CS 205

Lecture 2 6/1/21

Thm : The set $(0,1) = \{x \mid x \in \mathbb{R} \text{ and } 0<x<1\}$ is uncountable.

Proof : By contradiction.

$r_0, r_1, r_2, \ldots$

Every $x \in (0,1)$ can be expressed as an infinite decimal expansion

$0 . d_0 d_1 d_2 \ldots$

$1/3 = .333\ldots$

$\frac{3}{10} = .~~3000~~$

$= .2999\ldots$ ✓

0	$. d_{00}\ d_{01}\ d_{02} \ldots$
1	$. d_{10}\ d_{11}\ d_{12} \ldots$
2	$. d_{20}\ d_{21}\ d_{22} \ldots$
n	$. d_{n0}\ d_{n1}\ d_{n2} \ldots$
i	$. d_{ii}$

Diagonalization
Cantor

$r = . d_{00}\, d_{11}\, d_{22} \ldots d_{nn} \ldots$

$r' = . d'_{00}\, d'_{11}\, d'_{22} \ldots d'_{nn}$

Show

(1) The set of infinite sequences over $\{0,1\}$ is uncountable.

✓(2) The set of finite sequences over $\{0,1\}$ is countable.

(3) For any set S, there is no bijection between S and 2^S.

Q : The set of programs in a programming language : countable

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Q : The set of all fn. from $\mathbb{N} \to \{0,1\}$.
$\mathbb{N} \to \mathbb{N}$

Uncountable

Alphabets and Strings / Words

Alphabet : a finite n.e. set Σ of symbols / letters

Binary alphabet $\Sigma = \{0,1\}$

Roman letters $\Sigma = \{a, b, \ldots, z$

String : Finite sequence of symbols of Σ

denote u, v, w

e.g. 010011

abcda

empty sequence : ϵ

Length of a string w: $|w|$ no of positions in the string

$|\epsilon| = 0$

$|abc| = 3$

Concatenation : $u . v$ or sometimes uv

Σ alphabet

$\Sigma^* \triangleq$ set of all finite strings over Σ

~ infinite set

Ex $\Sigma = \{0,1\}$

$\Sigma^* = \{\epsilon, 0, 1, 00, 01, 10, 11, \ldots\}$

$\Sigma^+ \triangleq$ set of all finite n.e. strings over Σ

$\triangleq$ def
definition

$\Sigma^* \setminus \{\epsilon\}$ $\quad \setminus \to$ set difference

$\Sigma^* - \{\epsilon\}$

Language L over an alphabet Σ : $L \subseteq \Sigma^*$

i.e. a language is a set of strings over Σ

$\Sigma = \{0,1\}$

Ex (1) $L_1 = \{0, 10, 110, 1110, 11110, \ldots\}$

(2) $L_2 = \{w \in \Sigma^* \mid w \text{ ends with a } 0\}$

(3) $L_3 = \{w \in \Sigma^* \mid w \text{ is a palindrome}\}$

$= \{\epsilon, 0, 1, 00, 11, 010, 000, 111, 101, \ldots\}$

(4) $L_4 = \{w \in \Sigma^* \mid w \text{ contains an equal no. of 0's and 1's}\}$

Concatenation of languages

$L_1 L_2 = \{uv \mid u \in L_1, v \in L_2\}$

$L^n = \underbrace{L . L . \ldots L}_{n \text{ times}}$ for $n \in \mathbb{N}$

Inductively $L^0 = \{\epsilon\}$

$L^{n+1} = L^n . L$

Ex $L = \{\epsilon, 01, 10\}$

$L^0 = \{\epsilon\}$

$L^1 = L$

$L^2 = \{\epsilon, 01, 10, 0101, 0110, 1001, 1010\}$

$L^3 = \{\ldots\}$

ϵ is the unit of concatenation

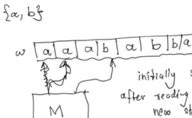
Monoid $(\Sigma^*, \cdot, \epsilon)$

Deterministic Finite Automaton (DFA)

$M = (Q, \Sigma, \delta, q_0, F)$ five-tuple

- Q = finite set of states
- Σ = finite alphabet
- $\delta : Q \times \Sigma \to Q$ transition function
- q_0 : initial state $q_0 \in Q$
- $F \subseteq Q$: set of final / accepting states

$\Sigma = \{a, b\}$



input tape

initially state $= q_0$

after reading 1st symbol new state $= \delta(q_0, a)$