Answer 1:

8. **EITHER**

$\left(\frac{\mathrm{d}V}{\mathrm{d}h}=\right)10\pi h-\pi h^2$ ***(A1)***

> **Note:** This ***A1*** may be implied by the value $\frac{\mathrm{d}V}{\mathrm{d}h}=76.5616...$.

attempt to use chain rule to find a relationship between $\frac{\mathrm{d}h}{\mathrm{d}t}$, $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}h}$ ***(M1)***

$$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}\left(=\frac{1}{\left(\frac{\mathrm{d}V}{\mathrm{d}h}\right)}\times\frac{\mathrm{d}V}{\mathrm{d}t}\right)$$

OR

attempt to differentiate $V=5\pi h^2-\frac{1}{3}\pi h^3$ throughout with respect to t ***(M1)***

$\frac{\mathrm{d}V}{\mathrm{d}t}=10\pi h\frac{\mathrm{d}h}{\mathrm{d}t}-\pi h^2\frac{\mathrm{d}h}{\mathrm{d}t}$ ***(A1)***

THEN

$\left(10\pi h-\pi h^2\right)\frac{\mathrm{d}h}{\mathrm{d}t}=2$ OR $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{2}{10\pi h-\pi h^2}$ ***(A1)***

> **Note:** Award this ***A1*** if the correct expression is seen with their h already substituted.

attempt to solve $200=5\pi h^2-\frac{1}{3}\pi h^3$ ***(M1)***

$h=4.20648...$ ***(A1)***

> **Note:** This ***(M1)(A1)*** can be awarded independently of all previous marks, and may be implied by the value $\frac{\mathrm{d}V}{\mathrm{d}h}=76.5616...$
> Ignore extra values of h -3.24 and 14.0.

$\frac{\mathrm{d}h}{\mathrm{d}t}=0.0261227...$

$\frac{\mathrm{d}h}{\mathrm{d}t}=0.0261\left(\mathrm{cms}^{-1}\right)$ ***A1***

[6 marks]

Answer 2:

13. (a) **EITHER**

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent) *M1A1*

Note: Accept $\theta = 180° - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent).

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$ (or equivalent) *M1A1*

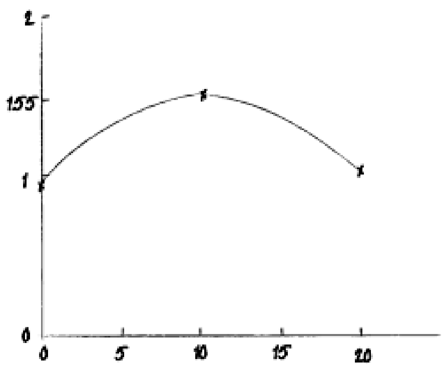
[2 marks]

(b) (i) $\theta = 0.994$ $\left(= \arctan\frac{20}{13}\right)$ *A1*

(ii) $\theta = 1.19$ $\left(= \arctan\frac{5}{2}\right)$ *A1*

[2 marks]

(c) correct shape. *A1*
correct domain indicated. *A1*



[2 marks]

(d) attempting to differentiate one $\arctan(f(x))$ term *M1*

EITHER

$$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$$

$$\frac{d\theta}{dx} = \frac{8}{x^2} \times \frac{1}{1+\left(\frac{8}{x}\right)^2} - \frac{13}{(20-x)^2} \times \frac{1}{1+\left(\frac{13}{20-x}\right)^2}$$ *A1A1*

OR

$$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$$

$$\frac{d\theta}{dx} = \frac{\frac{1}{8}}{1+\left(\frac{x}{8}\right)^2} + \frac{-\frac{1}{13}}{1+\left(\frac{20-x}{13}\right)^2}$$ *A1A1*

THEN

$$= \frac{8}{x^2+64} - \frac{13}{569-40x+x^2}$$ *A1*

$$= \frac{8(569-40x+x^2) - 13(x^2+64)}{(x^2+64)(x^2-40x+569)}$$ *M1A1*

$$= \frac{5(744-64x-x^2)}{(x^2+64)(x^2-40x+569)}$$ *AG*

[6 marks]

