Answer 1:

7. (a) $A\hat{B}C = 27°$ ***(A1)***

attempt to substitute into cosine rule ***(M1)***

$175^2 + 230^2 - 2(175)(230)\cos 27°$ ***(A1)***

$108.62308\ldots$

$AC = 109$ (m) ***A1***

[4 marks]

(b) correct substitution into area formula ***(A1)***

$\frac{1}{2} \times 175 \times 230 \times \sin 27°$

$9136.55...$

$\text{area} = 9140\left(\text{m}^2\right)$ ***A1***

[2 marks]

(c) attempt to substitute into sine rule or cosine rule ***(M1)***

$\frac{\sin 27°}{108.623...} = \frac{\sin \hat{A}}{175}$ OR $\cos A = \frac{(108.623...)^2 + 230^2 - 175^2}{2 \times 108.623... \times 230}$ ***(A1)***

$47.0049\ldots$

$C\hat{A}B = 47.0°$ ***A1***

[3 marks]

(d) **METHOD 1**

recognizing that for areas to be equal, $AD=DC$ ***(M1)***

$AD = \frac{1}{2} AC = 54.3115...$ ***A1***

attempt to substitute into cosine rule to find BD ***(M1)***

correct substitution into cosine rule ***(A1)***

$BD^2 = 230^2 + 54.3115^2 - 2(230)(54.3115)\cos 47.0049°$

$BD = 197.009...$

$BD = 197(\text{m})$ ***A1***

[5 marks]

METHOD 2

correct expressions for areas of triangle BDA and triangle BCD using BD ***A1***

$\frac{1}{2} \times \text{BD} \times 230 \times \sin x°$ and $\frac{1}{2} \times \text{BD} \times 175 \times \sin(27-x)°$ OR

$\frac{1}{2} \times \text{BD} \times 230 \times \sin(27-x)°$ and $\frac{1}{2} \times \text{BD} \times 175 \times \sin x°$

correct equation in terms of x ***(A1)***

$175\sin(27-x) = 230\sin x$ or $175\sin x = 230\sin(27-x)$

$x = 11.6326...$ or $x = 15.3673...$ ***(A1)***

substituting their value of x into equation to solve for BD ***(M1)***

$\frac{1}{2} \times \text{BD} \times 230 \times \sin 11.6326... = \frac{1}{2} \times \text{BD} \times 175 \times \sin 15.3673...$ or

$\frac{1}{2} \times \text{BD} \times 230 \times \sin 11.6326... = \frac{1}{2} \times 9136.55...$

$\text{BD} = 197(\text{m})$ ***A1***

[5 marks]

Total [14 marks]

Answer 2:

(a) **METHOD 1**

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}(6)(2\sqrt{3})\sin B$, $6\sqrt{3}\sin B$, $\frac{1}{2}(6)(2\sqrt{3})\sin B = 3\sqrt{3}$

correct working ***(A1)***

eg $6\sqrt{3}\sin B = 3\sqrt{3}$, $\sin B = \frac{3\sqrt{3}}{\frac{1}{2}(6)2\sqrt{3}}$

$\sin B = \frac{1}{2}$ ***(A1)***

$\frac{\pi}{6}$ $(30°)$ ***(A1)***

$\text{A}\hat{\text{B}}\text{C} = \frac{5\pi}{6}$ $(150°)$ ***A1*** ***N3***

METHOD 2

(using height of triangle ABC by drawing perpendicular segment from C to AD)

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}(2\sqrt{3})(h)=3\sqrt{3}$, $h\sqrt{3}$

correct working ***(A1)***

eg $h\sqrt{3}=3\sqrt{3}$

height of triangle is 3 ***A1***

$\text{C}\hat{\text{B}}\text{D}=\frac{\pi}{6}$ (30°) ***(A1)***

$\text{A}\hat{\text{B}}\text{C}=\frac{5\pi}{6}$ (150°) ***A1*** ***N3***

[5 marks]

(b) recognizing supplementary angle ***(M1)***

eg $\text{C}\hat{\text{B}}\text{D}=\frac{\pi}{6}$, sector $=\frac{1}{2}\left(180-\text{A}\hat{\text{B}}\text{C}\right)\left(6^2\right)$

correct substitution into formula for area of sector ***(A1)***

eg $\frac{1}{2}\times\frac{\pi}{6}\times 6^2$, $\pi\left(6^2\right)\left(\frac{30}{360}\right)$

area $=3\pi$ (cm^2) ***A1*** ***N2***

[3 marks]

Total [8 marks]

Answer 3:

(a) $\cos 105^\circ = \cos\left(180^\circ - 75^\circ\right) = -\cos 75^\circ$ ***R1***

$= -q$ ***AG***

Note: Accept arguments using the unit circle or graphical/diagrammatical considerations.

