Answer 1:

(a) $a^2 = 5 - 1$ **(M1)**

$a = 2$ **A1**

[2 marks]

(b) $2y\dfrac{dy}{dx} - \left(2x\dfrac{dy}{dx} + 2y\right) = -e^x$ **M1A1A1A1**

Note: Award ***M1*** for an attempt at implicit differentiation, ***A1*** for each part.

$\dfrac{dy}{dx} = \dfrac{2y - e^x}{2(y - x)}$ **AG**

[4 marks]

(c) at $x = 0$, $\dfrac{dy}{dx} = \dfrac{3}{4}$ **(A1)**

finding the negative reciprocal of a number **(M1)**

gradient of normal is $-\dfrac{4}{3}$

$y = -\dfrac{4}{3}x + 2$ **A1**

[3 marks]

(d) substituting linear expression **(M1)**

$\left(-\dfrac{4}{3}x + 2\right)^2 - 2x\left(-\dfrac{4}{3}x + 2\right) + e^x - 5 = 0$ or equivalent

$x = 1.56$ **(M1)A1**

$y = -0.0779$ **A1**

$(1.56, -0.0779)$

[4 marks]

(e) $\dfrac{dv}{dx} = 3y^2\dfrac{dy}{dx}$ **M1A1**

$\dfrac{dv}{dx} = 3 \times 4 \times \dfrac{3}{4} = 9$ **A1**

[3 marks]

Total [16 marks]

Answer 2:

(a) area of segment $= \frac{1}{2} \times 0.5^2 \times (\theta - \sin\theta)$ ***M1A1***

$V =$ area of segment $\times 10$

$V = \frac{5}{4}(\theta - \sin\theta)$ ***A1***

[3 marks]

(b) **METHOD 1**

$\frac{dV}{dt} = \frac{5}{4}(1 - \cos\theta)\frac{d\theta}{dt}$ ***M1A1***

$0.0008 = \frac{5}{4}\left(1 - \cos\frac{\pi}{3}\right)\frac{d\theta}{dt}$ ***(M1)***

$\frac{d\theta}{dt} = 0.00128 \text{ (rad } s^{-1})$ ***A1***

METHOD 2

$\frac{d\theta}{dt} = \frac{d\theta}{dV} \times \frac{dV}{dt}$ ***(M1)***

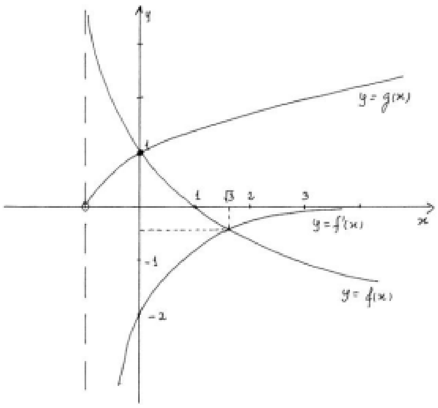
$\frac{dV}{d\theta} = \frac{5}{4}(1 - \cos\theta)$ ***A1***

$\frac{d\theta}{dt} = \frac{4 \times 0.0008}{5\left(1 - \cos\frac{\pi}{3}\right)}$ ***(M1)***

$\frac{d\theta}{dt} = 0.00128 \left(\frac{4}{3125}\right) \text{(rad } s^{-1})$ ***A1***

Answer 3:

3.



$f'(x) = \dfrac{-2}{(1+x)^2}$ ***M1A1***

Note: Alternatively, award ***M1A1*** for correct sketch of the derivative.

find at least one point of intersection of graphs ***(M1)***

$y = f(x)$ and $y = f'(x)$ for $x = \sqrt{3}$ or 1.73 ***(A1)***

$y = f(x)$ and $y = g(x)$ for $x = 0$ ***(A1)***

forming inequality $0 \le x \le \sqrt{3}$ (or $0 \le x \le 1.73$) ***A1A1*** ***N4***

Note: Award ***A1*** for correct limits and ***A1*** for correct inequalities.

[7 marks]

Answer 4:

(a) $y = 2$ (correct equation only) **A2** **N2**

[2 marks]

(b) valid approach **(M1)**

eg $(x-1)^{-1}+2$, $f'(x)=\dfrac{0(x-1)-1}{(x-1)^2}$

$-(x-1)^{-2}$, $f'(x)=\dfrac{-1}{(x-1)^2}$ **A1** **N2**

[2 marks]

(c) correct equation for the asymptote of g

eg $y = b$ **(A1)**

$b = 2$ **A1** **N2**

[2 marks]

(d) correct derivative of g (seen anywhere) **(A2)**

eg $g'(x) = -a\mathrm{e}^{-x}$

correct equation **(A1)**

eg $-\mathrm{e} = -a\mathrm{e}^{-1}$

7.38905

$a = \mathrm{e}^2$ (exact), 7.39 **A1** **N2**

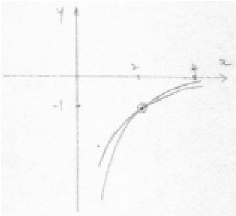
[4 marks]

(e) attempt to equate **their** derivatives *(M1)*

eg $f'(x) = g'(x)$, $\dfrac{-1}{(x-1)^2} = -ae^{-x}$

valid attempt to solve **their** equation *(M1)*

eg correct value outside the domain of f such as 0.522 or 4.51,



correct solution (may be seen in sketch) *(A1)*

eg $x = 2$, $(2, -1)$

gradient is -1 ***A1*** ***N3***

[4 marks]

Total [14 marks]

Answer 5:

