

Answer 1:

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(a) **EITHER**

$$2019: 2500 \times 0.93 + 250 = 2575$$

(M1)A1

$$2020: 2575 \times 0.93 + 250$$

M1

OR

$$2020: 2500 \times 0.93^2 + 250(0.93 + 1)$$

M1M1A1

Note: Award **M1** for starting with 2500, **M1** for multiplying by 0.93 and adding 250 twice. **A1** for correct expression. Can be shown in recursive form.

THEN

$$(\approx 2644.75) = 2645$$

AG

[3 marks]

(b) 2020: $2500 \times 0.93^2 + 250(0.93 + 1)$

$$2042: 2500 \times 0.93^{24} + 250(0.93^{23} + 0.93^{22} + \dots + 1)$$

(M1)(A1)

$$= 2500 \times 0.93^{24} + 250 \frac{(0.93^{24} - 1)}{(0.93 - 1)}$$

(M1)(A1)

$$= 3384$$

A1

Note: If recursive formula used, award **M1** for $u_n = 0.93 u_{n-1} + 250$ and u_0 or u_1 seen (can be awarded if seen in part (a)). Then award **M1A1** for attempt to find u_{24} or u_{25} respectively (different term if other than 2500 used) (**M1A0** if incorrect term is being found) and **A2** for correct answer.

Note: Accept all answers that round to 3380.

[5 marks]

Answer 2:

5. (a) $u_1 = S_1 = \frac{2}{3} \times \frac{7}{8}$ (M1)

$$= \frac{14}{24} \left(= \frac{7}{12} = 0.583333... \right) \quad \text{A1}$$

[2 marks]

(b) $r = \frac{7}{8} (= 0.875)$ (A1)

substituting their values for u_1 and r into $S_\infty = \frac{u_1}{1-r}$ (M1)

$$= \frac{14}{3} (= 4.66666...) \quad \text{A1}$$

[3 marks]

(c) attempt to substitute their values into the inequality or formula for S_n (M1)

$$\frac{14}{3} - \sum_{r=1}^n \frac{2}{3} \left(\frac{7}{8} \right)^r < 0.001 \quad \text{OR} \quad S_n = \frac{\frac{7}{12} \left(1 - \left(\frac{7}{8} \right)^n \right)}{\left(1 - \frac{7}{8} \right)}$$

attempt to solve their inequality using a table, graph or logarithms
(must be exponential)

(M1)

Note: Award (M0) if the candidate attempts to solve $S_\infty - u_n < 0.001$.

correct critical value or at least one correct crossover value (A1)

$$63.2675... \text{ OR } S_\infty - S_{63} = 0.001036... \text{ OR } S_\infty - S_{64} = 0.000906...$$

$$\text{OR } S_\infty - S_{63} - 0.001 = 0.0000363683... \text{ OR } S_\infty - S_{64} - 0.001 = -0.0000931777...$$

least value is $n = 64$

A1

[4 marks]

Total [9 marks]

Answer 3:

3. $86.4 = 50r^3$ (A1)

$r = 1.2 \left(= \sqrt[3]{\frac{86.4}{50}} \right)$ seen anywhere (A1)

$\frac{50(1.2^n - 1)}{0.2} > 33500$ OR $250(1.2^n - 1) = 33500$ (A1)

attempt to solve their geometric S_n inequality or equation (M1)

sketch OR $n > 26.9045$, $n = 26.9$ OR $S_{26} = 28368.8$ OR $S_{27} = 34092.6$ OR algebraic

manipulation involving logarithms

$n = 27$ (accept $n \geq 27$) A1

Total [5 marks]

Answer 4:

(a) $A_1 = 1.004x$ A1
 $A_2 = 1.004(1.004x + x)$ A1
 $= 1.004^2 x + 1.004x$ AG

Note: Accept an argument in words for example, first deposit has been in for two months and second deposit has been in for one month.

