Total Questions: 7
Total Marks: 77

Question 1:
Calculator Allowed: Yes

[Maximum mark: 16]

Consider the curve, C defined by the equation $y^2 - 2xy = 5 - e^x$. The point A lies on C and has coordinates $(0, a)$, $a > 0$.

(a) Find the value of a. [2]

(b) Show that $\frac{dy}{dx} = \frac{2y - e^x}{2(y - x)}$. [4]

(c) Find the equation of the normal to C at the point A. [3]

(d) Find the coordinates of the second point at which the normal found in part (c) intersects C. [4]

(e) Given that $v = y^3$, $y > 0$, find $\frac{dv}{dx}$ at $x = 0$. [3]

Question 2:
Calculator Allowed: Yes

[Maximum mark: 7]

A water trough which is 10 metres long has a uniform cross-section in the shape of a semicircle with radius 0.5 metres. It is partly filled with water as shown in the following diagram of the cross-section. The centre of the circle is O and the angle KOL is θ radians.

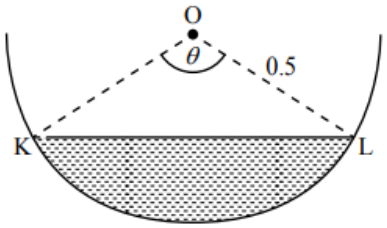


diagram not to scale

(a) Find an expression for the volume of water $V(m^3)$ in the trough in terms of θ. [3]

The volume of water is increasing at a constant rate of $0.0008\,\text{m}^3\,\text{s}^{-1}$.

(b) Calculate $\frac{d\theta}{dt}$ when $\theta = \frac{\pi}{3}$. [4]

Question 3:
Calculator Allowed: Yes

3. *[Maximum mark: 7]*

Let $f(x) = \frac{1-x}{1+x}$ and $g(x) = \sqrt{x+1}$, $x > -1$.

Find the set of values of x for which $f'(x) \le f(x) \le g(x)$.

. . . . . .

Question 4:
Calculator Allowed: Yes

[Maximum mark: 14]

Let $f(x)=\frac{1}{x-1}+2$, for $x>1$.

(a) Write down the equation of the horizontal asymptote of the graph of f. [2]

(b) Find $f'(x)$. [2]

Let $g(x)=ae^{-x}+b$, for $x\geq 1$. The graphs of f and g have the same horizontal asymptote.

(c) Write down the value of b. [2]

(d) Given that $g'(1)=-e$, find the value of a. [4]

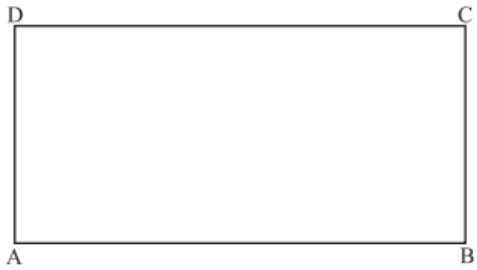
(e) There is a value of x, for $1<x<4$, for which the graphs of f and g have the same gradient. Find this gradient. [4]

Question 5:

Calculator Allowed: Yes

7. *[Maximum mark: 7]*

A farmer wishes to create a rectangular enclosure, ABCD, of area 525 m^2, as shown below.



The fencing used for side AB costs \$11 per metre. The fencing for the other three sides costs \$3 per metre. The farmer creates an enclosure so that the cost is a minimum. Find this minimum cost.

Question 6:

Calculator Allowed: Yes

[Maximum mark: 7]

Let $f(x) = \frac{\ln(4x)}{x}$, for $0 < x \leq 5$.

Points $\mathrm{P}(0.25, 0)$ and Q are on the curve of f. The tangent to the curve of f at P is perpendicular to the tangent at Q. Find the coordinates of Q.

Question 7:
Calculator Allowed: Yes

[Maximum mark: 19]

A curve C is given by the implicit equation $x + y - \cos(xy) = 0$.

(a) Show that $\frac{dy}{dx} = -\left(\frac{1 + y\sin(xy)}{1 + x\sin(xy)}\right)$. [5]

(b) The curve $xy = -\frac{\pi}{2}$ intersects C at P and Q.

(i) Find the coordinates of P and Q.

(ii) Given that the gradients of the tangents to C at P and Q are m_1 and m_2 respectively, show that $m_1 \times m_2 = 1$. [7]

(c) Find the coordinates of the three points on C, nearest the origin, where the tangent is parallel to the line $y = -x$. [7]