CS 205
Lecture 21
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PDA $\mathcal{P} = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Q: states; Σ: input alphabet; Γ: stack alphabet; δ: trans. fn.; q_0: init. state; Z_0: initial stack sym.; F: final states

$$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \to \underbrace{\mathcal{P}_f}_{\text{set of finite subsets of}} (Q \times \Gamma^*)$$

Meaning of
$$\delta(q, a, Z) = \{\langle p_1, \gamma_1\rangle, \cdots, \langle p_k, \gamma_k\rangle\}$$

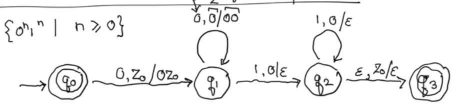
q: current state; a: input sym; Z: TOS; p_i: new state; γ_i: String to replace Z at TOS

Similarly $\delta(q, \varepsilon, Z) = \{\cdots\}$

input, TOS Z, γ string to replace Z: $\quad 0, 0/00$

Ex $\quad L = \{0^n 1^n \mid n \geqslant 0\}$

PDA $\mathcal{P}$



$\delta(q_0, 0, Z_0) = \{\langle q_1, 0Z_0\rangle\}$

$\delta(q_0, 1, Z_0) = \emptyset$

$\vdots$

Instantaneous Description (ID) : configuration / snapshot of a PDA

Three-tuple $\langle q, w, \gamma\rangle$ — q: current state; w: remaining input; γ: stack contents

Move of a PDA

$$\underbrace{\langle q, aw, Z\alpha\rangle}_{ID_1} \vdash \underbrace{\langle p, w, \beta\alpha\rangle}_{ID_2}$$

$q, p \in Q$
$a \in \Sigma \quad w \in \Sigma^*$
$Z \in \Gamma \quad \alpha \in \Gamma^*$
$\beta \in \Gamma^*$

iff

$$\langle p, \beta\rangle \in \delta(q, a, Z)$$

$\vdash^*$: reflexive, transitive closure of $\vdash$

Prop: For a PDA $\mathcal{P}$, if $\langle q, x, \alpha\rangle \vdash^* \langle p, y, \beta\rangle$ then for any $w \in \Sigma^*$ and $\gamma \in \Gamma^*$ we have
$$\langle q, xw, \alpha\gamma\rangle \vdash^* \langle p, yw, \beta\gamma\rangle.$$

Language accepted by a PDA $\mathcal{P} = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Two notions of acceptance :

(1) By final state : $L(\mathcal{P}) = \{w \in \Sigma^* \mid \exists p \in F.\ \exists \alpha \in \Gamma^*.\ \underbrace{\langle q_0, w, Z_0\rangle}_{\text{initial ID}} \vdash^* \langle p, \varepsilon, \alpha\rangle\}$

p: final state; ε: entire input has been read

(2) By empty stack : $N(\mathcal{P}) = \{w \in \Sigma^* \mid \exists p \in Q.\ \langle q_0, w, Z_0\rangle \vdash^* \langle p, \varepsilon, \varepsilon\rangle\}$

ε: no more input; ε: $\boxed{\text{empty stack}}$ — Not Z_0!

Equivalence of notions of acceptance

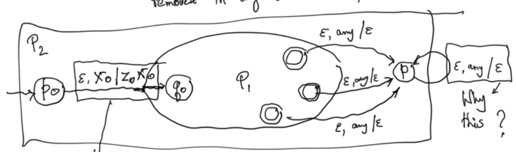
Thm 1. If $L = L(\mathcal{P}_1)$ for some PDA $\mathcal{P}_1$ then there is a PDA $\mathcal{P}_2$ s.t. $L = N(\mathcal{P}_2)$.

2. If $L = N(\mathcal{P}_1)$ for some PDA $\mathcal{P}_1$ then there is a PDA $\mathcal{P}_2$ s.t. $L = L(\mathcal{P}_2)$.

Final state to empty stack

Idea Let $L = L(\mathcal{P}_1)$. Construct a PDA $\mathcal{P}_2$ s.t. $L = N(\mathcal{P}_2)$ as follows.

- $\mathcal{P}_2$ simulates $\mathcal{P}_1$ on the input and when $\mathcal{P}_1$ enters a final state, $\mathcal{P}_2$ empties its stack.
  → $\boxed{\text{Will not work.}}$ Why? ($\mathcal{P}_1$ can empty its stack after reading the entire input and reaching a non-final state)
- Fix : $\mathcal{P}_2$ will use a marker X_0 on the bottom of its stack, which will never be removed in any simulation of $\mathcal{P}_1$.



Why ?

Empty stack to final state