Answer 1:

(a) $\cos\theta = \dfrac{\text{OC}}{r}$ **A1**

$\text{OC} = r\cos\theta$ **AG** **N0**

[1 mark]

(b) valid approach **(M1)**

eg $\dfrac{1}{2}\text{OC}\times\text{OB}\sin\theta$, $\text{BC} = r\sin\theta$, $\dfrac{1}{2}r\cos\theta\times\text{BC}$, $\dfrac{1}{2}r\sin\theta\times\text{OC}$

area $= \dfrac{1}{2}r^2\sin\theta\cos\theta \left(= \dfrac{1}{4}r^2\sin(2\theta)\right)$ (must be in terms of r and θ) **A1** **N2**

[2 marks]

(c) valid attempt to express the relationship between the areas (seen anywhere) **(M1)**

eg $\text{OCB} = \dfrac{3}{5}\text{OBA}$, $\dfrac{1}{2}r^2\sin\theta\cos\theta = \dfrac{3}{5}\times\dfrac{1}{2}r^2\theta$, $\dfrac{1}{4}r^2\sin 2\theta = \dfrac{3}{10}r^2\theta$

correct equation in terms of θ only **A1**

eg $\sin\theta\cos\theta = \dfrac{3}{5}\theta$, $\dfrac{1}{4}\sin 2\theta = \dfrac{3}{10}\theta$

valid attempt to solve **their** equation **(M1)**

eg sketch, $-0.830017, 0$

0.830017

$\theta = 0.830$ **A1** **N2**

Note: Do not award final ***A1*** if additional answers given.

[4 marks]

Answer 2:

(a) $\cos 105^\circ = \cos\left(180^\circ - 75^\circ\right) = -\cos 75^\circ$ **R1**

$= -q$ **AG**

Note: Accept arguments using the unit circle or graphical/diagrammatical considerations.

[1 mark]

(b) $\mathrm{AD} = \mathrm{CD} \Rightarrow \mathrm{C\hat{A}D} = 45^\circ$ **A1**

valid method to find $\mathrm{B\hat{A}C}$ ***(M1)***

for example: $\mathrm{BC} = r \Rightarrow \mathrm{B\hat{C}A} = 60^\circ$

$\Rightarrow \mathrm{B\hat{A}C} = 30^\circ$ **A1**

hence $\mathrm{B\hat{A}D} = 45^\circ + 30^\circ = 75^\circ$ **AG**

[3 marks]

(c) (i) $\mathrm{AB} = r\sqrt{3}$, $\mathrm{AD}(=\mathrm{CD}) = r\sqrt{2}$ **A1A1**

applying cosine rule ***(M1)***

$\mathrm{BD}^2 = \left(r\sqrt{3}\right)^2 + \left(r\sqrt{2}\right)^2 - 2\left(r\sqrt{3}\right)\left(r\sqrt{2}\right)\cos 75^\circ$ **A1**

$= 3r^2 + 2r^2 - 2r^2\sqrt{6}\cos 75^\circ$

$= 5r^2 - 2r^2 q\sqrt{6}$ **AG**

(ii) $\mathrm{B\hat{C}D} = 105^\circ$ ***(A1)***

attempt to use cosine rule on $\Delta\mathrm{BCD}$ ***(M1)***

$\mathrm{BD}^2 = r^2 + \left(r\sqrt{2}\right)^2 - 2r\left(r\sqrt{2}\right)\cos 105^\circ$

$= 3r^2 + 2r^2 q\sqrt{2}$ **A1**

[7 marks]

(d) $5r^2 - 2r^2 q\sqrt{6} = 3r^2 + 2r^2 q\sqrt{2}$ ***(M1)(A1)***

$2r^2 = 2r^2 q\left(\sqrt{6} + \sqrt{2}\right)$ **A1**

Note: Award ***A1*** for any correct intermediate step seen using only two terms.

$q = \dfrac{1}{\sqrt{6} + \sqrt{2}}$ **AG**

Note: Do not award the final ***A1*** if follow through is being applied.

[3 marks]

Total [14 marks]

Answer 3:

QUESTION 2

(a) (i) $\sin 140^{\circ} = p$ *A1* *N1*

(ii) $\cos 70^{\circ} = -q$ *A1* *N1*

(b) **METHOD 1**

evidence of using $\sin^2\theta + \cos^2\theta = 1$ *(M1)*

e.g. diagram, $\sqrt{1-p^2}$ (seen anywhere)

$\cos 140^{\circ} = \pm\sqrt{1-p^2}$ *(A1)*

$\cos 140^{\circ} = -\sqrt{1-p^2}$ *A1* *N2*

METHOD 2

evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ *(M1)*

$\cos 140^{\circ} = 2\cos^2 70 - 1$ *(A1)*

$\cos 140^{\circ} = 2(-q)^2 - 1 \ (= 2q^2 - 1)$ *A1* *N2*

(c) **METHOD 1**

$\tan 140^{\circ} = \frac{\sin 140^{\circ}}{\cos 140^{\circ}} = -\frac{p}{\sqrt{1-p^2}}$ *A1* *N1*

