Mark scheme

Answer 1:

2. (a) $BV = \sqrt{(8 \quad 4)^2 + (6 \quad 3)^2 + (0 \quad 10)^2}$ ***(A1)***

$= 11.1803...$

$= 11.2 \; (= \sqrt{125} = 5\sqrt{5})$ ***A1***

[2 marks]

(b) **METHOD 1**

$BV = VC$ AND $BC = 6$ (seen anywhere) ***(A1)***

attempt to use the cosine rule on triangle BVC for any angle ***(M1)***

Note: Recognition must be shown in context either in terms of labelled sides or side lengths.

$\cos B\hat{V}C = \dfrac{11.1...^2 + 11.1...^2 - 6^2}{2 \times 11.1... \times 11.1...}$ OR

$6^2 = 11.1....^2 + 11.1...^2 - 2 \times 11.1... \times 11.1... \cos B\hat{V}C$ ***(A1)***

$B\hat{V}C = 0.543314...$

$B\hat{V}C = 0.543$ (0.542 from 3 sf) (accept 31.1°) ***A1***

METHOD 2

let M be the midpoint of BC

$BM = 3$ (seen anywhere) ***(A1)***

attempt to use sine or cosine in triangle BMV or CMV ***(M1)***

$\arcsin \dfrac{3}{\sqrt{125}}$ OR $\dfrac{\pi}{2} - \arccos \dfrac{3}{\sqrt{125}}$ OR $0.271657...$ ***(A1)***

$B\hat{V}C = 0.543314...$

$B\hat{V}C = 0.543$ (0.542 from 3 sf) (accept 31.1°) ***A1***

[4 marks]

Total [6 marks]

Answer 2:

(a) valid approach (M1)

eg $\text{speed} = \frac{\text{distance}}{\text{time}}$, 6×1.5

$SL = 9$ (km) **A1** **N2**

[2 marks]

(b) evidence of choosing sine rule **(M1)**

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\sin\theta = \frac{(SL)\sin 20^\circ}{5}$

correct substitution **(A1)**

eg $\frac{\sin\theta}{9} = \frac{\sin 20^\circ}{5}$

37.9981

$S\hat{P}L = 38.0^\circ$ **A1** **N2**

recognition that second angle is the supplement of first **(M1)**

eg $180 - x$

142.001

$S\hat{Q}L = 142^\circ$ **A1** **N2**

[5 marks]

(c) (i) new store is at Q **A1** **N1**

(ii) **METHOD 1**

attempt to find third angle **(M1)**

eg $S\hat{L}P = 180 - 20 - 38$, $S\hat{L}Q = 180 - 20 - 142$

$S\hat{L}Q = 17.998^\circ$ (seen anywhere) **A1**

evidence of choosing sine rule or cosine rule **(M1)**

correct substitution into sine rule or cosine rule **(A1)**

eg $\frac{x}{\sin 17.998} = \frac{5}{\sin 20}\left(= \frac{9}{\sin 142}\right)$, $9^2 + 5^2 - 2(9)(5)\cos 17.998^\circ$

$4.51708\,\text{km}$

4.52 (km) **A1** **N3**

METHOD 2

evidence of choosing cosine rule **(M1)**

correct substitution into cosine rule **A1**

eg $9^2 = x^2 + 5^2 - 2(x)(5)\cos 142^\circ$

attempt to solve **(M1)**

eg sketch; setting quadratic equation equal to zero;

$0 = x^2 + 7.88x - 56$

one correct value for x **(A1)**

eg $x = -12.3973$, $x = 4.51708$

4.51708

4.52 (km) **A1** **N3**

[6 marks]

Total [13 marks]

Answer 3:

7. (a) $\frac{4.2}{60} \times 45$ *A1*

$AB = 3.15$ (km) *A1*

[2 marks]

(b) (i) 66° or $(180 - 114)$ *A1*

$35 + 66$ *A1*

$A\hat{B}C = 101^\circ$ *AG*

(ii) attempt to use cosine rule *(M1)*

$AC^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101^\circ$ (or equivalent) *A1*

$AC = 6.05$ (km) *A1*

[5 marks]

(c) valid approach to find angle BCA *(M1)*

eg sine rule

correct substitution into sine rule *A1*

eg $\frac{\sin(B\hat{C}A)}{3.15} = \frac{\sin 101}{6.0507...}$

$B\hat{C}A = 30.7^\circ$ *A1*

[3 marks]

