# MARKSCHEME

**1.** $81 = \frac{n}{2}(1.5 + 7.5)$ M1

$\Rightarrow\ n = 18$ A1

$1.5 + 17d = 7.5$ M1

$\Rightarrow d = \frac{6}{17}$ A1 N0

**[4]**

**2.** **METHOD 1**

If the areas are in arithmetic sequence, then so are the angles. (M1)

$\Rightarrow S_n = \frac{n}{2}(a+l) \Rightarrow \frac{12}{2}(\theta+2\theta) = 18\theta$ M1A1

$\Rightarrow 18\theta = 2\pi$ (A1)

$\theta = \frac{\pi}{9}$ (accept 20°) A1

**METHOD 2**

$a_{12} = 2a_1$ (M1)

$\frac{12}{2}(a_1 + 2a_1) = \pi r^2$ M1A1

$3a_1 = \frac{\pi r^2}{6}$

$\frac{3}{2}r^2\ \theta = \frac{\pi r^2}{6}$ (A1)

$\theta = \frac{2\pi}{18} = \frac{\pi}{9}$ (accept 20°) A1

**METHOD 3**

Let smallest angle = $a$, common difference = $d$

$a + 11d = 2a$ (M1)

$a = 11d$ A1

$S_n = \frac{12}{2}(2a + 11d) = 2\pi$ M1

$6(2a + a) = 2\pi$ (A1)

$18a = 2\pi$

$a = \frac{\pi}{9}$ (accept 20°) A1

**[5]**

**3.** **METHOD 1**

(a) $u_n = S_n - S_{n-1}$ (M1)

$= \dfrac{7^n - a^n}{7^n} - \dfrac{7^{n-1} - a^{n-1}}{7^{n-1}}$ A1

(b) **EITHER**

$u_1 = 1 - \dfrac{a}{7}$ A1

$u_2 = 1 - \dfrac{a^2}{7^2} - \left(1 - \dfrac{a}{7}\right)$ M1

$= \dfrac{a}{7}\left(1 - \dfrac{a}{7}\right)$ A1

common ratio $= \dfrac{a}{7}$ A1

**OR**

$u_n = 1 - \left(\dfrac{a}{7}\right)^n - 1 + \left(\dfrac{a}{7}\right)^{n-1}$ M1

$= \left(\dfrac{a}{7}\right)^{n-1}\left(1 - \dfrac{a}{7}\right)$

$= \dfrac{7-a}{7}\left(\dfrac{a}{7}\right)^{n-1}$ A1

$u_1 = \dfrac{7-a}{7}$, common ratio $= \dfrac{a}{7}$ A1A1

(c) (i) $0 < a < 7$ (accept $a < 7$) A1

(ii) 1 A1

**METHOD 2**

(a) $u_n = br^{n-1} = \left(\dfrac{7-a}{7}\right)\left(\dfrac{a}{7}\right)^{n-1}$ A1A1

(b) for a GP with first term $b$ and common ratio $r$

$S_n = \dfrac{b(1-r^n)}{1-r} = \left(\dfrac{b}{1-r}\right) - \left(\dfrac{b}{1-r}\right)r^n$ M1

as $S_n = \dfrac{7^n - a^n}{7^n} = 1 - \left(\dfrac{a}{7}\right)^n$

comparing both expressions M1

$\dfrac{b}{1-r} = 1$ and $r = \dfrac{a}{7}$

$b = 1 - \dfrac{a}{7} = \dfrac{7-a}{7}$

$u_1 = b = \dfrac{7-a}{7}$, common ratio $= r = \dfrac{a}{7}$ A1A1

**Note:** Award method marks if the expressions for $b$ and $r$ are deduced in part (a).

(c) (i) $0 < a < 7$ (accept $a < 7$) A1

(ii) 1 A1

**[8]**

**4.** **METHOD 1**

| | |
|---|---|
| $5(2a + 9d) = 60$ (or $2a + 9d = 12$) | M1A1 |
| $10(2a + 19d) = 320$ (or $2a + 19d = 32$) | A1 |
| solve simultaneously to obtain | M1 |
| $a = -3, d = 2$ | A1 |
| the 15th term is $-3 + 14 \times 2 = 25$ | A1 |

