} \cdot R_E \\ &= 0.99 \frac{(27 \parallel 10k)}{(27 \parallel 10k) + 100} \times \frac{1}{27} \times 5 \parallel 10^3 \\ &= 38.9 \approx 39 \text{ V/V} \end{aligned}$$

The amplifier clips when the peak output signal exceeds the difference between  $V_C$  (-5.4V) and the negative supply (-10V); see Fig. 5.56. That is when the peak output signal is 4.6V

This corresponds to a peak input signal of

$$\hat{V}_i = \frac{4.6}{39} = \underline{\underline{118 mV}}$$

5.120



$$I_E = \frac{25 \text{ mV}}{75 \mu \text{A}} = 0.33 \text{ mA}$$

$$R_E = \frac{10 - 0.7}{0.33} = 28 \text{ k}\Omega$$

$$n = \underline{\underline{2.8}}$$

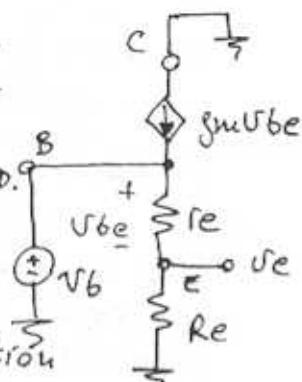
$$R_C = 14 \text{ k}\Omega$$

$$\frac{V_O}{V_i} = \alpha \frac{R_C}{r_e} = \frac{14}{0.075} = \underline{\underline{187 \text{ V/V}}}$$

5.121

(a) Using the voltage divider rule:

$$U_B = \frac{R_E}{R_E + r_e} Q.E.D.$$



(b) Node equation at B:

$$\begin{aligned} i_B &= \frac{U_B e}{r_e} - g_m U_B e \\ &= \frac{U_B e}{r_e} (1 - g_m r_e) \\ &= \frac{U_B e}{r_e} \left(1 - g_m \frac{\alpha}{g_m}\right) \\ &= \frac{U_B e}{r_e} (1 - \alpha) \\ &= \frac{U_B e}{r_e} \left(1 - \frac{\beta}{\beta+1}\right) \\ &= \frac{U_B e}{r_e} \cdot \frac{1}{\beta+1} \end{aligned}$$

But, from voltage-divider rule

$$U_B e = U_B \cdot r_e$$

$$\Rightarrow i_B = \frac{1}{(\beta+1)r_e} \cdot \frac{U_B \cdot r_e}{r_e + R_E}$$

from which we find

$$R_{in} = \frac{U_B}{i_B} = (\beta+1)(r_e + R_E)$$

Q.E.D.

For  $R_E = 1\text{ k}\Omega$ ,  $\beta = 100$  and

$$I_E = 1\text{ mA}$$

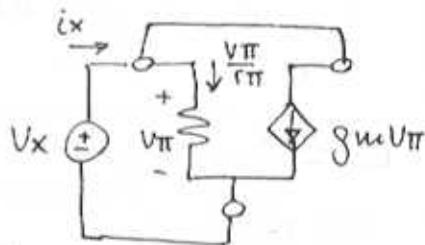
$$r_e = \frac{V_T}{I_E} = \frac{25\text{ mV}}{1\text{ mA}} = 25\text{ }\Omega$$

Thus,

$$\frac{U_B}{U_B} = \frac{1000}{1000 + 25} = 0.976 \text{ V/V}$$

$$\begin{aligned} R_{in} &= (100+1)(1000+25) \text{ }\Omega \\ &= 101 \times 1.025 \text{ k}\Omega \\ &= 103.5 \text{ k}\Omega \end{aligned}$$

5.122



$$\begin{aligned} i_x &= \frac{V_\pi}{r_\pi} + g_m V_\pi \\ &= \frac{V_\pi}{r_\pi} (1 + g_m r_\pi) \\ &= \frac{V_\pi}{r_\pi} (1 + \beta) \end{aligned}$$

$$\text{But } V_\pi = V_x$$

$$\rightarrow R_{in} = \frac{V_x}{i_x} = \frac{V_\pi}{i_x} = \frac{r_\pi}{\beta+1}$$

$$R_{in} = \underline{\underline{r_e}}$$

5.123

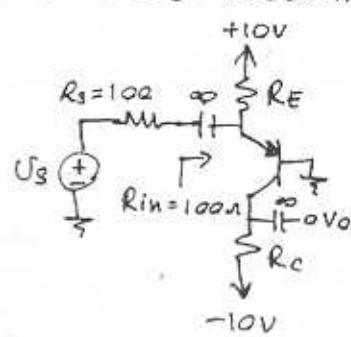
$$R_{in} = R_E // r_e \quad r_e \approx 100\text{ }\Omega$$

thus,  $\frac{V_T}{I_E} = 100 \rightarrow I_E = 0.25\text{ mA}$

$$V_E = 0.7\text{ V}$$

$$R_E = \frac{10 - 0.7}{1\text{ mA}}$$

$$= \underline{\underline{9.3\text{ k}\Omega}}$$



CONT.

## Selection of a value for $R_C$ :

The voltage gain is directly proportional to  $R_C$ ,

$$\begin{aligned}\frac{U_o}{U_s} &= \frac{U_e}{U_s} \cdot \frac{U_o}{U_e} \\ &= \frac{R_{in}}{R_S + R_{in}} \cdot \alpha \frac{R_C}{r_e} \\ &\approx \frac{100}{100+100} \cdot \frac{R_C}{0.1} \\ &= 5R_C, \quad R_C \text{ in } k\Omega.\end{aligned}$$

For an emitter-base signal as large as 10mV, the signal at the collector will be  $g_m R_C \times 0.01 \Omega$  volts. Thus the maximum collector voltage in the positive direction will be:

$$\begin{aligned}|U_{Cmax}| &= U_c + 0.01 g_m R_C \\ &= -10 + I_c R_C + 0.01 \times \frac{1}{0.1} \times R_C \\ &= -10 + 0.25 R_C + 0.1 R_C \\ &= -10 + 0.35 R_C\end{aligned}$$

To prevent saturation,  $|U_{Cmax}| \leq V_B$  which is 0V. Thus to obtain maximum gain while allowing an emitter-base signal as large as 10mV and at the same time keeping the transistor in the active mode we select  $R_C$  from:

$$\begin{aligned}-10 + 0.35 R_C &= 0 \\ \Rightarrow R_C &= \underline{28.6 \text{ k}\Omega}\end{aligned}$$

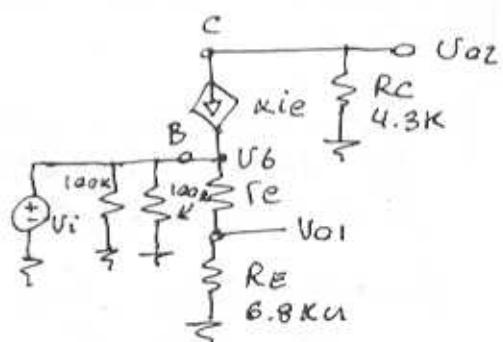
$$\text{Voltage gain} = \frac{U_o}{U_s} = 5R_C = 143 \text{ V/V}$$

15.124

Refer to Fig P5.124  
For large  $\beta$ , the DC base current will be  $\sim 0$   
Thus the DC voltage at the base can be found directly using the voltage divider rule

$$V_B = 15 \cdot \frac{100}{100+100} = 7.5 \text{ V}$$

$$\begin{aligned}V_{BE} &= 0.7 \\ V_E &= 7.5 - 0.7 = 6.8 \text{ V} \\ \rightarrow I_E &= \frac{6.8 \text{ V}}{6.8 \text{ k}\Omega} = \underline{1 \text{ mA}}\end{aligned}$$



$$\begin{aligned}U_6 &= U_i \\ \rightarrow \frac{U_{o1}}{U_i} &= \frac{R_E}{R_E + r_e} \quad \text{Q.E.D.}\end{aligned}$$

Also,

$$i_e = \frac{U_6}{r_e + R_E} = \frac{U_i}{r_e + R_E}$$

and,

$$\begin{aligned}U_{o2} &= -\alpha i_e R_C \\ &= -\frac{\alpha R_C U_i}{r_e + R_E}\end{aligned}$$

Thus,

$$\frac{U_{o2}}{U_i} = -\frac{\alpha R_C}{R_E + r_e} \quad \text{Q.E.D.}$$

CONT.

Substituting  $r_e = \frac{V_T}{I_E} = 25\text{k}\Omega$

and  $R_E = 6.8\text{k}\Omega$ ,  $R_C = 4.3\text{k}\Omega$   
and  $\alpha \approx 1$  gives

$$\frac{U_{o1}}{U_i} = \frac{6.8}{0.025 + 6.8} = 0.996 \text{ V/V}$$

$$\frac{U_{o2}}{U_i} = -\frac{4.3}{6.8 + 0.025} = 0.63 \text{ V/V}$$

If the node labeled  $U_{o2}$  is connected to ground:

$$R_E = 0$$

$$\frac{U_{o2}}{U_i} = -\alpha \frac{R_C}{r_e}$$

5.125

If:  $R_i = 10\text{k}\Omega$ ,  $A_{vo} = 100 \text{ V/V}$   
 $R_o = 100\Omega$ ,  $R_{sig} = 2\text{k}\Omega$  and  
 $R_{in} = 8\text{k}\Omega$  when  $R_L = 1\text{k}\Omega$   
then:

$$G_m = \frac{A_{vo}}{R_o} = \frac{100}{100} = 1 \text{ A/V}$$

$$A_v = A_{vo} \cdot \frac{R_L}{R_L + R_o} = \frac{100 \times 1\text{k}}{1\text{k} + 100} = 91 \text{ V/V}$$

$$G_{vo} = \frac{R_i}{R_i + R_{sig}} \cdot A_{vo} = \frac{10\text{k}}{10\text{k} + 2\text{k}} \cdot 100 = 83.3 \text{ V/V}$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \cdot A_{vo} \cdot \frac{R_L}{R_L + R_o}$$

$$= \frac{8\text{k}}{8\text{k} + 2\text{k}} \cdot \frac{100 \times 1\text{k}}{1\text{k} + 100} = \frac{72.7}{V}$$

$$R_{out} = \frac{G_{vo}}{G_v} \cdot R_L - R_L = \frac{8.3 \times 1\text{k} - 1\text{k}}{72.7} = 146 \Omega$$

$$A_i = \frac{U_o / R_L}{U_i / R_{in}} = A_v \cdot \frac{R_{in}}{R_L} = 91 \times \frac{8\text{k}}{1\text{k}}$$

$$= 728 \text{ AIA} //$$

5.126

Refer to Fig. PS.126

$$R_i = 900\text{k}\Omega$$

$$A_{vo} = 10 \text{ V/V}$$

$$R_o = 1.43\text{k}\Omega$$

$R_{in} = 400\text{k}\Omega$  when  $R_L = 10\text{k}\Omega$

$$(a) i_i = U_i / R_{in}$$

$$\Rightarrow \frac{U_i}{R_{in}} = \frac{U_i}{R_i} - f \cdot i_o$$

$$\text{where } i_o = \frac{A_{vo} U_i}{R_o + R_L}$$

$$\rightarrow \frac{U_i}{R_{in}} = \frac{U_i}{R_i} - f \cdot \frac{A_{vo} U_i}{R_o + R_L}$$

Solving for  $f$ :

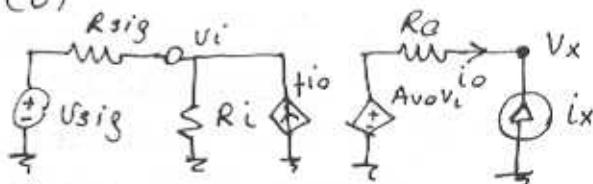
$$f = \left( \frac{1}{R_i} - \frac{1}{R_{in}} \right) \cdot \frac{R_o + R_L}{A_{vo}}$$

Thus,

$$f = \left( \frac{1}{900\text{k}} - \frac{1}{400\text{k}} \right) \cdot \frac{(1.43\text{k} + 10\text{k})}{10}$$

$$f = -1.6 \times 10^{-3}$$

(b)



$$R_{out} = \frac{V_x}{i_x} \quad | \quad U_{sig} = 0$$

If  $U_{sig} = 0$ : on the left hand side

$$U_i = f \cdot i_o \quad (R_{sig} // R_i)$$

$$U_i = (-1.6 \times 10^{-3} \times (100\text{k} // 900\text{k})) i_o$$

$$= -144 \cdot i_o$$

On the right hand side:

$$V_x = R_o i_x + A_{vo} \cdot U_i$$

$$= R_o i_x + A_{vo} (-144 i_o)$$

$$= R_o i_x + A_{vo} (+144) i_x$$

CONT.

$$\rightarrow V_x = i_x (R_o + A_{vo} \times 144)$$

$$R_{out} = \frac{V_x}{i_x} = 1.43K + 10 \times 144$$

$$R_{out} = \frac{i_x}{2.87Ku}$$

Which is the same value as in Example 5.17

5.127

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \cdot A_{vo} \cdot \frac{R_L}{R_L + R_o} = G_{vo} \cdot \frac{R_L}{R_L + R_{out}}$$

$$\text{but } G_{vo} = \frac{R_i}{R_i + R_{sig}} \cdot A_{vo}$$

Substituting  $G_{vo}$ :

$$\frac{R_{in}}{R_{in} + R_{sig}} \cdot A_{vo} \cdot \frac{R_L}{R_L + R_o} = \frac{R_i}{R_i + R_{sig}} \cdot A_{vo} \cdot \frac{R_L}{R_L + R_{out}}$$

$$\rightarrow \frac{R_{in}}{R_i} \cdot \frac{(R_{sig} + R_i)}{(R_{sig} + R_{in})} = \frac{R_L + R_o}{R_L + R_{out}}$$

Q.E.D

(a) for  $R_L = \infty$

$$\frac{R_{in}}{R_i} \cdot \frac{(R_{sig} + R_i)}{(R_{sig} + R_{in})} = 1$$

$$R_{in} R_{sig} + R_{in} R_i = R_i R_{sig} + R_i R_{in}$$

$$R_{in} \cdot R_{sig} = R_i \cdot R_{sig}$$

$$\rightarrow R_{in} = R_i$$

Q.E.D

(b) for  $R_{sig} = 0$

$$\frac{R_{in}}{R_i} \cdot \frac{R_i}{R_{in}} = \frac{R_L + R_o}{R_L + R_{out}}$$

$$\rightarrow R_{out} = R_o$$

Q.E.D.

(c) For  $R_{sig} = \infty$

$$\frac{R_{in}}{R_i} = \frac{R_L + R_o}{R_L + R_{out}}$$

$$R_{out} = \frac{(R_L + R_o) R_i}{R_i} - R_L // \frac{R_{in}}{R_i}$$

In Example 5.17:

$$R_{out} = (10K + 1.43K) \frac{900}{400} - 10 \\ = \underline{\underline{15.7Ku}}$$

5.128

Refer to Fig. 5.60(a)

$$I_c = 0.2mA \Rightarrow I_E = \frac{I_c}{\alpha}$$

$$I_E = \frac{0.2}{\beta} \cdot (\beta + 1) = 0.202mA$$

$$r_e = \frac{V_T}{I_E} = \frac{0.025}{0.202m} = 123.76uV$$

$$R_i = (\beta + 1) r_e = \underline{\underline{12.5Ku}}$$

$$\frac{U_o}{U_s} = -g_m (R_{e1} R_o) \cdot \frac{R_i}{R_i + R_S}$$

$$\approx -\frac{\alpha}{r_e} \cdot R_c \cdot \frac{R_i}{R_i + R_S}$$

$$= -\frac{100}{10} \times \frac{24K}{123.76} \times \frac{12.5K}{12.5K + 10K}$$

$$\frac{U_o}{U_s} = -\underline{\underline{106.7V/V}}$$

$$R_o = R_{e1} R_o \approx R_c = \underline{\underline{24Ku}}$$

$$\frac{U_o}{U_s} = \left. \frac{U_o}{U_s} \right|_{no load} \times \frac{10}{10 + 24}$$

$$= -106.7 \times \frac{10}{34} = -\underline{\underline{31.4V/V}}$$

5.129

$$I_C = 0.2 \text{ mA} \Rightarrow r_e = 123.76 \Omega$$

$$R_i = (\beta + 1)(r_e + R_E)$$

$$= (101)(123.76 + 125) = \underline{\underline{25.1 \text{ k}\Omega}}$$

$$\frac{U_o}{U_s} = -\frac{\alpha R_C}{r_e + R_E} \frac{R_i}{R_i + R_S}$$

$$= -\frac{100 \times 24 \text{ k}}{101(123.76 + 125)} \times \frac{25.1 \text{ k}}{25.1 \text{ k} + 10 \text{ k}}$$

$$= -\underline{\underline{68.30 \text{ V/V}}}$$

$$R_o = R_C = \underline{\underline{24 \text{ k}\Omega}}$$

With 10k $\Omega$  load

$$\frac{U_o}{U_s} = -68.30 \times \frac{10}{10+24} = -\underline{\underline{20 \text{ V}}}$$

Without  $r_e$   $V_{\pi} \leq 5 \text{ mV}$

$$V_{\pi} = \frac{(\beta + 1)r_e \cdot U_s}{(\beta + 1)r_e + R_S}$$

$$= \frac{(101) \cdot (123.76)}{(101) \cdot (123.76) + (10 \text{ k})} \cdot U_s \leq 5 \text{ mV}$$

$$\Rightarrow U_s \leq \underline{\underline{9 \text{ mV}}}$$

With  $r_e$

$$V_{\pi} = \frac{(\beta + 1)r_e + U_s}{(\beta + 1)(r_e + R_E) + R_S} \leq 5 \text{ mV}$$

$$U_s \leq 5 \text{ mV} \left[ \frac{(101)(123.76 + 125) + 10 \text{ k}}{101 \times 123.76} \right]$$

$$U_s \leq \underline{\underline{14 \text{ mV}}}$$

5.130

Refer to Fig. P5.130

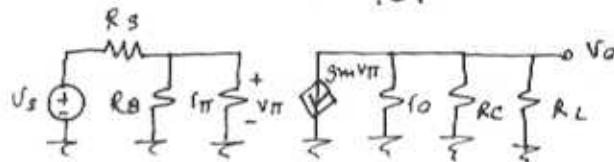
$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_B / (\beta + 1)}$$

where,  $V_{BB} = V_{cc} \cdot \frac{R_2}{R_1 + R_2}$

$$= 9 \cdot \frac{15}{27+15} = 3.21 \text{ V}$$

$$R_B = R_1 // R_2 = 15 // 27 = \underline{\underline{9.64 \text{ k}\Omega}}$$

Thus,  $I_E = \frac{3.21 - 0.7}{1.2 + \frac{9.64}{101}} = \underline{\underline{1.94 \text{ mA}}}$



$$g_m = \frac{I_C}{V_T} = \frac{0.99 \times 1.94}{0.025} = \underline{\underline{76.8 \text{ mA/V}}}$$

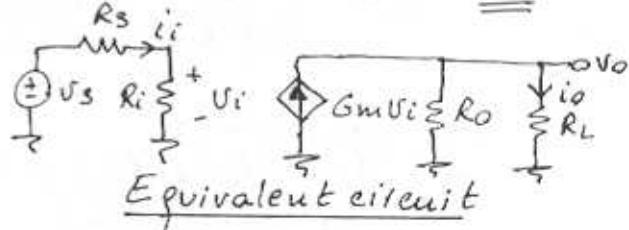
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{76.8} = \underline{\underline{1.3 \text{ k}\Omega}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.99 \times 1.94} = \underline{\underline{52.1 \text{ k}\Omega}}$$

$$R_i = R_B // r_\pi = 9.64 // 1.3 = \underline{\underline{1.15 \text{ k}\Omega}}$$

$$G_m = -g_m = -\underline{\underline{76.8 \text{ mA/V}}}$$

$$R_o = R_C // r_o = 2.2 // 52.1 = \underline{\underline{2.11 \text{ k}\Omega}}$$



$$A_U = \frac{U_o}{U_s} = \frac{U_i}{U_s} \cdot \frac{U_o}{U_i}$$

$$= \frac{R_i}{R_i + R_b} \cdot \frac{g_m (R_o // R_L) U_i}{U_i}$$

$$= \frac{-1.15}{10+1.15} \times 76.8 \times (2.11 // 2)$$

$$= \underline{\underline{-8.13 \text{ V/V}}}$$

$$A_i = \frac{i_o}{i_i} = \frac{U_o \cdot R_L}{U_s / (R_i + R_b)}$$

CONT.

$$\begin{aligned}\rightarrow A_i &= \frac{U_o}{U_s} \cdot \frac{R_{st} + R_i}{R_L} \\ &= -8.13 \times \frac{(10 + 1.15)}{2} \\ &= -\underline{45.3} \text{ A/A}\end{aligned}$$

5.131

Refer to Fig. P5.13Q.

$$V_{CC} = 9V, V_{BB} = \frac{1}{3} V_{CC} = 3V$$

Neglecting the base current,  
 $R_1 + R_2 = \frac{9}{0.2} = 45k\Omega$

$$\frac{R_2}{R_1 + R_2} = \frac{1}{3}$$

$$\Rightarrow R_2 = \underline{15k\Omega}, R_1 = \underline{30k\Omega}$$

$$R_B = \underline{R_1 \parallel R_2} = \frac{30 \times 15}{45} = 10k\Omega$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_B}$$

$$2 = \frac{3 - 0.7}{R_E + 10/101} \Rightarrow R_E = \underline{1.05k\Omega}$$

$$\text{Use } R_E = \underline{1k\Omega}$$

The resulting  $I_E$  will be

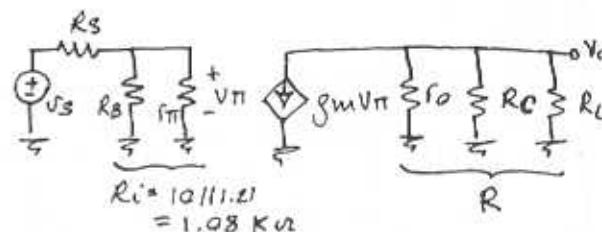
$$I_E = \frac{3 - 0.7}{1 + 10/101} = 2.09mA$$

$$I_C = \kappa I_E = 0.99 \times 2.09 = 2.07mA$$

$$g_m = \frac{I_C}{V_T} = \frac{2.07}{0.025} = 82.9 \frac{mA}{V}$$

$$r_{\pi} = \frac{\beta}{g_m} = \frac{100}{82.9} = 1.21k\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{2.07} = 48.3k\Omega$$



$$\begin{aligned}R_i &= 10/11.21 \\ &= 1.08k\Omega \\ \frac{U_o}{U_s} &= \frac{U_{pi}}{U_s} = \frac{R_i}{R_s + R_i} \cdot \frac{-g_m V_{pi} R}{U_{pi}} \\ &= \frac{-1.08}{10 + 1.08} \times 82.9 \times R\end{aligned}$$

To obtain  $\frac{U_o}{U_s} = -8$  we use:

$$R = \frac{8 \times 11.08}{1.08 \times 82.9} = 0.99k\Omega$$

$$R = r_o \parallel R_C \parallel R_L$$

$$0.99 = 48.3 \parallel R_C \parallel 2$$

$$\Rightarrow R_C = 2.04k\Omega$$

$$\text{use } R_C = 2k\Omega$$

Check:  $V_C = 9 - 2.07 \times 2 = 4.86V$   
 while  $V_B \approx 3V$ . Thus in active mode as assumed.

5.132

Refer to Fig. P5.13Q

$$V_{BB} = 9 \cdot \frac{47}{82+47} = 3.28V$$

$$R_B = 47 \parallel 82 = 29.88k\Omega$$

$$I_E = \frac{3.28 - 0.7}{3.6 + \frac{29.88}{101}} = 0.66mA$$

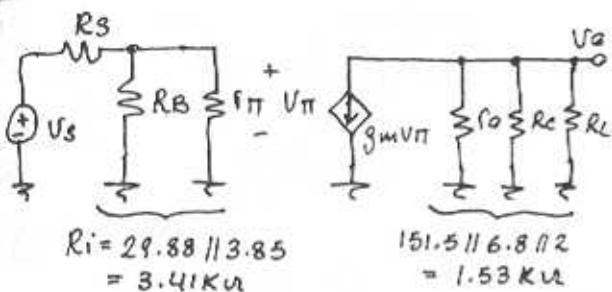
$$I_C = 0.99 \times 0.66 = 0.65mA$$

$$g_m = \frac{0.65}{0.025} = 26 \frac{mA}{V}$$

$$r_{\pi} = \frac{100}{26} = 3.85k\Omega$$

$$r_o = \frac{100}{0.66} = 151.5k\Omega$$

CONT.



$$A_V = \frac{U_o}{U_s} = \frac{3.41}{10 + 3.41} \times -26 \times 1.53$$

$$= -10.1 \text{ V/V}$$

Which is about 25%

higher than in the original design. The improvement is not as large as might have been expected because although  $R_i$  increases,  $g_m$  decreases by about the same factor.

Indeed most of the improvement is due to the increase in  $R_c$  and hence in the effective load resistance.

thus,  $r_{\pi} = 5250 \Omega$   
 then  $R_B = 105 \text{ k}\Omega$   
 choose  $R_B = \underline{\underline{100 \text{ k}\Omega}}$

$$\text{and } I_B = 4.76 \text{ mA}$$

$$I_E = (\beta + 1) I_B = 101 \times 4.76 \text{ mA}$$

$$I_E = 0.48 \text{ mA}$$

$$I = I_E \rightarrow I = \underline{\underline{0.5 \text{ mA}}}$$

To avoid saturation:

$$V_C - V_B \geq -0.5$$

$$V_C = 5 \text{ V} - R_C [I_C + g_m V_{BE}]$$

$$I_C = I \cdot \alpha = 0.5 \text{ mA} \times 100 / 101 = 0.49 \text{ mA}$$

$$g_m = \frac{V_T}{I_C} = \frac{25 \text{ mV}}{0.49 \text{ mA}} = 50 \text{ mV/V}$$

$$V_{BE} = 0.005 \text{ V}$$

$$\rightarrow V_C = 5 - R_C [0.49 \text{ mA} + 50 \text{ mV} \times 5 \text{ mV}] = 5 - 0.74 \times 10^{-3} \times R_C$$

Then:

$$V_C - V_B = (5 - 0.74 \text{ mA} \cdot R_C) - (-0.5 + V_{BE}) = 5.495 - 0.74 \text{ mA} \cdot R_C \geq -0.5 \\ R_C \leq \underline{\underline{8.1 \text{ k}\Omega}}$$

Base-to-Collector open circuit gain:

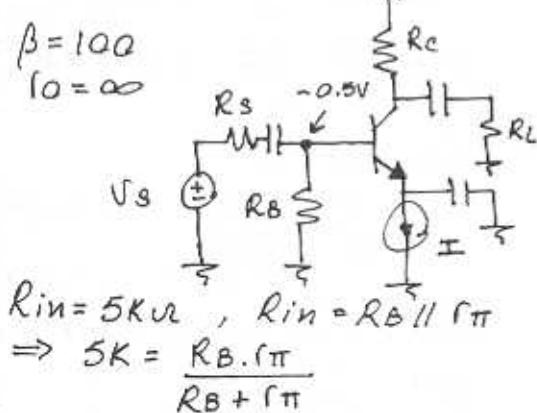
$$\frac{V_C}{V_B} = -g_m R_C = -50 \text{ mV} \times 8.1 \text{ k} = -\underline{\underline{405 \text{ V/V}}}$$

For  $R_s = 10 \text{ k}\Omega$ ,  $R_L = 10 \text{ k}\Omega$

$$\frac{U_o}{U_B} = -g_m (R_C \parallel R_L) = -50 \text{ mV} \times 4.47 \text{ k} = -223 \text{ V/V}$$

$$\frac{U_C}{U_s} = \frac{U_B}{U_s} \cdot \frac{U_o}{U_B} = \frac{5}{5+10} \times -223 = -\underline{\underline{74.3 \text{ V/V}}}$$

5.133



$$5 \text{ k}(\pi = R_B(r_{\pi} - 5 \text{ k}))$$

$$\text{but: } r_{\pi} = \frac{V_T}{I_B} \text{ and } R_B \cdot I_B = 0.5$$

$$\rightarrow 5 \text{ k} \cdot \frac{V_T}{I_B} = 0.5(r_{\pi} - 5 \text{ k})$$

5.134

Refer to Fig. P5.134

$$I_E = 0.5 \text{ mA}$$

$$(a) I_E = \frac{15 - 0.7}{R_E + R_S} \cdot \frac{1}{\beta + 1}$$

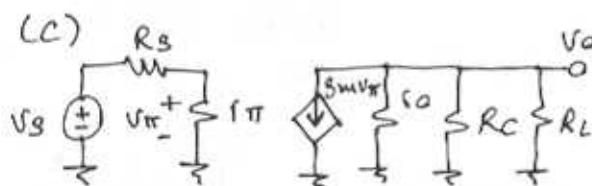
$$0.5 = \frac{14.3}{R_E + 2.5} \cdot \frac{1}{100}$$

$$\Rightarrow R_E = \underline{\underline{28.57 \text{ k}\Omega}}$$

$$(b) V_C = 15 - R_C \cdot I_C$$

$$5 = 15 - R_C \times 0.99 \times 0.5 \text{ mA}$$

$$\Rightarrow R_C = 20.2 \text{ k}\Omega \approx \underline{\underline{20 \text{ k}\Omega}}$$



$$R_L = 10 \text{ k}\Omega, R_S = 2.5 \text{ k}\Omega$$

$$f_T = 200 \text{ kHz}$$

$$g_m = \frac{I_C}{V_T} \approx \frac{0.5 \text{ mA}}{25 \text{ mV}} = 20 \text{ mA/V}$$

$$r_{\pi} = \frac{1}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$$\begin{aligned} A_V &= \frac{V_O}{V_S} = \frac{V_{\pi}}{V_S} \times \frac{V_O}{V_{\pi}} \\ &= \frac{r_{\pi}}{r_{\pi} + R_S} \times -g_m (R_C \parallel R_L \parallel R_C) \\ &= -\frac{5}{5+2.5} \times 20 (200 \parallel 20 \parallel 10) \\ &= \underline{\underline{-86 \text{ V/V}}} \end{aligned}$$

5.135

Refer to Fig. P5.135

(a) For each transistor

$$V_{BB} = 15 \times \frac{47}{100+47} = 4.8 \text{ V}$$

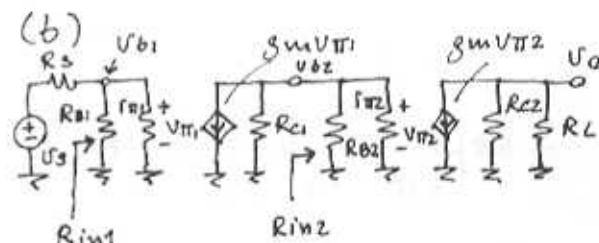
$$R_B = R_1 \parallel R_2 = 100 \parallel 47 = 32 \text{ k}\Omega$$

$$I_E = \frac{4.8 - 0.7}{3.9 + \frac{32}{101}} = 0.99 \text{ mA}$$

$$I_C = 0.99 \times 0.97 = \underline{\underline{0.96 \text{ mA}}}$$

$$V_C = V_{CC} - I_C \times R_C$$

$$= 15 - 0.96 \times 6.8 = \underline{\underline{8.5 \text{ V}}}$$



$$R_{B1} = R_{B2} = R_B = 32 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.96}{0.025} = 38.4 \text{ mA/V}$$

$$r_{\pi1} = r_{\pi2} = \frac{100}{38.4} = 2.6 \text{ k}\Omega$$

$$R_{C1} = R_{C2} = 6.8 \text{ k}\Omega$$

$$f_{o1} = f_{o2} = \infty$$

$$(c) R_{in1} = R_{B1} \parallel r_{\pi1}$$

$$= 32 \parallel 2.6 = \underline{\underline{2.4 \text{ k}\Omega}}$$

$$\frac{U_{B1}}{U_S} = \frac{R_{in1}}{R_S + R_{in1}}$$

$$= \frac{2.4}{5+2.4} = \underline{\underline{0.32 \text{ V/V}}}$$

$$(d) R_{in2} = R_{B2} \parallel r_{\pi2}$$

$$= 32 \parallel 2.6 = \underline{\underline{2.4 \text{ k}\Omega}}$$

CONT.

$$\begin{aligned} U_{62} &= -g_{m1} U_{\pi 1} (R_{C1} \parallel R_{in2}) \\ &= -38.4 U_{61} (6.8 \parallel 2.4) \end{aligned}$$

$$\frac{U_{62}}{U_{61}} = -68.1 \text{ V/V}$$

$$(e) U_o = -g_{m2} U_{\pi 2} (R_{C2} \parallel R_L) \\ = -38.4 U_{62} (6.8 \parallel 2)$$

$$\frac{U_o}{U_{62}} = -59.3 \text{ V/V}$$

$$(f) \frac{U_o}{U_s} = \frac{U_{61}}{U_s} \times \frac{U_{62}}{U_{61}} \times \frac{U_o}{U_{62}} \\ = 0.32 \times -68.1 \times -59.3 \\ = 12.92 \text{ V/V}$$

5.136

Refer to the circuit in Fig. P5.136

$$R_{in} = (\beta + 1)(r_e + 250)$$

$$\beta = 100 \quad r_e = \frac{V_T}{I_e} = \frac{0.025}{0.1} = 250\Omega$$

$$R_{in} = 101 \times (250 + 250) \\ = 50.5 \text{ k}\Omega$$

$$\frac{U_6}{U_s} = \frac{R_{in}}{R_s + R_{in}} = \frac{50.5}{20 + 50.5} \\ = 0.72 \text{ V/V}$$

$$\frac{U_o}{U_6} = -\alpha (20 \parallel 20) \\ = -\frac{0.99 \times 10}{0.250 + 0.250} = -19.8 \text{ V/V}$$

$$\text{Thus, } \frac{U_o}{U_s} = 0.72 \times -19.8 = -14.2 \text{ V/V}$$

For  $U_{6e} = 5 \text{ mV}$ ,  $U_e = 5 \text{ mV}$  also  
(since  $r_e = r_e = 250\Omega$ )

Thus,

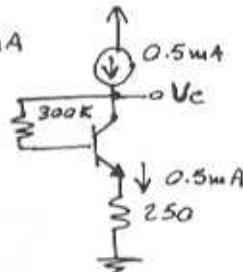
$$\begin{aligned} U_b &= 5 + 5 = 10 \text{ mV} \\ U_s &= \frac{10 \text{ mV}}{0.72} = 13.88 \text{ mV} \end{aligned}$$

$$U_o = 13.88 \times 14.2 = 197.2 \text{ mV}$$

5.137

$$(a) I_C = 0.99 \times 0.5 \text{ mA} \\ = 0.495 \text{ mA}$$

$$\begin{aligned} V_C &= I_E R_E + V_{BE} \\ &\quad + I_B R_B \\ &= 0.5 \times 0.175 + 0.7 \\ &\quad + 0.005 \times 300 \\ &= 2.28 \text{ V} \end{aligned}$$

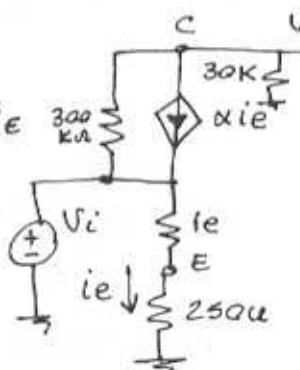


$$(b) i_e = \frac{U_i}{r_e + R_E}$$

$$r_e = \frac{V_T}{I_E} = 50\Omega$$

$$\rightarrow i_e = \frac{U_i}{50 + 250}$$

$$i_e = \frac{U_i}{300}$$



Node equation at C:

$$\frac{U_o - U_i}{300K} + \alpha i_e + \frac{U_o}{30K} = 0$$

$$\frac{U_o - U_i}{300K} + \frac{\alpha U_i}{(250 + 50)} + \frac{U_o}{30K} = 0$$

$$\Rightarrow \frac{U_o}{U_i} = -90 \text{ V/V}$$

5.138

(a) Without  $R_E$ ,

$$A_V \approx -\frac{\beta (R_C \parallel R_L \parallel r_o)}{(r_\pi + R_S)}$$

CONT.

Since no value is specified for  $\beta$  (or  $V_A$ ) we shall neglect its effect, thus

$$A_v = -\frac{\beta (R_c // R_L)}{r_\pi + R_s}$$

Substituting  $r_\pi = \beta/g_m$  yields

$$\begin{aligned} A_v &= -\frac{(R_c // R_L)}{\frac{\beta}{\beta+1} \cdot \frac{1}{g_m} + R_s / \beta} \\ &= -\frac{(R_c // R_L)}{r_e + \frac{R_s}{\beta}} \end{aligned}$$

Maximum gain is obtained at the high  $\beta$ ,

$$\begin{aligned} A_{v\max} &= -\frac{(R_c // R_L)}{r_e + \frac{0.050 + 10}{150}} \\ &= -8.57 (R_c // R_L) \end{aligned}$$

The minimum gain is obtained for  $\beta$  at its lowest value

$$\begin{aligned} A_{v\min} &= -\frac{(R_c // R_L)}{0.050 + \frac{10}{50}} \\ &= -4 (R_c // R_L) \end{aligned}$$

$$\text{Thus, } \frac{A_{v\max}}{A_{v\min}} = \frac{8.57}{4} = \underline{\underline{2.14}}$$

(b) With  $R_e$

$$\begin{aligned} A_v &\approx -\frac{\beta (R_c // R_L)}{r_\pi (1 + g_m R_e) + R_s} \\ &= -\frac{(R_c // R_L)}{\frac{1}{g_m} (1 + g_m R_e) + \frac{R_s}{\beta}} \end{aligned}$$

$$A_v = \frac{-(R_c // R_L)}{\frac{1}{g_m} + R_e + \frac{R_s}{\beta}}$$

$$\frac{A_{v\max}}{A_{v\min}} = \frac{\frac{1}{g_m} + R_e + \frac{R_s}{\beta_{\min}}}{\frac{1}{g_m} + R_e + \frac{R_s}{\beta_{\max}}}$$

Thus,

$$1.2 = \frac{50 + R_e + \frac{10,000}{50}}{50 + R_e + \frac{10,000}{150}}$$

$$\Rightarrow R_e = \underline{\underline{550 \Omega}}$$

(c) For  $\beta = 100$ :

(i) Without  $R_e$ ,

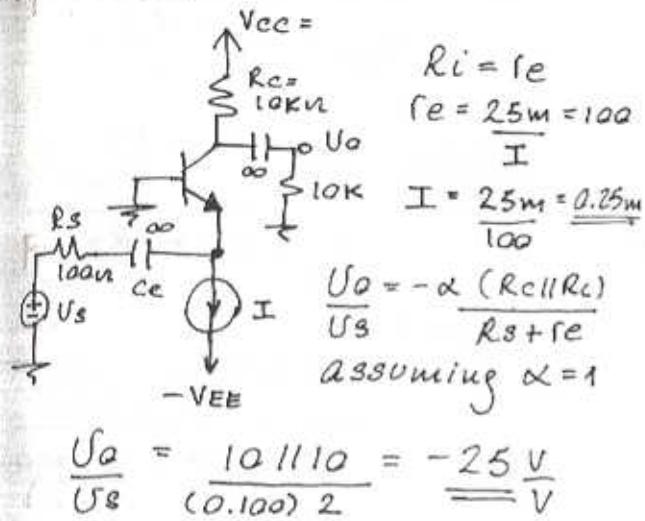
$$\begin{aligned} A_v &= -\frac{(R_c // R_L)}{r_e + \frac{R_s}{\beta}} \\ &= -\frac{(R_c // R_L)}{0.050 + 10/100} \\ &= -6.66 (R_c // R_L) \end{aligned}$$

(ii) With  $R_e = 550 \Omega$

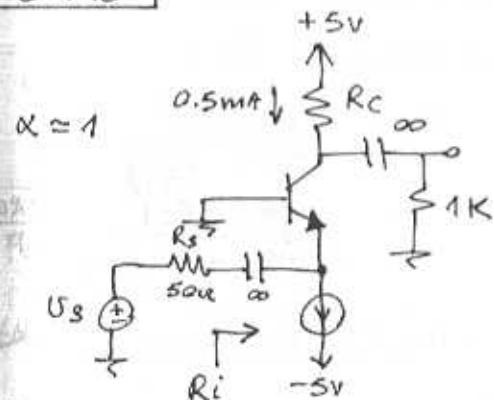
$$\begin{aligned} A_v &\approx -\frac{(R_c // R_L)}{0.050 + 0.550 + \frac{10}{100}} \\ &= -1.43 (R_c // R_L) \end{aligned}$$

Thus including  $R_e$  reduces the gain by a factor of  $\frac{6.66}{1.43} = \underline{\underline{4.6}}$

5.139



5.140



$$R_i = \frac{V_T}{I} = 50\Omega \Rightarrow I = 0.5mA$$

$$V_{Cmin} = V_C - 0.01gm(R_C // 1k)$$

To prevent saturation  $V_{Cmin} = 0$

$$\begin{aligned} \rightarrow 0 &= V_C - 0.01 \times 20 (R_C // 1) \\ &= 5 - 0.5 \frac{R_C}{R_C + 1} \end{aligned}$$

$$\begin{aligned} 5R_C + 5 - 0.5R_C^2 - 0.5R_C - 0.2R_C &= 0 \\ 0.5R_C^2 - 4.3R_C + 5 &= 0 \end{aligned}$$

$$R_C = \frac{4.3 + \sqrt{4.3^2 + 10}}{1}$$

$$= 9.64k\Omega$$

Select  $R_C = \underline{\underline{9.1k\Omega}}$

$$V_C = 0.45V$$

$$\begin{aligned} \frac{U_o}{U_s} &= \frac{R_i}{R_s + R_i} gm (R_C // 1) \\ &= \frac{50}{50+50} \times 20 \times (9.1 // 1) \\ &= \underline{\underline{9 \frac{V}{V}}} \end{aligned}$$

$$\text{For } U_{be(max)} = 10mV$$

$$U_{s(max)} = 20mV$$

$$V_{c(max)} = 180mV$$

Thus the collector voltage swings from

$$(0.45 - 0.18)V \text{ to } (0.45 + 0.18)V$$

i.e. from 0.27V to 0.63V

5.141

Refer to Fig P5.141

$$R_i = r_e = \frac{V_T}{I_E} = \frac{V_T}{0.5} = \underline{\underline{50\Omega}}$$

To find the voltage gain  $\frac{U_o}{U_s}$  first note that

$$\frac{U_e}{U_s} = \frac{R_i}{R_s + R_i} = \frac{50}{50+50} = 0.5$$

Then,

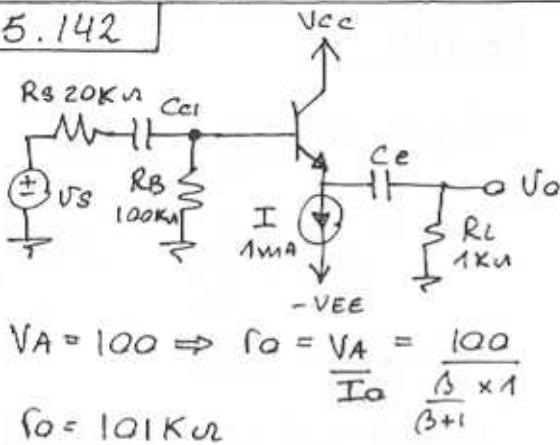
$$\frac{U_C}{U_e} = \frac{\alpha \times (\text{Total resistance at c})}{r_e}$$

$$\approx \frac{1 \times (100k\Omega // 1k\Omega)}{50\Omega}$$

$$= 19.8 \frac{V}{V}$$

$$\text{Thus, } \frac{U_o}{U_s} = 19.8 \times 0.5 = \underline{\underline{9.9 \frac{V}{V}}}$$

5.142



$$VA = 100 \Rightarrow r_o = \frac{V_A}{I_A} = \frac{100}{\frac{\beta+1}{\beta+1}} = 101 \text{ k}\Omega$$

$$(a) R_{in} = R_B \parallel [(\beta+1)(r_e + (R_L \parallel r_o))] = 100k \parallel [(101)(25 + (1k \parallel 101k))] = 50.6 \text{ k}\Omega$$

$$R_{in} = 50.6 \text{ k}\Omega$$

$$\frac{U_6}{U_S} = \frac{R_{in}}{R_{in} + R_s} = \underline{0.717} \text{ V/V}$$

$$\frac{U_o}{U_S} = \frac{U_6}{U_S} \cdot \frac{(R_L \parallel r_o)}{(R_L \parallel r_o) + r_e} = \underline{0.975} \text{ V/V}$$

(b) The peak value of  $U_o$  occurs when the current flowing from ground through  $R_C$  equals the bias current  $I$  on a negative signal swing.

$$0^o \quad \frac{\hat{U}_o}{R_L} = I = 1 \text{ mA}$$

$$\hat{U}_o = 1 \text{ V} \Rightarrow \hat{U}_S = \frac{1}{0.975} = \underline{1.025 \text{ V}}$$

$$U_{\pi} = \frac{U_S' \times (\beta+1) r_e}{(\beta+1) r_e + (\beta+1)(R_L \parallel r_o)}$$

: Where  $U_S'$  is

$$U_S' = U_S \times \frac{R_{in}}{R_{in} + R_s} = U_S \times 0.717$$

$$U_{\pi} = \frac{(1.025 \times 0.717) \times 101 \times 25}{101[25 + 990]} = \underline{18.10 \text{ mV}}$$

$$(c) \hat{U}_{\pi} = 10 \text{ mV} \Rightarrow$$

$$U_S = \frac{10 \text{ m} \times 101 [25 + 990]}{0.717 \times 101 \times 25} = \underline{0.566 \text{ V}}$$

$$U_o = \frac{U_o}{U_S} \times 0.566 = \underline{0.552 \text{ V}}$$

$$(d) R_o = \left( r_e + \frac{(R_s \parallel R_B)}{\beta+1} \right) \parallel r_o = \left( 25 + \frac{(20k \parallel 100k)}{101} \right) \parallel 101k = \underline{190 \text{ }\mu\Omega}$$

$$\text{Open circuit voltage gain } A_v \Big|_{R_L=\infty} = \frac{R_B}{R_s + R_B} \cdot \frac{r_o}{r_o + r_e + \frac{R_s \parallel R_B}{\beta+1}} = \frac{100}{20+100} \cdot \frac{101k}{101k + 25 + \frac{(100 \parallel 20)k}{101}} = \underline{0.832 \text{ V/V}}$$

$$A_v = A_v \Big|_{R_L=\infty} \times \frac{R_L}{R_L + r_o} = 0.832 \times \frac{500}{500 + 190} = \underline{0.603 \text{ V/V}}$$

5.143

Refer to Fig. P5.143

$$(a) I_E = \frac{9 - 0.7}{1 + 100 \parallel (\beta+1)} \text{ CONT.}$$

$$\text{for } \beta = 40, I_E = \frac{8.3}{1 + \frac{100}{40}} = \underline{\underline{2.41 \text{ mA}}}$$

$$V_E = 1 \times 2.41 = \underline{\underline{2.41 \text{ V}}}$$

$$V_B = 2.41 + 0.7 = \underline{\underline{3.11 \text{ V}}}$$

$$\text{for } \beta = 200, I_E = \frac{8.3}{1 + \frac{100}{200}} = \underline{\underline{5.54 \text{ mA}}}$$

$$V_E = + \underline{\underline{5.54 \text{ V}}}$$

$$V_B = + \underline{\underline{6.24 \text{ V}}}$$

$$(b) R_i = 100K\Omega // (\beta + 1)[r_e + (1//1)] \\ = 100 // (\beta + 1)[r_e + 0.5]$$

$$\text{For } \beta = 40, I_E = 2.41 \text{ mA}$$

$$\rightarrow r_e = 10.37 \Omega$$

$$\text{thus } R_i = 100 // 40 \times (0.01037 + 0.5) \\ = 100 // 21 \\ = \underline{\underline{17.30 \Omega}}$$

$$\text{For } \beta = 200, I_E = 5.54 \text{ mA}$$

$$\rightarrow r_e = 4.51 \Omega$$

$$\text{thus } R_i = 100 // 200 \times (0.0045 + 0.5) \\ = 100 // 101.4 \\ = \underline{\underline{50.3 K\Omega}}$$

$$(c) \frac{V_o}{V_s} = \frac{V_b}{V_s} \cdot \frac{V_o}{V_b} \\ = \frac{R_i}{R_s + R_i} \cdot \frac{(1//1)}{(1//1) + r_e}$$

$$\text{For } \beta = 40,$$

$$\frac{V_o}{V_s} = \frac{17.3}{10 + 17.3} \times \frac{0.5}{0.5 + 0.01037} \\ = \underline{\underline{0.621 \text{ V/V}}}$$

$$\text{For } \beta = 200,$$

$$\frac{V_o}{V_s} = \frac{50.3}{10 + 50.3} \cdot \frac{0.5}{0.5 + 0.0045} \\ = \underline{\underline{0.827 \text{ V/V}}}$$

5.144

Refer to Fig. P5.144

$$I_E = \frac{5 - 0.7}{3.3 + \frac{100}{101}} = \underline{\underline{1.00 \text{ mA}}}$$

$$r_e = \frac{25}{1.00} = 25 \Omega$$

$$R_i = (\beta + 1)[r_e + (3.3//1)] \\ = \underline{\underline{80.0 K\Omega}}$$

$$\frac{V_o}{V_s} = \frac{V_b}{V_s} \cdot \frac{V_o}{V_b} = \frac{R_i}{R_s + R_i} \frac{(3.3//1)}{r_e + (3.3//1)}$$

Thus,

$$\frac{V_o}{V_s} = \frac{80}{100 + 80} \times \frac{0.025 + (3.3//1)}{0.025 + (3.3//1)} \\ = \underline{\underline{0.430 \text{ V/V}}}$$

$$\frac{I_o}{I_i} = \frac{V_o / R_L}{V_s / (R_s + R_i)} \\ = \frac{V_o}{V_s} \cdot \frac{R_s + R_i}{R_L} \\ = 0.43 \times \frac{(100 + 80)}{1} \\ = \underline{\underline{77.4 \text{ A/A}}}$$

$$R_{out} = 3.3 // [r_e + \frac{100}{\beta + 1}]$$

$$= 3.3 // [0.025 + \frac{100}{101}] \\ = \underline{\underline{0.776 \text{ K}\Omega}}$$

5.145

Refer to Fig 5.63(a)

$$R_o = r_o // [r_e + 10K/\beta + 1]$$

CONT.

$$R_o = \frac{V_A}{I_c} \parallel \left[ \frac{V_T + 10k\mu}{\beta + 1} \right]$$

$$\begin{aligned} R_o &= \frac{125}{0.99 \times 2.5} \parallel \left[ \frac{0.025}{2.5} + \frac{10}{100+1} \right] \\ &= 50.5 \parallel [0.010 + 0.099] \\ &= 0.109 \text{ k}\Omega = \underline{\underline{109\mu}} \end{aligned}$$

With no load:

$$\begin{aligned} R_i &= (\beta + 1) [r_e + r_o] \\ \text{For } \beta = 100, r_e &= 10\mu\Omega, r_o = 50.5 \text{ k}\Omega \\ R_i &= 101 \times (0.010 + 50.5) \\ &= \underline{\underline{5.1 \text{ M}\Omega}} \end{aligned}$$

$$\begin{aligned} \frac{U_o}{U_s} &= \frac{U_B}{U_s} \cdot \frac{U_o}{U_B} = \frac{R_i}{R_s + R_i} \times \frac{r_o}{r_o + r_e} \\ &= \frac{5100}{10 + 5100} \cdot \frac{50.5}{50.5 + 0.01} \\ &= \underline{\underline{0.988 \text{ V/V}}} \end{aligned}$$

With  $R_L = 1 \text{ k}\Omega$ :

$$\begin{aligned} \frac{U_o}{U_s} &= 0.988 \times \frac{R_L}{R_L + R_o} \\ &= 0.988 \times \frac{1}{1 + 0.109} = \underline{\underline{0.890 \text{ V/V}}} \end{aligned}$$

Largest negative output signal occurs when the BJT cuts off, thus:

$$\begin{aligned} U_{o\min} &= -2.5 \text{ mA} \times 1 \text{ k}\Omega \\ &= \underline{\underline{-2.5 \text{ V}}} \end{aligned}$$

Largest positive output signal occurs when  $U_B = +3.4 \text{ V}$ . The corresponding value of  $V_E$  is  $\approx 3.4 - 0.4 = +2.7 \text{ V}$  (where we have neglected the signal component of  $U_{BE}$ )

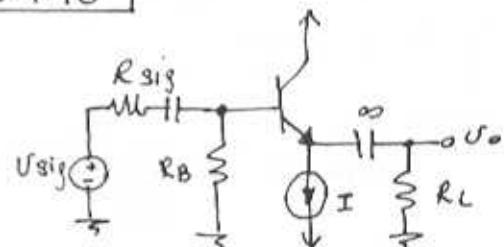
Now, the DC level at the emitter with zero input signal is

$$\begin{aligned} V_E &= -10k\mu \times I_B - 0.7 \\ &= -10 \times \frac{2.7}{101} - 0.7 \\ &\approx -0.97 \text{ V} \end{aligned}$$

Thus the signal component is:

$$\begin{aligned} U_{e\max} &= V_E - V_E \\ &= 2.5 - (-0.97) \\ &= \underline{\underline{3.47 \text{ V}}} \end{aligned}$$

5.146



$$\begin{aligned} G_o|_{R_L=\infty} &= \frac{R_B}{R_{sig} + R_B} \cdot \frac{r_o}{(R_{sig}||R_B + r_e + r_o)} = 0.99 \\ &\approx 1 \text{ for very large } r_o \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{R_B}{R_{sig} + R_B} &= 0.99 \\ \rightarrow \frac{R_B}{10k + R_B} &= 0.99 \Rightarrow R_B = 990 \text{ k}\Omega \end{aligned}$$

$$R_{out} = r_o \parallel \left( r_e + R_s \parallel R_B \right) \frac{1}{\beta + 1}$$

$$\text{for } r_o \text{ large: } R_{out} \approx \frac{r_e + R_s \parallel R_B}{\beta + 1}$$

$$r_e + \frac{(10 \parallel 990)k}{\beta + 1} = 200 \mu\Omega$$

$$r_e + \frac{(20 \parallel 990)k}{\beta + 1} = 300 \mu\Omega$$

CONT.

Then,

$$r_e + \frac{9.9K}{\beta+1} = 200 \quad (1)$$

$$r_e + \frac{19.6K}{\beta+1} = 300 \quad (2)$$

Solving Eqs. (1) and (2)

$$\beta+1 = 97 \Rightarrow \beta = 96$$

and  $r_e = 98 \Omega$

If:  $R_{sig} = 30K\Omega$  and  $R_L = 1K\Omega$

$$G_V = \frac{R_B}{R_{sig} + R_B} \cdot \frac{(r_o \parallel R_L)}{\frac{r_o \parallel R_B + r_e + (r_o \parallel R_L)}{\beta+1}}$$

If  $r_o$  is large:

$$\begin{aligned} G_V &= \frac{R_B}{R_{sig} + R_B} \cdot \frac{R_L}{\frac{R_{sig} \parallel R_B}{\beta+1} + r_e + R_L} \\ &= \frac{990}{30+990} \cdot \frac{1}{\frac{(30 \parallel 990)}{97} + 0.098 + 1} \\ &= \underline{0.7 \text{ V/V}} \end{aligned}$$

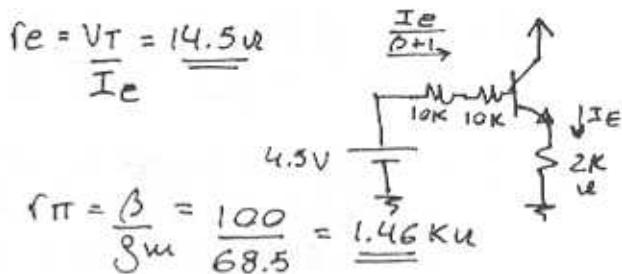
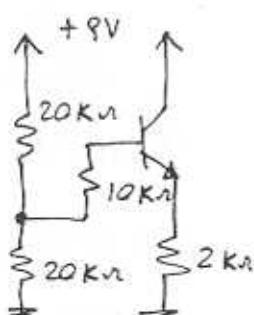
5.147

QDCA Analysis

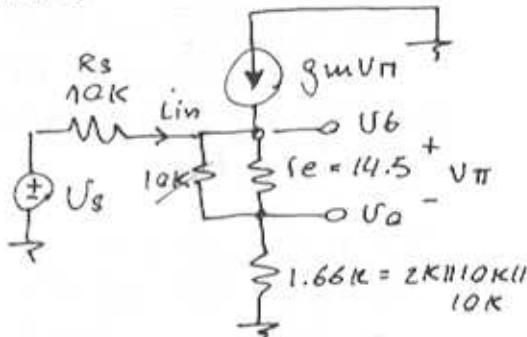
$$\begin{aligned} I_E &= \frac{4.5 - 0.7}{2 + 10 + 10} \\ &= \underline{1.73 \text{ mA}} \end{aligned}$$

$$I_C = 0.99 \times 1.73 = \underline{1.71 \text{ mA}}$$

$$g_m = \frac{I_c}{V_T} = 68.5 \text{ mA/V}$$



(b) Simplified equivalent circuit:



$$U_o \approx 1.66k \times \left( \frac{V_{\pi}}{14.5} \right) = \underline{114.48 \text{ V}}$$

$$\frac{V_{\pi}}{14.5} = i_{in} + 68 \times 10^{-3} \times V_{\pi}$$

$$\rightarrow V_{\pi} \left( \frac{1}{14.5} - 68 \times 10^{-3} \right) = i_{in}$$

$$i_{in} = \frac{V_{\pi}}{1035 \text{ K}}$$

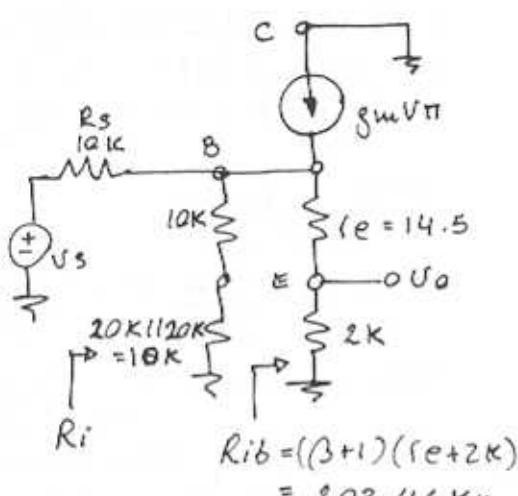
$$U_b = V_{\pi} + U_o = \underline{115.48 \text{ V}}$$

$$R_i = \frac{U_b}{i_{in}} = \frac{115.48 \text{ V}}{V_{\pi}/1.035} = \underline{120 \text{ k}\Omega}$$

$$\begin{aligned} U_o &= \frac{U_b}{U_s} \cdot \frac{U_o}{U_b} = \frac{R_i}{R_i + R_s} \cdot \frac{114.48 \text{ V}}{115.48 \text{ V}} \\ &= \frac{120}{120+10} \times \frac{114.48}{115.48} = \underline{0.92 \text{ V/V}} \end{aligned}$$

(c) With CB open-circuited so that bootstrapping is eliminated, we obtain the following equivalent circuit model.

CONT.



Much lower than the value obtained with bootstrapping. This is due to the lower  $R_i$ . Bootstrapping raises the component of input resistance due to the base biasing network.

5.148

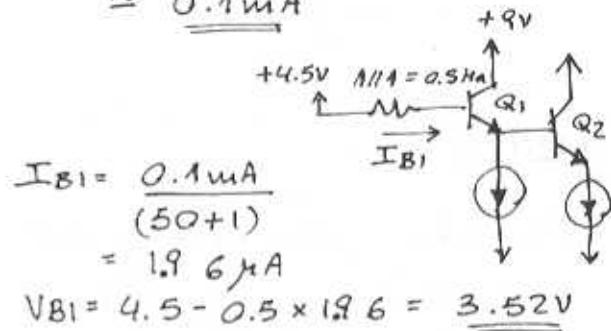
Refer to the circuit in Fig P5.148

(a)  $I_{E2} = \underline{5 \text{ mA}}$

$$I_{E1} = 50 \mu + I_{B2}$$

$$= 50 + \frac{I_{E2}}{\beta_2 + 1} = 50 + \frac{5000}{101}$$

$$\simeq \underline{0.1 \text{ mA}}$$



(b) Refer to Fig. P.5.148

$$\frac{U_o}{U_{b2}} = \frac{R_L}{R_L + r_{e2}}$$

$$R_L = 1 \text{ k}\Omega \quad r_{e2} = \frac{25}{5} = 5 \text{ }\Omega$$

$$\frac{U_o}{U_{b2}} = \frac{1}{1 + 0.005} = \underline{0.995 \text{ V/V}}$$

$$r_{ib2} = (\beta_2 + 1)(r_{e2} + R_L)$$

$$= (101) \times (1.005)$$

$$= \underline{101.5 \text{ k}\Omega}$$

(c)  $\frac{U_{e1}}{U_{b1}} = \frac{R_{ib2}}{R_{ib2} + r_{e1}}$

$$r_{e1} = \frac{V_T}{100 \mu A} = 250 \text{ }\Omega$$

$$\rightarrow \frac{U_{e1}}{U_{b1}} = \frac{101.5}{101.5 + 0.25} = \underline{0.997 \text{ V/V}}$$

$$R_i = 1 \text{ M}\Omega \parallel 1 \text{ M}\Omega \parallel (\beta_1 + 1)(r_{e1} + R_{ib2})$$

$$= 1 \parallel 1 \parallel 51 \times (0.25 + 101.5) \text{ k}\Omega$$

$$= 1 \parallel 1 \parallel 5.2 \text{ M}\Omega$$

$$= \underline{0.499 \text{ M}\Omega} = \underline{499 \text{ k}\Omega}$$

(d)  $\frac{U_{b1}}{U_s} = \frac{R_i}{R_s + R_i} = \frac{499}{100 + 499} = \underline{0.833 \text{ V/V}}$

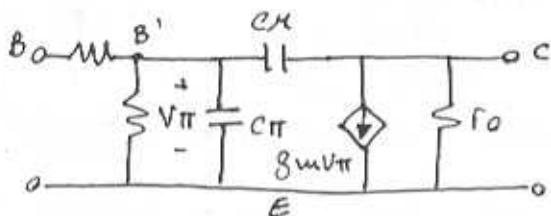
CONT.

$$(e) \frac{U_o}{U_S} = \frac{U_{B1}}{U_S} \cdot \frac{U_{E1}}{U_{B1}} \cdot \frac{U_o}{U_{E1}}$$

$$= 0.833 \times 0.997 \times 0.995$$

$$= \underline{\underline{0.826 \text{ V/V}}}$$

5.149



$$f_x = \underline{\underline{100 \text{ kHz}}}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{25 \text{ mV}} = \underline{\underline{20 \text{ mA/V}}}$$

$$C_{\pi} = \frac{\beta_0}{g_m} = \frac{100}{20} = \underline{\underline{5 \text{ pF}}}$$

$$f_0 = \frac{V_A}{I_C} = \frac{50 \text{ V}}{0.5 \text{ mA}} = \underline{\underline{100 \text{ kHz}}}$$

$$C_{\mu} = \frac{C_{\mu 0}}{\left(1 + \frac{V_{EB}}{V_{AC}}\right)^{0.5}} = \frac{30}{\left(1 + \frac{2}{0.75}\right)^{0.5}} = \underline{\underline{15.7 \text{ pF}}}$$

$$C_{je} \approx 2C_{jeo} = 2 \times 20 = 40 \text{ fF}$$

$$C_{de} = \gamma_F g_m = 30 \times 10^{-12} \times 20 \times 10^3 = 600 \text{ fF}$$

$$C_{\pi} = C_{je} + C_{de} = \underline{\underline{0.640 \text{ pF}}}$$

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})} = \frac{20 \times 10^{-3}}{2\pi(0.64 + 0.016) \times 10^{-12}} = \underline{\underline{4.85 \text{ GHz}}}$$

5.150

$$|h_{fe}| \approx f_T / f$$

At  $I_C = 0.2 \text{ mA}$ ,  $|h_{fe}| = 2.5$   
at  $f = 500 \text{ MHz}$ , thus:

$$f_T = 2.5 \times 500 = \underline{\underline{1.25 \text{ GHz}}}$$

At  $I_C = 1.0 \text{ mA}$ ,  $|h_{fe}| = 11.6$   
at  $f = 500 \text{ MHz}$ , thus:  
 $f_T = 11.6 \times 500 = \underline{\underline{5.8 \text{ GHz}}}$

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})}$$

$$C_{\pi} + C_{\mu} = \frac{g_m}{2\pi f_T} \rightarrow C_{\pi} = \frac{g_m}{2\pi f_T} - C_{\mu}$$

$$C_{\pi} (I_C = 0.2 \text{ mA}) = \frac{8 \times 10^{-3} - 0.05 \times 10^{-12}}{2\pi \times 1.25 \times 10^9} = 0.9686 \text{ pF}$$

$$C_{\pi} (I_C = 1.0 \text{ mA}) = \frac{40 \times 10^{-3} - 0.05 \times 10^{-12}}{2\pi \times 5.8 \times 10^9} = 1.0476 \text{ pF}$$

Since  $C_{\pi} = C_{je} + \gamma_F g_m$ ,  
 $C_{je} + 8 \times 10^{-3} \gamma_F = 0.9686 \times 10^{-12} \quad (1)$   
 $C_{je} + 40 \times 10^{-3} \gamma_F = 1.0476 \times 10^{-12} \quad (2)$   
 Solving Eqn. (1) and (2)  
 together yields,  
 $C_{je} = \underline{\underline{0.95 \text{ pF}}, \gamma_F = \underline{\underline{247 \text{ ps}}}}$

5.151

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})}$$

$$= \frac{80 \times 10^{-3}}{2\pi(10 + 1) \times 10^{-12}} = \underline{\underline{4.24 \text{ GHz}}}$$

$$f_B = f_T / \beta_0 = (4.24 / 150) \times 10^9 = \underline{\underline{28.26 \text{ MHz}}}$$

5.152

At  $I_C = 2 \text{ mA}$  the diffusion part of  $C_{\pi}$  is:  $10^{-2} = 8 \text{ pF}$

CONT.

At  $I_C = 0.2\text{mA}$  the diffusion capacitance becomes  $0.8\text{ pF}$  and thus,

$$C_{\pi} = 0.8 + 2 = 2.8 \text{ pF}$$

and:

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_R)}$$

$$= \frac{8 \times 10^{-3}}{2\pi(2.8 + 1) \times 10^{-12}} = \underline{\underline{335 \text{ MHz}}}$$

5.153

$$w_T = g_m / (C_{\pi} + C_R)$$

$$2\pi \times 5 \times 10^9 = \frac{20 \times 10^{-3}}{(C_{\pi} + 0.1) \times 10^{-12}}$$

$$C_{\pi} + 0.1 = \frac{20}{10\pi} = 0.64 \text{ pF}$$

$$C_{\pi} = \underline{\underline{0.54 \text{ pF}}}$$

$$g_m = \underline{\underline{20 \text{ mA/V}}}$$

$$f_{\pi} = \beta/g_m = 150/20 = \underline{\underline{7.5 \text{ KHz}}}$$

$$f_B = f_T/\beta = \frac{5 \times 10^9}{150} = \underline{\underline{33.3 \text{ MHz}}}$$

5.154

$|H_{FE}|$  becomes 20 at;

$$f_T/20 = \frac{1 \times 10^9}{20} = \underline{\underline{50 \text{ MHz}}}$$

$$f_B = f_T/\beta_0 = \frac{1000 \text{ MHz}}{200} = \underline{\underline{5 \text{ MHz}}}$$

5.155

$$Z = r_x + \frac{1}{\frac{1}{f_{\pi}} + j\omega c_{\pi}}$$

$$= r_x + \frac{f_{\pi}}{1 + j(\omega/\omega_p)f_{\pi}}$$

$$Z = r_x + \frac{f_{\pi}}{1 + j(\omega/\omega_p)}$$

$$= r_x + \frac{f_{\pi}(1 - j\omega/\omega_p)}{1 + (\frac{\omega}{\omega_p})^2}$$

$$= r_x + \frac{f_{\pi}}{1 + (\frac{\omega}{\omega_p})^2} - j \cdot \frac{f_{\pi}(\omega/\omega_p)}{1 + (\frac{\omega}{\omega_p})^2}$$

$$\text{Re}[Z] = r_x + \frac{f_{\pi}}{1 + (\frac{\omega}{\omega_p})^2}$$

For  $\text{Re}[Z]$  to be an estimate of  $r_x$  good to within 10% we must keep

$$\frac{f_{\pi}}{1 + (\frac{\omega}{\omega_p})^2} \leq \frac{r_x}{10}$$

But  $r_x \leq f_{\pi}/10$

Thus,

$$\frac{f_{\pi}}{1 + (\frac{\omega}{\omega_p})^2} \leq \frac{f_{\pi}}{100}$$

$$1 + (\frac{\omega}{\omega_p})^2 \geq 100$$

or  $\omega \geq 10\omega_p$  (approx.)

5.156

See completed table below

CONT.

	$I_E$ (mA)	$r_e$ ( $\mu V$ )	$g_m$ ( $\mu A/V$ )	$r_{\pi}$ ( $k\Omega$ )	$\beta_0$
(a)	1	25	40	2.5	100
(b)	1	25	40	3.13	125.3
(c)	0.99	25.3	39.6	2.525	100
(d)	10	2.5	400	0.25	100
(e)	0.1	250	4	25	100
(f)	1.0	25	40	0.25	10
(g)	1.25	20	50	0.20	10

	$f_T$ (MHz)	$C_{\pi}$ (pF)	$C_R$ (pF)	$f_B$ (MHz)
(a)	400	2	13.9	4
(b)	501.3	2	10.7	4
(c)	400	2	13.8	4
(d)	400	2	157	4
(e)	100	2	4.4	1
(f)	400	2	13.9	40
(g)	800	1	9	80

### 5.157

From Example 5.18:

$$I = 2 \text{ mA}, \beta = 100, f_T = 800 \text{ MHz}$$

$$R_B = 50 \text{ k}\Omega, R_C = 4 \text{ k}\Omega, r_x = 50 \Omega$$

$$V_A = 100, C_{\pi} = 1 \text{ pF}, R_{sig} = 5 \text{ k}\Omega$$

$$R_L = 5 \text{ k}\Omega$$

$$g_m = \frac{2 \text{ mA}}{25 \text{ mV}} = 80 \frac{\text{mA}}{\text{V}}$$

$$r_{\pi} = \frac{\beta_0}{g_m} = \frac{100}{80} = 1250 \Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{2 \text{ mA}} = 50 \text{ k}\Omega$$

$$C_{\pi} + C_R = \frac{g_m}{\omega_T} = \frac{80 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 16 \text{ pF}$$

$$C_R = 1 \text{ pF} \Rightarrow C_{\pi} = 15 \text{ pF}$$

$$AM = - \frac{R_B}{R_B + R_{sig}} \cdot \frac{r_{\pi} \times g_m R_L}{r_{\pi} + r_x + (R_B // R_{sig})}$$

$$\text{where } R_L' = r_0 // R_C // R_L$$

$$= (50 // 4 // 5) \text{ k}\Omega$$

$$= 2.1 \text{ k}\Omega$$

$$AM = - \frac{50}{50+5} \cdot \frac{1250 \times 168}{1250+50+(50//5)\text{k}\Omega}$$

$$\text{where } 168 = g_m \times R_L'$$

$$= 80 \times 10^{-3} \times 2.1 \cdot 10^3$$

$$\text{Then: } AM = -32.6 //$$

$$20 \log |AM| = \underline{\underline{30.3 \text{ dB}}}$$

$$C_{in} = C_{\pi} + C_R (1 + g_m R_L')$$

$$= 15 + 1 (1 + 168) = 184 \text{ pF}$$

$$R_{sig}' = r_{\pi} // [r_x + (R_B // R_{sig})]$$

$$= 1250 // [50 + (50k // 5k)]$$

$$= 983 \Omega$$

$$f_H = \frac{1}{2\pi C_{in} R_{sig}'} = \frac{1}{2\pi \times 184 \times 10^{-12} \times 983}$$

$$= \underline{\underline{880 \text{ kHz}}}$$

Gain-bandwidth product

$$GB = |AM| \times f_H = 32.6 \times 880 \times 10^3$$

$$= \underline{\underline{29 \times 10^6}}$$

Previously, in example 5.18

$$GB = 39 \times 754 \times 10^3 = 29 \times 10^6$$

Thus, the designer traded gain for bandwidth by increasing  $I$ . However, by doubling  $I$  the dissipation increased by a factor of 2, since:

$$\text{Power} = \frac{\overline{I} \times V_{\text{Supply}}}{2\overline{I}}$$

### 5.158

Refer to Fig. 5.71(a)

$$R_B \gg R_{sig}, r_x \ll R_{sig}$$

$$R_{sig} \gg r_{\pi}, g_m R_L' \gg 1$$

$$g_m R_L' C_R \gg C_{\pi}$$

CONT.

$$AM = \underbrace{\frac{-R_B}{R_B + R_{sig}}}_{(4)} \cdot \underbrace{\frac{f\pi \cdot g_m \cdot R_L'}{f\pi + f_x + (R_{sig} \parallel R_B)}}_{\substack{(1) \\ | \\ (2) \\ | \\ (3)}}$$

- (1) =  $R_{sig}$ , since  $R_B \gg R_{sig}$   
 (2) =  $R_{sig}$ , since  $R_{sig} \gg f_x$   
 (3) =  $R_{sig}$ , since  $R_{sig} \gg f\pi$   
 (4) = 1, since  $R_B \gg R_{sig}$

Thus,

$$AM = -\frac{f\pi \cdot g_m \cdot R_L'}{R_{sig}}$$

$$\text{but } f\pi = \frac{\beta}{g_m}$$

$$\Rightarrow AM = -\frac{\beta R_L'}{R_{sig}} \quad \text{Q.E.D.}$$

$$(b) f_H = \frac{1}{2\pi \cdot C_{in} \cdot R'_{sig}}$$

$$C_{in} = \underbrace{C\pi + C_R}_{\substack{| \\ (1)}} \underbrace{(1 + g_m R_L')}_{\substack{| \\ (2)}}$$

- (1) =  $g_m R_L'$ , since  $R_L' \gg 1$   
 (2) =  $C_R g_m R_L'$ , since  $C_R g_m R_L' \gg C\pi$

$$\Rightarrow C_{in} = C_R \cdot g_m \cdot R_L'$$

$$R'_{sig} = f\pi \parallel \underbrace{[f_x + (R_B \parallel R_{sig})]}_{(1)}$$

$$\underbrace{|}_{(2)}$$

$$\underbrace{|}_{(3)}$$

- (1) =  $R_{sig}$ , since  $R_B \gg R_{sig}$   
 (2) =  $R_{sig}$ , since  $R_{sig} \gg f_x$   
 (3) =  $f\pi$ , since  $R_{sig} \gg f\pi$

$$\Rightarrow R'_{sig} = f\pi$$

Thus,

$$f_H = \frac{1}{2\pi \cdot C_R \cdot g_m \cdot R_L' \cdot f\pi}$$

Q.E.D.

(c)

$$GB = |AM| \times f_H = \frac{\beta R_L'}{R_{sig}} \frac{1}{2\pi C_R \beta R_L'}$$

$$= \frac{1}{2\pi} \frac{1}{C_R \cdot R_{sig}} \quad \text{Q.E.D.}$$

$$\text{For: } C_R = 1 \mu F \text{ and } R_{sig} = 25 k\Omega$$

$$GB = \frac{1}{2\pi \times 10^{-12} \times 25 \times 10^3} = 6.36 \times 10^6$$

$$\text{For: } I_C = 1 \text{ mA}, \beta = 100, R_{sig} = 25 k\Omega$$

$$1) \text{ If } R_L' = 25 k\Omega$$

$$AM = -100 \cdot \frac{25k}{25k} = -100$$

$$20 \log |100| = 40 \text{ dB}$$

$$f_H = \frac{1}{2\pi \times 10^{-12} \times 100 \times 25 \times 10^3}$$

$$= \underline{\underline{63.6 \text{ kHz}}}$$

$$2) \text{ If } R_L' = 2.5 k\Omega$$

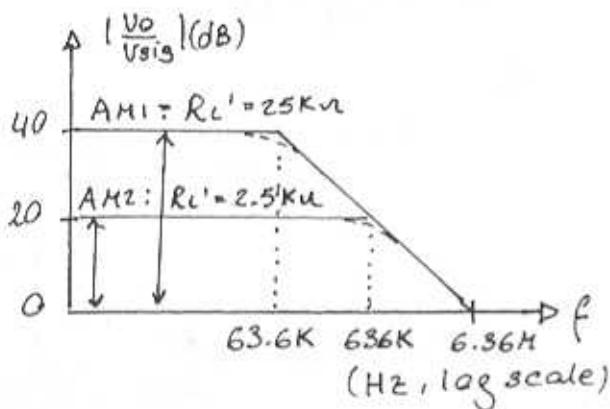
$$AM = -100 \cdot \frac{2.5k}{25k} = \underline{\underline{-10}}$$

$$20 \log |10| = 20$$

CONT.

$$f_H = \frac{1}{2\pi \times 10^{-12} \times 100 \times 2.5 \cdot 10^3}$$

$$f_H = \underline{\underline{636 \text{ kHz}}}$$



$$GP = 6.36 \times 10^6 = A_H \times f_H$$

when  $A_H = 1 \Rightarrow f_H = \underline{\underline{6.36 \text{ kHz}}}$

$$R_L' = \frac{1}{2\pi (6.36 \times 10^6) C_H \times \beta}$$

$\downarrow 1 \times 10^{-12} \quad \downarrow 100$

$$R_L' = \underline{\underline{250 \Omega}}$$

5.159

Refer to Fig. P5.159

$$R_{in} = R_1 \parallel R_2 \parallel (r_x + r_\pi)$$

$$\text{where } R_1 = 33 \text{ k}\Omega, R_2 = 22 \text{ k}\Omega$$

Next →

CONT.

$f_x = 50$  and,

$$f_{\pi} = \frac{\beta_0}{g_m} = \frac{120}{0.3 \times 40} = \frac{120}{12} = 10 \text{ KHz}$$

$$R_{in} = 33 \parallel 22 \parallel 10.05 = \underline{5.7 \text{ KHz}}$$

$$\begin{aligned} A_M &= - \frac{R_{in}}{R_{in} + R_S} \cdot \frac{f_{\pi}}{f_{\pi} + f_x} \cdot g_m (R_C \parallel R_L \parallel R_o) \\ &= - \frac{5.7}{5.7 + 5} \cdot \frac{10}{10 + 0.05} \cdot 12 (4.7 \parallel 5.6 \parallel 300) \\ &= - \underline{16.11 \text{ V/V}} \end{aligned}$$

$$\begin{aligned} R'_{sig} &= f_{\pi} \parallel [r_x + (R_1 \parallel R_2 \parallel R_{sig})] \\ &= 10 \text{ K} \parallel [50 + (33 \parallel 22 \parallel 5) \text{ K}] \\ &= \underline{2.69 \text{ KHz}} \end{aligned}$$

$$\begin{aligned} R'_L &= r_o \parallel R_C \parallel R_L = 300 \parallel 4.7 \parallel 5.6 \text{ (KHz)} \\ &= 2.53 \text{ KHz} \end{aligned}$$

$$C_{\pi} + C_R = \frac{g_m}{2\pi \cdot f_T} = \frac{12 \cdot 10^{-9}}{2\pi \times 700 \cdot 10^6} = 2.73 \text{ pF}$$

$$C_{\pi} = (2.73 - 1) \text{ pF} = 1.73 \text{ pF}$$

$$\begin{aligned} C_{in} &= C_{\pi} + C_R (1 + g_m R'_L) \\ &= 1.73 \text{ pF} + 1 \text{ pF} (1 + 12 \times 2.53) \\ &= 33 \text{ pF} \end{aligned}$$

$$\begin{aligned} f_H &= 1/2\pi C_{in} \cdot R'_{sig} \\ &= 1/2\pi \times 33 \cdot 10^{-12} \times 2.69 \cdot 10^3 \\ &= \underline{1.79 \text{ MHz}} \end{aligned}$$

5.160

$$R_{in} = R_1 \parallel R_2 \parallel f_{\pi}$$

$$\text{where } f_{\pi} = \frac{\beta}{g_m} \text{ and } g_m = \frac{I_C}{V_T}$$

$$g_m = \frac{0.8}{0.025} = 32 \text{ mA/V}$$

$$f_{\pi} = \frac{200}{32} = 6.25 \text{ KHz}$$

$$R_{in} = 68 \parallel 27 \parallel 6.25 = 4.72 \text{ KHz}$$

$$R'_L = R_C \parallel R_L = 4.7 \parallel 10 = 3.2 \text{ KHz}$$

$$A_M = \frac{R_{in}}{R_S + R_{in}} \times -g_m R'_L$$

$$= -\frac{4.72}{10 + 4.72} \times 32 \times 3.2$$

$$= \underline{-32.8 \text{ V/V}}$$

$$C_T = C_{\pi} + C_R (1 + g_m R'_L)$$

$$\text{where } C_{\pi} + C_R = \frac{g_m}{2\pi f_T} = \frac{32 \times 10^{-9}}{2\pi \times 10^9} = 5.1 \text{ pF}$$

$$C_{\pi} = 5.1 - 0.8 = 4.3 \text{ pF}$$

$$C_T = 4.3 + 0.8 (1 + 32 \times 3.2)$$

$$= 87 \text{ pF}$$

The resistance seen by  $C_T$  is  $R_T$ ,

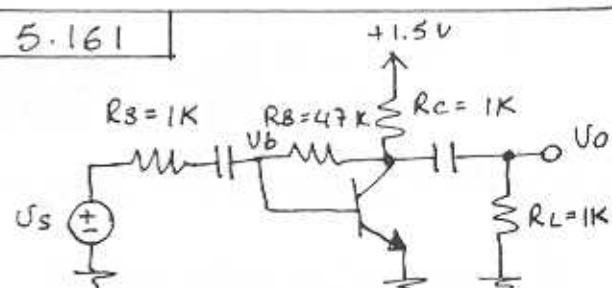
$$R_T = f_{\pi} \parallel R_1 \parallel R_2 \parallel R_S$$

$$= 6.25 \parallel 68 \parallel 27 \parallel 10 = 3.2 \text{ KHz}$$

Thus

$$\begin{aligned} f_H &\approx \frac{1}{2\pi C_T R_T} \\ &= \frac{1}{2\pi \times 87 \times 10^{-12} \times 3.2 \times 10^3} \\ &= \underline{572 \text{ KHz}} \end{aligned}$$

5.161



CONT.

$$(a) 1.5V = 1K(I_C + I_B) + 47K \cdot I_B + 0.7V$$

$$= I_C + \frac{I_C}{\beta} + 47 \frac{I_C}{\beta} + 0.7$$

$$I_C = \frac{0.8}{1 + \frac{48}{\beta}} = \frac{0.8}{1 + \frac{48}{100}} = \underline{\underline{0.54mA}}$$

$$R_{in} = R_i \parallel R_S = 4K \parallel 1K \\ = 0.8K\Omega$$

$$\Rightarrow f_H = \frac{1}{2\pi \times 14.37 \times 10^{-12} \times 0.8 \times 10^3} \\ f_H = \underline{\underline{13.84MHz}}.$$

$$(b) g_m = \frac{I_C}{V_T} = 40 \times 0.54 = \underline{\underline{21.6m\text{A}}} \text{ V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{21.6} = \underline{\underline{4.63K\Omega}}$$

$$(c) V_o \approx -g_m (R_C \parallel R_L) \cdot V_B \\ = -21.6 (1 \parallel 1) V_B \\ = -10.8 V_B$$

$$\Rightarrow \frac{V_o}{V_B} = \underline{\underline{-10.8}}$$

(d) Using Miller's theorem  
to find  $R_i$ :

$$R_i = \frac{47}{1 - \frac{V_o}{V_B}} = \frac{47}{1 + 10.8} = \underline{\underline{4K\Omega}}$$

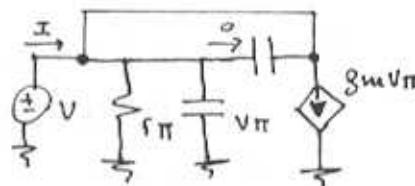
$$(e) AM = \frac{V_B}{V_S} \cdot \frac{V_o}{V_B} = \frac{R_i}{R_S + R_i} \times \frac{V_o}{V_B} \\ = \frac{4K}{1K + 4K} \times -10.8 = \underline{\underline{-8.64}}$$

$$(f) C_{\pi\pi} + C_R = \frac{g_m}{2\pi f_T} = \frac{21.6 \times 10^3}{2\pi \cdot 600 \times 10^6} \\ = 5.73 \text{ pF} \\ C_{\pi\pi} = 5.73 - 0.8 = 4.93 \text{ pF}$$

$$C_{in} = C_{\pi\pi} + C_R (1 + g_m R_i') \\ = 4.93 + 0.8 (1 + 21.6 \times (1 \parallel 1)) \\ = 14.37 \text{ pF}$$

$$(g) f_H = \frac{1}{2\pi \cdot C_{in} \cdot R_{in}}$$

5.162



$$I = \frac{V_S}{r_\pi} + sC_{\pi\pi}V_{\pi\pi} + g_m V_{\pi\pi}$$

$$V_{in} = (g_m + \frac{1}{r_\pi}) + sC_{\pi\pi}$$

$$Z_i = \frac{1}{(g_m + \frac{1}{r_\pi}) + sC_{\pi\pi}}$$

$$= \frac{1}{\frac{1}{r_e} + sC_{\pi\pi}} = \frac{r_e}{1 + sC_{\pi\pi}r_e} \quad //$$

$$f_T = \frac{g_m}{2\pi (C_{\pi\pi} + C_R)}$$

Since  $C_{\pi\pi}$  contains a component that is proportional to the bias current, it follows that at high currents  $C_{\pi\pi} \gg C_R$  and

$$f_T = \frac{g_m}{2\pi C_{\pi\pi}} \approx \frac{1}{2\pi C_{\pi\pi} r_e}$$

Thus,

$$Z_i = \frac{r_e}{1 + s/w_T} \quad (\text{at high currents})$$

The phase angle will be  $-45^\circ$  at  $w = w_T$ , or

$$f = f_T = \underline{\underline{400MHz}}$$

CONT.

For a lower bias current so that  $C_{\pi} = C_R$ ,

$$f_T = \frac{1}{4\pi C_{\pi} R_E}$$

$$\text{and } Z_i = \frac{r_e}{1 + \frac{s}{2\omega_T}}$$

$-45^\circ$  angle is obtained at  $\omega = 2\omega_T$  or  $f = 2f_T$   
 $= \underline{820 \text{ Hz}}$

(Assuming  $f_T$  remains constant which is not necessarily true)

5.163

Refer to Fig. P5.159 and Problem 5.159

$$\omega_L = \omega_{C1} + \omega_{C2} + \omega_{CE}$$

$$\omega_{C1} = \frac{1}{C_{C1} \cdot R_{C1}} \Rightarrow f_{C1} = \frac{1}{2\pi C_{C1} \cdot R_{C1}}$$

$$R_{C1} = R_S + [R_1 || R_2 || (r_x + r_{\pi})] \\ = 5K + [33K || 22K || (50 + 10K)] \\ = 10.7 \text{ k}\Omega$$

$$C_{C1} = 1 \mu F$$

$$\Rightarrow f_{C1} = \frac{1}{2\pi \cdot 1 \times 10^{-6} \times 10.7 \cdot 10^3} \\ = \underline{14.87 \text{ Hz}}$$

$$f_{C2} = 1 / 2\pi \cdot C_{C2} \cdot R_{C2}$$

$$R_{C2} = R_L + (R_C || r_o) \\ = 5.6K + (4.7K || 300K) \\ = \underline{10.23 \text{ k}\Omega}$$

$$C_{C2} = 1 \mu F$$

$$\Rightarrow f_{C2} = \frac{1}{2\pi \times 1 \cdot 10^{-6} \times 10.23 \cdot 10^3} \\ = \underline{15.55 \text{ Hz}}$$

$$f_{CE} = 1 / 2\pi C_E R_E' \\ R_E' = R_E \parallel \left[ \frac{r_{\pi} + r_x + (R_1 || R_2 || R_S)}{\beta_0 + 1} \right] \\ = 3.9 \parallel \left( \frac{10K + 50 + (33 || 22 || 5)K}{121} \right) \\ = 109.8 \text{ }\Omega$$

$$C_E = 10 \mu F$$

$$\Rightarrow f_{CE} = \frac{1}{2\pi \times 10 \times 10^6 \times 109.8} \\ = \underline{144.95 \text{ Hz}}$$

$$f_L = \frac{\omega_L}{2\pi} = 14.87 + 15.55 + 109.8$$

$$f_L = \underline{140.22 \text{ Hz}}$$

5.164

Refer to Fig P5.159 and Problem 5.163.

To select  $C_E$  so that it contributes 90% of the value of  $\omega_L$ :

$$\frac{1}{2\pi C_E \cdot R_E'} = 0.9 \times 100$$

$$R_E' = 109.8 \Omega \text{ (From problem 5.163)}$$

$$\Rightarrow C_E = \frac{1}{2\pi \cdot 109.8 \times 90}$$

$$= 16.1 \mu F \cancel{/}$$

To select  $C_1$  so that it contributes 5% of  $f_L$ :

$$R_{C1} = 10.7 \text{ k}\Omega \text{ (From P5.163)}$$

$$\Rightarrow C_1 = \frac{1}{2\pi \times 10.7 \cdot 10^3 \times 0.05 \times 10^3}$$

$$C_1 = 2.97 \mu F \cancel{/}$$

CONT.

To select  $C_2$  so that it contributes to 5% of  $f_L$ :

$$R_{C2} = 10.23 \text{ k}\Omega \text{ (From P5.163)}$$

$$\Rightarrow C_2 = \frac{1}{2\pi \times 10.23 \cdot 10^3 \times 0.05 \times 100}$$

$$= \underline{3.11 \mu F}$$

5.165

Refer to Fig. P5.159.

$$R_{C1} = R_s + [R_B \parallel (r_x + r_\pi)]$$

$$= 10 + [10 \parallel (0.1 + 1)]$$

$$= 10.99 \text{ k}\Omega$$

$$R_E' = R_E \parallel \frac{r_\pi + r_x + (R_B \parallel R_s)}{\beta_0 + 1}$$

$$\approx 1 \parallel \frac{1 + 0.1 + (10 \parallel 10)}{100 + 1}$$

$$= 57 \Omega$$

For  $CE$  and  $C_{c1}$  to contribute equally to the determination of  $f_L$ ,

$$CER_E' = C_{c1} \cdot R_{C1}$$

$$\Rightarrow \frac{CE}{C_{c1}} = \frac{R_{C1}}{R_E'} = \frac{10.99}{0.057} = \underline{193}$$

5.166

Refer to Fig P5.166.

(a) At midband frequencies

$$I_B = \frac{V_s}{R_s + r_\pi}$$

$$I_c = \beta \cdot I_B = \frac{\beta V_s}{R_s + r_\pi}$$

$$V_o = -I_c \cdot (R_C \parallel R_L)$$

$$= -\frac{\beta (R_C \parallel R_L) \cdot V_s}{R_s + r_\pi}$$

$$AM = \frac{V_o}{V_s} = -\frac{\beta (R_C \parallel R_L)}{R_s + r_\pi}$$

(b) Pole due to  $CE$ :

$$W_{PE} = \frac{1}{CE (r_e + \frac{R_s}{\beta + 1})} //$$

Pole due to  $CC$ :

$$W_{PC} = \frac{1}{C_C (R_C + R_L)} //$$

zeros are both at  $\underline{s=0}$

$$(c) A(s) = AM \cdot \frac{s^2}{(s + W_{PE})(s + W_{PC})}$$

$$A(s) = -\frac{\beta (R_C \parallel R_L)}{R_s + r_\pi} \cdot \frac{s^2}{\left[ s + \frac{1}{CE(r_e + \frac{R_s}{\beta + 1})} \right] \times }$$

$$\left[ s + \frac{1}{C_C(R_C + R_L)} \right]$$

$$(d) AM = -\frac{100 (10 \parallel 10)}{10 + \frac{100}{40}}$$

$$= \underline{-40} \text{ V/V}$$

(e) Since the resistance that forms the pole  $W_{PE}$  is very small, we choose to make  $W_{PE}$  the dominant pole, thus:

$$f_{PE} = f_L = 100 = \frac{1}{2\pi C_E (25 + 10k)}$$

$$\Rightarrow C_E = \frac{1}{2\pi \times 100 \times (0.025 + 0.100) \cdot 10^3}$$

$$= \underline{12.7 \mu F}$$

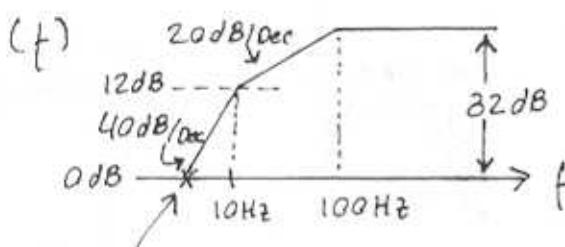
$$f_{PC} = 10 \text{ Hz} \Rightarrow$$

CONT.

$$10 \text{ Hz} = \frac{1}{2\pi C_C (R_L + R_C)}$$

$$\Rightarrow C_C = \frac{1}{2\pi \times 10 (10+10) \times 10^3}$$

$$= \underline{\underline{0.8 \mu F}}$$



Unity-gain frequency must be an octave lower than 10 Hz.  
i.e. at 5 Hz

$$(g) A(j\omega) = -A_M \frac{\omega^2}{(\omega_{PE} + j\omega)(\omega_{PC} + j\omega)}$$

$$= +40 \frac{\omega^2}{(\omega_{PE} + j\omega)(\omega_{PC} + j\omega)}$$

$$\text{Thus } \phi = \tan^{-1}\left(\frac{\omega}{\omega_{PE}}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{PC}}\right)$$

$$= -\left[\tan^{-1}\frac{f}{f_{PE}} + \tan^{-1}\frac{f}{f_{PC}}\right]$$

$$= -\left[\tan^{-1}\frac{f}{100} + \tan^{-1}\frac{f}{10}\right]$$

Thus at  $f = 100 \text{ Hz}$ ,

$$\phi = -[\tan^{-1} 1 + \tan^{-1} 10]$$

$$\simeq -\underline{\underline{129.3^\circ}}$$

5.167

Refer to Fig. P5.167

$$(a) I_e = \frac{V_s}{R_e + R_{et} + \frac{1}{sC_E}}$$

$$I_C \approx I_e$$

$$V_o = -R_C I_C = \frac{-R_C}{R_e + R_{et} + \frac{1}{sC_E}} \cdot V_s$$

$$A(s) \equiv \frac{V_o}{V_s} = \frac{-R_C}{R_e + R_{et} + \frac{1}{sC_E}}$$

$$= \frac{-R_C}{R_e + R_{et}} \cdot \frac{s}{s + \frac{1}{C_E(R_e + R_{et})}}$$

$$\text{Thus, } A_M = \frac{-R_C}{R_e + R_{et}}$$

$$W_L = \frac{1}{C_E(R_e + R_{et})}$$

$$(b) A_M \text{ is reduced by the factor } \frac{R_e}{R_e + R_{et}}$$

$$= 1 + \frac{R_{et}}{R_e}$$

$$(c) W_L \text{ is reduced by the factor } \left(1 + \frac{R_{et}}{R_e}\right)$$

which is the same as the gain reduction factor. Thus, the value of  $R_{et}$  can be used as the parameter for exercising the gain-bandwidth trade off.

$$(d) R_{et} = 0:$$

$$|A_M| = \frac{R_C}{R_e} = \frac{10,000}{25} = \underline{\underline{400 \text{ V/V}}}$$

$$f_L = \frac{1}{2\pi C_E R_e} = \frac{1}{2\pi \times 100 \times 10^{-6} \times 25}$$

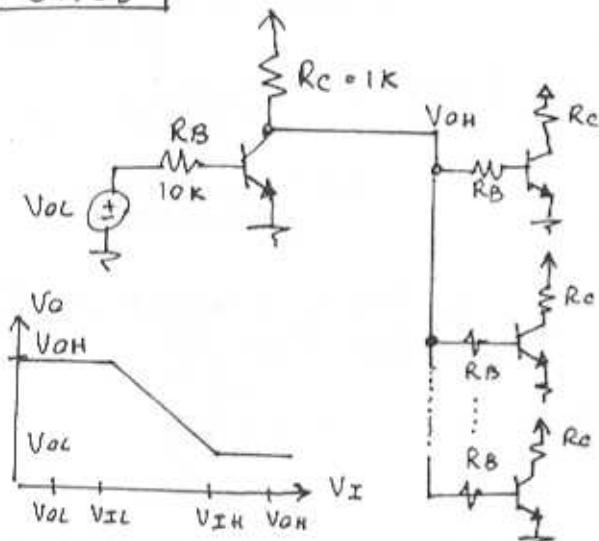
$$= \underline{\underline{63.7 \text{ Hz}}}$$

To lower  $f_L$  by a factor of 5 use:

CONT.

$R_e = 4R_e = 100\Omega$ . The gain is also lowered by a factor of 5 to 80 V/V

5.168



$$\begin{aligned} V_{IH} &= V_{BE} + I_B (EoS) R_B \\ &= 0.7 + \left[ \frac{(V_{cc} - V_{CEsat})}{R_C} \right] / 100 \cdot R_B \\ &= 0.7 + \frac{5 - 0.2}{100} \times 10 \\ &= 1.18V \end{aligned}$$

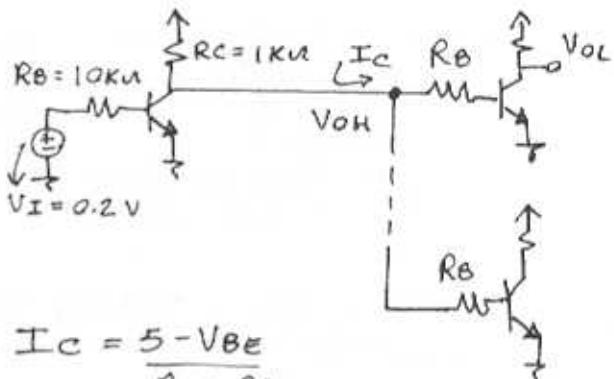
For  $NM_H = V_{OH} - V_{IH} = 1$   
 $V_{OH} = 1 + 1.18 = 2.18V$

$V_{OH} = V_{cc} - R_C \cdot \frac{V_{cc} - V_{BE}}{R_C + R_B}$   
 $= 5 - 1 \left( \frac{5 - 0.7}{1 + \frac{10}{N}} \right) > 2.18$

$$N < 19.05 \Rightarrow N = 19$$

5.169

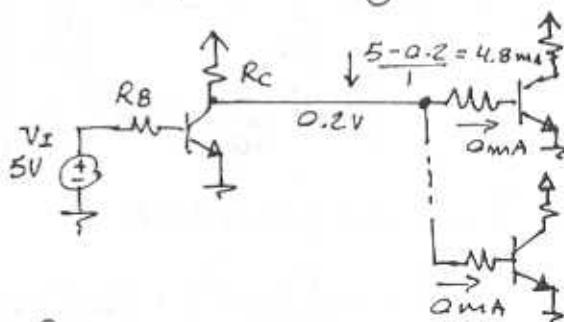
(a) Input is low:



$$\begin{aligned} I_C &= \frac{5 - V_{BE}}{R_C + R_B} \\ &= \frac{5 - 0.7}{1 + 10/10} = 2.15mA \end{aligned}$$

$$\begin{aligned} PD_{V_{ILOW}} &= I_C^2 \cdot R_C = 2.15^2 \times 1 \\ &= 4.62mW \end{aligned}$$

(b) Input is high:

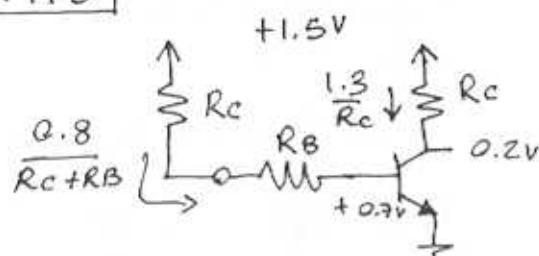


Power dissipated neglecting the base circuit

$$\begin{aligned} PD_{V_{IHIGH}} &= V_{cc} I_C \\ &= 5 \times 4.8 = 24mW \end{aligned}$$

(c)  $PD_{AVG} = \frac{1}{2} (4.62 + 24)$   
 $= 14.31mW$

5.170



$$\beta_{\text{Forced}} = 10 = \frac{1.3/R_C}{0.8/R_C + R_B} \quad ①$$

$$\frac{80}{13} = \frac{R_C + R_B}{R_C} = 1 + \frac{R_B}{R_C}$$

$$\frac{R_B}{R_C} = \frac{67}{13} \Rightarrow R_B = 5.15 R_C \quad ②$$

Total current from supply

$$= \frac{1.3}{R_C} + \frac{0.8}{R_C + R_B}$$

$$\text{From Eqn. ① } \frac{0.8}{R_C + R_B} = \frac{1.3/R_C}{10}$$

$$\begin{aligned} \text{Thus } I_{\text{TOTAL}} &= \frac{1.3}{R_C} + \frac{1}{10} \frac{1.3}{R_C} \\ &= \frac{1.1 \times 1.3}{R_C} \end{aligned}$$

$$\begin{aligned} P_{D\text{TOTAL}} &= I_{\text{TOTAL}} \times 1.5V \\ &= \frac{1.1 \times 1.3}{R_C} \times 1.5 = 1 \mu\text{W} \end{aligned}$$

$$R_C = \frac{1.1 \times 1.3 \times 1.5}{1} = 2.145 \text{ k}\Omega$$

$$\text{Choose } R_C = \underline{\underline{2.2 \text{ k}\Omega}}$$

$$R_B = 5.15 \times 2.145 = \underline{\underline{11 \text{ k}\Omega}}$$

5.171

Refer to Fig. P5.171

$V_X$	$V_Y$	$V_Z$
0.2	0.2	5
0.2	5	0.2
5	0.2	0.2
5	5	0.2

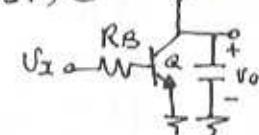
These are 4 input combinations. When any input is high, ( $V_X$  and/or  $V_Y$ ) high,  $V_Z = \underline{\underline{0.2V}}$

When both inputs are low, ( $V_X$  and  $V_Y$ ) low  $\rightarrow V_Z$  is high

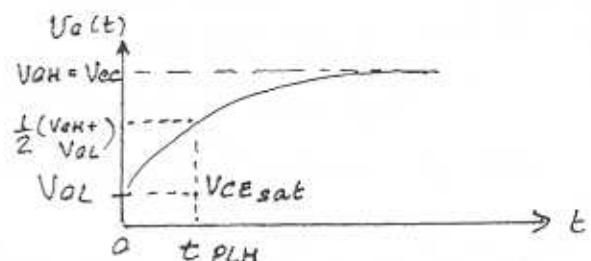
5.172

$$U_{o(t)} = U_{\infty} - (U_{\infty} - U_{o+}) e^{-t/\tau}$$

$U_{\infty} = V_{cc}$   
 $U_{o+} = V_{CE\text{sat}}$   
 $\tau = C \cdot R_C$   
 Thus,  
 $U_{o(t)} = V_{cc} - (V_{cc} - V_{CE\text{sat}}) e^{-t/\tau}$

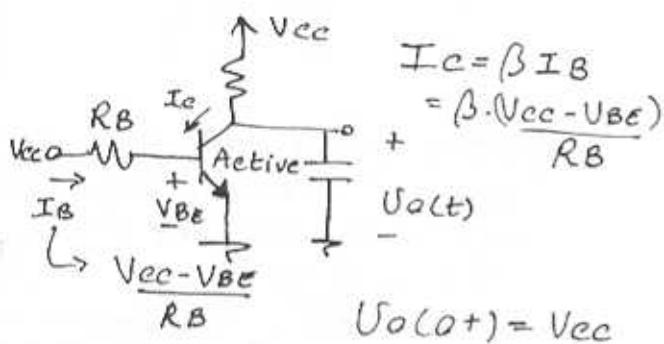


$$\begin{aligned} U_{o(t)} &= 5 - (5 - 0.2) e^{-t/(0.10 \times 10^{-12} \times 1 \times 10^3)} \\ \frac{1}{2} (V_{OH} + V_{OL}) &= 2.6V \\ 2.6 &= 5 - 4.8 e^{-t_{PLH}/10^{-8}} \\ t_{PLH} &= 0.69 \times 10^{-8} \text{ s.} \\ t_{PLH} &= 6.9 \text{ ns} \end{aligned}$$

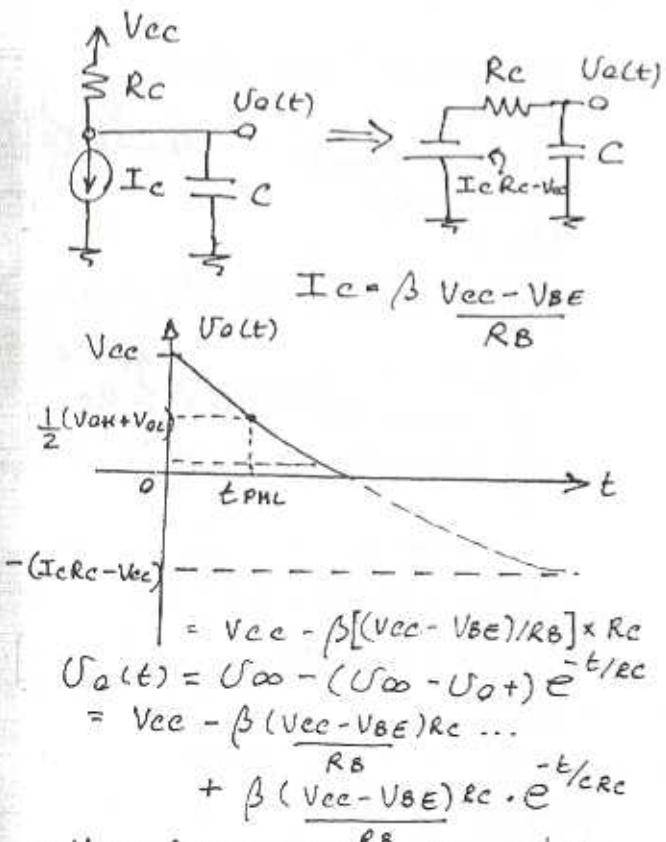


5.173

At  $t = 0^+$



Thus the equivalent circuit for the capacitor discharge will be:



$$U_o(t) = U_{\infty} - (U_{\infty} - U_{o+}) e^{-t/\tau_{RC}}$$

$$= U_{\infty} - \beta \frac{(V_{CC} - V_{BE})}{R_B} R_C e^{-t/\tau_{RC}}$$

$$= U_{\infty} - \beta \frac{(V_{CC} - V_{BE})}{R_B} \frac{R_B}{R_B + R_C} (1 - e^{-t/\tau_{RC}})$$

$$\text{At } t = t_{PHL}, U_o = \frac{1}{2} (U_{oH} + U_{oL})$$

$$= \frac{1}{2} (V_{CC} + V_{CEsat})$$

$$= \frac{1}{2} (5 + 0.2) = 2.6V$$

Thus,

$$2.6 = 5 - 50 (5 - 0.7) \frac{1}{10} (1 - e^{-t_{PHL}/\tau_{RC}})$$

$$1 - e^{-t_{PHL}/\tau_{RC}} = \frac{0.48}{4.3}$$

$$t_{PHL} = 0.118 \times 10 \times 10^{-12} \times 1 \times 10^3$$

$$= \underline{\underline{1.18 \text{ ns}}}$$

Comparing this value to that of  $t_{PLH}$  found in Problem 5.172 we observe that  $t_{PHL}$  is much smaller than  $t_{PLH}$ .

$$t_p = \frac{1}{2} (t_{PLH} + t_{PHL})$$

$$= \frac{1}{2} (6.9 + 1.18) = \underline{\underline{4 \text{ ns}}}$$

6.3

## Chapter 6 - Problems

6.1

Assume a 0.18- $\mu\text{m}$  CMOS process and refer to Table 6.1:

$$\mu_n C_{ox} = 387 \frac{\mu\text{A}}{\text{V}^2} \quad \text{Also assume } \frac{W}{L} = 10$$

Assuming operation in saturation mode:

$$V_{ov} = 0.15 \text{ V} \Rightarrow I_D = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 387 \times 0.15^2 \times 10$$

$$I_D = 43.5 \mu\text{A}$$

$$V_{ov} = 0.4 \text{ V} \Rightarrow I_D = 309.6 \mu\text{A}$$

Therefore:

$$0.15 \leq V_{ov} \leq 0.4 \Rightarrow 43.5 \mu\text{A} \leq I_D \leq 309.6 \mu\text{A}$$

Now if we consider the same range of current for  $I_C$  of a BJT and we assume an n-p-n transistor in a standard high-voltage process: (refer to Table 6.2)

$$I_S = 5 \times 10^{-15} \text{ A} = 5 \times 10^{-9} \mu\text{A}$$

$$I_C = I_S e^{\frac{V_{BE}}{V_T}} \Rightarrow V_{BE} = V_T \ln \frac{I_C}{I_S}$$

$$I_C = 43.5 \mu\text{A} \Rightarrow V_{BE} = 0.025 \ln \frac{43.5}{5 \times 10^{-9}} = 0.572 \text{ V}$$

$$I_C = 309.6 \mu\text{A} \Rightarrow V_{BE} = 0.621 \text{ V}$$

Therefore:

$$43.5 \mu\text{A} \leq I_C \leq 309.6 \mu\text{A} \Rightarrow 0.572 \leq V_{BE} \leq 0.621 \text{ V}$$

6.2

If the area of the emitter-base junction is changed by a factor of 10, then  $I_S$  is changed by the same factor. If  $V_{BE}$  is kept constant, then  $I_C$  is also changed by the same factor:

$$I_C = I_S e^{\frac{V_{BE}}{V_T}} \quad I_S \propto A, \quad I_C \propto I_S \Rightarrow I_C \propto A$$

$$A_2 = 10 A_1 \Rightarrow I_{C2} = 10 I_{C1}$$

If  $I_C$  is kept constant, then  $V_{BE}$  changes:

$$I_{S2} = 10 I_{S1} \Rightarrow I_S e^{\frac{V_{BE1}}{V_T}} = 10 I_S e^{\frac{V_{BE2}}{V_T}}$$

$$e^{\frac{V_{BE1} - V_{BE2}}{V_T}} = 10 \Rightarrow V_{BE1} - V_{BE2} = V_T \ln 10 = 0.058 \text{ V}$$

$$\text{or } 58 \text{ mV}$$

$$\frac{W}{L} = 10, \quad I_D = 100 \mu\text{A}, \quad I_D = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2$$

	0.8 $\mu\text{m}$ NMOS	0.5 $\mu\text{m}$ PMOS	0.25 $\mu\text{m}$ NMOS	0.18 $\mu\text{m}$ PMOS		
$V_{ov}$	0.4	-0.59	0.32	-0.54	0.27	-0.46
$V_{GS}$	1.1	-1.29	1.02	-1.34	0.7	-1.08

$$V_{ov} = \sqrt{\frac{2 I_D}{K_n \frac{W}{L}}} = \sqrt{\frac{2 \times 100}{K_n \times 10}} = \sqrt{\frac{20}{K_n}}$$

$$V_{GS} = V_E + V_{ov}$$

6.4

$$|V_{ov}| = 0.25 \text{ V}, \quad I_D = 100 \mu\text{A}$$

$$I_D = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2 \Rightarrow \frac{W}{L} = \frac{2 I_D}{K_n V_{ov}^2}$$

For NMOS:

$$K_n = 267 \frac{\mu\text{A}}{\text{V}^2} = \gamma \left( \frac{W}{L} \right)_n = \frac{2 \times 100}{267 \times 0.25^2} = 11.98 \text{ --}$$

For PMOS:

$$K_p = 93 \frac{\mu\text{A}}{\text{V}^2} \Rightarrow \left( \frac{W}{L} \right)_p = \frac{2 \times 100}{93 \times 0.25^2} = 34.4 \text{ --}$$

6.5

$$i_{on} = i_{Dp} \Rightarrow \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_p V_{ovp} = \frac{1}{2} \mu_p C_{ox} \left( \frac{W}{L} \right)_p V_{ovp}^2$$

We also have  $g_{mn} = g_{mp}$ :

$$g_m = \frac{2 I_D}{V_{ov}} \Rightarrow V_{ovn} = V_{ovp} \quad (2)$$

$$(1), (2) \Rightarrow \left( \frac{W}{L} \right)_p = \frac{\mu_n}{\mu_p} = \frac{460}{160} = 2.88$$

6.6

$$g_m = 10 \text{ mA/V} \Rightarrow V_{ov} = 0.2 \text{ V}$$

$$g_m = \frac{2 I_D}{V_{ov}} \Rightarrow I_D = \frac{g_m V_{ov}}{2} = \frac{10 \times 0.2}{2} = 1 \text{ mA}$$

$$I_D = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2 \Rightarrow \frac{W}{L} = \frac{2 I_D}{K_n V_{ov}^2} = \frac{2 \times 1 \times 10^3}{387 \times 0.2^2} = 12 \frac{W}{L}$$

$$W = 129 \text{ --}$$

$$\text{For an n-p-n transistor: } g_m = \frac{I_C}{V_T} \Rightarrow I_C = 10 \times 0.025 = 0.25 \text{ mA}$$

6.7

$$\frac{W}{L} = 10, I_D = 100 \text{ mA}$$

NMOS	PMOS	NMOS	PMOS	NMOS	PMOS	NMOS	PMOS	
0.8 μm	0.5 μm	0.25 μm	0.18 μm					
g <sub>m</sub> (mA/V)	0.5	0.34	0.62	0.37	0.73	0.43	0.88	0.41

$$g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} = \sqrt{\mu_n C_{ox}} \sqrt{2 \times 10 \times 100} = 44.7 \sqrt{\mu_n C_{ox} \frac{mA}{V}}$$

6.8

$$V_{ov} = 0.25 \text{ V}$$

For an npn transistor:  $g_m = \frac{I_c}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$   
 For an NMOS with the same  $g_m$ , i.e.  $g_m = 4 \text{ mA/V}$   
 we will have:  $g_m = \frac{2I_D}{V_{ov}} \Rightarrow I_D = g_m \times \frac{V_{ov}}{2} = 0.5 \text{ mA}$

6.9

Assuming large  $r_0$ . For both transistors, for case (a) we have  $r = \frac{1}{g_m} = \frac{1}{\sqrt{2\mu C_{ox} \frac{W}{L} I_D}}$

$$r = \frac{10^3}{\sqrt{2 \times 200 \times 10 \times 0.1 \times 10^3}} = 1.58 \text{ kΩ}$$

For case (b) we have  $r = r_{pi} \parallel \frac{1}{g_m} = \frac{\beta}{(\beta+1)g_m}$

$$r = \frac{\beta V_T}{(\beta+1) I_c} \approx \frac{V_T}{I_c} = \frac{0.025}{0.1} = 0.25 \text{ kΩ}$$

$$r = 250 \text{ Ω}$$

6.10

$$g_m = \frac{2I_D}{V_{ov}} = \frac{2 \times 100 \times 10^{-3}}{0.5} = 0.4 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_D} = \frac{V_A L}{I_D} = \frac{25 \times 1}{0.1} = 250 \text{ kΩ}$$

$$A_f = g_m r_0 = 0.4 \times 250 = 100 \text{ V/V}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{ov} \Rightarrow W = \frac{g_m \times L}{\mu_n C_{ox} \times V_{ov}} = \frac{0.4 \times 1}{12.7 \times 10^3 \times 0.5}$$

$$W = 6.3 \text{ μm}$$

6.11

$$L = 0.3 \text{ μm}, I_D = 100 \text{ mA}, V_{ov} = 0.2 \text{ V}$$

$$g_m = \frac{2I_D}{V_{ov}} = \frac{2 \times 100 \times 10^{-3}}{0.2} = 1 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_D} = \frac{V_A \times L}{I_D} = \frac{5 \times 0.3}{0.1} = 15 \text{ kΩ}$$

$$A_f = g_m r_0 = 1 \times 15 = 15 \text{ V/V}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{ov} \Rightarrow W = \frac{g_m \times L}{\mu_n C_{ox} V_{ov}} = \frac{1 \times 0.3}{12.7 \times 10^3 \times 0.2}$$

$$W = 3.88 \text{ μm}$$

6.12

$$\text{BJT: } \beta = 100, V_A = 100 \text{ V}$$

$$\text{MOSFET: } \mu_n C_{ox} = 200 \text{ μA/V}^2, \frac{W}{L} = 40, V_A = 10 \text{ V}$$

Device Bias current (mA)	BJT		MOSFET	
	0.1	I <sub>c</sub>	0.1	I <sub>D</sub>
g <sub>m</sub> (mA/V)	4	40	1.26	4
r <sub>0</sub> (kΩ)	1000	100	100	10
A <sub>f</sub> (V/V)	4000	4000	126	40
R <sub>i</sub> (kΩ)	25	2.5	∞	∞

$$R_i = r_{pi} = \frac{\beta}{g_m} \text{ for BJT}$$

6.13

$$L = 0.3 \text{ μm}, W = 6 \text{ μm}, V_{ov} = 0.2 \text{ V}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 12.7 \times \frac{6}{0.3} \times 0.2^2 = 155 \text{ mA}$$

$$I_D = 0.155 \text{ mA}$$

$$g_m = \frac{2I_D}{V_{ov}} = 1.55 \text{ mA/V}$$

$$C_{gs} = \frac{2}{3} \frac{W}{L} C_{ox} + C_{ov} = \frac{2}{3} W L C_{ox} + W L_{ov} C_{ox}$$

$$C_{gs} = \frac{2}{3} \times 6 \times 0.3 \times 8.6 + 6 \times 0.37 = 12.54 \text{ fF}$$

$$C_{gd} = C_{ov} W = 0.37 \times 6 = 2.22 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{1.55 \times 10^{-3}}{2\pi(12.54 + 2.22) \times 10^{-15}} = 16.7 \text{ GHz}$$

If we use the approximation formula:

$$f_T \approx \frac{1.5 \mu_n V_{ov}}{2\pi L^2} \text{ when } C_{gs} \gg C_{gd}, C_{gs} = \frac{2}{3} W L C_{ox}$$

$$f_T \approx \frac{1.5 \times 450 \times 10^{-4} \times 0.2}{2\pi \times 0.3^2 \times 10^{12}} = 23.9 \text{ GHz}$$

The approximation formula overestimates

Cont.

$f_T$  because it ignores  $W_{AV}C_{ox}$  or  $C_{ov}$  in  $C_{gs}$  and  $C_{g2}$  calculation.

6.14

$$L=0.3\mu m \quad W=6\mu m \quad V_{OV}=0.2 \quad C_L=100\text{fF}$$

$$A_0 = \frac{2VAL}{V_{OV}} = \frac{2 \times 5 \times 0.3}{0.2} = 15 \text{V/V}$$

$$I_D = \frac{1}{2} K_n \frac{W}{L} V_{OV}^2 = \frac{1}{2} \times 38 \times \frac{6}{0.3} \times 0.2^2 = 0.155 \text{mA}$$

$$r_o = \frac{V_{OL}}{I_D} = \frac{5 \times 0.3}{0.155} = 9.7 \text{k}\Omega$$

$$f_p = \frac{1}{2\pi C_L r_o} = \frac{1}{2\pi \times 9.7 \times 100 \times 10^{-12}} = 164.2 \text{MHz}$$

$$g_m = \frac{2I_D}{V_{OV}} = 1.55 \text{mA/V}$$

$$f_T = \frac{g_m}{2\pi C_L} = 2.5 \text{GHz}$$

In order to double  $f_T$  or equivalently double  $g_m \sqrt{I_D}$  has to be doubled or  $I_D$  has to be multiplied by 4:  $g_m = \sqrt{2K_n C_{ox} \frac{W}{L} I_D} \Rightarrow f_T \propto g_m \propto \sqrt{I_D}$

$$I_D = 4 \times 0.155 = 0.62 \text{mA}$$

In that case:  $A_0 \propto \frac{1}{V_{OV}} \propto \frac{1}{\sqrt{I_D}} \Rightarrow A_0 = 15 \times \frac{1}{2} = 7.5 \text{V/V}$

$$f_p \propto \frac{1}{r_o} \propto I_D \Rightarrow f_p = 164.2 \times 4 \Rightarrow$$

$$f_p = 656.8 \text{MHz}$$

6.15

$I_C = 10 \mu A$ , High-Voltage process:

$$g_m = \frac{I_C}{V_T} = \frac{10 \times 10^{-3}}{0.025} = 0.4 \text{mA/V}$$

$$C_{de} = C_F g_m = 0.35 \times 10^{-9} \times 0.4 \times 10^{-3} = 140 \times 10^{-15} \text{F} = 140 \text{fF}$$

$$C_{je} = 2C_{jeo} = 2 \times 1 = 2 \text{pF} = 2000 \text{fF}$$

$$C_{II} = C_{de} + C_{je} = 2140 \text{fF}$$

$$C_L = C_{de} = 0.3 \text{pF} = 300 \text{fF}$$

$$f_T = \frac{g_m}{2\pi(C_{II} + C_L)} = \frac{0.4 \times 10^{-3}}{2\pi(2140 + 300) \times 10^{-15}} = 26.1 \text{MHz}$$

$I_C = 100 \mu A$ , High-Voltage process:

$$g_m = 10 \times 0.4 = 4 \text{mA/V} \Rightarrow C_{de} = 10 \times 40 = 400 \text{fF}$$

$$C_{II} = 3400 \text{fF} \Rightarrow f_T = \frac{4 \times 10^{-3}}{2\pi(3400 + 300) \times 10^{-15}} = 172.1 \text{MHz}$$

$I_C = 10 \mu A$ , Low-Voltage process

$$g_m = \frac{10 \times 10^{-3}}{0.025} = 0.4 \text{mA/V}$$

$$C_{de} = 10 \times 10^{-12} \times 0.4 \times 10^{-3} = 4 \text{fF}$$

$$C_{je} = 2 \times 5 \text{fF} = 10 \text{fF}$$

$$C_{II} = C_{de} + C_{je} = 14 \text{fF}$$

$$C_L = C_{de} = 5 \text{fF}$$

$$f_T = \frac{g_m}{2\pi(C_{II} + C_L)} = \frac{0.4 \times 10^{-3}}{2\pi(14 + 5) \times 10^{-15}} = 3.35 \text{GHz}$$

$I_C = 100 \mu A$ , Low-Voltage process

$$g_m = \frac{100 \times 10^{-3}}{0.025} = 4 \text{mA/V}$$

$$C_{de} = 10 \times 4 = 40 \text{fF}$$

$$C_{II} = 40 + 10 = 50 \text{fF} \Rightarrow C_L = 5 \text{fF}$$

$$f_T = \frac{4 \times 10^{-3}}{2\pi(50 + 5) \times 10^{-15}} = 11.6 \text{GHz}$$

In Summary:

	Standard High-Voltage n-p-n	Standard low-Voltage n-p-n
$I_C = 10 \mu A$	$I_C = 100 \mu A$	$I_C = 10 \mu A$
$f_T = 26.1 \text{MHz}$	$f_T = 172.1 \text{MHz}$	$f_T = 3.35 \text{GHz}$

6.16

$L = 1 \mu m$ ,  $I_D = 100 \mu A$ ,  $0.8 \mu m$ -NMOS

a)  $V_{OV} = 0.25V$

$$I_D = \frac{1}{2} K_n \frac{W}{L} V_{OV}^2 \Rightarrow W = \frac{2L I_D}{M_n C_{ox} V_{OV}^2} = \frac{2 \times 1 \times 100}{127 \times 0.25^2} \Rightarrow W = 25.2 \mu m$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 100 \times 10^{-3}}{0.25} = 0.8 \text{mA/V}$$

$$r_o = \frac{V_{OL}}{I_D} = \frac{2.5 \times 1}{0.1} = 25 \text{k}\Omega$$

$$A_0 = g_m r_o = 20 \text{V/V}$$

$$C_{GS} = \frac{2}{3} WL C_{ox} + WL_{BV} C_{ox} = \frac{2}{3} \times 25.2 \times 1 \times 2.3 + 25.2 \times 0.1$$

$$C_{GS} = 43.68 \text{fF} \approx 44 \text{fF}$$

$$C_{GD} = WL_{BV} C_{ox} = 25.2 \times 0.2 = 5.04 \text{fF}$$

$$f_T = \frac{g_m}{2\pi(C_{GS} + C_{GD})} = \frac{0.8 \times 10^{-3}}{2\pi(44 + 5.04) \times 10^{-15}} = 2.6 \text{GHz}$$

b)  $f_T \propto g_m \propto V_{OV}$  therefore in order to double  $f_T$ ,

$V_{OV}$  has to be doubled:  $V_{OV} = 0.5V$ . Consequently,

$W = 25.2 / 4 = 6.3 \mu m$  and  $r_o, C_{GS}, C_{GD}$  unchanged

$$g_m = \frac{0.8}{2} = 0.4 \text{mA/V} \Rightarrow A_0 = \frac{20}{2} = 10 \text{V/V}$$

6.17

$$I_C = 1 \text{ mA} \Rightarrow g_m = \frac{I_C}{V_T} = 40 \text{ mA/V}$$

For pnp:

$$C_{de} = C_f g_m = 30^n \times 40^m = 1200 \text{ pF}$$

$$C_{je} = 2C_{je0} = 2 \times 0.3 = 0.6 \text{ pF}$$

$$C_H = 1200.6 \text{ pF}$$

$$C_M \approx 1 \text{ pF}$$

$$\left. \begin{aligned} \Rightarrow F_T &= \frac{g_m}{2\pi(C_H + C_M)} = \frac{40^m}{2\pi(1200.6 + 1) \text{ pF}} \\ F_T &= 5.3 \text{ MHz} \end{aligned} \right\}$$

For npn:

$$C_{de} = C_f g_m = 0.35^n \times 40^m = 14 \text{ pF}$$

$$C_{je} = 2 \times 1 = 2 \text{ pF}$$

$$C_M \approx 0.3 \text{ pF}$$

$$C_H = 14 + 2 = 16 \text{ pF}$$

$$\left. \begin{aligned} \Rightarrow F_T &= \frac{40^m}{2\pi(16 + 0.3) \text{ pF}} = 3.91 \text{ MHz} \end{aligned} \right\}$$

6.18

$$A_0 = g_m r_o = \frac{2 I_D}{V_{ov}} \times \frac{V_A}{I_D} = \frac{2 V_A}{V_{ov}} = \frac{2 V_A L}{V_{ov}}$$

Therefore  $A_0$  is only determined by setting values for  $L$  and  $V_{ov}$ .

$$F_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{2 I_D / V_{ov}}{2\pi(\frac{2}{3}WL_{ox} + C_{ov} + C_{ov})}$$

If we assume that  $C_{ov}$  is very small or equivalently  $C_{gs} \gg C_{gd}$  and  $C_{gs} = \frac{2}{3}WL_{ox}$ :

(replace  $I_D$  with  $\frac{1}{2}K_n \frac{W}{L} V_{ov}^2$ )

$$F_T \approx \frac{K_n \frac{W}{L} V_{ov}}{2\pi \times \frac{2}{3}WL_{ox}} = \frac{3}{4\pi} K_n \frac{V_{ov}}{L^2} = \frac{3}{4\pi} K_n \frac{V_{ov}}{L^2}$$

As we can see  $F_T$  can be determined after knowing  $V_{ov}$  and  $L$ , it is not dependent on either  $I_D$  or  $W$ .

6.19

$$V_{ov} = 0.2 \text{ V}, L = 0.2 \mu\text{m}, 0.3 \mu\text{m}, 0.4 \mu\text{m}$$

$$A_0 = g_m r_o = \frac{2 V_A}{V_{ov}} = \frac{2 V_A L}{V_{ov}} = \frac{2 \times 5 \times L}{0.2} \approx 50 L \text{ V/V}$$

$$F_T \approx \frac{1.5 K_n V_{ov}}{2\pi L^2} = \frac{1.5 \times 450 \times 10^{-4} \times 0.2}{2\pi \times 3.14 \times L^2 \times 10^{12}} = \frac{2.15}{L^2} \text{ GHz}$$

$L (\mu\text{m})$	0.2	0.3	0.4
$A_0 (\text{V/V})$	10	15	20
$F_T (\text{GHz})$	53.75	23.9	13.4

6.20

$$L = 0.5 \mu\text{m}, V_{ov} = 0.3 \text{ V}, C_L = 1 \text{ pF}, F_T = 100 \text{ MHz}$$

$$F_T = \frac{g_m}{2\pi C_L} \Rightarrow g_m = 2\pi C_L F_T = 2\pi \times 1 \times 100 \text{ MHz} = 628 \text{ mA/V}$$

$$g_m = \frac{2 I_D}{V_{ov}} \Rightarrow I_D = g_m \times V_{ov} / 2 = 628 \times \frac{0.3}{2} \text{ mA}$$

$$I_D = 94.2 \text{ mA}$$

$$I_D = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2 \Rightarrow W = \frac{2 L I_D}{K_n V_{ov}^2} = \frac{2 \times 0.5 \times 94.2}{190 \times 0.3^2} = 5.51 \mu\text{m}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A L}{I_D} = \frac{20 \times 0.5}{94.2 \times 10^3} = 106.2 \text{ k}\Omega$$

$$A_0 = g_m r_o = \frac{628}{1000} \times 106.2 = 66.7 \text{ V/V}$$

$$f_{3db} = \frac{1}{2\pi C_L r_o} = \frac{1}{2\pi \times 1^2 \times 106.2^2} = 1.5 \text{ MHz}$$

6.21

$$I_D = I_{REF} = 50 \text{ mA}, L = 0.5 \mu\text{m}, W = 5 \mu\text{m}, V_E = 0.5 \text{ V}$$

$$I_o = I_D = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2 \quad K_n' = 250 \text{ mA/V}^2$$

$$50 = \frac{1}{2} \times 250 \times \frac{5}{0.5} (V_{GS} - 0.5)^2 \Rightarrow V_{GS} = 0.7 \text{ V}, 0.3 \text{ V}$$

$V_{GS} = 0.3 \text{ V}$  ( $V_t$  is not acceptable, therefore)

$$V_{GS} = 0.7 \text{ V}$$

$$I_D = I_{REF} = \frac{V_{DD} - V_{GS}}{R} \Rightarrow \frac{1.8 - 0.7}{R} = 0.050 \Rightarrow R = 22 \text{ k}\Omega$$

$Q_1$  and  $Q_2$  have the same  $V_{GS}$ . The lowest value of  $V_0$  or  $V_{DS2}$  is when  $V_{DS} = V_{GS} - V_E = 0.7 - 0.5 = 0.2 \text{ V}$

hence  $V_{min} = 0.2 \text{ V}$

$$r_o = \frac{V_A}{I_D} = \frac{V_A L}{I_D} = \frac{20 \times 0.5}{0.05} = 200 \text{ k}\Omega$$

$$\Delta I_D \approx \frac{\Delta V_{GS}}{r_o} = \frac{1}{200} = 5 \text{ mA} \Rightarrow \Delta I_D = 5 \text{ mA}$$

6.22

$$\mu_n C_{ox} = 250 \text{ nF/V}^2, V_A = 20V, V_t = 0.6V$$

$$\frac{\Delta I_D}{I_0} = 5 \Rightarrow \Delta I_D = 5 \text{ mA} \quad \text{For } \Delta V_o = 1.8 - 0.25 = 1.55 \text{ V}$$

$$r_o = \frac{\Delta V_o}{\Delta I_D} = \frac{1.55}{5 \mu} = 310 \text{ k}\Omega$$

$$r_o = \frac{V_A L}{I_0} \Rightarrow L = I_D \times \frac{r_o}{V_A} = 0.1 \times \frac{310}{20} = 1.55 \mu\text{m}$$

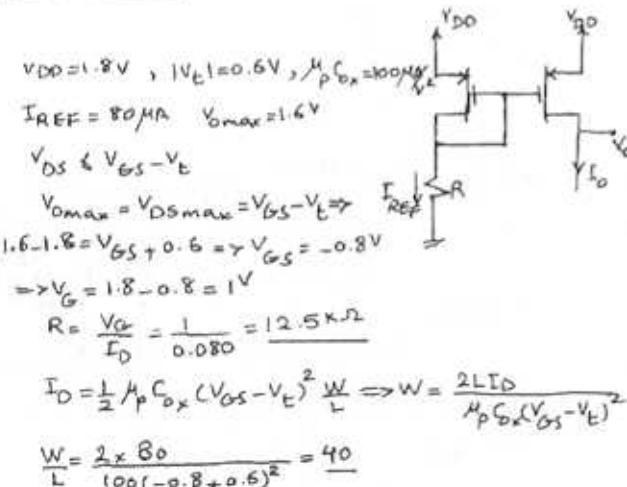
$$V_{omin} = V_{GS} - V_t = 0.25 \Rightarrow V_{GS} = 0.25 + 0.6 = 0.85 \text{ V}$$

$$R = \frac{V_{DD} - V_{GS}}{I_D} = \frac{1.8 - 0.85}{0.1} = 9.5 \text{ k}\Omega$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow W = \frac{2 L I_D}{\mu_n C_{ox} (V_{GS} - V_t)^2}$$

$$\Rightarrow W = \frac{2 \times 1.55 \times 100}{250 (0.85 - 0.6)^2} = 19.84 \mu\text{m}$$

6.23



6.24

$$W_2 = 4W_1, L_1 = L_2, V_{ov} = 0.3 \text{ V}, I_{REF} = 20 \mu\text{A}$$

$$I_0 = I_{REF} \frac{(W/L)_2}{(W/L)_1} = 20 \times 4 = 80 \mu\text{A}$$

$$V_{omin} = V_{ov} = 0.3 \text{ V}$$

$$V_t = 0.5 \text{ V}. \quad \text{According to Eq. 6.11} \quad I = \frac{(W/L)_2}{(W/L)_1} I_{REF} \frac{(1 + V_t - V_{GS})}{V_{A2}}$$

$$V_{ov} = V_{GS} - V_t \Rightarrow V_{GS} = 0.3 + 0.5 = 0.8 \text{ V}$$

$$1 + \frac{V_t - V_{GS}}{25} = 1 \Rightarrow V_t = 0.8 \text{ V}$$

Or we could simply say  $V_{DS1} = V_{DS2} = V_o$  and

$$\text{Since } V_{DS1} = V_{DS2} = 0.8 \text{ V} \Rightarrow V_o = 0.8 \text{ V}$$

$$r_{o2} = \frac{V_A}{I_{D2}} = \frac{25}{0.08} = 312.5 \text{ k}\Omega$$

$$r_{o2} = \frac{\Delta V_o}{\Delta I_o} = \frac{1}{\Delta I_o} \Rightarrow \Delta I_o = \frac{1}{312.5 \text{ k}} = 3.2 \mu\text{A}$$

6.25

Refer to Fig. P6.25

$$V_{GS1} = V_{GS2} \Rightarrow \frac{I_{D2}}{I_{D1}} = \frac{(W/L)_2}{(W/L)_1} \Rightarrow I_{D2} = I_{REF} \frac{(W/L)_2}{(W/L)_1}$$

$$I_{D2} = I_{D3}$$

$$V_{GS3} = V_{GS4} \Rightarrow \frac{I_{D3}}{I_{D4}} = \frac{(W/L)_2}{(W/L)_4} \Rightarrow \frac{I_{D2}}{I_{D4}} = \frac{(W/L)_3}{(W/L)_4}$$

$$I_{D4} = I_{REF} \frac{(W/L)_2}{(W/L)_1} \frac{(W/L)_4}{(W/L)_3} = I_o$$

6.26

Refer to Fig. P6.26:

$$V_{DS2} \leq V_{GS2} - V_t \Rightarrow V_{DSmax} = V_{GS2} - V_t$$

$$(1.3 - 1.5) = V_{GS2} - (-0.6) \Rightarrow V_{GS2} = -0.8 \text{ V} = V_{GS1}$$

For  $Q_1$ :

$$I_{D1} = 20 \text{ mA} = \frac{1}{2} \times 80 \times \frac{W_1}{0.8} \times (-0.8 + 0.6)^2$$

$$\Rightarrow W_1 = 10 \mu\text{m}$$

$$I_2 = 100 \mu\text{A} = 5 I_{REF} \Rightarrow W_2 = 5 \text{ V} = 50 \mu\text{m}$$

$$I_3 = I_{REF} \Rightarrow W_3 = W_1 = 10 \mu\text{m}$$

$$I_3 = I_4 \Rightarrow \frac{W_3}{W_4} = \frac{W_1}{W_2} \Rightarrow W_4 = 10 \times \frac{80}{200} = 4 \mu\text{m}$$

$$\text{For } Q_5: V_{DS5} = V_{GS5} - V_t \quad \text{For lowest } V_o \\ (-1.3 - (-1.5)) = V_{GS5} - 0.6 \Rightarrow V_{GS5} = 0.8 \text{ V}$$

$$I_5 = 50 \mu\text{A} = \frac{1}{2} \times 200 \times \frac{W_5}{0.8} \times (0.8 - 0.6)^2 \Rightarrow W_5 = 10 \mu\text{m}$$

Now we calculate  $R$ :

$$V_{GS2} = -0.8 \text{ V} \Rightarrow V_{G2} = 1.5 - 0.8 = 0.7 \text{ V}$$

$$R = V_{G2}/I_{REF}$$

$$R = \frac{0.7}{20 \mu\text{A}} = 35 \text{ k}\Omega$$

$$R = 35 \text{ k}\Omega$$

$$G_2 = \frac{V_A L}{I_2} = \frac{12 \times 0.8}{0.1} = 96 \text{ k}\Omega \Rightarrow r_{o5} = \frac{10 \times 0.8}{0.05} = 160 \text{ k}\Omega$$

6.27

IF the transistor with  $w=10$  is diode-connected,  
then:  $I_2 = 100 \times \frac{20}{10} = 200 \mu A$   
 $I_3 = 100 \times \frac{40}{10} = 400 \mu A$

IF the transistor with  $w=20$  is diode-connected  
then:  $I_2 = 100 \times \frac{10}{20} = 50 \mu A$   
 $I_3 = 100 \times \frac{40}{20} = 200 \mu A$

IF the transistor with  $w=40$  is diode-connected,  
then:  $I_2 = 100 \times \frac{10}{40} = 25 \mu A$   
 $I_3 = 100 \times \frac{20}{40} = 50 \mu A$

So for cases that only one transistor is diode connected, 4 different output currents are possible (depending on the configuration we choose).

IF 2 transistors are diode-connected, then they act as an equivalent transistor whose width is the sum of the widths of each transistor:

IF  $W_{eff} = 10+20$  then  $I_o = 100 \times \frac{40}{30} = 133 \mu A$

IF  $W_{eff} = 20+40$  then  $I_o = 100 \times \frac{10}{60} = 16.7 \mu A$

IF  $W_{eff} = 40+10$  then  $I_o = 100 \times \frac{20}{50} = 40 \mu A$

So 3 different output currents are possible depending on which two transistors are diode-connected. Now we calculate  $V_{SG}$ :

$100 = \frac{1}{2} \times 80 \times \frac{30}{1} (V_{SG} - 0.7)^2 \Rightarrow V_{SG} = 1^V$  for  $w=30$   
all have the same  $V_{SG}$  for any given configuration.

For  $W_{eff}=60 \Rightarrow 100 = \frac{1}{2} \times 80 \times \frac{60}{1} (V_{SG} - 0.7)^2$   
 $\Rightarrow V_{SG} = 0.9V$

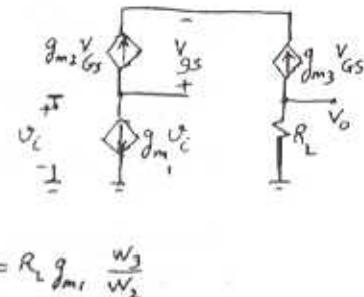
for  $W_{eff}=50 \Rightarrow 100 = \frac{1}{2} \times 80 \times \frac{50}{1} (V_{SG} - 0.7)^2$   
 $\Rightarrow V_{SG} = 0.93V$

6.28

$$-g_{m2} V_{GS} = g_{m1} V_i \\ \Rightarrow V_{GS} = -\frac{g_{m1}}{g_{m2}} V_i$$

$$V_o = R_L g_{m3} V_{GS} \\ V_o = R_L g_{m3} \frac{-g_{m1} V_i}{g_{m2}}$$

$$\frac{V_o}{V_i} = +R_L g_{m3} \frac{g_{m3}}{g_{m2}} = R_L g_{m3} \frac{w_3}{w_2}$$



Note that since  $Q_2, Q_3$  have the same  $V_{GS}$  and therefore the same  $V_{ov}$ :  $\frac{g_{m3}}{g_{m2}} = \frac{I_3}{I_2} = \frac{w_3}{w_2}$

6.29

$$I_s = 10^{-15} A$$

$$a) I_{REF} = I_s e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln \frac{I_{REF}}{I_s}$$

$$I_{REF} = 10 \mu A \Rightarrow V_{BE} = 0.025 \ln \frac{10 \times 10^{-6}}{10^{-15}} = 0.576 V$$

$$I_{REF} = 10 mA \Rightarrow V_{BE} = 0.025 \ln \frac{10 \times 10^{-3}}{10^{-15}} = 0.748 V$$

Therefore:

$$10 \mu A \leq I_{REF} \leq 10 mA \Rightarrow 0.576 V \leq V_{BE} \leq 0.748 V$$

Since  $\beta$  is very high,  $I_B$  is negligible and hence  $I_o \approx I_{REF}$ :  $10 \mu A \leq I_o \leq 10 mA$

$$b) I_o = I_{REF} \frac{1}{1 + 2/\beta} \quad (\text{Eq. 6.21})$$

For  $0.1 \leq I_o \leq 5 mA$ ,  $\beta$  remains constant at 100.

$$I_{REF} = 10 mA \Rightarrow I_o = \frac{10}{1 + 2/100} = 9.72 mA$$

$$I_{REF} = 0.1 mA \Rightarrow I_o = \frac{0.1}{1 + 2/100} = 0.098 mA$$

$$I_{REF} = 1 mA \Rightarrow I_o = \frac{1}{1 + 2/100} = 0.98 mA$$

$$I_{REF} = 10 \mu A \Rightarrow$$

6.30

$$I_{S2} = I_{S1} \times m, \quad I_{C1} = I_C \\ I_{REF} = I_C + \frac{I_C}{\beta} + \frac{I_O}{\beta} \quad (1)$$

$$V_{BE1} = V_{BE2} \Rightarrow \\ V_T \ln \frac{I_C}{I_{S1}} = V_T \ln \frac{I_O}{I_{S2}}$$

$$\Rightarrow \frac{I_O}{I_C} = \frac{I_{S2}}{I_{S1}} = m \Rightarrow I_C = I_O/m$$

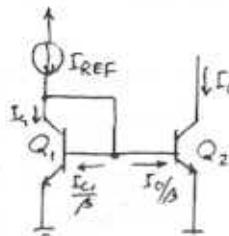
by substituting for  $I_C$  in (1):

$$I_{REF} = \frac{I_O}{m} + \frac{I_O}{m\beta} + \frac{I_O}{\beta} \Rightarrow \frac{I_O}{I_{REF}} = \frac{m}{1 + \frac{1}{\beta} + \frac{m}{\beta}} \\ \frac{I_O}{I_{REF}} = \frac{m}{1 + \frac{1+m}{\beta}}$$

This result is the same as Eq. 6.22.