(e) Maximum light intensity at P occurs when $\frac{d\theta}{dx} = 0$. *(M1)*

either attempting to solve $\frac{d\theta}{dx} = 0$ for x or using the graph of either θ or $\frac{d\theta}{dx}$ *(M1)*

$x = 10.05$ (m) *A1*

[3 marks]

(f) $\frac{dx}{dt} = 0.5$ *(A1)*

At $x = 10$, $\frac{d\theta}{dx} = 0.000453 \ \left(= \frac{5}{11\,029}\right)$. *(A1)*

use of $\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$ ***M1***

$\frac{d\theta}{dt} = 0.000227 \ \left(= \frac{5}{22058}\right)$ (rad s^{-1}) ***A1***

Note: Award *(A1)* for $\frac{dx}{dt} = -0.5$ and ***A1*** for $\frac{d\theta}{dt} = -0.000227 \ \left(= -\frac{5}{22058}\right)$.

Note: Implicit differentiation can be used to find $\frac{d\theta}{dt}$. Award as above.

[4 marks]

Total [19 marks]

Answer 3:

10. (a) let $\text{H}\hat{\text{P}}\text{Q} = \theta$

$\tan\theta = \frac{h}{40}$

$\sec^2\theta \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{40}\frac{\mathrm{d}h}{\mathrm{d}t}$ ***M1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{4\sec^2\theta}$ ***(A1)***

$= \frac{16}{4\times 25}$ $\left(\sec\theta = \frac{5}{4} \text{ or } \theta = 0.6435\right)$ ***A1***

$= 0.16$ radians per second ***AG***

(b) $x^2 = h^2 + 1600$, where $\text{PH} = x$

$2x\frac{\mathrm{d}x}{\mathrm{d}t} = 2h\frac{\mathrm{d}h}{\mathrm{d}t}$ ***M1***

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{h}{x}\times 10$ ***A1***

$= \frac{10h}{\sqrt{h^2+1600}}$ ***(A1)***

$h = 30$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 6\ \mathrm{m\,s^{-1}}$ ***A1***

Note: Accept solutions that begin $x = 40\sec\theta$ or use $h = 10t$.

[7 marks]

Answer 4:

7. (a) (i) $\frac{\text{AP}}{42}$ OR $\frac{215}{84}$ OR $\frac{65}{42}+\frac{215}{84}$ ***(M1)***

time = $4.10714\ldots$ (hours)

time = 4.11 (hours) ***A1***

(ii) $\text{AB}=\sqrt{215^2+65^2}\,(=224.610\ldots)$ ***(A1)***

time = $5.34787\ldots$ (hours)

time = 5.35 (hours) ***A1***

[4 marks]

(b) (i) $\text{AD}=\sqrt{(215-x)^2+65^2}$ ***(A1)***

$$t=\frac{\sqrt{(215-x)^2+65^2}}{42}$$ ***(A1)***

$$T=\frac{\sqrt{(215-x)^2+65^2}}{42}+\frac{x}{84}\left(=\frac{\sqrt{x^2-430x+50450}}{42}+\frac{x}{84}\right)$$ ***A1***

(ii) valid approach to find the minimum for T (may be seen in (iii)) ***(M1)***

graph of T OR $T'=0$ OR graph of T'

$x=177.472\ldots$ km

$x=177$ km ***A1***

(iii) $T=3.89980\ldots$

$T=3.90$ (hours) ***A1***

Note: Only allow ***FT*** in (b)(ii) and (iii) for $0<x<215$ and a function T that has a minimum in that interval.

[6 marks]

(c) (i) $C = 200 \cdot \frac{\sqrt{(215 - x)^2 + 65^2}}{42} + 150 \cdot \frac{x}{84}$ ***(A1)***

valid approach to find the minimum for $C(x)$ (may be seen in (ii)) ***(M1)***

graph of C OR $C' = 0$ OR graph of C'

$x = 188.706...$ km

$x = 189$ km ***A1***

Note: Only allow ***FT*** from (b) if the function T has a minimum in $0 < x < 215$.

