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Test

18.11.24

Name: Maanya
Start time: 9:01
End time: 9:54

1. Assume $\boxed{n=1}$. R1

LHS

$1 \cdot \left(\frac{1}{2}\right)^0$

$= 1$

RHS

$4 - \frac{3}{2^0} = 1$

hence true for $n=1$

Assume $\boxed{n=k}$ is true M1

$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \dots + k\left(\frac{1}{2}\right)^{k-1}$

$= 4 - \frac{k+2}{2^{k-1}}$

Assume $\boxed{n=k+1}$

$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \dots + k\left(\frac{1}{2}\right)^{k-1}$ A1

$+ (k+1)\left(\frac{1}{2}\right)^k = 4 - \frac{k+2}{2^{k-1}} + (k+1)\left(\frac{1}{2}\right)^k$

$4 - \frac{k+2}{2^{k-1}} + \frac{k+1}{2^k}$ M1A1

$= 4 - \frac{2(k+2)}{2^k} + \frac{k+1}{2^k}$ A1

$= 4 - \frac{2(k+2) - (k+1)}{2^k}$

$= 4 - \frac{k+3}{2^k}$

$= 4 - \frac{(k+1)+2}{2^{(k+1)-1}}$ A1

hence true for $n = k+1$, also true for $n=k$ and $n=1$. Hence true for all $n \in \mathbb{Z}^+$ R1

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2. Assume $\boxed{n=1}$

LHS RHS

$1(1!) = 1$ $(1+1)! - 1$

$= 1$

hence true for $n=1$ R1

Assume true for $\boxed{n=k}$

$$\sum_{r=1}^{k} r(r!) = (k+1)! - 1$$ M1

Assume $(n = k+1)$

$$\sum_{r=1}^{k+1} r(r!) = \sum_{r=1}^{k} r(r!) + (k+1)(k+1)!$$ M1A1

$$= (k+1)! - 1 + (k+1)(k+1)$$

$$= (k+2)(k+1)! - 1$$ A1

$$= (k+2)! - 1$$ A1

$$= ((k+1)+1)! - 1$$

hence true for $n = k+1$, so also true for $n=k$ and $n=1$. Hence true for all $n \in \mathbb{Z}^+$ R1 R1

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3. N/A What's conjecture?

4.

a) $S_6 = 81$

$S_{11} = 231$

$81 = 3(2a + 5d)$ M1

$81 = 6a + 15d$

$231 = \frac{11}{2}(2a + 10d)$ M1

$231 = 11a + 55d$

$81 = 6a + 15d$ (1) A1

$231 = 11a + 55d$ (2) A1

$a = \frac{231 - 55d}{11}$

$a = 21 - 5d$ (3)

sub (3) into (1)

$81 = 6(21 - 5d) + 15d$

$81 = 126 - 30d + 15d$ $-15d$

$d = 3$ (4) A1

Sub (4) into (1)

$81 = 6a + 15d \leftarrow 3$

$81 = 6a + 45$

$a = 6$ A1

b) $a + ar = 1$ A1

$a + ar + ar^2 + ar^3 = 5$ A1

$1 + ar^2 \cdot \frac{1}{a} = 5$

$r^2 - 4 = 0$ M1

$r = 2$ (positive) A1

$a + 2a = 1$

$3a = 1$

$a = \frac{1}{3}$ A1

c) Arithmetic series : $3r + 3$ A1

Geometric series : $\frac{1}{3} 2^{r-1}$ A1

$\cancel{3}(r+1) \times \frac{1}{\cancel{3}} 2^{r-1}$

$= (r+1)2^{r-1}$ AG

d) Assume for $\boxed{n=1}$ A1

LHS RHS

$2 \times 1 = 2$ $1 \times 2^1 = 2$

hence true for $\boxed{n=1}$

Assume true for $\boxed{n=k}$ M1

$$\sum_{r=1}^{k} (r+1)2^{r-1} = k2^k$$

Assume for $\boxed{n=k+1}$

$$\sum_{r=1}^{k+1} (r+1)2^{r-1} = k2^k + (k+2)2^k$$ M1A1

$= 2^k(k+k+2)$

$= 2^k(2k+2)$

$= 2(k+1)2^k$ A1

$= (k+1)2^{k+1}$ A1

hence true for $n=k+1$, so also $\forall n=k$ is ~~true~~ and also $n=1$. Hence true for $n \in \mathbb{Z}^+$ R1

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