Total Questions: 8

Total Marks: 85

Question 1:

Calculator Allowed: Yes

11. *[Maximum mark: 21]*

Tim throws two identical fair dice simultaneously. Each die has six faces: two faces numbered 1, two faces numbered 2 and two faces numbered 3. His score is the sum of the two numbers shown on the dice.

(a) (i) Calculate the probability that Tim obtains a score of 6.

(ii) Calculate the probability that Tim obtains a score of at least 3. *[3 marks]*

Tim plays a game with his friend Bill, who also has two dice numbered in the same way. Bill's score is the sum of the two numbers shown on his dice.

(b) (i) Calculate the probability that Tim and Bill **both** obtain a score of 6.

(ii) Calculate the probability that Tim and Bill obtain the same score. *[4 marks]*

(c) Let X denote the largest number shown on the four dice.

(i) Show that $\mathrm{P}(X \le 2) = \frac{16}{81}$.

(ii) Copy and complete the following probability distribution table.

x	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{81}$		

(iii) Calculate $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$ and hence find $\mathrm{Var}(X)$. *[10 marks]*

(d) Given that $X = 3$, find the probability that the sum of the numbers shown on the four dice is 8. *[4 marks]*

Question 2:

Calculator Allowed: Yes

A Chocolate Shop advertises free gifts to customers that collect three vouchers.
The vouchers are placed at random into 10% of all chocolate bars sold at this shop.
Kati buys some of these bars and she opens them one at a time to see if they contain a voucher. Let $\mathrm{P}(X=n)$ be the probability that Kati obtains her third voucher on the nth bar opened.

(It is assumed that the probability that a chocolate bar contains a voucher stays at 10% throughout the question.)

(a) Show that $\mathrm{P}(X=3)=0.001$ and $\mathrm{P}(X=4)=0.0027$. [3]

It is given that $\mathrm{P}(X=n)=\dfrac{n^2+an+b}{2000}\times 0.9^{n-3}$ for $n\geq 3$, $n\in\mathbb{N}$.

(b) Find the values of the constants a and b. [5]

(c) Deduce that $\dfrac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)}=\dfrac{0.9(n-1)}{n-3}$ for $n>3$. [4]

(d) (i) Hence show that X has two modes m_1 and m_2.

(ii) State the values of m_1 and m_2. [5]

Kati's mother goes to the shop and buys x chocolate bars. She takes the bars home for Kati to open.

(e) Determine the minimum value of x such that the probability Kati receives at least one free gift is greater than 0.5. [3]

Question 3:
Calculator Allowed: Yes

[Maximum mark: 5]

The age, L, in years, of a wolf can be modelled by the normal distribution $L\sim \mathrm{N}(8, 5)$.

(a) Find the probability that a wolf selected at random is at least 5 years old. [2]

Eight wolves are independently selected at random and their ages recorded.

(b) Find the probability that more than six of these wolves are at least 5 years old. [3]

Question 4:
Calculator Allowed: Yes

2. [Maximum mark: 6]

A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean 1000 and standard deviation 3.5. A bag of sugar is rejected for sale if its mass is less than 995 grams.

(a) Find the probability that a bag selected at random is rejected. [2]

(b) Estimate the number of bags which will be rejected from a random sample of 100 bags. [1]

(c) Given that a bag is not rejected, find the probability that it has a mass greater than 1005 grams. [3]

Question 5:
Calculator Allowed: Yes

[Maximum mark: 6]

The random variable X has a normal distribution with mean $\mu = 50$ and variance $\sigma^2 = 16$.

(a) Sketch the probability density function for X, and shade the region representing $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(b) Find the value of $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(c) Find the value of k for which $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0.5$. [2]

Question 6:
Calculator Allowed: No

1. [Maximum mark: 5]

Let A and B be events such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.4$ and $\mathrm{P}(A \cup B) = 0.6$.
Find $\mathrm{P}(A \mid B)$.

