Topic : Functions

Total Marks : 46   **Total Time : 60 mins**

GDC Allowed   :   Q5   **GDC not Allowed   :   Q1,   Q2,   Q3,   Q4,   Q6,   Q7**

Prior to starting the test, please take a moment to carefully review the following instructions:

1. **On the first page please mention your name, start time, and end time of the test and share your answer sheet as a single pdf.**

2. **To create a realistic test environment, ensure that you are live on Zoom during the test, for that you must use the Zoom app not web version. Keep your video camera turned on and share your entire desktop.**

3. **IMPORTANT: If you encounter any questions that haven't been covered in class yet or fall outside the test syllabus, no need to worry. Just skip that question and mention 'NA', and your grades will be based on the questions you attempted.**

4. **Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.**

5. **You are allowed to use the official IB formula booklet for all tests.**

1.   The real root of the equation $x^3 - x + 4 = 0$ is -1.796 to three decimal places.
        Determine the real root for each of the following.

        (a) $(x - 1)^3 - (x - 1) + 4 = 0$

        (2)

        (b) $8x^3 - 2x + 4 = 0$

        (3)
        (Total 5 marks)

2.   Let $f(x) = \dfrac{x+4}{x+1}$, $x \neq -1$ and $g(x) = \dfrac{x-2}{x-4}$, $x \neq 4$. Find the set of values of x such that $f(x) \leq g(x)$.

        (Total 6 marks)

3.   The functions f and g are defined by $f : x \mapsto e^x$, $g : x \mapsto x + 2$.

        Calculate

        (a) $f^{-1}(3) \times g^{-1}(3)$;

        (3)

        (b) $(f \circ g)^{-1}(3)$.

        (3)
        (Total 6 marks)

4. The polynomial $f(x) = x^3 + 3x^2 + ax + b$ leaves the same remainder when divided by $(x - 2)$ as when divided by $(x + 1)$. Find the value of a.

(Total 6 marks)

5. (a) Sketch the curve $y = |\ln x| - |\cos x| - 0.1$, $0 < x < 4$ showing clearly the coordinates of the points of intersection with the x-axis and the coordinates of any local maxima and minima.

(5)

(b) Find the values of x for which $|\ln x| > |\cos x| + 0.1$, $0 < x < 4$.

(2)

(Total 7 marks)

6. The functions f and g are defined as:

$$f(x) = e^{x^2}, \quad x \geq 0$$

$$g(x) = \frac{1}{x + 3}, x \neq -3.$$

(a) Find $h(x)$ where $h(x) = g \circ f(x)$.

(2)

(b) State the domain of $h^{-1}(x)$.

(2)

(c) Find $h^{-1}(x)$.

(4)

(Total 8 marks)

7. The cubic curve $y = 8x^3 + bx^2 + cx + d$ has two distinct points P and Q, where the gradient is zero.

(a) Show that $b^2 > 24c$.

(4)

(b) Given that the coordinates of P and Q are $\left(\frac{1}{2}, -12\right)$ and $\left(-\frac{3}{2}, 20\right)$, respectively, find the values of b, c and d.

(4)

(Total 8 marks)