Answer 1:

9. (a) attempt to expand binomial with negative fractional power ***(M1)***

$$\frac{1}{\sqrt{1+ax}}=(1+ax)^{-\frac{1}{2}}=1-\frac{ax}{2}+\frac{3a^2x^2}{8}+\dots$$ ***A1***

$$\sqrt{1-x}=(1-x)^{\frac{1}{2}}=1-\frac{x}{2}-\frac{x^2}{8}+\dots$$ ***A1***

$$\frac{1}{\sqrt{1+ax}}-\sqrt{1-x}=\frac{(1-a)}{2}x+\left(\frac{3a^2+1}{8}\right)x^2+\dots$$

attempt to equate coefficients of x or x^2 ***(M1)***

$$x:\quad \frac{1-a}{2}=4b;\ x^2:\ \frac{3a^2+1}{8}=b$$

attempt to solve simultaneously ***(M1)***

$$a=-\frac{1}{3},b=\frac{1}{6}$$ ***A1***

[6 marks]

(b) $|x|<1$ ***A1***

[1 mark]

Total [7 marks]

Answer 2:

(a) 1, 5, 10, 10, 5, 1 **A2** **N2**

[2 marks]

(b) evidence of binomial expansion with binomial coefficient **(M1)**

eg $\binom{n}{r}a^{n-r}b^r$, selecting correct term, $(2x)^5(3)^0+5(2x)^4(3)^1+10(2x)^3(3)^2+\dots$

correct substitution into correct term **(A1)(A1)(A1)**

eg $10(2)^3(3)^2$, $\binom{5}{3}(2x)^3(3)^2$

Note: Award ***A1*** for each factor.

$720x^3$ **A1** **N2**

Notes: Do not award any marks if there is clear evidence of adding instead of multiplying. Do not award final ***A1*** for a final answer of 720, even if $720x^3$ is seen previously.

[5 marks]

[Total 7 marks]

Answer 3:

5.

Note: Do not award any marks if there is clear evidence of adding instead of multiplying, for example ${}^9C_r+(ax)^{9-r}+(1)^r$.

valid approach for expansion (must be the product of a binomial coefficient with $n=9$ and a power of ax) **(M1)**

${}^9C_r(ax)^{9-r}(1)^r$ OR ${}^9C_{9-r}(ax)^r(1)^{9-r}$ OR ${}^9C_0(ax)^0(1)^9+{}^9C_1(ax)^1(1)^8+\dots$

recognizing that the term in x^6 is needed **(M1)**

$\frac{\text{Term in } x^6}{21x^2}=kx^4$ OR $r=6$ OR $r=3$ OR $9-r=6$

correct term or coefficient in binomial expansion (seen anywhere) **(A1)**

${}^9C_6(ax)^6(1)^3$ OR ${}^9C_3a^6x^6$ OR $84\left(a^6x^6\right)(1)$ OR $84a^6$

EITHER

correct term in x^4 or coefficient (may be seen in equation) ***(A1)***

$\frac{{}^9C_6}{21}a^6x^4$ OR $4a^6x^4$ OR $4a^6$

Set their term in x^4 or coefficient of x^4 equal to $\frac{8}{7}a^5x^4$ or $\frac{8}{7}a^5$ (do not accept other powers of *x*) ***(M1)***

$\frac{{}^9C_3}{21}a^6x^4 = \frac{8}{7}a^5x^4$ OR $4a^6 = \frac{8}{7}a^5$

OR

correct term in x^6 or coefficient of x^6 (may be seen in equation) ***(A1)***

$84a^6x^6$ OR $84a^6$

Set their term in x^6 or coefficient of x^6 equal to $24a^5x^6$ or $24a^5$
(do not accept other powers of *x*) ***(M1)***

$84a^6x^6 = 24a^5x^6$ OR $84a = 24$

THEN

$a = \frac{2}{7} \approx 0.286(0.285714...)$ ***A1***

Note: Award ***A0*** for the final mark for $a = \frac{2}{7}$ and $a = 0$.

Total [6 marks]

Answer 4:

1. attempt at binomial expansion, relevant row of Pascal's triangle or use of general term with binomial coefficient must be seen ***(M1)***

term independent of x is $\binom{10}{4}\left(2x^2\right)^6\left(\frac{1}{2x^3}\right)^4$ (or equivalent) ***(A1)(A1)(A1)***

> **Notes:** x's may be omitted.
> Also accept $\binom{10}{6}$ or 210.

$= 840$ ***A1***

[5 marks]

Answer 5:

5. attempt to find coefficients in binomial expansion ***(M1)***

coefficient of x^2 : $\binom{n}{2}\times 2^{n-2}$; coefficient of x^3 : $\binom{n}{3}\times 2^{n-3}$ ***A1A1***

> **Note:** Condone terms given rather than coefficients.
> Terms may be seen in an equation such as that below.

