Answer 1:

11.

+	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

(a) let T be Tim's score

(i) $P(T=6)=\frac{1}{9}$ $(=0.111 \text{ 3 sf})$ *A1*

(ii) $P(T \geq 3)=1-P(T \leq 2)=1-\frac{1}{9}=\frac{8}{9}$ $(=0.889 \text{ 3 sf})$ *(M1)A1*

[3 marks]

(b) let B be Bill's score

(i) $P(T=6 \text{ and } B=6)=\frac{1}{9}\times\frac{1}{9}=\frac{1}{81}$ $(=0.012 \text{ 3 sf})$ *(M1)A1*

(ii) $P(B=T)=P(2)P(2)+P(3)P(3)+\ldots+P(6)P(6)$

$$=\frac{1}{9}\times\frac{1}{9}+\frac{2}{9}\times\frac{2}{9}+\frac{3}{9}\times\frac{3}{9}+\frac{2}{9}\times\frac{2}{9}+\frac{1}{9}\times\frac{1}{9}$$ *M1*

$$=\frac{19}{81} \quad (=0.235 \text{ 3 sf})$$ *A1*

[4 marks]

(c) (i) **EITHER**

$P(X \le 2) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ ***M1A1***

because $P(X \le 2) = P\left((a, b, c, d) \middle| a, b, c, d = 1, 2\right)$ ***R1***

or equivalent

$P(X \le 2) = \frac{16}{81}$ ***AG***

OR

there are sixteen possible permutations which are

Combinations	Number
1111	1
1112	4
1122	6
1222	4
2222	1

M1A1

Note: This information may be presented in a variety of forms.

$P(X \le 2) = \frac{1+4+6+4+1}{81}$ ***A1***

$= \frac{16}{81}$ ***AG***

(ii)

x	1	2	3
$\mathrm{P}(X=x)$	$\frac{1}{81}$	$\frac{15}{81}$	$\frac{65}{81}$

A1A1

(iii) $\mathrm{E}(X)=\sum_{x=1}^{3} x\mathrm{P}(X=x)$ ***(M1)***

$=\frac{1}{81}+\frac{30}{81}+\frac{195}{81}$

$=\frac{226}{81}$ (2.79 to 3 sf) ***A1***

$\mathrm{E}(X^2)=\sum_{x=1}^{3} x^2\mathrm{P}(X=x)$

$=\frac{1}{81}+\frac{60}{81}+\frac{585}{81}$

$=\frac{646}{81}$ (7.98 to 3 sf) ***A1***

$\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$ ***(M1)***

$=0.191$ (3 sf) ***A1***

Note: Award ***M1A0*** for answers obtained using rounded values $(e.g.\ \mathrm{Var}(X)=0.196)$.

[10 marks]

(d)

Combinations	**Number**
3311	6
3221	12

$\mathrm{P}(\text{total is } 8 \cap (X=3))=\frac{18}{81}$ ***M1A1***

since $\mathrm{P}(X=3)=\frac{65}{81}$

$\mathrm{P}(\text{total is } 8 \mid (X=3))=\frac{\mathrm{P}((\text{total is } 8)\cap(X=3))}{\mathrm{P}(X=3)}$ ***M1***

$=\frac{18}{65}$ $(=0.277)$ ***A1***

[4 marks]

Total [21 marks]

Answer 2:

(a) $\mathrm{P}(X = 3) = (0.1)^3$ ***A1***

$= 0.001$ ***AG***

$\mathrm{P}(X = 4) = \mathrm{P}\left(VV\bar{V}V\right) + \mathrm{P}\left(V\bar{V}VV\right) + \mathrm{P}\left(\bar{V}VVV\right)$ ***(M1)***

$= 3 \times (0.1)^3 \times 0.9$ (or equivalent) ***A1***

$= 0.0027$ ***AG***

[3 marks]