For large  $\beta$ ,  $\frac{I_O}{I_{REF}} = m$ , with finite  $\beta$  this ratio drops to  $\frac{I_O}{I_{REF}} = \frac{m}{1 + \frac{1+m}{\beta}}$ . To keep the introduced error within 5%:  $0.95m = \frac{m}{1 + \frac{1+m}{\beta}}$

$$\beta_{min} = 80 \Rightarrow 0.95 = \frac{1}{1 + \frac{1+m}{80}} \Rightarrow m = 3.21$$

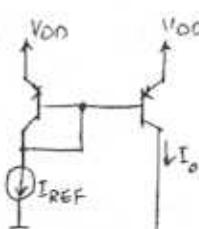


6.31

The transfer ratio is the same as Eq. 6.21:

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + \frac{2}{\beta}}$$

$$\beta = 20 \Rightarrow \frac{I_O}{I_{REF}} = \frac{1}{1 + \frac{2}{20}} = 0.91$$



6.32

$$I_O = I_{REF} = 2 \text{ mA}$$

$$r_{O2} = \frac{V_{A2}}{I_O} = \frac{90}{2} = 45 \text{ k}\Omega$$

$$r_{O2} = \frac{\Delta V_O}{\Delta I_O} \Rightarrow \frac{10 - 1}{\Delta I_O} = 45 \Rightarrow \underline{\Delta I_O = 0.2 \text{ mA}}$$

$$\frac{\Delta I_O}{I_O} = \frac{0.2}{2} = 10\% \text{ change}$$

6.33

$$I_S = 10^{-15} \text{ A}, \quad \beta = 50$$

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + \frac{2}{\beta}} \Rightarrow I_{REF} = I_O \left(1 + \frac{2}{\beta}\right) = I_O \left(1 + \frac{2}{50}\right) \\ I_{REF} = 1.02 \text{ mA}$$

$$V_{BE} = V_T \ln \frac{I_O}{I_S (1 + \frac{2}{\beta})} = 0.025 \frac{10^{-3}}{10^{15} (1 + \frac{2}{50})} = 0.689 \text{ V}$$

$$V_C = V_B = 5 - 0.689 = 4.31 \text{ V}$$

$$V_C = R \times I_{REF} \Rightarrow R = \frac{4.31}{1.02} = 4.2 \text{ k}\Omega$$

$$r_O = \frac{V_A}{I_O} = \frac{50}{1} = 50 \text{ k}\Omega$$

$V_{omax}$  occurs when  $Q_2$  is on the edge of saturation or  $V_{CE} = 0.3 \text{ V}$ . Therefore  $V_{omax}$  is  $5 - 0.3 = 4.7 \text{ V}$

$$r_O = \frac{\Delta V_O}{\Delta I_O} \Rightarrow \frac{4.7 - (-5)}{\Delta I_O} = 50 \text{ k} \Rightarrow \Delta I_O = 0.194 \text{ mA}$$

$$\frac{\Delta I_O}{I_O} \times 100 = \frac{0.194}{1} \times 100 = 19.4\% \text{ change in } I_O$$

6.34

$$I_{C1} = I_{C2} = I_{R1}$$

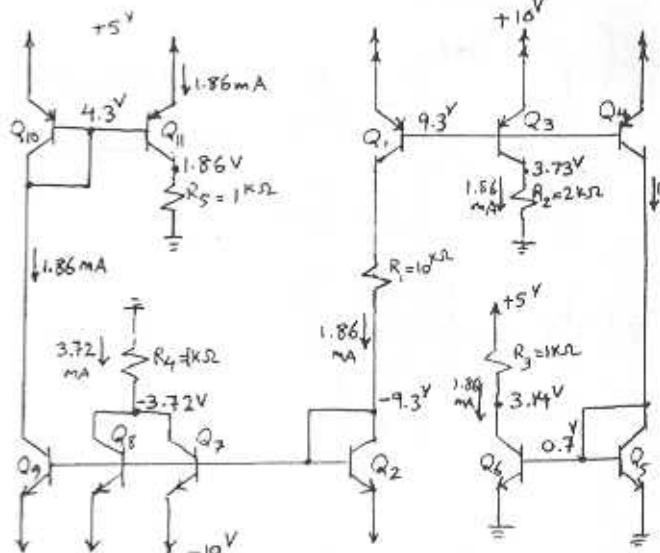
$$V_{B1} = 10 - 0.7 = 9.3 \text{ V}, \quad V_{B2} = -10 + 0.7 = -9.3 \text{ V}, \quad I_{R1} = \frac{9.3 + 9.3}{10} = 1.86 \text{ mA}$$

$$\Rightarrow I_{R1} = 1.86 \text{ mA} = I_{C1} = I_{C2} = I_{C3} = I_{C4} = I_{C5} = I_{C6}$$

$$V_{C3} = 1.86 \times 2^k = 3.72 \text{ V}, \quad V_{C5} = 0.7 \text{ V}$$

$$V_{C6} = 5 - 1.86 \times 1 = 3.14 \text{ V} \Rightarrow I_{C9} = I_{C8} = I_{C7} = I_{C2} = 1.86 \text{ mA}$$

$$I_{R4} = 2 \times 1.86 = 3.72 \text{ mA} \Rightarrow V_{C2} = -3.72 \times 1 = -3.72 \text{ V}$$



Cont.

$$I_{C10} = I_{C9} = 1.86 \text{ mA}$$

$$V_{C9} = V_{C10} = V_{B10} = 5 - 0.7 = 4.3 \text{ V}$$

$$I_{C11} = I_{C10} = 1.86 \text{ mA}$$

$$V_{C11} = 1.86 \times 1 = 1.86 \text{ V}$$

6.35

a) Refer to Fig. P6.35

$$R = 10 \text{ k}\Omega$$

$$V_i = -0.7 \text{ V} \Rightarrow I_{C1} = \frac{-0.7 - (-10.7)}{10^4} = 1 \text{ mA}$$

$$I_{C1} = 1 \text{ mA}$$

$$V_2 = 5.7 - 0.7 = 5 \text{ V}$$

$$I = I_{C3} + I_{C4}, I_{C3} = I_{C4} = I_{C1} \Rightarrow I = 2 \times 1 = 2 \text{ mA}$$

$$V_3 = 0 + 0.7 = 0.7 \text{ V}$$

$$V_4 = -10.7 + 1 \times 10^4 = -0.7 \text{ V}$$

$$V_5 = -10.7 + 1 \times \frac{10^4}{2} = -5.7 \text{ V}$$

b)  $R = 100 \text{ k}\Omega$

$$V_i = -0.7 \text{ V} \Rightarrow I_{C1} = \frac{-0.7 + 10.7}{100 \text{ k}} = 0.1 \text{ mA}$$

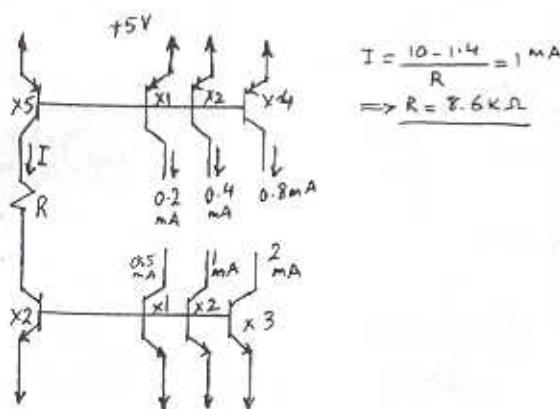
$$I = 2 I_{C1} = 0.2 \text{ mA}$$

$$V_3 = 0.7 \text{ V}, V_2 = 5.7 - 0.7 = 5 \text{ V}$$

$$V_4 = -10.7 + \frac{1}{10} \times 100 = -0.7 \text{ V}$$

$$V_5 = -10.7 + 0.1 \times \frac{100}{2} = -5.7 \text{ V}$$

6.36



6.37

a) Refer to Fig. P6.37.

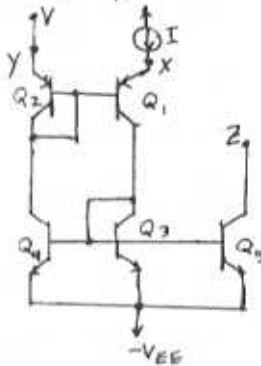
Since  $Q_1, Q_3$  have equal currents,  $I_1$ , and  $Q_3$  and  $Q_4$  have equal  $V_{BE}$  which results in equal currents; then  $Q_2$  has the same current as  $Q_1$ .

$$\text{Therefore: } V_{BE1} = V_{BE2} \rightarrow V_x = V_y = V$$

$$\text{Also, } V_{BE5} = V_{BE3}$$

$$\Rightarrow I_5 = I_3 = I$$

$$\text{or } I_2 = I.$$



$$\text{b) } I_2 = I_4, I_1 = I_3$$

$$\text{Since } V_{BE4} = V_{BE3} \text{ then } I_3 = I_4 \text{ and hence } I_1 = I_2.$$

$$\text{From } I_1 = I_2 \text{ we conclude}$$

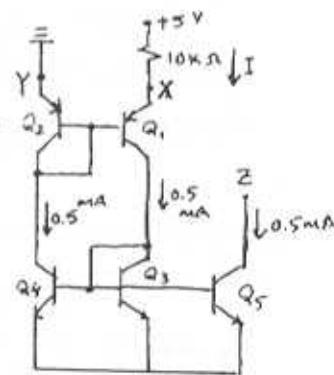
$$\text{that } V_{BE1} = V_{BE2} \text{ and}$$

$$\text{since } V_{B1} = V_{B2} \text{ then } V_{E1}$$

$$\text{has to be the same as}$$

$$V_{E2}: V_{E1} = V_{E2} = 0$$

$$V_X = 0$$



$$I = \frac{5.0}{10k} = 0.5 \text{ mA}$$

6.38

$$40 \text{ dB} = 20 \log A_0 \Rightarrow A_0 = 100 \text{ V/V}$$

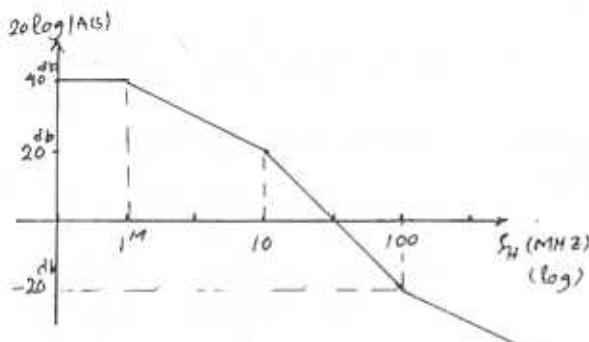
$$A(S) = +100 \frac{(1 + \frac{S}{100 \times 10^6 \times 2\pi})}{(1 + \frac{S}{2\pi \times 10^7})(1 + \frac{S}{2\pi \times 10^6})}$$

$$A(S) = +100 \frac{(1 + \frac{S}{2\pi \times 10^8})}{(1 + \frac{S}{2\pi \times 10^7})(1 + \frac{S}{2\pi \times 10^6})}$$

$$\text{Eq. 6.36: } \omega_H = \sqrt{\left(\frac{1}{2\pi \times 10^7}\right)^2 + \left(\frac{1}{2\pi \times 10^6} - 2\left(\frac{1}{2\pi \times 10^8}\right)^2\right)}$$

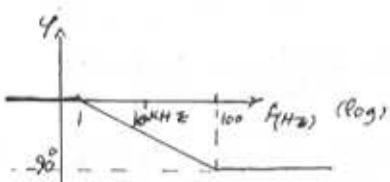
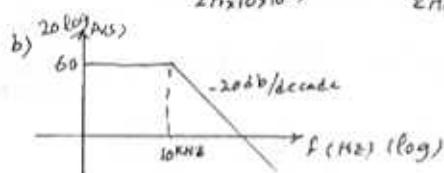
$$\omega_H = 0.995 \text{ MHz}$$

Cont.



6.39

$$a) A(s) = \frac{1000}{(1 + \frac{s}{2\pi \times 10^6})^2} = \frac{1000}{(1 + \frac{s}{2\pi \times 10^6})^2}$$



6.40

$$f_H(s) = \frac{1}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})} \quad \omega_p < \omega_{p2}$$

using dominant-pole approximator:  $\omega_H \approx \omega_{p1}$

using the root sum of squares formula:

$$\omega_H \approx \sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2}} = \sqrt{\frac{\omega_{p1}^2}{\omega_{p1}^2 + \omega_{p2}^2}}$$

The difference between the two estimates for  $\omega_H$  is:

$$\Delta \omega_H = \omega_{p1} - \sqrt{\frac{\omega_{p1}^2}{\omega_{p1}^2 + \omega_{p2}^2}}$$

IF  $n = \frac{\omega_{p2}}{\omega_{p1}}$ :

$$\frac{\omega_{p1} \Delta \omega_H}{\omega_{p1}} = 1 - \frac{1}{\sqrt{1 + n^2}}$$

$$\text{for } \frac{\Delta \omega_H}{\omega_{p1}} = 10\% = 0.1 \Rightarrow n = 2.07$$

$$\text{for } \frac{\Delta \omega_H}{\omega_{p1}} = 1\% = 0.01 \Rightarrow n = 7.02$$

6.41

$$A(s) = -100 \frac{1 + \frac{s}{10^6}}{(1 + \frac{s}{10^5})(1 + \frac{s}{10^7})}$$

$$a) \omega_H \approx 10^5 \text{ rad/s}$$

$$b) \omega_H \approx \sqrt{\frac{1}{(10^5)^2} + \left(\frac{1}{10^7}\right)^2 - 2\left(\frac{1}{10^5}\right)^2} = 101 \text{ Krad/s}$$

IF the pole at  $10^6 \text{ rad/s}$  is lowered to  $10^5 \text{ rad/s}$  the transfer function becomes:

$$A(s) = \frac{-100}{1 + \frac{s}{10^7}} \Rightarrow f_H = \frac{10^7}{2\pi} \text{ Hz}$$

6.42

$$30^\circ = 3 \tan^{-1} \frac{\omega}{\omega_p} = 3 \tan^{-1} \frac{10^6}{\omega_p} \Rightarrow \omega_p = 5.67 \times 10^6 \text{ rad/s}$$

6.43

$$\omega_H \approx \frac{1}{C_{gs} + C_{gd}} = \frac{1}{C_{gs}R_{gs} + C_{gd}R_{gd}}$$

$$\omega_H = \frac{1}{C_{gs}R' + C_{gd}(R' + R_L' + g_m R_L' R')} \quad (\text{From Example 6.1})$$

For  $C_{gs} = C_{gd} = 1 \text{ pF}$ ,  $R_L' = 3.33 \text{ k}\Omega$ ,  $g_m = 4 \text{ mA/V}$

$$\omega_H = \frac{1}{10^{12}R' + 10^{12}(R' + 3.33 \times 10^3 + 4 \times 3.33 \times R')}$$

To obtain  $\omega_H = 2\pi \times 150 \times 10^3$

$$2\pi \times 150 \times 10^3 = \frac{10^{12}}{3.33 \times 10^3 + 15.32 R'} \Rightarrow R' = 69.04 \text{ k}\Omega$$

$$R = R' // R_{in} = R' // 4.20^k = 69.04 \Rightarrow R = 82.6 \text{ k}\Omega$$

6.44

Pole at  $\frac{1}{2\pi \times 10^4 \times 5 \times 10^{12}} = 3.18 \text{ MHz}$  and  
cont

pole at  $\frac{1}{2\pi \times 20 \times 10^3 \times 2 \times 10^{12}} = 3.98 \text{ MHz}$

Since both poles are relatively close together, we use the root-sum-of-squares formula:

$$f_H = \frac{1}{\sqrt{\frac{1}{3.18^2} + \frac{1}{3.98^2}}} = 2.5 \text{ MHz}$$

Manufactured design:

Poles at  $\frac{1}{2\pi \times 10^4 \times 15 \times 10^{12}} = 1.06 \text{ MHz}$  and at

$$\frac{1}{2\pi \times 20 \times 10^3 \times 12 \times 10^{12}} = 0.66 \text{ MHz}$$

$$f_H = \frac{1}{\sqrt{\frac{1}{1.06^2} + \frac{1}{0.66^2}}} = 0.56 \text{ MHz}$$

6.45

Refer to solution of Example 6.6:

$$A_M = \frac{V_o}{V_i} = - \frac{R_{in}}{R_{in} + R} g_m R'_L = - \frac{1.2}{1.2 + 0.1} (2 \times 12)$$

$$A_M = -22.2 V/V$$

$$R_{gs} = R_{in} // R = 1.2 // 0.1 = 92.3 k\Omega$$

$$C_{gs} = R_{gs} C_{gs} = 1 \times 10^{-12} \times 92.3 \times 10^3 = 92.3 \text{ nS}$$

$$R_{gd} = R' + R'_L + g_m R'_L R' \text{ where } R' = R_{in} // R = 92.3 k\Omega$$

$$R_{gd} = 92.3 + 12 + 2 \times 12 \times 92.3 = 2.32 M\Omega$$

$$C_{gd} = C_{gd} R_{gd} = 1 \times 10^{-12} \times 2.32 \times 10^6 = 2320 \text{ nS}$$

$$\omega_H = \frac{1}{C_{gs} + C_{gd}} = \frac{1}{(92.3 + 2320) \times 10^{-9}} = 414.5 \text{ rad/s}$$

$$f_H = 66 \text{ kHz}$$

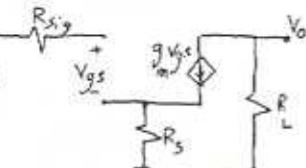
6.46

a)  $V_o = -g_m R_L V_{gs} \quad (1)$

$$V_{gs} = V_{sig} - R_S \times g_m V_{gs}$$

$$V_{gs}(1 + g_m R_S) = V_{sig}$$

$$(1) \Rightarrow V_o = \frac{-g_m R_L}{1 + g_m R_S} V_{sig} \Rightarrow \frac{V_o}{V_{sig}} = \frac{-g_m R_L}{1 + g_m R_S}$$



b)  $V_S = (g_m V_x - i_x) R_S \quad (2)$

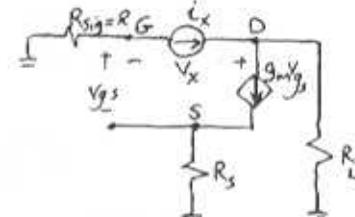
$V_G = V_S + V_x = (g_m V_x - i_x) R_S + V_x \quad (3)$

$\Rightarrow i_x = \frac{V_G}{R} = \frac{(1 + g_m R_S) V_x - i_x R_S}{R} \quad (4)$

$R_{gs} = \frac{V_x}{i_x} = \frac{1 + R_S/R}{1 + g_m R_S} = \frac{R + R_S}{1 + g_m R_S} \quad (R \text{ is } R_S/g)$

To calculate  $R_{gd}$ :

$$\begin{aligned} V_G &= -R i_x \\ V_S &= -R i_x - V_{gs} \\ V_S &= R_S \times g_m V_{gs} \end{aligned} \Rightarrow$$



$$R_S g_m V_{gs} = -R i_x - V_{gs} \Rightarrow V_{gs} = \frac{-R i_x}{1 + g_m R_S}$$

$$\text{At D: } i_x = g_m V_{gs} + \frac{V_X - R_S i_x}{R_L}$$

$$\text{Substitute } V_{gs}: i_x = -\frac{g_m R_S i_x}{1 + g_m R_S} + \frac{V_X}{R_L} - \frac{R_S i_x}{R_L}$$

$$i_x \left[ 1 + \frac{g_m R_S}{1 + g_m R_S} + \frac{R}{R_L} \right] = \frac{V_X}{R_L}$$

$$R_{gd} = \frac{V_X}{i_x} = R_L + R + \frac{g_m R_R L}{1 + g_m R_S} \quad (R \text{ is } R_S/g)$$

c)  $R_S = 0$ :

$$\frac{V_o}{V_{sig}} = \frac{-4 \times 5}{1 + 4 \times 0.1} = -20 V/V$$

$$R_{gs} = R = 100 k\Omega$$

$$R_{gd} = 5 + 100 + 4 \times 5 \times 100 = 2105 k\Omega$$

$$\omega_H \approx \frac{1}{C_{gs} R_{gs} + C_{gd} R_{gd}} = \frac{1}{10^{-12} \times 100 \times 10^3 + 10^{-12} \times 2105 \times 10^3}$$

$$\omega_H \approx 453.5 \text{ rad/s}$$

$$|\text{Gain}| \times \text{Bandwidth} = 20 \times 453.5 = 9.07 \text{ Mrad/s}$$

$R_S = 100 \Omega$ :

$$\frac{V_o}{V_{sig}} = \frac{-4 \times 5}{1 + 4 \times 0.1} = -14.3 V/V$$

$$R_{gs} = \frac{100 + 0.1}{1 + 4 \times 0.1} = 71.5 k\Omega$$

$$R_{gd} = 5 + 100 + \frac{4 \times 5 \times 100}{1 + 4 \times 0.1} = 1533.6 k\Omega$$

$$\omega_H = \frac{1}{10^{-12} \times 71.5 \times 10^3 + 10^{-12} \times 1533.6 \times 10^3} = 623 \text{ rad/s}$$

Cont.

$$|\text{Gain}| \times \text{Bandwidth} = 14.3 \times 623^k = 8.91 \text{ Mrad/s}$$

$$R_S = 250 \Omega$$

$$\frac{V_o}{V_{S,ig}} = \frac{-4 \times 5}{1 + 4 \times 0.25} = -10 \text{ V/V}$$

$$R_{gs} = \frac{100 + 0.25}{1 + 4 \times 0.25} = 50.1 \text{ k}\Omega$$

$$R_{gd} = 5 + 100 + \frac{4 \times 5 \times 100}{1 + 4 \times 0.25} = 1105 \text{ k}\Omega$$

$$\omega_H = \frac{1}{10^{-12} \times 50.1 \times 10^3 + 10^{-12} \times 1105 \times 10^3} = 865.7 \text{ rad/s}$$

$$|\text{Gain}| \times \text{Bandwidth} = 10 \times 865.7^k = 8.66 \text{ Mrad/s}$$

Summary table:

$R_S^{(n)}$	Gain (V/V)	$\omega$ (rad/s)	Gain-BW product (Mrad/s)
0	-20	453.5	9.07
100	-14.3	623.0	8.91
250	-10	865.7	8.66

The Gain x Bandwidth is approximately constant.

6.47

$$A_M = \frac{V_o}{V_{S,ig}} = -\frac{R_{in}}{R_{in} + R_{S,ig}} (g_m R'_L) = -\frac{5}{5+1} (0.3 \times 100^k)$$

$$A_M = -25 \text{ V/V} \quad \text{Now refer to Example 6.6.}$$

$$R_{gs} = R_{in} \parallel R_{S,ig} = 5 \text{ M}\Omega \parallel 1 \text{ M}\Omega = 0.83 \text{ M}\Omega$$

$$T_{gs} = R_{gs} C_{gs} = 0.2 \times 10^{-12} \times 0.83 \times 10^6 = 166.7 \text{ ns}$$

$$R_{gd} = R' + R'_L + g_m R'_L R' \\ R' = R_{in} \parallel R_{S,ig} = 0.83 \text{ M}\Omega \quad \Rightarrow R_{gd} = 0.83 + 0.1 + 0.83 \times 0.3 \times 100$$

$$R_{gd} = 25.92 \text{ M}\Omega$$

$$T_{gd} = C_{gd} R_{gd} = 25.92 \times 10^6 \times 0.1 \times 10^{-12} = 2592 \text{ ns}$$

$$\omega_H \approx \frac{1}{T_{gs} + T_{gd}} = \frac{1}{(166.7 + 2592) \times 10^{-9}} = 362.5 \text{ rad/s}$$

$$f_H = 57.7 \text{ kHz}$$

6.48

If we assume that capacitors are perfect open circuits for midband, then:

$$A_M = \frac{V_o}{V_{S,ig}} = \frac{-R_{in}}{R_{in} + R_{S,ig}} (g_m R'_L) = \frac{-650}{650 + 150} (5 \times 10) = 40.6 \text{ V/V}$$

$$C_{gs} = C_{gs} R_{gs} = C_{gs} (R_{in} \parallel R_{S,ig}) = 2^P \times (150^F / 1650 \text{ K})$$

$$T_{gs} = 243.75 \text{ ns}$$

$$T_{gd} = C_{gd} R_{gd}, \text{ Refer to Example 6.6}$$

$$R_{gd} = R' + R'_L + g_m R'_L R' \Rightarrow R_{gd} = 121.9 + 10 + 5 \times 10 \times \\ 121.9 \\ R' = 150^F / 1650 \text{ K} = 121.9 \text{ K}\Omega$$

$$R_{gd} = 6.2 \text{ M}\Omega$$

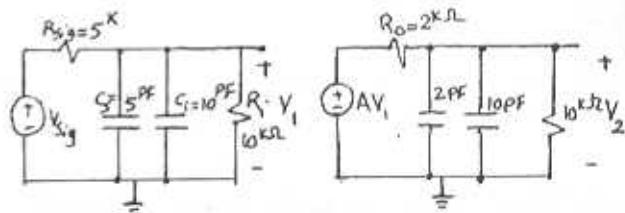
$$T_{gd} = C_{gd} R_{gd} = 0.5^P \times 6.2^M = 3100 \text{ ns}$$

$$T_L = R'_L C_L = 10^K \times 3^P = 30 \text{ ns}$$

$$\omega_H = \frac{1}{T_{gs} + T_{gd} + T_L} = \frac{1}{30^2 + 3100^2 + 243.75^2} = 296.4 \text{ rad/s}$$

$$f_H = 47.2 \text{ kHz}$$

6.49



$$T_1 = (5^P + 10^P) \times (5^K / 110^K)$$

$$T_1 = 50 \text{ ns}$$

$$\omega_1 = \frac{1}{T_1} = 20 \text{ rad/s}$$

$$T_2 = (2^P + 10^P) \times (10^K / 12^K)$$

$$T_2 = 20 \text{ ns}$$

$$\omega_2 = \frac{1}{T_2} = 50 \text{ rad/s}$$

$$T_3 = (2^P + 7^P) \times (1^K / 12^K)$$

$$T_3 = 6 \text{ ns}$$

$$\omega_3 = 166 \text{ rad/s}$$

$$\omega_H = \frac{1}{T_1 + T_2 + T_3} = \frac{1}{50^2 + 20^2 + 6^2} = 13.2 \text{ MHz}$$

$$f_H = 2.1 \text{ MHz}$$

6.50

$$\text{Eq. 6.41} \quad \omega_H = \frac{1}{\sum RC}$$

$$a) \omega_H = \frac{1}{20 + 5 + 100} = 38.46 \text{ rad/s} \Rightarrow f_H = 6.12 \text{ MHz}$$

$$b) \omega_H = \frac{1}{\frac{1}{w_{p1}} + \frac{1}{w_{p2}} + \frac{1}{w_{p3}}} \quad (\text{Eq. 6.38})$$

$$\omega_H = \frac{2\pi}{\frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}} = \frac{2\pi}{\frac{1}{50} + \frac{1}{200} + \frac{1}{1000}} = 241.5 \text{ rad/s}$$

$$f_H = 38.46 \text{ MHz}$$

$$c) \omega_H = \frac{1}{\frac{1}{50} + \frac{1}{200} + \frac{1}{1000}} = 38.46 \text{ rad/s}$$

$$f_H = 6.12 \text{ MHz}$$

$$d) \omega_H = \frac{1}{1000 + 200 + 200} = 714 \text{ rad/s} \Rightarrow f_H = 113.7 \text{ kHz}$$

$$e) \omega_H = \frac{1}{0.4 + 1} = 714 \text{ rad/s} \Rightarrow f_H = 113.7 \text{ kHz}$$

$$f) \omega_H = \frac{1}{1 + 0.2 + 0.15} = 741 \text{ rad/s} \Rightarrow f_H = 118 \text{ kHz}$$

$$g) \omega_H = \frac{2\pi}{\frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \frac{1}{f_4}} = \frac{2\pi}{\frac{1}{1} + \frac{1}{2} + \frac{1}{5} + \frac{1}{5}}$$

$$\omega_H = 3.36 \text{ rad/s} \Rightarrow f_H = 526 \text{ MHz}$$

6.51

$$R_{in} = \frac{R}{1 - \text{Gain}} = \frac{100}{1 - 0.95} = 2000 \text{ k}\Omega = 2 \text{ M}\Omega$$

6.52

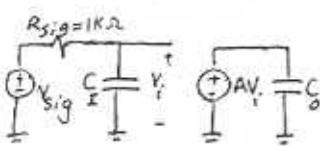
$$\text{Eq. 6.44a} \quad Z_I = Z_{i+K} \Rightarrow C_I = 0.1 \times (1 - (-1000))$$

$$\Rightarrow C_I = 100.1 \text{ pF}$$

$$C_o = 0.1 \times \left(\frac{-1000}{1000} + 1\right)$$

$$C_o = 99.9 \text{ pF}$$

(using Miller's Theorem)



$$V_0 = AV_i = A \times V_{sig} \frac{1/C_{sig}}{R_{sig} + 1/C_{sig}} \Rightarrow \frac{V_0}{V_{sig}} = \frac{A}{1 + C R_{sig}}$$

$$\omega_H = \frac{1}{C R_{sig}} = \frac{1}{100.1 \text{ pF} \times 1 \text{ k}\Omega} = 9.99 \text{ rad/s} \Rightarrow f_H = 1.59 \text{ MHz}$$

To calculate unity gain frequency:

$$|\text{Gain}| = 1$$

$$\frac{V_0}{V_i} = \frac{A}{1 + C R_{sig} s} = \frac{-1000}{1 + 100.1 \times 10^{-9} s} \quad (s = j\omega)$$

$$\frac{1000}{\sqrt{1 + (100.1 \times 10^{-9} \times \omega)^2}} = 1 \Rightarrow \omega = 10 \text{ rad/s}$$

$$f_T \approx 1.59 \text{ GHz}$$

As we can see  $f_T \approx f_H \times A$ 

6.53

Using Miller's Theorem, in each case the capacitance at the input is  $C(1-A)$  and the capacitance at the output is  $C(1-\frac{1}{A})$ . Thus:

$$a) A = -1000 \text{ V/V} \text{ and } C = 1 \text{ pF}$$

$$C_i = 1.001 \text{ nF} \text{ and } C_o = 1.001 \text{ pF}$$

$$b) A = -10 \text{ V/V} \text{ and } C = 10 \text{ pF}$$

$$C_i = 110 \text{ pF} \text{ and } C_o = 11 \text{ pF}$$

$$c) A = -1 \text{ V/V} \text{ and } C = 10 \text{ pF}$$

$$C_i = 20 \text{ pF} \text{ and } C_o = 20 \text{ pF}$$

$$d) A = 1 \text{ V/V} \text{ and } C = 10 \text{ pF}$$

$$C_i = 0 \text{ pF} \text{ and } C_o = 0 \text{ pF}$$

$$e) A = 10 \text{ V/V} \text{ and } C = 10 \text{ pF}$$

$$C_i = -9 \text{ pF} \text{ and } C_o = 9 \text{ pF}$$

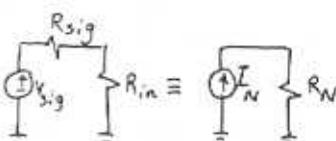
In (e) the negative capacitance at the input can be used to cancel the effect of the input capacitance of the amplifier.

6.54

$$a) R_{in} = \frac{R}{1 - A} = \frac{R}{1 - 2} = -R \quad (\text{Miller's theorem})$$

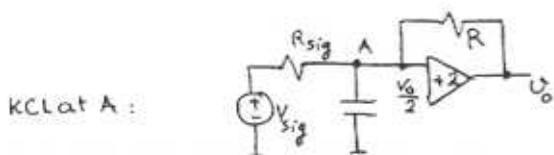
$$b) I_N = \frac{V_{sig}}{R_{sig}}$$

$$R_N = R_{sig} \parallel R_{in}$$

If  $R_{sig} = R$  then:

$$R_N = R \parallel (-R) = 0 \Rightarrow I_L = I_N = \frac{V_{sig}}{R_{sig}} = \frac{V_{sig}}{R}$$

Cont.



$$\frac{V_0 - V_{sig}}{R_{sig}} + \frac{V_0}{2} + CS + -\frac{V_0}{2R} = 0$$

$$\text{If } R_{sig} = R \Rightarrow +\frac{V_{sig}}{R} = \frac{V_0}{2} CS \Rightarrow \frac{V_0}{V_{sig}} = \frac{2}{RCS}$$

6.55

From Table 6.3 we have:

$$\text{Intrinsic gain} = g_m r_o = \frac{2VAL}{V_{ov}}$$

$$\Rightarrow \text{Intrinsic gain} = \frac{2 \times 10A}{0.2} = 100 \text{ V/V}$$

$$g_m r_o = 100 \text{ V/V}$$

$$g_m = \frac{2ID}{V_{ov}} \Rightarrow I_D = \frac{g_m V_{ov}}{2} = \frac{2 \times 0.2}{2} = 0.2 \text{ mA}$$

$$g_m = k_n C_{ox} \frac{W}{L} V_{ov} \Rightarrow W = \frac{g_m L}{k_n V_{ov}} = \frac{2 \times 10^3}{12.5 \times 0.2} = 80 \mu\text{m}$$

6.56

$$r_o = g_m r_o = 100 \text{ V/V} = \frac{2VAL}{V_{ov}}$$

$$I_D = \frac{1}{2} k_n \frac{W}{L} V_{ov}^2 = 100 \mu\text{A}$$

$$\frac{A_{o2}}{A_{o1}} = \frac{V_{ov1}}{V_{ov2}} = \sqrt{\frac{I_{D1}}{I_{D2}}} \Rightarrow A_{o2} = A_{o1} \sqrt{\frac{I_{D1}}{I_{D2}}}$$

$$\text{Now if } I_D \text{ is } 25 \mu\text{A} \text{ then: } A_{o2} = 100 \sqrt{\frac{100}{25}} = 200 \text{ V/V}$$

$$I_D = 400 \mu\text{A} \Rightarrow A_{o2} = 100 \sqrt{\frac{100}{400}} = 50 \text{ V/V}$$

$$g_m = \sqrt{2k_n C_{ox} \frac{W}{L} I_D} \Rightarrow \frac{g_m^2}{g_{m1}} = \sqrt{\frac{I_{D2}}{I_{D1}}} = \sqrt{\frac{25}{100}} = \sqrt{\frac{I_{D2}}{I_{D1}}} \cdot \sqrt{\frac{g_{m2}}{g_{m1}}}$$

$$\text{For } I_D = 25 \mu\text{A} \quad g_{m2} = g_{m1} \sqrt{\frac{25}{100}} \Rightarrow g_{m2} = g_{m1}/2$$

$$I_D = 400 \mu\text{A} \Rightarrow g_{m2} = g_{m1} \sqrt{\frac{400}{100}} \Rightarrow g_{m2} = 2g_{m1}$$

6.57

Refer to Fig. P6.57.

a) if we neglect the current in Feedback

$$\text{resistor: } I_D = 200 \mu\text{A} = \frac{1}{2} k_n \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 2 \times \frac{V_{ov}^2}{100} \Rightarrow$$

$$V_{ov}^2 = 200 \times 10^{-3} \Rightarrow V_{ov} = 0.45 \text{ V} \Rightarrow V_{GS} = V_t + V_{ov}$$

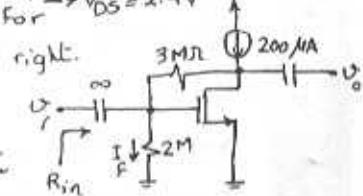
$$\Rightarrow V_{GS} = 0.5 + 0.45 = 0.95 \text{ V}$$

$$\Rightarrow V_G = 0.95 \text{ V} \Rightarrow I_F = \frac{0.95}{2M} = 0.48 \mu\text{A} \ll 200 \mu\text{A}$$

$$\Rightarrow V_{DS} = 0.48 \times 5 = 2.4 \text{ V} \Rightarrow V_{DS} = 2.4 \text{ V}$$

So our assumption for

neglecting  $I_F$  was right.



b) To find the small-signal gain we write a KCL

at the output node:

$$\frac{V_0}{r_o} + g_m V_{gs} + \frac{V_0 - V_i}{R_F} = 0 \quad V_i \quad 2M \quad V_{gs} \quad R_F = 3M \quad g_m V_{gs}$$

$$V_{gs} = V_i$$

$$\frac{V_0}{r_o} + g_m V_i + \frac{V_0}{R_F} - \frac{V_i}{R_F} = 0 \Rightarrow \frac{V_0}{V_i} = \frac{1/R_F - g_m}{\frac{1}{R_F} + \frac{1}{r_o}}$$

$$g_m = \frac{2ID}{V_{ov}} = \frac{2 \times 200 \mu\text{A}}{0.45} = 0.89 \mu\text{A/V}$$

$$r_o = \frac{VA}{ID} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

$$\frac{V_0}{V_i} = \frac{\frac{1}{3000} - 0.89}{\frac{1}{3000} + \frac{1}{100}} = -86.1 \text{ V/V}$$

$$V_{DS\min} = V_{GS} - V_t = V_{ov} = 0.45 \text{ V}$$

In our case  $V_{DS} = 2.4 \text{ V}$ , therefore the largest signal at the output is  $2.4 - 0.45 = 1.95 \text{ V}$ . The corresponding input signal is  $\frac{V_0}{A} = \frac{1.95}{-86.1} = 0.023 \text{ V}$  or  $23 \text{ mV}$ .

c)  $R_{in} = \frac{V_x}{I_x}$

$$I_x = \frac{V_x}{R_i} + \frac{V_x - V_o}{R_F} \quad ①$$

$$\frac{V_x - V_o}{R_F} = \frac{V_o}{r_o} + g_m V_x \Rightarrow V_o = V_x \frac{1/R_F - g_m}{\frac{1}{r_o} + \frac{1}{R_F}}$$

$$\text{② } I_x = \frac{V_x}{R_i} + \frac{V_x}{R_F} - \frac{V_x}{R_F} \frac{1/R_F - g_m}{\frac{1}{r_o} + \frac{1}{R_F}}$$

$$\frac{V_x}{r_o} = R_{in} = \frac{1}{\frac{1}{R_i} + \frac{1}{R_F} - \frac{1}{R_F} \frac{(R_F + r_o)}{r_o} + \frac{g_m}{(R_F + r_o)}}$$

$$R_{in} = R_i \parallel R_F \parallel \frac{1 + R_F/r_o}{g_m - 1/R_F}$$

$$R_{in} = 2M \parallel 3M \parallel \frac{1 + 3000/100}{0.89 - 1/3000} = 33.9 \text{ k}\Omega$$

6.58

Refer to Fig. 6.18a and assume  $Q_2$  and  $Q_3$  are matched:

$$K'_n = 2.5 K_p = 250 \text{ mA/V}^2, V_{A_n} = |V_{AP}| = 10V$$

$$R_{out} = 100 \text{ k}\Omega = r_{o1} \parallel r_{o2} = \frac{10}{I_{D1}} \parallel \frac{10}{I_{D2}}$$

$$\text{Since } I_{D1} = I_{D2} \Rightarrow \frac{10}{I_{D1}} = 200 \Rightarrow I_{D1} = I_{D2} = 0.05 \text{ mA}$$

$$I_{REF} = I_{D2} = 0.05 \text{ mA}$$

$$\text{Eq. 6.49: } A_v = -g_m (r_{o1} \parallel r_{o2}) \Rightarrow -40 = -g_m \times 100^k$$

$$\Rightarrow g_m = 0.4 \text{ mA/V}$$

$$g_m = \sqrt{2K_n(W/L)_1} I_{D1} \Rightarrow \left(\frac{W}{L}\right)_1 = \frac{0.4^2}{2 \times 150 \times 10^{-3} \times 0.05} = 6.4$$

$$\left(\frac{W}{L}\right)_1 = 6.4$$

$$\frac{g_m}{g_m} = \frac{2I_{D1}}{V_{OV1}} \Rightarrow V_{OV1} = \frac{2 \times 0.05}{0.4} = 0.25V$$

IF  $Q_2$  and  $Q_3$  have the same  $V_{OV}$  as  $Q_1$ , then  $|V_{OV2}| = 0.25V$

$$I_{D2} = \frac{1}{2} K_p \left(\frac{W}{L}\right)_2 V_{OV}^2 \Rightarrow \left(\frac{W}{L}\right)_2 = \frac{0.05 \times 2}{100 \times 10^{-3} \times 0.25^2} = 16$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = 16 \quad (Q_2 \text{ and } Q_3 \text{ are matched})$$

6.59

As discussed in Example 6.8, the transfer characteristic of the amplifier over the desired region (segment III) is quite linear. Therefore the DC bias component of the input signal (for maximum output swing) should be chosen at the midpoint between  $V_{IA}$  and  $V_{IB}$ , that is:

$$\text{input dc bias} = \frac{V_{IA} + V_{IB}}{2} = \frac{0.83 + 0.93}{2} = 0.905V$$

The corresponding amplitude of the resulting output sinusoid is:

$$\text{output sinusoid amplitude} = \frac{V_{OA} + V_{OB}}{2} = \frac{2.47 + 0.33}{2}$$

$$\text{output sinusoid amplitude} = 1.4V$$

6.60

As discussed in Example 6.8, the transfer characteristic of the amplifier over the region labeled as segment III, is quite linear.

$$V_{OA} = V_{DD} - V_{OV3} = 5 - 0.53 = 4.47V$$

Now to find the linear equation for segment III, we can write  $i_{D1} = i_{D2}$ :

$$\frac{1}{2} K_p \left(\frac{W}{L}\right)_1 (V_I - V_{In})^2 \left(1 + \frac{V_O}{V_{A_n}}\right) = \frac{1}{2} K_p \left(\frac{W}{L}\right)_2 (V_{SG} - V_{tp})^2 \left(1 + \frac{V_O}{V_{A_n}}\right)$$

$$\Rightarrow 200 (V_I - 0.6)^2 \left(1 + \frac{V_O}{10}\right) = 65 \times 0.53^2 \left(1 + \frac{V_{DD} - V_O}{10}\right) \quad \boxed{\frac{V_{DD} - V_O}{V_{A_n}}}$$

$$\frac{200}{65 \times 0.53^2} (V_I - 0.6)^2 = \frac{1.5 - V_O/10}{1 + \frac{V_O}{20}}$$

$$7.3 (V_I - 0.6)^2 = \frac{1 - V_O/15}{1 + \frac{V_O}{20}} = \frac{1 - 0.067 V_O}{1 + 0.05 V_O} \approx 1 - 0.117 V_O$$

$$\Rightarrow V_O = 8.57 - 62.57 (V_I - 0.6)^2 \quad (1)$$

IF we substitute for  $V_{A_n} = 4.47V$ , then  $V = 0.86V$

To determine coordinates of B, note that  $\frac{V_A}{V_A}$

$$V_{CB} - V_{In} = V_{OB} \quad \text{or} \quad V_{IB} - 0.6 = V_{OB}$$

Substitute in (1):

$$V_{OB} = 8.57 - 62.57 (V_{OB})^2 \Rightarrow V_{OB} = 0.36V$$

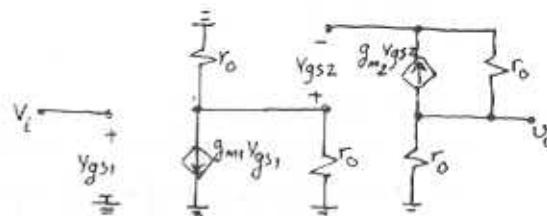
$$V_{CB} = 0.6 + 0.36 = 0.96V$$

Therefore the linear region is:

$$0.86V \leq V_I \leq 0.96V \quad \text{or} \quad 0.36V \leq V_O \leq 4.47V$$

6.61

Since  $V_{A_n} = |V_{AP}|$  and the drain current of both  $Q_1$  and  $Q_2$  is equal to  $I$ , their output resistances are equal. That is  $r_{o1} = r_{o2} = r_o$ . The small-signal model of this amplifier is:



Cont.

$$V_{GS2} = -g_m V_{GS1} (r_0 || r_o) \text{ and } V_{GS1} = V_i$$

$$V_{GS2} = -g_m \frac{r_o}{2} V_i$$

$$V_o = -g_m r_o V_{GS2} (r_0 || r_o) = -g_m r_o (-g_m \frac{r_o}{2} V_i) \frac{r_o}{2}$$

$$A_v = \frac{V_o}{V_i} = g_m r_o \frac{r_o^2}{4}$$

6.62

Refer to Fig. 6.18a.

Note that  $Q_2, Q_3$  are not matched:

$$I_{D1} = 100 \mu A$$

$$a) I_{D2} = I_{D1} = 100 \mu A \quad \frac{I_{D3}}{I_{D2}} = \frac{(W/L)_3}{(W/L)_2} = \frac{W_3}{W_2}$$

$$(Note that V_{SG2} = V_{SG3}) \Rightarrow I_{D3} = 100 \mu A \cdot \frac{10}{40} = 25 \mu A \Rightarrow I_{REF} = 25 \mu A$$

b) By referring to Fig. 6.18d, you notice that in Segment III, both  $Q_1$  and  $Q_2$  are in saturation and the transfer characteristic is quite linear. The output voltage in this segment is limited between  $V_{DA}$  and  $V_{DB}$ :

Coordinates of point A:  $V_{OA} = V_{DD} - V_{OV3}$

$$V_{OV3}^2 = \frac{I_{D3}}{\frac{1}{2} K_p (\frac{W}{L})_3} = \frac{25}{\frac{1}{2} \times 50 \times \frac{10}{1}} = 0.1 V \Rightarrow V_{OV3} = 0.32 V$$

$$V_{DA} = 3.3 - 0.32 = 2.98 V$$

$$At point B: V_{OB} = V_{IB} - V_{ta}$$

Now we find the transfer equation for the linear section: (Refer to Example 6.8)

$$i_{D1} = i_{D2} \Rightarrow (Note that V_{ta} = V_{OV3})$$

$$K_n (\frac{W}{L})_1 (V_I - V_{ta})^2 (1 + \frac{V_O}{V_{DD}}) = K_p (\frac{W}{L})_2 V_{OV3}^2 (1 + \frac{V_O - V_{ta}}{V_{AP}})$$

$$100 \times \frac{20}{1} (V_I - 0.8)^2 (1 + \frac{V_O}{100}) = 50 \times \frac{40}{1} \times 0.32^2 (1 + \frac{3.3 - 1.6}{50})$$

$$(V_I - 0.8)^2 = 0.32^2 (1.066 - \frac{V_O}{50}) / (1 + \frac{V_O}{100})$$

$$(V_I - 0.8)^2 = 0.11 \left( \frac{1 - 0.01945}{1 + 0.01V_O} \right) \approx 0.11 (1 - 0.03V_O)$$

$$(V_I - 0.8)^2 = 0.11 (1 - 0.03V_O) \quad (1)$$

Now if we solve for  $V_{OB} = V_{IB} - 0.8$

$$V_{OB}^2 + 0.0033V_{OB} - 0.11 = 0 \Rightarrow V_{OB} = 0.33 V$$

Therefore the extreme values of  $V_O$  for which  $Q_1$  and  $Q_2$  are in saturation:  $0.33 \leq V_O \leq 2.98$

c) From (b) we can find  $V_{IA}$  and  $V_{IB}$ :

$$V_{IB} = V_{OB} + V_t = 0.33 + 0.8 = 1.13 V$$

If we solve (1) for  $V_{IA} = 2.98 V$  then:

$$(V_{IA} - 0.8)^2 = 0.11 (1 - 0.03 \times 2.98) \Rightarrow V_{IA} = 1.11 V$$

$$\text{large-signal voltage gain} = \frac{\Delta V_O}{\Delta V_i} = \frac{2A_8 - 0.33}{1.13 - 1.11} = 189.3 V/V$$

$$d) V_o = \frac{V_{DD}}{2} = \frac{3.3}{2} = 1.65 V$$

Differentiating both sides of (1):  $\left( \frac{\partial}{\partial V_I} \right)$

$$2(V_I - 0.8) = 0.11 \times (-0.03) \frac{\partial V_O}{\partial V_I} \\ \Rightarrow \frac{\partial V_O}{\partial V_I} = -606.1 (V_I - 0.8)$$

For  $V_O = 1.65 V$ , from (1) we have:

$$(V_I - 0.8)^2 = 0.11 (1 - 0.03 \times 1.65) \Rightarrow V_I = 1.12 V$$

$$\left. \frac{\partial V_O}{\partial V_I} \right|_{V_I = 1.12} = -195.8 V/V$$

$$e) R_{out} = r_0 || r_{o2}$$

$$r_{o1} = \frac{V_{An}}{I_{D1}} = \frac{100}{0.1} = 1 M\Omega$$

$$r_{o2} = \frac{V_{Ap}}{I_{D2}} = \frac{50}{0.1} = 500 k\Omega \quad \left. \begin{array}{l} R_{out} = 500 k\Omega \\ R_{out} = 333 \end{array} \right\}$$

$$g_{m1} = \sqrt{2 K_n (\frac{W}{L})_1 I_{D1}} = \sqrt{2 \times 100 \times 32 \times 10^{-6}} = 0.632 m$$

$$A_{v2} = -g_{m1} (r_{o1} || r_{o2}) = -210.6 V/V$$

6.63

a) When D and G are open, since the gates don't draw any current, therefore no current through R and  $V_D = V_G$ ,  $I_{D1} = I_{D2} = 0$

$$\frac{1}{2} K_n (\frac{W}{L})_1 (V_G - (-1.5) - 0.5)^2 = \frac{1}{2} K_p (\frac{W}{L})_2 (1.5 - V_G)$$

$$(V_G + 1)^2 = (1 - V_G)^2 \Rightarrow V_G + 1 = 1 - V_G \Rightarrow V_G = 0$$

$$I_{D1} = \frac{1}{2} \times 1 \times (0 + 1.5 - 0.5)^2 = 0.5 mA$$

$$I_{D1} = I_{D2} = 0.5 mA$$

Cont

$$b) V_o = V_i - R(2g_m V_{gs})$$

$$V_{gs} = V_i$$

$$V_o = V_i - 2g_m R V_i$$

$$A_V = \frac{V_o}{V_i} = 1 - 2g_m R$$

$$g_m = \frac{2ID}{V_{GS} - V_E} = \frac{2 \times 0.5}{1.5 - 0.5} = 1 \text{ mA/V}$$

$$A_V = 1 - 2 \times 1 \times 1000 = -1999 \text{ V/V}$$

$$c) r_o = \frac{V_A}{I_D} = \frac{20}{0.5} = 40 \text{ k}\Omega$$

IF we write KCL at D:

$$\frac{V_i - V_o}{R} = 2g_m V_{gs} + \frac{V_o}{r_o/2}, V_{gs} = V_i$$

$$\frac{V_i}{R} - 2g_m V_i = \frac{V_o}{R} + \frac{2V_o}{r_o} \Rightarrow \frac{V_o}{V_i} \cdot A_V = \frac{1 - 2g_m R}{1 + \frac{2R}{r_o}}$$

$$\text{or } A_V = \frac{1 - 2 \times 1 \times 1000}{1 + \frac{2 \times 1000}{40}} = -39.2 \text{ V/V}$$

$$R_{in} = \frac{V_x}{i_x} : V_x - R i_x = \frac{r_o}{2} (i_x - 2g_m V_{gs})$$

$$V_{gs} = V_x$$

$$V_x + 2g_m \frac{r_o}{2} V_x = R i_x + \frac{r_o}{2} i_x \Rightarrow R_{in} = \frac{V_x}{i_x} = \frac{R + r_o/2}{1 + g_m r_o}$$

$$R_{in} = \frac{1000 + 40/2}{1 + 1 \times 40} = 24.9 \approx 25 \text{ k}\Omega$$

$$d) \frac{V_o}{V_{sig}} = A_V \frac{R_{in}}{R_{in} + R_{sig}} = -39.2 \times \frac{25}{100 + 25} \text{ V/V}$$

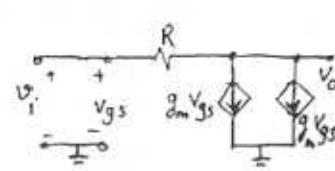
$$\frac{V_o}{V_{sig}} = -7.84 \text{ V/V}$$

e) In order for both  $Q_1$  and  $Q_2$  to remain in the saturation region:

$$V_{DS} \geq V_{GS} - V_E \Rightarrow V_D + 1.5 \geq V_G + 1.5 - V_E \\ \Rightarrow V_D \geq V_G - 0.5$$

$$\text{For } Q_2: V_{SD} \geq V_{SG} - V_E \Rightarrow 1.5 - V_D \geq 1.5 - V_G - 0.5 \\ \Rightarrow V_D \leq V_G + 0.5$$

In (a) we showed that the gate is at 0 volt, therefore  $V_D \geq -0.5$  and  $V_D \leq 0.5$   
 $\Rightarrow -0.5 \leq V_o \leq 0.5$



6.64

$$R_i = r_H = \frac{\beta}{g_m} = \frac{\beta}{I_C/V_T} = \frac{100 \times 0.025}{1} = 2.5 \text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = 40 \text{ mA/V} \Rightarrow r_o = \frac{V_A}{I_C} = 100 \text{ k}\Omega$$

$$A_{VD} = -g_m r_o = -40 \times 100 = -4000 \text{ V/V}$$

$$R_o = r_o = 100 \text{ k}\Omega$$

If  $R_i$  is multiplied by 4, since  $R_i \propto L$ , then  $I_C$  has to be divided by 4:  $I_C = \frac{1 \text{ mA}}{4} = 0.25 \text{ mA}$

$$\text{For } I_C = 0.25 \text{ mA}: g_m = \frac{0.25}{0.025} = 10 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.25} = 400 \text{ k}\Omega$$

$$A_{VD} = -10 \times 400 = -4000 \text{ V/V}$$

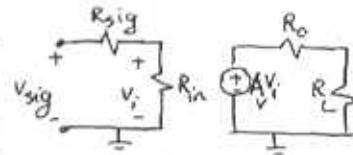
In general  $A_{VD} = -g_m r_o = -\frac{I_C}{4} \times \frac{V_A}{I_C} = -\frac{V_A}{V_T}$   
 and  $A_{VD}$  is not dependent on  $I_C$ .

$$R_o = r_o = 400 \text{ k}\Omega, R_i = 4 \times 2.5 = 10 \text{ k}\Omega$$

For  $I_C = 1 \text{ mA}$  and  $R_{sig} = 5 \text{ k}\Omega, R_L = 500 \text{ k}\Omega$

$$V_i = V_{sig} \frac{R_{in}}{R_{in} + R_{sig}}$$

$$G_V = \frac{R_{in}}{R_{in} + R_{sig}} A_V \frac{R_L}{R_L + R_o}$$



$$G_V = -4000 \times \frac{2.5 \text{ k}\Omega}{5 + 2.5} \times \frac{500}{500 + 100} = -1111 \text{ V/V}$$

$$G_V = -1111 \text{ V/V}$$

For  $I_C = 0.25 \text{ mA}$ :  $R_{in} = 10 \text{ k}\Omega, R_o = 400 \text{ k}\Omega$

$$G_V = \frac{10}{10 + 5} \times (-4000) \times \frac{500}{500 + 400} = -1481.5 \text{ V/V}$$

$$G_V = -1481.5 \text{ V/V}$$

6.65

$$\text{a) } I_{REF} = I_{C3} = \frac{3 - V_{BE3}}{23k\Omega}$$

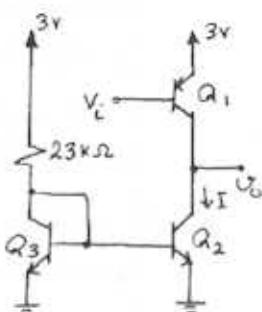
$$I_{REF} = \frac{3 - 0.7}{23}$$

$$I_{REF} = 0.1 \text{ mA}$$

$$\frac{I_{C2}}{I_{C3}} = \frac{\text{Area of } Q_2}{\text{Area of } Q_3} = 5$$

$$\Rightarrow I_{C2} = 5 I_{C3}$$

$$I_{C2} = I = 0.5 \text{ mA} \Rightarrow I = 0.5 \text{ mA}$$



$$\text{b) } |V_A| = 50 \text{ V} \Rightarrow r_{O1} = \frac{|V_A|}{I} = \frac{50}{0.5} = 100 \text{ k}\Omega$$

$$r_{O2} = \frac{50}{0.5} = 100 \text{ k}\Omega$$

Total resistance at the collector of  $Q_1$  is equal to  $r_{O1} \parallel r_{O2}$ , thus:  $r_{tot} = 100 \text{ k}\Omega \parallel 100 \text{ k}\Omega = 50 \text{ k}\Omega$

$$r_{tot} = 50 \text{ k}\Omega$$

$$\text{c) } g_{m1} = \frac{f_{C1}}{V_F} = \frac{0.5}{0.025} = 20 \text{ mA/V}$$

$$r_{m1} = \frac{\beta}{g_m} = \frac{50}{20} = 2.5 \text{ k}\Omega$$

$$\text{d) } R_{in} = r_{m1} = 2.5 \text{ k}\Omega$$

$$R_o = r_{O1} \parallel r_{O2} = 100 \text{ k}\Omega \parallel 100 \text{ k}\Omega = 50 \text{ k}\Omega$$

$$A_V = -g_{m1} R_o = -20 \times 50 = -1000 \text{ V/V}$$

6.66

$$A_H = -g_m R'_L = -5 \times 20 = -100 \text{ V/V}$$

$$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L) \quad (\text{Eq. 6.55})$$

$$C_{in} = 2 + 0.1(1 + 5 \times 20) = 12.1 \text{ pF}$$

$$f_H \leq \frac{1}{2\pi C_{in} R_{sig}} \quad (\text{Eq. 6.54})$$

$$f_H \leq \frac{1}{2\pi \times 12.1 \times 10^{-12} \times 20k} = 658 \text{ kHz}$$

6.67

$$\text{Eq. 6.57: } T_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R_{CL}$$

$$T_H = C_{gs} R_{sig} + C_{gd} [R_{sig}(1 + g_m R'_L) + R'_L] + C_L R'_L$$

$$T_H = 2^P \times 20^K + 0.1 [20^K (1 + 5 \times 20) + 20^K] + 1^P \times 20^K$$

$$T_H = 264 \text{ ns}$$

$$f_H = \frac{1}{2\pi T_H} = 603 \text{ kHz}$$

$$A_H = -g_m R'_L = -5 \times 20 = -100 \text{ V/V}$$

$$C_{gs} : 15.1\%$$

$$C_{gd} : 77.3\%$$

Contribution of each time-constant to the overall  $T_H$ .

$$C_L : 7.6\%$$

If we compare  $f_H$  to the one obtained in Problem 6.66, we notice that Problem 6.66 has a larger  $f_H$  due to neglecting the time constants of  $C_L$  and  $C_{gs}$ .

6.68

From Eq. 6.60 we have:  $\omega_Z = 9m/C_{gd}$

$$\Rightarrow f_Z = \frac{9m}{2\pi C_{gd}} = \frac{5^m}{2\pi \times 0.1^P} = 7.966 \text{ Hz}$$

$f_{p1}$  and  $f_{p2}$  are the poles of the transfer function of equation (6.60), whose denominator is a quadratic polynomial with coefficients  $s$ :

$$\begin{aligned} &= [C_{gs} + C_{gd}[1 + g_m R'_L]] R_{sig} + (C_L + C_{gd}) R'_L \\ &= (2 + 0.1(1 + 5 \times 20)) 20 + (1 + 0.1) \times 20 \\ &= 264 \text{ ns} = 264 \times 10^{-9} \text{ sec} \end{aligned}$$

Coefficient of  $s^2$ :

$$\begin{aligned} &= [(C_L + C_{gd}) C_{gs} + C_L C_{gd}] R_{sig} R'_L \\ &= [(1 + 0.1) 2 + 1 \times 0.1] 20^K \times 20^K = \\ &= 920 \times 10^{-18} (\text{sec})^2 \end{aligned}$$

Therefore the quadratic equation is:

$$1 + 264 \times 10^{-9} s + 920 \times 10^{-18} s^2 = 0$$

Denoting the frequencies of the roots of this equation with  $\omega_{p1}$  and  $\omega_{p2}$ , we have:

$$\omega_{p1} = 3.84 \times 10^6 \text{ rad/s} \Rightarrow f_{p1} = \frac{\omega_{p1}}{2\pi} = 611.15 \text{ kHz}$$

$$\omega_{p2} = 283.12 \times 10^6 \text{ rad/s} \Rightarrow f_{p2} = \frac{\omega_{p2}}{2\pi} = 45.06 \text{ MHz}$$

Cont.

Since  $f_{p_1} \ll f_{p_2}$  and  $f_{p_1} \ll f_2$ , a good estimate for  $f_H$  is  $f_{p_1}$ :

$$f_H \approx f_{p_1} = 611.15 \text{ kHz}$$

Approximate value of  $f_{p_1}$  obtained using (Eq. 6.66) is:

$$f_{p_1} = \frac{1}{2\pi[(C_{gs} + C_{gd}(1+g_m R'_L))R_{sig} + (C_L + C_{gd})R'_L]} = 603.16 \text{ kHz}$$

$$f_{p_1} = 603.16 \text{ kHz}$$

Approximate value of  $f_{p_2}$  obtained using (Eq. 6.67) is:

$$f_{p_2} = \frac{[C_{gs} + C_{gd}(1+g_m R'_L)]R_{sig} + (C_L + C_{gd})R'_L}{2\pi [(C_L + C_{gd})C_{gs} + C_L C_{gd}]R'_L R_{sig}} = 45.67 \text{ MHz}$$

$$f_{p_2} = 45.67 \text{ MHz}$$

The estimate of  $f_{p_1}$  using Eq. 6.66 is 1.3% lower than the exact value, while the estimate of  $f_{p_2}$  is about 1.3% higher than its exact value.

6.69

$$R'_L = 5 \text{ k}\Omega :$$

$$A_M = -g_m R'_L = -5 \times 5 = -25 \text{ V/V}$$

using (Eq. 6.66):

$$f_{p_1} = \frac{1}{2\pi[(C_{gs} + C_{gd}(1+g_m R'_L))R_{sig} + (C_L + C_{gd})R'_L]} = 603.16 \text{ kHz}$$

$$f_{p_1} = \frac{1}{2\pi[(2+0.1 \times (1+5 \times 5))20 + (1+0.1) \times 5]} = 603.16 \text{ kHz}$$

$$f_{p_1} = 603.16 \text{ kHz}$$

using (Eq. 6.67):

$$f_{p_2} = \frac{[C_{gs} + C_{gd}(1+g_m R'_L)]R_{sig} + (C_L + C_{gd})R'_L}{[(C_L + C_{gd})C_{gs} + C_L C_{gd}]R'_L R_{sig} \times 2\pi} = 67.5 \text{ MHz}$$

$$f_{p_2} = \frac{(2+0.1(1+5 \times 5))20 + (1+0.1)5}{((1+0.1) \times 2 + 1 \times 0.1)5 \times 20 \times 2\pi} = 67.5 \text{ MHz}$$

$$f_{p_2} = 67.5 \text{ MHz}$$

$$\text{Eq. 6.63: } S_Z = \frac{g_m}{C_{gd}} \Rightarrow f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{5 \text{ m}}{2\pi 0.1^2} = 7.96 \text{ GHz}$$

$f_{p_1} \ll f_{p_2}$  and  $f_{p_1} \ll f_Z \rightarrow f_{p_1}$  is the dominant pole.

$$f_H = f_{p_1} = 1.63 \text{ MHz}$$

$$\text{Gain} / \times \text{Bandwidth} = 25 \times 1.63 = 40.75 \text{ MHz}$$

$$F_L = |A_M| / f_H = 40.75 \text{ MHz}$$

Since  $f_{p_1} \ll f_{p_2}$  and  $f_{p_1} \ll f_Z$ , a dominant pole exists.

$$R'_L = 10 \text{ k}\Omega :$$

$$A_M = -5 \times 10 = -50 \text{ V/V}$$

$$f_{p_1} = \frac{1}{2\pi[(2+0.1(1+5 \times 10))20 + (1+0.1) \times 10]} = 1.04 \text{ MHz}$$

$$f_{p_2} = \frac{(2+0.1(1+5 \times 10))20 + (1+0.1) \times 10}{[(1+0.1)2 + 1 \times 0.1]10 \times 20 \times 2\pi} = 52.96 \text{ MHz}$$

$$f_2 = \frac{5}{2\pi \times 0.1} = 7.96 \text{ GHz}$$

$f_{p_1} \ll f_{p_2}$  and  $f_{p_1} \ll f_2 \rightarrow f_{p_1}$  is the dominant pole and therefore  $f_H \approx f_{p_1} = 1.04 \text{ MHz}$

$$|A_M| \cdot f_H = 50 \times 1.04 = 52 \text{ MHz}$$

Since  $f_{p_2}$  is still slightly greater than  $|A_M| \cdot f_H$ , therefore:

$$f_T = 52 \text{ MHz}$$

$$A_M = -5 \times 20 = -100 \text{ V/V}, \text{ from Problem 6.68 we have:}$$

$$f_{p_1} = 603.16 \text{ kHz}$$

$$f_{p_2} = 45.67 \text{ MHz}$$

$$f_2 = 7.96 \text{ GHz}$$

Again  $f_{p_1} \ll f_{p_2}$  and  $f_{p_1} \ll f_2$ , therefore  $f_{p_1}$  is the dominant pole and  $f_H$  can be approximated by  $f_{p_1}$ .  $f_H \approx f_{p_1} = 603.16 \text{ kHz}$

$$|A_M| \cdot f_H = 60.32 \text{ MHz}$$

Since  $f_{p_2} < |A_M| \cdot f_H$ , therefore  $f_T$  is smaller than  $|A_M| \cdot f_H$ .

The results are summarized in this table:

$R'_L$	5 k $\Omega$	10 k $\Omega$	20 k $\Omega$
$A_M (\text{V/V})$	-25	-50	-100
$f_{p_1} (\text{MHz})$	1.63	1.04	0.60
$ A_M  \cdot f_H (\text{MHz})$	40.75	52.00	60.32

6.70

$$\text{Using Eq. 6.70: } A_H = -\frac{r_{\pi}}{R_S g + r_x + r_{\pi}} (g_m R'_L)$$

$$r_H = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega \Rightarrow A_H = -\frac{5}{1 + 0.2 + 5} (20 \times 5)$$

$$A_H = 80.65 \text{ V/V}$$

Using Miller's Theorem and Eq. 6.71:

$$C_{in} = C_{\pi} + C_{\mu} (1 + g_m R'_L) = 10 + 0.5(1 + 20 \times 5) = 60.5 \text{ PF}$$

$$\text{Eq. 6.69: } R'_S g = r_{\pi} / (R_S g + r_x) = 5 / (1 + 0.2)$$

$$R'_S g = 0.97 \text{ k}\Omega$$

$$\text{Eq. 6.72: } f_H \approx \frac{1}{2\pi C_{in} R'_S g} = \frac{1}{2\pi \times 60.5 \times 0.97} \Rightarrow$$

$$f_H = 2.71 \text{ MHz}$$

6.71

From Problem 6.70 we have:  $r_{\pi} = 5 \text{ k}\Omega$  and  $A_H = -80.65 \text{ V/V}$ ,  $R_S g = 0.97 \text{ k}\Omega$

$$\text{Eq. 6.75: } f_Z = \frac{j_m}{2\pi C_{\mu}} = \frac{20 \text{ M}}{2\pi \times 0.5 \text{ P}} = 6.37 \text{ GHz}$$

$$\text{Eq. 6.76: } f_P \approx \frac{1}{2\pi [(C_{\pi} + C_{\mu}(1 + g_m R'_L)) R'_S g + (C_{\pi} + C_{\mu}) R'_L]}$$

$$f_P = \frac{1}{2\pi [(10 + 0.5(1 + 20 \times 5)) 0.97 + 2.5 \times 5]}$$

$$f_P = 2.24 \text{ MHz}$$

$$\text{Eq. 6.77: } f_{P2} = \frac{(C_{\pi} + C_{\mu}(1 + g_m R'_L)) R'_S g + (C_{\pi} + C_{\mu}) R'_L}{2\pi [C_{\pi}(C_{\mu} + C_{\mu}) + C_{\mu}^2] R'_S g R'_L}$$

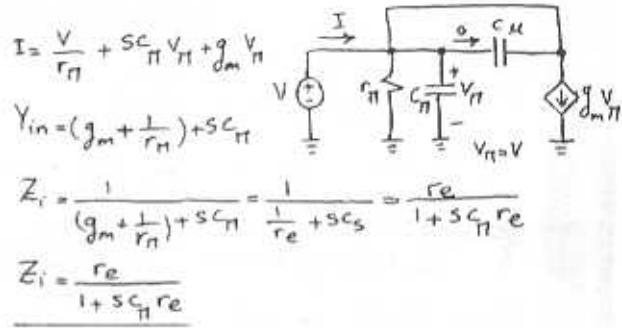
$$f_{P2} = \frac{(10 + 0.5(1 + 20 \times 5)) 0.97 + 2.5 \times 5}{2\pi (10(2 + 0.5) + 2 \times 0.5) 0.97 \times 5}$$

$$f_{P2} = 89.89 \text{ MHz}$$

Since  $f_P \ll f_{P2}$  and  $f_P \ll f_Z$ , we can approximate  $f_H$  by  $f_P$ :  $f_H \approx f_P = 2.24 \text{ MHz}$

If we compare  $f_H$  to the results obtained from applying Miller's Theorem in Problem 6.70, then our results are 17% lower.

6.72



$$Z_i = \frac{V}{I} = \frac{V}{\frac{V}{r_{\pi}} + sC_{\pi}V_H + g_m V} = \frac{r_{\pi}}{(g_m + \frac{1}{r_{\pi}}) + sC_{\pi}} = \frac{r_e}{\frac{1}{r_e} + sC_{\pi}r_e} = \frac{r_e}{1 + sC_{\pi}r_e}$$

$$Z_i = \frac{r_e}{1 + sC_{\pi}r_e}$$

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})} \quad \text{Since } C_{\pi} = C_{de} + C_{je} = C_F g_m + C_{je}$$

Therefore  $C_{\pi}$  has a component that depends on the bias current and at high currents  $C_{\pi} \gg C_{je}$  and  $f_T = \frac{g_m}{2\pi C_{\pi}} = \frac{1}{2\pi C_{\pi} r_e}$

Thus:  $Z_i \approx \frac{r_e}{1 + \frac{s}{\omega_T}}$  at high currents

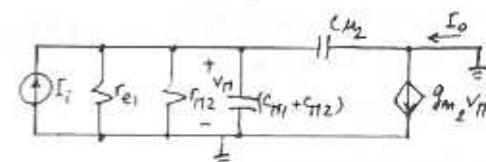
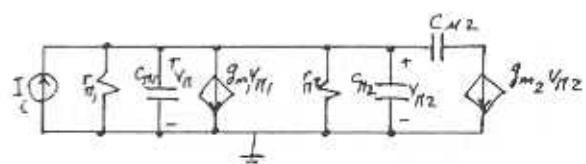
The phase angle will be  $-45^\circ$  at  $\omega = \omega_T$  or  $f = f_T = 400 \text{ MHz}$ .

For lower bias currents, so that  $C_{\pi} = C_{je}$ ,  $f_T \approx \frac{1}{4\pi C_{je} r_e}$  and  $Z_i = \frac{r_e}{1 + \frac{s}{2\omega_T}}$  and  $-45^\circ$

phase is obtained at  $\omega = 2\omega_T$  or  $f = 2f_T = 800 \text{ MHz}$

(Assuming that  $f_T$  remains constant which is not necessarily true!!)

6.73



$$V_H = \frac{I_i}{(\frac{1}{r_{e1}} + \frac{1}{r_{e2}}) + s(C_{M1} + C_{M2} + C_{H2})}$$

Cont.

$$I_o = g_{m2} V_{\pi} - c_M s V_{\pi} = \frac{(g_{m2} - c_M s) I_i}{(\frac{1}{r_{e1}} + \frac{1}{r_{\pi2}}) + s(c_{\pi1} + c_{\pi2} + c_M)}$$

$$\frac{I_o}{I_i} = \frac{g_{m2} - c_M s}{(\frac{1}{r_{e1}} + \frac{1}{r_{\pi2}}) + s(c_{\pi1} + c_{\pi2} + c_M)}$$

$$I_{C1} = I_{C2} \Rightarrow r_{\pi1} = r_{\pi2}, g_{m1} = g_{m2}, c_{\pi1} = c_{\pi2}$$

$$\frac{I_{o1}}{I_i} = \frac{g_m - c_M s}{(\frac{1}{r_e} + \frac{1}{r_{\pi}}) + (c_M + 2c_{\pi})s} = \frac{1 - \frac{c_M}{g_m} s}{(\frac{1}{g_m r_e} + \frac{1}{g_m r_{\pi}}) + s \frac{c_M + 2c_{\pi}}{g_m}}$$

$$g_m r_e = \frac{I_C}{V_T} \frac{V_T}{I_E} = \alpha = \frac{\beta}{\beta + 1}$$

$$g_m r_{\pi} = \beta$$

$$\Rightarrow \frac{I_o}{I_i} = \frac{1 - \frac{c_M}{g_m} s}{1 + \frac{1}{\beta} + \frac{1}{\beta} + s(2c_{\pi} + c_M)/g_m}$$

$$\frac{I_o}{I_i} = \frac{1}{1 + 2/\beta} \frac{1 - s \frac{c_M}{g_m}}{1 + s \frac{(2c_{\pi} + c_M)}{g_m(1 + \frac{2}{\beta})}}$$

If the circuit is biased at 1mA and  $\beta=00$ ,

$$f_T = 400 \text{ MHz} \text{ and } c_M = 2 \text{ pF};$$

$$g_m = \frac{1}{0.025} = 40 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(c_{\pi} + c_M)} \Rightarrow c_{\pi} + c_M = \frac{40^M}{2\pi \times 400 \text{ Hz}} = 15.9 \text{ pF}$$

$$c_{\pi} = 15.9 - 2 = 13.9 \text{ pF}$$

$$\text{Pole frequency: } f_p = \frac{g_m}{2\pi(2c_{\pi} + c_M)} = \frac{40 \times 10^{-3}}{2\pi(2 \times 13.9 + 2)} \text{ p} \\ f_p = 213.74 \text{ MHz}$$

$$\text{Zero frequency: } f_z = \frac{g_m}{2\pi c_M} = \frac{40^M}{2\pi \times 2^P} = 3.18 \text{ GHz}$$

6.74

$$A_M = -g_m R'_L = -5 \times 20 = -100 \text{ V/V}$$

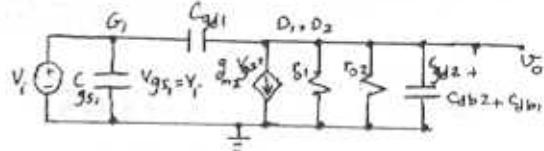
$$f_H = \frac{1}{2\pi(c_L + c_{gd})R'_L} = \frac{1}{2\pi(1+0.1) \times 20} = 7.23 \text{ MHz}.$$

(Note that in this case there is no  $R_s$  and we used Eq. 6.79)

$$f_{gd} = f_H = 7.23 \text{ MHz}$$

$$f_E = |A_M|, f_H = 7.23 \text{ MHz}$$

6.75



$$g_m = \sqrt{2N c_{\pi} W L / D} = \sqrt{2 \times 90 \times 100 \times 100} = 1060 \text{ mA/V}$$

$$g_m = 1.06 \text{ mA/V}$$

$$r_{o1} = \frac{V_A1}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_A2|}{I_{D2}} = \frac{19.2}{0.1} = 192 \text{ k}\Omega$$

$$\text{DC-gain} = -g_m (r_{o1} || r_{o2}) = -1.06 \times (128 || 192) \\ = -81.4 \text{ V/V}$$

$$\text{Total capacitance between output node and ground} = c_{gd2} + c_{db1} + c_{ab2} = 0.015 + 0.020 + 0.036$$

$$C_L = 0.071 \text{ pF}$$

Write a KCL at output:

$$sC_{gd1}(V_i - V_o) = g_m V_i + \frac{V_o}{r_{o1}} + \frac{V_o}{r_{o2}} + V_o sC_L$$

$$\frac{V_o}{V_i} = -\frac{g_m + sC_{gd1}}{\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + (C_L + C_{gd1})s}$$

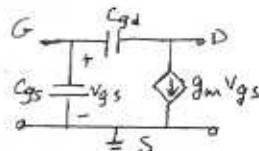
$$\text{Thus: } f_Z = \frac{g_m}{2\pi C_{gd1}} = \frac{1.06^M}{2\pi \times 0.015} = 11.36 \text{ GHz}$$

$$f_P = \frac{1}{2\pi} \frac{\frac{1}{r_{o1}} + \frac{1}{r_{o2}}}{C_L + C_{gd1}} = \frac{\frac{1}{128} + \frac{1}{192}}{2\pi (0.071 + 0.015)} \text{ P}$$

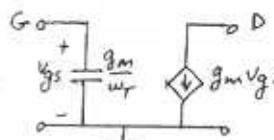
$$f_P = 24.1 \text{ MHz}$$

6.76

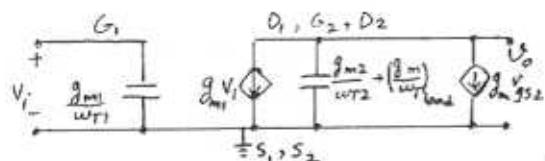
a) For small  $C_{gd}$  and low gain from G to D, we can neglect the Miller effect and  $C_{gd}$ .



$$w_T = \frac{g_m}{C_{gs} + C_{gd}} = \frac{g_m}{C_{gs}} \text{ Thus } C_{gs} \approx \frac{g_m}{w_T}$$



b) replace the controlled source  $g_{m2} V_{gs2}$  with a resistance  $\frac{1}{g_{m2}}$ . (Source absorption theory)  
Cont.



$$V_o = -g_{m1} \frac{V_i}{g_{m2} + S \left( \frac{g_{m3}}{w_{T2}} + \frac{g_{m4}}{w_{T2} \text{load}} \right)}$$

Since the load device is identical to  $Q_1$ ,

$$g_{\text{load}} = g_{m1} \quad \text{and} \quad w_{T\text{load}} = w_{T1} = w_T.$$

Thus:

$$\frac{V_o}{V_i} = \frac{-g_{m1}/g_{m2}}{1 + \frac{S}{w_T} (1 + \frac{g_{m1}}{g_{m2}})}$$

$$\frac{g_{m1}}{g_{m2}} = \frac{\mu_n C_{ox}(\frac{W}{L})_1 V_{ov}}{\mu_n C_{ox}(\frac{W}{L})_2 V_{ov}} = \frac{W_1}{W_2}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{-A_0}{1 + \frac{S}{w_T} (1 + A_0)} \quad \text{where } A_0 = \frac{W_1}{W_2} = \frac{g_{m1}}{g_{m2}}$$

C)  $A_0 = 3 \text{ V/V} \rightarrow W_2 = 25 \mu\text{m}$

$$A_0 = \frac{W_1}{W_2} \Rightarrow W_1 = 3 \times 25 = 75 \mu\text{m}$$

$$I = \frac{1}{2} \mu_n C_{ox} \frac{W_1}{L} (V_{G3} - V_t)^2 = \frac{1}{2} \times 200 \times \frac{75}{0.5} \times 0.3^2$$

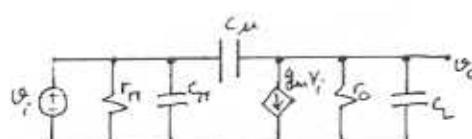
$$I = 1.35 \text{ mA}$$

$$I_{D2} = \frac{1}{2} \times 200 \times \frac{W_2}{L} \times \frac{25}{0.5} \times 0.3^2 = 0.45 \text{ mA}$$

Thus:  $I = I_{D1} + I_{D2} = 1.35 + 0.45 = 1.8 \text{ mA}$

$$f_{3\text{dB}} = \frac{f_T}{1 + A_0} = \frac{12 \times 10^9}{1 + 3} = 3 \text{ GHz}$$

6.77



writing a node equation at the output yields:

$$sC_F(V_i - V_o) = g_m V_i + \frac{V_o}{r_o} + V_o C_S$$

$$\frac{V_o}{V_i} = \frac{C_F s - g_m}{\frac{1}{r_o} + (C_L + C_F)s} = -g_m r_o \frac{1 - sC_F g_m}{1 + s(C_L + C_F)r_o}$$

$$\text{for small } C_F, \omega C_F \ll g_m: \frac{V_o}{V_i} = \frac{-g_m r_o}{1 + s(C_L + C_F)r_o}$$

$$\text{For } I_C = 200 \text{ mA}, V_A = 100 \text{ V}: g_m = \frac{200}{0.025} = 8 \text{ mA/V}$$

$$r_o = \frac{100}{200} = 0.5 \text{ M}\Omega$$

Thus DC-Gain =  $-g_m r_o = -8 \times 0.5 \times 10^3 = -4000 \text{ V/V}$

For  $C_L = 1 \text{ pF}$ ,  $C_F = 0.2 \text{ pF}$ :

$$w_{3\text{dB}} = \frac{1}{(C_L + C_F)r_o} = \frac{1}{(1+0.2) \times 0.5 \times 10^3} = 1.67 \text{ MHz}$$

$$f_{3\text{dB}} = 265.4 \text{ kHz} = f_H$$

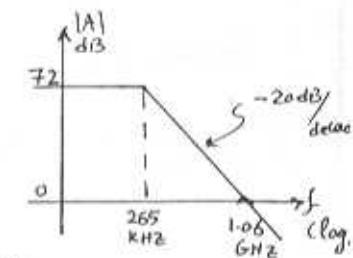
$$f_T = |A_0| f_H = 4000 \times 265.4 = 1.06 \text{ GHz}$$

Bode Plot for  $|A_f|$ :

$$4000 \text{ V/V} = 72 \text{ dB}$$

$$\text{Check } f_Z = \frac{g_m}{2\pi C_F}$$

$$f_Z = \frac{8 \times 10^9}{2\pi \times 0.2 \times 10^{-12}} = 6.46 \text{ Hz}$$



6.78

$$f_T = \frac{g_m}{2\pi(C_L + C_{gd})} \rightarrow g_m = 1 \text{ mA/V}, f_T = 2 \text{ GHz}$$

$$\rightarrow C_L + C_{gd} = \frac{1 \times 10^3}{2\pi \times 2 \times 10^9} = 79.61 \text{ fF}$$

To have  $f_{T2} = 1 \text{ GHz}$ , we need:

$$C_L + C_{gd} = \frac{1 \times 10^3}{2\pi \times 10^9} = 159.23 \text{ fF}$$

Thus we need an additional capacitance of

$$159.23 - 79.61 = 79.61 \text{ fF}$$

6.79

$$g_m = \sqrt{2k' n \frac{W}{L} I_D} = \sqrt{2 \times 160 \times \frac{50}{0.5} \times 0.5} = 4 \text{ mA/V}$$

$$g_{mb} = \lambda g_m = 0.2 \times 4 = 0.8 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 0.5} = 20 \text{ k}\Omega$$

$$R_o = r_o = 20 \text{ k}\Omega$$

$$A_{V_o} = 1 + (g_m + g_{mb})r_o = 1 + (4 + 0.8) \times 20 = 97 \text{ V/V}$$

$$R_{out} = r_o + A_{V_o} R_S \quad (\text{Eq. 6.102})$$

$$R_{out} = 20 + 97 \times 20 = 1.96 \text{ M}\Omega$$

$$R_{in} = \frac{r_o + R_L}{A_{V_o}} \quad (\text{Eq. 6.86})$$

$$R_{in} = \frac{20 + 20}{97} = 0.41 \text{ k}\Omega$$

$$A_u = A_{V_o} \frac{R_L}{R_L + r_o} = 97 \frac{20}{20 + 20} = 48.5 \text{ V/V}$$

Cont.

$$G_V = \frac{V_O}{V_S \cdot g} = A_{V_0} \frac{R_L}{R_L + r_o + A_{V_0} R_S} \quad (\text{Eq. 6.95})$$

$$G_V = \frac{97 \times 20 \text{ k}}{20 \text{ k} + 20 \text{ k} + 97 \times 20 \text{ k}} = 0.98 \text{ V/V}$$

$$G_{IS} = A_{V_0} \frac{R_S}{R_{out}} = 97 \times \frac{20}{1960} = 0.99 \text{ A/A}$$

$$G_i = G_{IS} \frac{R_{out}}{R_{out} + R_i} = 0.99 \times \frac{1960}{1960 + 20} = 0.98 \text{ A/A}$$

6.80

$$R_L = r_o$$

$$A_{V_0} = 100 \text{ V/V} = \frac{V_O}{V_i} = A_{V_0} \frac{R_L}{R_L + r_o} = A_{V_0} \frac{r_o}{r_o + r_o}$$

$$\Rightarrow A_{V_0} = A_{V_0}/2 = 100 \Rightarrow A_{V_0} = 200 \text{ V/V}$$

$$R_{in} = 2 \text{ k}\Omega$$

$$R_{in} = \frac{r_o + R_L}{A_{V_0}} = \frac{r_o + r_o}{A_{V_0}} \Rightarrow 2^k = \frac{2r_o}{200} \Rightarrow r_o = 100 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_D} \Rightarrow I_D = \frac{V_A}{r_o} = \frac{20}{100} = 0.2 \text{ mA}$$

$$A_{V_0} = 1 + (g_m + g_{mb}) r_o = 1 + (1 + \chi) g_m r_o$$

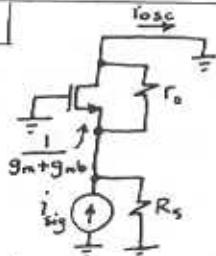
$$200 = 1 + (1 + 0.2) g_m \times 100 \Rightarrow g_m = 1.66 \text{ m A/V}$$

$$g_m = \frac{2I_D}{V_{ov}} \Rightarrow V_{ov} = \frac{2 \times 0.2}{1.66} = 0.24 \text{ V}$$

$$I_D = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2 \Rightarrow \frac{W}{L} = \frac{2I_D}{K_n V_{ov}^2} = \frac{2 \times 0.2}{0.1 \times 0.24^2}$$

$$\frac{W}{L} = 69.4$$

6.81



$$i_{osc} = \frac{R_s i_{sig}}{R_s + (r_o \parallel \frac{1}{g_m + g_{mb}})} = \frac{R_s}{R_s + \frac{r_o}{1 + (g_m + g_{mb}) r_o}} i_{sig}$$

$$G_{IS} = \frac{i_{osc}}{i_{sig}} = \frac{R_s}{R_s + \frac{r_o}{A_{V_0}}} = \frac{A_{V_0} R_s}{r_o + A_{V_0} R_s}$$

Using (6.102) we have:  $G_{IS} = \frac{A_{V_0} R_s}{R_{out}}$

If  $A_{V_0} R_s \gg r_o$  that is  $[1 + (g_m + g_{mb}) r_o] R_s \gg r_o$  or  $(g_m + g_{mb}) R_s \gg 1$  then  $G_{IS} \approx 1$

6.82

$$R_{in} = \frac{r_o + R_L}{A_{V_0}}, \quad A_{V_0} = 1 + (g_m + g_{mb}) r_o$$

$$A_{V_0} \leq (g_m + g_{mb}) r_o \leq A_o$$

$$\Rightarrow R_{in} \approx \frac{1}{g_m + g_{mb}} + \frac{R_L}{A_o}$$

$$\text{If } R_L = A_o r_o \text{ then } R_{in} = \frac{1}{g_m + g_{mb}} + r_o$$

6.83

$$g_m = \frac{2I_D}{V_{ov}} = \frac{2 \times 1^m}{0.8 - 0.55} = 8 \text{ m A/V}$$

$$g_{mb} = \chi g_m = 0.2 \times 8 = 1.6 \text{ m A/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20}{1^m} = 20 \text{ k}\Omega$$

$$\text{From Eq. 6.101 we have: } R_{out} = r_o + [1 + (g_m + g_{mb}) r_o] R_S$$

$$R_{out} = 20 + (1 + 8 + 1.6) \times 20 R_S = 200 \text{ k}\Omega$$

$$R_S = 932.64 \text{ }\Omega$$

$$V_{BIAS} = I_D R_S + V_{GS} = 1 \times 932.64 \times 10^{-3} + 0.8 = 1.73 \text{ V}$$

$$V_{BIAS} = 1.73 \text{ V}$$

6.84

$$V_S \cdot (g_m + g_{mb}) = i_{osc} - \frac{V_S}{r_o}$$

$$\Rightarrow i_{osc} = V_S \left( \frac{1}{r_o} + g_m + g_{mb} \right) \quad (1)$$

$$V_S = V_{sig} - R_S i_{osc}$$

If we substitute  $V_S$  in (1):

$$i_{osc} = (V_{sig} - R_S i_{osc}) \left( \frac{1}{r_o} + g_m + g_{mb} \right)$$

$$i_{osc} (1 + \frac{R_S}{r_o} + R_S (g_m + g_{mb})) = V_{sig} \frac{(1 + (g_m + g_{mb}) r_o)}{r_o}$$

$$i_{osc} = V_{sig} \frac{1 + (g_m + g_{mb}) r_o}{r_o + R_S + R_S r_o (g_m + g_{mb})}$$

Since  $A_{V_0} = G_{V_0} = 1 + (g_m + g_{mb}) r_o$  (Eq. 6.88)

and  $R_{out} = r_o + A_{V_0} R_S$  (Eq. 6.102), then

$$i_{osc} = V_{sig} \frac{A_{V_0}}{R_{out}}$$

6.85

a)  $V_{GS2} = V_{GS3} \Rightarrow I_{D2} = I_{D3}$ . Also  $I_{D1} = I_{D2}$   
 $\Rightarrow I_{D1} = 100 \mu A$ .

$$I_{D1} = \frac{1}{2} K_n \frac{W}{L} (V_{GS1} - V_t)^2 \Rightarrow 0.1 = \frac{1}{2} \times 4 \times (V_{GS1} - 0.8)^2$$
 $V_{GS1} - 0.8 = 0.224 \Rightarrow V_{GS1} = 1.024 \text{ V}, V_{OV} = V_{VR2} = 0.224 \text{ V}$

$$V_{BIAS} = R_S I_{D1} + V_{GS1} = 0.05 \times 0.1 + 1.024 = 1.029 \text{ V}$$

b)  $g_{m1} = \frac{2 I_{D1}}{V_{OV}} = \frac{0.1 \times 2}{0.224} = 0.89 \text{ mA/V}$

$$g_{m2} = \sqrt{2 K_n \frac{W}{L} I_D} = \sqrt{2 \times 4 \times 0.1} = 0.89 \text{ mA/V}$$

$$g_{m3} = g_{m2} = 0.89 \text{ mA/V}$$

$$g_{mb} = \lambda g_m \Rightarrow g_{mb1} = g_{mb2} = g_{mb3} = 0.2 \times 0.89 = 0.18 \text{ mA/V}$$

$$r_{o1} = r_{o2} = r_{o3} = \frac{V_A}{I_D} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

c)  $R_{in} = \frac{r_o + R_L}{A_{V0}}$  where  $A_{V0} = 1 + (g_m + g_{mb}) r_o$   
 $A_{V0} = 1 + (0.89 + 0.18) 200$   
 $A_{V0} = 215 \text{ V/V}$

In this case  $R_L$  is in fact  $R_{out}$  of the active load in the drain of  $Q_1$ , which is  $r_{o2} = 200 \text{ k}\Omega$ .

Thus:  $R_{in} = \frac{200 + 200}{215} = 1.86 \text{ k}\Omega$

d)  $R_{out} = r_o + A_{V0} R_S = 200 + 215 \times 0.05 = 210.75 \text{ k}\Omega$

e)  $A_V = \frac{V_o}{V_i} = A_{V0} \frac{R_L}{R_L + r_o} = 215 \frac{200}{200 + 200} = 107.5 \text{ V/V}$   
 $G_V = A_{V0} \frac{R_L}{R_L + R_{out}} = 215 \frac{200}{200 + 210.75} = 104.7 \text{ V/V}$

f) For  $Q_2$  to stay in saturation region,  $V_o$  can go as high as  $V_{DD} - V_{VR2}$ , thus  $V_o \leq 3.3 - 0.224$  or  $V_o \leq 3.076 \text{ V}$ .

For  $Q_1$  to stay in saturation region, we need to have  $V_{GD1} \leq V_t \Rightarrow V_{BIAS} - V_o \leq 0.8 \Rightarrow V_o \geq 1.029 - 0.8 \text{ or } V_o \geq 0.229 \text{ V}$   
 hence  $0.229 \leq V_o \leq 3.076 \text{ V}$ . This implies that the peak-to-peak output swing is  $3.076 - 0.229 = 2.847 \text{ V}$ .

Since  $G_V = \frac{V_o}{V_{sig}} = 104.7$ , then the maximum peak-to-peak value of  $V_{sig}$  is  $\frac{2.847}{104.7} = 27 \text{ mV}$ .

6.86

From Fig. 6.31b the low-frequency gain  $\frac{V_o}{V_{sig}}$  can be written as:

$$\frac{V_o}{V_{sig}} = \frac{1}{R_S + \frac{1}{g_m + g_{mb}}} \times (g_m + g_{mb}) \times R'_L$$

$$\frac{V_o}{V_{sig}} = \frac{(g_m + g_{mb}) R'_L}{1 + (g_m + g_{mb}) R_S} = \frac{(5 + 0.2 \times 5) \times 20}{1 + (5 + 0.2 \times 5) \times 1} = 17.14$$

$$\frac{V_o}{V_{sig}} = 17.14 \text{ V/V}$$

From Eq. 6.105 we have  $f_{p1} = \frac{1}{2\pi C_{gs} (R_{sig} \parallel \frac{1}{g_{mb}})}$

$$f_{p1} = \frac{1}{2\pi \times 2pf(1k \parallel \frac{1}{5+0.2\times 5})} = 557 \text{ MHz}$$

From Eq. 6.106 we have:

$$f_{p2} = \frac{1}{2\pi (C_{gd} + C_L) R'_L} = \frac{1}{2\pi (0.1^p + 2^p) 20^k} = 3.79 \text{ MHz}$$

Since  $f_{p2} \ll f_{p1}$ , then  $f_{p2}$  is the dominant pole and  $f_H \approx f_{p2} = 3.79 \text{ MHz}$

6.87

$$A_{V0} = 1 + (g_m + g_{mb}) r_o = 1 + (5 + 0.2 \times 5) 20^k = 121 \text{ V/V}$$

$$R_{out} = r_o + A_{V0} R_S = 20 + 121 \times 1 = 141 \text{ k}\Omega$$

$$G_V = G_{V0} \frac{R_L}{R_L + R_{out}} = A_{V0} \frac{R_L}{R_L + R_{out}} = 121 \frac{20^k}{20^k + 141^k}$$

$$G_V = \frac{V_o}{V_{sig}} = 15 \text{ V/V}$$

$$R_{in} = \frac{r_o + R_L}{A_{V0}} = \frac{20 + 20}{121} = 0.33 \text{ k}\Omega$$

$$R_{gs} = R_S \parallel R_{in} = 1^k \parallel 0.33^k = 0.25 \text{ k}\Omega = 250 \text{ }\Omega$$

$$R_{gd} = R_L \parallel R_{out} = 20^k \parallel 141^k = 17.5 \text{ k}\Omega$$

$$T_H = C_{gs} R_{gs} + (\xi + \zeta) R_{gd} = 2^p \times 0.25^k + (2 + 0.1) 17.5^k = 37.2 \text{ ns}$$

$$f_H \approx \frac{1}{2\pi T_H} = 4.27 \text{ MHz}$$

Comparing with results in Problem 6.86, we notice that gain is reduced by 12.5%, while  $f_H$  is increased by 12.7%, when  $r_o$  is taken into account.

6.88

$$i_o = i_c - \frac{v_i}{r_n} \quad \text{Eq. 6.110}$$

If we write a KCL at the emitter node, we will have:  $\frac{v_i - i_o R_L}{r_o} + \frac{v_i}{r_e} = i_c$

Substitute for  $i_o$  from Eq. 6.110, then:

$$\frac{v_i - R_L i_c + R_L r_n v_i}{r_o} + \frac{v_i}{r_e} = i_c$$

$$v_i \left( \frac{1}{r_o} + \frac{R_L}{r_n r_o} + \frac{1}{r_e} \right) = i_c \left( 1 + \frac{R_L}{r_o} \right)$$

$$R_{in} = \frac{v_i}{i_c} = \frac{1 + R_L/r_o}{\frac{1}{r_o} + \frac{1}{r_e} + \frac{R_L}{r_n r_o}} = \frac{r_o + R_L}{1 + r_o/r_e + R_L/r_n}$$

$$r_H = (\beta + 1) r_e \quad \text{therefore:}$$

$$R_{in} = \frac{r_o + R_L}{1 + \frac{r_o}{r_e} + \frac{R_L}{(\beta + 1)r_e}}$$

This is the same as equation 6.111.

6.89

$$\beta = 100, \quad R_{in} = r_e \frac{r_o + R_L}{r_o + R_L/(\beta + 1)}$$

$$R_{in} = r_e \frac{1 + R_L/r_o}{1 + \frac{R_L}{r_o} \frac{1}{\beta + 1}}$$

$R_L/r_o$	0	1	10	100	1000
$R_{in}/r_e$	1	1.98	10	50.75	91.83

6.90

$$I = 1 \text{ mA}, \text{ Intrinsic gain } A_0 = \frac{V_A}{V_T} = 2000 \text{ V/V}$$

$$V_A = 2000 \text{ V}_T = 50 \text{ V}$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{1} = 50 \text{ k}\Omega$$

From Eq. 6.112 we have:  $R_{in} \approx r_e \frac{r_o + R_L}{r_o + R_L/(\beta + 1)}$

Assuming  $\beta$  is very large so that:

$$\frac{R_L}{\beta + 1} \ll r_o, \text{ we have: } R_{in} \approx r_e \frac{r_o + R_L}{r_o}$$

For  $R_{in}$  to be  $2r_e$ , we need:

$$2r_e = r_e \frac{r_o + R_L}{r_o} \Rightarrow r_o = R_L \Rightarrow R_L = 50 \text{ k}\Omega$$

6.91

Refer to Fig. 6.34:

write KCL at the emitter:

$$\frac{v}{r_e} + \frac{v}{R_e} = i_x + g_m v \Rightarrow v = \frac{i_x}{\frac{1}{r_e} + g_m + \frac{1}{R_e}}$$

$$\text{Note that } \frac{1}{r_e} + g_m = \frac{\beta + 1}{r_n} - g_m = \frac{\beta + 1 - \beta}{r_n} = \frac{1}{r_n}$$

$$\text{therefore: } v = \frac{i_x}{\frac{1}{r_n} + \frac{1}{R_e}} = (r_n \parallel R_e) i_x$$

$$v = R'_e i_x \quad \text{where } R'_e = r_n \parallel R_e$$

Now we write the equation for  $v$ :

$$v = v_x - (i_x + g_m v) r_o$$

if we substitute  $v$  from ①:

$$R'_e i_x = v_x - i_x r_o - g_m R'_e i_x r_o$$

$$i_x (R'_e + r_o + g_m R'_e r_o) = v_x$$

$$R_{in} = \frac{v_x}{i_x} = r_o + (1 + g_m r_o) R'_e \quad \text{where } R'_e = R_e \parallel r_n$$

(same as Eq. 6.117)

6.92

$$\text{Eq. 6.118: } R_{out} \approx (1 + g_m R'_e) r_o$$

$$\Rightarrow \frac{R_{out}}{r_o} \approx 1 + g_m R'_e \quad \text{where } R'_e = R_e \parallel r_n$$

$$R'_e = \frac{r_n R_e}{r_n + R_e} = \frac{(\beta + 1) r_e R_e}{(\beta + 1) r_e + R_e} = \frac{(\beta + 1) r_e}{(\beta + 1) \frac{r_e}{m} + 1}$$

$$m = \frac{r_e}{r_o} \Rightarrow R'_e = \frac{(\beta + 1) r_e}{\frac{\beta + 1}{m} + 1}$$

$$\frac{R_{out}}{r_o} = 1 + g_m \frac{r_e}{r_o} \frac{(\beta + 1)}{\frac{\beta + 1}{m} + 1} = 1 + \frac{g_m \frac{r_n}{r_o} (\beta + 1)}{\frac{\beta + 1}{m} + 1}$$

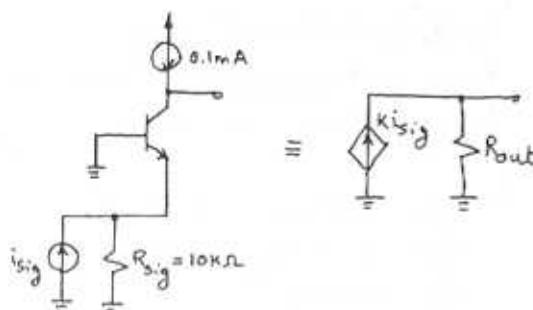
$$\frac{R_{out}}{r_o} = 1 + \frac{\beta}{\frac{\beta + 1}{m} + 1} = \frac{(\beta + 1)(m + 1)}{\beta + 1 + m} = \frac{101(m + 1)}{101 + m}$$

$m = \frac{r_e}{r_o}$	1	2	10	$\beta/2$	1000
$\frac{R_{out}}{r_o}$	1.98	2.94	10.01	$\beta/3$	91.74

6.93

$$R_{sig} = 10 \text{ k}\Omega, \beta = 100, V_A = 50 \text{ V}, I_C = 0.1 \text{ mA}$$

Cont.



$$r_o = \frac{V_A}{I_C} = \frac{50}{0.1} = 500 \text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$r_{in} = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

The Thevenin equivalent circuit of the source is:

$$\text{V}_{\text{sig}} = R_{\text{sig}} i_{\text{sig}}$$

Thus using equations 6.121 and 6.114 we can write:

$$\frac{V_{\text{out}}}{V_{\text{sig}}} = \frac{r_{in}}{r_{in} + R_{\text{sig}}} A_{v0} = \frac{r_{in}}{r_{in} + R_{\text{sig}}} (1 + g_m r_o)$$

$$i_{\text{sig}} = \frac{r_{in} R_{\text{sig}}}{r_{in} + R_{\text{sig}}} (1 + g_m r_o) = (r_{in} \| R_{\text{sig}}) (1 + g_m r_o)$$

$$\frac{V_{\text{out}}}{i_{\text{sig}}} = 14.29 \times 10^6 \text{ V/A}$$

From Eq. 6.117a we have  $R_{\text{out}} = r_o + (1 + g_m r_o) R'_e$  where  $R'_e = r_{in} \| R_{\text{sig}}$ .

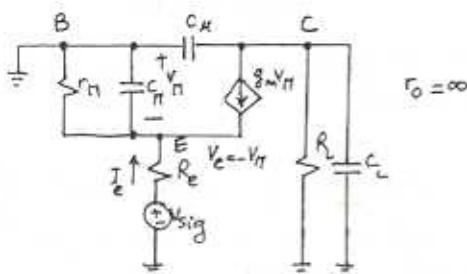
$$R_{\text{out}} = 500 + (1 + 4 \times 500) (25 \| 10^4) = 14.79 \text{ M}\Omega$$

$$R_{\text{out}} = 14.79 \text{ M}\Omega$$

$$A_{v0} = K * i_{\text{sig}} R_{\text{out}} \Rightarrow K = \frac{V_{\text{out}}}{i_{\text{sig}}} \times \frac{1}{R_{\text{out}}} = \frac{14.29 \times 10^6}{14.79 \times 10^6}$$

$$= 0.97$$

6.94

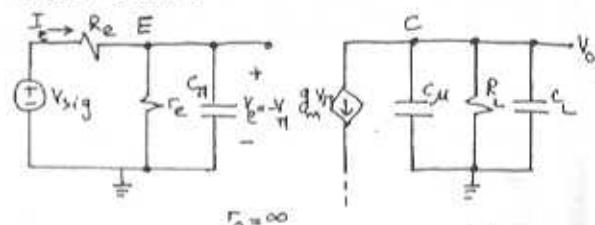


We observe that  $V_e$ , the voltage at the emitter is equal to  $-V_H$ . We can write a node equation at the emitter:

$$I_e = -V_H \left( \frac{1}{r_{in}} + g_m \right) - g_m V_H = V_e \left( \frac{1}{r_{in}} + g_m + g_m \right)$$

Thus, the input admittance looking into the emitter is:  $\frac{I_e}{V_e} = \frac{1}{r_{in}} + g_m + g_m = \frac{1}{r_e} + g_m$

Therefore we can replace the transistor at the input of the circuit by this admittance as shown below:



a) As we can see above, the circuit can be separated into two parts with each part having its own pole:

$$f_{p1} = \frac{1}{2\pi C_M (R_L \| r_e)} \quad (\text{input part})$$

$$f_{p2} = \frac{1}{2\pi (C_M + C_L) R_L} \quad (\text{output part})$$

If we compare the poles to Eq. 6.105 and 6.106 for MOSFETs, we observe that these equations are the bipolar counterparts of those ones.

$$f_{p1} = \frac{1}{2\pi C_M (R_L \| \frac{1}{g_m + g_m})} \quad (6.105)$$

$$f_{p2} = \frac{1}{2\pi (C_M + C_L) R_L} \quad (6.106)$$

b) For  $C_M = 14 \text{ pF}$ ,  $C_L = 2 \text{ pF}$ ,  $C_e = 1 \text{ pF}$ ,  $I_c = 1 \text{ mA}$ ,

$$R_{\text{sig}} = 1 \text{ k}\Omega, R_L = 10 \text{ k}\Omega \Rightarrow g_m = \frac{1}{0.025} = 40 \text{ mA/V}$$

$$f_{p1} = \frac{1}{2\pi \times 14 \times (1 \text{ k} \| \frac{10}{40})} \quad (\text{Assuming } \beta = 100)$$

$$f_{p1} = 15.9 \text{ MHz}$$

$$f_{p2} = \frac{1}{2\pi (2+1) \times 10} = 5.3 \text{ MHz}$$

$$f_T = \frac{g_m}{2\pi (C_M + C_L) R_L} = \frac{40 \text{ m}}{2\pi (14+2) \times 10} = 398.1 \text{ MHz}$$

$f_T$  is much greater than the poles.  $f_T \gg f_{p1}$   
 $f_T \gg f_{p2}$

6.95

In Fig. 6.32, if we replace the NMOS with an npn transistor, then we can write:

$$\begin{aligned} R_{be} &= R_e \parallel R_{in} \quad \text{where } R_{in} = r_e \frac{r_o + R_L}{1 + \frac{r_o}{r_e} + \frac{R_L}{r_e}} \\ R_{bc} &= R_L \parallel R_{out} \quad \text{where } R_{out} = (1 + g_m R_e) r_o \end{aligned}$$

using the open-circuit time Constants method to evaluate  $t_H$ :

$$t_H = \frac{1}{2\pi(C_{pi}(R_e/R_{in}) + C_C + C_{mu})(R_L/R_{out})}$$

6.96

$$\beta = 100, V_A = 100V$$

$$I_C \approx I_E = \frac{5 - V_{BE}}{4.3k} = \frac{5 - 0.3}{4.3k} = 1mA$$

From Eq. 6.117a we have:

$$R_{out} = r_o + (1 + g_m r_o)(r_H \parallel R_e)$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{1} = 100k\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{1}{0.025} = 40mA/V$$

$$r_H = \frac{\beta}{g_m} = 2.5k\Omega, R_e = 4.3k\Omega$$

$$\text{Thus: } R_{out} = 100 + (1 + 40 \times 100)(2.5 \parallel 4.3k)$$

$$R_{out} = 6.43M\Omega$$

IF the collector voltage undergoes a change of 10V while the BJT remains in the active mode, the corresponding change in the collector current is:  $\Delta I = \frac{\Delta V}{R_{out}} = \frac{10}{6.43M} = 1.56\mu A$

6.97

$$a) I = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2 = \frac{1}{2} \times 160 \mu \times 100 \times 0.2^2 = 320\mu A$$

$$b) g_m = \frac{2 I_D}{V_{ov}} = \frac{2 \times 0.32}{0.2} = 3.2mA/V = g_{m1} = g_{m2}$$

$$g_{mb} = \lambda g_m = 0.2 \times 3.2 = 0.64mA/V = g_{mb1} = g_{mb2}$$

$$r_o = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 0.32} = 625k\Omega = r_{o1} = r_{o2}$$

$$A_o = g_m r_o = 3.2 \times 62.5 = \underline{200V/V}$$

$$A_{vo2} = 1 + (g_{m2} + g_{mb2}) r_{o2} \quad (\text{Eq. 6.123})$$

$$A_{vo2} = 1 + (3.2 + 0.64) \times 62.5 = \underline{241 V/V}$$

$$c) A_{vo} = -A_{o1} A_{vo2} \quad (\text{Eq. 6.128})$$

$$A_{vo} = -g_{m1} r_{o1}, A_{vo2} = -3.2 \times 62.5 \times 241 = \underline{48200V/V}$$

$$A_{vo} = \underline{48200 V/V}$$

$$d) G_m = \frac{A_{o1} A_{vo2}}{r_{o2} + A_{vo2} r_{o1}} \quad (\text{Eq. 6.130})$$

$$G_m = \frac{200 \times 241}{62.5 + 241 \times 62.5} = \underline{3.19 mV/V \approx g_m}$$

$$R_{out} = r_{o2} + [1 + (g_{m2} + g_{mb2}) r_{o2}] r_{o1} \quad (\text{Eq. 6.126})$$

$$R_{out} = 62.5 + [1 + (3.2 + 0.64) 62.5] 62.5$$

$$R_{out} = \underline{15.125 M\Omega}$$

$$e) A_{vo} = -A_{o1} \frac{R_L}{R_L + A_{o1} r_o} \quad (\text{Eq. 6.131})$$

$$A_{vo} = -(200)^2 \frac{10M}{10^4 + 200 \times \frac{62.5}{1000}} = \underline{17778 V/V}$$

f)  $V_{DS} > V_{ov}$  for  $Q_1$ . Therefore  $V_{DSmin} = V_{ov}$

$$V_{DSmin} = 0.2V$$

$$V_{BIAS} = V_{GS2} + V_{DSmin} = (0.2 + 0.6) + 0.2 = 1V$$

$$V_{BIAS} = 1V$$

6.98

$$V_y = i_x \times r_{o1} = \frac{V_x}{R_{out}} \times r_{o1} \Rightarrow \frac{V_y}{V_x} = \frac{r_{o1}}{R_{out}}$$

$$\frac{V_y}{V_x} = \frac{r_{o1}}{r_{o2} + [1 + (g_{m2} + g_{mb2}) r_{o2}] r_{o1}}$$

IF we use equation 6.127 to approximate  $R_{out}$ , then

$$R_{out}, \text{ then } \frac{V_y}{V_x} \approx \frac{r_{o1}}{\frac{g_m}{2} r_{o2} r_{o1}} = \frac{1}{g_{m2} r_{o2}}$$

6.99

$$a) I = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2 \Rightarrow \text{For same } I: \frac{V_{ovB}}{V_{ova}}^2 = \frac{(\frac{W}{L})_B}{(\frac{W}{L})_a}$$

For same  $I$ , if  $\frac{W}{L}$  is divided by 4,  $\frac{V_{ovB}}{V_{ova}}^2$

then  $\frac{V_{ovB}}{V_{ova}}$  is multiplied by 4, or equivalently

cont.

$V_{ov}$  is doubled.

$g_m = \mu_n C_ox \frac{W}{L} V_{ov}$ . Thus  $g_m$  for circuit (b) is half of the one for circuit (a).

$A_o = g_m r_o = \frac{2ID}{V_{ov}} \times \frac{VA}{ID} = \frac{2VA_L}{V_{ov}}$ . Thus if  $L$  is multiplied by 4 and  $V_{ov}$  is halved, then  $A_o$  is doubled for circuit (b).

In summary, for circuit (b),  $V_{ov}$  is doubled,  $g_m$  is halved,  $A_o$  is doubled.

b) Each transistor in circuit (c) has the same overdrive voltage as the one in circuit (a).

Referring to Eq. 6.129 and 6.130:

$$A_{v_o} = -A_o = -(g_m r_o)^2$$

$$G_m \approx g_{m1} = g_m \text{ (same as circuit (a))}$$

Note that for the transistors in circuit (c) the  $g_m$  and  $r_o$  are the same as the ones in circuit (a). Thus the intrinsic gain for circuit (c),  $A_{v_o} = -A_o^2$  where  $A_o$  is the intrinsic gain for circuit (a).

In general, circuit (c) has a higher output resistance and for the same  $V_{ov}$  of transistors it has lower output swing. The output swing is limited to  $2V_{ov}$  on the low side for circuits (b) and (c) while it is only limited to  $V_{ov}$  for circuit (a).

### 6.100

$$a) A_H = -g_m R'_L = -5 \times (20^k || 20^k) = -50 \frac{V}{V}$$

$$R_{gs} = R_{sig} = 20 k\Omega$$

$$R_{gL} = R_{sig} (1 + g_m R'_L) + R'_L = 20^k (1 + 5 \times 20 || 20) + 20 || 20^k$$

$$R_{gd} = 10.30 k\Omega = 1.03 \text{ MHz}$$

We use Eq. 6.57:

$$T_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R'_L$$

$$T_H = 2^P \times 20^k + 0.2^P \times 1.03 \text{ MHz} + 1^P \times (20^k || 20^k) = 256 \text{ ns}$$

$$f_H = \frac{1}{2\pi T_H} = 622 \text{ kHz}$$

$$|A_H| \cdot f_H = 31.1 \text{ MHz}$$

b) For the cascode amplifier:

$$A_{o1} = g_m r_o = 5 \times 20 = 100 \frac{V}{V}$$

$$A_{v_{o2}} = 1 + (g_{m2} + g_{mb2}) r_{o2} = 1 + (5 + 0.2 \times 5) \times 20^k$$

$$A_{v_{o2}} = 121 \frac{V}{V}$$

$$R_{out} = r_{o2} + A_{v_{o2}} r_{o1} = 20^k + (121 \times 20^k) = 2.44 \text{ M}\Omega$$

$$A_o = A_{v_{o2}} \frac{R_L}{R_L + R_{out}} = 121 \times 100 \times \frac{20}{20 + 2.44 \times 10^6} = 98.4 \frac{V}{V}$$

Using Eq. 6.137,

$$T_H = R_{sig} [C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})] + R_{d1} (C_{gd1} + C_{ab1} + C_{gs2}) + (R_L || R_{out})(C_L + C_{gd2})$$

$$R_{d1} = r_o || \left( \frac{1}{g_{m2} + g_{mb2}} + \frac{R_L}{A_{v_{o2}}} \right) \quad (\text{Eq. 6.124})$$

$$R_{d1} = 20^k || \left( \frac{1}{5 + 0.2 \times 5} + \frac{20}{121} \right) = 0.327 \text{ k}\Omega$$

$$T_H = 20^k [2 + 0.2(1 + 5 \times 0.327)] + 0.327 (0.2 + 0.2 + 2) + (20^k || 2.44 \text{ M}\Omega)(1 + 0.2)$$

$$T_H = 75.1 \text{ ns}$$

$$f_H = \frac{1}{2\pi T_H} = 2.12 \text{ MHz}$$

$$|A_H| \cdot f_H = 208.61 \text{ MHz}$$

### 6.101

$$A_o = 6.6 \text{ dB} = 1995 \frac{V}{V}$$

$$A_o = A_{v_{o2}} \frac{R_L}{R_L + R_{out}} \quad \text{and} \quad R_L = R_{out} \Rightarrow A_o = A_{v_{o2}} \times \frac{1}{2}$$

$$A_{v_{o2}} = (1 + g_{m2} r_{o2}) g_{m1} r_{o1} \approx g_m^2 r_o^2 = \left( \frac{2ID}{V_{ov}} \times \frac{VA}{ID} \right)^2 = \left( \frac{2VA}{V_{ov}} \right)^2$$

$$\Rightarrow 1995 = \frac{1}{2} \times \left( \frac{2 \times 10}{V_{ov}} \right)^2 \Rightarrow V_{ov} = 0.317 \text{ V}$$

$$\Rightarrow I_D = \frac{1}{2} \times \sqrt{\frac{W}{L} V_{ov}^2} = \frac{1}{2} \times 200 \times 10 \times 10 \times 0.317^2 = 0.1 \text{ mA}$$

Since  $R_{sig}$  is small:

$$T_H = (C_L + C_{gd})(R_L || R_{out})$$

$$f_H = \frac{VA}{ID} = \frac{10}{0.1} = 100 \text{ kHz} \Rightarrow g_m = \frac{2ID}{V_{ov}} = 0.631 \text{ mA/V}$$

$$R_{out} = A_{v_{o2}} r_{o1} + r_{o2} = (1 + g_{m2} r_{o2}) r_o + r_o$$

$$R_{out} = 6510 \text{ k}\Omega \approx 6.5 \text{ M}\Omega \quad , \quad R_L = R_{out} \quad .$$

$$T_H \approx (1 \text{ pF} + 0.1 \text{ pF}) \frac{6510}{2} = 3580.5 \text{ ns}$$

$$f_H = 44.5 \text{ kHz}$$

$$f_T \approx |A_H| \cdot f_H = 1995 \times 44.5 = 88.8 \text{ MHz}$$

If the cascode transistor is removed, then we have a common-source configuration.

Cont.

$$A_H = -g_m (r_0 || R_L) = -0.637 (100^k || 6510^k)$$

$$A_H = -62.74 \text{ V/V}$$

$$f_H = \frac{1}{2\pi(C_L + C_D) R'_L} = \frac{1}{2\pi(1+0.1)(100^k || 6510^k)} = 1.47 \text{ MHz}$$

$$f_H = 1.47 \text{ MHz}$$

$$|A_H| \cdot f_H = 92.2 \text{ MHz} \approx f_T$$

Note that the unity-gain stays nearly unchanged. The result is the same at Fig. 6.39.

6.102

$$R_L = \beta r_0, \beta = 100, |V_A| = 100 \text{ V}, I = 0.1 \text{ mA}$$

From Fig. 6.40, we can write:

$$R_{in} = r_{H1} = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{100 \times 0.025}{0.1} = 2.5 \text{ k}\Omega$$

$$R_{in} = 2.5 \text{ k}\Omega$$

$$R_{out} \approx \beta r_{O2}, r_{O2} = \frac{V_A}{I_C} = \frac{100}{0.1} = 1 \text{ M}\Omega$$

$$R_{out} = 100 \times 10^M = 100 \text{ M}\Omega$$

$$A_{V_{O2}} = -\beta A_{V_{B2}} = -\beta g_m r_{O2} = -100 \times \frac{0.1}{0.025} \times 1 \Rightarrow$$

$$A_{V_B} = -400 \times 10^3 \text{ V/V}$$

$$G_m = \frac{A_{V_B}}{R_{out}} = \frac{400 \times 10^3}{100 \times 10^8} = 4 \text{ mA/V} \approx g_m$$

$$G_m = 4 \text{ mA/V}$$

Since  $R_{out} = R_L = \beta r_0$ , we have

$$\frac{V_o}{V_i} = -G_m (R_L || R_{out}) = -G_m \frac{\beta r_0}{2} = -4 \frac{100 \times 1}{2}$$

$$\frac{V_o}{V_i} = -200 \times 10^3 \text{ V/V}$$

From Fig. 6.41 the gain of the CE stage is

$$A_{CE} = -g_{m1} (r_{o1} || r_{e2} \frac{r_{o2} + R_L}{r_{o2} + \frac{R_L}{\beta+1}})$$

$$r_{o1} = r_{o2} = r_0, R_L = \beta r_0$$

$$A_{CE} = -g_{m1} (r_0 || r_{e2} \frac{r_0 + \beta r_0}{r_0 + \frac{\beta r_0}{r_0 + 1}})$$

$$A_{CE} = -g_{m1} (r_0 || r_{e2} \frac{(\beta + 1)r_0}{2r_0})$$

$$A_{CE} = -g_{m1} (r_0 || \frac{r_n}{2}) = -\frac{I}{V_T} (1^M || \frac{25^k}{2})$$

$$A_{CE} = -50 \text{ V/V}$$

6.103

$$R_{sig} = 4 \text{ k}\Omega, R_L = 2.4 \text{ k}\Omega, I = 1 \text{ mA}, \beta = 100, r_0 = 100 \text{ k}\Omega$$

Refer to Fig. 6.42:

$$A_M = -\frac{r_H}{r_H + r_x + R_{sig}} \times g_m (\beta r_0 || R_L)$$

$$r_H = \frac{\beta}{g_m} = \frac{100}{1/0.025} = 2.5 \text{ k}\Omega \quad g_m = \frac{I_C}{V_T} = 40 \text{ mA/V}$$

$$A_M = -\frac{2.5}{2.5 + 0.05 + 4} \times 40 \times (100 \times 100 || 2.4^k)$$

$$A_M = -36.6 \text{ V/V}$$

$$R'_{sig} = r_H || (r_x + R_{sig}) = 2.5^k || (0.05 + 4^k)$$

$$R_{sig} = 1.55 \text{ k}\Omega$$

$$R_{T1} = R'_{sig} = 1.55 \text{ k}\Omega$$

$$R_{H1} = R'_{sig} (1 + g_m R_{e1}) + R_{e1}$$

$$R_{e1} = r_{o1} || r_{e2} \left( \frac{r_0 + R_L}{r_0 + R_L / \beta + 1} \right) = 100^k || \frac{100^k}{101} \left( \frac{100 + 2.4}{100 + 2.4} \right)$$

$$R_{e1} = 1 \text{ k}\Omega$$

$$R_{H1} = 1.55 (1 + 40 \times 1) + 1 = 64.55 \text{ k}\Omega$$

$$R_{out} = \beta r_0 = 10 \text{ M}\Omega$$

$$T_H = C_{H1} R_{H1} + C_{H1} R_{H1} + (C_{S1} + C_{S2}) R_{e1} + (C_L + C_{CS2} + C_{H2}) (R || R_{out})$$

$$T_H = 14 \times 1.55 + 2 \times 64.55 + (0 + 14) \times 1 + (0 + 2) (2.4^k || 10^M)$$

$$T_H = 169.6 \text{ ns}$$

$$f_H = 939 \text{ kHz}$$

6.104

a) If we employ Miller's theorem to  $C_{H1}$ :

$$\frac{1}{C_{H1} s} \frac{1}{1-A} = \frac{1}{C_{H1} s} \frac{1}{1-(\beta+1)} = \frac{1}{2C_{H1} s}$$

or  $2C_{H1}$  appears in parallel with  $C_H$ . Thus the time constant due to  $(C_H + 2C_{H1})$  is:

$R'_{sig} (C_H + 2C_{H1})$  which results in:

$$f_{P1} = \frac{1}{2\pi R'_{sig} (C_H + 2C_{H1})}$$

If we refer to Fig. 6.42, we'll see that the output pole is:

$$f_{P2} = \frac{1}{2\pi (C_L + C_{CS2} + C_{H2}) R_L}$$

$$b) R_{sig} = 1 \text{ k}\Omega \Rightarrow R'_{sig} = r_H || R_{sig} = \frac{100}{1/0.025} || 1^k$$

$$\Rightarrow R'_{sig} = 0.714 \text{ k}\Omega$$

$$f_{P1} = \frac{1}{2\pi \times 0.714^k (5 + 2 \times 1)^k} = 31.85 \text{ MHz}$$

$$f_{P2} = \frac{1}{2\pi (0 + 0 + 1) \times 10} = 15.9 \text{ MHz}$$

(Assume  $R_L = 10 \text{ k}\Omega$ )

Cont.

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{F_{P_1}}\right)^2 + \left(\frac{1}{F_{P_2}}\right)^2}} = 14.2 \text{ MHz}$$

IF  $R_{\text{sig}} = 10 \text{ k}\Omega$ :

$$R'_{\text{sig}} = 2.5 \times 10^3 \parallel 10^3 = 2 \times 10^3$$

$$F_{P_1} = \frac{1}{2\pi(5+2)2} = 11.4 \text{ MHz}$$

$F_{P_2}$  is the same;  $F_{P_2} = 15.9 \text{ MHz}$

$$f_H = 9.26 \text{ MHz}$$

6.105

Refer to Fig. 6.43.

$$I = \frac{1}{2} \mu_F g_m \frac{W}{L} V_{ov}^2 \Rightarrow 100 \mu\text{A} = \frac{1}{2} \times 60 \frac{W}{L} 0.2^2 \Rightarrow \frac{W}{L} = 83.3$$

$$V_{SG1} = V_{DD} - V_{BIAS1} = 3.3 - V_{BIAS1}$$

$$V_{ov} = V_{SG1} - |V_{tp}| = 0.2 \Rightarrow 0.2 = 3.3 - V_{BIAS1} - 0.8$$

$$\Rightarrow V_{BIAS1} = 2.3 \text{ V}$$

$$\text{For Maximum swing: } V_{SG1} = V_{ov} \Rightarrow V_{D1} = 3.3 - 0.2 = 3.1 \text{ V}$$

$$\Rightarrow V_{D1} = 3.1 \text{ V}$$

$$\text{then: } V_{SG2} - |V_{tp}| = V_{ov} \Rightarrow 3.1 - V_{BIAS2} - 0.8 = 0.2$$

$$V_{BIAS2} = 2.1 \text{ V}$$

$$\text{The highest allowable voltage at the output is } V_{DD} - V_{ov} - V_{ov} = 3.3 - 0.2 - 0.2 = 2.9 \text{ V}$$

$$R_o = g_m r_{o2} r_{o1} \quad (\text{Eq. 6.141})$$

$$g_m = \frac{I_c}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V} \quad r_o = \frac{V_A}{I_c} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$R_o = 4 \times 50 \times 50 = 10 \text{ M}\Omega$$

$$\underline{R_o = 10 \text{ M}\Omega}$$

6.106

$$I_D = 0.2 \text{ mA} \Rightarrow g_m = \frac{2I_D}{V_{ov}} = \frac{2 \times 0.2}{0.25} = 1.6 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$R_{out} = A_o^2 r_o \Rightarrow A_o = g_m r_o = 1.6 \times 25 = 40 \text{ V/V}$$

$$R_{out} = 40 \times 40 \times 25 = 40 \text{ M}\Omega$$

6.107

$$I = 100 \mu\text{A} \Rightarrow V_{BIAS} = 1 \text{ V}$$

$$a) I_{C1} = 2I - I_{E2} = 2I - I = 100 \mu\text{A}$$

$$b) V_X = V_{BIAS} + V_{BE} = 1 + 0.7 = 1.7 \text{ V}$$

$$c) g_m = \frac{I}{V_T} = \frac{0.1}{0.025} = 4 \text{ mA/V} \quad g_m = 4 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_c} = \frac{100}{0.1} = 1 \text{ M}\Omega$$

$$d) V_{max} = V_X - V_{CESat}$$

$$V_{max} = 1.7 - 0.2 = 1.5 \text{ V}$$

$$e) R_i = r_{in} = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

f) Since the output resistance of current source is equal to  $R_o$ , then the overall output resistance  $R_{out} = \frac{R_o}{2}$ .

$$R_o = \beta r_o = 100 \times 1 \text{ M} = 100 \text{ M}\Omega \Rightarrow R_{out} = 50 \text{ M}\Omega$$

$$g) A_M \approx g_m (\beta r_o \parallel R_L) \text{ where } R_L = \beta r_o$$

$$A_M = g_m \frac{\beta r_o}{2} = 4 \times 100 \times \frac{1}{2} = 200 \times 10^6 \text{ V/V}$$

The current source  $2I$  should ideally have infinite output resistance.

$$a) I_{C1} = 2I - I = 100 \mu\text{A}$$

$$b) V_X = V_{BIAS} + V_{SG}$$

$$I = \frac{1}{2} \mu_F g_m \frac{W}{L} V_{ov}^2$$

$$0.1 = \frac{1}{2} \times 2 \times \frac{W}{L} V_{ov}^2 \Rightarrow V_{ov} = 0.316 \text{ V}$$

$$V_{SG} = |V_E| + V_{ov} = 0.6 + 0.316$$

$$V_{SG} = 0.916 \text{ V} \Rightarrow V_X = 1 \text{ V} + 0.916 = 1.916 \text{ V}$$

$$c) g_{m1} = \frac{0.1}{0.025} = 4 \text{ mA/V}$$

$$g_{m2} = \frac{2I_D}{V_{ov}} = \frac{2 \times 0.1}{0.316} = 0.633 \text{ mA/V}$$

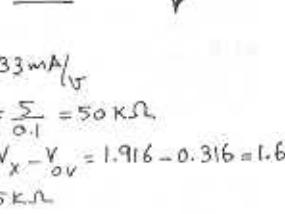
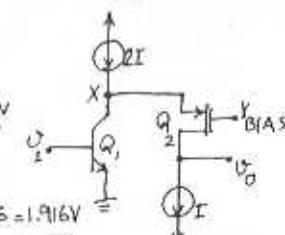
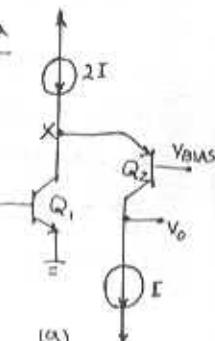
$$r_{o1} = \frac{V_A}{0.1} = 1 \text{ M}\Omega, \quad r_{o2} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$d) V_{max} = V_X - V_{SDmin} = V_X - V_{ov} = 1.916 - 0.316 = 1.6 \text{ V}$$

$$e) R_i = r_{in} = \frac{\beta}{g_{m1}} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$f) R_{out} = R_{out1} + [(1 + g_{m1} g_{m2}) R_{out1}] r_{o2}$$

Cont.



$$R_{out1} = r_{o1}$$

$$R_{out} = r_{o2} + [1 + g_{m2} r_{o2}] r_{o1} = 50 + 0.633 \times 1 \times 50 \text{ K}$$

$$R_{out} = 31.7 \text{ M}\Omega$$

f) First note that  $R_{L2}$  or the output resistance of the current source is equal to  $R_{out}$  or  $32.6 \text{ M}\Omega$  or  $R_{L2} \approx g_{m2} r_{o1} r_{o2}$

$$\frac{V_x}{V_i} = -g_{m1} (r_{o1} \parallel R_{in2})$$

$$R_{in2} = \frac{1}{g_{m2}} + \frac{R_{L2}}{A_{o2}} = \frac{1}{g_{m2}} + \frac{g_{m2} r_{o1} r_{o2}}{g_{m2} r_{o2}} = \frac{1}{g_{m2}} + r_{o1}$$

$$\frac{V_x}{V_i} = -g_{m1} (r_{o1} \parallel (r_{o1} + \frac{1}{g_{m2}})) \approx -g_{m1} \frac{r_{o1}}{2} = -2000 \text{ V/V}$$

$$\frac{V_o}{V_x} = (1 + g_{m2} r_{o2}) \frac{R_{L2}}{R_{L2} + R_{bat}} = (1 + 0.633 \times 50) \frac{1}{2} = 16.3 \text{ V/V}$$

$$\frac{V_o}{V_i} = -2000 \times 16.3 = -32600 \text{ V/V}$$

In order for the output resistance of the current-source to reduce the gain by  $\beta$ :

$$\frac{V_x}{V_i} = -g_{m1} (r_{o1} \parallel (r_{o1} + \frac{1}{g_{m2}}) \parallel R) = -g_{m1} \left( \frac{r_{o1}}{2} \parallel R \right)$$

For  $\frac{V_x}{V_i} = -\frac{99}{100} \times 2000 = 1980$  we should have:

$$r_{o1} \parallel R = \frac{1980}{4} = 495 \text{ K} \Rightarrow 500 \text{ K} \parallel R = 495 \text{ K} \Rightarrow$$

$R = 49.5 \text{ M}\Omega$  (This is the output resistance

of the  $2I$  current-source)

(Note that  $\frac{V_o}{V_x}$  did not depend on  $R$ )

a)  $I_1 = 2I - I = 100 \mu\text{A}$

b)  $V_x = V_{B1AS} + V_{SG}$

$$I = \frac{1}{2} \frac{V_x - V_{SG}}{R_L} \Rightarrow V_{OV} = 0.316 \text{ V}$$

$$V_{SG} = 0.6 + 0.316 = 0.916 \text{ V}$$

$$V_x = 1 + 0.916 = 1.916 \text{ V}$$

c)  $g_{m1} = g_{m2} = \frac{2I_0}{V_{OV}} = \frac{0.1 \times 2}{0.316} = 0.633 \text{ mA/V}$

$$r_{o1} = r_{o2} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

d)  $V_{Omax} = V_x - V_{DSMin} = V_x - V_{OV} = 1.916 - 0.316 = 1.6 \text{ V}$

e)  $R_{in} = \infty$

f)  $R_{out} = r_{o2} + (1 + g_{m2} r_{o2}) r_{o1} = 50 + (1 + 0.633 \times 50) \times 50$

$$R_{out} = 1682.5 \text{ K} = 1.68 \text{ M}\Omega = R_{L2} \approx g_{m2} r_{o2} r_{o1}$$

g)  $\frac{V_x}{V_i} = -g_{m1} (r_{o1} \parallel R_{in2})$

$$R_{in2} = \frac{1}{g_{m2}} + \frac{R_{L2}}{A_{o2}} \approx \frac{1}{g_{m2}} + \frac{g_{m2} r_{o2} r_{o1}}{g_{m2} r_{o2}} = \frac{1}{g_{m2}} + r_{o1}$$

Note that  $R_{L2}$  is the output resistance of the current source  $I$ :

$$\frac{V_x}{V_i} = -g_{m1} (r_{o1} \parallel (\frac{1}{g_{m2}} + r_{o1})) = -0.633 \times (50 \parallel (\frac{1}{0.633} + 50))$$

$$\frac{V_x}{V_i} = 16.07 \text{ V/V}$$

$$\frac{V_o}{V_x} = (1 + g_{m2} r_{o2}) \frac{R_{L2}}{R_{L2} + R_{out}} = (1 + 0.633 \times 50) \times \frac{1}{2}$$

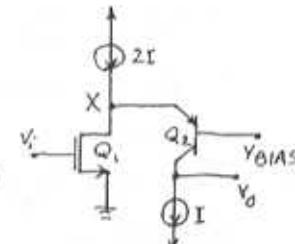
$$\frac{V_o}{V_x} = 16.32 \text{ V/V}$$

$$\frac{V_o}{V_i} = -262.34 \text{ V/V}$$

In order to reduce the gain by  $\beta$  by introducing non-ideal current-source  $2I$  with output resistance  $R$ :

$$\frac{V_x}{V_i} = -g_{m1} (r_{o1} \parallel R \parallel (\frac{1}{g_{m2}} + r_{o1})) = \frac{90}{100} \times 16.07 \approx 14.46$$

$$\Rightarrow (50 \text{ K} \parallel R \parallel (51.58 \text{ K})) = \frac{14.46}{0.633} \Rightarrow R = 227.5 \text{ K}$$



a)  $I_1 = 2I - I = I = 0.1 \text{ mA}$

b)  $V_x = V_{B1AS} + 0.7 = 1.7 \text{ V}$

c)  $g_{m1} = \frac{2I_0}{V_{OV}}$

$$\frac{V_{OV}}{2 \times \frac{1}{2}} = 2V_{OV} = 0.316 \text{ V}$$

$$g_{m1} = 0.633 \text{ mA/V}$$

$$r_{o1} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$g_{m2} = \frac{0.1}{0.025} = 4 \text{ mA/V} \Rightarrow r_{o2} = \frac{V_A}{I_C} = \frac{100}{0.1} = 1 \text{ M}\Omega$$

d)  $V_{Omax} = V_x - V_{ESat} = V_x - 0.2 = 1.5 \text{ V}$

e)  $R_{in} = \infty$

f)  $R_{out} = r_{o2} (1 + g_{m2} (r_{o1} \parallel r_{H1}))$

$$R_{out} = 1 \text{ M} (1 + 4 (50 \text{ K} \parallel \frac{100}{4})) = 67.67 \text{ M}\Omega$$

$R_{L2} = R_{out}$

g)  $\frac{V_x}{V_i} = -g_{m1} (r_{o1} \parallel R_{in2}) = -g_{m1} (r_{o1} \parallel \frac{r_{o2} + R_{L2}}{r_{o2} + R_{L2} / \beta + 1})$

$$r_{in2} = \frac{r_{o2}}{\beta + 1} = \frac{\beta}{\beta + 1} g_{m2} = 3.96 \text{ k}\Omega$$

$$\frac{V_x}{V_i} = -4 (50 \text{ K} \parallel \frac{1 \text{ M} + 67.67 \text{ M}}{1 + 67.67} \times 3.96) = -152.86 \text{ V/V}$$

$$\frac{V_o}{V_x} = (1 + g_{m2} r_{o2}) \frac{R_{L2}}{R_{L2} + R_{out}} = (1 + g_{m2} r_{o2}) \frac{1}{2}$$

$$\frac{V_o}{V_x} = (1 + 4 \times 1000) \frac{1}{2} = 2000.5 \text{ V/V}$$

$$\frac{V_o}{V_i} = -152.86 \times 2000.5 = -305.8 \times 10^3 \text{ V/V}$$

To reduce the gain by  $\beta$ , the effect of  $R_{in}$

is considered:  $0.99 \frac{V_x}{V_i} = 0.99 \times 152.86$

Cont.

$$0.99 \times 152.86 = 4 \times (50^k \parallel \frac{1+67.67}{1+67.67} \parallel R) \\ \Rightarrow R = 3.74 \text{ M}\Omega$$

6.108

$$g_m = 2 \text{ mA/V} \Rightarrow g_{mb} = g_m = 0.2 \times 2 = 0.4 \text{ mA/V}$$

$$R_{out} = r_o [1 + (g_m + g_{mb}) R_s] = 50^k [1 + (2 + 0.4) \times 50^k]$$

$$\underline{R_{out} = 110 \text{ k}\Omega}$$

$$A_{Vd} = -g_m r_o = -2 \times 50 = -100 \text{ V/V}$$

$$A_V = -A_{Vd} \frac{R_L}{R_L + R_{out}} = -100 \times \frac{50}{50 + 110} = -31.25 \text{ V/V}$$

Using Eq. 6.144:

$$G_m = \frac{g_m}{1 + (g_m + g_{mb}) R_s} = \frac{2}{1 + (2 + 0.2 \times 2) 50^k}$$

$$\Rightarrow G_m = 0.91 \text{ mA/V}$$

$$\frac{V_{gs}}{V_i} = \frac{1}{1 + (g_m + g_{mb}) R_s} \frac{R_L \parallel R_{out}}{R_L \parallel r_o} \quad (\text{Eq. 6.144})$$

$$\underline{\frac{V_{gs}}{V_i} = 0.625 \text{ V} \Rightarrow V_{gs} = 0.625 \text{ V}}$$

6.109

$$\frac{V_{gs}}{V_i} = \frac{1}{3} \quad \text{using Eq. 6.144:}$$

$$\frac{V_{gs}}{V_i} = \frac{1}{1 + (g_m + g_{mb}) R_s} \frac{R_L \parallel R_{out}}{R_L \parallel r_o}$$

$$\Rightarrow \frac{1}{3} = \frac{1}{1 + (2 + 0.2 \times 2) R_s} \times \frac{50^k \parallel R_{out}}{50^k \parallel 50^k} \quad \textcircled{1}$$

$$R_{out} = r_o [1 + (g_m + g_{mb}) R_s] = 50 (1 + 2.4 R_s)$$

$$R_{out} = 50 (1 + 2.4 \times R_s)$$

Substitute in \textcircled{1}:

$$\frac{1}{3} = \frac{1}{1 + 2.4 R_s} \frac{50^k \times 50 (1 + 2.4 R_s)}{50 + 50 (1 + 2.4 R_s)} \times \frac{1}{25}$$

$$\Rightarrow R_s = 1.67 \text{ k}\Omega$$

$$\Rightarrow R_{out} = 50 (1 + 2.4 \times 1.67) = 250.4 \text{ k}\Omega$$

$$A_V = -A_{Vd} \frac{R_L}{R_L + R_{out}} = -g_m r_o \frac{R_L}{R_L + R_{out}}$$

$$A_V = -2 \times 50 \times \frac{50}{50 + 250.4} = -16.64 \text{ V/V}$$

6.110

$$\text{a) } A_H = -A_{Vd} \frac{R_L}{R_L + R_{out}} = -g_m r_o \frac{R_L}{R_L + r_o}$$

$$A_H = -5 \times 40 \times \frac{40}{40 + 40} = -100 \text{ V/V}$$

$$R'_L = R_L \parallel R_{out} = R_L \parallel r_o = 20 \text{ k}\Omega$$

$$R_{gd} = R_{sig} (1 + G_m R'_L) \quad \text{where } G_m = g_m \quad (\text{Eq. 6.144})$$

$$\Rightarrow R_{gd} = 20^k (1 + 5 \times 20) = 2020 \text{ k}\Omega = 2.02 \text{ M}\Omega$$

$$R_{gd} = 2.02 \text{ M}\Omega$$

$$R_s = 0 \Rightarrow R_{gs} = R_{sig} = 20 \text{ k}\Omega$$

$$R_{CL} = R'_L = 20 \text{ k}\Omega$$

$$C_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R'_L$$

$$\tau_H = 2 \times 20^k + 0.1 \times 2.02 \text{ M} + 1 \times 20^k = 262 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H} = \underline{607.8 \text{ kHz}}$$

$$|A_H| \cdot f_H = +100 \times 607.8 = 6078 \times 10^3 = \underline{60.78 \text{ MHz}}$$

b)  $R_s = 500 \Omega$

$$R_{out} = r_o [1 + (g_m + g_{mb}) R_s] = 40 [1 + (5 + 1) 0.5] = 160 \text{ k}\Omega$$

$$A_M = -g_m r_o \frac{R_L}{R_L + R_{out}} = -5 \times 40 \times \frac{40}{40 + 160} = -40 \text{ V/V}$$

$$R'_L = R_L \parallel R_{out} = 40 \text{ k} \parallel 160 \text{ k} = 32 \text{ k}\Omega$$

$$R_{gd} = R_{sig} (1 + G_m R'_L)$$

$$G_m = \frac{g_m r_o}{r_o [1 + (g_m + g_{mb}) R_s]} \quad (\text{Eq. 6.144})$$

$$G_m = \frac{5 \times 40}{40 [1 + (5 + 1) 0.5]} = 1.25 \text{ mA/V}$$

$$R_{gd} = 20^k (1 + 1.25 \times 32^k) = 820 \text{ k}\Omega$$

$$R_{gs} = \frac{R_{sig} + R_s}{1 + (g_m + g_{mb}) R_s} \frac{r_o}{r_o + R_L} \quad (\text{Eq. 6.151})$$

$$R_{gs} = \frac{20^k + 0.5^k}{1 + (5 + 1) 0.5} \frac{40}{40 + 40} = 8.2 \text{ k}\Omega$$

$$R_{CL} = R_L \parallel R_{out} = R'_L = 32 \text{ k}\Omega$$

$$C_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R'_L = 2 \times 8.2 + 0.1 \times 820 + 32 \times$$

$$\tau_H = 130.4 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H} = \underline{1.22 \text{ MHz}}$$

$$|A_H| \cdot f_H = 48.8 \text{ MHz}$$

6.111

$$\text{Using Eq. 6.151: } R_{gs} = \frac{R_{sig} + R_s}{1 + (g_m + g_{mb}) R_s} \frac{r_o}{r_o + R_L}$$

$$\text{If we define: } K = (g_m + g_{mb}) R_s \quad \text{and} \quad R_{sig} \gg R_s$$

$$\text{then: } R_{gs} = \frac{R_{sig}}{1 + K \frac{r_o}{r_o + R_s}} = \frac{R_{sig}}{1 + K/2}$$

$$\text{Using Eq. 6.144: } G_m = \frac{g_m}{1 + (g_m + g_{mb}) R_s} = g_m / (K + 1)$$

Cont.

$$R'_L = R_L || R_{out}$$

$$R_{out} = r_o [1 + (g_m + g_{mb}) R_S] = r_o (1+K)$$

$$R'_L = r_o || r_o (1+K) = r_o \frac{(1+K)}{2+K}$$

Using Eq. 6.148:

$$R_{gd} = R_S \cdot g (1 + G_m R'_L) + R'_L$$

$$R_{gd} = R_S \cdot g (1 + \frac{g_m}{1+K} \times r_o \frac{1+K}{2+K}) + r_o \frac{1+K}{2+K}$$

$$R_{gd} = R_S \cdot g (1 + \frac{A_o}{2+K}) + r_o \frac{1+K}{2+K}$$

$$R_{C_2} = R'_L = r_o \frac{1+K}{2+K}$$

$$T_H = R_S C_{gs} + R_{gd} C_{gd} + R_{C_2} C_2$$

$$T_H = \frac{R_S \cdot g}{1+K/2} C_{gs} + R_S \cdot g (1 + \frac{A_o}{2+K}) C_{gd} + (C_L + C_{gd}) r_o \frac{1+K}{2+K}$$

6.112

$$R_{out} = r_o [1 + (g_m + g_{mb}) R_S] = r_o (1+K)$$

$$R_{out} = 40(1+K)$$

$$|A_H| = -g_m r_o \frac{R_o}{R_o + R_{out}} = -5 \times 40 \times \frac{40}{40 + 40(1+K)}$$

$$|A_H| = -\frac{200}{2+K}$$

$$T_H = \frac{C_{gs} R_S \cdot g}{1+K/2} + C_{gd} R_S \cdot g (1 + \frac{A_o}{2+K}) + (C_L + C_{gd}) r_o \frac{1+K}{2+K}$$

(From problem 6.111)

$$T_H = \frac{2k 20k}{1+K/2} + 0.1 \times 20k (1 + \frac{5 \times 40}{2+K}) + (1 + 0.1) 40 \frac{1+K}{2+K}$$

$$T_H = \frac{80}{2+K} + 2(1 + \frac{200}{2+K}) + 44 \frac{1+K}{2+K}$$

$$T_H = \frac{528 + 46k}{2+K} \text{ nS}$$

$$f_H = \frac{1}{2\pi T_H} = \frac{(2+K) \times 10^3}{2\pi (528 + 46k)} \text{ MHz}$$

$$f_T = |A_H| \cdot f_H$$

K	$ A_H  V/V$	$f_H \text{ (MHz)}$	$ A_H  \cdot f_H \text{ (MHz)}$
0	-100	0.603	60.3
1	-66.67	0.832	55.47
2	-50.00	1.027	51.35
3	-40.00	1.195	47.8
4	-33.33	1.342	44.73
5	-28.57	1.471	42.03
6	-25.00	1.584	39.6
7	-22.22	1.686	37.46
8	-20.00	1.777	35.54
9	-18.18	1.859	33.8
10	-16.67	1.934	32.24
11	-15.38	2.002	30.79

K	$A_H (V/V)$	$f_H (\text{MHz})$	$ A_H  \cdot f_H$
12	-14.28	2.064	29.47
13	-13.33	2.121	28.27
14	-12.5	2.174	26.75
15	-11.76	2.223	26.14

IF  $f_H = 2 \text{ MHz}$ , then by looking at the table,

$K=11$ . Therefore:  $K=11=(g_m + g_{mb}) R_S \Rightarrow$

$$R_S = \frac{11}{5+1} = 1.83 \text{ k}\Omega$$

From the table:  $A_H = -15.38$

6.113

a) Eq. 6.156: Gain Bandwidth product =  $|A_H| \cdot f_H$

$$|A_H| \cdot f_H = \frac{1}{2\pi C_{gd} R_S \cdot g}$$

For  $C_{gd} = 0.1 \text{ pF}$ ,  $R_S \cdot g = 10 \text{ k}\Omega \cdot \text{A/V}$ :

$$|A_H| \cdot f_H = \frac{159.2}{20}$$

b) IF  $|A_H| = 20 \text{ V/V}$ , then  $f_T = 159.2 \text{ MHz}$ ,

$$f_H = \frac{159.2}{20} = 7.96 \text{ MHz}$$

$$c) g_{ds} = 5 \text{ mA/V} \quad X = 0.2 \Rightarrow g_{mb} = 1 \text{ mA/V}$$

$$A_D = 100 V/V \quad R_L = 20 \text{ k}\Omega$$

$$r_o = \frac{A_o}{g_m} = \frac{100}{5} = 20 \text{ k}\Omega = R_L$$

$$A_D = A_o \frac{R_L}{R_L + R_{out}} \Rightarrow 20 = -100 \frac{20}{R_{out} + 20}$$

$$\Rightarrow R_{out} = 80 \text{ k}\Omega$$

$$R_{out} = r_o [1 + (g_m + g_{mb}) R_S] = 20 [1 + (5+1) R_S] = 80$$

$$\Rightarrow 1 + 6 R_S = 4 \Rightarrow R_S = 0.5 \text{ k}\Omega = 500 \text{ }\Omega$$

$$R_S = 500 \text{ }\Omega$$

6.114

$R_E = 100 \text{ }\Omega$ ,  $I = 0.5 \text{ mA}$ ,  $\beta = 100$ ,  $V_A = 100 \text{ V}$ ,  $R_L = r_o$

$R_{in} = (\beta + 1) r_e + (\beta + 1) R_E \frac{1}{1 + R_E / r_b}$  when  $R_E \ll r_b$   
(Eq. 6.158)

$$r_{in} = \frac{\beta}{g_m} = \frac{100}{0.5/0.025} = 5 \text{ k}\Omega$$

$$g_m = \frac{I}{V_T} = 20 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$r_E = \frac{r_{in}}{(\beta + 1)} = \frac{5}{101} = 49.5 \text{ }\Omega$$

Cont.

$$R_{in} = (1+100) \cdot 0.0495 + (1+100) \times 0.1 \frac{1}{1+1}$$

$$R_{in} = 10.05 \text{ k}\Omega$$

$$R_o \approx r_o (1 + g_m R'_e) \quad \text{where } R'_e = R_e \parallel r_n \quad (\text{Eq. 6.160})$$

$$R_o = 200 (1 + 20 \cdot 0.1^k / (5K)) = 592.2 \text{ k}\Omega$$

$$R_o = 592.2 \text{ k}\Omega$$

$$A_{v_o} = -g_m r_o = -20 \times 200 = -4000 \text{ V/V}$$

$$G_m = -\frac{A_{v_o}}{R_o} = 6.75 \text{ mA/V}$$

$$A_{v_o} = A_{v_o} \frac{R_L}{R_L + R_o} = -4000 \times \frac{200}{200 + 592.2} = -1009.8 \text{ V/V}$$

$$AV = -1009.8 \text{ V/V}$$

$$G_v = \frac{V_o}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} \quad A_{v_o} \frac{R_L}{R_L + R_o} = \frac{R_{in}}{R_{in} + R_{sig}} A_{v_o}$$

$$G_v = \frac{10.05}{10.05 + 10} \times (-1009.8) = -506.2 \text{ V/V}$$

6.115

$$g_m = \frac{I}{V_T} = 20 \text{ mA/V}$$

$$r_n = \frac{\beta}{g_m} = 5 \text{ k}\Omega$$

$$r_e = \frac{5 \text{ k}}{100+1} = 49.5 \text{ }\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$R_{in} = (\beta + 1) r_e + (\beta + 1) R_o \frac{1}{1 + R_o/r_o} \quad (\text{Eq. 6.158})$$

$$R_{in} = 101 \times 49.5 + 101 \times 100 \times \frac{1}{2} = 10049.5 \frac{\Omega}{\text{k}\Omega} = 10.05 \text{ k}\Omega$$

$$R_{in} = 10.05 \text{ k}\Omega$$

$$R_i = (\beta + 1) (r_e + R_o) = 101 \times (49.5 + 100) = 15.1 \text{ k}\Omega$$

$$R_i = 15.1 \text{ k}\Omega$$

$$A_{v_o} = -g_m r_o = -20 \times 200 = -4000 \text{ V/V}$$

$$G_{v_o} = \frac{R_i}{R_i + R_{sig}} A_{v_o} = \frac{15.1}{15.1 + 10} \times 4000 = -2406.4 \text{ V/V}$$

$$R_o = r_o (1 + g_m R'_e)$$

$$R'_e = R_e \parallel r_H = 0.1^k / 5^k = 0.1 \text{ k}\Omega$$

$$R_o = 200 (1 + 20 \times 0.1) = 600 \text{ k}\Omega$$

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} \quad A_{v_o} = \frac{-10.05}{10.05 + 10} \times 4000 \times \frac{200}{200 + 600}$$

$$(\text{Note that } A_v = A_{v_o} \frac{R_L}{R_L + R_o})$$

$$G_v = 501.25 \text{ V/V}$$

$$G_v = G_{v_o} \frac{R_L}{R_L + R_{out}} \Rightarrow 501.25 = 2406.4 \frac{200}{200 + R_{out}}$$

$$R_{out} = 760.16 \text{ k}\Omega$$

6.116

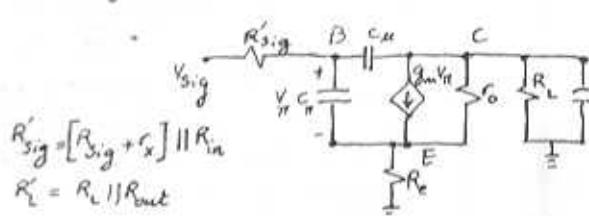
$$\text{a) Eq. 6.148: } R_{jd} = R_{sig} (1 + G_m R'_L) + R_L$$

If we try to adapt the above formula for BJTs:

Using Miller's Theorem for  $C_M$ , we

would have  $G_M (1 + G_m R'_L)$  hanging from the base to the ground.

