MARKSCHEME

1. prove that $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}$

for $n = 1$

LHS = 1, RHS = $4 - \frac{1+2}{2^0} = 4 - 3 = 1$

so true for $n = 1$ R1

assume true for $n = k$ M1

so $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} = 4 - \frac{k+2}{2^{k-1}}$

now for $n = k + 1$

LHS: $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} + (k+1)\left(\frac{1}{2}\right)^k$ A1

$= 4 - \frac{k+2}{2^{k-1}} + (k+1)\left(\frac{1}{2}\right)^k$ M1A1

$= 4 - \frac{2(k+2)}{2^k} + \frac{k+1}{2^k}$ (or equivalent) A1

$= 4 - \frac{(k+1)+2}{2^{(k+1)-1}}$ (accept $4 - \frac{k+3}{2^k}$) A1

Therefore if it is true for $n = k$ it is true for $n = k + 1$. It has been shown to be true for $n = 1$ so it is true for all n ($\in \mathbb{Z}^+$). R1

Note: To obtain the final *R* mark, a reasonable attempt at induction must have been made.

[8]

2. let $n = 1$

LHS = $1 \times 1! = 1$

RHS = $(1 + 1)! - 1 = 2 - 1 = 1$

hence true for $n = 1$ R1

assume true for $n = k$

$\sum_{r=1}^{k} r(r!) = (k + 1)! - 1$ M1

$\sum_{r=1}^{k+1} r(r!) = (k + 1)! - 1 + (k + 1) \times (k + 1)!$ M1A1

$= (k + 1)!(1 + k + 1) - 1$

$= (k + 1)!(k + 2) - 1$ A1

$= (k + 2)! - 1$ A1

hence if true for $n = k$, true for $n = k + 1$ R1

since the result is true for $n = 1$ and P $(k) \Rightarrow$ P$(k + 1)$ the result is proved by mathematical induction $\forall n \in \mathbb{Z}^+$ R1

[8]

3. (a) (i) $1 \times 2 + 2 \times 3 + \ldots + n(n + 1) = \frac{1}{3}n(n + 1)(n + 2)$ R1

(ii) LHS = 40; RHS = 40 A1

(b) the sequence of values are:
5, 7, 11, 19, 35 … or an example A1

35 is not prime, so Bill′s conjecture is false R1AG

(c) $P(n) : 5 \times 7^n + 1$ is divisible by 6
P(1): 36 is divisible by 6 $\Rightarrow$ P(1) true A1
assume $P(k)$ is true $(5 \times 7^k + 1 = 6r)$ M1

Note: Do **not** award M1 for statement starting 'let $n = k$'.
Subsequent marks are independent of this M1.

consider $5 \times 7^{k+1} + 1$ M1
$= 7(6r - 1) + 1$ (A1)
$= 6(7r - 1) \Rightarrow P(k + 1)$ is true A1
P(1) true and $P(k)$ true $\Rightarrow$ $P(k + 1)$ true, so by MI $P(n)$ is true for all $n \in \mathbb{Z}^+$ R1

Note: Only award R1 if there is consideration of P(1), $P(k)$ and $P(k + 1)$ in the final statement.

Only award R1 if at least one of the two preceding A marks has been awarded.

[10]

4. (a) $S_6 = 81 \Rightarrow 81 = \frac{6}{2}(2a + 5d)$ M1A1
$\Rightarrow 27 = 2a + 5d$
$S_{11} = 231 \Rightarrow 231 = \frac{11}{2}(2a + 10d)$ M1A1
$\Rightarrow 21 = a + 5d$
solving simultaneously, $a = 6$, $d = 3$ A1A1

(b) $a + ar = 1$ A1
$a + ar + ar^2 + ar^3 = 5$ A1
$\Rightarrow (a + ar) + ar^2(1 + r) = 5$
$\Rightarrow 1 + ar^2 \times \frac{1}{a} = 5$
obtaining $r^2 - 4 = 0$ M1
$\Rightarrow r = \pm 2$
$r = 2$ (since all terms are positive) A1
$a = \frac{1}{3}$ A1

(c) AP r^{th} term is $3r + 3$ A1
GP r^{th} term is $\frac{1}{3}2^{r-1}$ A1
$3(r + 1) \times \frac{1}{3}2^{r-1} = (r + 1)2^{r-1}$ M1AG

(d) prove: $P_n : \sum_{r=1}^{n}(r+1)2^{r-1} = n2^n, n \in \mathbb{Z}^+$
show true for $n = 1$, i.e.
LHS $= 2 \times 2^0 = 2 =$ RHS A1
assume true for $n = k$, i.e. M1
$\sum_{r=1}^{k}(r+1)2^{r-1} = k2^k, k \in \mathbb{Z}^+$
consider $n = k + 1$

$\sum_{r=1}^{k+1}(r+1)2^{r-1} = k2^k + (k+1)2^k$ M1A1

$= 2^k(k + k + 2)$

$= 2(k + 1)2^k$ A1

$= (k + 1)2^{k+1}$ A1

hence true for $n = k + 1$

P_{k+1} is true whenever P_k is true, and P_1 is true, therefore P_n is true for $n \in \mathbb{Z}^+$ R1

[21]