CS 205: Formal Languages, Automata Theory and Computation
Homework # 1, Winter 2020-21
Due Date: End of Wednesday 10-02-2021

Important

1. Typeset your answers using LaTeX or MS Word. Upload a pdf file to TurnItIn as your submission. Also submit a copy to Teams, so that we can assign marks.
2. Identical answers by two students on the same problem will incur zero marks for both students for the problem.
3. Copying answers from the Internet will also be penalized by awarding zero marks.
4. Turnitin will be used to detect all types of copying. You must submit your answers in Turnitin.
5. Include your name and roll number at the top of your answer script.
6. Late submissions will incur 10% deduction for each day of delay from the total marks obtained.

1. Let the alphabet Σ be the set of all column vectors of length three where each entry is a bit. We consider each string $w \in \Sigma^*$ as containing three rows, where each row is just the concatenation of the corresponding rows from each symbol in Σ. Then we consider each row in the string w as a binary number shown from the least significant bit on the left to the most significant bit on the right, *i.e.*, just the reverse of the way numbers are usually written. For example, the string

$$\begin{bmatrix}0\\1\\0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

is viewed as consisting of the rows of the column vector

$$\begin{bmatrix}011\\110\\001\end{bmatrix}$$

where the three rows represent the binary numbers 110, 011 and 100 from top to bottom. Consider the following language

$$L = \{w \in \Sigma^* \mid \text{ the bottom row of } w \text{ is the sum of the other two rows }\},$$

where we ignore a carry out of the most significant bit. Show that L is regular.

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2. Let L be a regular language over the alphabet $\Sigma = \{0, 1\}$. Define a new language L' over the same alphabet obtained from L as follows:

$$L' = \{w \mid w \in L \text{ and } w \text{ contains an even number of 0's }\}.$$

Show that L' is regular.

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