The resistance seen by  $G_M (1 + G_m R'_L)$  is:  $R'_M = (R_{sig} + r_H) \parallel R_{in}$



From the collector side, the resistance seen by  $C_M$  is  $R_{out} \parallel R_L$  or  $R'_L$ . Therefore the total resistance seen by  $C_M$  is  $R_M$ :

$$R_M = [(R_{sig} + r_H) \parallel R_{in}] (1 + G_m R'_L) + R'_L$$

The resistance seen by  $C_L$  is again  $R_L \parallel R'_L$  or  $R'_L \parallel R_{out}$   $\Rightarrow R'_L = R_L$  (Same as Eq. 6.150 for

Now the resistance seen by  $C_H$  is  $r_H$  para to

then we can write:  $C_H = C_H R_H + C_M R_M + C_L R_L$

$$\text{b) i) } R_e = 0, R_{out} = R_o, r_H = \frac{\beta}{g_m} = 5 \text{ k}\Omega$$

$$A_H = \frac{r_H}{r_H + R_{sig} + r_H} \times g_m R'_L \quad \text{Eq. (6.70)}$$

$R'_L = R_L \parallel R_{out}$ , since  $R_{out} \leq R_o$  then  $R'_L = R_L \parallel R_o$

$$R_o = r_o (1 + g_m R'_e) = r_o (1 + g_m r_H) = (\beta + 1) r_o = 10.1 \text{ M}\Omega$$

$$R'_L = 5.3 \text{ k} \parallel 10.1 \text{ M} = 5.3 \text{ k} \Rightarrow A_H = \frac{-5}{5 + 10.1} \times 20 \times 5.3$$

Now using formulas given in part (a) we can't

$$S_H = \frac{1}{2\pi C_H}. \text{ Since } R_{out} = R_o, \text{ then } R'_L = 5.3 \text{ k}\Omega =$$

Note that for  $R_e = 0$ , we have  $R_{in} = r_H = 5$  and  $G_m = g_m = 20 \text{ mA/V}$

Cont

$$\begin{aligned}
 R_H &= ((R_{sig} + r_X) || r_\pi) (1 + g_m R'_L) + R'_L \\
 R_H &= ((1+0.2) || 5) (1 + 20 \times 5.3) + 5.3 = 108.85 \text{ k}\Omega \\
 R_\pi &= r_\pi || (R_{sig} + r_X) = 5^k || 1.2^k = 0.97 \text{ k}\Omega \\
 r_H &= R_\pi C_H + R_H C_H + R_L' C_L = 10 \times 0.97 + 0.5 \times 108.85 \times 2 \times 5.3 \\
 f_H &= 74.73 \text{ n}s \Rightarrow f_H = 2.13 \text{ MHz} \Rightarrow A_H = -85.5 \text{ V/V}
 \end{aligned}$$

(ii)  $R_e = 200 \text{ }\Omega$ .

$$A_{V_0} \equiv -g_m r_o \quad (\text{Eq. 6.159})$$

$$A_{V_0} = -20 \times 100 = -2000 \text{ V/V}$$

$$R_{in} = (\beta + 1) r_e + (\beta + 1) \frac{R_e}{1 + R_e / r_o} \frac{1}{1 + R_L / R_o}$$

$$R_{in} = 5^k + 101 \times 0.2 \times \frac{1}{1 + \frac{5.3}{100}} = 24.18 \text{ k}\Omega$$

$$R_o \equiv (1 + g_m R'_e) r_o \quad R'_e = R_e || r_\pi$$

$$R_o = (1 + 20(5^k || 0.2^k)) \times 100 \text{ k} = 484.6 \text{ k}\Omega$$

$$G_V = \frac{R_{in}}{R_{in} + R_{sig}} \approx \frac{r_o}{R_L + R_o} = \frac{24.18}{24.18 + 1} \frac{2000 \times 5.3}{5.3 + 484.6}$$

$$G_V = -20.78 \text{ V/V} = A_H$$

Now to calculate  $f_H$ :

$$R'_L = R_L || R_{out} \Rightarrow R_{out} = R_o = 484.6 \text{ k}\Omega$$

$$R'_L = 5.3 || 484.6 = 5.24 \text{ k}\Omega = R_{CL}$$

$$G_m = \frac{g_m}{1 + g_m R_e} = \frac{20}{1 + 20 + 0.2} = 4 \text{ mA/V}$$

$$R_{H'} = (R_{sig} + r_X) || R_{in} (1 + G_m R'_L) + R'_L$$

$$R_{H'} = (1.2^k || 24.18^k) (1 + 4 \times 5.24) + 5.24 = 30.35$$

$$R_{H'} = 30.35 \text{ k}\Omega$$

$$R_H = r_\pi || \frac{\frac{R_{sig} + r_X + R_o}{1 + g_m R_e} = 5^k || \frac{1 + 0.2 + 0.2}{1 + 20 \times 0.2} \frac{100}{105.3}}$$

$$R_H = 0.275 \text{ k}\Omega$$

$$C_H = 0.275 \times 10 + 30.35 \times 0.5 + 5.24 \times 2 = 28.42 \text{ n}s$$

$$f_H = 5.6 \text{ MHz} \Rightarrow A_H = -20.78 \text{ V/V}$$

6.117

$$\text{a)} I = \frac{1}{2} K_s \frac{W}{L} V_{DD}^2 = \frac{1}{2} \times 160 \times 100 \times (0.5)^2 = 2 \text{ mA}$$

$$\text{b)} g_m = \frac{2I}{V_{DD}} = \frac{2 \times 2}{0.5} = 8 \text{ mA/V}$$

$$g_{mb} = X g_m = 0.2 \times 8 = 1.6 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.05 \times 2} = 10 \text{ k}\Omega$$

$$\text{c)} \text{ Using Eq. 6.167: } A_{V_0} = \frac{g_m r_o}{1 + (g_m + g_{mb}) r_o}$$

$$A_{V_0} = \frac{8 \times 10}{1 + (8 + 1.6) \times 10} = 0.82 \text{ V/V}$$

If we use the approximation formula

$$\begin{aligned}
 \text{Eq. 6.168: } A_{V_0} &= \frac{1}{1 + X} = 0.83 \text{ V/V} \\
 R_o &= \frac{1}{g_m + g_{mb}} || r_o = \frac{1}{8 + 1.6} || 10^k = 10 \text{ k}\Omega
 \end{aligned}$$

d) with  $R_L = 1 \text{ k}\Omega$

$$A_{V_0} = \frac{g_m R'_L}{1 + g_m R'_L} \quad (\text{Eq. 6.166})$$

$$R'_L = R_L || 10^k \parallel \frac{1}{g_{mb}} = 1^k || 10^k \parallel \frac{1}{1.6} = 370 \text{ }\Omega$$

$$A_{V_0} = \frac{8 \times 370 \times 10^{-3}}{1 + 8 \times 0.370} = 0.75 \text{ V/V}$$

6.118

$$R_o = r_o || \frac{1}{g_m + g_{mb}} = 20^k + \frac{1}{5+1} = 20.17 \text{ k}\Omega$$

$$A_H = \frac{V_o}{V_{DD}} = G_V = A_{V_0} \frac{R_L}{R_o + R_L} = \frac{g_m r_o}{1 + (g_m + g_{mb}) r_o} \frac{R_L}{R_o + R_L}$$

$$A_H = \frac{5 \times 20}{1 + (5+1) \times 20} \times \frac{20^k}{20^k + 20.17^k} = 0.41 \text{ V/V}$$

$$f_Z = \frac{2\pi}{2\pi C_{GS}} \quad (\text{Eq. 6.172}) \Rightarrow f_Z = \frac{5}{2\pi \times 2} = 3.98 \text{ MHz}$$

$$R_{gd} = R_{sig} = 20 \text{ k}\Omega$$

$$R_{GS} = \frac{R_{sig} + R'_L}{1 + g_m R'_L}, \quad R'_L = R_L || r_o || \frac{1}{g_{mb}} = 20 || 20 || \frac{1}{1} \quad (\text{Eq. 6.175}) \quad R'_L = 0.91 \text{ k}\Omega$$

$$R_{GS} = \frac{20 + 0.91}{1 + 5 \times 0.91} = 3.77 \text{ k}\Omega$$

$$R_{CL} = R_L || R_o = 20 \text{ k} \parallel 20.17 = 10.04 \text{ k}\Omega$$

$$C_H = C_{gd} R_{gd} + C_{GS} R_{GS} + C_L R_{CL}$$

$$C_H = 0.1 \times 20^k + 2 \times 3.77 + 1 \times 10.04 = 19.58 \text{ n}s$$

$$f_H = \frac{1}{2\pi C_H} = 8.13 \text{ MHz}$$

Contribution of each capacitor time constant to the overall  $C_H$  is:

$\underline{10.2\%}$  for  $C_{gd}$ ,  $\underline{38.5\%}$  for  $C_{GS}$ ,  $\underline{51.3\%}$  for  $C_L$ .

6.119

$$R_{gd} = R_{sig}, \quad R_{GS} = \frac{R_{sig} + R'_L}{1 + g_m R'_L} \approx \frac{R_{sig}}{1 + g_m R'_L} \text{ if } R \gg R'_L$$

$$C_H = R_{gd} C_{gd} + R_{GS} C_{GS} + C_L R_{CL}$$

$$C_H = R_{sig} (C_{gd} + \frac{C_{GS}}{1 + g_m R'_L}) \Rightarrow f_H = \frac{1}{2\pi R_{sig} (C_{gd} + \frac{C_{GS}}{1 + g_m R'_L})}$$

$R_{CL}$  is ignored and  $R'_L = R_L || r_o || \frac{1}{g_{mb}}$

$f_H \propto (1 + g_m R'_L)$ . Therefore  $f_H$  can be increased

Cont.

by increasing  $g_m R'_L$ . We also know that  $R'_L = R_L || r_o || \frac{1}{g_{mb}}$  and hence  $R'_L < \frac{1}{g_{mb}}$  or  $g_m R'_L < \frac{g_m}{g_{mb}} = \frac{1}{x} \Rightarrow g_m R'_L < \frac{1}{x}$ . Therefore maximum  $f_H$  can be achieved when we have  $g_m R'_L = \frac{1}{x}$ :

$$f_H = \frac{1}{2\pi R_S g (g_d + \frac{C_{GS}}{1+x})} = \frac{1}{2\pi R_S g (g_d + \frac{x C_{GS}}{x+1})}$$

For the source follower specified in Problem

$$6.118: f_H = \frac{1}{R_{max}^2 2\pi \times 20^6 (0.1 + \frac{0.2 \times 2}{1+0.2})} = 18.37 \text{ MHz}$$

$$x = \frac{g_{mb}}{g_m} = 0.2$$

6.120

Using Eq. 6.178:  $f_Z = \frac{1}{2\pi C_A R_C}$

$$g_m = \frac{I_C}{V_T} = \frac{5}{0.025} = 200 \text{ mA/V}$$

$$r_H = \frac{\beta}{g_m} = \frac{100}{200} = 0.5 \text{ k}\Omega$$

$$r_C = \frac{r_H}{\beta+1} = \frac{0.5 \text{ k}}{100+1}$$

$$f_Z = \frac{g_m}{2\pi(C_H + C_\mu)} \Rightarrow (C_H + C_\mu) = \frac{200 \text{ m}}{2\pi \times 800 \text{ M}} = 39.8 \text{ pF}$$

$$C_H = 39.8 - 2 = 37.8 \text{ pF}$$

$$f_Z = \frac{1}{2\pi \times 37.8 \times 4.95 \times 10^{-12}} = 851 \text{ MHz}$$

$$\text{From Table 5.6: } A_H = \frac{r_o || R_L}{R_S g + r_C + r_H + r_o || R_L}$$

$$A_H = \frac{2.0 \text{ k} || 1 \text{ k}}{10 + 0.2 + 0.5 + (2.0 \text{ k} || 1 \text{ k})}$$

$$A_H = \frac{0.95}{0.11 + 0.95} = 0.9 \text{ V/V}$$

$$R_\mu = R_S g || (r_H + (\beta+1) R'_L) \quad (\text{Eq. 6.179})$$

$$R'_S g = R_S g + r_C = 10 + 0.2 = 10.2 \text{ k}\Omega$$

$$R'_L = R_L || r_o = 1 \text{ k} || 20 \text{ k} = 0.95 \text{ k}\Omega$$

$$R_\mu = 10.2 \text{ k} || (0.5 + 101 \times 0.95) = 9.22 \text{ k}\Omega$$

$$R_H = \frac{R_S g + R'_L}{1 + \frac{R_S g + R'_L}{r_H}} = \frac{10.2 \text{ k} + 0.95 \text{ k}}{1 + \frac{10.2}{0.5} + \frac{0.95 \times 101}{0.5}} = 52.3 \text{ }\Omega$$

$$f_H = \frac{1}{2\pi(C_\mu R_\mu + C_H R_H)} = \frac{1}{2\pi(2 \times 9.22 + 37.8 \times 0.0523)} = 7.8 \text{ MHz}$$

6.121

$$g_m = \frac{I_C}{V_T} = \frac{1}{0.025} = 40 \text{ mA/V}, \quad \beta = \frac{\beta}{(\beta+1)g_m} = 25 \text{ }\Omega$$

$$f_T = \frac{g_m}{2\pi(C_H + C_\mu)} = 2 \text{ GHz} \Rightarrow C_H + C_\mu = 3.18 \text{ pF}$$

$$C_H = 0.1 \text{ pF} \Rightarrow C_H = 3.08 \text{ pF}$$

$$r_H = \frac{\beta}{g_m} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{20}{1} = 20 \text{ k}\Omega$$

$r_o$  is in effect parallel to  $R_L$ , so  $R'_L = R_L ||$

$$R'_L = 1 \text{ k} || 20 \text{ k} = 0.95 \text{ k}\Omega. \quad \text{From Table 5.6:}$$

$$A_H = \frac{R'_L}{R_S g + r_H + r_C + R'_L} = \frac{0.95}{1 + 2.5 + 0.1 + 0.95} = 0.96$$

$$R_\mu = R'_S g || (r_H + (\beta+1) R'_L) \quad (\text{Eq. 6.179})$$

$$R'_S g = R_S g + r_C = 1 + 0.1 = 1.1 \text{ k}\Omega$$

$$R_\mu = 1.1 \text{ k} || (2.5 \text{ k} + 101 \times 0.95) = 1.08 \text{ k}\Omega$$

$$R_H = \frac{R'_S g + R'_L}{1 + \frac{R'_S g + R'_L}{r_H}} = \frac{1.1 + 0.95}{1 + \frac{1.1}{2.5} + \frac{0.95}{0.025}} = 0.052 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi C_H} = \frac{1}{2\pi(R_H C_H + R_\mu C_\mu)} = \frac{1}{2\pi(0.052 \times 3.08 + 1.08)}$$

$$f_H = 593.8 \text{ MHz}$$

6.122

$$I = 2 \text{ mA} \Rightarrow g_m = \frac{2}{0.025} = 80 \text{ mA/V} \Rightarrow r_H = \frac{\beta}{g_m} = 1.$$

$$r_C = \frac{r_H}{\beta+1} = 12.4 \text{ }\Omega$$

$$f_T = \frac{g_m}{2\pi(C_H + C_\mu)} \Rightarrow C_H + C_\mu = \frac{80 \text{ m}}{2\pi \times 400 \times 10^6} = 31.85 \text{ pF}$$

$$\Rightarrow C_H = 31.85 - 2 = 29.85 \text{ pF}$$

$$A_H = \frac{R_L}{R_S g + r_H + R_L} = \frac{1}{R_S g + \frac{0.0124}{101} + 1} = \frac{1}{1.0124} =$$

$$R'_L = R_L = 1 \text{ k}\Omega \Rightarrow R'_S g = R_S g + r_C = R_S g$$

$$R_\mu = R'_S g || (r_H + (\beta+1) R'_L) \quad (\text{Eq. 6.179})$$

$$R_\mu = R'_S g || (1.25 + 101 \times 1) = R'_S g || 102.25 \text{ k}\Omega$$

$$R_H = \frac{R'_S g + R'_L}{1 + \frac{R'_S g + R'_L}{r_H}} = \frac{R'_S g + 1}{1 + \frac{R'_S g + 1}{12.4}} =$$

$$R_H = \frac{R'_S g + 1 \text{ k}}{1 + 0.8 R'_S g + 80} = \frac{R'_S g + 1}{0.8 R'_S g + 81}$$

$$f_H = \frac{1}{2\pi(R_H C_H + R_\mu C_\mu)} = \frac{1}{2\pi(29.85 R_H + 2 R_\mu)} =$$

$$a) R'_S g = 1 \text{ k}\Omega: \quad A_H = 0.998 \text{ V/V}$$

$$R_\mu = 0.99 \text{ k}\Omega, R_H = 24.4 \text{ }\Omega \Rightarrow f_H = 58.8 \text{ MHz}$$

$$b) R'_S g = 10 \text{ k}\Omega: \quad A_H = 0.9 \text{ V/V}$$

$$R_\mu = 9.11 \text{ k}\Omega, R_H = 124 \text{ }\Omega \Rightarrow f_H = 7.27 \text{ MHz}$$

$$c) R'_S g = 100 \text{ k}\Omega: \quad A_H = 0.499 \text{ V/V}$$

$$R_\mu = 50.6 \text{ k}\Omega, R_H = 627 \text{ }\Omega \Rightarrow f_H = 1.34 \text{ MHz}$$

6.123

Refer to Fig. P6.123:

Each of the transistors is operating at a bias current of approximately  $100\mu A$ . Thus:

$$g_m = \frac{0.1}{0.025} = 4 \text{ mA/V} \Rightarrow r_{\pi} = \frac{100}{4} = 25 \text{ k}\Omega$$

$$r_e \approx 250 \text{ }\Omega \Rightarrow r_o = \frac{100}{0.1} = 1 \text{ M}\Omega$$

$$C_{\pi} + C_{\mu} = \frac{g_m}{2\pi f_T} = \frac{4}{2\pi \times 4000} = 1.39 \text{ pF} \Rightarrow C_{\pi} = 1.39 \text{ pF}$$

a)  $R_{in} = (\beta+1) [r_{e1} + (r_{n2} || r_{o1})]$

$$R_{in} = 101 [250 \times 10^{-3} + 25 \text{ k}\Omega || 1 \text{ M}\Omega] \approx 2.5 \text{ k}\Omega$$

$$A_M = -\frac{R_{in}}{R_{in} + R_{sig}} \times \frac{r_{n2} || r_{o1}}{r_{e1} + (r_{n2} || r_{o1})} \times g_{m2} r_{o2}$$

$$A_M = -\frac{2.5}{2.5 + 0.01} \times \frac{25 \text{ k}\Omega || 1 \text{ M}\Omega}{0.25 + (25 \text{ k}\Omega || 1 \text{ M}\Omega)} \times 4 \times 10^4$$

$$A_M = -3943.6 \text{ V/V}$$

b) To calculate  $f_H$ , refer to Example 6.13:

$$R_{H1} = R_{sig} || R_{in} = 10^4 || 2.5 \text{ M} \approx 10 \text{ k}\Omega$$

$$R_{in2} = r_{n2} || r_{o1}$$

$$R_{in2} = 25 \text{ k}\Omega || 1 \text{ M}\Omega$$

$$R_{in2} = 24.4 \text{ k}\Omega$$

$$R_{H1} = \frac{R_{sig} + R_{in2}}{1 + \frac{R_{sig}}{R_{H1}} + \frac{R_{in2}}{r_{e1}}}$$

$$R_{H1} = \frac{10 + 24.4}{1 + \frac{10}{25} + \frac{24.4}{0.25}} = 0.35 \text{ k}\Omega$$

$$R_T = 2.5 \text{ k}\Omega || 10^4 \text{ k}\Omega || \frac{2.5 + 10}{101} = 117.8 \text{ }\Omega$$

$$R_T \text{ is almost reduced by a factor of 3.}$$

$$R_{in2} = r_{n2} || r_{o1} = 2.44 \text{ k}\Omega$$

$$R_{H1} = \frac{10^4 + 2.44}{1 + \frac{10}{2.5} + \frac{2.44}{0.025}} = 120 \text{ }\Omega$$

$$R_{H1} \text{ is almost decreased by a factor of 3.}$$

$$R_{H2} = r_{o2} = 100 \text{ k}\Omega \quad (\text{decreased by a factor of } 10)$$

$$f_H = 0.2 \times 9.61 + 6.7 \times 0.120 + 806.9 \times 0.118 + 1.2 \times 100$$

$$f_H = 1.92 + 0.8 + 95.2 + 120 = 217.92 \text{ ns}$$

$$\text{Thus the dominant effect, that of the output pole, is reduced by a factor of 10.}$$

$$\text{This occurs because } (C_L + C_{H2}) \text{ remains}$$

$$\text{constant while } r_{o2} \text{ decreases by a factor of 10.}$$

$$\text{The second most significant factor (that due to } C_T \text{ or } C_{H2} \text{ with Miller Effect) also decreases, but only by a factor of 3. The overall result is an increase in } f_H.$$

$$R_{H2} = r_{o2} = 1000 \text{ k}\Omega$$

$$C_H = C_{H1} R_{H1} + C_{H2} R_{H2} + C_T R_T + (C_{H2} + C_L) R_{H2}$$

$$C_H = 0.2 \times 10 + 1.39 \times 0.35 + 806.6 \times 0.342 + (0.2 + 1) \times 1000$$

$$C_H = 2 + 0.49 + 274.15 + 1200 = 1274.6 \text{ ns}$$

Thus  $(C_L + C_{H2}) R_{H2}$  is the dominating term, The second most significant term is  $C_T R_T$ .So  $(C_L + C_{H2})$  dominates and then  $C_T$  or equivalently  $C_{H2}$ .

$$f_H = \frac{1}{2\pi C_H} = \frac{1}{2\pi \times 1274.6} = 107.8 \text{ MHz}$$

c) Increasing the bias currents by a factor of 10:

$$g_m = 40 \text{ mA/V} \Rightarrow r_{\pi} = 2.5 \text{ k}\Omega$$

$$r_e \approx 25 \text{ }\Omega \Rightarrow r_o = 100 \text{ k}\Omega$$

$$C_{\pi} = C_{je} + C_{do} \times 10 = 0.8 + 0.59 \times 10 = 6.7 \text{ pF}$$

$$C_{\mu} = 0.2 \text{ pF}$$

$$R_{in} = 101 [0.025 + (2.5 \text{ k}\Omega // 100 \text{ k}\Omega)] = 249 \text{ k}\Omega$$

R<sub>in</sub> is almost decreased by a factor of 10.

$$A_M = -\frac{249}{249 + 10} \times \frac{2.5 \text{ k}\Omega // 100 \text{ k}\Omega}{0.025 + (2.5 \text{ k}\Omega // 100 \text{ k}\Omega)} \times 4000$$

$$A_M = -3807 \text{ V/V}$$

A<sub>M</sub> remains almost constant.

$$C_T = 6.7 + 0.2(1 + 40 \times 100) = 806.9 \text{ (almost constant)}$$

$$R_{H1} = R_{sig} || R_{in} = 10^4 || 249 \text{ k}\Omega = 9.61 \text{ k}\Omega$$

R<sub>H1</sub> stays almost the same.

$$R_T = 2.5 \text{ k}\Omega // 10^4 \text{ k}\Omega // \frac{2.5 + 10}{101} = 117.8 \text{ }\Omega$$

R<sub>T</sub> is almost reduced by a factor of 3.

$$R_{in2} = r_{n2} || r_{o1} = 2.44 \text{ k}\Omega$$

$$R_{H1} = \frac{10^4 + 2.44}{1 + \frac{10}{2.5} + \frac{2.44}{0.025}} = 120 \text{ }\Omega$$

R<sub>H1</sub> is almost decreased by a factor of 3.

$$R_{H2} = r_{o2} = 100 \text{ k}\Omega \quad (\text{decreased by a factor of } 10)$$

$$f_H = 0.2 \times 9.61 + 6.7 \times 0.120 + 806.9 \times 0.118 + 1.2 \times 100$$

$$f_H = 1.92 + 0.8 + 95.2 + 120 = 217.92 \text{ ns}$$

Thus the dominant effect, that of the output pole, is reduced by a factor of 10.

This occurs because  $(C_L + C_{H2})$  remainsconstant while  $r_{o2}$  decreases by a factor of 10.The second most significant factor (that due to  $C_T$  or  $C_{H2}$  with Miller Effect) alsodecreases, but only by a factor of 3. The overall result is an increase in  $f_H$ .

Cont.

$$f_H = \frac{1}{2\pi T_A} = \frac{1}{2\pi \times 217.9 \times 10^3} = 730.3 \text{ kHz}$$

$f_H$  has increased by a factor of nearly 7.  
Significant increase!

6.124

Note: Although rather long, this is an excellent problem with considerable educational value.

a) DC-Bias

$$\text{For } Q_1: I_{D1} = \frac{1}{2} K_n' W (V_{GS} - V_t)^2$$

$$0.1 = \frac{1}{2} \times 2 (V_{GS} - 1)^2$$

$$\rightarrow V_{GS} = 1.316 \text{ V}$$

$$I_{D1} = 0.1 \text{ mA}$$

$$I_{C2} = 1 \text{ mA}$$

See analysis ↗

$$\text{b) For } Q_1: g_{m1} = \sqrt{2\pi K_n' W L I_D} = \sqrt{2 \times 2 \times 0.1} = 0.63 \text{ mA/V}$$

$$\text{For } Q_2: g_{m2} = 40 \text{ mA/V} \quad r_{\pi 2} = \frac{200}{40} = 5 \text{ k}\Omega$$

$$C_{\pi 1} + C_{\mu 1} = \frac{g_m}{2\pi f_T} = \frac{40 \times 10^3}{2\pi \times 600 \times 10^6} = 10.6 \text{ pF}$$

$$\text{Since } C_{\mu 1} = 0.8 \text{ pF} \rightarrow C_{\pi 1} = 9.8 \text{ pF}$$

c) at midband:

$$\frac{V_{\pi 1}}{V_i} = \frac{6.8/15}{(6.8/5) + \frac{1}{g_{m1}}}$$

$$\frac{V_{\pi 1}}{V_i} = \frac{2.88}{2.88 + \frac{1}{0.63}}$$

$$\frac{V_o}{V_i} = 0.64 \text{ V/V}$$

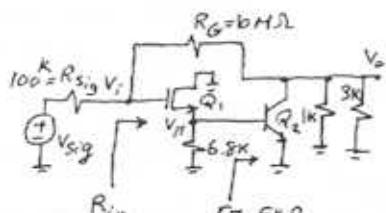
$$\frac{V_o}{V_{\pi 1}} = -g_{m2} Y_{\pi 1} (1K/13K) \quad \text{where we have neglected the effect of } R_G.$$

$$\frac{V_o}{V_{\pi 1}} = -40 \times \frac{3}{4} Y_{\pi 1} = -30 Y_{\pi 1} \Rightarrow \frac{V_o}{V_i} = 0.64 \times (-30) = -19.2 \text{ V/V}$$

$$\frac{V_o}{V_i} = -19.2 \text{ V/V}$$

$$R_{in} = \frac{R_G}{1 - \frac{V_o}{V_i}} = \frac{10M}{1 - (-19.2)} = 495 \text{ k}\Omega$$

$$\frac{V_o}{V_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}} \times \frac{V_o}{V_i} = \frac{495}{495+100} \times 19.2 = -16 \text{ V/V}$$



$R_{in} = 10M \parallel 6.8k = 495 \text{ k}\Omega$

$R_{out} = 10M \parallel 10M = 5k\Omega$

$A_v = -19.2 \text{ V/V}$

d) At low frequencies:

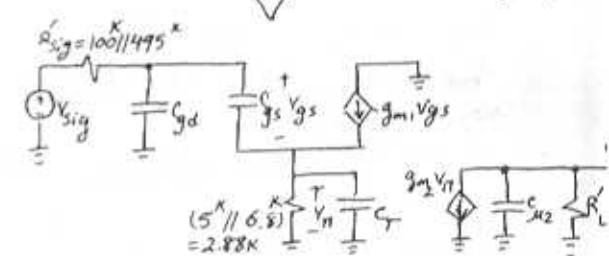
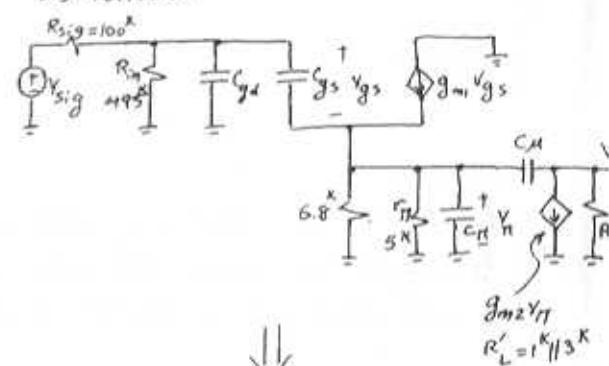
$$C_1 \rightarrow f_{p1} = \frac{1}{2\pi \times 0.1 \times 10^6 \times (100+495) \times 10^3} = 2.7 \text{ MHz}$$

$$C_2 \rightarrow f_{p2} = \frac{1}{2\pi \times 1 \times 10^{-6} \times (3+1) \times 10^{-3}} = 40 \text{ Hz}$$

Thus  $f_L = 40 \text{ Hz}$

e) At high frequencies:

The high frequency equivalent circuit is as follows:



$$C_T = C_{\pi 1} + C_{\mu 1} (1 + g_{m2} R'_L) = 9.8 + 0.8 (1 + 10 \times \frac{3}{4}) = 34.6 \text{ pF}$$

$$R_{gd} = R'_L = 100 \text{ k} \parallel 495 \text{ k} = 83.2 \text{ k}\Omega$$

$$R_{gs} = \frac{R_{sig} + (6.8 \text{ k} \parallel 15 \text{ k})}{1 + g_{m1} (6.8 \text{ k} \parallel 15 \text{ k})} = \frac{83.2 + 2.88}{1 + 0.63 \times 2.88} = 30.6 \text{ k}\Omega$$

$$R_T = 6.8 \text{ k} \parallel \frac{1}{g_{m1}} = 1 \text{ k}\Omega$$

$$R'_L = 0.75 \text{ k}\Omega$$

$$C_H = C_{gd} R_{gd} + C_{gs} R_{gs} + C_T R_T + C_{\mu 2} R'_L$$

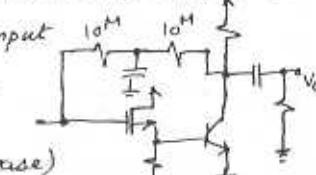
$$C_H = 1 \times 83.2 + 1 \times 30.6 + 34.6 \times 1 + 0.8 \times 0.75 = 149 \text{ nS}$$

$$f_H = \frac{1}{2\pi C_H} = 1.07 \text{ MHz}$$

f) There will no longer be a signal feedback. The left-hand side 10M resistor will in effect appear between the input terminal and ground,

$$\text{Thus: } R_{in} = 10 \text{ M}\Omega$$

(a factor of 20 increase)



Cont.

and correspondingly  $A_H$  becomes:

$A_H = \frac{10}{10.1} \times (-19.2) = -19 \text{ V/V}$  (an increase from  $-16 \text{ V/V}$ )  
 Now  $R_{\text{sig}}$  becomes approximately  $100 \text{ k}\Omega$ , and correspondingly  $R_{\text{gd}}$  becomes  $100 \text{ k}\Omega$ , and  $R_{\text{gs}}$  becomes  $36.6 \text{ k}\Omega$  while  $R_T$  and  $R_L$  remain practically unchanged. Thus  $C_H$  becomes  $172.5 \text{ nF}$  and  $f_H$  decreases from  $1.07 \text{ MHz}$  to  $0.92 \text{ MHz}$ .

6.125

Refer to Fig. P6.125.

$$I_{E2} = 10 \text{ mA} \Rightarrow r_{e2} = 2.5 \Omega, r_{n2} = 253 \Omega$$

$$I_{E1} = \frac{10}{101} \approx 0.1 \text{ mA} \Rightarrow r_{e1} = 250 \Omega, r_{n1} = 25.3 \text{ k}\Omega$$

$$R_{in} = 101 \times [0.25 + 101(0.0025 + 1)] = 10.3 \text{ M}\Omega$$

$$R_{in} = 10.3 \text{ M}\Omega$$

$$R_{out} = r_{e2} + \frac{1}{\beta_2 + 1} [r_{e1} + \frac{R_{\text{sig}}}{\beta_1 + 1}]$$

$$R_{out} = 2.5 + \frac{1}{101} [250 + \frac{100000}{101}] = 14.8 \Omega$$

Neglecting  $r_o$ :

$$A_V = 1000 \text{ V/V}, A_V = \frac{1 \times 1000}{14.8 + 1000} = 0.985 \text{ V/V}$$

6.126

$$I_1 = I_2 = I = 1 \text{ mA} \Rightarrow g_m = 40 \text{ mA/V}, r_{n1} = \frac{120}{40} = 3 \text{ k}\Omega$$

$$r_e = \frac{3}{121} \approx 25 \Omega, C_{n1} + C_{n2} = \frac{g_m}{2\pi f_T} = \frac{40}{2\pi \times 700 \text{ MHz}} = 9.1 \text{ pF}$$

Using Eq. 6.185:

$$A_H = \frac{V_O}{V_{\text{sig}}} = \frac{1}{2} \left( \frac{R_{in}}{R_{in} + R_{\text{sig}}} \right) g_m R_L \quad C_{n1} = 8.6 \text{ pF}$$

$$R_{in} = 2r_n = 2 \times 3 \text{ k}\Omega = 6 \text{ k}\Omega$$

$$A_H = \frac{1}{2} \times \frac{6}{6+20} \times 40 \times 10^6 = 46.15 \text{ V/V}$$

$$f_{p1} = \frac{1}{2\pi \left( \frac{C_{n1}}{2} + C_{n2} \right) (R_{\text{sig}} || 2r_n)} = \frac{1}{2\pi \left( \frac{8.6}{2} + 0.5 \right) (20 || 6\text{k})} \text{ Hz}$$

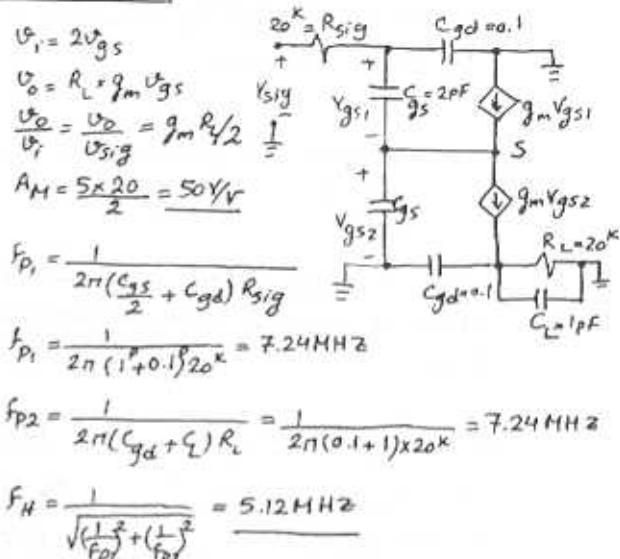
$$f_{p1} = 7.19 \text{ MHz}$$

$$f_{p2} = \frac{1}{2\pi C_{n2} R_L} = \frac{1}{2\pi \times 0.5 \times 10^6} = 31.8 \text{ MHz}$$

$$f_{\#} = \frac{1}{\sqrt{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 7.01 \text{ MHz}$$

(For  $f_{p1}$  and  $f_{p2}$  formulas, refer to Eq. 6.186 and 6.187)

6.127



6.128

All the transistors in this problem are operating at a bias current of  $0.5 \text{ mA}$  and thus have:

$$r_e = 50 \Omega, g_m = 20 \text{ mA/V}, r_{n1} = 5 \text{ k}\Omega$$

$$C_{n1} + C_{n2} = \frac{20^6}{2\pi \times 400 \text{ MHz}} = 8 \text{ pF}$$

$$\text{Since } C_{n1} = 2 \text{ pF} \Rightarrow C_{n1} = 6 \text{ pF} \Rightarrow r_o = \infty, r_x = 0$$

a) Common-Emitter Amplifier:

$$R_{S1} = 10 \text{ k}\Omega, R_C = 10 \text{ k}\Omega$$

$$A_H = -\frac{r_{n1}}{R_{S1} + r_{n1}} g_m R_C = -\frac{5}{10+5} 20 \times 10 = -66.7 \text{ V/V}$$

$$f_H = \frac{1}{2\pi(R_{S1} || r_{n1})(C_{n1} + (1 + g_m R_C) C_{n2})} \Rightarrow$$

$$f_H = \frac{1}{2\pi(10 || 5)(6 + (1 + 20 \times 10) 2)} = 117 \text{ kHz}$$

b) Cascode:

$$A_H = -\frac{\beta_2 \alpha_2 R_C}{R_{S2} + r_{n2}} = -\frac{100 \times 0.99 \times 10}{10+5} = -66 \text{ V/V}$$

$$\text{Input pole: } f_{p1} = \frac{1}{2\pi(R_{S2} || r_{n2})(C_{n1} + 2C_{n2})} \text{ Hz}$$

$$f_{p1} = \frac{1}{2\pi(10 || 5)(6+4)} = 4.77 \text{ MHz}$$

$$\text{Output pole: } f_{p3} = \frac{1}{2\pi C_{n2} R_C} = \frac{1}{2\pi \times 20 \times 10^6} = 7.96 \text{ MHz}$$

Pole at midband node:

$$f_{p2} = \frac{1}{2\pi C_{n2} R_C} = \frac{1}{2\pi \times 6 \times 50} = 530.5 \text{ Hz}$$

Very high  
Cont.

$$f_H = \sqrt{\frac{1}{\left(\frac{1}{f_{p1}}\right)^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 4.1 \text{ MHz}$$

c) CC-CB Cascade (Modified diff. amplifier)

$$A_M = \frac{\beta_2 R_C}{R_S g + 2r_{n1}} = \frac{100 \times 10}{10 + 10} = 50 \text{ V/V}$$

$$\text{Input pole: } f_{p1} = \frac{1}{2\pi(R_S g + 2r_{n1})(C_{n1/2} + C_{\mu 1})}$$

$$f_{p1} = \frac{1}{2\pi(10^k/10^k)(3+2)^2} = 6.4 \text{ MHz}$$

$$\text{Output pole: } f_{p2} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2 \times 10^k} = 7.96 \text{ MHz}$$

$$\text{Thus: } f_H = \frac{1}{\sqrt{\frac{1}{f_{p1}}^2 + \left(\frac{1}{f_{p2}}\right)^2}} = 5 \text{ MHz}$$

d) CC-CE Cascade:

$$A_M = -\frac{(\beta_1 + 1)\beta_2 R_C}{R_S g + r_{n1} + (\beta_1 + 1)r_{n2}} = -\frac{101 \times 100 \times 10}{10 + 5 + 101 \times 5} = -194 \text{ V}$$

Refer to Example 6.13 in:

$$R_{M1} = (R_S g || R_{in}) = 10^k // (\beta + 1)[r_{e1} + r_{n2}]$$

$$R_{M1} = 10^k // 101 \times [0.05 + 5] = 9.81 \text{ k}\Omega$$

$$R_{n1} = r_{n1} // \frac{R_S + r_{n2}}{1 + g_m r_{n2}} = 5 // \frac{10 + 5}{1 + 20 \times 5} = 1.44 \text{ }\Omega$$

$$R_T = r_{n2} // \frac{r_{n1} + R_S g}{\beta + 1} = 5^k // \frac{5 + 10}{101} = 1.44 \text{ }\Omega$$

$$\text{where } C_T = C_{n2} + C_{n2}(1 + g_m r_{n2}) R_C = 6 + 2(1 + 200) \\ C_T = 408 \text{ pF}$$

$$R_{\mu 2} = R_C = 10 \text{ k}\Omega$$

$$C_{\mu 2} = C_{n1} R_{\mu 1} + C_{n1} R_{n1} + C_T R_T + C_{\mu 2} R_{\mu 2}$$

$$T_A = 2 \times 9.81 + 6 \times 0.144 + 408 \times 0.144 + 2 \times 10$$

$$T_A = 19.62 + 0.86 + 58.75 + 20 = 99.2 \text{ ns}$$

$$f_H = \frac{1}{2\pi T_A} = \frac{1}{2\pi \times 99.2 \text{ ns}} = 1.6 \text{ MHz}$$

e) Folded Cascode:

$$A_M = -\frac{\beta_1 \alpha_2 R_C}{R_S g + r_{n1}} = -\frac{100 \times 0.99 \times 10}{10 + 5} = -66 \text{ V/V}$$

Input pole:

$$f_{p1} = \frac{1}{2\pi(R_S g || r_{n1})(C_{n1} + 2g_{m1})} = \frac{1}{2\pi(10/5)(6+4)}$$

$$f_{p1} = 4.77 \text{ MHz}$$

$$\text{At middle: } f_{p2} = \frac{1}{2\pi C_{n2} r_{e2}} = \frac{1}{2\pi \times 6 \times 0.05} = 530 \text{ kHz}$$

$$\text{At output: } f_{p3} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 2 \times 10} \Rightarrow \text{very high!}$$

$$f_{p3} = 7.96 \text{ MHz}$$

$$\text{Thus: } f_H = \frac{1}{\sqrt{\frac{1}{4.77^2} + \frac{1}{7.96^2}}} = 4.1 \text{ MHz}$$

f) CC-CB Cascade:

$$A_M = \frac{(\beta_1 + 1)\alpha_2 R_C}{R_S g + (\beta_1 + 1)2r_{n1}} = \frac{101 \times 0.99 \times 10}{10 + 101 \times 0.1} \approx 50 \text{ V/V}$$

$$\text{Input pole: } f_{p1} = \frac{1}{2\pi(R_S g || 2r_{n1})(C_{n1/2} + C_{\mu 1})}$$

$$f_{p1} = \frac{1}{2\pi(10^k/10^k)(3+2)^2} = 6.4 \text{ MHz}$$

$$\text{Output pole: } f_{p2} = \frac{1}{2\pi C_{\mu 2} R_C} = \frac{1}{2\pi \times 10^k \times 2^2} = 7.96 \text{ MHz}$$

$$f_H = \frac{1}{\sqrt{\frac{1}{6.4^2} + \frac{1}{7.96^2}}} = 5 \text{ MHz}$$

Summary of results:

Configuration	$A_H (\text{V/V})$	$f_H (\text{MHz})$	G.B.
a) CE	-66.7	0.117	7.8
b) Cascode	-66	4.1	271
c) CC-CB cascade	+50	5.0	250
d) CC-CE cascade	-194	1.6	310
e) Folded cascode	-66	4.1	271
f) CC-CB cascade	+50	5.0	250

6.129

Refer to Fig. 6.58.

$$I_{REF} = 20 \mu A = I_A = I_1 = I_2 = I_3$$

All transistors have the same  $g_m$ ,  $r_o$ ,  $V_{OV}$  values.

$$I = \frac{1}{2} K_A \frac{W}{L} V_{OV}^2 \Rightarrow 0.08 = \frac{1}{2} \times 4 \times V_{OV}^2 \Rightarrow V_{OV} = 0.2$$

$$V_{GS} = V_{OV} + V_E = 0.2 + 0.5 = 0.7 \text{ V}$$

$$V_{G1} = V_{GS} = 0.7 \text{ V} = V_{S4} \Rightarrow V_{G4} = 0.7 + V_{GS4} = 1.4 \text{ V}$$

$$\Rightarrow V_{G3} = 1.4 \text{ V} \Rightarrow V_{S3} = 1.4 - V_{GS} = 0.7 \text{ V}$$

$$\Rightarrow V_{Q3} = V_{O3} = V_{S3} + V_{OV} = 0.9 \text{ V}$$

As explained, the voltage at the gate of  $Q_3$  is  $2V_{GS}$  which implies voltage of  $V_{GS} = V_{OV} + V_E$  at the source of  $Q_3$ . For minimum allowable voltage at the output,  $V_{DS} = V_{OV}$  or equivalently  $V_{min} = V_{OV}$ .

$$V_{min} = V_{OV} + V_E + V_{C} = 2V_{OV} + V_E$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.08}{0.2} = 0.8 \text{ mA/V} \quad r_o = \frac{V_A}{I_D \cdot 0.08} = \frac{8}{0.8 \times 0.08} = 100$$

$$\text{Using Eq. 6.189: } R_o = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}]^{-1} \cdot 10^2$$

$$R_o = 100 \text{ k} + [1 + 0.8 \times 100] \times 100 = 8.2 \text{ M}\Omega$$

6.130

$$I_{REF} = 25 \mu A, \text{ Refer to Fig. 6.58}$$

$$I_H = 25 \mu A = I_1, \quad W_1 = W_4 = 2 \mu m, \quad W_2 = W_3 = 4 \mu m$$

$$I_1 = \frac{1}{2} K' \frac{W_1}{L_1} \frac{V_{OV}}{V_T^2} \Rightarrow 25 = \frac{1}{2} \times 200 \times \frac{2}{1} \frac{V_{OV}^2}{V_T^2} \Rightarrow V_{OV} = 0.354 V$$

$$V_{OV1} = V_{OV2} \Rightarrow \frac{I_2}{I_1} = \frac{(W_2/L_2)^2}{(W_1/L_1)} \Rightarrow I_2 = 25 \times \frac{4}{2} = 500 \mu A$$

$$I_2 = 0.5 mA = I_3, \quad I_0 = 0.5 mA$$

$$V_{GS1} = V_{OV1} + V_t = 0.354 + 0.6 = 0.954 V$$

$$V_{G1} = 0.954 V$$

$V_{G4} = V_{GS1} + V_{GS4}$ , Since  $I_1 = I_H$  and  $W_1 = W_4$  then

$$V_{GS1} = V_{GS4} \Rightarrow V_{G4} = 2V_{GS1} = 1.91 V = V_{G3}$$

The lowest possible voltage for the output is

when  $Q_3$  has  $V_{DS3} = V_{OV3}$  or  $V_{min} = V_{G3} - V_{DS3} + V_{OV3}$

Since  $V_{GS1} = V_{GS2}$  and  $I_2 = I_3$  then  $V_{GS3} = V_{GS1}$

$$\Rightarrow V_{min} = 1.91 - 0.954 + 0.354 = 1.31 V$$

$$g_{m2} = g_{m3} = \frac{2ID}{V_{OV}} = \frac{2 \times 0.5}{0.354} = 2.82 mA/V$$

$$r_{o2} = r_{o3} = \frac{V_A}{ID} = \frac{20}{0.5} = 40 k\Omega$$

$$Eq. 6.189: R_o = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}]r_{o2}$$

$$R_o = 40 + [1 + 2.82 \times 40] \times 40 k = 4.6 M\Omega$$

6.131

In the gate of  $Q_3$  we see a small incremental resistance of approximately  $\frac{1}{g_m}$  and since the incremental voltages across  $Q_4, Q_5, Q_6$  will be small, we can assume that the gates are grounded. Thus the output resistance will be that of the CG transistor  $Q_3$  which has a resistance  $R_o = r_{o2} + (1 + (g_{m2} + g_{mb2})r_{o2})r_{o1}$  in its source. We use Eq. 6.101:

$$R_o = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}]R_{o2}$$

$$R_o = r_{o3} + [1 + (g_{m3} + g_{mb3})r_{o3}] [r_{o2} + (1 + (g_{m2} + g_{mb2})r_{o2})r_{o1}]$$

$$R_o = r_{o3} + [g_{m3}r_{o3}] [g_{m2}r_{o2}r_{o1}]$$

$$R_o = r_{o3} + g_{m2}g_{m3}r_{o1}r_{o2}r_{o3} = g_{m2}g_{m3}r_{o1}r_{o2}r_{o3}$$

6.132

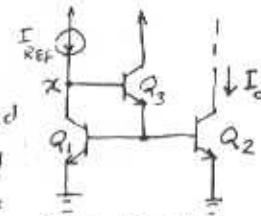
$$V_{DD} = V_{BE3} + V_{BE1} = 1.4 V$$

If  $I_{REF}$  is increased to  $1 mA$  or equivalently multiplied by  $10$ , then:

$$\frac{I_{C2}}{I_{C1}} = \frac{I_0 e^{V_{BE2}/V_T}}{I_0 e^{V_{BE1}/V_T}} \Rightarrow 10 = e^{(V_{BE2} - V_{BE1})/V_T}$$

$$V_{BE2} - V_{BE1} = \Delta V_{BE} = V_T \ln 10 = 0.058 V$$

$$\Delta V_{BE} = 0.058 V \Rightarrow \Delta V_x = 2 \Delta V_{BE} = 0.116 V$$



Now we calculate  $I_o$  for  $V_o = V_x$ :

$$I_{REF} = I_C = 100 \mu A \Rightarrow V_{BE1} =$$

The actual value of  $I_o = \frac{I_{REF}}{1 + \frac{2}{200 + 200}}$

$$\Rightarrow I_o = \frac{100 \mu A}{1 + \frac{2}{400}} = 99.995 \mu A = 99.995 \frac{1}{\beta^2 + \beta}$$

$$\frac{\Delta I_o}{I_o} = \frac{0.005}{100} = 5 \times 10^{-5} = 0.005 \frac{1}{\beta}$$

6.133

Since  $Q_{21}, Q_{22}, \dots, Q_{2n}$  are all matched to  $Q_1$ :

$$I_{o1} = I_{o2} = \dots = I_{on} = I_o$$

The emitter of  $Q_3$  supplies the basecurrent for all transistors, so  $I_{e3} = \frac{(n+1)I_o}{\beta}$

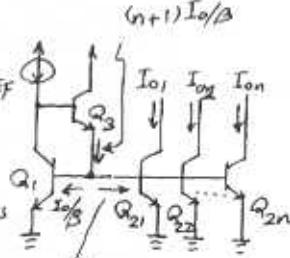
A node equation at the base of  $Q_3$  yields:

$$I_{REF} = I_o + \frac{(n+1)I_o}{\beta(\beta+1)}, \text{ Thus: } \frac{I_o}{I_{REF}} = \frac{1}{1 + \frac{n+1}{\beta^2}}$$

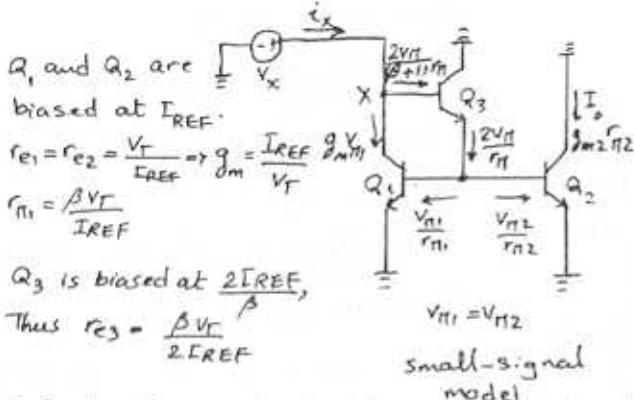
For a deviation from unity of less than  $0.1\%$ :

$$\frac{99.9}{100} = \frac{1}{1 + \frac{n+1}{\beta^2}} \Rightarrow \frac{n+1}{\beta^2} = \frac{1}{99.9}$$

$$\Rightarrow n = \frac{\beta^2}{99.9} - 1 \Rightarrow n \leq 9$$



6.134



Refer to the small-circuit analysis performed directly on the circuit. Since the current in the emitter of  $Q_3$  is  $\frac{2V_{T1}}{r_{st1}}$ , the voltage  $V_{T3}$  will be:  $V_{T3} = \frac{2V_{T1}}{r_{st1}} \times r_{c3}$ .

$$V_X = V_{R12} + V_{n1} = \frac{2V_{REF}r_{e3}}{r_{n1}} + V_{n1} = V_{n1} \left(1 + 2 \frac{r_{e3}}{r_{n1}}\right)$$

$$V_X = V_{n1} \left(1 + 2 \frac{\beta_{VT}}{2\epsilon_{REF}} \frac{1}{\beta_{VT}}\right) = 2V_{n1}$$

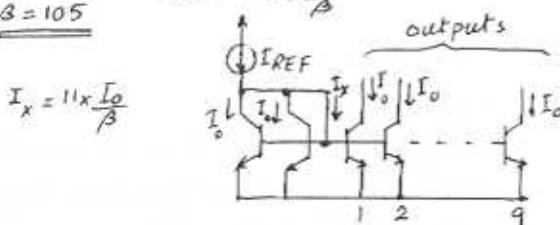
$$\text{and } i_x \approx g_m V_{R1} \text{ . Thus: } R_{in} = \frac{V_x}{i_x} = \frac{2}{g_m} = \frac{2 V_r}{I_{REF}}$$

For  $I_{REF} = 100 \mu A \Rightarrow R_{in} = \frac{2 \times 0.025}{0.1} = 0.5 k\Omega$

All the output currents are equal to  $I_o$ , then we have:  $I_{REF} = 2I_o + \frac{1}{\beta} I_o \Rightarrow I_o = \frac{1}{2 + \frac{1}{\beta}}$

$$I_o \text{ is ideally } I_{REF}/2, \text{ For } 5\% \text{ lower } I_o: \\ 0.95 \times \frac{I_{REF}/2}{I_{REF}} = \frac{1}{2 + \frac{11}{A}} \Rightarrow \beta = 104.5 \approx 105$$

$$\beta = 105$$



6.136

(c) See the analysis on the circuit.

$$I_{REF} = I + \frac{\beta+2}{\beta(\beta+1)} \quad I = I \cdot \frac{\beta^2+2\beta+2}{\beta(\beta+1)}$$

$$I_{O1} = I_{O2} = \frac{1}{2} \cdot \frac{\beta+2}{\beta+1} \cdot T$$

$$\frac{I_{O1}}{I_{REF}} = \frac{I_{O2}}{I_{REF}} = \frac{1}{2} \cdot \frac{\beta(\beta+2)}{\beta^2 + 2\beta + 2} = \frac{1}{2} \times \frac{1}{1 + 2/\beta^2 + 2\beta}$$

$$\frac{I_{O_1}}{I_{REF}} \approx \frac{1}{2} \frac{1}{1 + 2/\beta^2}$$

Observe that the deviation factor  $\frac{1}{\beta^2}$  is independent of the number of outputs or the value of each output, i.e.:

The current  $I_{REF}$  can be split into any number of outputs through an appropriate combination of parallel-connected transistors. (Q<sub>3</sub> and Q<sub>4</sub> in this case) The reason the error factor remains unchanged at  $\frac{1}{1 + 2/\beta^2}$  is that the base current that need to be supplied by  $I_{REF}$  (subt from  $I_{REF}$ ) remains unchanged.

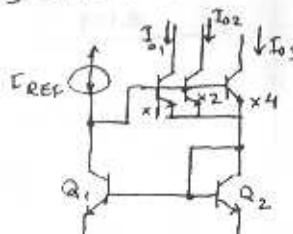
b) The TMA reference current can be used to generate three output currents of  $1, 2, 4 \text{ mA}$  by using 3 transistors in parallel having relative area ratios of  $1, 2, 4$  as shown.

$$\frac{I_{OL}}{I_{REF}} = \frac{1}{7} \cdot \frac{1}{1 + 2/\beta^2} \Rightarrow I_{OL} = 0.998 \text{ mA}$$

(1 mA ideally)

$$\frac{I_{O2}}{I_{REF}} = \frac{2}{7} \frac{1}{1 + 2/A^2} \Rightarrow I_{O2} = 1.996 \text{ mA} \quad (2 \text{ mA ideally})$$

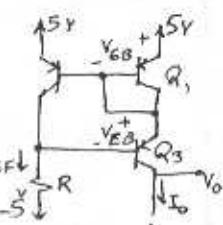
$$\frac{I_{O3}}{I_{REF}} = \frac{4}{7} \cdot \frac{1}{1 + 2/\beta^2} \Rightarrow I_{O3} = 3.992 \text{ mA (4mA ideal)}$$



6.137

$$I_{REF} = 0.1mA = \frac{5 - 0.7 - 0.7 - (-5)}{R}$$

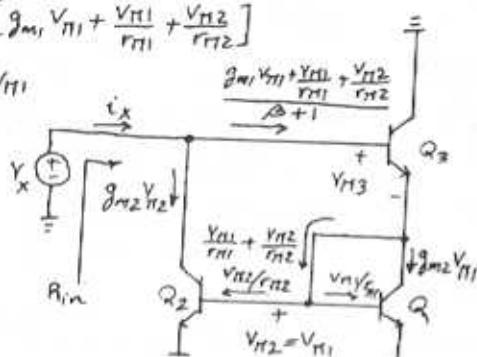
$V_{max}$  is obtained when  $S$  is saturated:  $V_m = 5 - 0.7 - 0.1$



6.138

$$V_{H3} = r_{e3} \left[ g_m V_{H1} + \frac{V_{H1}}{r_{m1}} + \frac{V_{H2}}{r_{m2}} \right]$$

$$V_X = V_{H3} + V_{H1}$$



Small-signal analysis

Note that all 3 transistors are biased at  $I_{REF}$

$$V_X = V_{H3} + V_{H1} = r_{e3} \left[ g_m V_{H1} + \frac{V_{H1}}{r_{m1}} + \frac{V_{H2}}{r_{m2}} \right] + V_{H1}$$

$$V_X = (r_e g_m + \frac{2r_e}{r_m} + 1) V_{H1}$$

$$V_X = (\frac{\beta}{\beta+1} + \frac{2}{\beta+1} + 1) V_{H1} = 2 V_{H1} \left( 1 + \frac{1}{\beta+1} \right) \approx 2 V_{H1}$$

$$i_X = g_{m2} V_{H2} + \frac{1}{\beta+1} (g_m V_{H1} + \frac{V_{H1}}{r_{m1}} + \frac{V_{H2}}{r_{m2}})$$

$$i_X = V_{H1} \left[ g_m + \frac{g_m}{\beta+1} + \frac{g_m}{\beta(\beta+1)} + \frac{g_m}{\beta^2(\beta+1)} \right]$$

$$i_X \approx g_m V_{H1}$$

$$\text{Thus: } R_{in} = \frac{V_X}{i_X} = \frac{2 V_{H1}}{g_m V_{H1}} = \frac{2}{g_m} = \frac{2 V_r}{I_{REF}}$$

For  $I_{REF} = 100 \mu A$ :

$$R_{in} = \frac{2 \times 0.025}{0.1} = 0.5 k\Omega$$

6.139

$$R_o = \frac{\Delta V_o}{\Delta I_o} = \frac{\beta}{2} \frac{V_A}{I_c} = \frac{100}{2} \times \frac{100}{1} = 5 M\Omega$$

$$\Delta I_o = \frac{\Delta V_o}{R_o} = \frac{10}{5} = 2 \mu A$$

$$\frac{\Delta I_o}{I_o} = \frac{2}{10} = 0.2$$

6.140

$$I_{REF} = 25 \mu A \Rightarrow I_2 = I_{REF} = 25 \mu A$$

$$V_{OV1} = V_{OV2} \Rightarrow \frac{I_1}{I_2} = \frac{W_1}{W_2} \Rightarrow I_1 = 25 \times \frac{2}{40} = 1.25 \mu A$$

$$I_o = I_3 = I_1 = 1.25 \mu A$$

Refer to Fig. 6.61a

$$V_{OV2}^2 = \frac{I_2}{\frac{1}{2} K_n \frac{W}{L}} = \frac{25}{\frac{1}{2} \times 200 \times \frac{40}{1}} = \frac{1}{160} \Rightarrow V_{OV2} = 0.08 V$$

$$\Rightarrow V_{GS2} = V_{GS1} = 0.08 + 0.6 = 0.86 V \text{ therefore}$$

$$V_{G1} = V_{G2} = 0.86 V$$

$$V_{OV3}^2 = \frac{I_3}{\frac{1}{2} K_n \frac{W}{L}} = \frac{1.25}{\frac{1}{2} \times 200 \times \frac{40}{1}} = \frac{1}{3200} \Rightarrow V_{OV3} = 0.018 V$$

$$\text{Therefore } V_{GS3} = 0.018 + 0.6 = 0.618 V$$

$$\Rightarrow V_{G3} = V_{S3} + V_{GS3} = 0.86 + 0.618 = 1.48 V$$

For  $V_{min}$ , we have to have  $V_{DS3} = V_{OV3} = 0.018$   
then:  $V_{min} = V_{GS1} + V_{DS3min} = 0.86 + 0.018 = 0.88 V$

$$\text{For } Q_2: g_{m2} = \frac{2I}{V_{ov}} = \frac{2 \times 25 \mu A}{0.08} = 625 \mu A/V = 0.625 mA/V$$

$$r_{o2} = \frac{V_A}{I} = \frac{20}{0.025} = 800 k\Omega$$

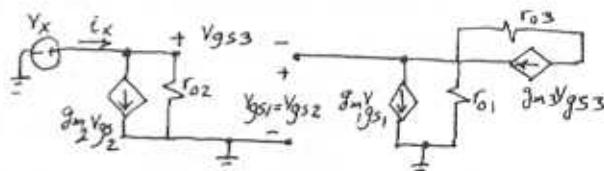
$$\text{For } Q_3: g_{m3} = \frac{2 \times 1.25}{0.018} = 139 \mu A/V = 0.139 mA/V$$

$$r_{o3} = \frac{20}{0.00125} = 16 M\Omega$$

Refer to section 6.12.4 in textbook:

$$R_o = r_{o3} (g_{m3} r_{o2} + 2) = 16 (0.139 \times 800 k\Omega + 2) = 1811.2 M\Omega$$

6.141



$$i_X = g_{m2} V_{gs2} \quad (1)$$

$$V_{gs2} + V_{gs3} = V_X$$

Since  $Q_2$  and  $Q_3$  have the same parameters and same current, therefore  $V_{gs2} = V_{gs3}$

$$V_X = 2 V_{gs2} \Rightarrow V_{gs2} = \frac{V_X}{2}$$

Substitute for  $V_{gs2}$  in (1):

$$i_X = g_{m2} \times \frac{V_X}{2}$$

$$R_{in} = \frac{V_X}{i_X} = \frac{2}{g_{m2}}$$

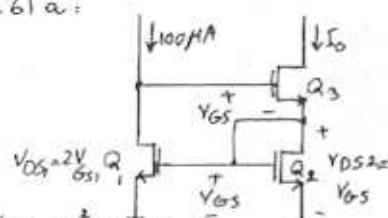
6.142

Refer to Fig. 6.61a:

$$I_{REF} = 100 \mu A$$

$$V_{DS1} = 2V_{GS}$$

$$V_{DS2} = V_{GS}$$



$$I_D = I_{REF} \frac{L}{2} \mu \left( \frac{W}{L} (V_{GS} - V_T)^2 \right) \left( 1 + \frac{V_{DS1}}{V_A} \right)$$

$$100 = \frac{1}{2} \times 2000 (V_{GS} - 0.6)^2 \left( 1 + \frac{2V_{GS}}{20} \right)$$

$$1 = (V_{GS} - 0.6)^2 (10 + V_{GS})$$

$$V_{GS} \approx 0.91 V \text{ (by iteration)}$$

$$I_D = I_{D2} = \frac{1}{2} \times 2000 (V_{GS} - 0.6)^2 \left( 1 + \frac{V_{GS}}{20} \right)$$

$$I_D = 100.47 \mu A$$

Thus there is  $\frac{0.47}{100}$  or  $0.5\%$  error. Modifying the circuit as Fig. 6.61c ensures that  $Q_1$  and  $Q_2$  have the same  $V_{DS}$  and thus eliminate the above error.

6.143

Refer to Fig. 6.62:  $I_{REF} = 100 \mu A$   $I_o = 10 \mu A$ 

$$(a) V_{BE1} = 0.7 + V_T \ln \frac{100}{1000} = 0.642 V$$

$$V_{BE2} = 0.7 + V_T \ln \frac{10}{1000} = 0.585 V$$

$$I_o = \frac{V_{BE1} - V_{BE2}}{R_E} = 10 \mu A \Rightarrow R_E = 5.7 k\Omega$$

$$(b) r_{T2} = (\beta + 1) \frac{V_T}{I_o / \mu A} = 503 k\Omega \gg R_E$$

$$r_{o2} = \frac{V_A}{I_o} = 10 M\Omega \Rightarrow R_o = (1 + g_m R_E) r_{o2} = 33 M\Omega$$

$$R_o = 33 M\Omega$$

$$\Delta I_o = \frac{\Delta V_o}{R_o} = \frac{5}{R_o} = 0.15 \mu A$$

6.144

$$(a) \frac{I_o}{I_{REF}} = 0.9 \Rightarrow I_o = 90 \mu A$$

 $I_{REF}$ 

$$V_{RE} = V_T \ln \frac{1}{0.9} = 2.63 mV$$

$$R_E = \frac{2.63 mV}{90 \mu A} = 29.3 \Omega$$

$$r_o = \frac{V_A}{I_o} = 1.11 M\Omega$$

$$g_m = 3.6 mA/V$$

$$R_o = (1 + g_m R_E) r_o = 1.23 M\Omega \text{ Compare to } r_o = 1.11 M\Omega$$

$$(b) \frac{I_o}{I_{REF}} = 0.1 \Rightarrow I_o = 10 \mu A$$

$$V_{RE} = V_T \ln 10 = 57.56 mV$$

$$R_E = \frac{57.56 mV}{10 \mu A} = 5.76 k\Omega$$

$$r_o = \frac{100}{10 \mu A} = 10 M\Omega$$

$$g_m = 0.4 mA/V$$

$$R_o = (1 + g_m R_E) r_o = 33 M\Omega \text{ Compare to } r_o = 10 M\Omega$$

$$(c) \frac{I_o}{I_{REF}} = 0.01 \Rightarrow I_o = 1 \mu A$$

$$V_{RE} = V_T \ln 100 = 115 mV$$

$$R_E = \frac{115}{1} = 115 k\Omega$$

$$r_o = \frac{100}{1} = 100 M\Omega$$

$$g_m = 0.04 mA/V$$

$$R_o = (1 + g_m R_E) r_o = 560 M\Omega \text{ Compare to } r_o = 10 M\Omega$$

6.145

$$R_o = [1 + g_m (R_E || r_{in})] r_o$$

$$I_E = \frac{-0.7 - (-5)}{R_E} = 0.43 mA$$

$$g_m = \frac{I_C}{V_T} = \frac{0.43}{0.025} = 17.2 mA/V$$

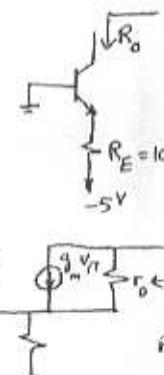
$$r_o = \frac{V_A}{I_C} = \frac{100}{0.43} = 232.6 k\Omega$$

$$r_{in} = \frac{\beta}{g_m} = \frac{100}{17.2} = 5.8 k\Omega$$

$$R_E = 10 k\Omega$$

$$R_o = [1 + (10^k || 5.8^k) \times 17.2] \times 232.6$$

$$R_o = 14.92 M\Omega$$



6.146

$$(a) I_o = \frac{V_{BE1} + V_{BE2} - V_{BE3}}{R}$$

$$\text{For } I_o = I_{REF} : V_{BE1} = V_{BE2} = V_{BE3} \Rightarrow I_o = \frac{V_B}{R}$$

$$\text{For } I_o = 10 \mu A : V_{BE} = 0.585 \text{ therefore } R = \frac{V_B}{K}$$

$$(b) g_m = \frac{I_o}{V_T} = \frac{10 \mu A}{0.025} = 0.4 mA/V \Rightarrow r_{in} = \frac{\beta}{g_m} = 250 k\Omega$$

$$r_o = \frac{V_A}{I_o} = 10 M\Omega$$

$$R_o = [1 + g_m (R || r_{in})] r_o$$

$$R_o = [1 + 0.4 (58.5 / 250^k)] 10^M = 199.6 M\Omega$$

$$R_o \approx 200 M\Omega$$

6.147

$Q_1$  and  $Q_2$  are matched and  $I_{D1} = I_{D2} = I$   
therefore  $V_{GS1} = V_{GS2}$  and this implies that  
the source of  $Q_1$  and the source of  $Q_2$  have  
the same voltage. Hence :  $IR = V_{BE} = V_T \ln \frac{I}{I_S}$   
 $IR = V_T \ln \frac{I}{I_S}$  ①

To find the value of  $R$ , first we have to  
calculate  $I_S$ .

$$\text{For } I_E = 1 \text{ mA}, V_{BE} = 0.7 \text{ V} \Rightarrow I = I_S e^{0.7/0.025}$$
$$\Rightarrow I_S = 6.9 \times 10^{-13} \text{ mA}$$

$$\text{From ① : } R = \frac{V_T \ln \frac{I}{I_S}}{I} = \frac{0.025}{0.010 \text{ m}} \ln \frac{0.010}{6.9 \times 10^{-13}}$$
$$R = 58.49 \text{ k}\Omega \approx 58.5 \text{ k}\Omega$$

## Chapter 7 - Problems

7.1

$$V_{DD} = V_{SS} = 2.5V$$

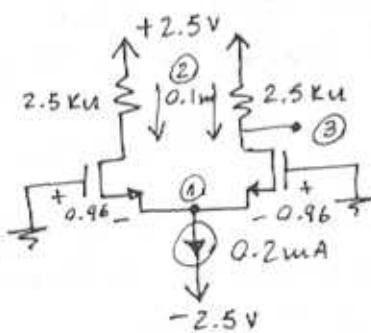
$$K_n' \frac{W}{L} = 3 \frac{mA}{V^2}; V_{tH} = 0.7V$$

$$I = 0.2mA; R_D = 5k\Omega$$

$$(a) V_{ov} = \sqrt{\frac{I}{K_n' W L}} = \sqrt{0.2 / 3} = 0.26V$$

$$V_{GS} = V_{ov} + V_t = 0.26 + 0.7 = 0.96V$$

(b)



$$\textcircled{1} \quad V_{S1} = V_{S2} = V_{CM} - V_{GS} = 0 - 0.96 = -0.96V$$

$$\textcircled{2} \quad I_{D1} = I_{D2} = \frac{I}{2} = 0.1mA$$

$$\textcircled{3} \quad V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} \times R_D = +2.5 - 0.1 \times 2.5 = 2.25V$$

(c) If  $V_{CM} = +1V$

$$V_{S1} = V_{S2} = +1 - 0.96 = 0.04V$$

$$I_{D1} = I_{D2} = 0.1mA$$

$$V_{D1} = V_{D2} = 2.25V$$

(d) If  $V_{CM} = -1V$

$$V_{S1} = V_{S2} = -1 - 0.96 = -1.96V$$

$$I_{D1} = I_{D2} = 0.1mA$$

$$V_{D1} = V_{D2} = 2.25V$$

$$(e) V_{CHMAX} = V_t + V_{DD} - \frac{I}{2} R_D$$

$$= 0.7 + 2.5 - 0.1 \times 2.5 = +2.95V$$

$$(f) V_{CHMIN} = -V_{SS} + V_{GS} + V_t + V_{ov}$$

$$= -2.5 + 0.3 + 0.7 + 0.26$$

$$= -1.24V$$

$$V_{Smin} = V_{CHmin} - V_{GS}$$

$$= -1.24 - 0.96 = -2.2V$$

7.2

$$(a) V_{ov} = -\sqrt{\frac{I}{K_p' (W/L)}} = -\sqrt{0.7 / 3.5} = -0.45V$$

$$V_{GS} = V_{ov} + V_t = -0.45 - 0.8 = -1.25V$$

$$V_{S1} = V_{S2} = V_G - V_{GS} = 0 + 1.25 = +1.25V$$

$$V_{D1} = V_{D2} = \frac{I}{2} \times R_D - V_{DD} = \frac{0.7}{2} \times 2 - 2.5 = -1.8V$$

(b) For  $Q_1$  and  $Q_2$  to remain in saturation:

$$V_{DS} \leq V_{GS} - V_t \rightarrow V_{CM} > \left( \frac{I}{2} R_D - V_{DD} \right) + V_t$$

$$V_{CMmin} = \frac{0.7}{2} \times 2 - 2.5 - 0.8 = -2.6V$$

To allow sufficient voltage for the current source to operate properly:

$$V_{CM} \leq V_{SS} - V_{GS} + (V_{tH} + V_{ov})$$

$$\rightarrow V_{CMmax} = 2.5 - 0.5 - 1.25 = 0.75V$$

7.3

(a)  $V_{G2} = 0$ ,  $V_{G1} = U_{id}$   
if  $i_{D1} = i_{D2} = I/2 = 0.1 \text{ mA}$   
then  $V_{G1} = V_{G2}$   
thus  $U_{id} = 0V$ .

$$(b) i_{D1} = \frac{I}{2} + \frac{I}{V_{ov}} \cdot \frac{U_{id}}{2}$$

$$\rightarrow U_{id} = \left( \frac{2i_{D1}}{I} - 1 \right) \cdot V_{ov}$$

For  $i_{D1} = 0.15 \text{ mA}$

$$U_{id} = \left( \frac{2 \times 0.15 \text{ mA}}{0.2 \text{ mA}} - 1 \right) \times 0.26$$

$$= \underline{0.13 \text{ V}}$$

(c)  $i_{D1} = 0.2 \text{ mA}$  and  
 $i_{D2} = 0$  (when  $Q_2$  just  
cuts off)  
this occurs at

$$U_{id} = +\sqrt{2} \times V_{ov}$$

$$= +\underline{0.367 \text{ V}}$$

(d)  $i_{D1} = 0.05 \text{ mA}$  } Opposite  
 $i_{D2} = 1.50 \text{ mA}$  } same to (b)  
 $\rightarrow U_{id} = -\underline{0.13 \text{ V}}$

(e)  $i_{D1} = 0$ ,  $i_{D2} = 0.2 \text{ mA}$  when  
 $Q_1$  just cuts off.  
This occurs at  $U_{id} = -\sqrt{2} \times V_{ov}$   
=  $-\underline{0.367 \text{ V}}$

For each case find  $U_s$ ,  $U_{D1}$ ,  
 $U_{D2}$ ,  $U_{D2} - U_{D1}$

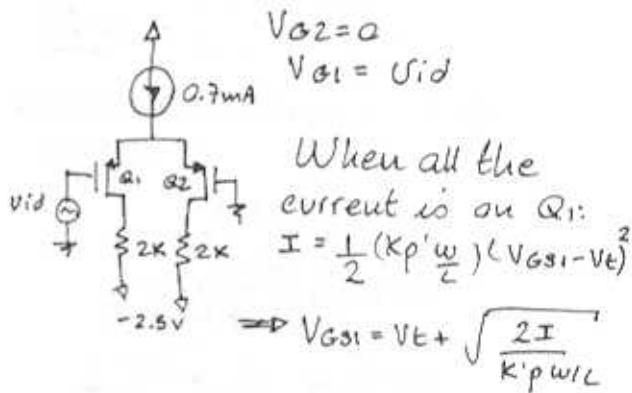
$$U_{GS} = \sqrt{\frac{2 \times I_D}{K_n W/L}} + V_t$$

then  $U_s = U_g - U_{GS}$

Solving for each case we  
obtain the following results

$U_{id}$ (v)	$i_{D1}$ (mA)	$i_{D2}$ (mA)	$U_s$ (v)	$U_{D1}$ (v)	$U_{D2}$ (v)	$U_{D2} - U_{D1}$ (v)
(c) +.37	0.2	0	-0.7	1.5	2.5	-1.0
(b) +.13	0.15	0.05	-0.88	1.75	2.25	-0.5
(a) 0	0.1	0.1	-0.96	2	2	0
(d) -.13	0.05	0.15	-1.01	2.25	1.75	+0.5
(e) -.37	0	0.2	-1.07	2.5	1.5	+1.0

7.4



When all the current is on  $Q_1$ :

$$I = \frac{1}{2} (K_p' w_L) (V_{GS1} - V_t)^2$$

$$\Rightarrow V_{GS1} = V_t + \sqrt{\frac{2I}{K_p' w_L}}$$

$$= V_t + \sqrt{2} V_{ov}$$

and  $V_{GS2}$  is reduced to  $V_t$   
thus  $U_s = -V_t$ .

Then  $U_{id} = U_{GS1} + U_s$   
=  $V_t + \sqrt{2} V_{ov} - V_t = \sqrt{2} V_{ov}$

In a similar manner as for the NMOS Differential Amplifier, as  $U_{id}$  reaches  $-\sqrt{2} V_{ov}$   
 $Q_1$  turns off and  $Q_2$  on.  
Thus the steering range is  
 $\sqrt{2} V_{ov} \leq U_i \leq -(\sqrt{2} V_{ov})$

For this particular case

$$\sqrt{2} \times -0.45 \leq U_{id} \leq \sqrt{2} \times 0.45$$

$$-0.63 \leq U_{id} \leq 0.63$$

When  $U_{id} = -0.63 \text{ V}$ ,

$$i_{D1} = 0.7 \text{ mA}, i_{D2} = 0$$

$$U_s = -V_t = +0.8 \text{ V}$$

$$U_{D1} = 2k \times 0.7 \text{ mA} - 2.5 = -1.1 \text{ V}$$

CONT.

$$V_{D2} = 0 - 2.5V = \underline{-2.5V}$$

When  $V_{id} = +0.63$

$$I_{D1} = 0 ; I_{D2} = 0.7mA$$

$$\begin{aligned} V_S &= V_{id} - V_{GS1} = V_{id} - V_{T1} \\ &= 0.63 + 0.8 = \underline{1.43} \end{aligned}$$

$$V_{D1} = -2.5V //$$

$$V_{D2} = -1.1V //$$

7.5

$$V_{G1} = V_{id} \quad I_{D1} = 0.11mA$$

$$V_{G2} = 0 \quad I_{D2} = 0.09mA$$

$$I_D = \frac{1}{2} K_w \frac{W}{L} (V_{GS} - V_T)^2$$

For Q1:

$$0.11m = \frac{1}{2} 3m (V_{GS1} - 0.7)^2$$

$$\rightarrow V_{GS1} = \underline{0.97V}$$

For Q2:

$$0.09m = \frac{1}{2} 3m (V_{GS2} - 0.7)^2$$

$$\rightarrow V_{GS2} = \underline{0.94V}$$

$$V_S = -V_{GS2} = -\underline{0.94V}$$

$$\begin{aligned} V_{id} &= V_S + V_{GS1} = -0.94 + 0.97 \\ &= \underline{0.03V} \end{aligned}$$

$$V_{D2} - V_{D1} = 5K(I_{D1} - I_{D2})$$

$$= 5K(0.11 - 0.09)m$$

$$= \underline{0.1V}$$

thus

$$\frac{V_{D2} - V_{D1}}{V_{id}} = \frac{0.1}{0.03} = \underline{3.33}$$

When  $I_{D1} = 0.09mA$  and  
 $I_{D2} = 0.11mA$

is the reverse condition  
from the case we just

studied, thus  $V_{id} = \underline{-0.03V}$

7.6

We know that there is a linear relationship between  $V_{ov}$  &  $V_{id}$  since:

$$V_{ov} = V_{id} \frac{W}{L}$$

Then from the data in table 7.3 we can tell that for

$$V_{imax} = 150mV$$

$$V_{ov} = 0.2 \times \frac{150}{126} = \underline{0.238V}$$

$$\text{For } W/L: \frac{W}{L} = \frac{1}{(V_{ov})^2} \cdot \frac{I}{K}$$

where  $I$  and  $K$  are constant  
thus, for  $W/L$ :

$$\left(\frac{W}{L}\right)_2 = \frac{50}{\left(\frac{150}{126}\right)^2} = 35.3$$

For gm:  $g_m = \frac{I}{V_{ov}}$  where  $I$   
is constant

$$\rightarrow g_{m2} = \frac{g_{m1}}{\left(\frac{150}{126}\right)} = \frac{2}{\frac{150}{126}} = \underline{1.68}$$

7.7

$$\left(\frac{V_{idmax}/2}{V_{ov}}\right)^2 = K$$

$$\Rightarrow \underline{2V_{ov}\sqrt{K}} = V_{idmax} \quad \text{Q.E.D}$$

$$I_{D1} = \frac{I}{2} + \left(\frac{I}{V_{ov}}\right) \frac{V_{id}}{2} \sqrt{1-K}$$

CONT.

$$I_{D1} = \frac{I}{2} \pm \frac{I}{V_{ov}} \cdot \frac{2V_{ov}\sqrt{K}}{2} \cdot \sqrt{1-K}$$

$$\rightarrow I_{D1} = \frac{I}{2} \pm I \sqrt{K(1-K)}$$

thus  $\Delta I = 2I\sqrt{K(1-K)}$   
Q.E.D.

For  $K = 0.01$

$$\Delta I = 2I\sqrt{0.01(1-0.01)}$$

$$= 0.198 \times I$$

$$V_{idmax} = 2V_{ov}\sqrt{0.01} = \underline{\underline{0.2V_{ov}}}$$

For  $K = 0.1$

$$\Delta I = 2I\sqrt{0.1(1-0.1)} = \underline{\underline{0.8I}}$$

$$V_{idmax} = 2V_{ov}\sqrt{0.2} = \underline{\underline{0.894V_{ov}}}$$

7.8

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$$

$$\frac{200}{2} = \frac{1}{2} \times 90 \times \frac{100}{1.6} (V_{GS} - 0.8)^2$$

$$\Rightarrow V_{GS} = \underline{\underline{1.19V}}$$

$$g_m = \frac{2I_D}{V_{GS} - V_t} = \frac{2 \times 100}{(1.19 - 1)} = \underline{\underline{1.06mA/V}}$$

$$V_{id} \Big|_{\substack{\text{full current} \\ \text{switching}}} = \sqrt{2} (V_{GS} - V_t) \\ = \underline{\underline{0.27V}}$$

To double this value,  $V_{GS} - V_t$  must be doubled which means that  $I_D$  should be quadrupled. i.e.  $I$  changed to:

$$\underline{\underline{800mA}}$$

7.9

$$g_m = \frac{2I_D}{V_{ov}} \rightarrow I_m = \frac{I}{0.2}$$

$$\rightarrow I = \underline{\underline{0.2mA}}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{ov}^2$$

$$100 = \frac{1}{2} \times 90 \times \frac{W}{L} \times (0.2)^2$$

$$\Rightarrow \frac{W}{L} = \underline{\underline{55.6}}$$

7.10

$$I_D = \frac{1}{2} \mu_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$50 = \frac{1}{2} \times 400 (V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 1.5V$$

For  $V_{G1} = V_{G2} = 0$ ,  $V_s = -1.5V$

For  $V_{G1} = V_{G2} = 2V$ ,  $V_s = +\underline{\underline{0.5V}}$

The drain currents are equal in both cases.

For  $V_{G2} = 0$ :

To reduce  $I_{D2}$  by 10% ,

$$I_{D2} = 0.9 \times 50 = 45mA$$

$$I_{D1} = \underline{\underline{55mA}}$$

$$V_{GS2} = \sqrt{\frac{2I_{D2}}{400}} + 1 = 1.47V$$

$$V_{GS1} = \sqrt{\frac{2 \times 55}{400}} + 1 = 1.52V$$

$$\text{Thus, } V_{G1} = V_{GS1} - V_{GS2} = \underline{\underline{0.05V}}$$

To increase  $I_{D2}$  by 10%

$$I_{D2} = \underline{\underline{55mA}}$$

$$I_{D1} = \underline{\underline{45mA}}$$

CONT.

$$V_{GS2} = 1.52V$$

$$V_{GS1} = 1.47V$$

$$\Rightarrow V_{G1} = \underline{-0.05V}$$

$i_{D2}/i_{D1}$	$i_{D2}$	$i_{D1}$	$V_{GS2}$	$V_{GS1}$	$V_G - V_{G1}$
(mA)	(mA)	(V)	(V)	(V)	(V)
1	50	50	1.5	1.5	0
0.5	33.3	66.7	1.408	1.577	-0.17
0.9	47.4	52.6	1.487	1.513	-0.026
0.99	47.75	50.25	1.4886	1.5012	-0.013

For  $i_{D1}/i_{D2} = 20 \Rightarrow i_{D2} = 4.76mA$

$$i_{D1} = 95.24mA$$

$$V_{GS2} = 1.154V, V_{GS1} = 1.690$$

$$\text{Thus } V_{G1} - V_{G2} = \underline{0.536V}$$

7.11

$$V_{ov} = \sqrt{\frac{I}{Kn'w}} = \sqrt{\frac{0.5}{0.25 \times 50}} = \underline{0.2V}$$

$$g_m = \frac{I}{V_{ov}} = \frac{0.5mA}{0.2V} = \underline{2.5mA/V}$$

$$f_0 = \frac{V_A}{I_D} = \frac{10}{(0.5mA/2)} = \underline{40KHz}$$

$$A_d = g_m \times (R_D \parallel f_0)$$

$$= \frac{2.5mA}{V} (40KHz \parallel 40KHz)$$

$$= \underline{9.09 V/V}$$

7.12

From Eqn. (7.23)

$$\left(\frac{V_{id}/2}{V_{ov}}\right)^2 = 0.1 \rightarrow \left(\frac{0.2/2}{V_{ov}}\right)^2 = 0.1$$

$$\rightarrow V_{ov} = \sqrt{0.1} = \underline{0.316V}$$

$$g_m = \frac{I}{V_{ov}} \rightarrow 3 \frac{mA}{V} = \frac{I}{0.316}$$

$$\rightarrow I = \underline{0.95mA}$$

$$\text{also: } V_{ov} = \sqrt{\frac{I}{Kn'w/L}}$$

$$\Rightarrow (0.316)^2 = \frac{0.95mA}{0.1mA \times \frac{w}{L}}$$

$$\rightarrow \frac{w}{L} = \underline{95}$$

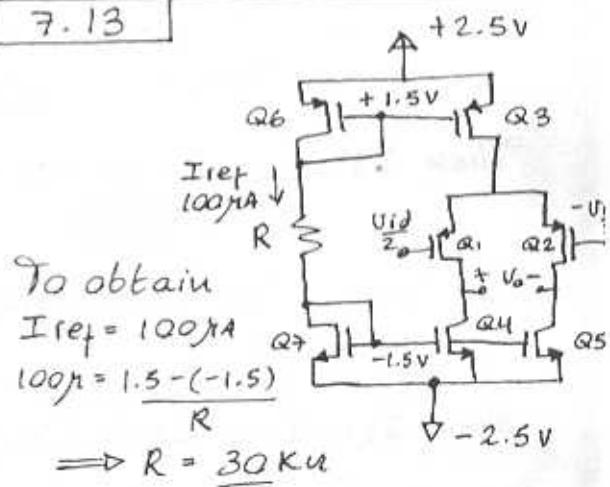
$$\text{If } R_o = 5K\Omega \Rightarrow$$

$$A_d = g_m R_o = \frac{3mA}{V} \times 5K\Omega = \underline{15V}$$

$$\text{if } V_{id} = 0.2 \Rightarrow V_{od} = V_{id} \times A_d$$

$$= 0.2 \times 15 = \underline{3V}$$

7.13



To obtain

$$I_{ref} = 100mA$$

$$100mA = \frac{1.5 - (-1.5)}{R}$$

$$\Rightarrow R = \underline{30K\Omega}$$

$$V_{GS7,4,5} = -1.5 + 2.5 = \underline{1V}$$

$$V_{ov7,4,5} = V_{GS} - V_{tn} = 1 - 0.7 = \underline{0.3V}$$

$$V_{GS6,3} = 1.5 - 2.5 = \underline{-1V}$$

$$V_{ov6,3} = -1 - (-0.7) = \underline{-0.3V}$$

The differential half circuit is an active-loaded common-source amplifier

CONT

thus, for Q<sub>1</sub>, Q<sub>4</sub>:

$$U_{O^+} = \frac{V_{id}}{2} \times g_m (\tau_{01} \parallel \tau_{04})$$

For Q<sub>2</sub>, Q<sub>5</sub>:

$$U_{O^-} = -\frac{V_{id}}{2} \times g_m (\tau_{02} \parallel \tau_{05})$$

Since  $\tau_{01} = \tau_{02} = \tau_{04} = \tau_{05} \equiv \tau_0$

$$U_{O^+} - U_{O^-} = V_{id} \times g_m \times \frac{\tau_0}{2}$$

$$\rightarrow \frac{U_{O^+} - U_{O^-}}{V_{id}} = A_d = g_m \times \frac{\tau_0}{2}$$

$$= \frac{g_{m_{1,2}}}{2} \times \frac{V_{An}}{I_{D_{1,2}}}$$

$$= \frac{1}{2} \times 2 \frac{I_{D_{1,2}}}{V_{ov}} \times \frac{V_{An}}{I_{D_{1,2}}} = \frac{V_{An}}{V_{ov_{1,2}}}$$

thus:

$$80 = \frac{20}{|V_{ov_{1,2}}|} \rightarrow V_{ov_{1,2}} = -0.25 \text{ V}_{\text{MOS}}$$

$$\text{Then } V_{GS1,2} = -0.25 - 0.7 \\ = -0.95 \text{ V}$$

We have:  $I_{D7} = I_{D6} = \underline{100 \mu A}$

If we choose:

$$I_{D3} = I_{D6} = 100 \mu A \parallel$$

$$\text{then } I_{D1} = I_{D2} = I_{D4} = I_{D5} = \underline{50 \mu A}$$

To obtain w/L ratios:

$$I_D = \frac{1}{2} \mu_{nCox} (W/L) V_{ov}^2$$

$$\Rightarrow \frac{w}{l} = \frac{2ID}{\mu_{nCox} V_{ov}^2}$$

where:

$$\mu_{nCox} = 90 \mu A/V^2$$

$$\mu_{pCox} = 30 \mu A/V^2$$

For Q<sub>7</sub>:

$$\left(\frac{w}{l}\right)_7 = \frac{2 \times 100 \mu A}{90 \mu A \times (0.3)^2} = \underline{24.7}$$

For Q<sub>4</sub> and Q<sub>5</sub>:

$$\left(\frac{w}{l}\right)_{4,5} = \frac{2 \times (100 \mu A/2)}{90 \mu A \times (0.3)^2} = \underline{12.3}$$

For Q<sub>1</sub> and Q<sub>2</sub>:

$$\left(\frac{w}{l}\right)_{1,2} = \frac{2 \times (100 \mu A/2)}{30 \mu A \times (0.25)^2} = \underline{53.3}$$

For Q<sub>6</sub> and Q<sub>3</sub>:

$$\left(\frac{w}{l}\right)_{6,3} = \frac{2 \times 100 \mu A}{30 \mu A \times (0.3)^2} = \underline{74.1}$$

In summary, the results are:

	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	Q <sub>5</sub>	Q <sub>6</sub>	Q <sub>7</sub>	
$\mu_{nCox}$	30	30	30	90	90	30	90	$\mu A/V^2$
$I_D$	50	50	100	50	50	100	100	$\mu A$
$V_{ov}$	-0.25	-0.25	-0.3	0.3	0.3	-0.3	0.3	V
$W/L$	53.3	53.3	74.1	12.3	12.3	74.1	24.7	
$V_{GS}$	-0.95	-0.95	-1	1	1	-1	1	

7.14

$$(a) I_{D1} = \frac{1}{2} \mu_n' \frac{w}{l} (V_{GS1} - V_t)^2$$

$$I_{D2} = \frac{1}{2} \mu_n' \left(2 \times \frac{w}{l}\right) (V_{GS2} - V_t)^2$$

Since  $V_{GS} - V_t$  is equal for both transistors:

$$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{1}{2}$$

$$\text{but } I = I_{D1} + I_{D2}$$

$$\Rightarrow 2I_{D1} = I - I_{D2}$$

$$I_{D1} = \underline{\underline{I/3}}$$

$$I_{D2} = \underline{\underline{2I/3}}$$

$$(b) V_{ov} = V_{GS} - V_t$$

CONT.

$$V_{ov1} = V_{ov2} = V_{ov}$$

$$\text{For Q1: } \frac{I}{3} = \frac{1}{2} K_{n'} \left( \frac{w}{L} \right) V_{ov}^2$$

$$\Rightarrow V_{ov} = \sqrt{\frac{2}{3} \frac{I}{K_{n'} w L}}$$

$$(c) g_m = \frac{2 I_D}{V_{ov}} \rightarrow g_{m1} = \frac{2 I}{3 V_{ov}}$$

$$g_{m2} = \frac{4}{3} \frac{I}{V_{ov}}$$

$$U_{o1} = -g_{m1} \times \frac{U_{id}}{2} \cdot R_D$$

$$= -\frac{2}{3} \frac{I}{V_{ov}} \cdot R_D \cdot U_{id}$$

$$U_{o2} = +g_{m2} \times \frac{U_{id}}{2} \cdot R_D$$

$$= \frac{4}{3} \frac{I}{V_{ov}} \cdot R_D \cdot U_{id}$$

$$\Rightarrow \frac{U_{o2} - U_{o1}}{U_{id}} = \left( \frac{4}{3} + \frac{2}{3} \right) \frac{I}{V_{ov}} \cdot R_D$$

$$= 2 \times \frac{I}{V_{ov}} \cdot R_D$$

7.15

$$V_{ov} = \sqrt{\frac{I}{K_{n'} w L}} = \sqrt{\frac{0.2}{3}} = 0.26V$$

$$g_m = \frac{I}{V_{ov}} = \frac{0.2mA}{0.26V} = 0.77mA/V$$

(a) Single-ended output:

From Eqn. (7.42)

$$|Ad| = \frac{1}{2} g_m \times R_D = \frac{0.77 \times 10}{2}$$

$$= 3.85 V/V$$

From Eqn. (7.41)

$$|A_{cm}| = \frac{R_D}{2 R_{ss}} = \frac{10}{2 \times 100} = 0.05 V/V$$

$$CMRR = \left| \frac{Ad}{A_{cm}} \right| = \frac{3.85}{0.05} = 77$$

i.e. 37.7 dB

(b) Differential output, and 1% mismatch in  $R_D$ 's:

$$\text{Eqn (7.52)} |Ad| = g_m R_D$$

$$= 0.77 \times 10 = 7.7 V/V$$

$$\text{Eqn (7.51)} |A_{cm}| = \frac{R_D}{2 R_{ss}} \times \left( \frac{\Delta R_D}{R_D} \right)$$

$$= \frac{10}{2 \times 100} \times 0.01 = 0.5 mV/V$$

$$CMRR = \left| \frac{Ad}{A_{cm}} \right| = \frac{7.7}{0.5 \times 10^{-3}} = 15,400$$

i.e. 83.7 dB

7.16

$$V_{ov} = -\sqrt{\frac{I}{K_{n'} w L}} = -\sqrt{\frac{0.7mA}{3.5mA/V^2}}$$

$$= -0.45V$$

$$g_m = \frac{I}{[V_{ov}]} = \frac{0.7mA}{0.45V} = 1.56mA/V$$

$$|Ad| = g_m R_D = 1.56 \times 2 = 3.12 V/V$$

$$|A_{cm}| = \frac{R_D}{2 R_{ss}} \times \left( \frac{\Delta R_D}{R_D} \right) = \frac{2}{2 \times 30} \times 0.02$$

$$= 6.7 \times 10^{-4}$$

$$CMRR = \frac{3.12}{6.7 \times 10^{-4}} = 4680 \rightarrow 73.4$$

7.17

$$(a) I_{D1} = I_{D2} = \frac{1 \text{ mA}}{2} = 0.5 \text{ mA}$$

$$I_D = \frac{1}{2} K_n \frac{W}{L} V_{ov}^2$$

$$\Rightarrow 0.5 \text{ mA} = \frac{1}{2} \times 2.5 \text{ m} \times V_{ov}^2$$

$$\rightarrow V_{ov} = 0.632 \text{ V}$$

$$V_{ov} = V_{GS} - V_t = V_{GS} - 0.7$$

$$\rightarrow V_{GS} = 0.632 + 0.7 \\ = 1.332 \text{ V}$$

To obtain 1mA over  $R_{SS} = 1 \text{ k}\Omega$

$$V_s = 1 \text{ mA} \times 1 \text{ k} = 1 \text{ V}$$

$$\rightarrow V_{CM} = V_s + V_{GS} = 1 + 1.332 \\ = \underline{\underline{2.332 \text{ V}}}$$

$$(b) g_m = \frac{I}{V_{ov}} = \frac{1 \text{ mA}}{0.632 \text{ V}} = 1.6 \frac{\text{mA}}{\text{V}}$$

$$\text{Eqn. (7.45): } A_d = g_m \cdot R_o \\ \text{for } A_d = 8 \text{ V/V } R_o = \frac{8}{1.6 \text{ mA}} = \underline{\underline{5 \text{ k}\Omega}}$$

(c) At the drains:

$$V_{D1} = V_{D2} = 5 \text{ V} - \frac{1 \text{ mA}}{2} \times 5 \text{ k}\Omega \\ = \underline{\underline{+2.5 \text{ V}}}$$

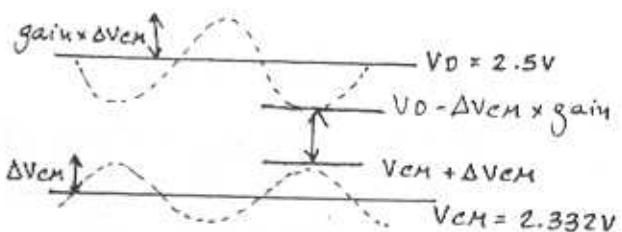
(d) Eqn. (7.39):

$$\frac{V_{oi}}{V_{iem}} = \frac{-R_D}{\frac{1}{g_m} + 2R_{SS}}$$

$$\Rightarrow |A_{cm}| = \left| \frac{\Delta V_{oi}}{\Delta V_{em}} \right| = \frac{5 \text{ k}}{\frac{1}{1.6 \text{ mA}} + 2 \times 1 \text{ k}} \\ = \underline{\underline{1.9 \text{ V/V}}}$$

(e) On the edge of the triode region:

$$V_G - V_D = V_t$$



$$\text{If: } V_G - V_D = V_t \\ \Rightarrow V_{CM} + \Delta V_{CM} - V_D + \Delta V_{CM} (\text{A}_{cm}) \\ = V_t.$$

$$\rightarrow 2.332 + \Delta V_{CM} - 2.5 + \Delta V_{CM} \cdot 1.9 \\ = 0.7$$

$$2.9 \Delta V_{CM} = 0.868 \\ \Delta V_{CM} = \underline{\underline{0.3 \text{ V}}}$$

7.18

$$(a) R_{D1} = R_D + \Delta R/2$$

$$R_{D2} = R_D - \Delta R/2$$

$$g_{m1} = g_m + \Delta g_m/2$$

$$g_{m2} = g_m - \Delta g_m/2$$

The following Eqs are still valid:

$$\text{Eqn. (7.62)} \quad i_{d1} = \frac{g_{m1} V_{iem}}{2 g_m R_{SS}}$$

$$\text{Eqn. (7.63)} \quad i_{d2} = \frac{g_{m2} V_{iem}}{2 g_m R_{SS}}$$

From which:

$$i_{d1} - i_{d2} = (g_{m1} - g_{m2}) \frac{V_{iem}}{2 g_m R_{SS}} \\ = \Delta g_m \frac{V_{iem}}{2 g_m R_{SS}} \quad ①$$

$$i_{d1} + i_{d2} = (g_{m1} + g_{m2}) \frac{V_{iem}}{2 g_m R_{SS}}$$

Assuming that  $\Delta g_m$  is small, then

$$g_{m1} + g_{m2} \approx 2 g_m$$

$$\rightarrow i_{d1} + i_{d2} = \frac{2 g_m \cdot V_{iem}}{2 g_m R_{SS}} \rightarrow \text{CONT.}$$

$$i_{d1} + i_{d2} = \frac{V_{icm}}{R_{SS}} \quad (2)$$

The differential output is:

$$\begin{aligned} V_{o2} - V_{o1} &= -i_{d2} \cdot R_{D2} + i_{d1} \cdot R_{D1} \\ &= -i_{d2} \left( R_D - \frac{\Delta R_D}{2} \right) + i_{d1} \left( R_D + \frac{\Delta R_D}{2} \right) \\ &= R_D \cdot (i_{d1} - i_{d2}) + \frac{\Delta R_D}{2} (i_{d1} + i_{d2}) \end{aligned}$$

Substituting (1) & (2):

$$= R_D \cdot \frac{\Delta g_m}{2 g_m R_{SS}} \cdot V_{icm} + \frac{\Delta R_D}{2} \frac{V_{icm}}{R_{SS}}$$

From which the common-mode gain is:

$$\begin{aligned} A_{cm} &= \frac{V_{o2} - V_{o1}}{V_{icm}} = \frac{R_D}{2 R_{SS}} \cdot \frac{\Delta g_m}{g_m} \\ &\quad + \frac{\Delta R_D}{2 R_{SS}} \\ &\approx \frac{R_D}{2 R_{SS}} \left[ \frac{\Delta g_m}{g_m} + \frac{\Delta R_D}{R_D} \right] \end{aligned}$$

Q.E.D.

(b) For  $R_D = 5 Ku$ ,

$$R_{SS} = 25 Ku$$

$$\rightarrow A_{cm} = 0.002 V/V$$

For identical  $R_D$ 's, this common-mode gain is caused by a mismatch in  $g_m$ .

Thus, from Eqn. (7.64)

$$A_{cm} = \left( \frac{R_D}{2 R_{SS}} \right) \cdot \left( \frac{\Delta g_m}{g_m} \right)$$

$$0.002 = \frac{5}{2 \times 25} \cdot \frac{\Delta g_m}{g_m}$$

$$\rightarrow \frac{\Delta g_m}{g_m} = 0.02$$

To reduce  $A_{cm}$  to zero, create a mismatch in  $R_D$  such as:

$$\begin{aligned} 0 &= \frac{R_D}{2 R_{SS}} \left( \frac{\Delta g_m}{g_m} + \frac{\Delta R_D}{R_D} \right) \\ \Rightarrow \frac{\Delta R_D}{R_D} &= -\frac{\Delta g_m}{g_m} = -0.02 \\ \text{i.e. } &\underline{-2\%} \text{ of } R_D \end{aligned}$$

7.19

$$g_m = K_n' \left( \frac{w}{l} \right) \cdot (V_{GS} - V_t)$$

Recalling from calculus that for a function  $u = f(x, y)$  the total derivative is:

$$du = \frac{\partial u}{\partial x} \cdot dx + \frac{\partial u}{\partial y} \cdot dy$$

Thus,

$$\begin{aligned} \Delta g_m &= \frac{\partial g_m}{\partial (w/l)} \Delta(w/l) + \frac{\partial g_m}{\partial V_t} \Delta V_t \\ &= K_n' (V_{GS} - V_t) \Delta(w/l) \\ &\quad - K_n' (w/l) \Delta V_t \end{aligned}$$

thus,

$$\begin{aligned} \frac{\Delta g_m}{g_m} &= \frac{K_n' (V_{GS} - V_t) \Delta(w/l)}{K_n' (w/l) (V_{GS} - V_t)} \\ &\quad - \frac{K_n' (w/l) \cdot \Delta V_t}{K_n' (w/l) (V_{GS} - V_t)} \end{aligned}$$

$$\frac{\Delta g_m}{g_m} = \frac{\Delta(w/l)}{(w/l)} + \frac{-\Delta V_t}{(V_{GS} - V_t)}$$

$$\text{but: } V_{GS} - V_t = V_{ov}$$

$$\Rightarrow \frac{\Delta g_m}{g_m} = \frac{\Delta(w/l)}{(w/l)} + \frac{-\Delta V_t}{V_{ov}}$$

Q.E.D.

CONT.

If  $\frac{\Delta(W/L)}{W/L} = \pm 0.01$ ;  $\Delta Vt = \pm 5 \text{ mV}$

$$V_{ov} = 0.25 \text{ V}$$

Then, the worst-case fractional mismatch in  $g_m$  is:

$$\begin{aligned}\Delta g_m &= \left| \frac{\Delta W/L}{W/L} \right| + \left| \frac{\Delta Vt}{V_{ov}} \right| \\ &= 0.01 + \frac{0.005}{0.25} = \underline{\underline{0.03}}\end{aligned}$$

If:  $R_D = 5 \text{ k}\Omega$ ,  $R_{SS} = 25 \text{ k}\Omega$   
 $I = 1 \text{ mA}$

From Eqn. (7.64)

$$A_{cm} = \frac{R_D}{2R_{SS}} \cdot \frac{\Delta g_m}{g_m}$$

$$A_{cm} = \frac{5}{2 \times 25} \times 0.03 = \underline{\underline{0.003 \text{ V/V}}}$$

From Eqn. (7.66)  
 $CMRR = 2 g_m R_{SS}$

$$\left( \frac{\Delta g_m}{g_m} \right)$$

where:  $g_m = I / V_{ov}$   
 $= 1 \text{ mA} / 0.25 \text{ V} = \underline{\underline{4 \text{ mA/V}}}$

$$\begin{aligned}CMRR &= \frac{2 \times 4 \times 25}{0.03} = 6,666.7 \\ \rightarrow \text{i.e. } &\underline{\underline{76.5 \text{ dB}}}\end{aligned}$$

7.20

$$U_{BE} = 0.7 \text{ V} \quad i_e = 1 \text{ mA}$$

$$\rightarrow \text{at } i_e = 0.5 \text{ mA} \quad V_{cm} = -2 \text{ V}$$

$$U_{BE} = 0.7 + 25 \ln \left( \frac{0.5}{1} \right) = 0.683 \text{ V}$$

Thus,

$$\begin{aligned}U_E &= U_{cm} - U_{BE} \\ &= -2 - 0.683 = \underline{\underline{-2.683 \text{ V}}}\end{aligned}$$

$$i_{c1} = i_{c2} = \alpha \times 0.5 = \frac{100}{101} \times 0.5$$

$$= 0.495 \text{ mA}$$

$$U_{c1} = U_{c2} = V_{cc} - i_c R_c$$

$$= 5 - 0.495 \times 3$$

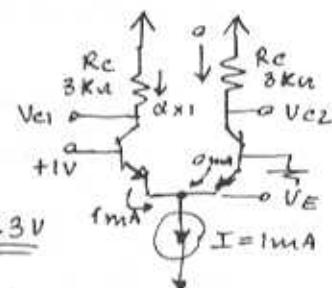
$$= + \underline{\underline{3.515 \text{ V}}}$$

7.21

$$\begin{aligned}i_{c1} &= \alpha I \\ &= 1 \text{ mA}\end{aligned}$$

$$U_{BE1} = 0.7 \text{ V}$$

$$U_E = 1 - 0.7 = + \underline{\underline{0.3 \text{ V}}}$$

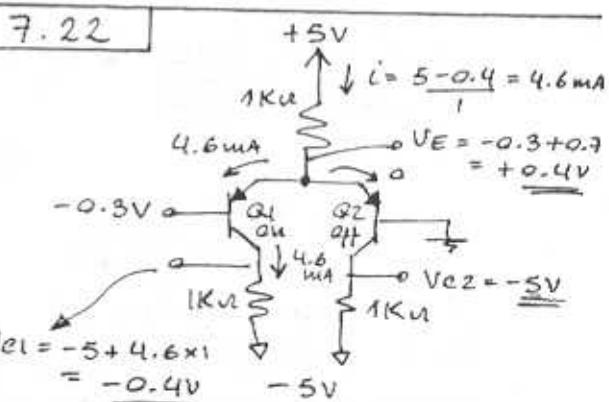


$$U_{c1} = V_{cc} - i_{c1} R_c$$

$$\approx 5 - 1 \times 3 = \underline{\underline{2 \text{ V}}}$$

$$U_{c2} = V_{cc} - \alpha \times i_{c1} R_c = \underline{\underline{5 \text{ V}}}$$

7.22



7.23

$$i_{E1} = \frac{I}{1 + e^{(UB2 - VBD)/VT}} = 0.8 I$$

$$\rightarrow U_{B2} - U_{B1} = VT \ln 0.25 = -34.6 \text{ mV}$$

$$\text{Thus, } U_{B1} - U_{B2} = \underline{\underline{34.6 \text{ mV}}}$$

7.24

$$(a) U_{CMmax} = U_{C1,2} = \\ V_{CC} - \frac{I}{2} \cdot R_C$$

(b) If the current is steered to  $Q_1$ , then

$$U_{C1} = V_{CC} - I R_C, \text{ a change of: } -\frac{I}{2} R_C$$

$$U_{C2} = \underline{\underline{V_{CC}}}, \text{ a change of} \\ +\frac{I}{2} R_C$$

$$(c) U_{CMmax} = 3 = 5 - \frac{I}{2} R_C \\ \Rightarrow I R_C = \underline{\underline{4V}}$$

$$(d) \frac{I/2}{\beta+1} \leq 2mA$$

$$\Rightarrow I \leq 4(\beta+1)mA$$

$$\text{Thus, } I = 4 \times 10mA = 0.404mA$$

$$\text{Select } I = \underline{\underline{0.4mA}}$$

$$R_C = \frac{4V}{I} = \frac{4V}{0.4mA} = \underline{\underline{10k\Omega}}$$

7.25

$$I_{E1} = \frac{I}{1 + e^{-\frac{Ud}{VT}}}, Ud = V_{B1} - V_{B2}$$

$$\frac{\Delta I_{E1}}{I} \equiv \frac{I_{E1} - I/2}{I} = \frac{I_{E1}}{I} - 0.5$$

Define normalized Gain

$$G_n = \frac{\Delta I_{E1}}{I} \frac{I}{Ud}$$

Ud(mV)	5	10	20	30	40
Gn	9.97	9.87	9.50	8.95	8.30

Observe that the gain stays relatively constant up to Ud nearly 20 mV. Then it decreases significantly with the increase in signal level. Whenever gain depends on signal level, nonlinear distortion occurs.

7.26

With:

$$V_{B1} - V_{B2} = 10mV$$

$$I_{E1} = \frac{I}{1 + e^{-10/25}} = 0.598I$$

$$\text{Since } I_{E1} + I_{E2} = I$$

$$I_{E2} = 0.402I$$

For a collector resistance  $R_C$

$$\begin{aligned} U_o &= U_{C1} - U_{C2} = (U_{CC} - i_{C1} R_C) \\ &\quad - (U_{CC} - i_{C2} R_C) \\ &= -(i_{C2} - i_{C1}) R_C \\ &= -\alpha (I_{E2} - I_{E1}) R_C \\ &\approx -0.196 I R_C \end{aligned}$$

Thus, for

$$U_o = 1V; 0.196 I R_C = 1$$

$$I R_C = 5.102$$

$$\text{Now } I = 2mA, \text{ thus}$$

$$R_C = \underline{\underline{2.5k\Omega}}$$

DC (bias) voltage at each collector

$$= V_{CC} - \frac{I}{2} R_C = 10 - 1 \times 2.5 \\ = 7.5V$$

For a  $-1V$  output swing, the minimum voltage at each collector is:

CONT.

$$7.5 - 0.5 = 7.0V$$

Thus,  $V_{ICM}/_{max} = \underline{\underline{7V}}$

$$= V_{CC} - 0.55 \frac{AV}{20}$$

$$= \underline{\underline{V_{CC} - 0.0275 AV}}$$

7.27

$$(a) i_{E1} = \frac{I}{1 + e^{(VB_2 - VB_1)/VT}}$$

$$i_{E2} = \frac{I}{1 + e^{(VB_1 - VB_2)/VT}}$$

For  $VB_1 - VB_2 = 5mV$ ,  $VT = 25mV$

$$\left. \begin{aligned} i_{E1} &= 0.550I \\ i_{E2} &= 0.450I \end{aligned} \right\} \text{Note that } i_{E1} + i_{E2} = I$$

as should be the case.

$\alpha \approx 1$ , thus  $i_{C1} = 0.55I$   
 $i_{C2} = 0.45I$

$$V_o = U_{C2} - U_{C1} \Rightarrow$$

$$= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C)$$

$$= (i_{C1} - i_{C2})R_C$$

$$= 0.10 IR_C$$

Voltage gain  $AV = \frac{V_o}{U_{B1} - U_{B2}} = \frac{0.1IR_C}{0.005}$   
 $= \underline{\underline{20 IR_C V/V}}$

(b) Bias voltage at each collector is:  $\underline{\underline{V_{CC} - \frac{1}{2}IR_C}}$

For an output differential voltage of  $0.1R_C$ , each collector should be allowed to fall by  $(0.05IR_C)$  below its bias value, thus

$$V_{Cmin} = V_{CC} - 0.5IR_C - 0.05IR_C$$

$$= V_{CC} - 0.55IR_C$$

Thus for operation in saturation with

$$U_{CB} = 0 \text{ (minimum)}$$

$$V_{ICM/max} = V_{Cmin} = V_{CC} - 0.55IR_C$$

Obviously for a given  $V_{CC}$ , increasing  $AV$  reduces the maximum allowable  $V_{ICM}$

$AV (V/V)$	100	200	300	400
$V_{ICMmax} (V)$	$V_{CC} - 2.75$	$V_{CC} - 5.5$	$V_{CC} - 8.75$	$V_{CC} - 11$
$IR_C (V)$	5	10	15	20
$R_C (k\Omega)$	5	10	15	20

Example: For  $V_{CC} = 10V$ , a gain of 200 can be achieved using  $R_C = 10k\Omega$ . The corresponding maximum input common-mode voltage is 4.5V. If a gain of 300 is required, it can be achieved by increasing  $R_C$  to 15k $\Omega$ , however the maximum permitted input common-mode voltage then becomes 1.75V only.

7.28

$$i_{C1} = \alpha i_{E1} \approx i_{E1} = I / (1 + e^{-UD/VT})$$

$$i_{C2} = \alpha i_{E2} \approx i_{E2} = I / (1 + e^{UD/VT})$$

$$V_o = U_{C2} - U_{C1} = (V_{CC} - i_{C2}R_C) \dots$$

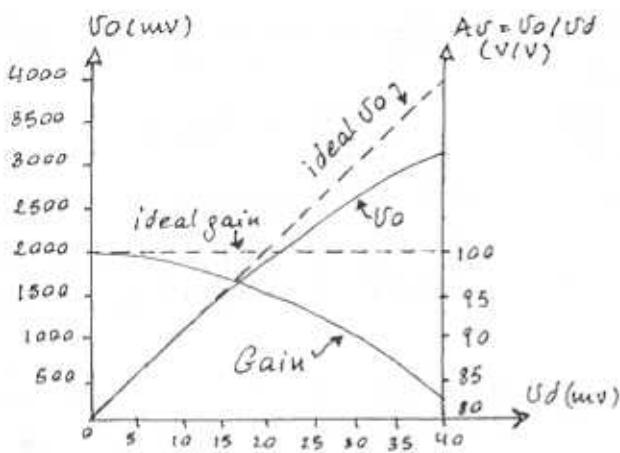
$$- (V_{CC} - i_{C1}R_C)$$

$$= IR_C \left( \frac{1}{1 + e^{-UD/VT}} - \frac{1}{1 + e^{UD/VT}} \right)$$

For  $IR_C = 5V$ :

$UD(mV)$	5	10	15	20	25	30	35	40
$V_o(V)$	4.98	9.87	14.57	19.00	23.11	26.85	30.22	33.20
$\frac{V_o}{U_d} (V)$	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9

CONT.



7.30

Refer to Fig. P7.30

(a) For  $U_I$  sufficiently low so that  $Q_1$  is cut off:

$$\begin{aligned} V_{BE1|Q_3} &= 0.7 + VT \ln \frac{10}{1} \\ &= 0.76 \text{ V} \\ V_{OH} &= U_o = 5 - \frac{10}{101} \times 1 - V_{BE1|Q_3} \\ &= 5 - \frac{10}{101} - 0.76 \\ &= 4.14 \text{ V} \end{aligned}$$

4.14 V

7.29

$$I = 6 \text{ mA}$$

The current will divide between the two transistors in proportion to their emitter areas. Thus with no input,

$$IE_1 = 1.5 IE_2$$

$$IE_1 + IE_2 = 2.5 IE_2 = 6 \text{ mA}$$

$$IE_2 = 2.4 \text{ mA}$$

$$IE_1 = 3.6 \text{ mA}$$

For  $\alpha \approx 1$

$$I_{C1} = 3.6 \text{ mA}$$

$$I_{C2} = 2.4 \text{ mA}$$

To equalize the collector currents we apply a difference signal  $U_d = U_{B2} - U_{B1}$  whose value can be determined as follows:

$$IE_1 = I_{SE1} e^{((UB_1 - U_E)/VT)}$$

$$IE_2 = I_{SE2} e^{((UB_2 - U_E)/VT)}$$

where  $I_{SE1}/I_{SE2} = 1.5$

Now,  $IE_1 = IE_2$  when

$$1 = 1.5 e^{(UB_1 - UB_2)/VT}$$

$$U_d = UB_2 - UB_1 = VT \ln 1.5 = 10.1 \text{ mV}$$

(b) For  $U_I$  sufficiently high so that  $Q_1$  is conducting all the current  $I$ :

$$\begin{aligned} V_{BE1|Q_3} &\approx 0.76 \text{ V} \\ V_{OL} &= U_o = 5 - \left( \frac{10}{101} + 0.99 \right) \times 1 \\ &\quad - 0.76 \\ &= 3.15 \text{ V} \end{aligned}$$

$$(c) IE_1 = I / (1 + e^{(UB_2 - UB_1)/VT})$$

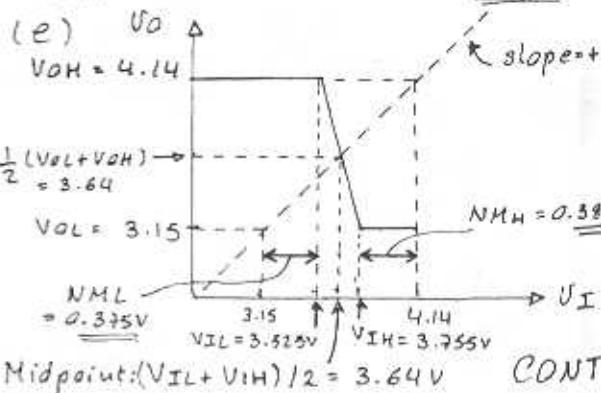
$$e^{(3.64 - U_I)/VT} = 99$$

$$U_I = 3.64 - VT \ln 99 = 3.525 \text{ V} = V_{IL}$$

$$(d) IE_1 = 0.99 I = \frac{I}{1 + e^{(3.64 - V_{IH})/VT}}$$

$$1 + e^{(3.64 - V_{IH})/VT} = 1.01$$

$$V_{IH} = 3.64 + VT \ln 1.01 = 3.755 \text{ V}$$



Since the mid-point of the output voltage swing is equal to 3.64V, the output voltage swing is centred on the mid-point of the input range; an ideal choice for it equalizes the noise margins.

7.31

Each device is operating at a current of  $150\mu A = 0.15mA$ . Thus,

$$g_m = \frac{0.15mA}{25mV} = \frac{6mA}{V}$$

$$R_{id} = 2(\beta + 1) r_e = 2\pi\beta \\ = 2 \times \frac{150}{4} = \underline{\underline{75k\Omega}}$$

7.32

$R_{id} > 10k\Omega$ ;  $A_d = 200 V/V$ ;  $\beta > 100$ ;  $V_{ce} = 10V$

$$R_{id} = 10^4 = 2\pi\beta = 2 \times \frac{100}{g_m}$$

$$\Rightarrow g_m = 20mA/V$$

Thus each device is operating at  $0.5mA$  and  $I = \underline{\underline{1mA}}$

Voltage gain =  $g_m \cdot R_c$

$$200 = 20 R_c$$

$$\Rightarrow R_c = \underline{\underline{10k\Omega}}$$

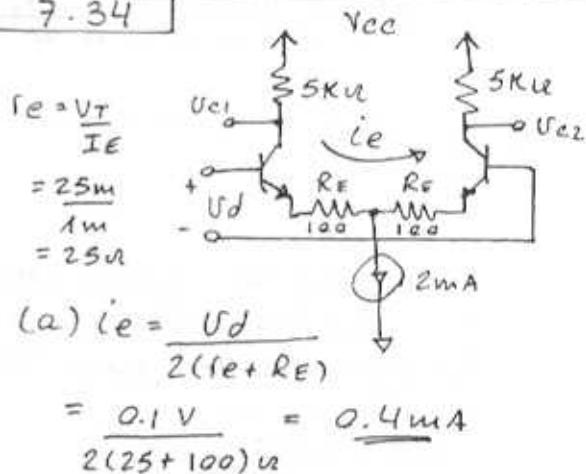
7.33

$$\frac{5mV}{I/2} = \frac{25mV}{50mA} = \underline{\underline{500\Omega}}$$

$$\text{Half-circuit gain} = \frac{\alpha R_c}{r_e} \approx \frac{R_c}{r_e} \\ = \frac{10k}{500} = \underline{\underline{20V/V}}$$

At one collector we expect a signal of (+100mV) and at the other a signal of (-100mV)

7.34



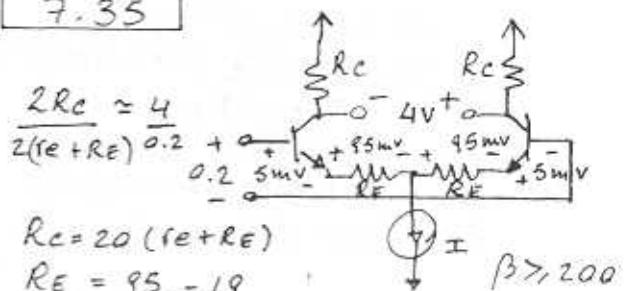
$$(a) i_e = \frac{U_d}{2(r_e + R_E)} \\ = \frac{0.1V}{2(25 + 100)\mu A} = \underline{\underline{0.4mA}}$$

$$(b) i_{E1} = 1 + 0.4 = \underline{\underline{1.4mA}} \\ i_{E2} = 1 - 0.4 = \underline{\underline{0.6mA}}$$

$$(c) U_{ce1} = -i_e R_c \approx -0.4 \times 5 = -\underline{\underline{2V}} \\ U_{ce2} = +\underline{\underline{2V}}$$

$$(d) U_{od} = 4V \\ A_d = U_{od} / U_{id} = \frac{4}{0.1} = \underline{\underline{40V/V}}$$

7.35



$$\frac{2R_c}{2(r_e + R_E)} \approx \frac{4}{0.2} = \underline{\underline{20}}$$

$$R_E = \frac{95}{5} = 19$$

$$\beta > 200 \\ R_{in} > 80k\Omega$$

CONT.

$$\begin{aligned} R_{in} &= 2(\beta+1)(r_e + R_E) \\ &= 2 \times 201 \times 20 r_e = 80 \text{ k}\Omega \\ \Rightarrow r_e &\approx \frac{80000}{8000} = 10 \mu\Omega \end{aligned}$$

Thus each device is operating at a current of  $\frac{25 \text{ mA}}{10 \mu\Omega} = 2.5 \text{ mA}$

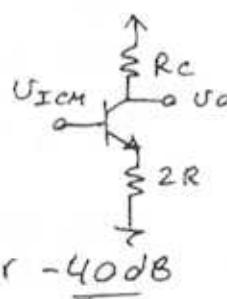
$$\Rightarrow I = 5 \text{ mA}$$

$$R_E = 19 \times 10 = 190 \mu\Omega$$

$$R_C = 20 \times 200 = 4 \text{ k}\Omega$$

7.36

$$\begin{aligned} \underline{\underline{2 \text{ M}\Omega}} \\ \frac{U_o}{U_{ICM}} &\approx \frac{R_C}{2R} = \frac{20 \text{ k}\Omega}{2 \text{ M}\Omega} \\ &= 0.01 \text{ V/V} \quad \text{or } -40 \text{ dB} \end{aligned}$$



7.37

Refer to Fig. P7.37

$$\frac{U_o}{U_i} = \frac{\alpha \times 20 \text{ k}\Omega}{(2r_e + 2 \times 200) \text{ }\mu\Omega}$$

$$\text{Where } r_e = \frac{V_T}{0.5/2} = \frac{0.05 \text{ V}}{0.5 \text{ mA}} = 100 \mu\Omega$$

$$\frac{U_o}{U_i} \approx \frac{20000}{600} = \underline{\underline{33.3 \text{ V/V}}}$$

$$\begin{aligned} R_i &= (\beta+1)(2r_e + 2 \times 200) \\ &= 101 \times 2 \times 300 \approx 60 \text{ k}\Omega \end{aligned}$$

7.38

Refer to Fig. P7.38

Each transistor is operating at  $I_E = 1 \text{ mA}$ , thus  $r_e = 25 \mu\Omega$  and  $r_{pi} = 101 \times 25 = 2525 \mu\Omega$

$$\frac{U_o}{U_i} = \frac{\alpha \times 7.5 \text{ k}\Omega}{(2r_e + 200) \mu\Omega} = \frac{7500}{250} = \underline{\underline{30}}$$

$$R_i = (\beta+1)(r_e + 200 + r_{pi}) \approx \underline{\underline{25 \text{ k}\Omega}}$$

7.39

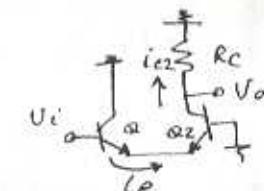
Refer to Fig. P7.39

(a) As a differential amplifier the gain is given by

$$\frac{U_o}{U_i} = \frac{\alpha R_C}{2r_e} \quad (1)$$

(b) Transistor  $Q_1$  can be considered as a common-collector stage. It is biased at  $I/2$  and has a resistance  $r_{e2}$  in its emitter, thus.

$$\begin{aligned} i_e &= \frac{U_i}{r_{e1} + r_{e2}} \\ &= \frac{U_i}{2r_e} \quad (2) \end{aligned}$$



Now,  $Q_2$  is connected in the common-base configuration. It has an input signal current  $i_{e2}$ . Thus its collector signal current (in the direction indicated) will be  $i_{c2} = \alpha i_{e2}$  (3)

The output voltage will be

$$U_o = i_{c2} \cdot R_C \quad (4)$$

Combining equations (2), (3) and (4) provides:

CONT.

$\frac{U_o}{U_i} = \frac{\alpha R_c}{2r_e}$ , which is identical to the result found above in part (a)

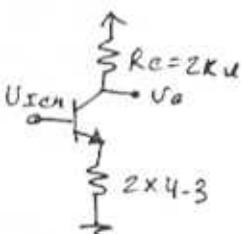
7.40

$$(a) V_E = -0.7V$$

$$\text{Thus, } I = \frac{-0.7 - (-5)}{4.3} = 1mA \quad r_e = 50\Omega$$

$$A_d = \frac{U_o}{U_d} = \frac{\alpha \times 2}{2r_e} \approx \frac{2000}{2 \times 50} = \underline{\underline{20}} \text{ V/V}$$

$$(b) A_{cm} = \frac{\alpha R_c}{8.6 + r_e} = \frac{2}{8.65} = \underline{\underline{0.23}} \text{ V/V}$$



$$(c) CMRR = 20 \log \left| \frac{A_d}{A_{cm}} \right| = 20 \log \frac{20}{0.23} = \underline{\underline{38.8}} \text{ dB}$$

$$(d) U_d = 2 \times 0.005 \sin(2\pi \times 1000t)$$

$$U_{ICM} = 0.1 \sin(2\pi \times 60t)$$

$$\begin{aligned} \text{Thus, } U_o &= A_d \cdot U_d + A_{cm} \cdot U_{ICM} \\ &= 20 \times 0.01 \sin 2\pi \times 1000t \\ &\quad + 0.23 \times 0.1 \sin(2\pi \times 60t) \\ &= \underline{\underline{0.2 \sin 2\pi \times 1000t + 0.023 \sin 2\pi \times 60t}} \end{aligned}$$

7.41

Each transistor is biased at 1mA. Thus,  $r_e = 25\Omega$ ,  $g_m = 40\text{mA/V}$ ,  $r_o = 100/\beta = 100\text{K}\Omega$ . The differential half-circuit is

$$\begin{aligned} A_d &= \frac{U_o}{U_i} = \alpha \left[ \frac{R_c || (R_L || 2r_e)}{r_e + RE/2} \right] \\ &\approx \frac{10/15}{0.025 + 0.100} = \underline{\underline{26.7}} \text{ V/V} \end{aligned}$$

$$\begin{aligned} R_{id} &= 2 [RB || (\beta+1)(r_e + RE/2)] \\ &= 2 [30 || 101 (0.025 + 0.100)] \\ &= \underline{\underline{17.8}} \text{ k}\Omega \end{aligned}$$

The common-mode half-circuit

$$\begin{aligned} A_{cm} &= \frac{U_o}{U_{ICM}} \approx \frac{10}{300} \\ &= \frac{1}{30} = \underline{\underline{0.033}} \text{ V/V} \end{aligned}$$

$$\begin{aligned} 2R_{icm} &= 30k \parallel 7.5k\Omega \\ &= 30k\Omega \quad 2R_{icm} \\ R_{icm} &= \underline{\underline{15k\Omega}} \quad (\beta+1)(300) \parallel (\beta+1)r_o \end{aligned}$$

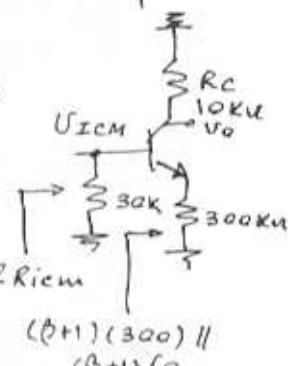
Without the  $R_B$  resistors  $R_{icm} = \underline{\underline{3.75}} \text{ M}\Omega$

$$\approx 30 \parallel 10 = 7.5 \text{ M}\Omega$$

$\underline{\underline{3.75}}$  MΩ

$$\approx 30 \parallel 10 = 7.5 \text{ M}\Omega$$

$\underline{\underline{3.75}}$  MΩ



$$(a) A_d |_{\text{single-ended output}} = \alpha \frac{(R_c || r_o)}{2r_e}$$

$$\text{where } r_e = \frac{0.025V}{0.25mA} = 100\Omega$$

$$r_o = \frac{200V}{0.25mA} = 800\text{K}\Omega$$

$$A_d |_{\text{single ended}} \approx \frac{20}{2 \times 0.1} = \underline{\underline{100}} \text{ V/V}$$

$$(b) A_d |_{\text{diff output}} = 2 \times A_d |_{\text{single ended}} = \underline{\underline{200}} \text{ V/V}$$

$$(c) R_{id} = 2\pi = 2 \times 20 \times 100 = \underline{\underline{40.2}} \text{ K}\Omega$$

CONT.

$$(d) A_{cm} \left|_{\text{single-ended output}} \right. = \frac{R_C}{2R}$$

$$= \frac{20}{2000} = \underline{\underline{0.1 \text{ V/V}}}$$

$$(e) A_{cm} \left|_{\text{diff out}} \right. = 0$$

7.43

The output resistance of the current source ( $R$ ) =  $r_o = \frac{V_A}{0.1 \text{ mA}}$

$$= \frac{200}{0.1} = 2 \text{ M}\Omega$$

( $r_o$  for each transistor is the pair) =  $200 / 0.05 = 4 \text{ M}\Omega$

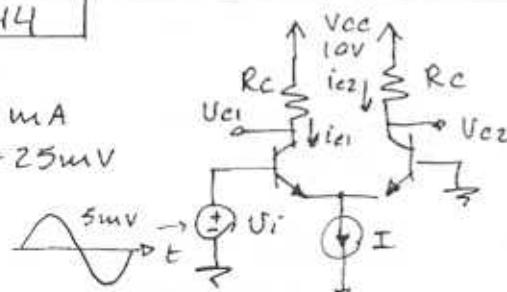
Thus,

$$R_{iem} = \frac{(\beta+1)r_o}{2} \parallel (\beta+1) R$$

$$= \frac{51 \times 4}{2} \parallel 51 \times 2 = \underline{\underline{51 \text{ M}\Omega}}$$

7.44

$$I_{in \text{ mA}} \\ V_T = 25 \text{ mV}$$



$$i_{c1} = \frac{I}{2} + \left( \frac{I}{2} \right) \left( \frac{5}{2} \right) \sin \omega t$$

$$i_{c2} = \frac{I}{2} - \left( \frac{I}{2} \right) \left( \frac{5}{2} \right) \sin \omega t$$

$$U_{c1} = V_{cc} - \frac{I}{2} R_C - \frac{I}{2} R_C \frac{5}{2} \sin \omega t$$

$$U_{c2} = V_{cc} - \frac{I}{2} R_C - \frac{I}{2} R_C \frac{5}{2} \sin \omega t$$

$$U_{c1}, U_{c2} \geq 0$$

$$\Rightarrow 10 - 5I - 0.5I = 0$$

$$I = \underline{\underline{1.8 \text{ mA}}}$$

$$V_{c1} = V_{c2} = 1 \text{ V}$$

$$A_d = \frac{20 \text{ k}\Omega}{2r_e} \text{, where } r_e = \frac{25}{0.9} = 27.8 \text{ }\Omega$$

$$\text{Thus, } A_d = \underline{\underline{360 \text{ V/V}}}$$

$$U_{c2} - U_{c1} = \underline{\underline{1.8 \sin \omega t, v}}$$

7.45

$$\text{At } I_C = 1 \text{ mA, } r_e = 25 \text{ }\Omega$$

$$R_{id} = (\beta+1) 2r_e = 5.05 \text{ k}\Omega < 1 \text{ G}$$

$\Rightarrow$  need emitter resistors  
In this case:

$$R_{id} = (\beta+1) (2r_e + 2R_E) = 10 \text{ k}\Omega$$

$$\text{Thus, } R_E = \underline{\underline{25 \Omega}}$$

$$A_d = 100 = \frac{R_C}{2(R_E + r_e)}$$

$$\Rightarrow R_C = \underline{\underline{10 \text{ k}\Omega}}$$

$$A_{cm} = 0.1 \geq \frac{R_C}{2R_o + R_E + r_e}$$

$$\Rightarrow R_o \geq \underline{\underline{50 \text{ k}\Omega}}$$

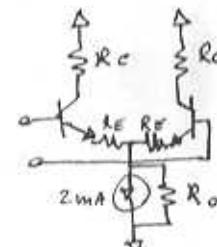
$$\text{For } \pm 2 \text{ swing } V_{c1} = V_{c2} = V_{cc} - \frac{I}{2} R_C = 2$$

$$\Rightarrow V_{cc} = 2 + 10^3 \times 10^4 = 12 \text{ V}$$

Choose  $V_{cc} = \pm 15 \text{ V}$  although 12V is OK.

$$2R_{iem} = (\beta+1)(2R_o + R_E + r_e)$$

$$\Rightarrow R_{iem} = \underline{\underline{5 \text{ M}\Omega}}$$



7.46

Taken single-endedly

$$A_{cmS} = \frac{\alpha R_c}{2R_o}$$

Let collector resistors be  $R_c$  &  $R_c + \Delta R_c$ , then

$$A_{cm} = \frac{\alpha}{2R_o} (R_c + \Delta R_c - R_c)$$

$$= \frac{\alpha \Delta R_c}{2R_o}$$

Which can be written as

$$A_{cmS} = \frac{\alpha R_c}{2R_o} \cdot \frac{\Delta R_c}{R_c} = A_{cmS} \frac{\Delta R_c}{R_c}$$

$$\text{CMRR} = \frac{Ad}{A_{cmS}} = \frac{2 \cdot A_S}{A_{cmS} \frac{\Delta R_c}{R_c}}$$

$$= \frac{A_S}{A_{cmS}} \cdot \frac{2}{\frac{\Delta R_c}{R_c}}$$

$$\text{Thus, } 20 \log \frac{2}{\frac{\Delta R_c}{R_c}} = 40 \text{ dB}$$

$$\rightarrow \Delta R_c / R_c = \underline{\underline{2\%}}$$

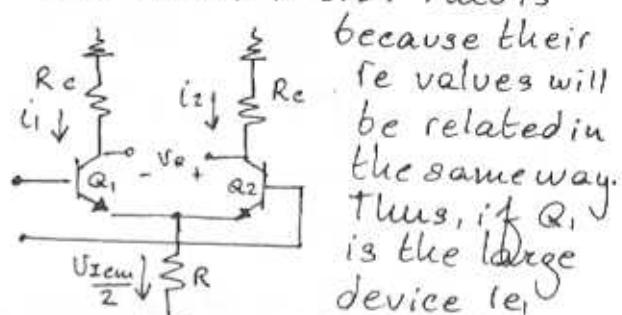
7.47

The bias current will split between the two transistors according to their area ratio. Thus the large-area device will carry twice the current of the other device.

