MARKSCHEME

1. (a) $18n - 10$ (or equivalent) A1

(b) $\sum_{1}^{n}(18r - 10)$ (or equivalent) A1

(c) by use of GDC or algebraic summation or sum of an AP (M1)

$\sum_{1}^{15}(18r - 10) = 2010$ A1

[4]

2. $2 \times 1.05^{n-1} > 500$ M1

$n - 1 > \dfrac{\log 250}{\log 1.05}$ M1

$n - 1 > 113.1675...$ A1

$n = 115$ (A1)

$u_{115} = 521$ A1 N5

Note: Accept graphical solution with appropriate sketch.

[5]

3. **METHOD 1**

constant term: $\binom{5}{0}(-2x)^0\binom{7}{0}x^0 = 1$ A1

term in x: $\binom{7}{1}x + \binom{5}{1}(-2x) = -3x$ (M1)A1

term in x^2: $\binom{7}{2}x^2 + \binom{5}{2}(-2x)^2 + \binom{7}{1}x\binom{5}{1}(-2x) = -9x^2$ M1A1 N3

METHOD 2

$(1-2x)^5(1+x)^7 = \left(1 + 5(-2x) + \dfrac{5\times 4(-2x)^2}{2!} + \ldots\right)\left(1 + 7x + \dfrac{7\times 6}{2}x^2 + \ldots\right)$ M1M1

$= (1 - 10x + 40x^2 + ...)(1 + 7x + 21x^2 + ...)$

$= 1 + 7x + 21x^2 - 10x - 70x^2 + 40x^2 + ...$

$= 1 - 3x - 9x^2 + ...$ A1A1A1 N3

[5]

4. **METHOD 1**

$r = 2, \theta = -\dfrac{\pi}{3}$ (A1)(A1)

$\therefore (1 - \mathrm{i}\sqrt{3})^{-3} = 2^{-3}\left(\cos\left(-\dfrac{\pi}{3}\right) + \mathrm{i}\sin\left(-\dfrac{\pi}{3}\right)\right)^{-3}$ M1

$= \frac{1}{8}(\cos\pi + \mathrm{i}\sin\pi)$ (M1)

$= -\frac{1}{8}$ A1

METHOD 2

$(1 - \mathrm{i}\sqrt{3})(1 - \mathrm{i}\sqrt{3}) = 1 - 2\mathrm{i}\sqrt{3} - 3\ (= -2 - 2\mathrm{i}\sqrt{3})$ (M1)A1

$(-2 - 2\mathrm{i}\sqrt{3})(1 - \mathrm{i}\sqrt{3}) = -8$ (M1)(A1)

$\therefore \frac{1}{(1-\mathrm{i}\sqrt{3})^3} = -\frac{1}{8}$ A1

METHOD 3

Attempt at Binomial expansion M1

$(1 - \mathrm{i}\sqrt{3})^3 = 1 + 3(-\mathrm{i}\sqrt{3}) + 3(-\mathrm{i}\sqrt{3})^2 + (-\mathrm{i}\sqrt{3})^3$ (A1)

$= 1 - 3\mathrm{i}\sqrt{3} - 9 + 3\mathrm{i}\sqrt{3}$ (A1)

