Assignment

A2

Q. 1. Prove that either graphically or analytically

$$\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$$

2. Prove that $\omega(g(n)) \cap o(g(n)) = \Phi$.

3. Show that for any real constants a and b, where $b > 0$

$(n+a)^b = \Theta(n^b)$

4. Show that

(i) $\log(\lfloor n)) = \Theta(n \log n)$ (ii) $\lfloor n = \omega(2^n)$

(iii) $\lfloor n = o(n^n)$

5. Solve the recurrence $T(n) = 2T(\sqrt{n}) + 1$. Solution should be asymptotically true

6. Express following in terms of Θ notation.

1. $1 + 4 + 9 + 16 + 25 + \cdots$

2. $1.1 + 3.8 + 5.27 + 7.64 + 9.125 \cdots$

7. Consider sorting 'n' numbers in an array by first finding the smallest element of A and exchanging it with A[1], then finding second smallest element and exchanging it with A[2] and continue in this manner for the first n-1 elements. Write a pseudocode of this algorithm. Why does the algorithm need to run for only n-1 elements? Give the best case and worst case running time in terms of Θ notation.

8. Compute the solution of the following recurrence.

1. $T(n) = \begin{cases} T(n-1) + 1 & n \geq 2 \\ 1 & n = 1 \end{cases}$

2. $T(n) = T(n/3) + T(2n/3) + cn$

3. $T(n) = 5T\left(\frac{n}{5}\right) + \frac{n}{\log n}$

4. $T(n) = T(n-1) + \frac{1}{n}$

5. $T(n) = T(n-2) + 2\log n$

6. $T(n) = 13246\, T(\frac{n}{68}) + n^2 \lg$

7. $T(n) = 7T(\frac{n}{2}) + cn^2$

8. $T(n) = \sqrt{2}\, T(\frac{n}{2}) + \lg n$

9. $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

10. $T(n) = T(\frac{n}{2}) + T(\frac{n}{4}) + T(\frac{n}{8}) + n$

11. $T(n) = \sqrt{n}\, T(\sqrt{n}) + n$

9.a. In implementation of a priority queue where following operation takes what amount of time in O representation.

i. insert ii. extractMin - $O(n^2)$ iii. decrease key.

b. what is the run time of Dijkstra's algorithm ~~n the the number of items in a priority~~ using priority queue implementation

10. Given two matrices of A and B of order $n \times n$, compute $C = AB$ where $c_{ij} = \sum_{k=1} a_{ik} b_{kj}$ for every i and j using Strassen's Algo.

11. Find the Complexity of given algo. -

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Euclid-GCD(a, b)
    if b == 0 return a

    else return Euclid-GCD(b, a mod b)
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12. Show that, let x and y be nodes in Breadth first spanning tree belonging to layers L_i and L_j of BFT(S) respectively for $i \le j$, and let $e(x, y)$ be an edge of a graph G. Then $0 \le (i - j) \le 1$

13. Show that, In an undirected graph, each edge joining two node in the same layer in Breadth-first-tree (BFT) can not be Bipartite.

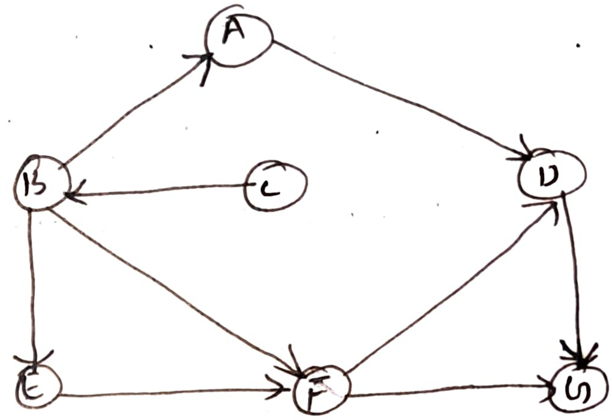
14. with respect to Depth first Traversal (G), define following terms with respect to time stamp.

i. Tree Edge ii. Back Edge iii. Forward Edge

iv. Cross Edge.

15. For a given graph, represent its adjacency matrix and adjacency list representation and then run topological sort. Your answer should consists of a list of the vertices in a particular order.

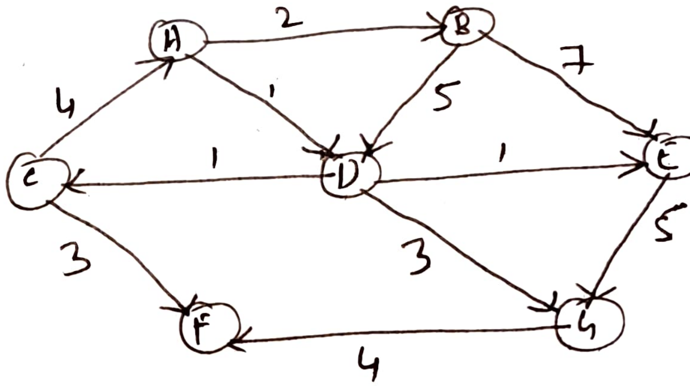
fig-1



16. Show that a graph is DAG iff it does not have back edge.

17. In a given graph, find shortest distance from vertex ~~G~~ 'G' to each vertex.

fig. 2



18. Assume graph of fig-2 is undirected and then find minimum weight spanning tree using prims, kurskal and Reverse Delete algorithm.

19. We have to schedule following lecture, find the minimum number of class room required. Assume time between 9AM to 4:30PM

lecture	a	b	c	d	e	f	g	h	i	j
Start time	9:00	9:00	9:00	11:00	11:00	01:00	01:00	02:15	3:00	3:[illegible]
Finish time	10:30	12:30	10:30	12:30	02:00	02:30	02:30	04:30	4:30	4:3[illegible]

20: For a given sequence analyze which one will be time and space efficient in the case of insertion sort, Quick sort and merge sort.

sequence a: 1, 2, 3, 4, 5

sequence b: 5, 4, 3, 2, 1.

21. Show that the second smallest element can be found with $n + \log n - 2$ comparision in the worst case. where 'n' is total number of elements.