That is, the bias currents will be  $2I/3$  and  $I/3$ .

Now with  $V_{ICM}$  applied, the CM signal current will  $\rightarrow V_{ICM}/R$

split between  $Q_1$  and  $Q_2$  in the same ratio. This is



because their beta values will be related in the same way. Thus, if  $Q_1$  is the large device i.e.,

will be half the value of  $i_{Q2}$ .

The result will be that

$$i_1 = \frac{2}{3} \frac{V_{ICM}}{R} \quad \text{and} \quad i_2 = \frac{1}{3} \frac{V_{ICM}}{R}$$

Thus the differential output voltage  $V_O$  will be

$$V_O = (i_1 R_c - (-i_2 R_c)) = (i_1 - i_2) R_c$$

$$= \frac{1}{3} \frac{V_{ICM}}{R} \cdot R_c$$

$$A_{cm} = \frac{1}{3} \frac{R_c}{R} = \frac{1}{3} \times \frac{12}{1000} = \underline{\underline{0.004 \text{ V/V}}}$$

7.48

For  $I = 200 \mu A$ :

$$g_m = \sqrt{2 K_n W_L I_D} = \sqrt{2 \times 4 \times 0.1} \\ = 0.89 \text{ mA/V}$$

$$R_D = 10K\Omega$$

$$\text{Thus, } Ad = g_m R_D = 10 \times 0.89 = \underline{\underline{8.9 \text{ V/V}}}$$

$$V_{os} = \frac{(V_{GS} - V_t)}{2} \cdot \frac{\Delta R_D}{R_D}$$

where  $\frac{\Delta R_D}{R_D} = 0.02$  (worst case)

$$\text{and } V_{GS} - V_t = \sqrt{\frac{2 I_D}{K_n W_L}} = \sqrt{\frac{2 \times 0.1}{4}} \\ = \underline{\underline{0.223 \text{ V}}}$$

CONT.

$$\text{Thus, } V_{OS} = \frac{1}{2} \times 0.223 \times 0.02 \\ = \underline{\underline{2.23 \mu V}}$$

For  $I = 400 \mu A$ :

$$g_m = \sqrt{2 \times 4 \times 0.2} = 1.265 \mu A/V \\ A_d = \underline{\underline{12.65 \mu V/V}}$$

$$V_{OV} = V_{GS} - V_t = 0.316 V$$

$$V_{OS} = \frac{1}{2} \times 0.316 \times 0.02 = \underline{\underline{3.16 \mu V}}$$

Thus both  $A_d$  and  $V_{OS}$  increase by the same ratio since both are proportional to  $\sqrt{I}$

7.49

Worst cases:  $\Delta V_t = 10 \mu V$

$$\frac{\Delta R_D}{R_D} = 0.04; \frac{\Delta (W/L)}{(W/L)} = 0.04$$

$$V_{OS_1} (\text{due to } \Delta R_D) = \frac{V_{OV}}{2} \frac{\Delta R_D}{R_D} = \frac{0.3}{2} \times 0.04 \\ = 6 \mu V//$$

$$V_{OS_2} (\text{due to } \Delta W/L) = \frac{V_{OV}}{2} \frac{\Delta W/L}{W/L} = \frac{0.3}{2} \times 0.04 \\ = 6 \mu V//$$

$$V_{OS_3} (\text{due to } \Delta V_t) = \Delta V_t = \underline{\underline{10 \mu V}}$$

Since these offsets are not correlated

$$V_{OS} = \sqrt{V_{OS_1}^2 + V_{OS_2}^2 + V_{OS_3}^2}$$

$$V_{OS} = \sqrt{6^2 + 6^2 + 10^2} = \underline{\underline{13.11 \mu V}}$$

The major contribution is due to the threshold mismatch  $\Delta V_t$ .

To find the required mismatch  $\Delta R_D$  that can correct for  $V_{OS}$

$$13.11 \mu V = \frac{V_{OV}}{2} \cdot \frac{\Delta R_D}{R_D}$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = \frac{2 \times 13.11 \mu V}{0.3 V}$$

$$= 0.087 \text{ or } \underline{\underline{8.7\%}}$$

If  $\Delta V_t$  is reduced by a factor of 10 to  $1 \mu V$ ,  $V_{OS}$  reduces to:

$$\sqrt{6^2 + 6^2 + 1^2} = 8.54 \mu V$$

$$\text{and } \frac{\Delta R_D}{R_D} = \frac{2 \times 8.54 \mu V}{0.3 V} = \underline{\underline{5.6\%}}$$

7.50

$$V_{OV} = \sqrt{\frac{I}{K_n W/L}} = \sqrt{\frac{100}{100 \times 20}} = 0.223$$

Using Eqn. (7.112) we obtain  $V_{OS}$  due to  $\Delta R_D / R_D$  as:

$$V_{OS} = \frac{V_{OV}}{2} \frac{\Delta R_D}{R_D} = \frac{0.223}{2} \times 0.04 \\ = 5.57 \mu V$$

From Eqn. (7.117),  $V_{OS}$  due to  $\Delta (W/L) / (W/L)$  is:

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \frac{\Delta W/L}{W/L} = \frac{0.223}{2} \times 0.04 \\ = 5.57 \mu V$$

The offset arising from  $\Delta V_t$  (Eqn. (7.120)):

$$V_{OS} = \Delta V_t = \underline{\underline{5 \mu V}}$$

Worst case offset is:

$$5.57 + 5.57 + 5 = 16.15 \mu V$$

Applying the root-sum-of-squares (Eq. (7.121))

$$V_{OS} = \sqrt{2(5.57 \mu V)^2 + 5 \mu V^2} = \underline{\underline{9.33 \mu V}}$$

7.51

$$\Delta U_C = \Delta R_C \cdot \frac{I}{2}$$

$$A_d = \frac{R_C}{r_e} = \frac{R_C}{V_T / \frac{I}{2}} = \frac{I R_C}{2 V_T}$$

$$\Rightarrow V_{OS} = \frac{\Delta U_C}{A_d} = \frac{\Delta R_C}{R_C} \cdot V_T \\ = 0.1 \times 25 = \underline{\underline{2.5 \mu V}}$$

$$= \frac{I}{2} R_C \left( \frac{1}{\beta_2} - \frac{1}{\beta_1} \right)$$

$$A_d = \frac{R_C}{r_e} = \frac{I R_C}{2 V_T}$$

$$V_{OS} = \frac{\Delta U_C}{A_d} = V_T \left( \frac{1}{\beta_2} - \frac{1}{\beta_1} \right) \quad Q.E.D.$$

For  $\beta_1 = 100$  and  $\beta_2 = 200$

$$V_{OS} = 25 \left( \frac{1}{200} - \frac{1}{100} \right)$$

$$= \underline{\underline{-125 \mu V}}$$

7.52

$$Eq (7.130) V_{OS} = V_T \cdot \frac{\Delta I_S}{I_S}$$

$$= 25 \times 0.1 = \underline{\underline{2.5 \mu V}}$$

7.53

$$\Delta U_C = \Delta R_C \frac{I}{2}$$

$$A_d = \frac{R_C}{r_e + R_E} = \frac{R_C}{2 V_T + R_E} = \frac{I R_C}{2 V_T + I R_E}$$

$$V_{OS} = \frac{\Delta U_C}{A_d} = \frac{\Delta R_C}{R_C} \left( V_T + \frac{I R_E}{2} \right)$$

7.54

$$\Delta U_C = \alpha_1 \frac{I}{2} R_C - \alpha_2 \frac{I}{2} R_C$$

$$= \frac{I}{2} R_C (\alpha_1 - \alpha_2)$$

$$= \frac{I}{2} R_C \left( \frac{\beta_1}{\beta_1 + 1} - \frac{\beta_2}{\beta_2 + 1} \right)$$

For  $\beta_1, \beta_2 \gg 1$

$$\Delta U_C = \frac{I}{2} R_C \cdot \frac{\beta_1 - \beta_2}{\beta_1 \cdot \beta_2}$$

7.55

Since the two transistors are matched except for their  $V_A$  value, we can express the collector currents when the input terminals are grounded as,

$$I_{C1} = I_C \left( 1 + \frac{V_{CE}}{V_{A1}} \right)$$

$$I_{C2} = I_C \left( 1 + \frac{V_{CE}}{V_{A2}} \right)$$

where  $I_C$  can be determined from

$$I_{C1} + I_{C2} = I$$

$$\Rightarrow I_C = \frac{I}{2 + \frac{V_{CE}}{V_{A1}} + \frac{V_{CE}}{V_{A2}}}$$

Note that for  $V_{CE} \ll V_{A1}, V_{A2}$ ,  $I_C \approx \frac{I}{2}$ . Thus, the differential gain  $A_d$  can still be written as

$$A_d \approx \frac{R_C}{r_e} = \frac{I R_C}{2 V_T}$$

The offset voltage at the output can be found from

$$\Delta V_C = V_{C2} - V_{C1} = (I_{C1} - I_{C2}) R_C$$

CONT.

$$= I_C R_C \left( \frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}} \right)$$

$$\approx \frac{I}{2} R_C \left( \frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}} \right)$$

Thus,  $V_{OS} = \frac{\Delta V_C}{Ad}$

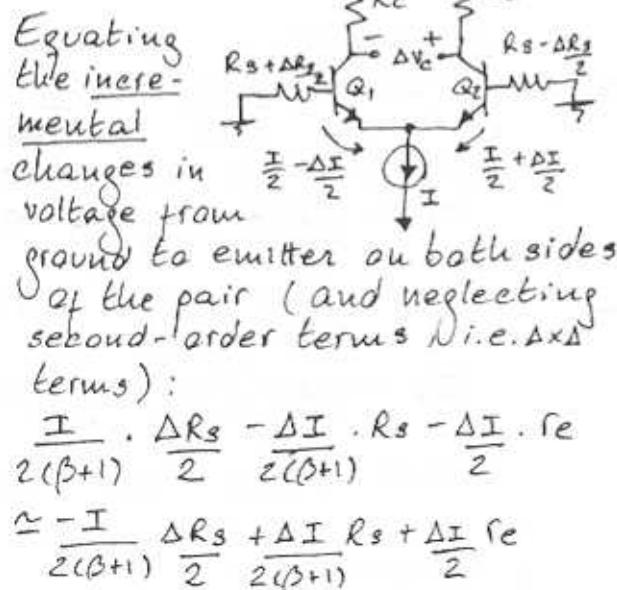
$$V_{OS} = V_T \left( \frac{V_{CE}}{V_{A1}} - \frac{V_{CE}}{V_{A2}} \right)$$

For  $V_{CE} = 10V$ ,  $V_{A1} = 100V$  and  
 $V_{A2} = 300V$

$$V_{OS} = 25 \left( \frac{10}{100} - \frac{10}{300} \right)$$

$$= \underline{\underline{1.7 \text{ mV}}}$$

**7.56**



$$\Delta I \left[ R_e + \frac{R_s}{\beta+1} \right] = \frac{I}{2(\beta+1)} \cdot \Delta R_s$$

$$\Delta I = \frac{I \Delta R_s}{2(\beta+1)}, \quad \frac{1}{R_e + \frac{R_s}{\beta+1}}$$

$$\Delta V_C = -\Delta I \cdot R_C$$

$$= -\frac{I R_C \Delta R_s}{2(\beta+1)} \cdot \frac{1}{R_e + \frac{R_s}{\beta+1}}$$

$$Ad = R_C / r_e$$

Thus,  $V_{OS} = \Delta V_C / Ad$

$$= -\frac{I \Delta R_s}{2(\beta+1)} \cdot \frac{r_e}{r_e + R_s} \frac{1}{\beta+1}$$

For  $\frac{R_s}{\beta+1} \ll r_e$  and  $\beta \gg 1$ ,

$$|V_{OS}| \approx \frac{I}{2\beta} (\Delta R_s) \quad Q.E.D.$$

**7.57**

Refer to Fig. P7.57.

(a)  $R_{C1} = 5 \times 1.05 = 5.25 \text{ k}\Omega$   
 $R_{C2} = 5 \times 0.95 = 4.75 \text{ k}\Omega$

Perfect offset nulling will be achieved when  $x \cdot Y_3$  such that  
 $R_{C1} + (x \cdot 1 \text{ k}\Omega) = R_{C2} + ((1-x) \cdot 1 \text{ k}\Omega)$   
 $\Rightarrow 5.25 + x = 4.75 + 1 - x$   
 $\Rightarrow x = \underline{\underline{0.25}}$

(b)  $I_{C1} = 1.05 \text{ mA}$   
 $I_{C2} = 0.95 \text{ mA}$

Offset nulling is achieved when  $x$  is such that  
 $1.05(x+5) = 0.95((1-x)+5)$   
 $x = \underline{\underline{0.225}}$

**7.58**

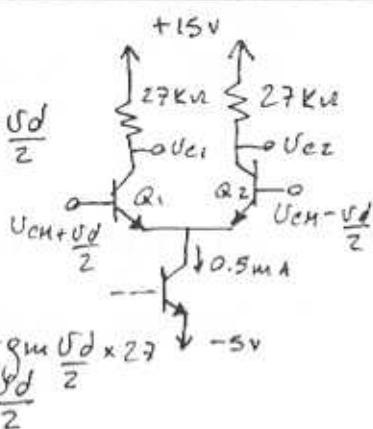
$$I_{B\max} = \frac{I/2}{\beta_{\min}+1} = \frac{300}{80+1} = \underline{\underline{3.7 \text{ mA}}}$$

$$I_{B\min} = \frac{I/2}{\beta_{\max}+1} = \frac{300}{200+1} = \underline{\underline{1.5 \text{ mA}}}$$

$$I_{OS} = I_{B\max} - I_{B\min} = \underline{\underline{2.2 \text{ mA}}}$$

7.59

$$U_{C1} \geq U_{CM} + \frac{U_d}{2}$$



$$15 - 0.25 \times 27 - g_m \frac{U_d}{2} \times 27 \geq U_{CM} + \frac{U_d}{2}$$

$$\begin{aligned} \text{Thus } U_{CM} &= 15 - 0.25 \times 27 + \\ &- 100 \times 0.01 \times 27 \\ &= -0.01 \\ &= +5.54V \end{aligned}$$

To find  $U_{CM\min}$  we observe that to keep the current-source transistor in the active region, the voltage at its collector should exceed the collector-emitter saturation voltage ( $0.2 - 0.3V$ ). Thus,  $U_{EMin} = -4.7V$  and correspondingly  $U_{CM\min} = -4V$

7.60

$$I_{E1} = \frac{2}{3} I \text{ and } I_{E2} = \frac{1}{3} I$$

(Q<sub>1</sub> twice the area of Q<sub>2</sub>)

$$\Delta U_C = U_{C2} - U_{C1} \approx \frac{1}{3} I R_C$$

Nominally,

$$A_d = \frac{V_{Rc}}{T_e} = \frac{I R_c}{2 V_T}$$

$$V_{os} = \frac{\Delta U_C}{A_d} = \frac{2}{3} V_T = 16.7mV$$

Thus, small-signal analysis predicts that a 16.7mV DC

voltage applied as  $U_{B2} - U_{B1} = 16.7V$  would restore the current balance in the pair and reduce  $\Delta U_C$  to zero.

Using large-signal analysis:

$$i_{E1} = I_{S1} \cdot e^{\frac{U_{B1} - U_E}{V_T}}$$

$$i_{E2} = I_{S2} \cdot e^{\frac{U_{B2} - U_E}{V_T}}$$

Thus,

$$\frac{i_{E1}}{i_{E2}} = \frac{I_{S1}}{I_{S2}} \cdot e^{\frac{U_{B1} - U_{B2}}{V_T}}$$

To restore balance,  $i_{E1} = i_{E2}$ , thus

$$1 = 2 e^{\frac{U_{B1} - U_{B2}}{V_T}}$$

$$\Rightarrow U_{B1} - U_{B2} = -V_T \ln 2$$

$$V_{B2} - V_{B1} = 17.3mV$$

Nominally

$$I_B = \frac{I/2}{\beta + 1} \approx \frac{100}{2 \times 100} = 0.5mA$$

But with the imbalance,

$$I_{B1} \approx \frac{2I/3}{\beta} = \frac{2 \times 100}{300} = 0.67mA$$

$$I_{B2} = \frac{I/3}{\beta} = \frac{100}{300} = 0.33mA$$

$$I_B = \frac{I_{B1} + I_{B2}}{2} = 0.5mA$$

$$I_{os} = |I_{B1} - I_{B2}| = 0.34mA$$

7.61

$$R_C = 20k\Omega ; A_d = 90V/V$$

$$V_{os} = \pm 3mV$$

Worst case  $|V_{os}|$  is 3mV

$$|V_{os}| = V_T \left( \frac{\Delta R_C}{R_C} \right) \quad |V_{os}| = 3mV$$

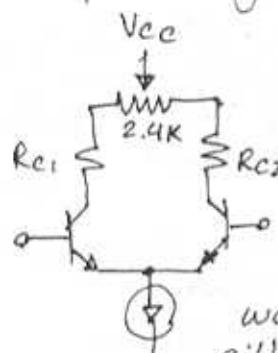
$$\Rightarrow \frac{3m \times 20k}{25m} = 24k, \Delta R_C = 2.4k\Omega$$

CONT.

This is the maximum mismatch that occurs in  $R_c$ .

Thus, if the lowest collector resistor is adjusted from  $R_{Cmin} + \Delta R$  with  $\Delta R$  varying between zero and  $2.4k\Omega$ , then the offset would be eliminated!

This can be achieved with the following circuit:



The diagram shows a circuit with a voltage source  $V_{cc}$  at the top. A potentiometer is connected between  $V_{cc}$  and ground. The wiper of the potentiometer is connected to a node. From this node, a 2.4K resistor goes to ground, and another resistor, labeled  $R_{c2}$ , goes to the bottom terminal of the potentiometer. The top terminal of the potentiometer is connected to  $V_{cc}$ . The bottom terminal of the potentiometer is connected to ground.

$$\rightarrow V_{AV} = 20/80 = 0.25V$$

Finally,

$$= 3.2 \frac{m}{V^2} (0.25V)^2 = 0.2$$

7.63

For all transistors  $I_D = I/2$   
and all  $r_0$ 's are equal.

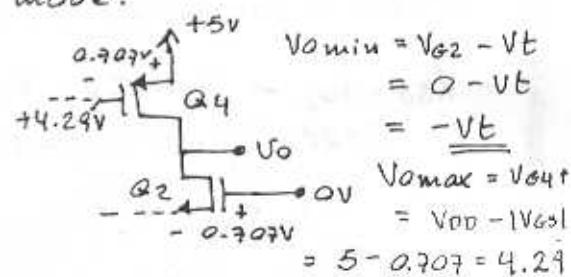
$$V_{OU} = \sqrt{\frac{2ID}{K'wL}} = \sqrt{\frac{100mA}{0.2mA/\sqrt{2}}} = 0.7$$

$$V_{NO_2} = \sqrt{\frac{400RA}{0.2MAV_2}} = 1.414V$$

(a) For  $I = 100mA$ :

Range of the differential mo  
is  $-\sqrt{2} V_0 \leq U_{id} \leq \sqrt{2} V_0$   
(as in Eqn. (7.10))

But the range of  $V_o$  is limited by the requirement of keeping the transistors in saturation mode.



$$g_m = \frac{2ID}{V_{DS}} \rightarrow g_{m1} = g_{m2} = \frac{100M}{0.707} = 0.1414mA/V$$

$$f_0 = \frac{V_A}{I_0} \Rightarrow f_{02} = f_{04} = \frac{20}{50\pi} = \underline{\underline{400 \text{ KI}}}$$

$$R_0 = f_{02} / f_{04} = \frac{1}{2} \times 400K = \underline{\underline{200K}}$$

$$Ad = \frac{1}{2} g_{mf0} = \frac{1}{2} \times (0.1414m)(400k)$$

$$\begin{aligned} \text{but } g_m &= \frac{2I_D}{V_{DD}} \quad \text{and } f_0 = \frac{VA}{I_D} \\ \Rightarrow Ad &= \frac{1}{2} \left( \frac{2I_D}{V_{DD}} \right) \frac{VA}{I_D} = \frac{VA}{V_{DD}} \\ \rightarrow 8QV/V &= 20V/V_{DD} \end{aligned}$$

$$A_d = \underline{28.28} \text{ V/V}$$

(b) For  $I = 400 \mu\text{A}$ :  
Linear range of  $V_{o3}$

$$V_{omin} = -V_t //$$

$$V_{omax} = (5V - V_{os}) + |Vt| \\ = \underline{\underline{5 - 1 = 4V}}$$

$$g_m = \frac{2 \times 200R}{1.414} = 0.28 \mu\text{A/V}$$

$$f_0 = 20/(400/2) = \underline{100 \text{ KHz}}$$

$$R_o = 1/2 \cdot 100 \text{ KHz} = \underline{50 \text{ KHz}}$$

$$A_d = \frac{1}{2} (0.28 \mu\text{A} \times 100 \text{ KHz}) = \underline{14 \text{ V/V}}$$

7.64

Recall from Eqn. (7.155)

$$\text{CMRR} = (g_m r_o)(g_m R_{SS})$$

(a) For a simple current mirror

$$R_{SS} = r_{os} \Rightarrow (\text{for } I_D = I/2)$$

$$\text{CMRR} = (g_m r_o)(g_m r_{os})$$

$$= \left( \frac{2I_D}{V_{ou}} \cdot \frac{V_A}{I_D} \right) \cdot \left( \frac{2I_D}{V_{ou}} \cdot \frac{V_A}{2I_D} \right) \\ = 2 \cdot \frac{V_A}{V_{ou}} \cdot \frac{V_A}{V_{ou}} \\ = 2 \left( \frac{V_A}{V_{ou}} \right)^2 //$$

Q.E.D.

(b) for the modified Wilson current source of Fig. P7.64

$$R_{SS} \approx g_m r_o r_{os}$$

$$\Rightarrow \text{CMRR} = (g_m r_o)(g_m \cdot g_m r_{os} r_{os})$$

For  $Q_5, 6, 7, 8$ :

$$V_{ou3} = \sqrt{\frac{2I}{K'W/L}}$$

while for  $Q_1, 2, 3, 4$ :

$$V_{ou} = \sqrt{\frac{I}{K'W/L}}$$

$$\Rightarrow V_{ou3} = \sqrt{2} V_{ou}$$

Thus, (for  $I = 2I_D$ )

$$\begin{aligned} \text{CMRR} &= \frac{I}{V_{ou}} \cdot \frac{V_A}{(I/2)} \cdot \frac{I}{V_{ou}} \cdot \frac{2I}{\sqrt{2} V_{ou}} \cdot \frac{V_A}{I} \cdot \frac{V_A}{I} \\ &= \frac{4}{\sqrt{2}} \frac{V_A^3}{V_{ou}^3} = \underline{\underline{2 \cdot \sqrt{2} \frac{V_A^3}{V_{ou}^3}}} \end{aligned}$$

For  $K'W/L = 10 \mu\text{A/V}^2$

$$I = 1 \mu\text{A}$$

$$|V_A| = 10V$$

$$V_{ou} = \sqrt{\frac{1 \mu\text{A}}{10 \mu\text{A/V}^2}} = 0.316 \text{ V}$$

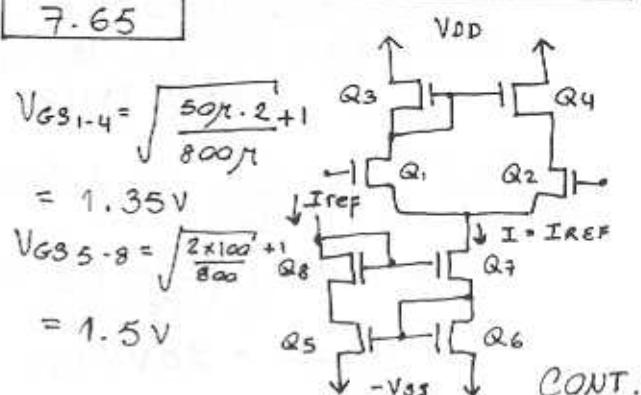
$\Rightarrow$  For the simple current mirror case:

$$\text{CMRR} = 2 \left( \frac{10}{0.316} \right)^2 = 2000 \rightarrow \underline{66 \text{ dB}}$$

For the Wilson source:

$$\text{CMRR} = 2\sqrt{2} \left( \frac{10}{(0.316)^3} \right)^3 = 89442 \rightarrow \underline{99 \text{ dB}}$$

7.65



For  $V_{DS} = V_{GS}$

$$-V_{SS} + 2V_{GS_{5-8}} + 2V_{GS_{1-4}} = V_{DD}$$

Thus,

$$V_{DD} + V_{SS} = 2(1.5) + 2(1.35) = \underline{\underline{5.7V}}$$

$$A_d = G_m R_o = 2 \times 1600 = \underline{\underline{3200 \text{ V/V}}}$$

With a subsequent stage having a  $100\text{ k}\Omega$  input resistance,

$$A_d = G_m (R_o / 100\text{ k}\Omega) = \underline{\underline{188.2 \text{ V/V}}}$$

7.66

(b)

$$R_{o4} = (g_{m4} r_{o4}) r_{o2} = g_{m4} r_o^2$$

$$R_{o6} = (g_{m6} r_{o6}) r_{o8} = g_{m6} r_o^2$$

$$A_d = g_{m4} (R_{o4} \parallel R_{o6}) = g_{m4} \cdot \frac{1}{2} g_{m6} r_o^2$$

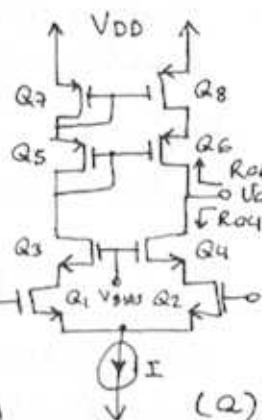
$$g_{m4} = \frac{2I_D}{V_{ov}} \quad r_o = \frac{V_A}{I_D}$$

$$\text{thus, } g_{m4} r_o = 2V_A / V_{ov}$$

$$\Rightarrow A_d = 2(V_A / V_{ov})^2 \quad \text{Q.E.D.}$$

For  $V_{ov} = 0.25V \times V_A = 20V$

$$A_d = 2(20/0.25)^2 = \underline{\underline{12800 \text{ V/V}}}$$



7.68

$$G_m = I_c / V_T = 5 \text{ mA} / V$$

$$I_c = \underline{\underline{250 \mu\text{A}}}$$

$$R = \frac{5 - (-5) - V_{BE}}{I_c}$$

$$= \frac{9.3}{0.25} = \underline{\underline{37.2 \text{ k}\Omega}}$$

$$R_{id} = (\beta + 1) 2r_e \text{ where,}$$

$$r_e = \frac{V_T}{I_c} = \frac{25 \text{ mV}}{0.125 \text{ mA}} = 200 \text{ m}\Omega$$

$$\Rightarrow R_{id} = 151 \times 2 \times 0.2 = \underline{\underline{60.4 \text{ k}\Omega}}$$

$$r_o = \frac{V_A}{I_c} = \frac{100}{0.125} = 800 \text{ k}\Omega$$

$$R_o = \frac{r_o}{2} = \underline{\underline{400 \text{ k}\Omega}}$$

$$A_d = g_m R_o = 5 \times 400 = \underline{\underline{2000 \text{ V/V}}}$$

$$I_B = \frac{I_c / 2}{\beta + 1} = \frac{125}{151} = \underline{\underline{0.83 \text{ mA}}}$$

$$V_{IC1/\max} = V_{C1} + 0.4V$$

$$= 5 - 0.7 + 0.4 = \underline{\underline{4.7V}}$$

$$V_{IC1/\min} = V_{BS} - 0.4 + 0.7$$

$$= -5 - 0.4 + 0.7$$

$$= -4V$$

Thus, the input common-mode range is  $-4V$  to  $+4.7V$

(where we have assumed that a transistor remains active)

CONT.

7.67

$$R_{id} = (\beta + 1) 2r_e ; r_e = \frac{25 \text{ mV}}{50 \mu\text{A}} = 500 \text{ m}\Omega$$

$$\Rightarrow R_{id} = 101 \times 1000 = \underline{\underline{101 \text{ k}\Omega}}$$

$$R_o = r_{o4} \parallel r_{o2} = \frac{r_o}{2} ; r_o = \frac{V_A}{I_c}$$

$$\Rightarrow r_o = \frac{160V}{50 \mu\text{A}} = 3.2 \text{ M}\Omega$$

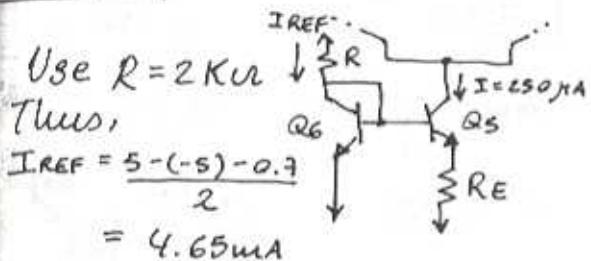
$$\text{Thus, } R_o = \underline{\underline{1.6 \text{ M}\Omega}}$$

$$G_m = g_{m1} = g_{m2} = \frac{50 \mu\text{A}}{25 \text{ mV}} = \underline{\underline{2 \text{ mA/V}}}$$

even through its base-collector junction is forward biased by 0.4V)

$$R_{i\text{cm}} \approx (\beta+1) [r_{05} \parallel r_{01} \parallel r_{02}] \\ = 151 \times (400 \parallel 800 \parallel 800) = \underline{\underline{30\text{k}\Omega}}$$

7.69



$$V_{\text{BEG}} - V_{\text{BE5}} = V_T / k (4.65 / 0.25) = 73.1\text{mV}$$

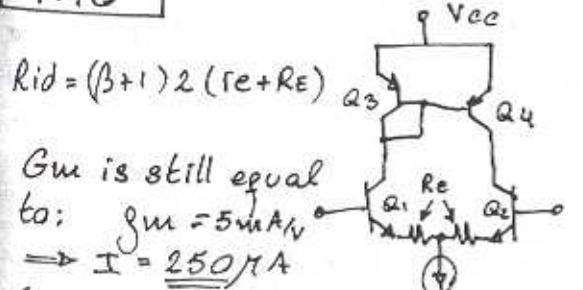
$$R_E = 73.1\text{mV} / 0.25\text{mA} = \underline{\underline{292\text{k}\Omega}}$$

The only amplifier parameter that changes is  $R_{i\text{cm}}$ . This is because now  $r_{05}$  is:

$$r_{05} \approx (1 + g_m R_E) r_{05} \\ = (1 + 10 \times 0.292) \times 400 = \underline{\underline{1.57\text{M}\Omega}}$$

$$\text{Thus, } R_{i\text{cm}} = (\beta+1) [r_{05} \parallel r_{01} \parallel r_{02}] \\ = 151 [1.57 \parallel 0.80 \parallel 0.8] = \underline{\underline{48\text{k}\Omega}}$$

7.70



$$\text{and } r_e = \frac{V_T}{I/2} = 200\text{k}\Omega, r_0 = 800\text{k}\Omega$$

If  $R_{id} = 100\text{k}\Omega \Rightarrow$

$$100\text{k} = 151 \times 2 \times (200 + R_E) \\ \rightarrow R_E = 131\text{k}\Omega$$

To obtain  $A_d$ :

$$A_d = G_m \cdot R_o \quad (\text{Eqn. 7.165})$$

As in the derivation of  $R_{o2}$  in Eqn. (7.162),  $R_{o2}$  can be found using Eqn. (6.159), but this time noting that  $r_e$  at the emitter of  $Q_2$  is:

$$r_{e1} + 2R_E$$

Thus,

$$R_{o2} = r_{02} [1 + g_m ((r_{e1} + 2R_E) \parallel r_{112})]$$

$$R_{o2} = 800\text{k} [1 + 5m ((200 + 2 \times 131) \parallel 30.2\text{k})]$$

$$R_{o2} = 2620\text{k}\Omega \quad (\beta+1)r_e$$

$$R_o = R_{o2} \parallel r_{04} \\ = (2620 \parallel 800)\text{k} = 613\text{k}\Omega \\ \Rightarrow A_d = 5m \times 613\text{k} = \underline{\underline{3065\text{V/V}}}$$

7.71

$$G_m = g_m = \frac{I/2}{V_T} = \frac{0.5\text{mA}}{25\text{mV}}$$

$$g_m = \underline{\underline{10\text{mA/V}}}$$

$$R_o = r_{02} \parallel r_{04} = \frac{V_A}{I_{C2}} \parallel \frac{V_A}{I_{C4}} = \frac{1}{2} \frac{V_A}{I/2} \\ = \frac{120}{0.5\text{m}} = \underline{\underline{240\text{k}\Omega}}$$

$$A_d = G_m R_o = 10 \times 240 = \underline{\underline{2400\text{V/V}}}$$

$$R_{id} = 2r_{11} \approx 2 \frac{V_T}{I/2} \beta = \frac{25\text{mV} \times 150}{0.5\text{m}}$$

$$R_{id} = \underline{\underline{7.5\text{k}\Omega}}$$

For a simple current mirror  
CONT.

the output resistance (thus  $R_{EE}$ ) is  $r_o$

$$\Rightarrow R_{EE} = \frac{V_A}{I} = \frac{120}{0.5\text{mA}} = \underline{\underline{240\text{K}\Omega}}$$

$$A_{cm} = \frac{-f_{04}}{\beta_3 R_{EE}} = \frac{-(2 \times 240\text{K})}{150 \times 240\text{K}} = -13.3 \text{mV/V}$$

$$\text{and, } CMRR = \frac{2400}{-13.3\text{mV}} = \underline{\underline{180,451}}$$

i.e.  $\underline{\underline{105 \text{dB}}}$

$$\frac{U_i}{V_s} = \frac{R_{id}}{R_{id} + R_s} = \frac{7.5\text{K}}{7.5\text{K} + 10\text{K}} = 0.43 \frac{\text{V}}{\text{V}}$$

$\Rightarrow$  Overall gain A:

$$A = \frac{U_o}{V_s} \frac{U_o}{U_i} = 0.43 \times 2400 \\ = \underline{\underline{1032 \text{V/V}}}$$

7.72

$$i_o = \frac{U_{icm} - U_{icm} \cdot \frac{1}{2R_{EE}} \frac{1+z}{1+z + \beta_p}}{2R_{EE}} \frac{1}{1+z + \beta_p} \\ = \frac{U_{icm} + 2/\beta_p}{2R_{EE}} \frac{1}{1+2/\beta_p} \\ \approx \frac{U_{icm}}{\beta_p R_{EE}} (\beta_p \gg 2) \approx \frac{U_{icm}}{R_{EE}}$$

Q.E.D.

$$\text{Thus, } G_{mcm} = \frac{i_o}{U_{icm}} = \frac{1}{\beta_p R_{EE}}$$

Q.E.D.

$$CMRR = \frac{G_{md}}{G_{mcm}} = \frac{g_m}{1/\beta_p R_{EE}} \\ = g_m \beta_p R_{EE} = \frac{\beta_p \cdot I_{REE}}{2VT}$$

For  $I = 0.2\text{mA}$ ,  $R_{EE} = 1\text{M}\Omega$  and  $\beta_p = 25$ :

$$CMRR = \frac{\beta_p \cdot I_{REE}}{2VT} = \frac{25 \times 0.2 \times 1000}{2 \times 0.025} \\ = \underline{\underline{10^5}} \text{ i.e. } \underline{\underline{100 \text{dB}}}$$

7.73

$$i_o = \frac{U_{icm} - U_{icm} \cdot \frac{1}{2R_{EE}} \frac{1+z}{1+z + \beta_p}}{2R_{EE}} \frac{1}{1+z + \beta_p} \\ \approx \frac{U_{icm}}{2R_{EE}} \left( 1 - \left( \frac{1+z}{\beta_p} \right) \right) \\ = \frac{U_{icm}}{\beta_p^2 R_{EE}}$$

$$G_{mch} = \frac{i_o}{U_{icm}} \\ = \frac{1}{\beta_p^2 \cdot R}$$

$$CMRR = \frac{G_{md}}{G_{mch}} = \frac{\beta_p^2 g_m R_{EE}}{G_{mch}} = \frac{\beta_p^2 I_{REE}}{2VT}$$

Thus,  
 $CMRR = (25)^2 \times 0.2 \times 1000 = 25 \times 10^5$   
or  $\underline{\underline{128 \text{dB}}}$

7.74

$$A_{vo} = G_m R_o$$

$G_m = g_m$ .

$R_o = R_{oc} \parallel R_{ow}$   
where  $R_{oc}$  is  
the output  
resistance  
of the cascode  
stage K  
and the output  
resistance of the  
wilson stage

CONT.

$$R_{oc} = \beta r_o \quad \& \quad R_{ow} = \frac{\beta r_o}{2}$$

$$\Rightarrow R_o = \beta r_o \parallel \frac{\beta r_o}{2} = \frac{\beta r_o}{2} \cdot \frac{\beta r_o}{\beta r_o + \frac{\beta r_o}{2}} = \frac{\beta r_o (1 + \frac{1}{2})}{3}$$

$$= \frac{\beta r_o}{2 \times \frac{3}{2}} = \frac{\beta r_o}{3}$$

$$\Rightarrow A_{vo} = G_m R_o = \frac{g_m \beta r_o}{3} \quad Q.E.D.$$

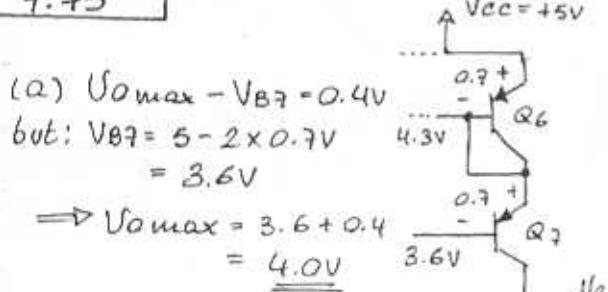
For:  $I = 0.4 \text{ mA}$ ,  $\beta = 100$ ,  $V_A = 120 \text{ V}$

$$A_{vo} = \frac{I/2}{V_T} \cdot \frac{\beta}{3} \cdot \frac{V_A}{I/2} = \frac{\beta}{3} \frac{V_A}{V_T}$$

$$= \frac{100}{3} \times \frac{120 \text{ V}}{25 \text{ mV}} = \underline{\underline{160000}}$$

i.e. 104 dB

7.75



(b) For a 1.5V max. positive swing the DC bias at the output should be:

$$V_{o \text{ DC}} = 4 - 1.5 = \underline{\underline{2.5 \text{ V}}}$$

(c) In the edge of saturating  $Q_4$

$$V_{BIAS} - V_{min} = 0.4 \text{ V}$$

$$\rightarrow V_{min} = V_{BIAS} - 0.4 \text{ V}$$

If  $V_{min}$  is:  $V_{o \text{ DC}} - 1.5$

$$2.5 - 1.5 = 1 \text{ V}$$

$$\Rightarrow V_{BIAS} = \underline{\underline{1.4 \text{ V}}}$$

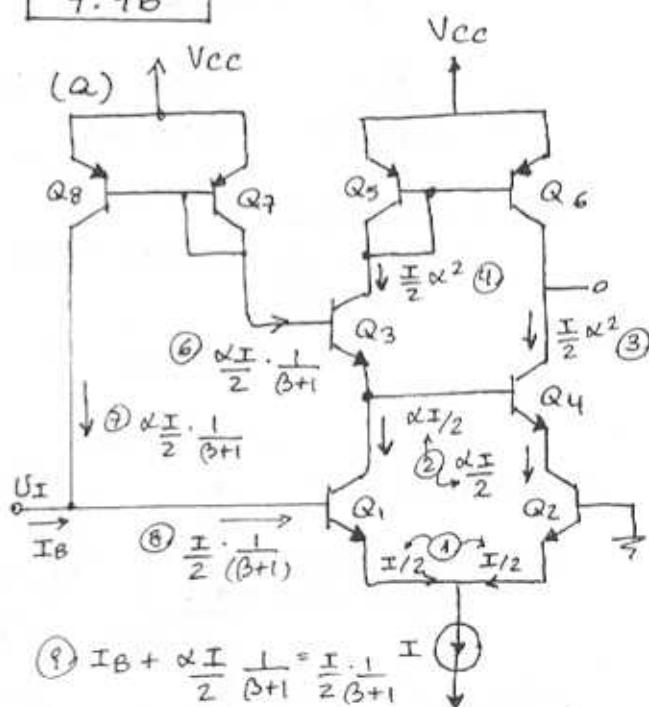
(d) If  $V_{BIAS} = 1.4 \text{ V}$   
 $\Rightarrow V_{C2} = 1.4 - 0.7 = 0.7 \text{ V}$

The upper limit of  $V_{iem}$  is such that brings  $Q_2$  to the edge of saturating.

$$V_{iem \min} - V_{C2} = 0.4 \text{ V}$$

$$\Rightarrow V_{iem \min} = 0.4 + 0.7 = \underline{\underline{1.1 \text{ V}}}$$

7.76



Without  $Q_8 \& Q_7$

$$I_B = \frac{I}{2} \frac{1}{(\beta+1)}$$

thus, the input dc current is reduced by  $\times 1/(\beta+1)$

(b) Without  $Q_8 \& Q_7$ :

$$R_{in} = 2 \pi f_{1,2} ; \quad f_{1,2} = \frac{V_T}{I_B} = \frac{V_T}{\frac{I}{2} (\beta+1)}$$

$$\Rightarrow R_{in} = \frac{4 V_T (\beta+1)}{I}$$

CONT.

With  $Q_8 \times Q_7$ :  $R_{in} = 2(\pi \parallel r_{08})$   
where  $r_{08} = \frac{V_A}{I} \cdot 2(\beta + 1)$

$$\Rightarrow R_{in} = \frac{4V_T(\beta + 1)}{I} \parallel \frac{V_A}{I} \cdot 2(\beta + 1)$$

Solving, yields:

$$R_{in} = \frac{4V_T(\beta + 1) \cdot V_A}{I \cdot 2\alpha V_T + V_A}$$

$$\text{but } \frac{V_A}{2\alpha V_T + V_A} \approx 1 \Rightarrow R_{in} = \frac{4V_T(\beta + 1)}{I}$$

Just like when  $Q_8 \times Q_7$  are not present.

7.77

For the Wilson current source

$$\frac{I_4}{I_3} = \frac{1}{1 + 2/\beta_p^2}$$

Since  $I_3 = \alpha I/2$

$$\Rightarrow I_4 = \frac{\alpha I/2}{1 + 2/\beta_p^2}$$

$$\Delta i = \frac{\alpha I}{2} - \frac{\alpha I/2}{1 + 2/\beta_p^2}$$

$$= \frac{\alpha I}{2} \cdot \frac{2/\beta_p^2}{1 + 2/\beta_p^2} = \alpha I \cdot \frac{1}{\beta_p^2 + 2}$$

$$\approx \frac{\alpha I}{\beta_p^2}$$

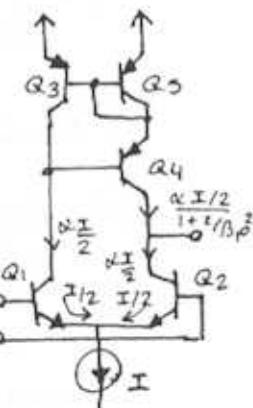
thus.  $V_{os} = \frac{-\Delta i}{G_m}$  with  $G_m = g_m$

$$= \frac{\alpha I}{\beta_p^2}$$

$$\Rightarrow V_{os} = -\frac{\alpha I / \beta_p^2}{\alpha I / 2 V_T} = -\frac{2 V_T}{\beta_p^2}$$

For  $\beta = 50$ :

$$V_{os} = -\frac{2 \times 25 \mu V}{(50)^2} = -\underline{20 \mu V}$$



$V_{bias}$  must be as low as possible  
To keep the top current sources out of saturation:

$$V_{cc} - 0.2 - 0.7 = V_{bias \max}$$

$$V_{bias \max} = 4.1 V$$

And:  $V_o - V_{bias \min} = +0.4 V$

Since  $V_o \sim 0 \Rightarrow V_{bias \min} = -0.4 V$

$\Rightarrow$  Range of  $V_{bias}$  is:

$$(-0.4 \leq V_{bias} \leq 4.1) V$$

For:  $I = 0.4 \mu A$ ,  $\beta_p = 50$ ,  $\beta_N = 150$   
 $\times V_A = 120$

$$G_m = g_m = \frac{0.2 \mu A}{25 \mu V} = \frac{8 \mu A}{V}$$

For the folded cascode:  $R_{o4} = \beta_4 r_{04}$

For the Wilson mirror:  $R_{o5} = \beta_5 \frac{r_{05}}{2}$

$$\Rightarrow R_o = [\beta_4 \cdot r_{04} \parallel \beta_5 \cdot \frac{r_{05}}{2}]$$

$$(r_{04} = r_{05} = 120 / 0.2 \mu A)^2 = 600 \text{ k}\Omega$$

$$\Rightarrow R_o = [50 \times 600 \text{ k}\Omega \parallel 150 \times \frac{600 \text{ k}\Omega}{2}]$$

$$= [30 \text{ M} \parallel 45 \text{ M}]$$

$$= 18 \text{ M}\Omega$$

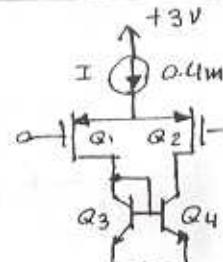
$$A_d = G_m R_o = \frac{8 \mu A}{V} \times 18 \text{ M}\Omega = \underline{144 \text{ d}}$$

7.78

$$K_p' W/L = 6.4 \text{ mA/V}^2$$

$$|V_{Apl}| = 10 \text{ V}$$

$$V_{ANPN} = 120 \text{ V}$$



$$R_o = r_{02} \parallel r_{04} = \frac{V_{Apl}}{I/2} \parallel \frac{120}{I/2} = -3 \text{ V}$$

$$R_o = (10 / 0.2 \mu A) \parallel (120 / 0.2 \mu A) = \underline{4 \text{ k}\Omega}$$

$$G_m = g_m = \sqrt{I \times K_p' W/L}$$

$$= \sqrt{0.4 \mu A \times 6.4 \text{ mA/V}^2}$$

$$\Rightarrow G_m = \frac{1.6 \mu A}{V}$$

$$A_d = G_m \times R_o = \frac{1.6 \mu A}{V} \times 4 \text{ k}\Omega$$

$$\rightarrow A_d = \underline{73.6 \text{ V/V}}$$

7.78

To obtain maximum positive swing

7.80

$$Ku'(w/L) = 128 \frac{VA}{V^2} \times 25 = 3.2 \text{ mA/V}$$

$$(a) V_{ov} = \sqrt{\frac{I}{Ku' w/L}} = \sqrt{\frac{0.2}{3.2}} = \underline{0.25V}$$

$$g_m = \frac{I}{V_{ov}} = \frac{0.2 \text{ mA}}{0.25V} = \underline{0.8 \text{ mA/V}}$$

$$(b) Ad = g_m (R_O || R_o)$$

$$\text{where } R_o = \frac{V_A}{I/2} = \frac{20}{0.2/2} = 200 \text{ k}\Omega$$

$$\Rightarrow Ad = 0.8 \text{ mA} \times (20 \text{ k}\Omega || 20 \text{ k}\Omega) \\ = \underline{14.54 \text{ V/V}}$$

(c) For a CS amplifier when  $R_{sig}$  is low:

$$f_H = \frac{1}{2\pi (C_L + C_{gd}) R_L'}$$

$$\text{where } R_L' = R_O || R_o \\ = 20 \text{ k}\Omega || 200 \text{ k}\Omega \\ = 18.18 \text{ k}\Omega$$

$$\text{and } C_L' = C_L + C_{db}$$

Since for a grounded source terminal  $C_{db}$  is in parallel with the load.

$$\rightarrow C_L' = 90 + 5 = 95 \text{ pF}$$

thus,

$$f_H = \frac{1}{2\pi (95+5) 10^{15} \times 18.18 \text{ k}\Omega} \\ = \underline{87.54 \text{ MHz}}$$

(d) Using the open-circuit time-constants method for  $R_S = 20 \text{ k}\Omega$

$$f_H = \frac{1}{2\pi \gamma_H}$$

$$\text{where } \gamma_H = C_{gs} \cdot R_S$$

$$+ C_{gd} [R_S (1 + g_m R_L') + R_L'] \\ + C_L \cdot R_L'$$

thus,

$$\gamma_H = 30_f \times 20 \text{ k}\Omega \\ + 5_f [20 \text{ k}\Omega (1 + 0.8 \times 18.18) + 18.18 \text{ k}\Omega] \\ + (90_f + 5_f) \times 18.18 \text{ k}\Omega$$

$$\gamma_H = 0.6 \text{ nS} + 1.64 \text{ nS}$$

$$+ 1.72 \text{ nS} = 3.96 \text{ nS}$$

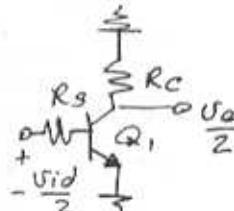
$$\Rightarrow f_H = \frac{1}{2\pi 3.96 \text{ nS}} = \underline{40.2 \text{ MHz}}$$

7.81

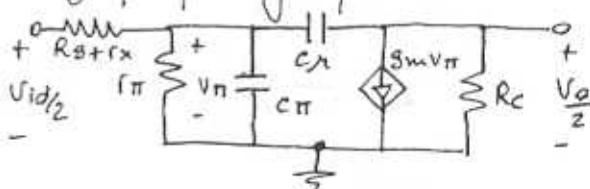
$$f_Z = \frac{1}{2\pi C_{ss} R_{ss}} = \frac{1}{2\pi (0.2p)(100 \text{ k}\Omega)} = \underline{7.95 \text{ MHz}}$$

7.82

(a) Differential half-circuit



High-frequency equivalent circuit



$$(b) I_E = 0.5 \text{ mA} \rightarrow g_m = \frac{0.5}{25}$$

$$g_m = 20 \text{ mA/V} \\ C_{\pi} = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$$C_{\pi} + C_d\pi = \frac{20 \times 10^{-3}}{2\pi \times 600 \times 10^6} = 5.3 \text{ pF}$$

$$C_r = 0.5 \text{ pF} \Rightarrow C_{\pi} = 4.8 \text{ pF}$$

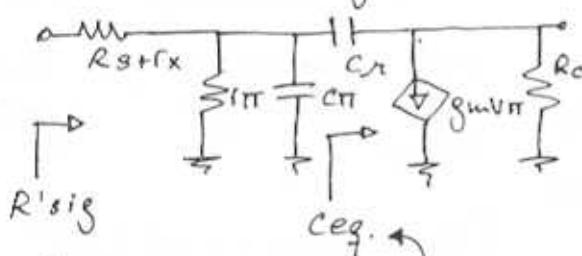
Now,  $R_s = 10 \text{ k}\Omega$ ,  $R_c = 10 \text{ k}\Omega$ ,  $r_x = 100$   
For this common-emitter amplifier (Refer to Section..  
CONT.

5.9 Eqn. (5.164))

$$\begin{aligned} \frac{V_o}{V_i} &= \frac{-f\pi}{f\pi + (R_s + r_x)} \cdot g_m R_c \\ &= \frac{-5K}{5K + (10K + 100)} \cdot 20m \times 10K \\ &= -\underline{\underline{66.22}} \text{ V/V} \end{aligned}$$

(c) Refer to Section 5.9,  
Eqns (5.176), (5.173), (5.168)  
From Eqn. (5.176):

$$f_H = \frac{1}{2\pi C_{in} R'sig}$$



where:  $C_{in} = C_{\pi} + C_{eq}$

Using Miller's theorem

$$C_{eq} = C_R (1 + g_m R_c)$$

and

$$R'sig = (R_s + r_x) \parallel f\pi$$

Thus,

$$\begin{aligned} f_H &= \frac{1}{2\pi [(R_s + r_x) \parallel f\pi] [C_{\pi} + C_R (1 + g_m R_c)]} \\ &= \frac{1}{2\pi ((10K + 100) \parallel 5K) (4.8p + 0.5p(1 + 20 \times 10))} \\ &= \underline{\underline{452 \text{ KHz}}} \end{aligned}$$

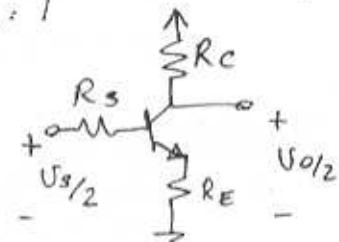
$$GBW = 66.22 \times 452K = \underline{\underline{30 \text{ MHz}}}$$

7.83

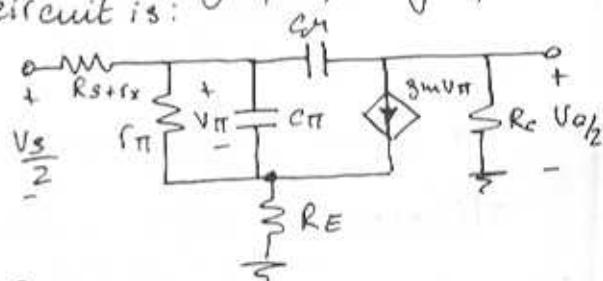
From problem 7.82 above:

$$\begin{aligned} g_m &= 20mA/V \quad f\pi = 5K \Omega \quad C_{\pi} = 4.8pF \\ C_R &= 0.5pF \quad R_S = 5K \Omega \quad R_E = 10K \Omega \\ r_x &= 100 \Omega \end{aligned}$$

If we add an emitter resistor,  
then the equivalent half-  
circuit is:



And the high-frequency equivalent  
circuit is:



For this circuit the low-  
frequency gain is:

$$A_0 = \frac{V_o}{V_s} = \frac{-(\beta + 1) \cdot (r_e + R_E) \times R_c}{(R_s + r_x) + (\beta + 1)(r_e + R_E) \underbrace{R_E + f_e}_{R_E + 50 \Omega}}$$

obtained by  
reflecting emitter  
resistance to the  
base

$$\text{where } r_e = \frac{V_T}{I_E} = \frac{25}{0.5} = 50 \Omega$$

$$\begin{aligned} A_0 &= \frac{-101 \times (50 + 100)}{(10K + 100) + 101 \times (50 + 100)} \times \frac{100}{101} \cdot \frac{10K}{100 + 101} \\ &= \underline{\underline{-39.6 \text{ V/V}}} \end{aligned}$$

Using the method of open-  
circuit time constants:

$$f_H = 1 / (2\pi (C_{\pi} R_{\pi} + C_R R_E))$$

CONT.

Referring to the high-frequency equivalent circuit:

$$\begin{aligned} R_{\pi} &= r_{\pi} \parallel \frac{R_s + r_x + R_e}{1 + g_m R_e} \\ &= 5K \parallel \frac{10K + 100 + 100}{1 + 20m \times 100} \\ &= 5K \parallel 3.4K = 2.02K\Omega \end{aligned}$$

To obtain  $R_r$ :

$$\begin{aligned} R_{in} &= (\beta + 1)(R_e + r_e) \\ &= 101 \times (100 + 50) = 15.15K\Omega \\ G_m &= \frac{g_m}{1 + g_m R_e} = \frac{20m}{1 + 20m \times 100} \\ &= 6.67m. \end{aligned}$$

Thus,

$$\begin{aligned} R_r &= [(R_s + r_x) \parallel R_{in}] (1 + G_m R_c) + R_c \\ &= [(10K + 100) \parallel 15.15K] \cdot (1 + 6.67 \times 10) + 10K \\ &= 420.26K\Omega \end{aligned}$$

Thus,  $C_{\pi} R_{\pi} + C_r R_r =$

$$\begin{aligned} &= 4.8pF \times 2.02K\Omega + 0.5pF \times 420.26K \\ &= 219.8nS \approx 220nS. \end{aligned}$$

$$\Rightarrow f_H = \frac{1}{2\pi \times 220nS} = 723.4\text{ KHz}$$

$$GBW = 39.6 \times 723.4\text{ K} = 28.6\text{ MHz}$$

Approx. the same as in problem 7.82 above.

7.84

From the solution of problem 7.81:

$$\begin{aligned} g_m &= 20mA/V \quad r_e = 50\Omega \quad r_{\pi} = 5K\Omega \\ C_{\pi} &= 4.8pF \quad C_H = 0.5pF \quad R_s = 10K\Omega \\ R_c &= 10K\Omega \quad r_x = 100 \end{aligned}$$

$$\begin{aligned} f_H &= 1/(2\pi C_{\pi}) = 1 \times 10^6 \text{ Hz} \\ \Rightarrow C_{\pi} &= 1/(2\pi \times 10^6) \\ &= 159.15\text{ nS} \end{aligned}$$

But:  $C_{\pi} = C_{\pi} R_{\pi} + C_r R_r$   
where

$$\begin{aligned} R_{\pi} &= r_{\pi} \parallel \left( \frac{R_s + r_x + R_e}{1 + g_m R_e} \right) \\ &= \frac{1}{\frac{1}{r_{\pi}} + \frac{1 + g_m R_e}{R_s + r_x + R_e}} \\ &= \frac{1}{\frac{1}{5K} + \frac{1 + g_m R_e}{10.1K + R_e}} \end{aligned}$$

$$R_r = [(R_s + r_x) \parallel R_{in}] (1 + G_m R_c) + R_c$$

$$\begin{aligned} \text{with: } R_{in} &= (\beta + 1)(R_e + r_e) \\ G_m &= g_m / (1 + g_m R_e) \\ \text{thus,} \end{aligned}$$

$$\begin{aligned} R_r &= \frac{1}{\frac{1}{R_s + r_x} + \frac{1}{(\beta + 1)(R_e + r_e)}} \cdot \left( \frac{1 + g_m R_c}{1 + g_m R_e} \right) + R_c \\ &= \frac{1 + g_m R_c + g_m R_e}{\frac{1 + g_m R_e}{R_s + r_x} + \underbrace{\frac{1 + g_m R_e}{(\beta + 1)(R_e + r_e)}}_{= 1 + g_m R_e}} + R_c \\ &= \frac{1 + g_m R_e}{(\beta + 1)(R_e + \frac{1}{g_m})} \end{aligned}$$

and, since  $r_{\pi} = \beta/g_m$

$$\Rightarrow \frac{g_m}{\beta + 1} \approx \frac{1}{r_{\pi}}$$

thus,

$$R_r = \frac{1 + g_m R_c + g_m R_e}{\frac{1 + g_m R_e}{R_s + r_x} + \frac{1}{r_{\pi}}} + R_c$$

CONT.

$$159.15 \text{ ns} = \frac{4.8 \text{ pF}}{\frac{1}{5K} + \frac{1+gmRe}{10.1K+Re}}$$

$$+ \frac{0.5 \text{ pF}(1+200+gmRe)}{\frac{1+gmRe}{10.1K} + \frac{1}{5K}}$$

$$+ \underbrace{0.5 \text{ pF} \times 10 \text{ kV}}_{5 \text{ ns}}$$

Assuming that  $Re \ll 10.1 \text{ kV}$

$$154.15 \text{ ns} = \frac{4.8 \text{ pF} + 0.5 \text{ pF}(1+200+gmRe)}{\frac{1}{5K} + \frac{1+gmRe}{10.1K}}$$

$$(154.15 \text{ ns}) \left( \frac{1}{5K} + \frac{1+gmRe}{10.1K} \right) =$$

$$= 4.8 \text{ p} + 0.5 \text{ p}(1+gmRe) + 100 \text{ p}$$

Solving for  $1+gmRe$  yields:  
 $1+gmRe = 5 \Rightarrow Re = \underline{200 \text{ kV}}$

The DC gain is:

$$A_o = \frac{-101 \times (50+200)}{(10K+100) + 101 \times (50+200)} \times \frac{100}{101} \times \frac{10K}{(200+50)}$$

$$= -\underline{28.28 \text{ V/V}}$$

$$R_{in} = 101 \times (200+50) = 25.25 \text{ kV}$$

$$G_m = 20 \mu \text{A} / (1+20 \mu \text{A} \times 200) = 4 \mu \text{A}$$

$$R_M = ((10K+100) \parallel 25.25 \text{ kV}) (1+4 \times 10) + 10K$$

$$= 305,785 \text{ kV}$$

$$R_{pi} = 5 \text{ kV} \parallel \frac{10K+100+200}{1+20 \mu \text{A} \times 200} = 145 \text{ kV}$$

$$\gamma_H = 145 \text{ k} \times 4.8 \text{ p} + 305,785 \times 0.5 \text{ p}$$

$$= 159.8 \text{ ns}$$

$$f_H = \frac{1}{2\pi\gamma_H} = 0.99 \text{ MHz} \approx \underline{1 \text{ MHz}}$$

$$GBW = 28.28 \times 1 = \underline{28.28 \text{ MHz}}$$

7.85

$$I = 0.6 \text{ mA}$$

$$V_{ovn} = 0.3 \text{ V}$$

$$V_{ovp} = 0.5 \text{ V}$$

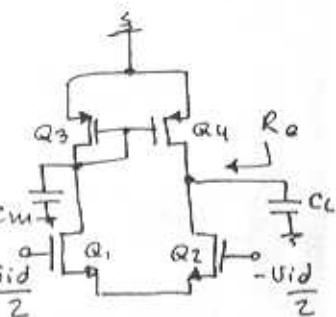
$$V_{An} = |VA_{pl}| = 9 \text{ V}$$

$$C_m = 0.1 \text{ pF}$$

$$C_L = 0.2 \text{ pF}$$

$$+ \frac{U_{id}}{2}$$

$$- \frac{U_{id}}{2}$$



All  $r_o$ 's are identical:

$$r_o = \frac{V_A}{I_D} = \frac{9}{0.3 \text{ mA}} = 30 \text{ kV}$$

$$g_m = \frac{2I_D}{V_{ov}} \approx g_{m1,2} = \frac{0.6 \text{ mA}}{0.3 \text{ V}} = 2 \text{ mA/V}$$

$$g_{m3,4} = \frac{0.6 \text{ mA}}{0.5 \text{ V}} = 1.2 \text{ mA/V}$$

The low frequency differential gain is:

$$Ad = g_{m1,2} (f_{o2} \parallel f_{o4})$$

$$= 2 \text{ mA} \parallel (30 \text{ kV} \parallel 30 \text{ kV}) = \underline{30!}$$

From Eqn. (7.192)

$$f_{p1} = 1/(2\pi C_L R_o)$$

$$\text{where } R_o = f_{o2} \parallel f_{o4} = 15 \text{ kV}$$

$$f_{p1} = \frac{1}{2\pi \times 0.2 \text{ pF} \times 15 \text{ kV}} = \underline{53 \text{ MHz}}$$

$$(7.193) f_{p2} = \frac{g_{m3}}{2\pi C_m} = \frac{1.2 \text{ mA/V}}{2\pi \times 0.1 \text{ pF}}$$

$$= \underline{1.9 \text{ GHz}}$$

$$(7.194) f_2 = \frac{2g_{m3}}{2\pi C_m} = \frac{2 \times 1.2 \text{ mA}}{2\pi \times 0.1 \text{ pF}}$$

$$= \underline{3.8 \text{ GHz}}$$

7.86

The CMRR will have.. CON1

poles at 500 kHz and at

$$\frac{1}{2\pi \times 10^6 \times 10 \times 10^{-12}} = 15.9 \text{ kHz}$$

7.87

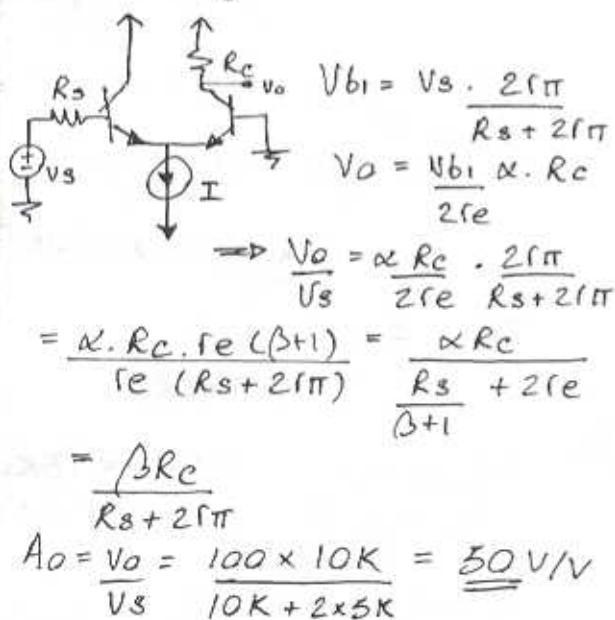
From the solution to Problem 7.82 above:

$$g_m = 20 \text{ mA/V} \quad r_e = 50 \Omega \quad f_T = 5 \text{ kHz}$$

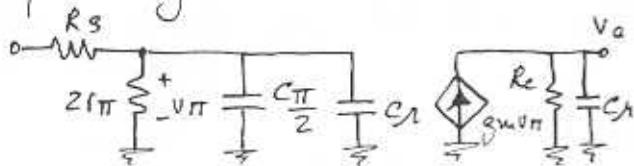
$$C_{\pi} = 4.8 \text{ pF} \quad C_R = 0.5 \text{ pF} \quad R_s = 10 \text{ k}\Omega$$

$$R_C = 10 \text{ k}\Omega$$

Here we neglect  $r_x$ , i.e.  $r_x = 0$ .



Using the equivalent circuit of Fig 6.57, i.e.



thus,

$$f_{p1} = \frac{1}{2\pi (R_s || 2fT) (\frac{C_{\pi}}{2} + C_R)}$$

$$= \frac{1}{2\pi (10K || 10K) (\frac{4.8p}{2} + 0.5p)}$$

$$= \underline{11 \text{ MHz}}$$

$$\text{and, } f_{p2} = \frac{1}{2\pi R_C C_R}$$

$$= \frac{1}{2\pi \times 10K \times 0.5p} = 31.8 \sim \underline{32 \text{ MHz}}$$

$$\text{thus, } f_H = \frac{1}{\sqrt{\left(\frac{1}{11}\right)^2 + \left(\frac{1}{32}\right)^2}} = \underline{10.4 \text{ MHz}}$$

7.88

$$V_{G1} = \frac{V_s \cdot 2/g_m}{2/g_m + R_s} \quad I = \frac{V_{G1}}{2/g_m}$$

$$V_{o1} = I R_D = \frac{V_{G1} \times R_D}{2/g_m}$$

$$= \frac{V_s \times 2/g_m}{2/g_m + R_s} \cdot \frac{R_D}{2/g_m}$$

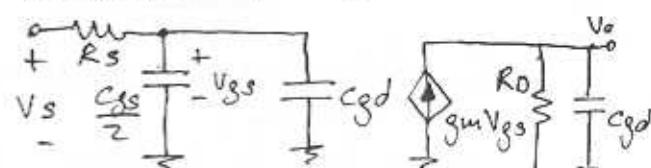
$$= \frac{V_s \cdot R_D}{2/g_m + R_s}$$

$$\Rightarrow A_{o1} = \frac{V_o}{V_s} = \frac{g_m R_D}{2 + g_m R_s}$$

$$g_m = \frac{200 \mu A}{0.25V} = 0.8 \text{ mA/V}$$

$$\Rightarrow A_{o1} = \frac{0.8 \times 50}{2 + 0.8 \times 200} = 0.24 \text{ V/V}$$

The high-frequency equivalent circuit is:



Thus, the pole at the input has a frequency  $f_{p1}$ :

CONT.

$$f_{P1} = \frac{1}{2\pi R_3 \times \left( \frac{C_{GS}}{2} + C_{GD} \right)}$$

$$= \frac{1}{2\pi \times 200K \times (1/2 + 1)p}$$

$$= \underline{\underline{530 \text{ KHz}}}$$

and the pole at the output has a frequency  $f_{P2}$ :

$$f_{P2} = \frac{1}{2\pi R_0 C_{GD}} = \frac{1}{2\pi \times 50K \times 1p}$$

$$= \underline{\underline{3.18 \text{ MHz}}}$$

Thus  $f_H \approx \frac{1}{\sqrt{\left(\frac{1}{530K}\right)^2 + \left(\frac{1}{3.18M}\right)^2}}$

$$= \underline{\underline{523 \text{ KHz}}}$$

Notice that this low value of  $f_H$  is due to the large value of  $R_3$ .

7.89

Refer to Fig. P7.89.  
It can be shown that this circuit is equivalent to that of Problem 7.87 above.

$$g_m = \frac{0.1mA}{25mV} = 4mA/V \quad r_e = 250\Omega$$

$$r_{\pi} = \frac{B}{g_m} = \frac{200}{4mA/V} = 50K\Omega$$

$$A_o = \frac{\alpha R_c}{\frac{R_s}{\beta+1} + 2r_e} = \frac{\frac{200}{201} \times 50K}{\frac{50K}{201} + 2 \times 250}$$

$$= \underline{\underline{66.44 \text{ V/V}}}$$

$$2\pi f_T = \frac{g_m}{C_{\pi} + C_R} \Rightarrow C_{\pi} + C_R = \frac{4m}{2\pi 600}$$

$$C_{\pi} + C_R = 1.08pF$$

$$\text{Since } C_R = 0.2pF \Rightarrow C_{\pi} = 0.86pF$$

$$f_{P1} = \frac{1}{2\pi (R_3 || 2r_{\pi}) (\frac{C_{\pi}}{2} + C_R)}$$

$$= \frac{1}{2\pi (50K || 2 \times 50K) (\frac{0.86}{2} + 0.2)p}$$

$$= \underline{\underline{7.58 \text{ MHz}}}$$

$$f_{P2} = \frac{1}{2\pi R_0 C_R} = \frac{1}{2\pi 50K \times 0.2p}$$

$$= \underline{\underline{15.9 \text{ MHz}}}$$

Thus the estimate of  $f_H$  is:

$$f_H = \frac{1}{\sqrt{\left(\frac{1}{7.58}\right)^2 + \left(\frac{1}{15.9}\right)^2}}$$

$$= \underline{\underline{6.84 \text{ MHz}}}$$

7.90

To ensure that the op amp will not have a systematic offset voltage, use Eqn. 7.201

$$(W/L)_6 = 2 (W/L)_7$$

$$(W/L)_4 = \frac{(W/L)_5}{(W/L)_6}$$

$$\Rightarrow (W/L)_6 = 2 \times \frac{(60/0.5)}{(10/0.5)}$$

$$\Rightarrow (W/L)_6 = \underline{\underline{(20/0.5)}}$$

Since  $Q_8$  and  $Q_5$  are matched  $I = I_{REF} = 225\mu A$   
Thus  $Q_1, Q_2, Q_3$  and  $Q_4$

CONT.

each conducts a current equal to  $I/2 = 112.5\text{mA}$ .

Since  $Q_7$  is matched to  $Q_5$  and  $Q_8$ , the current in  $Q_7$  is equal to  $I_{REF} = 225\text{mA}$ .

Finally  $Q_6$  conducts an equal current of  $225\text{mA}$ . Thus,

$$\begin{aligned} I_{D1} &= 112.5\text{mA} & I_{D5} &= 225\text{mA} \\ I_{D2} &= 112.5\text{mA} & I_{D6} &= 225\text{mA} \\ I_{D3} &= 112.5\text{mA} & I_{D7} &= 225\text{mA} \\ I_{D4} &= 112.5\text{mA} & I_{D8} &= 225\text{mA} \end{aligned}$$

To find  $|V_{ov1}|$  we use:

$$I_D = \frac{1}{2} \mu_{COX} (\omega/L) V_{ov}^2$$

then we find  $|V_{GS1}|$  from:

$$|V_{GS1}| = |V_t| + |V_{ov1}|$$

this results in:

$$\begin{array}{ll} |V_{ov1}| = 0.25\text{V} & |V_{GS1}| = 1\text{V} \\ |V_{ov2}| = 0.25\text{V} & |V_{GS2}| = 1\text{V} \\ |V_{ov3}| = 0.25\text{V} & |V_{GS3}| = 1\text{V} \\ |V_{ov4}| = 0.25\text{V} & |V_{GS4}| = 1\text{V} \\ |V_{ov5}| = 0.25\text{V} & |V_{GS5}| = 1\text{V} \\ |V_{ov6}| = 0.25\text{V} & |V_{GS6}| = 1\text{V} \\ |V_{ov7}| = 0.25\text{V} & |V_{GS7}| = 1\text{V} \\ |V_{ov8}| = 0.25\text{V} & |V_{GS8}| = 1\text{V} \end{array}$$

To find  $g_m$ :  $g_m = 2ID / |V_{ov1}|$   
and  $r_o$ :  $r_o = |V_A| / I_D$

Thus:

$$\begin{array}{ll} g_{m1} = 0.9\text{mA/V} & r_{o1} = 80\text{K}\Omega \\ g_{m2} = 0.9\text{mA/V} & r_{o2} = 80\text{K}\Omega \\ g_{m3} = 0.9\text{mA/V} & r_{o3} = 80\text{K}\Omega \\ g_{m4} = 0.9\text{mA/V} & r_{o4} = 80\text{K}\Omega \\ g_{m5} = 1.8\text{mA/V} & r_{o5} = 40\text{K}\Omega \\ g_{m6} = 1.8\text{mA/V} & r_{o6} = 40\text{K}\Omega \\ g_{m7} = 1.8\text{mA/V} & r_{o7} = 40\text{K}\Omega \\ g_{m8} = 1.8\text{mA/V} & r_{o8} = 40\text{K}\Omega \end{array}$$

$$\begin{aligned} A_1 &= -g_{m1} (r_{o2} || r_{o4}) \\ &= -0.9 \frac{\text{mA}}{\text{V}} (80 || 80) \text{K}\Omega \end{aligned}$$

$$= -36 \text{V/V}$$

$$\begin{aligned} A_2 &= -g_{m6} (r_{o6} || r_{o7}) \\ &= -1.8 \frac{\text{mA}}{\text{V}} (40 || 40) \text{K}\Omega \\ &= -36 \text{V/V} \end{aligned}$$

Thus, the DC open-loop gain is:

$$A_0 = A_1 A_2 = (-36) \times (-36)$$

$$= 1296 \text{V/V}$$

$$\rightarrow 62.25 \text{dB}$$

Input common-mode range:  
lower limit is when the input is such that  $Q_1$  &  $Q_2$  leave the saturation region.

$$\begin{aligned} V_{DI} &= -V_{SS} + V_{GS3} \\ &= -1.5 + 1 = -0.5\text{V} \end{aligned}$$

$$\begin{aligned} V_{in\min} &= -0.5 - V_{tp1} \\ &= -0.5 - 0.75 = -1.25\text{V} \end{aligned}$$

The upper limit is the value of  $V_{in\max}$  at which  $Q_5$  leaves the saturation region.

$$\begin{aligned} V_{GS4\max} &= +1.5 - |V_{ov4}| \\ &= 1.25\text{V} \end{aligned}$$

$$\rightarrow V_{in\max} = 1.25 - 1 = +0.25\text{V}$$

Input range is:  $-1.25$  to  $0.25\text{V}$

Output voltage range:

$V_{max}$  is the value at which  $Q_7$  leaves saturation.

$$\begin{aligned} V_{DD} - |V_{ov7}| &= +1.5 - 0.25 \\ &= 1.25\text{V} \end{aligned}$$

The lowest  $V_o$  is the value at which  $Q_6$  leaves saturation i.e.  $-V_{SS} + V_{ov6} = -1.5 + 0.25 = -1.25\text{V}$

The output range is

$$-1.25 \text{ to } +1.25\text{V}$$

7.91

$$I = \frac{1}{2} K V_{ov}^2$$

$$(a) V_{ov} = \sqrt{\frac{2I}{K}}$$

If  $K$  increases by 4  $\rightarrow V_{ov}$  decreases by  $\frac{1}{2}$

$$g_m = 2I/V_{ov} = K \cdot V_{ov}$$

$\rightarrow$  If  $K$  increases by 4  
 $g_m$  increases by  $\underline{\underline{\times 2}}$

$$(b) A_i = g_m R_{o1}$$

$\Rightarrow A_i$  increases  $\times 2$  and so  $A_o$

(c) Offsets due to  $V_t$  mismatches are unaffected. Others reduced  $\times \frac{1}{2}$  since  $A_o$  increases  $\times 2$

$$(d) A_o f_p = f_T \Rightarrow f_p = \frac{f_T}{A_o}$$

and is reduced  $\times \frac{1}{2}$   
 $\Rightarrow C_c$  must be doubled.

7.93

$$\text{Offset current} = I_{D2} - I_{D4}$$

$$= I_{D3} - I_{D4}$$

$$I_{D3} = \frac{K}{2} (V_{GS} - V_t)^2$$

$$I_{D4} = \frac{K}{2} (V_{GS} - (V_t + \Delta V_t))^2$$

$$I_o = I_{D3} - I_{D4}$$

$$= \frac{K}{2} [(V_{GS} - V_t - V_{GS} + V_t + \Delta V_t) \times$$

$$(V_{GS} - V_t + V_{GS} - V_t - \Delta V_t)]$$

$$= \Delta V_t \cdot \frac{K}{2} (2V_{GS} - 2V_t - \Delta V_t)$$

$$\simeq K (V_{GS} - V_t) \cdot \Delta V_t$$

$$I_o = \underline{\underline{g_{m3} \Delta V_t}}$$

$$\text{Recall } I_o = G_{m1} \cdot V_{os}$$

$$\text{and } G_{m1} = g_{m1}$$

$$\Rightarrow V_{os} = \underline{\underline{g_{m3} \cdot \Delta V_t}} / g_{m1}$$

$$\text{For } \Delta V_t = 2\text{mV}$$

$$V_{os} = \frac{0.3\text{m}}{0.3\text{m}} \times 2\text{m} = \underline{\underline{2\text{mV}}}$$

7.92.

$$I_{D7} = \frac{W_7}{W_8} I_{REF} = \frac{50}{40} \times 80\text{mA}$$

$$= 112.5\text{nA} //$$

$$\text{Output offset current} = I_{D7} - I_{D6}$$

$$= 112.5 - 90 = 22.5\text{nA}$$

$$\Rightarrow V_o = 22.5\text{n} (I_{D6} // I_{D7})$$

$$I_{D7} = \frac{10}{112.5} = 88.9\text{k}\Omega$$

$$\Rightarrow V_o = 22.5\text{n} (11\text{k} // 88.9\text{k})$$

$$= 1.11\text{V}$$

$$V_{os} = \underline{\underline{\frac{V_o}{A_o}}} = \frac{1.11\text{V}}{1109} = \underline{\underline{1\text{mV}}}$$

7.94

$$\text{From Eqn. (7.211)} \quad W_t = \frac{G_{m1}}{C_C}$$

$$G_{m1} = g_{m1} = 1\text{mA/V}$$

$$\text{For } f_T = 50\text{MHz}$$

$$C_C = \frac{1\text{mA/V}}{2\pi \times 50\text{MHz}} = \underline{\underline{3.18\text{pF}}}$$

$$\text{From Eqn. (7.206)}$$

$$f_2 = \frac{G_{m2}}{2\pi C_C}$$

$$G_{m2} = g_{m6} = 3\text{mA/V}$$

CON

$$f_2 = \frac{3mA/V}{2\pi \times 3.18\text{ pF}} = 150.14 \text{ MHz}$$

From Eqn. (7.210)

$$f_{p2} = \frac{Gm2}{2\pi C2} = \frac{3mA/V}{2\pi \times 3\text{ pF}} = 160 \text{ MHz}$$

7.95

$$(a) I_{E1} = I_{E2} = 0.1 \text{ mA} \approx I_{E3}, I_{E4}$$

I<sub>E5</sub> ≈ 1 mA and since the output is held at 0V  
I<sub>E6</sub> = 2 mA

$$(b) f_{e1} = f_{e2} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \text{ Hz}$$

$$f_{e5} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \text{ Hz}$$

$$f_{e6} = \frac{25 \text{ mV}}{2 \text{ mA}} = 12.5 \text{ Hz}$$

For the active loaded differential pair; Recall from Eqn.

$$(7.161) \quad Gm_1 = g_{m1} \approx \frac{1}{I_{e1}} = \frac{1}{250} = 4 \text{ mA/V}$$

$$R_{o1} = (\beta + 1)f_{e5} \quad \text{Since all } r_o's = \infty$$

$$R_{o1} = 101 \times 25 = 2525 \text{ ohms}$$

$$\Rightarrow A_1 = Gm_1 R_{o1} = \frac{4 \text{ mA} \times 2525 \text{ ohms}}{10^3} = 10.1 \text{ V/V}$$

For the common-emitter:

$$A_5 = -g_{m5} \cdot R_{cs} \approx -\frac{\beta R_L}{f_{e5}} = -\frac{100 \times 10 \text{ k}}{25} = -40,000 \text{ V/V}$$

For the emitter follower:  
A<sub>6</sub> ≈ 1

$$A_{2\text{nd-stage}} = A_5, A_6 = -40,000 \text{ V/V}$$

$$A = A_1, A_{2\text{nd-stage}} = 10.1 \times -40,000 = -404,000 \text{ V/V}$$

(c) Since the dominant low-frequency pole is set by  $C_C \times \frac{1}{2\pi f_p}$

$$f_p = \frac{1}{2\pi \cdot R_{o1} \cdot \underbrace{(A_5 + 1)}_{\text{by Miller effect}} C_C} = 100 \text{ Hz}$$

$$\Rightarrow C \equiv 1 / (2\pi \times 2525 \times 40 \text{ k} \times 100) = 15.76 \text{ pF}$$

7.96

$$I_B = 22.5 \mu\text{A}$$

$$\mu_n C_{ox} = 180 \mu\text{A/V}^2$$

$$\mu_p C_{ox} = 60 \mu\text{A/V}^2$$

$$\text{For } Q_8 \& Q_9: W/L = 60/0.5$$

$$\Rightarrow |V_{ovl}| = \sqrt{\frac{2 \cdot I_D}{K_p'(W/L)}}$$

$$|V_{ovl_{8,9}}| = \sqrt{\frac{2 \times 22.5 \mu\text{A}}{60 \mu\text{A} \times 120}} = 0.25 \text{ V}$$

$$\text{then } g_{m_{8,9}} = \frac{2 \cdot I_D}{|V_{ovl}|} = \frac{2 \times 22.5 \mu\text{A}}{0.25 \text{ V}} = 1.8 \text{ mA/V}$$

Since g<sub>m</sub> of Q<sub>10</sub>, Q<sub>11</sub> & Q<sub>13</sub> are identical to g<sub>m</sub> of Q<sub>8</sub> & Q<sub>9</sub> then V<sub>ovl</sub><sub>13</sub> = 0.25 V

Thus for Q<sub>13</sub>

$$(0.25)^2 = \frac{2 \times 22.5 \mu\text{A}}{180 \mu\text{A} \times (W/L)_{13}}$$

$$\Rightarrow (W/L)_{13} = 40 \text{ i.e. } (20/0.5)$$

Since Q<sub>12</sub> is 4 times as wide as Q<sub>13</sub>, then

CONT.

$$(W/L)_{12} = \frac{4 \times 20}{0.5} = 80/0.5$$

$$R_B = \frac{2}{\sqrt{2Ku'(W/L)_{12}} I_B} \cdot \left( \sqrt{(W/L)_{13}} - 1 \right)$$

$$= \frac{2}{\sqrt{2 \times 180 \mu A \times \frac{80}{0.5} \times 225 \mu A}} \cdot \left( \sqrt{\frac{80/0.5}{20/0.5}} - 1 \right)$$

$$\rightarrow R_B = \underline{555.6 \Omega}$$

The voltage drop on  $R_B$  is:

$$555.6 \times 225 \mu A = 0.125 V.$$

To obtain the gate voltages:

(assume  $|V_{t_h}| = |V_{t_p}| = 0.7 V$ )

$$V_{OV_{12}} = \sqrt{\frac{2 \times 225 \mu A}{180 \mu A \times \frac{80}{0.5}}} = 0.125 V$$

$$V_{OV_{12}} = V_{GS12} - V_{th}$$

$$\rightarrow V_{GS12} = 0.125 + 0.7 = 0.825 V$$

thus,

$$V_{G_{12,13}} = V_{GS12} + I_B R_B - V_{SS}$$

$$= 0.825 + 0.125 - 1.5$$

$$= -0.55 V$$

$$V_{OV_{11}} = |V_{OV_8}| = 0.25 V$$

$$\Rightarrow V_{GS_{11}} = 0.25 + 0.7 = 0.95 V$$

$$V_{G_{11}} = -0.55 + 0.95$$

$$V_{G_{11}} = V_{GS10} = 0.4 V$$

$$V_{GS8} = V_{DD} - V_{SG8} = 1.5 + (-0.25 - 0.7)$$

$$= +0.55 V$$

Finally, from the results above:

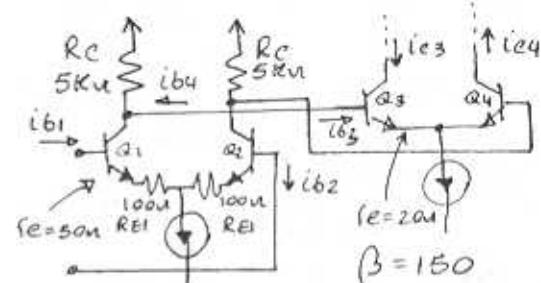
$$(W/L)_{10} = 20/0.5$$

$$(W/L)_{11} = 20/0.5$$

$$(W/L)_{12} = 80/0.5$$

$$(W/L)_{13} = 20/0.5$$

7.97



$$A_1 = \frac{2R_E \parallel R_{id2}}{2(R_{E1} + r_{e1})}$$

$$R_{id2} = (\beta + 1)(2r_{e2}) = 6.04 k\Omega$$

$$\Rightarrow A_1 = \frac{12.5 V/V}{2}$$

$$A_1 = \frac{i_{e4}}{i_{b1}} = \beta_1 \cdot \frac{2R_C}{R_{id2} + 2R_C} \beta_4$$

$$= 1.4 \times 10^4 \text{ A/A}$$

7.98

$$R_{id1} = (\beta + 1)(R_{E1} + r_{e1}) 2$$

$$= 101 \times (100 + 100) 2 = \underline{40.4 k\Omega}$$

increase!

$$R_{id2} = (\beta + 1)(2)(R_{E2} + r_{e2})$$

$$= 101 \times (25 + 25) 2 = 10.1 k\Omega$$

$$\therefore A_1 = \frac{2(20 k\Omega) \parallel R_{id2}}{2(R_{E1} + r_{e1})} = \underline{20.2 V/V}$$

$$A_2 = -\frac{R_3 \parallel R_{i3}}{2(R_{E2} + r_{e2})} = -28.6 V/V$$

Observe that  $A_3$  and  $A_4$  are unchanged.

$$A = \underline{A_1 A_2 A_3 A_4}$$

$$= (20.2)(28.6)(6.42)(0.998)$$

$$= \underline{3823 V/V}$$

decrease.

7.99

$$R_O \approx \frac{R_S}{\beta + 1} + r_{eg} = R_C'$$

CONT.

Thus  $R_5$  affects  $R_o$

$$\text{We want } R_o' \parallel 3K = 76$$

$$\Rightarrow R_o' = 78 \Omega$$

$$\Rightarrow R_5 = (78 - r_e) (\beta + 1)$$

$$= 7.37 \text{ k}\Omega$$

$$A_3 = -\frac{R_5 \parallel R_{i4}}{r_{e4} + R_4}; \quad R_{i4} \approx 304 \text{ k}\Omega$$

$$\text{and } A_3 = -3.09 \text{ V/V}$$

$$\text{and } A = 8513 \cdot \frac{3.09}{6.42} = 4104 \text{ V/V}$$

The gain has been reduced by a factor of 2.07 and can be restored by reducing  $R_4$  by this same factor to increase  $A_3$ . Thus  $R_4 = 1.11 \text{ k}\Omega$   
(Note that this is a first order approximation).

### 7.100

$$(a) A_3 = \frac{-R_{i4}}{2.325 \text{ k}\Omega} = \frac{-303.5}{2.325}$$

$$= -130.5 \text{ V/V}$$

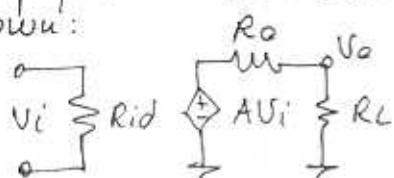
i.e.  $A_3$  is increased by  $\frac{130.5}{6.42}$   
= 20.33

$$\Rightarrow A = 8513 \times 20.33$$

$$= 173.1 \times 10^3 \text{ V/V}$$

(b) Let the output resistance of the current source be  $R_o \rightarrow \infty$   
 $R_o = 3K \parallel \left( \frac{R}{\beta+1} + r_e \right)$

$= 3K$   
The amplifier can be modelled as shown:



Thus,

$$A_{LOAD} = \frac{A \cdot R_L}{R_L + R_o}$$

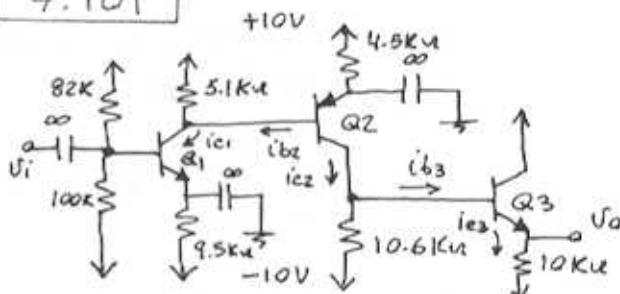
$$= 173.1 \times 10^3 \frac{100}{100 + 3000}$$

$$= 5583 \text{ V/V}$$

For the original amplifier:

$$A_{LOAD} = 8513 \times \frac{100}{100 + 152} = 3378 \text{ V/V}$$

### 7.101



$$(a) I_{E1} = \frac{20V \times 100k}{82k + 100k} - 0.7$$

$$= \frac{9.5k + (82k \parallel 100k)}{\beta + 1}$$

$$\beta = 100 \Rightarrow I_{E1} = 1.03 \text{ mA}$$

$$\alpha = \frac{100}{101} \Rightarrow I_{C1} = 1.02 \text{ mA}$$

$$V_{C1} \approx 10V - 1.02 \text{ mA} \times 5.1 \text{ k}\Omega = 4.8 \text{ V}$$

$$I_{E2} = \frac{(10 - 0.7 - 4.8)V}{4.5 \text{ k}\Omega} = 1 \text{ mA}$$

$$\rightarrow I_{C2} = 0.99 \text{ mA}$$

$$V_{C2} \approx 0.99 \text{ mA} \times 10.6 \text{ k}\Omega - 10 = 0.5 \text{ V}$$

$$\Rightarrow U_{Odc} = 0.5 - 0.7 = -0.2 \text{ V}$$

$$I_{E3} = \frac{-0.2 - (-10)}{10k} = 0.98 \text{ mA}$$

$$\rightarrow I_{C3} = 0.97 \text{ mA}$$

Thus all transistors are operating at  $I_c \approx 1 \text{ mA}$

CONT.

$$(b) R_{in} = 82K \parallel 100K \parallel r_{\pi 1}$$

where  $r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{40m} = 2.5K\Omega$

$$\Rightarrow R_{in} = (82 \parallel 100 \parallel 2.5)K = \underline{2.37K\Omega}$$

$$R_{out} = 10K \parallel [r_{e3} + \frac{10.6K}{\beta+1}]$$

$$= 10K \parallel [25 + \frac{10.6K}{101}]$$

$$= \underline{128\Omega}$$

$$(c) \frac{i_{c1}}{V_i} = g_{m1} = 40mA/V$$

$$\frac{i_{b2}}{i_{c1}} = \frac{5.1K}{5.1K + r_{\pi 2}} = \frac{5.1}{5.1 + 2.5} = 0.671A/A$$

$$\frac{i_{e2}}{i_{b2}} = \beta_2 = 100 A/A$$

$$i_{b2}$$

$$\frac{i_{b3}}{i_{c2}} = \frac{10.6K}{10.6K + (\beta+1)(r_{e3} + 10K)}$$

$$= 0.01036 A/A$$

$$\frac{i_{e3}}{i_{b3}} = \beta_3 + 1 = 101$$

$$i_{b3}$$

$$V_o = i_{e3} \times 10K$$

Thus,

$$\frac{V_o}{V_i} = 10 \times 101 \times 0.01036 \times 100 \times 0.671 \times 40$$

$$= \underline{2.81 \times 10^4 V/V}$$

$$(d) f_{p2} = 1 / (2\pi C_2 \cdot R_2)$$

where:  $R_2 = 5.1K \parallel r_{\pi 2}$   
 $= 5.1K \parallel 2.5K = 1.68K\Omega$

$$C_2 = C_{\pi 2} + C_{L2} (1 + g_{m2} R_{L2})$$

with:

$$R_{L2} = 10.6K \parallel (\beta+1)(r_{e3} + 10K)$$

$$= 10.6K \parallel 101 \times (25 + 10K)$$

$$= 10.5K\Omega$$

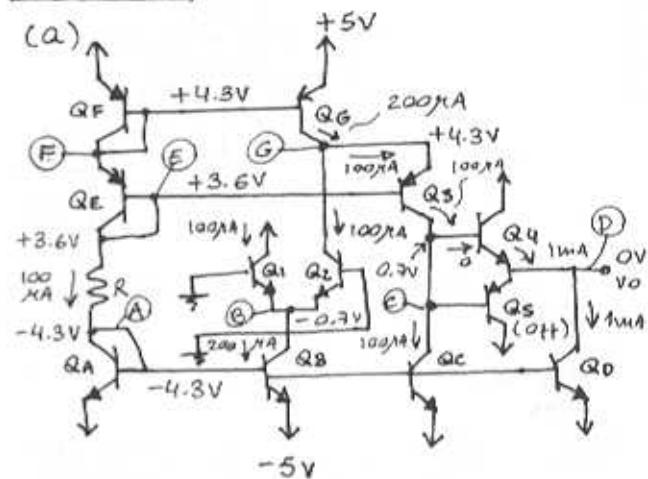
$$\Rightarrow C_2 = 10p + 2p (1 + 40m \times 10.5K)$$

$$= 852pF$$

$$\Rightarrow f_{p2} = \frac{1}{2\pi \times 852p \times 10.5K}$$

$$= \underline{17.8 \text{ KHz}}$$

### 7.102



DC Analysis

$$R = \frac{3.6 - (-4.3)}{100\mu A} = \underline{79K\Omega}$$

Node voltages:

$$V_A = \underline{-4.3V}, \quad V_B = \underline{-0.7V}$$

$$V_C = \underline{+0.7V} \quad V_D = \underline{0V}$$

$$V_E = \underline{+3.6V} \quad V_F = \underline{+4.3V}$$

$$V_G = \underline{+4.3V}$$

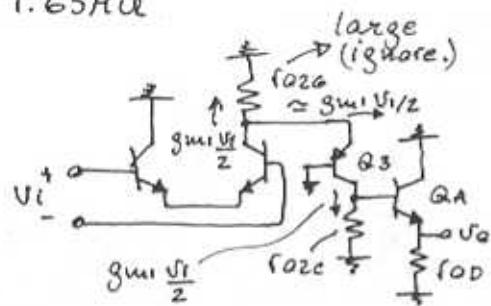
(b)

Transistor	$I_c$ (mA)	$g_m$ (mA/V)	$f_0$ (MHz)
Q <sub>1</sub>	0.1	4	2
Q <sub>2</sub>	0.1	4	2
Q <sub>3</sub>	0.1	4	2
Q <sub>4</sub>	1.0	40	0.2
Q <sub>5</sub>	0	0	$\infty$
Q <sub>A</sub>	0.1		
Q <sub>B</sub>	0.2		
Q <sub>C</sub>	0.1		2
Q <sub>D</sub>	1.0		0.2
Q <sub>E</sub>	0.1		
Q <sub>F</sub>	0.1		
Q <sub>G</sub>	0.2		1

CONT.

(c) Total resistance at collector  $Q_3$  is

$$\approx \beta_3 r_{03} \parallel r_{0c} \parallel (\beta_4 + 1) (r_{04} \parallel r_{0D}) \\ = 100 \times 2 \parallel 2 \parallel 101 (0.2 \parallel 0.2) \\ = 1.65 \text{ k}\Omega$$



$$\frac{U_{C3}}{U_i} = + g_{mu} \times \frac{1}{2} \times 1.65 \times 10^3 = 3300 \frac{\text{V}}{\text{V}}$$

$$\frac{U_o}{U_{C3}} \approx 1$$

Thus,  $\frac{U_o}{U_i} \approx 3300 \frac{\text{V}}{\text{V}}$  (polarity correct)

(d)  $R_{in} = 2 \text{ M}\Omega$

$$= 2 \times \frac{100}{4} = 50 \text{ k}\Omega$$

$$R_{out} = r_{0D} \parallel r_{04} \parallel [r_{e4} + r_{02c} \parallel \beta_3 r_{03}] \\ = 0.2 \parallel 0.2 \parallel \left[ 25.10^6 + \frac{2 \parallel 100 \times 2}{101} \right] \\ \approx 16.4 \text{ k}\Omega$$

$$(e) |U_{ICM}|_{\min} = -4.3 - 0.4 + 0.7 \\ = -4 \text{ V}$$

$$|U_{ICM}|_{\max} = V_G + 0.4 = +4.7 \text{ V}$$

(f) The voltage at the base of  $Q_4$  can rise to  $V_{B3}$ :

$$(V_E) + 0.4 = +4 \text{ V}$$

before  $Q_3$  saturating. Thus  $U_o$  can go up to  $+3.3 \text{ V}$

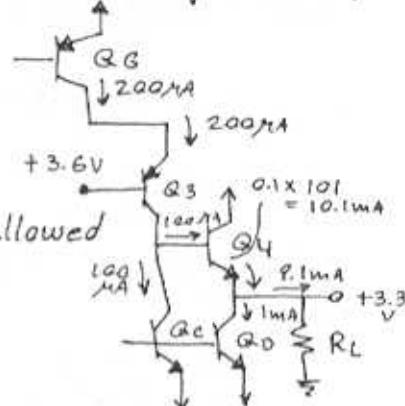
The voltage at the output can go down to  $V_{base\ of\ Q_0} + 0.4$   
 $= V_A - 0.4 = -4.3 - 0.4 = -4.7 \text{ V}$

Thus the linear range at the output is -4.7 V to +3.3 V

(g) At the positive limit of  $U_o$ , i.e.  $U_o = +3.3 \text{ V}$  and  $Q_2$  just cut off

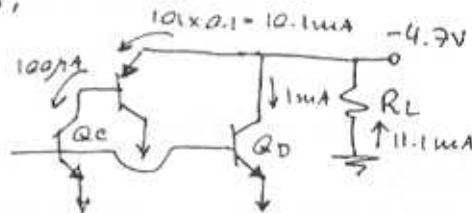
$$R_L = \frac{3.3 \text{ V}}{9.1 \text{ mA}} \\ = 363 \Omega$$

(This is the minimum allowed  $R_L$  for +3.3 V output)



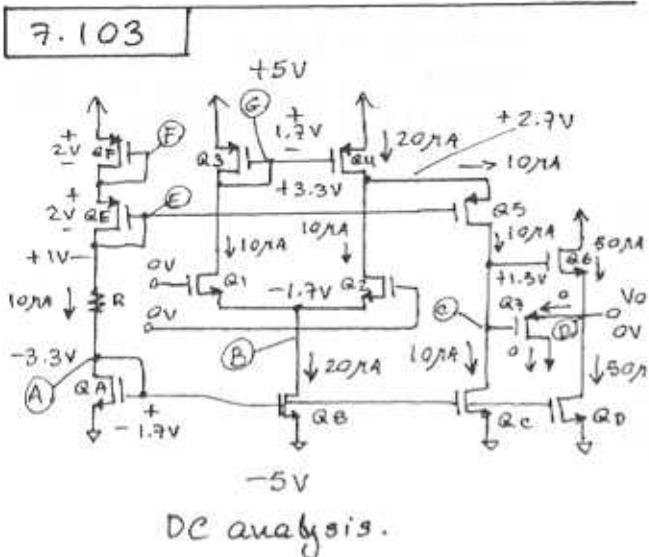
At the negative limit of  $U_o$ , i.e.  $U_o = -3.3 \text{ V}$  and  $Q_1$  has cut-off.  $Q_3$  will also be cut-off, and  $Q_4$  will cut-off.

Thus,



$$R_L = \frac{4.7}{11.1 \text{ mA}} = 423 \Omega$$

This is the minimum allowed  $R_L$  for a -4.7 V output.



## DC analysis.

$$(a) I_{REF} = 10mA = \frac{1}{2} \times 40 \times \frac{5}{S} (V_{GS_A} - V_t)^2$$

$$\Rightarrow V_{GS_A} = 1.71V \approx 1.7V$$

$$10 = \frac{1}{2} \times 20 \times \frac{5}{5} (V_{GS,E,F} - 1)^2$$

$$\Rightarrow V_{GSE,E} = 2V$$

$$R = \frac{3 - (-3.3)}{10\pi A} = \underline{\underline{660 \text{ kN}}}$$

(b) See figure above

$$V_{GS1} = V_{GS2} = V_{GSA} \approx 1.7V$$

$$V_{GS3} = \sqrt{\frac{2 \times 10}{20 \times 10}} + 1 = 1.71V \approx 1.7V$$

$$V_{GS5} = V_{GS3} = 1.7 \text{ V}$$

$$\text{For Q6: } 50 = \frac{1}{2} \times 40 \times \frac{50}{5} (V_{GS6} - Vt)^2$$

$$\Rightarrow V_{GS6} = 1.50V$$

$$V_A = -3.3V \quad V_B = -1.7V$$

$$V_C = +1.5V$$

$$V_E = +1V \quad V_F = +3V$$

$$V_G = +3.3V$$

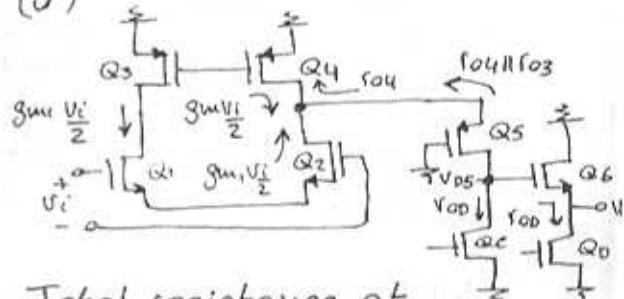
— 11 —

(C) Transistor	$I_D$ (mA)	$V_{DS}$ (V)	$\beta_M$ (mA/V)	$f_0$ (MHz)
Q1	10	1.7	28.3	5
Q2	10	1.7	28.3	5
Q3	10	1.7	28.3	5
Q4	20	1.7	56.6	2.5

Q5	10	1.7	28.3	5
Q6	50	1.5	200	1
Q7	0	-1.5	0	$\infty$
QA	10	1.7	28.3	5
QB	20	1.7	56.6	2.5
QC	10	1.7	28.3	5
QD	50	1.7	141.4	1
QE	10	2	20	5
QF	10	2	20	5

\* cut-off.

(d)



Total resistance at  
the drain of Q5, R, is

$$R = (g_{\text{m}1} f_{\text{o}1})(f_{\text{o}4} \parallel f_{\text{o}2}) \parallel f_{\text{o}3}$$

$$= [(2.3 \times 5)(2.5 \parallel 2)] \parallel 5$$

$$= 4.9 \mu\text{H}$$

$$\text{Thus, } \frac{Uds}{U_i} = g_{mi} R$$

$$= 28.3 \times 4.9 = 138.7 \text{ V/V}$$

$$\text{and } \frac{U_0}{U_{ds}} = \frac{(f_{OD} || f_{OB})}{(f_{OD} || f_{OB}) + \frac{1}{g_{m6}}}$$

$$= \frac{(1/1)}{(1/1) + \frac{1}{200}} \approx 1$$

$$\frac{U_o}{U_i} = \underline{\underline{138.7}} \text{ V/V}$$

$$R_{in} = \infty$$

Rout = f00 // f06 // 1 / gme

$$= 1 // 1 // \sqrt{200} \text{ Hz}$$

$$\approx \frac{5K\Omega}{2}$$

CONT.

$$(e) V_{Icm} |_{max} = V_G + V_t \\ = + \underline{4.3V}$$

$$V_{Icm} |_{min} = V_{GS1} + V_{Bmin} \\ = V_{GS1} + V_A - V_t \\ = 1.7 - 3.3 - 1 = - \underline{2.6V}$$

$\Rightarrow \hat{V}_o = 1.45V$   
That is, the range of  $V_o$  is  
 $-1.45V \leq o + 0.17V$

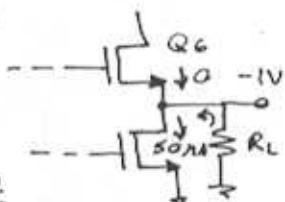
$$(f) V_{o\max} = V_{Cmax} - V_{GS6} \\ = V_E + |V_t| - V_{GS6} \\ = +1 + 1 - 1.5 = +0.5V \\ V_{o\min} = V_A - V_t = -3.3 - 1 = - \underline{4.3V}$$

(g)  $Q_6$  cuts off

thus,

$$\frac{1V}{R_L} = 50\mu A$$

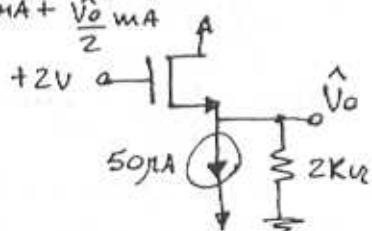
$$R_L = \frac{1V}{50\mu A} = 20K\Omega$$



(h) Maximum possible voltage at drain of  $Q_5$  is  $+2V$ .

At this value we have:

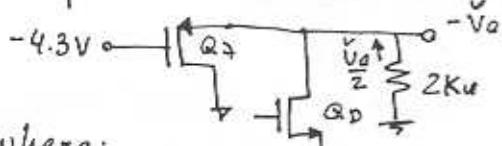
$$I_D = 50\mu A + \frac{\hat{V}_o}{2} mA$$



$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2 - V_o - V_t)^2$$

$$\Rightarrow \hat{V}_o \approx 0.17V$$

For the lowest possible output, the circuit becomes



where:

$Q_6$  cuts off and  $Q_7$  conducts

$$I_D = \frac{\hat{V}_o}{2} - 0.05mA$$

$\downarrow (w/L)_7$

$$= 1/2 \mu_p C_{ox} \left( \frac{100}{5} \right) (-\hat{V}_o + 4.3 - 1)^2$$

## Chapter 8 - Problems

**8.1**

$$A_f = \frac{A}{1+A\beta} = 100$$

$$A\beta = \frac{10^5}{100} - 1 = 999$$

$$\Rightarrow \beta = \frac{999}{10^5} = 9.99 \times 10^{-3}$$

$$A = 10^3, A_f = \frac{10^3}{1 + 10^3(9.99 \times 10^{-3})} = 90.99$$

$$\frac{\Delta A_f}{A_f} = \frac{90.99 - 100}{100} \Rightarrow -9\%$$

**8.2**

$$(b) A_f = 10 = \frac{100}{1+100\beta} \Rightarrow \beta = 90 \times 10^{-3}$$

$$\beta = \frac{R_1}{R_1+R_2} \Rightarrow \frac{R_2}{R_1} = \frac{1}{\beta} - 1 = 10.11$$

$$(c) \text{amount of feedback} = 1 + A\beta = 1 + 100(90 \times 10^{-3}) = 10 \equiv 20 \text{dB}$$

$$(d) V_o = 10V_s = 10V$$

$$V_f = \beta V_o = 90 \times 10^{-3} \times 10 = 0.9V$$

$$V_i = V_s - V_f = 1 - 0.9 = 0.1V$$

$$(e) A_f = \frac{80}{1 + 80(90 \times 10^{-3})} = 9.756$$

$$\frac{\Delta A_f}{A_f} = \frac{9.756 - 10}{10} \Rightarrow -2.44\%$$

**8.3**

$$(b) A_f = 10^3 = 10^3 / (1 + 10^3 \beta)$$

$$\Rightarrow \beta = 900 \times 10^{-6}$$

$$\frac{R_2}{R_1} = \frac{1}{\beta} - 1 = 1110$$

$$(c) 1 + A\beta = 1 + 10^3(900 \times 10^{-6}) = 10 \equiv 20 \text{dB}$$

$$(d) V_s = 0.01V$$

$$V_o = 10^3 V_s = 10V$$

$$V_f = \beta V_o = 900 \times 10^{-6} \times 10 = 9mV$$

$$V_i = V_s - V_f = 1mV$$

$$(e) A = 8000$$

$$A_f = \frac{8000}{1 + 8000(900 \times 10^{-6})} = 975.6$$

$$\frac{\Delta A_f}{A_f} = \frac{975.6 - 1000}{1000} \equiv -2.44\%$$

**8.4**

All output voltage is fed back  $\therefore \beta = 1$

$$A_f = \frac{100}{1 + 100 \times 1} = 0.99$$

$$1 + A\beta = 1 + 100 \times 1 = 101 \equiv 40.1 \text{dB}$$

$$V_o = 0.99V_s = 0.99V$$

$$V_i = V_s - V_o \beta = 1 - 0.99 = 10mV$$

$$A = 90 \Rightarrow A_f = \frac{90}{1 + 90 \times 1} \approx 0.989$$

$$\frac{\Delta A_f}{A_f} = \frac{0.989 - 0.99}{0.99} \equiv -0.1\%$$

**8.5**

$$V_s = 1V, V_i = 10mV, V_o = 10V$$

$$A_o = (V_o/V_i) = 10V/10mA = 1000$$

$$V_i = V_s - V_f = V_s - \beta V_o = 1 - \beta 10$$

$$\Rightarrow \beta = (1 - 10^2)10 = 0.099$$

**8.6**

$$A_f = \frac{A_o}{1 + A_o \beta} = \frac{1}{1/A_o + \beta} = \frac{1}{\beta(1 + 1/A_o \beta)}$$

so  $A_f + 1/\beta$  will be within  $x\%$  when

$$1/(A_o \beta) = 0.01 \times x$$

$$(a) \text{For } 1\%: A_o \beta = 1/0.01 = 100$$

Many possible solutions.

$$\text{Let } A_o = 10^5 \text{ & } A_o \beta = 100 \Rightarrow \beta = 10^{-3}$$

$$(b) \text{For } 5\%: A_o \beta = 1/0.05 = 20$$

$$\text{Let } A_o = 10^5 \text{ & } A_o \beta = 20 \Rightarrow \beta = 2 \times 10^{-4}$$

$$(c) \text{For } 10\%: A_o \beta = 1/0.1 = 10$$

$$\text{Let } A_o = 10^5 \text{ & } A_o \beta = 10 \Rightarrow \beta = 10^{-4}$$

$$(d) \text{For } 50\%: A_o \beta = 1/0.5 = 2$$

Let  $A_0 = 10^5$ ;  $A_0\beta = 2 \Rightarrow \beta = 2 \times 10^{-5}$

% error	$A_0$	$A_0\beta$	$1+A_0\beta$
1	$10^5$	100	101
5	$10^5$	20	21
10	$10^5$	10	11
50	$10^5$	2	3

8.7.

$0 \leq \beta \leq 1$  Linear

$$(a) \text{ For } A_0 = 1: A_{f1} = \frac{A_0}{1+A_0\beta} = \frac{1}{1+0} = 1 \text{ V/V}$$

$$A_{f2} = \frac{1}{1+1 \times 0.5} = 0.667 \text{ V/V}$$

$$A_{f3} = \frac{1}{1+1 \times 1} = 0.5 \text{ V/V}$$

$$(b) \text{ For } A_0 = 10: A_{f1} = \frac{10}{1+0} = 10 \text{ V/V}$$

$$A_{f2} = \frac{10}{1+10/2} = 1.6 \text{ V/V}$$

$$A_{f3} = \frac{10}{1+10 \times 1} = 0.909 \text{ V/V}$$

$$(c) \text{ For } A_0 = 100: A_{f1} = \frac{100}{1+0} = 100 \text{ V/V}$$

$$A_{f2} = \frac{100}{1+100/2} = 1.96 \text{ V/V}$$

$$A_{f3} = \frac{100}{1+100} = 0.99 \text{ V/V}$$

$$(d) \text{ For } A_0 = 10^4: A_{f1} = \frac{10^4}{1+0} = 10^4 \text{ V/V}$$

$$A_{f2} = \frac{10^4}{1+10^4/2} = 1.99 \text{ V/V}$$

$$A_{f3} = \frac{10^4}{1+10^4} = 0.9999 \text{ V/V}$$

8.8

$A_0: 2 \text{ mV} \rightarrow 10 \text{ V}$

$$A_0 = 10 \text{ V}/(2 \times 10^{-3}) \text{ V} = 5000 \equiv 74 \text{ dB}$$

$A_F: 200 \text{ mV} \rightarrow 10 \text{ V}$

$$A_F = (10)/(200) = 500 \equiv 54 \text{ dB}$$

$$A_F = \frac{A_0}{1+A_0\beta} = \frac{5000}{1+5000\beta} = 500$$

$$\Rightarrow 1 + 5000\beta = 10$$

$$\Rightarrow \beta = 9/5000 = 0.0018 \equiv -54 \text{ dB}$$

$$(1+A_0\beta) = 10 \equiv 20 \text{ dB}$$

$$A_0\beta = 5000(9/5000) = 9 \equiv 19.08 \text{ dB}$$

8.9

$$\frac{dA_f}{A_f} = \frac{1}{1+\lambda\beta} \cdot \frac{d\lambda}{\lambda}$$

$$\frac{dA_f/A_f}{d\lambda/\lambda} = \frac{1}{1+\lambda\beta} \equiv -20 \text{ dB}$$

$$\Rightarrow 1 + \lambda\beta = +20 \text{ dB} \equiv 10$$

$$\therefore \lambda\beta = 9$$

$$\text{Require } \frac{1}{1+\lambda\beta} = \frac{1}{2} \Rightarrow \lambda\beta = 1$$

8.10

$A_0 \equiv 1000 \pm 30\%$  want  $A_f = 100 \pm 1\%$

To reduce % change in  $A_0$  we need

$$\frac{1}{1+\lambda\beta} \equiv \frac{1}{30} \Rightarrow A_f = \frac{1000}{30} < 100$$

$$\text{For single stage } A_F = \frac{A}{1+\lambda\beta} \Rightarrow \frac{1}{1+\lambda\beta} = \frac{100}{1000} = \frac{1}{10}$$

$$\text{For two stages } A_2 = \frac{A}{1+\lambda\beta_2} \Rightarrow 10$$

$$\Rightarrow (1+\lambda\beta_2) = 1000/10 = 100$$

Thus each stage has  $\pm 30/100 \% = \pm 0.3\%$

But two such stages may give  $\pm 0.6\% \text{ OK}$

$$[ \text{3 stages } A_3 = A''_3 = 100^{1/3} \\ (1+\lambda\beta_3) = 1000/100^{1/3} \approx 21.5 ]$$

Now each stage has  $\pm 0.14\%$

8.11

Gain desensitivity factor =  $[1+A_0\beta]$

$$A_F = \frac{A_0}{1+A_0\beta} = \frac{10^5}{1+10^5\beta} = 10^3$$

$$\Rightarrow \beta = 0.99 \times 10^{-3}$$

$$\text{From } \frac{dA_F}{A_F} = \frac{1}{1+A_0\beta} \cdot \frac{dA}{A}$$

(a) When A drops 10%

$$\text{then } A_F \text{ drops by } \frac{10\%}{1+A_0\beta} = \frac{10\%}{10^5/10^3} = 0.1\%$$

$$\text{But using } A_F = \frac{A}{1+\lambda\beta} = \frac{0.9 \times 10^5}{1+(0.9 \times 10^5)(99/10^3)} \\ = 998.89$$

$$\text{Corresponding \% } \frac{1000 - 998.89}{1000} = 0.11\% \quad (\text{close})$$

(b) When A drops 30%

$$\text{then } A_F \text{ drops by } \frac{30\%}{100} = 0.3\%$$

$$\text{while } A_F = \frac{0.7 \times 10^5}{1+(0.7 \times 10^5)(99/10^3)} = 995.7$$

$$\text{and } \frac{1000 - 995.7}{1000} = 0.43\% \quad (\text{not so close})$$

8.12

$$A(s) = A_m \frac{s}{s+WL}$$

$$A_f(s) = \frac{A_m \frac{s}{s+WL}}{1 + \frac{A_m s}{s+WL} \beta} = \frac{A_m s}{s+WL+A_m \beta s} \\ = \frac{A_m}{1+A_m \beta} \cdot \frac{s}{s+WL \frac{1}{1+A_m \beta}}$$

Thus

$$A_{mf} = \frac{A_m}{1+A_m \beta}$$

$$w_{Lf} = \frac{WL}{1+A_m \beta}$$

Both decreased by same amount

8.13

$$\text{Worst case: } A_{f1} = \frac{A_0}{1+A_0\beta} = 9.8 \quad (\text{down 2\%})$$

$$\text{full battery: } A_{f2} = \frac{2A_0}{1+2A_0\beta} = 10$$

$$\text{from } A_{f1}: \quad 1+A_0\beta = A_0/9.8$$

$$\therefore \beta = \frac{1}{9.8} - \frac{1}{A_0}$$

$$\text{then } A_{f2} = \frac{2A_0}{1+2A_0\left[\frac{1}{9.8} - \frac{1}{A_0}\right]} = 10$$

$$\Rightarrow 1+2A_0\left[\frac{1}{9.8} - \frac{1}{A_0}\right] = \frac{2A_0}{10}$$

$$\Rightarrow 2A_0\left[\frac{1}{9.8} - \frac{1}{A_0}\right] = 2-1$$

$$\therefore 2A_0 = 490$$

$$\left[ \begin{array}{l} \text{Check } \frac{2A_0}{1+2A_0\left[\frac{1}{9.8} - \frac{2}{2A_0}\right]} = 10 \\ \qquad \qquad \qquad \boxed{\beta \text{ const}} \\ \frac{A_0}{1+A_0\left[\frac{1}{9.8} - \frac{1}{A_0}\right]} = 9.8 \end{array} \right]$$

8.14

$$A_f = \frac{A_0}{1+A_0\beta} = 10 = \frac{100}{1+100\beta}$$

$$\therefore (1+A_0\beta) = 100/10 = 10$$

$$f_L' = f_L / (1+A_0\beta) = 100/10 = 10 \text{ Hz}$$

$$f_H' = f_H (1+A_0\beta) = 100K \times 10 = 1000 \text{ kHz}$$

8.15

Need pole at 0.5 MHz, any others should be well above for stability.

$$f_H = 10 \text{ kHz} \quad \text{and} \quad f_{Hf} = 500 \text{ kHz}$$

$$\therefore \text{need } (1+A\beta) = 500/10 = 50$$

$$\text{thus } A_{f1} = \frac{A}{1+A\beta} = \frac{1000}{50} = 20$$

$$\text{and } \beta = (50-1)/1000 = 4.9 \times 10^{-3}$$

Subsequent stages must provide gain

$$A_{\text{total}} / A_{f1} = 1000/20 = 50$$

$$\text{Let } f_{H2} = 3 \times f_{H1} = 3 \times 500 = 1500 \text{ kHz}$$

$$\text{needs } (1+A\beta_2) = \frac{1500}{10} = 150$$

$$\text{then } A_{f2} = \frac{1000}{150} = 6.67$$

$$\text{and } \beta_2 = \frac{150-1}{1000} = 0.149$$

This leaves stage 3 with  $A_{f3} = \frac{50}{6.67} = 7.5$   
this gain needs  $1 + A_3\beta = \frac{1000}{7.5} = 133.3$

and  $f_{Hf3} = 10k(1 + A_3\beta) = 1.33 \text{ MHz}$

this is safely well above 500 kHz

and  $\beta_3 = 0.132$

The actual  $f_{Hf}$  is given by

$$f_{Hf} = \frac{1}{[(\frac{1}{0.5})^2 + (\frac{1}{1.5})^2 + (\frac{1}{1.33})^2]}$$

$= 0.447 \text{ MHz}$  close!

Using a 4 stage amplifier would allow lower stage gains and higher  $f_H$ 's

8.16

$$V_o = V_s \frac{A_1 A_2}{1 + A_1 A_2 \beta} + V_N \frac{A_1}{1 + A_1 A_2 \beta}$$

$$\text{and } \frac{S}{N} = \frac{V_s}{V_N} A_2$$

if we raise  $V_s$  by factor  $A_2$  then

S/N is improved by  $A_2$  also

To reduce  $V_N$  from  $\pm 1\text{v}$  to  $\pm 1\text{mV}$  we must have  $A_2 = 1000$ , then

$$V_{os} = \frac{A_1 A_2}{1 + A_1 A_2 \beta} = \frac{1000 \times 0.9}{1 + 1000 \times 0.9 \beta} = 10$$

$$\Rightarrow \beta = 0.099$$

if  $A_2 = 100$ ;  $V_N = \pm 10\text{mV}$

$$V_{os} = \frac{100 \times 0.9}{1 + 100 \times 0.9 \beta} = 10$$

$$\Rightarrow \beta = 0.089$$

if  $A_2 = 10$ ;  $V_N = \pm 100\text{mV}$

$$A_1 A_2 = 0.9 \times 10 \rightarrow V_o = 9\text{v} < 10\text{v}!!$$

8.17

$$A_F = \frac{A_U}{1 + A_U \beta}$$

$$\frac{\Delta A_F / A_F}{\Delta A_U / A_U} = \frac{1}{1 + A_U \beta}$$

then for 1% change in  $A_F$  for 90% change in  $A_U$

$$1 + A_U \beta = 90 \Rightarrow A_U \beta = 89$$

$$\text{Then } A_F = 100 = \frac{A_U}{90} \Rightarrow A_U = 9000$$

Let  $A_U \rightarrow 10 \times A_U$  then for  $\beta = 89/A_U$

$$A_F = \frac{9000 \times 100}{1 + (9000 \times 10)(89/9000)} = 101.12$$

Therefore, select  $A_U \geq 9 \times 10^4$

$$\text{and } \beta = 89/A_U$$

If  $A_U \rightarrow 100 A_U$  + same  $\beta$ .

$$A_F = \frac{9000 \times 100}{1 + (9000 \times 100)(89/9000)} \approx \frac{9000 \times 100}{8900} \approx 101.12$$

if  $A_U \rightarrow \infty$

$$A_F \Rightarrow A_U / (A_U \beta) \Rightarrow 1/\beta$$

$$\approx 9000/89 = 101.12$$

Ideally select  $A_U$  as high as possible

8.18

$A_1$  has  $f_{H1}$  high,  $A_2$  has  $A_m = 10 \text{ v/V}$  with  $f_L = 80 \text{ Hz}$ ,  $f_H = 8 \text{ kHz}$ .

$$A_F = \frac{A_1 A_2}{1 + A_1 A_2 \beta} = 100$$

$$\text{Require } f_{Hf} = 40 \text{ kHz} = 8(1 + A_1 A_2 \beta)$$

$$\therefore 1 + A_1 A_2 \beta = 40/8 = 5$$

$$\text{and } A_F = \frac{A_1 A_2}{5} = 100 \Rightarrow A_1 A_2 = 500$$

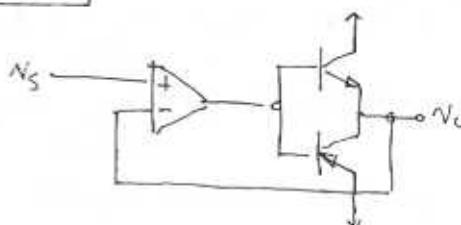
$$\Rightarrow A_1 = 500/A_2 = 500/10 = 50$$

$$1 + A_1 A_2 \beta = 5 \Rightarrow \beta = 4/A_1 A_2 = 4/500$$

$$\therefore \beta = 0.08$$

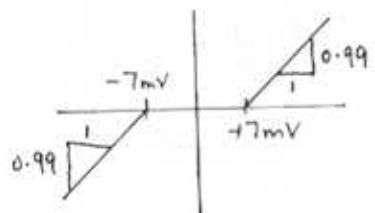
$$f_{Lf} = f_L / (1 + A_1 A_2 \beta) = 80/5 = 16 \text{ Hz}$$

8.19



Dead band will be narrowed by the factor  $1 + A\beta = 1 + A$  since  $\beta = 1$  and since  $A \gg 1$ ,  $1 + A \rightarrow A$   
 $\therefore$  new limits are  $\pm \frac{0.7}{A} = \pm \frac{0.7}{100}$

$$= \pm 7 \text{ mV}$$



$$\text{New slope} \equiv \text{gain} = A_f = \frac{A}{1+A}$$

$$\Rightarrow \frac{100}{1+100} = 0.99$$

8.20

For  $A = N_0/N_1 = 10^2$  (select lowest  $N_0$ ) to reduce % change in gain by factor of 10

$$1 + A\beta = 10 \Rightarrow \beta = 9/10^2$$

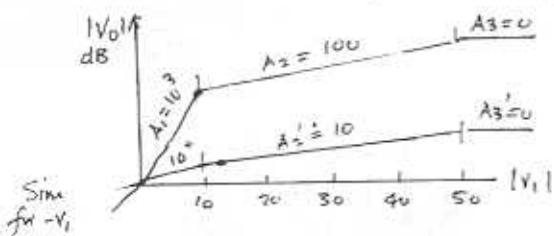
For  $A_2 = 10^2$ :  $A_{2F} = 10^3/10 = 10$

For  $A_1 = 10^3$ :  $A_{1F} = \frac{A}{1 + A\beta}$

$$\therefore A_{1F} = \frac{10^3}{1 + 10^3(9/10^2)} = \frac{10^3}{91} = 10.98$$

For  $A_3 = 0$ : stays saturated

[For 10mV in and  $A_1 = 10^3$ ,  $V_0 = 10 \text{ V}$ ?]  
[For 10mV in and  $A_2 = 10^2$ ,  $V_0 = 1 \text{ V}$ ?]