[2 marks]

(b) (i) $A_3 = 1.004(1.004^2 x + 1.004x + x) = 1.004^3 x + 1.004^2 x + 1.004x$ (M1)A1
 $A_4 = 1.004^4 x + 1.004^3 x + 1.004^2 x + 1.004x$ A1

(ii) $A_{120} = (1.004^{120} + 1.004^{119} + \dots + 1.004)x$ (A1)

$= \frac{1.004^{120} - 1}{1.004 - 1} \times 1.004x$ M1A1

$= 251(1.004^{120} - 1)x$ AG

[6 marks]

(c) $A_{216} = 251(1.004^{216} - 1)x \left(= x \sum_{t=1}^{216} 1.004^t \right)$ A1

[1 mark]

(d) $251(1.004^{216} - 1)x = 20000 \Rightarrow x = 58.22\dots$ (A1)(M1)(A1)

Note: Award (A1) for $251(1.004^{216} - 1)x > 20000$, (M1) for attempting to solve and (A1) for $x > 58.22\dots$

$x = 59$

A1

Note: Accept $x = 58$. Accept $x \geq 59$.

[4 marks]

(e) $r = 1.004^{12} (=1.049\dots)$ (M1)

$15000 r^n - 1000 \frac{r^n - 1}{r - 1} = 0 \Rightarrow n = 27.8\dots$ (A1)(M1)(A1)

Note: Award (A1) for the equation (with their value of r), (M1) for attempting to solve for n and (A1) for $n = 27.8\dots$

$n = 28$

A1

Note: Accept $n = 27$.

[5 marks]

Total [18 marks]

Answer 5:

10. Attempting to solve $|0.1x^2 - 2x + 3| = \log_{10} x$ numerically or graphically.

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(M1)

$x = 1.52, 1.79$

(A1)(A1)

$x = 17.6, 19.1$

(A1)

$(1.52 < x < 1.79) \cup (17.6 < x < 19.1)$

A1A1

N2

[6 marks]

Answer 6:

(a) (i) **METHOD 1**

$$\frac{v_{n+1}}{v_n} = \frac{2^{u_{n+1}}}{2^{u_n}} \\ = 2^{u_{n+1} - u_n} = 2^d$$

M1**A1****METHOD 2**

$$\frac{v_{n+1}}{v_n} = \frac{2^{a+nd}}{2^{a+(n-1)d}} \\ = 2^d$$

M1**A1**(ii) 2^a **A1****Note:** Accept 2^{u_1} .(iii) **EITHER** v_n is a GP with first term 2^a and common ratio 2^d

$$v_n = 2^a (2^d)^{(n-1)}$$

OR $u_n = a + (n-1)d$ as it is an AP**THEN**

$$v_n = 2^{a+(n-1)d}$$

A1**[4 marks]**

$$(b) \quad (i) \quad S_n = \frac{2^a ((2^d)^n - 1)}{2^d - 1} = \frac{2^a (2^{dn} - 1)}{2^d - 1}$$

M1A1**Note:** Accept either expression.(ii) for sum to infinity to exist need $-1 < 2^d < 1$ **R1**

$$\Rightarrow \log 2^d < 0 \Rightarrow d \log 2 < 0 \Rightarrow d < 0$$

(M1)A1**Note:** Also allow graph of 2^d .

$$(iii) \quad S_\infty = \frac{2^a}{1-2^d}$$

A1

$$(iv) \frac{2^a}{1-2^d} = 2^{a+1} \Rightarrow \frac{1}{1-2^d} = 2$$

M1

$$\Rightarrow 1 = 2 - 2^{d+1} \Rightarrow 2^{d+1} = 1$$

$$\Rightarrow d = -1$$

A1**[8 marks]****(c) METHOD 1**

$$w_n = pq^{n-1}, z_n = \ln pq^{n-1}$$

(A1)

$$z_n = \ln p + (n-1) \ln q$$

M1A1

$$z_{n+1} - z_n = (\ln p + n \ln q) - (\ln p + (n-1) \ln q) = \ln q$$

which is a constant so this is an AP

(with first term $\ln p$ and common difference $\ln q$)

$$\sum_{i=1}^n z_i = \frac{n}{2} (2 \ln p + (n-1) \ln q)$$

M1

$$= n \left(\ln p + \ln q^{\left(\frac{n-1}{2}\right)} \right) = n \ln \left(pq^{\left(\frac{n-1}{2}\right)} \right)$$