**Note:** FT the final A1 on the values found in the penultimate line.

**METHOD 2**

| | |
|---|---|
| with an AP the mean of an even number of consecutive terms equals the mean of the middle terms | (M1) |
| $\frac{a_{10} + a_{11}}{2} = 16$ (or $a_{10} + a_{11} = 32$) | A1 |
| $\frac{a_5 + a_6}{2} = 6$ (or $a_5 + a_6 = 12$) | A1 |
| $a_{10} - a_5 + a_{11} - a_6 = 20$ | M1 |
| $5d + 5d = 20$ | |
| $d = 2$ and $a = -3$ (or $a_5 = 5$ or $a_{10} = 15$) | A1 |
| the 15th term is $-3 + 14 \times 2 = 25$ (or $5 + 10 \times 2 = 25$ or $15 + 5 \times 2 = 25$) | A1 |

**Note:** FT the final A1 on the values found in the penultimate line.

**[6]**

**5.** (a)

| | | |
|---|---|---|
| $0 < 2^x < 1$ | (M1) | |
| $x < 0$ | A1 | N2 |

(b)

| | |
|---|---|
| $\frac{35}{1-r} = 40$ | M1 |
| $\Rightarrow 40 - 40 \times r = 35$ | |
| $\Rightarrow -40 \times r = -5$ | (A1) |
| $\Rightarrow r = 2^x = \frac{1}{8}$ | A1 |
| $\Rightarrow x = \log_2 \frac{1}{8}$ $(= -3)$ | A1 |

**Note:** The substitution $r = 2^x$ may be seen at any stage in the solution.

**[6]**

**6.** (a)

| | |
|---|---|
| $u_1 = 27$ | |
| $\frac{81}{2} = \frac{27}{1-r}$ | M1 |
| $r = \frac{1}{3}$ | A1 |

(b)

| | |
|---|---|
| $v_2 = 9$ | |
| $v_4 = 1$ | |
| $2d = -8 \Rightarrow d = -4$ | (A1) |
| $v_1 = 13$ | (A1) |

$\frac{N}{2}(2 \times 13 - 4(N - 1)) > 0$ (accept equality) M1

$\frac{N}{2}(30 - 4N) > 0$

$N(15 - 2N) > 0$

$N < 7.5$ (M1)

$N = 7$ A1

**Note:** $13 + 9 + 5 + 1 - 3 - 7 - 11 > 0 \Rightarrow N = 7$ or equivalent receives full marks.

**[7]**

**7.** (a) $S_6 = 81 \Rightarrow 81 = \frac{6}{2}(2a + 5d)$ M1A1

$\Rightarrow 27 = 2a + 5d$

$S_{11} = 231 \Rightarrow 231 = \frac{11}{2}(2a + 10d)$ M1A1

$\Rightarrow 21 = a + 5d$

solving simultaneously, $a = 6$, $d = 3$ A1A1

(b) $a + ar = 1$ A1

$a + ar + ar^2 + ar^3 = 5$ A1

$\Rightarrow (a + ar) + ar^2(1 + r) = 5$

$\Rightarrow 1 + ar^2 \times \frac{1}{a} = 5$

obtaining $r^2 - 4 = 0$ M1

$\Rightarrow r = \pm 2$

$r = 2$ (since all terms are positive) A1

$a = \frac{1}{3}$ A1

(c) AP $r^{\text{th}}$ term is $3r + 3$ A1

GP $r^{\text{th}}$ term is $\frac{1}{3}2^{r-1}$ A1

$3(r + 1) \times \frac{1}{3}2^{r-1} = (r + 1)2^{r-1}$ M1AG

**[14]**