8.21

$$V_S = 100 \text{ mV}, V_f = 95 \text{ mV}, V_0 = 10 \text{ V}$$

$$V_i = V_S - V_f = 100 - 95 = 5 \text{ mV}$$

$$AV = \frac{V_0}{V_i} = \frac{10 \text{ V}}{5 \text{ mV}} = 2 \times 10^3 \text{ V/V}$$

$$\beta = \frac{V_f}{V_0} = \frac{95 \times 10^{-3}}{10} = 9.5 \times 10^{-3} \text{ V/V}$$

$$= 0.0095$$

8.22

$$I_S = 100 \mu\text{A}, I_F = 90 \mu\text{A}, I_O = 10 \text{ mA}$$

$$A = \frac{I_O}{I_S - I_F} = \frac{10 \times 10^{-3}}{(100 - 90) \times 10^{-6}} = 10^3 \text{ A/A}$$

$$= 10 \text{ mA/}\mu\text{A}$$

$$\beta = \frac{I_F}{I_O} = \frac{90 \mu\text{A}}{10 \text{ mA}} = 9 \mu\text{A/mA}$$

8.23

For  $r_{01}, r_{02}, P_S$  very large in Fig 8.5

At G1:  $I_S = I_F$  since  $I_{G1} = 0$

$$V_{G1} = [R_1 + R_2] I_S + I_O R$$

$$V_{D1} = -g_{m1} V_{G1} R_{L1}$$

$$= -g_{m1} R_{L1} [I_S (R_1 + R_2) + I_O R_1]$$

$$I_O = g_{m2} (V_{D1} - V_{S2})$$

$$= g_{m2} [(-g_{m1} R_{L1})] [I_S (R_1 + R_2)]$$

$$- I_S R_1 - I_O R_1$$

$$\therefore \frac{I_O}{I_S} = - \frac{g_{m2} [g_{m1} R_{L1} (R_1 + R_2)]}{1 + g_{m2} g_{m1} R_{L1} R_1 + g_{m2} R_1}$$

$$A_f = - \frac{R_1 + g_{m1} R_{L1} (R_1 + R_2)}{R_1 + 1/g_{m2} + g_{m1} R_{L1} R_1}$$

QED

$$R_{IN} \equiv \frac{V_S}{I_S} = I_S [R_1 + R_2] + I_O R_1$$

$$I_S = I_S [R_1 + R_2 + A_f R_1]$$

$$\therefore R_{IN} = R_1 + R_2 + A_f R_1$$

QED

(b) When  $g_{m1} R_{L1} = 100$ ,  $R_L = 10 \text{ k}$ ,  $R_2 = 90 \text{ k}$  and  $g_{m2} = 5 \text{ mA/V}$

$$A_f = - \frac{R_1 + g_{m1} R_{L1} (R_1 + R_2)}{R_1 + 1/g_{m2} + g_{m1} R_{L1} R_1}$$

$$= - \frac{10^4 + 100(100 \times 10^3)}{10^4 + 1000/5 + 100 \times 10^4}$$

$$= - 9.91 \text{ A/A} \approx - 10 \text{ A/A}$$

$$R_{IN} = 10k + 90k - 10k \times 10k \approx 0$$

$$\text{or } 10k + 90k - 9.91 \times 10k$$

$$= 100 - 99.1k = 0.9k$$

$$(c) \beta \equiv \frac{I_F}{I_o} = \frac{R_1}{R_1 + R_2} \Rightarrow \beta = \frac{10}{10+90} = \frac{1}{10}$$

from above  $A_F \approx -10 \equiv -1/\beta$

8.24

series-series

$$I_S = 100 \mu A, V_f = 95 mV, I_o = 10 mA$$

$$V_i = V_S - V_f = 100 - 95 = 5 mV$$

$$V_f = \beta I_o \Rightarrow \beta = \frac{95 mV}{10 mA} = 9.5 V/A$$

$$A \equiv \frac{I_o}{V_i} = \frac{10 mA}{5 mV} = 2 mA/V$$

$$A_F \equiv \frac{I_o}{V_S} = \frac{A}{1 + AB} = \frac{2}{1 + 2(9.5)} = 0.1 mA/V$$

8.25

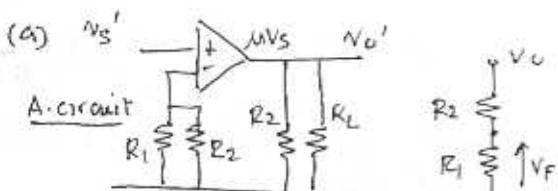
shunt-shunt

$$I_S = 100 \mu A, I_F = 95 mA, V_o = 100V$$

$$A \equiv \frac{V_o}{I_i} = \frac{V_o}{I_S - I_F} = \frac{10}{(100 - 95) \times 10^6} = 2 \times 10^6 V/A$$

$$\beta \equiv \frac{I_F}{V_o} = \frac{95 mA}{100 V} = 9.5 \mu A/V$$

8.26



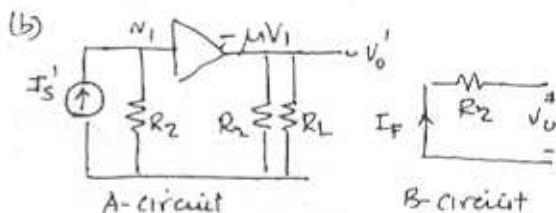
series-shunt; sample  $V_o$ , return  $V_F$

$$\beta \equiv \frac{V_F}{V_o} = \frac{R_1}{R_1 + R_2}$$

$$A \equiv \frac{V_o'}{V_S'} = \mu$$

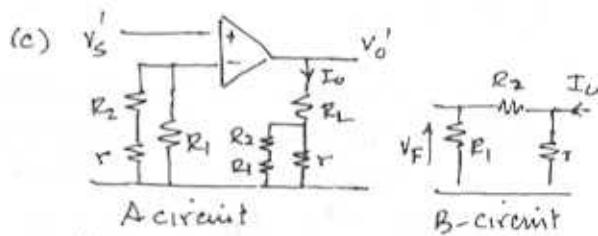
$$A_F \equiv \frac{V_o}{V_S} = \frac{A}{1 + AB} = \frac{\mu}{1 + \mu \frac{R_1}{R_1 + R_2}}$$

$$\Rightarrow \frac{R_1 + R_2}{R_1} = 1/\beta$$



$$\beta \equiv \frac{I_F}{V_o} = -\frac{1}{R_2} \quad A \equiv \frac{V_o}{I_S'} = -\mu R_2$$

$$A_F \equiv \frac{V_o}{V_S} = \frac{A}{1 + AB} = \frac{-\mu R_2}{1 + \mu R_2/R_2} = -\frac{\mu R_2}{1 + \mu} \Rightarrow -R_2 = -1/\beta$$

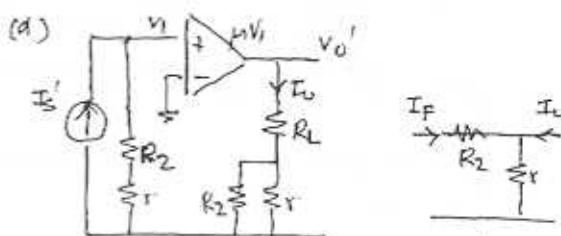


series-series.

$$\beta \equiv \frac{V_F}{I_o} = \frac{R_1 r}{R_1 + R_2 + r} \quad \text{let } r^* = r/(R_1 + R_2)$$

$$A \equiv \frac{I_o}{I_S} = \frac{\mu}{R_L + r^*/(R_1 + R_2)} = \frac{\mu}{R_L + r^*}$$

$$A_F \equiv \frac{A}{1 + AB} = \frac{\mu}{R_L + r^*} \cdot \frac{R_1 r}{1 + \frac{\mu}{R_L + r^*} \cdot \frac{R_1 r}{R_1 + R_2 + r}}$$



shunt-series: return  $I_F$  sample  $I_o$

$$\beta \equiv \frac{I_F}{I_o} = \frac{r}{R_2 + r}, \quad A \equiv \frac{V_o}{I_S'} = \frac{\mu(R_2 + r)}{R_L + r/(R_2 + r)}$$

$$\frac{I_o}{I_s} = \frac{\frac{A(R_2+r)}{R_L+r||R_2}}{1 + \frac{A(R_2+r)}{R_L+r||R_2} \cdot \frac{r}{r+R_2}}$$

$$\Rightarrow \frac{1}{A \gg 1} \cdot \frac{1}{r/(r+R_2)} = \frac{1}{\beta}$$

### Section 8.4 The Series-Shunt Feedback Amplifier

8.27

$$A_F = \frac{A}{1+A\beta} = \frac{10^3 \times 2}{1+2 \cdot 10^3 \times 0.1} = 9.95 \text{ V}$$

$$R_{if} = R_i(1+A\beta) = 1/201 = 201 \text{ k}\Omega$$

$$R_{of} = R_o/(1+A\beta) = 1/201 = 4.975 \text{ k}\Omega$$

8.28

Here  $R_o$  is lowered by amount of feedback

$$\text{i.e., } (1+A\beta) = 80$$

$$\Rightarrow A\beta = 79$$

$$R_o = R_{of}(1+A\beta) = 100 \times 80 = 8 \text{ k}\Omega$$

8.29

$$A_o = 10^4 \text{ V/V} \quad f_t = 1 \text{ MHz}$$

$$20 \text{ dB/dec roll off} \therefore f_p = f_t/A_o = 100 \text{ Hz}$$

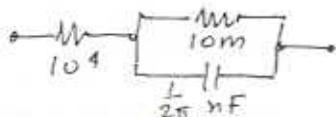
$$A(f) = \frac{A_o}{1 + jf/f_p} = \frac{10^4}{1 + jf/100}$$

$$\text{loop gain} = A(f)\beta = \frac{10^4 (0.1)}{1 + jf/100} = \frac{10^5}{jf + 100}$$

$$Z_{if} = R_i(1+A\beta) = 10^4 \left(1 + \frac{10^3}{1 + jf/100}\right)$$

$$= 10^4 + \frac{10^7}{1 + jf/100} \equiv R_A + \frac{E_B}{1 + jR_E C}$$

Thus



$$Z_{of} = \frac{R_o}{(1+A\beta)} = \frac{10^3}{1 + 10^3/(1+jf/100)}$$

$$\therefore Z_{of} = \frac{1}{\frac{1}{10^3} + \frac{1}{1+jf/100}}$$

$$= \frac{1}{\frac{1}{R_A} + \frac{1}{R_B + j\omega L}}$$

At  $f = 10^3$

$$|Z_{if}(10^3)| = \left| \frac{10^4 + 10^7}{1 + j \frac{10^3}{100}} \right| = 1 \text{ M}\Omega$$

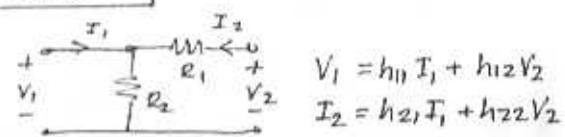
$$|Z_{of}(10^3)| = \left| \frac{1}{\frac{1}{10^3} + \frac{1}{1 + j 10^2}} \right| = 10 \text{ }\Omega$$

At  $f = 10^5$

$$|Z_{if}(10^5)| = \left| \frac{10^4 + 10^7}{1 + j \frac{10^5}{100}} \right| = 14.1 \text{ k}\Omega$$

$$|Z_{of}(10^5)| = \left| \frac{1}{\frac{1}{10^5} + \frac{1}{1 + j 10^4}} \right| = 700 \text{ }\mu\text{V}$$

8.30



$$(a) h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} \text{ }\mu\text{V}$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \frac{R_2}{R_1 + R_2} \text{ }\mu\text{V/V}$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} = -\frac{R_2}{R_1 + R_2} \text{ A/A}$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \frac{1}{R_1 + R_2} \text{ }\text{--V--}$$

$$(b) \beta = \left. \frac{V_1}{V_2} \right|_{I_1=0} = h_{12} = \frac{R_2}{R_1 + R_2}$$

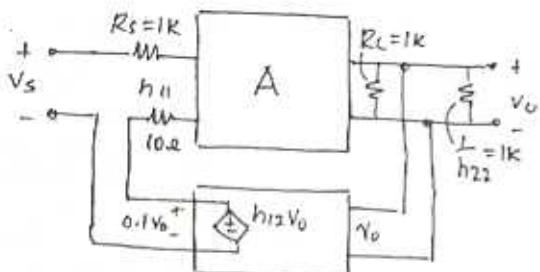
$$\Rightarrow \frac{R_1}{R_2} = \frac{1}{\beta} - 1 = 99$$

$$\Rightarrow R_2 = R_1/99 = 10.1 \text{ }\Omega$$

$$\text{Thus } h_{11} = 10 \text{ }\mu\text{V} ; \quad h_{12} = 0.01 \text{ }\mu\text{V/V}$$

$$h_{21} = -0.01 \text{ }\mu\text{A/V} ; \quad h_{22} = 0.99 \times 10^{-3}$$

(c)



8.31

$$R_{of} = \frac{R_o}{1+AB} = \frac{1000}{1+AB} = 100\Omega$$

$$\Rightarrow 1+AB = 1000/100 = 10$$

$$\Rightarrow A_F = \frac{A}{1+AB} = \frac{100}{10} = 10 \text{ V/V}$$

$$\text{If } B=1: R_{of} \Rightarrow \frac{R_o}{1+A} = \frac{1000}{1+10^4} = 9.9\Omega$$

$$R_i = R_s + r_e + \frac{(R_1||R_2)}{1+\beta}$$

$$= 100 + 250 + 909/101 = 700\Omega$$

$$R_o = R_L || (R_1+R_2) || (r_e + r_{e1}/\beta+1) \\ = 2k || 10k || 4 = 1.67\text{ k}\Omega$$

$$A = \frac{r_{e1}(\beta+1)(r_{e3} + 10k||2k)}{R_s + r_{e1}} \times \frac{2k||10k}{2k||10k+r_{e3}}$$

$$\beta = \frac{R_1}{R_1+R_2} = \frac{1}{1+10} = 0.0909 \text{ V/V}$$

$$A_F = \frac{A}{1+\beta} \doteq \frac{10 \cdot 4}{0.7} \cdot \frac{1.67}{1 + 23.15 \times 0.0909} \\ = 7.45 \text{ V/V}$$

$$R_{if} = R_{in}(1+\beta) = 700(3.1) = 2170\Omega$$

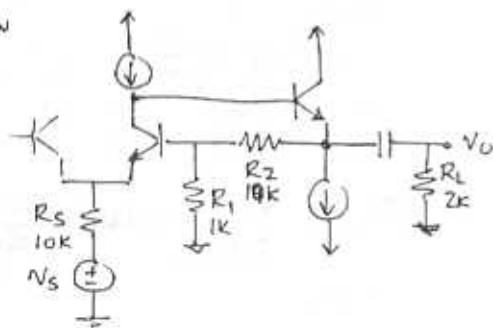
Neglecting base currents since  $\beta \gg 1$

DC voltage at input =  $I_B R_S = 0.01\text{ V}$

DC voltage at output = 0 V

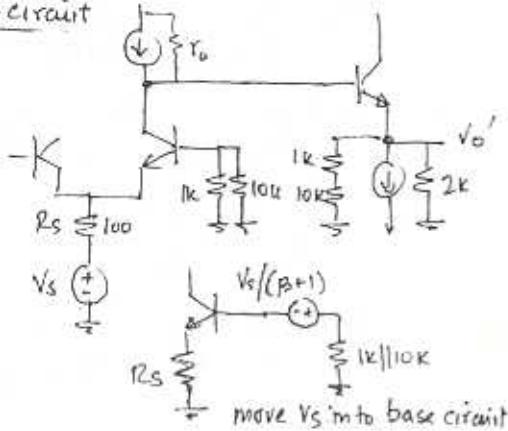
8.32

Redraw



$$r_{e1} = V_T/I_E = 25\text{ mV}/0.1\text{ mA} \approx 250\Omega$$

$$r_{e2} = 25/1 = 25\Omega$$

A-circuit

8.33

$Q_3 + Q_4$  form current multiplier  
 $\times 120/40 = \times 3$

$$g_{m1} = 2\sqrt{\frac{1}{2}120(20/1)100} \doteq 693\mu\text{A/V}$$

$$g_{m5} = 2\sqrt{\frac{1}{2}60(20/1)1000} \doteq 1550\mu\text{A/V}$$

$$g_{m3} = 2\sqrt{\frac{1}{2}60(40/1)100} \doteq 693\mu\text{A/V}$$

$$g_{m4} = 2\sqrt{\frac{1}{2}60(120/1)300} \doteq 2078\mu\text{A/V}$$

$$g_{m2} = g_{m1} = 693\mu\text{A/V}$$

$$r_{o1} = 24/100 \Rightarrow 240\text{ k}\Omega$$

$$r_{o2} = 24/100 \Rightarrow 240\text{ k}\Omega$$

$$r_{o3} = 24/100 \Rightarrow 240\text{ k}\Omega$$

$$r_{o4} = 24/300 \Rightarrow 80\text{ k}\Omega$$

$$r_{o5} = 24/1000 \Rightarrow 24\text{ k}\Omega$$

(c) Open-loop gain  $A_\beta \approx g_{m1}(r_{o1}||r_{o3}) \times 1$

$$\left[ \frac{(3 \times g_{m1})}{3} \left( \frac{r_{o4}}{3} \right) = g_{m1} r_{o1} \right]$$

$$\beta = 1: \therefore A = g_{m1}(r_{o2}||r_{o3})$$

$$\Rightarrow A \approx 693 \times 120 \times 10^{-3} = 84$$

$$(d) A_F = \frac{A}{1+AB} = \frac{84}{1+84} = 0.988 \text{ v/v}$$

$$R_o = r_{os} \parallel r_{os} = 12 \text{ k}\Omega$$

$$R_{of} = R_o / (1+AB) = 12/85 = 140 \text{ }\Omega$$

$$R_{if} = \infty \therefore R_{IN} = \infty$$

$$R_{of} = \frac{R_o}{1+AB} = \frac{622.2}{1+3.13} = 150.6 \text{ }\Omega$$

$$R_{of} = R_{out} \parallel R_L \Rightarrow R_{out} = 163 \text{ }\Omega$$

(e) To obtain  $V_o/V_s = 5$  we could change direct connection from  $Q_{SS}$  to  $Q_{2G}$  by voltage divider  $R_1/(R_1+R_2)$  & change  $\beta$  from 1 to  $1/5 \cdot 3$ . Then

$$A_F = \frac{84}{1+84 \times 1/5 \cdot 3} = \frac{84}{16.8} = 5$$

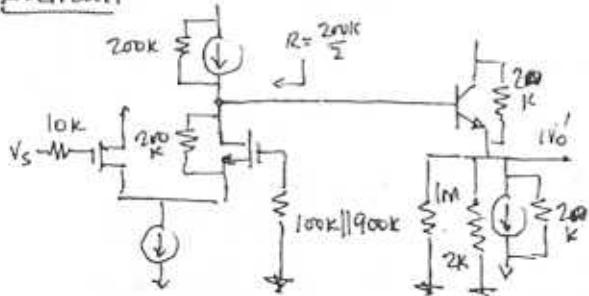
Now  $1+AB = 16.8$

$$R_{of}'' = R_o / (1+AB'') = 12/16.8 = 714 \text{ }\Omega$$

### 8.34

Since  $V_{G1} = 0 = V_{G2}$   
 $\Rightarrow V_{E3} = V_o = 0 \text{ and } V_{B3} = +0.7 \text{ V}$   
 $g_{m1} = g_{m2} = 2\sqrt{\frac{1}{2}k'(W/L)}I = 2\sqrt{\frac{1}{2}1(0.5)} = 1 \text{ mA/V}$   
 $r_{e3} = V_T/SmA = 5 \text{ }\Omega$   
 $r_o = V_A/I = 100/0.5 = 200 \text{ k}\Omega$

A-circuit



$$R_i = \infty$$

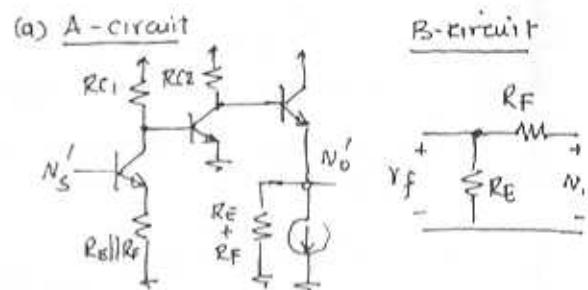
$$R_o = 1 \text{ M} \parallel 2 \text{ k} \parallel \frac{20 \text{ k}}{2} \parallel (r_{e3} + \frac{200 \text{ k}}{\beta+1}) = 622.2 \text{ }\Omega$$

$$A = \frac{100}{(\beta+1)(r_{e3} + 1 \text{ M} \parallel 10 \text{ k} \parallel 2 \text{ k})} \frac{2/g_m}{2/g_m} = 31.3 \text{ V/V}$$

$$\beta = \frac{100}{100 + 900} = 0.1 \text{ V/V}$$

$$A_F = \frac{A}{1+AB} = \frac{31.3}{1+31.3 \times 0.1} = 7.58 \text{ V/V}$$

### 8.35



(b) For  $AB \gg 1$ :  $A_F = \frac{A}{1+AB} \Rightarrow \frac{1}{\beta}$

$$\beta = \frac{R_E}{R_E + R_F} \Rightarrow A_f \approx \frac{R_E + R_F}{R_E} = 1 + \frac{R_F}{R_E}$$

(c)  $R_E = 50 \text{ }\Omega$   
 $\Rightarrow A_F = 1 + R_F/R_E = 25 \text{ V/V}$   
 $\Rightarrow R_F/R_E = 24 \text{ and } R_F = 24R_E = 1200 \text{ }\Omega$

(d)  $I_{Q1} = 1 \text{ mA}, I_{Q2} = 2 \text{ mA}, I_3 = 5 \text{ mA}$   
 $\beta = 100$

$$r_{e1} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \text{ }\Omega, r_{e2} = 12.5 \text{ }\Omega, r_{e3} = 1$$

$$A_1 = \frac{-R_{C1} \parallel r_{e2}}{r_{e1} + R_E \parallel R_F} = -10$$

$$\Rightarrow R_{C1} \parallel r_{e2} = 10(25 + 50 \parallel 12.5) = 730 \text{ }\Omega$$

$$\Rightarrow R_{C1} = 1.75 \text{ k}\Omega$$

$$A_2 = -R_{C2} \parallel [\beta(r_{e1} + R_E + R_F)] = -$$

$$\Rightarrow R_{C2} \parallel 125 \cdot 5 \text{ k} = 5 \times 12.5 = 625 \text{ }\Omega$$

$$\Rightarrow R_{C2} = 628.1 \text{ }\Omega$$

$$A_3 = \frac{R_E + R_F}{r_{e3} + R_E + R_F} = \frac{1.25}{1.255} = 0.996$$

(e)  $\therefore A_1 A_2 A_3 = 10 \times 50 \times 0.996 = 498 \text{ V}$

$$AB = 498(50/1250) = 19.92$$

$$A_F = \frac{A}{1+AB} = 23.8 \text{ V/V}$$

(f)  $R_i = (\beta+1)(r_{e1} + R_E \parallel R_F)$

$$\begin{aligned} \therefore R_i &= 101(25 + 4.8) = 7.37 \text{ k}\Omega \\ R_{if} &= R_i(1 + 19.92) = 154 \text{ k}\Omega \\ R_o &= 1.25k \parallel (r_{e3} + R_{c2} \parallel 101) = 11.12 \text{ }\Omega \\ R_{of} &= \frac{R_o}{1 + 19.92} = 0.53 \text{ }\Omega \end{aligned}$$

### Section 8.5 Series-Series Feedback

8.36

$$\text{For } AB \gg 1 : A_F = \frac{A}{1 + AB} \Rightarrow \frac{1}{\beta} = A_F^*$$

$$\begin{aligned} \text{Here, } \beta &= \frac{V_f}{I_o} = \frac{R_{E2} \times R_{E1}}{R_{E2} + R_E + R_{E1}} \\ &= \frac{100 \times 100}{100 + 640 + 100} = 11.9 \text{ }\Omega \end{aligned}$$

$$\frac{I_o}{V_s} \approx A_F^* \equiv \frac{1}{\beta} = 84 \mu\text{A/V} \quad (\text{cf. 8.3.7})$$

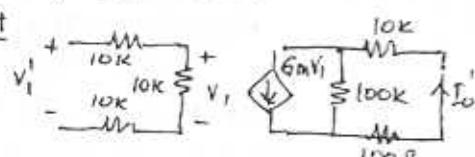
$$\begin{aligned} \frac{V_o}{V_s} &= -\frac{I_o R_{C3}}{V_s} = -84 \times 0.6 \\ &= -50.4 \text{ V/V} \quad (\text{cf. 8.3.2}) \end{aligned}$$

8.37

$$A = G_m = 100 \text{ mA/V} \quad \beta = 0.1 \text{ V/mA}$$

$$r_{in} = 10 \text{ k}\Omega, r_{out} = 100 \text{ k}\Omega$$

A-circuit



$$V_i = V_i' \frac{10}{10+10+10} = V_i'/3$$

$$I_o' = \frac{G_m V_i' 100}{100 + 10 + 0.1} = 30.28 V_i' \text{ mA}$$

$$A = \frac{I_o'}{V_i} = 30.28 \text{ mA/V}$$

$$A_F = \frac{A}{1 + AB} = \frac{30.28}{1 + 30.28(0.1)} = 7.52 \text{ mA/V}$$

$$\begin{aligned} R_i &= R_S + R_{1a} + R_1 = 30 \text{ k}\Omega \\ R_{if} &= R_i(1 + AB) = 120.8 \text{ k}\Omega \\ R_{in} &= R_{if} - R_S = 110.8 \text{ k}\Omega \end{aligned}$$

$$\begin{aligned} R_o &= R_L + R_{op} + R_2 = 110.1 \text{ k}\Omega \\ R_{of} &= R_o(1 + AB) = 443.4 \text{ k}\Omega \\ R_{out} &= R_{of} - R_L = 433.4 \text{ k}\Omega \end{aligned}$$

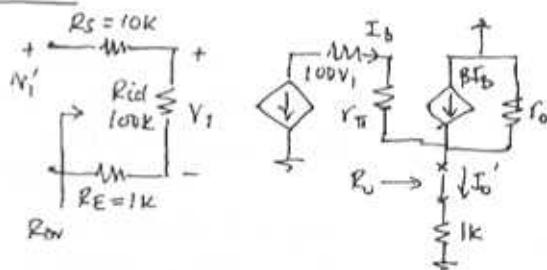
8.38

$$\text{For } AB \gg 1 : I_o/V_s \approx 1/\beta$$

$$R_E \text{ samples } I_o/d \text{ and } V_E = I_o R_E/d$$

$$\beta = \frac{V_f}{I_o} = R_E \Rightarrow A_F \rightarrow \frac{1}{\beta} = \frac{1}{R_E} \text{ qed}$$

A-circuit



$$\text{For } \frac{I_o}{V_s} = \frac{1 \text{ mA}}{V} \approx \frac{1}{\beta} \approx \frac{1}{R_E} \Rightarrow R_E = 1 \text{ k}\Omega$$

$$V_i' = V_i \frac{R_{id}}{R_S + R_{id} + R_E} = \frac{100 V_i}{100 + 10 + 1} = 0.9 V_i'$$

$$I_b = \frac{G_m V_i}{1 + r_\pi + (1 + \beta)(r_o \parallel R_E)}$$

$$I_o' = \frac{(\beta + 1) I_b (R_E \parallel r_o)}{R_E}$$

$$\frac{I_o'}{V_i'} = \frac{1}{R_E} \frac{(\beta + 1)(R_E \parallel r_o) 100 \times 0.9}{1 + r_\pi + (1 + \beta)(R_E \parallel r_o)} \rightarrow \frac{1}{R_E}$$

$$A = \frac{I_o'}{V_i} = \frac{1}{1} \frac{1.101(1 \times 100) \times 9}{1 + 2.5 + 1.01(1 \parallel 100)} = 87 \text{ mA/V}$$

$$A_F = \frac{I_E}{V_s} = \frac{87}{1 + 87} = 0.989 \text{ mA/V}$$

$$\text{Thus } \frac{I_o}{V_s} = \frac{\beta}{\beta + 1} \cdot \frac{I_E}{V_s} = 0.989 \times 0.99$$

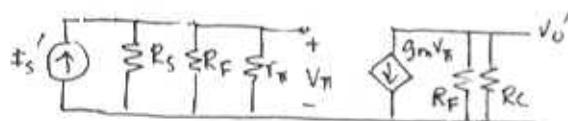
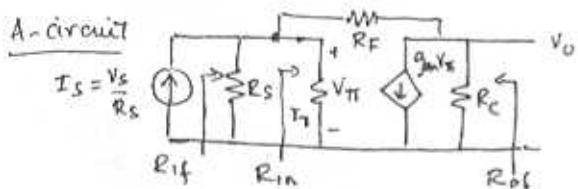
$$= 0.98 \text{ mA/V}$$



## Section 8.6 The Shunt-shunt and Shunt Series Feedback Amplifiers

8.41

A-circuit

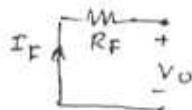


$$V_{T\pi} = I_s (R_s \parallel R_F \parallel Y_{T\pi})$$

$$A = \frac{V_o'}{I_s'} = -g_m (R_s \parallel R_F \parallel Y_{T\pi}) (R_F \parallel R_c)$$

B-circuit

$$\beta \equiv \frac{I_F}{V_o} = -\frac{1}{R_F}$$



$$A_F \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta} = \frac{-g_m R^*}{1 + g_m R^*/R_F}$$

$$\rightarrow -\frac{g_m R^*}{g_m R^*} \cdot R_F = -R_F$$

$$A_V = \frac{V_o}{I_s R_s} \approx -\frac{R_F}{R_s} \quad \text{QED}$$

For  $R_s = 10k$ ,  $R_F = 47k$ ,  $R_c = 4.7k$

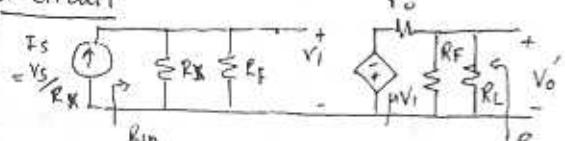
$$A_V \approx -\frac{47k}{10} = -4.7 \text{ v/v}$$

For  $A_V = -7.5 \text{ v/v}$

$$R_F = A_V R_s = 7.5 \times 10 = 75k$$

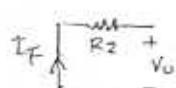
8.42

A-circuit



B-circuit

$$\beta \equiv \frac{I_F}{V_o} = -\frac{1}{R_F}$$



Here let  $R_x = R_s + R_i$  and  $\mu = g_m r_o$

$$A \equiv \frac{V_o'}{I_s'} = -[R_x \parallel R_2] [R_2 \parallel r_o] g_m$$

$$A_F = \frac{V_o}{I_s} = \frac{A}{1 + AB} = -\frac{g_m [R_x \parallel R_2] [R_2 \parallel r_o]}{1 + (\Delta) (-1/R_2)}$$

$$= -g_m \frac{R_x R_2}{R_x + R_2} \cdot \frac{R_2 r_o}{R_2 + r_o}$$

$$1 + g_m \frac{R_x R_2}{R_x + R_2} \cdot \frac{R_2 r_o}{R_2 + r_o} \left[ \frac{-1}{R_2} \right]$$

$$\xrightarrow{\text{large } R_2} -\frac{R_2}{R_2} = -\frac{1}{\beta}$$

$$\text{Thus } \frac{V_o}{I_s R_x} = \frac{V_o}{V_i} = \frac{1}{\beta R_x} = -\frac{R_2}{R_i + R_2}$$

$$R_{in} = (R_s + R_i) \parallel R_2 = (10k + 1M) \parallel 4.7M$$

$$= 1.01M \parallel 4.7M = 0.83M \text{ \Omega}$$

$$\text{Now } g_m = \frac{2 I_b}{V_{ov}} = \frac{2 \times 1}{0.8 - 0.6} = \frac{2}{0.2} = 10 \text{ mA/V}$$

$$r_o = \frac{V_A}{I} = \frac{30V}{1 \text{ mA}} = 30 \text{ k}\Omega$$

$$\text{Hence } A = -[(R_s + R_i) \parallel R_2] [R_2 \parallel r_o] g_m$$

$$= -10 [1.01M \parallel 4.7M] [4.7M \parallel 0.03M]$$

$$= -10 [831k \parallel 29k] \text{ v/mA}$$

$$= -24100$$

$$1 + AB = 1 + (24100 \times \frac{1}{4700}) = 6.13$$

$$A_F = \frac{-24100}{6.13} = -3931 \text{ k}\Omega$$

$$\text{Hence } \frac{V_o}{V_i} = \frac{V_o}{I_s (R_s + R_i)} = -\frac{3931k}{1010} \approx -3.89 \text{ v/v}$$

$$R_o' = R_2 \parallel r_o = 4.7M \parallel 30k = 29.8k\Omega$$

$$R_{of}' = R_o' / (1 + AB) = 29.8 / 6.13 = 4.86k\Omega$$

$$R_i' = (R_s + R_i) \parallel R_2 = 1010 \parallel 4.7 = 0.83M\Omega$$

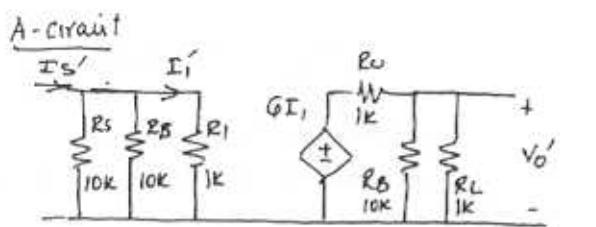
$$R_{if}' = R_i' / (1 + AB) = 0.83 / 6.13 = 136k\Omega$$

$$R_{in}' = R_{if}' - R_i = 136k - 10k = 126k$$

8.43

$$G_m = 100 \text{ mA/v}, R_{input} = 1k, R_{output} = 1k$$

$$R_i = 10k, R_2 = 100, R_s = 10k, R_L = 1k$$



$$\beta = 0.1$$

$$A \equiv \frac{V_O'}{I_S'} = \frac{(R_S || R_B) G}{(R_S || R_B) + R_1} \cdot \frac{(R_B || R_L)}{(R_B || R_L) + R_0}$$

$$= \frac{(10 || 10) 100}{(10 || 10) + 1} \cdot \frac{(10 || 1)}{(10 || 1) + 1} = 75.75 \text{ V/mA.}$$

$$A_F \equiv \frac{A}{1+AB} = \frac{75.75}{1+75.75 \times 0.1} = 8.83 \text{ V/mA}$$

$$(1+AB) = 8.575$$

$$R_i = R_S || R_B || R_1 = 833 \Omega$$

$$R_{if} = R_i / (1+AB) = 97.1 \Omega$$

$$R_{in} || R_S = R_{if} \rightarrow R_{in} \approx 89.7 \Omega$$

$$R_o' = (R_0 || R_B || R_L) = 476 \Omega$$

$$R_{of} = R_o' / (1+AB) = 55.5 \Omega$$

$$R_{out} || R_L = R_{of} \rightarrow R_{out} \approx 58.76 \Omega$$

$$AB = 56 \Rightarrow 1+AB = 57$$

$$A_F = \frac{-555.5}{1+555.5 \times 0.1} = -9.8 \text{ mA}$$

$$R_i' = R_F + R_S = 100 \Omega$$

$$R_{if} = R_i' / (1+AB) = 1.58 \text{ k}\Omega$$

$$R_{of} = (R_F || R_S) + R_{o2}$$

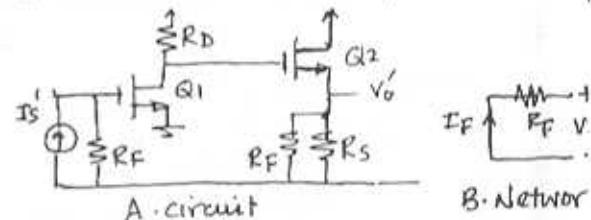
$$= 9\Omega + 20\Omega = 29 \text{ k}\Omega$$

$$R_o \approx r_o (1 + g_m R_{of}) =$$

$$= 20 \Omega (1 + 5 \times 29) = 2.92 \text{ M}\Omega$$

8.45

Here shunt-shunt : sample  $V_o$  return  $I_f$



$$A = \frac{V_O'}{I_S'} = -g_m R_D R_F \text{ (neglecting } R_i)$$

$$\beta \equiv \frac{I_F}{V_o} = -1/R_F, \quad AB = g_m R_D$$

$$A_F \equiv \frac{V_o}{I_S} = \frac{A}{1+AB} = \frac{-g_m R_D R_F}{1 + g_m R_D R_F / R}$$

$$= \frac{-5 \times 10 \times 90}{1 + 5 \times 10} = -4500 / 51 = -88.2$$

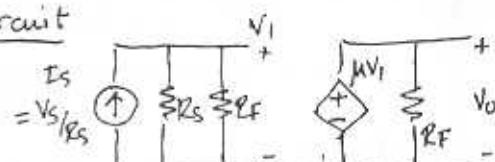
$$R_i' = R_F = 90 \Omega \rightarrow R_{if} = R_i' / (1+AB) = 1.76$$

$$R_o' = R_o || R_F || R_S = 20 || 90 || 20 = 6.2 \text{ k}\Omega$$

$$\Rightarrow R_{of} = R_o' / (1+AB) = 121.6 \Omega$$

8.46

A-circuit



$$A \equiv \frac{I_o'}{I_S'} = \frac{-g_m R_D (R_S + R_F)}{(R_S || R_F)} = -555.5$$

$$\beta \equiv \frac{I_F}{I_o} = \frac{R_S}{R_F + R_S} = \frac{10}{100} = 0.1$$

$$A_F \equiv \frac{I_o}{I_S} = \frac{-g_m R_D (R_F + R_S)}{(R_S || R_F)}$$

$$\frac{1 + g_m R_D (R_F + R_S) / R_S}{(R_S || R_F) (R_F + R_S)}$$

$$\rightarrow \frac{R_F + R_S}{R_S} = 1 + R_F / R_S$$

$$\begin{aligned}\beta &= -1/R_F \\ V_D &= -\mu V_I R_F \\ V_I &= I_S (R_S \parallel R_F) \\ \therefore V_D &= -\mu I_S (R_S \parallel R_F) R_F \\ A &\equiv \frac{V_O}{I_S} = -\mu \frac{R_S R_F}{(R_S + R_F)} R_F\end{aligned}$$

$$\begin{aligned}A_F &= \frac{A}{1 + A\beta} = -\frac{\mu (R_S \parallel R_F) R_F}{1 + \mu (R_S \parallel R_F) R_F / R_F} \\ &= -\frac{\mu (R_S \parallel R_F) R_F}{1 + \mu (R_S \parallel R_F)} \\ \frac{V_O}{V_S} &= \frac{V_O}{I_S R_S} = -\frac{\mu (R_S \parallel R_F) R_F}{[(R_S + R_F) + \mu (R_S R_F)] R_S} \\ \Rightarrow & -\frac{\mu (R_S R_F)}{\mu (R_S R_F)} \cdot \frac{R_F}{R_S} = -\frac{R_F}{R_S} \\ M \gg 1 &\quad \text{QED}\end{aligned}$$

(b) For circuit of Fig P8.46(b)

$$\begin{aligned}I_B &= V_{cc} \frac{R_o}{10+15} - V_{BE} \\ &= \frac{(15)(10) - 0.7}{(15)(10) + (4.7 \times 10)} = \frac{5.3V}{480.7K} \approx 0.011 \text{ mA}\end{aligned}$$

$$I_C = 100 I_B = 1.1 \text{ mA}$$

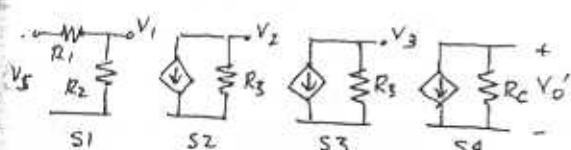
$$r_e = V_T / I = 22.6 \Omega$$

$$r_{\pi} = (\beta + 1) r_e = 2.286 K$$

$$g_m = \beta / r_{\pi} = 100 / 2.286 K \approx 43.7 \text{ mA/V}$$

$$R_{IN} = (15 \parallel 10 \parallel 2.286) K \approx 1.5 K$$

$$R_C \parallel R_B / r_{\pi} = (7.5 \parallel 6 \parallel 2.286) \approx 1.35 K$$



$$\text{For } S_1: R_1 = R_S, R_2 = R_{IN}$$

$$\therefore V_1 = 1.5 / 11.5 \times V_S = 0.13 V_S$$

$$\text{For } S_2: R_3 = R_C \parallel R_B \parallel r_{\pi}$$

$$\therefore \frac{V_2}{V_1} = -g_m R_3 \approx -43.7 \times 1.35 \approx -59 \text{ V/V}$$

$$\text{For } S_3: \text{Same as } S_2 \quad \therefore \frac{V_3}{V_2} = -59 \text{ V/V}$$

$$\text{For } S_4: \frac{V_0}{V_3} = -g_m R_C = -43.7 \times 7.5$$

$$\frac{V_0}{V_S} = -327.75 \text{ V/V}$$

$$\therefore \frac{V_0}{V_S} = -0.13 \times 59 \times 59 \times 327.75$$

$$\rightarrow \frac{V_0}{V_S} = -1.488 \times 10^5$$

Because we have ignored  $r_o$  etc let us estimate  $V_0/V_S = -1 \times 10^5$  which is quite large.

$$\begin{aligned}\text{Then } A_F &= \frac{A}{1 + A\beta} \approx 100 \text{ needed} \\ &= \frac{10^5}{1 + 10^5 \beta} \approx \frac{1}{\beta} = 100\end{aligned}$$

Select  $R_F$  so that  $R_F / R_S = 100$   
 $\rightarrow R_F = 100 \times 10K = 1M\Omega$   
 We can ignore loading effect of  $R_F$  in A-circuit.  $R_L$  will cause loading  
 $\text{of } R_C \quad \therefore V_L = (R_L / (R_C + R_L)) V_0$   
 $= (1/8.5) = 0.11 V_0$

Now  $A_0 \approx 10^4$ .

$$A'_F = \frac{10^4}{1 + 10^4 / 10^4} \approx 99.00$$

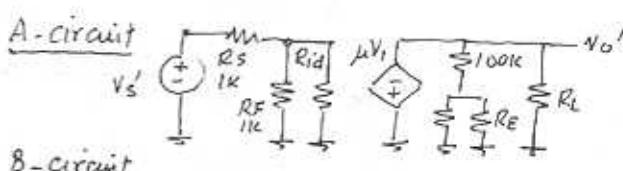
### 8.47

(a) To lower  $R_{IN}$  and raise  $R_{out}$   
 SHUNT - SERIES

(b) To raise  $R_{IN}$  and  $R_{out}$   
 SERIES - SERIES

(c) To lower  $R_{IN}$  and  $R_{out}$   
 SHUNT - SHUNT

### 8.48



$$\begin{aligned}\text{A-circuit: } I_F &\approx 100K \quad \phi = \frac{I_F}{V_0} = \frac{100K \parallel 1K}{100K \parallel 1K + 1K} \frac{1}{100K} \\ &= -4.98 \mu A/V\end{aligned}$$

$$V_1 = (1 \parallel 100 \parallel 1.99) \times 10^3 I_S' = 661.2 I_S'$$

$$\frac{V_o'}{I_S'} = -661.2 (10)^4 \frac{2K \parallel 100.5K}{2K \parallel 100.5K + 1K}$$

$$= -4.38 \times 10^6 = -4.38 M\Omega$$

$$\Rightarrow \frac{V_o}{V_S} = \frac{A}{1+AB} = \frac{-4.38 \times 10^6}{1 + 4.38 \times 4.98}$$

$$= -191.86 \times 10^3 \text{ V/A}$$

$$\frac{V_o}{V_S} = \frac{V_o}{I_S R_S} = -191.86 \text{ V/V}$$

$$R_i = (1 \parallel 100 \parallel 1.99) K = 661.2 K$$

$$R_{if} = R_i / 22.8 = 29 = R_{in} \parallel R_S$$

$$\Rightarrow R_{in} = 29.9 \Omega$$

$$R_o = 1K \parallel 100.5K \parallel 2K = 662.2 \Omega$$

$$R_{of} = R_{out} \parallel R_L = R_o / 22.8 = 29 \Omega$$

$$\Rightarrow R_{out} = 29.5 \Omega$$

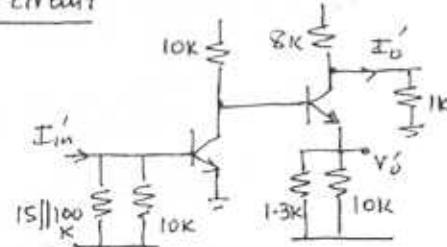
8.49

$$\beta = -1/R_F = -10^{-4}$$

$$I_{E1} = I_{E2} = 1 \text{ mA} \rightarrow r_{e1} = r_{e2} = 25 \Omega$$

$$r_{\pi 1} = r_{\pi 2} = 2.5 \text{ k}\Omega, r_o = 100 \text{ k}\Omega$$

A - circuit



$$\mu = \frac{V_{E2}}{V_{B1}} = \frac{-10K \parallel r_o \parallel \beta (1.3K \parallel 100K)}{25} \times \frac{1.3K \parallel 10K}{25 + 1.3K \parallel 10K}$$

$$= -337 (0.979) = -330 \text{ V/V}$$

$$\frac{V_o'}{I_{in}} = (15K \parallel 100K \parallel 10K \parallel 2.5K)(-330)$$

$$= -572 \text{ k}\Omega$$

$$\Rightarrow \frac{V_o}{I_{in}} = \frac{-572}{1 + 572(0.1)} = -9.83 \text{ k}\Omega$$

$$R_{if} = 15K \parallel 100K \parallel 10K \parallel 2.5K = 1.73K$$

$$R_{in} = R_{if} = 1.73K / 58.2 = 29.7 \Omega$$

$$I_{out} = \frac{8K}{1K + 8K} \cdot \frac{V_o}{1.3K \parallel 10K} = 0.773 \times 10^3 V_o$$

$$\Rightarrow \frac{I_o}{I_{in}} = -9.83 \times 0.773 = -7.6 \text{ A/A}$$

$$\text{cf. Ex 8.4} \quad \frac{I_o}{I_{in}} = -7.57 \text{ A/A}, R_{in} = 3 \Omega$$

8.50

$$I_o/I_S = 100 \text{ A/A}, R_{in} = 1K, R_{out} = 10K$$

$$\beta = 0.1 \quad \text{shunt-series topology}$$

$$A_P = \frac{I_o'}{I_S} = 100$$

$$A_F = \frac{A_U}{1 + A_U \beta} = \frac{100}{1 + 100(0.1)} = 9.09 \text{ A/A}$$

$$R_i = 1K \Omega$$

$$\Rightarrow R_{if} = R_i / (1 + AB) = 90.9 \Omega$$

$$R_o = 10K \Omega$$

$$\Rightarrow R_{of} = R_o (1 + AB) = 110K \Omega$$

8.51

$$\text{Neglect } I_{B2}; I_{B1} \approx \frac{200}{100} = 2 \mu\text{A}$$

$$V_{BE} = 0.7V \quad \therefore V_{B1} = +0.7V$$

But no d.c. component in  $V_S$

$$\therefore I_{RS} (\text{into } V_S) = 0.7 / 10K = 0.07 \mu\text{A}$$

$$\text{Thus } I_F = I_{RS} + I_{B1} = 0.07 + 0.002 = 0.072 \mu\text{A}$$

$$V_{E2} = 0.7 + 10 \times 0.072 = 0.7 + 0.72 = 1.42$$

$$I_{C2} = 1.42 / 140 + 0.072 = 10.2 \mu\text{A}$$

$$I_{B2} = I_{E2} / (\beta + 1) = 0.1 \mu\text{A} \approx \frac{1}{2} 200 \mu\text{A}$$

Iterate:

$$I_{B1} = \frac{200 \mu\text{A} - 100 \mu\text{A}}{100} = 0.001 \mu\text{A}$$

$$V_{E2} = 0.7 + 10 \times 0.001 = 1.41V$$

$$I_{B2} = 1.41 / 140 + 0.001 = 10.1 \mu\text{A}$$

$$I_{C2} = 10.1 / 140 = 100 \mu\text{A} \quad \therefore I_{C2} = 100 \mu\text{A}$$

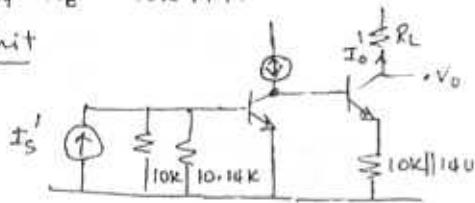
$$V_{BL} = V_{E2} + V_{BE} = 1.41 + 0.7 = 2.11V$$

$$V_o = 10 - 10 \times 500 \Omega = +5V$$

$$r_{\pi} = \frac{V_T}{I_E} = \frac{25mV}{0.1mA} = 250\Omega, r_{e2} = 25\Omega$$

$$\beta = \frac{R_E}{R_F + R_E} = \frac{140}{10k + 140} \approx 0.0138$$

A-circuit



$$V_{B1} = 10.14k \parallel 10k \parallel \beta(250) I_s'$$

$$= 4.2 \times 10^3 I_s'$$

$$\Rightarrow \frac{I_o'}{I_s'} = \frac{4.2 \times 10^3 (\beta+1)(r_{e2} + 10k \parallel 140)}{250} \times \frac{1}{(r_{e2} + 10k \parallel 140)}$$

$$= 1.69 \times 10^3 A/A$$

$$A_F = \frac{I_o}{I_s} = \frac{A}{1+A\beta} = \frac{1.69 \times 10^3}{1 + 1.69 \times 10^3 \times 0.0138} = 69.6$$

$$\Rightarrow \frac{V_o}{V_s} = \frac{I_o R_L}{I_s R_S} = \frac{500 \cdot 69.6}{10^4} = 3.5V/V$$

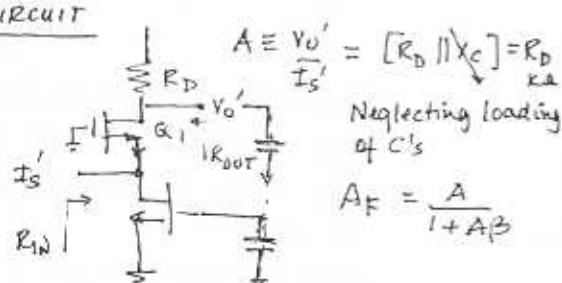
$$R_i = 10k \parallel 10.14k \parallel 25k = 4.2k\Omega$$

$$R_{if} = R_i / (1+AB) = \frac{4.2}{1+23.3} = 172.8\Omega$$

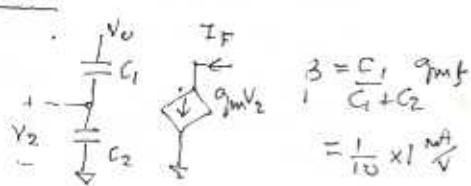
$$\Rightarrow R_{in} = R_{in} \parallel R_S \Rightarrow R_{in} = 175.8\Omega$$

**8.52**

A-circuit



B-circuit



Here,  $g_{m1} = 5mA/V$ ,  $g_{mf} = 1mA/V$

$$R_D = 10k$$

$$\text{Thus } A = 10k\Omega$$

$$A_F = \frac{10k}{1 + 10k(0.01)} = 5k\Omega$$

$$R_{in} = (R_D \parallel r_o) \rightarrow R_D$$

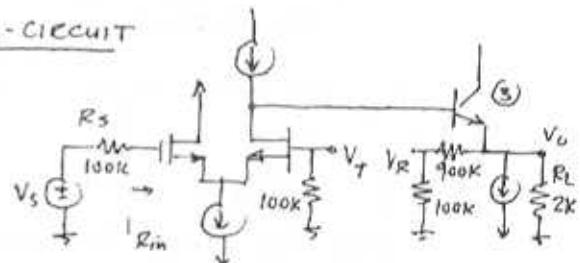
$$R_{if} = R_D / (1+AB) \text{ shunt} = \frac{R_D}{2} = 5k\Omega$$

$$R_{out} = R_D / (1+AB) = \frac{R_D}{2} = 5k\Omega$$

### Section 8.7 Determining the Loop Gain

**8.53**

A-circuit



$$V_o = -\frac{100k / ((\beta+1)(r_{e2} + (1M \parallel 20k) \parallel R_L))}{2/g_m} \times \frac{r_{e2}}{r_{e2} + (1M \parallel 20k) \parallel R_L}$$

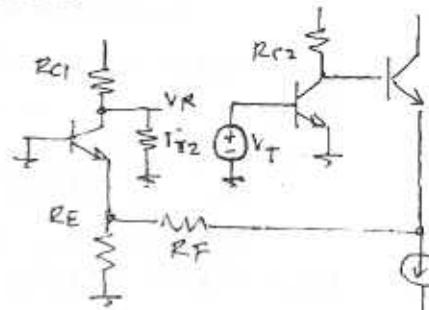
$$= -\frac{100k / 166.5}{2} \times \frac{5}{1665} = -31.25$$

$$R_o = [1M \parallel \frac{20k}{2} \parallel R_L] \parallel [\frac{200k}{2} / (\beta + 1) r_{e2}]$$

$$= 623\Omega$$

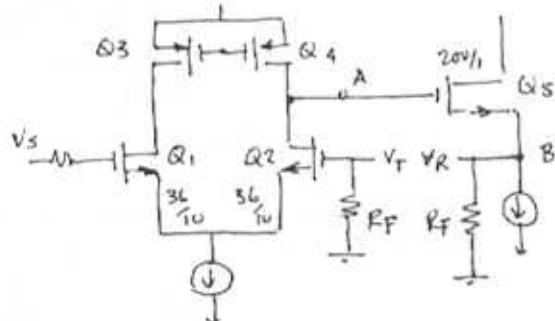
which agree with previous solution

**8.54**



$$\begin{aligned}
 V_R &= V_{C1} \\
 V_{C1} &= -I_{C1}(R_C \parallel r_{\pi}) \\
 I_{C1} &= (\beta/\beta+1) I_{E1} = I_F R_E / (R_E + r_{e1}) \\
 I_F &= I_2 = (\beta+1) I_{B1} \\
 I_{B1} &= \frac{R_C I_{C2}}{R_C + (\beta+1)[r_{e3} + R_F + (R_E \parallel r_{e1})]} \\
 I_{C2} &= g_m V_T \\
 \text{Combining} \\
 V_R &= \frac{g_m^2 R_C (\beta+1)}{R_C + (\beta+1)[r_{e3} + R_F + (R_E \parallel r_{e1})]} \\
 &\quad \times \frac{\alpha R_E (R_C \parallel r_{e2})}{R_E + r_{e1}} \quad \text{QED}
 \end{aligned}$$

8.55



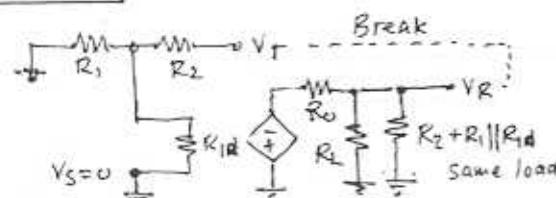
$$V_T = V_{Q2} \text{ and } V_S \rightarrow 0$$

$$V_A = -g_m^2 (r_{Q2} \parallel r_{Q4})$$

$$V_B = V_A \frac{(R_F \parallel r_{Q5})}{(R_F \parallel r_{Q5}) + 1/g_{m5}}$$

$$\begin{aligned}
 AB &= -\frac{V_T}{V_R} = +g_m^2 \frac{(r_{Q2} \parallel r_{Q4}) / (R_F \parallel r_{Q5})}{(R_F \parallel r_{Q5}) + 1/g_{m5}} \\
 &\quad \text{QED}
 \end{aligned}$$

8.56

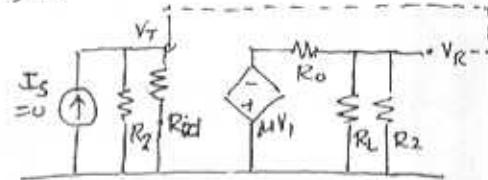


$$\begin{aligned}
 V_R &= -\frac{\mu R_L \parallel (R_2 + R_1 \parallel R_{id})}{R_0 + R_L \parallel (R_2 + R_1 \parallel R_{id})} \cdot \frac{R_1 \parallel R_{id}}{R_2 + R_1 \parallel R_{id}}
 \end{aligned}$$

For  $(R_1 + R_2) \gg R_L$  and  $R_{id} \gg R_1$   
and  $R_0 \ll R_L$

$$\begin{aligned}
 \frac{V_R}{V_T} &\approx -\mu \cdot 1 \cdot \frac{R_1}{R_1 + R_2} = -\frac{\mu R_1}{R_1 + R_2} \\
 &= -AB.
 \end{aligned}$$

(b)  $I_S = 0$

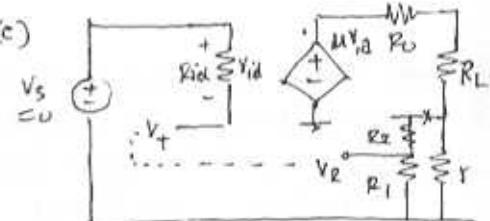


$$\frac{V_R}{V_T} = -\mu \frac{R_L \parallel R_2}{R_0 + R_L \parallel R_2}$$

For  $R_0 \ll R_L$

$$\frac{V_R}{V_T} \approx -\mu = -AB.$$

(c)

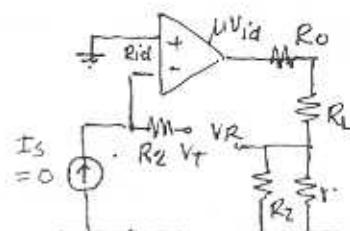


$$\frac{V_R}{V_T} = \frac{-\mu r}{R_L + r} \cdot \frac{R_1}{R_1 + R_2} = -AB$$

Note: breaking at  $R_2 + r$  is simpler

$$\frac{V_R}{V_T} = -\frac{\mu R_1}{R_{id} \parallel R_1 + R_2} \cdot \frac{r}{R_0 + R_L + r}$$

(d)



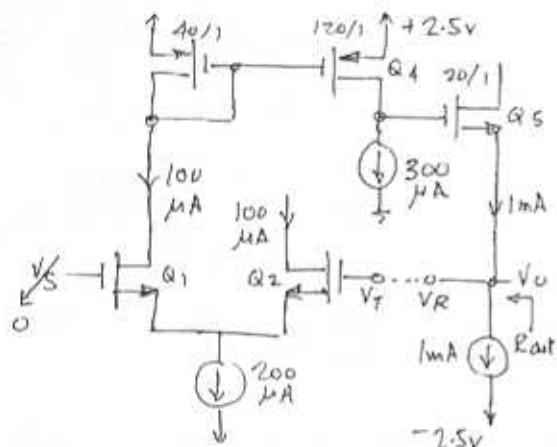
$$V_{id} = V_T \frac{R_{id}}{R_2 + R_{id}}$$

$$\frac{V_E}{V_T} = - \frac{R_{\text{in}}}{R_{\text{in}} + R_2} \mu \frac{(r || R_2)}{(r || R_2) + R_L + R_0}$$

$$AB = M \frac{r}{r + R_L} \quad \text{for } R_{\text{in}} \gg R_1, R_2$$

$$R_0 \ll R_L$$

6.57



From PB.33  $g_{m1} = g_{m2} = 693 \mu A/V$

$$\sigma_{PM3} = 693 \mu\text{A/V}, \sigma_{PM4} = 2078 \mu\text{A/V}$$

$$g_{MS} = 1550 \mu A/V, \quad r_{05} = 24 \text{ k}\Omega$$

$$T_{01} = T_{02} = T_{03} = 240 \text{ K}, T_{04} = 80 \text{ K}$$

$$B_{\text{out}} \equiv (E - H^2) = 12.5 \text{ G}$$

$$R_{\text{out}} = (105 \parallel 105) = 12 \text{ k}\Omega$$

$$c_1 = -\alpha_{\text{gen}} \sqrt{T}$$

$$c_4 = -3g_{\mu\nu}\sqrt{g}$$

$$V_{C4} = -3q_{pm}V_T(r_{04}||r_{04})$$

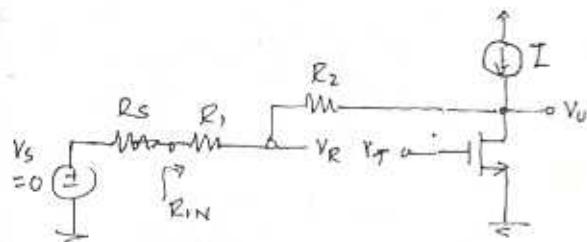
$$V_{S5} = N_{C4} = V_R$$

$$\therefore \frac{V_R}{V_T} = -3 \operatorname{gml} \left( \frac{r_{01}/(1r_{01})}{3} \right) = -\frac{\operatorname{gml}_1 r_{01}}{2}$$

$$\Delta \beta = -\frac{V_R}{V_f} = \frac{g_{m1} V_o}{2} = 83.16$$

$$R_0 = (\gamma_{05} \parallel \gamma_{05}) = 12 \text{ k}\Omega$$

8.58



Apply V<sub>t</sub> with N<sub>s</sub> = 0

$$\frac{N_0}{N_T} = - g_m [R_2 + R_1 + R_S] / |V_0|$$

$$\frac{N_R}{N_T} = - \frac{g_m [R_2 + R_1 + R_S] \| r_o }{[R_2 + R_1 + R_S]} \\ = - \frac{g_m (R_1 + R_S) r_o}{R_1 + R_2 + R_S + r_o}$$

$$V_A = 30V, P_S = 10W, R_1 = 10\Omega, R_2 = 4.7\Omega$$

$$g_m = \frac{2E}{V_{ov}} = \frac{2}{0.8 - 0.6} = \frac{2}{0.2} = 10 \text{ mA/V}$$

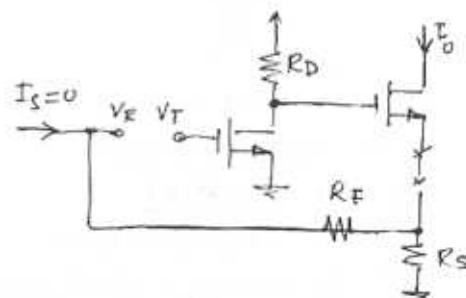
$$f_0 = V_A/I = 30V/1mA = 30k\Omega$$

$$\text{then } AB = \frac{10 \times 10^{-3} (10k + 10^3 k) 30k}{1M + 4.7M + 10k + 30k}$$

$$= \frac{10 \times 10 \times 30 \times 10^{-3}}{5740}$$

$$= 52.8 \times 10^{-3}$$

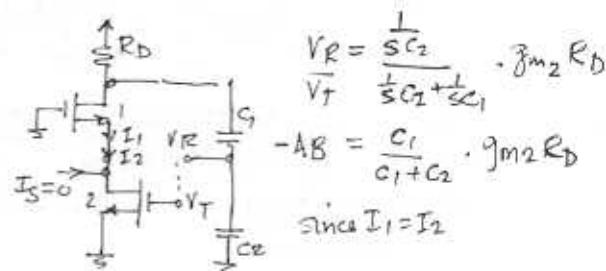
8.59



$$\frac{V_B}{V_T} = (g_{m2} R_S) (-g_{m1} R_D)$$

$$= - g_{m1} g_{m2} R_D R_S. = -AB.$$

8.60



Section 8.8  
The Stability Problem

8.61

$$A(s) = \frac{10^5}{1+s/100}$$

$$\text{Ang}(A) = -\tan^{-1} \frac{\omega}{100} - 2\tan^{-1} \frac{\omega}{10^4}$$

at  $\omega_{180}$ :  $\text{Ang}(A) = -180^\circ$  for  $\omega_{180} \gg 100$

$$\Rightarrow 180^\circ = 90^\circ + 2\tan^{-1} \left[ \frac{\omega_{180}}{10^4} \right]$$

$$\text{hence } \tan^{-1} \frac{\omega_{180}}{10^4} = \frac{90^\circ}{2}$$

$$\text{i.e. } \frac{\omega_{180}}{10^4} = \tan(45^\circ) = 1$$

$$\therefore \omega_{180} = 10^4 \text{ rad/s}$$

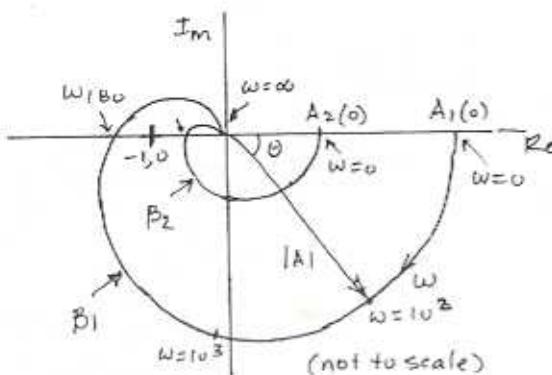
$$|AB| = \frac{10^5 \beta}{\sqrt{1+(10^4/10^2)^2}} \cdot \frac{1}{(\sqrt{1+1})^2} = 1$$

$$\Rightarrow \beta = 0.002$$

$$A_f(0) = \frac{10^5}{1+10^5(0.002)} \approx 500 \text{ V/V}$$

8.62

$\omega$	$\text{Ang}(A)$	$ A/B_1 $	$ A/B_2 $
0	0	$10^5$	$10^2$
$10^2$	45	$7.07 \times 10^4$	707
$10^3$	95.7	$9.85 \times 10^3$	9.85
$10^4$	180	500	0.5
$\infty$	0	0	0



8.63

$$A(s) = \frac{10^3}{1+s/10^4}$$

$$\beta(s) = \frac{k}{(1+s/10^4)^2}$$

$$\text{Ang}(AB) = -\tan^{-1} \frac{\omega}{10^4} - 2\tan^{-1} \frac{\omega}{10^4}$$

$$= -3\tan^{-1} \frac{\omega}{10^4}$$

$$\text{For } 180^\circ: \omega_{180} = \sqrt{3} \times 10^4 \text{ rad/s}$$

$$\text{For } |AB(\omega_{180})| < 1$$

$$\frac{10^3}{\sqrt{1+(\sqrt{3})^2}} \cdot \frac{k}{1+(\sqrt{3})^2} < 1$$

$$\Rightarrow k < 0.008$$

8.64

$$A(s) = \frac{1000}{(1+s/10^4)(1+s/10^5)}$$

and  $\beta$  is independent of frequency

$$\text{Ang}(A) = -\tan^{-1} \frac{\omega}{10^4} - 2\tan^{-1} \frac{\omega}{10^5}$$

$$\text{try } \omega = 10^4: \theta = 45^\circ + 2 \times 5.7 = 56^\circ$$

$$\text{try } \omega = 10^5: \theta = 84.2^\circ + 2 \times 45 = 174^\circ$$

Iteration yields  $\omega \approx 1.1 \times 10^5 \text{ rad/s}$

For oscillations:  $|AB(\omega_{180})| \geq 1$

$$\frac{\beta \cdot 10^3}{(\sqrt{1+11^2})(\sqrt{1+11^2})} \geq 1$$

$$\Rightarrow \beta \geq 0.0244$$

Section 8.9  
Effect of Feedback on Amplifier Poles

8.65

$$A(jf) = \frac{(10 \times 10^6)/10^4}{1+jf/10^4}$$

$$\therefore A(jf) = \frac{10^3}{1 + jf/10^4}$$

$\beta = 0.1$  independent of frequency

$$A_f(jf) = \frac{10^3}{1 + 10^3(0.1)} \cdot \frac{1}{1 + \frac{jf}{10^4(1 + 10^3(0.1))}}$$

$$= \frac{9.9}{1 + jf/(101 \times 10^4)}$$

$$A_f(0) = 9.9 \text{ v/v}$$

$$f_{pf} = 10^4 (101) = 1.01 \text{ MHz}$$

$$\text{for } f \gg 1 : A_f \approx 9.9 \frac{10^4(101)}{f}$$

$$\text{for } A_f = 1 : f = f_t = 10 \text{ MHz}$$

Pole is shifted by  $(1 + A(0)\beta) = 101$

8.66

$$A(jf) = \frac{10^3}{(1 + jf/10^4)(1 + jf/10^5)}$$

(a) closed-loop poles given by

$$1 + A(f)B = 0$$

$$\text{using } p = jf$$

$$p^2 + p(10^4 + 10^5) + (1 + 10^3\beta)10^4 \cdot 10^5 = 0$$

$$\text{i.e. } p^2 + (1.1 \times 10^5)p + 10^9(1 + 10^3\beta) = 0$$

compare terms with

$$(p + f_{pf})^2 = p^2 + 2f_{pf}p + f_{pf}^2$$

$$2f_{pf} = (1.1 \times 10^5)$$

$$(1 + 10^3\beta) \times 10^9 = f_{pf}^2$$

$$\Rightarrow f_{pf} = 5.5 \times 10^5$$

$$\text{and } (1 + 10^3\beta) = 3.025 \Rightarrow \beta = 2.025 \times 10^{-3}$$

$$(b) A_f(0) = \frac{10^3}{1 + 10^3(2.025 \times 10^{-3})} = 330.6 \text{ v/v}$$

$$\text{at } 55 \text{ kHz} : |A(f)\beta| = 1$$

$$\Rightarrow A_f(55 \text{ kHz}) = \frac{A_f(0)}{1 + |AB|} = 165.3 \text{ v/v}$$

(c) from  $s^2 + (\omega_0/Q)s + \omega_0^2$  cf. above

$$Q = \frac{f_{pf}}{2f_{pf}} = \frac{1}{2}$$

$$(d) p^2 + 1.1 \times 10^5 p + (1 + 10^3\beta) = 0$$

$$\Rightarrow p^2 + 1.1 \times 10^5 p + 21.25 \times 10^9 = 0$$

$$\therefore p = \frac{-1.1 \times 10^5 \pm \sqrt{(1.1 \times 10^5)^2 - 4(1)(21.25 \times 10^9)}}{2}$$

$$= \frac{-1.1 \times 10^5 \pm j 2.7 \times 10^5}{2}$$

$$= -5.5 \times 10^4 \pm j 1.35 \times 10^5 \text{ Hz}$$

$$Q = \frac{|P|}{2(5.5 \times 10^4)} = \frac{\sqrt{(5.5 \times 10^4)^2 + (1.35 \times 10^5)^2}}{1.1 \times 10^5}$$

$$= 1.23$$

8.67

$$A(jf) = \frac{10^3}{(1 + jf/10)(1 + jf/f_{pf})}$$

$$A_f(0) = \frac{10^3}{1 + 10^3\beta} = 100$$

$$\Rightarrow \beta = 9 \times 10^{-3} \text{ v/v}$$

Maximally flat when  $Q = 0.707 = 1/\sqrt{2}$  from

$$p^2 + p(f_1 + f_2) + (1 + A_0\beta)(f_1 f_2) = 0$$

$$Q = \sqrt{\frac{(1 + A_0\beta)f_1 f_2}{f_1 + f_2}}$$

$$\Rightarrow \sqrt{\frac{(1 + 10^3\beta)10^3 f_{pf}}{10^3 + f_{pf}}} = \frac{1}{\sqrt{2}}$$

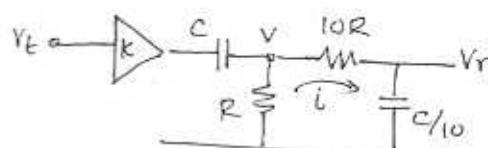
$$\Rightarrow f_{pf}^2 + (2 \times 10^3)f_{pf} + 10^6$$

$$= 2(1 + 10^3\beta)^2 10^6$$

$$\Rightarrow f_{pf} = \frac{18 \times 10^3 \pm \sqrt{(18 \times 10^3)^2 - 4(10^6)}}{2}$$

$$= 17.94 \text{ kHz.}$$

8.68



$$i = \frac{SCV_r}{10} \Rightarrow V = V_r + SCR V_r$$

$$= V_r(1 + SCR)$$

By KCL:  $\sum i = 0$  at V

$$\Rightarrow \frac{KV_f - V}{1/SC} = \frac{V}{R} + i = \frac{V}{R} + \frac{SCV_r}{10}$$

$$KSCV_f - SCV_r(1+SCR) =$$

$$= V_r \left( \frac{1+SCR}{R} + \frac{SC}{10} \right)$$

Collecting terms:

$$KSCV_f R = V_r [(SCR)^2 + 2 \cdot 1 SCR + 1]$$

$$\text{Thus } L(s) \triangleq -V_r/V_f$$

$$= -\frac{K/CR}{s^2 + \frac{2 \cdot 1 S}{CR} + \frac{1}{CR^2}}$$

$$= -\frac{K/CR}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

from which

$$\omega_0 = 1/CR$$

$$Q = \frac{1}{2 \cdot 1 - K}$$

Poles coincide when  $Q = 1/2$

$$\Rightarrow K = 2 \cdot 1 - 2 = 0 \cdot 1$$

maximally flat when  $Q = 1/\sqrt{2}$

$$\Rightarrow K = 2 \cdot 1 - 1.414 = 0.686$$

Oscillates when  $Q \rightarrow \infty$

$$\Rightarrow K = 2 \cdot 1$$

8.69

$$A(jf) = \frac{K}{1 + jf/10^7}$$

$$\text{for } \beta = 1: AB = \frac{k^3 \beta}{(1 + jf/10^7)^3} = \frac{k^3}{(1 + jf/10^7)^3}$$

For oscillations:

$$|AB| > 1 \text{ at } \arg(AB) = 180^\circ$$

$$\text{i.e., } +3 + \tan^{-1} \left( \frac{f_{180}}{10^7} \right) = 60^\circ$$

$$\Rightarrow f_{180} = \sqrt{3} \times 10^7 \text{ Hz}$$

$$\therefore \left[ \frac{K}{\sqrt{1 + (\sqrt{3})^2}} \right]^3 > 1 \Rightarrow K > 2$$

$$f_{osc} = f_{180} = 17.3 \text{ MHz.}$$

## Section 8.10

Stability Study using Bode Plots

8.70

$$A(f) = \frac{10^5}{1 + jf/10}$$

$$\text{for } \beta = 1: A(f)\beta = \frac{10^5}{1 + jf/10}$$

$$\text{for } f \gg 10: |AB| \approx 10^5 \cdot \frac{10}{f},$$

$$\Rightarrow f_1 = 1 \text{ MHz}$$

$$\text{at } f_1: \text{phase margin} = 180^\circ - \tan^{-1} \frac{10^5}{10} = 90^\circ$$

8.71

$$A(f) = \frac{10^5}{(1 + jf/10)(1 + jf/10^4)}$$

$$AB(0) = 10^5 \beta$$

$$A_F(0) = 10^5 = \frac{10^5}{1 + 10^5 \beta} \Rightarrow \beta \approx 0.01$$

$$|AB| = 1 \Rightarrow |1 + jf/10| \cdot |1 + jf/10^4| = 10^5 \beta = 10^3$$

$$(1 + \frac{f^2}{10^2})(1 + f^2/10^8) = 10^6$$

$$f^4 + f^2 (10^8 + 10^2) - (10^8)(10^2)10^6 = 0$$

$$f^2 \approx -\frac{10^8 + \sqrt{10^{16} + 4 \times 10^6}}{2}$$

$$\Rightarrow 61.8 \times 10^6 \Rightarrow f = 7.86 \text{ kHz}$$

Phase margin

$$= 180 - (\tan^{-1} \frac{7.86 \times 10^3}{10} + \tan^{-1} \frac{7.86}{10})$$

$$\approx 180 - 90^\circ - 38.16^\circ = 51.8^\circ$$

For PM  $> 45^\circ$ :  $\tan^{-1} f_1/10^4 \leq 45^\circ$

$$\Rightarrow f_1 \leq 10^4$$

thus

$$|AB| = 1 = \sqrt{(1 + 10^3)^2 + 1^2}$$

$$\Rightarrow \beta = \frac{\sqrt{2}}{100} = 0.0141$$

8.72

$$\begin{aligned} |1 + e^{-j\theta}| &= |1 + \cos\theta - j\sin\theta| \\ &= [(1 + \cos\theta)^2 + (\sin\theta)^2]^{1/2} \\ &= [1 + 2\cos\theta + \cos^2\theta + 1 - \cos^2\theta]^{1/2} \\ &= \sqrt{2}(1 + \cos\theta)^{1/2} \end{aligned}$$

for 5% :  $1 + \cos\theta = \frac{1}{1.05^2(2)} = 0.4535$

$\theta = 123.13^\circ$  and  $PM = 180 - \theta = 56.87^\circ$

for 10% :  $1 + \cos\theta = \frac{1}{1.1^2(2)} = -0.586$

$\theta = 125.93^\circ$  and  $PM = 54.07^\circ$

for 0.1 dB  $\equiv 10^{0.1/20} = 1.0116$

$\cos\theta = \frac{1}{2(1.0116)^2} - 1 = -0.5114$

$\theta = 120.76^\circ$  and  $PM = 59.24^\circ$

for 1 dB  $\equiv 10^{1/20} = 1.122$

$\cos\theta = \frac{1}{2(1.122)^2} - 1 = -0.6028$

$\theta = 127.07^\circ$  and  $PM = 52.93^\circ$

8.73

$$A(jf) = \frac{10^5}{(1 + j\frac{f}{10^5})(1 + j\frac{f}{3.16 \times 10^5})(1 + j\frac{f}{10^6})}$$

Assume  $B$  independent of frequency

For  $45^\circ$  PM :  $\theta = 180 - 45$

$$\tan^{-1}\frac{f_1}{10^5} + \tan^{-1}\frac{f_1}{3.16 \times 10^5} + \tan^{-1}\frac{f_1}{10^6} = 135^\circ$$

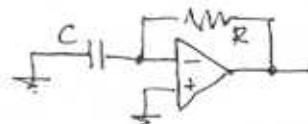
Solve  $\Rightarrow f_1 = 3.16 \times 10^5 \text{ Hz}$

$$|AB(f_1)| = 1 = \frac{10^5 B}{\sqrt{1 + (3.16)^2} \cdot \sqrt{2} \cdot \sqrt{1 + (0.316)^2}}$$

$$\Rightarrow B = 49 \times 10^{-6}$$

$$Af(0) = \frac{10^5}{1 + 10^5(4.9 \times 10^{-6})} = 16.9 \times 10^3$$

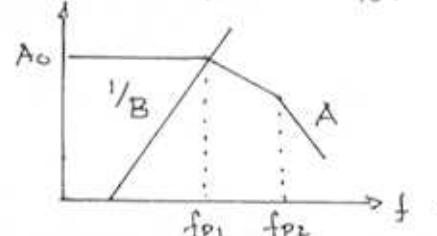
8.74



$$\beta = \frac{1/SC}{R + 1/SC} = \frac{1}{1 + SCR}$$

$$\beta(j) = \frac{1}{1 + j2\pi f CR}$$

$$A(jf) = \frac{10^3}{(1 + j\frac{f}{10^4})(1 + j\frac{f}{10^7})}$$



From sketch, we need

$$A_0 \frac{1}{2\pi f_p R} = 1 = AB$$

$$\Rightarrow RC = \frac{Av}{2\pi f_p} = \frac{10^3}{2\pi \times 10^6} = 159.2 \mu s$$

At 1 MHz  $\text{Ang}(\beta) = -90^\circ$

$$\begin{aligned} \text{Ang}(A) &= -\tan^{-1} 1 - \tan^{-1} 0.1 \\ &= -45 - 5.7 = -50.7^\circ \end{aligned}$$

$$\therefore PM = 180 - (90 + 50.7) = 39.3^\circ$$

Gain margin exists at  $\omega_{180}$

$$\text{Item } \tan^{-1}\frac{f_1}{10^5} + \tan^{-1}\frac{f_1}{10^6} \Rightarrow 90^\circ$$

$$\therefore f_{180} = \sqrt{10^5 \cdot 10^6} = \text{geometric mean} \\ = 3.16 \text{ MHz}$$

$$|AB(f_{180})| = 20 \log|A| - 20 \log|1/\beta|$$

(A) has fallen 10 dB, (B) has risen 10 dB

$$\text{thus GM} = +10 - (-10) = 20 \text{ dB}$$

8.75

For  $90^\circ$  PM :

$$\tan^{-1}\frac{f_1}{10^5} + \tan^{-1}\frac{f_1}{10^6} + \tan^{-1}\frac{f_1}{10^7} = 90^\circ$$

From graph  $f_1 = 3 \times 10^5 \text{ Hz}$   
 $\text{Thus } 71.6 + 16.7 + 1.72 = 89.9^\circ \text{ (close)}$

 $|A(f_1)| = \frac{10^5}{\sqrt{1+3^2} \cdot \sqrt{1+0.3^2} \cdot \sqrt{1+0.03^2}} = 30.28 \times 10^3$ 
 $|AB| = 1 \Rightarrow \beta = 33.0 \times 10^{-6}$ 
 $\therefore A_f(0) = \frac{10^5}{1+10^5\beta} = 2.32 \times 10^4$

For PM = 45°  $f_1 \approx 10^6 \text{ Hz}$  from graph  
 $\text{Thus } 84.3 + 45 + 5.7 = 135^\circ \text{ (OK)}$

 $|A(f_2)| = \frac{10^5}{\sqrt{1+10^2} \cdot \sqrt{2} \cdot \sqrt{1+0.1^2}} = 7 \times 10^3$ 
 $|AB| = 1 \Rightarrow \beta = 1.43 \times 10^{-4}$ 
 $\therefore A_f(0) = \frac{10^5}{1+10^5\beta} = 6.54 \times 10^3$

### Section 8.11 Frequency Compensation

8.76

$f_1 = 2 \text{ MHz}$ 
 $A_o = 80 \text{ dB} = 10^4$ 
 $\Rightarrow f_p = f_1/A = (2 \times 10^6)/10^4 = 200 \text{ Hz}$

8.77

$f_{p1} = 2 \text{ MHz}, f_{p2} = 10 \text{ MHz}$ 
 $A_o = 80 \text{ dB} = 10^4$ 
 $f_p = \frac{f_p}{A_o} = \frac{10 \times 10^6}{10^4} = 10^3 \text{ Hz}$ 
 $f'_p = 1/(C_x + C_c) 2\pi R_x \rightarrow C \times \frac{2 \times 10^6}{10^3} = 2000 \text{ pF}$

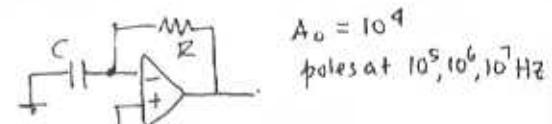
8.78

$W_{p1} = \frac{1}{10CR}, W_{p2} = \frac{1}{CR}$ 
 $W'_{p1} \approx \frac{1}{g_m R_2 C_f R_1} = \frac{1}{100 \cdot R \cdot C_f R} = \frac{1}{100 C_f R}$ 
 $W'_{p2} \approx \frac{g_m C_f}{C_2 + C_f(C_1 + C_2)} = \frac{g_m C_f}{C_1(C_2 + C_f) + C_f C_2}$ 

for  $C_f \gg C_2 = C$

$W'_{p2} \approx \frac{g_m}{C_1 + C_2} = \frac{100}{R(C_1 + C_2)} = \frac{9 \cdot 1}{CR}$

8.79



$A_o = 10^4$ 

poles at  $10^5, 10^6, 10^7 \text{ Hz}$

For  $\beta = 1$ ,  $f_p$  must be kept  $\times 10^4$  lower than lowest amplifier pole at  $10^5 \text{ Hz}$

 $\Rightarrow f_p = \frac{10^5}{10^4} = 10 \text{ Hz}$ 

$f_p = \frac{1}{2\pi CR} \text{ and } R = 1 \text{ M}\Omega$

$\Rightarrow C = \frac{1}{2\pi \cdot 10^6 (10)} = 15.9 \text{ nF}$

8.80

$A_o = 80 \text{ dB} = 10^4$

$f_{p1} = 10^5 = \frac{1}{2\pi C_1 R_1} \Rightarrow R_1 = \frac{1}{2\pi f_{p1} C_1}$

$\Rightarrow R_1 = \frac{1}{2\pi \cdot 10^5 \cdot (150 \times 10^{-12})} = 10.62 \text{ k}\Omega$

$f_{p2} = 10^6 = \frac{1}{2\pi C_2 R_2}$

$\Rightarrow R_2 = \frac{1}{2\pi \cdot 10^6 \cdot (5 \times 10^{-12})} = 31.85 \text{ k}\Omega$

Assuming  $f'_{p2} \gg f_{p3}$

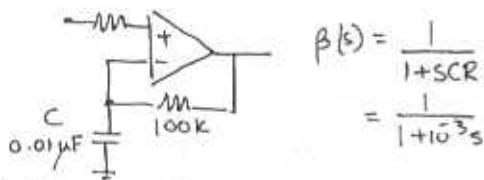
$f'_{p1} = \frac{f_{p3}}{10^4} = \frac{2 \times 10^6}{10^4} = 200 \text{ Hz}$

$\text{and } f'_{p1} = \frac{1}{2\pi g_m R_1 R_2 C_f} \Rightarrow C_f = \frac{1}{2\pi g_m R_1 R_2 f'_{p1}}$

$\therefore C_f = \frac{1}{2\pi (40 \times 10^{-3}) (10.62 \times 10^3) (31.85 \times 10^3) 200} = 58.8 \text{ pF}$

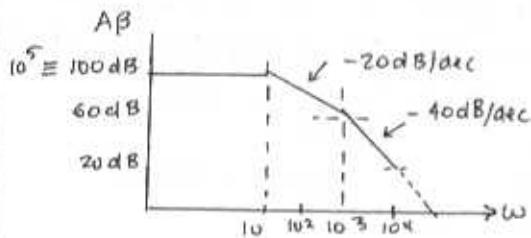
$f'_{p2} = \frac{1}{2\pi} \frac{g_m C_f}{C_1 C_2 + C_f (C_1 + C_2)} = \frac{40 \times 10^{-3} (58.8 \times 10^{-12})}{2\pi (150 \times 5) 10^{-24} + 58.8 (155) 10^{-24}} = 38.8 \text{ MHz}$

8.81



$$\beta(s) = \frac{1}{1+SCR} = \frac{1}{1+10^3 s}$$

$$(a) A_B(s) = \frac{10^5}{1+s/10} \cdot \frac{1}{1+s/10^3}$$



(b) From plot  $|A_B| = 20 \text{ dB}$  at  $10^4 \text{ rad/s}$   
Hence  $|A\beta| = 1$  at  $31.6 \text{ krad/s}$  ( $\pm \text{dec}$ )

$$(c) A_f(s) = \frac{10^5}{1+s/10} \cdot \frac{1}{1+s/10^3} = \frac{10^5(1+s/10^3)}{(1+s/10)(1+s/10^3) + 10^5}$$

$$\therefore A_f(s) = \frac{1+s/10^3}{1+s/10^6 + s^2/10^9} = \frac{10^6 s + 10^9}{s^2 + 10^2 s + 10^9}$$

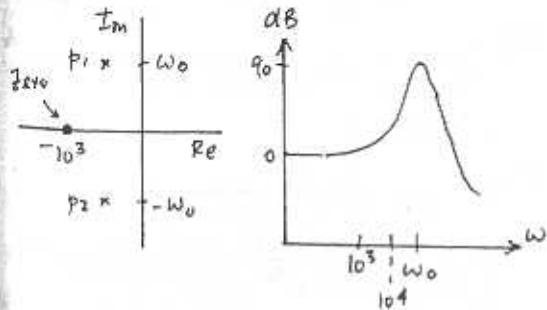
Zero at  $s = -10^3 \text{ rad/s}$

$$\begin{aligned} \text{Poles at } & -10^3 \pm \sqrt{10^6 - 4 \times 10^9} \\ & = -10^3 \pm j 63.2 \times 10^3 \end{aligned}$$

$$= -500 \pm j 31.6 \times 10^3 \text{ rad/s}$$

$$\omega_0 = 31.6 \text{ krad/s}$$

$$Q = 31.6$$



## Chapter 9 - Problems

### Section 9.1: The two-stage CMOS op. Amp

9.1

$$\begin{aligned} V_{ICM(\max)} &\leq V_{DD} - |V_{tp1}| - |V_{ov1}| - |V_{ov5}| \\ &\leq +2.5 - 0.7 - 0.3 - 0.3 \\ &\leq +1.2 \text{V} \end{aligned}$$

$$\begin{aligned} V_{ICM(\min)} &\geq -V_{SS} + V_{OV3} + V_{tn} - |V_{tp1}| \\ &\geq -2.5 + 0.3 (+0.7 - 0.7) \\ &\geq -2.2 \text{V} \end{aligned}$$

$$\begin{aligned} -V_{SS} + V_{OV6} &\leq V_o \leq V_{DD} - |V_{OV7}| \\ -2.5 + 0.3 &\leq V_o \leq +2.5 - 0.3 \\ -2.2 \text{V} &\leq V_o \leq +2.2 \text{V} \end{aligned}$$

9.2

$$V_A' = 25 \text{V}/\mu\text{m}, |V_P'| = 20 \text{V}/\mu\text{m}, L = 0.8 \mu\text{m}$$

$$\text{Hence } V_A = 20 \text{V} \text{ and } |V_P| = 16 \text{V}$$

$$\text{For all devices } V_{ov} = 0.25 \text{V}$$

$$A = A_1 A_2 = Gm_1 (r_{o2} || r_{o4}) Gm_2 (r_{o6} || r_{o7})$$

$$[r_{oP} || r_{on}] = \left[ \frac{V_A}{I} \times \frac{V_P}{I} \right] \times \frac{I}{V_A + V_P} = \left[ \frac{V_A || V_P}{I} \right]$$

$$\text{For } A_2: R_o = \frac{8.89}{I} \rightarrow \frac{8.89 \text{V}}{0.4 \text{mA}} = 22.2 \text{k}\Omega$$

To avoid systematic output dc. offset

$$\frac{(W/L)_6}{(W/L)_4} = \frac{2(W/L)_7}{(W/L)_5}$$

Since Q5, Q6, Q7 carry I and Q4 only I/2 satisfy requirement by making Q4 have  $(W/L)/2$

$$\text{Since } g_m = \sqrt{2(\mu C_{ox} X (W/L)) I} = 2K(V_{ov})$$

$$g_{m1} = 2I_1/V_{ov} = 0.4 \text{mA}/0.25 \text{V} = 1.6 \text{mA/V}$$

$$g_{m6} = 2I_6/V_{ov} = 3.2 \text{mA/V}$$

$$\therefore A = (1.6)(44.4)(3.2)(22.2) = 5047 \text{v/v}$$

For unity gain amplifier

$$A_f = \frac{A}{1 + A\beta} = \frac{5047}{1 + 5047\beta} = 1$$

$$\text{Thus } (1 + A\beta) = 5047$$

$$\text{Then } R_{of} = R_o / (1 + A\beta)$$

$$= 22.2 \text{k} / 5047 \approx 4.4 \text{n}\Omega$$

9.3

$$\begin{aligned} A = A_1 A_2 &= Gm_1 (r_{o2} || r_{o4}) Gm_2 (r_{o6} || r_{o7}) \\ &= \frac{2I_1}{V_{ov}} \cdot \frac{1}{2} \frac{V_A}{I_1} \cdot \frac{2I_2}{V_{ov}} \cdot \frac{1}{2} \frac{V_A}{I_2} \\ &= \left[ \frac{V_A}{V_{ov}} \right]^2 = 2500 \end{aligned}$$

$$\text{where } V_A = 10 \text{V}/\mu\text{m} \times 1 \mu\text{m} = 10 \text{V}$$

$$\text{Hence } V_{ov} = V_A / 50 = 10 / 50 = 0.2 \text{V}$$

9.4

$$\text{From } \frac{(W/L)_6}{(W/L)_4} = \frac{2(W/L)_7}{(W/L)_5}$$

$$(W/L)_6 = \frac{2(60/0.5)(10/0.5)}{(60/0.5)} = (20/0.5)$$

$$I_{Dp} = \frac{1}{2} [60] (W/L) (V_{GS} - V_T)^2$$

$$\text{Given } I_{ref} = 22.5 \mu\text{A} = \frac{1}{2} 60 (60/0.5) [V_{ov}]^2 \Rightarrow V_{ov} = 15/60 = 0.25 \text{V}$$

$$\text{Given } V_T = 0.75 \text{V}$$

$$\Rightarrow V_{GS} = V_T + V_{ov} = 0.25 + 0.75 = 1 \text{V}$$

Since Q5, Q7, Q8 have same  $(W/L)$

$$\Rightarrow I_1 = I_2 = I_3 = I_4 = \frac{1}{2} I_5 = \frac{1}{2} I_8$$

$$\text{For Q8: } I_1 = \frac{1}{2} 60 (30/0.5) (V_{ov})^2 = 112.5 \text{mA}$$

$$\Rightarrow V_{ov1} = 1.0 \text{V}$$

$$\Rightarrow V_{GS1} = V_T + V_{ov1} = 1.0 \text{V}$$

$$\text{For Q3 Q4: } I_3 = \frac{1}{2} 180 (10/0.5) (V_{ov})^2 = 112.5 \text{mA}$$

$$\Rightarrow V_{ov3} = 0.25 \text{V}$$

$$\Rightarrow V_{GS3} = 0.25 + 0.75 = 1 \text{V}$$

$$\text{For Q6: } I_6 = \frac{1}{2} 180 (20/0.5) (V_{ov})^2 = 225 \text{mA}$$

$$\Rightarrow V_{ov6} = 0.35 \text{V}$$

$$\Rightarrow V_{GS6} = 0.35 + 0.75 = 1.1 \text{V}$$

$$R_o = \frac{V_A}{I_{mA}} = r_o K_{SR} \quad \text{and } |V_A| = 9 \text{V}$$

$$g_m = \frac{2I}{V_{ov}}$$

and  $|V_{ov}| = 9 \text{V}$

$Q$	$K$	$I$	$V_{ov}$	$V_{GS}$	$g_m$	$r_o$
	(mA)	( $\mu A$ )	(V)	(V)	(mA/V)	(k $\Omega$ )
Q1	60	112.5	0.25	0.81	0.9	80
Q2	60	112.5	0.25	0.81	0.9	80
Q3	180	180	0.25	1.0	0.9	80
Q4	180	180	0.25	1.0	0.9	80
Q5	60	225	0.25	1.0	1.8	40
Q6	180	225	0.35	1.1	1.285	40
Q7	60	60	0.25	1.0	1.8	40
Q8	60	60	0.25	1.0	1.8	40

$$A_1 = Gm_1 (r_{o2} \parallel r_{o4}) = 0.9 \times 40 = 36 \text{ V/V}$$

$$A_2 = Gm_2 (r_{o6} \parallel r_{o7}) = 1.285 \times 20 = 25.7 \text{ V/V}$$

$$A = A_1 A_2 = 36 \times 25.7 = 925 \text{ V/V}$$

$$\begin{aligned} V_{ICM(\max)} &\leq V_{DD} - |V_{tp}| - |V_{ov1}| - |V_{ov5}| \\ &\leq 1.5 - 0.75 - 0.25 - 0.25 \\ &\leq +0.25 \text{ V} \end{aligned}$$

$$\begin{aligned} V_{ICM(\min)} &\geq -V_{SS} + V_{ov3} + V_{en} - |V_{tp}| \\ &\geq -1.5 + 0.25 = -1.25 \text{ V} \\ -V_{SS} + V_{ov6} &\leq V_o \leq +V_{DD} - |V_{ov7}| \\ -1.5 + 0.35 &\leq V_o \leq +1.5 - 0.25 \\ -1.15 \text{ V} &\leq V_o \leq +1.25 \text{ V} \end{aligned}$$

### 9.5

$$Gm_1 = 0.3 \text{ mA/V} \quad Gm_2 = 0.6 \text{ mA/V} \quad C_2 = 1 \text{ pF}$$

$$r_{o2} = r_{o4} = 222 \text{ k}\Omega \quad r_{o6} = r_{o7} = 111 \text{ k}\Omega$$

$$(i) f_{p2} = \frac{Gm_2}{2\pi C_2} = \frac{0.6 \times 10^{-3}}{2\pi 10^{-12}} = 95.5 \text{ MHz}$$

$$(ii) R = \frac{1}{Gm_2} = \frac{1}{0.6} = 1.66 \text{ k}\Omega$$

$$(iii) \text{For } PM = 10^\circ: \quad \tan^{-1} \frac{f_t}{f_{p2}} = 10^\circ$$

$$\begin{aligned} f_t &= f_{p2} \tan 10^\circ \\ &= 95.5 \times 0.176 = 16.84 \text{ MHz} \end{aligned}$$

$$C_c = \frac{Gm_1}{2\pi f_t} = \frac{0.3 \times 10^{-3}}{2\pi 16.84 \times 10^6} \Rightarrow 2.83 \text{ pF}$$

$$\begin{aligned} A &= A_1 A_2 = Gm_1 (r_{o2} \parallel r_{o4}) Gm_2 (r_{o6} \parallel r_{o7}) \\ &= 0.3 \times 111 \times 0.6 \times 55.5 \\ &= 1109 \equiv 60.8 \text{ dB} \end{aligned}$$

$$\text{Dominant pole } f_{p1} = f_t / 2A$$

Thus  $f_{p1}$  is approx 3 decades below  $f_t$   
i.e. at  $16.84 \text{ kHz}$  providing uniform  
20 dB/dec slope down to  $f_t$ .

$$(d) f_t = \frac{Gm_1}{2\pi C_c} \quad \therefore \text{to double } f_t, \text{ halve } C_c$$

$$C_{(new)} = 1.4 \text{ pF}$$

$$\tan^{-1} \frac{f_t}{f_p} = \tan^{-1} \frac{33.7}{95.5} = 19.4^\circ$$

The zero must be moved to reduce  
the  $19.4 - 10 = 9.4^\circ$

$$\tan^{-1} \frac{f_t}{f_z} = 9.4^\circ \rightarrow \frac{f_t}{f_z} = 0.16$$

$$\Rightarrow f_z = 0.16 f_t = 0.16 \times 33.7$$

$$= 5.6 \text{ MHz}$$

$$f_z = \frac{1}{2\pi C_c [R - \frac{1}{Gm_1}]} \rightarrow [R - \frac{1}{Gm_1}] = \frac{1}{2\pi f_z C_c}$$

$$\text{Hence } [R - \frac{1}{Gm_1}] = \frac{10^{12} \cdot 10^{-6}}{2\pi 5.6 \times 1.4}$$

$$R = 1.67 + 20.3 = 21.97 \text{ k}\Omega$$

### 9.6

Two-stage amp with  $C_2 = 1 \text{ pF}$

$$f_t = 100 \text{ MHz}, \quad PM = 75^\circ$$

$$\text{For } PM = 75^\circ: \quad \tan^{-1} \frac{f_t}{f_{p2}} = 15^\circ$$

$$\therefore f_{p2} = f_t + \tan 15^\circ = 3.73 f_t = 373 \text{ MHz}$$

$$f_{p2} = \frac{Gm_2}{2\pi C_2} = \frac{1}{2\pi R_2 10^{-12}} = 373 \text{ MHz}$$

$$\Rightarrow R_2 = \frac{10^{12}}{2\pi (373 \times 10^6)} = 426 \Omega$$

$$\Rightarrow Gm_2 = \frac{1}{R_2} = 2.35 \times 10^{-3} \text{ mA/V}$$

$$\text{To move zero to infinity, } R = \frac{1}{Gm_2} = 426 \Omega$$

$$SR = \frac{I}{C_c} = \frac{200 \mu A}{C_c}$$

$$SR = 2\pi f_t V_{ov1} = 2\pi 10^8 \times 0.2 = 1.26 \times 10^8$$

$$\Rightarrow C_c = \frac{200 \times 10^{-6}}{1.26 \times 10^8} \Rightarrow 1.6 \text{ pF}$$

9.7

$$Gm_1 = 1mA/V, Gm_2 = 2mA/V, R = 500\Omega$$

$$f_t \approx \frac{Gm_1}{2\pi C_C} \Rightarrow C_C = \frac{Gm_1}{2\pi f_t}$$

$$= \frac{1 \times 10^{-3}}{2\pi \cdot 100 \times 10^6} \Rightarrow 1.59 \mu F$$

$$\text{For } \frac{1}{Gm_2} - R = \frac{10^3}{2} - 500 = 0$$

zero has been moved to  $\infty$

$$\text{For } PM = 60: f_t = f_{p_2} \tan(90 - 30)^\circ$$

$$\Rightarrow f_{p_2} = f_t / \tan 30^\circ = 173 \text{ MHz}$$

$$C_2 \approx \frac{Gm_2}{2\pi f_{p_2}} = \frac{2 \times 10^{-3}}{2\pi \cdot 173 \times 10^6} = 1.84 \mu F$$

9.8

$$SR = 60V/\mu s \quad f_t = 50 \text{ MHz}$$

$$(a) SR = 2\pi f_t V_{ov_1}$$

$$\Rightarrow V_{ov_1} = SR / 2\pi f_t$$

$$= \frac{60 \times 10^6}{2\pi (50 \times 10^6)} \approx 0.2V$$

$$(b) SR = \frac{I}{C_C} \Rightarrow C_C = \frac{100 \mu A}{60 \times 10^6} = 1.67 \mu F$$

$$(c) I = \frac{1}{2} \mu C_{ox} (W/L) [V_{ov}]^2$$

$$\Rightarrow \left(\frac{W}{L}\right) = \frac{2I}{50[0.2]^2} = \frac{100}{1}$$

9.9

Invert circuit leaving  $V_{DD} + V_{SS}$   
and reverse all arrows on FETs.

### Section 9.2 The Folded-Cascode OP AMP

9.10

From circuit: Current drawn from  $V_{DD}$  rail  
 $= 2 I_B$  = Current returned to  $V_{SS}$  rail

$$\therefore \text{Power} = (V_{DD} + V_{SS}) \times 2I_B \Rightarrow$$

$$\text{Im W} = (1.65 + 1.65) \times 2 \times 8$$

$$\therefore I_B = \frac{1 \text{ mA}}{4 \times 1.65 \text{ V}} = 151.5 \mu A$$

$$\Rightarrow I = I_B / 1.2 = 126.3 \mu A$$

9.11

$V_{BIAS1}$ :  $V_S$  can rise to  $V_{DD} + V_T - V_{ov}$   
 $V_{D3}$  can rise to  $V_{S3} - V_{ov}$

$$\therefore V_{BIAS1} = V_{DD} - V_{ov_{10}} - V_{ov_4} + V_T$$

$$= 1.65 - 0.2 - 0.2 + 0.5$$

$$= 1.75V$$

$$V_{BIAS2} = V_{DD} - V_{ov_{10}}$$

$$= +1.65 - 0.2 = +1.45V$$

$$V_{BIAS3} = -V_{SS} + V_{ov_{11}}$$

$$= -1.65 + 0.2 = -1.45V$$

$$V_{ICM(max)} = V_{DD} - |V_{ov_9}| + V_{fn}$$

$$= +1.65 - 0.2 + 0.5$$

$$= +1.75V$$

$$V_{ICM(min)} = -V_{SS} + V_{ov_{11}} + V_{ov_1} + V_T$$

$$= -1.65 + 0.2 + 0.2 + 0.5$$

$$= -0.75V$$

$$-V_{SS} + 2V_{ov} + V_T \leq V_o \leq +V_{DD} - 2V_{ov}$$

$$-1.65 + 0.4 + 0.5 \leq V_o \leq +1.65 - 0.4$$

$$-0.75V \leq V_o \leq +1.25V$$

9.12

$$I = 125 \mu A, I_B = 150 \mu A, V_T = 0.2V$$

$$\text{For } Q_9, Q_{10}: I_B = 150 \mu A$$

$$I = \frac{1}{2} (\mu C_{ox})(W/L)(V_{ov})^2$$

$$150 = \frac{1}{2} 90 (W/L)(0.2)^2$$

$$\Rightarrow (W/L)_{9,10} = (83.33/1)$$

$$\text{For } Q_1, Q_2: I = 125 \mu A / 2$$

$$\frac{1}{2} 125 = \frac{1}{2} 250 (W/L)(0.2)^2$$

$$\Rightarrow (W/L)_{1,2} = (12.5/1)$$

For Q<sub>11</sub>:  $I = 125 \mu A$

$$125 = \frac{1}{2} 250 (W/L)(0.2)^2$$

$$\Rightarrow (W/L)_{11} = (25/1)$$

For Q<sub>3</sub>, Q<sub>4</sub>:  $I = 125 \mu A/2$

$$\frac{1}{2} 125 = \frac{1}{2} 60 (W/L)(0.2)^2$$

$$\Rightarrow (W/L)_{3,4} = (52/1)$$

For Q<sub>5</sub>, Q<sub>6</sub>, Q<sub>7</sub>, Q<sub>8</sub>:  $I = 125 \mu A/2$

$$\frac{1}{2} 125 = \frac{1}{2} 250 (W/L)(0.2)^2$$

$$\Rightarrow (W/L)_{5,6,7,8} = (12.5/1)$$

9.13

$$(1) SR = \frac{I}{C_L} \Rightarrow I = SR \times C_L \\ = 10 \times 10^6 \times 10 \times 10^{-12} \\ = 100 \mu A$$

$$(2) I_B = 1.2 \mu A = 120 \mu A$$

$$(3) f_p = \frac{1}{2\pi C_L R_0} \quad . \quad f_t = \frac{G_m}{2\pi C_L} \\ G_m = \frac{2I}{V_{DD}} = \frac{100 \mu A}{0.2V} = 0.5mA/V \\ f_t = \frac{0.5 \times 10^{-3}}{2\pi \times 10 \times 10^{-12}} = 7.96 \text{ MHz}$$

$$(4) R_{o3} \approx \frac{1}{G_m} = \frac{1000}{0.5} = 2k\Omega$$

$$Av = f_t/f_p = G_m R_o$$

$$\text{Bmt } R_{o3} = \frac{R_o}{1+G_m R_o} \Rightarrow R_o = 2M\Omega$$

$$\therefore Av = 0.5 \times 10^{-3} \times 2 \times 10^6 = 1000$$

$$f_p = f_t/1000 = 7.96 \text{ kHz}$$

$$(5) \theta = -\tan^{-1} \frac{f_t}{f_p} - 2(\tan^{-1} \frac{f_t}{f_{p2}})$$

$$P_m @ f_{p2} = 90^\circ - \tan^{-1} \frac{f_t}{f_{p2}} \\ = 90^\circ - \tan^{-1} \left[ \frac{7.96 \text{ MHz}}{25 \text{ MHz}} \right]$$

$$= 90^\circ - 17.7^\circ = 72.3^\circ$$

$$(6) \text{ For } P_m = 75^\circ: \tan^{-1} \frac{f_t}{f_{p2}} = 15^\circ$$

$$\text{thus } f_t^* = f_{p2} \tan 15^\circ$$

$$= 25 \text{ MHz} \times 0.27 = 6.7 \text{ MHz}$$

$$\frac{f_t^*}{f_t} = \frac{C_L}{C_L^*} \Rightarrow \frac{6.7}{7.96} = \frac{10 \text{ pF}}{C_L^*}$$

$$\Rightarrow C_L^* = C_L \frac{7.96}{6.7} = C_L \times 1.19$$

$\therefore$  Increase C<sub>L</sub> by 19%.

$$(7) SR^* = \frac{I}{C_L^*} \Rightarrow \frac{SR}{1.19} = 8.4 \text{ V}/\mu s$$

9.14

$$G_m = \frac{I}{V_{DD}} \quad r_o = \frac{V_A}{I}$$

$$R_o = R_{o4} \parallel R_{o6}$$

$$R_{o4} = g_{m4} r_{o4} (r_{o2} \parallel r_{o10})$$

$$R_{o6} = g_{m6} (r_{o6} \parallel r_{o8})$$

$$A = G_m R_o$$

$$Q_{10}: I = I_B \Rightarrow g_{m10} = 0.75 \text{ mA/V}$$

$$r_{o10} = 66.6 \text{ k}\Omega$$

$$Q_x: I = 125 \mu A \Rightarrow g_x = 0.31 \text{ mA/V}$$

$$r_{ox} = 160 \text{ k}\Omega$$

$$\therefore R_{o4} = 0.31 \times 160 \times 47k = 2359k$$

$$R_{o6} = 0.31 \times 160 \times 160k = 8000k$$

$$R_o = 2.35 \parallel 8.0 = 1.8 \text{ M}\Omega$$

$$A = g_{m1} R_o = 0.31 \times 1800 = 558$$

$$\frac{V_o}{V_i} = 1 + \frac{(1/\beta)}{(1/g_{m1})} = 1 + \frac{g_{m1}}{\beta} = 10$$

$$\therefore \beta = 1/10 = 0.1$$

$$A_F = \frac{A}{1+A\beta} = \frac{558}{1+558 \times 0.1} = 9.8$$

$$R_{oF} = \frac{R_o}{1+A\beta} = \frac{1.8}{56.8} = 31.69 \text{ k}\Omega$$

9.15

## Rail-to-Rail OP Amp

All  $V_{OV} = 0.2V$  All  $V_T = 0.5V$

$V_{DD} = V_{SS} = 1.65V$

$V_{ICM(\max)} = V_{DD} - |V_{OVg}| + V_T$  $= 1.65 - 0.2 + 0.5 = +1.95V$

$V_{ICM(\min)} = -V_{SS} + V_{OV_{II}} + V_{OV_I} + V_T$  $= -1.65 + 0.2 + 0.2 + 0.5$  $= -0.75V$

(a) For NMOS stage

$-0.75V \leq V_{ICM} \leq +1.95V$

(b) For PMOS stage (symmetrical)

$-1.95V \leq V_{ICM} \leq +0.75V$

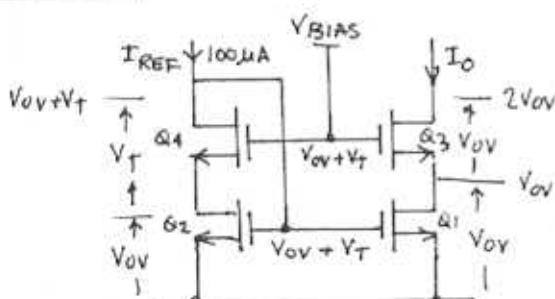
(c) For both active

$-0.75V \leq V_{ICM} \leq +0.75V$

(d) Overall

$-1.95V \leq V_{ICM} \leq +1.95V$

9.16

All same  $K(W/L)$   $\therefore I_0 \approx I_{REF}$ 

All same  $r_o = V_A/I = 10V/100\mu A = 100k\Omega$   
 $I = \frac{1}{2} K [W/L] [V_{OV}]^2$

$V_{D3} = V_O : V_O(\min) = 2V_{OV}$

R<sub>O</sub> (looking into Q<sub>3D</sub> and assuming  
I<sub>O</sub> current source is ideal). ( $r_o = \infty$ )

$= r_o(1 + g_m r_o) \approx g_m r_o^2$

$g_m = \frac{2I}{V_{OV}} = \frac{2 \times 100}{0.2} \Rightarrow 1mA/V$

Then  $R_o \approx g_m r_o^2 = 1 \times 10^5 \times 10^5$   
 $\approx 10^4 M\Omega$

9.17

$A = 80 \text{ dB}$   $f_t = 10 \text{ MHz}$   $C_L = 10 \text{ pF}$   
 $I_B = I$  All same  $|V_{OV}|$ ,  $L = 0 \mu \text{m}$   
 $|VA| = 20V$

$g_m = \frac{2I}{V_{OV}}$  and  $f_t = \frac{g_m}{2\pi C_L}$

$A = g_m [g_m + r_o (r_o || r_{o2})] [g_m r_o r_{o2} r_o]$

Consider Q<sub>1</sub>:  $I_1 = \frac{1}{2} K_n [W/L]_1 [V_{OV}]^2$   
 $= \frac{1}{2} 200 [W/L]_1 [V_{OV}]^2$

Q<sub>1</sub> Q<sub>2</sub> Q<sub>5</sub> Q<sub>6</sub> Q<sub>7</sub> Q<sub>8</sub> are same

$g_{m1} = \frac{2I_1}{V_{OV}}$  and  $r_{o1} = \frac{V_A}{I_1}$

Consider Q<sub>3</sub>, Q<sub>4</sub>:  $I_3 = \frac{1}{2} K_p [W/L]_3 [V_{OV}]^2$

$I_1 = \frac{1}{2} \frac{200}{2.5} [W/L]_3 [V_{OV}]^2$   
 $\Rightarrow [W/L]_3 = 2.5 [W/L]_1$

$g_{m3,4} = g_{m1}$  and  $r_{o3,4} = r_{o1}$

Consider Q<sub>9</sub>, Q<sub>10</sub>:  $I_{10} = \frac{1}{2} K_p [W/L]_{10} [V_{OV}]^2$

$2I_1 = \frac{1}{2} \frac{200}{2.5} [W/L]_{10} [V_{OV}]^2$   
 $\Rightarrow [W/L]_{10} = 5 [W/L]_1$

$g_{m9,10} = 2g_{m1}$  and  $r_{o9,10} = r_{o1}/2$

Consider Q<sub>11</sub>:  $I_{11} = \frac{1}{2} K_n [W/L]_{11} [V_{OV}]^2$

$2I_1 = \frac{1}{2} 200 [W/L]_{11} [V_{OV}]^2$   
 $\Rightarrow [W/L]_{11} = 2 [W/L]_1$

$g_{m11} = 2g_{m1}$  and  $r_{o11} = r_{o1}/2$

Thus  $A = g_{m1} [g_{m1} r_{o1} (r_{o1} || \frac{r_{o1}}{2})] [g_{m1} r_{o1} r_{o1}]$   
 $= g_{m1} [g_{m1} r_{o1}] (r_{o1} || \frac{r_{o1}}{2} || r_{o1})$

$10^4 = \frac{1}{4} g_{m1} g_{m1} r_{o1} r_{o1}$   
 $\Rightarrow g_{m1} r_{o1} = 200$

Now  $g_{m1} r_{o1} = \frac{2I}{V_{OV}} \cdot \frac{V_A}{I}$

$\Rightarrow V_{OV} = 2V_A/200 = 2(10)/200 = 0.2V$

Hence  $g_{m1} = \frac{1}{2\pi f_t C_L} = 0.628 \text{ mA/V}$

$\Rightarrow r_{o1} = 200/g_{m1} = 318 \text{ k}\Omega$

$$g_{m1} = \frac{2I}{V_{ov}} \Rightarrow I = \frac{g_{m1} V_{ov}}{2}$$

$$\Rightarrow I_1 = \frac{g_{m1} V_{ov}}{2} = \frac{0.628 \text{ mA/V} \times 0.2 \text{ V}}{2} = 62.8 \mu\text{A}$$

$$SR = 2\pi f_t V_{ov} = 2\pi 10 \times 10^4 \times 0.2 = 12.5 \text{ V/}\mu\text{s}$$

$Q_1, Q_2, Q_5, Q_6, Q_7, Q_8$ :

$$I = \frac{1}{2} K_n [W/L] [V_{ov}]^2$$

$$62.8 = \frac{1}{2} 200 [W/L] [V_{ov}]^2$$

$$\Rightarrow [W/L]_1 = 15.7$$

$$\text{For } Q_3, Q_4: I = \frac{1}{2} \frac{200}{2.5} [W/L] [V_{ov}]^2$$

$$62.8 = \frac{1}{2} \frac{200}{2.5} [W/L] [0.2]^2$$

$$\Rightarrow [W/L]_3 = 2.5 [W/L]_1 = 39.25$$

For  $Q_9, Q_{10}$ :

$$[W/L]_9 = 5 [W/L]_1 = 78.5$$

For  $Q_{11}$ :

$$[W/L]_{11} = 2 [W/L]_1 = 31.4$$

For  $L = 1 \mu\text{m}$ :  $W_x = [W/L]_x \mu\text{m}$

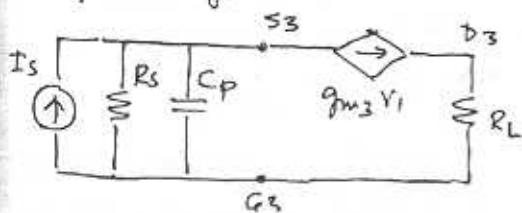
$$\begin{aligned} \text{Width for } Q_1, Q_2, Q_5, Q_6, Q_7, Q_8 &= 15.7 \mu\text{m} \\ \text{for } Q_3, Q_4 &= 39.25 \mu\text{m} \\ \text{for } Q_9, Q_{10} &= 78.5 \mu\text{m} \\ \text{for } Q_{11} &= 31.4 \mu\text{m} \end{aligned}$$

9.18

Simply invert circuit relative to  $V_{DD}, V_{SS}$  and reverse all arrows on FETs

9.19

Model for CG stage:



$$\text{at } S_3: Is - g_{m3} V_1 - V_1 \frac{(1 + sC_p R_s)}{R_s} = 0$$

$$\begin{aligned} \Rightarrow \frac{V_1}{I_s} &= \frac{1}{g_{m3} + (1 + sC_p R_s)/R_s} \\ &= \frac{1}{(g_{m3} + 1/R_s) + sC_p R_s / R_s} \end{aligned}$$

3dB down when

$$sC_p = (g_{m3} + 1/R_s)$$

$$\Rightarrow f_{p3} = \frac{(g_{m3} + 1/R_s)}{2\pi C_p} \approx \frac{g_{m3}}{2\pi C_p}$$

Q.E.D.

$$\text{Also } f_t \approx \frac{g_{m1}}{2\pi C}$$

$$PM = 180 - \phi_{\text{total}} = 90^\circ - \tan^{-1} \frac{f_t}{f_2}$$

$$\text{For } PM = 75^\circ: \frac{f_t}{f_2} = \tan 15^\circ = 0.27$$

$$\Rightarrow \frac{f_t}{f_2} = \frac{C_p}{C_L} = 0.27$$

$$\Rightarrow C_p = 0.27 C_L$$

20  $V_{BE} = V_T \ln \frac{I_S}{I_0}$

$$I_0 = \frac{2k}{V_T}$$

$$r_e = \frac{1}{I_0}$$

$$r_K = \beta r_e$$

$$r_o = \frac{V_A}{I_C}$$

$$r_H = \beta \frac{10V_A}{I_C}$$

DEVICE	$V_{BE}$ (mV)	$I_0$ (mA)	$r_e$ (Ω)	$r_h$ (kΩ)	$r_o$ (MΩ)	$r_H$ (kΩ)
Q1-A	517	0.38	2632	524	13.2	263
Q1-B	530	0.448	1543	309	7.72	154
Q1-C	618	22	456	9.09	0.227	0.46

21  $I_S = I_L \sqrt{\frac{I_{S1} I_{S2}}{I_{S1} + I_{S2}}}$

$$= 154 \sqrt{\frac{10^{14} \cdot 10^{14}}{3 \times 10^9 \cdot 6 \times 10^9}}$$

$$= \underline{\underline{36.3 \mu A}}$$

22  $I_{Etot} = 0.73 \text{ mA}$

$$I_{EA} = 0.26(0.73) = 0.1825 \text{ mA}$$

$$I_{EB} = 0.75(0.73) = 0.5475 \text{ mA}$$

$$V_{EB_A} = V_T \ln \frac{0.1825 \times 10^{-3}}{0.26 \times 10^{-3}} = \underline{\underline{0.625 \text{ V}}}$$

$$g_mA = \frac{I_C}{V_T} = \frac{I_E}{V_T} = \underline{\underline{7.3 \text{ mA/V}}}$$

$$r_{eA} = \frac{\alpha}{g_mA} = \underline{\underline{134.3 \Omega}}$$

$$r_{KA} = (\beta + 1)r_{eA} = \underline{\underline{6.95 \text{ k}\Omega}}$$

$$r_{oA} = \frac{V_A}{r_{eA}} = \underline{\underline{274 \text{ k}\Omega}}$$

$$V_{EB_B} = V_{EB_A} = \underline{\underline{0.625 \text{ V}}}$$

$$g_mB = \frac{0.5475}{26} = \underline{\underline{21.9 \text{ mA/V}}}$$

$$r_{eB} = \frac{\alpha}{g_mB} = \underline{\underline{44.7 \Omega}}$$

$$r_{KB} = (\beta + 1)r_{eB} = \underline{\underline{2.22 \text{ k}\Omega}}$$

$$r_{oB} = \underline{\underline{91.3 \text{ k}\Omega}}$$

23

Let  $V_{BE} = 0$

$$\text{For breakdown } V_{ID} = V_{BE} - V_{BE2} \\ > V_{BE1} + V_{BE2} + \gamma + S_A$$

$$\text{or } V_{ID} \geq \underline{\underline{58.4 \text{ V}}}$$

24

$$V_{G11} + V_{G22} = V_{G33} + V_{G44}$$

since  $V_G$ 's are equal

$$\sqrt{\frac{I_1}{K_1}} + \sqrt{\frac{I_2}{K_2}} = \sqrt{\frac{I_3}{K_3}} + \sqrt{\frac{I_4}{K_4}}$$

$$\sqrt{I_1} \left[ \frac{1}{\sqrt{K_1}} + \frac{1}{\sqrt{K_2}} \right] = \sqrt{I_3} \left[ \frac{1}{\sqrt{K_3}} + \frac{1}{\sqrt{K_4}} \right]$$

$$\text{or } \sqrt{\frac{I_1}{I_3}} = \frac{\sqrt{\frac{1}{K_2}} + \sqrt{\frac{1}{K_4}}}{\sqrt{\frac{1}{K_1}} + \sqrt{\frac{1}{K_2}}}$$

$$K_1 = K_2 \quad ; \quad K_3 = K_4 = 16 \cdot K_1$$

$$\sqrt{I_1} = \sqrt{I_3} \sqrt{\frac{K_2}{K_4}}$$

$$\text{or } I_1 = I_3 \frac{K_2}{K_4} \\ = \frac{I_3}{16} = \underline{\underline{100 \mu A}}$$

25

$$\text{As } V_{BE} = 0.7$$

$$I_{ref} = \frac{S - 1.9 - 0.57}{R_S}$$

$$= \underline{\underline{220.5 \mu A}}$$

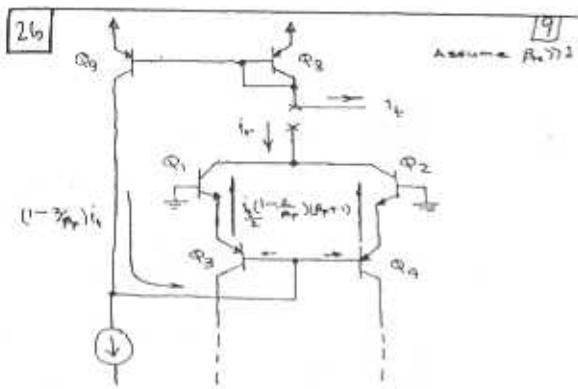
At this current level

$$V_{BE} = V_T \ln \frac{220.5 \times 10^{-3}}{10^{-6}} = 595 \text{ mV}$$

$$\Rightarrow I_{ref} = \frac{10 - 2(0.595)}{39 \text{ k}} = \underline{\underline{22.6 \mu A}}$$

For  $I_{ref} = 0.73 \text{ mA}$ ,  $V_{BE} = 0.625 \text{ V}$

$$R_S = \frac{10 - 2(0.625)}{0.73 \times 10^{-3}} = \underline{\underline{12 \text{ k}\Omega}}$$



29 In this case

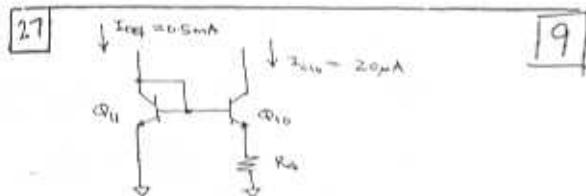
$$\frac{4I}{1 + \gamma_{pp}} + \frac{2I}{B_p} = I_{C10}$$

For  $B_p \gg 1$

$$I_{C10} \approx 4I \text{ or } I = \underline{\underline{475 \mu A}}$$

To correct we need  $I_{C10} = 58 \mu A$

$$\Rightarrow R_+ = \frac{V_T}{I_{C10}} \ln \frac{0.73mA}{I_{C10}} \approx \underline{\underline{1.94 k\Omega}}$$



$$\begin{aligned} I_{C10} R_4 &= V_{BE10} - V_{BE10} \\ &= V_T \ln \frac{I_{ref}}{I_{C10}} \\ \therefore R_4 &= \frac{26 \times 10^3}{20 \times 10^4} \ln \frac{0.5 \times 10^{-3}}{20 \times 10^{-6}} = \underline{\underline{4.02 k\Omega}} \end{aligned}$$

$$\begin{aligned} V_{BE11} &= V_T \ln \frac{0.5 \times 10^{-3}}{10^{-4}} = \underline{\underline{0.76 mV}} \\ V_{BE10} &= V_T \ln \frac{20 \times 10^{-6}}{10^{-4}} = \underline{\underline{5.35 mV}} \end{aligned}$$

28 Assume  $\beta_p \gg 1$

$$I_{C10} \doteq \frac{2I}{1 + \gamma_{pp}} + \frac{2I}{B_p}$$

$$\doteq 2I \left( 1 - \frac{2}{B_p} + \frac{1}{B_p} \right)$$

$$= 2I \left( 1 - \gamma_{pp} \right)$$

$$\Rightarrow I \doteq \frac{I_{C10}}{2} \left( 1 + \frac{1}{B_p} \right)$$

$$\text{Thus } \frac{1}{B_p} = 0.1 \Rightarrow \underline{\underline{\beta_p = 10}}$$

Without the above assumption and using the exact relationship  $\beta_p = 774$ .

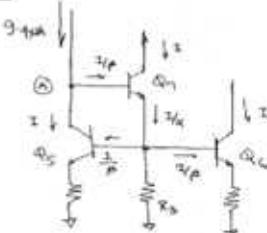
30 At  $I = 9.5 \mu A$

$$\begin{aligned} V_{BES} &= V_{BE1} = 517 mV \\ \text{and } V_{BE} &= V_{BES} + I R_2 \\ &= 526.5 mV \end{aligned}$$

$$\text{If } R_2 \text{ is shorted } V_{BES} = V_{BE} = 526.5 mV$$

$$\text{and } I_{C6} = I_S e^{\frac{V_{BES}}{V_T}} = \underline{\underline{14 \mu A}}$$

31



$$2I \text{ or } I + I_{p1} = 9.4 \mu A$$

$$\Rightarrow I = \frac{9.4}{1 + \gamma_p} = \underline{\underline{9.353 \mu A}}$$

$$I_{R_3} = \frac{I}{\alpha} = \frac{2I}{\beta} = 9.357 \mu A$$

$$V_{BS} = I_{R_3} R_3 = V_{BE1} + \frac{I}{\alpha} R$$

$$V_{BES} = V_T \ln \frac{9.353 \mu A}{10^{-4}} = 516.4 mV$$

$$\text{Thus } V_{BS} = 525.8 mV$$

$$\text{and } R_3 = \frac{V_{BS}}{I_{R_3}} = \underline{\underline{56.5 k\Omega}}$$

32 Assume equal collector current

$$I_A = I_{C2} = 9.5 \mu A$$

$$\begin{aligned} I_{C1} + I_{B1} - I_{B2} &\approx \frac{I_{C1}}{\beta_1} - \frac{I_{C2}}{\beta_2} \\ &= \underline{\underline{15.7 \mu A}} \end{aligned}$$

$$\begin{aligned} I_B &= \frac{1}{2} (I_{B1} + I_{B2}) \\ &= \underline{\underline{55.4 \mu A}} \end{aligned}$$

[33]  $I_B = 40\text{nA}$ ,  $I_{OB} = 4\text{nA}$

Thus, base currents are

$$I_{B1} = (I_B \pm \frac{I_{OB}}{2})$$

$$= \frac{9.5}{19} \mu\text{A}$$

$$\hat{B}_1 = \frac{9.5 \mu\text{A}}{38 \mu\text{A}} = \underline{\underline{250}}$$

$$\hat{B}_2 = \frac{9.5 \mu\text{A}}{42 \mu\text{A}} = \underline{\underline{225}}$$

$$\therefore \alpha \hat{B}_2 = 2.4$$

$$\overline{B}_2 = \frac{\hat{B}_1 + \hat{B}_2}{2} = 235$$

[34]  $I_{C1} + I_{C2} = 19\mu\text{A}$

Mirror forces  $I_{C2} = 0.9 I_{C1}$

$$\text{Thus } I_{C1} = \frac{19}{1.9} \mu\text{A} = 10\mu\text{A}$$

$$\text{and } I_{C2} = 9\mu\text{A}$$

$$V_{DS} = \frac{a V_{BE}}{2}$$

$$= V_{BE1} - V_{BE2}$$

$$= V_T \ln \frac{10}{9} = \underline{\underline{2.68 \text{ mV}}}$$

[35] At  $I_{BE} = 550\text{nA}$ ,  $V_{BE17} = 618\text{mV}$

$$I_{BE17} = \frac{550}{2.68} \approx 2.05 \mu\text{A}$$

$$\Rightarrow I_{C17} = 9.5\mu\text{A} = I_{BE17} + \frac{I_{BE17} R_E + V_{BE17}}{R_E}$$

$$\text{or } R_E = 95.7 \text{ k}\Omega$$

[36] Neglecting base currents

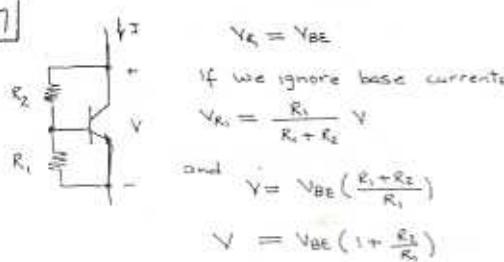
$$I_{C18} = I_{C19} = \frac{180}{2} = 90\mu\text{A}$$

$$V_{BE18} = V_T \ln \frac{90 \times 10^3}{154} = 575\text{mV}$$

$$\text{Thus } R_{in} = \frac{V_{BE18}}{I_{C19}} = \underline{\underline{5.57 \text{ k}\Omega}}$$

$$I_{C14} = 2 \times 10^{-4} e^{\frac{575 \times 10^3}{R_{in}}} = \underline{\underline{2.70 \mu\text{A}}} = I_{C20}$$

[37]



[38]  $I_{Vce} = I_{C12} + I_{C13A} + I_{C13B} + I_{C17} + I_{C19} + I_{C20}$

$$+ I_{C17} + I_{C19}$$

$$= (730 + 180 + 550 + 154 + 19 + 19 + 10.5 + 16.2) \mu\text{A}$$

$$= \underline{\underline{1.63 \mu\text{A}}}$$

$$P_{Diss} = P_D = I_{Vce} (V_C + V_{BE})$$

$$= 1.62 (15 + 1.5) \text{ mW}$$

$$= \underline{\underline{50.4 \text{ mW}}}$$

9

[39] Series connection of devices assures the same bias currents

$$R_{in} = (\beta + 1) (4.7\text{k}\Omega)$$

$$r_e = \frac{V_T}{9.5 \mu\text{A}} = 2.68 \text{ k}\Omega$$

$$R_{out} = \underline{\underline{3.17 \text{ M}\Omega}}$$

$$i_E = \frac{V_{out}}{4.7\text{k}\Omega}; i_O = 2i_E$$

$$\therefore G_m = \frac{i_E}{V_{in}} = \frac{2}{4.7\text{k}\Omega} = \frac{2}{3.54} = \underline{\underline{127 \mu\text{A/V}}}$$

$$R_{out} = r_o (1 + g_m (R_{in} r_o))$$

$$g_m \approx r_o$$

$$R_o = 2r_o = 5.24 \text{ k}\Omega$$

$$r_o = (\beta + 1) r_o = 134 \text{ k}\Omega$$

$$\text{Thus } R_{out} = \underline{\underline{15.4 \text{ M}\Omega}}$$

$$R_{in} = 18.2 \text{ M}\Omega \text{ (from text)}$$

$$R_{in} = R_{in} \parallel R_{out} = \underline{\underline{8.34 \text{ M}\Omega}}$$

$$G_m R_{in} = 127 \times 8.34 = 1059 \text{ V/V}$$

See gain decreases due to negative feedback.

[40]

$$R_o = r_o (1 + g_m (R_2 \parallel r_{re}))$$

- need to double the second factor

Since  $r_{re} \gg R_2$

$$R_{out} \approx r_o (1 + g_m R_2)$$

$$\text{Thus } 1 + g_m R_2 = 2 (1 + g_m R_2)$$

$$g_m = \frac{1}{2.63 \times 10^3}, R_2 = 1\text{k}$$

$$\Rightarrow R_o = \underline{\underline{4.63 \text{ k}\Omega}}$$

[41]  $I_{ES} = I_{CC} = 2mA$

$$\Rightarrow r_{es} = r_{ce} = r_{em} = 2.63k\Omega$$

(a)  $V_{be} \Rightarrow (V_{cc} + R_2)I_e = \underline{4.63k\Omega \times I_e}$

(b)  $R_B = 50k\Omega \parallel r_{es} \parallel r_{re}$

$$= 4.5 + 1k\Omega$$

$$\Rightarrow I_{re} = \frac{V_{be}}{R_B} = \underline{0.163I_e}$$

(c)  $I_{bT} = \frac{I_{re}}{\beta_p+1} = \frac{0.163}{2.61} I_e = \underline{0.062I_e}$

(d)  $V_{bT} = V_{be} + r_{es} I_{re}$

$$= (4.63k\Omega + 2.63k\Omega \times 0.062)I_e$$

$$= \underline{4.9k\Omega \times I_e}$$

(e)  $R_{in} = \frac{V_{bT}}{I_e} = \underline{4.9k\Omega}$

[42] From Eqn 9.85

$$\frac{\Delta I}{I} = \frac{\Delta R}{R + \Delta R + r_e}$$

$$\Delta I \approx G_m V_{os} = \frac{V_{os}}{2r_e}$$

Thus

$$\frac{V_{os}}{2r_e I} = \frac{\Delta R}{R + \Delta R + r_e} ; \quad r_e I = V_T$$

$$\frac{V_{os}}{2V_T} = \frac{\Delta R}{R} \left[ \frac{1}{1 + \frac{r_e}{R} + \frac{\Delta R}{R}} \right] \quad (*)$$

$$\frac{V_{os}}{2V_T} \left( 1 + \frac{r_e}{R} \right) = \frac{\Delta R}{R} \left( 1 - \frac{V_{os}}{2V_T} \right)$$

$$\frac{\Delta R}{R} = \frac{V_{os}}{2V_T} \frac{1 + r_e/R}{1 - \frac{V_{os}}{2V_T}}$$

(b)  $V_{os} = 5mV, \quad r_e = 2.63k\Omega, \quad R = 1k\Omega$

$$\frac{\Delta R}{R} = \frac{5}{2(2.5)} \frac{1 + 2.63}{1 - \frac{5}{2(2.5)}} = \underline{0.40}$$

(c)  $r_2$  completely shorted

$$\Rightarrow \frac{\Delta R}{R} = -1$$

From (b)  $\frac{V_{os}}{2V_T} = -1 \frac{1}{r_e/R}$

$$\Rightarrow V_{os} = \underline{-19mV} \quad (\text{or } 19mV)$$

[43] Current in the collector of  $T_3$  remains unchanged at  $9.5\mu A$

Thus  $I_{E3} = I_{E4} = \frac{E_1}{2\theta} 9.5\mu A = 9.69\mu A$

$$I_{C4} = \frac{2\theta}{2\theta} I_{E4} = 3.317\mu A$$

$$\Rightarrow \Delta I = 9.5 - 3.317 = 6.183\mu A$$

$$V_{os} = \frac{\Delta I}{G_m} = 2r_e \Delta I = 2(2.63k\Omega)(6.183\mu A)$$

$$= \underline{0.36mV}$$

[44]

[49]

[45] Working with Common-mode half circuits

$$I_2 = \frac{\beta_p}{\beta_p + 1} \frac{V_{cm}}{r_{e1} + r_{e2} + \frac{2R_o}{\beta_p + 1}}$$

$$= \frac{\beta_p V_{cm}}{(\beta_p + 1)(r_{e1} + r_{e2}) + 2R_o}$$

Similarly

$$I_4 = \frac{\beta_p V_{cm}}{(\beta_p + 1)(r_{e1} + r_{e2}) + 2R_o}$$

[46] Recall that for a resistor degenerated mirror the current gain is given, approximately, by

$$\frac{I_2}{I_1} = \frac{R_{e1}}{R_{e2}} \quad \text{where } R_{e1} \text{ is the total emitter resistance.}$$

(a)  $r_1$  shorted

$$\frac{I_2}{I_1} = \frac{r_{e1}}{r_{e2} + R_2} = \frac{2.63}{2.63 + 1} = 0.72$$

Thus  $I_2 = 0.72 I_1 \quad (\text{cf. z}_1)$

Since the output impedance is unaffected, the gain is thus reduced by  $\underline{14\% \left( \frac{1-0.72}{0.72} \right)}$

(b)  $r_2$  shorted

$$\frac{I_2}{I_1} = \frac{r_{e1} + R_2}{r_{e2}} = \frac{2.63}{2.63} = 1.33$$

$$\Rightarrow I_2 = (1 + 1.33) I_1 = 2.33 I_1$$

Note that the output of the mirror decreases because of the lack of degeneration from  $r_2$ . Neglecting this, since  $R_{e1}$  is largely determined by  $R_{e2}$ , the gain increases by  $\underline{\frac{2.33-1}{1} = 19\%}$ .

(c) current gain remains at unity. Thus  $I_2 = 2 I_1$  and gain is unaffected.

[47]

Since  $r_o$  is typically very large we will ignore its effect

$$r_{o1} = \frac{125}{9.5 \mu A} = 13.16M\Omega$$

$$r_{o3} = \frac{50}{9.5 \mu A} = 5.26M\Omega$$

$$R_L = (\beta_p + 1) \left[ r_{e1} + \left( r_{e2} + \frac{2R_o}{\beta_p + 1} \right) \parallel r_{o1} \parallel r_{o3} \right]$$

$$= 2.63 \left[ 2.63 + \left( 2.63 + \frac{2 \cdot 5.26}{1.33} \right) \parallel 13.16M \parallel 5.26M \right]$$

$$= \underline{19.5M\Omega}$$

Since  $R_L = 2 R_{in}$

$$R_{in} = \frac{R_L}{2} (1 + \beta_p) \quad \text{See problem #8}$$

$$= \underline{49.7M\Omega}$$

48  $R_{12} = (\beta + 1) [R_{16} + R_{17} \parallel R_9]$

$R_{16} = 1.54 k\Omega$

$R_{17} = 45.5 \Omega$

$R_{17} = 201 (45.5 + 50) = 19.2 k\Omega$

$\Rightarrow R_{12} = 201 (1.54 + 19.2 \parallel 50) \Omega = 3.1 M\Omega$

$V_{167} = \frac{R_{17} \parallel R_9}{R_{16} + R_{17} \parallel R_9} V_{12}$

$= 0.9 V_{12}$

$I_{17} = \frac{\alpha}{R_{17} + R_9} 0.9 V_{12}$

$\Rightarrow g_{m12} = \frac{\alpha (0.9)}{45.5 + 50} = 9.38 mA/V$

49  $R_{167} = 7.87 k\Omega$

$I_{16B} = 650 \mu A$

$g_{m13B} = 22 mA/V ; R_{13B} = (\beta + 1) / g_m = 2.32 k\Omega$

$R_E = \frac{50}{650 \mu A} = 76.9 k\Omega$

$R_{13B} = R_E (1 + g_m (R_E \parallel r_o))$

$= 76.9 [1 + 22 (R_E \parallel 2.32)]$

$= 7.87$

$\Rightarrow R_E \parallel 2.32 = 0.34\%$

$\text{and } \frac{1}{R_E} = 2.44 \text{ or } R_E = 0.415 k\Omega$

$R_E = 410 \Omega$

$\text{Current } \frac{R_{E12}}{R_E} = \frac{650 \mu A}{730 \mu A} \Rightarrow R_{E12} = 3.05 \Omega$

$\frac{R_{E13A}}{R_E} = \frac{550}{180} = 3.05 k\Omega$

50  $\hat{V}_o = V_{ce} - V_{ce, saturation} - V_{BE14}$   
 $= 4.2 V$

$V_o \approx -V_{EE} + V_{sat,17} + V_{BE23} + V_{BE20}$ 
 $= -5 - 0.2 + 0.4 + 0.4$ 
 $= -3.4 V$

9

51 With Q<sub>23</sub> removed, current in Q<sub>11</sub> increases to 730 μA. This changes R<sub>12</sub>.

$r_{117} = \frac{V_T}{730 \mu A} = 34.2 \Omega$

$\Rightarrow g_{m2} = \frac{\alpha}{100 + 34.2} = 6.3 mA/V$

Because  $r_{117} \gg r_{13B}$ , R<sub>12</sub> remains virtually unchanged at 3.1 MΩ.

$R_3 = (\beta + 1) R_E \parallel R_{13A} = 74 k\Omega$

$\Rightarrow A_Z = -6.3 (81) \frac{74}{74 + 7.81} = -2.63 V/V$

52 Ignore base currents of Q<sub>11</sub>.

$180 \mu A = I_{C16} + \frac{I}{\beta + 1}, \text{ where } I = 3 R_E$

$I_{C16} = I_S e^{\frac{V_{BE}}{V_T}}, \text{ where } V_{BE} = I R_E$

$\text{Thus } I = \frac{V_T}{2.7} \ln \left[ \frac{180 \mu A - \frac{I}{\beta + 1}}{I_S} \right]$

$= 191.422 V/V$

$\text{or } 105.648$

OUTPUT CURRENT IS LIMITED TO 120mA (SEE PROBLEMS 35 AND 36)

$\Rightarrow |V_o| < 200 mV (2nd)$

$|V_o| < 44$

To obtain a seed solution, let I = 0

$\text{on RHS } \Rightarrow I = \frac{V_T}{2.7} \ln \frac{180 \mu A}{120} = 21.9 mA$

$\text{iterating } I = 21.0 mA$

53 Maximum output current of the 1st stage = 19mA

$\Rightarrow I_{C22} = 19 \mu A \Rightarrow V_{BE22} = V_{BE24} = 534 mV$

$\Rightarrow I_{R11} = \frac{534}{50} = 10.7 \mu A$

$\therefore I_{C21} = (19 + 10.7) \mu A = 29.7 \mu A$

$\text{and } V_{BE21} = 545.3 mV$

$V_{BE21} = I R_4 \Rightarrow I = 20.2 mA$

A simple doubling of R<sub>1</sub>

54  $\frac{V_o}{V_i} = \frac{243,147}{5.97} = 250,667 \text{ V/V} = \underline{\underline{103 \text{ dB}}}$

$$\frac{R_L}{R_L + R_o} = 0.2 \implies R_o = R_L \left( \frac{1}{0.2} - 1 \right)$$

$$\text{or } R_o = \underline{\underline{61.9 \Omega}}$$

$$\frac{V_o}{V_i} \Big|_{R_o=260} = 250,667 \cdot \frac{260}{260 + 61.9}$$

58 dominant pole  $f_p = \frac{1}{2\pi R(A_C)} ; A = 1000$

with single pole response

$$A_{fp} = f_L \rightarrow f_p = \frac{SR}{1000} = 5 \text{ Hz}$$

$$\Rightarrow R = \frac{1}{2\pi(5)1000(50\pi)} = \underline{\underline{637 \text{ k}\Omega}}$$

55 86° PM says that 2nd pole introduces 10° of phase shift at unity gain

$$\text{i.e. } \tan^{-1} \frac{f_L}{f_{p2}} = 10^\circ$$

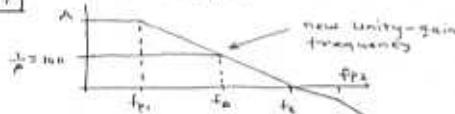
$$\text{or } f_{p2} = \underline{\underline{5.67 \text{ MHz}}}$$

56 Each pole adds 5° of phase shift

$$\tan^{-1} \frac{10^6}{f_{p1,2}} = 5^\circ$$

$$\Rightarrow f_{p1,2} = \underline{\underline{11.4 \text{ MHz}}}$$

57 Consider Bode plot



85° of closed-loop phase margin

$$\Rightarrow \tan^{-1} \frac{f_L}{f_{p2}} = 5^\circ$$

$$\text{or } f_B = \underline{\underline{437 \text{ kHz}}}$$

Recalling the "broadbanding" effect of negative feedback, we get

$$f_B = f_p(1 + AB) \approx f_p, AB$$

$$\text{Loop gain } AB = 2.43 \times 10^5 \cdot \frac{1}{100} = 2.43 \times 10^3$$

$$\Rightarrow f_B = \underline{\underline{180 \text{ Hz}}}$$

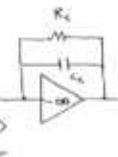
$$f_B = \frac{G_m}{2\pi C_L} = 437 \text{ kHz}$$

$$\Rightarrow C_L = \frac{1}{5.26 \times 10^5 (2\pi) 437 \times 10^3} = 0.69 \text{ pF}$$

59

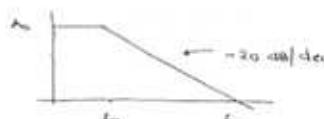
dc. gain is

$$A_{dc} = G_m R_C \\ = 10 \times 10^3 \times 10^3 \\ = 10^6 \text{ V/V} = \underline{\underline{10^6}}$$



$$f_p = \frac{1}{2\pi R_C} = \frac{1}{2\pi(10^3)10^3} \\ \approx 31.8 \text{ Hz}$$

$$f_L = A_{dc} f_p \\ = 31.8 \text{ MHz}$$



$$SR = \frac{2I}{C_L}$$

$$G_m = \frac{I}{2V_T} \Rightarrow 2I = 4G_m V_T$$

$$SR = \frac{4G_m V_T}{C_L} = \frac{4(10 \times 10^3)(25 \times 10^{-12})}{50 \times 10^{-12}} \\ = \underline{\underline{20 \text{ V/}\mu\text{s}}}$$

60 SR = 10V/μs

$$|V_{max}| = 10V$$

For sine wave of amplitude  $V_{max}$ , maximum rate of change

$$\left. \frac{dV}{dt} \right|_{max} = \omega_m V_0 = 2f_m V_0$$

Thus

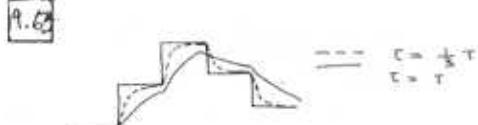
$$2f_m V_0 = SR$$

$$\Rightarrow f_m = \frac{SR}{2\pi(4V_T)} = \frac{10 \times 10^3}{2\pi(4 \times 25 \times 10^{-12})} = \underline{\underline{159.2 \text{ kHz}}}$$

$$f_c = \frac{SR}{2\pi(4V_T)} = \frac{10 \times 10^3}{2\pi(4 \times 25 \times 10^{-12})} \\ = \underline{\underline{159 \text{ mHz}}}$$

[9.61]  $I_{E1} = I_{E2} = 59 \mu A \geq I_{E3} = I_{E4}$   
 $I_{E5} = I_{mA} ; V_{BE5} = V_{BE4}$   
 $\therefore I_{E5} = I_{mA} = I_{E3}$   
 $r_{e1} = r_{e2} = 56 \Omega$   
 $r_{e3} = r_{e4} = r_{e7} = 25 \Omega$   
 $G_m1 = 2 \left( \frac{1}{r_{e1}} \right) = 2mA/V$   
 $R_{in} = (1 + 1) (r_{e5} \parallel r_{e6})$   
 $= 125 k\Omega$   
 and  $A_1 = G_m R_{in} = 2.5 V/V$

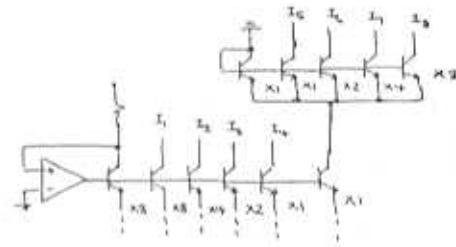
[9.62]  $\pm \frac{1}{2} LSA$  must be less than 1%  
 $i.e. \frac{1}{2} \frac{1}{2^N} \leq \frac{1}{100} \Rightarrow N \geq 6.4$   
 $\therefore N = 6.67$   
 Resolution  $\frac{10V}{2^6} = 0.15625V$   
 For some resolution need 7 bits  
 Still 7 bits  
 Resolution  $= \frac{10}{2^7} = 0.117V$   
 $\therefore \pm \frac{1}{2} LSA = \pm \frac{10}{2^7} = \pm 0.055V$



[9.64] Require error in MSB  $\leq \pm \frac{1}{2} LSA$   
 $\frac{V}{R} - \frac{V}{R(1 + \frac{X}{100})} \leq \pm \frac{1}{2} \frac{V}{2^{N+1} R}$   
 $\frac{1 + \frac{X}{100} - 1}{1 + \frac{X}{100}} \leq \frac{1}{2^{N+1}} \text{ or } \frac{X}{100} (2^N - 1) \leq 1$   
 $\Rightarrow X = \frac{1}{2^N - 1} \times 100$   
 $N=2 \quad X = 33.3\%$   
 $N=4 \quad X = 6.67\%$   
 $N=8 \quad X = 0.39\%$

[9.65] Since  $V_{BE}$ 's are equal, collector currents are scaled with respect to emitter areas  
 $I_1 + I_2 + I_3 + I_4 = I$   
 $I_1 (1 + 1 + 2 + 4 + 8) = I \Rightarrow I_1 = \frac{I}{16}$   
 $I_2 = \frac{I}{8}$   
 $I_3 = \frac{I}{4}$        $I_4 = \frac{I}{2}$

[9.66] Circuit is sketched below



$$A_{out} = 2 + 2 + 4 + 2 + 1 + 1 + 1 + 2 + 4 + 8 \\ = 40$$

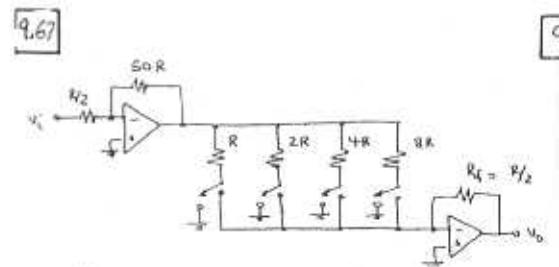
For n-bit binary weighted

$$A_{out} = (1 + 1 + 2 + 4 + \dots + 2^{n-1} + 2^n)$$

$$= 2^{n+1} + 1 \quad \{ 1 + 2 + 4 + \dots + 2^{n-1} \}$$

$$= 2^{n+1} + 1 + 2^n - 1 = 2^{n+1} + 2^n$$

$$N=3 \quad A_{out} = 2^7 + 2^6 = 384$$



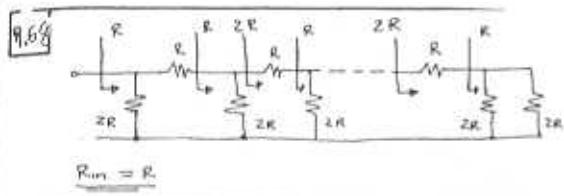
$$2^N - 1 \text{ discrete outputs} = 2^4 - 1 \\ = 15$$

$$\text{Smallest sine wave} = \frac{10}{2^4} = 0.425V$$

$$\text{Largest} = 10 \times \frac{G_E}{G_B} = 10 \cdot \frac{1 + 2 + 4 + 8}{2} \\ = 5 \cdot \frac{15}{2} = 37.5V$$

$$10V \text{ pp-pk} \Rightarrow 5V \text{ pk} \text{ or } \frac{1}{2} \text{ FS}$$

$$\therefore D = 1000$$



9.69  $T_C = \frac{1}{f_{cav}} = 1\mu s$

$$T_1 = 2^{12} T_C = 4.096 ms$$

$$T = T_1 + T_2 = T_1 \left(1 + \frac{V_h}{V_{ref}}\right)$$

$$= 2 T_1 = 8.19 ms$$

$$V_{peak} = 10 \Rightarrow \frac{V_h}{V_{peak}} T_1$$

$$\Rightarrow T = \frac{V_h}{V_{peak}} T_1 = 4.096 ms$$

$\Delta T = -1\%$  and causes a  $-1\%$  change in  $V_{peak}$

$$\Rightarrow V_{peak} = 9.9 V$$

No. : Final count does not depend on  $T$

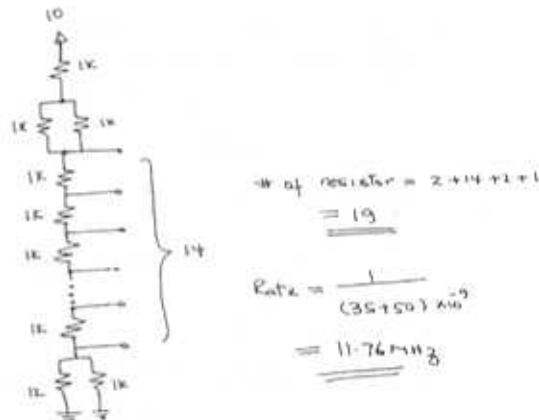
9.70  $2^{n-1}$  comparators = 15

Comparators are biased 1LSB apart starting from  $\frac{1}{2}$  LSB.

$$\text{Thus, } V_{refn} = \left(\frac{2^n-1}{2}\right) L_{SB}$$

$$1 L_{SB} = \frac{10}{2^8} = 0.625 V$$

and references are  $0.625, 0.9375, 1.5625, 2.1875, 2.8125, \dots, 9.84375$



(a) (000...000)  $L_5$  in all  
and  $(0000)_2$

(b) all comparators with references less than 5IV will produce ones  
Let's find 'em

$$\left(\frac{2^n-1}{2}\right) 0.625 \geq 5.1 \Rightarrow n \geq 8.4$$

Thus the lowest 8 outputs will be ones:  
 $(000000001111)_2$  and  $(0100)_2$

(c) Full scale input  $(111\dots111)$   
and  $(1111)_2$

[10]

## Chapter 10 - Problems

1 Ideal 3V logic implies:

$$V_{OH} = V_{DD} = \underline{3.0V}, \quad V_{OL} = \underline{0.0V};$$

$$V_{IH} = V_{DD}/2 = 3.0/2 = \underline{1.5V};$$

$$V_{IL} = V_{DD}/2 = \underline{1.5V}; \quad V_{IH} = V_{DD}/2 = \underline{1.5V}$$

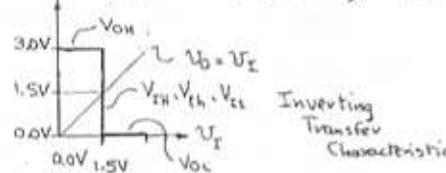
$$NM_H = V_{OH} - V_{IH} = 3.0 - 1.5 = \underline{1.5V}$$

$$NM_L = V_{IL} - V_{OL} = 1.5 - 0.0 = \underline{1.5V}$$

The gain in the transition region is:

$$(V_{OH} - V_{OL}) / (V_{IH} - V_{IL}) =$$

$$(3.0 - 0.0) / (1.5 - 1.5) = 3/0 = \underline{\infty V/V}$$



2 Nearly ideal 3.3V logic, assumed ideal: [10]

$$\rightarrow V_{OH} = \underline{3.3V}, \quad V_{OL} = \underline{0.0V}, \quad V_{IH} = 0.7(3.3) = \underline{2.32V}$$

Now, at  $V_{IH}$ ,  $V_O = V_I$ , so to reach  $V_O = 1.32V$

the required input is  $1.32/(-50) = -26.4mV$

$$\text{Thus, } V_{IL} = 1.32 - 26.4 \times 10^{-3} = \underline{1.294V}$$

$$\text{Likewise, } V_{IH} = 1.32 + (3.3 - 1.32)/50 = \underline{1.360V}$$

Best possible noise margins are:

$$NM_H = V_{OH} - V_{IH} = 3.3 - 1.360 = \underline{1.940V}$$

$$NM_L = V_{IL} - V_{OL} = 1.294 - 0.0 = \underline{1.294V}$$

For noise margins only 7/10 of these, and

$V_{OH}, V_{OL}$  still ideal:

$$V_{IH} = 3.3 - 0.7(1.940) = \underline{1.942V}, \quad \text{and}$$

$$V_{IL} = 0.0 + 0.7(1.294) = \underline{0.906V}$$

Correspondingly, the large-signal voltage gain is:  
 $G = (3.3 - 0.0) / (0.906 - 1.942) = \underline{-3.18V/V}$

[3]

Here,  $V_{OH} = 1.2V$ , and  $V_{OL} = 0.0V$ .

$$\text{Also, } V_{IH} - V_{IL} \leq 1.2/3 = 0.4V \quad \dots (1)$$

Now, the noise margins are "within 30% of one other". Thus  $NM_H = (1 \pm 0.3)NM_L$  or

$$NM_L = (1 \pm 0.3)NM_H. \quad \text{Thus to remain "within" either } NM_H = 1.3NM_L \text{ or } NM_L = 1.3NM_H,$$

in which case, either  $NM_L = 0.769 NM_H$ , or  $NM_H = 0.769 NM_L$

$$\text{For the former case: } 0.769(V_{OH} - V_{IH}) = (V_{IL} - V_{OL})$$

$$\text{or } 0.769(1.2 - V_{IH}) = V_{IL} - 0, \quad \text{whence}$$

$$V_{IL} = 0.923 - 0.769 V_{IH}$$

$$\text{Now, from (1), } V_{IH} = V_{IL} + 0.4.$$

$$\text{Thus, } V_{IL} = 0.923 - 0.769(V_{IL} + 0.4) = 0.615 - 0.769V_{IL}$$

$$\text{and } V_{IL} = 0.615/1.769 = \underline{0.349V},$$

$$\text{whence } V_{IH} = 0.4 + 0.349 = \underline{0.749V}. \quad [10]$$

Alternatively,  $NM_H = 0.769 NM_L$ , and

$$(V_{OH} - V_{IH}) = 0.769(V_{IL} - V_{OL}), \quad \text{or}$$

$$1.2 - V_{IH} = 0.769 V_{IL} - 0, \quad \text{and } V_{IH} = 1.2 - 0.769 V_{IL}$$

$$\text{With (1), } V_{IL} + 0.4 = 1.2 - 0.769 V_{IL}, \quad \text{and}$$

$$1.769 V_{IL} = 0.8, \quad \text{whence } V_{IL} = \underline{0.452V}$$

$$\text{and } V_{IH} = 0.4 + 0.452 = \underline{0.852V}$$

Thus, overall,  $V_{OH} = 1.2V$ ,  $V_{OL} = 0.0V$ ,

$V_{IH}$  ranges from  $\underline{0.749V}$  to  $\underline{0.852V}$ , and

$V_{IL}$  ranges from  $\underline{0.349V}$  to  $\underline{0.452V}$ , in

which case, margins can be as low as :

$$NM_L = V_{IL} - V_{OL} = \underline{0.349V}, \quad \text{and}$$

$$NM_H = V_{OH} - V_{IH} = 1.2 - 0.852 = \underline{0.348V},$$

and as high as  $\underline{0.452V}$ , and  $\underline{0.451V}$ .

[4] [10] a) Generally,  $t_p = (t_{PHL} + t_{PLH})/2$ ,

but due to current ratio,  $t_{PHL} = 0.5t_{PLH}$ .

Thus  $1.5t_{PLH} = 2(1.2\text{ ns})$ , whence

$$t_{PLH} = 2.4/1.5 = 1.6\text{ ns}, \text{ and } t_{PHL} = 0.8\text{ ns}$$

Check:  $t_p = (1.6 + 0.8)/2 = 1.2\text{ ns}$

b) Generally,  $t_p = CV/I = kC$

Originally,  $1.2 = kC \quad \dots (1)$

$$\text{Then, } 1.7(1.2) = k(C+1) \quad \dots (2)$$

$$\text{Dividing (2)/(1)}: 1.7 = (C+1)/C$$

$$\text{Thus, } 1.7C = C+1, 0.7C = 1, C = 1.43\text{ pF}$$

(the combined load and output capacitance)

c) With the load inverter removed:

$$0.6(1.2) = k(1.43 - C_{in}) \quad \dots (3)$$

$$\text{Dividing (3)/(1)}: 0.6 = (1.43 - C_{in})/1.43$$

$$\text{Thus, } C_{in} = 1.43(1 - 0.6) = 0.57\text{ pF}, C_{out} = 1.43 - 0.57 = 0.86\text{ pF}$$

[5] [10] Average static current (at 50% duty cycle)

$$\text{is } (0+40)/2 = 20\text{ mA}$$

When switching at 100 MHz, current is 150 mA

Dynamic current is  $(150 - 20) = 130\text{ mA}$ , and

dynamic power is  $P_d = (3.3V)(130\text{ mA}) = 429\text{ mW}$ .

But,  $P_d = fCV_{DD}^2$ , and  $C = 429 \times 10^{-6} / (100 \times 10^6 \times 3.3)$

$$\text{Thus } C_{eq} = C = 0.394\text{ pF}$$

[6] [10] Now,  $P_d = fCV_{DD}^2$ . Thus for  $V_{DD}$  reduced

from 5V to 3.3V, power reduces by a factor  $(3.3/5)^2 = 0.436$ .

For frequency reduced by a factor  $(3.3/5)$ ,

additional power saved is  $((5-3.3)/5) \times 10^{-3} \times 0.436 =$

$$1.48\text{ mW}$$

Check: First reduction is to  $10/0.436 = 4.36\text{ mW}$ , and the second to  $4.36(2.3/5) = 2.31\text{ mW}$ , by  $4.36 - 2.31 = 1.48\text{ mW}$

[7] [10] a) For current proportional to  $V_{DD}$ ,

reduction in current (and frequency) is to  $3.3/5 = 0.66$  or 66% of previous (or by 34%)

b) For current proportional to  $V_{DD}^2$ ,

reduction is to  $(3.3/5)^2 = 0.436$  or 44% of previous (or by 56%).