QUESTION 7

METHOD 1

correct expression for **second** side, using area $= 525$ *(A1)*

e.g. let $\text{AB} = x$, $\text{AD} = \frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side *(M1)*

e.g. $3(\text{AD} + \text{BC} + \text{CD}) + 11\text{AB}$

correct expression for cost ***A2***

e.g. $\frac{525}{x} \times 3 + \frac{525}{x} \times 3 + 11x + 3x$, $\frac{525}{\text{AB}} \times 3 + \frac{525}{\text{AB}} \times 3 + 11\text{AB} + 3\text{AB}$, $\frac{3150}{x} + 14x$

EITHER

sketch of cost function *(M1)*

identifying minimum point *(A1)*

e.g. marking point on graph, $x = 15$

minimum cost is 420 (dollars) ***A1*** ***N4***

OR

correct derivative (may be seen in equation below) *(A1)*

e.g. $C'(x) = \frac{-1575}{x^2} + \frac{-1575}{x^2} + 14$

setting their derivative equal to 0 (seen anywhere) *(M1)*

e.g. $\frac{-3150}{x^2} + 14 = 0$

METHOD 2

correct expression for **second** side, using area $= 525$ *(A1)*

e.g. let $\text{AD} = x$, $\text{AB} = \frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side *(M1)*

e.g. $3(\text{AD} + \text{BC} + \text{CD}) + 11\text{AB}$

correct expression for cost ***A2***

e.g. $3\left(x + x + \frac{525}{x}\right) + \frac{525}{x} \times 11$, $3\left(\text{AD} + \text{AD} + \frac{525}{\text{AD}}\right) + \frac{525}{\text{AD}} \times 11$, $6x + \frac{7350}{x}$

EITHER

sketch of cost function *(M1)*

identifying minimum point *(A1)*

e.g. marking point on graph, $x = 35$

minimum cost is 420 (dollars) ***A1*** ***N4***

OR

correct derivative (may be seen in equation below) *(A1)*

e.g. $C'(x) = 6 - \frac{7350}{x^2}$

setting their derivative equal to 0 (seen anywhere) *(M1)*

e.g. $6 - \frac{7350}{x^2} = 0$

minimum cost is 420 (dollars) ***A1*** ***N4***

[7 marks]

Answer 6:

recognizing that the gradient of tangent is the derivative *eg* f'	*(M1)*	
finding the gradient of f at P *eg* $f'(0.25)=16$	*(A1)*	
evidence of taking negative reciprocal of **their** gradient at P *eg* $\frac{-1}{m}, -\frac{1}{f'(0.25)}$	*(M1)*	
equating derivatives *eg* $f'(x)=\frac{-1}{16},\ f'=-\frac{1}{m},\ \frac{x\left(\frac{1}{x}\right)-\ln(4x)}{x^2}=16$	***M1***	
finding the x-coordinate of Q, $x=0.700750$ $x=0.701$	***A1***	***N3***
attempt to substitute **their** x into f to find the y-coordinate of Q *eg* $f(0.7)$	*(M1)*	
$y=1.47083$ $y=1.47$	***A1***	***N2***
		[7 marks]

Answer 7:

(a) attempt at implicit differentiation **M1**

$1+\frac{\mathrm{d}y}{\mathrm{d}x}+(y+x\frac{\mathrm{d}y}{\mathrm{d}x})\sin(xy)=0$ **A1M1A1**

Note: Award **A1** for first two terms. Award **M1** for an attempt at chain rule **A1** for last term.

$(1+x\sin(xy))\frac{\mathrm{d}y}{\mathrm{d}x}=-1-y\sin(xy)$ or equivalent **A1**

$\frac{\mathrm{d}y}{\mathrm{d}x}=-\left(\frac{1+y\sin(xy)}{1+x\sin(xy)}\right)$ **AG**

[5 marks]

(b) (i) **EITHER**

when $xy=-\frac{\pi}{2}$, $\cos xy=0$ **M1**

$\Rightarrow x+y=0$ **(A1)**

OR

$x-\frac{\pi}{2x}-\cos\left(\frac{-\pi}{2}\right)=0$ or equivalent **M1**

$x-\frac{\pi}{2x}=0$ **(A1)**

THEN

therefore $x^2=\frac{\pi}{2}\left(x=\pm\sqrt{\frac{\pi}{2}}\right)(x=\pm1.25)$ **A1**

$\mathrm{P}\left(\sqrt{\frac{\pi}{2}},-\sqrt{\frac{\pi}{2}}\right),\mathrm{Q}\left(-\sqrt{\frac{\pi}{2}},\sqrt{\frac{\pi}{2}}\right)$ or $P(1.25,-1.25)$, $Q(-1.25,1.25)$ **A1**

(ii) $m_1 = -\left(\dfrac{1 - \sqrt{\dfrac{\pi}{2}} \times -1}{1 + \sqrt{\dfrac{\pi}{2}} \times -1}\right)$ ***M1A1***

$m_2 = -\left(\dfrac{1 + \sqrt{\dfrac{\pi}{2}} \times -1}{1 - \sqrt{\dfrac{\pi}{2}} \times -1}\right)$ ***A1***

$m_1 m_2 = 1$ ***AG***

Note: Award ***M1A0A0*** if decimal approximations are used.

Note: No ***FT*** applies.

[7 marks]

(c) equate derivative to -1 ***M1***

$(y - x)\sin(xy) = 0$ ***(A1)***

$y = x$, $\sin(xy) = 0$ ***R1***

in the first case, attempt to solve $2x = \cos\left(x^2\right)$ ***M1***

$(0.486, 0.486)$ ***A1***

in the second case, $\sin(xy) = 0 \Rightarrow xy = 0$ and $x + y = 1$ ***(M1)***

$(0,1)$, $(1,0)$ ***A1***

[7 marks]

Total [19 marks]