(d) $B\hat{A}C = 48.267$ (seen anywhere) *A1*

valid approach to find correct bearing *(M1)*

eg $48.267 + 35$

bearing $= 83.3^\circ$ (accept 083°) *A1*

[3 marks]

(e) attempt to use time $= \frac{\text{distance}}{\text{speed}}$ *M1*

$\frac{6.0507}{3.9}$ or 0.065768 km/min *(A1)*

$t = 93$ (minutes) *A1*

[3 marks]

Total [16 marks]

Answer 4:

$AC^2 = 7.8^2 + 10.4^2$ *(M1)*

$AC = 13$ *(A1)*

use of cosine rule *eg*, $\cos(A\hat{B}C) = \frac{6.5^2 + 9.1^2 - 13^2}{2(6.5)(9.1)}$ *M1*

$A\hat{B}C = 111.804...^\circ$ $(= 1.95134...)$ *(A1)*

$= 112^\circ$ *A1*

[5 marks]

Answer 5:

determines $\frac{\pi}{4}$ (or $45°$) as the first quadrant (reference) angle **(A1)**

attempts to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}$ **(M1)**

Note: Award ***M1*** for attempting to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4},\frac{7\pi}{4}(,...)$

$\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}\Rightarrow x<0$ and so $\frac{\pi}{4}$ is rejected **(R1)**

$\frac{x}{2}+\frac{\pi}{3}=2\pi-\frac{\pi}{4}\left(=\frac{7\pi}{4}\right)$ ***A1***

$x=\frac{17\pi}{6}$ (must be in radians) ***A1***

[5 marks]

Answer 6:

(a) $\cos 105^\circ=\cos\left(180^\circ-75^\circ\right)=-\cos 75^\circ$ ***R1***

$=-q$ ***AG***

Note: Accept arguments using the unit circle or graphical/diagrammatical considerations.

[1 mark]

(b) $\mathrm{AD}=\mathrm{CD}\Rightarrow \mathrm{C\hat{A}D}=45^\circ$ ***A1***

valid method to find $\mathrm{B\hat{A}C}$ ***(M1)***

for example: $\mathrm{BC}=r\Rightarrow \mathrm{B\hat{C}A}=60^\circ$

$\Rightarrow \mathrm{B\hat{A}C}=30^\circ$ ***A1***

hence $\mathrm{B\hat{A}D}=45^\circ+30^\circ=75^\circ$ ***AG***

[3 marks]

(c) (i) $\mathrm{AB}=r\sqrt{3}$, $\mathrm{AD}(=\mathrm{CD})=r\sqrt{2}$ ***A1A1***

applying cosine rule ***(M1)***

$\mathrm{BD}^2=\left(r\sqrt{3}\right)^2+\left(r\sqrt{2}\right)^2-2\left(r\sqrt{3}\right)\left(r\sqrt{2}\right)\cos 75^\circ$ ***A1***

$=3r^2+2r^2-2r^2\sqrt{6}\cos 75^\circ$

$=5r^2-2r^2q\sqrt{6}$ ***AG***

(ii) $\mathrm{B\hat{C}D}=105^\circ$ ***(A1)***

attempt to use cosine rule on $\Delta\mathrm{BCD}$ ***(M1)***

$\mathrm{BD}^2=r^2+\left(r\sqrt{2}\right)^2-2r\left(r\sqrt{2}\right)\cos 105^\circ$

$=3r^2+2r^2q\sqrt{2}$ ***A1***

[7 marks]

(d) $5r^2 - 2r^2 q\sqrt{6} = 3r^2 + 2r^2 q\sqrt{2}$ *(M1)(A1)*

$2r^2 = 2r^2 q\left(\sqrt{6} + \sqrt{2}\right)$ ***A1***

Note: Award ***A1*** for any correct intermediate step seen using only two terms.

$q = \dfrac{1}{\sqrt{6} + \sqrt{2}}$ ***AG***

Note: Do not award the final ***A1*** if follow through is being applied.