For case a), delay increases by  $1/0.66 = 1.515$

times, and power reduces (from Eq 13.4) to  $0.66 \times (3.3/5)^2 = 0.287$  of previous, for a net change in DP to  $1.515 \times 0.287 =$

$$0.435 \text{ of previous, a net decrease by 56.5\%}$$

For case b), delay increases by  $1/0.436 = 2.29$

times, and power reduces to  $0.436(3.3/5)^2 = 0.190$  of previous, for a net change in DP to  $2.29 \times 0.190 = 0.435$  of previous (by 56.5%)

[8] [10] a) For current proportional to  $(V_{DD} - V_t)$

i) For  $V_t = 1V$ : Change in current to  $(3.3-1)/(5.0-1) = 0.515$ , or 51.5% of previous, a decrease of 42.5%.

Change in delay to  $1/0.515 = 1.74$  or

an increase of 74%.

Change in frequency to 0.515 or 51.5% of previous.

Change in dynamic power due to change in both voltage and frequency is to  $0.515 \times (3.3/5)^2 = 0.250$  or 25% of previous.

Change in DP is to  $1.74 \times 0.25 = 0.435$ , or

43.5% of previous, a decrease of 56.5%.

ii) For  $V_t = 0.5V$ :

Current: to  $(3.3-0.5)/(5.0-0.5) = 0.622$ , to 62.2% (next)

[8]

[10]

$$\text{Delay: to } \sqrt{0.622} = 1.61, \text{ to } \underline{\underline{161\%}}$$

$$\text{Frequency: to } \underline{\underline{62.2\%}}$$

$$\text{Dynamic Power: to } 0.622 (3.3/5.0)^2 = 0.271, \text{ to } \underline{\underline{27.1\%}}$$

$$\text{Delay-Power (DP): to } 1.61 \times 0.271 = 0.436, \text{ to } \underline{\underline{43.6\%}}$$

(iii) For  $V_t = 0V$  (as in P10.7)

$$\text{Current: to } 3.3/5.0 = 0.66, \text{ to } \underline{\underline{66\%}}$$

$$\text{Delay: to } \sqrt{0.66} = 1.515, \text{ to } \underline{\underline{151.5\%}}$$

$$\text{Frequency: to } \sqrt{1.515} = 0.66, \text{ to } \underline{\underline{66\%}}$$

$$\text{Dynamic Power: to } 0.66 (3.3/5.0)^2 = 0.287, \text{ to } \underline{\underline{28.7\%}}$$

$$\text{Delay-Power (DP): to } 1.515 \times 0.287 = 0.435, \text{ to } \underline{\underline{43.5\%}}$$

b) For current proportional to  $(V_{DD} - V_t)^2$ i) For  $V_t = 1V$ :

$$\text{Current: to } [(3.3-1)/(5.0-1)]^2 = 0.331, \text{ to } \underline{\underline{33.1\%}}$$

$$\text{Delay: to } \sqrt{0.331} = 3.02, \text{ to } \underline{\underline{302\%}} \text{ (next)}$$

[11]

[10]

$$\text{Frequency: to } \sqrt{3.02} = 0.331, \text{ to } \underline{\underline{33.1\%}}$$

$$\text{Dynamic Power: to } 0.331 (3.3/5.0)^2 = 0.144, \text{ to } \underline{\underline{14.4\%}}$$

$$\text{Delay-Power (DP): to } 3.02 \times 0.144 = 0.435, \text{ to } \underline{\underline{43.5\%}}$$

ii) For  $V_t = 0.5V$ :

$$\text{Current: to } [(3.3-0.5)/(5.0-0.5)]^2 = 0.387, \text{ to } \underline{\underline{38.7\%}}$$

$$\text{Delay: to } \sqrt{0.387} = 2.58, \text{ to } \underline{\underline{258\%}}$$

$$\text{Frequency: to } \sqrt{2.58} = 0.387, \text{ to } \underline{\underline{38.7\%}}$$

$$\text{Dynamic Power: to } 0.387 (3.3/5.0)^2 = 0.168, \text{ to } \underline{\underline{16.8\%}}$$

$$\text{Delay-Power (DP): to } 2.58 \times 0.168 = 0.435, \text{ to } \underline{\underline{43.5\%}}$$

(iii) For  $V_t = 0V$  (as in P10.7):

$$\text{Current: to } [3.3/5.0]^2 = 0.436, \text{ to } \underline{\underline{43.6\%}}$$

$$\text{Delay: to } \sqrt{0.436} = 2.30, \text{ to } \underline{\underline{230\%}}$$

$$\text{Frequency: to } \sqrt{2.30} = 0.436, \text{ to } \underline{\underline{43.6\%}}$$

$$\text{Dynamic Power: to } 0.436 (3.3/5.0)^2 = 0.190, \text{ to } \underline{\underline{19.0\%}}$$

$$\text{Delay-Power (DP): to } 2.30 \times 0.190 = 0.437, \text{ to } \underline{\underline{43.7\%}}$$

[9]

Chip area changes to  $0.9 \times 0.9 = \underline{\underline{0.81}}$  of prevCurrent changes only due to oxide change, since  
change cancels, to  $0.9 = \underline{\underline{1.11}}$  of previousEffective Capacitance: half changes by  $0.9^2$  to C  
half changes by  $0.9/0.9 = 1$ Net Capacitance changes by  $(0.81 + 0.90)/2 = \underline{\underline{0}}$ Propagation Delay changes by  $(1/1.11)0.855 = \underline{\underline{0.770}}$ Max. Operating Frequency changes by  $1/0.77 = \underline{\underline{1.31}}$ Dyn. Power Diss. (at max. freq.) changes by  $1.30 \times 0.855$ Delay-Power Product changes by  $0.770 \times 1.11 = \underline{\underline{0.86}}$ Performance (f/A) changes by  $1.30/0.81 = \underline{\underline{1.60}}$ 

Now, if the supply voltage is reduced by 1

(but  $V_t$  is not remaining at  $0.2V_{DD}$ ) $(V_{DD} - V_t)$  changes to  $(0.9V_{DD} - 0.2V_{DD}) = 0.7$   
or to  $(0.7V_{DD})/0.8V_{DD} = \underline{\underline{0.875}}$  of previous

[9]

[10]

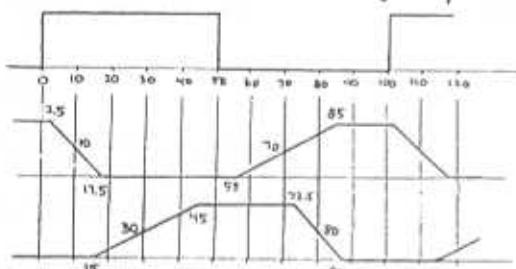
Thus, current changes by  $(1/0.9)(0.875)^2 = \underline{\underline{0.858}}$ 

Assume Capacitances are voltage-independent.

Thus, propagation delay changes by  $(1/0.855)0.855 = \underline{\underline{1.005}}$ Maximum frequency changes by  $1/1.005 = \underline{\underline{0.995}}$ Dyn. Power changes by  $(0.995)(0.855)(0.9)^2 = \underline{\underline{0.689}}$ Delay-Power product changes by  $1.005(0.689) = \underline{\underline{0.693}}$ Performance (f/A) changes by  $0.995/0.81 = \underline{\underline{1.23}}$

**[10]** The results depend on whether the gates are inverting or non-inverting.

For inverting gates, the timing diagram is:



Note: For simplicity, 0% to 100% (rather than 10% to 90%) both in the diagram above and calculation to follow:

For inverting gates (as shown, above):

- For a rising input, time to 90% change of output of second gate is  $10 + 20 + 30/2 = \underline{45 \text{ ns}}$
- For a falling input, time to 90% change

**[10]** of output of 2nd gate is  $20 + 10 + 15/2 = \underline{37.5 \text{ ns}}$

For non-inverting gates:

- Time to 90% rise is  $10 + 10 + 15/2 = \underline{27.5 \text{ ns}}$
- Time to 90% fall is  $20 + 20 + 30/2 = \underline{55 \text{ ns}}$

The propagation delay for these gates is

$$t_p = (t_{PNU} + t_{PLH})/2 = (10 + 20)/2 = \underline{15 \text{ ns}}$$

**[11]** Note that this question ignores the

possibility of dynamic power dissipation:

Average propagation delay is  $t_p = (50+70)/2 = 60 \text{ ns}$

Average power loss at 50% duty cycle =  $(1+0.5)/2 = 0.75 \text{ mW}$

Delay-Power product is  $DP = 60 \times 10^{-9} \times 0.75 \times 10^{-3}$

$$\text{or } DP = 45 \times 10^{-12} \text{ J} = \underline{45 \text{ pJ}}$$

10.12

10.13

**[14]** For  $V_I = 1.5 \text{ V}$ , the NMOS operates in

triode mode while the PMOS is cut off.

$$r_{DSN} = [k_n(V_I - V_t)]^{-1} = [100 \times 10^{-6} (1.5 - 0.5)]^{-1} = 10 \text{ k}\Omega$$

$$\text{Thus } V_o = 100 \times 10^3 \times 10^2 / (10^4 + 10^3) = \underline{9.09 \text{ mV}}$$

For  $V_I = -1.5 \text{ V}$ , the PMOS operates with

$$r_{DSP} = [k_p(|V_I| - V_t)]^{-1} = [100 \times 10^{-6} / 10 (1.5 - 0.5)]^{-1} = 10 \text{ k}\Omega$$

$$\text{Thus } V_o = 100 \times 10^3 \times 10^2 / (10^5 + 10^3) = \underline{50 \text{ mV}}$$

**[15]** At  $V_{th}$ , both transistors operate in

saturation with  $V_{th} = V_I = V_D = v$ ,

$$\text{at which } I_D = \frac{k_n}{2} (v - V_{th})^2 = \frac{k_p}{2} (V_{DD} - v - |V_{tp}|)^2$$

$$\text{Thus, } V_{DD} - v - |V_{tp}| = \sqrt{k_n/k_p} (v - V_{th})$$

$$\approx (1 + (k_n/k_p)^{1/2}) v = V_{DD} - |V_{tp}| + V_{th} (k_n/k_p)^{1/2}$$

$$\text{Thus } v = V_{th} = \frac{V_{DD} - |V_{tp}| + V_{th} (k_n/k_p)^{1/2}}{1 + (k_n/k_p)^{1/2}}$$

as presented in Equation 10.8

10.16

10.17

10.18

**[19]** NMOS width is 8  $\mu\text{m}$ ; PMOS width is 8/(25) = 20  $\mu\text{m}$

Total output capacitance  $\approx (20+8)2 + 50 = \underline{106 \text{ fF}}$

$$\text{Now, } t_p = 1.6C / [k'(\omega L) V_{DD}]$$

$$\text{Thus } t_p = 1.6(106)10^{-15} / [75 \times 10^6 (8/0.8) 3.3] = \underline{68.5 \text{ ps}}$$

[20] From Eq. 10.14, 10.15, 10.16:

$$\begin{aligned} I_{DnAV} &= \frac{1}{2} \left[ k_n (W/L)_n (V_{DD}(1-\alpha))^2 \right. \\ &\quad + k_n (W/L)_n (V_{DD}(1-\alpha) V_{DD}/2 - \frac{1}{2} (V_{DD}/2)^2) \\ &= \frac{1}{2} k_n (W/L)_n [V_{DD}^2 (1-\alpha)/2 + V_{DD}^2 (1-\alpha)/2 - V_{DD}^2/8] \\ &= k_n k_n (W/L)_n V_{DD}^2 [(1-\alpha)^2 + (1-\alpha) - \frac{1}{4}] \end{aligned}$$

Now, see the term [ ] becomes:

$$[(1-\alpha)^2 + (1-\alpha) - \frac{1}{4}] = \alpha^2 - 3\alpha + 1.75$$

$$\text{Thus, } t_{PHL} = (CV_{DD}/2) / I_{DnAV} = \frac{4C/2}{k_n (W/L)_n V_{DD} (\alpha^2 - 3\alpha + 1.75)}$$

or  $t_{PHL} = k_n (W/L)_n V_{DD} / (\alpha^2 - 3\alpha + 1.75)$

$$\text{Check for } \alpha = 0.2: (\alpha^2 - 3\alpha + 1.75) = 0.2^2 - 3(0.2) + 1.75 = 1.14$$

and the multiplier in the numerator is  $\frac{2}{1.14} = 1.82 \approx 1.75$

[20]

$$\text{For } \alpha = 0.1, \alpha^2 - 3\alpha + 1.75 = 0.1^2 - 3(0.1) + 1.75 = 1.46,$$

$$\text{and the multiplier becomes } \frac{2}{1.46} = \underline{1.37}$$

$$\text{For } \alpha = 0.5, \alpha^2 - 3\alpha + 1.75 = 0.5^2 - 3(0.5) + 1.75 = 0.50$$

$$\text{and the multiplier becomes } \frac{2}{0.5} = \underline{4.00}$$

10.21

[22]

Dynamic Power is  $P_D = fCV_{DD}^2$ ; Static Power is  $P_S$ .

$$\text{Now, } 9.0 = P_S + 120 \times 10^6 C S^2$$

$$\text{and } 4.7 = P_S + 50 \times 10^6 C S^2$$

$$\text{Subtracting, } 4.3 = 70 \times 10^6 C (25)$$

$$\text{Whence } C = 4.3 / (25 \times 70 \times 10^6) = \underline{2457 \text{ pF}}$$

$$\text{and } P_S = 9.0 - 120 \times 10^6 (25) 2457 \times 10^{-12} = 9.0 - 7.37 = \underline{1.63 \text{ W}}$$

For 70% of the gates active, total gates =  $0.7 \times 10^6$

$$\text{Capacitance per gate is } \underline{2457 \times 10^{-12} / (0.7 \times 10^6)} = 3.55 \text{ fF}$$

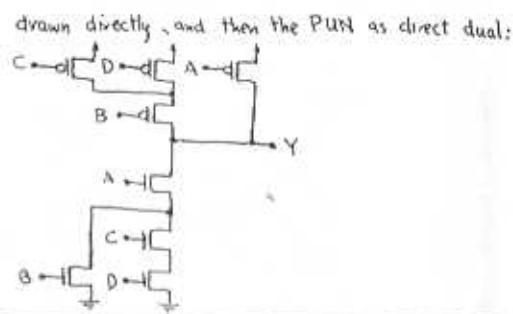
10.23

10.24

[10]

[25]

For  $Y = \overline{A} + B(C+D)$ , the PDN can be



[10]

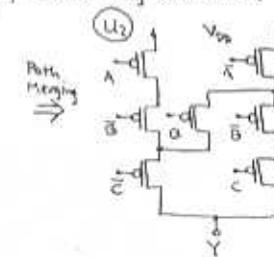
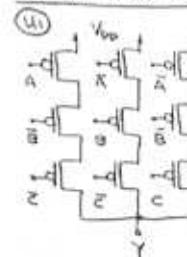
[26]

$$Y = \overline{ABC} + \overline{ABC} + ABC$$

A very direct implementation would need:

$2(3 \times 3) = 18$  MOS for the gate itself, plus  
 $3 \times 2 = 6$  for the required inverters, for  
a total of 24 transistors.

For the PLUN directly: (Inverting variables)



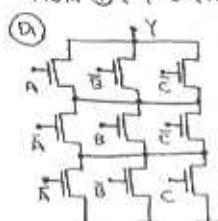
For the PDN,  $Y = \overline{ABC} + \overline{ABC} + ABC$ .

$$\begin{aligned} \text{Correspondingly: } Y &= \overline{\overline{ABC} + \overline{ABC} + ABC} \\ &= \overline{\overline{ABC} \cdot \overline{ABC} \cdot ABC} = (A+B+C)(\overline{A}+\overline{B}+\overline{C}) \text{ (1)} \\ &= ABC + \overline{ABC} + \overline{ABC} + \overline{ABC} + ABC, \text{ and replicating } \overline{ABC}, \\ Y &= ABC + \overline{AB} + AC + BC \text{ --- (2)} \\ &\quad \text{(next)} \end{aligned}$$

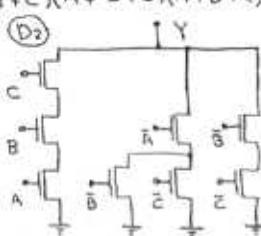
26

For the PDN directly:

$$\text{From } \textcircled{1}, Y = (A+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$



10

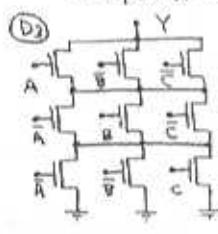


$$\text{From } \textcircled{2}, Y = ABC + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}, \text{ see } \textcircled{D}_2$$

where path merging is included.

For the PDN from the PUN  $\textcircled{U}_1$ 

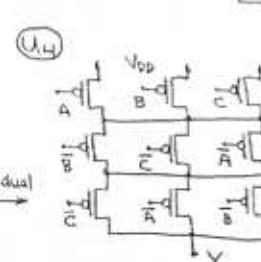
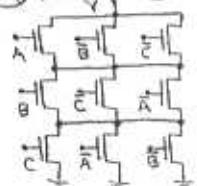
Simply, replacing series connections with parallel connections:



See that  $\textcircled{D}_3$  is identical to  $\textcircled{D}_1$

in detail!

(next)

 $\textcircled{D}_4$  (reordered  $\textcircled{D}_1$ )

10

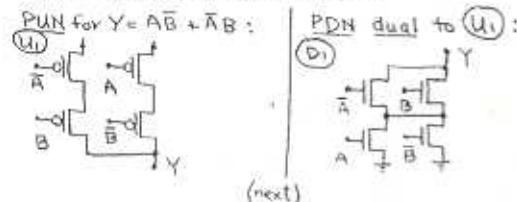
See that  $\textcircled{U}_4$ , while not the same as  $\textcircled{U}_1$ , is highly related, having some variable exchange in the middle and right columns.  
Clearly, there are lots of variations of the completely-connected array.

27

$$Y = AB + \bar{A}\bar{B} \rightarrow \bar{Y} = \overline{AB + \bar{A}\bar{B}} = \overline{AB} \cdot \overline{\bar{A}\bar{B}}$$

10

$$\text{or } \bar{Y} = (\bar{A}+B)(A+\bar{B}) = AB + \bar{A}\bar{B}$$



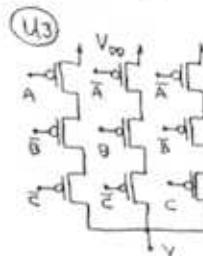
27

For the PUN from the PDN  $\textcircled{D}_1$ 

10

See that both parallel and series connections can be identified for transformation, but that the series (vertical) ones (as drawn) contain variables and their complements and are therefore always open.

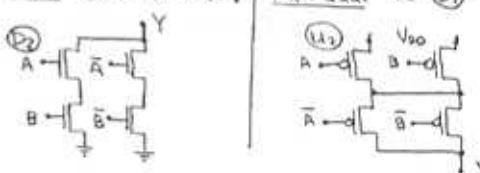
Thus converting parallel paths to serial ones



See this is the same as  $\textcircled{U}_1$

Note, however that  $\textcircled{D}_1$  can be redrawn as shown, then its columns (series links) converted to rows (parallel links of a PUN):

PDN for  $\bar{Y} = AB + \bar{A}\bar{B}$ : PUN dual to  $\textcircled{D}_1$

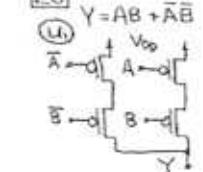


10

The two circuits required are  $\textcircled{U}_1$  with  $\textcircled{D}_1$  and  $\textcircled{U}_2$  with  $\textcircled{D}_2$

28

$Y = AB + \bar{A}\bar{B}$ . Directly, the PUN is as follows:

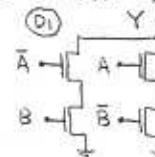


10

$$\text{Now, } \bar{Y} = \overline{AB + \bar{A}\bar{B}} = \overline{AB} \cdot \overline{\bar{A}\bar{B}} = (\bar{A}+B)(A+\bar{B})$$

or  $\bar{Y} = \bar{A}B + A\bar{B}$ .

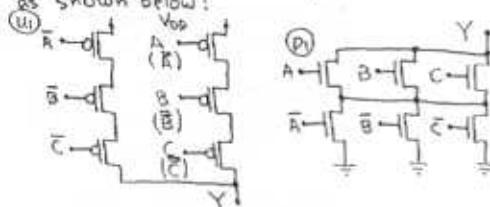
Directly, the PDN is:



29

10

$Y = ABC + A\bar{B}\bar{C}$ . Directly, the PUN is shown below:



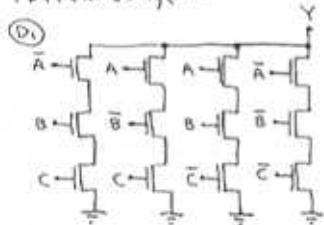
The corresponding dual PUN is shown above right.

30

10

Even-parity circuit:  $\bar{Y} = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + A\bar{B}C$

PUN, directly, is:



It uses 12 transistors

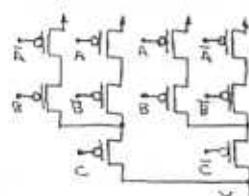
31

10

For output high with odd parity:

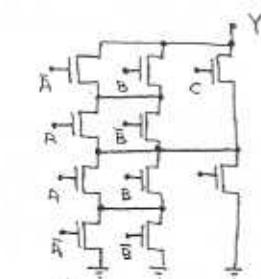
$$Y = A\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}C + ABC = (A\bar{B} + AB)\bar{C} + (\bar{A}B + AC)C$$

Directly, the PUN is:



[Note that two transistors could be eliminated by combining the  $A$  and  $\bar{A}$  inputs in a non-planar topology.]

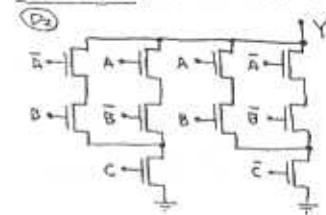
Using duality, the PUN becomes:



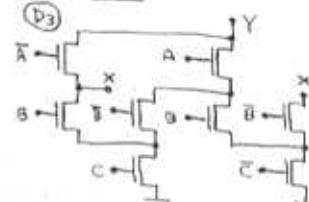
31

10

PUN reduced to 10 transistors:



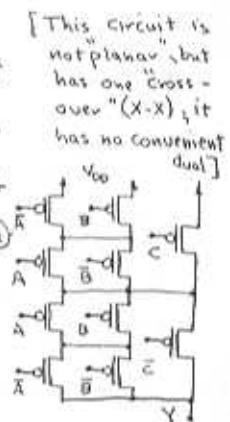
PUN reduced to 8 transistors: ( $X$  max are joined)



PUN as the dual of U2:

[Think of the structure of the dual of U2 when constructing this.]

The complete circuit, using U6 and U7 has 20 transistors



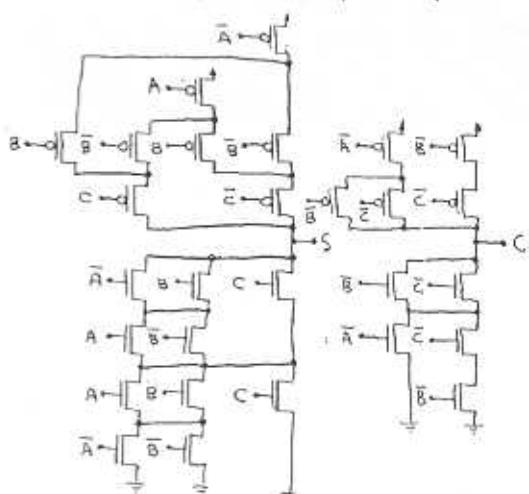
32

10

$$\text{Sum, } S = A\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}C + ABC$$

$$\text{Carry, } C_0 = A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}C + ABC \\ = AB + AC + BC = A(B+C) + BC$$

Create the PUNs directly, simplifying that for  $S$  as in P10.20 above, as  
 $S = \bar{A}(B\bar{C} + BC) + A(\bar{B}\bar{C} + BC)$



33

10

For matched-inverter equivalence of the circuit in Fig. 13.14 :  $P_A = P$ ;  $P_B = P_C = P_D = 2P$ ;  
and  $n_A = n_B = 2n$ ;  $n_C = n_D = 2(2n) = 4n$ .

10.34

35

10

Ignore the capacitances of the transistors themselves: For the matched NMHD,  $t_{PLH} = t_{PHL} = t_P$ .  
For the "uncompensated" NMHD,  $t_{PLH} = t_P, t_{PHL} = t_P/4$ .  
Thus  $t_{PLH}$  are the same, but  $t_{PHL}$  is 4 times greater with no matching.

36

For design a), there are  $2(6) + 2 = 14$  transistors:

All 7 NMOS use  $(W/L)_n = n$   
1 PMOS uses  $(W/L)_p = p$   
6 PMOS use  $(W/L)_p = 6p$   
Total Area =  $7(1.2)0.8 + 1(3.6)0.8 + 6(6)(3.6)0.8 = 113.3 \mu\text{m}^2$

For design b), there are  $2(3)2 + 1(2)2 = 16$  transistors:  
6 NMOS use  $(W/L)_n = n$   
6 PMOS use  $(W/L)_p = 3p$   
2 PMOS use  $(W/L)_p = p$   
2 NMOS use  $(W/L)_n = 2n$   
Total equivalent devices is  $6n + 18p + 2p + 2n = 10n + 20p$   
Total equivalent area is  $[10 + 3(10)]n = 70n$ , and  
Total Area =  $70(1.2)0.8 = 67.2 \mu\text{m}^2$ , or 59% of a)

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10

Corresponding to a matched inverter characterized by  $n$  and  $p$  where  $k_p = k_n = k$ , the two-input NOR uses transistors  $n_1$  and  $2p$  where  $k_p = 2k_n$ .

a) For A grounded,  $V_{FB}$  occurs near  $V_{DD}/2$ , with  $Q_{PB}$  and  $Q_{NB}$  in saturation and  $Q_{PA}$  in triode. Let  $V_{fb} = v$ , and the voltage across  $Q_{PA}$  be  $x$ .

$$\begin{aligned} I_D &= k_p[(5-v)x - x^2/2] \\ \text{and } I_D &= k_p k_p (5-x-v-1)^2 \\ \text{and } I_D &= k_p k_p (v-1)^2 \end{aligned}$$

$$\text{For } k_p = 2k_n, I_D = 2k_n(4x - x^2/2) = k_n(8x - x^2) \quad \dots (1)$$

$$\text{and } I_D = k_n(4-x-v) \quad \dots (2)$$

$$\text{and } I_D = k_n(v-1)^2 \quad \dots (3)$$

$$\text{From } (2) \text{ and } (3): \frac{t}{2}(v-1)(0.707) = 4-x-v$$

$$\text{Thus, } \begin{aligned} 1.707v &= 4.707 - x & (4) \\ \text{or } x &= 4.707 - 1.707v & 0.293v = 3.293 - x \\ \text{or } x &= 3.293 - 0.293v & \text{or } x = 3.293 - 0.293v \end{aligned}$$

$$\text{Now } x \approx 0, \text{ in which case, } v = \frac{4.707}{1.707} = 2.78 \text{ (ok)} \\ \text{or } v = 3.293/0.293 = 11.2 \text{ (Clearly too large)}$$

$$\text{Thus } x = 4.707 - 1.707v \quad \dots (4)$$

$$\text{Now, from } (1) \text{ and } (3): (v-1)^2 = 2(8x - x^2) \quad \dots (5)$$

$$\begin{aligned} \text{With } (4), v^2 - 2v + 1 &= 16(4.707 - 1.707v) - 2(4.707 - 1.707v)^2 \\ \text{or } v^2 - 2v + 1 &= 15.32 - 27.32v - 44.31 + 32.13v - 5.83v^2 \\ \text{or } 6.83v^2 + 27.32v - 44.31 + (-15.32 + 44.31) &= 0 \\ \text{or } 6.83v^2 - 6.81v - 30.01 &= 0 \end{aligned}$$

$$\text{whence } v = \frac{(-6.81 \pm \sqrt{(6.81)^2 - 4(6.83)(-30.01)})}{2(6.83)} \\ = (6.81 \pm 29.43)/13.66 = 2.65V$$

Check: [ $> 2.5V$  probably OK since one PMOS is full on]

Thus  $V_{th} = 2.65V$

b) For A and B joined, the PMOS can be approximated as a single device with twice the length, for which the width is twice that in a matched inverter. Thus, for the equivalent PMOS device,  $(W/L)_{pAB} = p$  and  $k_p = k$ . For each of the two NMOS,  $(W/L)_n = n$  and  $k_n = k$ .

Thus at  $V_{th} = v$  with all devices in saturation:

(next)

$$i_D = k_p/2 (V_{DS} - V_t)^2 = k_p/2 (5 - V - 1)^2, \quad [10]$$

$$2(V-1)^2 = (4-V)^2, \text{ and } \pm\sqrt{2}(V-1) = (4-V)$$

Thus,  $1.414V - 1.414 \approx 4 - V$ ,  $2.414V \approx 5.414$ ,  
whence  $V_{th} = V = \underline{2.24V}$

See this is reduced from the single-input value (of 2.65V)!

Note that this fact can be used to control the relative threshold of multiple gates connected to a single fanout node, in order to guarantee operation sequence for slowly changing signals.

[38]

For the resistor load, the output voltage rising is  $V_o = V_{DD}(1 - e^{-t/R_o C})$ . This reaches  $V_{DD}/2$  when  $(1 - e^{-t/R_o C}) = 0.5$ , or  $e^{-t/R_o C} = 0.5$ , and  $t = -R_o C \ln 0.5 = 0.693 R_o C$ .

Thus  $t_{PLH} = \underline{0.693 R_o C}$

For the current load,  $I = V_{DD}/R_o$ , the output reaches  $V_{DD}/2$  when  $t = (C V_{DD}/2)/I$ , or  $t = (C V_{DD}/2)/(V_{DD}/R_o) = 0.5 R_o C$  (next)

$$\text{Now, } k_n = k_n' (W/L)_n = 75 \times 10^{-6} (1.2/0.8) = \underline{112.5 \mu A/V^2} \quad [10]$$

$$\text{and } k_p = 0.256 \times 112.5 = \underline{28.8 \mu A/V^2}$$

$$\text{Now, } k_p' (W/L)_p = 28.8; \text{ Thus } \frac{V_L}{V} = 28.8 / (15) + \underline{1.152}$$

$$\text{For } L_p = 0.8 \mu m, W_p = 1.152(0.8) = \underline{0.922 \mu m}$$

$$\text{In summary, for this inverter: } k_n = \underline{112.5 \mu A/V^2}$$

$$k_p = \underline{28.8 \mu A/V^2}$$

$$\text{and } V = k_n/k_p = \underline{3.91}$$

$$\text{Now, } V_{OH} = \underline{+5.0V}$$

$$\text{For } V_{OL} = V \approx 0, i_D = k_p/2 (V_{DS} - V_t)^2$$

$$\text{or } i_D = (28.8/2)(5.0 - 0.8)^2 = \underline{254 \mu A}$$

$$\text{For the NMOS, } 254 = 112.5 [5 - 0.8] V - \underline{V^2/2}$$

$$\text{or } V^2 - 8.4V + 4.52 = 0$$

$$\text{whence } V = (-8.4 \pm (8.4^2 - 4(1)(4.52))^{1/2})/2$$

$$= (8.40 \pm 2.24)/2 = \underline{0.58V}$$

$$\text{Thus, } V_{OL} = \underline{0.58V}$$

$$\text{Thus } t_{PLH} = \underline{0.5 R_o C} \rightarrow \text{a reduction to}$$

$$(0.50/0.61) \times 100 = \underline{72.5 \%} \text{ or by } \frac{0.69 - 0.59}{0.69} \times 100 = \underline{27.8 \%}$$

[39] Here  $V_{DD}/4 = 5/4 = 1.25V$

Now, for  $V_o$  rising, the NMOS is cutoff, and the PMOS is in triode mode with:

$$i_{Dp} = k_p [(V_{DS} - V_t) V_{SD} - V_{SO}^2/2], \text{ and here}$$

$$i_D = k_p [(5 - 0.8)(5 - 1.25) - (5 - 1.25)^2/2]$$

$$\Rightarrow k_p (15.75 - 7.03) = \underline{8.72 k_p}$$

Now, for  $V_o$  falling, the net current extracted from the load is  $i_{Dn} - i_{Dp}$  which should be  $i_{pp}$ . Thus  $i_{Dn} = 2i_{Dp} = 2(8.72)k_p$ , for triode operation where  $i_{Dn} = k_n [(5 - 0.8)1.25 - 1.25^2/2]$

$$\text{Overall, } i_{Dn} = 2(8.72)k_p = k_n (5.25 - 0.78) = 4.47k_n$$

$$\text{Thus } k_n = (4.47 / (2(8.72)))k_n = \underline{0.256 k_n}$$

Check using Eq 10.39, where  $r = k_n/k_p = 3.91$ :

$$V_{OL} = (V_{DD} - V_t) [1 - (r - V_r)V_r]$$

$$= (5 - 0.8) [1 - (1 - 1/3.91)V_r] = \underline{0.571V}$$

$$\text{From Eq 13.35, } V_{IL} = V_t + (V_{DD} - V_t) / (r(r+1))^{1/2}$$

$$= 0.8 + 4.2 / (3.91 \times 4.91)^{1/2} = \underline{1.16V}$$

$$\text{From Eq 13.38, } V_{EL} = V_t + (2/\sqrt{r}) (V_{DD} - V_t)$$

$$= 0.8 + (2/\sqrt{3.91}) 4.2 = \underline{3.25V}$$

$$\text{From Eq 13.36, } V_M = V_t + (V_{DD} - V_t) / (r+1)^{1/2}$$

$$= 0.8 + 4.2 / (4.91)^{1/2} = \underline{2.70V}$$

$$\text{Now, } NM_{SH} = V_{OH} - V_{IH} = 5.00 - 3.25 = \underline{1.75V}$$

$$\text{and } NM_{SL} = V_{EL} - V_{OL} = 3.25 - 0.58 = \underline{2.67V}$$

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Note that a design with  $r=2$  sacrifices  $V_{DD}$  for improved propagation-delay symmetry.

$$\text{From Eq. 10.39, } V_{OL} = (V_{DD}-V_t)[1 - (1 - 1/r)^{1/2}] \\ = (5 - 0.8)[1 - (1 - 1/2)^{1/2}] = \underline{1.7V}$$

$$\text{Now, } k_n'(W/L)_m = 75(1.2/0.8) = r k_p = 2(75/3)(W/L)_p \\ \text{Thus, } (WL)_p = (70/25) \times 1/2 \times (1.2/0.8) = \underline{1.2/0.8}$$

$$\text{Now } C_{GSN} = 1.2(1.5) = 1.8fF; C_{GDN} = 1.2(0.5) = 0.6fF \\ \text{Thus } C_{in} = 1.8 + 0.6 = \underline{2.4fF}.$$

Now, the output capacitance includes  $C_{DSS}$  and  $C_{DS}$  of the load plus  $C_{DSS}$  and twice  $C_{DS}$  of the switch.

Thus, the total capacitance of the output and the loading gate is:

$$C_{out} = 1.8(0.5+2) + 1.2(2.0+2(0.5)) + 2.4 = \underline{10.5fF}$$

$$\text{Now, from Eq. 10.43, } t_{PLH} = 1.7C/(k_p V_{DD}), \\ \text{and } t_{PUL} = 1.7 \times 10.5 \times 10^{-15} / [(75/3) 10^{-4} (1.2/0.8) 5] = \underline{41.2ps}$$

$$\text{Now, from Eq. 10.44, } t_{PHL} = 1.7C/[k_m(1 - 0.46/r)k_p] \\ \text{and } t_{PHU} = 1.7 \times 10.5 \times 10^{-15} / [15 \times 10^4 (1.2/0.8)(1 - 0.46/2) 5] = \underline{41.2ps}$$

$$\text{Thus } t_p = (t_{PLH} + t_{PHL})/2 = (41.2 + 41.2)/2 = \underline{\pm 2.4ps}$$

$$\text{Thus } (2r+1)^2(r-1) = (r+1)^2 \\ \text{and } (4r^2+4r+1)(r-1) = (r^2+2r+1)(r+1) \\ 4r^3+4r^2+r-4r^2-4r-1 = r^3+2r^2+r^2+r+1 \\ 4r^3-3r^2-4r-1 = r^3+3r^2+2r+1 \\ 3r^3-3r^2-6r-2 = 0 \\ r^3-r^2-2r-2/3 = 0$$

Test  $r^3-r^2-2r-2/3$

$$r=2 \rightarrow 8-4-4-2/3 = -2/3$$

$$r=3 \rightarrow 27-9-6-2/3 = 11/3$$

$$r=2.1 \rightarrow 2.1^3 - 2.1^2 - 4.2 - 4.7 = 9.26 - 4.4 - 4.2 - 4.7 = -0.04$$

Thus  $r = \underline{2.1}$ , for which:

$$NM_L = 0.8 - (5.0 - 0.8)[1 - (1 - 1/2.1)^{1/2} - (2.1(2.1+1))^{1/2}] \\ = 0.8 - 4.2[1 - 0.724 - 0.342] = \underline{1.29V}$$

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$$\text{From Eq. 10.41, } NM_C = V_t - (V_{DD}-V_t)[1 - (1 - \frac{1}{r})^{1/2} - (r(r+1))^{1/2}]$$

$$\text{Now, } \Delta NM_C / \Delta r = -(V_{DD}-V_t)[\frac{1}{2}(1 - \frac{1}{r})^{1/2}(-\frac{1}{r^2}) - (\frac{1}{2}(r(r+1))^{1/2})]$$

Maximum occurs where:

$$-\frac{1}{2}(1 - \frac{1}{r})^{1/2}(\frac{1}{r^2}) = -\frac{1}{2}(r(r+1))^{-1/2}(2r+1)$$

$$\text{Square both sides: } (1 - \frac{1}{r})^{1/2}r^4 = r^{-3}(r+1)^{-1/2}(2r+1)^2$$

$$\text{or } \frac{1}{1-r} = \frac{r^5}{(r+1)^2} (2r+1)^2 = \frac{r^5}{r-1}$$

10.42

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$$\text{From Eq. 10.41, } NM_H = (V_{DD}-V_t)(1 - 2/\sqrt{3r})$$

This is zero when  $1 - 2/\sqrt{3r} = 0$ ,

$$\text{or } \sqrt{3r} = 2, \text{ or } 3r = 4, \text{ or } r = \underline{1.33}$$

$$\text{For } r=1, NM_H = 4.2(1 - 2/\sqrt{3}) = \underline{-0.65V}$$

$$\text{For } r=2, NM_H = 4.2(1 - 2/\sqrt{6}) = \underline{0.17V}$$

$$\text{For } r=4, NM_H = 4.2(1 - 2/\sqrt{12}) = \underline{1.78V}$$

$$\text{For } r=8, NM_H = 4.2(1 - 2/\sqrt{24}) = \underline{2.48V}$$

$$\text{For } r=16, NM_H = 4.2(1 - 2/\sqrt{48}) = \underline{2.99V}$$

But, what about  $NM_L$ ? (For  $r=16$ , it is  $0.92V$ )

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From Eq. 10.41 and 10.42, noise margins are equal when

$$V_t - (V_{DD}-V_t)[1 - (1 - \frac{1}{r})^{1/2}/(r(r+1))^{1/2}] = (V_{DD}-V_t)[1 - 2/\sqrt{3r}]$$

$$\text{or } V_t/(V_{DD}-V_t) = 2 - 2/\sqrt{3r} = (1 - \frac{1}{r})^{1/2} - 1/(r(r+1))^{1/2}$$

$$\text{Here } V_t/(V_{DD}-V_t) = 0.8/(5.0-0.8) = \underline{0.1404}$$

Try various values of  $r$  to solve (1):

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$$\text{For } r=2, f(r) = 2 - 2/\sqrt{4} = (1 - \frac{1}{2})^{1/2} - 1/(2(2))^{1/2} \\ = 2 - 0.86 - 0.707 = 0.408 = \underline{0.064}$$

$$\text{For } r=3, f(r) = 2 - 2/\sqrt{9} = (1 - \frac{1}{3})^{1/2} - 1/(3(3))^{1/2} \\ = 2 - 0.67 - 0.86 = 0.228 = \underline{0.228}$$

$$\text{Try } r=2.8, f(r) = 2 - 2/\sqrt{7.84} = (1 - \frac{1}{2.8})^{1/2} - 1/(2.8(2.8))^{1/2} \\ = 2 - 0.690 - 0.802 = 0.307 = \underline{0.307}$$

$$\text{Try } r=2.7, f(r) = 2 - 2/\sqrt{7.29} = (1 - \frac{1}{2.7})^{1/2} - 1/(2.7(2.7))^{1/2} \\ = 2 - 0.703 - 0.713 = 0.316 = \underline{0.316}$$

Conclude  $r = \underline{2.72}$ , for which the margins are:

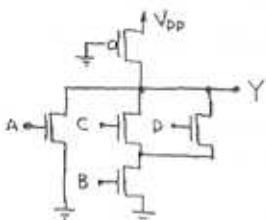
$$NM_H = NM_L = (V_{DD}-V_t)(1 - 2/\sqrt{3r}) \\ = 4.2(1 - 2/\sqrt{3(2.72)}) = \underline{1.26V}$$

10.45

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$$Y = \overline{A + B(C+D)}, \text{ whence } \bar{Y} = A + B(C+D)$$

Thus the PDN can be formed directly as shown:



$$\text{Now, } t_{PHL}/t_{TTL} = (k_n/k_p)(1 - 0.45/v) \\ = v(1 - 0.45/v) = 2.24 - 0.45 = \underline{\underline{2.24}}$$

$$\text{Now, } t_{TTL} = 1.7(1 \times 10^{-12}) / (25 \times 10^{-12}(1.3V_0/2)) = \underline{\underline{8.24\mu s}}$$

$$\text{and } t_{PLH} = 8.24/2.24 = \underline{\underline{3.65\mu s}}$$

$$\text{and } t_P = (8.24 + 3.65)/2 = \underline{\underline{5.95\mu s}}$$

Now, dynamic power is approximately  $fCV_0^2$ , since the output swing is not quite  $V_0$ .

For equal static and dynamic power

$$f \times 1 \times 10^{-12} \times 5^2 = 1.82 \times 10^{-3} \\ \text{whence } f = 1.82 \times 10^{-3} / (25 \times 10^{-12}) = \underline{\underline{72.8\text{MHz}}}, \\ \text{for which the period is } 1/(72.8 \times 10^6) = \underline{\underline{13.7\mu s}}$$

Now, for transition times in the same proportion as propagation delays  $t_{PLH}/t_{TTL} = 8.24/3.65 = 2.24$

Now, for full output swing, there must be time for 2 full transitions in each cycle:

$$\text{Thus } t_{TTL} \approx 13.7 / (1 + 2.24) = \underline{\underline{4.19\mu s}} \text{ and} \\ t_{PLH} \approx 4.19 / 2.24 = \underline{\underline{1.87\mu s}}$$

Since these values are of the same order as the propagation delays, full swing operation is likely not possible at 72.8MHz.

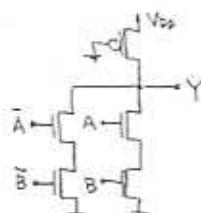
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For an Exclusive OR,  $Y = \overline{AB} + \overline{A}\overline{B}$ , and

$$\bar{Y} = \overline{\overline{AB} + \overline{A}\overline{B}} = \overline{\overline{A}\overline{B}} \cdot \overline{\overline{A}\overline{B}} = (\overline{A} + B)(A + \overline{B})$$

$$\text{or } Y = \overline{A}\overline{B} + AB$$

The PDN results directly:



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For a pseudo-NMOS NOR gate, independent of the number of inputs, the worst-case value of  $V_{OL}$  occurs for one input high (and all NMOS conducting)

$$\text{From Eq. 10.39, } V_{OL} = (V_{DD} - V_t)[1 - (1 - V_r)^{1/2}] \\ \text{for which } 0.2 = (5.0 - 0.8)[1 - (1 - V_r)^{1/2}]$$

$$\text{and } (1 - V_r)^{1/2} = 1 - 0.2/4.2 = 0.952$$

$$\text{Thus } 1 - V_r = 0.907$$

$$V_r = 0.093 \text{, and } V = 10.76$$

$$\text{Thus } k_n/k_p = 10.76 = 75(1.8/1.2)/(25(W/L)_P)$$

$$\text{Thus } (W/L)_P = (75/25)(1.8/1.2)/10.76 = \underline{\underline{0.418}}$$

$$\text{Thus for } W_p = 1.84\mu m, L_p = 1.8/0.418 = \underline{\underline{4.31\mu m}}$$

$$\text{and } (W/L)_P = (1.8/4.31)$$

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For a), see directly that  $X=1, \bar{A} = \bar{A}$

$$\text{and } Y = X \cdot \bar{B} = \underline{\underline{\bar{A} \cdot \bar{B}}}$$

For b), see directly that  $Y = \underline{\underline{\bar{A} \cdot \bar{B}}}$

For each circuit, node Y nominally satisfies both conditions. However in a), with A high and B low, Y is not pulled down completely to ground, but remains at  $V_{Tn}$ , due to the PMOS threshold. Circuit b) does not have this problem, but node X is floating for A,B both high. However, X is not an output node. The body effect makes this worse. Notice that b) is exactly a complementary CMOS NOR gate for which  $Y = \bar{A} \cdot \bar{B} = \underline{\underline{\bar{A} \cdot \bar{B}}}$

For  $V_{DD}$  replaced by an inverter driven by C,  $Y = \bar{C}(\bar{A} \cdot \bar{B}) = \bar{A} \cdot \bar{B} \cdot \bar{C} = \underline{\underline{\bar{A} \cdot \bar{B} \cdot \bar{C}}}$ , a 3-input NOR (for both a) and b)).

Practically speaking, however, there is a problem because, as noted above, the series PMOS do not operate well with a low input. In fact Y is pulled down only to one threshold drop below ground, when C is high.

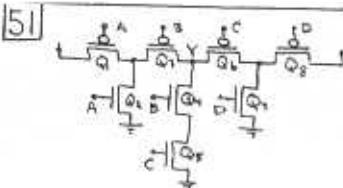
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With these reversals, the output is high when B is low or when A is low.

Thus,  $Y = \bar{A} + \bar{B}$ , or  $\underline{\underline{Y = \bar{A} \cdot \bar{B}}} \text{ is a NAND function}$

Circuit b), being a fully complementary CMOS gate, functions ideally. However circuit b) provides a relatively low high output for A low with B high. In this case, the output becomes  $V_{DD} - V_{Th}$ , where  $V_{Th}$  is raised from  $V_{Thn}$  by the body effect.

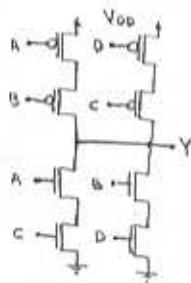


There are several deficiencies :

First, there are two possibilities for short-circuits when B and C are low and A and D are complementary, in which case current flows (for example) through  $Q_1, Q_3, Q_6, Q_7$ . This problem occurs for  $\bar{A}B\bar{C}\bar{D}$  and  $A\bar{B}\bar{C}\bar{D}$ .

Second, node Y is pulled down to  $V_t$  (increased by the body effect) when B or C are low with A or D high, respectively. In this respect, this circuit is like that in Fig P13.4(a).

With respect to Y as a function of A,B,C,D, the result will depend on device sizing. For  $Q_2, Q_7$ , relatively weaker than the PMOS,  $Y = \bar{A}\bar{B} + \bar{C}\bar{D}$ . A better circuit is shown below:



$$\text{For } Q_3, Q_6 \text{ relatively weaker, } Y = A + D + BC \\ \text{and } Y = \bar{A} + D + \bar{B}C \\ = \bar{A}\bar{B} + \bar{C}\bar{D} \\ = AB\bar{B} + \bar{A}\bar{C}\bar{D}$$

With input E driving an inverter to replace both  $V_{DD}$  connections,  $Y = \bar{E}(\bar{A}\bar{B} + \bar{C}\bar{D})$  is possible, bypassing all the other problems this circuit brings.

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For the switch gate and the input both at 3.3V, the switch output is

$$V_{OH} = 3.3 - V_t, \text{ where } V_t = V_{DD} + Y(\sqrt{V_{DD}/2} + \sqrt{V_t})$$

$$\text{Thus } V_{OH} = 3.3 - 0.8 - 0.5(V_{OH} + 0.6)^{1/2} + 0.5(0.6)^{1/2} \\ = 2.88 - 0.5(V_{OH} + 0.6)^{1/2} \\ \text{or } 2V_{OH} - 5.77 = (V_{OH} + 0.6)^{1/2}$$

$$\text{Squaring, } V_{OH} + 0.6 = 4V_{OH}^2 - 23.1V_{OH} + 32.29 \\ 4V_{OH}^2 - 24.1V_{OH} + 31.69 = 0 \quad \text{and}$$

$$V_{OH} = (-24.1 \pm \sqrt{24.1^2 - 4(4)(31.69)})/8 \\ \text{or } V_{OH} = (24.1 \pm 8.39)/8 = 1.94V \quad (\text{and a long one})$$

Now, for input low and switch gate high,  $V_{OL} = 0V$

For the inverter current: For  $V_{OH} = 1.94V$ , the

PMOS is in saturation with

$$i_D = \frac{1}{2}A_s C_{ox}(2V_A) \cdot (3.3 - 1.94 - 0.8)^2 = \frac{1}{2} \left( 25 \times 10^{-6} \right) \left( \frac{1}{2} \right) \left( 0.5 \right)^2$$

Thus, the static inverter current is  $17.64 \mu A$

For  $t_{PLH}$ : Here,  $V_{OL} = 0$ ,  $V_t = V_{DD} = 0.8V$ ,  $V_{SS} = V_{DD} = 3.3V$ .

Thus, initially,  $i_{D1} = \frac{1}{2}A_s \left( 1.2 / 0.8 \right) (3.3 - 0.8)^2 = 351.5 \mu A$

At  $V_D = 3.3/2 = 1.65V$ ,

$$V_t = 0.8 + 0.5(1.65 + 0.6)^{1/2} - 0.5(0.6)^{1/2} = 1.16V$$

$$\text{and } i_{D2}(t_p) = \frac{1}{2} \left( 1.2 \right) \left( 1.2 / 0.8 \right) \left( 3.3 - 1.65 - 1.16 \right)^2 = 13.5 \mu A$$

$$\text{Thus } i_{D_{av}} = (351.5 + 13.5)/2 = 183.5 \mu A$$

$$\text{and } t_{PLH} = (CV_{DD}/2) / i_{D_{av}} = 100(10^{-15}) 1.65 / (183 \times 10^{-6}) = 0.909ns$$

For  $t_{PHL}$ : Here,  $V_t = V_{DD}$ , and the initial current is

$$i_{D1} = \frac{1}{2} \left( 1.2 \right) \left( 1.2 / 0.8 \right) \left( 3.3 - 0.8 \right)^2 = 352.4 \mu A$$

At half swing, operation is in triode mode, and

$$i_D(t_p) = 1.2 \left( 1.2 / 0.8 \right) [ (3.3 - 0.8) 1.65 - 1.65 \cdot \frac{1}{2} ] = 311 \mu A$$

$$\text{Thus, the average current is } (352 + 311)/2 = 331.5 \mu A$$

$$\text{and } t_{PHL} = 100(10^{-15}) 1.65 / 311 \times 10^{-6} = 0.53ns$$

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a) For the inverter, with  $V_{O1} = 3.3 - 0.8 = 2.5V$ ,  
 $Q_H$  is in saturation and the current is  
 $i_{D_H} = \frac{1}{2}(75)(1.2/0.8)(3.3 - 0.8)^2 = 56.25(1.2 - 0.8)$

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and  $Q_P$  is in triode mode with current

$$i_{D_P} = (\frac{15}{3})(3.4/0.8)[(3.3 - V_{O1})0.8 - 0.8^2/2] \\ \text{or } i_{D_P} = 100[(2.5 - V_{O1})0.8 - 0.8^2/2] = 80(2.5 - V_{O1}) - 32$$

$$\text{Now } i_{D_H} = i_{D_P} \text{ or } 56.25(1.2 - 0.8)^2 = 80(2.5 - V_{O1}) - 32$$

$$\text{Thus, } V_{O1}^2 - 1.6V_{O1} + 0.64 = 3.56 - 1.422V_{O1} - 0.564$$

$$\text{or } V_{O1}^2 - 0.178V_{O1} - 2.35 = 0$$

$$\text{whence } V_{O1} = [-0.178 \pm \sqrt{0.178^2 + 2.35}] / 2 \\ = [0.178 \pm 3.07] / 2 = 1.62V$$

$$\text{For } Q_1, V_g = 0.8 + 0.5[(1.62 + 0.6)^2 - 0.4^2] = 1.158V$$

Capacitor charging current :

$$\text{At } V_{D1}; i_D = \frac{1}{2}(75)(1.2/0.8)(3.3 - 1.62 - 1.158)^2 = 15.3\mu A$$

$$\text{At } V_{D2}; i_D = \frac{1}{2}(75)(1.2/0.8)(3.3 - 0.8)^2 = 351.6\mu A$$

$$\text{Average, } i_D = (15.3 + 351.6) / 2 = 183\mu A$$

$$\text{Now, } t_{PHL} = 20 \times 10^{-15} \times 1.62 / 183 \times 10^{-6} = 177.95$$

b) For the inverter,  $V_{IH} = V_B (5(1.3) - 2(0.8)) + 1.86V$

$$\text{For this value, } i_{D1} = 75(1.2/0.8)[(3.3 - 0.8)1.86 - 1.2^2/2] = 324\mu A$$

$$\text{Current in } Q_H = \frac{1}{2}(75)(1.2/0.8)(3.3 - 0.8)^2 = 352/2 \\ \text{for which } (W/L)_H = 1.05, \text{ and for } L_x = 0.8\mu m \\ (W/L)_H = (0.84/0.8)$$

For  $t_{PHL}$ : For  $Q_1$  with source grounded and gate at  $V_{DD}$  while  $Q_H$  has a source-to-gate voltage  $V_{DD}$ :

$$\text{Initially, at } V_{DD} = V_{DD}, i_{D_H} = 0, \text{ since } V_{SD_H} = 0V, \\ \text{and } i_{D_P} = \frac{1}{2}(75)(1.2/0.8)(3.3 - 0.8)^2 = 352\mu A$$

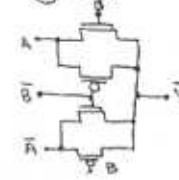
$$\text{At } V_{DD} = V_{IH}, i_{D_H} = 25(0.84/0.80)[(3.3 - 0.8)(3.3 - 1.2) - 1.2^2/2] \\ = 67.3\mu A$$

$$\text{and } i_{D_P} = 75(1.2/0.8)[(3.3 - 0.8)(1.16 - 1.2)^2/2 = 328\mu A \\ \text{where average current for capacitor charging is} \\ i_{D_{AV}} = (352 + 328 - 67.3)/2 = 306\mu A$$

$$\text{Thus, } t_{PHL} = 20 \times 10^{-15} (3.3 - 1.86) / (306 \times 10^{-6}) = 942$$

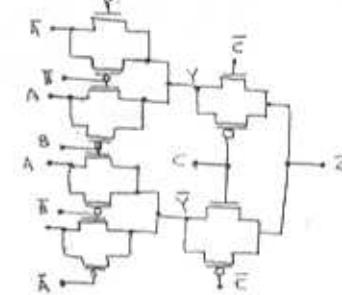
54

a) Need  $\bar{Y} = AB + \bar{A}\bar{B}$ . In direct analogy to Fig 10.31:



(next)

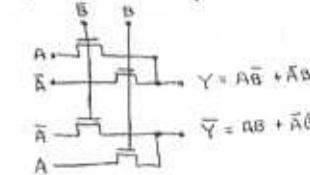
b) Need  $Z = \bar{Y}C + Y\bar{C}$ , where  $Y = \bar{Y} = (AB + \bar{A}\bar{B})$ , whence  $Y = \bar{A}\bar{B} + \bar{A}B = (A+B)(\bar{A}+\bar{B}) = A\bar{B} + \bar{A}B$ .



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Need a CPL circuit for  $Y = AB + \bar{A}\bar{B}$  and

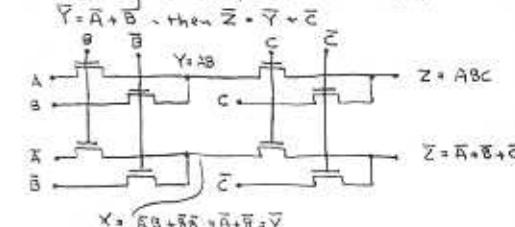
$$\bar{Y} = A\bar{B} + \bar{A}B. \quad \{\text{See Exercise 13.9b}\}$$

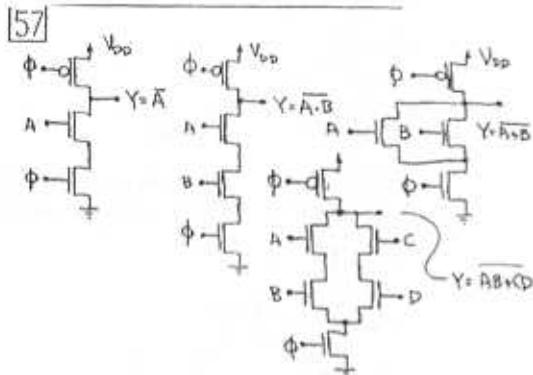


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Require a CPL for  $Z = ABC$  and  $\bar{Z} = \bar{ABC} = \bar{A} + \bar{B} + \bar{C}$

Extend Fig 10.32 to 3 variables by dealing in pairs, creating  $Y = AB$ , then  $Z = YC$  with  $\bar{Y} = \bar{A} + \bar{B}$ . Then  $\bar{Z} = \bar{Y} + \bar{C}$





[58] At  $V_Y = 0.3V$ ,  $i_{DP} = \gamma_2(75/3)(2.4/0.8)(3.0-0.8) = 181.5\mu A$  [10]

At  $V_Y = 2.7V$ ,  $i_{DP} = (75/3)(2.4/0.8)[(3.0-0.8)0.3 - 0.3/2] = 46.1\mu A$

Thus  $i_{Dav} = (181.5 + 46.1)/2 = 114\mu A$

and  $t_{PLH} = t_p = 15 \times 10^{-15}(2.7-0.3)/(114 \times 10^6) = 216\text{ps}$

[59] For 3 NMOS in series, the equivalent length is  $3L_n = 3(0.8) = 2.4\mu m$

Thus for  $V_Y = 3.0V$ ,  $i_D = \gamma_2(75)(1.2/2.4)(3.0-0.8) = 90.8\mu A$

and for  $V_Y = 1.5V$ ,  $i_D = 15(1.2/2.4)[(3.0-0.8)1.5 - 1.5/2] = 81.6\mu A$

Thus  $i_{Dav} = (90.8 + 81.6)/2 = 86.2\mu A$

and  $t_{PLC} = 15 \times 10^{-15}(3.0/2)/(86.2 \times 10^6) = 261\text{ps}$

[60] a)  $C_s = 3FF$ ; [10]

Now, for  $V_C$  rising to  $V_{DD} - V_t = 5 - 1.4V$ , and assuming  $Q_1$  continues to conduct,  $V_Y$  will fall by an amount  $(C_s/C_i)(\Delta V C_i) = 5/30(4) = 0.67V$  to  $5.0 - 0.67 = 4.33V$ . Since this exceeds  $4.0$ , the assumption that  $Q_1$  continues to conduct is verified. Thus  $V_Y$  drops by  $0.67V$ .

Note that if the body effect is included, it will likely to be impossible to raise  $V_C$  to  $4V$ . Thus  $0.67V$  is the largest possible change.

b)  $C_s = 10FF$ :

In view of the previous analysis, assume that ultimately  $V_Y = V_C = V$ . Now, the charge change in each capacitor is the same:  $Q = CV \rightarrow 10(V - 0) = 30(5 - V)$  and  $10V = 150 - 30V \rightarrow 40V = 150$ , and  $V = 3.75V$

Thus  $V_Y$  drops by  $5 - 3.75 = 1.25V$  to  $3.75V$

[61] For a  $0.5V$  change,  $t = \frac{CV}{I_p} = 30(10^{-15})0.5/10 \cdot 15\text{ns}$  [10]

Since the precharge interval is much shorter than the evaluate, the period of the minimum clocking frequency can be as great as  $15\text{ms}$ , for which  $f_{min} = 1/(15 \times 10^3) = 67\text{Hz}$

[62] In each cycle, the output must charge completely, then discharge to at least the threshold of the succeeding gate.

From Exercise 10.10, the precharge time is about  $t_p = 0.4\mu s$ . From Exercise 10.11, the discharge time is about  $t_{PLH} = 0.5\mu s$ .

Thus, the maximum allowed clocking frequency can be as high as  $1/(0.4 + 0.5) \times 10^3 = 1.1\text{GHz}$

## Chapter 11 Problems

[1] For the inverters, using Eq 13.8, :

$$V_{th} = [V_{DD} - |V_T| + (k_n/k_p)^{1/2} V_{in}] / (1 + (k_n/k_p)^2) \\ = (5 - 1 + \sqrt{2.5} \cdot 1) / (1 + \sqrt{2.5}) = 5.58 / 2.58 = 2.1V$$

Current from the PMOS, for which  $V_{DS} = 5V$  with  $V_D = 2.1V$ , is  $I_D = (100/2.5)(2/1)[(5-1)(5-2.16) - (5-2.16)^2/2]$ , or  $i_D = 58\mu A$

To sustain this from the NMOS,

$$58\mu A = 100(W/L)_{Q1} [(5-1)2.16 - 2.16^2/2] \\ \text{whence } (W/L)_{Q1} = 5.8V/6.31 = 0.939$$

Thus, for  $Q_5, Q_6$  in series,  $(W/L)_{Q5,6} = 2(0.939) = 1.878$   
and  $W_5 \cdot W_6 = 1.878\mu m$  for  $L_{Q5,6} = 3\mu m$ .

[2] Initially, the current from the PMOS (say  $Q_6$ ) is essentially zero, since  $V_{SD}$  is 0.

$$\text{Thus, for } V_D = 5V, i_{D,6} = \frac{1}{2}(100)(4/1)(5-1)^2 = 160\mu A$$

Now, for  $V_D = V_{DD}/2 = 2.5V$ :

$$i_{D,6} = 100(4/2)[(5-1)2.5 - 2.5^2/2] = 137.5\mu A \\ i_{D,4} = (100/2.5)(2/1)[(5-1)2.5 - 2.5^2/2] = 550\mu A$$

Thus, the average discharge current is  $(160 - 0 + 137.5 - 550)/2 = 12.375\mu A$   
and  $t_{PHL} = 30 \times 10^{-15} \times 2.5 / 12.375 \times 10^{-15} = 61.9\mu s$

$$\text{Now, } t_{PLH} = 1.7C/(k_p(W/L)_{Q1} V_{DD}) = 1.7 \times 30 \times 10^{-15} / (100/2.5)(2/1)5 = 127.5\mu s$$

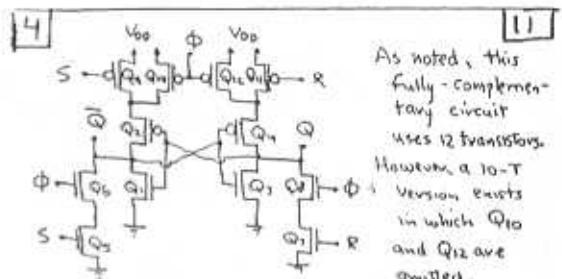
Thus, the minimum set/reset pulse length is

$$127.5 + 61.9 = 189\mu s$$

[3] Assume that  $Q_2$  is conducting,  $Q_1$  is cut off, and  $Q_5$  is conducting. Now to lower  $\bar{Q}$  to  $V_{DD}/2 = 2.5V$ ,

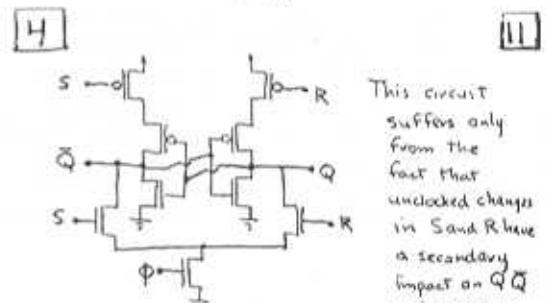
$Q_2$  and  $Q_5$  must be matched, since equivalent control voltages are applied to each.

$$\text{Thus } k_2 = k_5 \text{ and } 25(W/L)_2 = 75/2, \text{ whence } (W/L)_2 = (4/1) \times \frac{5}{6}$$

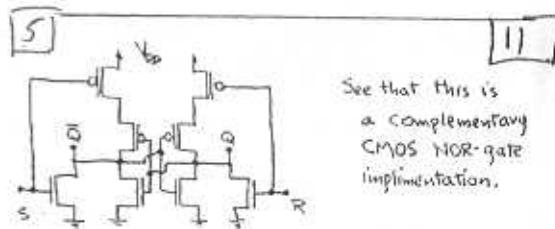


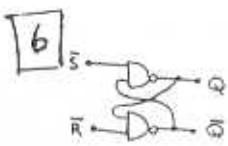
Note further that an effective 9-T version exists, in which  $Q_6$  and  $Q_8$  are moved below  $Q_5$  and  $Q_7$ , then merged into a single grounded-source device. Note that all of the designs can employ the latter idea to reduce the transistor count by 1. See the sketch following:

(next)



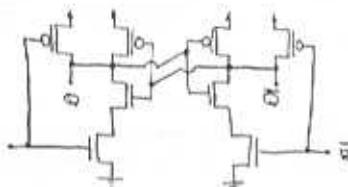
[11] This circuit suffers only from the fact that unclocked changes in  $S$  and  $R$  have a secondary impact on  $Q_1$  and  $Q_2$ . Since raising  $S$  or  $R$  disconnects  $Q_1$  and  $Q_2$  from  $V_{DD}$ . In some applications this may lead to system noise sensitivity in which case one or the other or both of  $Q_{10}, Q_{12}$  (in the previous sketch) may be added.





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R	S	$\bar{R}$	$\bar{S}$	$Q_{out}$
0	0	1	1	$Q_n$
0	1	1	0	1
1	0	0	1	0
1	1	0	0	not used



In this circuit,  $\bar{R}$  and  $\bar{S}$  rest high. Either going low forces the corresponding output to go high. [Thus,  $\bar{S} \rightarrow 0 \rightarrow S_1 \rightarrow Q = 1$ ]

If both  $\bar{S}$  and  $\bar{R}$  are low, the outputs  $Q$  and  $\bar{Q}$  are no longer complementary, but are both high. Following this, the output retains the state defined by the last of  $\bar{R}, S$  to go high. If both  $\bar{R}$  and  $\bar{S}$  go high simultaneously, the outputs  $Q$  and  $\bar{Q}$  oscillate, initially in phase, with a phase difference, initiated by noise, which

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grows increasingly rapidly until the outputs "snap" into a complementary state of 0/1+1/0. During the oscillation, the period is somewhat less than twice the propagation delay of the basic inverter, and the amplitude is a fraction of the full logic swing. This fraction depends on the ratio of propagation time to transition time of the inverters, being larger for faster logic transitions and longer pure propagation delays. Generally speaking, the outputs are usually nearly sinusoidal.

11

7

Note that the devices are matched, with  $k_n = k_p = 20(1/2) = 40 \mu A/V^2$ , and  $|V_t| = 1V$ .

For  $V_x = 0V, 5V$ : one device is on, one off;  $V_o = 5V, 0V$

For  $V_x = 1V, 4V$ : one on, one off;  $V_o = 5V, 0V$

For  $V_x = 1.5V, 3.5V$ : one in saturation, one in triode mode.

$$I_o = V_x(40)(1.5-1)^2 = 40[(1.5-1)V_o - V_o^2/2]$$

$$\text{thus } 0.125 = 2.5V_o - V_o^2/2$$

$$\text{or } V_o^2 - 5V_o + 0.25 = 0$$

$$\text{and } V_o = [(-5 \pm \sqrt{24.75})]/2$$

$$= (5 \pm 4.948)/2 = 0.05V$$

$$\text{thus } V_o = 0.05V \text{ or } 4.95V$$

For  $V_x = 2.0V, 3.0V$ :

$$V_x(2-1)^2 = (2-1)V_o - V_o^2/2$$

$$\text{or } (2-1)^2 = 2 \times 2V_o - V_o^2$$

$$\text{and } V_o^2 - 4V_o + 1 = 0$$

$$\text{whence } V_o = [(-4 \pm \sqrt{15.75})]/2$$

$$= (4 \pm 3.944)/2 = 0.27V$$

$$\text{thus } V_o = 0.27V \text{ or } 4.73V$$

For  $V_x = 2.25V$  or  $2.75V$ :

$$(2.25-1)^2 = 2(2.25-1)V_o - V_o^2$$

$$1.5625 = 3.5 - V_o - V_o^2$$

$$V_o^2 - 3.5V_o + 1.5625 = 0$$

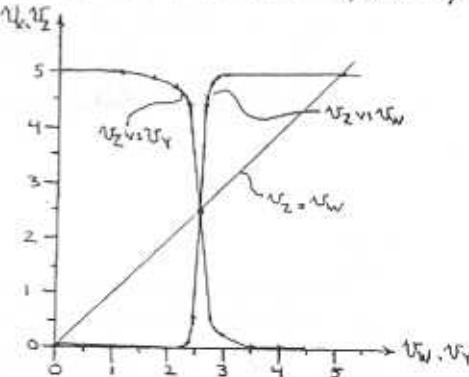
7

$$\text{whence } V_o = (-3.5 \pm \sqrt{3.5^2 - 4(1.5625)})/2$$

$$= (3.5 \pm 2.45)/2 = 0.525$$

$$\text{thus } V_o = 0.525V \text{ or } 4.475V$$

b. For  $V_x = 2.5V$ ,  $V_o = 2.5V$  by symmetry.



Now, having plotted  $V_z$  versus  $V_y$  (or  $V_x$  versus  $V_w$ ),

use the graph to find  $V_z$  versus  $V_w$ ;

Work backwards: first  $V_z$ , then  $V_y$  &  $V_x$ , then  $V_w$ .

For  $V_z = 2.5V$ ,  $V_y = V_x + V_w = 2.5V$

For  $V_z = 4.4V$ ,  $V_y = 2.25V$ ; for  $V_x = 2.25V$ ,  $V_w = 2.55V$

For  $V_z = 4.9V$ ,  $V_y = 1.50V$ ; for  $V_x = 1.50V$ ,  $V_w = 3.65V$

(next)

[7]

- Plot these on the sketch above. See that the  $V_Z$  versus  $V_W$  characteristic crosses the  $V_Z = V_W$  line at:

point A : (0,0)  
point B : (2.5, 2.5)  
point C : (5, 5)

C. At point B, the current flow in each inverter is:  
 $I_p = V_2(40)(2.5-1)^2 = 45\text{mA}$   
 where for each transistor,  $r_0 = 100/(45 \times 10^3) = 2.22\text{mV}$   
 and  $g_m = 2(1/2)40(2.5-1) = 60\mu\text{A/V}$

Thus for each inverter operating at (2.5, 2.5),  
 the voltage gain is  $(g_m + g_m)(r_{OL}r_0) =$   
 $-g_m r_0 = -60 \times 10^{-6} \times 2.22 \times 10^3 = 133\text{V/V}$   
 Thus an estimate of the slope of the  $V_Z$  versus  $V_W$  curve at B is  $(133)^2 = 17.7 \times 10^3\text{V}$

Correspondingly a lower bound on the width of the transition region is  $(5-0)/(2.7 \times 10^3)$ , or  $0.28\text{mV}$ , that is  $\pm 0.14\text{mV}$  around 2.5V.

Approximately, a more detailed analysis can be done in a number of ways. For example, consider a sample-point analysis:  
 From earlier work above, for  $V_Z = 4.95\text{V}$ ,  $V_I = 1.5\text{V}$ . Now to produce  $V_X = V_Y = 1.5\text{V}$ ,  $V_W$  must be around 2.5V with a value determined by

[7]

the PMOS in saturation and the NMOS in triode mode sharing the current  $I_D$ :  
 $I_D = V_2(40)(5-V-1)^2 = 40[(V-1)1.5-1.5^2/2]$   
 or  $(4-V)^2 = 2(V-1)1.5 - 1.5^2$   
 or  $16 - 8V + V^2 = 3V - 3 - 2.25$   
 or  $V^2 - 11V + 21.25 = 0$   
 whence  $V = (-11 \pm \sqrt{11^2 - 4(21.25)})/2$   
 $= (11 \pm 6)/2 = 2.5\text{V}$

While this seems strange at first, it is simply an indicator that for  $V_X = 1.5\text{V}$ , the NMOS is actually just at the edge of saturation, such that the saturation-current expression still applies, for which the saturation currents of both devices are equal when  $V_W = 2.5\text{V}$ . Thus the output resistance and finite current gain must be considered (as we did earlier).

For the gain of  $-133\text{V/V}$  found there, to produce a 1V change in  $V_Z$  (from 2.5 to 1.5V) would require  $1/133\text{V}$  or  $7.5\text{mV}$  (or  $15\text{mV}$  total). Recall that this  $15\text{mV}$  change on  $V_W$  would  $V_Z$  from 0.05 to 4.95V! We conclude that the transition region is more like  $15\text{mV}$  than a fraction of a mV.

Note that this approach uses a combination of large- and small-signal modelling to better approximate the large-signal non-linearities of logic.

[11]

[11] The approximate transfer characteristic of each inverter passes through points: (0.5)(2.0, 4.6); (2.42, 0.4); (5, 0).

For the linear centre segment between (2.0, 4.6), (2.42, 0.4)

an equation is  $V_0 = a - bV_T$

Here:  $4.6 = a - 2.0b$ , and

$0.4 = a - 2.42b$

Subtract:  $4.6 = 0.4 \rightarrow b = 10$

Now:  $4.6 = a - 2(10) \rightarrow a = 4.6 + 20 = 24.6$

Check:  $0.4 = 24.6 - 2(24.6) \checkmark$

Thus the middle part of the characteristic is

$V_0 = 24.6 - 10V_T$

For each device,  $V_0 = V_Z = V_T$  when

$V_T = 24.6 - 10V_T \rightarrow 11V_T = 24.6, \text{ or } V_T = 2.236\text{V}$

where the gain is  $\Delta V_0/\Delta V_T = -b = -10\text{V/V}$

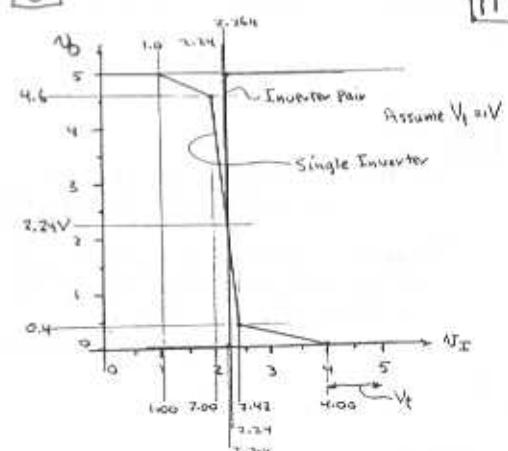
This point B on the open-loop characteristic is  $V_T = V_Z = 2.236\text{V}$ , where the loop gain can be approximated to be at least  $(-10)^2 = 100\text{V/V}$ .

The open-loop characteristic reaches  $V_0 = 5\text{V}$ ,

where  $V_T = 2.236 + (5-2.236)/100 = 2.264\text{V}$

and it reaches 0V where  $V_T = 2.236 - 2.236/100 = 2.214\text{V}$

[8]



[9]

[11] From Exercise,  $T = 0.69\text{RC}$ . From the equation just preceding the exercise, a 2% error introduced by Ron implies that:

$$R + R_{on} = 1.02R$$

$$\text{and } R = R_{on}/0.02 = 1k\Omega/0.02 = 50k\Omega,$$

$$\text{for which } C \cdot T/0.69R = 1 \times 10^{-3}/(0.69 \times 50 \times 10^3) = 30\text{nF}$$

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From page 1025, for the monostable:

$$T = C(R + R_{on}) \ln \left[ R/(R + R_{on}) \times V_{DD}/(V_{DD} - V_{th}) \right]$$

$$\text{Neglecting } R_{on}, T = CR \ln \left( V_{DD}/(V_{DD} - V_{th}) \right)$$

$$\text{Now, } V_{th} = (0.5 \pm 0.1)V_{DD} \text{ and}$$

$$\ln(1/(1-0.5)) = 0.693, \ln(1/(1-0.4)) = 0.511, \text{ and}$$

$$\ln(1/(1-0.6)) = 0.916$$

$$\text{Thus } T \text{ can vary from } (0.511/0.693)100 = 73.7\%$$

$$\text{to } (0.916/0.693)100 = 132.2\% \text{ of nominal}$$

or by  $-26.3\%$  to  $+32.2\%$  of nominal.

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$$T = C(R + R_{on}) \ln \left[ R/(R + R_{on}) \times V_{DD}/(V_{DD} - V_{th}) \right]$$

$$= 0.001 \times 10^{-6} (10^4 + 200) \ln [10/(10^4 + 200) \times 10/(10-5)]$$

$$= 1.02 \times 10^{-7} \ln 1.16 = 1.04 \times 10^{-7} = 10.4\mu s$$

$$\text{Now, } \Delta V_1 = V_{DD}R/(R + R_{on}) = 10 \times 10^4 / (1.02 \times 10^4) = 9.80V$$

$$\text{and } \Delta V_2 = V_{DD} \times (V_{DD} - V_{th}) = 10 + 0.5 - 5 = 5.5V$$

Change in  $V_{DD}$  is due to the variable current in  $R$  flowing in  $R_{on}$ .

Initially, the voltage across  $R_{on}$  is  $V_{DD}R_{on}/(R + R_{on})$   
Finally, it is  $(V_{DD} - V_{th})R_{on}/(R + R_{on})$

13

$$\text{a) From Ex 13.17, } T = CR \ln \left[ V_{DD}/(V_{DD} - V_{th}) \times V_{DD}/V_{th} \right]$$

$$\text{Now, for } V_{th} = V_{DD}/2, T = CR \ln(2 \times 2) = 1.39 CR$$

$$\text{b) For } f_0 = 100\text{kHz, } T = 10^3 = 1.39 CR$$

$$\text{whence } CR = 0.721 \times 10^{-3}$$

$$\text{For a choice of } R = 10k\Omega, C = 0.721 \times 10^{-3} \times \frac{1}{10k\Omega} = 721\text{pF}$$

14

$$\text{From Exercise 13.17, } T = CR \ln \left( \frac{V_{DD}}{V_{DD} - V_{th}} \times \frac{V_{DD}}{V_{th}} \right)$$

$$\text{For } V_{th} = 0.5V_{DD}, f_0 = 1/f = [CR \ln 4]^{-1} = 0.721/CR$$

$$\text{For } V_{th} = 0.6V_{DD}, f_0 = [CR \ln (10.4 \times 10.4)]^{-1} = 0.704/CR$$

$$\text{For } V_{th} = 0.4V_{DD}, f_0 = [CR \ln (10.4 \times 10.4)]^{-1} = 0.704/CR$$

Thus, for  $V_{th}/V_{DD}$  over the range 0.4 to 0.6,  $f_0$  varies from  $(0.704/0.721)100 = 97.2\%$  of nominal through nominal to 97.2% of nominal again.

Note that the frequency is relatively constant, independent of  $V_{th}$  variation. This is due to compensation of one half-cycle by the next, each varying quite strongly but in opposite directions.

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$$\text{Thus, the total change is } (V_{DD}/2)R_{on}/(R + R_{on})$$

$$\text{or } (10/2)200/(10^4 + 200) = 0.098V$$

The peak sinking current through  $G_1$  occurs as  $V_{DD}$  rises and current flows through  $R_{on}$  and the diode. Its peak value is

$$(V_{DD} - V_{th} - V_0)/R_{on} = (10 - 5 - 0.7)/200 = 21.5mA$$

12

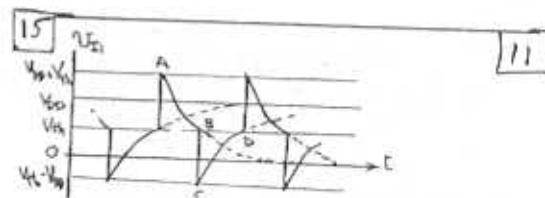
$$T = C(R + R_{on}) \ln \left[ R/(R + R_{on}) \times V_{DD}/(V_{DD} - V_{th}) \right]$$

$$\text{Thus } 1 = 10^6 (R + 100) \ln \left[ R/(R + 100) \times 1/(1 - 0.4) \right]$$

Now for  $R_{on}$  ignored:

$$R + 10^6 \ln [1/0.6] = 1.958mA$$

Clearly, for this  $R$ ,  $R_{on}$  can be neglected, and  $R = 1.96M\Omega$



$$\text{At Point A: } V_D = V_{DD} + V_{th} \text{, and thereafter}$$

$$V_D = (V_{DD} + V_{th}) e^{-t/RC} + V_{th}$$

$$\text{At Point B: } V_D = V_{th} \text{ and } t = t_1, \text{ where}$$

$$V_{th} = (V_{DD} - V_{th}) e^{-t_1/RC}$$

$$\text{or } t_1 = -RC \ln [V_{th}/(V_{DD} + V_{th})]$$

$$\text{or } t_1 = RC \ln [(V_{DD} + V_{th})/V_{th}]$$

$$\text{At Point C: } V_D = V_{th} - V_{DD} \text{, and thereafter}$$

$$V_D = (V_{th} - V_{DD} - V_{DD}) e^{-t/RC} + V_{DD}$$

$$\text{At Point D: } V_D = V_{th} \text{ at } t = t_2, \text{ where}$$

$$V_{th} = (V_{th} - 2V_{DD}) e^{-t_2/RC} + V_{DD}$$

$$\text{or } t_2 = -RC \ln [(V_{th} - 2V_{DD})/(2V_{DD} - V_{th})]$$

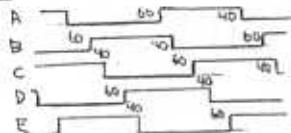
$$\text{or } t_2 = RC \ln [(2V_{DD} - V_{th})/(V_{th} - 2V_{DD})]$$

$$\text{Thus, the period } T = t_1 + t_2$$

$$\text{whence } T = RC \left[ \ln \frac{V_{DD} + V_{th}}{V_{th}} + \ln \frac{2V_{DD} - V_{th}}{V_{th} - 2V_{DD}} \right]$$

$$\text{or } T = RC \ln \left[ \frac{V_{DD} + V_{th}}{V_{th}} \times \frac{2V_{DD} - V_{th}}{V_{th} - 2V_{DD}} \right] \text{, as suggested.}$$

16



17

16

17

Waveform A is low for  $60+40+60+40+60 = 260\text{ns}$   
and high for  $40+60+40+60+40 = 240\text{ns}$

Period is  $(260+240) = 500\text{ns}$   
Frequency is  $1/500 = 2\text{MHz}$

Percentage of cycle for which output is high is 48%  
Check:  $t_p = (60+40)/2 = 50\text{ns}$

17

17

For 11 inverters, there are  $2(11)+22$  transitions  
whose average length is  $t_p = (1.2 \times 10^{-9})/11 = 22\text{ns}$

18

A 1Mb array requires  $n$  address bits where  
 $2^n = 10^6$ , or  $n \log_{10} 2 = 6$ ,  $n = 6/\log_{10} 2 = 19.93$   
Thus, 20 bits are needed to address every cell.

For 16-bit words,  $2^4 = 16$  and 4 bits are not needed.  
Thus  $20-4 = 16$  bits of address are sufficient.

Check:  $m = \log_{10}(10^6/16)/\log_{10}(2) = 4.74/0.30 = 15.83$   
Use 16

Note: A "1Mb array" actually holds  $2^{10} = 1024 = 1048576$  cells.

19

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Since the array is square, there are about  $(10^6)^{1/2} = 1000$  word lines. More precisely, there are  $(2^{10})^{1/2} = 2^5 = 32$  word lines.

For a 1-Mb square array, there are also 1024 bit lines.

Thus a straightforward design would use 1024 sense amplifiers/drivers.

Now for dynamic storage on C operating at f in voltage V, the dynamic power is  $P_D = fCV^2$ .  
Thus  $C = P_D/(fV^2) = 500 \times 10^{-9}/(1.2 \times 10^9 \cdot 3^2)$   
or  $C = 4000\text{PF}$

10% of the dynamic loss is in the array - the array capacitance is  $4000(0.1) = 400\text{PF}$ .  
For 16 bit lines selected and active at a time, the capacitance per bit line could be  $300/16 = 22.5\text{PF}$ , which is  $22.5/1024 = 22\text{fF/bit}$ .  
For voltage reduction to 3V, power is reduced to a factor  $3^2/2^2 = 0.75$ . For the same power level, the array could be larger by a factor  $0.75 = 2.8$  times.

20

20

$$\text{Cell area} = \frac{1}{2}(21 \times 10^{-3} \times 31 \times 10^{-3})/10^4 = 0.326\mu\text{m}^2$$

For 2 cells, the area is  $0.65\mu\text{m}^2$ , for which (if square) the side length is  $\sqrt{0.65} = 0.802\mu\text{m}$  and the cell dimension is about  $0.4\mu\text{m}$  by  $0.8\mu\text{m}$ .

21

21

$$\text{The cell area is } 10^4 \times 0.38 \times 10^{-3} \times 0.76 \times 10^{-3} = 0.289 \times 10^{-3}\mu\text{m}^2$$

$$\text{The chip area is } 19 \times 10^{-3} \times 38 \times 10^{-3} = 0.722 \times 10^{-3}\mu\text{m}^2$$

Thus the peripheral circuits and interconnect occupy  $(0.722 - 0.289) \times 10^{-3} = 0.433\mu\text{m}^2$ , or  $(433/722) \times 100 = 60\%$  of the chip area.

22

22

For 16 blocks, 4 bits of block address are needed. The arrays are each of  $256\text{M}/16 = 16\text{Mb}$  size. If each block array is square, it has 4096 rows and 4096 columns. It needs  $\log_2 4096 = 12$  bits each of row and column address bits, for a total of  $2(12) + 4 = 28$  bits total.

Check:  $2^{28} = 268 \times 10^6$ , or 256M bits.

[23]

- a) For W high and B high, with Q originally low and the gate of Q<sub>1</sub> high:

$$\text{At } V_Q = 2.5V, \text{ from Eq. 5.30, } V_t V_{t0} + \gamma [V_{BQ} + 2\phi_f - \sqrt{2\phi_f}] \\ \text{or } V_t = 0.8 + 0.5 [\sqrt{2.5+0.6} - \sqrt{0.6}] = 1.293V$$

$$\text{Now, for } I_S = I_{t1}, V_{tBQ}(W_{S55}-V_t)^2 = \frac{1}{2} k_m (W_{S55}-V_t)^2 \\ \text{or } (W/L)_S (2.5-1.293)^2 = n (5-0.8)^2 \\ \text{or } (W/L)_S = n (4.2)^2 / 1.207^2 = 12.1 \text{ n.}$$

- b) For W high and B low, with Q originally high and the gate of Q<sub>6</sub> high:

$$\text{At } V_Q = 2.5V, \text{ with } Q_6 \text{ in saturation and } Q_5 \text{ in mode:} \\ \frac{1}{2} k_{D4} (5-0.8)^2 = k_{D6} [(5-0.8)(2.5-2.5)/2] \\ \text{or } V_t n (4.2)^2 = (W/L)_6 [7.775] \\ \text{or } (W/L)_6 = n (4.2)^2 / (2(7.775)) = 1.20n$$

- c) Clearly, for a very conservative design, one in which both write mechanisms work, the larger of the choices in a) b) would be used, namely  $(W/L)_S = (W/L)_6 = 12.1 \text{ n, very large!}$

In practice, this design needs a lot of area, and instead, a conservative version of b) would likely be used, with  $(W/L)_S = (W/L)_6$  larger than 1.2n, say by a factor of 2, perhaps 2.4n

d.

$$\text{For } (W/L)_6 = 12.1 \text{ n and matched inverters,} \\ k_{D4} = 50 \times 10^{-6} n = 50 \times 10^{-6} \times 2 = 100 \mu A/V^2, \\ \text{and } k_D = 1210 \mu A/V^2$$

$$\text{For } V_Q = 5V: V_{S04} = 0 \text{ and } i_{D4} = 0 \text{, but} \\ i_{D6} = \frac{1}{2} (1210 \times 10^{-6}) (5-0.8)^2 = 10.7 \mu A$$

$$\text{For } V_Q = 2.5V: \\ i_{D6} = 1210 \times 10^{-6} [(5-0.8)(2.5-2.5)/2] = 8.97 \mu A \\ \text{and } i_{D4} = 100 \times 10^{-6} [(5-0.8)(2.5-2.5)/2] = 0.74 \mu A$$

Thus, the average discharge current is  $(10.7 + 0 + 0.74)/2 = 5.7 \mu A$

Thus, the discharge time is  $t = 50 \times 10^{-15} \times 2.5 / 9.44 \times 10^{-7} = 13.295 \text{ ns}$

For a minimum-size design with  $(W/L)_6 = 2.4 \text{ n, the average current would be:}$

$$[(2.4/1.2) [10.7 - 0 + 0.74] - 0.74]/2 = 1.58 \mu A$$

$$\text{And the time to } V_{DD}/2 \text{ would be} \\ t = 50 \times 10^{-15} \times 2.5 / 1.58 \times 10^{-7} = 79 \text{ ps}$$

[24]

$$\text{a) Initially, } I_S = \frac{1}{2} k_m (W_{S55}-V_t)^2 = \frac{1}{2} (50) 10^{-6} (10/2)(5-0.8)^2 \\ \text{or } I_S = 2.205 \mu A, \text{ and } I_t = 0 \text{ since } W_{S55} = 0. \\ \text{Thus, } I_{CQ} = 2.205 - 0 = 2.205 \mu A$$

$$\text{b) At the end of the interval, } V_Q = 2.5V, \text{ and} \\ V_{t5} = V_t + \gamma [\sqrt{V_{BQ} + 2\phi_f} - \sqrt{2\phi_f}] \\ = 1.0 + 0.5 [\sqrt{2.5+0.6} - \sqrt{0.6}] = 1.49V$$

$$\text{Correspondingly, } I_S = V_t (50) 10^{-6} (10/2)(2.5-1.49) \text{ A}$$

$$\text{Now, } Q_1 \text{ is in triode mode, with} \\ I_t = 50 \times 10^{-6} (4/2) [(5-1) \times 5 - 2.5] \text{ A} = 687.5 \mu A$$

$$\text{Thus } I_{CQ} = 127.5 - 687.5 = -560 \mu A \text{ (negative!)}$$

Thus we see that Q<sub>5</sub> is incapable of raising Q to 2.5V. This is the case even if the body effect is ignored (check this for interest!).

(you may also be interested in calculating the highest voltage to which Q can be raised.)

c) The average value is not relevant.

d) It is 0A as defined here!

Note that in Exercise 11.9  $\Delta t = 19.4 \text{ ps}$ , during which time the voltage on the gate of Q<sub>1</sub> reduces, the current in Q<sub>1</sub> reduces and the current in Q<sub>2</sub> increases, allowing Q<sub>5</sub> to become more effective.

[25]

$$\text{For } V_W = 2.5V, V_B = 2.5V, V_Q = 0V: \\ (5-1) V_{t0} (W_{S55}-V_t)^2 = V_t (50) 10^{-6} (10/2)(2.5-1)^2 = 28 \mu A$$

However, the voltage at Q will rise due to this current where  $28 = 50(4/2) [(5-1)V_Q - V_{Q1}/2]$ .

$$\text{Thus } V_{Q1}^2 - 8V_{Q1} + 5.62 = 0 \\ \text{whence } V_{Q1} = (-8 \pm \sqrt{64-4(5.62)})/2 \\ = (8 \pm 6.44)/2 = 0.74 \mu V$$

This reduces i<sub>S</sub> to  $i_S = \frac{1}{2} (50) (10/2) (2.5-0.74-1)^2 = 64.3 \mu A$

$$\text{for which } V_{Q1}^2 - 8V_{Q1} + (0.48 \times 2) = 0 \\ \text{whence } V_{Q1} = (-8 \pm \sqrt{64-4(0.48 \times 2)})/2 \\ = 0.165 \mu V$$

$$\text{Try } V_{Q1} = (0.165 + 0.74)/2 = 0.47 \mu V$$

$$\text{for which } i_S = 125 (1.5-0.47)^2 = 133 \mu A \\ \text{and } V_{Q1} = (8 \pm \sqrt{64-4(1.33 \times 2)})/2 = 0.347 \mu V$$

$$\text{Try } V_{Q1} = (0.347 + 0.47)/2 = 0.41 \approx 0.40 \mu V$$

$$\text{for which } i_S = 125 (1.5-0.40)^2 = 157.4 \mu A \\ \text{and } V_{Q1} = (8 \pm \sqrt{64-4(1.57 \times 2)})/2 = 0.397 \approx 0.40 \mu V \\ \text{with } i_S = 150 \mu A$$

Now, for the bit-line voltage to reduce by 0.2V, it will take

$$t = (1 \times 10^{-12} \times 0.2) / 150 \times 10^{-6} = 1.33 \text{ ns}$$

[26]

The bit-line capacitance is  $[n(z) + z_0] \times 2n + 20\text{fF}$

For a 5V supply and precharge to  $V_{DD}/2$ , there is a 2.5V change on the 50fF cell capacitor to produce a 0.1V bit-line signal.

$$\text{Thus } 0.1 = 2.5 / (50 / (2n + 20 + 50))$$

$$\text{or } 2n + 20 = 2.5(50) / 0.1 = 1250$$

$$\text{or } 2n = 1180 \text{ and } n = 590 > 512$$

Thus we can use 512 rows for which  $\log_{2} 512 = 9$   
or 9 bits addressing is needed.

For a sense amplifier of 5x greater  $g_m$

$$0.1/5 = 125 / (2n + 20)$$

$$2n + 20 = 5(125) / 0.1 = 6250$$

$$2n = 6180 \text{ and } n = 3090 > 2048$$

To address 2048 rows requires 11 bits

[27]

If the memory array has  $n$  columns, it has  $2n$  rows and  $2n^2$  cells

$$\text{Refresh time is } 2n(z_0)10^{-9} = (1.00 - 0.48)8 \times 10^{-9}$$

$$\text{whence } n = 0.02 = 8 \times 10^{-9} / 400 \times 10^{-9} = 4000$$

The corresponding memory capacity is  $2n^2 \times 2 / 4000^2$   
or 32 Mb

[28]

For a 1Mb square array there are 1024 rows and 1024 columns.

Thus the bit-line capacitance is  $10^5 (1024 \times 1024)$

or 1.036 pF

When storing a '1', the voltage on  $C_S$  is  $(V_{DD} - V_t)$

or  $(5 - 1.5) = 3.5V$ . With precharge to  $V_{DD}/2 = 2.5V$ , the change in voltage on  $C_S = 3.5 - 2.5 = 1.0V$ .

For  $C_S = 25\text{fF}$ , the bit-line voltage resulting is  $25 / (25 + 1036) \times 1 = 23.6\text{mV}$

When storing a '0', the voltage on  $C_S$  is 0V and the change is  $2.5 - 0 = 2.5V$  with a resulting bit-line signal of  $25 / (25 + 1036) \times 2.5 = 58\text{mV}$

[29]

[11]

For leakage current  $I$ , the voltage change on  $C$  in time  $T$  is  $V = IT/C$

Correspondingly,  $\Delta V = I \times 10 \times 10^{-3} / 20 \times 10^{-15}$ , and the maximum leakage is  $I = 20 \times 10^{15} / 10 \times 10^{-3} = 2\text{pA}$

[30]

Pattern the solution after the approach used in the solution of Example 11.3.

For the bit-line output to reach  $0.4V_{DD} = 2.7V$

from  $V_{DD}/2 = 1.5V$  in 2ns for an initial bit-line

signal of  $0.1/2 = 0.05V$ :

$$2.7 = 1.5 + 0.05e^{t/0.629}$$

$$\text{whence } 2/3 = \ln [(2.7 - 1.5) / 0.05] = 3.178$$

$$\text{and } t = 2/3.178 = 0.629\text{ ns}$$

Thus  $C/G_m = 0.629 \times 10^{-9}$ , and  $G_m = 1 \times 10^{-13} / (0.629 \times 10^{-10}) = 1.589\text{ mS}$

For matched inverters,  $g_m = g_{op} = G_m/2 = 1.545/2 = 0.773\text{ mS}$

Now,  $g_m = k'(W/L)(V_{GS} - V_t)$

$$\text{and } 0.773 \times 10^{-9} = 100 \times 10^{-6} (W/L) [3.0/2 - 0.8]$$

$$\text{Thus } (W/L)_n = 0.773 \times 10^{-9} / (100 \times 10^{-6}) / 1.2 = 11.36$$

Now, for devices assumed to have length  $L/10\mu\text{m}$  (or, alternatively, for each micron of device length)

$$W_n = 11.36 \mu\text{m} \text{ and } W_p = 3(11.36) = 34.1 \mu\text{m}$$

Now, for a differential input signal of 0.2V (and 0.1V on each bit-line), the response time is  $t$ , where  $2.7 = 1.5 + 0.1 e^{t/0.629}$ ,

$$\text{whence } t = 0.629 \ln (2.7 - 1.5) / 0.1 = 1.56\text{ ns}$$

31

11

Note that for the inverters,  
 $k_n = k_n'(W/L)n = 100(6/1.5) = 400 \mu A/V^2$   
 $k_p = k_p'(W/L)p = 100(2.5)(15/1.5) = 400 \mu A/V^2$   
 Thus we see that the inverters are matched.  
 Generally,  $i_D = \frac{1}{2}k_n(V_{GS}-V_t)^2$  and  
 $g_m = 260/6V_0 = k_n(V_{GS}-V_t)$

Now, at  $V_{GS}=V_B=V_{DD}/2 = 3.3/2 = 1.65V$ ,  
 $g_m = 400(1.65 - 0.8) = 340 \mu A/V$   
 Thus  $G_m = g_m + g_{mp} = 2(340) = 680 \mu A/V$

For a bit-line capacitance of  $0.8 \text{ pF}$ ,  $\beta = C/G_m$   
 or  $T = 0.8 \times 10^{-12} / 680 \times 10^{-6} = 1.17 \mu s$

Now, for  $0.4V_{DD}$  reached in  $2\tau_{in}$  for a signal  $\Delta V$ ,  
 $0.9(3) = 1.65 + \Delta V e^{2/T}$   
 or  $\Delta V = (2.97 - 1.65)/5.478 = 0.241V$   
 Thus the initial voltage between B lines  
 must be  $z(0.241) = 0.482V$

If an additional  $\tau_{in}$  is allowed :  $T = 2\tau_{in} = 3\tau_{in}$   
 and  $\Delta V = (2.97 - 1.65)/e^{3/T} = 0.103V$   
 allowing a signal to be used of  $2(0.103) = 0.206V$

Now, with the original bit-line signal of  $0.241V$ ,  
 and a delay of  $3\tau_{in}$  :

31

11

$$2.97 = 1.65 + 0.241 e^{3/T}$$

$$\text{and } e^{3/T} = (2.97 - 1.65)/0.241 = 5.477$$

$$3/T = \ln(5.477) = 1.7006$$

$$\text{whence } T = 3/1.7006 = 1.764 \mu s$$

Thus  $C = G_m T = 680 \times 10^{-6} \times 1.764 \times 10^{-6} = 1.20 \text{ pF}$   
 This is an increase (from  $0.8 \text{ pF}$ ) of  $(1.2 - 0.8) / 0.8 = 50\%$

For the longer line, the initial delay to establish  
 a suitable signal becomes  $150\% \text{ of } 5\tau_{in} = 7.5\mu s$

32

11

a) For an initial difference between bit lines  
 of  $\Delta V$ , each bit-line signal is  $\Delta V/2$ .

For the rising line :  $V_B = V_{DD}/2 + (\Delta V/2) e^{t/(C_0/G_m)}$   
 whence  $e^{t/(C_0/G_m)} = (2/\Delta V)(0.9 - 0.5)V_{DD}$   
 $= 0.8V_{DD}/\Delta V$

Taking base-e logarithms and cross-multiplying,

$$t = \tau_{in} = (k_p/G_m) \ln(0.8V_{DD}/\Delta V) \text{ as stated.}$$

b) For a reduction of  $\tau_{in}$  to  $1/2$  the original value,  $G_m$  must be doubled. To do this,  
double the width of all transistors (or  
 increase by a factor of  $2.0x$ )  
 (next)

32

11

c) Now, for  $V_{DD}=5V$ ,  $\Delta V=0.2V$ , with the  
 original design,  $t_{in} = C_0/G_m \ln(0.8/0.5)/0.2$   
 or  $t_{in} = 3.00 \text{ } \mu s/C_0/G_m$

For the modified design,  $\Delta V=0.2/4=0.05V$   
 and  $t_{in} = C_0/G_m \ln(0.8/0.5)/0.05 = 4.38 \text{ } \mu s/C_0/G_m$

For these to be equal,  $3.00/G_m = 4.38/G_m$   
 and  $G_m' = 1.46 G_m$ .

Thus the transistors must be made 4.6% wider  
 (or be increased by a factor of 1.46x)

33

11

For the DRAM arrangement, the signal is  
 applied to only one side. Thus in comparison  
 to the SRAM treatment, the applied signal  
 is only half as large.

Now, the specification must be met for  
 either a '0' or a '1' stored. The worst  
 case is a differential signal of  $40mV$   
 (corresponding to a single-side signal of  $20mV$ )

Thus  $2.0 = 20 \times 10^{-3} e^{5/T}$ , and  $5 = 3 \ln(2/20 \times 10^{-3})$ ,  
 or  $5 = 3 \ln(100) + 4.605 \text{ } \mu s$ , whence  $T = 1.086 \text{ } \mu s$   
 (next)

33

11

For a  $1pF$  bit-line capacitance,  $G_m = C/\beta$   
 or  $G_m = 1 \times 10^{12} / 1.086 \times 10^{-9} = 0.921 \text{ mA/V}$ ,  
 with  $0.921 = 0.46 \text{ mA/V}$  from each transistor.

Now, for the n-channel device,  $g_m = k_n'(W/L)n(V_{GS}-V_t)$   
 or  $0.46 \times 10^{-3} = 100 \times 10^{-6} (W/L)_n (2.5 - 1)$

$$\text{Thus } (W/L)_n > (0.46/0.1) / 1.5 = 3.07$$

$$\text{For matched inverters, } (W/L)_p = 2.5(3.07) = 7.68$$

When a '1' is read, the response time will be  
 $t = 3 \ln(2/20 \times 10^{-3}) = 1.086 \ln 100 = 5 \mu s$   
 (Note: this is as designed!)

When a '0' is read,  
 $t = 1.086 \times 10^{-3} \ln(2/100/2) \times 10^{-3} = 4.0 \mu s$

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11

Here  $2^n = 512$ ,  $n \log_{10} 2 = \log_{10} 512$ ,  $n = 2.709 / 0.301 = 9.00$   
 Thus the number of bits is 9.  
 The decoder has 512 output lines, one of  
 which is active (high). The NOR array requires  
 true and complement input lines for each bit:  $2 \times 9 = 18$ .  
 Each row uses 9 NMOS for a total of  $9 \times 512 = 4608$  NMOS  
 and 512 PMOS, for a total of 5120 transistors.

35

For a 256Kbit square array, there are  
 $(256 \times 1024)^{1/2} = 512$  rows and columns.

Number of column-address bits is  $\log_2 512 = 9$

Two multiplexors are needed - since both true and complement bit lines are required. For each multiplexor, there are 512 output lines.

For each (half) multiplexor, 512 NMOS are needed for a total of 1024 NMOS pass gates.

For the 512 output NOR decoder itself,  $512 \times 9 + 4608$   
 NMOS and 512 PMOS are needed.

The address-bit inverters need 9 NMOS and 9 PMOS

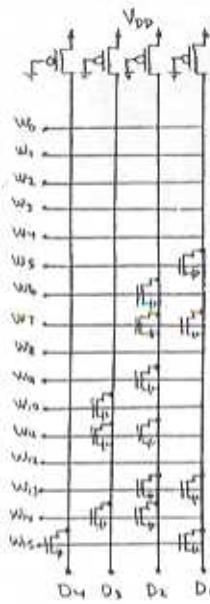
Overall, the need is for  $1024 + 4608 + 9 = 5741$  NMOS  
 and  $512 + 9 = 521$  PMOS, for a total of 5762 transistors.

11

38

Need  $Z = X \cdot Y$

X	Y	Z
00	00	0000
00	01	0000
00	10	0000
00	11	0000
01	00	0000
01	01	0001
01	10	0010
01	11	0011
10	00	0000
10	01	0010
10	10	0100
10	11	0110
11	00	0000
11	01	0011
11	10	0110
11	11	1001



Note that a total of 14 NMOS and 4 PMOS are used.

$$\frac{1}{H_{10}} = \frac{1}{H_{11}}$$

36

11

From the solution to P11.35 above, a square 256Kbit array has 512 rows and 512 columns, for which 9 row and 9 column address bits are needed.

$$\text{Check: } 2^{9+9} = 2^18 = 262144$$

For the tree of Fig 11.28, 9 levels of pass gates are needed.

36

11

The total number of pass gates is

$$N = 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512$$

$$\text{See that } N = 2 + 2(N - 512), \text{ or } N = 2 + 2N - 1024, \text{ whence } N = 1022$$

Thus a tree column decoder for 9 bits needs 1022 pass transistors.

For true and complement bit lines, a total of  $2(1022) = 2044$  pass transistors are needed. Compare this with the number required beyond the input inverters in P11.35 namely  $6162 - 18 = 6144$

37

11

Note from Fig 11.29 that the output is high if no word is selected. Thus logically, high must correspond to logic 0 (and no transistor was noted in the text).

Correspondingly, the words stored in Fig 11.29 are 0100, 0000, 1000, 1001, 0101, 0001, 0110, and 0010.

39

11

a) For the PMOS, with  $V_D = 2.5V$

$$I_D = (90/3)10^4(12/1.2)[(5-1)2.5 - 2.5/2] = 30 \times 10^4(10)[4(2.5) - 2.5/2] = 2.06$$

Thus the average charging current is 2.06mA

Time for precharge,  $t = CV/I$

$$\text{whence } t = 1 \times 10^{-10}(5-0)/(2.06 \times 10^{-3}) = 2.42$$

b) For the word-line rise,  $I = RC = 5 \times 10^{-3} \times 2 \times 10^{-12}$

$$\text{Here, } V_W = 5(1 - e^{-t/10})$$

Thus the rise time (10% to 90%) is essentially the time  $t$  to 90%, where

$$0.9(5) = 5(1 - e^{-t/10})$$

$$e^{-t/10} = 0.1$$

$$\text{and } t = -10 \ln(0.1) = 23ns$$

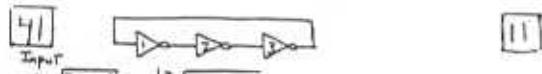
At the end of one time constant,  $t = 10ns$ , and  $V_W = 5(1 - e^{-10/10}) = 3.16V$

For discharge,  $I_{D_{out}} = V_L k_n^2 (W/L)_n (V_{GS} - V_t)^2$

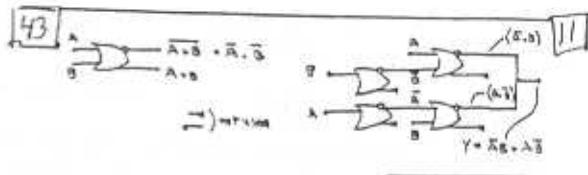
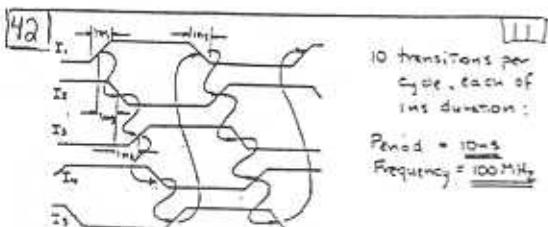
$$= V_L (90)(3/1.2)(3.16)^2 = 525$$

Thus, the bit-line voltage will lower by  $IV$  in about  $\Delta t = CAV/I_{D_{out}} = 1 \times 10^{-12}/(525 \times 10^{-9}) = 1.904ns$

40. (a)  $V_{OH} = 0 - 0.75 = -0.75V$       [1]  
 $V_{OL} = 0 - 0.75 - IR = -(0.75 + IR)$   
 (b)  $V_{th} = -(IR/2 + 0.75) = -(0.75 + IR/2)$   
 (c) For  $i = 0.99I$ ,  $V_{ce} = 750 + 25L \cdot 0.99 = 750 \text{ mV}$   
 $i = 0.01I$ ,  $V_{ce} = 750 + 25L \cdot 0.01 = 675 \text{ mV}$   
 For  $0.99I$  in  $Q_R$ ,  $\Delta V = -(0.75 + \frac{IR}{2}) - (0.750 - 0.635)$   
 $= -(0.275 + IR/2)$   
 (d) For  $0.01I$  in  $Q_R$ ,  $\Delta V = -(0.75 + IR/2) + 0.05$   
 $= -(0.635 + IR/2)$   
 (e)  $V_{IH} = -(0.635 + IR/2)$   
 $V_{IL} = -(0.275 + IR/2)$   
 (f)  $NM_{H} = -0.75 - -(0.635 + IR/2)$   
 $= IR/2 - 0.115$   
 $NM_{L} = -(0.275 + IR/2) - -(0.75 + IR)$   
 $= IR/2 - 0.115$   
 (g)  $V_{IH} - V_{IL} = -(0.635 + IR/2) - -(0.275 + IR/2)$   
 That is,  $IR/2 - 0.115 = 0.320$   
 And  $IR = 2(0.320) = 0.640V$   
 (h)  $V_{OH} = -0.75V$ ;  $V_{OL} = -0.75 - 0.115 = -1.86V$ ;  
 $V_{IL} = \underline{\underline{-(0.275 + 0.345)}} = -1.22V$ ;  
 $V_{IH} = \underline{\underline{-(0.635 + 0.345)}} = \underline{\underline{-0.98V}}$ ;  
 $V_E = \underline{\underline{-(0.750 + 0.345)}} = \underline{\underline{-1.095V}}$ .



See that once started, the process continues; that is we have an oscillation.  
 In each cycle each grid output rises and falls.  
 Thus the period is  $2(3+7) = 20\text{ns}$   
 Frequency is  $f_{osc} = 50.0 \text{ MHz}$ .  
 Any output is high for  $3+7+2 = 12\text{ns}$   
 and low for  $7+3+7 = 17\text{ns}$   
 Check:  $20\text{ns}$ .



43. For  $V_C = V_{IL} = -1.435V$ ,  $I_E = 3.97$ ,  $V_B = -1.77V$   
 For  $V_C = V_E = -1.77V$ ,  $I_E = 4.00\text{mA}$ ,  $V_B = -1.71V$   
 For  $V_C = V_{IE} = -(20\text{mV})$ ,  $I_E = 4.12\text{mA}$ ,  $V_B = -0.33V$   
At 1:  $I_A = 0.01(3.97) = 3.74\text{mA} \rightarrow V_E = \frac{25\text{mV}}{3.74 \times 10^3} = 630\text{mV}$   
 $I_B = 0.01(3.97) = 3.74\text{mA} \rightarrow V_E = \frac{25\text{mV}}{3.74 \times 10^3} = 630\text{mV}$   
 $I_{E2} = \frac{2-1.77}{50} = 2.4\text{mA} \rightarrow V_E = \frac{25\text{mV}}{2.4 \times 10^3} = 5.4\text{mV}$   
 and  $R_{E2} = 10(50 + 5.4) = 5.12\text{k}\Omega$   
 $\therefore \text{gain} = \frac{50}{50 + 5.4} \times (2.4 \times 10^3) \times \frac{100}{10(50 + 5.4)} = 0.273$   
At 2:  $I_A = I_B = 2-A \rightarrow V_E = \frac{25\text{mV}}{2-A} = 12.5\text{mV}$   
 $I_{E2} = \frac{2-A}{50} = 12.5\text{mA} \rightarrow V_E = 1.25\text{mV}$   
 and  $R_{E2} = 10(50 + 1.25) = 5.25\text{k}\Omega$   
 $\therefore \text{gain} = \frac{50}{50 + 1.25} \times (2.4 \times 10^3) \times \frac{100}{10(50 + 1.25)} = 0.24$   
At 3:  $I_A = 0.94(4.12) = 4.08\text{mA} \rightarrow V_E = \frac{25\text{mV}}{4.08 \times 10^3} = 6.12\text{mV}$   
 $I_B = 0.01(4.12) = 0.412\text{mA} \rightarrow V_E = \frac{25\text{mV}}{0.412 \times 10^3} = 4.10\text{mV}$   
 $I_{E2} = \frac{2-0.412}{50} = 22.4\text{mA} \rightarrow V_E = 1.12\text{mV}$   
 and  $R_{E2} = 10(50 + 1.12) = 5.12\text{k}\Omega$   
 $\therefore \text{gain} = \frac{50}{50 + 1.12} \times (2.4 \times 10^3) \times \frac{100}{10(50 + 1.12)} = 0.248$

45. Assume  $I_B$  is constant at  $4\text{mA}$ .      [1]  
 (a) Currents are:  $3.6\text{mA}$  and  $0.4\text{mA}$   
 $\therefore$  Emitter-Base voltage difference =  $V_{BE} = \frac{2.6}{0.4} = 6.5\text{V}$   
 $\text{or } 25L = 6.5 \Rightarrow 54.9 \text{ mV}$   
 Thus  $V_{IL} = -1.32 - 0.55 = -1.875V$   
 $\therefore V_E = -1.72 + 0.55 = -1.245V$   
 (b) Currents are:  $4(0.999) = 3.996\text{mA}$  and  $0.01 \times 4 = 4\text{uA}$   
 $\therefore$  Emitter-Base voltage difference =  $V_{BE} = \frac{2.6}{0.004} = 650\text{mV}$   
 $\text{or } 25L = 650 \Rightarrow 173 \text{ mV}$   
 Thus  $V_{IL} = -1.11 - 0.173 = -1.493V$   
 $V_E = -1.32 + 0.173 = -1.147V$

[46]  $I_{OA} = \frac{2-1.77}{50} = 4.6 \mu A$  [11]

$P_D = 2(4.6) = 9.2 \mu W$

$I_{OB} = \frac{2-0.77}{50} = 22.4 \mu A$

$P_D = 2(22.4) = 44.8 \mu W$

For the logic-gate core  $P_D = 4 \mu A \times 5.2 V = 20.8 \mu W$ .  
Thus total  $P_D = 9.2 + 44.8 + 20.8 = 74.2 \mu W$   
(ignoring  $R_S$ )

[50]  $V_{OH} = 0.7V$ ;  $V_{OL} = +5.0V$

More precisely, for  $V_{OH} = V$ ,

$$V = 2.5k\Omega \quad V = \frac{0.753}{2.5 + 0.753} \cdot (5 - 0.7) + 0.7 = 1.27V$$

$$\frac{V-0.7}{2.5k\Omega} \quad \text{i.e. } V_{OH} = 1.27V$$

Logically : A is high if one of A or B or  
one of C or D are high.

$\text{That is } A = (A+B) \cdot (C+D)$

[47]  $NMOS = 0.325V$ , of which 50% is 162mV,  
for  $\beta = 100$  and  $V_{BEZ} = 0.23V$ ,  $I_{DS} = 22.4 \mu A$ . [11]

Approximately :

$-2 + \frac{50}{50+2k\Omega} \cdot (2-0.23) = -0.33 - 0.112$

$\text{or } \frac{50(1.77)}{50+2k\Omega} = 0.452$

$50 + \frac{2k\Omega}{2k\Omega+1} = 61.02$

$\text{whence } \mu = \frac{2k\Omega}{61.02-1} = 21.2$

$\text{Check: For } V_{DS} = -1.12 - 0.12 = -1.042V$

$I_{DS} = \frac{2-1.042}{50} = 19.2 \mu A$

$\text{and } V_{DS} \frac{2k\Omega}{2k\Omega+1} = 2.25 - V = \text{OK, since small.}$

Can ignore.

[48]  $V = -0.23V$  [11]

$R = 50k\Omega \quad C$ 
 $-2V \quad -1.77V \quad -2V$ 
 $\text{or } V = -0.23 + (-1.77-2)(1-e^{-t/50C})$ 
 $\text{or } V = -2 + 1.12 e^{-t/50C}$

After 1ms,  $V = -1.77V$

$\text{ie } -1.77 = -2 + 1.12 e^{-1000/50}$

$\text{or } e^{-1000/50} = \frac{-2-1.77}{1.12} \text{ and } -1/50C = -1.573$

$\text{Thus } C = \frac{10^{-9}}{50(-1.573)} = 12.5 \times 10^{-12} F = 12.5 pF$

[51]

For  $V_T = V_D = V_{DD}/2 = 5/2 = 2.5V$ ,

$i_{DN} = \frac{1}{2} k_m (W/L)_D (V_{GS} - V_T)^2$ 
 $= V_2(100)(2/1)(2.5 - 0.7 - 1)^2 = 64 \mu A$

Now, the collector current of  $Q_2$  is  $i_{C2} = i_{DN}$   
 $= 100(64 \times 10^{-6}) = 6.4 \mu A$

Correspondingly, the totem-pole current is  
 $i_{EQ2} = (6400 + 64) \times 10^{-6} = 6.456 \mu A$

Now, for  $i_{EQ2} = i_{EQ3}$ ,  $i_{DP} = i_{DN} = 64 \mu A$

$\text{Thus } 64 = V_2(100)(2/1)(W/L)_P (5 - 2.5 - 0.7 - 1)^2$

$\text{whence } (W/L)_P = 2.5(2/1) = \frac{5.0 \mu m/\mu m}{}$

Note that the latter could be seen directly!

[52]

At the threshold  $V_{th}$ ,  $V_G = V_D = V_{th} = V$ , and  
the two Mos operate in saturation with equal currents.

$\text{Thus } V_2(100)(2/1)(5-V-0.7-1)^2$

$V_2(100)(2/1)(V_2 - 0.7 - 1)^2$

$(3.3-V)^2 = 2.5(V_2 - 1.7)^2$

$\text{and } (3.3-V) = \pm \sqrt{2.5}(V_2 - 1.7)$

$\text{Usefully, } (3.3-V) = (1.52V - 2.69)$

$\text{whence } 2.58V^2 = 5.99 \text{, and } V = V_{th} = 2.32V$

For this value,  $i_{DH} = V_2(100)(2/1)(2.32 - 0.7 - 1)^2 = 58.4 \mu A$

$\text{and the totem-pole current is } (\beta + 1)i_{DN}$

$\text{or } 101(38.4) \times 10^{-6} = 3.88 \mu A$

[49]  $\mu = 2/3 \times 30 \text{ cm}^2/\text{Ns} = 20 \text{ cm}^{-1}/\text{ns}$

$\text{Ratio, } \frac{\text{Rise Time}}{\text{Fall Time}} = \frac{5/1}{2L/20} = \frac{3.5}{2L}$

$L = \frac{3.5 \text{ ns} \times 20 \text{ cm}^2/\text{Ns}}{5 \times 2} = 7 \text{ cm}$

[53]

The problem as stated is very general, and correspondingly, its solution can be long and complex. Assume for simplicity that the specifications of Problem 11.57e apply, with matched MOS having  $(W/L)_P = 2.5(W/L)_N$ .

$$\text{For } R_2: \text{With } V_{D1} = V_t/2 = V_D + 0.333V$$

$$I_{D_N} = 100(10^4)(2/1)[(5-0.7-1)0.33 - 0.75/2] = 209\mu A$$

Now, if 50% of this is lost in  $R_2$ ,

$$R_2 = 0.7/(0.50 \times 209) = 6.10 k\Omega$$

Now if 20% is lost in  $R_2$ ,

$$R_2 = 0.7/(0.20 \times 209) = 16.7 k\Omega$$

For  $R_1$ :

$$I_{D_P} = (100/2.5)10^{-4}[2.5(2/1)][(5-0-1)0.33 - 0.75/2] = 256\mu A$$

Now, if 50% of this is lost in  $R_1$ ,

$$R_1 = (5-0.33)/(0.50 \times 256) = 36.5 k\Omega$$

Now, if 20% is lost in  $R_1$ ,

$$R_1 = 2.5(36.5) = 91.1 k\Omega$$

In comparison:

$$\text{For the 50% case: } R_1/R_2 = 36.5/6.10 = 5.45$$

$$\text{For the 20% case: } R_1/R_2 = 91.1/16.7 = 5.45$$

(Why should their equality be obvious?)

Thus, in general,  $R_1/R_2 = 5.45$

[54]

For  $t_{PLH}$ :

At  $V_D = 0V$ ,  $I_{D_N} = 1/2(100/2.5)(2/1)(5-0-1)^2 = 640\mu A$   
 At  $V_D = 2.5V$ ,  $I_{D_N} = 100(2/1)(5-0.7-1/2.5-0.7) = 550\mu A$

Thus  $I_{D_{PAV}} = (640 + 550)/2 = 595\mu A$   
 and  $I_{O_{AV}} = (100+1)595 = 60.1 \text{ mA}$

Thus  $t_{PLH} = CV/I = 2 \times 10^{-13} \times 2.5 / (60.1 \times 10^{-3}) = 83.2 \text{ ps}$

For  $t_{PHL}$ :

At  $V_D = 5.0V$ ,  $I_{D_N} = 1/2(100)(2/1)(5-0.7-1)^2 = 1.04 \text{ mA}$   
 At  $V_D = 2.5V$ ,  $I_{D_N} = 100(2/1)[(5-0.7-1)(2.5-0.7) - (2.5-0.7)/2] = 864\mu A$

Thus  $I_{D_{PAV}} = (1.04 + 864)/2 = 977\mu A$   
 and  $I_{O_{AV}} = 101(977 \times 10^{-6}) = 9.86 \text{ mA}$

Thus  $t_{PHL} = CV/I = 2 \times 10^{-13}(2.5) / (9.86 \times 10^{-3}) = 50.7 \text{ ps}$   
 Thus  $t_p = (83.2 + 50.7)/2 = 67.0 \text{ ps}$

Note that this solution embodies two assumptions:

- 1) Internal capacitances can be neglected.
- 2) Transitions are from ideal 0V and 5V output-signal levels.

If outputs of  $(5-0.7) = 4.3V$  and  $(0+0.7) = 0.7V$  apply,  $t_p$  becomes about  
 $67 \times (2.5-0.7)/2.5 = 48 \text{ ps}$

[11]

[55]

For the circuit in Fig 11.45e,  $R_1 = R_2 = 5k\Omega$  sets the base of some of its drive current, namely  $0.7/5 \times 10^{-3} = 140\mu A$ . Using results from the solution of P11-52 above:

$$\text{For } t_{PSL}: I_{O_{AV}} = 595 - 140 = 455\mu A$$

and  $I_{O_{AV}} = 101(455 \times 10^{-6}) = 46.0 \text{ mA}$   
 Thus  $t_{PLH} = 2 \times 10^{-13} \times 2.5 / 4.6 \times 10^{-3} = 108.7 \text{ ps}$

$$\text{For } t_{PHL}: I_{O_{AV}} = 977 - 140 = 837\mu A$$

and  $I_{O_{AV}} = 101(837 \times 10^{-6}) = 84.5 \text{ mA}$   
 Thus  $t_{PHL} = 2 \times 10^{-13} \times 2.5 / 84.5 \times 10^{-3} = 59.3 \text{ ps}$

Thus  $t_p = (59.3 + 108.7)/2 = 84 \text{ ps}$

[56]

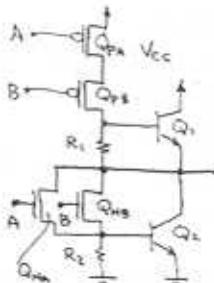
For the BiCMOS NAND of Fig 11.45f to have a dynamic response somewhat like that of the inverter of Fig 11.45e:

$$(W/L)_P_A \cdot (W/L)_P_B = (W/L)_P$$

$$\text{and } (W/L)_{N_A} \cdot (W/L)_{N_B} = 2(W/L)_N$$

[57]

A BiCMOS 2-input NOR is as shown:



In terms of the basic matched inverter:

$$(W/L)_P_A \cdot (W/L)_P_B = 2(W/L)_P$$

$$(W/L)_{N_A} \cdot (W/L)_{N_B} = (W/L)_N$$

where  $(W/L)_P$  and  $(W/L)_N$  characterize the inverter.

CHAPTER 12 - PROBLEMS

12.1

$$T(s) = \frac{\omega_0}{s + \omega_0} \quad T(j\omega) = \frac{\omega_0}{j\omega + \omega_0}$$

$$|T(j\omega)| = \frac{\omega_0}{\sqrt{\omega_0^2 + \omega^2}}$$

$$\begin{aligned}\phi(\omega) &\triangleq \tan^{-1} \left[ \frac{\text{Im}(T(j\omega))}{\text{Re}(T(j\omega))} \right] \\ &= -\tan^{-1} \frac{\omega}{\omega_0}\end{aligned}$$

$$G = 20 \log_{10} |T(j\omega)|$$

$$A = -20 \log_{10} |T(j\omega)|$$

$\omega$	$ T(j\omega) $ [V/V]	$G$ [dB]	$A$ [dB]	$\phi$ °
0	1	0	0	0
$0.5\omega_0$	0.8944	-0.97	0.97	-26.57
$\omega_0$	0.7071	-3.01	3.01	-45.0
$2\omega_0$	0.4472	-6.99	6.99	-63.43
$5\omega_0$	0.1961	-14.1	14.1	-78.69
$10\omega_0$	0.0995	-20.0	20.0	-84.29
$100\omega_0$	0.010	-40.0	40.0	-89.43

12.2

$$T(s) = \frac{1}{(s+1)(s^2 + s + 1)}$$

$$= \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$T(j\omega) = \left[ j(2\omega - \omega^3) + (1 - 2\omega^2) \right]^{-1}$$

$$\begin{aligned}|T(j\omega)| &= \left[ (2\omega - \omega^3)^2 + (1 - 2\omega^2)^2 \right]^{-1/2} \\ &= \left[ 4\omega^2 - 4\omega^4 + \omega^6 + 1 - 4\omega^2 + 4\omega^4 \right]^{-1/2} \\ &= \frac{1}{\sqrt{1 + \omega^6}}\end{aligned}$$

For Phase Angle :

$$\begin{aligned}\phi(\omega) &= \tan^{-1} \left[ \frac{\text{Im}[T(j\omega)]}{\text{Re}[T(j\omega)]} \right] \\ &= -\tan^{-1} \left[ \frac{2\omega - \omega^3}{1 - 2\omega^2} \right]\end{aligned}$$

For  $\omega = 0.1$ :

$$\begin{aligned}|T(j\omega)| &= (1 + 0.1^6)^{-1/2} \approx 1 \\ \phi(\omega) &= -11.5^\circ = -0.20 \text{ rad}\end{aligned}$$

For  $\omega = 1 \text{ rad/s}$ :

$$\begin{aligned}|T(j\omega)| &= (1 + 1^6)^{-1/2} = 1/\sqrt{2} = 0.7 \\ \phi &= -\tan^{-1} (1/-1) = -135^\circ = 2.356\text{ rad}\end{aligned}$$

Note :  $G = -3 \text{ dB}$

$$\begin{aligned}\text{Also: } \tan^{-1}(-1) &= -45^\circ \text{ or } -135^\circ \\ \tan^{-1}(-1/1) &= -45^\circ \\ \tan^{-1}(1/-1) &= -135^\circ\end{aligned}$$

For  $\omega = 10 \text{ rad/s}$ :

$$\begin{aligned}|T(j\omega)| &= (1 + 10^6)^{-1/2} = 0.001 \\ \phi &= -\tan^{-1} \left[ \frac{2(10) - 10^3}{1 - 2(10^2)} \right]\end{aligned}$$

CONT.

$$\begin{aligned}
 &= -\tan^{-1} \left[ \frac{-980}{-199} \right] \\
 &= -\left[ 180^\circ + \tan^{-1} \left( \frac{980}{199} \right) \right] \\
 &= -258.5^\circ \\
 &= \underline{\underline{4.512 \text{ rad}}}
 \end{aligned}$$

Now consider an input of  $A \sin \omega t$  to  $T(s)$ . The output is then given by:

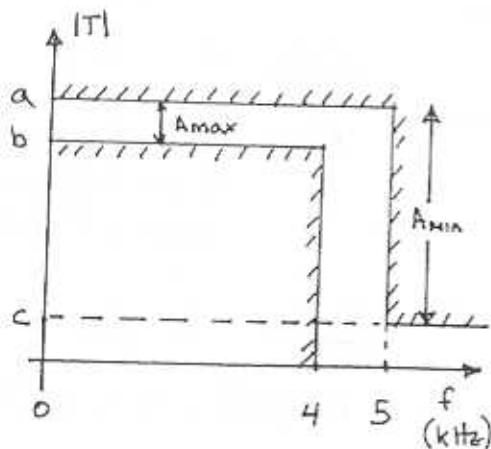
$$A |T(j\omega)| \sin(\omega t + \phi(\omega))$$

Using this result, the output to each of the following inputs will be:

INPUT	OUTPUT
$2 \sin(0.1t)$	$2 \sin(0.1t - 0.2)$ i.e. $2 \times 1 = 2$
$2 \sin(1t)$	$\sqrt{2} \sin(t - 2.356)$ i.e. $2 \times \sqrt{2} = \sqrt{2}$
$2 \sin(10t)$	$2 \times 10^{-3} \sin(10t - 4.512)$

12.3

12.4



Note  $|T(f)|$  is shown in a linear scale but  $A_{\max}$  and  $A_{\min}$  are in dB

From the problem

$$\frac{a}{b} = 1.1, \quad c = 0.1\% a \text{ or } \frac{c}{a} = 0.001$$

$$A_{\max} = 20 \log_{10} a - 20 \log_{10} b$$

$$= 20 \log_{10} \frac{a}{b}$$

$$= 20 \log_{10}(1.1)$$

$$= \underline{\underline{0.83 \text{ dB}}}$$

$$A_{\min} = 20 \log_{10} a - 20 \log_{10} c$$

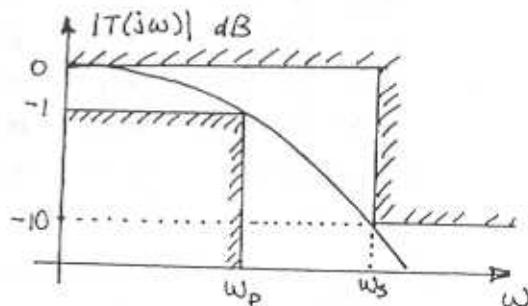
$$= 20 \log_{10} \left( \frac{a}{c} \right)$$

$$= 20 \log_{10}(0.001)$$

$$= \underline{\underline{60 \text{ dB}}}$$

$$\text{Selectivity} = \frac{\omega_s}{\omega_p} = \frac{2\pi 5}{2\pi 4} = \underline{\underline{1.25}}$$

12.5



$$\begin{aligned}
 T(s) &= \frac{k}{1+sT} \quad \text{If } T=1s \text{ & the DC gain = 1 then} \\
 &= \frac{1}{1+1s} \quad \underline{\underline{k=1}}
 \end{aligned}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

At the passband edge :

CONT.

$$|T(j\omega_p)| = \frac{1}{\sqrt{1+\omega_p^2}} = 10^{-1/20}$$

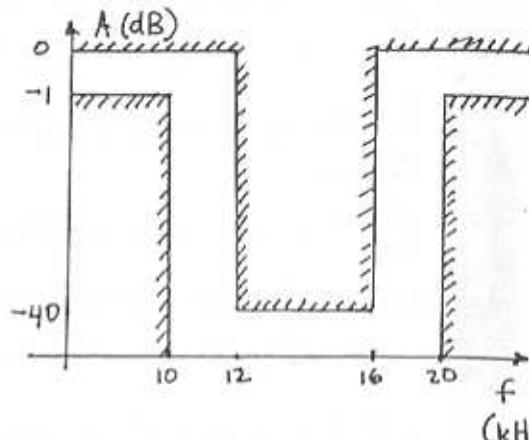
$$A_{\max} = 1 \text{ dB}, A_{\min} = 40 \text{ dB}$$

$$\therefore \omega_p = \underline{0.5088 \text{ rad/s}}$$

At the stopband edge:

$$|T(j\omega_s)| = \frac{1}{\sqrt{1+\omega_s^2}} = 10^{-10/20}$$

$$\therefore \omega_s = \underline{3 \text{ rad/s}}$$

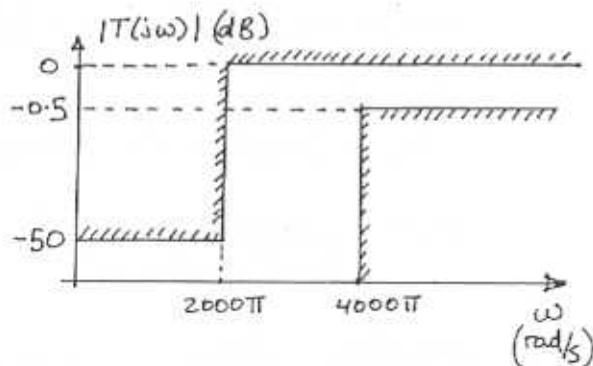


$$\text{Selectivity} = \frac{\omega_s}{\omega_p} = \frac{3}{0.5088} = \underline{5.9}$$

12.6

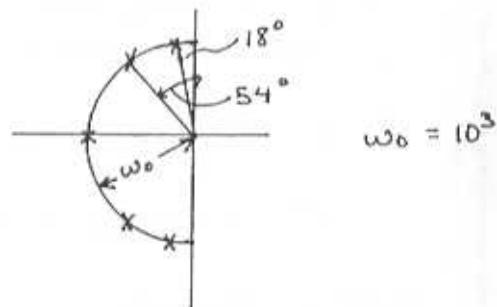
Passband is defined by:  $f \geq 2 \text{ kHz}$   
 $\Rightarrow \omega_p = 2\pi(2000) \text{ rad/s}$

Stopband is defined by:  $f \leq 1 \text{ kHz}$   
 $\Rightarrow \omega_s = 2\pi(1000) \text{ rad/s}$



Note we assumed a maximum transmission of 0 dB.

12.8



Poles at 18°:

$$\begin{aligned} p_1 &= \omega_0 (-\cos(90^\circ - 18^\circ) \pm j \sin(90^\circ - 18^\circ)) \\ &= \omega_0 (-\cos 72^\circ \pm j \sin 72^\circ) \\ &= \omega_0 (-0.309 \pm j 0.951) \end{aligned}$$

Poles at 54°

$$\begin{aligned} p_2 &= \omega_0 (-\cos 36^\circ \pm j \sin 36^\circ) \\ &= \omega_0 (-0.809 \pm j 0.588) \end{aligned}$$

Poles on Real Axis

$$p_3 = -\omega_0$$

CONT.

Note: Given a pair of poles

$$P_i = \omega_0 (-\cos\alpha \pm j\sin\alpha)$$

introduces a second order term as follows:

$$\begin{aligned} & (s + \omega_0 \cos\alpha - j\omega_0 \sin\alpha)(s + \omega_0 \cos\alpha + j\omega_0 \sin\alpha) \\ &= s^2 + s(\omega_0 \cos\alpha - j\omega_0 \sin\alpha + \omega_0 \cos\alpha + j\omega_0 \sin\alpha) \\ &\quad + \omega_0^2 [\cos^2\alpha + j\cos\alpha \sin\alpha - j\cos\alpha \sin\alpha + \sin^2\alpha] \\ &= s^2 + s(2\omega_0 \cos\alpha) + \omega_0^2 \end{aligned}$$

So for  $P_1$  we get a term:

$$\begin{aligned} s^2 + s(2\omega_0 \cdot 0.309) + \omega_0^2 \\ = s^2 + 0.618\omega_0 s + \omega_0^2 \end{aligned}$$

For  $P_2$  we get:

$$s^2 + 1.618\omega_0 s + \omega_0^2$$

For  $P_3$ :  $(s + \omega_0)$

∴ The denominator of  $T(s)$  is given by

$$D(s) = (s + \omega_0)(s^2 + 0.618\omega_0 s + \omega_0^2) \times \underline{(s^2 + 1.618\omega_0 s + \omega_0^2)}$$

Case (a) - If all the zeros are  $\pm\infty$ , the numerator is a constant

$$|T(0)| = \frac{k}{\omega_0^5} = 1 \text{ for unity gain at DC}$$

$$\therefore k = \omega_0^5$$

$$T(s) = \frac{k}{D(s)} = \frac{\omega_0^5}{\underline{D(s)}}$$

where  $D(s)$  is given above.

Case (b) - For all zeros at 0, the numerator is given by  $ks^5$

$$\text{At } s=j\infty \quad |T(s \rightarrow j\infty)| = \frac{k}{1} = 1$$

$$\therefore T(s) = \frac{s^5}{\underline{D(s)}}$$

12.9

Poles at  $-1$  and  $-0.5 \pm j0.8$  gives a denominator:

$$\begin{aligned} D(s) &= (s+1)(s+0.5-j0.8)(s+0.5+j0.8) \\ &= (s+1)(s^2 + 2(0.5)s + 0.5^2 + 0.8^2) \\ &= (s+1)(s^2 + s + 0.89) \end{aligned}$$

Zeros at  $\infty$  and  $\pm j2$  give a numerator:

$$N(s) = k(s+j2)(s-j2) = k(s^2+4)$$

Note there is one zero at  $\infty$  because  $\text{Degree}(D(s)) - \text{Degree}(N(s)) = 1$

$$T(s) = \frac{k(s^2+4)}{(s+1)(s^2+s+0.89)}$$

$$|T(j0)| = \frac{k(4)}{0.89} = 1 \because \text{DC gain} = 1$$

$$\Rightarrow k = 0.2225$$

$$\therefore T(s) = \frac{0.2225(s^2+4)}{(s+1)(s^2+s+0.89)}$$

12.10

Numerator is given by

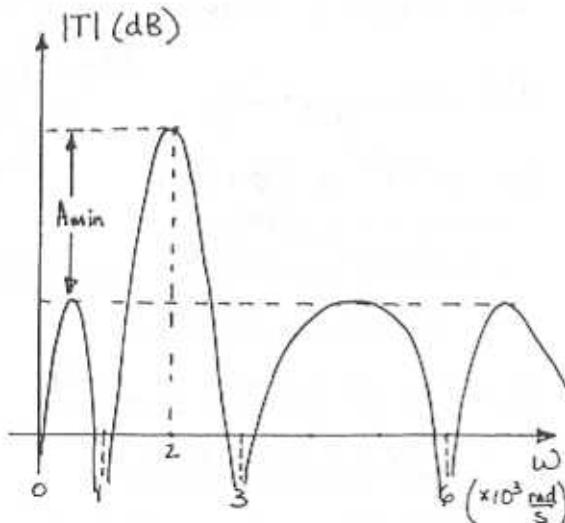
$$a_7 (s-0) (s^2 + (10^3)^2) (s^2 + (3 \times 10^3)^2) \times \\ (s^2 + (6 \times 10^3)^2)$$

$$= a_7 s (s^2 + 10^6) (s^2 + 9 \times 10^6) (s^2 + 36 \times 10^6)$$

Degree of Numerator  $\hat{=} M = 7$ Degree of Denominator  $\hat{=} N$ Given that there is one zero at  $\infty$ :

$$N - M = 1 \Rightarrow \underline{N = 8}$$

$$\therefore T(s) = \frac{a_7 s (s^2 + 10^6) (s^2 + 9 \times 10^6) (s^2 + 36 \times 10^6)}{s^8 + b_7 s^7 + b_6 s^6 + \dots + b_0}$$



The easiest way to solve the circuit is to use nodal analysis at nodes ①, ②, ③

At node ③  $\sum I = 0$ 

$$\frac{V_0}{1} + \frac{V_0}{Y_S} + \frac{V_0 - V_1}{2S} = 0$$

$$\therefore V_1 = V_0 (2s^2 + 2s + 1) \quad \text{Eq.(a)}$$

At node ②  $\sum I = 0$ 

$$\frac{V_1 - V_2}{1} + \frac{V_1}{Y_S} + \frac{V_1 - V_0}{2S} = 0$$

$$\therefore V_1 (2s^2 + 2s + 1) = V_0 + 2s V_1 \quad \text{Eq.(b)}$$

(a)  $\rightarrow$  (b)

$$V_0 (2s^2 + 2s + 1)^2 = V_0 + 2s V_1$$

$$V_0 (4s^4 + s^3(4+4) + s^2(2+4+2) + s(2+2)+1) = V_0 + 2s V_1$$

$$\frac{V_0(s)}{V_i(s)} \triangleq T(s) = \frac{2s}{4s^4 + 8s^3 + 8s^2 + 4s}$$

$$T(s) = \frac{0.5}{s^3 + 2s^2 + 2s + 1}$$

Poles are given by:

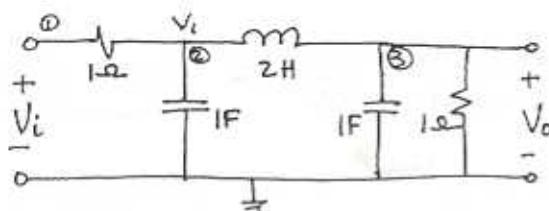
$$s^3 + 2s^2 + 2s + 1 = 0$$

$$(s+1)(s^2 + s + 1) = 0$$

 $\therefore$  Poles are  $s = -1$  and

$$s = \frac{-1}{2} \pm j\sqrt{3}/2$$

12.11



12.12

$$A_{max} = 1 \text{ dB}, A_{min} = 20 \text{ dB}, \omega_S/\omega_p = 1.3$$

Using:

$$A(\omega_S) = 10 \log \left[ 1 + e^2 \left( \frac{\omega_S}{\omega_p} \right)^{2N} \right] \quad \text{Eq.(12.15)}$$

$$= A_{MIN}$$

CONT

$$E = \left[ 10^{\frac{A_{min}}{10}} - 1 \right]^{\frac{1}{N}} = 0.5088$$

$$A_{min} = 10 \log \left[ 1 + \epsilon^2 \left( \frac{\omega_s}{\omega_p} \right)^{2N} \right]$$

$$10^{\frac{A_{min}/10}{10}} - 1 = \epsilon^2 \left( \frac{\omega_s}{\omega_p} \right)^{2N}$$

$$\log \left( 10^{\frac{A_{min}/10}{10}} - 1 \right) = \log \left( \epsilon^2 \left( \frac{\omega_s}{\omega_p} \right)^{2N} \right)$$

$$N = \frac{\log \left[ \left( 10^{\frac{A_{min}/10}{10}} - 1 \right) / \epsilon^2 \right]}{2 \log \left( \frac{\omega_s}{\omega_p} \right)}$$

$= 11.3 \Rightarrow \text{choose } \underline{N=12}$

The actual value of stopband attenuation can be calculated using the integer value of  $N$ :

$$A(\omega_s) = 10 \log \left[ 1 + \epsilon^2 \left( \frac{\omega_s}{\omega_p} \right)^{2N} \right]; N=12$$

$= \underline{27.35 \text{ dB}}$  actual attenuation

If the stopband specs are to be met exactly we need to find  $A_{max}$ .

