Answer 1:

5. (a) $X \sim N(4, 0.25^2)$

EITHER

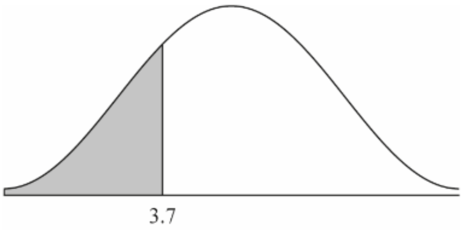
correct probability expression ***(M1)***

$P(X < 3.7)$

Note:	Accept a weak or strict inequality, and any label instead of X, e.g. length or L.

OR

normal curve with vertical line, left of mean, labelled 3.7, and shaded region ***(M1)***



THEN

0.115 (0.115069…, 11.5%) ***A1***

Note:	Award ***M1A0*** for 0.12 if no previous working.

[2 marks]

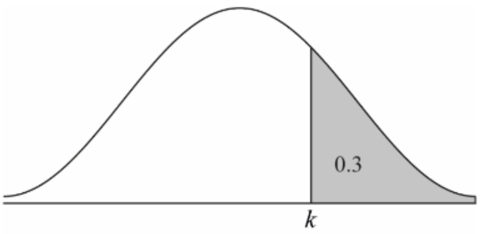
(b) **EITHER**

Correct probability expression ***(M1)***

$(P(X < k) = 0.7$ **OR** $P(X > k) = 0.3$

Note:	Accept a weak or strict inequality, and any label instead of X ***e.g.***, length or L.

OR
normal curve with vertical line to the right of the mean and shaded region, correctly labelled either 0.3 or 0.7 ***(M1)***



THEN
$(k =)\ 4.13\ \ (4.13110\ldots)$ ***A1***

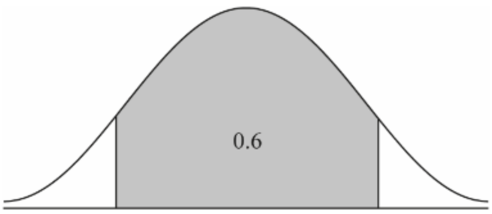
Note: Award ***M1A0*** for 4.1 if no previous working.

[2 marks]

(c) **EITHER**
correct probability equation ***(M1)***
$\mathrm{P}(\text{length} < 4+m) = 0.8$ **OR** $\mathrm{P}(\text{length} < 4-m) = 0.2$

Note: Accept any letter instead of "length" ***e.g.***, X or L.

OR
normal curve with vertical lines symmetrical about the mean line with a correct indication of an area of 0.6 or 0.2 or 0.8 ***(M1)***



THEN
$0.210\ (0.210405\ldots)$ ***A1***

Note: Award ***(M1)A0*** for an answer of 3.7895 or 4.2105 seen without working. Condone 0.21 seen and award ***(M1)A1***.

[2 marks]
Total [6 marks]

Answer 2:

7. (a) $(56 \times 0.86) = 48.2 \quad (48.16)$ ***A1***

Note: Accept 48.

[1 mark]

(b) recognizing binomial distribution (may be seen in (a)) ***(M1)***

e.g. $X \sim \mathrm{B}(56,\, 0.86)$

$\left(\mathrm{P}(X \geq 50) =\right)\ 0.316$ ***A2***

[3 marks]

(c) $\mathrm{P}(X \leq n) \geq 0.25$

$n = 46$ ***A2***

[2 marks]

Total [6 marks]

Answer 3:

5. (a) $0.5 \times 0.1 + 0.4 \times 0.4 + 0.1 \times 0.5$ ***(M1)(M1)(M1)***

Note: Award ***M1*** for 0.5×0.1 or 0.1×0.5, ***M1*** for 0.4×0.4, ***M1*** for adding three correct products.

0.26 ***A1***

[4 marks]

(b) $0 = -8 \times 0.5 + 4 \times 0.4 + 0.1k$ ***(M1)(M1)***

Note: Award ***M1*** for correct substitution into the formula for expected value, award ***M1*** for the expected value formula equated to zero.