(M1)

$$= \ln \left(p^n q^{\frac{n(n-1)}{2}} \right)$$

A1**METHOD 2**

$$\sum_{i=1}^n z_i = \ln p + \ln pq + \ln pq^2 + \dots + \ln pq^{n-1}$$

(M1)A1

$$= \ln \left(p^n q^{(1+2+3+\dots+(n-1))} \right)$$

(M1)A1

$$= \ln \left(p^n q^{\frac{n(n-1)}{2}} \right)$$

(M1)A1**[6 marks]****Total [18 marks]****Answer 7:**

8. Assume that a and b are both odd.

M1

Note: Award **M0** for statements such as “let a and b be both odd”.

Note: Subsequent marks after this **M1** are independent of this mark and can be awarded.

Then $a = 2m + 1$ and $b = 2n + 1$

A1

$$a^2 + b^2 \equiv (2m + 1)^2 + (2n + 1)^2$$

$$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$$

A1

$$= 4(m^2 + m + n^2 + n) + 2$$

(A1)

$(4(m^2 + m + n^2 + n))$ is always divisible by 4 but 2 is not divisible by 4. (or equivalent)

R1

$\Rightarrow a^2 + b^2$ is not divisible by 4, a contradiction. (or equivalent)

R1

hence a and b cannot both be odd.

AG

Note: Award a maximum of **M1A0A0(A0)R1R1** for considering identical or two consecutive odd numbers for a and b .

[6 marks]

Answer 8:

METHOD 1 (rearranging the equation)

assume there exists some $\alpha \in \mathbb{Z}$ such that $2\alpha^3 + 6\alpha + 1 = 0$

M1

Note: Award **M1** for equivalent statements such as ‘assume that α is an integer root of $2\alpha^3 + 6\alpha + 1 = 0$ ’. Condone the use of x throughout the proof.

Award **M1** for an assumption involving $\alpha^3 + 3\alpha + \frac{1}{2} = 0$.

Note: Award **M0** for statements such as “let’s consider the equation has integer roots...”, “let $\alpha \in \mathbb{Z}$ be a root of $2\alpha^3 + 6\alpha + 1 = 0$...”

Note: Subsequent marks after this **M1** are independent of this **M1** and can be awarded.

attempts to rearrange their equation into a suitable form

M1**EITHER**

$$2\alpha^3 + 6\alpha = -1$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow 2\alpha^3 + 6\alpha \text{ is even}$$

R1

$$2\alpha^3 + 6\alpha = -1 \text{ which is not even and so } \alpha \text{ cannot be an integer}$$

R1

Note: Accept ‘ $2\alpha^3 + 6\alpha = -1$ which gives a contradiction’.

OR

$$1 = 2(-\alpha^3 - 3\alpha)$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow (-\alpha^3 - 3\alpha) \in \mathbb{Z}$$

R1

$$\Rightarrow 1 \text{ is even which is not true and so } \alpha \text{ cannot be an integer}$$

R1

Note: Accept ‘ $\Rightarrow 1$ is even which gives a contradiction’.

OR

$$\frac{1}{2} = -\alpha^3 - 3\alpha$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow (-\alpha^3 - 3\alpha) \in \mathbb{Z}$$

R1

$-\alpha^3 - 3\alpha$ is not an integer $\left(= \frac{1}{2} \right)$ and so α cannot be an integer

R1

Note: Accept ' $-\alpha^3 - 3\alpha$ is not an integer $\left(= \frac{1}{2} \right)$ which gives a contradiction'.