Eq. 12.15 can be rearranged to give

$$\epsilon^2 = \frac{10^{\frac{A_{min}/10}{10}} - 1}{\left( \frac{\omega_s}{\omega_p} \right)^{2N}} \quad A_{min} = 20 \quad N = 12$$

$= 0.1824$

$$\therefore A_{max} = 10 \log (1 + \epsilon^2)$$

$= \underline{0.73 \text{ dB}}$

12.13

$$N = 7, A_{max} = 3 \text{ dB}$$

We want attenuation at

$$\omega = 1.6 \omega_p \text{ or } \frac{\omega}{\omega_p} = 1.6$$

$$\epsilon = \sqrt{10^{\frac{A_{max}/10}{10}} - 1} = 0.998$$

$$A(\omega) = 10 \log \left[ 1 + \epsilon^2 \left( \frac{\omega}{\omega_p} \right)^{2N} \right]$$

$$= 10 \log \left[ 1 + 0.998^2 (1.6)^4 \right]$$

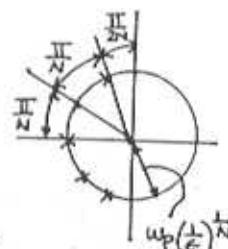
$$= \underline{24.8 \cdot 56 \text{ dB}}$$

12.14

$$\omega_p = 10^3 \text{ rad/s}, N=5$$

$$A_{max} = 1 \text{ dB} \Rightarrow \epsilon = 0.5088$$

Find solution graphically



$$\begin{aligned} P_1 &= \omega_p \left( \frac{1}{\epsilon} \right)^{1/N} \neq \left( \frac{\pi}{2} + \frac{\pi}{2N} \right) \\ &= 873.59 \neq (6\pi/10) \\ &= 873.59 \left[ \cos(6\pi/10) \pm j \sin(6\pi/10) \right] \\ &= -269.96 \pm j 830.84 \end{aligned}$$

$$\begin{aligned} P_2 &= 873.59 \neq \left[ \frac{\pi}{2} + \frac{\pi}{2N} + \frac{\pi}{N} \right] \\ &= -706.75 \pm j 513.49 \end{aligned}$$

$$P_3 = 873.59 \neq \pi = -873.59$$

12.15

$$\begin{aligned} f_p &= 10 \text{ kHz} & \frac{\omega_s}{\omega_p} &= 1.5 & A_{min} &= 15 \text{ dB} \\ f_s &= 15 \text{ kHz} & & & A_{max} &= 2 \text{ dB} \end{aligned}$$

$$\epsilon^2 = 10^{\frac{A_{max}/10}{10}} - 1 \Rightarrow \epsilon = 0.76478$$

CONT.

Manipulation Eq (12.15) we get:

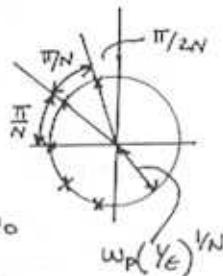
$$N = \frac{\log \left[ (10^{\frac{A_{min}/10} - 1}) / \epsilon^2 \right]}{2 \log (\omega_0 / \omega_p)} = 4.88$$

$\therefore$  Use  $N = 5$

Finding natural modes graphically :-

$$\text{radius} = \omega_p \left( \frac{1}{\epsilon} \right)^{1/N} \triangleq \omega_0$$

$$\omega_0 = 6.629 \times 10^4$$



$$\begin{aligned} P_1 &= \omega_0 \angle (\pi/2 + \pi/2N) = \omega_0 \angle (6\pi/10) \\ &= \omega_0 \left( \cos \left( \frac{6\pi}{10} \right) \pm j \sin \left( \frac{6\pi}{10} \right) \right) \\ &= \underline{\omega_0 (-0.309 \pm j 0.951)} \end{aligned}$$

$$\begin{aligned} P_2 &= \omega_0 \left( \cos \frac{3\pi}{10} \pm j \sin \frac{3\pi}{10} \right) \\ &= \underline{\omega_0 (-0.809 \pm j 0.588)} \end{aligned}$$

$$P_3 = \omega_0 \left( \cos \pi \pm j \sin \pi \right) = \underline{-\omega_0}$$

Given a natural mode  $-\alpha \pm j\beta$ , the following term results

$$\begin{aligned} &(s + \alpha + j\beta)(s + \alpha - j\beta) \\ &= s^2 + 2\alpha s + \alpha^2 + \beta^2 \\ &= \underline{s^2 + 2\operatorname{Re}[P]s + |P|^2} \end{aligned}$$

Also, note that for a Butterworth, all natural modes have a magnitude of  $\omega_0$ .

$$P_1 \text{ yields: } s^2 + 0.618\omega_0 s + \omega_0^2$$

$$P_2 \text{ yields: } s^2 + 1.618\omega_0 s + \omega_0^2$$

$$P_3 \text{ yields: } s + \omega_0$$

$$\begin{aligned} \therefore T(s) &= \frac{k}{(s + \omega_0)(s^2 + 1.618\omega_0 s + \omega_0^2)} \\ &\times \frac{1}{s^2 + 1.618\omega_0 s + \omega_0^2} \end{aligned}$$

For unity dc gain

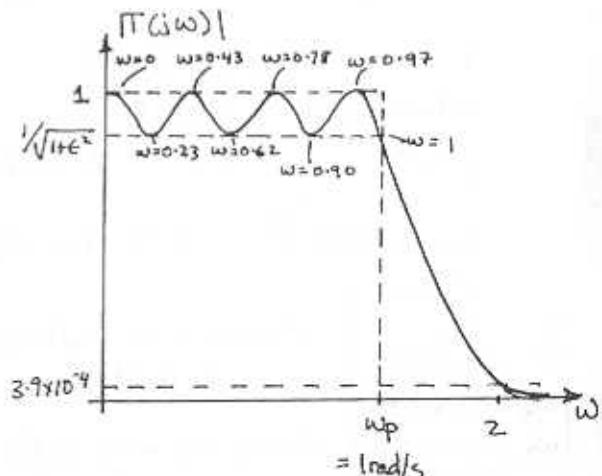
$$|T(j0)| = \frac{k}{\omega_0^2} = 1 \Rightarrow k = \omega_0^5$$

$$\therefore T(s) = \frac{\omega_0^5}{(s + \omega_0)(s^2 + 1.618\omega_0 s + \omega_0^2)} \times \frac{1}{s^2 + 1.618\omega_0 s + \omega_0^2}$$

For attenuation at 20 kHz use Eq (12.15) with  $\frac{\omega_2}{\omega_p} = \frac{20}{10} = 2$

$$\begin{aligned} A(\omega_s) &= 10 \log \left[ 1 + \epsilon^2 \left( \frac{\omega_s}{\omega_p} \right)^{2N} \right] \\ &= \underline{27.8 \text{ dB}} \end{aligned}$$

12.16



Given  $A_{max} = 1 \text{ dB} \Rightarrow \epsilon = 0.5088$

CONT.

Using Eq 12.18

$$|T(j\omega)| = \left[ 1 + \epsilon^2 \cos^2(N \cos^{-1}(\frac{\omega}{\omega_p})) \right]^{-\frac{1}{2}}$$

for  $\omega \leq \omega_p$

If  $|T(j\omega)| = 1$

$$1 = 1 + \epsilon^2 \cos^2(N \cos^{-1}(\omega/\omega_p)) \quad \omega_p = 1$$

$$N \cos^{-1}(\omega_p) = \cos^{-1}(0)$$

$$\cos^{-1}(\omega) = \frac{2i+1}{2N}\pi$$

ω's repeat  
 after this  
 value

$$\therefore \omega_i = \cos\left[\frac{2i+1}{2N}\pi\right] \quad i=0, 1, \dots, \frac{N-1}{2}$$

$\omega_0 = 0.9749$
$\omega_1 = 0.7818$
$\omega_2 = 0.4339$
$\omega_3 = 0$

ω values at which  
 $|T| = 1$   
 note  $\omega_4 = -0.4339$   
 $= -\omega_2$ .

If  $|T| = \sqrt{1+\epsilon^2}$ , then

$$\sqrt{1+\epsilon^2} = \left[ 1 + \epsilon^2 \cos^2(N \cos^{-1}(\omega/\omega_p)) \right]^{-\frac{1}{2}}$$

$$1 = \cos(N \cos^{-1}(\frac{\omega}{\omega_p}))$$

$$N \cos^{-1}(\omega) = \cos^{-1}(0)$$

$$= i\pi \quad i=0, 1, 2, \dots$$

$$\omega_i = \cos\left[\frac{i\pi}{N}\right] \quad i=0, 1, 2, \dots, \frac{N}{2}$$

$\omega_0 = 1.0$
$\omega_1 = 0.9010$
$\omega_2 = 0.6235$
$\omega_3 = 0.2252$

ω values at which  
 $|T| = (1+\epsilon^2)^{-1/2}$   
 Note  $\omega_4 = -0.2252$   
 $= -\omega_3$ !

To find  $|T(j\omega)|$  use Eq(12.19)  
 since  $\omega > \omega_p$

$$|T(j\omega)| = \left[ 1 + \epsilon^2 \cosh^2(N \cosh^{-1}(\frac{\omega}{\omega_p})) \right]^{-\frac{1}{2}}$$

$$= \left[ 1 + 0.5088^2 \cosh^2(7 \cosh^{-1} 2) \right]^{-\frac{1}{2}}$$

$$= \underline{\underline{3.898 \times 10^{-4} \text{ V/V}}}$$

$$|T|_{dB} = \underline{\underline{-68.2 \text{ dB}}}$$

For roll-off consider

$$T(s) = \frac{k}{s^2 + b_1 s^4 + \dots + b_0}$$

$$\text{for } \omega \gg \omega_p \quad T(j\omega) \cong \frac{k}{\omega^2}$$

∴ Rolloff is  $\frac{1}{2}$  or  $20 \log(\frac{1}{2})$   
 per octave = -42 dB/octave.

12.17

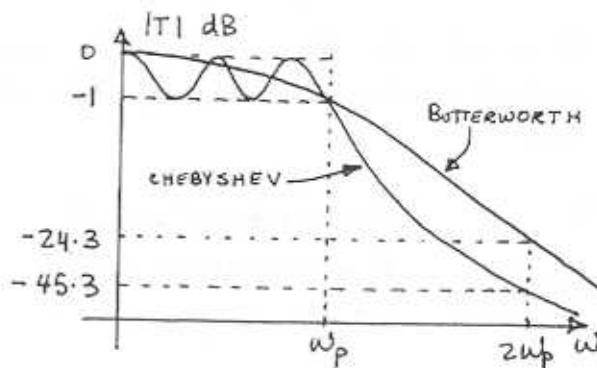
$$\omega_s/\omega_p = 2 \quad A_{max} = 1 \text{ dB} \Rightarrow \epsilon = \sqrt{10^{\frac{A_{max}}{20}}} = 0.5088$$

$$|T_B| = \left[ 1 + \epsilon^2 (\omega_s/\omega_p)^{2N} \right]^{-\frac{1}{2}}$$

$$|T_C| = \left[ 1 + \epsilon^2 \cosh^2(N \cosh^{-1}(\frac{\omega_s}{\omega_p})) \right]^{-\frac{1}{2}}$$

$$|T_B| = 6.13 \times 10^{-2} \Rightarrow \underline{\underline{-24.3 \text{ dB}}}$$

$$|T_C| = 5.43 \times 10^{-3} \Rightarrow \underline{\underline{-45.3 \text{ dB}}}$$



12.18

$$\begin{aligned} f_p &= 3.4 \text{ kHz} & A_{\max} = 1 \text{ dB} \Rightarrow \epsilon = 0.5088 \\ f_s &= 4 \text{ kHz} & A_{\min} = 35 \text{ dB} \\ \omega_s/\omega_p &= 1.174 \end{aligned}$$

Using Eq (12.22) :

$$A(\omega_s) = 10 \log \left[ 1 + \epsilon^2 \cosh^2 \left( N \cosh^{-1} \left( \frac{\omega_s}{\omega_p} \right) \right) \right]$$

by trying different values for N

N	A( $\omega_s$ )
8	28.8 dB
9	33.9 dB
10	38.98 dB

$\therefore \text{Use } \underline{N=10}$

$$\text{Excess attenuation} = 39 - 35 = \underline{4 \text{ dB}}$$

Poles are given by:

$$P_k = -\omega_p \sin \left( \frac{2k-1}{N} \cdot \frac{\pi}{2} \right) \operatorname{Sinh} \left( \frac{1}{N} \operatorname{Sinh}^{-1} \left( \frac{1}{\epsilon} \right) \right) + j \omega_p \cos \left( \frac{2k-1}{N} \cdot \frac{\pi}{2} \right) \operatorname{Cosh} \left( \frac{1}{N} \operatorname{Sinh}^{-1} \left( \frac{1}{\epsilon} \right) \right)$$

for  $k = 1, 2, \dots, N$ .

Since  $\epsilon = 0.5088$  and  $N = 10$

$$\operatorname{Sinh} \left( \frac{1}{N} \operatorname{Sinh}^{-1} \left( \frac{1}{\epsilon} \right) \right) = 0.1433$$

$$\operatorname{Cosh} \left( \frac{1}{N} \operatorname{Sinh}^{-1} \left( \frac{1}{\epsilon} \right) \right) = 1.010$$

$$\therefore P_1 = \omega_p \left[ -0.1433 \sin \left( \frac{\pi}{20} \right) + j 1.010 \cos \left( \frac{\pi}{20} \right) \right] = \omega_p (-0.0224 + j 0.9978)$$

$$P_2 = \omega_p (-0.0650 + j 0.900)$$

$$P_3 = \omega_p (-0.1013 + j 0.7143)$$

$$P_4 = \omega_p (-0.1277 + j 0.4586)$$

$$P_5 = \omega_p (-0.1415 + j 0.1580)$$

Now it should be realized that the remaining poles are complex conjugates of the above.

Pole-pair  $P_i$  &  $P_i^*$  give a factor:

$$s^2 + 2(0.0224)\omega_ps + \omega_p^2 (0.0224^2 + 0.9978^2) = s^2 + 0.0448\omega_ps + 1.023\omega_p^2$$

i.e. this factor is from  $(s-P_i)(s-P_i^*)$

$$P_2 \text{ yields: } s^2 + 0.130\omega_ps + 0.902\omega_p^2$$

$$P_3 \text{ yields: } s^2 + 0.203\omega_ps + 0.721\omega_p^2$$

$$P_4 \text{ yields: } s^2 + 0.265\omega_ps + 0.476\omega_p^2$$

$$P_5 \text{ yields: } s^2 + 0.283\omega_ps + 0.212\omega_p^2$$

Now  $T(s)$  is given by

$$T(s) = \frac{k \omega_p^{10}}{s^2 + 0.0448\omega_ps + 1.023\omega_p^2 (s-P_1)(s-P_1^*) \cdots (s-P_5)(s-P_5^*)}$$

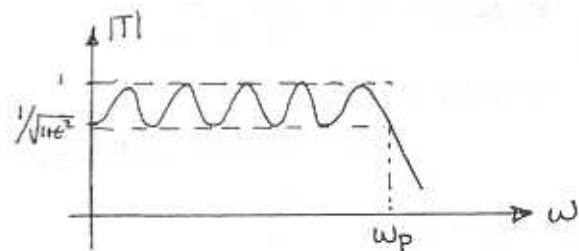
where the second order terms of the denominator are given above.

$k$  is the dc gain

$\therefore$  we want the dc gain to be

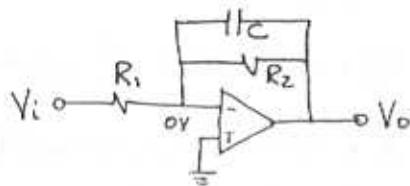
$$k = \frac{1}{1+\epsilon^2} = \underline{0.8913}$$

$$\omega_p = \underline{\frac{2\pi \times 3400}{1+\epsilon^2}}$$



12.19

$$f_0 = 10\text{ kHz} \quad \text{DC gain} = 10 \quad R_{in} = 10\text{k}\Omega$$



$$R_{in} = R_1 = \underline{10\text{k}\Omega}$$

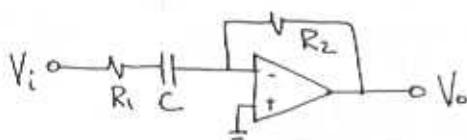
$$\begin{aligned} \text{DC gain} &= -R_2/R_1 = -10 \\ R_2 &= 10R_1 = \underline{100\text{k}\Omega} \end{aligned}$$

$$R_2 C = \underline{\gamma \omega_0}$$

$$\begin{aligned} C &= \frac{1}{\omega_0 R_2} = \frac{1}{2\pi \cdot 10^4 \times 100 \times 10^3} \\ &\approx \underline{0.159\text{nF}} \end{aligned}$$

12.20

$$f_0 = 100\text{ kHz} \quad R_{in}(\infty) = 100\text{k}\Omega \quad |T(\infty)| = 1$$



$$R_{in}(\infty) = R_1 = \underline{100\text{k}\Omega}$$

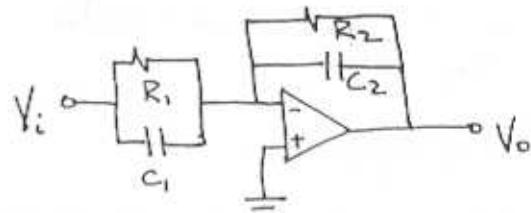
$$\begin{aligned} |T(\infty)| &= R_2/R_1 = 1 \\ R_2 &= R_1 = \underline{100\text{k}\Omega} \end{aligned}$$

$$CR_1 = \underline{\gamma \omega_0}$$

$$\begin{aligned} C &= \frac{1}{\omega_0 R_1} = \frac{1}{2\pi \cdot 100 \times 10^3 \times 100 \times 10^3} \\ &\approx \underline{15.9\text{nF}} \end{aligned}$$

12.21

Use general first-order circuit:



-Zero at 1kHz; Pole at 100kHz  
-|T(0)| = 1;  $R_{in}(0) = 1\text{k}\Omega$

$$\text{Thus: } R_{in}(\text{DC}) = R_1 = \underline{1\text{k}\Omega}$$

$$\begin{aligned} T(\text{DC}) &= -R_2/R_1 = -1 \\ R_2 &= R_1 = \underline{1\text{k}\Omega} \end{aligned}$$

For a pole at 100kHz

$$\begin{aligned} C_2 R_2 &= \frac{1}{\omega_0} \Rightarrow C_2 = \frac{1}{2\pi f_0 R_2} \\ &= \underline{1.59\text{nF}} \end{aligned}$$

For the circuit  $T(s) = \frac{a_1 s + a_0}{s + \omega_0}$

Thus the zero at  $-\frac{a_0}{a_1} = -2\pi \cdot 10^3$

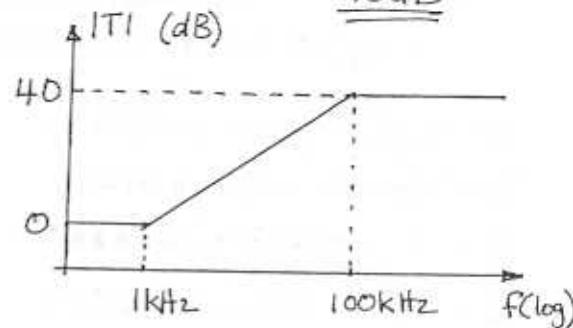
$$C_1 R_1 = \frac{a_0}{a_1}$$

$$C_1 = \frac{1}{2\pi \cdot 10^3 R_1}$$

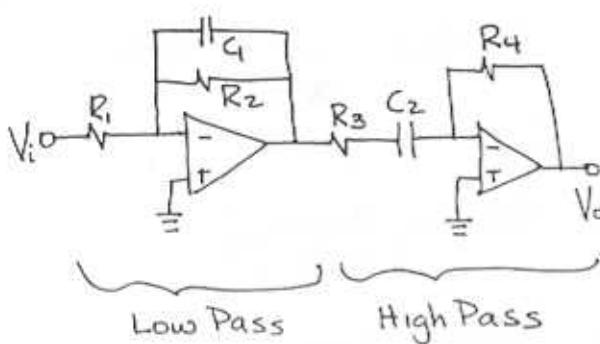
$$= \underline{15.9\text{nF}}$$

$$\text{High freq gain} = -\frac{C_1}{C_2} = \underline{-100}$$

$$= \underline{40\text{dB}}$$



12.22



$$\text{gain} = 10^{12/20} = 3.98 \approx 4$$

want  $R_1 = R_2$  large  
 $\therefore R_1 = \underline{100\text{k}\Omega}$

$$\text{Total gain} = A_{LP} A_{HP} = 4$$

$$A_{LP} = -R_2/R_1 \Rightarrow R_2 = -A_{LP} R_1 \text{ and}$$

$$R_2 \leq 100\text{k}\Omega$$

$\therefore$  make  $A_{LP} = -1$     $A_{HP} = -4$   
 $R_2 = \underline{100\text{k}\Omega}$

$$R_2 C_1 = \frac{1}{\omega_{0,LP}}$$

$$C_1 = \frac{1}{2\pi f_{0,LP} R_2} = \frac{1}{2\pi (10 \times 10^3) 100 \times 10^3}$$

$$= \underline{0.159\text{nF}}$$

$$A_{HP} = -R_4/R_3 = -4$$

$$R_4 = 4R_3$$

$\left. \right\} \text{make } R_4 = \underline{100\text{k}\Omega}$

$$R_3 = \underline{25\text{k}\Omega}$$

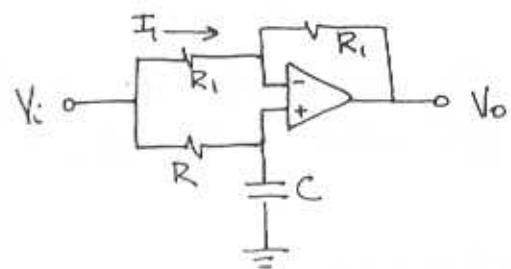
$$\text{now } R_3 C_2 = \frac{1}{\omega_{0,HP}}$$

$$C_2 = \frac{1}{2\pi f_{0,HP} R_3}$$

$$= \frac{1}{2\pi (100 \times 10^3) 25 \times 10^3}$$

$$= \underline{63.7\text{nF}}$$

12.23



At +ve terminal

$$V_r = \frac{V_s C}{1/sC + R_1} V_i$$

$$= \frac{1}{1+s\tau} V_i \quad \tau = RC$$

$V_- = V_+$  due to virtual short between terminals.

$$\therefore I_1 = \left( V_i - \frac{1}{1+s\tau} V_i \right) \frac{1}{R_1}$$

$$V_o = V_- - I_1 R_1$$

$$= \frac{V_i}{1+s\tau} - \left( V_i - \frac{V_i}{1+s\tau} \right) \frac{R_1}{R_1}$$

$$\frac{V_o}{V_i} = \frac{1 - (1+s\tau) + 1}{1+s\tau} = \frac{1-s\tau}{1+s\tau}$$

$$= \frac{\omega_0 - s}{\omega_0 + s} \quad \omega_0 = \frac{1}{\tau}$$

$$= \frac{s - \omega_0}{s + \omega_0} = T(s)$$

$$T(j\omega) = -\frac{j\omega - \omega_0}{j\omega + \omega_0}$$

$$\phi(\omega) = 180^\circ + \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= 360^\circ - 2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= -2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\stackrel{0^\circ \tan^{-1}\left(\frac{\omega}{-\omega_0}\right)}{=} 180^\circ - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Now this equation can be rearranged:

$$\frac{\omega}{\omega_0} = \tan(-\phi/2) \Leftrightarrow \omega_0 = \frac{1}{2} = \frac{1}{RC}$$

$$RC\omega = \tan(-\phi/2)$$

CONT.

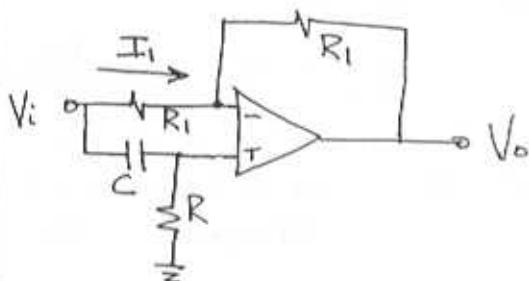
$$\therefore R = \frac{\tan(-\phi/2)}{c\omega} = 10^4 \tan(\phi/2)$$

12.25

$$\phi = -30^\circ, -60^\circ, -90^\circ, -120^\circ, -150^\circ$$

$$R = 2.68k\Omega, 5.77k\Omega, 10k\Omega, 17.32k\Omega, 37.32k\Omega$$

12.24



$$V_t = \frac{R}{R + 1/sC} V_i \quad V_i = \frac{s}{s + \omega_0} V_i$$

$$\text{where } \omega_0 = \frac{1}{RC}$$

$$I_1 = \frac{V_t - (s/s + \omega_0)V_i}{R_1}$$

$$V_0 = \frac{s}{s + \omega_0} V_i - I_1 R_1$$

$$= \frac{s}{s + \omega_0} V_i - V_i \left( 1 - \frac{s}{s + \omega_0} \right)$$

$$\frac{V_0}{V_i} = \frac{2s - s - \omega_0}{s + \omega_0} = \frac{s - \omega_0}{s + \omega_0}$$

Now:

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

NOTE  
 $\tan^{-1}\frac{\omega}{-\omega_0} = 180^\circ - \tan^{-1}\frac{\omega}{\omega_0}$

$$= 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= 180 - 2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\text{Clearly } \phi(0) = 180^\circ \text{ & } \phi(\omega \rightarrow \infty) = 0^\circ$$

Low Pass  $\omega_0 = 10^3 \text{ rad/s}$

$$Q = 1$$

DC gain = 1

$$T(s) = \frac{a_0}{s^2 + s \frac{a_0}{Q} + \omega_0^2}$$

$$T(0) = \frac{a_0 / \omega_0^2}{1} = 1$$

$$\underline{a_0 = \omega_0^2 = 10^6}$$

$$\therefore T(s) = \frac{10^6}{s^2 + 10^3 s + 10^6}$$

$$\begin{aligned} \omega_{\max} &= \omega_0 \sqrt{1 - \frac{1}{2} Q^2} \\ &= \frac{\omega_0}{\sqrt{2}} \\ &= \underline{0.707 \text{ rad/s}} \end{aligned}$$

$$\begin{aligned} |T_{\max}| &= \frac{|a_0| Q}{\omega_0^2 \sqrt{1 - \frac{1}{4} Q^2}} \quad \in a_0 = \omega_0^2 \\ &= \frac{|a_0|^2}{\omega_0^2 \sqrt{3/4}} \\ &= 2/\sqrt{3} \\ &= 1.15 \text{ V/V} \\ &= \underline{1.21 \text{ dB}} \end{aligned}$$

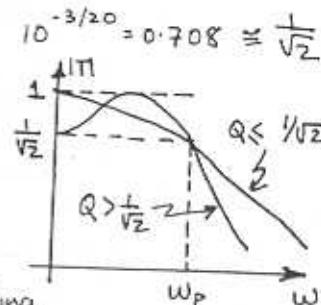
12.26

$$\omega_p = 1 \text{ rad/s}$$

$$A_{\max} = 3 \text{ dB}$$

There are many Q-values which may be used

$Q \leq \frac{1}{\sqrt{2}}$  - no peaking  
 $Q > \frac{1}{\sqrt{2}}$  - peaking



CONT.

Solution 1  $Q \leq \sqrt{2}$

For  $Q = \sqrt{2}$  the response is maximally flat. Because this is desirable, use:  $\underline{Q = \frac{1}{\sqrt{2}}}$

$$T(s) = \frac{\omega_0}{s^2 + s\omega_0\sqrt{2} + \omega_0^2}$$

$$|T(\omega)| = \frac{\omega_0}{\omega_0^2} = 1$$

$$\underline{\omega_0 = \omega_0^2}$$

$$|T(j\omega)|^2 = \frac{\omega_0^2}{(\omega_0^2 - 1) + 2\omega_0^2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\omega_0 = 1 \text{ rad/s}$$

$$\therefore \underline{T_1(s) = \frac{1}{s^2 + \sqrt{2}s + 1}}$$

Solution 2  $Q > \sqrt{2}$

From the figure:  $|T(\omega)| = \sqrt{2} = \frac{\omega_0}{\omega_0^2}$

$$\therefore \underline{\omega_0 = \omega_0^2/\sqrt{2}}$$

$$\text{Now } |T|_{\max} = \frac{|a_0| Q}{\omega_0^2 \sqrt{1 - \frac{1}{4}Q^2}} = 1$$

$$\frac{Q}{\sqrt{2} \sqrt{1 - \frac{1}{4}Q^2}} = 1$$

$$Q = \sqrt{2} \sqrt{1 - \frac{1}{4}Q^2}$$

$$\text{ASIDE: } \because Q > \sqrt{2} \quad \therefore Q = 2\left(1 - \frac{1}{4}Q^2\right)$$

$$Q^2 > \frac{1}{2}$$

$$4Q^2 > 2$$

$$\frac{1}{4Q^2} < \frac{1}{2}$$

$$\therefore 1 - \frac{1}{4Q^2} > \frac{1}{2}$$

$$\therefore \left|1 - \frac{1}{4Q^2}\right| = 1 - \frac{1}{4Q^2} \quad Q^2 = 1 \pm \sqrt{2}$$

$$\Rightarrow Q = 0.5412 \text{ or } 1.3066$$

$$\therefore Q > \frac{1}{\sqrt{2}} \quad \text{use } \underline{Q = 1.3066}$$

Now at the passband edge

$$|T(j\omega)| = \sqrt{2}$$

$$|T(j\omega)|^2 = \frac{(\omega_0^2/\sqrt{2})^2}{(\omega_0^2 - 1)^2 + \frac{\omega_0^2}{Q^2}} = \frac{1}{2}$$

$$\frac{\omega_0^4}{2} = \frac{1}{2} \left[ \omega_0^4 - 2\omega_0^2 + 1 + \frac{\omega_0^2}{Q^2} \right]$$

$$\omega_0^2 (2 - \frac{1}{Q^2}) = 1$$

$$\underline{\omega_0 = 0.841}$$

$$\therefore T_2(s) = \frac{\omega_0^2/\sqrt{2}}{s^2 + \omega_0/Qs + \omega_0^2}$$

$$= \frac{0.5}{s^2 + 0.644s + 0.707}$$

$$\text{If } \omega_0 = 2$$

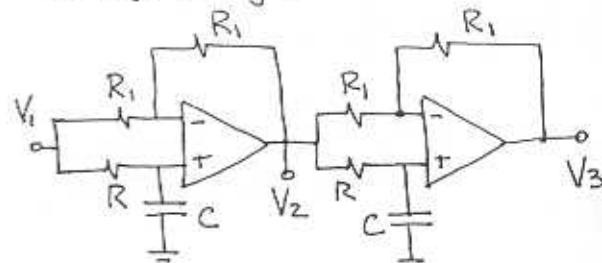
$$|T_1(j\omega)| = 0.242 \quad |T_2(j\omega)| = 0.1414$$

$$\therefore \underline{A_{min,1} = -12.3 \text{ dB}} \quad \underline{A_{min,2} = -17 \text{ dB}}$$

12.27

$V_2$  lags  $V_1$  by  $120^\circ$

$V_3$  lags  $V_2$  by  $120^\circ$



$$\omega = 2\pi 60 \quad \text{if } C = 1 \mu F$$

$$T(s) = \frac{s - \omega_0}{s + \omega_0} \quad \omega_0 = \frac{1}{RC}$$

CONT.

$$\phi(\omega) = 180^\circ + \tan^{-1}\left(\frac{\omega}{\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\text{Sub: } \tan\left(\frac{\omega}{\omega_0}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\Rightarrow \phi(\omega) = -2 \tan\left(\frac{\omega}{\omega_0}\right)$$

Now  $\phi = -120^\circ$  at  $\omega = 2\pi 60$

$$-120 = -2 \tan(\omega_{RL})$$

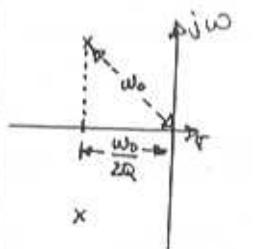
$$-60 = -\tan^{-1}(2\pi 60 \times R \times 10^{-6})$$

$$\underline{R = 4.59 \text{ k}\Omega}$$

$R_1$  can be arbitrarily chosen

$$\text{use } \underline{R_1 = 10 \text{ k}\Omega}$$

12.28



Natural modes:

$$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$\omega_0 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \underline{1.0}$$

$$\frac{\omega_0}{2Q} = \frac{1}{2} \Rightarrow \frac{\omega_0}{Q} = 1$$

$$T(s) = \frac{a_2 s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} = \frac{a_2 s^2}{s^2 + s + 1}$$

$$|T(j\infty)| = a_2 = 1$$

$$\therefore T(s) = \underline{\underline{\frac{s^2}{s^2 + s + 1}}}$$

12.29

For a 2nd-order bandpass

$$T(s) = \frac{a_1 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{j\omega a_1}{(\omega_0^2 - \omega^2) + j\omega \frac{\omega_0}{Q}}$$

$$|T(j\omega)| = \frac{a_1 \omega}{\left[(\omega_0^2 - \omega^2)^2 + \frac{\omega^2 \omega_0^2}{Q^2}\right]^{1/2}}$$

Part (a):

$$|T(j\omega_1)| = |T(j\omega_2)|$$

$$\frac{a_1 \omega_1}{\sqrt{(\omega_0^2 - \omega_1^2)^2 + \left(\frac{\omega_1 \omega_0}{Q}\right)^2}} = \frac{a_1 \omega_2}{\sqrt{(\omega_0^2 - \omega_2^2)^2 + \left(\frac{\omega_2 \omega_0}{Q}\right)^2}}$$

$$\omega_1^2 \left[ (\omega_0^2 - \omega_1^2)^2 + \left(\frac{\omega_2 \omega_0}{Q}\right)^2 \right] = \omega_2^2 \left[ (\omega_0^2 - \omega_2^2)^2 + \left(\frac{\omega_1 \omega_0}{Q}\right)^2 \right]$$

$$\omega_1^2 \left( \omega_0^4 - 2\omega_0^2 \omega_2^2 + \omega_2^4 \right) =$$

$$\omega_2^2 \left( \omega_0^4 - 2\omega_0^2 \omega_1^2 + \omega_1^4 \right)$$

$$\omega_1^2 \omega_0^4 + \omega_1^2 \omega_2^4 = \omega_2^2 \omega_0^4 + \omega_2^2 \omega_1^4$$

$$\omega_0^4 (\omega_1^2 - \omega_2^2) = \omega_2^2 \omega_1^4 - \omega_1^2 \omega_2^4$$

$$\omega_0^4 (\omega_1^2 - \omega_2^2) = \omega_2^2 \omega_1^2 (\omega_1^2 - \omega_2^2)$$

$$\omega_0^4 = \omega_1^2 \omega_2^2$$

$$\underline{\underline{\omega_0^2 = \omega_1 \omega_2}} \quad Q.E.D.$$

CONT.

For Fig. 12.4:  $\omega_{p1} = 8100 \text{ rad/s}$   
 $\omega_{p2} = 10000 \text{ rad/s}$   
 $A_{\max} = 1 \text{ dB}$

$$\omega_0^2 = (8100)(10000)$$

$$\omega_0 = \underline{9000 \text{ rad/s}}$$

$$|T(j\omega_{p1})| = |T(j\omega_{p2})| = 10^{-1/20} = \underline{0.8913}$$

$$|T(j\omega_0)| = \frac{\omega_0 a_1}{\sqrt{(\omega_0^2 - \omega_0^2)^2 + \left(\frac{\omega_0^2}{Q}\right)^2}} = 1 \\ \Rightarrow \frac{\omega_0 a_1}{\omega_0^2/Q} = 1$$

$$\therefore \frac{Q a_1}{\omega_0} = 1 \Rightarrow a_1 = \frac{\omega_0}{Q}$$

$$|T(j\omega_{p1})|^2 = |T(j0.9\omega_0)|^2 = 0.8913^2 \\ \frac{(\omega_0/Q)^2 (0.9\omega_0)^2}{(\omega_0^2 - (0.9\omega_0)^2)^2 + \left(\frac{0.9\omega_0}{Q}\right)^2} = 0.8913^2$$

$$\left(\frac{\omega_0}{Q} (0.9\omega_0)\right)^2 = 0.8913^2 \left[ (\omega_0^2 - (0.9\omega_0)^2)^2 + \left(\frac{0.9\omega_0}{Q}\right)^2 \right]$$

$$\frac{0.81\omega_0^4}{Q^2} = 0.8913^2 \left[ \omega_0^4 (1-0.81)^2 + \frac{0.81\omega_0^4}{Q^2} \right]$$

$$\frac{0.81\omega_0^4}{Q^2} (1-0.8913^2) = 0.8913^2 \omega_0^4 \times \frac{(1-0.81)^2}{(1-0.81)^2}$$

SUB  $\omega_0 = 9000$  gives

$$\underline{Q = 2.41}$$

$$\text{Now } a_1 = \frac{\omega_0}{Q} = 0.415 \omega_0$$

$$\therefore T(s) = \frac{0.415 \omega_0 s}{s^2 + 0.415 \omega_0 s + \omega_0^2}$$

If  $\omega_{s1} = 3000 \text{ rad/s}$

$$|T(j3000)| = \frac{0.415 \omega_0 (3000)}{\sqrt{(\omega_0^2 - 3000^2)^2 + (\omega_0 3000)^2 \times 0.415}} \\ = 0.1537 \\ \therefore A_{\min} = -20 \log(0.1537) \\ = \underline{16.3 \text{ dB}}$$

Now  $\omega_{s1}$  and  $\omega_{s2}$  are geometrically symmetrical about  $\omega_0$ :

$$\omega_{s1} \omega_{s2} = \omega_0^2 \\ \omega_{s2} = \frac{9000^2}{3000} \\ = \underline{27000 \text{ rad/s}}$$

### 12.30

From exercise 12.15

$$Q = \frac{\omega_0}{BW \sqrt{10^{A/10} - 1}} \quad \begin{cases} \omega_0 = 2\pi(60) \\ BW = 2\pi/6 \\ A = 20 \text{ dB} \end{cases} \\ = \underline{1.005}$$

$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$|T(0)| = a_2 \frac{\omega_0^2}{\omega_0^2} = 1 \leftarrow \text{DC Gain}$$

CONT.

$$\underline{a_2 = 1}$$

$$T(s) = \frac{s^2 + (2\pi 60)^2}{s^2 + s \frac{2\pi 60}{1.005} + (2\pi 60)^2}$$

$$\underline{T(s) = \frac{s + 1.421 \times 10^5}{s^2 + 375.1s + 1.421 \times 10^5}}$$

12.31

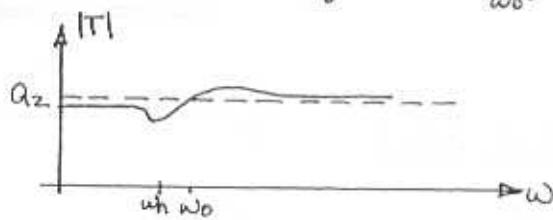
For All Pass:

$$\underline{T(s) = a_2 \frac{s^2 - s \omega_0/Q + \omega_0^2}{s^2 + s \omega_0/Q + \omega_0^2}}$$

If zero frequency < pole frequency

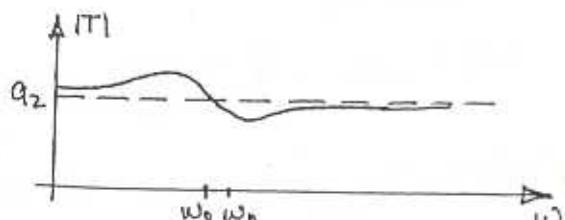
$$T(s) = a_2 \frac{s^2 - s \omega_n/Q + \omega_n^2}{s^2 + s \omega_0/Q + \omega_0^2} \quad \omega_n < \omega_0$$

$$\text{At DC: } |T| = a_2 \frac{\omega_n^2}{\omega_0^2} \quad \text{where } \frac{\omega_n^2}{\omega_0^2} < 1$$



If zero frequency > pole frequency  
then  $\omega_n > \omega_0$

$$\text{At DC: } |T| = a_2 \frac{\omega_n^2}{\omega_0^2} \quad \text{where } \frac{\omega_n^2}{\omega_0^2} > 1$$



12.32

$$\underline{T(s) = \frac{s^2 - s \omega_0/Q' + \omega_0^2}{s^2 + s \omega_0/Q_0 + \omega_0^2} a_2}$$

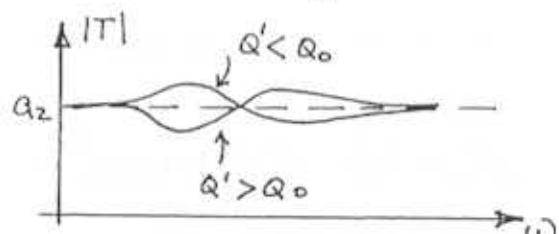
Zero  $Q <$  Pole  $Q \Rightarrow Q' < Q_0$

At  $\omega = \omega_0$ :

$$|T| = \frac{a_2 \frac{\omega_0^2/Q'}{\omega_0^2/Q_0}}{\omega_0^2/Q_0} = a_2 \frac{Q_0}{Q'} > a_2$$

If  $Q' > Q_0$

$$|T(i\omega_0)| = \frac{a_2 Q_0}{Q'} < a_2$$



12.33

$$\omega_0 = 10^4 \text{ rad/s}, \quad Q = 2, \quad R = 10 \text{ k}\Omega$$

$$\omega_0 > \frac{1}{\sqrt{LC}} \Rightarrow \omega_0^2 = \frac{1}{LC} \Leftrightarrow L = \frac{R^2 C}{Q^2} \\ = \frac{Q^2}{R^2 C^2}$$

$$C = \frac{Q}{R \omega_0} = \underline{20 \text{nF}}$$

$$L = \frac{1}{C \omega_0^2} = \underline{500 \text{mH}}$$

12.34

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

If  $L' = 1.01L$ 

$$\begin{aligned}\omega'_0 &= (1.01LC)^{-1/2} \\ &= 0.9950 \frac{1}{\sqrt{LC}} \\ &= 0.9950 \omega_0\end{aligned}$$

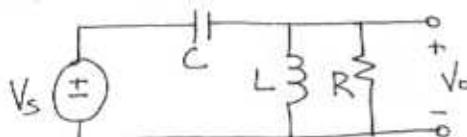
$$\therefore \underline{\Delta \omega_0 = -0.5\%}$$

If  $C' = 1.01C$ 

$$\begin{aligned}\omega'_0 &= 0.9950 \omega_0 \\ \underline{\Delta \omega'_0} &= -0.5\%\end{aligned}$$

Changing R has no effect on  $\omega_0$ 

12.35



Use voltage divider rule:

$$V_o = \frac{Z_R H L}{Z_{R H L} + Z_C} V_s$$

$$\frac{V_o}{V_s} = \frac{\left(\frac{1}{R} + \frac{1}{sL}\right)^{-1}}{\left(\frac{1}{R} + \frac{1}{sL}\right)^{-1} + \frac{1}{sC}}$$

$$= \frac{sC}{\left(\frac{1}{sC} + \frac{1}{R}\right) + sC}$$

$$\therefore T(s) = \frac{V_o(s)}{V_s(s)} = \frac{s^2}{s^2 + s/R + 1/C}$$

12.36

$$\text{Low Pass: } \omega_0 = 10^5, C = 0.1 \mu F, Q = \frac{1}{\sqrt{2}}$$

$$Q = \omega_0 CR$$

$$R = \frac{Q}{\omega_0 C}$$

$$= \frac{1}{\sqrt{2} \times 10^5 \times 0.1 \times 10^{-6}}$$

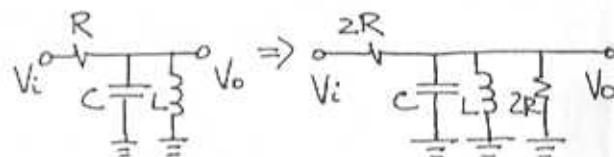
$$= \underline{70.7 \Omega}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$L = \frac{1}{\omega_0^2 C}$$

$$= \underline{\frac{1}{\omega_0^2 C}}$$

12.37



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 CR$$

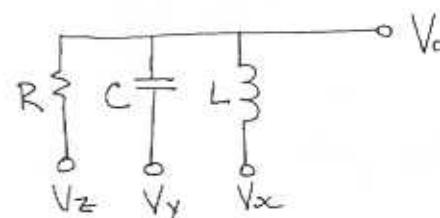
$$A_{MID} = 1$$

$$Q = \omega_0 C (2R/1/2R)$$

$$= \omega_0 CR$$

$$A_{MID} = \frac{2R}{2R+2R} = 1/2$$

12.38



$$\left. \frac{V_o}{V_s} \right|_{V_y = V_x = 0} = T_{BP}(s)$$

$$\left. \frac{V_o}{V_y} \right|_{V_x = V_z = 0} = T_{HP}(s)$$

CONT.

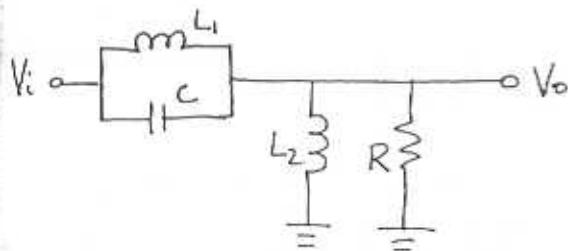
$$\frac{V_o}{V_{x_c}} \Big|_{V_y=V_z=0} = T_{LP}(s)$$

Using superposition

$$\begin{aligned} V_o &= \frac{V_o}{V_{x_c}} V_{x_c} + \frac{V_o}{V_y} V_y + \frac{V_o}{V_z} V_z \\ &= T_{LP} V_{x_c} + T_{HP} V_y + T_{BP} V_z \\ &= \frac{\frac{1}{L_1} V_{x_c}}{s^2 + s/R_C + 1/L_C} + \frac{s^2 V_y}{s^2 + s/R_C + 1/L_C} + \frac{\frac{s}{R_C} V_z}{s^2 + s/R_C + 1/L_C} \end{aligned}$$

$$\begin{aligned} \therefore V_o &= V_{x_c} \frac{1/L_1}{s^2 + s/R_C + 1/L_C} + \\ &\quad V_y \frac{s^2}{s^2 + s/R_C + 1/L_C} + \\ &\quad \underline{V_z \frac{s/R_C}{s^2 + s/R_C + 1/L_C}} \end{aligned}$$

12.39



From Eq 12.46

$$T(s) = \frac{s^2 + 1/L_1 C}{s^2 + s(1/C R) + 1/(L_1 L_2) C}$$

$$\text{Required notch } \omega_n^2 = \frac{1}{L_1 C} = (0.9\omega_0)^2$$

but:

$$\begin{aligned} \omega_0^2 &= \frac{1}{(L_1 L_2) C} \quad \text{where } L_1 || L_2 = \frac{1}{4^2 L_2} \\ &= \frac{L_1 + L_2}{4 L_2 C} \\ &= \frac{L_1 + L_2}{L_2} (0.9\omega_0)^2 \end{aligned}$$

$$1 = \left(\frac{L_1}{L_2} + 1\right) 0.9^2$$

$$\therefore L_1/L_2 = \frac{1}{0.9^2 - 1} = \underline{\underline{0.2346}}$$

For  $\omega \ll \omega_0$  :-

$$|T| \approx \frac{1/L_1 C}{1/(L_1 L_2) C} = \frac{L_2}{L_1 + L_2}$$

i.e. inductors dominate!

For  $\omega \gg \omega_0$   $L_1 \not\parallel L_2$  are "open"  
C is shorted

$$\underline{|T| \approx 1}$$

12.40

$$L = C_4 R_1 R_3 R_5 / R_2$$

$$\text{Choose } R_1 = R_2 = R_3 = R_5 = 10k\Omega$$

$$\therefore L = C_4 \times 10^8$$

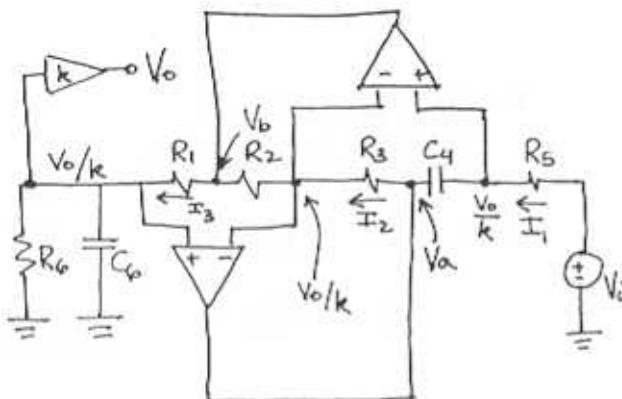
FOR:

$$L = 10H = C_4 \times 10^8 \Rightarrow C_4 = 100nF$$

$$L = 1H \Rightarrow C_4 = 10nF$$

$$L = 0.1H \Rightarrow C_4 = 1nF$$

12.41



Because of the virtual short at opamp input terminals, the voltages are:  $V_o/k$

$$I_1 = \frac{V_i - V_o/k}{R_5}$$

$$V_a = V_o/k - I_1/C_4$$

$$= V_o/k - \frac{V_i - V_o/k}{C_4 R_5}$$

$$= \frac{V_o (C_4 R_5 + 1)}{C_4 R_5 k} - k V_i$$

$$I_2 = \frac{V_a - V_o/k}{R_3}$$

$$= -\frac{V_i}{C_4 R_3 R_5} + \frac{V_o/k}{C_4 R_3 R_5}$$

$$V_b = \frac{V_o}{k} - I_2 R_2$$

$$= \frac{V_o}{k} - \frac{V_o/k - V_i}{C_4 R_3 R_5 / R_2}$$

$$I_3 = \frac{V_b - V_o/k}{R_1} = \frac{V_i - V_o/k}{C_4 R_1 R_3 R_5 / R_2}$$

Now,  $I_3$  flows only into  $R_6 \& C_6$  since for ideal opamps,  $R_{IN} = \infty$

$$\therefore \frac{V_o}{k} = I_3 (R_6 \parallel \frac{1}{C_6})$$

$$\text{Let } L \triangleq \frac{R_1 R_3 R_5 C_4}{R_2}$$

$$\text{So: } I_3 = \frac{V_i - V_o/k}{S L}$$

$$V_o/k = I_3 (R_6 \parallel \frac{1}{C_6})$$

$$\frac{V_o}{k} = \frac{V_i - V_o/k}{S L} \cdot \frac{1}{1/R_6 + 1/C_6}$$

$$= \frac{V_i - V_o/k}{S L / R_6 + S^2 L C_6}$$

$$V_o/k (1 + S L / R_6 + S^2 L C_6) = V_i$$

$$\frac{V_o}{V_i} = \frac{k / L C_6}{S^2 + \frac{S}{R_6 C_6} + \frac{1}{L C_6}}$$

Recall  $L = R_1 R_3 R_5 C_4 / R_2$

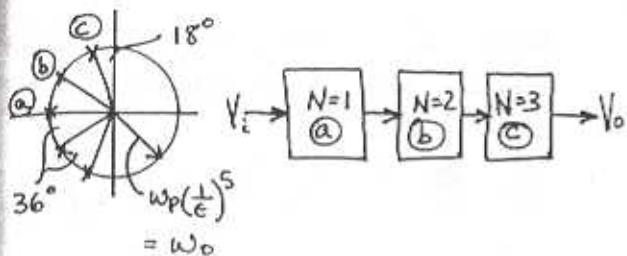
$$\therefore \frac{V_o}{V_i} = \frac{\frac{k R_2}{C_4 R_1 R_3 R_5}}{S^2 + \frac{S}{R_6 C_6} + \frac{R_2}{C_6 C_4 R_1 R_3 R_5}}$$

12.42

$$A_{max} = 10 \log(1 + e^2) = 3 \text{ dB}$$

$$\therefore G = 0.998 \approx 1$$

$$\omega_0 = \omega_p = 10^4$$



For circuit (c) use Fig 12.22 (a)

$\omega_0 = 10^4$  which is the same as for circuit (b).

$$\therefore \underline{C_4 = C_6 = 100 \text{nF}}$$

$$\underline{R_1 = R_2 = R_3 = R_5 = 1 \text{k}\Omega}$$

$$\text{Now: } Q = \frac{1}{2 \cos 72^\circ} = 1.618$$

$$R_4 = \frac{Q}{\omega_0 C_4} = \underline{1.618 \text{k}\Omega}$$

For circuit (a) use fig 12.13 (a)

$$\text{DC Gain} = 1 = \underline{R_1 / R_2} \Rightarrow \underline{R_1 = R_2 = 10 \text{k}\Omega}$$

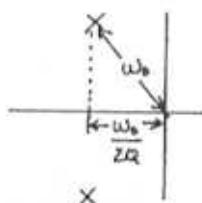
$$CR_2 = \frac{1}{\omega_0} \Rightarrow C = \frac{1}{R_2 \omega_0} = \frac{1}{10^4 \cdot 10^4} = \underline{10 \text{nF}}$$

For circuit (b) use Fig. 12.22 (a)

$$\omega_0 = 10^4 \text{ rad/s}$$

$$\frac{\omega_0}{2Q} = \omega_0 \cos 36^\circ$$

$$Q = \frac{1}{2 \cos 36^\circ} = \underline{0.618}$$



From Table 12.1

$$T(s) = \frac{\frac{kR_2}{C_4 C_6 R_1 R_3 R_5}}{s^2 + s/C_4 R_6 + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$$

$$\omega_0^2 = \frac{R_2}{C_4 C_6 R_1 R_3 R_5} \quad \text{Let } R_1 = R_3 = R_5 = R_2 = R$$

$$C_4 = C_6 = C$$

$$\omega_0^2 = \frac{1}{R^2 C^2} \quad \text{use } \underline{C_4 = C_6 = 100 \text{nF}}$$

$$\therefore R = \frac{1}{\omega_0 C} \Rightarrow \underline{R_1 = R_2 = R_3 = R_5 = 1 \text{k}\Omega}$$

Now using:

$$\frac{\omega_0}{Q} = \frac{1}{C_6 R_6} \quad \& \quad Q = 0.618$$

$$R_6 = \frac{Q}{C_6 \omega_0} = \underline{618 \Omega}$$

$$\omega_0 = 4 \text{ kHz} \quad f_N = 5 \text{ kHz} \quad Q = 10$$

now  $C_4 = 10 \text{nF}$  and  $k = 1 \equiv \text{dc gain}$

$$\omega_0 = \left[ C_4 (C_{41} + C_{42}) R_1 R_3 R_5 / R_2 \right]^{-1/2}$$

$$C_{41} + C_{42} = C_4$$

$$\text{Choose } \underline{C_4 = C_6 = 10 \text{nF}}$$

$$R_1 = R_3 = R_5 = R_2 = R$$

$$\therefore \omega_0 = (C_4 C_6 R^2)^{-1/2}$$

$$R = \frac{1}{\omega_0 C_4}$$

$$\Rightarrow \underline{R_1 = R_3 = R_5 = R_2 = 3.979 \text{k}\Omega}$$

$$\omega_n = (C_4 C_{41} R^2)^{-1/2}$$

$$C_{41} = \frac{1}{\omega_n^2 R^2 C_4} \Rightarrow \underline{C_{41} = 6.4 \text{nF}}$$

$$\& \quad \underline{C_{42} = 3.6 \text{nF}}$$

$$Q = R_6 \sqrt{\frac{C_{41} + C_{42}}{C_4} \cdot \frac{R_2}{R_1 R_3 R_5}}$$

$$= R_6 \sqrt{\frac{1}{R_1^2}} = R_6 / R_1 \Rightarrow \underline{R_6 = 39.79 \text{k}\Omega}$$

12.44

From Fig. 12.16 (g)  $\phi = 180^\circ$  at  $f_0$ !

$\therefore$  Use  $f_0 = 1\text{kHz}$   $Q = 1$

$$\omega_0^2 = \frac{R_2}{C_4 C_6 R_1 R_3 R_5} \quad \begin{matrix} \text{LET} \\ C = C_4 = C_6 = 1\text{nF} \\ R_1 = R_3, R_5 = R_2 = R \end{matrix}$$

$$= \frac{1}{C^2 R^2}$$

$$R = \sqrt{\omega_0 C} = \sqrt{15.9 \cdot 16 \text{k}\Omega} = R_1 = R_3 = R_5 = R_2$$

$$\frac{\omega_0}{Q} = \frac{1}{R_6 C_6} \Rightarrow R_6 = \frac{Q}{C_6 \omega_0}$$

$$= \frac{1}{10^{-9} 2\pi 10^3}$$

$$\therefore R_6 = 159.16 \text{k}\Omega$$

$$V_a = V_2 - I_1 R_1$$

$$= V_2 (1 + s C_6 R_1)$$

$$I_2 = (V_a - V_2) \frac{1}{R_2}$$

$$= \frac{s C_6 R_1}{R_2}$$

$$V_b = V_2 - I_2 R_3$$

$$= V_2 - \frac{s C_6 R_1 R_3}{R_2}$$

$$I_3 = (V_b - V_2) S C_4$$

$$= -\frac{s^2 C_4 C_6 R_1 R_3}{R_2} V_2$$

Now the voltage source sees an input impedance given by :

$$Z_{in} = -V_2/I_3 = \frac{R_2}{S^2 C_4 C_6 R_1 R_3}$$

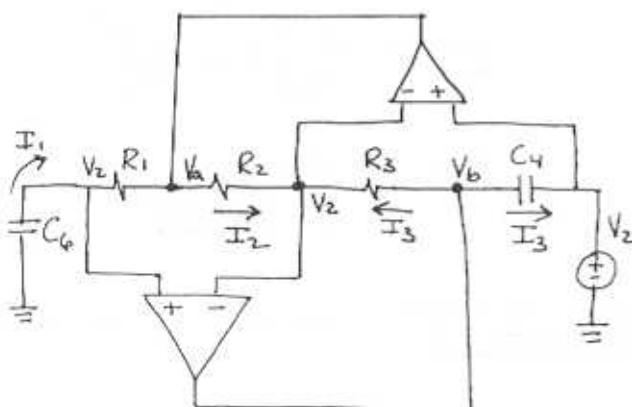
As required.

$$\text{for } s = j\omega \Rightarrow s^2 = -\omega^2$$

$$Z_{in}(j\omega) = \frac{-R_2}{C_4 C_6 R_1 R_3} \cdot \frac{1}{\omega^2}$$

$$= -R(\omega) \text{ i.e. A pure negative resistance!}$$

12.45



Because of virtual short at opamp input terminals all nodes are at  $V_2$ !

$$I_1 = -s C_6 V_2$$

Since no current goes into the opamp input terminals we have:

For LPN: See Table 12.1 & Fig. 12.22(e)

$$T(s) = k \frac{C_{61}}{C_{61} + C_{62}} \cdot \frac{\frac{R_2}{s^2 + \frac{R_2}{C_4 C_{61} R_1 R_3 R_5}}}{s^2 + s/C_6 R_6 + \frac{R_2}{C_4 (C_{61} + C_{62}) R_1 R_3 R_5}}$$

At DC  $\rightarrow s=0$

$$T(0) = k \frac{C_{61}}{C_{61} + C_{62}} \cdot \frac{\frac{R_2}{C_4 C_{61} R_1 R_3 R_5}}{\frac{R_2}{C_4 (C_{61} + C_{62}) R_1 R_3 R_5}}$$

CONT.

$$\Rightarrow T(0) = k \triangleq \text{DC Gain!}$$

Note that  $C_{G1} + C_{G2}$  is the total capacitance across  $R_G$

$$\therefore C_G = C_{G1} + C_{G2}$$

$$\frac{\omega_n^2}{\omega_0^2} = \frac{R_2/C_4 C_{G1} R_1 R_3 R_5}{R_2/C_4 (C_{G1} + C_{G2}) R_1 R_3 R_5}$$

$$= \frac{C_{G1}}{C_{G1} + C_{G2}}$$

$$\frac{\omega_n^2}{\omega_0^2} = \frac{C_{G1}}{C_G}$$

$$\therefore C_{G1} = C_G \left( \frac{\omega_n}{\omega_0} \right)^2 = C \left( \frac{\omega_n}{\omega_0} \right)^2$$

Clearly from  $T(s)$  above :

$$\omega_n^2 = R_2/C_4 C_{G1} R_1 R_3 R_5$$

$$\Rightarrow \omega_n = \sqrt{\frac{R_2}{C_4 C_{G1} R_1 R_3 R_5}}$$

$$\omega_0^2 = R_2/C_4 C_G R_1 R_3 R_5$$

$$\Rightarrow \omega_0 = \sqrt{\frac{R_2}{C_4 (C_{G1} + C_{G2}) R_1 R_3 R_5}}$$

12.47

For HPN : See Table 12.1 & Fig 11.22(f)

$$T(s) = k \frac{s^2 + \left( R_2/C_4 C_{G1} R_1 R_3 R_5 \right)}{s^2 + \frac{s/C_{G1}}{C_4 R_4} + \left( \frac{R_2}{C_4 C_{G1} R_1 R_3} \right) \left( \frac{1}{R_{S1}} + \frac{1}{R_{S2}} \right)}$$

$$\text{clearly: } \omega_n = \sqrt{\frac{R_2}{C_4 C_{G1} R_1 R_3 R_{S1}}}$$

$$\omega_0 = \sqrt{\frac{R_2}{C_4 C_{G1} R_1 R_3} \left( \frac{1}{R_{S1}} + \frac{1}{R_{S2}} \right)}$$

At high frequencies  $s \rightarrow \infty$

$$T(\infty) = k \triangleq \text{high freq gain.}$$

Observe that the equivalent resistance at the non terminal of  $A_1$  is :

$$\frac{1}{R_S} = \frac{1}{R_{S1}} + \frac{1}{R_{BL}} \quad \text{AND}$$

for the resonator (table 12.1)

$$R_S = 1/\omega_0 C \Rightarrow \frac{1}{R_S} = \omega_0 C$$

$$\frac{\omega_0^2}{\omega_n^2} = \frac{R_2/C_4 C_{G1} R_1 R_3 R_5}{R_2/C_4 C_G R_1 R_3 R_{S1}} \Rightarrow R_{S1} = R_S \frac{\omega_0^2}{\omega_n^2}$$

$$\text{Now } \frac{1}{R_S} = \frac{1}{R_S \frac{\omega_0^2}{\omega_n^2}} + \frac{1}{R_{S2}}$$

$$\frac{1}{R_{S2}} = \frac{1}{R_S} \left[ 1 - \frac{\omega_n^2}{\omega_0^2} \right]$$

$$R_{S2} = \frac{R_S}{1 - \omega_n^2/\omega_0^2}$$

12.48

$$T(s) = \frac{0.4508 (s^2 + 1.6996)}{(s + 0.7294)(s^2 + 0.2786s + 1.0504)}$$

PART (a) Replace  $s$  with  $s/w_p$

$$T(s) = \frac{0.4508 \left( \frac{s^2}{w_p^2} + 1.6996 \right)}{\left( \frac{s}{w_p} + 0.7294 \right) \left[ \frac{s^2}{w_p^2} + \frac{0.2786s}{w_p} + 1.0504 \right]}$$

CONT.

$$T(s) = \frac{0.4508 w_p (s^2 + 1.6996 w_p^2)}{(s + 0.7294 w_p)(s^2 + 0.2786 w_p s + 1.0504 w_p^2)}$$

Sub  $w_p = 10^4$  rad/s

$$T(s) = \frac{4508 (s^2 + 1.6996 \times 10^8)}{(s + 7294)(s^2 + 2786s + 1.0504 \times 10^8)}$$

Part (b)

First decompose  $T(s)$  into 1st-and 2nd-order sections with unity DC gain!

$$T_1(s) = \frac{k_1}{s + 7294} \quad T_1(0) = \frac{k_1}{7294} = 1$$

$$\Rightarrow \underline{\underline{k_1 = 7294}}$$

$$\text{Now } k_1 k_2 = 4508 \Rightarrow \underline{\underline{k_2 = 0.6180}}$$

$$\therefore T_2(s) = \frac{0.6180 (s^2 + 1.6996 \times 10^8)}{s^2 + 2786s + 1.0504 \times 10^8}$$

As a check:

$$T_2(0) = \frac{0.6180 (1.6996 \times 10^8)}{1.0504 \times 10^8} = 1.000$$

As EXPECTED!

$$\therefore T(s) = T_1(s) \cdot T_2(s)$$

For first-order section use Fig 12.13(a)

$$w_0 = 7294 \text{ rad/s} \quad \text{DC Gain} = 1$$

$$\text{let } C = 10 \text{nF}$$

$$R_1 = R_2 = \frac{1}{w_0 C} \Rightarrow \underline{\underline{R_1 = R_2 = 13.71 \text{k}\Omega}}$$

For second-order section

$$w_n^2 = 1.6996 \times 10^8 \Rightarrow w_n = 13.037 \times 10^3$$

$$w_0^2 = 1.0504 \times 10^8 \Rightarrow w_0 = 10.249 \times 10^3$$

$$\frac{w_0}{Q} = 2786 \Rightarrow Q = 3.6787$$

For LPN use table 12.1 and Fig 12.22(e)

Make  $R_1 = R_2 = R_3 = R_B = R$  and

$$C = \underline{\underline{C_4 = C_6 = 10 \text{nF}}}$$

$$R = \frac{1}{w_0 C} = \underline{\underline{9.757 \text{k}\Omega = R_1 = R_2 = R_3 = R_S}}$$

$$w_n^2 = \frac{1}{C R^2 C_{G1}} \Rightarrow \underline{\underline{C_{G1} = 6.18 \text{nF}}}$$

$$C_{G2} = C - C_{G1} \\ = \underline{\underline{3.82 \text{nF}}}$$

$$Q = R_S \sqrt{\frac{C_6}{C_4} \cdot \frac{R_2}{R_1 R_3 R_S}} = \frac{R_S}{\sqrt{R^2}}$$

$$= \frac{R_S}{R}$$

$$\therefore R_S = R Q = (9.757 \times 10^3) (3.6787) \\ = \underline{\underline{35.89 \text{k}\Omega}}$$

12.49

$$f_0 = 1 \text{ kHz}$$

The 3dB bandwidth for a 2<sup>nd</sup> order filter is given by:

$$B = \frac{w_0}{Q} \Rightarrow Q = \frac{2\pi 10^3}{2\pi 50} = \underline{\underline{20}}$$

Choose  $C = 10 \text{nF}$

$$R = \frac{1}{w_0 C} = \underline{\underline{15.92 \text{k}\Omega}}$$

$$\text{use } \underline{\underline{R_1 = R_F = 10 \text{k}\Omega}}$$

CONT.

$$\frac{R_3}{R_L} = 2Q - 1 = 39$$

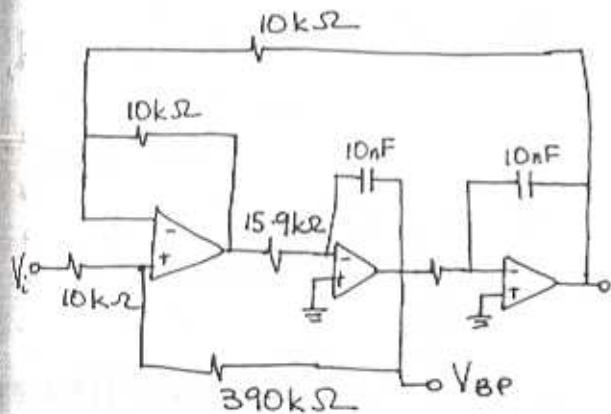
choose  $R_L = 10k\Omega$   $R_3 = 390k\Omega$

$$\text{Now } T(s) = \frac{-k\omega_0 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\Rightarrow |T(j\omega_0)| = \frac{k\omega_0^2}{\omega_0^2/Q} = kQ$$

$$\text{but } k = 2 - Y_Q = 1.95$$

$$\therefore \text{Centre-freq gain} = kQ = 39$$



12.50

$$R_L = R_H = R_B/Q \Rightarrow R_B = Q R_H$$

$$R_L = R_H$$

Using Eq 12.66 :

$$\begin{aligned} \frac{V_o}{V_i} &= -k \frac{\frac{R_F}{R_H} s^2 - s \left( \frac{R_F}{R_B} \right) \omega_0 + \left( \frac{R_F}{R_L} \right) \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \\ &= -k \frac{\frac{R_F}{R_H}}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \end{aligned}$$

$$\text{Flat Gain} = -k \frac{R_F}{R_H}$$

Part (b) -  $\omega_0 = 10^4 \text{ rad/s}$   $Q = 2$  Flat Gain = 10

$$\text{choose } C = 10nF \Rightarrow R = \frac{1}{\omega_0 C} = 10k\Omega$$

$$\text{choose } R_F = R_L = 10k\Omega$$

$$\frac{R_3}{R_2} = 2Q - 1 = 3 \Rightarrow \frac{R_2}{R_3} = 10k\Omega$$

$$\text{Now } k = 2 - Y_Q = 1.5$$

$$\therefore \text{Flat Gain} = 10 = (1.5) \frac{R_F}{R_H}$$

$$\therefore \frac{R_H}{R_F} = 0.15$$

$$\text{choose } R_F = 100k\Omega$$

$$R_H = R_L = 15k\Omega$$

$$R_B = Q R_H = 30k\Omega$$

12.51

Note  $\omega_n$  does not depend on  $R$  or  $C$   
From eq. 12.67 :

$$\frac{R_H}{R_L} = \left( \frac{\omega_n}{\omega_0} \right)^2$$

$$\therefore \omega_n = \omega_0 \sqrt{\frac{R_H}{R_L}} \quad \text{Nominally } R_H = R_L \pm 1\%$$

Thus :

$$\begin{aligned} \omega_n' &= \omega_0 \sqrt{\frac{1.01}{0.99}} & \omega_n'' &= \omega_0 \sqrt{\frac{0.99}{1.01}} \\ &= 1.01 \omega_0 & &= 0.99 \omega_0 \end{aligned}$$

$\therefore \omega_n$  can deviate from  $\omega_0$  by  $\pm 1\%$

12.52

Use Tow Thomas to realize a LPN  
(Fig 12.26)

$$\omega_0 = 10^4 \quad \omega_n = 1.2\omega_0 \quad Q = 10 \quad DC\text{Gain} = 1$$

$$C = 10\text{nF} \quad r = 20\text{k}\Omega$$

$$R = \frac{1}{\omega_0 C} = \underline{\underline{10\text{k}\Omega}}$$

from PB 16(e):

$$DC\text{ Gain} = \alpha_2 \frac{\omega_n^2}{\omega_0^2} = 1$$

$$\alpha_2 \frac{1.2^2 \omega_0^2}{\omega_0^2} = 1$$

$$\alpha_2 = \frac{1}{1.2^2} = HF\text{. Gain}$$

$$C_1 = C \alpha_2 = \frac{10 \times 10^{-9}}{1.2^2} = \underline{\underline{6.94\text{nF}}}$$

$$R_2 = \frac{R \left( \frac{\omega_0}{\omega_n} \right)^2}{HF\text{ Gain}} = R \left( \frac{1}{1.2} \right)^2 \times (1.2)^2$$

$$= R = \underline{\underline{10\text{k}\Omega}}$$

$$\underline{\underline{R_1 = R_3 = \infty}}$$

12.53

For all pass:

$$T(s) = - \frac{s^2 \left( \frac{C_1}{C} \right) + s \frac{1}{C} \left( \frac{1}{R_1} - \frac{r}{RR_3} \right) + \frac{1}{C^2 RR_2}}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_z^2 = \frac{1}{C^2 RR_2} \cdot \frac{C}{C_1} \Rightarrow \omega_z = \frac{1}{C \sqrt{RR_2}} \cdot \underline{\underline{\sqrt{\frac{C}{C_1}}}}$$

$$Q_Z = \frac{\omega_z}{\frac{1}{C} \left( \frac{1}{R_1} - \frac{r}{RR_3} \right) \frac{C}{C_1}}$$

$$Q_Z = \frac{\frac{1}{C^2 RR_2} \frac{C}{C_1}}{\frac{1}{C} \left( \frac{1}{R_1} - \frac{r}{RR_3} \right) \left( \frac{C}{C_1} \right)}$$

$$= \frac{1}{\sqrt{RR_2} \left( \frac{1}{R_1} - \frac{r}{RR_3} \right) \sqrt{\frac{C}{C_1}}}$$

For All Pass  $R_1 \rightarrow \infty$

To adjust  $Q_Z$ , trim  $r$  or  $R_2$   
(independent of  $\omega_z$ !)

Now  $\omega_0 = \frac{1}{CR}$  so do not trim  $R$  or  $C$ !

Note if we trim  $R_2$  or  $C_1$  to adjust  $\omega_z$ ,  
this will also affect  $Q_Z$ . So the  
options are:

For  $\omega_z$ : (a) trim  $R_2$  AND ( $r$  or  $R_3$ ) to  
maintain the value of  $Q_Z$   
OR

(b) trim  $C_1$ , and  $r$  or  $R_3$

Prefer not to trim a capacitor so  
use (a)!

12.54

$$T(s) = \frac{0.4508 (s^2 + 1.6996)}{(s + 0.7294)(s^2 + 0.2786s + 1.0504)}$$

Part (a) Replace  $s$  with  $s/w_p$   
 $w_p = 10^4 \text{ rad/s}$ .

$$T(s) = \frac{0.4508 (s^2/w_p^2 + 1.6996)}{\left( \frac{s}{w_p} + 0.7294 \right) \left( \frac{s^2}{w_p^2} + \frac{0.2786s}{w_p} + 1.0504 \right)}$$

CONT.

$$T(s) = \frac{0.4608 w_p (s^2 + 1.6996 w_p^2)}{(s + 0.7294 w_p)(s^2 + 0.2786 w_p s + 1.0504 w_p^2)}$$

$$= \frac{4508 (s^2 + 1.6996 \times 10^8)}{(s + 72.94)(s^2 + 2786s + 1.0504 \times 10^8)}$$

For FIRST ORDER SECTION use Fig 12.13(g)

$$\omega_0 = 72.94 \quad \text{Dc gain} = 1$$

$$\text{choose } C = 10\text{nF}$$

$$R_1 = R_2 = \frac{1}{\omega_0 C} \Rightarrow R_1 = R_2 = 13.71\text{k}\Omega$$

FOR SECOND ORDER SECTION - use Fig 12.26

$$\omega_n^2 = 1.6996 \times 10^8 \Rightarrow \omega_n = 13.037 \times 10^3$$

$$\omega_0^2 = 1.0504 \times 10^8 \Rightarrow \omega_0 = 10.249 \times 10^3$$

$$\frac{\omega_0}{Q} = 2786 \Rightarrow Q = 3.6787$$

$$\text{Dc gain} = 1$$

For Tow Thomas LPN use table 12.2  
choose  $C = 10\text{nF}$

$$R = \frac{1}{\omega_0 C} = \frac{1}{10.249 \times 10^3 \times 10 \times 10^{-9}}$$

$$= 9.757\text{k}\Omega$$

$$\text{choose } r = 20\text{k}\Omega$$

Now from Fig 12.16 (e):

$$T(0) = a_2 \frac{\omega_n^2}{\omega_0^2} = 1 \Rightarrow a_2 = \frac{\omega_0^2}{\omega_n^2} = 0.618$$

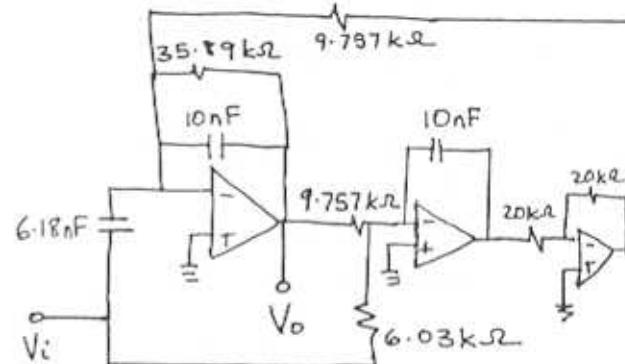
$$\therefore \text{HF gain} = a_2 = 0.618$$

$$C_1 = C \times \text{HF gain} \Rightarrow C_1 = 6.18\text{nF}$$

$$R_2 = R \left( \frac{\omega_0}{\omega_n} \right)^2$$

$$R_2 = 0.618 R \Rightarrow R_2 = 6.03\text{k}\Omega$$

$$R_1 = R_3 = \infty \quad Q.R = 35.89\text{k}\Omega$$



12.55

$$\text{Make } C_1 = C_2 = 1\text{nF} = C$$

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}} \quad Q = \left[ \frac{\sqrt{C_1 C_2 R_3 R_4}}{R_3} \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \right]$$

$$\text{Let } R_3 = R \quad m = 4Q^2$$

$$R_4 = \frac{R}{m} \quad = 4/2 = 2.$$

$$\therefore \omega_0 = \frac{1}{\sqrt{C^2 R^2 / 2}} = \frac{\sqrt{2}}{RC} = 10^4$$

$$\Rightarrow R = \frac{\sqrt{2}}{10^4 \times 10^{-9}} = 141.42\text{k}\Omega$$

$$R_3 = 141.4\text{k}\Omega$$

$$R_4 = \frac{R_3}{2} \Rightarrow R_4 = 70.7\text{k}\Omega$$

12.56

For Fig 12.2B(a)

$$t(s) = \frac{s^2 + s \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}}{s^2 + s \left( \frac{1}{C_1 R_3} + \frac{1}{C_1 R_4} + \frac{1}{C_2 R_3} \right) + \frac{1}{C_1 C_2 R_3 R_4}}$$

But  $C_1 = C_2 = C$  &  $R_3 = R_4 = R$ ,  $RC = \tau$ 

$$\begin{aligned} \therefore t(s) &= \frac{s^2 + s^2/RC + 1/R^2 C^2}{s^2 + s^3/RC + 1/R^2 C^2} \\ &= \frac{s^2 + s^2/\tau + 1/\tau^2}{s^2 + s^3/\tau + 1/\tau^2} \end{aligned}$$

Zeros defined by  $\omega_z = 1/\tau$   
 $Q_z = \frac{1}{2}$

 $\Rightarrow$  Double Root at  $s = -1/\tau$ Poles of  $t(s)$  are given by the quadratic formula:

$$s = \frac{-3}{2\tau} \pm \frac{\sqrt{5}}{2\tau} = \frac{-3 \pm \sqrt{5}}{2\tau}$$

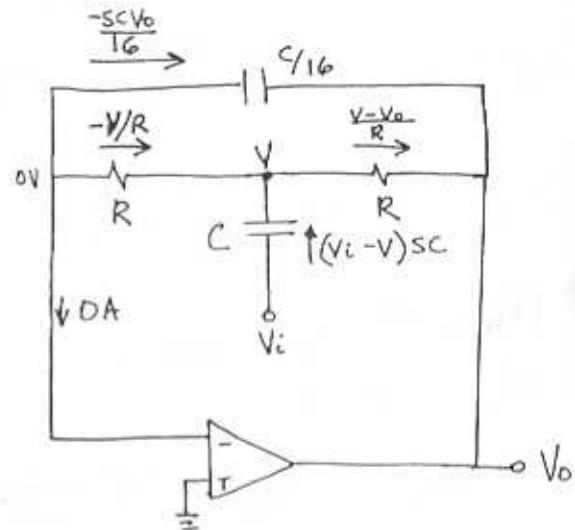
i.e. two roots on the negative real axis

If the network is placed in the negative feedback path of an ideal amplifier ( $A = \infty$ ) then the poles are given by the zeros of  $t(s)$ :

Closed loop poles:

$$s = -1/\tau \quad (\text{multiplicity } 2)$$

12.57



Note first  $\frac{-SCV_0}{16} = -\frac{V}{R}$   
 $V = -\frac{SCRV_0}{16}$

 $\therefore I$  at  $V$ 

$$-\frac{V}{R} + SC(V_i - V) - \frac{V - V_0}{R} = 0$$

$$\begin{aligned} \frac{SCV_0 R}{R 16} + SCV_i + S^2 \frac{C^2 R V_0}{16} + \frac{SCV_0}{16} \\ + \frac{V_0}{R} = 0 \end{aligned}$$

mult by:  $16R$  and let  $RC = \tau$ 

$$\tau V_0 + 16\tau V_i s + s^2 \tau^2 V_0 + SCV_0 + 16V_0 = 0$$

$$V_0 [s^2 \tau^2 + s \cdot 2\tau + 16] = -16\tau V_i$$

$$\therefore \frac{V_0}{V_i} = -\frac{16\tau}{s^2 \tau^2 + 2\tau s + 16}$$

$$\therefore T(s) = \frac{s^{16}/RC}{s^2 + s^2/RC + 16/RC^2}$$

CONT.

$$\text{Let } \omega_0^2 = \frac{16}{(RC)^2} \Rightarrow \omega_0 = \frac{4}{RC}$$

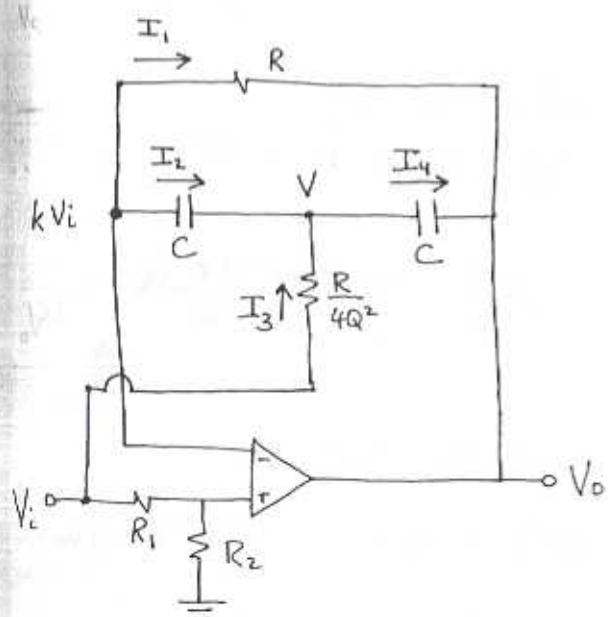
$$\frac{\omega_0}{Q} = \frac{2}{RC} \Rightarrow Q = \frac{RC\omega_0}{2} = \underline{\underline{2}}$$

$$\therefore \frac{V_o}{V_i} = \frac{-4\omega_0 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\left. \begin{array}{l} |T|_{s=0} = 0 \\ |T|_{s=\infty} = 0 \end{array} \right\} \text{Bandpass}$$

$$|T(j\omega_0)| = 4/V_2 = \underline{\underline{8 \frac{\text{CENTRE FREQ}}{\text{GAIN}}}}$$

12.58



$$RC = 2Q/\omega_0$$

$$k = \frac{R_2}{R_1 + R_2}$$

$$V_{+ve} = V_{-ve} = kV_i \quad \text{due to virtual short}$$

$$I_1 = -I_2$$

$$\frac{kV_i - V_0}{R} = \frac{V - kV_i}{1} (\text{sc})$$

$$V = \frac{1}{\text{sc}R} (kV_i - V_0 + \text{sc}RkV_i)$$

$$\sum I \text{ at } V = 0$$

$$I_2 + I_3 - I_4 = 0$$

$$\text{sc}(kV_i - V) + \frac{4Q^2}{R} (V_i - V) - \text{sc}(V - V_0) = 0$$

$$\text{sc}\left(kV_i - \frac{kV_i}{\text{sc}R} + \frac{V_0}{\text{sc}R} - kV_i\right)$$

$$+ \frac{4Q^2}{R} \left(V_i - \frac{kV_i}{\text{sc}R} + \frac{V_0}{\text{sc}R} - kV_i\right)$$

$$- \text{sc}\left(\frac{kV_i}{\text{sc}R} - \frac{V_0}{\text{sc}R} + kV_i - V_0\right) = 0$$

$\Rightarrow$

$$-\frac{kV_i}{R} + \frac{V_0}{R}$$

$$+ \frac{4Q^2}{R} \left(V_i - \frac{kV_i}{\text{sc}R} + \frac{V_0}{\text{sc}R} - kV_i\right)$$

$$- \frac{\text{sc}}{R} \left(\frac{kV_i}{\text{sc}} - \frac{V_0}{\text{sc}} + kRV_i - V_0 R\right) = 0$$

$$\Rightarrow \text{SUB CR} = \frac{2Q}{\omega_0} \quad \& \quad R = \frac{2Q}{C\omega_0}$$

$$-kV_i + V_0 + 4Q^2V_i - \frac{2Q^2kV_i \omega_0}{S^2R} + \frac{V_0 \omega_0^2}{S^2R} = 0$$

$$-kV_i + V_0 - \frac{sk2Q}{\omega_0} V_i + \frac{sv_0}{\omega_0} = 0$$

$$V_o \left[ 1 + \frac{2Q\omega_0}{s} + 1 + \frac{2QS}{\omega_0} \right]$$

$$= V_i \left[ k - 4Q^2 + \frac{2kQ\omega_0}{s} + 4Q^2k + k + \frac{2kQS}{\omega_0} \right]$$

$$\Rightarrow V_o \left[ s^2 \frac{2Q}{\omega_0} + 2s + 2Q\omega_0 \right] = V_i \left[ s^2 \frac{2kQ}{\omega_0} + \right.$$

$$\left. s(4Q^2k - 4Q^2 + 2k) + 2kQS \right]$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{s^2 \frac{2kQ}{\omega_0} + s(4Q^2k - 4Q^2 + 2k) + 2kQS}{s^2 + \frac{2Q}{\omega_0} + 2s + 2Q\omega_0}$$

$$= k \frac{s^2 + s \frac{\omega_0}{Q} \left( 2Q^2 - \frac{2Q^2}{k} + 1 \right) + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\text{Recall } k = \frac{R_2}{R_1 + R_2} \text{ and } \frac{1}{k} = 1 + \frac{R_1}{R_2}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{\left( \frac{R_2}{R_1 + R_2} \right) s^2 + s \frac{\omega_0}{Q} \left( 1 - \frac{R_1}{R_2} \cdot 2Q^2 \right) + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\therefore T(s) = \frac{R_2}{R_1 + R_2} \frac{s^2 + s \frac{\omega_0}{Q} \left( 1 - \frac{2Q^2 R_1}{R_2} \right) + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

For All Pass

$$\text{we want } T(s) \propto \frac{s^2 + \frac{\omega_0}{Q} (-i)s + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$\Rightarrow 1 - \frac{2Q^2 R_1}{R_2} = -1$$

$$2Q^2 \frac{R_1}{R_2} = 2$$

$$\frac{R_1}{R_2} = \frac{1}{Q^2}$$

$$\therefore \frac{R_2}{R_1} = Q^2 \quad \frac{R_2}{R_1 + R_2} = \frac{R_2 / R_1}{1 + R_2 / R_1} = \frac{Q^2}{1 + Q^2}$$

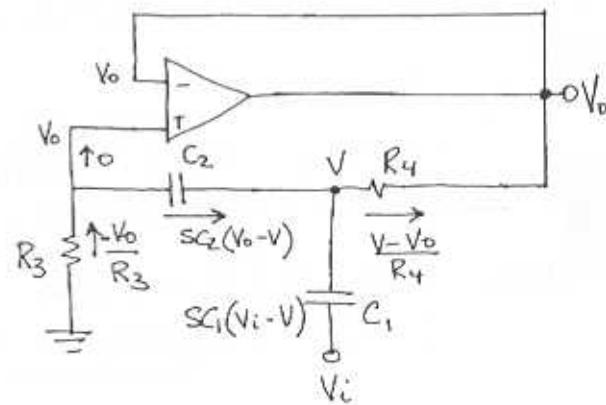
For Notch:

$$1 - 2Q^2 \frac{R_1}{R_2} = 0$$

$$\frac{R_1}{R_2} = \frac{1}{2Q^2}$$

$$\frac{R_2}{R_1} = 2Q^2 \quad \frac{R_2}{R_1 + R_2} = \frac{2Q^2}{1 + 2Q^2}$$

12.59



°° No current can't flow into the terminal

$$-\frac{V_o}{R_3} = SC_2(V_o - V)$$

$$V = V_o \left( 1 + \frac{1}{SC_2 R_3} \right)$$

CONT.

$$\sum I @ V = 0$$

$$-\frac{V_o}{R_3} + \frac{V_i - V}{1} S C_1 = \frac{V - V_o}{R_4}$$

$$V_o \left[ -\frac{1}{R_3} + \frac{1}{R_4} \right] + V \left[ -S C_1 - \frac{1}{R_4} \right] = -S C_1 V_i$$

$$V_o [R_4 - R_3] + V [S C_1 R_3 R_4 + R_3] = \sqrt{S C_1 R_3 R_4} V_i$$

$$V_o (R_4 - R_3) + V_o \left( 1 + \frac{1}{S C_2 R_3} \right) (S C_1 R_3 R_4 + R_3) \\ = S C_1 R_3 R_4 V_i$$

$$V_o \left( R_4 - R_3 + S C_1 R_3 R_4 + R_3 + \frac{C_1}{C_2} R_4 + \frac{1}{S C_2} \right) \\ = S C_1 R_3 R_4 V_i$$

$$V_o \left( S^2 C_1 C_2 R_3 R_4 + S C_1 R_4 + S C_2 R_4 + 1 \right) \\ = S^2 C_1 R_3 R_4 C_2 V_i$$

$$\therefore \frac{V_o}{V_i} = \frac{S^2 C_1 C_2 R_3 R_4}{S^2 C_1 C_2 R_3 R_4 + S C_1 R_4 + S C_2 R_4 + 1} \\ = \frac{S^2}{S^2 + S \left( \frac{1}{C_2 R_3} + \frac{1}{C_1 R_3} \right) + \frac{1}{C_1 C_2 R_3 R_4}}$$

Note  $|T(0)| = 0$  }  $\therefore$  High Pass  
 $|T(\infty)| = 1$  } High Freq Gain =  $1 \frac{V_o}{V_i}$

$$3\text{dB freq} = 10^3 \text{ rad/s}, Q = \frac{1}{\sqrt{2}} \text{ for max flat.}$$

$$\therefore \omega_o = 10^3 \frac{\text{rad}}{\text{s}} \quad C_1 = C_2 = 10 \text{nF}$$

$$\text{clearly } \omega_o^2 = \frac{1}{C_1 C_2 R_3 R_4} \quad \text{and}$$

$$\frac{\omega_o}{Q} = \frac{1}{C_2 R_3} + \frac{1}{C_1 R_3} = \frac{C_1 + C_2}{C_1 C_2 R_3}$$

$$= \frac{2C}{C^2 R_3} = \frac{2}{C R_3} = \sqrt{2} \times 10^3$$

$$R_3 = \frac{2}{10 \times 10^{-9} \times 10^3 \times \sqrt{2}}$$

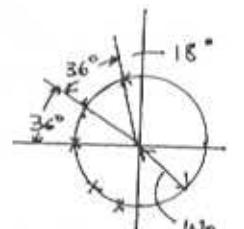
$$\underline{R_3 = 141.4 \text{ k}\Omega}$$

$$R_4 = \frac{1}{\omega_o^2 C_1 C_2 R_3} \Rightarrow \underline{R_4 = 70.7 \text{ k}\Omega}$$

12.60

$$A_{\max} = 3 \text{ dB}$$

$$\epsilon = \left( 10^{\frac{3}{10}} - 1 \right)^{-\frac{1}{2}} \approx 1$$



$$\omega_o = \omega_p \left( \frac{1}{\epsilon} \right)^{1/2} = \omega_p = 2\pi 5000 = 10^4 \pi$$

$$Q_1 = \frac{1}{2 \cos 36^\circ} = 0.618$$

$$Q_2 = \frac{1}{2 \cos 72^\circ} =$$

For first order section:

$$\omega_o = 10^4 \pi \quad \text{dc gain} = 1$$

From 12.13 (a)

$$\underline{R_1 = R_2 = 10 \text{ k}\Omega}$$

$$C = \frac{1}{\omega_o R_2} = \frac{1}{10^4 \pi \times 10^4} = \underline{3.18 \text{ nF}}$$

Second-Order Section  $Q = 0.618$ :

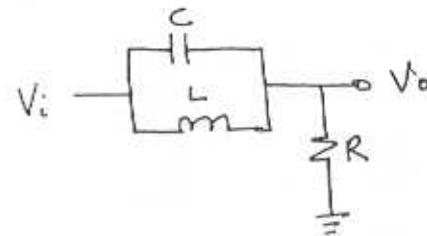
$$\text{from 12.34(c): } m = 4Q^2 = 1.528$$

CONT.

$$RC = \frac{2Q}{\omega_0} \quad \text{let } R_1 = R_2 = 10k\Omega$$

$$C = \frac{2Q}{\omega_0 R} \Rightarrow C_4 = C = 3.93nF$$

$$\underline{C_3 = \frac{C}{m} = 2.57nF}$$



this is the same as Fig 12.18(e)

Second Order Section  $Q = 1.618$ :

$$C = \frac{2Q}{\omega_0 R} \quad m = 4Q^2 = 10.47^2$$

$$= 10.3nF \Rightarrow R_1 = R_2 = 10k\Omega$$

$$\underline{C_4 = C = 10.3nF}$$

$$\underline{C_3 = \frac{C}{m} = 0.984nF}$$

12.62

for Fig 12.18(d):

$$T(s) = \frac{s/RC}{s^2 + s/RC + 1/C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad Q = R \sqrt{\frac{C}{L}}$$

12.61

For a bandpass filter

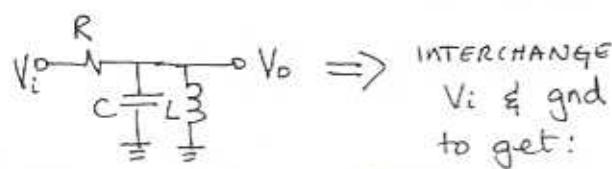
$$T(s) = \frac{\omega_0/Qs}{s^2 + s\omega_0/Q + \omega_0^2}$$

centre freq. gain = 1

complementary transfer function:

$$\begin{aligned} z^2 &= 1 - \frac{1}{z} \\ &= \frac{s^2 + \omega_0^2}{s^2 + s\omega_0/Q + \omega_0^2} \equiv \text{NOTCH!} \end{aligned}$$

From Fig 12.18(d)



FOR  $\omega_0$

$$\frac{\partial \omega_0}{\partial L} = \frac{\partial (LC)^{-1/2}}{\partial L} = -\frac{1}{2} L^{-3/2} C^{-1/2} = -\frac{\omega_0}{2L}$$

$$\frac{\partial \omega_0}{\partial C} = -\frac{\omega_0}{2C}$$

$$\frac{\partial \omega_0}{\partial R} = 0$$

$$\therefore S_L^{\omega_0} = \frac{\partial \omega_0}{\partial L} \frac{L}{\omega_0} = -\frac{1}{2}$$

$$S_C^{\omega_0} = \frac{\partial \omega_0}{\partial C} \times \frac{C}{\omega_0} = -\frac{1}{2}$$

$$S_R^{\omega_0} = \frac{\partial \omega_0}{\partial R} \frac{R}{\omega_0} = 0$$

FOR Q

$$\frac{\partial Q}{\partial L} = \frac{R\sqrt{C}}{L\sqrt{L}} \left( -\frac{1}{2} \right)$$

$$\frac{\partial Q}{\partial C} = \frac{1}{2} \frac{R}{\sqrt{LC}} = \frac{1}{2} \frac{R\sqrt{C}}{C\sqrt{L}} = \frac{Q}{2C}$$

$$\frac{\partial Q}{\partial R} = \sqrt{C/L} = \frac{R}{R} \sqrt{Q_L} = Q/R$$

CONT.

$$S_L^Q = \frac{-Q}{2L} \times \frac{L}{Q} = -\frac{1}{2}$$

$$S_C^Q = \frac{Q}{2C} \times \frac{C}{Q} = \frac{1}{2}$$

$$S_R^Q = \frac{Q \cdot R}{R \cdot Q} = 1$$

12.63

$$y = uv$$

$$\begin{aligned} S_x^y &= \frac{\partial(uv)}{\partial x} \frac{x}{uv} \\ &= v \frac{\partial u}{\partial x} \frac{x}{uv} + u \frac{\partial v}{\partial x} \frac{x}{uv} \\ &= \frac{\partial u}{\partial x} \frac{x}{u} + \frac{\partial v}{\partial x} \frac{x}{v} \\ &= \underline{\underline{S_x^u}} + \underline{\underline{S_x^v}} \end{aligned}$$

$$\text{Part (b)} \quad y = u/v$$

$$\begin{aligned} S_x^y &= \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\partial(u/v)}{\partial x} \frac{xv}{u} \\ &= \frac{1}{v} \frac{\partial u}{\partial x} \frac{vx}{u} + -\frac{u}{v^2} \frac{\partial v}{\partial x} \cdot \frac{xv}{u} \\ &= \frac{\partial u}{\partial x} \frac{x}{u} - \frac{\partial v}{\partial x} \frac{x}{v} \\ &= \underline{\underline{S_x^u}} - \underline{\underline{S_x^v}} \end{aligned}$$

$$\text{Part (c)} \quad y = ku$$

$$\begin{aligned} S_x^y &= \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\partial(ku)}{\partial x} \frac{x}{ku} \\ &= k \frac{\partial u}{\partial x} \frac{x}{ku} \\ &= \frac{\partial u}{\partial x} \frac{x}{u} \\ &= \underline{\underline{S_x^u}} \end{aligned}$$

$$\text{Part (d)} \quad y = u^n$$

$$\begin{aligned} S_x^y &= \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\partial(u^n)}{\partial x} \frac{x}{u^n} \\ &= n u^{n-1} \frac{\partial u}{\partial x} \cdot \frac{x}{u^n} \\ &= n \frac{\partial u}{\partial x} \frac{x}{u} = \underline{\underline{n S_x^u}} \end{aligned}$$

$$\text{Part (e)} \quad y = f_1(u) \quad u = f_2(x)$$

$$\begin{aligned} S_x^y &= \frac{\partial y}{\partial x} \frac{x}{y} = -\frac{\partial f_1(u)}{\partial x} \frac{x}{f_1(u)} \\ &= \frac{\partial f_1(u)}{\partial x} \frac{\partial u}{\partial x} \cdot \frac{x}{f_1(u)} \\ &= \frac{\partial f_1}{\partial u} \cdot \frac{\partial f_2}{\partial x} \cdot \frac{\partial u}{\partial x} \cdot \frac{u}{f_1} \end{aligned}$$

$$\text{But } u = f_2$$

$$\begin{aligned} \therefore S_x^y &= \frac{\partial f_1}{\partial u} \cdot \frac{\partial f_2}{\partial x} \cdot \frac{x}{f_1} \cdot \frac{u}{f_2} \\ &= \frac{\partial f_1}{\partial u} \cdot \frac{u}{f_1} \cdot \frac{\partial f_2}{\partial x} \frac{x}{f_2} \\ &= \underline{\underline{S_u^{f_1} \cdot S_x^{f_2}}} \\ &= \underline{\underline{S_u^u S_x^u}} \end{aligned}$$

12.64

Since the characteristic equation of Fig. 12.33(b) is the same as for Fig. 12.29 the poles are given by Eq 12.86:

$$s^2 + s \frac{\omega_0}{Q} \left[ 1 + \frac{2Q^2}{A+1} \right] + \omega_0^2 = 0$$

This is because 12.33(b) is based on the complementary transform of 12.29 and hence the pole

locations are preserved!

Now the actual  $\omega_0$  and  $Q$  are given by:

$$\omega_{0,a} = \omega_0 \quad \text{and} \quad Q_a = \frac{Q}{1 + \frac{2Q^2}{(A+1)}}$$

Thus from Ex. 12.3

$$S_A^{\omega_0,a} = \underline{\underline{0}}$$

$$S_A^Q_a = \frac{A}{A+1} \cdot \frac{2Q^2/(A+1)}{1 + 2Q^2/(A+1)}$$

$$\therefore S_A^Q_a \approx \underline{\underline{\frac{2Q^2}{A}}}$$

$$\frac{\partial Q}{\partial C_4} = \frac{Q}{2C_4} \Rightarrow S_{C_4}^Q = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial R_1} = \frac{\sqrt{R_1} - \sqrt{R_2}/R_2}{R_1(\frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}})} \cdot \frac{Q}{2}$$

$$= \frac{\sqrt{R_2}/R_1 - \sqrt{R_1}/R_2}{R_1(\sqrt{\frac{R_2}{R_1}} + \sqrt{\frac{R_1}{R_2}})} \cdot \frac{Q}{2}$$

$$\therefore S_{R_1}^Q = \frac{\sqrt{R_2}/R_1 - \sqrt{R_1}/R_2}{\sqrt{R_2}/R_1 + \sqrt{R_1}/R_2}$$

$$\text{if } R_1 = R_2 \Rightarrow S_{R_1}^Q = \underline{\underline{0}} \quad \underline{\underline{S_{R_2}^Q = 0}}$$

12.65

If  $R_1 = R_2$ , then from (12.77) & (12.78)

$$\omega_0 = \frac{1}{\sqrt{C_3 C_4 R_1 R_2}}$$

$$Q = \frac{1}{\sqrt{C_3 C_4 R_1 R_2} \left( \frac{1}{C_4} \right) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)}$$

$$\frac{\partial \omega_0}{\partial C_3} = \frac{-1}{2C_3 \sqrt{C_3 C_4 R_1 R_2}}$$

$$\frac{\partial \omega_0}{\partial C_3} = \frac{-1}{2C_3 \sqrt{C_3 C_4 R_1 R_2}} = \frac{-\omega_0}{2C_3}$$

$$S_{C_3}^{\omega_0} = \frac{\partial \omega_0}{\partial C_3} \cdot \frac{C_3}{\omega_0} = \underline{\underline{-\frac{1}{2}}}$$

$$\text{clearly } S_{C_3}^{\omega_0} = S_{C_4}^{\omega_0} = S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = -\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_3} = \frac{-1}{2C_3 \sqrt{C_3 C_4 R_1 R_2} \left( \frac{1}{C_4} \right) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} = \frac{-Q}{2C_3}$$

$$\therefore S_{C_3}^Q = -\frac{1}{2}$$

12.66

From table 12.1

$$\omega_0 = \frac{1}{\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}}$$

$$Q = R_4 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$$

$$\frac{\partial \omega_0}{\partial C_4} = \frac{-\omega_0}{2C_4}$$

$$\therefore S_{C_4}^{\omega_0} = \frac{-\omega_0}{2C_4} \times \frac{C_4}{\omega_0} = \underline{\underline{-\frac{1}{2}}}$$

$$\text{Similarly } S_{C_6}^{\omega_0} = S_{R_1}^{\omega_0} = S_{R_3}^{\omega_0} = S_{R_5}^{\omega_0} = \underline{\underline{-\frac{1}{2}}}$$

$$\frac{\partial \omega_0}{\partial R_2} = \frac{\omega_0}{2R_2} \Rightarrow S_{R_2}^{\omega_0} = \underline{\underline{\frac{1}{2}}}$$

Now for  $Q$ :

$$\frac{\partial Q}{\partial R_6} = \frac{Q}{R_6} \Rightarrow S_{R_6}^Q = \frac{\partial Q}{\partial R_6} \frac{R_6}{Q} = +1$$

CONT.

$$\frac{\partial Q}{\partial C_6} = \frac{Q}{2C_6} \Rightarrow S_{C_6}^Q = S_{R_2}^Q = \pm \frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = -\frac{Q}{2C_4} \Rightarrow S_{C_4}^Q = S_{R_1, R_3, R_5}^Q = \mp \frac{1}{2}$$

12.67

$$R_{eq} = \frac{T_c}{C_1} = \frac{100 \times 10^3}{C_1}$$

$$\text{for } 1\text{pF} \rightarrow R_{eq} = 10^5 / 10^{-12} = 10\text{M}\Omega$$

$$\text{for } 10\text{pF} \rightarrow R_{eq} = 10^5 / 10 \times 10^{-12} = 1\text{M}\Omega$$

12.68

charge transferred  $\Rightarrow Q = CV$

$$= 10^{-12}(1) \\ = 1\text{pC}$$

For  $f_o = 100\text{kHz}$ , average current is given by :

$$I_{AVE} = \frac{Q}{T} = 1\text{pC} \times \frac{1}{100 \times 10^3} \\ = 0.1\text{mA}$$

For each clock cycle, the output will change by the same amount as the change in voltage across  $C_2$ !

$$\therefore \Delta V = Q/C_2 = \frac{1\text{pC}}{10\text{pF}} = 0.1\text{V}$$

For  $\Delta V = 0.1\text{V}$  for each clock cycle, the amplifier will saturate in

$$\# \text{cycles} = \frac{10\text{V}}{0.1\text{V}} = 100 \text{cycles}$$

$$\text{slope} = \frac{\Delta V}{\Delta t} = \frac{10\text{V}}{(100 \text{cycles}) (1/100 \times 10^3)} \\ = \underline{\underline{10^4 \frac{\text{V}}{\text{s}}}}$$

12.69

$$f_c = 400\text{kHz} \quad f_o = 10\text{kHz} \quad Q = 20$$

$$C_1 = C_2 = 20\text{pF} = C$$

$$C_3 = C_4 = \omega_0 T_c C = 2\pi(10^4) \frac{1}{400 \times 10^3} 20 \times 10^{-12} \\ = \underline{\underline{3.14\text{pF}}}$$

$$C_5 = \frac{\omega_0 T_c C}{Q}$$

$$= \frac{C_3}{Q} = \underline{\underline{0.157\text{pF}}}$$

$$C_6 = \frac{\omega_0 T_c C}{Q} \times \text{center frequency gain} \\ = \underline{\underline{0.157\text{pF}}}$$

Note that the clock frequency has doubled. Hence the period,  $T_c$ , is halved. Therefore, for the same integrating capacitors, the resistors (switched capacitors) will change by the factor of 2. So compensate for this by changing the switched caps by a factor of 1/2.

12.70

Ex 12.31 for  $Q = 40 \quad f_c = 200\text{kHz}$

$$f_o = 10\text{kHz}$$

$$C_1 = C_2 = 20\text{pF} = C$$

$$C_3 = C_4 = \omega_0 T_c C \\ = 2\pi(10^4) \left(\frac{1}{200 \times 10^3}\right) 20 \times 10^{-12} \\ = \underline{\underline{6.28\text{pF}}}$$

CONT.

$$C_5 = \frac{\omega_0 T_c C}{Q} = \frac{C_3}{Q} = \underline{\underline{0.157 \text{ pF}}}$$

$$C_6 = \frac{\omega_0 T_c C}{Q} = C_5 = \underline{\underline{0.157 \text{ pF}}}$$

12.71

$$\omega_0 = 10^4, Q = \sqrt{2}, f_c = 100 \text{ kHz}$$

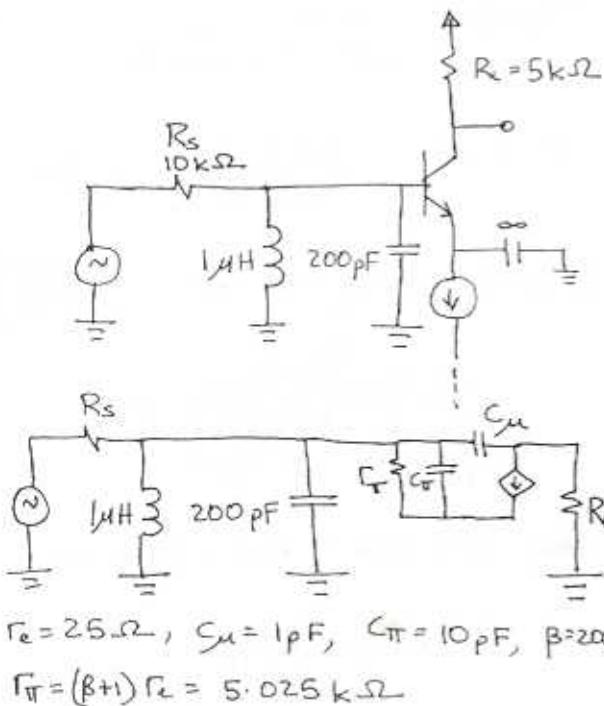
$$\text{DC gain} \Rightarrow \frac{R_4}{R_6} \Rightarrow \frac{C_6}{C_4} = 1$$

$$C_1 = C_2 = 10 \text{ pF}$$

$$C_3 = C_4 = C_6 = \omega_0 T_c C \\ = 10^4 \left( \frac{1}{100 \times 10^3} \right) 10 \times 10^{-12} \\ = \underline{\underline{1 \text{ pF}}}$$

$$C_B = C_4/Q = \underline{\underline{1.41 \text{ pF}}}$$

12.72



From base to collector

$$\frac{V_C}{V_B} = -\frac{\beta}{\beta+1} \cdot \frac{R_L}{r_e} = -199 = k$$

Total capacitance at base

$$C_T = C_\pi + 200 \text{ pF} + C_MU (1-k) \quad \xrightarrow{\text{Miller Effect}}$$

$$= 10 + 200 + 1(1+199) \\ = 416 \text{ pF}$$

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}} \\ = \frac{1}{\sqrt{10^{-4} \times 416 \times 10^{-12}}} \\ = \underline{\underline{49.4 \times 10^6 \text{ rad/s}}}$$

Centre frequency gain =

$$\frac{\Gamma_\pi}{R_s + \Gamma_\pi} \cdot k \\ = \frac{5.025}{10 + 5.025} \times -199 \\ = \underline{\underline{-66 \cdot 6 \text{ V/V}}}$$

$$\text{BW} = \frac{1}{RC} \\ = \frac{1}{(R_s || \Gamma_\pi) 416 \text{ pF}} \\ = \underline{\underline{7.29 \times 10^3 \text{ rad/s}}}$$

$$Q = \frac{\omega_0}{\text{BW}} \\ = \frac{49.4}{0.7293} \\ = \underline{\underline{67.7}}$$

12.73

$$Q_0 = \frac{R_p}{\omega_0 L} \Rightarrow R_p = Q_0 \omega_0 L \\ = 200 (2\pi 10^6) (10 \times 10^{-6}) \\ = \underline{12.57 \text{ k}\Omega}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_0^2 L} \\ = \frac{1}{(2\pi 10^6)^2 10 \times 10^{-6}} \\ = \underline{2.533 \text{nF}}$$

$$B = \frac{1}{RC} \quad R_r = \frac{1}{(2\pi \times 10 \times 10^3)(2.533 \times 10^{-9})} \\ = 6.283 \text{k}\Omega$$

$$\therefore \frac{1}{R_i} + \frac{1}{R_p} = \frac{1}{R_r}$$

$$\Rightarrow R_i = \underline{12.57 \text{k}\Omega} \quad \text{i.e. } R_i \parallel R_p = R_r$$

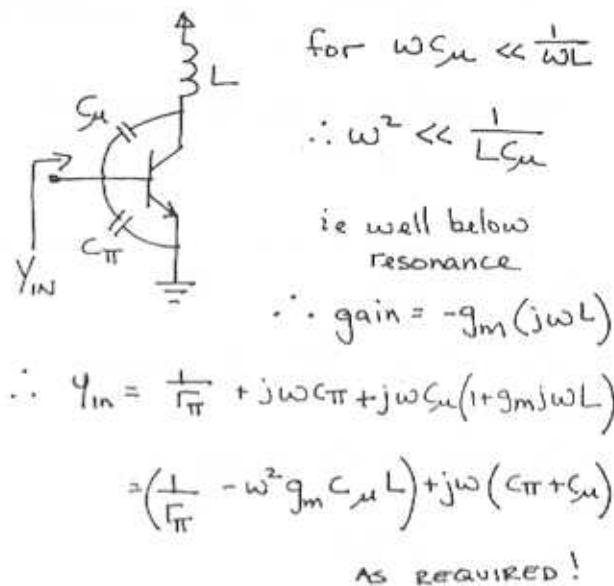
12.74

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \\ = (2\pi(36 \times 10^{-4}) 10^{-9})^{-1} \\ = \underline{838.8 \text{ kHz}}$$

$$R_p = n^2 R \\ = 9(1 \text{k}\Omega) \\ = 9 \text{k}\Omega$$

$$Q = \frac{R_p}{\omega_0 L} \\ = \frac{9 \times 10^3}{2\pi 838.8 \times 10^3 \times 36 \times 10^{-4}} \\ = \underline{47.4}$$

12.75



12.76

From Fig. 12.16 (c):

$$T(s) = \frac{a_1 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{j a_1 \omega}{\omega_0^2 - \omega^2 + j \frac{\omega \omega_0}{Q}}$$

$$T(j\omega_0) = \frac{j a_1 \omega_0}{j \omega_0^2 / Q} = \frac{a_1 Q}{\omega_0}$$

$$|T(j\omega)| = a_1 \omega \left[ (\omega_0^2 - \omega^2)^2 + \left( \frac{\omega \omega_0}{Q} \right)^2 \right]^{-\frac{1}{2}}$$

$$= \frac{a_1 \omega \frac{Q \omega \omega_0}{\omega_0}}{\sqrt{1 + Q^2 \left( \frac{\omega_0^2 - \omega^2}{\omega_0 \omega} \right)^2}}$$

$$\text{Now } \omega = \omega_0 + \delta \omega, \frac{\delta \omega}{\omega_0} \ll 1$$

$$\text{and } \omega^2 \approx \omega_0^2 \left( 1 + 2 \frac{\delta \omega}{\omega_0} \right)$$

CONT.

$$so \omega_0^2 - \omega^2 = -2\delta\omega\omega_0$$

12.77

$$\therefore |T(j\omega)| \approx \frac{\omega_0 Q / \omega_0}{\sqrt{1 + Q^2 \left(\frac{2\delta\omega}{\omega}\right)^2}}$$

$$\text{for } Q \gg 1 : Q^2 \left(\frac{2\delta\omega}{\omega}\right)^2 \approx Q^2 \left(\frac{2\delta\omega}{\omega_0}\right)^2 \\ \therefore \omega \approx \omega_0 !$$

$$\Rightarrow |T(j\omega)| \approx \frac{|T(j\omega_0)|}{\sqrt{1 + Q^2 \left(\frac{2\delta\omega}{\omega_0}\right)^2}} \\ = \frac{|T(j\omega_0)|}{\sqrt{1 + 4Q^2 \frac{\delta\omega}{\omega_0}}}$$

For N bandpass sections, synchronously tuned in cascade, half power is given by:

$$\left( \frac{1}{\sqrt{1 + 4Q^2 \left(\frac{\delta\omega}{\omega_0}\right)^2}} \right)^N = \frac{1}{\sqrt{2}}$$

$$\left( 1 + 4Q^2 \left(\frac{\delta\omega}{\omega_0}\right)^2 \right)^N = 2$$

$$4Q^2 \left(\frac{\delta\omega}{\omega_0}\right)^2 = 2^{N-1}$$

$$\delta\omega = \frac{\omega_0}{2Q} \sqrt{2^{N-1}}$$

$\therefore$  Bandwidth:

$$B = 2\delta\omega = \frac{\omega_0}{Q} \sqrt{2^{N-1}}$$

For first order low pass:

$$T(s) = \frac{\omega_0'}{s + \omega_0'} \quad |T(j\omega)| = \frac{\omega_0'}{\sqrt{\omega^2 + \omega_0'^2}}$$

for a bandpass response around  $\omega_0$ , with  $\omega_0' = \frac{\omega_0}{2Q}$ :

$$|T(j\omega)| \approx \frac{\omega_0/2Q}{(\delta\omega)^2 + \left(\frac{\omega_0}{2Q}\right)^2} \\ = \frac{\omega_0/2Q}{\frac{\omega_0}{2Q} \sqrt{\left(\frac{2Q}{\omega_0}\right)^2 (\delta\omega)^2 + 1}} \\ = \frac{1}{\sqrt{1 + 4Q^2 \left(\frac{\delta\omega}{\omega_0}\right)^2}}$$

Now at  $\omega = \omega_0$  or  $\delta\omega = 0$   
 $|r(j\omega_0)| = 1$ , then

$$T(j\omega) \approx \frac{|T(j\omega_0)|}{\sqrt{1 + 4Q^2 \left(\frac{\delta\omega}{\omega_0}\right)^2}}$$

Part (b)

For N synchronously tuned sections in cascade; 3dB bandwidth is given by:

$$\left( \frac{|T|}{|T_0|} \right)^N = \frac{1}{\sqrt{2}}$$

$$\left( \frac{|T|}{|T_0|} \right)^2 = \frac{1}{2^N} \quad \text{or}$$

$$1 + 4Q^2 \left(\frac{\delta\omega}{\omega_0}\right)^2 = 2^N \quad \text{or}$$

$$2\delta\omega = \frac{\omega_0}{Q} \sqrt{2^N - 1} \quad (12.110)$$

CONT.

$$\text{Thus: } |T(j\omega)|_{\text{overall}} = |T(j\omega)|^N$$

$$= \frac{|T(j\omega_0)|_{\text{overall}}}{\left[1 + 4Q^2 \left(\frac{\delta\omega}{\omega_0}\right)^2\right]^{N/2}}$$

NOTE  
 $Q = \frac{\omega_0}{B} \sqrt{2^{N-1}}$

$$= \frac{|T(j\omega_0)|_{\text{overall}}}{\left(1 + 4 \frac{\omega_0^2}{B^2} (2^{N-1}) \left(\frac{\delta\omega}{\omega_0}\right)^2\right)^{N/2}}$$

$$= \frac{|T(j\omega_0)|_{\text{overall}}}{\left[1 + 4 (2^{N-1}) \left(\frac{\delta\omega}{B}\right)^2\right]^{N/2}}$$

Part (c) (i)

for bandwidth  $= 2B$ , i.e.  $\delta\omega = \pm B$

$$\text{Att} = -20 \log \left(1 + 4 (2^{N-1}) (1)\right)^{-N/2}$$

$$= -10 N \log (1 + 2^{2+N} - 4)$$

$$= 10 N \log (2^{2+N} - 3)$$

$N =$	1	2	3	4	5
Att (dB)	6.70	8.49	9.28	9.79	10.13

Part (c) (ii)

3 dB bandwidth  $\delta\omega = \pm B/2$

30dB bandwidth  $\frac{\delta\omega}{B} = x$

$$-30 = -20 \frac{N}{2} \log \left(1 + 4 (2^{N-1}) x^2\right)$$

$$3 = N \log (1 + 4 (2^{N-1}) x^2)$$

$$x = \left[ \frac{10^{3/N} - 1}{4(2^{N-1})} \right]^{1/2}$$

Ratio of 30 dB to 3dB

$$\text{BW} = \frac{2Bx}{B} = 2x$$

$N =$	1	2	3	4	5
Ratio =	31.6	8.6	5.7	4.5	

12.78

See fig 12.48 and Eq 12.115 and 12.116

(a) For the narrowband approximation, variation of  $\omega$  around 0 is equivalent to  $\delta\omega$  around  $\omega_0$ . Thus, a low-pass maximally flat filter of bandwidth  $B/2$  and order  $N$  for which  $|T| = \left[ \left(1 + \frac{\delta\omega}{B/2}\right)^{2N} \right]^{-1/2}$  is transformed to a band-pass maximally flat filter of bandwidth  $B/2$  and order  $2N$ , and centre frequency  $\omega_0$ , for which:

$$|T| = \left(1 + \left(\frac{\delta\omega}{B/2}\right)^{2N}\right)^{-1/2}$$

(b) For bandwidth  $2B$ ,  $\delta\omega = B$  &

$$|T| = \left(1 + \left(\frac{B}{B/2}\right)^{2N}\right)^{-1/2}$$

$$= \left(1 + 2^{2N}\right)^{-1/2} \text{ thus:}$$

$N =$	1	2	3	4	5
$ T $	0.447	0.242	0.124	0.062	0.031
$ T _{\text{dB}}$	-6.99	-12.3	-18.1	-24.1	-30.1

For 30 dB bandwidth,

$$-30 = 20 \log \alpha \Rightarrow \alpha = 10^{-3/2} = \frac{1}{31.6}$$

$$\therefore 1 + \left( \frac{s^1 \omega}{B/2} \right)^{2N} = (31.6)^2$$

$$\left( \frac{s^1 \omega}{B/2} \right)^{2N} = 999 - 1 = 998$$

Now the ratio of 30dB to 3dB bandwidths is

$$\text{ratio} = \frac{2s^1 \omega}{B} = \frac{s^1 \omega}{B/2} = 998^{\frac{1}{2N}}$$

N	1	2	3	4	5
ratio	31.6	5.62	3.16	2.37	1.99

12.79

$$A_{\max} = 3 \text{dB} \Rightarrow \epsilon = \sqrt{10^{A_{\max}/10} - 1} \approx 1$$

Poles of lowpass prototype are given by FIG 12.10(c)

$$\text{Poles: } -\omega_p, \omega_p (-\frac{1}{2} \pm j\frac{\sqrt{3}}{2})$$

$$\text{Make } \omega_p = B/2$$

$$\Rightarrow \text{poles: } \left\{ -\frac{B}{2}, \frac{+B}{2} \left( -\frac{1}{2} \pm j\frac{\sqrt{3}}{2} \right) \right\}$$

Using the low-pass to bandpass transformation:

Poles of the bandpass filter:

$$-\frac{B}{2} \pm j\omega_0,$$

$$-\frac{B}{4} \pm j\left(\frac{\sqrt{3}}{4}B + \omega_0\right) \quad \text{and}$$

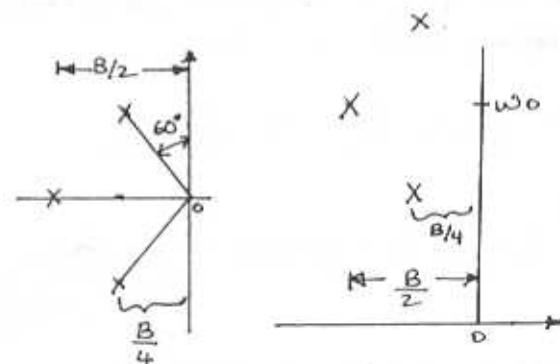
$$-\frac{B}{4} \pm j\left(\frac{\sqrt{3}}{4}B - \omega_0\right)$$

For the three circuits:

$$\textcircled{1} \omega_{01} = \omega_0 \quad B_1 = B \quad Q_1 = \frac{\omega_0}{B}$$

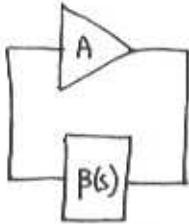
$$\textcircled{2} \omega_{02} \approx \frac{\sqrt{3}}{4}B + \omega_0 \quad B_2 = \frac{B}{2} \quad Q_2 \approx \frac{2\omega_0}{B}$$

$$\textcircled{3} \omega_{03} \approx \frac{\sqrt{3}}{4}B - \omega_0 \quad B_3 = \frac{B}{2} \quad Q_3 \approx \frac{2\omega_0}{B}$$



## CHAPTER 13 - PROBLEMS

13.1



$$A = A_0 > 0$$

$$\beta(s) = \frac{k \frac{\omega_0}{Q} s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

$$(a) \text{ for oscillations } 1 - A\beta(s) = 0$$

$$\therefore A_0 K \frac{\omega_0}{Q} s = s^2 + s \frac{\omega_0}{Q} + \omega_0^2$$

$$\omega_0^2 - \omega^2 = j\omega \left(\frac{\omega_0}{Q}\right)(A_0 K - 1)$$

at the freq of oscillation, both Real & Imaginary parts are 0.

$$\therefore \underline{\omega = \omega_0} \quad \& \quad \underline{A_0 K = 1}$$

(b)

$$L(j\omega) \triangleq A \beta(j\omega) = \frac{A K \frac{\omega_0}{Q} j\omega}{(\omega_0^2 - \omega^2) + j\omega \left(\frac{\omega_0}{Q}\right)}$$

$$\therefore \phi(\omega) = 90^\circ - \tan^{-1} \left( \frac{\omega \omega_0 / Q}{\omega_0^2 - \omega^2} \right)$$

$$\text{Now } \frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\therefore \frac{d\phi}{d\omega} = \frac{1}{1 + \left( \frac{\omega \omega_0 / Q}{\omega_0^2 - \omega^2} \right)^2} \cdot \frac{d}{d\omega} \left( \frac{\omega \omega_0 / Q}{\omega_0^2 - \omega^2} \right)$$

$$= \frac{-(\omega_0^2 - \omega^2)^2}{(\omega_0^2 - \omega^2)^2 + (\omega \omega_0 / Q)^2} \cdot \frac{\frac{\omega_0}{Q}(\omega_0^2 - \omega^2) - 2\omega \frac{\omega_0 \omega}{Q}}{(\omega_0^2 - \omega^2)^2}$$

$$\begin{aligned} \frac{d\phi}{d\omega} \Big|_{\omega=\omega_0} &= \frac{-1}{\omega_0^4/Q^2} \cdot \frac{2\omega_0^3}{Q} \\ &= \underline{\underline{\frac{-2Q}{\omega_0}}} \end{aligned}$$

$$\begin{aligned} (c) \Delta \omega_0 &= \frac{\Delta \phi}{\frac{\partial \phi}{\partial \omega}} = \frac{\Delta \phi}{-2Q/\omega_0} \\ &= \underline{\underline{-\frac{\Delta \phi \omega_0}{2Q}}} \end{aligned}$$

$\therefore$  Per unit change in  $\omega_0$  is given by

$$\underline{\underline{\frac{\Delta \omega_0}{\omega_0} = -\frac{\Delta \phi}{2Q}}}$$

13.2

For the circuit of problem 13.1, the poles, which are the zeros of the characteristic equation, are given by:

$$1 - L(s) = 0$$

$$L(s) = 1$$

$$\frac{A K \left(\frac{\omega_0}{Q}\right) s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} = 1$$

$$s^2 + s \frac{\omega_0}{Q} (1 - A K) + \omega_0^2 = 0$$

$\therefore$  Poles are at :

$$s = \frac{-\omega_0 (1 - A K) \pm \sqrt{\left(\frac{\omega_0}{Q}\right)^2 (1 - A K)^2 - 4\omega_0^2}}{2}$$

$$= -\omega_0 \left[ \frac{1 - A K}{2Q} \pm \sqrt{\left(\frac{1 - A K}{2Q}\right)^2 - 1} \right]$$

$$= -\omega_0 \left( \frac{1 - A K}{2Q} \right) \left[ 1 \pm j \sqrt{\left(\frac{2Q}{1 - A K}\right)^2 - 1} \right]$$

CONT.

Radial distance of  $\omega_0 \Rightarrow$

$$|s^2| = \omega_0^2 \left( \frac{1-AK}{2Q} \right)^2 \left[ 1 + \left( \frac{2Q}{1-AK} \right)^2 - 1 \right] \\ = \omega_0^2$$

$\therefore |s| = \omega_0 \sim \text{independent of } K!$

(a) For poles on  $j\omega$ -axis  $\Rightarrow$  real part = 0

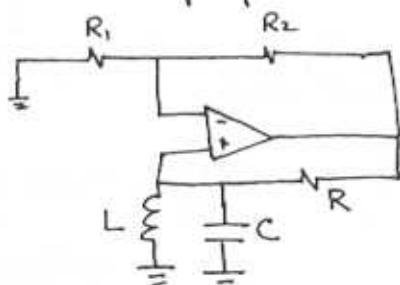
$$\therefore -(1-AK) = 0 \Rightarrow \underline{AK=1}$$

(b) For poles in RHS  $\rightarrow$  Real Part > 0

$$-(1-AK) > 0 \\ \underline{AK > 1}$$

13.3

Assume ideal opamp.



$$\text{At resonance } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$K = 1$$

Thees A must also be 1 (or slightly higher).

For  $R_1 = 10k\Omega$ ,  $R_2 = 100\Omega$

$$A = 1 + \frac{R_2}{R_1} = \underline{1.01}$$

(a) If  $L' = 1.01L$

$$\omega_0' = \frac{1}{\sqrt{1.01}} \omega_0 = 0.995\omega_0 \\ \text{or } \underline{-\frac{1}{2}\%}$$

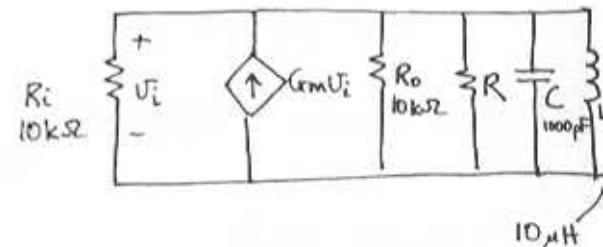
(b) If  $C$  changes by  $+1\%$

$\Rightarrow \omega_0$  changes by  $\underline{-\frac{1}{2}\%}$

(c) If  $R$  changes by  $+1\%$

$\Rightarrow \omega_0 \sim \underline{\text{unchanged}}$

13.4



$$\text{For resonator: } \omega_0 = \frac{1}{\sqrt{LC}} = 10^7 \frac{\text{rad}}{\text{s}}$$

$$\frac{\omega_0}{Q} = \frac{1}{RC}$$

$$R = \frac{Q}{\omega_0 C} = \frac{100}{10^7 \times 1000 \times 10^{-12}} = 10k\Omega$$

Oscillations will occur at  $\omega_0 = 10^7 \frac{\text{rad}}{\text{s}}$

when  $G_m (R_1 || R_0 || R) = 1$  i.e. gain = 1

$$\therefore G_m = \frac{1}{10k \parallel 10k \parallel 10k} = \frac{3}{10^9} = 300 \frac{\mu\text{V}}{\text{V}}$$

13.5

At  $\omega_0$   $A\beta = 1$

If  $\beta(\omega_0)$  is  $-20 \text{ dB}$  with a phase

CONT.

shift of  $180^\circ$  then clearly A should have a gain of 20dB  
(i.e.  $A(\omega_0) = 10$ ) with a phase shift of  $\pm 180^\circ$   
i.e.  $A = -10$

$$\frac{\Delta U_I}{R_1} = -\frac{V_B}{R_B}$$

$$\therefore \Delta U_I = -\frac{R_1}{R_B} V_B$$

$$U_D = 0 \sim \text{assumed}$$

13.6

From Eq(13.8)

$$L_- = -V \frac{R_3}{R_2} - V_D \left( 1 + \frac{R_3}{R_2} \right)$$

$$0 = 10 \frac{R_3}{R_2} + 0.7 \left( 1 + \frac{R_3}{R_2} \right)$$

$$= 10 \cdot 7 \frac{R_3}{R_2} + 0.7$$

$$\frac{R_3}{R_2} = 0.495 \quad \text{By symmetry } \frac{R_4}{R_5} = 0.495$$

$$\text{Use } R_2 = R_5 = 10k\Omega$$

$$\therefore R_2 = R_4 \approx 5k\Omega$$

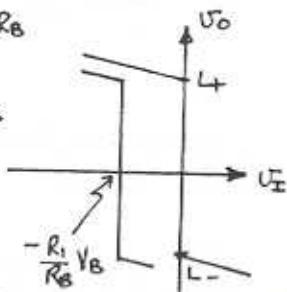
Slope of limiting characteristic

$$= \frac{R_4}{R_5} = 0.1$$

$$\therefore R_1 = \frac{1}{0.1} R_4 = 50k\Omega$$

13.7

For  $V_B$  connected via  $R_B$  to the virtual ground, a current  $= \frac{V_B}{R_B}$  flows into the node. To compensate,  $U_I$  must be moved by  $\Delta U_I$ , in a direction opposite to  $V_B$  to produce a current  $\Rightarrow$



$$L_- = -5 = -15 \frac{R_3}{R_2}$$

$$\frac{R_3}{R_2} = \frac{1}{3} = \frac{R_4}{R_5}$$

$$\text{Given } R_m = 100k\Omega \Rightarrow R_1 = 100k\Omega$$

$$\text{Slope} = \frac{R_4}{R_1} \leq 0.05$$

$$R_4 \leq R_1 \times 0.05$$

$$R_4 \leq 5k\Omega \Rightarrow \text{let } R_4 = 4.3k\Omega$$

$$\therefore R_3 = R_4 \Rightarrow R_3 = 4.3k\Omega$$

$$R_2 = R_5 = 3R_4 = 12.9k\Omega$$

For standard resistance values:

$$R_2 = R_5 = 13k\Omega$$

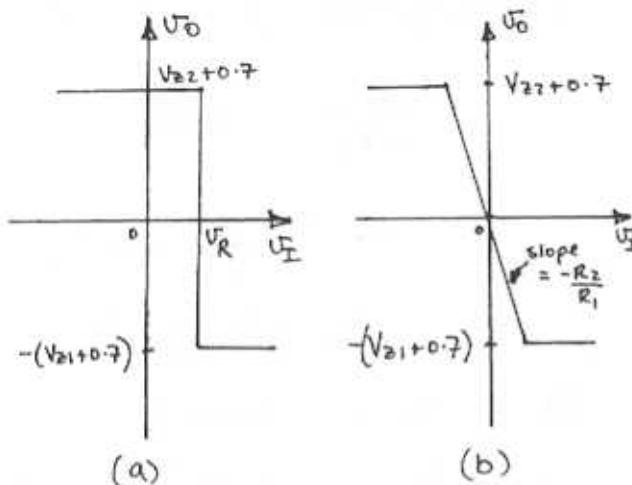
$$\therefore L_- = -15 \frac{R_3}{R_2} = -15 \times \frac{4.3}{12.9} = -4.96V \approx -5V$$

Offset is +5V  $\Rightarrow$  then  $V_B = -15V$

$$\text{and } 5 = R_1/R_B \text{ is}$$

$$\therefore R_B = 3R_1 = 300k\Omega$$

13.8



$$\omega_0^2 = \frac{1}{RC^2} \Rightarrow \underline{\underline{\omega_0 = \frac{1}{RC}}}$$

$$\frac{\omega_0}{Q} = \frac{3}{RC} \Rightarrow \underline{\underline{Q = \frac{1}{3}}}$$

For centre frequency gain:

$$S = j\omega_0 = j/RC$$

$$\therefore \left. \frac{V_a}{V_o} \right|_{S=j/RC} = \frac{\frac{1}{RC} j/RC}{-\frac{1}{R^2C^2} + \frac{3}{RC} \left( \frac{j}{RC} \right) + \frac{1}{R^2C^2}}$$

$$= \underline{\underline{\frac{1}{3}}} = \text{centre freq. gain}$$

13.9

$$\begin{aligned} \frac{V_a}{V_o} &= \frac{\frac{1}{SC} || R}{\frac{1}{SC} || R + \frac{1}{SC} + R} \\ &= \frac{\left( \frac{1}{SC} \right) / \left( \frac{1}{SC} + R \right)}{\left( \frac{1}{SC} + R \right) + \frac{1}{SC} + R} \\ &= \frac{\frac{R}{SC}}{\frac{R}{SC} + \left( \frac{1}{SC} + R \right)^2} \times \frac{S^2 C^2}{S^2 C^2} \\ &= \frac{SCR}{SCR + (1+SCR)^2} \\ &= \frac{SCR}{SCR + 1 + 2SCR + S^2 C^2 R^2} \\ &= \frac{\frac{1}{RC} S}{S^2 + S \frac{3}{RC} + \frac{1}{R^2 C^2}} \end{aligned}$$

Note  $\frac{V_a}{V_o}$  has zeros at 0 and  $\infty$

i.e. A Bandpass!

13.10

$$L(j\omega) = \frac{1 + R_2/R_1}{3 + j(\omega CR - \frac{1}{\omega CR})} \quad \text{Eq}(13.11)$$

$$\phi(\omega) = -\tan^{-1} \left( \frac{\omega CR - \frac{1}{\omega CR}}{3} \right)$$

$$\text{using } \frac{d \tan^{-1} U}{dX} = \frac{1}{1+U^2} \frac{dU}{dX}$$

$$\frac{d\phi}{d\omega} = \frac{-1}{1 + \left( \frac{\omega CR - \frac{1}{\omega CR}}{3} \right)^2} \cdot \frac{1}{3} \left( CR + \frac{1}{\omega^2 CR} \right)$$

$$\left. \frac{d\phi}{d\omega} \right|_{\omega=\frac{1}{RC}} = -\frac{1}{3} \left( CR + CR \right) = -\frac{2}{3} CR$$

for  $\Delta\phi = -0.1 \text{ rad}$

$$\Delta\omega_0 = \frac{\Delta\phi}{\frac{d\phi}{d\omega}} = \frac{-0.1}{-2/3 \frac{1}{\omega_0}}$$

$$= 0.15 \omega_0$$

$\therefore$  New frequency of oscillation

$$= 1.15 \omega_0 = \underline{\underline{\frac{1.15}{RC}}}$$

13.11

Using Eq (13.10)

$$L(s) = \frac{1 + R_2/R_1}{3 + SCR + \gamma SCR}$$

Poles of closed loop given by:  $L(s)=1$

$$1 + R_2/R_1 = 3 + SCR + \frac{1}{SCR}$$

$$0 = s^2 + \frac{s}{RC} \left( 2 - \frac{R_2}{R_1} \right) + \frac{1}{R^2 C^2}$$

$$\underline{Q = \frac{1}{\left( 2 - \frac{R_2}{R_1} \right)}}$$

for  $Q = \infty \sim$  poles on jw axis

$$\sim R_2/R_1 > 2$$

For poles in R.H.P.  $R_2/R_1 > 2$

Using (2)  $R_S = 1k\Omega$

$$5 - \left( \frac{1}{1+R_G} \cdot (V_o + 15) \right) - 0.7 = 1.65$$

$$\frac{20}{1+R_G} = 2.65$$

$$R_G = \frac{20}{2.65} = 1$$

$$\underline{R_G = 6.5 k\Omega = R_3}$$

If  $R_3 = R_G = \infty$  from (2)

$$V_o - \left( \frac{1}{1+\infty} (V_o + 15) \right) - 0.7 = \frac{V_o}{3.03}$$

$$V_o - 0.7 = \frac{V_o}{3.3}$$

$$V_o = 1.04 V$$

Output is  $2V_o = \underline{2.08 V_{p-p}}$ .

13.12

From Fig(13.5) assuming resistance of limiting network is very low

At positive peak

$$V_o = \left( 1 + \frac{20.3k}{10k} \right) V_s = 3.03 V_s \quad (1)$$

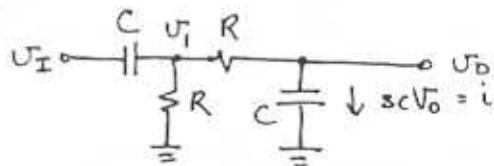
$$V_o = \left[ \frac{R_S}{R_S + R_G} \cdot (V_o - (-15)) \right] - 0.7 = V_I \quad (2)$$

Now for  $10V_{p-p}$  out

$$\hat{V}_o = 5V$$

$$\hat{V}_I = \frac{5}{3.03} = 1.65 V$$

13.13



$$\frac{V_I - V_o}{R} = SCR \Rightarrow V_I = V_o (1 + SCR)$$

$\sum I$  at  $V_I$

$$\frac{V_I}{R} + SCR(V_I - V_o) + SCR V_o = 0$$

$$V_o (1 + SCR) + SCR(V_o + V_o SCR) - SCR V_I + SCR V_o = 0$$

$$V_o (1 + SCR + SCR + SCR^2 + SCR) = SCR V_I$$

CONT.

$$\beta(s) \triangleq \frac{V_o}{V_I} = \frac{sCR}{s^2C^2R^2 + 3SCR + 1}$$

$$= \frac{1}{s^2C^2R^2 + 3SCR + 1}$$

From Fig 13.13)  $A = 1 + \frac{R_2}{R_1}$

$$\beta(j\omega) = \frac{1}{3 + j(\omega CR - \frac{1}{\omega CR})}$$

Zero phase when  $\omega CR = \frac{1}{\omega CR}$

$$\underline{\underline{\omega = \frac{1}{CR}}}$$

$$|\beta(\omega = \frac{1}{RC})| = \frac{1}{3}$$

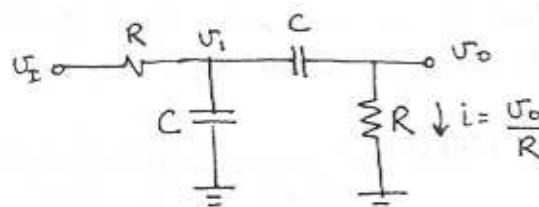
for oscillations  $1 + \frac{R_2}{R_1} \geq 3$   
 $\Rightarrow \underline{\underline{\frac{R_2}{R_1} \geq 2}}$

$$L(s) = AB = \frac{1 + \frac{R_2}{R_1}}{3 + SCR + \frac{1}{SCR}}$$

$$L(j\omega) = \frac{1 + \frac{R_2}{R_1}}{3 + j(\omega CR - \frac{1}{\omega CR})}$$

$$\underline{\underline{}}$$

13.14



$$SC(V_I - V_O) = \frac{V_O}{R}$$

$$V_I = V_O \left( 1 + \frac{1}{SRC} \right)$$

$\Sigma I$  at  $V_I$ :

$$\frac{-V_I + V_I}{R} + SCV_I + \frac{V_O}{R} = 0 \quad \leftarrow \text{SUB FOR } V_I \text{ and mult by } \frac{1}{SC}$$

$$\frac{1}{SCR} \left[ -V_I + V_O \left( 1 + \frac{1}{SRC} \right) \right] + V_O \left( 1 + \frac{1}{SRC} \right) + \frac{V_O}{SCR} = 0$$

$$V_O \left[ \frac{1}{SCR} + 1 + \frac{1}{SRC} + \frac{1}{S^2C^2R^2} + \frac{1}{SCR} \right] = \frac{V_I}{SCR}$$

$$\frac{V_O}{V_I} \triangleq \beta(s) = \frac{1}{3 + SCR + \frac{1}{SCR}}$$

$$\beta(j\omega) = \frac{1}{3 + j(\omega CR - \frac{1}{\omega CR})}$$

phase is zero when  $\omega CR = \frac{1}{\omega CR}$

$$\underline{\underline{\omega_0 = \frac{1}{RC}}}$$

$$\beta(j\omega_0) = \frac{1}{3}$$

$\therefore$  For oscillations

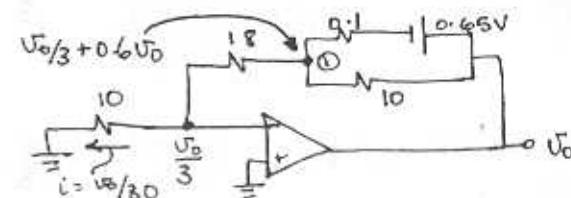
$$A = 1 + \frac{R_2}{R_1} \geq 3$$

$$\underline{\underline{\frac{R_2}{R_1} \geq 2}}$$

$$L(s) = \frac{1 + \frac{R_2}{R_1}}{3 + SCR + \frac{1}{SCR}}$$

$$L(j\omega) = \frac{1 + \frac{R_2}{R_1}}{3 + j(\omega CR - \frac{1}{\omega CR})}$$

13.15



CONT.

