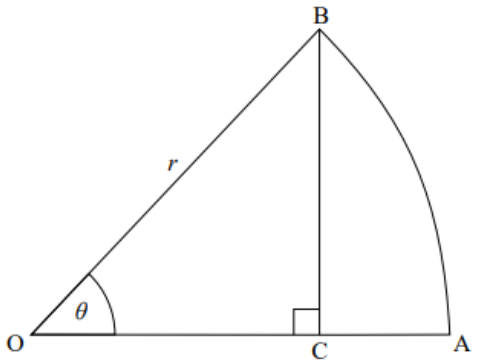
Total Questions: 9
Total Marks: 75

Question 1:
Calculator Allowed: Yes

[Maximum mark: 7]

OAB is a sector of the circle with centre O and radius r, as shown in the following diagram.

diagram not to scale



The angle AOB is θ radians, where $0 < \theta < \frac{\pi}{2}$.

The point C lies on OA and OA is perpendicular to BC.

(a) Show that $OC = r\cos\theta$. [1]

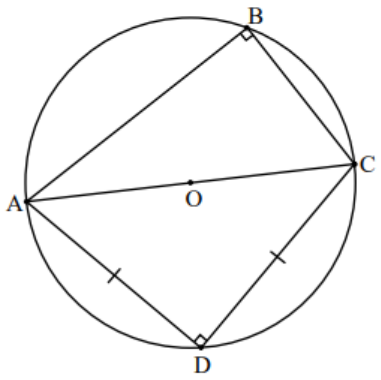
(b) Find the area of triangle OBC in terms of r and θ. [2]

(c) Given that the area of triangle OBC is $\frac{3}{5}$ of the area of sector OAB, find θ. [4]

Question 2:
Calculator Allowed: No

(a) Given that $\cos 75^\circ = q$, show that $\cos 105^\circ = -q$. [1]

In the following diagram, the points A, B, C and D are on the circumference of a circle with centre O and radius r. [AC] is a diameter of the circle. $BC = r$, $AD = CD$ and $A\hat{B}C = A\hat{D}C = 90^\circ$.



(b) Show that $B\hat{A}D = 75^\circ$. [3]

(c) (i) By considering triangle ABD, show that $BD^2 = 5r^2 - 2r^2 q\sqrt{6}$.

(ii) By considering triangle CBD, find another expression for BD^2 in terms of r and q. [7]

(d) Use your answers to part (c) to show that $\cos 75^\circ = \frac{1}{\sqrt{6}+\sqrt{2}}$. [3]

Question 3:
Calculator Allowed: No

2. *[Maximum mark: 6]*

Let $p = \sin 40^\circ$ and $q = \cos 110^\circ$. Give your answers to the following in terms of p and/or q.

(a) Write down an expression for

(i) $\sin 140^\circ$;

(ii) $\cos 70^\circ$. *[2 marks]*

(b) Find an expression for $\cos 140^\circ$. *[3 marks]*

(c) Find an expression for $\tan 140^\circ$. *[1 mark]*

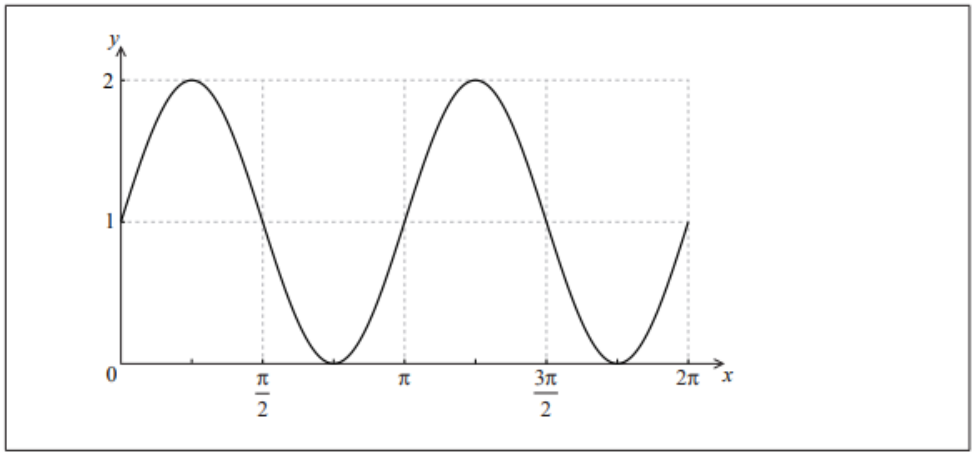
Question 4:
Calculator Allowed: No

7. *[Maximum mark: 6]*

Let $f(x) = (\sin x + \cos x)^2$.