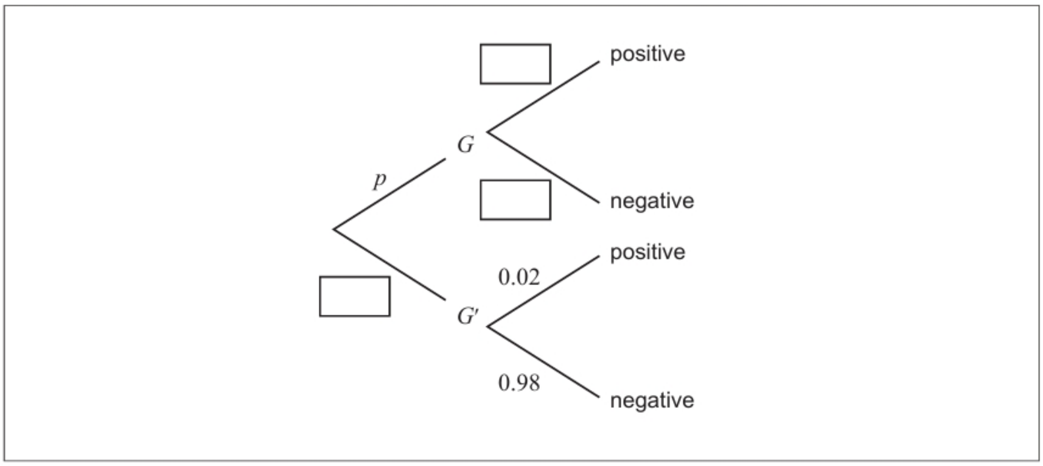
Question 7:
Calculator Allowed: Yes

7. [Maximum mark: 6]

A new test has been developed to identify whether a particular gene, G, is present in a population of parrots. The test returns a correct positive result 95% of the time for parrots with the gene, and a false positive result 2% of the time for parrots without the gene.

The proportion of the population with the gene is p.

(a) Complete the tree diagram below. [2]



(b) A random sample of the population was tested. It was found that 150 tests returned a positive result. Out of the 150 parrots with a positive test result, 18 did not actually have the gene. Find an estimate for p. [4]

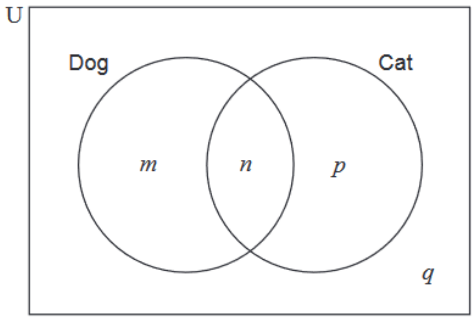
Question 8:

Calculator Allowed: Yes

4. [Maximum mark: 16]

At Mirabooka Primary School, a survey found that 68% of students have a dog and 36% of students have a cat. 14% of students have both a dog and a cat.

This information can be represented in the following Venn diagram, where m, n, p and q represent the percentage of students within each region.



(a) Find the value of

(i) m.

(ii) n.

(iii) p.

(iv) q. [4]

(b) Find the percentage of students who have a dog or a cat or both. [1]

(c) Find the probability that a randomly chosen student

(i) has a dog but does not have a cat.

(ii) has a dog given that they do not have a cat. [3]

Each year, one student is chosen randomly to be the school captain of Mirabooka Primary School.

Tim is using a binomial distribution to make predictions about how many of the next 10 school captains will own a dog. He assumes that the percentages found in the survey will remain constant for future years and that the events "being a school captain" and "having a dog" are independent.

Use Tim's model to find the probability that in the next 10 years

(d) (i) 5 school captains have a dog.

(ii) more than 3 school captains have a dog.

(iii) exactly 9 school captains in succession have a dog. [7]

John randomly chooses 10 students from the survey.

(e) State why John should not use the binomial distribution to find the probability that 5 of these students have a dog. [1]