(b) **METHOD 1**

attempting to form equations in a and b ***M1***

$\dfrac{9 + 3a + b}{2000} = \dfrac{1}{1000}$ $(3a + b = -7)$ ***A1***

$\dfrac{16 + 4a + b}{2000} \times \dfrac{9}{10} = \dfrac{27}{10000}$ $(4a + b = -10)$ ***A1***

attempting to solve simultaneously ***(M1)***

$a = -3,\ b = 2$ ***A1***

METHOD 2

$\mathrm{P}(X = n) = \begin{pmatrix} n-1 \\ 2 \end{pmatrix} \times 0.1^3 \times 0.9^{n-3}$ ***M1***

$= \dfrac{(n-1)(n-2)}{2000} \times 0.9^{n-3}$ ***(M1)A1***

$= \dfrac{n^2 - 3n + 2}{2000} \times 0.9^{n-3}$ ***A1***

$a = -3,\ b = 2$ ***A1***

Note: Condone the absence of 0.9^{n-3} in the determination of the values of a and b.

[5 marks]

(c) **METHOD 1**

EITHER

$$\mathrm{P}(X = n) = \frac{n^2 - 3n + 2}{2000} \times 0.9^{n-3}$$ ***(M1)***

OR

$$\mathrm{P}(X = n) = \binom{n-1}{2} \times 0.1^3 \times 0.9^{n-3}$$ ***(M1)***

THEN

$$= \frac{(n-1)(n-2)}{2000} \times 0.9^{n-3}$$ ***A1***

$$\mathrm{P}(X = n-1) = \frac{(n-2)(n-3)}{2000} \times 0.9^{n-4}$$ ***A1***

$$\frac{\mathrm{P}(X = n)}{\mathrm{P}(X = n-1)} = \frac{(n-1)(n-2)}{(n-2)(n-3)} \times 0.9$$ ***A1***

$$= \frac{0.9(n-1)}{n-3}$$ ***AG***

METHOD 2

$$\frac{\mathrm{P}(X = n)}{\mathrm{P}(X = n-1)} = \frac{\frac{n^2 - 3n + 2}{2000} \times 0.9^{n-3}}{\frac{(n-1)^2 - 3(n-1) + 2}{2000} \times 0.9^{n-4}}$$ ***(M1)***

$$= \frac{0.9\left(n^2 - 3n + 2\right)}{\left(n^2 - 5n + 6\right)}$$ ***A1A1***

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

$$= \frac{0.9(n-1)(n-2)}{(n-2)(n-3)}$$ ***A1***

$$= \frac{0.9(n-1)}{n-3}$$ ***AG***

[4 marks]

(d) (i) attempting to solve $\frac{0.9(n-1)}{n-3}=1$ for n **M1**

$n=21$ **A1**

$\frac{0.9(n-1)}{n-3}<1 \Rightarrow n>21$ **R1**

$\frac{0.9(n-1)}{n-3}>1 \Rightarrow n<21$ **R1**

X has two modes **AG**

Note: Award ***R1R1*** for a clearly labelled graphical representation of the two inequalities (using $\frac{\mathrm{P}(X=n)}{\mathrm{P}(X=n-1)}$).

(ii) the modes are 20 and 21 **A1**

[5 marks]

(e) **METHOD 1**

$Y \sim \mathrm{B}(x, 0.1)$ ***(A1)***

attempting to solve $\mathrm{P}(Y \geq 3) > 0.5$ (or equivalent ***eg*** $1-\mathrm{P}(Y \leq 2) > 0.5$) for x ***(M1)***

Note: Award ***(M1)*** for attempting to solve an equality (obtaining $x = 26.4$).

$x = 27$ ***A1***

METHOD 2

$\sum_{n=0}^{x} \mathrm{P}(X=n) > 0.5$ ***(A1)***

attempting to solve for x ***(M1)***

$x = 27$ ***A1***

[3 marks]

Total [20 marks]

Answer 3:

(a) $P(L \geq 5) = 0.910$ ***(M1)A1***

[2 marks]

(b) X is the number of wolves found to be at least 5 years old
recognising binomial distribution ***M1***
$X \sim B(8, 0.910\ldots)$
$P(X > 6) = 1 - P(X \leq 6)$ ***(M1)***
$= 0.843$ ***A1***

Note: Award ***M1A0*** for finding $P(X \geq 6)$.

