Answer 1:

7. (a) $y = \dfrac{\sin x}{\cos x}$

$\dfrac{dy}{dx} = \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$ ***M1A1***

$= \dfrac{1}{\cos^2 x}$ ***A1***

$= \sec^2 x$ ***AG***

(b) $y = \arctan x$

$\Rightarrow x = \tan y$ ***(M1)***

$\dfrac{dx}{dy} = \sec^2 y$ ***A1***

EITHER

$\dfrac{dx}{dy} = 1 + \tan^2 y$ ***(A1)***

$= 1 + x^2$ ***A1***

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1+x^2}$ ***AG***

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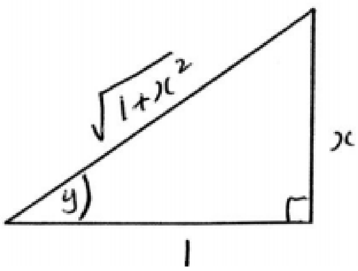
OR

$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} = \cos^2 y$ ***(A1)***



A1

$= \left(\dfrac{1}{\sqrt{1+x^2}}\right)^2 = \dfrac{1}{1+x^2}$ ***AG***

[7 marks]

Answer 2:

9. (a) $f(0)=\frac{100}{51}$ (exact), 1.96 *A1* *N1*

[1 mark]

(b) setting up equation *(M1)*

eg $95=\frac{100}{1+50e^{-0.2x}}$, sketch of graph with horizontal line at $y=95$

$x=34.3$ *A1* *N2*

[2 marks]

(c) upper bound of y is 100 *(A1)*

lower bound of y is 0 *(A1)*

range is $0<y<100$ *A1* *N3*

[3 marks]

(d) **METHOD 1**

setting function ready to apply the chain rule *(M1)*

eg $100\left(1+50e^{-0.2x}\right)^{-1}$

evidence of correct differentiation (must be substituted into chain rule) *(A1)(A1)*

eg $u' = -100\left(1+50e^{-0.2x}\right)^{-2}$, $v' = \left(50e^{-0.2x}\right)(-0.2)$

correct chain rule derivative *A1*

eg $f'(x) = -100\left(1+50e^{-0.2x}\right)^{-2}\left(50e^{-0.2x}\right)(-0.2)$

correct working clearly leading to the required answer *A1*

eg $f'(x) = 1000e^{-0.2x}\left(1+50e^{-0.2x}\right)^{-2}$

$$f'(x) = \frac{1000e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$$ *AG* *N0*

METHOD 2

attempt to apply the quotient rule (accept reversed numerator terms) *(M1)*

eg $\dfrac{vu' - uv'}{v^2}$, $\dfrac{uv' - vu'}{v^2}$

evidence of correct differentiation inside the quotient rule *(A1)(A1)*

eg $f'(x) = \dfrac{\left(1+50e^{-0.2x}\right)(0) - 100\left(50e^{-0.2x} \times -0.2\right)}{\left(1+50e^{-0.2x}\right)^2}$, $\dfrac{100(-10)e^{-0.2x} - 0}{\left(1+50e^{-0.2x}\right)^2}$

any correct expression for derivative (0 may not be explicitly seen) *(A1)*

eg $\dfrac{-100\left(50e^{-0.2x} \times -0.2\right)}{\left(1+50e^{-0.2x}\right)^2}$

correct working clearly leading to the required answer *A1*

eg $f'(x) = \dfrac{0 - 100(-10)e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$, $\dfrac{-100(-10)e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$

$$f'(x) = \frac{1000e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$$ *AG* *N0*

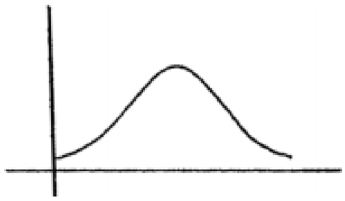
[5 marks]

continued …

(e) **METHOD 1**

sketch of $f'(x)$ *(A1)*

eg



recognizing maximum on $f'(x)$ *(M1)*

eg dot on max of sketch

finding maximum on graph of $f'(x)$ ***A1***

eg $(19.6, 5)$, $x = 19.560\ldots$

maximum rate of increase is 5 ***A1*** ***N2***

[4 marks]

