

## Answer 1:

1. (a) Let N be North

$$\hat{N}\hat{J}D = 34^\circ \text{ OR } \hat{D}\hat{J}L = 56^\circ \text{ (must be labelled or indicated in diagram):}$$

**(A1)**

$$\hat{J}\hat{D}L = 99^\circ$$

**A1**

**Note:** Accept  $\frac{11\pi}{20}$ , 1.73 (radians).

**[2 marks]**

- (b) attempt to apply the sine rule

**(M1)**

$$\frac{DL}{\sin 56^\circ} = \frac{500}{\sin 99^\circ} \text{ OR } \frac{DL}{\sin 0.977384\dots} = \frac{500}{\sin 1.72787\dots}$$

**(A1)**

$$419.685\dots$$

$$DL = 420 \text{ (km)}$$

**A1**

**Note:** Award **M1A1A0** for 261 (km) from use of degrees with GDC set in radians (with or without working).

**[3 marks]**

**Total [5 marks]**

## Answer 2:

9. (a) (i) attempt to use the cosine rule (M1)

$$AC = \sqrt{2^2 + 4^2 - 2(2)(4)\cos\alpha} (= \sqrt{20 - 16\cos\alpha} = 2\sqrt{5 - 4\cos\alpha}) \quad \text{A1}$$

$$(ii) \quad AC = \sqrt{6^2 + 8^2 - 2(6)(8)\cos\beta} (= \sqrt{100 - 96\cos\beta} = 2\sqrt{25 - 24\cos\beta}) \quad \text{A1}$$

$$(iii) \quad 5 - 4\cos\alpha = 25 - 24\cos\beta$$

$$\alpha = \arccos(6\cos\beta - 5) \quad \text{A1}$$

[4 marks]

- (b) attempt to find the sum of two triangle areas using  $A = \frac{1}{2}ab\sin C$  (M1)

**Note:** Do not award this **M1** if the triangle is assumed to be right angled.

$$\text{Area} = \frac{1}{2}(8)\sin\alpha + \frac{1}{2}(48)\sin\beta \quad \text{(A1)}$$

attempt to express the area in terms of one variable only (M1)

$$= 4\sqrt{1 - (6\cos\beta - 5)^2} + 24\sin\beta \text{ or } 4\sin(\arccos(6\cos\beta - 5)) + 24\sin\beta \text{ OR}$$

$$4\sin\alpha + 24\sqrt{1 - \left(\frac{5 + \cos\alpha}{6}\right)^2} \text{ or } 4\sin\alpha + 24\sin\left(\arccos\left(\frac{5 + \cos\alpha}{6}\right)\right)$$

$$\text{Max area} = 19.5959\dots$$

$$= 19.6 \quad \text{A1}$$

[4 marks]

**Total [8 marks]**

Answer 3:

3. (a)  $(f \circ g)(x) = f(2x)$  (A1)  
 $f(2x) = \sqrt{3} \sin 2x + \cos 2x$  A1  
[2 marks]

(b)  $\sqrt{3} \sin 2x + \cos 2x = 2 \cos 2x$   
 $\sqrt{3} \sin 2x = \cos 2x$   
 recognizing to use  $\tan$  or  $\cot$  M1  
 $\tan 2x = \frac{1}{\sqrt{3}}$  OR  $\cot 2x = \sqrt{3}$  (values may be seen in right triangle) (A1)  
 $\left( \arctan\left(\frac{1}{\sqrt{3}}\right) = \right) \frac{\pi}{6}$  (seen anywhere) (accept degrees) (A1)  
 $2x = \frac{\pi}{6}, \frac{7\pi}{6}$   
 $x = \frac{\pi}{12}, \frac{7\pi}{12}$  A1A1

**Note:** Do not award the final **A1** if any additional solutions are seen.  
 Award **A1A0** for correct answers in degrees.  
 Award **A0A0** for correct answers in degrees with additional values.

[5 marks]  
 Total [7 marks]

Answer 4:

9.  $(\sin 2x - \sin x) - (\cos 2x - \cos x) = 1$   
 attempt to use both double-angle formulae, in whatever form **M1**  
 $(2\sin x \cos x - \sin x) - (2\cos^2 x - 1 - \cos x) = 1$   
 or  $(2\sin x \cos x - \sin x) - (2\cos^2 x - \cos x) = 0$  for example **A1**

**Note:** Allow any rearrangement of the above equations.

$$\sin x(2\cos x - 1) - \cos x(2\cos x - 1) = 0$$

$$(\sin x - \cos x)(2\cos x - 1) = 0$$

$$\tan x = 1 \text{ and } \cos x = \frac{1}{2}$$

**(M1)****A1A1**

**Note:** These **A** marks are dependent on the **M** mark awarded for factorisation.

$$x = -\frac{3\pi}{4}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{4}$$

**A2**

**Note:** Award **A1** for two correct answers, which could be for both tan or both cos solutions, for example.

**[7 marks]**

Answer 5:

3.  $\tan(x + \pi) = \tan x \left( = \frac{\sin x}{\cos x} \right)$

**(M1)A1**

$$\cos\left(x - \frac{\pi}{2}\right) = \sin x$$

**(M1)A1**

**Note:** The two **M1**'s can be awarded for observation or for expanding.

$$\tan(x + \pi) \cos\left(x - \frac{\pi}{2}\right) = \frac{\sin^2 x}{\cos x}$$

**A1****[5 marks]**

Answer 6:

(a) **METHOD 1**

$$\begin{aligned}
 \text{LHS} &= \frac{1 + \sin 2x}{\cos 2x} = \frac{1 + 2 \sin x \cos x}{\cos^2 x - \sin^2 x} \\
 &= \frac{(\cos^2 x + \sin^2 x) + 2 \sin x \cos x}{\cos^2 x - \sin^2 x} \\
 &= \frac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)} \\
 &= \frac{\cos x + \sin x}{\cos x - \sin x} \\
 &= \frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}} \\
 &= \frac{1 + \tan x}{1 - \tan x}
 \end{aligned}$$

**M1****M1****A1****A1****AG**

**Note:** Candidates may start with RHS, apply MS in reverse.

**[4 marks]****METHOD 2**

$$\begin{aligned}
 \text{LHS} &= \frac{1 + \sin 2x}{\cos 2x} = \frac{1 + 2 \sin x \cos x}{\cos^2 x - \sin^2 x} \\
 &\text{dividing numerator and denominator by } \cos^2 x \\
 &= \frac{\sec^2 x + 2 \tan x}{1 - \tan^2 x} \\
 &= \frac{1 + \tan^2 x + 2 \tan x}{1 - \tan^2 x} \\
 &= \frac{(\tan x + 1)^2}{(1 - \tan x)(1 + \tan x)} \\
 &= \frac{1 + \tan x}{1 - \tan x}
 \end{aligned}$$

**M1****M1****A1****A1****AG**

**Note:** Candidates may start with RHS; apply MS in reverse.

**[4 marks]**

(b) valid attempt to solve  $\frac{1 + \tan x}{1 - \tan x} = \sqrt{3}$

**(M1)**

$$\tan x = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$x = 0.262 \left( = \frac{\pi}{12} \right), x = 3.40 \left( = \frac{13\pi}{12} \right)$$

**A1**

**Note:** Award **M1A0** if only one correct solution is given.

**[2 marks]****Total [6 marks]**

Answer 7:

5. (a)  $\cos x = 0, \sin x = 0$

(M1) [leadib.com](https://www.leadib.com)

$$x = \frac{n\pi}{2}, n \in \mathbb{Z}$$

AI

[2 marks]

(b) EITHER

$$\frac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x}$$

M1A1

$$= \frac{\sin(3x - x)}{\frac{1}{2} \sin 2x}$$

A1A1

$$= 2$$

AI

OR

$$\frac{\sin 2x \cos x + \cos 2x \sin x}{\sin x} - \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$$

M1

$$= \frac{2 \sin x \cos^2 x + 2 \cos^2 x \sin x - \sin x}{\sin x} - \frac{2 \cos^3 x - \cos x - 2 \sin^2 x \cos x}{\cos x}$$

A1A1

$$= 4 \cos^2 x - 1 - 2 \cos^2 x + 1 + 2 \sin^2 x$$

AI

$$= 2 \cos^2 x + 2 \sin^2 x$$

AI

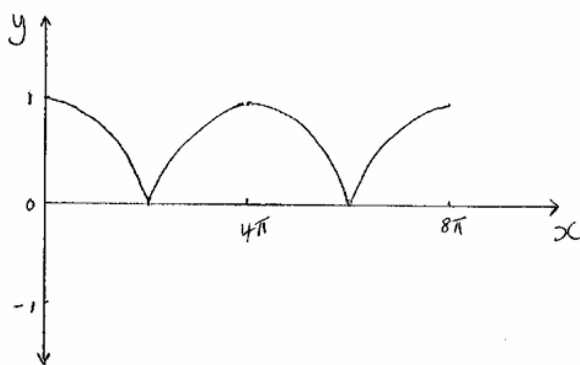
$$= 2$$

[5 marks]

Total [7 marks]

Answer 8:

5. (a)



*AI AI*

**Note:** Award *AI* for correct shape and *AI* for correct domain and range.

[2 marks]

(b)  $\left| \cos\left(\frac{x}{4}\right) \right| = \frac{1}{2}$   
 $x = \frac{4\pi}{3}$

*AI*

attempting to find any other solutions

*MI*

**Note:** Award (*MI*) if at least one of the other solutions is correct (in radians or degrees) or clear use of symmetry is seen.

$$x = 8\pi - \frac{4\pi}{3} = \frac{20\pi}{3}$$

$$x = 4\pi - \frac{4\pi}{3} = \frac{8\pi}{3}$$

$$x = 4\pi + \frac{4\pi}{3} = \frac{16\pi}{3}$$

*AI*

**Note:** Award *AI* for all other three solutions correct and no extra solutions.

**Note:** If working in degrees, then max *A0M1A0*.

[3 marks]

Total [5 marks]