# Answer 1:

**Note:** Award as above if the series $1+p+\frac{1}{3}+\ldots$ is considered leading to

$\frac{n}{2}\left(\frac{7}{3}-\frac{n}{3}\right)=-3$.

attempt to form a quadratic $=0$ ***(M1)***

$n^2-7n-18=0$

attempt to solve their quadratic ***(M1)***

$(n-9)(n+2)=0$

$n=9$ ***A1***

**10.** (a) (i) **EITHER**

attempt to use a ratio from consecutive terms ***M1***

$\frac{p\ln x}{\ln x}=\frac{\frac{1}{3}\ln x}{p\ln x}$ OR $\frac{1}{3}\ln x=(\ln x)r^2$ OR $p\ln x=\ln x\left(\frac{1}{3p}\right)$

**Note:** Candidates may use $\ln x^1+\ln x^p+\ln x^{\frac{1}{3}}+\ldots$ and consider the powers of $x$ in geometric sequence.

Award ***M1*** for $\frac{p}{1}=\frac{\frac{1}{3}}{p}$.

**OR**

$r=p$ and $r^2=\frac{1}{3}$ ***M1***

**THEN**

$p^2=\frac{1}{3}$ OR $r=\pm\frac{1}{\sqrt{3}}$ ***A1***

$p=\pm\frac{1}{\sqrt{3}}$ ***AG***

**Note:** Award ***M0A0*** for $r^2=\frac{1}{3}$ or $p^2=\frac{1}{3}$ with no other working seen.

(iii) **METHOD 1**

$$S_n = \frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right]$$

attempt to substitute into $S_n$ and equate to $\ln\left(\frac{1}{x^3}\right)$ ***(M1)***

$$\frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right] = \ln\left(\frac{1}{x^3}\right)$$

$\ln\left(\frac{1}{x^3}\right) = -\ln x^3 \left(= \ln x^{-3}\right)$ ***(A1)***

$= -3\ln x$ ***(A1)***

correct working with $S_n$ (seen anywhere) ***(A1)***

$\frac{n}{2}\left[2\ln x - \frac{n}{3}\ln x + \frac{1}{3}\ln x\right]$ OR $n\ln x - \frac{n(n-1)}{6}\ln x$ OR $\frac{n}{2}\left(\ln x + \left(\frac{4-n}{3}\right)\ln x\right)$

correct equation without $\ln x$ ***A1***

$\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$ OR $n - \frac{n(n-1)}{6} = -3$ (or equivalent)

**METHOD 2**

attempt to use arithmetic mean $u_2 = \frac{u_1 + u_3}{2}$ ***M1***

$p\ln x = \frac{\ln x + \frac{1}{3}\ln x}{2}$ ***A1***

$2p\ln x = \frac{4}{3}\ln x \quad \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

(b) (i) **METHOD 1**

attempt to find a difference from consecutive terms or from $u_2$ ***M1***

correct equation ***A1***

$p\ln x - \ln x = \frac{1}{3}\ln x - p\ln x$ OR $\frac{1}{3}\ln x = \ln x + 2\left(p\ln x - \ln x\right)$

**Note:** Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \dots$ and consider the powers of $x$ in arithmetic sequence.

Award ***M1A1*** for $p - 1 = \frac{1}{3} - p$.

$2p\ln x = \frac{4}{3}\ln x \quad \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

**METHOD 2**

$\ln\left(\frac{1}{x^3}\right) = -\ln x^3 \left(= \ln x^{-3}\right)$ ***(A1)***

$= -3\ln x$ ***(A1)***

listing the first 7 terms of the sequence ***(A1)***

$\ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln x + 0 - \frac{1}{3}\ln x - \frac{2}{3}\ln x - \ln x + \dots$

recognizing first 7 terms sum to 0 ***M1***

$8^{th}$ term is $-\frac{4}{3}\ln x$ ***(A1)***

$9^{th}$ term is $-\frac{5}{3}\ln x$ ***(A1)***

sum of $8^{th}$ and $9^{th}$ terms $= -3\ln x$ ***(A1)***

$n = 9$ ***A1***

***[12 marks]***

***Total [18 marks]***

(ii) **EITHER**

since, $|p| = \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}} < 1$ ***R1***

**OR**

since, $|p| = \frac{1}{\sqrt{3}}$ and $-1 < p < 1$ ***R1***

**THEN**

$\Rightarrow$ the geometric series converges. ***AG***

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| **Note:** Accept $r$ instead of $p$.<br>Award ***R0*** if both values of $p$ not considered. |

(iii) $\dfrac{\ln x}{1-\frac{1}{\sqrt{3}}} \quad \left(=3+\sqrt{3}\right)$ ***(A1)***

$\ln x = 3 - \frac{3}{\sqrt{3}} + \sqrt{3} - \frac{\sqrt{3}}{\sqrt{3}}$ OR $\ln x = 3 - \sqrt{3} + \sqrt{3} - 1 \quad (\Rightarrow \ln x = 2)$ ***A1***

$x = e^2$ ***A1***

***[6 marks]***

**METHOD 3**

attempt to find difference using $u_3$ ***M1***

$\frac{1}{3}\ln x = \ln x + 2d \quad \left(\Rightarrow d = -\frac{1}{3}\ln x\right)$