$(k =)\ 24$ (points) ***A1***

[3 marks]

Total [7 marks]

Answer 4:

10. (a) attempt to use the symmetry of the normal curve *(M1)*

eg diagram, $0.5-0.1446$

$\mathrm{P}(24.15 < X < 25) = 0.3554$ ***A1***

[2 marks]

(b) (i) use of inverse normal to find z score *(M1)*

$z = -1.0598$

correct substitution $\frac{24.15-25}{\sigma} = -1.0598$ *(A1)*

$\sigma = 0.802$ ***A1***

(ii) $\mathrm{P}(X > 26) = 0.106$ ***(M1)A1***

[5 marks]

(c) recognizing binomial probability *(M1)*

$\mathrm{E}(Y) = 10 \times 0.10621$ *(A1)*

$= 1.06$ ***A1***

[3 marks]

(d) $\mathrm{P}(Y=3)$ *(M1)*

$= 0.0655$ ***A1***

[2 marks]

(e) recognizing conditional probability *(M1)*

correct substitution ***A1***

$$\frac{0.3554}{1-0.10621}$$

$= 0.398$ ***A1***

[3 marks]

Total [15 marks]

Answer 5:

11. (a) $f(x) \geq \frac{1}{25}$ *A1*

$g(x) \in \mathbb{R}, g(x) \geq 0$ *A1*

[2 marks]

(b) $f \circ g(x) = \frac{2\left(\frac{3x-4}{10}\right)^2 + 3}{75}$ *M1A1*

$= \frac{\frac{2(9x^2-24x+16)}{100} + 3}{75}$ *(A1)*

$= \frac{9x^2-24x+166}{3750}$ *A1*

[4 marks]

(c) (i) **METHOD 1**

$y = \frac{2x^2+3}{75}$

$x^2 = \frac{75y-3}{2}$ *M1*

$x = \sqrt{\frac{75y-3}{2}}$ *(A1)*

$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}}$ *A1*

Note: Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***.
Award the ***A1*** only if written in the form $f^{-1}(x) =$.

METHOD 2

$y = \frac{2x^2+3}{75}$

$x = \frac{2y^2+3}{75}$ ***M1***

$y = \sqrt{\frac{75x-3}{2}}$ ***(A1)***

$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}}$ ***A1***

Note:	Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***. Award the ***A1*** only if written in the form $f^{-1}(x) =$.

(ii) domain: $x \geq \frac{1}{25}$; range: $f^{-1}(x) \geq 0$ ***A1***

[4 marks]

continued …

(d) probabilities from $f(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{3}{75}$	$\frac{5}{75}$	$\frac{11}{75}$	$\frac{21}{75}$	$\frac{35}{75}$

A2

Note: Award ***A1*** for one error, ***A0*** otherwise.

probabilities from $g(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{8}{10}$

A2

Note: Award ***A1*** for one error, ***A0*** otherwise.

only in the case of $f(x)$ does $\sum P(X=x)=1$, hence only $f(x)$ can be used as a probability mass function ***A2***

[6 marks]

(e)

$$E(x)=\sum x\cdot \mathrm{P}(X=x)$$ ***M1***

$$=\frac{5}{75}+\frac{22}{75}+\frac{63}{75}+\frac{140}{75}=\frac{230}{75}\left(=\frac{46}{15}\right)$$ ***A1***

[2 marks]

Total [18 marks]

Answer 6:

3. (a) (i) Let X be the random variable "distance from O".

$X \sim \mathrm{N}(10,\, 3^2)$

$\mathrm{P}(X < 13) = 0.841 \;\; (0.841344\ldots)$ ***(M1)A1***

(ii) $(\mathrm{P}(X > 15) =)\; 0.0478 \;\; (0.0477903)$ ***A1***

[3 marks]

(b) $\mathrm{P}(X > 15) \times \mathrm{P}(X > 15)$ ***(M1)***

$= 0.00228 \;\; (0.00228391\ldots)$ ***A1***

[2 marks]

(c) $1 - (0.8143)^3$ ***(M1)***

$= 0.460 \;\; (0.460050\ldots)$ ***A1***

[2 marks]

(d) (i) let Y be the random variable "number of points scored"

evidence of use of binomial distribution ***(M1)***

$Y \sim \mathrm{B}(10,\, 0.539949\ldots)$ ***(A1)***

$(\mathrm{E}(Y) =) 10 \times 0.539949\ldots$ ***(M1)***

$= 5.40$ ***A1***

(ii) $(\mathrm{P}(Y \geq 5) =)\; 0.717 \;\; (0.716650\ldots)$ ***A1***

(iii) $\mathrm{P}(5 \leq Y < 8)$ ***(M1)***

$= 0.628 \;\; (0.627788\ldots)$ ***A1***

Note: Award ***M1*** for a correct probability statement or indication of correct lower and upper bounds, 5 and 7.

