

## Answer 1:

1. (a) Let N be North

$\hat{NJD} = 34^\circ$  OR  $\hat{DJL} = 56^\circ$  (must be labelled or indicated in diagram): **(A1)**

$$\hat{JDL} = 99^\circ \quad \text{spanning from } 180^\circ - 56^\circ = 124^\circ \quad \text{spanning from } 180^\circ - 34^\circ = 146^\circ$$
**A1**

**Note:** Accept  $\frac{11\pi}{20}, 1.73$  (radians).

**[2 marks]**

- (b) attempt to apply the sine rule

**(M1)**

$$\frac{DL}{\sin 56^\circ} = \frac{500}{\sin 99^\circ} \quad \text{OR} \quad \frac{DL}{\sin 0.977384...} = \frac{500}{\sin 1.72787...}$$
**(A1)**

$$419.685\dots$$

$$DL = 420 \text{ (km)} \quad \text{spanning from } 419.685\dots \text{ to } 420 \quad \text{spanning from } 419.685\dots \text{ to } 420$$
**A1**

**Note:** Award **M1A1A0** for 261 (km) from use of degrees with GDC set in radians (with or without working).

**[3 marks]**

**Total [5 marks]**

## Answer 2:

9. (a) (i) attempt to use the cosine rule **(M1)**

$$AC = \sqrt{2^2 + 4^2 - 2(2)(4)\cos\alpha} \left( = \sqrt{20 - 16\cos\alpha} = 2\sqrt{5 - 4\cos\alpha} \right) \quad \text{A1}$$

$$(ii) \quad AC = \sqrt{6^2 + 8^2 - 2(6)(8)\cos\beta} \left( = \sqrt{100 - 96\cos\beta} = 2\sqrt{25 - 24\cos\beta} \right) \quad \text{A1}$$

$$(iii) \quad 5 - 4\cos\alpha = 25 - 24\cos\beta$$

$$\alpha = \arccos(6\cos\beta - 5) \quad \text{A1}$$

**[4 marks]**

- (b) attempt to find the sum of two triangle areas using  $A = \frac{1}{2}ab\sin C$  **(M1)**

**Note:** Do not award this **M1** if the triangle is assumed to be right angled.

$$\text{Area} = \frac{1}{2}(8)\sin\alpha + \frac{1}{2}(48)\sin\beta \quad \text{A1}$$

attempt to express the area in terms of one variable only **(M1)**

$$= 4\sqrt{1 - (6\cos\beta - 5)^2} + 24\sin\beta \text{ or } 4\sin(\arccos(6\cos\beta - 5)) + 24\sin\beta \text{ OR}$$

$$4\sin\alpha + 24\sqrt{1 - \left(\frac{5 + \cos\alpha}{6}\right)^2} \text{ or } 4\sin\alpha + 24\sin\left(\arccos\left(\frac{5 + \cos\alpha}{6}\right)\right)$$

Max area = 19.5959...

$$= 19.6 \quad \text{A1}$$

**[4 marks]**

**Total [8 marks]**

**Answer 3:**

3. (a)  $(f \circ g)(x) = f(2x)$  **(A1)**

$f(2x) = \sqrt{3} \sin 2x + \cos 2x$  **A1**

**[2 marks]**

(b)  $\sqrt{3} \sin 2x + \cos 2x = 2 \cos 2x$

$\sqrt{3} \sin 2x = \cos 2x$

recognizing to use tan or cot **M1**

$\tan 2x = \frac{1}{\sqrt{3}}$  OR  $\cot 2x = \sqrt{3}$  (values may be seen in right triangle) **(A1)**

$\left( \arctan \left( \frac{1}{\sqrt{3}} \right) = \right) \frac{\pi}{6}$  (seen anywhere) (accept degrees) **(A1)**

$2x = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$  **A1A1**

**Note:** Do not award the final **A1** if any additional solutions are seen.

Award **A1AO** for correct answers in degrees.

Award **A0AO** for correct answers in degrees with additional values.

**[5 marks]**

**Total [7 marks]**

Answer 4:

9.  $(\sin 2x - \sin x) - (\cos 2x - \cos x) = 1$   
attempt to use both double-angle formulae, in whatever form  
 $(2\sin x \cos x - \sin x) - (2\cos^2 x - 1 - \cos x) = 1$   
or  $(2\sin x \cos x - \sin x) - (2\cos^2 x - \cos x) = 0$  for example

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M1

A1

**Note:** Allow any rearrangement of the above equations.

$$\sin x(2\cos x - 1) - \cos x(2\cos x - 1) = 0$$

$$(\sin x - \cos x)(2\cos x - 1) = 0$$

$$\tan x = 1 \text{ and } \cos x = \frac{1}{2}$$

(M1)

A1A1

**Note:** These A marks are dependent on the M mark awarded for factorisation.

$$x = -\frac{3\pi}{4}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{4}$$

A2

**Note:** Award A1 for two correct answers, which could be for both tan or both cos solutions, for example.

[7 marks]

## Answer 5:

3.  $\tan(x + \pi) = \tan x \left( = \frac{\sin x}{\cos x} \right)$
- $$\cos\left(x - \frac{\pi}{2}\right) = \sin x$$