[3 marks]

Total [14 marks]

Answer 7:

(a) correct substitution *(A1)*

eg $10(1.2)$

$\text{ACB is } 12 \text{ (cm)}$ ***A1*** ***N2***

[2 marks]

(b) valid approach to find major arc *(M1)*

eg circumference $-\text{AB}$, major angle $\text{AOB} \times$ radius

correct working for arc length *(A1)*

eg $2\pi(10) - 12$, $10(2 \times 3.142 - 1.2)$, $2\pi(10) - 12 + 20$

perimeter is $20\pi + 8 \ \ (= 70.8) \text{ (cm)}$ ***A1*** ***N2***

[3 marks]

Total [5 marks]

Answer 8:

(a) evidence of valid approach *(M1)*

eg right triangle, $\cos^2\theta = 1 - \sin^2\theta$

correct working *(A1)*

eg missing side is 2, $\sqrt{1-\left(\frac{\sqrt{5}}{3}\right)^2}$

$\cos\theta = \frac{2}{3}$ **A1 N2**

[3 marks]

(b) correct substitution into formula for $\cos 2\theta$ *(A1)*

eg $2\times\left(\frac{2}{3}\right)^2 - 1,\ 1-2\left(\frac{\sqrt{5}}{3}\right)^2,\ \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2$

$\cos 2\theta = -\frac{1}{9}$ **A1 N2**

[2 marks]

[Total 5 marks]

Answer 9:

5. (a) valid approach to find p *(M1)*

eg amplitude $= \frac{\max - \min}{2}$, $p = 6$

$p = 3$ ***A1 N2***

[2 marks]

(b) valid approach to find q *(M1)*

eg period $= 4$, $q = \frac{2\pi}{\text{period}}$

$q = \frac{\pi}{2}$ ***A1 N2***

[2 marks]

(c) valid approach to find r *(M1)*

eg axis $= \frac{\max + \min}{2}$, sketch of horizontal axis, $f(0)$

$r = 2$ ***A1 N2***

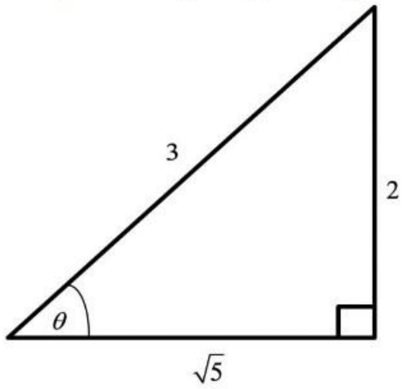
[2 marks]

Total [6 marks]

Answer 10:

METHOD 1

attempt to use a right angled triangle *M1*



correct placement of all three values and θ seen in the triangle *(A1)*

$\cot\theta < 0$ (since $\text{cosec}\,\theta > 0$ puts θ in the second quadrant) *R1*

$\cot\theta = -\frac{\sqrt{5}}{2}$ *A1*

Note: Award ***M1A1R0A0*** for $\cot\theta = \frac{\sqrt{5}}{2}$ seen as the final answer

The ***R1*** should be awarded independently for a negative value only given as a final answer.

[4 marks]

METHOD 2

Attempt to use $1+\cot^2\theta=\text{cosec}^2\theta$ ***M1***

$1+\cot^2\theta=\frac{9}{4}$

$\cot^2\theta=\frac{5}{4}$ ***(A1)***

$\cot\theta=\pm\frac{\sqrt{5}}{2}$

$\cot\theta<0$ (since $\text{cosec}\theta>0$ puts θ in the second quadrant) ***R1***

$\cot\theta=-\frac{\sqrt{5}}{2}$ ***A1***

Note: Award ***M1A1R0A0*** for $\cot\theta=\frac{\sqrt{5}}{2}$ seen as the final answer

The ***R1*** should be awarded independently for a negative value only given as a final answer.

METHOD 3

$\sin\theta=\frac{2}{3}$

attempt to use $\sin^2\theta+\cos^2\theta=1$ ***M1***

$\frac{4}{9}+\cos^2\theta=1$

$\cos^2\theta=\frac{5}{9}$ ***(A1)***

$\cos\theta=\pm\frac{\sqrt{5}}{3}$

$\cos\theta<0$ (since $\text{cosec}\theta>0$ puts θ in the second quadrant) ***R1***

$\cos\theta=-\frac{\sqrt{5}}{3}$

$\cot\theta=-\frac{\sqrt{5}}{2}$ ***A1***

Note: Award ***M1A1R0A0*** for $\cot\theta=\frac{\sqrt{5}}{2}$ seen as the final answer

The ***R1*** should be awarded independently for a negative value only given as a final answer.