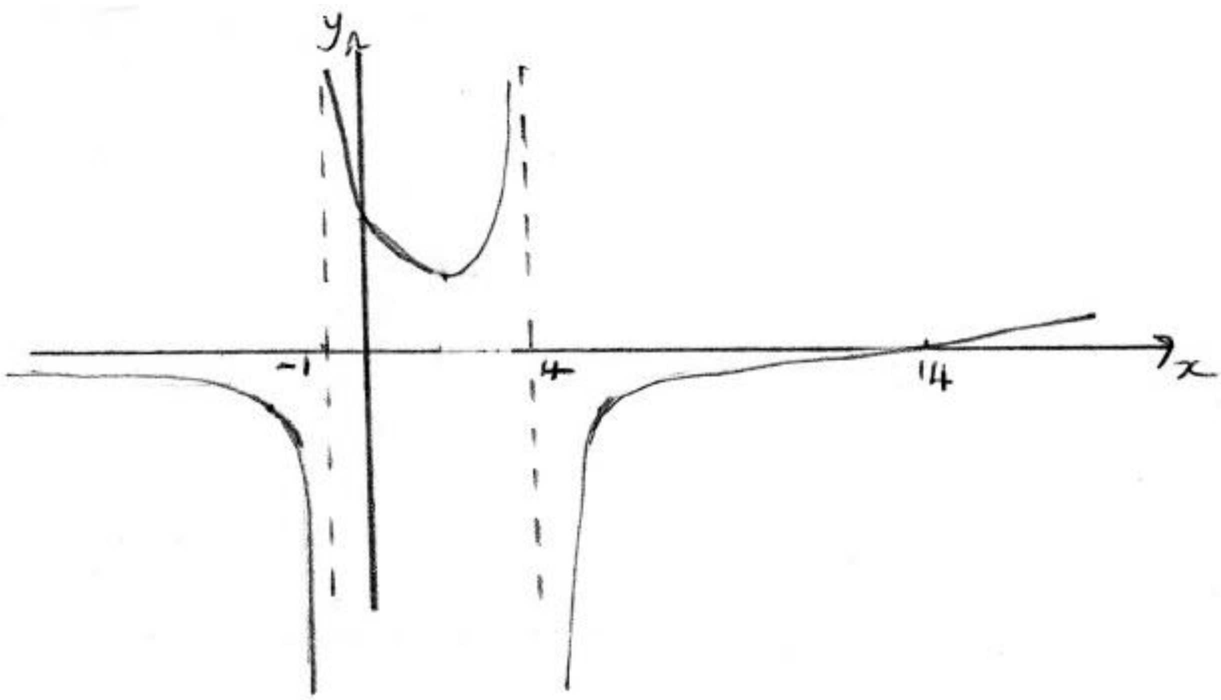
$= -8$ A1

$\therefore \frac{1}{(1-\mathrm{i}\sqrt{3})^3} = -\frac{1}{8}$ M1

[5]

5. **METHOD 1**

Graph of $f(x) - g(x)$ M1



A1A1A1

Note: Award A1 for each branch.

$x < -1$ or $4 < x \leq 14$ A1A1 N3

Note: Each value and inequality sign must be correct.

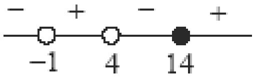
METHOD 2

$\frac{x+4}{x+1} - \frac{x-2}{x-4} \le 0$ M1

$\frac{x^2 - 16 - x^2 + x + 2}{(x+1)(x-4)} \le 0$

$\frac{x-14}{(x+1)(x-4)} \le 0$ A1

Critical value of $x = 14$ A1
Other critical values $x = -1$ and $x = 4$ A1



$x < -1$ or $4 < x \le 14$ A1A1 N3

Note: Each value and inequality sign must be correct.

[6]

6. (a) coefficient of x^3 is $\binom{n}{3}\left(\frac{1}{2}\right)^3 = 70$ M1(A1)

$\frac{n!}{3!(n-3)!} \times \frac{1}{8} = 70$ (A1)

$\Rightarrow \frac{n(n-1)(n-2)}{48} = 70$ (M1)

$n = 16$ A1

(b) $\binom{16}{2}\left(\frac{1}{2}\right)^2 = 30$ A1

[6]

7. (a) $5000(1.063)^n$ A1 N1

(b) Value = \$ $5000(1.063)^5$ (= \$ 6786.3511…)
= \$ 6790 to 3 s.f. (accept \$ 6786, or \$ 6786.35) A1 N1

(c) (i) $5000(1.063)^n > 10\,000$ (or $(1.063)^n > 2$) A1 N1

(ii) Attempting to solve the above inequality $n \log(1.063) > \log 2$ (M1)
$n > 11.345...$ (A1)
12 years A1 N3

Note: Candidates are likely to use TABLE or LIST on a GDC to find n. A good way of communicating this is suggested below.

