

## MARKSCHEME

1. (a) **EITHER**

graph of the cubic is shifted horizontally one unit to the right (M1)  
 $\Rightarrow x = -0.796$  A1

**OR**

$(x - 1) = -1.796$  (M1)  
 $x = -0.796$  A1

(b) **EITHER**

stretch factor of 0.5 in the  $x$ -direction (M1)  
 $\Rightarrow 2x = -1.796$  (M1)

**Note:** At least one of the above lines must be seen to award the  $M$  marks.

$\Rightarrow x = -0.898$  A1

**OR**

$8x^3 - 2x + 4 = (2x)^3 - 2x + 4 = 0$  (M1)  
 $\Rightarrow 2x = -1.796$  (M1)

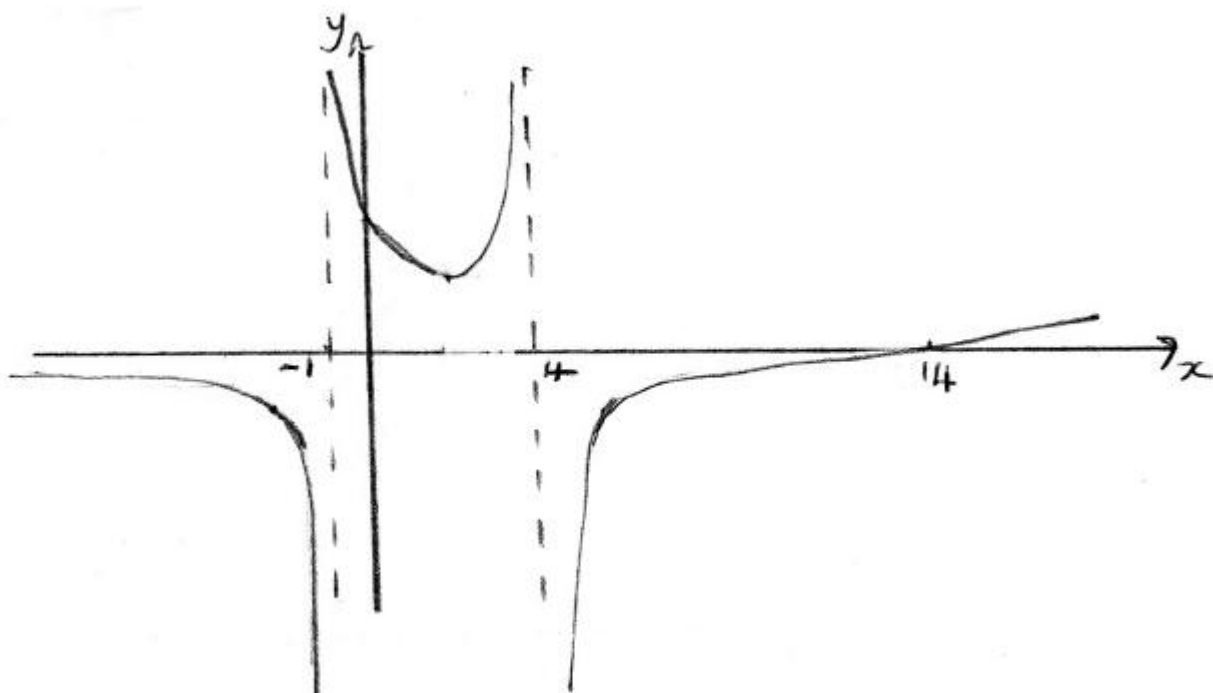
**Note:** At least one of the above lines must be seen to award the  $M$  marks.

$\Rightarrow x = -0.898$  A1

[5]

2. **METHOD 1**

Graph of  $f(x) - g(x)$  M1



A1A1A1

**Note:** Award A1 for each branch.

$$x < -1 \text{ or } 4 < x \leq 14$$

A1A1 N3

**Note:** Each value and inequality sign must be correct.

### METHOD 2

$$\frac{x+4}{x+1} - \frac{x-2}{x-4} \leq 0$$

M1

$$\frac{x^2 - 16 - x^2 + x + 2}{(x+1)(x-4)} \leq 0$$

$$\frac{x-14}{(x+1)(x-4)} \leq 0$$

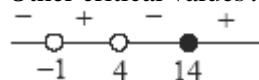
A1

Critical value of  $x = 14$

A1

Other critical values  $x = -1$  and  $x = 4$

A1



$$x < -1 \text{ or } 4 < x \leq 14$$

A1A1 N3

**Note:** Each value and inequality sign must be correct.

[6]

