Test   12.11.24
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Start : 4:06
END : 5:05

1.
a) (i)

$$\frac{p \cdot \ln x}{\ln x} = \frac{\frac{1}{3} \ln x}{p \cdot \ln x}$$ M1

$$p^2 = \frac{1}{3}$$ A1

$$p = \pm \frac{1}{\sqrt{3}}$$ AG ✓

(ii) $p = r$

$$|p| = \frac{1}{\sqrt{3}}$$ R1 ✓

$$\frac{1}{\sqrt{3}} < 1$$

thus the series is
convergent ✓

(iii)

$$S_\infty = \frac{u_1}{1-r}$$

$$3 \cdot \sqrt{3} = \frac{\ln x}{1 - \frac{1}{\sqrt{3}}}$$ ✓ A1

$$\ln x = \left(1 - \frac{1}{\sqrt{3}}\right)\left(3 + \sqrt{3}\right)$$ ✓

$$= 3 + \sqrt{3} - \frac{3}{\sqrt{3}} - 1$$

$$= 3 + \sqrt{3} - \sqrt{3} - 1$$ ✓ A1

$$\ln x = 2$$

$$x = e^2$$ ✓ A1

b) (i) $p \ln x - \ln x = \frac{1}{3} \ln x - p \ln x$ M1

$2p \ln x = \frac{4}{3} \ln x$ A1

$2p = \frac{4}{3}$ ✓

$p = \frac{4}{6}$

$p = \frac{2}{3}$ ✓   AG

(ii) $\frac{3}{3} \ln x + \frac{2}{3} \ln x + \frac{1}{3} \ln x$

$d = -\frac{1}{3} \ln x$ ✓   A1

REDO

(iii) $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ ✓

$\ln\left(\frac{1}{x^3}\right) = \frac{n}{2}\left(2\ln x + (n-1)\left(-\frac{1}{3}\ln x\right)\right)$ ✓   M1

$\ln\left(\frac{1}{x^3}\right) = \frac{n}{2}\left(2\ln x - \frac{n}{3}\ln x + \frac{1}{3}\ln x\right)$

$\ln\left(\frac{1}{x^3}\right) = n\ln x - \frac{n^2 \ln x}{6} + \frac{n\ln x}{6}$

$\ln\left(\frac{1}{x^3}\right) = n\ln x - \frac{n^2 \ln x + n\ln x}{6}$

$\ln\left(\frac{1}{x^3}\right) = n\ln x - \frac{n\ln x(n+1)}{6}$

$\therefore$

?

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2.

a) $A = A_0 e^{-kt}$

$t = 0$ ✓

$A = 100$

$100 = A_0 e^0$

$A_0 = 100$ ✓ AG

b) $50 = 100 e^{-5730k}$ M1

$\frac{1}{2} = e^{-5730k}$ ✓

$\ln \frac{1}{2} = -5730k$ ✓ $\div = -1$ A1

$\ln 2 = 5730k$

$k = \frac{\ln 2}{5730}$ ✓ AG

c) $75 = 100 e^{-\frac{\ln 2}{5730} t}$ ✓ A1

$\ln 0.75 = \frac{-\ln 2}{5730} t$ ✓ M1

$t = -\frac{\ln 0.75 \cdot 5730}{\ln 2}$

$t = 2378.164871$ ✓

(nearest 10 years)

$\rightarrow$ 2380 years ✓ A1

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3. $(e^x)^2 - 3(e^x) + \ln k = 0$ ✓ M1

$(-3)^2 - 4(1)(\ln k) \geq 0$ ✓ M1

$9 - 4(\ln k) \geq 0$ A1

$\ln k \leq \frac{9}{4}$ ✓ A1

$\ln k \le \dfrac{9}{4}$

$e^{\frac{9}{4}} \le k$ $\quad$ A1 $\qquad$ 5/6

4. $\quad \left| 0.1x^2 - 2x + 3 \right| < \log_{10} x$ $\quad$ M1

(using GDC)

$x = 1.52, 1.79$ $\quad$ A1A1

$x = 17.6, 19.1$ $\quad$ A1

$(1.52 < x < 1.79) \cup (17.6 < x < 19.1)$

A1A1

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5. $\quad d = \dfrac{1}{\log_8 x} - \dfrac{1}{\log_2 x}$ $\quad$ M1

$= \dfrac{1}{\dfrac{\log_2 x}{\log_2 8}} - \dfrac{1}{\log_2 x}$

$= \dfrac{\log_2 8^{\,3}}{\log_2 x} - \dfrac{1}{\log_2 x}$ $\quad$ M1

$= \dfrac{2}{\log_2 x}$ $\quad$ A1

$100 = \dfrac{20}{2}\left(2 \times \dfrac{1}{\log_2 x} + 19\left(\dfrac{2}{\log_2 x} \right) \right)$

$= \dfrac{20}{2}\left(\dfrac{40}{\log_2 x} \right)$ $\quad$ M1

$100 = \dfrac{400}{\log_2 x}$ $\quad$ A1

$$\log_2 x = 4 \checkmark$$

$$2^4 = 16 \checkmark$$

$$\underline{x = 16} \checkmark \quad \text{A1}$$

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