METHOD 2

recognizing $f''(x) = 0$ *(M1)*

finding any correct expression for $f''(x)$ *(A1)*

eg $$\frac{\left(1+50\mathrm{e}^{-0.2x}\right)^2\left(-200\mathrm{e}^{-0.2x}\right)-\left(1000\mathrm{e}^{-0.2x}\right)\left(2\left(1+50\mathrm{e}^{-0.2x}\right)\left(-10\mathrm{e}^{-0.2x}\right)\right)}{\left(1+50\mathrm{e}^{-0.2x}\right)^4}$$

finding $x = 19.560\ldots$ ***A1***

maximum rate of increase is 5 ***A1*** ***N2***

[4 marks]

Total [15 marks]

Answer 3:

$\frac{dy}{dx} = 8x^3 + 18x^2 + 7x - 5$	***A1***
when $x = -1, \frac{dy}{dx} = -2$	***A1***
$8x^3 + 18x^2 + 7x - 5 = -2$	***M1***
$8x^3 + 18x^2 + 7x - 3 = 0$	
$(x + 1)$ is a factor	***A1***
$8x^3 + 18x^2 + 7x - 3 = (x + 1)(8x^2 + 10x - 3)$	***(M1)***

Note: ***M1*** is for attempting to find the quadratic factor.

$(x + 1)(4x - 1)(2x + 3) = 0$	
$(x = -1)$, $x = 0.25$, $x = -1.5$	***(M1)A1***

Note: ***M1*** is for an attempt to solve their quadratic factor.

[7 marks]

Answer 4:

8. **EITHER**

differentiating implicitly:

$1\times e^{-y} - xe^{-y}\frac{dy}{dx} + e^{y}\frac{dy}{dx} = 1$ *M1A1*

at the point $(c, \ln c)$

$\frac{1}{c} - c\times\frac{1}{c}\frac{dy}{dx} + c\frac{dy}{dx} = 1$ *M1*

$\frac{dy}{dx} = \frac{1}{c} \quad (c \neq 1)$ *(A1)*

OR

reasonable attempt to make expression explicit *(M1)*

$xe^{-y} + e^{y} = 1 + x$

$x + e^{2y} = e^{y}(1 + x)$

$e^{2y} - e^{y}(1 + x) + x = 0$

$(e^{y} - 1)(e^{y} - x) = 0$ *(A1)*

$e^{y} = 1,\ e^{y} = x$

$y = 0,\ y = \ln x$ *A1*

Note: Do not penalize if $y = 0$ not stated.

$\frac{dy}{dx} = \frac{1}{x}$

gradient of tangent $= \frac{1}{c}$ *A1*

Note: If candidate starts with $y = \ln x$ with no justification, award ***(M0)(A0)A1A1.***

THEN

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the equation of the normal is

$y - \ln c = -c(x - c)$ *M1*

$x = 0,\ y = c^2 + 1$

$c^2 + 1 - \ln c = c^2$ *(A1)*

$\ln c = 1$

$c = e$ *A1*

[7 marks]

Answer 5:

1. $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 12x + k$ *M1A1*

For use of discriminant $b^2 - 4ac = 0$ or completing the square $3(x-2)^2 + k - 12$ *(M1)*

$144 - 12k = 0$ *(A1)*

Note:	Accept trial and error, sketches of parabolas with vertex (2,0) or use of second derivative.

$k = 12$ *A1*

[5 marks]

Answer 6:

7. (a) $x^3 + 1 = \frac{1}{x^3 + 1}$

$(-1.26, -1)$ $\left(= \left(-\sqrt[3]{2}, -1\right)\right)$ *A1*

(b) $f'(-1.259...) = 4.762...$ $(3 \times 2^{\frac{2}{3}})$ *A1*

$g'(-1.259...) = -4.762...$ $(-3 \times 2^{\frac{2}{3}})$ *A1*

required angle $= 2\arctan\left(\frac{1}{4.762...}\right)$ *M1*

$= 0.414$ (accept $23.7°$) *A1*

Note:	Accept alternative methods including finding the obtuse angle first.

[5 marks]

Answer 7:

12. (a) (i) $f'(x) = \dfrac{x\dfrac{1}{x} - \ln x}{x^2}$ ***M1A1***

$= \dfrac{1 - \ln x}{x^2}$

so $f'(x) = 0$ when $\ln x = 1$, *i.e.* $x = \mathrm{e}$ ***A1***

(ii) $f'(x) > 0$ when $x < \mathrm{e}$ and $f'(x) < 0$ when $x > \mathrm{e}$ ***R1***

hence local maximum ***AG***

Note: Accept argument using correct second derivative.

