Total Questions: 6
Total Marks: 72

Question 1:
Calculator Allowed: Yes

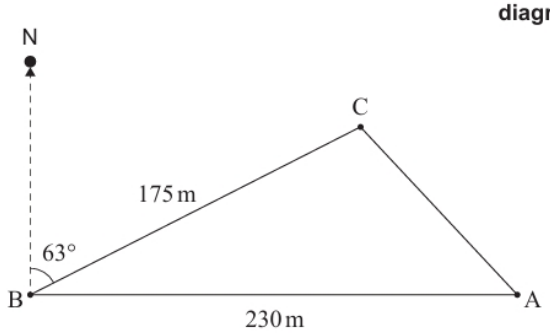
7. [Maximum mark: 14]

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of $063°$ to reach point C.

This is shown in the following diagram.

diagram not to scale



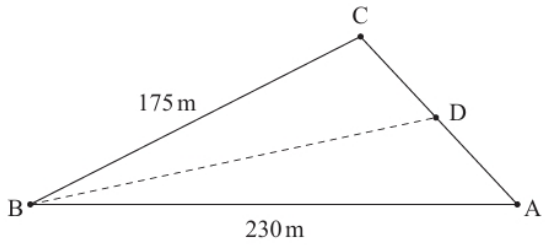
(a) Find the distance from point A to point C. [4]

(b) Find the area of this piece of land. [2]

(c) Find $C\hat{A}B$. [3]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.

diagram not to scale



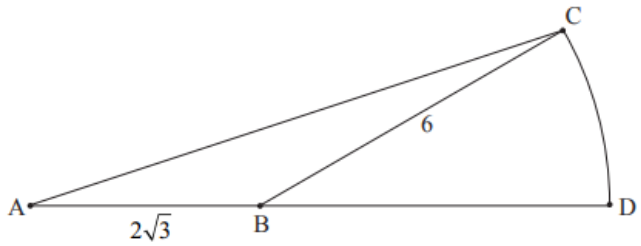
(d) Find the distance from point B to point D. [5]

Question 2:
Calculator Allowed: No

[Maximum mark: 8]

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius 6 cm. The points A, B and D are on the same line.

diagram not to scale



$AB = 2\sqrt{3}\,\text{cm}$, $BC = 6\,\text{cm}$, area of triangle $ABC = 3\sqrt{3}\,\text{cm}^2$, $A\hat{B}C$ is obtuse.

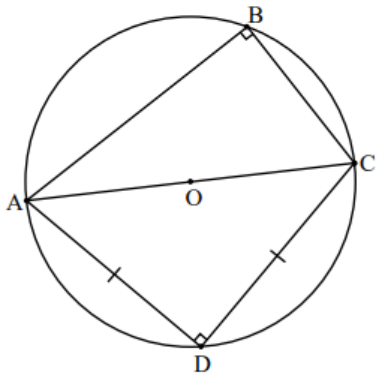
(a) Find $A\hat{B}C$. [5]

(b) Find the exact area of the sector BDC. [3]

Question 3:
Calculator Allowed: No

(a) Given that $\cos 75^\circ = q$, show that $\cos 105^\circ = -q$. [1]

In the following diagram, the points A, B, C and D are on the circumference of a circle with centre O and radius r. [AC] is a diameter of the circle. $BC = r$, $AD = CD$ and $A\hat{B}C = A\hat{D}C = 90^\circ$.



(b) Show that $B\hat{A}D = 75^\circ$. [3]

(c) (i) By considering triangle ABD, show that $BD^2 = 5r^2 - 2r^2q\sqrt{6}$.

(ii) By considering triangle CBD, find another expression for BD^2 in terms of r and q. [7]

(d) Use your answers to part (c) to show that $\cos 75^\circ = \frac{1}{\sqrt{6}+\sqrt{2}}$. [3]

Question 4:
Calculator Allowed: No

8. [Maximum mark: 14]

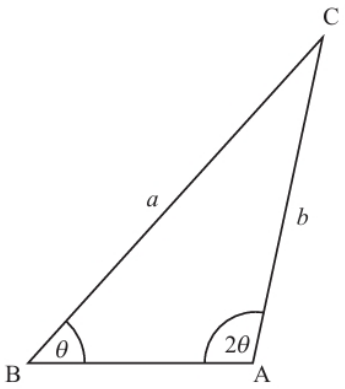
Consider an acute angle θ such that $\cos\theta = \frac{2}{3}$.

(a) Find the value of

(i) $\sin\theta$;

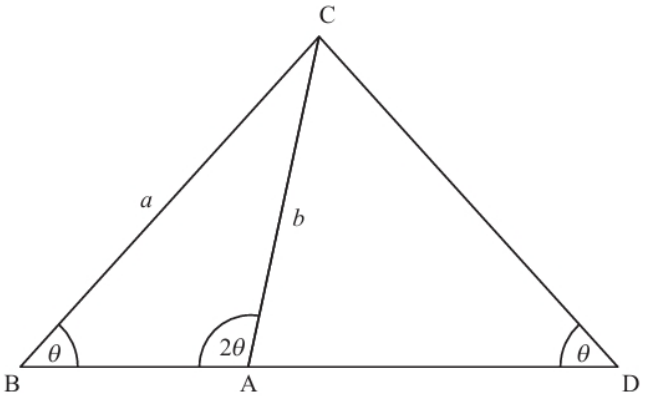
(ii) $\sin 2\theta$. [4]

The following diagram shows triangle ABC, with $\hat{B} = \theta$, $\hat{A} = 2\theta$, $BC = a$ and $AC = b$.



(b) Show that $b = \frac{3a}{4}$. [2]

$[BA]$ is extended to form an isosceles triangle DAC, with $\hat{D} = \theta$, as shown in the following diagram.



(c) Find the value of $\sin C\hat{A}D$. [3]

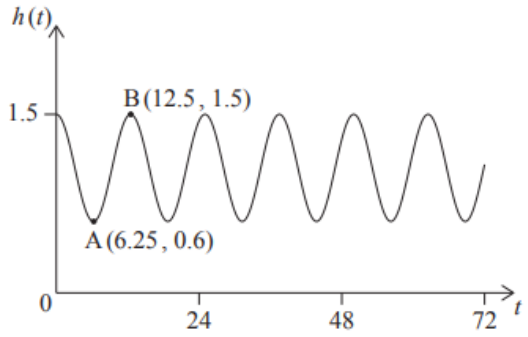
(d) Find the area of triangle DAC, in terms of a. [5]

Question 5:

Calculator Allowed: Yes

[Maximum mark: 14]

At Grande Anse Beach the height of the water in metres is modelled by the function $h(t) = p\cos(q \times t) + r$, where t is the number of hours after 21:00 hours on 10 December 2017. The following diagram shows the graph of h, for $0 \le t \le 72$.



The point $A(6.25, 0.6)$ represents the first low tide and $B(12.5, 1.5)$ represents the next high tide.

(a) (i) How much time is there between the first low tide and the next high tide?

(ii) Find the difference in height between low tide and high tide. [4]

(b) Find the value of

(i) p;

(ii) q;

(iii) r. [7]

(c) There are two high tides on 12 December 2017. At what time does the second high tide occur? [3]

Question 6:

Calculator Allowed: No

[Maximum mark: 8]

Let $f(x)=4\cos\left(\frac{x}{2}\right)+1$, for $0\le x\le 6\pi$. Find the values of x for which $f(x)>2\sqrt{2}+1$.