[1 mark]

(b) $\text{AD} = \text{CD} \Rightarrow \text{C}\hat{\text{A}}\text{D} = 45^\circ$ ***A1***

valid method to find $\text{B}\hat{\text{A}}\text{C}$ ***(M1)***

for example: $\text{BC} = r \Rightarrow \text{B}\hat{\text{C}}\text{A} = 60^\circ$

$\Rightarrow \text{B}\hat{\text{A}}\text{C} = 30^\circ$ ***A1***

hence $\text{B}\hat{\text{A}}\text{D} = 45^\circ + 30^\circ = 75^\circ$ ***AG***

[3 marks]

(c) (i) $\text{AB} = r\sqrt{3}$, $\text{AD}\left(= \text{CD}\right) = r\sqrt{2}$ ***A1A1***

applying cosine rule ***(M1)***

$\text{BD}^2 = \left(r\sqrt{3}\right)^2 + \left(r\sqrt{2}\right)^2 - 2\left(r\sqrt{3}\right)\left(r\sqrt{2}\right)\cos 75^\circ$ ***A1***

$= 3r^2 + 2r^2 - 2r^2\sqrt{6}\cos 75^\circ$

$= 5r^2 - 2r^2 q\sqrt{6}$ ***AG***

(ii) $\text{B}\hat{\text{C}}\text{D} = 105^\circ$ ***(A1)***

attempt to use cosine rule on ΔBCD ***(M1)***

$\text{BD}^2 = r^2 + \left(r\sqrt{2}\right)^2 - 2r\left(r\sqrt{2}\right)\cos 105^\circ$

$= 3r^2 + 2r^2 q\sqrt{2}$ ***A1***

[7 marks]

(d) $5r^2 - 2r^2 q\sqrt{6} = 3r^2 + 2r^2 q\sqrt{2}$ ***(M1)(A1)***

$2r^2 = 2r^2 q\left(\sqrt{6} + \sqrt{2}\right)$ ***A1***

Note: Award ***A1*** for any correct intermediate step seen using only two terms.

$q = \dfrac{1}{\sqrt{6} + \sqrt{2}}$ ***AG***

Note: Do not award the final ***A1*** if follow through is being applied.

[3 marks]

Total [14 marks]

Answer 4:

8. (a) (i) attempt to use Pythagoras ***(M1)***

$\sin^2\theta + \left(\frac{2}{3}\right)^2 = 1$ OR $x^2 + 2^2 = 3^2$ OR right triangle with side 2 and hypotenuse 3

$\sin\theta = \frac{\sqrt{5}}{3}$ ***A1***

(ii) attempt to substitute into double-angle identity using their value of $\sin\theta$ ***(M1)***

$\sin 2\theta = 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$

$\sin 2\theta = \frac{4\sqrt{5}}{9}$ ***A1***

[4 marks]

(b) **METHOD 1 (using values from part (a))**

$$\frac{b}{\sin\theta} = \frac{a}{\sin 2\theta}$$

attempt to use sine rule with their values from part (a) ***(M1)***

$$\frac{b}{\left(\frac{\sqrt{5}}{3}\right)} = \frac{a}{\left(\frac{4\sqrt{5}}{9}\right)} \quad \text{OR} \quad \frac{\left(\frac{\sqrt{5}}{3}\right)}{b} = \frac{\left(\frac{4\sqrt{5}}{9}\right)}{a}$$

correct working that leads to ***AG*** ***A1***

$\frac{\sqrt{5}}{3}a = \frac{4\sqrt{5}}{9}b$ OR $\frac{3b}{\sqrt{5}} = \frac{9a}{4\sqrt{5}}$ OR $\frac{a}{3} = \frac{4b}{9}$ (or equivalent)

$b = \frac{3a}{4}$ ***AG***

METHOD 2 (double-angle identity)

$$\frac{b}{\sin\theta} = \frac{a}{\sin 2\theta}$$

using double-angle identity **(A1)**

$\frac{b}{\sin\theta} = \frac{a}{2\sin\theta\cos\theta}$ OR $b = \frac{a\sin\theta}{2\sin\theta\cos\theta}$ OR $b = \frac{a}{2\cos\theta}$

correct working (involving substituting $\cos\theta = \frac{2}{3}$) that leads to ***AG*** **A1**

$b = \frac{a\sin\theta}{2\sin\theta\left(\frac{2}{3}\right)}$ OR $b = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)}$ OR $b = \frac{a}{2\left(\frac{2}{3}\right)}$ (or equivalent)

$b = \frac{3a}{4}$ ***AG***

[2 marks]