(iii) $y \le \dfrac{1}{\mathrm{e}}$ ***A1***

[5 marks]

(b) $f''(x) = \dfrac{x^2\dfrac{-1}{x} - (1 - \ln x)2x}{x^4}$ ***M1***

$= \dfrac{-x - 2x + 2x\ln x}{x^4}$

$= \dfrac{-3 + 2\ln x}{x^3}$ ***A1***

Note: May be seen in part (a).

$f''(x) = 0$ *(M1)*

$-3 + 2\ln x = 0$

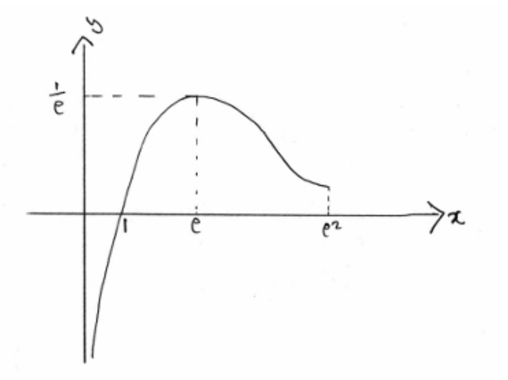
$x = e^{\frac{3}{2}}$

since $f''(x) < 0$ when $x < e^{\frac{3}{2}}$ and $f''(x) > 0$ when $x > e^{\frac{3}{2}}$ ***R1***

then point of inflexion $\left(e^{\frac{3}{2}}, \dfrac{3}{2e^{\frac{3}{2}}}\right)$ ***A1***

[5 marks]

(c)

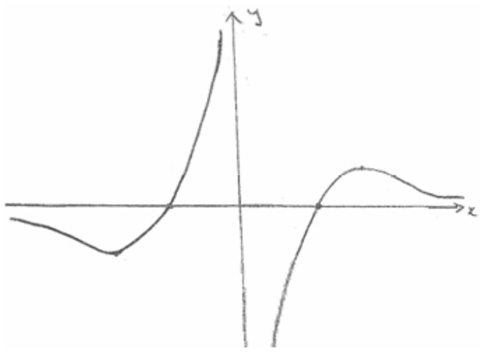


A1A1A1

Note:	Award ***A1*** for the maximum and intercept, ***A1*** for a vertical asymptote and ***A1*** for shape (including turning concave up).

[3 marks]

(d) (i)

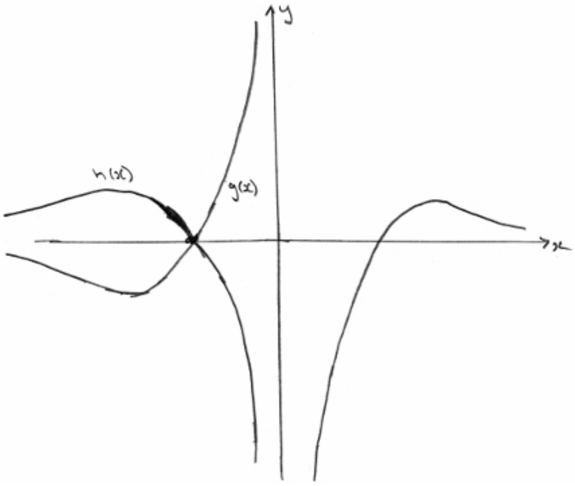


A1A1

Note:	Award ***A1*** for each correct branch.

(ii) all real values ***A1***

(iii)



(M1)(A1)

Note: Award ***(M1)(A1)*** for sketching the graph of h, ignoring any graph of g.

$-e^2 < x < -1$ (accept $x < -1$) ***A1***

[6 marks]

Total [19 marks]

Answer 8:

(b) $3-2y^2=2 \Rightarrow y^2=\frac{1}{2} \Rightarrow y=\pm\sqrt{\frac{1}{2}}$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3-2\cdot\frac{1}{2}}{\pm 4\sqrt{\frac{1}{2}}}=\pm\frac{\sqrt{2}}{2}$ ***M1***

at $\left(1, \sqrt{\frac{1}{2}}\right)$ the tangent is $y-\sqrt{\frac{1}{2}}=\frac{\sqrt{2}}{2}(x-1)$ and ***A1***

at $\left(1, -\sqrt{\frac{1}{2}}\right)$ the tangent is $y+\sqrt{\frac{1}{2}}=-\frac{\sqrt{2}}{2}(x-1)$ ***A1***

Note: These equations simplify to $y=\pm\frac{\sqrt{2}}{2}x$.

Note: Award ***A0M1A1A0*** if just the positive value of y is considered and just one tangent is found.

[4 marks]

Total [9 marks]