OR

$$\alpha = -\frac{1}{2(\alpha^2 + 3)}$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow -\frac{1}{2(\alpha^2 + 3)} \in \mathbb{Z}$$

R1

$-\frac{1}{2(\alpha^2 + 3)}$ is not an integer and so α cannot be an integer

R1

Note: Accept $-\frac{1}{2(\alpha^2 + 3)}$ is not an integer which gives a contradiction'.

THEN

so the equation $2x^3 + 6x + 1 = 0$ has no integer roots

AG

[5 marks]

METHOD 2

assume there exists some $\alpha \in \mathbb{Z}$ such that $2\alpha^3 + 6\alpha + 1 = 0$

M1

Note: Award **M1** for statements such as ‘assume that α is an integer root of $2\alpha^3 + 6\alpha + 1 = 0$ ’. Condone the use of x throughout the proof. Award **M1** for an assumption involving $\alpha^3 + 3\alpha + \frac{1}{2} = 0$ and award subsequent marks based on this.

Note: Award **M0** for statements such as “let’s consider the equation has integer roots...”, “let $\alpha \in \mathbb{Z}$ be a root of $2\alpha^3 + 6\alpha + 1 = 0 \dots$ ”

Note: Subsequent marks after this **M1** are independent of this **M1** and can be awarded.

let $f(x) = 2x^3 + 6x + 1$ (and $f(\alpha) = 0$)

$f'(x) = 6x^2 + 6 > 0$ for all $x \in \mathbb{R} \Rightarrow f$ is a (strictly) increasing function

M1A1

$f(0) = 1$ and $f(-1) = -7$

R1

thus $f(x) = 0$ has only one real root between -1 and 0 , which gives a contradiction

(or therefore, contradicting the assumption that $f(\alpha) = 0$ for some $\alpha \in \mathbb{Z}$),

R1

so the equation $2x^3 + 6x + 1 = 0$ has no integer roots

AG

[5 marks]

Answer 9:

9. (a) attempt to expand binomial with negative fractional power (M1)

$$\frac{1}{\sqrt{1+ax}} = (1+ax)^{-\frac{1}{2}} = 1 - \frac{ax}{2} + \frac{3a^2x^2}{8} + \dots \quad \text{A1}$$

$$\sqrt{1-x} = (1-x)^{\frac{1}{2}} = 1 - \frac{x}{2} - \frac{x^2}{8} + \dots \quad \text{A1}$$

$$\frac{1}{\sqrt{1+ax}} - \sqrt{1-x} = \frac{(1-a)}{2}x + \left(\frac{3a^2+1}{8}\right)x^2 + \dots$$

attempt to equate coefficients of x or x^2 (M1)

$$x: \frac{1-a}{2} = 4b; x^2: \frac{3a^2+1}{8} = b$$

attempt to solve simultaneously (M1)

$$a = -\frac{1}{3}, b = \frac{1}{6} \quad \text{A1}$$

[6 marks]

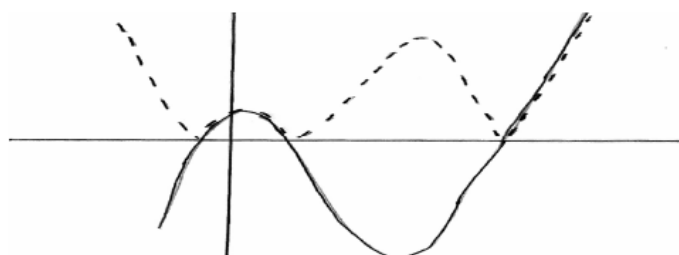
(b) $|x| < 1$ A1

[1 mark]

Total [7 marks]

Answer 10:

10. (a)



as roots of $f(x) = 0$ are $-1, 1, 5$

(M1)

solution is $]-\infty, -1[\cup]1, 5[$ ($x < -1$ or $1 < x < 5$)