Let $y = 1.063^x$ (M1)
When $x = 11$, $y = 1.9582$, when $x = 12$, $y = 2.0816$ (A1)
$x = 12$ i.e. 12 years A1 N3

[6]

8. Attempting to find $f(2)$ $= 8 + 12 + 2a + b$ (M1)
$= 2a + b + 20$ A1
Attempting to find $f(-1)$ $= -1 + 3 - a + b$ (M1)
$= 2 - a + b$ A1

Equating $2a + 20 = 2 - a$ A1

$a = -6$ A1 N2

[6]

9. (a) $f : x \ \alpha \ \ \mathrm{e}^x \Rightarrow f^{-1} : x \ \alpha \ \ \ln x$

$\Rightarrow f^{-1}(3) = \ln 3$ A1

$g : x \ \alpha \ \ x + 2 \Rightarrow g^{-1} : x \ \alpha \ \ x - 2$

$\Rightarrow g^{-1}(3) = 1$ A1

$f^{-1}(3) \times g^{-1}(3) = \ln 3$ A1 N1

(b) **EITHER**

$f \circ g(x) = f(x + 2) = \mathrm{e}^{x+2}$ A1

$\mathrm{e}^{x+2} = 3 \Rightarrow x + 2 = \ln 3$ M1

$x = \ln 3 - 2$ A1 N0

OR

$f \circ g(x) = \mathrm{e}^{x+2}$

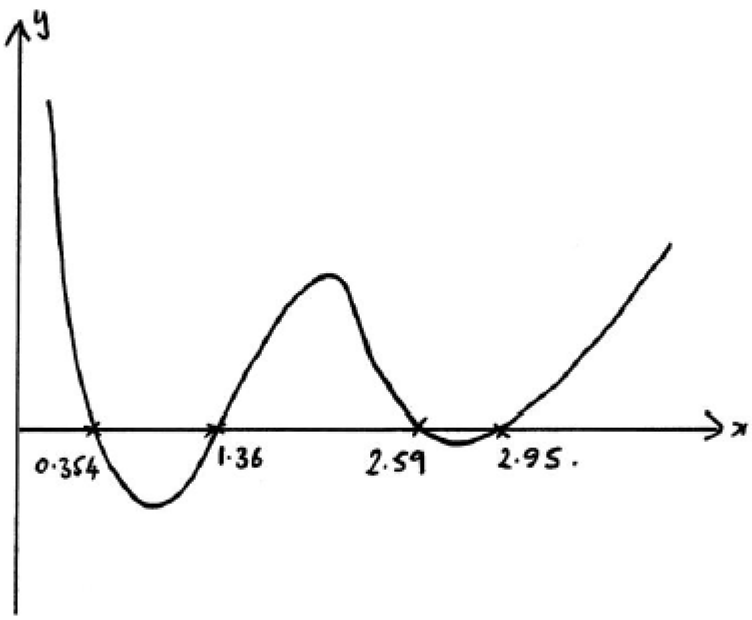
$f \circ g^{-1}(x) = \ln(x) - 2$ A1

$f \circ g^{-1}(3) = \ln(3) - 2$ M1

$x = \ln 3 - 2$ A1 N0

[6]

10. (a)



A1

Note: Award A1 for shape.

x intercepts 0.354, 1.36, 2.59, 2.95 A2

Note: Award A1 for three correct, A0 otherwise.

maximum = (1.57, 0.352) = $\left(\frac{\pi}{2}, 0.352\right)$ A1

minimum = (1, –0.640) and (2.77, –0.0129) A1

(b) $0 < x < 0.354$, $1.36 < x < 2.59$, $2.95 < x < 4$ A2

Note: Award A1 if two correct regions given.

[7]