$u_2 = \ln x + \frac{1}{2}\left(\frac{1}{3}\ln x - \ln x\right)$ OR $p\ln x - \ln x = -\frac{1}{3}\ln x$ ***A1***

$p\ln x = \frac{2}{3}\ln x$ ***A1***

$p = \frac{2}{3}$ ***AG***

(ii) $d = -\frac{1}{3}\ln x$ ***A1***

Answer 2:

**5.** (a) $100 = A_0 e^0$ *A1*

$A_0 = 100$ *AG*

***[1 mark]***

(b) correct substitution of values into exponential equation *(M1)*

$50 = 100e^{-5730k}$ OR $e^{-5730k} = \frac{1}{2}$

**EITHER**

$-5730k = \ln\frac{1}{2}$ *A1*

$\ln\frac{1}{2} = -\ln 2$ OR $-\ln\frac{1}{2} = \ln 2$ *A1*

**OR**

$e^{5730k} = 2$ *A1*

$5730k = \ln 2$ *A1*

**THEN**

$k = \frac{\ln 2}{5730}$ *AG*

| |
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| **Note:** There are many different ways of showing that $k = \frac{\ln 2}{5730}$ which involve showing different steps. Award full marks for at least two correct algebraic steps seen. |

***[3 marks]***

(c) if $25\%$ of the carbon-14 has decayed, $75\%$ remains ie, $75$ units remain

$75 = 100e^{-\frac{\ln 2}{5730}t}$ *(A1)*

**EITHER**

using an appropriate graph to attempt to solve for $t$ *(M1)*

**OR**

manipulating logs to attempt to solve for $t$ *(M1)*

$\ln 0.75 = -\frac{\ln 2}{5730}t$

$t = 2378.164...$

**THEN**

$t = 2380$ (years) (correct to the nearest 10 years) *A1*

***[3 marks]***
***Total [7 marks]***

Answer 3:

**4.** recognition of quadratic in $e^x$ *(M1)*

$\left(e^x\right)^2 - 3e^x + \ln k\left(=0\right)$ OR $A^2 - 3A + \ln k\left(=0\right)$

recognizing discriminant ≥ 0 (seen anywhere) *(M1)*

$\left(-3\right)^2 - 4(1)(\ln k)$ OR $9 - 4\ln k$ *(A1)*

$\ln k \le \frac{9}{4}$ *(A1)*

$e^{9/4}$ (seen anywhere) *A1*

$0 < k \le e^{9/4}$ *A1*

*[6 marks]*

Answer 4:



**10.** Attempting to solve $\left|0.1x^2 - 2x + 3\right| = \log_{10} x$ numerically or graphically. *(M1)*

$x = 1.52, 1.79$ *(A1)(A1)*

$x = 17.6, 19.1$ *(A1)*

$(1.52 < x < 1.79) \cup (17.6 < x < 19.1)$ *A1A1* *N2*

*[6 marks]*

Answer 5:

**8.** **METHOD 1**

$$d = \frac{1}{\log_8 x} - \frac{1}{\log_2 x}$$ ***(M1)***

$$= \frac{\log_2 8}{\log_2 x} - \frac{1}{\log_2 x}$$ ***(M1)***

**Note:** Award this ***M1*** for a correct change of base anywhere in the question.

$$= \frac{2}{\log_2 x}$$ ***(A1)***

$$\frac{20}{2}\left(2 \times \frac{1}{\log_2 x} + 19 \times \frac{2}{\log_2 x}\right)$$ ***M1***

$$= \frac{400}{\log_2 x}$$ ***(A1)***

$$100 = \frac{400}{\log_2 x}$$

$$\log_2 x = 4 \Rightarrow x = 2^4 = 16$$ ***A1***

**METHOD 2**

$$20^{\text{th}} \text{ term } = \frac{1}{\log_{2^{39}} x}$$ ***A1***

$$100 = \frac{20}{2}\left(\frac{1}{\log_2 x} + \frac{1}{\log_{2^{39}} x}\right)$$ ***M1***

$$100 = \frac{20}{2}\left(\frac{1}{\log_2 x} + \frac{\log_2 2^{39}}{\log_2 x}\right)$$ ***M1(A1)***

**Note:** Award this ***M1*** for a correct change of base anywhere in the question.



$$100 = \frac{400}{\log_2 x}$$ ***(A1)***

$$\log_2 x = 4 \Rightarrow x = 2^4 = 16$$ ***A1***

**METHOD 3**

$$\frac{1}{\log_2 x}+\frac{1}{\log_8 x}+\frac{1}{\log_{32} x}+\frac{1}{\log_{128} x}+\ldots$$

$$\frac{1}{\log_2 x}+\frac{\log_2 8}{\log_2 x}+\frac{\log_2 32}{\log_2 x}+\frac{\log_2 128}{\log_2 x}+\ldots \qquad \textbf{\textit{(M1)(A1)}}$$

| **Note:** Award this ***M1*** for a correct change of base anywhere in the question. |
|---|

$$=\frac{1}{\log_2 x}(1+3+5+\ldots) \qquad \textbf{\textit{A1}}$$

$$=\frac{1}{\log_2 x}\left(\frac{20}{2}(2+38)\right) \qquad \textbf{\textit{(M1)(A1)}}$$

$$100=\frac{400}{\log_2 x}$$

$$\log_2 x=4 \Rightarrow x=2^4=16 \qquad \textbf{\textit{A1}}$$

***[6 marks]***