(iv) $\dfrac{\mathrm{P}(5 \leq Y < 8)}{\mathrm{P}(Y \geq 5)} \left(= \dfrac{0.627788\ldots}{0.716650\ldots} \right)$ ***(M1)***

$= 0.876 \;\; (0.876003\ldots)$ ***A1***

[9 marks]

Total: [16 marks]

Answer 7:

(a) **METHOD 1**

$T \sim \mathrm{N}(35, \sigma^2)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

attempt to solve for σ graphically or numerically using the GDC ***(M1)***

graph of normal curve $T \sim \mathrm{N}(35, \sigma^2)$ for $\mathrm{P}(T > 40)$ and $y = 0.25$ OR $\mathrm{P}(T < 40)$ and $y = 0.75$ OR table of values for $\mathrm{P}(T < 40)$ or $\mathrm{P}(T > 40)$

$\sigma = 7.413011...$

$\sigma = 7.41$ (min) ***A2***

METHOD 2

$T \sim \mathrm{N}(35, \sigma^2)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

$z = 0.674489...$ ***(A1)***

valid equation using their z-score (clearly identified as z-score and not a probability) ***(M1)***

$\frac{40-35}{\sigma} = 0.674489...$ OR $5 = 0.674489...\sigma$

$7.413011\ldots$

$\sigma = 7.41$ (min) ***A1***

[4 marks]

(b) $\mathrm{P}(T > 45)$ ***(M1)***

$= 0.0886718...$

$= 0.0887$ ***A1***

[2 marks]

(c) recognizing binomial probability **(M1)**

$L \sim \mathrm{B}(5, 0.0886718...)$

$\mathrm{P}(L \geq 1) = 1 - \mathrm{P}(L = 0)$ OR

$\mathrm{P}(L \geq 1) = \mathrm{P}(L = 1) + \mathrm{P}(L = 2) + \mathrm{P}(L = 3) + \mathrm{P}(L = 4) + \mathrm{P}(L = 5)$ **(M1)**

$0.371400\ldots$

$\mathrm{P}(L \geq 1) = 0.371$ **A1**

[3 marks]

(d) recognizing conditional probability in context **(M1)**

finding $\{L < 3\} \cap \{L \geq 1\} = \{L = 1, L = 2\}$ (may be seen in conditional probability) **(A1)**

$\mathrm{P}(L = 1) + \mathrm{P}(L = 2) = 0.36532...$ (may be seen in conditional probability) **(A1)**

$\mathrm{P}(L < 3 \mid L \geq 1) = \dfrac{0.36532...}{0.37140...}$ **(A1)**

$0.983636\ldots$

0.984 **A1**

[5 marks]

(e) **METHOD 1**

recognizing that Suzi can be late no more than once (in the remaining six days) ***(M1)***

$\mathrm{X} \sim \mathrm{B}(6, 0.0886718...)$, where X is the number of days late ***(A1)***

$\mathrm{P}(X \le 1) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1)$ ***(M1)***

$= 0.907294...$

$\mathrm{P}(\text{Suzi gets a bonus}) = 0.907$ ***A1***

Note: The first two marks may be awarded independently.

METHOD 2

recognizing that Suzi must be on time at least five times (of the remaining six days) ***(M1)***

$\mathrm{X} \sim \mathrm{B}(6, 0.911328...)$, where X is the number of days on time ***(A1)***

$\mathrm{P}(X \ge 5) = 1 - \mathrm{P}(X \le 4)$ OR $1 - 0.0927052...$OR $\mathrm{P}(X = 5) + (X = 6)$ OR
$0.334434... + 0.572860...$ ***(M1)***

$= 0.907294...$

$\mathrm{P}(\text{Suzi gets a bonus}) = 0.907$ ***A1***

Note: The first two marks may be awarded independently.

[4 marks]

Total [18 marks]