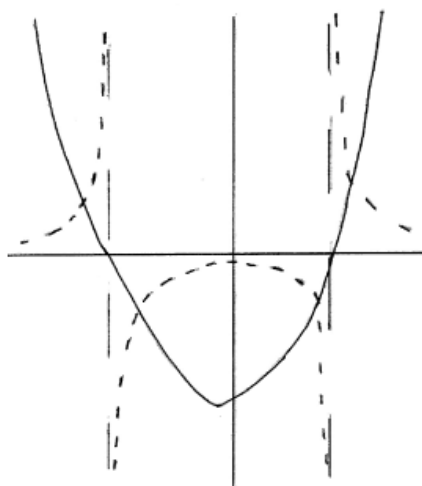
A1A1

Note: Award A1A0 for closed intervals.

[3 marks]

(b) METHOD 1

(graphs of $g(x)$ and $\frac{1}{g(x)}$)



roots of $g(x) = 0$ are -3 and 2

(M1)(A1)

Notes: Award **M1** if quadratic graph is drawn or two roots obtained.
 Roots may be indicated anywhere eg asymptotes on graph or in inequalities below.

the intersections of the graphs $g(x)$ and of $1/g(x)$
 are $-3.19, -2.79, 1.79, 2.19$

(M1)(A1)

Note: Award **A1** for at least one of the values above seen anywhere.

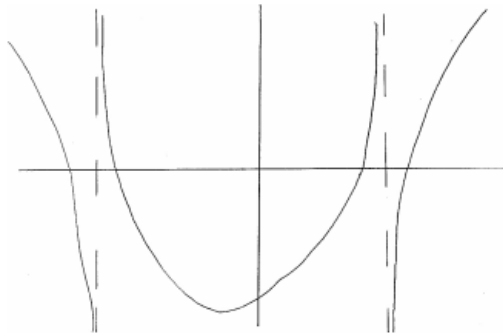
solution is $]-3.19, -3[\cup]-2.79, 1.79[\cup]2, 2.19[$
 $(-3.19 < x < -3 \text{ or } -2.79 < x < 1.79 \text{ or } 2 < x < 2.19)$

A1A1A1

Note: Award **A1A1A0** for closed intervals.

METHOD 2

(graph of $g(x) - \frac{1}{g(x)}$)



asymptotes at $x = -3$ and $x = 2$

(M1)(A1)

Note: May be indicated on the graph.

roots of graph are $-3.19, -2.79, 1.79, 2.19$

(M1)(A1)

Note: Award **A1** for at least one of the values above seen anywhere.

solution is (when graph is negative)

$] -3.19, -3[\cup] -2.79, 1.79[\cup] 2, 2.19[$

$(-3.19 < x < -3 \text{ or } -2.79 < x < 1.79 \text{ or } 2 < x < 2.19)$

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A1A1A1

Note: Award **A1A1A0** for closed intervals.

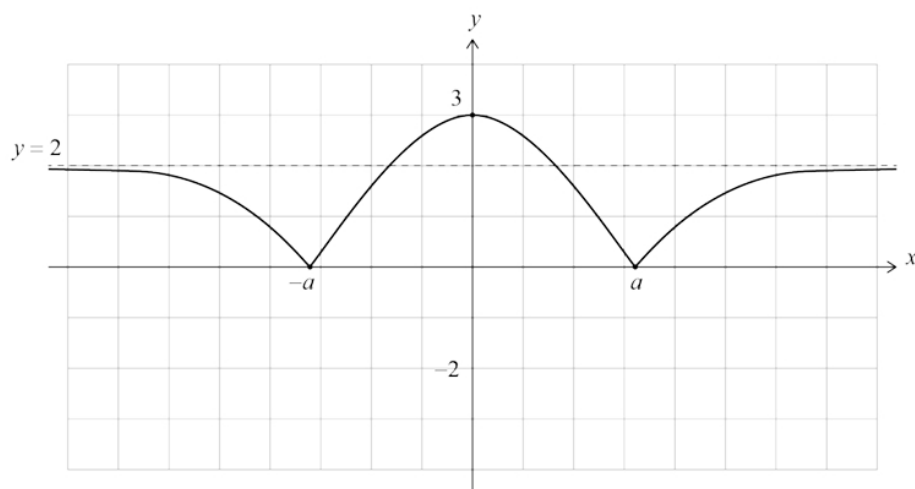
[7 marks]

