

Topic:

Total Time:

Name of student

General Instructions

- On the first page, write your **name**, **start time**, and **end time** clearly.
- Submit your answers as a **single PDF file**.
- During the test, stay **live on Zoom** and **share your entire desktop**.
- If a question hasn't been covered in class or is outside the syllabus, just skip it and write '**NA**'. It won't affect your grade.
- Unless mentioned otherwise, give all numerical answers **exactly** or correct to **three significant figures**.
- You may use the **official IB formula booklet** during the test.

<https://leadib.com>

Total Questions: 8

Total Marks: 74

Question 1:

Calculator Allowed: No

7. [Maximum mark: 7]

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- (a) Use the derivatives of $\sin x$ and $\cos x$ to show that the derivative of $\tan x$ is $\sec^2 x$.

[3 marks]

- (b) Hence by using $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$, show that the derivative of $\arctan x$ is $\frac{1}{1+x^2}$.

[4 marks]

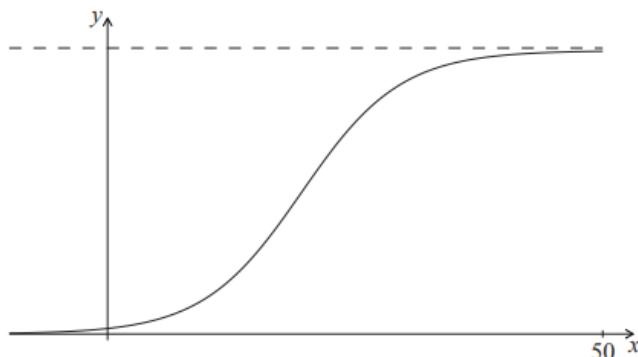
Question 2:

Calculator Allowed: Yes

9. [Maximum mark: 15]

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Let $f(x) = \frac{100}{(1+50e^{-0.2x})}$. Part of the graph of f is shown below.



- (a) Write down $f(0)$.

[1 mark]

- (b) Solve $f(x) = 95$.

[2 marks]

- (c) Find the range of f .

[3 marks]

- (d) Show that $f'(x) = \frac{1000e^{-0.2x}}{(1+50e^{-0.2x})^2}$.

[5 marks]

- (e) Find the maximum rate of change of f .

[4 marks]

Question 3:

Calculator Allowed: No

[Maximum mark: 7]

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Find the x -coordinates of all the points on the curve $y = 2x^4 + 6x^3 + \frac{7}{2}x^2 - 5x + \frac{3}{2}$ at which the tangent to the curve is parallel to the tangent at $(-1, 6)$.

Question 4:

Calculator Allowed: No

8. [Maximum mark: 7]

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The normal to the curve $xe^{-y} + e^y = 1+x$, at the point $(c, \ln c)$, has a y -intercept $c^2 + 1$.

Determine the value of c .

Question 5:

Calculator Allowed: Yes

1. [Maximum mark: 5]

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Given that the graph of $y = x^3 - 6x^2 + kx - 4$ has exactly one point at which the gradient is zero, find the value of k .

Question 6:

Calculator Allowed: Yes

7. [Maximum mark: 5]

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Consider the functions $f(x) = x^3 + 1$ and $g(x) = \frac{1}{x^3 + 1}$. The graphs of $y = f(x)$ and $y = g(x)$ meet at the point $(0, 1)$ and one other point, P.

- (a) Find the coordinates of P.

[1 mark]

- (b) Calculate the size of the acute angle between the tangents to the two graphs at the point P.

[4 marks]

Question 7:

Calculator Allowed: No

12. [Maximum mark: 19]

Consider the function $f(x) = \frac{\ln x}{x}$, $0 < x < e^2$.

- (a) (i) Solve the equation $f'(x) = 0$.

- (ii) Hence show the graph of f has a local maximum.

- (iii) Write down the range of the function f .

[5 marks]

- (b) Show that there is a point of inflexion on the graph and determine its coordinates.

[5 marks]

- (c) Sketch the graph of $y = f(x)$, indicating clearly the asymptote, x -intercept and the local maximum.

[3 marks]

- (d) Now consider the functions $g(x) = \frac{\ln|x|}{x}$ and $h(x) = \frac{\ln|x|}{|x|}$, where $0 < |x| < e^2$.

- (i) Sketch the graph of $y = g(x)$.

- (ii) Write down the range of g .

- (iii) Find the values of x such that $h(x) > g(x)$.

[6 marks]

Question 8:

Calculator Allowed: No

A curve has equation $3x - 2y^2e^{x-1} = 2$.

- (a) Find an expression for $\frac{dy}{dx}$ in terms of x and y . [5]
- (b) Find the equations of the tangents to this curve at the points where the curve intersects the line $x = 1$. [4]