METHOD 2

$\tan 140^{\circ} = \frac{p}{2q^2 - 1}$ *A1* *N1*

Answer 4:

7. (a) attempt to expand *(M1)*

e.g. $(\sin x+\cos x)(\sin x+\cos x)$; at least 3 terms

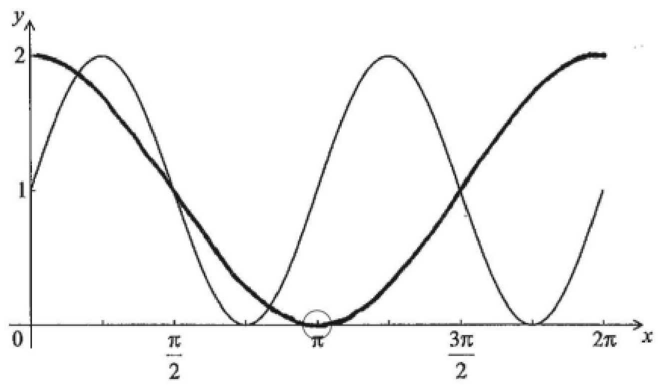
correct expansion *A1*

e.g. $\sin^2 x+2\sin x\cos x+\cos^2 x$

$f(x)=1+\sin 2x$ *AG* *N0*

[2 marks]

(b)



A1A1 *N2*

Note: Award ***A1*** for correct sinusoidal shape with period 2π and range [0, 2], ***A1*** for minimum in circle.

[2 marks]

(c) $p=2,\ k=-\frac{\pi}{2}$ *A1A1* *N2*

[2 marks]

Total [6 marks]

Answer 5:

(a) **EITHER**

$\text{LHS} = \dfrac{\sqrt{3}-1}{\dfrac{\sqrt{6}-\sqrt{2}}{4}} + \dfrac{\sqrt{3}+1}{\dfrac{\sqrt{6}+\sqrt{2}}{4}}$ ***M1***

$= \dfrac{\sqrt{3}-1}{\dfrac{\sqrt{3}-1}{2\sqrt{2}}} + \dfrac{\sqrt{3}+1}{\dfrac{\sqrt{3}+1}{2\sqrt{2}}}$ ***A1***

$= 2\sqrt{2} + 2\sqrt{2}$ ***A1***

$\text{LHS} = 4\sqrt{2} \Rightarrow x = \dfrac{\pi}{12}$ is a solution ***AG***

OR

$\text{LHS} = \dfrac{\sqrt{3}-1}{\dfrac{\sqrt{6}-\sqrt{2}}{4}} + \dfrac{\sqrt{3}+1}{\dfrac{\sqrt{6}+\sqrt{2}}{4}}$ ***M1***

$= \dfrac{\left(\sqrt{3}-1\right)\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)+\left(\sqrt{3}+1\right)\left(\dfrac{\sqrt{6}-\sqrt{2}}{4}\right)}{\left(\dfrac{\sqrt{6}-\sqrt{2}}{4}\right)\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)}$ ***A1***

$= 2\sqrt{18} - 2\sqrt{2}$ (or equivalent) ***A1***

$\text{LHS} = 4\sqrt{2} \Rightarrow x = \dfrac{\pi}{12}$ is a solution ***AG***

[3 marks]

(b) $\frac{\sqrt{2}}{4}\left(\frac{\sqrt{3}-1}{\sin x}+\frac{\sqrt{3}+1}{\cos x}\right)=2 \Rightarrow \frac{\sin\frac{\pi}{12}}{\sin x}+\frac{\cos\frac{\pi}{12}}{\cos x}=2$ **M1**

$\frac{\sin\frac{\pi}{12}\cos x+\cos\frac{\pi}{12}\sin x}{\sin x\cos x}=2$ **M1**

$\sin\frac{\pi}{12}\cos x+\cos\frac{\pi}{12}\sin x=2\sin x\cos x$

$\sin\left(\frac{\pi}{12}+x\right)=\sin 2x$ **A1**

$\frac{\pi}{12}+x=\pi-2x$ or $\pi-\left(\frac{\pi}{12}+x\right)=2x$ ***(M1)***

$x=\frac{11\pi}{36}$ **A1**

[5 marks]

Total [8 marks]

Answer 6:

(a) evidence of choosing sine rule *(M1)*

eg $\dfrac{\text{AC}}{\sin \text{C}\hat{\text{B}}\text{A}} = \dfrac{\text{AB}}{\sin \text{A}\hat{\text{C}}\text{B}}$

correct substitution *(A1)*

eg $\dfrac{\text{AC}}{\sin 44^\circ} = \dfrac{15}{\sin 83^\circ}$

10.4981

$\text{AC} = 10.5$ (cm) ***A1*** ***N2***

[3 marks]

(b) finding $\text{C}\hat{\text{A}}\text{B}$ (seen anywhere) *(A1)*

eg $180^\circ - 44^\circ - 83^\circ$, $\text{C}\hat{\text{A}}\text{B} = 53^\circ$

correct substitution for area of triangle ABC ***A1***

eg $\dfrac{1}{2} \times 15 \times 10.4981 \times \sin 53^\circ$

62.8813

area = 62.9 (cm^2) ***A1*** ***N2***

[3 marks]

(c) correct substitution for area of triangle DAC *(A1)*

eg $\dfrac{1}{2} \times 6 \times 10.4981 \times \sin\theta$

attempt to equate area of triangle ACD to half the area of triangle ABC *(M1)*

eg area $\text{ACD} = \dfrac{1}{2} \times$ area ABC; $2\text{ACD} = \text{ABC}$

correct equation ***A1***

eg $\dfrac{1}{2} \times 6 \times 10.4981 \times \sin\theta = \dfrac{1}{2}(62.9)$, $62.9887 \sin\theta = 62.8813$, $\sin\theta = 0.998294$

$86.6531, 93.3468$

$\theta = 86.7^\circ$, $\theta = 93.3^\circ$ ***A1A1*** ***N2***

[5 marks]

(d)

Note:	Note: If candidates use an acute angle from part (c) in the cosine rule , award ***M1A0A0*** in part (d).

evidence of choosing cosine rule *(M1)*

eg $CD^2 = AD^2 + AC^2 - 2 \times AD \times AC \times \cos\theta$

correct substitution into rhs *(A1)*

eg $CD^2 = 6^2 + 10.498^2 - 2(6)(10.498)\cos 93.336^\circ$

12.3921

12.4 (cm) ***A1*** ***N2***

[3 marks]

Total [14 marks]

Answer 7:

5. (a) evidence of valid approach *(M1)*

e.g. $\dfrac{\text{max } y \text{ value} - \text{min } y \text{ value}}{2}$, distance from $y = -1$

$a = 3$ ***A1*** ***N2***

[2 marks]

(b) (i) evidence of valid approach *(M1)*

e.g. finding difference in x-coordinates, $\dfrac{\pi}{2}$

evidence of doubling ***A1***

e.g. $2 \times \left(\dfrac{\pi}{2}\right)$

period $= \pi$ ***AG*** ***N0***

(ii) evidence of valid approach *(M1)*

e.g. $b = \dfrac{2\pi}{\pi}$

$b = 2$ ***A1*** ***N2***

[4 marks]

(c) $c = \dfrac{\pi}{4}$ ***A1*** ***N1***

[1 mark]

Total [7 marks]

Answer 8:

4. (a) $(f \circ g)(x) = f(2x)$ ***(A1)***

$f(2x) = \sqrt{3}\sin 2x + \cos 2x$ ***A1***

[2 marks]

(b) $\sqrt{3}\sin 2x + \cos 2x = 2\cos 2x$

$\sqrt{3}\sin 2x = \cos 2x$

recognising to use $\tan$ or $\cot$ ***M1***

$\tan 2x = \frac{1}{\sqrt{3}}$ OR $\cot 2x = \sqrt{3}$ (values may be seen in right triangle) ***(A1)***

$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) = \right) \frac{\pi}{6}$ (seen anywhere) (accept degrees) ***(A1)***

$2x = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$ ***A1A1***

Note: Do not award the final ***A1*** if any additional solutions are seen.
Award ***A1A0*** for correct answers in degrees.
Award ***A0A0*** for correct answers in degrees with additional values.

[5 marks]

Total [7 marks]

Answer 9:

(a) valid approach *(M1)*

eg $\frac{\text{max} - \text{min}}{2}$, sketch of graph, $9.7 = p\cos(0) + 7.5$

$p = 2.2$ **A1** **N2**

[2 marks]

(b) valid approach *(M1)*

eg $B = \frac{2\pi}{\text{period}}$, period is 14, $\frac{360}{14}$, $5.3 = 2.2\cos 7q + 7.5$

0.448798

$q = \frac{2\pi}{14}\left(\frac{\pi}{7}\right)$, 0.449 (do not accept degrees) **A1** **N2**

[2 marks]

(c) valid approach *(M1)*

eg $d(10)$, $2.2\cos\left(\frac{20\pi}{14}\right) + 7.5$

7.01045

7.01 (m) **A1** **N2**

[2 marks]

[Total 6 marks]