(c) **METHOD 1 (using supplementary angles)**

recognizing $\mathrm{C\hat{A}D}$ and $\mathrm{B\hat{A}C}$ are supplementary ***(M1)***

recognizing supplementary angles have the same sine value ***(A1)***

$\sin \mathrm{C\hat{A}D} = \sin 2\theta$

$\sin \mathrm{C\hat{A}D} = \frac{4\sqrt{5}}{9}$ ***A1***

METHOD 2 (using sine rule)

recognizing $\mathrm{CD} = a$ ***(M1)***

$$\frac{a}{\sin \mathrm{C\hat{A}D}} = \frac{b}{\sin \theta}$$

correct substitution of $\sin \theta = \frac{\sqrt{5}}{3}$ and $b = \frac{3a}{4}$ into sine rule ***(A1)***

$$\frac{a}{\sin \mathrm{C\hat{A}D}} = \frac{\left(\frac{3a}{4}\right)}{\left(\frac{\sqrt{5}}{3}\right)} \quad \textbf{OR} \quad \sin \mathrm{C\hat{A}D} = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{\left(\frac{3a}{4}\right)} \quad \text{(or equivalent)}$$

$\sin \mathrm{C\hat{A}D} = \frac{4\sqrt{5}}{9}$ ***A1***

[3 marks]

(d) **METHOD 1 (using $\mathrm{C\hat{A}D}$ in area formula)**

recognizing $\mathrm{D\hat{C}A} = \theta$ ***(A1)***

recognizing $\mathrm{AD} = b \left(= \frac{3a}{4}\right)$ ***(A1)***

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin \mathrm{C\hat{A}D}$) ***(M1)***

$$\text{area} = \frac{1}{2}(b)(b)\left(\frac{4\sqrt{5}}{9}\right) \quad \textbf{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin 2\theta \quad \textbf{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin \mathrm{C\hat{A}D}$$

correct substitution in terms of a ***(A1)***

$$\text{area} = \frac{1}{2}\left(\frac{3a}{4}\right)\left(\frac{3a}{4}\right)\left(\frac{4\sqrt{5}}{9}\right)$$

$\text{area} = \frac{\sqrt{5}\,a^2}{8}$ ***A1***

METHOD 2 (using $\hat{ACD}$ or $\hat{ADC}$ in area formula)

recognizing $CD = a$ ***(A1)***

recognizing $AD = b \left(= \frac{3a}{4}\right)$ and/or $D\hat{C}A = \theta$ ***(A1)***

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin A\hat{D}C$ or $\sin A\hat{C}D$) ***(M1)***

$\text{area} = \frac{1}{2}(a)(b)\left(\frac{\sqrt{5}}{3}\right)$ OR $\text{area} = \frac{1}{2}(a)(b)\sin\theta$ OR $\text{area} = \frac{1}{2}(a)(b)\sin A\hat{D}C$

OR $\text{area} = \frac{1}{2}(a)(b)\sin A\hat{C}D$

correct substitution in terms of a ***(A1)***

$\text{area} = \frac{1}{2}(a)\left(\frac{3a}{4}\right)\left(\frac{\sqrt{5}}{3}\right)$

$\text{area} = \frac{\sqrt{5}\,a^2}{8}$ ***A1***

[5 marks]

Total [14 marks]

Answer 5:

(a)	(i)	attempt to find the difference of x-values of A and B	*(M1)*	
		eg $6.25–12.5$		
		6.25 (hours), (6 hours 15 minutes)	***A1***	***N2***
	(ii)	attempt to find the difference of y-values of A and B	*(M1)*	
		eg $1.5–0.6$		
		0.9 (m)	***A1***	***N2***
				[4 marks]
(b)	(i)	valid approach	*(M1)*	
		eg $\frac{\max - \min}{2}$, $0.9 \div 2$		
		$p = 0.45$	***A1***	***N2***
	(ii)	**METHOD 1**		
		period $= 12.5$ (seen anywhere)	*(A1)*	
		valid approach (seen anywhere)	*(M1)*	
		eg $\text{period} = \frac{2\pi}{b}$, $q = \frac{2\pi}{\text{period}}$, $\frac{2\pi}{12.5}$		
		0.502654		
		$q = \frac{4\pi}{25}, 0.503 \left(\text{or } -\frac{4\pi}{25}, -0.503\right)$	***A1***	***N2***

	METHOD 2		
	attempt to use a coordinate to make an equation	*(M1)*	
	e.g. $p\cos(6.25q) + r = 0.6$, $p\cos(12.5q) + r = 1.5$		
	correct substitution	*(A1)*	
	eg $0.45\cos(6.25q) + 1.05 = 0.6$, $0.45\cos(12.5q) + 1.05 = 1.5$		
	0.502654		
	$q = \frac{4\pi}{25}, 0.503 \left(\text{or } -\frac{4\pi}{25}, -0.503\right)$	***A1***	***N2***
(iii)	valid method to find r	*(M1)*	
	eg $\frac{\max + \min}{2}$, $0.6 + 0.45$		
	$r = 1.05$	***A1***	***N2***
			[7 marks]

