Topic : Sequence and Series
Total Marks : 50 Total Time : 60 mins
Prior to starting the test, please take a moment to carefully review the following instructions:

1. On the first page please mention your name, start time, and end time of the test and share your answer sheet as a single pdf.
2. To create a realistic test environment, ensure that you are live on Zoom during the test, for that you must use the Zoom app not web version. Keep your video camera turned on and share your entire desktop.
3. **IMPORTANT:** If you encounter any questions that haven't been covered in class yet or fall outside the test syllabus, no need to worry. Just skip that question and mention 'NA', and your grades will be based on the questions you attempted.
4. Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
5. You are allowed to use the official IB formula booklet for all tests
6. Use of GDC is not allowed.

1. An 81 metre rope is cut into n pieces of increasing lengths that form an arithmetic sequence with a common difference of d metres. Given that the lengths of the shortest and longest pieces are 1.5 metres and 7.5 metres respectively, find the values of n and d.

(Total 4 marks)

2. A circular disc is cut into twelve sectors whose areas are in an arithmetic sequence. The angle of the largest sector is twice the angle of the smallest sector.

Find the size of the angle of the smallest sector.

(Total 5 marks)

3. The sum, S_n, of the first n terms of a geometric sequence, whose n^{th} term is u_n, is given by

$$S_n = \frac{7^n - a^n}{7^n}, \text{ where } a > 0.$$

(a) Find an expression for u_n.

(2)

(b) Find the first term and common ratio of the sequence.

(4)

(c) Consider the sum to infinity of the sequence.

(i) Determine the values of a such that the sum to infinity exists.

(ii) Find the sum to infinity when it exists.

(2)
(Total 8 marks)

4. The mean of the first ten terms of an arithmetic sequence is 6. The mean of the first twenty terms of the arithmetic sequence is 16. Find the value of the 15^{th} term of the sequence.

(Total 6 marks)

5. The common ratio of the terms in a geometric series is 2^x.

(a) State the set of values of x for which the sum to infinity of the series exists.

(2)

(b) If the first term of the series is 35, find the value of x for which the sum to infinity is 40.

(4)

(Total 6 marks)

6. A geometric sequence $u_1, u_2, u_3, \ldots$ has $u_1 = 27$ and a sum to infinity of $\frac{81}{2}$.

(a) Find the common ratio of the geometric sequence.

(2)

An arithmetic sequence $v_1, v_2, v_3, \ldots$ is such that $v_2 = u_2$ and $v_4 = u_4$.

(b) Find the greatest value of N such that $\sum_{n=1}^{N} v_n > 0$.

(5)

(Total 7 marks)

7. (a) The sum of the first six terms of an arithmetic series is 81. The sum of its first eleven terms is 231. Find the first term and the common difference.

(6)

(b) The sum of the first two terms of a geometric series is 1 and the sum of its first four terms is 5. If all of its terms are positive, find the first term and the common ratio.

(5)

(c) The r^{th} term of a new series is defined as the product of the r^{th} term of the arithmetic series and the r^{th} term of the geometric series above. Show that the r^{th} term of this new series is $(r + 1)2^{r-1}$.

(3)

(Total 14 marks)