Total [10 marks]

Answer 11:

8. (a) attempt to reflect f in the x OR y axis

(M1)



A1A1A1

Note: For a curve with an approximately correct shaped right-hand branch, award:

A1 for correct asymptotic behaviour at $y = 2$ (either side)

A1 for correctly reflected RHS of the graph in the y -axis with smooth maximum at $(0, 3)$.

A1 for labelled x -intercept at $(-a, 0)$ and labelled asymptote at $y = 2$ with sharp points (cusps) at the x -intercepts.

[4 marks]

- (b) $k = 0$

A1

$$4 \leq k < 9$$

A2

Note: If final answer incorrect, award **A1** for critical values 4 and 9 seen anywhere.

Exception to FT:

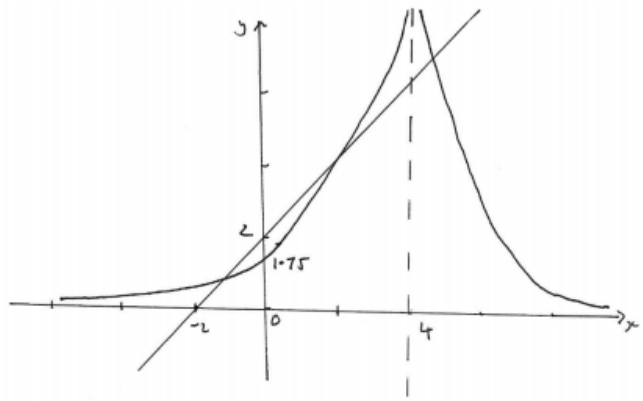
Award a maximum of **A0A2FT** if their graph from (a) is not symmetric about the y -axis.

[3 marks]

Total [7 marks]

Answer 12:

(a)



A1 for vertical asymptote and for the y -intercept $\frac{7}{4}$

A1 for general shape of $y = \left| \frac{7}{x-4} \right|$ including the x -axis as asymptote

A1 for straight line with y -intercept 2 and x -intercept of -2

A1A1A1

[3 marks]

(b) **METHOD 1**for $x > 4$

$$(x+2)(x-4) = 7$$

(M1)

$$x^2 - 2x - 8 = 7 \Rightarrow x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

(as $x > 4$ then) $x = 5$ **A1**
Note: Award **A0** if $x = -3$ is also given as a solution.
for $x < 4$

