CS 205, Formal Languages, Automata Theory and Computation
Quiz 1 Solutions, Winter 2020-21
Department of CSE, IIT Guwahati

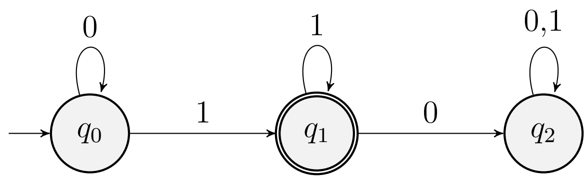
1. Suppose $(S, \leq)$ is a partial order. Denote by $Up(S)$ the set of all non-empty up-closed subsets of S, where a subset A of S is up-closed if $x \in A$ and $x \leq y$ imply $y \in S$.. Let Nat be the set of natural numbers, with $\leq$ the usual "less than or equal" relation. Indicate true or false for the following claim: $Up(Nat)$ a countable set. Justify your answer by either giving an appropriate bijection or argue that such a bijection cannot exist.

> **Solution:** True. The function $f : Nat \to \mathrm{Up}(Nat)$ defined by $f(n) = \{p \in Nat \mid p \geq n\}$ is a bijection.

2. Suppose $(S, \leq)$ is a partial order. Denote by $Up(S)$ the set of all non-empty up-closed subsets of S, where a subset A of S is up-closed if $x \in A$ and $x \leq y$ imply $y \in S$.. Let Rat be the set of rational numbers, with $\leq$ the usual "less than or equal" relation. Indicate true or false for the following claim: $Up(Rat)$ a countable set. Justify your answer by either giving an appropriate bijection or argue that such a bijection cannot exist.

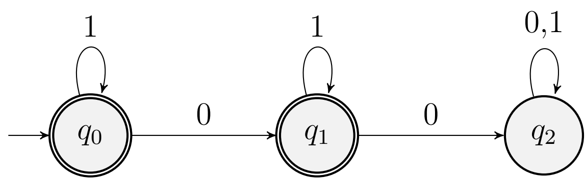
> **Solution:** False. The function $f : Real \to \mathrm{Up}(Rat) \setminus \{Rat\}$ defined by $f(x) = \{p \in Rat \mid p \geq x\}$ is a bijection from the real numbers to a subset of $Up(Rat)$. Since the real numbers form an uncountable set, $Up(Rat)$ is uncountable as well.

3. Describe the language over the alphabet 0,1 accepted by the following finite automaton in English.



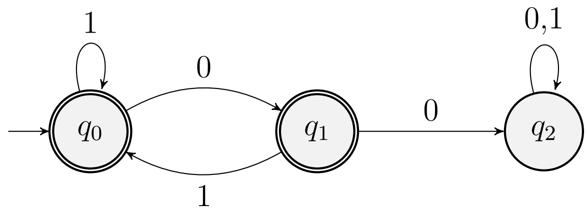
> **Solution:** The set of all strings that begin with a possibly empty block of 0's followed by a non-empty block of 1's.

4. Describe the language over the alphabet 0,1 accepted by the following finite automaton in English.



Solution: The set of all strings with at most one 0.

5. Describe the language over the alphabet 0,1 accepted by the following finite automaton in English.



Solution: The set of all strings such that any two 0's are separated by at least one 1.