Question 1:
Calculator Allowed: No

10. [Maximum mark: 18]

Consider the series $\ln x + p\ln x + \frac{1}{3}\ln x + \ldots$, where $x \in \mathbb{R}, x > 1$ and $p \in \mathbb{R}, p \neq 0$.

(a) Consider the case where the series is geometric.

(i) Show that $p = \pm\frac{1}{\sqrt{3}}$.

(ii) Hence or otherwise, show that the series is convergent.

(iii) Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x. [6]

(b) Now consider the case where the series is arithmetic with common difference d.

(i) Show that $p = \frac{2}{3}$.

(ii) Write down d in the form $k\ln x$, where $k \in \mathbb{Q}$.

(iii) The sum of the first n terms of the series is $\ln\left(\frac{1}{x^3}\right)$.

Find the value of n. [12]

Question 2:
Calculator Allowed: No

5. [Maximum mark: 7]

All living plants contain an isotope of carbon called carbon-14. When a plant dies, the isotope decays so that the amount of carbon-14 present in the remains of the plant decreases. The time since the death of a plant can be determined by measuring the amount of carbon-14 still present in the remains.

The amount, A, of carbon-14 present in a plant t years after its death can be modelled by $A = A_0 e^{-kt}$ where $t \geq 0$ and A_0, k are positive constants.

At the time of death, a plant is defined to have 100 units of carbon-14.

(a) Show that $A_0 = 100$. [1]

The time taken for half the original amount of carbon-14 to decay is known to be 5730 years.

(b) Show that $k = \frac{\ln 2}{5730}$. [3]

(c) Find, correct to the nearest 10 years, the time taken after the plant's death for 25% of the carbon-14 to decay. [3]

Question 3:
Calculator Allowed: No

4. [Maximum mark: 6]

Find the range of possible values of k such that $e^{2x} + \ln k = 3e^{x}$ has at least one real solution.

Question 4:
Calculator Allowed: Yes

10. *[Maximum mark: 6]*

Find the set of values of x for which $\left|0.1x^2 - 2x + 3\right| < \log_{10} x$.

Question 5:
Calculator Allowed: No

8. *[Maximum mark: 6]*

The first terms of an arithmetic sequence are $\frac{1}{\log_2 x}, \frac{1}{\log_8 x}, \frac{1}{\log_{32} x}, \frac{1}{\log_{128} x}, \ldots$

Find x if the sum of the first 20 terms of the sequence is equal to 100.