[3 marks]

Total [5 marks]

Answer 4:

2.

Note: In this question, do not penalise incorrect use of strict inequality signs.

Let X = mass of a bag of sugar

(a) evidence of identifying the correct area ***(M1)***

$P(X < 995) = 0.0765637\ldots$

$= 0.0766$ ***A1***

[2 marks]

(b) 0.0766×100
≈ 8 ***A1***

[1 mark]

Note: Accept 7.66 .

(c) recognition that $P(X > 1005 | X \geq 995)$ is required ***(M1)***

$$\frac{P(X \geq 995 \cap X > 1005)}{P(X \geq 995)}$$

$$\frac{P(X > 1005)}{P(X \geq 995)}$$ ***(A1)***

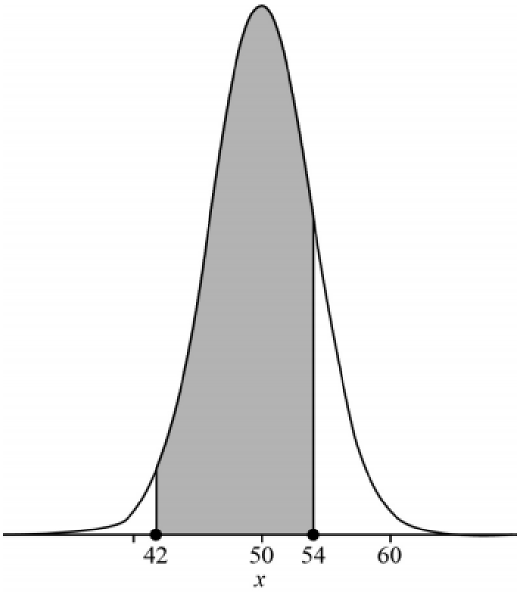
$$\frac{0.07656\ldots}{1 - 0.07656\ldots}\left(= \frac{0.07656\ldots}{0.9234\ldots}\right)$$

$= 0.0829$ ***A1***

[3 marks]
Total [6 marks]

Answer 5:

(a)



normal curve centred on 50 *A1*

vertical lines at $x = 42$ and $x = 54$, with shading in between *A1*

[2 marks]

(b) $\mathrm{P}(42 < X < 54)$ $\left(= \mathrm{P}(-2 < Z < 1)\right)$ ***(M1)***

$= 0.819$ ***A1***

[2 marks]

(c) $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0.5 \Rightarrow \mathrm{P}(X < \mu + k\sigma) = 0.75$ ***(M1)***

$k = 0.674$ ***A1***

Note: Award ***M1A0*** for $k = -0.674$.

[2 marks]

Total [6 marks]

Answer 6:

1. attempt to substitute into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

Note: Accept use of Venn diagram or other valid method.

$0.6 = 0.5 + 0.4 - \mathrm{P}(A \cap B)$ ***(A1)***

$\mathrm{P}(A \cap B) = 0.3$ (seen anywhere) ***A1***

attempt to substitute into $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ ***(M1)***

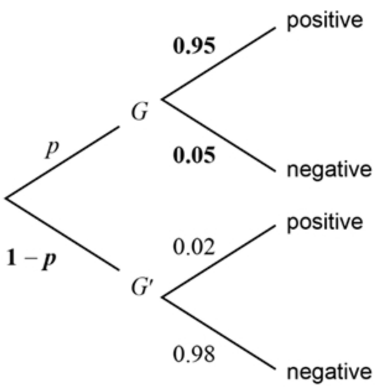
$= \dfrac{0.3}{0.4}$

$\mathrm{P}(A \mid B) = 0.75 \left(= \dfrac{3}{4} \right)$ ***A1***

Total [5 marks]