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$. *[2 marks]*

The graph of f is shown below for $0 \le x \le 2\pi$.



(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$. *[2 marks]*

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$.

(c) Write down the value of p and a possible value of k. *[2 marks]*

Question 5:
Calculator Allowed: No

[Maximum mark: 8]

Consider the equation $\frac{\sqrt{3}-1}{\sin x} + \frac{\sqrt{3}+1}{\cos x} = 4\sqrt{2}$, $0 < x < \frac{\pi}{2}$. Given that $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$ and $\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$

(a) verify that $x = \frac{\pi}{12}$ is a solution to the equation; [3]

(b) hence find the other solution to the equation for $0 < x < \frac{\pi}{2}$. [5]

Question 6:
Calculator Allowed: Yes

[Maximum mark: 14]

The following diagram shows the quadrilateral ABCD.

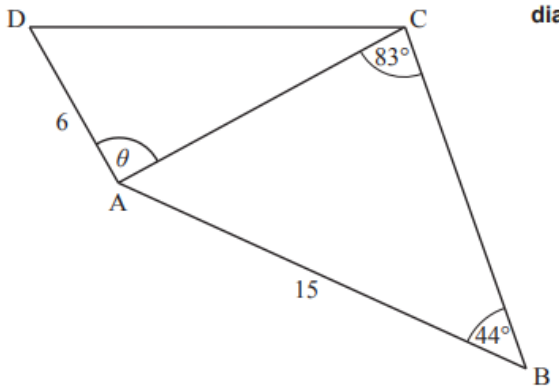


diagram not to scale

$AD = 6\,\text{cm}$, $AB = 15\,\text{cm}$, $A\hat{B}C = 44^\circ$, $A\hat{C}B = 83^\circ$ and $D\hat{A}C = \theta$

(a) Find AC. [3]

(b) Find the area of triangle ABC. [3]

The area of triangle ACD is half the area of triangle ABC.

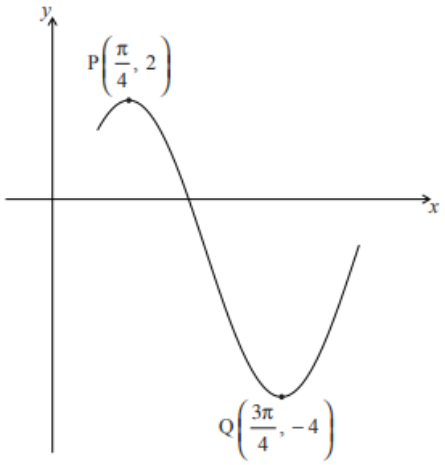
(c) Find the possible values of θ. [5]

(d) Given that θ is obtuse, find CD. [3]

Question 7:
Calculator Allowed: No

5. *[Maximum mark: 7]*

The diagram below shows part of the graph of $f(x) = a\cos(b(x-c)) - 1$, where $a > 0$.



The point $P\left(\frac{\pi}{4}, 2\right)$ is a maximum point and the point $Q\left(\frac{3\pi}{4}, -4\right)$ is a minimum point.

(a) Find the value of a. *[2 marks]*

(b) (i) Show that the period of f is π.

(ii) Hence, find the value of b. *[4 marks]*

(c) Given that $0 < c < \pi$, write down the value of c. *[1 mark]*

Question 8:
Calculator Allowed: No

4. [Maximum mark: 7]

Consider the functions $f(x) = \sqrt{3}\sin x + \cos x$ where $0 \le x \le \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

(a) Find $(f \circ g)(x)$. [2]

(b) Solve the equation $(f \circ g)(x) = 2\cos 2x$ where $0 \le x \le \pi$. [5]

Question 9:
Calculator Allowed: Yes

[Maximum mark: 6]

The depth of water in a port is modelled by the function $d(t) = p\cos qt + 7.5$, for $0 \le t \le 12$, where t is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

At low tide, which is 7 hours later, the depth is 5.3 metres.

(a) Find the value of p. [2]

(b) Find the value of q. [2]

(c) Use the model to find the depth of the water 10 hours after high tide. [2]