(c) **METHOD 1**

attempt to find start or end t-values for 12 December **(M1)**

eg $3+24,\ t=27,\ t=51$

finds t-value for second max **(A1)**

$t=50$

23:00 (or 11 pm) **A1** **N3**

METHOD 2

valid approach to list either the times of high tides after 21:00 or the t-values of high tides after 21:00, showing at least two times **(M1)**

eg $21{:}00+12.5,\ 21{:}00+25,\ 12.5+12.5,\ 25+12.5$

correct time of first high tide on 12 December **(A1)**

eg 10:30 (or 10:30 am)

time of second high tide = 23:00 **A1** **N3**

METHOD 3

attempt to set **their** h equal to 1.5 **(M1)**

eg $h(t)=1.5,\ 0.45\cos\left(\frac{4\pi}{25}t\right)+1.05=1.5$

correct working to find second max **(A1)**

eg $0.503t=8\pi,\ t=50$

23:00 (or 11 pm) **A1** **N3**

[3 marks]

Answer 6:

METHOD 1 – FINDING INTERVALS FOR x

$4\cos\left(\frac{x}{2}\right)+1>2\sqrt{2}+1$

correct working **(A1)**

eg $4\cos\left(\frac{x}{2}\right)=2\sqrt{2},\ \cos\left(\frac{x}{2}\right)>\frac{\sqrt{2}}{2}$

recognizing $\cos^{-1}\frac{\sqrt{2}}{2}=\frac{\pi}{4}$ **(A1)**

one additional correct value for $\frac{x}{2}$ (ignoring domain and equation/inequalities) **(A1)**

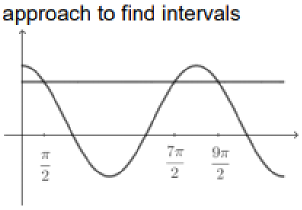
eg $-\frac{\pi}{4},\ \frac{7\pi}{4},\ 315°,\ \frac{9\pi}{4},\ -45°,\ \frac{15\pi}{4}$

three correct values for x ***A1A1***

eg $\frac{\pi}{2},\ \frac{7\pi}{2},\ \frac{9\pi}{2}$

valid approach to find intervals **(M1)**

eg



correct intervals (must be in radians) ***A1A1*** ***N2***

$0\le x<\frac{\pi}{2},\ \frac{7\pi}{2}<x<\frac{9\pi}{2}$

Note:	If working shown, award ***A1A0*** if inclusion/exclusion of endpoints is incorrect. If no working shown award ***N1***. If working shown, award ***A1A0*** if both correct intervals are given, **and** additional intervals are given. If no working shown award ***N1***. Award ***A0A0*** if inclusion/exclusion of endpoints are incorrect **and** additional intervals are given.

METHOD 2 – FINDING INTERVALS FOR $\frac{x}{2}$

$4\cos\left(\frac{x}{2}\right)+1>2\sqrt{2}+1$

correct working **(A1)**

eg $4\cos\left(\frac{x}{2}\right)=2\sqrt{2},\ \cos\left(\frac{x}{2}\right)>\frac{\sqrt{2}}{2}$

recognizing $\cos^{-1}\frac{\sqrt{2}}{2}=\frac{\pi}{4}$ **(A1)**

one additional correct value for $\frac{x}{2}$ (ignoring domain and equation/inequalities) **(A1)**

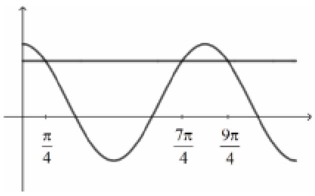
eg $-\frac{\pi}{4}, \frac{7\pi}{4}, 315°, \frac{9\pi}{4}, -45°, \frac{15\pi}{4}$

three correct values for $\frac{x}{2}$ **A1**

eg $\frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$

valid approach to find intervals **(M1)**

eg



one correct interval for $\frac{x}{2}$ **A1**

eg $0\le\frac{x}{2}<\frac{\pi}{4}, \frac{7\pi}{4}<\frac{x}{2}<\frac{9\pi}{4}$

correct intervals (must be in radians) **A1A1** **N2**

$0\le x<\frac{\pi}{2}, \frac{7\pi}{2}<x<\frac{9\pi}{2}$

Note: If working shown, award ***A1A0*** if inclusion/exclusion of endpoints is incorrect. If no working shown award ***N1***.
If working shown, award ***A1A0*** if both correct intervals are given, **and** additional intervals are given. If no working shown award ***N1***.
Award ***A0A0*** if inclusion/exclusion of endpoints are incorrect **and** additional intervals are given.

Total [8 marks]