Answer 7:

7. (a)



A1A1

Note: award ***A1*** for branch correctly labelled $1 - p$
award ***A1*** for branches correctly labelled 0.95 and 0.05
award ***A0*** for G' branch labelled p'
award ***A0*** for G' branch labelled q unless explicitly defined as $1 - p$

[2 marks]

(b) **METHOD 1**

recognizing conditional probability *(M1)*

$\mathrm{P}\left(G'\middle|pos\right)$ OR $\mathrm{P}\left(G\middle|pos\right)$

$\dfrac{0.02(1-p)}{0.95p+0.02(1-p)}\left(=\dfrac{18}{150}\right)$ OR $\dfrac{0.95p}{0.95p+0.02(1-p)}\left(=\dfrac{132}{150}\right)$ *(A1)(A1)*

Note: Award ***A1*** for a correct numerator and ***A1*** for a correct denominator.

$p=0.133738$

$p=0.134$ ***A1***

METHOD 2

attempt to set up a system of equations (S = sample size) *(M1)*

$p(0.95S)=132$ and $(1-p)(0.02S)=18$ *(A1)*

attempt to solve for p or S *(M1)*

$\dfrac{0.95p}{0.02(1-p)}=\dfrac{132}{18}$

OR $S=pS+(1-p)S=\dfrac{132}{0.95}+\dfrac{18}{0.02}=138.947\ldots+900=1038.94\ldots$

$p=0.133738\ldots$

$p=0.134$ ***A1***

METHOD 3

attempt to find the number of parrots with the gene and the number without ***(M1)***

number of parrots with the gene $\approx \dfrac{132}{0.95} = 138.947...$ AND

number of parrots without the gene $\approx \dfrac{18}{0.02} = 900$ ***(A1)***

number of parrots in the sample $\approx 138.947... + 900 = 1038.94...$

attempt to find proportion of sample with the gene ***(M1)***

$$p \approx \frac{138.947...}{1038.94...} = 0.133738...$$

$p = 0.134$ ***A1***

[4 marks]

Total [6 marks]

Answer 8:

4. (a) (i) $(m=)\ 54(\%)$ ***A1***

(ii) $(n=)\ 14(\%)$ ***A1***

(iii) $(p=)\ 22(\%)$ ***A1***

(iv) $(q=)\ 10(\%)$ ***A1***

> **Note:** Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[4 marks]

(b) $90\ (\%)$ ***A1***

> **Note:** Award ***A0*** for a decimal answer.

[1 mark]

(c) (i) $0.54\ \left(\frac{54}{100}, \frac{27}{50}, 54\%\right)$ ***A1***

(ii) $\frac{54}{64}\left(0.844, \frac{27}{32}, 84.4\%, 0.84375\right)$ ***A1A1***

> **Note:** Award ***A1*** for a correct denominator (0.64 or 64 seen), ***A1*** for the correct final answer.

[3 marks]

(d) (i) recognizing Binomial distribution with correct parameters ***(M1)***

$X \sim \mathrm{B}(10,\ 0.68)$

$\left(\mathrm{P}(X=5)=\right)\ 0.123\ \ (0.122940\ldots,\ 12.3\%)$ ***A1***

(ii) $1-\mathrm{P}(X \le 3)$ **OR** $\mathrm{P}(X \ge 4)$ **OR** $\mathrm{P}(4 \le X \le 10)$ ***(M1)***

$0.984\ (0.984497\ldots,\ 98.4\%)$ ***A1***

(iii) $(0.68)^9 \times 0.32$ ***(M1)***

recognition of two possible cases ***(M1)***

$2\times\left((0.68)^9 \times 0.32\right)$

$0.0199\ \ (0.0198957\ldots,\ 1.99\%)$ ***A1***

[7 marks]

(e) **EITHER**

the probability is not constant ***A1***

OR

the events are not independent ***A1***

OR

the events should be modelled by the hypergeometric distribution instead ***A1***

[1 mark]

Total [16 marks]