$$(x+2)(x-4) = -7$$

M1

$$\Rightarrow x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

(M1)A1
Note: Second **M1** is dependent on first **M1**.
[5 marks]**METHOD 2**

$$(x+2)^2 = \frac{49}{(x-4)^2}$$

M1

$$x^4 - 4x^3 - 12x^2 + 32x + 15 = 0$$

A1

$$(x+3)(x-5)(x^2 - 2x - 1) = 0$$

$$x = 5$$

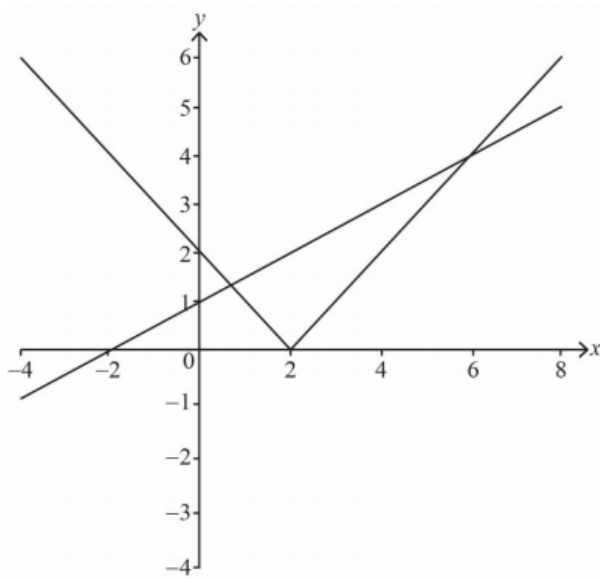
A1
Note: Award **A0** if $x = -3$ is also given as a solution.

$$x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

(M1)A1**[5 marks]****Total [8 marks]**

Answer 13:

(a)



straight line graph with correct axis intercepts
 modulus graph: V shape in upper half plane
 modulus graph having correct vertex and y -intercept

A1**A1****A1****[3 marks]**

(b) **METHOD 1**attempt to solve $\frac{x}{2} + 1 = x - 2$ **(M1)**

$$x = 6$$

A1**Note:** Accept $x = 6$ using the graph.attempt to solve (algebraically) $\frac{x}{2} + 1 = 2 - x$ **M1**

$$x = \frac{2}{3}$$

A1**[4 marks]****METHOD 2**

$$\left(\frac{x}{2} + 1\right)^2 = (x - 2)^2$$

M1

$$\frac{x^2}{4} + x + 1 = x^2 - 4x + 4$$

$$0 = \frac{3x^2}{4} - 5x + 3$$

$$3x^2 - 20x + 12 = 0$$

attempt to factorise (or equivalent)

M1

$$(3x - 2)(x - 6) = 0$$

$$x = \frac{2}{3}$$

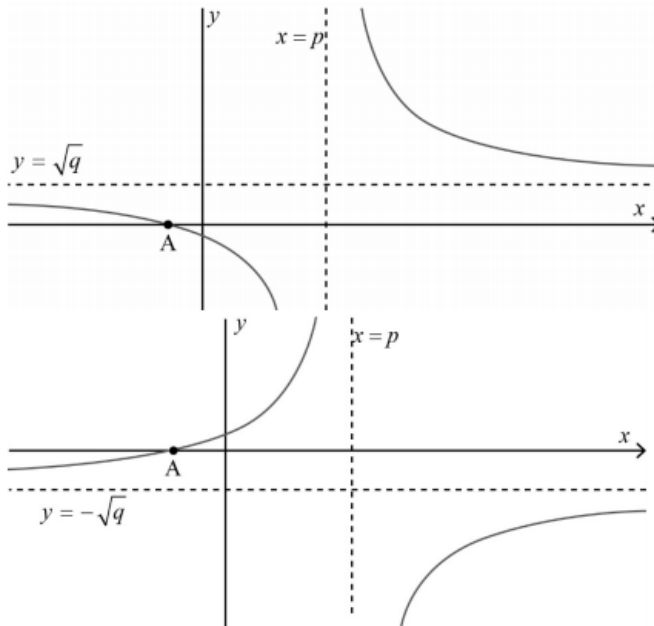
A1

$$x = 6$$

A1

Answer 14:

(a)



either graph passing through (or touching) A
 correct shape and vertical asymptote with correct equation for either graph
 correct horizontal asymptote with correct equation for either graph
 two completely correct sketches

A1**A1****A1****A1****[4 marks]**

(b) $a\left(-\frac{1}{2}\right) + 1 = 0 \Rightarrow a = 2$

A1

from horizontal asymptote, $\left(\frac{a}{b}\right)^2 = \frac{4}{9}$

(M1)

$\frac{a}{b} = \pm \frac{2}{3} \Rightarrow b = \pm 3$

A1

from vertical asymptote, $b\left(\frac{4}{3}\right) + c = 0$

$b = 3, c = -4$ or $b = -3, c = 4$

A1**[4 marks]****Total [8 marks]**