$\Sigma I$  at node ①

$$\begin{aligned}\frac{\dot{U}_0}{30} &= \frac{\dot{U}_0 - \frac{\dot{U}_0}{3} - 0.6\dot{U}_0}{10} \\ &\quad + \frac{\dot{U}_0 - 0.65 - \frac{\dot{U}_0}{3} - 0.6\dot{U}_0}{0.1} \\ &= 0.00666\dot{U}_0 + 0.666\dot{U}_0 - 0.65\end{aligned}$$

$$\dot{U}_0 = 10.156\text{ V}$$

$$\therefore \text{max. output} = \underline{20.3\text{ V p-p}}$$

13.16

$$w_0 = \frac{1}{RC} = 2\pi 10^4 \quad R = 10\text{k}\Omega$$

$$C = \frac{1}{10^4 \times 2\pi \times 10^4} \Rightarrow C \approx 1.6\text{nF}$$

Now from Eq (13.11)

$$\beta(j\omega) = \left[ 3 + j \left( \omega CR - \frac{1}{\omega CR} \right) \right]^{-1}$$

$$\therefore \phi(\omega) = -\tan^{-1} \left( \frac{\omega CR - \frac{1}{\omega CR}}{3} \right)$$

Using  $\frac{d \tan^{-1} v}{d x} = \frac{1}{1+v^2} \frac{dv}{dx}$  we get

$$\frac{d\phi(\omega)}{dw} = \frac{-1}{1 + \left( \frac{\omega CR - \frac{1}{\omega CR}}{3} \right)^2} \left[ \frac{RC + \frac{1}{\omega^2 RC}}{3} \right]$$

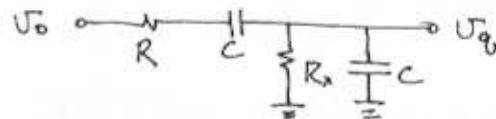
$$\text{At } w = w_0 = \frac{1}{RC} \quad \frac{d\phi(w)}{dw} = -\frac{2}{3} \frac{1}{RC}$$

Now  $5.7^\circ \approx 0.1\text{ rad}$  (lag =  $-0.1\text{ rad}$ )

$$\therefore \Delta w_0 = \frac{-0.1}{-2/3 RC} = 0.15 w_0 = 1.5\text{ kHz}$$

$\therefore$  New frequency of oscillation = 8.5kHz

To restore operation :



$$\beta(s) = \frac{R_x \parallel \frac{1}{sc}}{R_x \parallel \frac{1}{sc} + R + \frac{1}{sc}}$$

$$= \frac{\frac{R_x}{sc}}{\frac{R_x}{sc} + R + \frac{1}{sc}}$$

$$= \frac{R_x / sc}{R_x / sc + RR_x + \frac{R}{sc} + \frac{R_{xc}}{sc} + \frac{1}{sc^2}}$$

$$\therefore \beta(s) = \frac{1}{2 + \frac{R}{R_{xc}} + scR + \frac{1}{scR_{xc}}}$$

$$\phi = \tan^{-1} \left( \frac{\omega CR - \frac{1}{\omega CR}}{2 + R/R_{xc}} \right)$$

Now it is required that  $\phi = 5.7^\circ$  at  $w = w_0$  ! where  $w_0 = 1/RC$

$$\therefore w_0 RC - \frac{1}{w_0 R_{xc}} = \left( 2 + \frac{R}{R_{xc}} \right) \tan(57)$$

$$1 - \frac{1}{w_0 R_{xc}} = \left( 2 + \frac{R}{R_{xc}} \right) (-0.1)$$

$$1 + 0.2 = \frac{1}{w_0 R_{xc}} - 0.1 \frac{R}{R_{xc}}$$

CONT.

$$R_x = \frac{1/\omega_0 C - 0.1 R}{1.2}$$

given:  
 $\omega_0 = 2\pi \times 10^4$   
 $C = 1.6 \times 10^{-9}$   
 $R = 10^4$

$$R_x = \underline{7.5 \text{ k}\Omega}$$

Now:

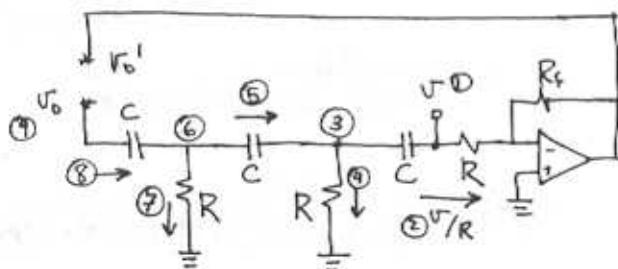
$$\beta(j\omega_0) = \frac{1}{2 + 10(7.5 + j(1 - \frac{1}{\omega_0 C}))} \\ = (3.333 - j0.326)^{-1}$$

$$|\beta(j\omega_0)| = \frac{1}{3.35}$$

$\therefore 1 + \frac{R_2}{R_1} = 3.35$  for oscillations

$$\frac{R_2}{R_1} = \underline{\underline{2.35}} \quad (\text{not } 2 \text{ as before})$$

13.17



$$③ \quad V_o + \frac{1}{sC} \frac{V_o}{R} = V_o \left( 1 + \frac{1}{sCR} \right)$$

$$④ \quad \frac{V_o \left( 1 + \frac{1}{sCR} \right)}{R} = \frac{V_o}{R} + \frac{V_o}{sCR^2}$$

$$⑤ \quad i_s = \frac{2V_o}{R} + \frac{V_o}{sCR^2}$$

$$⑥ \quad ③ + ⑤ / sC$$

$$= V_o + \frac{V_o}{sCR} + \frac{1}{sC} \left( 2 \frac{V_o}{R} + \frac{V_o}{sCR^2} \right) \\ = V_o + \frac{3V_o}{sCR} + \frac{V_o}{s^2 C^2 R^2}$$

$$⑦ \quad \frac{V_o}{R} + \frac{3V_o}{sCR^2} + \frac{V_o}{s^2 C^2 R^3}$$

$$⑧ = ⑤ + ⑦$$

$$= \frac{3V_o}{R} + \frac{4V_o}{sCR^2} + \frac{V_o}{s^2 C^2 R^3}$$

$$⑨ \quad V_o = ⑥ + \frac{1}{sC} ⑧$$

$$= V_o + \frac{3V_o}{sCR} + \frac{V_o}{s^2 C^2 R^2} + \frac{3V_o}{sCR} + \frac{4V_o}{s^2 C^2 R^3} \\ + \frac{V_o}{s^3 C^3 R^3}$$

$$= V_o + \frac{6V_o}{sCR} + \frac{5V_o}{s^2 C^2 R^2} + \frac{V_o}{s^3 C^3 R^3}$$

Now loop gain  $\equiv$

$$L(s) = - \frac{V_o}{V_o}$$

$$\therefore L(s) = \frac{R_f / R \cdot V_o}{V_o \left( 1 + \frac{6}{sCR} + \frac{5}{s^2 C^2 R^2} + \frac{1}{s^3 C^3 R^3} \right)} \\ = \frac{s^3 R_f / R}{s^3 + \frac{6s^2}{RC} + \frac{5s}{C^2 R^2} + \frac{1}{C^3 R^3}}$$

$$L(j\omega) = \frac{-j\omega^3 R_f / R}{\frac{1}{C^3 R^3} - \frac{6\omega^2}{RC} + j \left( \frac{5\omega}{C^2 R^2} - \omega^3 \right)}$$

CONT.

$L(j\omega)$  is real if

$$\frac{6w_0^2}{RC} > \frac{1}{R^3 C^3}$$

$$w_0 = \frac{1}{\sqrt{6} RC}$$

$$\begin{aligned} L(jw_0) &= \frac{w_0^2 R_f / R}{-w_0^2 + 5/R^2 C^2} \\ &= \frac{R_f / R \ w_0^2}{-w_0^2 + 30w_0^2} \\ &= \frac{R_f / R}{29} \end{aligned}$$

Now Loop Gain = 1 if  $R_f = 29R$

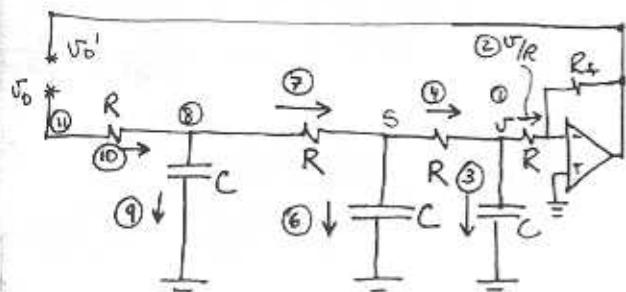
∴ Minimum value for  $\underline{R_f = 29R}$

Given  $C = 16nF$ ,  $R = 10k\Omega$

$$\begin{aligned} f_0 &= \frac{1}{2\pi\sqrt{6} \cdot 16 \times 10^{-9} \cdot 10 \times 10^3} \\ &= \underline{406.1 \text{ Hz}} \end{aligned}$$

$$\text{or } f_0 = \frac{0.065}{RC}$$

B-18



$$\textcircled{3} i = sCV \quad \textcircled{4} sCV + V/R$$

$$\textcircled{5} V + (sCV + \frac{V}{R})R = 2V + 8SCRV$$

$$\textcircled{6} 2sCV + s^2 C^2 RV$$

$$\textcircled{7} = \textcircled{6} + \textcircled{4} = 3sCV + s^2 C^2 RV + V/R$$

$$\begin{aligned} \textcircled{8} 2V + SCRV + V + 3SCRV + s^2 C^2 R^2 V \\ = 3V + 4SCRV + s^2 C^2 R^2 V \end{aligned}$$

$$\textcircled{9} 3sCV + 4s^2 C^2 RV + s^3 C^3 R^2 V$$

$$\textcircled{10} = \textcircled{7} + \textcircled{9}$$

$$= 6sCV + 5s^2 C^2 RV + \frac{V}{R} + s^3 C^3 R^2 V$$

$$\textcircled{11} = \textcircled{8} + \textcircled{10} \times R$$

$$V_o = 4V + 10SCRV + 6s^2 C^2 R^2 V + s^3 C^3 R^3 V$$

Now:

$$\begin{aligned} L(s) &= -\frac{V_o}{V_o} = \frac{V \frac{R_f / R}{s^3 C^3 R^3 + 6s^2 C^2 R^2 + 10SCR + 4}}{V (s^3 C^3 R^3 + 6s^2 C^2 R^2 + 10SCR + 4)} \\ &= \frac{R_f / R}{s^3 C^3 R^3 + 6s^2 C^2 R^2 + 10SCR + 4} \end{aligned}$$

$$L(j\omega) = \frac{R_f / R}{(4 - 6\omega^2 C^2 R^2) + j(10\omega CR + \omega^3 R^3)}$$

$L(j\omega)$  is purely real if

$$10w_0 CR = w_0^3 R^3 C^3$$

$$w_0 = \frac{1}{\sqrt{10}} \frac{1}{RC}$$

Given  $R = 10k\Omega$ ,  $f_0 = 10 \text{ kHz}$

$$C = \frac{1}{\sqrt{10} \times 10^4 \times 2\pi \times 10^4}$$

$$= \underline{0.503 \text{ nF}}$$

CONT.

Now,

$$|L(j\omega_0)| = \frac{R_f/R}{4 - 6\omega_0^2 R^2 C^2}$$

Sub for  $\omega_0$

$$= \frac{R_f/R}{4 - 6 \frac{1}{10R^2} R^2 C^2}$$

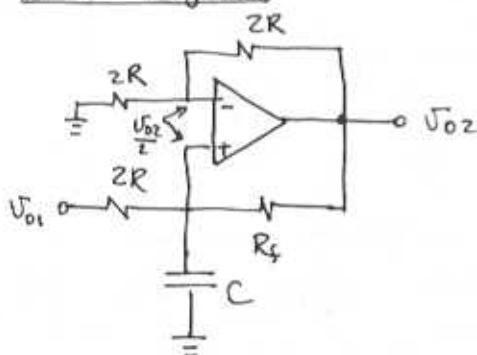
$$= \frac{R_f/R}{4 - 6/10} \gg 1$$

$$\therefore R_f/R \gg 3.4$$

$$\underline{R_f \geq 34 k\Omega}$$

13.19

for 2<sup>nd</sup> integrator



From the voltage divider around the upper branch:  $V_+ = V_- = \frac{1}{2}V_{o2}$

$\sum I = 0$  at +ve input

$$\frac{\frac{1}{2}V_{o2} - V_{o1}}{2R} + SC \frac{V_{o2}}{2} + \frac{\frac{V_{o2}}{2} - V_{o2}}{R_f} = 0$$

$$\frac{V_{o2} - 2V_{o1}}{2R} + SC V_{o2} - \frac{V_{o2}}{R_f} = 0 \quad R_f = \frac{2R}{1+\Delta}$$

$$V_{o2} \left( \frac{1}{2R} + SC - \frac{1+\Delta}{2R} \right) = \frac{V_{o1}}{R}$$

$$V_{o2} \left( SCR - \frac{\Delta}{2} \right) = V_{o1}$$

$$\therefore \frac{V_{o2}}{V_{o1}} = \frac{1}{SCR - \Delta/2}$$

$$\text{Now: } \frac{V_{o1}}{V_x} = \frac{-1}{SCR} \quad \therefore L(s) = \frac{-1/SCR}{SCR - \Delta/2}$$

Characteristic equation  $L(s) = 1$

$$\therefore s^2 C^2 R^2 - \frac{SCR\Delta}{2} + 1 = 0$$

$\therefore$  Poles are

$$s_p = \frac{RC\Delta}{2} \pm \sqrt{\frac{R^2 C^2 \Delta^2}{4} - 4C^2 R^2}$$

$$= \frac{\Delta/2 \pm 2j\sqrt{1 - (\Delta/4)^2}}{2RC}$$

$$\text{for } \Delta \ll 1 \quad (1 - (\frac{\Delta}{4})^2)^{1/2} \approx \left(1 - \frac{1}{2}(\frac{\Delta}{4})^2\right)$$

$$\therefore s_p \approx \left[ \frac{\Delta/2 \pm j2(1 - \frac{1}{2}(\frac{\Delta}{4})^2)}{2RC} \right] \frac{1}{2RC}$$

$$= \frac{\Delta}{2} \pm j \left( 2 - \frac{(\Delta/4)^2}{2RC} \right)$$

Now:

$\text{Re}[s_p] > 0 \Rightarrow$  Poles in R.H.P.!

for  $\Delta \ll 1$

$$s_p \approx \frac{\Delta/2 \pm j2}{2RC} = \frac{1}{RC} \left( \frac{\Delta}{4} \pm j \right)$$

Q.E.D.

13.20

The transmission of the filter normalized to the centre frequency,  $\omega_0$  is:

$$|T(j\omega)| = \frac{\omega\omega_0/Q}{(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}}$$

$$= \frac{\gamma_Q \left( \frac{\omega_0}{\omega} \right)}{\left( \left( \frac{\omega_0}{\omega} \right)^2 - 1 \right)^2 + \frac{1}{Q^2} \left( \frac{\omega_0}{\omega} \right)^2}$$

Relative to the amplitude of the fundamental

(a) The second harmonic = 0

(b) The third harmonic

$$= \frac{1}{3} \frac{\gamma_{20} \times \gamma_3}{\left( \frac{1}{9} - 1 \right)^2 + \left( \frac{1}{20} \right)^2 \left( \frac{1}{9} \right)} = 6.25 \times 10^{-3}$$

(c) The fifth harmonic

$$= \frac{1}{5} \frac{\frac{1}{20} \times \frac{1}{5}}{\left( \frac{1}{25} - 1 \right)^2 + \left( \frac{1}{20} \right)^2 \left( \frac{1}{25} \right)} = 2.08 \times 10^{-3}$$

(d) The 4<sup>th</sup> harmonic = 6<sup>th</sup> = 10<sup>th</sup> = 0

7<sup>th</sup> harmonic =  $1.04 \times 10^{-3}$

9<sup>th</sup> " =  $0.625 \times 10^{-3}$

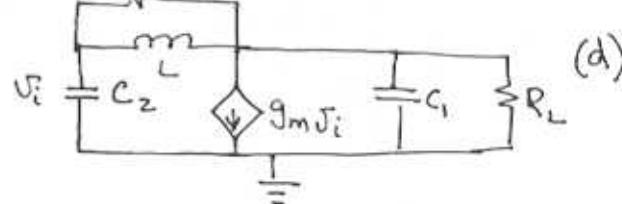
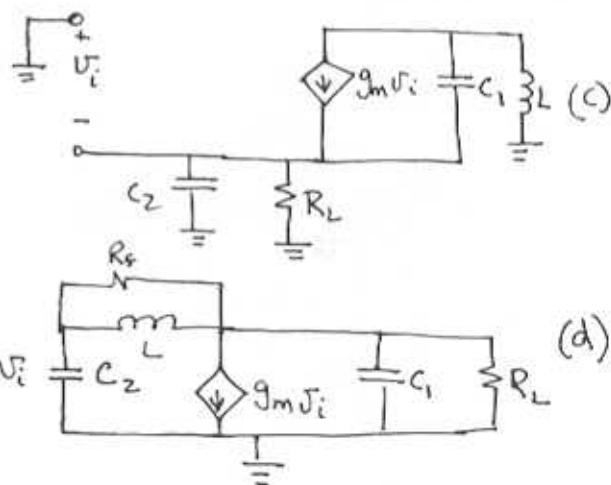
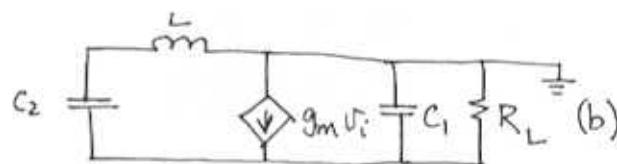
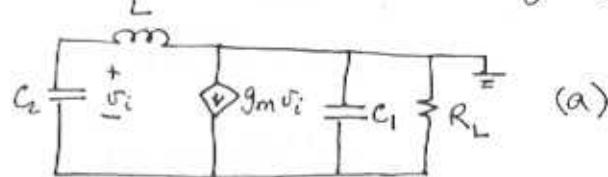
$\therefore$  RMS of 2<sup>nd</sup> to 10<sup>th</sup> harmonic is  
RMS of fundamental

$$\left[ 6.25^2 + 2.08^2 + 1.04^2 + 0.625^2 \right]^{1/2} \times 10^{-3}$$

$$= \underline{6.7 \times 10^{-3}} \text{ OR } \underline{0.7\%}$$

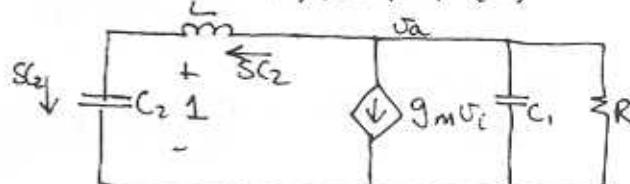
13.21

Consider the small signal models for each circuit. Assume  $R_f$  very large:



Given  $R_f \gg \omega_0 L$ , circuits (a), (b) and (d) are the same except for the reference (ground) node.

For circuits (a), (b) & (d)



- Break the loop at  $V_i$  and assume unit return.

CONT.

$$U_a = 1 + SC_2 SL$$

$$= 1 + S^2 C_2 L$$

$\sum I = 0$  at  $U_a$

$$g_m + SC_2 + SC_1 \left( 1 + S^2 C_2 L \right) + \frac{(1 + S^2 C_2 L)}{R} = 0$$

$$\therefore g_m + \frac{1}{R} + S(C_1 + C_2) + \frac{S^2 C_2 L}{R} + \frac{S^3 C_2 L}{R} = 0$$

This is the characteristic equation.  
For  $S = j\omega$ :

$$g_m + \frac{1}{R} - \frac{\omega^2 C_2 L}{R} + j(C_1 + C_2)\omega - \frac{S^3 C_2 L}{R} = 0$$

IMAGINARY PART = 0:

$$C_1 + C_2 = \frac{\omega^2 C_1 C_2 L}{R}$$

$$\omega = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}} \equiv \text{Frequency of Oscillation}$$

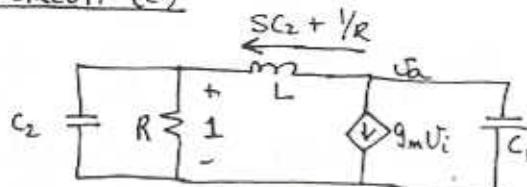
REAL PART = 0

$$g_m + \frac{1}{R} = \frac{\omega^2 C_2 L}{R}$$

$$g_m R = \left( \frac{C_1 + C_2}{C_1 C_2 L} \right) C_2 L - 1$$

$$g_m R = \frac{C_2}{C_1} \equiv \text{LIMIT ON GAIN}$$

FOR CIRCUIT (c)



$$U_a = \left( SC_2 + \frac{1}{R} \right) SL + 1$$

$\sum I = 0$  at  $U_a$ ,  $U_i = 1$

$$g_m + SC_2 + \frac{1}{R} + SG \left[ SL \left( SC_2 + \frac{1}{R} \right) + 1 \right] = 0$$

$$g_m + \frac{1}{R} + SC_2 + S^3 C_1 C_2 L + \frac{S^2 C_2 L}{R} + SC_1 = 0$$

THE CHARACTERISTIC EQUATION  $\equiv$

$$g_m + \frac{1}{R} + S(C_1 + C_2) + \frac{S^2 C_2 L}{R} + S^3 C_1 C_2 L = 0$$

Note this is the same as above,  
with  $C_1 \leftrightarrow C_2$

$$\therefore \omega_0 = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}} \quad \text{and} \quad g_m R = \frac{C_1}{C_2}$$

13.22

(a) frequency of oscillations  $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\begin{aligned} \text{gain} > 1 &\quad \text{gain} = \frac{R_c}{2R_e} = \frac{R_c}{2U_T/I/2} \\ &= \frac{IR_c}{4U_T} \end{aligned}$$

for  $U_T = 0.025V$  then

$$IR_c > 4 U_T$$

$$R_c > 0.1/I \quad \text{for oscillations to start.}$$

(b) for  $R_c = \frac{1}{I}$  ( $k\Omega$ ) we have

$$\text{gain} = \frac{IR_c}{2\left(\frac{2U_T}{I}\right)} = \frac{1}{4 \times 0.025} = 10$$

CONT.

Oscillations will start ( $10 > 1$ ) and grow until  $Q_1, Q_2$ , go into cut off. Out put will go from  $V_{cc}$  to  $V_{cc} - IR_c = V_{cc} - I$ . Therefore, output will be  $1V$  p-p. Fundamental has a p-p amplitude.

$\int \frac{4}{\pi} = 1.27 \text{ V p-p.}$

13.23

From Exercise B.10,

$$L = 0.52 \text{ H}$$

$$C_s = 0.012 \mu\text{F}$$

$$C_p = 4 \mu\text{F}$$

From (B.27)

$$C_{eq} = \frac{C_s \left( C_p + \frac{C_1 C_2}{C_1 + C_2} \right)}{C_s + C_p + \frac{C_1 C_2}{C_1 + C_2}}$$

$$C_2 = 10 \mu\text{F} \quad C_1 = 1 \text{ to } 10 \mu\text{F}$$

$$C_s = \frac{0.012 \left( 4 + \frac{10 \times 1}{10+1} \right)}{\left( 0.012 + 4 + \frac{10}{11} \right)} = 0.01197 \mu\text{F}$$

$$C_p = \frac{0.012 \left( 4 + \frac{10 \times 10}{10+10} \right)}{\left( 0.012 + 4 + \frac{100}{20} \right)} = 0.01198 \mu\text{F}$$

$$\therefore f_{out} = \frac{1}{2\pi \left[ 0.52 \times 0.01197 \times 10^{-12} \right]^{1/2}}$$

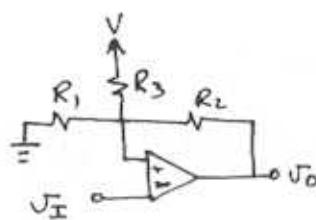
$$= 2.0172 \text{ MHz}$$

$$f_{out} = \left[ 2\pi \left( 0.52 \times 0.01198 \times 10^{-12} \right)^{1/2} \right]^{-1}$$

$$= 2.01612 \text{ MHz}$$

$$\text{Difference} = 1 \text{ kHz!}$$

13.24



$\sum I$  at +ve node:

$$\frac{V_{TH}}{R_1} = \frac{V - V_{TH}}{R_3} + \frac{L - V_{TH}}{R_2}$$

$$V_{TH} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{V}{R_3} + \frac{L}{R_2}$$

$$\begin{aligned} V_{TH} &= \left( \frac{V}{R_3} + \frac{L}{R_2} \right) \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} \\ &= \left( \frac{V}{R_3} + \frac{L}{R_2} \right) R_1 \parallel R_2 \parallel R_3 \end{aligned}$$

Similarly

$$V_{RL} = \left( \frac{V}{R_3} + \frac{L}{R_2} \right) \left( R_1 \parallel R_2 \parallel R_3 \right)$$

(b) Now

$$V_{TH} = 5.1 = \left( \frac{15}{R_3} + \frac{13}{R_2} \right) \left( \frac{1}{10} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

$$\frac{5.1}{10} + \frac{5.1}{R_2} + \frac{5.1}{R_3} = \frac{15}{R_3} + \frac{13}{R_2}$$

$$0.51 = \frac{7.9}{R_2} + \frac{9.9}{R_3} \quad (1)$$

AND

$$V_{RL} = 4.9 = \left( \frac{15}{R_3} - \frac{13}{R_2} \right) \left( \frac{1}{10} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

$$0.49 = -\frac{17.9}{R_2} + \frac{10.1}{R_3} \quad (2)$$

$$(1) \times \frac{10.1}{9.9} \Rightarrow 0.52 = \frac{8.06}{R_2} + \frac{10.1}{R_3} \quad \left. \begin{array}{l} \text{SUBTRACT TO GET} \\ \text{DOWN} \end{array} \right\}$$

$$(2) \Rightarrow 0.49 = -\frac{17.9}{R_2} + \frac{10.1}{R_3} \quad \left. \begin{array}{l} \text{SUBTRACT TO GET} \\ \text{DOWN} \end{array} \right\}$$

CONT.

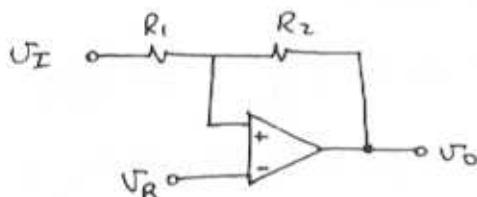
$$0.62 - 0.49 = \frac{8.06 + 17.9}{R_2}$$

$$R_2 = \frac{23.96}{0.0303} = \underline{\underline{856.8 \text{ k}\Omega}}$$

$$\frac{10.1}{R_3} = 0.49 + \frac{17.9}{856.8}$$

$$R_3 \approx \underline{\underline{19.8 \text{ k}\Omega}}$$

13.25



for  $U_I = U_{TL}$  and  $U_O = L+$  initially

$$\frac{L+ - U_R}{R_2} = \frac{U_R - U_{TL}}{R_1}$$

$$U_{TL} = U_R - \frac{R_1}{R_2} U_R - \frac{R_1}{R_2} L+$$

$$\therefore U_{TL} = \underline{\underline{U_R \left( 1 - \frac{R_1}{R_2} \right) - \frac{R_1}{R_2} L_+}}$$

Similarly

$$\frac{L- - U_R}{R_2} = \frac{U_R - U_{TH}}{R_1}$$

$$U_{TH} = \underline{\underline{U_R \left( 1 + \frac{R_2}{R_1} \right) - \frac{R_1}{R_2} L_-}}$$

(b) Given  $L_+ = -L_- = V$

$$R_1 = 10 \text{ k}\Omega$$

$$U_{TL} = 0$$

$$U_{TH} = \frac{V}{10}$$

Substituting these values we get:

$$0 = U_R \left( 1 + \frac{10}{R_2} \right) - \frac{10}{R_2} V \quad (1)$$

$$\frac{V}{10} = U_R \left( 1 + \frac{10}{R_2} \right) + \frac{10}{R_2} V \quad (2)$$

$$(1) - (2) \quad -\frac{V}{10} = -\frac{20}{R_2} V$$

$$R_2 = \underline{\underline{200 \text{ k}\Omega}}$$

$$0 = U_R \left( 1 + \frac{10}{200} \right) - \frac{10}{200} V$$

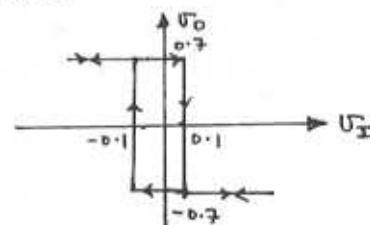
$$U_R = \frac{\frac{10}{200} V}{1 + \frac{10}{200}} = \underline{\underline{47.62 \text{ mV}}}$$

13.26

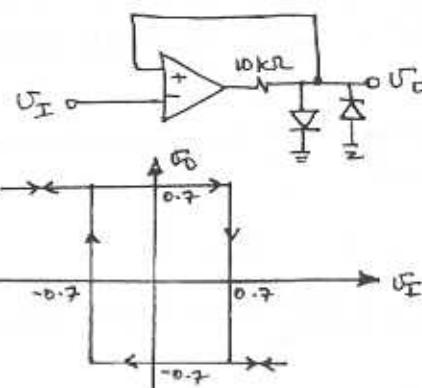
Output levels =  $\pm 0.7 \text{ V}$

Threshold levels =  $\pm \frac{10}{10+60} \times 0.7 = 0.1 \text{ V}$

$$i_{D,\max} = \frac{12 - 0.7}{10} = \frac{0.7}{10+60} = \underline{\underline{1.12 \text{ mA}}}$$



13.27



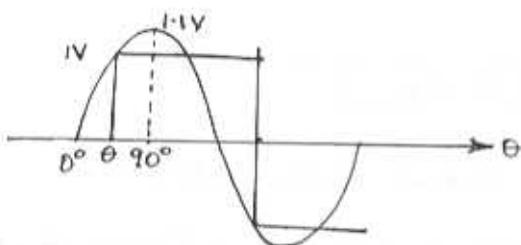
13.28

(a) A 0.5 V peak sine wave, is not large enough to change the state of the circuit. Hence, the output will be either +12V or -12V at DC.

(b) The 1.1 V peak will change the state when

$$1.1 \sin \theta = 1$$

$$\theta = 65.4^\circ$$



∴ The output is a symmetric square wave at frequency  $f$ , and lags the sine wave by an angle of  $65.4^\circ$ . The square wave has a swing of  $\pm 12V$ .

Since  $V_{TH} = -V_{TL} = 1V$ , if the average shift by an amount so either the +ve or -ve swing is  $< 1V$ , then no change of state will occur. Clearly, if the shift is 0.1V, the output will be a DC voltage.

13.29

For  $L_+ = -L_- = 7.5V$

$\underline{V_Z = 6.8V}$  with  $V_D = 0.7V$ .

For  $V_{TH} = -V_{TL} = 7.5V \Rightarrow R_1 = R_2$

$$\text{for } V_D = 0 \quad I_{R_2} = 0.1mA = \frac{7.5}{R_1 + R_2}$$

$$\Rightarrow R_1 = R_2 = \underline{37.5 \text{ k}\Omega}$$

$$I_D = 1mA = \frac{12 - 7.5}{R} - \frac{7.5}{2R_1}$$

$$I = \frac{4.5}{R} + 0.1$$

$$R = \underline{4.1 \text{ k}\Omega}$$

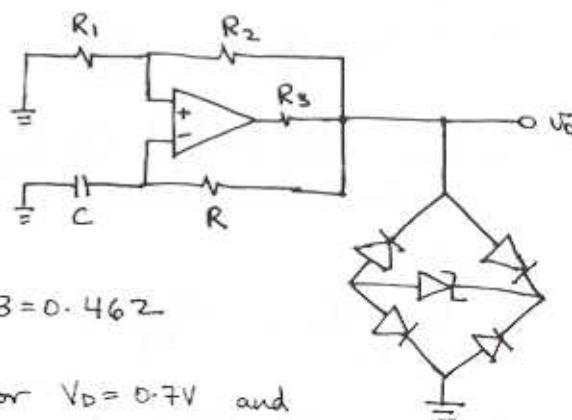
13.30

$$T = 2\tau \ln \frac{1+\beta}{1-\beta} \quad \beta = \frac{R_1}{R_1 + R_2} = \frac{10}{26}$$

$$T = 2(10 \times 10^{-9})(6.2 \times 10^3) \ln \left( \frac{1+10/26}{1-10/26} \right)$$

$$T = 1.006 \text{ ms} \Rightarrow f = \underline{994.5 \text{ Hz}}$$

13.31



$$\beta = 0.462$$

for  $V_D = 0.7V$  and

$$V_0 = \pm 6V$$

$$V_Z = 5 - 2V_D$$

$$\underline{V_Z = 3.6V}$$

CONT.

$$T = 2\tau \ln \left( \frac{1+B}{1-B} \right)$$

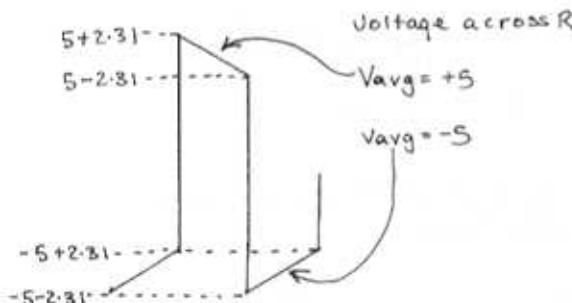
$$10^{-3} = 2\tau \ln \left( \frac{1.462}{1-0.462} \right) \Rightarrow \tau = 0.5 \text{ msec}$$

$$\tau = RC \quad \Rightarrow R = \frac{\tau}{C} = \underline{50 \text{ k}\Omega}$$

$$\text{Thresholds} = \pm 0.462 \times 5 = \pm 2.31 \text{ V}$$

Average current in R in  $\frac{1}{2}$  cycle:

$$I = \frac{1}{R} \left( \frac{5 - 2.31 + 2.31 + 5}{2} \right) \\ = \frac{5}{R} = \frac{5}{50 \text{ k}} = \underline{0.1 \text{ mA}}$$



$$R_1 + R_2 = \frac{5 \text{ V}}{0.1 \text{ mA}} = 50 \text{ k}\Omega$$

$$\frac{R_1}{R_1 + R_2} = 0.462 \rightarrow R_1 = 50(0.462) \\ = 23.1 \text{ k}\Omega$$

$$I = \frac{13 - 5}{R_3} \cdot 0.1 - 0.1 \quad \therefore R_2 = 26.9 \text{ k}\Omega \\ \text{use } \underline{R_1 = 23 \text{ k}\Omega} \\ \underline{R_2 = 27 \text{ k}\Omega} \\ I = I_{\text{DIODE}}$$

$$R_3 = \frac{8}{1.2} \\ = \underline{6.67 \text{ k}\Omega}$$

13.32

From Fig 13.23(b), for  $\pm 5 \text{ V}$  outputs  
 $V_Z = 5 - 2V_{\text{DIODE}} = 5 - 1.4 = \underline{3.6 \text{ V}}$

For  $\pm 5 \text{ V}$  out:

$$R_1 = R_2 \quad L_+ = -L_- = 5 \text{ V} \\ V_{TH} = -V_{TL} = 5 \text{ V}$$

Max current in feedback network =  $0.2 \text{ mA}$

$$\therefore 0.2 = \frac{5}{R_1 + R_2} \Rightarrow R_1 = R_2 = \underline{25 \text{ k}\Omega}$$

Max diode current =  $1 \text{ mA}$

$$\therefore \frac{13 - 5}{R_2} = (0.2 + 1) \text{ mA} \\ R_2 = \frac{8}{1.2} = \underline{6.67 \text{ k}\Omega}$$

Now from Fig 13.25(c)

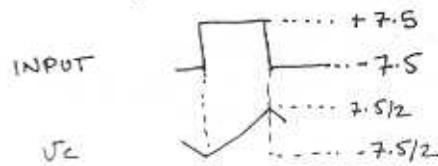
$$\text{slope} = \frac{-L_-}{RC} \Rightarrow \frac{V_{TH} - V_{TL}}{T/2} \quad \text{for } f = 1 \text{ kHz} \\ T = 10^{-3} \text{ s} \\ C = \underline{0.01 \mu\text{F}}$$

$$\frac{5}{RC} = \frac{10}{10^{-3}/2} \Rightarrow R = \underline{25 \text{ k}\Omega}$$

13.33

$$\text{For } 15 \text{ Vpp output } V_Z = \frac{15}{2} - 0.7 \\ = \underline{6.8 \text{ V}}$$

For the integrator:



i.e.  $V_c$  should ramp between  $V_{TH}$  &  $V_{TL}$ !

CONT.

$$V_C(t_1) = \frac{1}{RC} \int_{t_0}^{t_1} v dt + V_c(t_0)$$

-  $v$  is a square wave

$$\frac{7.5}{2} = \frac{1}{RC} (t_1 - t_0) (7.5 - (-7.5)) - \frac{7.5}{2}$$

$(t_1 - t_0) = \frac{T}{2}$

$$7.5 = \frac{1}{RC} \frac{T}{2} (15)$$

$$I = \frac{T}{RC} \Rightarrow R = \frac{I}{C} = \frac{1}{f_C}$$

$$= \frac{1}{10^4 (0.5 \times 10^{-9})}$$

$$\therefore \underline{R = R_{1-6} = 200k\Omega}$$

Minimum zero current = 1mA

$$\frac{13-7.5}{R_p} = 1 + \frac{7.5}{R_1+R_2} + \frac{2.5 - V_c}{R_3}$$

Maximum current into the integrator when

$$V_c = -\frac{7.5}{2}$$

$$\therefore \frac{5.5}{7.5} = 1 + \frac{7.5}{400} + \frac{11.25}{200}$$

$$\therefore R_p = 5.12k\Omega \xrightarrow{\text{use}} \underline{R_p = 5.1k\Omega}$$

Integrator output is triangular, with period = 100μs and ±7.5V peaks.  
(i.e. 2x voltage at capacitor)

**13.34**

See sketches that follow:

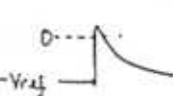
$$V_A(t=T) = -V_{ref} = -(L_+ - L_-) e^{-T/RC}$$

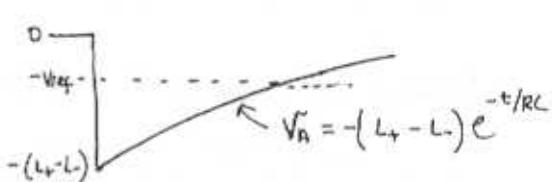
$$\frac{V_{ref}}{L_+ - L_-} = e^{-T/RC}$$

$$T = -RC \ln \left( \frac{V_{ref}}{L_+ - L_-} \right) = RC \ln \left( \frac{L_+ - L_-}{V_{ref}} \right)$$

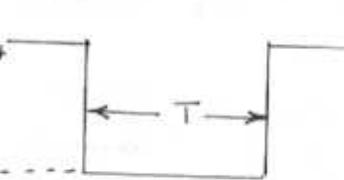
Q.E.D.

Trigger: 

$v_B$ : 

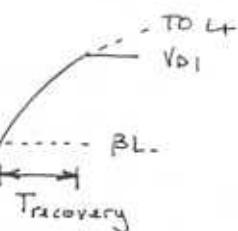
$v_A$ : 

$v_A = -(L_+ - L_-) e^{-t/RC}$

$v_O$ : 

**13.35**

For recovery,  $v_B$  goes from  $\beta L_-$  to  $L_+$  until  $D_1$  conducts at  $V_{D1} = 0.7V$

$v_B$ : 

TD L+

$v_{D1}$

$\beta L_-$

Recovery

For recovery

$$v_B = -0.1(12) + (12 + 1.2)(1 - e^{-t/\tau})$$

$$= 12 - 13.2 e^{-t/\tau}$$

At Recovery:

$$v_{D1} = 12 - 13.2 e^{-\tau_r/\tau} \quad \tau = R_3 C_1$$

$$\tau_r = -R_3 C_1 \ln \left( \frac{V_{D1}-12}{13.2} \right)$$

$$= -(617)(0.1 \times 10^{-6}) \ln \left( \frac{11.3}{13.2} \right)$$

$$= \underline{96 \mu s}$$

13.36

$$\text{choose } C_1 = \text{InF} \quad C_2 = 0.1\text{InF}$$

$$R_1 = R_2 = 100\text{k}\Omega \Rightarrow \beta = \frac{1}{2}$$

$$T \approx C_1 R_3 \ln \left( \frac{0.7 + 13}{-13(0.5 - 1)} \right)$$

$$10^{-4} = 10^{-9} R_3 \ln \left( \frac{13 + 7}{13(0.5)} \right)$$

$$\underline{R_3 = 134.1\text{k}\Omega}$$

Need  $R_4 \gg R_1 \Rightarrow \text{choose } R_4 = 470\text{k}\Omega$

$$\begin{aligned} \text{Min trigger voltage} &= \beta L + -V_{O2} + V_{O1} \\ &= \underline{6.5V} \end{aligned}$$

For recovery

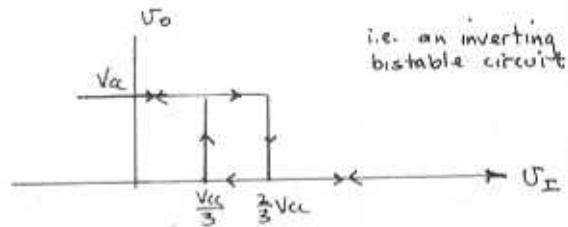
$$\begin{aligned} V_B &= 13 - (13 - \beta L) e^{-t/\tau} \\ &= 13 - 19.5 e^{-t/10} = 0.7 \end{aligned}$$

$$\begin{aligned} \therefore t_{\text{recovery}} &= -\tau \ln \left( \frac{12.3}{19.5} \right) \\ &= (134.1 \times 10^3 \times 10^{-9}) (-0.4608) \\ &= \underline{61.8\mu s} \end{aligned}$$

13.37

For  $V_I > \frac{2}{3}V_{cc}$  comp-1 = "1" and comp-2 = "0" and flip flop is reset. I.E.  $V_o = 0V$ . Now  $V_o$  will not change until  $V_I = \frac{1}{3}V_{cc}$ , when comp-2 = "1" and comp-1 = "0" and FF is set: I.E.  $V_o = V_{cc}$

For  $\frac{1}{3}V_{cc} < V_I < \frac{2}{3}V_{cc}$ , comp-1 = comp-2 = "0" and no change of state will occur.



13.38

$$(a) C = \text{InF} \quad V_C = V_{cc} (1 - e^{-t/\tau}) \quad \text{where } \tau = RC$$

Pulse width of  $10\mu s$  when  $V_C = V_{TH}$   
 $= \frac{2}{3}V_{cc}$

$$\begin{aligned} \therefore \frac{2}{3} &= 1 - e^{-t/RC} \quad t = T = 10\mu s \\ -\frac{T}{RC} &= \ln \left( \frac{1}{3} \right) \Rightarrow R = \frac{-T}{C \ln(1/3)} \\ &= \underline{9.1\text{k}\Omega} \end{aligned}$$

(b) for  $T = 20\mu s \quad R = 9.1\text{k}\Omega, C = \text{InF}$

$$\begin{aligned} \therefore V_{TH} &= 15 \left( 1 - e^{-T/RC} \right) \\ &= 15 \left( 1 - e^{-\frac{20 \times 10^{-6}}{9.1 \times 10^3 \times 10^{-9}}} \right) \\ &= \underline{13.3V} \end{aligned}$$

13.39

$$C = 680\text{pF} \quad f = 50\text{kHz}$$

$$T = 20\mu s = T_H + T_L$$

$$\text{For } 75\% \text{ Duty} \quad T_H = 15\mu s \quad T_L = 5\mu s$$

From Eq (13.43) we have:

CONT.

$$T_L = CR_B \ln 2$$

$$\therefore R_B = \frac{5 \times 10^{-6}}{680 \times 10^{-12} \ln(2)} = \underline{\underline{10.6 \text{ k}\Omega}}$$

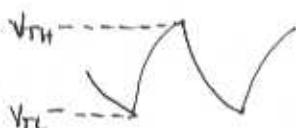
From Eq (13.41)

$$T_H = C(R_A + R_B) \ln(2)$$

$$R_A = \frac{15 \times 10^{-6}}{680 \times 10^{-12} \ln(2)} = 10.6 \times 10^3$$

$$= \underline{\underline{21.2 \text{ k}\Omega}}$$

13.40



For the rise:

$$V_C = V_{CC} - (V_{CC} - V_{TL}) e^{-t/C(R_A + R_B)}$$

$$V_{TH} = V_{CC} - (V_{CC} - V_{TL}) e^{-T_H/C(R_A + R_B)}$$

$$\frac{V_{CC} - V_{TH}}{V_{CC} - V_{TL}} = e^{-T_H/C(R_A + R_B)}$$

$$\underline{T_H = C(R_A + R_B) \ln \left( \frac{V_{CC} - V_{TL}}{V_{CC} - V_{TH}} \right)}$$

For exponential fall:

$$V_C = V_{TH} e^{-t/R_B}$$

$$\therefore V_{TL} = V_{TH} e^{-T_L/C R_B}$$

$$T_L = C R_B \ln \left( \frac{V_{TH}}{V_{TL}} \right)$$

$$\text{for } V_{TH} = 2 V_{TL} \Rightarrow T_L = C R_B \ln(2)$$

$$(b) C = 1nF, R_A = 7.2k\Omega, R_B = 3.6k\Omega$$

$$V_{CC} = 6V \quad V_{TH, ext} = 0$$

$$\therefore T_H + T_L = T = \ln 2 (R_A + 2R_B) C$$

$$T = 9.98 \mu s \rightarrow f = \underline{\underline{100 \text{ kHz}}}$$

$$\text{Duty cycle} = \frac{T_H}{T_H + T_L} = \frac{R_A + R_B}{R_A + 2R_B} = 0.75$$

$$\Rightarrow \underline{\underline{75 \%}}$$

$$(c) V_{CC} = 5V, V_{TH} = \frac{2}{3} \times 5 = \frac{10}{3} = 3.33V$$

$$\text{for IV input } V_{TH}^I = 4.33V$$

$$V_{TL}^I = \frac{1}{2} V_{TH}^I = 2.17V$$

$$T_H^I = 10^{-9} (3.6 + 7.2) \times 10^3 \ln \left( \frac{5 - 2.17}{5 - 4.33} \right)$$

$$= \underline{\underline{15.6 \mu s}}$$

$$T_L^I = 10^{-9} \times 3.6 \times 10^3 \ln 2 = \underline{\underline{2.5 \mu s}}$$

$$\therefore f = \frac{1}{(15.6 + 2.5) 10^{-9}} = \underline{\underline{55.2 \text{ kHz}}}$$

$$\text{duty cycle} = \frac{15.6}{2.5 + 15.6} = \underline{\underline{86.2 \%}}$$

$$\text{for IV input } V_{TH}^{II} = 2.33$$

$$V_{TL}^{II} = 1.17$$

$$\therefore T_H^{II} = 10^{-9} (3.6 + 7.2) \times 10^3 \ln \left( \frac{5 - 1.17}{5 - 2.33} \right)$$

$$= 3.92 \mu s$$

$$T_L^{II} + T_L^I = 2.5 \mu s$$

$$\therefore f = \frac{10^6}{(3.92 + 2.5)} = \underline{\underline{156 \text{ kHz}}}$$

$$\text{duty cycle} = \frac{3.92}{2.5 + 3.92} = \underline{\underline{61 \%}}$$

13.41

$$\text{for sine wave: } V_o = 0.7 \sin \omega t$$

slope at zero crossing

$$0.7\omega \cos \omega t \mid t=0 = \frac{1}{4}$$

$$= 0.7\omega = 0.7(2\pi f)$$

Slope of triangular wave with peak of  $V$  volts and period  $T_f$  is:

$$\frac{V - 0}{T_f/2} = \frac{2V}{\frac{T_f}{2}} = 4Vf$$

Equating the slopes:

$$4Vf = 0.7(2\pi f)$$

$$V = 0.7 \times \frac{2\pi}{4} = \underline{\underline{1.0996}}$$

$$\text{Now } R = \frac{1.0996 - 0.7}{1 \text{ mA}} = 399.6 \Omega \\ \approx \underline{\underline{400 \Omega}}$$

$r_D$  changes by  $0.1 \text{ V}$  per decade change in current.

$$\therefore V_o = 0.7 + 0.1 \log \left( \frac{I_x}{I_x} \right)$$

$$\Rightarrow I_x = 10^{\frac{V_o - 0.7}{0.1}} \text{ mA}$$

For output of  $0.7 \text{ V}$   $I_x = 1 \text{ mA}$  and  
 $\theta = 90^\circ$   
Error =  $0\%$

$$\text{output} = 0.65 \text{ } i_x = 0.316 \text{ mA}$$

$$\therefore V_I = 0.65 + 0.316 \times 0.4$$

$$= 0.7765 \text{ V}$$

$$\theta = \frac{0.7765}{1.0996} \times 90^\circ = 63.6^\circ$$

$$\text{output} = 0.6 \text{ V } i_x = 0.1 \text{ mA}$$

$$V_I = 0.6 + 0.1 \times 0.4 = 0.640 \text{ V}$$

$$\theta = \frac{0.640}{1.0996} \times 90 = 52.4^\circ$$

$$\text{out} = 0.55 \text{ V } i_x = 0.0316 \text{ mA}$$

$$V_I = 0.55 + 0.0316 \times 0.4 \\ = 0.563 \text{ V}$$

$$\theta = \frac{0.563}{1.0996} \times 90 = 46.1^\circ$$

$$\text{out} = 0.5 \text{ V } i_x = 0.01 \text{ mA}$$

$$V_I = 0.5 + 0.01 \times 0.4 = 0.504 \text{ V}$$

$$\theta = \frac{0.504}{1.0996} \times 90^\circ = 41.3^\circ$$

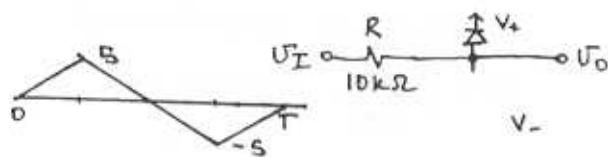
OUT (V)

0.4	32.8°
0.3	24.6°
0.2	16.4°
0.1	8.2°
0	0°

Summarizing in Table Form:

VO(V)	θ	0.7 sin θ	% Error
0.7	90	0.7	0
0.65	63.6	0.627	3.7
0.6	52.4	0.554	8.2
0.55	46.1	0.504	9.1
0.50	41.3	0.462	8.2
0.40	32.8	0.379	5.5
0.30	24.6	0.291	3.0
0.20	16.4	0.198	1.2
0.10	8.2	0.0998	0.1
0.00	0	0.0	0.0

13.42



$$V_D = A \sin \frac{2\pi}{T} t$$

Slope of  $V_D$  at  $t = 0$ ,

$$\begin{aligned}\frac{dV_D}{dt} &= A \frac{2\pi}{T} \cos\left(\frac{2\pi}{T} t\right) \Big|_{t=0} \\ &= \frac{A \cdot 2\pi}{T} \quad \text{SLOPE AT ZERO CROSSING}\end{aligned}$$

$$\text{Slope of } \Delta\text{-wave} = \frac{5}{T/4} = \frac{20}{T}$$

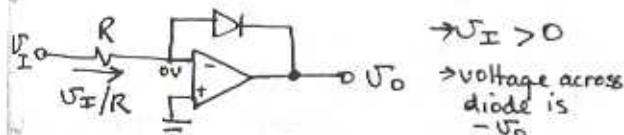
$$\therefore \frac{20}{T} = \frac{A \cdot 2\pi}{T}$$

$$A = 3.18 \text{ V}$$

• Clamp voltage:

$$\begin{aligned}V_+ &= -V_- = 3.18 - 0.7 \\ &= 2.48 = \underline{\underline{2.5 \text{ V}}}\end{aligned}$$

13.43

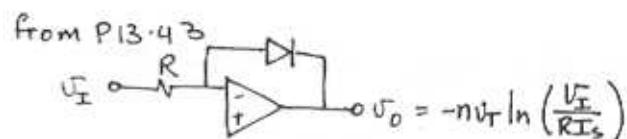


$$i_D = \frac{V_I}{R} = I_s e^{-V_O/nV_T}$$

$$-\frac{V_O}{nV_T} = \ln\left(\frac{V_I}{RI_s}\right)$$

$$V_O = -nV_T \ln\left(\frac{V_I}{RI_s}\right), \quad V_I > 0 \quad \underline{\underline{\text{Q.E.D.}}}$$

13.44



Now,

$$V_A = -nV_T \ln \frac{V_1}{RI_s} \quad R = 1k\Omega$$

$$V_B = -nV_T \ln \frac{V_2}{RI_s} \quad V_1, V_2 > 0$$

$$V_C = +nV_T \ln \frac{1}{RI_s}$$

$$V_D = -(V_A + V_B + V_C)$$

$$= nV_T \left( \ln \left[ \frac{V_1}{RI_s} \times \frac{V_2}{RI_s} \times \frac{RI_s}{1} \right] \right)$$

$$= nV_T \ln \left( \frac{V_1 V_2}{RI_s} \right)$$

$$i_{D4} = I_s e^{V_D/nV_T}$$

$$= I_s \times \frac{V_1 V_2}{RI_s} = \frac{V_1 V_2}{R}$$

$$V_O = -i_{D4} R = -\frac{V_1 V_2}{R} \times R$$

$$\therefore \underline{\underline{V_O = -V_1 V_2}}$$

ANALOG  
MULTIPLIER

To check  $\underline{\underline{V_1 = 0.5, \quad V_2 = 2}}$

$$\begin{aligned}I_{D1} &= 0.5 \text{ mA} \rightarrow V_A = -0.7 + nV_T \ln\left(\frac{0.5}{1}\right) \\ &= 0.7 + 2(0.025) \ln\left(\frac{1}{2}\right) \\ &= -0.6653 \text{ V}\end{aligned}$$

$$\begin{aligned}I_{D2} &= 2 \text{ mA} \rightarrow V_B = (0.7 + 0.05 \ln(2))(-1) \\ &= -0.7347 \text{ V}\end{aligned}$$

$$I_{D3} = 1 \text{ mA} \rightarrow V_C = 0.700 \text{ V}$$

$$V_D = -(-0.6653 - 0.7347 + 0.7) = 0.7 \text{ V}$$

CONT.

$$V_D = V_{D4} = 0.7V \Rightarrow I_{D4} = 1mA$$

$$\therefore V_O = -1V \text{ i.e. } 2 \times 0.5 = 1$$

For  $V_1 = 3, V_2 = 2$ :

$$I_{D1} = 3mA \rightarrow V_A = -(0.7 + 0.05 \ln 3) = -0.7549V$$

$$I_{D2} = 2mA \rightarrow V_A = -(0.7 + 0.05 \ln 2) = -0.7347V$$

$$I_{D3} = 1mA \rightarrow V_C = 0.7V$$

$$\therefore V_D = V_{D4} = -(V_A + V_B + V_C) = +0.7896V$$

$$\therefore \frac{I_{D4}}{1mA} = \frac{I_S e^{\frac{V_D}{0.05}}}{I_S e^{\frac{0.7+0.05}{0.05}}} = 6$$

$$I_{D4} = e^{\frac{0.7896 - 0.7}{0.05}} = 6mA$$

$$\therefore V_O = -6V \text{ i.e. } 2 \times 3 = 6$$

For squarer:  $V_1 = 2$  through  $\frac{1}{2}k\Omega$  resistor

$$I_{D1} = 4mA \rightarrow V_A = -(0.7 + 0.05 \ln 4) = -0.7693$$

$$V_D = -(-0.7693) = 0.7693V$$

$$I_{D4} = e^{\frac{0.7693 - 0.7}{0.05}} = 3.999mA$$

$$\therefore V_D = -3.999V \text{ i.e. } 2^2 = 4$$

13. 45

$$\text{Say } V_{BE} = \tilde{V}_D @ I \quad n=1$$

for  $V_D = 0.25V_T$ :

$$I_R = \frac{0.25V_T}{R} = \frac{0.25V_T}{2.5V_T} = \frac{I}{10}$$

$$V_{BE1} = \tilde{V}_D + nV_T \ln\left(\frac{I + I/10}{I}\right) \approx \tilde{V}_D + V_T \ln(1.1)$$

$$V_{BE2} = \tilde{V}_D + nV_T \ln\left(\frac{I - I/10}{I}\right) \approx \tilde{V}_D + V_T \ln(0.9)$$

$$V_I = -V_{BE2} + V_D + V_{BE1}$$

$$= V_T [ \ln(1.1) + 0.25 - \ln(0.9) ]$$

$$= \underline{0.451V_T}$$

$$\text{For } V_O = 0.5V_T$$

$$I_R = \frac{0.5 I}{2.5} = 0.2 I$$

$$V_I = V_T [ \ln(1.2) + 0.5 - \ln(0.8) ]$$

$$= \underline{0.905V_T}$$

$$V_D = V_T \quad I_R = 0.4 I$$

$$V_I = V_T [ \ln 1.4 + 1 - \ln 0.6 ]$$

$$= \underline{1.847V_T}$$

$$V_D = 1.5V_T \quad I_R = 0.6 I$$

$$V_I = V_T [ \ln 1.6 + 1.5 - \ln 0.4 ]$$

$$= \underline{2.886V_T}$$

$$V_D = 2V_T \quad I_R = 0.8 I$$

$$V_I = V_T [ \ln 1.8 + 2 - \ln 0.2 ] = \underline{4.197V_T}$$

$$V_D = 2.4V_T \quad I_R = 0.96 I$$

$$V_I = V_T [ \ln 1.96 + 2.4 - \ln 0.04 ] = \underline{6.292V_T}$$

$$V_D = 2.42V_T \quad I_R = 0.968 I$$

$$V_I = V_T [ \ln 1.968 + 2.42 - \ln 0.032 ]$$

$$= \underline{6.519V_T}$$

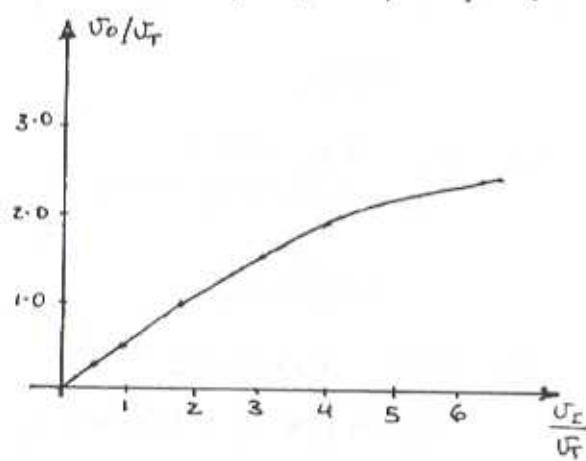
CONJ.

Ideal curve given by

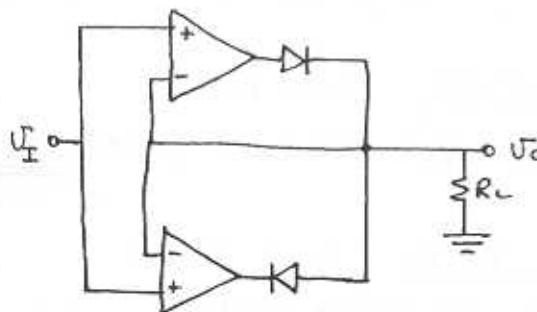
$$U_o = 2.42 V_T \sin\left(\frac{U_I}{6.6 V_T} \times 90^\circ\right)$$

$$\frac{U_I}{V_T} = \frac{6.6}{90} \sin^{-1}\left(\frac{U_o}{2.42 V_T}\right)$$

$\frac{U_o}{V_T}$	0.25	0.60	1.00	1.50	2.00	2.40	2.42
$\frac{U_I}{V_T}$	0.451	0.905	1.85	2.89	4.20	6.29	6.52
$\frac{U_I}{V_T}$ (ideal)	0.435	0.874	1.79	2.81	4.09	6.06	6.60



13.46



for high opamp gain  $U_o = U_I$

∴  $U_o$  is also a 10Vpp sine wave.

for  $U_I > 0$

$$U_o = U_I - \frac{0.7}{A}$$

$$\approx U_I$$

with  $D_1$  "on"

$D_2$  "off"

for  $U_I < 0$

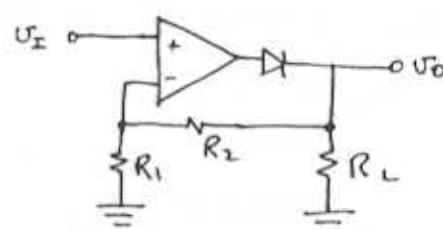
$$U_o = U_I + \frac{0.7}{A}$$

$$\approx U_I$$

with  $D_1$  "off"

$D_2$  "on"

13.47

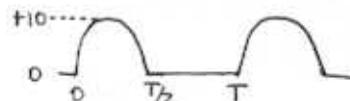


for  $U_I \geq 0$

$$U_o = \left(1 + \frac{R_2}{R_1}\right) U_I$$

for a gain of 2  $\underline{R_1 = R_2 = 10k\Omega}$

for  $U_I = 10V_{pp}$  Sine wave  $U_o = ?$



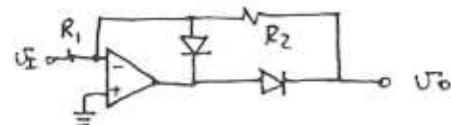
$$Avg = \frac{1}{T} \int_0^{T/2} 10 \sin \frac{2\pi}{T} t dt$$

$$= \frac{1}{T} \frac{1}{2\pi} \cos \frac{2\pi}{T} t \Big|_{t=0}^{T/2}$$

$$= \frac{-10}{2\pi} (\cos \pi - \cos 0)$$

$$= 10/\pi = \underline{3.18V}$$

13.48



CONT.

for  $V_I < 0 \Rightarrow V_O = -R_2/R_1$

$$R_{in} = R_1 = 100k\Omega \quad \therefore R_2 = 200k\Omega$$

13.49

for high  $R_{in}$ , use  $R_1 = 1M\Omega$

AC gain is given by  $R_2/R_1$

$$\Rightarrow R_2 = 1M\Omega$$

Now for 1Vrms sine, peak is 1.414V.  
The value of  $V_i$  is then  $\frac{1.414}{\pi} = 0.450V$

For 10V out at second stage

$$\text{gain (dc)} = \frac{10}{0.450} = 22.2$$

$$\therefore R_4/R_3 = 22.2$$

& choose  $\frac{1}{2\pi R_4 C} = 10\text{Hz}$  (i.e. corner frequency)

To make  $C$  small, make  $R_4 = 1M\Omega$

$$\therefore C = 15.9\text{nF}$$

$$R_3 = \frac{1M\Omega}{22.2} = 45k\Omega$$

13.50

At the +ve terminal  $V_+ = -5V$

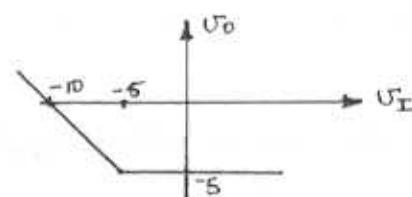
for  $V_I > -5$   $D_1$  is "on" and faces virtual short.  $\therefore V_- = -5$ , and no current will flow in feedback resistor  $R$ .

$$\therefore V_O = -5V$$

for  $V_I < -5 \sim D_1$  "off" and

$$\frac{V_O}{V_I} = -\frac{-5 - V_I}{R} = \frac{V_O + 5}{R}$$

$$\Rightarrow V_O = -V_I -$$



13.51

for  $V_I < 0$

$D_2$  "off"

$$\frac{V_{O1}}{V_I} = -1$$

$$\frac{V_{O2}}{V_I} = 0$$

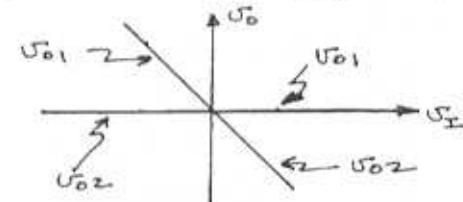
$$\frac{V_{O2}}{V_I} = -1$$

for  $V_I > 0$

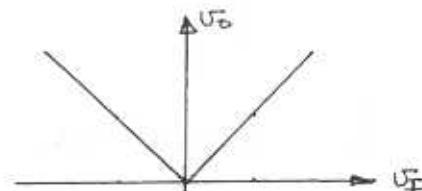
$D_1$  ~ "off"

$$\frac{V_{O1}}{V_I} = 0$$

$$\frac{V_{O2}}{V_I} = -1$$



13.52



For  $V_I < 0 \sim$  Diode is on, and cathode is forced to  $\approx 0V$ .

$$\therefore V_O/V_I = -1$$

For  $V_I > 0 \sim$  Diode is off, and the cathode now follows  $V_I$  since no current flows in Resistor. So  $V_O$  must follow.

CONT.

$V_I = 0$  so that no current flows in feedback resistor.

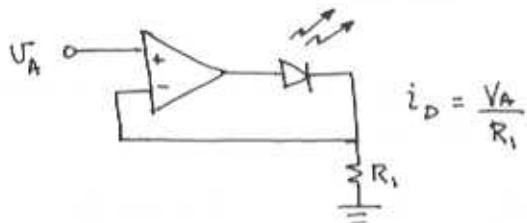
$$\therefore \frac{V_O}{V_I} = +1$$

$$\frac{1}{10^3} |\sin 2\pi 60t| = 2\pi 60 V_C |\cos 2\pi 60t|$$

$$\therefore C = \frac{1}{2\pi 60 \cdot 10^3} = \underline{2.65 \mu F}$$

13.53

Simply place the LED in the feedback path.



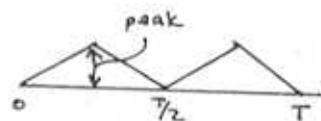
$$\text{At } 120\text{Hz: } i_m = 2\pi 120 V_C |\cos 2\pi 60t|$$

$$i_{m_{120}} = 2 i_{m_{60}}$$

$$\text{At } 180\text{Hz: } i_m = 3 i_{m_{60}}$$

For D-wave

$$\text{with } R_1 \\ i_m = 1\text{mA}, R = 1\text{k}\Omega$$



$\therefore$  Full wave rectified wave has average voltage =  $V$ .  $\therefore V_{peak} > 2V$

$$\text{with } C: \text{slope} = \frac{V_{peak}}{T/4} = 4V_{peak} f$$

$$= 4 \times 2 \times 60 = 480$$

Now, current through the capacitor will be a square wave (50% duty cycle)

$$\text{Peak current} = 2.65 \times 10^{-6} \times 480 \\ = 1.27 \text{mA}$$

$$\therefore i_m = i_{avg} = \underline{1.27 \text{mA}}$$

13.54

$$i_m = |i_c| = C \frac{dV}{dt} \quad \text{using } R = 1\text{k}\Omega$$

$$i_m = |i_R| = \frac{|V|}{R} = \frac{|V|}{1\text{k}\Omega} \Rightarrow i_m = |V| \text{ mA}$$

$$\text{Now } V = V \sin 2\pi 60t$$

$$\Rightarrow i_m = C \times 2\pi 60 |\cos(2\pi 60t)|$$

for equivalence:

10V pulses of  $10\mu s$ , and large  $C_{load}$ , will cause the op-amp to current limit.

Charge transferred in one pulse:

$$Q = (10\text{mA})(10\mu s) \\ = 10^{-7} C$$

CONT

Voltage change per pulse:

$$\Delta V = Q/C = \frac{10^{-7}}{10 \times 10^{-6}} = 10 \text{ mV}$$

After: 1 pulse  $V_c = 10 \text{ mV}$

2 pulses  $20 \text{ mV}$

10 pulses  $100 \text{ mV}$

To reach  $0.5 \text{ V}$  require 50 pulses

$1.0 \text{ V}$  " 100 "

$2.0 \text{ V}$  " 200 "

13.56

For  $1V_{pp}$ , peak detector output =

$$V_o = 0.5 \text{ V}.$$

$$\text{Ripple Voltage} = (10\%) 0.5 = 5 \text{ mV}$$

$$\text{Total leakage} = 10 + 1 = 11 \text{nA}$$

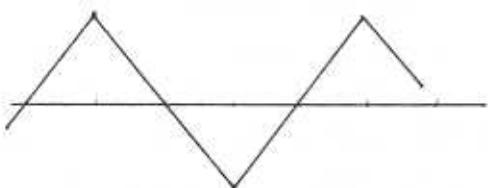
$\therefore$  total charge lost:

$$\Delta Q = 11 \text{nA} \times 1 \text{ ms} = 11 \text{ pC}$$

$\therefore$  Required capacitance:

$$C = \frac{Q}{\Delta V} = \frac{11 \times 10^{-12}}{5 \times 10^{-3}} = \underline{\underline{2.2 \text{nF}}}$$

$\xleftarrow[1 \text{ ms}]{} \rightarrow$



## Chapter 14 - Problems

[14]

[14.1] Refer to Fig. 14.2 (Page 123). The upper limit of the output voltage is determined by the saturation of  $Q_1$ . ~

$$V_{O\max} = V_{CC} - V_{CE\text{sat}}$$

$$= 5 - 0.3 = \underline{4.7 \text{ V}}$$

The corresponding input is

$$V_I = 4.7 + 0.7 = \underline{5.4 \text{ V}}$$

The bias current  $I$  is

$$I = \frac{0 - (-V_{CC} + V_{BE2})}{R}$$

$$= \frac{5 - 0.7}{1} = 4.3 \text{ mA}$$

The lower limit of  $V_O$  is determined by either  $Q_1$  cutting off,

$$\frac{-V_O}{R_L} = I \Rightarrow V_O = -4.3 \text{ V}$$

or by  $Q_2$  saturating,

$$V_O = -V_{CC} + V_{CE\text{sat}} = -4.7 \text{ V}$$

Obviously,  $V_{O\min} = \underline{-4.5 \text{ V}}$

and the corresponding input is

$$V_I = -4.3 + 0.7 = \underline{-3.6 \text{ V}}$$

If the emitter-base junction area of  $Q_2$  is [14] made twice as large as that of  $Q_1$ ,  $I$  becomes one half its previous value,

$$I = \frac{4.3}{2} = 2.15 \text{ mA}$$

and thus the lower limit of  $V_O$  changes to

$$V_{O\min} = -IR_L = \underline{-2.15 \text{ V}}$$

The corresponding value of  $V_I$  is

$$V_I = -2.15 + 0.7 = \underline{-1.45 \text{ V}}$$

The upper limit does not change.

[14.2] First we determine the

bias current  $I$  as

follows:

$$I = \frac{1}{2} I_{C\text{sat}} (V_{GS} - V_t)^2$$

$$\text{But } V_{GS} = 5 - IR \\ = 5 - I$$

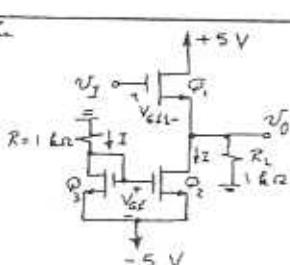
Thus

$$I = \frac{1}{2} I_{C\text{sat}} (5 - I - V_t)^2$$

$$I = 10 (5 - I - 1)^2$$

$$\Rightarrow I^2 - 8I + 16 = 0$$

$$I = 3.416 \text{ mA} \quad \text{and } V_{GS} = 1.584 \text{ V}$$



The upper limit on  $V_O$  is determined [14] by  $Q_1$  leaving the saturation region (and entering the triode region). This occurs when  $V_I$  exceeds  $V_{D1}$  by  $V_t$  volts,

$$V_{I\max} = 5 + 1 = \underline{+6 \text{ V}}$$

To obtain the corresponding value of  $V_O$  we must find the corresponding value of  $V_{GS1}$ , as follows:

$$V_O = V_I - V_{GS1}$$

$$I_L = \frac{V_O}{R_L} = \frac{V_I - V_{GS1}}{1}$$

$$= V_I - V_{GS1} = 6 - V_{GS1}$$

$$i_1 = I + i_L$$

$$= 3.416 + 6 - V_{GS1}$$

$$= 9.416 - V_{GS1}$$

$$\text{But } i_1 = \frac{1}{2} I_{C\text{sat}} (V_{GS1} - V_t)^2$$

$$\text{Thus, } 9.416 - V_{GS1} = 10 (V_{GS1} - 1)^2$$

$$\Rightarrow V_{GS1}^2 - 19 V_{GS1} + 9.0584 = 0$$

$$V_{GS1} = 1.869 \text{ V}$$

$$V_{O\max} = 6 - 1.869$$

$$= \underline{+4.131 \text{ V}}$$

The lower limit of  $V_O$  is determined either by  $Q_1$  cutting off,

$$V_O = -IR_L = -3.416 \times 1 = -3.416 \text{ V}$$

or by  $Q_2$  leaving saturation,

$$V_O = V_{GS} - V_t$$

$$= -5 + 1.584 - 1 = -4.416 \text{ V}$$

$$\text{Thus, } V_{O\min} = \underline{-3.416 \text{ V}}$$

The corresponding value of  $V_I$  is determined by noting that since  $Q_1$  is on the verge of cut-off,  $V_{GS1} = V_t = 1 \text{ V}$  and

$$V_I = -3.416 + 1 = \underline{-2.416 \text{ V}}$$

[14.3] Refer to Fig. 14.2. With  $V_{CC} = +9 \text{ V}$ , the upper limit on  $V_O$  is  $+8.7 \text{ V}$ , which is greater than the required value of  $+7 \text{ V}$ . To obtain a lower limit of  $-7 \text{ V}$  we select  $I$  so that

$$IR_L = 7$$

$$\Rightarrow I = 7 \text{ mA}$$

Since we are provided with four devices, we can minimize the total supply current by paralleling two devices to form  $Q_2$  as shown below.

The resulting supply current will be

$3 \times \frac{I}{2}$  rather than  $2I$  which is the value obtained

in the circuit of Fig. 14.2.

Thus the supply current is  $10.5 \text{ mA}$ . The value of  $R$  is found from

$$R = \frac{8.3 \text{ V}}{3.5 \text{ mA}} = 2.37 \text{ k}\Omega$$

In a practical design we would select a standard value for  $R$  that results in  $I$  somewhat larger than  $7 \text{ mA}$ . Say,  $R = 2.6 \text{ k}\Omega$

14.4 Refer to Fig 14.2. For a load resistance of  $100 \text{ }\Omega$  and  $V_O$  ranging between  $-5 \text{ V}$  and  $+5 \text{ V}$ , the maximum current through  $Q_1$  is

$$I + \frac{5}{0.1} = I + 50, \text{ mA}$$

the minimum current is

$$I - \frac{5}{0.1} = I - 50, \text{ mA}$$

For a current ratio of 10,

$$\frac{I+50}{I-50} = 10$$

$$\Rightarrow I = 61.1 \text{ mA}$$

$$R = \frac{9.3 \text{ V}}{61.1 \text{ mA}} = 152 \text{ }\Omega$$

The incremental voltage gain is

$$A_V = \frac{R_L}{R_L + R_{\text{el}}}$$

For  $R_L = 100 \text{ }\Omega$ :

$$\text{At } V_O = +5 \text{ V}, I_{EL} = 61.1 + 50 = 111.1 \text{ mA}$$

$$R_{\text{el}} = \frac{25}{111.1} = 0.225 \text{ }\Omega$$

$$A_V = \frac{100}{100 + 0.225} = 0.998 \text{ V/V}$$

14

$$\text{At } V_O = 0 \text{ V}, I_{EL} = 61.1 \text{ mA}$$

$$R_{\text{el}} = \frac{25}{61.1} = 0.40952$$

$$A_V = \frac{100}{100 + 0.40952} = 0.996 \text{ V/V}$$

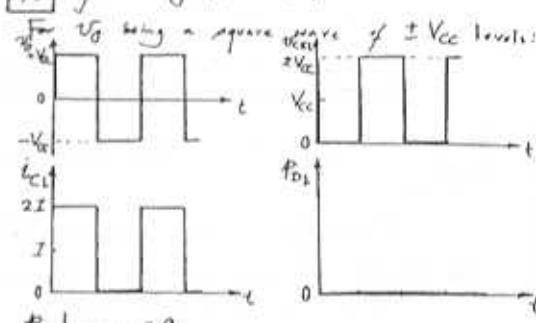
$$\text{At } V_O = -5 \text{ V}, I_{EL} = 61.1 - 50 = 11.1 \text{ mA}$$

$$R_{\text{el}} = \frac{25}{11.1} = 2.25 \text{ }\Omega$$

$$A_V = \frac{100}{100 + 2.25} = 0.978 \text{ V/V}$$

Thus the incremental gain changes by  $0.998 - 0.978 = 0.02$  or about  $\underline{\underline{2\%}}$  over the range of  $V_O$ .

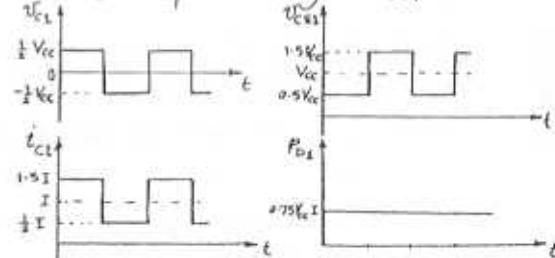
14.5 Refer to Fig 14.2 and 14.4



$$P_{D1/\text{average}} = 0$$

For the corresponding sine-wave case [Fig. 14.4],  $P_{D1/\text{avg}} = \frac{1}{2}V_{CC}I$

For  $V_O$  a square wave of  $\pm V_{CC}/2$  levels:



$$P_{D1/\text{average}} = 0.75 V_{CC} I$$

for a sine-wave output of peak-to-peak amplitude

$$V_{CL} = \frac{1}{2}V_{CC} \sin \theta$$

$$I_{C1} = I + \frac{1}{R_L} V_{CC} \sin \theta = I + \frac{1}{2} I \sin \theta$$

$$V_{CEL} = V_{CC} - \frac{1}{2} V_{CC} \sin \theta$$

$$P_{D1} = (V_{CC} - \frac{1}{2} V_{CC} \sin \theta)(I + \frac{1}{2} I \sin \theta)$$

$$= V_{CC} I - \frac{1}{4} V_{CC} I \sin^2 \theta$$

$$= V_{CC} I - \frac{1}{4} V_{CC} I \times \frac{1}{2}(1 - \cos 2\theta)$$

$$= \frac{7}{8} V_{CC} I + \frac{1}{8} V_{CC} I \cos 2\theta$$

$$P_{D1/\text{average}} = \frac{7}{8} V_{CC} I$$

[14.6] In all cases, the average voltage [14] across  $Q_2$  is equal to  $V_{CC}$ . Thus, since  $Q_2$  conducts a constant current  $I$ , its average power dissipation is  $\underline{V_{CC} I}$ .

[14.7]  $V_{CC} = 16, 12, 10 \text{ and } 8 \text{ V}$   
 $I = 100 \text{ mA} \quad R_L = 100 \Omega$

$$\hat{V}_o = 8 \text{ V}$$

$$I = \frac{1}{4} \left( \frac{\hat{V}_o}{R_L} \right) \left( \frac{\hat{V}_o}{V_{CC}} \right)$$

$$= \frac{1}{4} \left( \frac{8}{100} \right) \left( \frac{8}{V_{CC}} \right) = \frac{1.6}{V_{CC}}$$

$V_{CC}$	16	12	10	8
$\eta$	10%	13.3%	16%	20%

[14.8] Refer to Fig. P14.8. The bias current  $I$  is

$$I = \mu_{FE,0} (0 - (-2))^2 = 10 \times 4 = 40 \text{ mA}$$

For  $R_L = \infty$ , the maximum  $\hat{V}_o$  is limited by the saturation of  $Q_1$  to  $V_{CC} - V_{CE,0} = 19.7 \text{ V}$ , and the minimum  $\hat{V}_o$  is limited by  $Q_2$  leaving saturation to  $-V_{CC} + |V_t| = -3 \text{ V}$ .

For  $R_L = 100 \Omega$ , the maximum  $\hat{V}_o$  is [14] still  $+4.7 \text{ V}$ , and the minimum  $\hat{V}_o$  is still  $-3 \text{ V}$  because at this level the current in  $R_L$  is  $-30 \text{ mA}$  and thus  $Q_1$  is still conducting  $10 \text{ mA}$ . (That is, the cut off limit of  $-40 \text{ mA} \times 100 \Omega = -4 \text{ V}$  is not applicable since  $Q_2$  would have entered the triode region.)

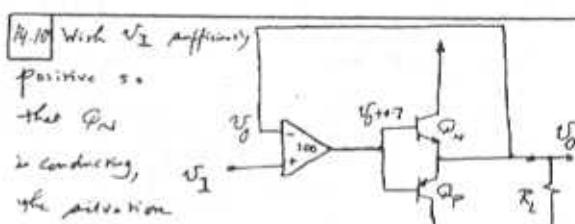
To obtain a 1-V peak sine-wave output,  $R_L$  must be limited to ( $\leq$ )

$$R_L = \frac{1 \text{ V}}{40 \text{ mA}} = \underline{25 \Omega}$$

For this case,

$$\begin{aligned} \eta &= \frac{1}{4} \left( \frac{\hat{V}_o}{R_L} \right) \left( \frac{\hat{V}_o}{V_{CC}} \right) \\ &= \frac{1}{4} \left( \frac{1}{0.04 \times 25} \right) \left( \frac{1}{2} \right) \\ &= 5 \% \end{aligned}$$

[14.9] Refer to Figs. 14.6 and 14.7. A 10% [14] loss in peak amplitude is obtained when the amplitude of the input signal is 5 V.



$$(V_I - V_O) \times 100 = V_O + 0.7$$

$$\Rightarrow V_O = \frac{1}{1.01} (V_I - 0.007)$$

This relationship applies for  $V_I \geq 0.007$ .

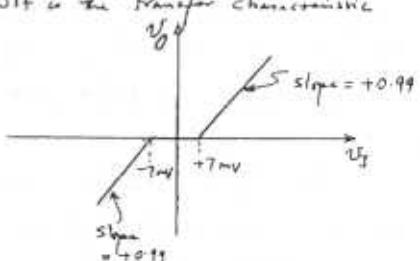
Similarly, for  $V_I$  sufficiently negative so that  $Q_P$  conducts, the voltage at the output of the amplifier becomes  $V_O - 0.7$ , then

$$(V_I - V_O) \times 100 = V_O - 0.7$$

$$\Rightarrow V_O = \frac{1}{1.01} (V_I + 0.007)$$

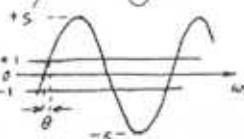
This relationship applies for  $V_I \leq -0.007$  [14].

The result is the transfer characteristic



Without the feedback arrangement, the deadband becomes  $\pm 700 \text{ mV}$  and the slope change is little (to  $\pm 1 \text{ V/V}$ ).

[14.11] With  $R_L = \infty$  and  $V_I = +5\text{ V}$ ,  $\hat{V}_O$  will be  $V_I - V_{GSL} = V_I - V_t = 4\text{ V}$  (since the current is virtually zero and thus  $V_{GS} \approx V_t$ ). Thus the resulting peak output voltage will be  $4\text{ V}$ .  
 $\sin \theta = \frac{1}{5}$   
 $\Rightarrow \theta = 11.54^\circ$   
 Cross-over interval =  $180^\circ$   
 Fraction of cycle =  $\frac{180}{360} = \underline{12.8\%}$



For  $V_I = +5\text{ V}$  and  
 $\hat{V}_O = +2.5\text{ V}$ ,  
 $V_{GS} = 2.5\text{ V}$ , then  
 $\ell_D = M_A V_s (V_{GS} - V_t)^2$   
 $= 0.1 (2.5 - 1)^2 = 0.225 \text{ mA}$   
 Then,  $R_L = \frac{2.5}{0.225} = \underline{11.1 \Omega}$

[14.12] For  $V_{CC} = 10\text{ V}$  and  $R_L = 100\Omega$ , the maximum sine-wave output power occurs when  $\hat{V}_o = V_{CC}$  and is

$$P_{max} = \frac{1}{2} \frac{\hat{V}_o^2}{R_L} = \frac{1}{2} \times \frac{100}{100} = \underline{0.5 \text{ W}}$$

Correspondingly,

$$P_{S+} = P_{S-} = \frac{1}{\pi} \frac{\hat{V}_o}{R_L} V_{CC} = \frac{1}{\pi} \times \frac{10}{100} \times 10 = 0.318 \text{ W}$$

for a total supply power of

$$P_S = 2 \times 0.318 = \underline{0.637 \text{ W}}$$

The power conversion efficiency  $\eta$  is

$$\eta = \frac{P_L}{P_S} \times 100 = \frac{0.5}{0.637} \times 100 = \underline{78.5\%}$$

For  $\hat{V}_o = 5\text{ V}$ ,

$$P_L = \frac{1}{2} \frac{\hat{V}_o^2}{R_L} = \frac{1}{2} \times \frac{25}{100} = \frac{1}{8} \text{ W}$$

$$P_{S+} = P_{S-} = \frac{1}{\pi} \frac{\hat{V}_o}{R_L} V_{CC} = \frac{1}{\pi} \times \frac{5}{100} \times 10 = \frac{1}{2\pi} \text{ W}$$

$$P_S = \frac{1}{\pi} \text{ W} = \underline{0.318 \text{ W}}$$

$$\eta = \frac{1/8}{1/\pi} \times 100 = \frac{\pi}{8} \times 100 = \underline{39.3\%}$$

[14.13]  $V_{CC} = 5\text{ V}$   
 For maximum  $\eta$ ,  
 $\hat{V}_o = V_{CC} = \underline{5\text{ V}}$   
 The output voltage that results in maximum device dissipation is given by Eq.(7.20),  
 $\hat{V}_o = \frac{2}{\pi} V_{CC}$   
 $= \frac{2}{\pi} \times 5 = \underline{3.18 \text{ V}}$   
 If operation is always at full output voltage,  $\eta = 78.5\%$  and thus  
 $P_{dissipation} = (1-\eta) P_S$   
 $= (1-\eta) \frac{P_L}{2} = \frac{1-0.785}{2} \frac{P_L}{0.785} = 0.214 P_L$   
 $P_{dissipation}/device = \frac{1}{2} \times 0.214 P_L = 0.107 P_L$

For a rated device dissipation of  $1\text{ W}$ , and using a factor of 2 safety margin,  
 $P_{dissipation}/device = 0.5 \text{ W}$   
 $= 0.107 P_L$

$$\Rightarrow P_L = \underline{3.65 \text{ W}}$$

$$3.65 = \frac{1}{2} \times \frac{25}{R_L}$$

$$\Rightarrow R_L = \underline{3.425 \Omega} \quad (\text{i.e. } R_L \geq 3.425 \Omega)$$

The corresponding output power (i.e. greatest possible output power) is  $\underline{3.65 \text{ W}}$ .

If operation is allowed at  $\hat{V}_o = \frac{1}{2} V_{CC} = 2.5\text{ V}$ ,

$$\ell = \frac{\pi}{4} \frac{\hat{V}_o}{V_{CC}} \quad (\text{Eq. 7.15})$$

$$= \frac{\pi}{4} \times \frac{1}{2} = 0.393$$

$$P_{dissipation}/device = \frac{1}{2} \frac{1.7}{\eta} P_L = 0.772 P_L$$

$$0.5 = 0.772 P_L$$

$$\Rightarrow P_L = \frac{0.647}{0.772} \text{ W}$$

$$= \frac{1}{2} \frac{2.5}{R_L} \text{ W}$$

$$\Rightarrow R_L = \underline{4.83 \Omega} \quad (\text{i.e. } \geq 4.83 \Omega)$$

$$[14.14] P_L = \frac{1}{2} \frac{\hat{V}_o^2}{R_L}$$

$$100 = \frac{1}{2} \frac{\hat{V}_o^2}{16}$$

$$\hat{V}_o = 56.6 \text{ V}$$

$$V_{CC} = 56.6 + 6 = 60.6 \rightarrow \underline{61 \text{ V}}$$

$$\text{Peak current from each supply} = \frac{\hat{V}_o}{R_L} = \frac{56.6}{16} = 3.54 \text{ A}$$

$$P_{S+} = P_{S-} = \frac{1}{2} \times 3.54 \times 61$$

$$\text{Thus, } P_S = \frac{2}{\pi} \times 3.54 \times 61$$

$$= 137.4 \text{ W}$$

$$\eta = \frac{P_L}{P_S} = \frac{100}{137.4} = \underline{73\%}$$

Using Eq. (14.22),

$$P_{DNmax} = P_{DPmax} = \frac{V_{CC}^2}{\pi^2 R_L} = \frac{61^2}{\pi^2 \times 16}$$

$$= \underline{23.6 \text{ W}}$$

$$[14.15] P_L = \frac{\hat{V}_o^2}{R_L}$$

$$P_{S+} = P_{S-} = \frac{1}{2} \left( \frac{\hat{V}_o}{R_L} \right) V_{SS}$$

$$P_S = \frac{\hat{V}_o}{R_L} V_{SS}$$

$$\eta = \frac{P_L}{P_S} = \frac{\hat{V}_o^2 / R_L}{\hat{V}_o V_{SS} / R_L} = \frac{\hat{V}_o}{V_{SS}}$$

$\eta_{max} = \pm 100\%$ , obtained for  $\hat{V}_o = V_{SS}$

$$P_{Lmax} = \frac{\hat{V}_o^2}{R_L}$$

[14]

$$P_{Dissipation} = P_S - P_L$$

$$= \frac{\hat{V}_o}{R_L} V_{SS} - \frac{\hat{V}_o^2}{R_L}$$

$$\frac{\partial P_{Dissipation}}{\partial \hat{V}_o} = \frac{V_{SS}}{R_L} - \frac{2\hat{V}_o}{R_L}$$

$$= 0 \quad \text{for } \underline{\hat{V}_o = \frac{V_{SS}}{2}}$$

$$\text{Correspondingly, } \eta = \frac{V_{SS}/2}{V_{SS}} = \frac{1}{2} \text{ or } \underline{50\%}$$

$$[14.16] A_V = \frac{R_L}{R_L + R_{out}}$$

$$\text{where, } R_{out} = \frac{r_e}{2} = \frac{V_T}{2 I_Q}$$

For  $A_V \geq 0.99$  with  $R_L \geq 100 \Omega$ ,

$$0.99 = \frac{100}{100 + R_{out}} \Rightarrow R_{out} = 1 \Omega$$

$$\frac{V_T}{2 I_Q} = 1 \Rightarrow I_Q = \underline{12.5 \text{ mA}}$$

$$V_{BB} = 2 V_{BE}$$

$$= 2 \left[ 0.7 + V_T \ln \frac{12.5}{100} \right]$$

$$= \underline{1.296 \text{ V}}$$

[14]

$$[14.17] R_{out} = \frac{1}{g_m} \parallel \frac{1}{g_m} = \frac{1}{2g_m} = 10 \Omega$$

$$\Rightarrow g_m = \frac{1}{20} = 50 \text{ mA/V}$$

$$\text{But, } g_m = k_C \frac{V}{L} (V_{GS} - V_F)$$

$$50 = 2 \times 100 (V_{GS} - 1)$$

$$\Rightarrow V_{GS} = 1.25 \text{ V}$$

$$V_{GG} = \underline{2.5 \text{ V}}$$

[14]

$$[14.18] I_Q = 1 \text{ mA}$$

$$\text{For output of } -1 \text{ V, } i_L = -\frac{1}{100} = -10 \text{ mA}$$

Using Eq. (14.27),

$$i_N^2 - L_L i_N - I_Q^2 = 0$$

$$i_N^2 + 10 i_N - 1 = 0$$

$$i_N = 0.1 \text{ mA}$$

$$i_P = \underline{10.1 \text{ mA}}$$

Then  $V_{BB}$  increases by  $V_T \ln \frac{10.1}{1} = 0.06 \text{ V}$

and the input step must be  $\underline{-1.06 \text{ V}}$

Largest possible positive output from 6 to 10,  
i.e.  $\underline{4 \text{ V}}$ .

Largest negative output from 6 to 0, i.e.  $\underline{6 \text{ V}}$

$$[14.19] R_{out} = r_e/2 = 10 \Omega$$

$$\Rightarrow r_e = 20 \Omega$$

$$I_Q = \frac{V_T}{r_e} = \frac{25}{20} = 1.25 \text{ mA}$$

$$\text{Thus, } M = \frac{1.25}{0.1} = \underline{12.5}$$

[14]

[14.20]  $I_Q \approx I_{bias} = 0.5 \text{ mA}$ , neglecting the base current of  $\beta_N$ . More precisely,

$$I_Q = I_{bias} - \frac{I_Q}{\beta+1}$$

$$\Rightarrow I_Q = \frac{I_{bias}}{1 + \frac{1}{\beta+1}} \approx 0.18 \times 0.5 = \underline{0.49 \text{ mA}}$$

The largest positive output is obtained when all of  $I_{bias}$  flows into the base of  $\beta_N$ , resulting in

$$V_O = (\beta_N + 1) I_{bias} R_L$$

$$= 51 \times 0.5 \times 100 \Omega = \underline{2.55 \text{ V}}$$

The largest possible negative output voltage is limited by the saturation of  $Q_P$  to  $-10 + V_{CE(sat)} = \underline{-10 \text{ V}}$ .

To achieve a maximum positive output of 10 V without changing  $I_{bias}$ ,  $\beta_N$  must be

$$10 = (\beta_N + 1) \times 0.5 \times 100 \Omega$$

$$\Rightarrow \beta_N = \underline{199}$$

[14]

Alternatively, if  $\beta_N$  is held at 50,  $I_{bias}$  must be increased so that

$$10 = 51 \times I_{bias} \times 100 \Omega$$

$$\Rightarrow I_{bias} = \underline{1.96 \text{ mA}}$$

for which,

$$I_Q = \frac{I_{bias}}{1 + \frac{1}{\beta+1}} = \underline{1.92 \text{ mA}}$$

[14.21] At  $20^\circ\text{C}$ ,  $I_Q = 1 \text{ mA} = I_S e^{(0.6/0.025)}$

$$\Rightarrow I_S (\text{at } 20^\circ\text{C}) = 3.78 \times 10^{-11} \text{ mA}$$

$$\text{At } 70^\circ\text{C}, I_S = 3.78 \times 10^{-11} \times (1.14)^{50}$$

$$= 2.64 \times 10^{-8} \text{ mA}$$

$$\text{At } 70^\circ\text{C}, V_T = 25 \frac{273+70}{273+20} = 29.3 \text{ mV}$$

$$\text{Thus, } I_Q (\text{at } 70^\circ\text{C}) = 2.64 \times 10^{-8} e^{(0.6/0.0293)}$$

$$= \underline{20.7 \text{ mA}}$$

$$\text{Additional current} = 20.7 - 1 = 19.7 \text{ mA}$$

$$\text{Additional power} = 2 \times 20 \times 19.7 = \underline{788 \text{ mW}}$$

$$\text{Additional temperature rise} = 10 \times 0.788 = \underline{7.9^\circ\text{C}}$$

$$\text{At } 77.7^\circ, V_T = \frac{25}{293} (273 + 77.7) = 29.1 \text{ mV}$$

$$I_Q = 3.78 \times 10^{-11} \times (1.14)^{57.4} e^{(0.6/0.0293)}$$

$$= \underline{37.6 \text{ mA}}$$

etc., etc.

[14.22] Refer to Fig. P 14.22.

$$0.100 = 1 (V_{GS3} - 1)^2$$

$$V_{GS3} = 1.316 \text{ V} = |V_{GS1}|$$

$$A_V = \frac{R_L}{R_L + R_{out}} = 0.99$$

$$\frac{1}{1 + R_{out}} = 0.99$$

$$\Rightarrow R_{out} = 0.01 \text{ k}\Omega = 10 \Omega$$

$$\text{But } R_{out} = (1/g_m) // (1/g_m)$$

$$= \frac{1}{2g_m} \Rightarrow g_m = 50 \text{ mA/V}$$

$$50 = \beta_1 (V_{GS1} - V_t)$$

$$= \beta_1 (1.316 - 1)$$

$$\Rightarrow \beta_1 = \frac{50}{1.316} = 158.2 \text{ mA/V}^2$$

$$M = \frac{158.2}{2} = \underline{79.1}$$

[14.23] Since the peak positive output current is 200 mA, the base current of  $\beta_N$  can be as high as  $\frac{200}{\beta_N + 1} = \frac{200}{51} \approx 4 \text{ mA}$ . We select  $I_{bias} = 5 \text{ mA}$ , thus providing the multiplier with a minimum current of 1 mA.

Under quiescent conditions ( $V_O = 0$  and  $i_L = 0$ ) the base current of  $\beta_N$  can be neglected. Selecting  $I_R = 0.5 \text{ mA}$  leaves  $I_{C1} = 4.5 \text{ mA}$ .

To obtain a quiescent current of 2 mA in the output transistors,  $V_{BB}$  should be

$$V_{BB} = 2 V_T \ln \frac{2 \times 10^{-3}}{10^{-13}} = 1.19 \text{ V}$$

thus

$$R_1 + R_2 = \frac{V_{BB}}{I_R} = \frac{1.19}{0.5} = 2.38 \text{ k}\Omega$$

At a collector current of 4.5 mA,  $\beta_1$  has

$$V_{BEL} = V_T \ln \frac{4.5 \times 10^{-3}}{10^{-14}} = 0.671 \text{ V}$$

The value of  $R_1$  can now be determined as

$$R_1 = \frac{0.671}{0.5} = \underline{1.34 \text{ k}\Omega}$$

and

$$R_2 = 2.38 - 1.34 = \underline{1.04 \text{ k}\Omega}$$

$$[14.24] (a) V_{BE} = 0.7 \text{ V at } 1 \text{ mA}$$

[14]

$$\text{At } 0.5 \text{ mA}, V_{BE} = 0.7 + 0.025 \ln \frac{0.5}{1} = 0.683 \text{ V}$$

$$\text{Thus } R_1 = \frac{0.683}{0.5} = 1.365 \text{ k}\Omega$$

$$\text{and } R_2 = 1.365 \text{ k}\Omega$$

(b) For  $I_{B10} = 2 \text{ mA}$ ,  $I_C$  increases to nearly 1.5 mA for which

$$V_{BE} = 0.7 + 0.025 \ln \frac{1.5}{1} = 0.710 \text{ V}$$

Note that  $I_R = \frac{0.710}{1.365} = 0.52 \text{ mA}$  is very nearly equal to the assumed value of 0.50 mA, thus no further iterations are required.

$$V_{BB} = 2 V_{BE} = 1.420 \text{ V}$$

(c) For  $I_{B10} = 10 \text{ mA}$ , assume that  $I_R$  remains constant at 0.5 mA, then  $I_C = 9.5 \text{ mA}$

$$\text{and } V_{BE} = 0.7 + 0.025 \ln \frac{9.5}{1} = 0.756 \text{ V}$$

$$\text{at which } I_R = \frac{0.756}{1.365} = 0.554 \text{ mA}$$

$$\text{Thus, } I_{C1} = 10 - 0.554 = 9.45 \text{ mA}$$

$$\text{and } V_{BE} = 0.7 + 0.025 \ln \frac{9.45}{1} = 0.756 \text{ V}$$

$$\text{Thus } V_{BB} = 2 \times 0.756 = 1.512 \text{ V}$$

(d) Now for  $\beta = 100$ ,

$$I_{R1} = \frac{0.756}{1.365} = 0.554 \text{ mA}$$

$$I_{R2} = 0.554 + \frac{9.45}{100} = 0.648 \text{ mA}$$

$$I_C = 10 - 0.648 = 9.352 \text{ mA}$$

$$\text{Thus, } V_{BE} = 0.7 + 0.025 \ln \frac{9.352}{1} = 0.756 \text{ V}$$

$$\begin{aligned} V_{BB} &= 0.756 + I_{R2} R_2 \\ &= 0.756 + 0.648 \times 1.365 \\ &= 1.641 \text{ V} \end{aligned}$$

$$[14.25] \text{ Power rating} = \frac{130 - 30}{2} = 50 \text{ W}$$

$$I_{Cav} \leq \frac{50}{20} = 2.5 \text{ A}$$

$$[14.26] \theta_{JA} = \frac{130 - 25}{0.2} = 625 \text{ }^{\circ}\text{C/W} = 0.625 \text{ }^{\circ}\text{C/mW}$$

$$\text{At } 70^{\circ}\text{C, Power rating} = \frac{150 - 70}{0.625} = 128 \text{ mW}$$

$$T_J = 50 + 0.625 \times 100 = 112.5 \text{ }^{\circ}\text{C}$$

$$[14.27] T_J \leq 50 + 3 \times 30 = 140 \text{ }^{\circ}\text{C}$$

$$\begin{aligned} V_{BE} &= 800 - 2 \times (140 - 25) = 570 \text{ mV} \\ &= 0.57 \text{ V} \end{aligned}$$

$$[14.28] (a) \theta_{JA} = \frac{T_{Jmax} - T_A}{P_D}$$

$$= \frac{100 - 25}{2} = 37.5 \text{ }^{\circ}\text{C/W}$$

$$(b) \text{ At } T_A = 50 \text{ }^{\circ}\text{C},$$

$$P_{Dmax} = \frac{T_{Jmax} - T_A}{\theta_{JA}}$$

$$= \frac{100 - 50}{37.5} = 1.33 \text{ W}$$

$$(c) T_J = 25 + 37.5 \times 1 = 62.5 \text{ }^{\circ}\text{C}$$

$$[14.29] T_C - T_A = \theta_{CA} P_D$$

$$= (\theta_{CS} + \theta_{SA}) P_D$$

$$\Rightarrow P_D = \frac{T_C - T_A}{\theta_{CS} + \theta_{SA}} = \frac{90 - 30}{0.5 + 0.1} = 100 \text{ W}$$

$$T_J - T_C = \theta_{JC} P_D$$

$$130 - 90 = \theta_{JC} \times 100$$

$$\Rightarrow \theta_{JC} = 0.4 \text{ }^{\circ}\text{C/W}$$

$$[14.30] \theta_{JC} = \frac{T_J - T_S}{P_D} = \frac{180 - 50}{50} = 2.6 \text{ }^{\circ}\text{C/W}$$

$$T_J - T_S = \theta_{JS} P_D$$

$$180 - 50 = (\theta_{JC} + \theta_{CS}) P_D$$

$$\Rightarrow T_S = 180 - (2.6 + 0.4) \times 30 = 84 \text{ }^{\circ}\text{C}$$

$$T_S - T_A = \theta_{SA} P_D$$

$$84 - 39 = \theta_{SA} \times 30$$

$$\Rightarrow \theta_{SA} = 1.5 \text{ }^{\circ}\text{C/W}$$

$$\text{Required heat-sink length} = \frac{4.5 \text{ }^{\circ}\text{C/W/cm}}{1.5 \text{ }^{\circ}\text{C/W}} = \underline{\underline{3 \text{ cm}}}$$

$$[14.31] \tau_T = \frac{n V_T}{I_B} = \frac{2 \times 25 \times 10^{-3}}{0.5} = 0.1 \text{ s}$$

$$\tau_x \approx \tau_T - \tau_T = 0.98 - 0.1 = 0.85 \text{ s}$$

$$[14.32] V_{BE'E} = V_{BE} - I_B \tau_x$$

$$= 1.05 - 0.19 \times 0.8$$

$$= 0.898 \text{ V} \quad (\text{at } I_C = 5 \text{ A})$$

For  $I_C = 2 \text{ A}$ ,

$$V_{BE'E} = 0.898 + M V_T \ln \frac{2}{5}$$

$$= 0.898 + 2 \times 0.025 \ln \left( \frac{2}{5} \right)$$

$$= 0.852 \text{ V}$$

$$V_{BE} = 0.852 + I_B \tau_x$$

$$= 0.852 + (0.19 \times \frac{2}{5}) \times 0.8$$

$$= 0.913 \text{ V}$$

14.33 (a) For  $R_L = \infty$ :

At  $V_I = 0$  V,

$$I_{B1} = I_{B2} = \frac{2.87}{200}$$

$$I_I = I_{B2} - I_{B1} = 0$$

At  $V_I = +10$  V,

$$I_{B1} = \frac{0.88}{200} \text{ mA} = 4.4 \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200} \text{ mA} = 24.4 \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = 20 \mu\text{A}$$

At  $V_I = -10$  V,

$$I_{B1} = \frac{4.87}{200} \text{ mA} = 24.4 \mu\text{A}$$

$$I_{B2} = \frac{0.88}{200} \text{ mA} = 4.4 \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = -20 \mu\text{A}$$

(b) For  $R_L = 100 \Omega$ :

At  $V_I = 0$ ,  $I_I = 0$

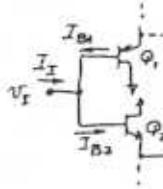
At  $V_I = +10$  V,

$$I_{B1} = \frac{0.38}{200} = 1.9 \mu\text{A}$$

$$I_{B2} = \frac{4.87}{200} = 24.4 \mu\text{A}$$

$$I_I = I_{B2} - I_{B1} = 22.5 \mu\text{A}$$

At  $V_I = -10$  V,  $I_I = -22.5 \mu\text{A}$



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Output Resistance at  $V_I = 0$ :

As a result of 2 paths to output,

$$R_{out} = \frac{1}{2} \left( R_3 + V_{BE3} + \frac{V_{EL}}{\beta_3 + 1} \right)$$

$$\text{where } V_{EL} \approx V_{CE3} = \frac{25 \text{ mV}}{40 \text{ mA}} = 0.625 \Omega$$

$$\text{Thus, } R_{out} = \frac{1}{2} \left( 0.74 + 0.625 + \frac{0.625}{51} \right) = 0.69 \Omega$$

Output Voltage for  $V_I = +1$  V and  $R_L = 2.5 \Omega$

To start the solution, let  $V_O = 1$  V,

$$\text{then } I_L \approx \frac{1}{2} = 0.5 \text{ A} = 500 \mu\text{A}$$

$$I_{B3} = \frac{500}{50} = 10 \text{ mA}$$

$$I_{E1} \approx \frac{10 - 1 - 0.7}{0.215} - 10 = 28.6 \text{ mA}$$

for which,

$$V_{EB1} = 0.7 + 0.025 \ln \frac{350}{10} = 0.726 \text{ V}$$

$$V_{B3} = 1 + 0.726 = 1.726 \text{ V}$$

Assuming  $I_{E4} \approx 0$ ,

$$V_{BE3} \approx 700 + 25 \ln \frac{500}{3 \times 10} = 770 \text{ mV}$$

$$\text{Thus, } I_L = \frac{1.726 - 0.770}{0.74 + 2} = 0.349 \text{ A}$$

for which

$$V_O = 2 \times 0.349 = 0.698 \text{ V}$$

Then, the voltage drop across the series combination of  $R_4$  and the EB junction of  $Q_4$  can be found as follows,

$$V_{B4} = V_{E4} \approx 1 - 0.74 = 0.26 \text{ V}$$

leaving a drop across the series combination of  $R_4$  and  $EBJ4$  of  $0.698 - 0.26 = 0.438$ . It follows that  $I_{E4} \approx 0$ , as assumed.

Iterate again:

$$I_L \approx 0.35 \text{ A}$$

$$I_{B3} \approx \frac{0.35}{51} \approx 7 \text{ mA}$$

$$I_{E1} \approx \frac{10 - 1 - 0.73}{0.215} - 7 = 31.5 \text{ mA}$$

$$V_{EB3} = 0.7 + 0.025 \ln \frac{31.5}{10} = 0.729 \text{ V}$$

$$V_{B3} = 1 + 0.729 = 1.729 \text{ V}$$

$$V_{BE3} = 0.7 + 25 \ln \frac{350}{31.5} = 0.761 \text{ V}$$

$$I_L = \frac{1.729 - 0.761}{0.74 + 2} = 0.353 \text{ A}$$

$$V_O = 2 \times 0.353$$

$$= 0.706 \text{ V}$$

14.34 For the design process:

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At  $V_I = +5$  V, the voltage across

$R_1$ ,  $V_{R1}$ , is

$$V_{R1} = 10 - 5 - 0.7 = 4.3 \text{ V}$$

and,  $I_{R1} = 2 \times 10 \text{ mA}$  (to allow for  $I_{B3}$  of as much as 10 mA and only a 2 to 1 variation in  $I_{E1}$ )

$$\text{Thus, } R_1 = \frac{4.3}{20} = 215 \Omega$$

Correspondingly,

$$R_2 = 215 \Omega$$

Now, for  $V_I = 0$  and  $V_{EB1} \approx 0.7$  V

$$I_{R2} = \frac{10 - 0.7 - 0}{215} = 43.3 \text{ mA}$$

$$\text{for which } V_{EB1} = 700 + 25 \ln \frac{43.3}{10} = 736.6 \text{ mV}$$

Since  $I_Q = 40 \text{ mA}$  and  $I_{S3} = 3 I_{S1}$ ,

$$V_{BE3} = 700 + 25 \ln \frac{40}{3 \times 10} = 707.2 \text{ mV}$$

$$\text{Thus, } R_3 = \frac{736.6 - 707.2}{40 \text{ mA}} = 0.74 \Omega$$

Correspondingly,

$$R_4 = 0.74 \Omega$$

**A.35**  $I_{E1} = I_{E2} \approx 2 \text{ mA}$   
 $I_Q = I_{E3} = I_{E4} = I_{E1} = I_{E2}$

Thus,  $I_Q \approx 2 \text{ mA}$

More precisely,  $I_Q = \frac{2}{\beta_1 + \beta_2} = \frac{1.96 \text{ mA}}{51} \approx 0.04 \text{ mA}$

$I_{B1} = I_{B2} = \frac{2}{51} \approx 0.04 \text{ mA}$

The net input bias current is ideally zero.

For a  $\beta$  mismatch of 10%:

$I_I = 0.10 \times 0.04 \text{ mA}$

$\approx 4 \mu\text{A}$

For  $R_L = 100 \Omega$ , the equivalent half circuit becomes

Thus,  $2R_L = (\beta_1 + 1) [r_{e1}(\beta_1 + 1)]$

where  $r_{e1} = r_{e2} = \frac{2}{\beta_1 + 1}$

$= 12.5 \Omega$

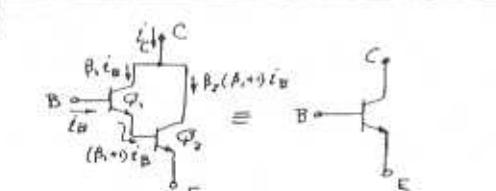
$R_L = \frac{51}{2} [0.0125 + 51(0.0125 + 0.200)]$

$\approx 276.7 \Omega$

The gain can be determined using the equivalent half-circuit, as follows:

$$A_V = \frac{V_o}{V_i} = \frac{2R_L}{2R_L + r_{e1} + r_{e2}}$$

$$= \frac{2 \times 100}{2 \times 100 + 12.5 + \frac{12.5}{51}} = 0.94 \text{ V/V}$$



$$I_C = \beta_1 i_B + \beta_2 (\beta_1 + 1) i_B$$

$$\beta_{eq} = \frac{i_C}{i_B} = \frac{\beta_1 + \beta_2 (\beta_1 + 1)}{\beta_1}$$

$$\approx \beta_1 \beta_2 \quad (\text{for } \beta_1 \gg 1 \text{ and } \beta_2 \gg 1)$$

If the composite transistor is operated at a collector current  $I_C$ , then

$$I_B = \frac{I_C}{\beta_1 + \beta_2 (\beta_1 + 1)}$$

$$I_{C1} = \frac{\beta_1}{\beta_1 + \beta_2 (\beta_1 + 1)} I_C \approx I_C / \beta_1$$

**A.37**  $I_{C2} = \frac{\beta_2 (\beta_1 + 1) I_C}{\beta_1 + \beta_2 (\beta_1 + 1)} \approx I_C$

Then,

$$V_{BE2} = V_{BE} (1 \text{ mA}) + V_T \ln \left( \frac{I_C}{1 \text{ mA}} \right)$$

$$V_{BE1} = V_{BE} (1 \text{ mA}) + V_T \ln \left( \frac{I_C}{\beta_1 \times 1 \text{ mA}} \right)$$

$$V_{BEq} = V_{BE1} + V_{BE2}$$

$$= 2V_{BE} (1 \text{ mA}) + V_T \ln \left( \frac{I_C^2}{\beta_2} \right)$$

$$r_{e2} = \frac{V_T}{I_{C2}} \approx \frac{V_T}{I_C} \approx \frac{V_T}{I_C}$$

$$r_{e1} = \frac{V_T}{I_{C1}} \approx \frac{V_T}{I_C} \approx \frac{V_T}{I_C / \beta_1} = \beta_1 r_{e2}$$

$$r_{\pi eq} = (\beta_1 + 1) [r_{e1} + (\beta_2 + 1) r_{e2}]$$

$$\approx \beta_1 [\beta_2 r_{e2} + \beta_2 r_{e2}]$$

$$= 2\beta_1 \beta_2 r_{e2} \approx 2\beta_1 \beta_2 \left( \frac{V_T}{I_C} \right)$$

To determine  $g_{m eq}$ , apply a signal to the base and find the corresponding current  $i_C$ :

$$i_C = i_{C1} + i_{C2} = g_{m1} V_{be1} + g_{m2} V_{be2}$$

$$= g_{m1} \frac{r_{e1}}{r_{e1} + (\beta_1 + 1) r_{e2}} + g_{m2} \frac{(\beta_1 + 1) r_{e2}}{r_{e1} + (\beta_1 + 1) r_{e2}}$$

$$i_C \approx V_{be} \frac{1}{2\beta_2 r_{e2}} + \frac{\beta_1}{2\beta_1 r_{e2}} V_{be}$$

$$g_{m eq} \equiv \frac{i_C}{V_{be}} \approx \frac{1}{2 r_{e2}} = \frac{1}{2} \frac{I_C}{V_T}$$

For operation at  $I_C = 10 \text{ mA}$ :

$$\beta_{eq} = \beta_1 \beta_2 = 50^2 = 2500$$

$$V_{BEq} = 1.4 + 0.025 \ln \frac{10^2}{50}$$

$$\approx 1.42 \text{ V}$$

$$r_{\pi eq} \approx 2 \times 50 \times 50 \times \frac{25}{10} = 12.5 \Omega$$

$$g_{m eq} \approx \frac{1}{2} \times \frac{10}{12.5} = 200 \text{ mA/V}$$

**A.38** DC analysis:

$$5 = I_{C2} \times 1 \text{ k}\Omega$$

$$+ \frac{I_{C2}}{\beta_2} \times 1 \text{ M}\Omega$$

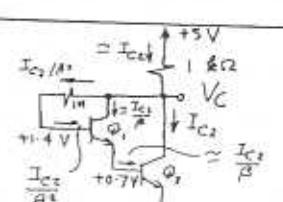
$$+ 1.4$$

$$I_{C2} = \frac{3.6}{1 + \frac{1000}{100 \times 10^3}}$$

$$= 3.3 \text{ mA}$$

$$I_{C1} = \frac{3.3}{100} = 0.033 \text{ mA}$$

$$V_C \approx 1.7 \text{ V}$$



[14.37 CONT'D]

$$I_c = g_m V_{ce_1} \\ = g_m \frac{(\beta_1 + 1) r_{e_1}}{r_{e_1} + (\beta_1 + 1) r_{e_2}} V_i$$

$$\text{But } r_{e_1} = \frac{V_T}{I_{E1}} = \frac{V_T}{I_{E2}(\beta_1)} \quad r_{e_2}(\beta_1)$$

$$\text{Then, } I_c \approx \frac{V_i}{2 r_{e_2}}$$

$$g_{m_{eq}} \approx \frac{I_c}{V_i} = \frac{1}{2 r_{e_2}} \\ = \frac{1}{2 \times (25/1.5)} = \underline{66 \text{ mA/V}}$$

$$V_o \approx - I_c \times 1 k\Omega$$

$$= - g_{m_{eq}} V_i \times 1 k\Omega$$

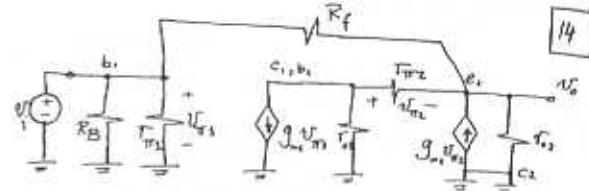
$$\frac{V_o}{V_i} = - 66 \times 1 = - 66 \text{ V/V}$$

To find  $R_{in}$ ,

$$I_i = I_{B1} + I_{1H\Omega_2 \text{ common}} \\ \approx \frac{I_c}{\beta_1^2} + \frac{V_i - V_b}{1M\Omega_2} \\ = \frac{V_i}{2 \beta_1^2 r_{e_2}} + \frac{67 V_i}{1M\Omega_2} \\ = V_i \left[ \frac{1}{2 \times 10^2 \times 33} + \frac{67}{1M\Omega_2} \right] = 73.6 V_i \quad (14)$$

$$R_{in} = \frac{V_i}{I_i} = \frac{V_i}{73.6} M\Omega_2 = \underline{13.6 k\Omega}$$

[14]



[14]

Node equation at  $b_2$ :

$$g_{m_1} V_{\pi_1} + \frac{V_{\pi_1}}{r_{\pi_1}} + \frac{V_{\pi_2}}{r_{\pi_2}} = 0$$

But  $V_{\pi_1} = V_o + V_{\pi_2}$ , then

$$g_{m_1} V_{\pi_1} + \frac{V_o + V_{\pi_1}}{r_{\pi_1}} + \frac{V_{\pi_2}}{r_{\pi_2}} = 0$$

$$\Rightarrow V_{\pi_2} \left( \frac{1}{r_{\pi_1}} + \frac{1}{r_{\pi_2}} \right) = - \left( \frac{V_o}{r_{\pi_1}} + g_{m_1} V_{\pi_1} \right)$$

$$\text{or, } V_{\pi_2} = - \frac{\frac{V_o}{r_{\pi_1}} + g_{m_1} V_{\pi_1}}{\frac{1}{r_{\pi_1}} + \frac{1}{r_{\pi_2}}}$$

Node equation at output:

$$\frac{V_o}{r_{\pi_2}} + \frac{V_o - V_{\pi_1}}{R_f} = g_{m_1} V_{\pi_1} + \frac{1}{r_{\pi_2}} V_{\pi_2}$$

$$= \left( g_{m_1} + \frac{1}{r_{\pi_2}} \right) V_{\pi_2}$$

$$= - \frac{\left( g_{m_1} + \frac{1}{r_{\pi_1}} \right) \left[ \frac{V_o}{r_{\pi_1}} + g_{m_1} V_{\pi_1} \right]}{\frac{1}{r_{\pi_1}} + \frac{1}{r_{\pi_2}}}$$

Substituting  $V_{\pi_1} = V_o$  and collecting terms, results in

$$V_o \left[ \frac{1}{r_{\pi_2}} + \frac{1}{R_f} + \frac{\left( g_{m_1} + \frac{1}{r_{\pi_2}} \right)}{r_{\pi_1} \left( \frac{1}{r_{\pi_1}} + \frac{1}{r_{\pi_2}} \right)} \right]$$

$$= - V_i \left[ \frac{g_{m_1} \left( g_{m_1} + \frac{1}{r_{\pi_2}} \right)}{\frac{1}{r_{\pi_2}} + \frac{1}{r_{\pi_1}}} - \frac{1}{R_f} \right]$$

$$\frac{V_o}{V_i} = - \frac{\frac{g_{m_1} \left( g_{m_1} + \frac{1}{r_{\pi_2}} \right)}{\frac{1}{r_{\pi_2}} + \frac{1}{r_{\pi_1}}} - \frac{1}{R_f}}{\frac{1}{r_{\pi_2}} + \frac{1}{R_f} + \frac{g_{m_1} + \frac{1}{r_{\pi_2}}}{r_{\pi_1} \left( \frac{1}{r_{\pi_1}} + \frac{1}{r_{\pi_2}} \right)}}$$

$$\text{Since } r_{\pi_2} \ll r_{\pi_1}, \quad \frac{V_o}{V_i} \approx - \frac{g_{m_1} \left( g_{m_1} r_{\pi_2} + 1 \right) - \frac{1}{R_f}}{\frac{1}{r_{\pi_2}} + \frac{1}{R_f} + \frac{1}{r_{\pi_1}} \left( g_{m_1} r_{\pi_2} + 1 \right)}$$

$$= - \frac{g_{m_1} \left( \beta_2 + 1 \right) - \frac{1}{R_f}}{\frac{1}{r_{\pi_2}} + \frac{1}{R_f} + \frac{1}{r_{\pi_1}} \left( \beta_2 + 1 \right)}$$

$$\text{Since } \frac{1}{R_f} \ll g_{m_1} \left( \beta_2 + 1 \right), \quad \frac{V_o}{V_i} \approx - \frac{g_{m_1} \left( \beta_2 + 1 \right)}{\left( \frac{1}{r_{\pi_2}} + \frac{1}{R_f} \right) + \frac{1}{r_{\pi_1}} \left( \beta_2 + 1 \right)}$$

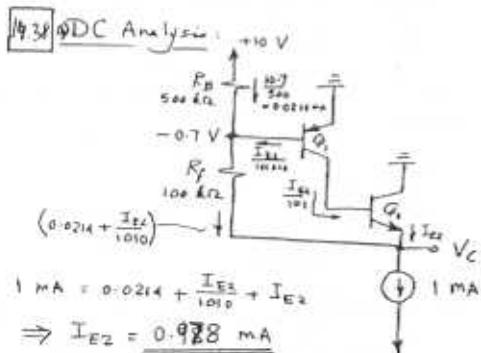
Substituting  $\beta_2 = \beta_N$  and noting that  $\beta_N \gg 1$ ,

$$\frac{V_o}{V_i} \approx - g_{m_1} \frac{1}{\beta_N \left( \frac{1}{r_{\pi_2}} + \frac{1}{R_f} \right) + \frac{1}{r_{\pi_1}}}$$

$$= - g_{m_1} \left[ r_{\pi_1} // \beta_N \left( r_{\pi_2} // R_f \right) \right]$$

G.E.D.

[14.38] QDC Analysis:



[14]

$$1 \text{ mA} = 0.0214 + \frac{I_{E2}}{1010} + I_{E2} \\ \Rightarrow I_{E2} = \underline{0.978 \text{ mA}} \\ I_{C2} = 0.99 \times 0.978 = \underline{0.97 \text{ mA}} \\ I_{C1} = \frac{0.978}{101} = \underline{9.7 \text{ mA}} \\ V_C = -0.7 - 100 \left( 0.0214 + \frac{0.978}{1010} \right) \\ = -2.94 \text{ V}$$

(b) Small-signal parameters:

$$g_{m1} = \frac{9.7 \times 10^{-6}}{2.5 \times 10^{-3}} = 0.388 \text{ mA/V}$$

$$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = 25.77 \text{ k}\Omega$$

$$r_{\pi_2} = \frac{|V_A|}{I_{C2}} = \frac{100}{9.7 \text{ mA}} = 10.31 \text{ M}\Omega$$

$$g_{m2} = \frac{0.97 \times 10^{-3}}{2.5 \times 10^{-3}} = 38.8 \text{ mA/V}$$

$$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = 2.58 \text{ k}\Omega$$

$$r_{\pi_1} = |V_A| / I_{C2} = 103.1 \text{ k}\Omega$$

14.38 CONT'D

$$(c) \frac{V_0}{V_T} = -0.388 \left[ 10.31 \times 10^3 // 100 (103.1 // 100) \right] [14]$$

$$= -1320 \text{ V/V}$$

$$\begin{aligned} R_{in} &= R_B // R_{T1} // \left[ \frac{R_T}{1 - \frac{V_T}{V_0}} \right] \\ &= 500 // 25.77 // \left[ \frac{R_T}{1 - \frac{V_T}{V_0}} \right] \\ &= 500 // 25.77 // \frac{100}{1 + 1320} \\ &= 500 // 25.77 // 0.0757 \\ &= 75.5 \Omega \end{aligned}$$

[14.39] Refer to Fig. 14.27 (p.1258).

The quiescent current through  $Q_2$  and  $Q_3$  is to be 2 mA. Then

$$V_{BE2} = V_{BEA} = 0.7 + 0.025 \ln \left( \frac{2}{10} \right) = 0.660 \text{ V}$$

For  $Q_1$  and  $Q_2$ ,  $I_C \approx \frac{2}{100} = 0.02 \text{ mA}$ , thus

$$V_{BE1} = V_{EB3} = 0.7 + 0.025 \ln \frac{0.02}{1} = 0.602 \text{ V}$$

$$I_{BL} = \frac{20 \text{ mA}}{100} = 0.2 \mu\text{A}$$

$$I_{bias} = 100 \times 0.2 = 20 \mu\text{A}$$

$$I_{R_1 R_2} = \frac{1}{10} \times 20 \mu\text{A} = 2 \mu\text{A}$$

$$I_{CA} = 20 - 2 = 18 \mu\text{A}$$

$$V_{BES} = 0.7 + 0.025 \ln \frac{0.018}{1} = 0.600 \text{ V}$$

$$V_{BB} = V_{BE1} + V_{BE2} + V_{EB3} = 1.864 \text{ V}$$

$$R_1 + R_2 = \frac{1.864}{2 \mu\text{A}} = 932 \text{ k}\Omega$$

$$R_1 = \frac{0.600}{2 \mu\text{A}} = 300 \text{ k}\Omega$$

$$R_2 = 932 - 300 = 632 \text{ k}\Omega$$

For  $V_0 = -10 \text{ V}$  and  $R_L = 1 \text{ k}\Omega$ :

$$i_L = \frac{-10}{1} = -10 \text{ mA}$$

Assume that the current through  $Q_2$  becomes almost zero, thus

$$I_{CA} = 10 \text{ mA}$$

i.e. the current through  $Q_4$  increases by a factor of 5. It follows that the current through  $Q_3$  must increase by the same factor, thus  $V_{EB3}$  becomes:

$$\begin{aligned} V_{EB3} &= 0.602 + 0.025 \ln 5 \\ &= 0.642 \text{ V} \quad (\text{an increase of } 0.04 \text{ V}) \end{aligned}$$

Let us check the current through  $Q_2$ . Since we assumed  $Q_1$  and  $Q_2$  to be almost cut off, all of  $I_{bias}$  now flows through the  $V_{BE}$  multiplier, an increase of 0.2  $\mu\text{A}$ . Assuming that most of this increase occurs in  $I_{CS}$ ,  $V_{BES}$  becomes:

$$V_{BES} = 0.7 + 0.025 \ln \frac{0.018}{1} = 0.600 \text{ V}$$

Thus the voltage across the  $V_{BE}$ -multiplier remains approximately constant and the voltage ( $V_{BE1} + V_{BE2}$ ) decreases by the same value that  $V_{EB3}$  increases. That is:

$$V_{BE1} + V_{BE2} = 0.660 + 0.602 - 0.04$$

Since the current through each of  $Q_1$  and  $Q_2$  decreases by the same factor (call it  $m$ ),

$$0.025 \ln m + 0.025 \ln m = -0.04 \text{ V}$$

$$\Rightarrow m = 0.45$$

$$\text{Thus } I_{C2} = 0.45 \times 2 = 0.9 \text{ mA}$$

New iteration:  $I_{CA} = 10.9 \text{ mA}$  (an increase by a factor  $\approx 5.5$ ).

$$\begin{aligned} V_{EB3} &= 0.602 + 0.025 \ln 5.5 \\ &= 0.645 \text{ V} \end{aligned}$$

$$V_I \approx -10.645 \text{ V}$$

For  $V_0 = +10 \text{ V}$  and  $R_L = 1 \text{ k}\Omega$ :

Assume that  $Q_4$  is now conducting a negligible current. Then  $I_{CA} \approx I_L = 10 \text{ mA}$ , i.e. the current through each of  $Q_1$  and  $Q_2$

increases by a factor of 5. Thus

$$V_{BE2} = 0.66 + 0.025 \ln 5$$

$$= 0.700 \text{ V}$$

$$V_{BE1} = 0.602 + 0.025 \ln 5 = 0.642 \text{ V}$$

$$I_{BL} = 5 \times 0.2 = 1 \mu\text{A}$$

Thus the current through the multiplier becomes 19  $\mu\text{A}$ , and assuming that most of the decrease occurs in  $I_{CS}$ ,

$$\begin{aligned} V_{BES} &= 0.7 + 0.025 \ln \frac{0.017}{1} \\ &= 0.598 \text{ V} \end{aligned}$$

Thus the voltage across the multiplier becomes

$$V_{BB} = 0.598 \times \frac{932}{300} = 1.858 \text{ V}$$

It follows that  $V_{EB3}$  becomes

$$V_{EB3} = 1.858 - 0.700 - 0.642 = 0.516 \text{ V}$$

i.e.  $V_{EB3}$  decreases by  $0.600 - 0.516 = 0.084 \text{ V}$

and corresponding  $I_{CA}$  decreases by a factor of  $\frac{0.084}{0.516} = 0.035$ .

$I_{CA}$  becomes  $0.035 \times 2 = 0.07 \text{ mA}$ , close to the zero value assumed. Thus no further iteration are required and

$$\begin{aligned} V_I &\approx 10 + 0.7 + 0.642 - 1.858 \\ &= +9.484 \text{ V} \end{aligned}$$

[14.40] Refer to Exercise 14.13 and Fig. 14.28.

Now Q<sub>S</sub> has  $I_S = 10^{-13} \text{ A}$ . Then,

$$2 \times 10^{-3} = 10^{-13} e^{V_{BE}/V_T}$$

$$V_{BE} = 0.025 \ln \frac{2 \times 10^{-3}}{10^{-13}}$$

$$= 0.593 \text{ V}$$

$$R_{E1} = \frac{0.593}{150 \mu\text{A}} \approx 4.5 \Omega$$

For a nominal peak current of 100 mA, the voltage drop across  $R_{E1}$  is 400 mV and its collector current is  $10^{-13} e^{400/25} = 0.89 \mu\text{A}$

[14.41] Refer to Exercise 14.13 and Fig. 14.28.

$$2 \times 10^{-3} = 10^{-14} e^{V_{BE}/V_T}$$

$$\Rightarrow V_{BE} = 0.650 \text{ V}$$

$$R_{E1} = \frac{0.650 \text{ V}}{50 \mu\text{A}} = 13 \Omega$$

For a peak output current of 33.3 mA,

$$V_{BE} = 13 \times 33.3 = 433 \text{ mV}$$

$$I_{C5} = 10^{-14} e^{433/25} = 0.33 \mu\text{A}$$

[14.42] Refer to Fig. P.14.42.

$$2 \times 10^{-3} = 10^{-14} e^{V_{EB5}/V_T}$$

$$V_{EB5} = 0.025 \ln (2 \times 10^{-11})$$

$$= 0.650 \text{ V}$$

$$R = \frac{0.650 \text{ V}}{150 \mu\text{A}} = 4.3 \Omega$$

For a peak output current of 100 mA,

$$V_{EB5} = 430 \text{ mV}$$

$$I_{C5} = 10^{-14} e^{430/25} = 0.3 \mu\text{A}$$

[14.43] Refer to Fig. 14.29 (P.1260)

$$\text{At } 125^\circ\text{C}, V_Z = 6.8 + (125 - 25) \times 2 = 7.0 \text{ V}$$

$$V_{E1} = 7.0 - (0.7 - 100 \times 0.002)$$

$$= 6.5 \text{ V}$$

$$V_{BE2} = 0.5 \text{ V}$$

$$R_2 = \frac{0.5 \text{ V}}{100 \mu\text{A}} = 5 \text{ k}\Omega$$

$$R_1 = \frac{6.5 - 0.5}{100 \mu\text{A}} = 60 \text{ k}\Omega$$

$$\text{At } 25^\circ\text{C}, V_Z = 6.8 \text{ V} \Rightarrow V_{E1} = 6.8 - 0.7 = 6.1 \text{ V}$$

$$V_{B2} = 6.1 \times \frac{5}{60+5} = 0.469 \text{ V}$$

$$I_{C2} = 100 e^{(469-700)/25} = 0.01 \mu\text{A}$$

[14.44] Refer to the circuit of Fig. 14.30 (P.1262).

Resistors  $R_2$  and  $R_3$  control the gain.

$$A_V = -\frac{2R_3}{R_2} \quad (\text{See analysis on pages 681-682})$$

Resistor  $R_3$  controls the gain alone. Resistor  $R_2$  affects both the gain and the dc output level.

To set the bias point, Equate  $I_3$  and  $I_4$  from equations (9.43) and (9.44) to obtain

$$\frac{V_S - 3V_{EB}}{R_1} = \frac{V_O - 2V_{EB}}{R_3}$$

$$\Rightarrow V_O = 2V_{EB} + \frac{R_3}{R_1} V_S - \frac{3R_3}{R_1} V_{EB}$$

$$= \frac{R_3}{R_1} V_S + (2 - \frac{3R_3}{R_1}) V_{EB}$$

$$\text{For } V_O = \frac{1}{2} V_S, \text{ select } \frac{R_3}{R_1} = \frac{1}{3}$$

$$R_3 = \frac{R_1}{3} = \frac{50}{3} = 16.7 \text{ k}\Omega$$

To keep the gain unchanged, we must change  $R_3$  so that  $\frac{2R_3}{R_1} = 50$

$$R_3 = \frac{2 \times (50/3)}{50} = \frac{2}{3} = 0.67 \text{ k}\Omega$$

[14.45] Refer to the circuit in Fig. 14.30

$$V_{B1} \approx 0$$

$$V_{E1} \approx +0.7 \text{ V}$$

$$V_{E3} \approx +1.4 \text{ V}$$

$$V_{C10} = 20 - 0.7 = 19.3 \text{ V}$$

$$I_{E3} = \frac{19.3 - 1.4}{50} = 0.358 \text{ mA}$$

$$I_{B3} = I_{E1} = \frac{0.358}{21} = 17.05 \mu\text{A}$$

$$I_{A1} = \frac{17.05}{21} = 0.81 \text{ mA}$$

$$V_{B_L} = 0.81 \text{ mA} \times 150 \text{ k}\Omega = 0.122 \text{ V} \approx 0 \text{ V}$$

$$\text{i.e. } I_{E1} = I_{E2} \approx 17 \mu\text{A}$$

$$I_{E3} = I_{E4} = 358 \mu\text{A}$$

$$I_{E5} \approx I_{E6} = \frac{20}{21} \times 358 = 341 \mu\text{A}$$

$$I_{R_3} = I_{P_L} = 358 \mu\text{A}$$

$$V_O = 0.12 + 1.1 + 25 \text{ k}\Omega \times 0.358 \text{ mA}$$

$$= 10.5 \text{ V}$$

[14.46] The equivalent common-mode half circuit is approximately   
From the results of Problem 14.45  
Above:  $I_{E1} = 17 \mu A$   
 $I_{E3} = 0.758 \mu A$

$$T_{ot} = \frac{IV_A}{I_C}$$

Assume  $|V_A|$  for all transistors = 100 V.

$$T_{ot} \approx \frac{100}{17 \mu A} = 5.9 \text{ M}\Omega$$

$$T_{\mu 3} = \frac{100}{0.758} = 279.3 \text{ k}\Omega$$

Assume  $T_{\mu 1} = 10 \beta T_{\mu 3}$

$$T_{\mu 1} = 10 \times 20 \times 3.9 = 1180 \text{ M}\Omega$$

$$T_{\mu 3} = 10 \times 20 \times 279.3 = 55.9 \text{ M}\Omega$$

$$\begin{aligned} \text{Now, } 2R_{in} &= T_{\mu 1} \parallel (\beta + 1) [T_{\mu 3} \parallel (\beta + 1) (25 \parallel G_o)] \\ &= 1180 \text{ M}\Omega \parallel 21 [54 \parallel 55.9 \parallel 21(250 \text{ M}\Omega \parallel 25)] \text{ M}\Omega \\ &\approx 21 \times 21 \times (25 \parallel 279.3) \text{ k}\Omega \\ &= 10.1 \text{ M}\Omega \end{aligned}$$

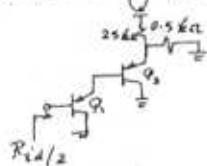
If we neglect all  $T_{\mu}$ 's and  $T_{ot}$ ,

$$2R_{in} \approx (\beta + 1)^2 \times 25 \text{ k}\Omega$$

= 11 MΩ (close enough!)

$$R_{in} = 5.5 \text{ M}\Omega$$

To find  $R_{id}$  we use  
the equivalent differential  
half-circuit  $\rightarrow$



$$R_{id}/2 = (\beta + 1) [T_{ot} + (\beta + 1)(r_{e3} + 0.5k\Omega)]$$

$$T_{ot} = \frac{25 \text{ mV}}{17 \mu A} = 1.47 \text{ k}\Omega$$

$$r_{e3} = \frac{25 \text{ mV}}{0.758 \mu A} = 69.8 \text{ }\Omega$$

$$\begin{aligned} R_{id} &= 2 \times 21 [1.47 + 21 (0.0698 + 0.49)] \\ &= 556 \text{ k}\Omega \end{aligned}$$

To find  $G_{m1} = \frac{2t}{V_{td}}$

$$\begin{aligned} i_{c3} &= \beta_3 i_{e3} \\ &= \beta_3 \frac{V_{td}/2}{(\beta_3/2) + G_{m1} + \frac{r_{e3}}{\beta_3}} - V_{td}/2 \\ &= \frac{20}{21} \times \frac{1}{2} \frac{V_{td}}{(\beta_3/2) + 0.468 + \frac{1.47}{21}} \end{aligned}$$

$$= 0.757 V_{td}$$

$$t = \frac{i_{c3}}{1 + \frac{2}{\beta_3 \mu_B}} = 0.98 i_{c3} = 0.98 \times 0.757 V_{td}$$

$$G_{m1} = \frac{2t}{V_{td}} = 2 \times 0.98 \times 0.757 = 1.5 \text{ mA/V}$$

[14.47] Using Fig. 14.33 (p1269) for 8 Ω load, we see  
that  $V_S = 14 \text{ V}$  allows more than 1.5 W power  
dissipation for some input signals. Thus we use

$$V_S = 14 \text{ V}$$

For THD = 2%,  $P_{max} = 1.9 \text{ W}$

$$1.9 = V_o^2 / R_L = V_o^2 / 8$$

$$V_o = \sqrt{8 \times 1.9}$$

$$\text{Peak-to-peak output sinusoid} = 2\sqrt{2} \sqrt{8 \times 1.9} = 11 \text{ V}$$

$$[14.48] f_L = \frac{G_m}{2\pi C}$$

$$= \frac{1.6 \times 10^{-3}}{2\pi \times 10 \times 10^{-12}} = 25.5 \text{ MHz}$$

With feedback that results in a closed-loop

gain of 50,

$$f_L = \frac{25.5}{50} = 509 \text{ kHz}$$

[14.49] Refer to Fig. 14.33

For  $I_L = 1 \text{ A}$ ,  $i_{C5} \approx 1 \text{ A}$  and  $i_{B5} = \frac{1 \text{ A}}{50} = 20 \text{ mA}$

For  $I_L = 20 \text{ mA}$ ,  $i_{E3} = 0.9 \times 20 = 18 \text{ mA}$ ,

$$i_{C5} = 2 \text{ mA}$$

$$i_{C3} = \frac{50}{\beta_1} \times 18 = 17.65 \text{ mA}$$

$$i_{R3} = i_{C3} - i_{B5} = 17.65 - 20 = -2.35 \text{ mA}$$

$$\text{Thus, } R_3 = \frac{0.7}{17.65} \times 31.8 = 40 \text{ }\Omega$$

$$\text{Similarly, } R_4 = 40 \text{ }\Omega$$

$$\text{Since } i_{B5} \leq 20 \text{ mA}, i_{C3} \leq \frac{0.7 \text{ V}}{40 \text{ }\Omega} + 20 \text{ mA}$$

i.e.  $i_{C3} \leq 37.5 \text{ mA}$

$$i_{B3} \leq \frac{37.5}{50} = 0.75 \text{ mA}$$

Allowing for a factor of safety of 2, we select  $R_1$  so that the current through it

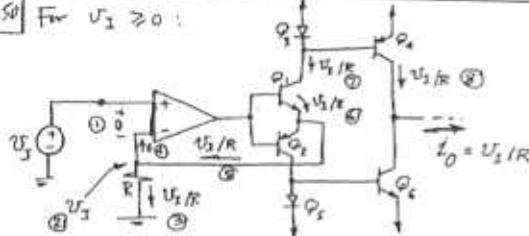
is 1.5 mA. Now, for  $V_o = 11 \text{ V}$ ,  $V_{E1} = 10.7 \text{ V}$ ,

$$R_1 = \frac{15 - 10.7}{1.5} = 2.2 \text{ k}\Omega$$

Similarly,

$$R_2 = 2.2 \text{ k}\Omega$$

[14.50] For  $V_2 \geq 0$ :



For  $V_2 \leq 0$ ,  $Q_3$ ,  $Q_5$  and  $Q_6$  take the roles of  $Q_1$ ,  $Q_2$  and  $Q_4$ , respectively.

For  $\beta = 100$ ,  $I_{C1} = 0.99 V_2 / R$  and

$$I_{C4} = I_{C1} \frac{1}{1 + \frac{1}{\beta}} = 0.99 \frac{V_2}{R} \frac{1}{1 + \frac{1}{100}} = 0.97 \frac{V_2}{R}$$

That is, the current is lower than the ideal value by approximately 3%.

For  $\frac{V_2}{V_1} = 10 = 1 + \frac{R_2 + R_3}{R_1}$   
and selecting  $R_1 = 1 k\Omega$

$$R_2 + R_3 = 9 k\Omega$$

Now to keep the outputs complementary

$$\frac{R_3}{R_1} = 1 + \frac{R_2}{R_1}$$

$$\Rightarrow R_1 = R_2$$

$$\text{Thus } R_2 = \underline{1 k\Omega} \text{ & } R_3 = \underline{5 k\Omega}$$

[14]

[14.51] Refer to Fig. 14.34

$$\frac{V_2}{V_1} = 2K = 2(1 + \frac{R_2}{R_1}) = 10$$

$$\Rightarrow \frac{R_2}{R_1} = 4$$

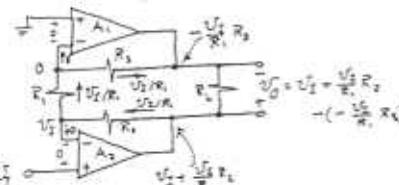
$$R_2 = \underline{40 k\Omega}$$

$$\text{Also, } K = \frac{R_4}{R_3} = 5$$

$$\Rightarrow R_4 = \underline{50 k\Omega}$$

[14]

[14.52]



$$\frac{V_2}{V_1} = 1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}$$

The largest sine-wave output is obtained when the voltage at the output of one op-amp (say  $A_2$ ) is  $+13$  V and that at the output of the other op-amp ( $A_1$ ) is  $-13$  V, resulting in a 26-V peak output (52 V peak-to-peak).

[14.53] Normally, (Eq. 14.46)

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$$

For velocity saturation, (Eq. 14.47)

$$I_D = \frac{1}{2} C_{ox} W V_{sat} (V_{GS} - V_t)$$

Equating at the two expressions (at the boundary)

$$\therefore \mu_n \frac{1}{L} (V_{GS} - V_t) = V_{sat}$$

$$\Rightarrow L = \frac{\mu_n (V_{GS} - V_t)}{V_{sat}}$$

For  $\mu_n = 500 \text{ cm}^2/\text{V.s.}$ ,  $V_{GS} = 5 \text{ V}$ ,  $V_t = 2 \text{ V}$ ,

$$V_{sat} = 5 \times 10^6 \text{ cm/s},$$

$$L = \frac{500 (5 - 2)}{5 \times 10^6} = 3 \times 10^{-4} \text{ cm} = \underline{3 \mu\text{m}}$$

Velocity saturation begins at:

$$I_D = \frac{1}{2} C_{ox} W V_{sat} (V_{GS} - V_t)$$

$$= \frac{1}{2} \times 400 \times 10^{-6} \times 10^5 \times 10^{-6} \times 5 \times 10^6 \times 10^{-2} (5 - 2) \\ = \underline{3 \text{ A}}$$

At high currents,

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2} C_{ox} W V_{sat}$$

$$= \frac{1}{2} \times 400 \times 10^{-6} \times 10^5 \times 10^{-6} \times 5 \times 10^6 \times 10^{-2}$$

$$= \underline{1 \text{ A/V}}$$

[14]

Q54 Refer to the circuit in Fig. 14.38

For  $I_{Q_1} = I_{Q_2} = 10 \text{ mA} = C_{ox}W(V_{GS} - V_t)^2$

$$10 = 100 (V_{GS} - 2)^2$$

$$\Rightarrow V_{GS} = 2.32 \text{ V}$$

$$V_R = 2 |V_{GS}| = 4.63 \text{ V}$$

For  $I_R = 10 \text{ mA}$ ,

$$R = \frac{4.63 \text{ V}}{10 \text{ mA}} = 463 \text{ } \Omega$$

$$V_{BB} = 4.63 + 4 \times 0.7 = 7.43 \text{ V}$$

$$I_{R_2} = I_{R_4} = \frac{100}{2} = 50 \mu\text{A}$$

$$R_2 = R_4 = \frac{700 \text{ mV}}{50 \mu\text{A}} = 14 \text{ k}\Omega$$

Now, since  $V_{GS}$  changes by  $2 \times 3 \text{ mV}/^\circ\text{C}$  [4]

$= -6 \text{ mV}/^\circ\text{C}$  while  $V_{BE1}, V_{BE2}, V_{BE3}$

and  $V_{BE4}$  remain constant,  $V_{BB}$  changes by  $-6 \text{ mV}/^\circ\text{C}$ . But the voltage across the  $Q_3$  multiplier remains constant.

Thus the voltage across the  $Q_2$  multiplier should be made to change by  $-6 \text{ mV}/^\circ\text{C}$  which can be achieved by making

$$1 + \frac{R_3}{R_4} = 3$$

$$\Rightarrow R_3 = 2 R_4 = 28 \text{ k}\Omega$$

The voltage across the  $Q_3$  multiplier is

$$V_{BB} - 3 V_{BEC} = 7.43 - 2.1 = 5.33 \text{ V}$$

Thus  $5.33 = (1 + \frac{R_1}{R_2}) \times 0.7$

$$\Rightarrow \frac{R_1}{R_2} = 6.61$$

But  $R_2 = 14 \text{ k}\Omega$ , thus

$$R_1 = 92.6 \text{ k}\Omega$$