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(M1)A1

(M1)A1

**Note:** The two M1's can be awarded for observation or for expanding.

$$\tan(x + \pi) \cos\left(x - \frac{\pi}{2}\right) = \frac{\sin^2 x}{\cos x}$$

A1

[5 marks]

## Answer 6:

(a) **METHOD 1**

$$\begin{aligned}
 \text{LHS} &= \frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x \cos x}{\cos^2 x - \sin^2 x} && \text{M1} \\
 &= \frac{(\cos^2 x + \sin^2 x) + 2\sin x \cos x}{\cos^2 x - \sin^2 x} && \text{M1} \\
 &= \frac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)} && \text{A1} \\
 &= \frac{\cos x + \sin x}{\cos x - \sin x} \\
 &= \frac{\cos x + \sin x}{\cos x - \sin x} + \frac{\cos x - \sin x}{\cos x - \sin x} && \text{A1} \\
 &= \frac{\cos x - \sin x}{\cos x - \sin x} \\
 &= \frac{1 + \tan x}{1 - \tan x} && \text{AG}
 \end{aligned}$$

**Note:** Candidates may start with RHS, apply MS in reverse.**[4 marks]****METHOD 2**

$$\begin{aligned}
 \text{LHS} &= \frac{1+\sin 2x}{\cos 2x} = \frac{1+2\sin x \cos x}{\cos^2 x - \sin^2 x} && \text{M1} \\
 \text{dividing numerator and denominator by } \cos^2 x && \text{M1} \\
 &= \frac{\sec^2 x + 2\tan x}{1 - \tan^2 x} \\
 &= \frac{1 + \tan^2 x + 2\tan x}{1 - \tan^2 x} && \text{A1} \\
 &= \frac{(\tan x + 1)^2}{(1 - \tan x)(1 + \tan x)} && \text{A1} \\
 &= \frac{1 + \tan x}{1 - \tan x} && \text{AG}
 \end{aligned}$$

**Note:** Candidates may start with RHS; apply MS in reverse.**[4 marks]**

(b) valid attempt to solve  $\frac{1+\tan x}{1-\tan x} = \sqrt{3}$  **(M1)**

$$\begin{aligned}
 \tan x &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \\
 x &= 0.262\left(=\frac{\pi}{12}\right), \quad x = 3.40\left(=\frac{13\pi}{12}\right) && \text{A1}
 \end{aligned}$$

**Note:** Award **M1A0** if only one correct solution is given.**[2 marks]****Total [6 marks]**

## Answer 7:

5. (a)  $\cos x = 0, \sin x = 0$

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**(M1)**

$$x = \frac{n\pi}{2}, n \in \mathbb{Z}$$

**AI**

[2 marks]

(b) EITHER

$$\frac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x}$$

**MIAI**

$$= \frac{\sin(3x-x)}{\frac{1}{2} \sin 2x}$$

**AIAI**

$$= 2$$

**AI**

OR

$$\frac{\sin 2x \cos x + \cos 2x \sin x}{\sin x} - \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$$

**MI**

$$= \frac{2\sin x \cos^2 x + 2\cos^2 x \sin x - \sin x}{\sin x} - \frac{2\cos^3 x - \cos x - 2\sin^2 x \cos x}{\cos x}$$

**AIAI**

$$= 4\cos^2 x - 1 - 2\cos^2 x + 1 + 2\sin^2 x$$

**AI**

$$= 2\cos^2 x + 2\sin^2 x$$

**AI**

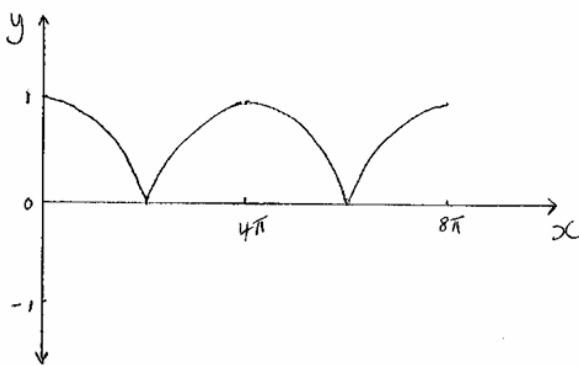
$$= 2$$

[5 marks]

Total [7 marks]

## Answer 8:

5. (a)

*AIAI*

**Note:** Award *A1* for correct shape and *A1* for correct domain and range.

*[2 marks]*

$$(b) \quad \left| \cos\left(\frac{x}{4}\right) \right| = \frac{1}{2}$$

$$x = \frac{4\pi}{3}$$

*A1*

attempting to find any other solutions

*MI*

**Note:** Award *(M1)* if at least one of the other solutions is correct  
(in radians or degrees) or clear use of symmetry is seen.

$$x = 8\pi - \frac{4\pi}{3} = \frac{20\pi}{3}$$

$$x = 4\pi - \frac{4\pi}{3} = \frac{8\pi}{3}$$

$$x = 4\pi + \frac{4\pi}{3} = \frac{16\pi}{3}$$

*A1*

**Note:** Award *A1* for all other three solutions correct and no extra solutions.

**Note:** If working in degrees, then max *A0M1A0*.

*[3 marks]**Total [5 marks]*