CS 205, Formal Languages, Automata Theory and Computation
Quiz 2 Solutions, Winter 2020-21
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1. Consider the following languages

$$L_1 = \{0^i 1^i \mid i \geq 0\} \text{ and } L_2 = \{0^{2k} 1^{2k} \mid k \geq 0\}.$$

Without using the Pumping Lemma or its proof, assuming that L_1 is not regular show that L_2 is also not regular. You can only use results that have been proved in the class.

> **Solution:** Consider the homomorphism $h : \Sigma \to \Sigma^*$ defined by $h(0) = 00$ and $h(1) = 11$. Then $L_1 = h^{-1}(L_2)$. Since regular languages are closed under inverse homomorphisms, if L_2 is regular then so is L_1. Contradiction.

2. Give an example of a language L over some alphabet that is not regular but L* is regular. Justify your answer. You can only use results (including examples of non-regular languages) that have been proved in the class.

> **Solution:** Let $L = \{0^p \mid p \text{ is prime }\}$. We know that L is not regular. Now,
>
> $$L^* = \{0, 1\}^* \setminus \{0\}$$
>
> since every even number is a sum (possibly the empty sum) of primes (since 2 is a prime) and every odd number $n \geq 3$ is an even number plus 3 and hence is also a sum of primes. Also note that 1 is neither a prime nor a sum of primes. Thus L^*, being the set difference of two regular sets, is also regular.

3. Consider the following language over the alphabet $\{0, 1\}$:

$$L = \{0^i 1^j \mid j \geq 0 \text{ and } 0 \leq i \leq 2j\}.$$

Show that L is not regular.

> **Solution:** Suppose L is regular. Let n be the constant in the Pumping Lemma. Let $w = 0^{2n}1^n$. Since $w \in L$ and $|w| \geq 0$, w can be written as xyz where $xy \leq n$, $|y| \geq 0$ and $xy^kz \in L$ for all $k \geq 0$. This implies that $x = 0^r$ and $y = 0^s$ where $|s| \geq 1$. Take $k = 2$. Then $xy^2z = 0^{2n+s}1^n \notin L$ since $s \geq 2$. Contradiction.

4. Given two regular languages L_1 and L_2 over the same alphabet $\{0, 1\}$, show that the following language is also regular:

$$L = \{xy \mid x \in L_1, y \in L_2 \text{ and } |x| - |y| \text{ is even}.$$

Here $|x|$ denotes the length of string x.

Solution: Let $L_e = L(((0+1)(0+1))^*))$ be the set of even length strings over $\{0,1\}$. L_e is a regular language since it is denoted by a regular expression. Then, since $|x| - |y|$ is even iff $|x+y|$ is even,

$$L = L_1 L_2 \cap L_e$$

which is a regular language, since regular languages are closed under concatenation and intersection.