3. (a)  $f: x \mapsto e^x \Rightarrow f^{-1}: x \mapsto \ln x$

$$\Rightarrow f^{-1}(3) = \ln 3$$

A1

$$g: x \mapsto x + 2 \Rightarrow g^{-1}: x \mapsto x - 2$$

$$\Rightarrow g^{-1}(3) = 1$$

A1

$$f^{-1}(3) \times g^{-1}(3) = \ln 3$$

A1 N1

(b) **EITHER**

$$f \circ g(x) = f(x + 2) = e^{x+2}$$

A1

$$e^{x+2} = 3 \Rightarrow x + 2 = \ln 3$$

M1

$$x = \ln 3 - 2$$

A1 N0

**OR**

$$f \circ g(x) = e^{x+2}$$

$$f \circ g^{-1}(x) = \ln(x) - 2$$

A1

$$f \circ g^{-1}(3) = \ln(3) - 2$$

M1

$$x = \ln 3 - 2$$

A1 N0

[6]

4. Attempting to find  $f(2) = 8 + 12 + 2a + b$

(M1)

$$= 2a + b + 20$$

A1

Attempting to find  $f(-1) = -1 + 3 - a + b$

(M1)

$$= 2 - a + b$$

A1

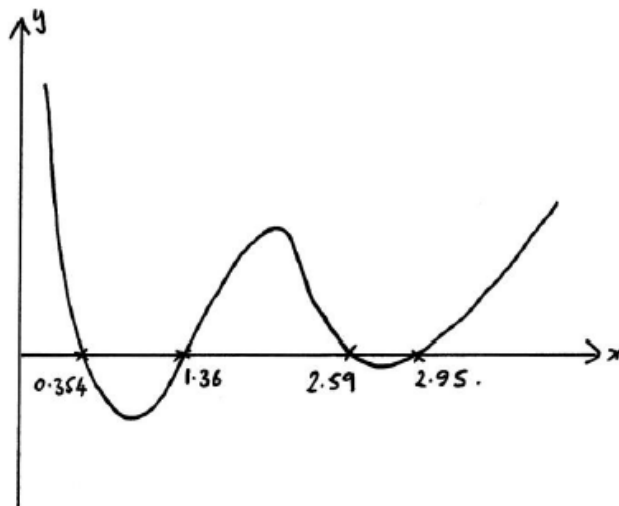
Equating  $2a + 20 = 2 - a$

$$a = -6$$

A1 N2

[6]

5. (a)



A1

**Note:** Award A1 for shape.

$x$  intercepts 0.354, 1.36, 2.59, 2.95

A2

**Note:** Award A1 for three correct, A0 otherwise.

$$\text{maximum} = (1.57, 0.352) = \left(\frac{\pi}{2}, 0.352\right)$$

A1

$$\text{minimum} = (1, -0.640) \text{ and } (2.77, -0.0129)$$

A1

(b)  $0 < x < 0.354, 1.36 < x < 2.59, 2.95 < x < 4$

A2

**Note:** Award A1 if two correct regions given.

[7]

6. (a)  $h(x) = g \circ f(x) = \frac{1}{e^{x^2} + 3}, (x \geq 0)$

(M1)A1

(b)  $0 < x \leq \frac{1}{4}$

A1A1

**Note:** Award A1 for limits and A1 for correct inequality signs.

(c)  $y = \frac{1}{e^{x^2} + 3}$

$$ye^{x^2} + 3y = 1$$

M1

$$e^{x^2} = \frac{1-3y}{y}$$

A1

$$x^2 = \ln \frac{1-3y}{y}$$

M1

$$x = \pm \sqrt{\ln \frac{1-3y}{y}}$$

$$\Rightarrow h^{-1}(x) = \sqrt{\ln \frac{1-3x}{x}} \quad \left( = \sqrt{\ln \left( \frac{1}{x} - 3 \right)} \right)$$

A1

[8]

$$7. \quad (a) \quad \frac{dy}{dx} = 24x^2 + 2bx + c \quad (A1)$$

$$24x^2 + 2bx + c = 0 \quad (M1)$$

$$\Delta = (2b)^2 - 96(c) \quad (A1)$$

$$4b^2 - 96c > 0 \quad A1$$

$$b^2 > 24c \quad AG$$

$$(b) \quad 1 + \frac{1}{4}b + \frac{1}{2}c + d = -12$$

$$6 + b + c = 0$$

$$-27 + \frac{9}{4}b - \frac{3}{2}c + d = 20$$

$$54 - 3b + c = 0 \quad A1A1A1$$

**Note:** Award A1 for each correct equation, up to 3, not necessarily simplified.

$$b = 12, c = -18, d = -7 \quad A1$$

[8]