(ii) $C = 670.864$

$C = \$671$ ***A1***

Note: Only allow ***FT*** from (c)(i) if the function C has a minimum in $0 < x < 215$.

[4 marks]

Total [14 marks]

Answer 5:

17. (a) attempt to use the chain rule to set up a related rate ***(M1)***

correct expression ***A1***

$$\frac{dx}{d\theta} = \frac{dx}{dt} \div \frac{d\theta}{dt} \quad \textbf{OR} \quad \frac{-250}{0.075}$$

$$= -\frac{10000}{3}$$ ***AG***

[2 marks]

(b) $x(\theta) = \dfrac{3000}{\tan\theta}$ ***A1***

[1 mark]

(c) attempt to use chain rule **OR** quotient rule ***(M1)***

$$\frac{-3000}{\tan^2\theta \times \cos^2\theta}, \ \frac{-3000\left(\sin\theta(-\sin\theta) - \cos^2\theta\right)}{\sin^2\theta}$$ ***(A1)***

$$= -\frac{3000}{\sin^2\theta}$$ ***A1***

[3 marks]

(d) setting their equation in part (c) equal to the given expression in part (a) ***(M1)***

$$-\frac{3000}{\sin^2\theta} = -\frac{10000}{3}$$

$\theta = 1.24904...$ ***(A1)***

$x(1.24904...) = 1000\,\text{m}$ ***A1***

[3 marks]

[Total: 9 marks]

Answer 6:

5. (a) (i) $x-3$ ***A1***

(ii) attempt to use 1200 to find width of park in terms of only x ***(M1)***

$\frac{1200}{x}$ (seen) **OR** $1200 = x \times \text{park width}$ **OR** $1200 = x \times (\text{garden width} + 4)$

$\frac{1200}{x} - 4$ ***A1***

(iii) $A = (x-3)\times\left(\frac{1200}{x} - 4\right)$ ***A1***

$= 1200 - 4x - \frac{3600}{x} + 12$ ***A1***

Note: Award first ***A1FT*** for multiplying *their* garden length and width and second ***A1*** for a simplified (parentheses removed) expression for A that leads to the given answer. The given answer must be shown for the second ***A1*** mark to be awarded

$= 1212 - 4x - \frac{3600}{x}$ ***AG***

[5 marks]

(b) setting $1212 - 4x - \frac{3600}{x} = 800$ (accept a sketch) ***(M1)***

$x = 9.64$ $(9.64011\ldots)$ (m) **OR** $x = 93.4$ $(93.3598\ldots)$ (m) ***A1***

(width $=$) 124 $(124.479\ldots)$ (m) ***A1***

(width $=$) 12.9 $(12.8534\ldots)$ (m) ***A1***

Note: To award the final ***A1*** both values of x **and** both values of the width must be seen. Accept 12.8 for second value of width from candidate dividing 1200 by 3 sf value of 93.4.

[4 marks]

(c) $\left(\frac{\mathrm{d}A}{\mathrm{d}x} = \right) -4 + \frac{3600}{x^2}$ **OR** $-4 + 3600x^{-2}$ ***A1A1A1***

Note: Award ***A1*** for -4, ***A1*** for $+3600$, and ***A1*** for x^{-2} or x^2 in denominator.

[3 marks]

(d) setting *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ equal to 0 **OR** sketch of *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ with x-intercept highlighted ***M1***

$(x =)$ 30 (m) ***A1***

Note: To award ***A1FT*** the candidate's value of x must be within the domain given in the problem $(3 < x < 300)$.

[2 marks]

(e) **EITHER**

evidence of using GDC to find maximum of graph of $A = 1212 - 4x - \dfrac{3600}{x}$ ***(M1)***

OR

substitution of *their* x into A ***(M1)***

OR

dividing 1200 by *their* x to find width of park **and** subtracting 3 from *their* x and 4 from the width to find park dimensions ***(M1)***

Note: For the last two methods, only follow through if $3 < \text{their } x < 300$.

THEN

$(A =)\ 972\ (\mathrm{m}^2)$ ***A1***

[2 marks]

Total [16 marks]