$\binom{n}{3}\times 2^{n-3} = 4\binom{n}{2}\times 2^{n-2}$ ***(A1)***

attempt to solve equation using GDC or algebraically ***(M1)***

$\binom{n}{3} = 8\binom{n}{2}$

$\frac{n!}{3!(n-3)!} = \frac{8n!}{2!(n-2)!}$

$\frac{1}{3} = \frac{8}{n-2}$

$n = 26$ ***A1***

[6 marks]

Answer 6:

6. **EITHER**

attempt to obtain the general term of the expansion

$T_{r+1} = {}^nC_r\left(8x^3\right)^{n-r}\left(-\frac{1}{2x}\right)^r$ OR $T_{r+1} = {}^nC_{n-r}\left(8x^3\right)^r\left(-\frac{1}{2x}\right)^{n-r}$ ***(M1)***

OR

recognize power of x starts at $3n$ and goes down by 4 each time ***(M1)***

THEN

recognizing the constant term when the power of x is zero (or equivalent) ***(M1)***

$r = \frac{3n}{4}$ or $n = \frac{4}{3}r$ or $3n - 4r = 0$ OR $3r - (n-r) = 0$ (or equivalent) ***A1***

r is a multiple of 3 $(r = 3, 6, 9, \ldots)$ or one correct value of n (seen anywhere) ***(A1)***

$n = 4k, k \in \mathbb{Z}^+$ ***A1***

Note: Accept n is a (positive) multiple of 4 or $n = 4, 8, 12, \ldots$

Do not accept $n = 4, 8, 12$

Note: Award full marks for a correct answer using trial and error approach showing $n = 4, 8, 12, \ldots$ and for recognizing that this pattern continues.

[5 marks]

Answer 7:

QUESTION 2

evidence of using binomial expansion ***(M1)***

e.g. selecting correct term, $a^8b^0 + \binom{8}{1}a^7b + \binom{8}{2}a^6b^2 + \ldots$

evidence of calculating the factors, in any order ***A1A1A1***

e.g. 56, $\frac{2^3}{3^3}$, -3^5, $\binom{8}{5}\left(\frac{2}{3}x\right)^3(-3)^5$

$-4032x^3$ (accept $= -4030x^3$ to 3 s.f.) ***A1*** ***N2***

[5 marks]

Answer 8:

6. product of a binomial coefficient, a power of ax^3 and a power of b seen ***(M1)***

evidence of correct term chosen

for $n=8$: $r=2$ (or $r=6$) OR for $n=10$: $r=2$ (or $r=8$) ***(A1)***

correct equations (may include powers of x) ***A1A1***

$${}^{8}C_2 a^2 b^6 = 448 \;\left(28a^2b^6 = 448 \Rightarrow a^2b^6 = 16\right),\; {}^{10}C_2 a^2 b^8 = 2880 \;\left(45a^2b^8 = 2880 \Rightarrow a^2b^8 = 64\right)$$

attempt to solve their system in a and b algebraically or graphically ***(M1)***

$b=2$; $a=\frac{1}{2}$ ***A1A1***

Note: Award a maximum of ***(M1)(A1)A1A1(M1)A1A0*** for $b=\pm 2$ and/or $a=\pm\frac{1}{2}$.

[7 marks]

Answer 9:

8. (a) the three girls can sit together in $3!=6$ ways ***(A1)***

this leaves 4 'objects' to arrange so the number of ways this can be done is $4!$ ***(M1)***

so the number of arrangements is $6\times 4! = 144$ ***A1***

[3 marks]

(b) Finding more than one position that the girls can sit ***(M1)***

Counting exactly four positions ***(A1)***

number of ways $= 4\times 3!\times 3! = 144$ ***M1A1*** ***N2***

[4 marks]

Total [7 marks]

Answer 10:

4. (a) number of arrangements of boys is 15! and number of arrangements of girls is 10! *(A1)*

total number of arrangements is $15! \times 10! \times 2 (= 9.49 \times 10^{18})$ *M1A1*

Note: If 2 is omitted, award *(A1)M1A0*.

[3 marks]

(b) number of ways of choosing two boys is $\binom{15}{2}$ and the number of ways of choosing three girls is $\binom{10}{3}$ *(A1)*

number of ways of choosing two boys and three girls is $\binom{15}{2} \times \binom{10}{3} = 12600$ *M1A1*

[3 marks]

Total [6 marks]

Answer 11:

3. (a) $\binom{10}{6} = 210$ *(M1)A1*

[2 marks]

(b) $2 \times \binom{8}{5} = 112$ *(M1)A1A1*

Note: Accept $210 - 28 - 70 = 112$

[3 marks]

(c) $\frac{112}{210} \left(= \frac{8}{15} = 0.533 \right)$ *(M1)A1*

[2 marks]

Total [7 marks]