& Computation

CS 205

Formal Languages, Automata

Theory & Computation

(1) Course Web Page : link from

my home page

Text : - Hopcroft , Motwani & Ullman
2007

- Hopcroft & Ullman 1979

Grading Policy : 60% Quizzes
40% HW

Interactions : – Teams (preferred)
– Email responses will take more time

Doubts / Queries ?

Q : What is the course about ?

A : Fundamental capabilities & limitations of computers.

– Models of computation

Automata : computers with constrained

memories

Finite Automata → Pushdown automata → Turing Machines

Language acceptance

→ set of strings / words / finite sequences of symbols

Complexity theory : Computations with bounded space & time

Cardinality of Sets

Q : When do two sets A and B have the same size?

the same size ;

A : When there is a bijection between them

$f : A \to B$ injective (one-to-one)

surjective (onto)

$\forall x \forall y \in A$

<u>injective</u> : $f(x) = f(y) \Rightarrow x = y$

$x \neq y \Rightarrow f(x) \neq f(y)$

<u>Surjective</u> $\forall y \in B, \exists x \in A$

$f(x) = y$

<u>Q</u> (1) $\mathbb{N} = \{0, 1, 2, \ldots\}$

$\mathbb{E} = \{0, 2, 4, \dots\}$

Same size ?

$f : \mathbb{N} \to \mathbb{E} \quad f(x) = 2x$

(2) $\mathbb{N}$ and $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

$$\boxed{\mathbb{Z} \quad 0, -1, 1, -2, 2, -3, 3, \dots}$$

$$\mathbb{N} \quad 0, 1, 2, 3, 4, 5, 6$$

- Denumerable / Countably infinite
- countable : finite or countably

infinite

Q: $\mathbb{N} \times \mathbb{N} = \{(i,j) \mid i, j \in \mathbb{N}\}$

Denumerable ?

$$\begin{array}{cccc}
(0,0) & (0,1) & (0,2) & \cdots \\
(1,0) & (1,1) & (1,2) & \cdots \\
(2,0) & (2,1) & (2,2) &
\end{array}$$

$\underbrace{(0,0)}, \underbrace{(1,0), (0,1)}, \underbrace{(2,0), (1,1), (0,2)}_{uoo}$

A

h

$\mathbb{Q}$ = set of rat no.s

rat no — of the form p/q $\quad p, q$ integers $q \neq 0$

$$\begin{array}{ccccc} 0/1 & 1/1 & 2/1 & 3/1 & \cdots \\ \cancel{0/2} & 1/2 & 2/2 & 3/2 & \cdots \\ 0/3 & 1/3 & 2/3 & & \end{array}$$

Real no.s. $\mathbb{R}$

Denumerable ?

No.

Cantor: Diagonalization Technique

$(0,1)$: set of all reals x

$0 < x < 1$

not countable

Proof by contradiction