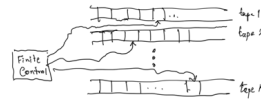


TM Programming Techniques & TM Variants

Multitape TM



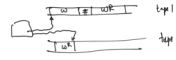
- k tapes with a head for each tape
- move: for each state and the symbol scanned by each head
 - new state
 - new symbol for each tape
 - head motion for each tape

Utility

Ex: $L = \{w \in \{0,1\}^* \mid w \in \{0,1\}^*\}$

Single tape TM: move back and forth
No. of steps: $O(n^2)$

Multitape TM: Copy the portion after ϵ on the second tape
(2 tapes)



No. of steps: $O(n)$

Many tapes & single tape simulation

To simulate TM M with k tapes

- Use TM M' with a single tape with $2k$ tracks
- one track for each tape
- another track for a mark $\#$ representing the head position



To simulate one move of the multitape TM M ,

1. M' moves left and then right visiting all the $\#$'s to see what each tape head in M is scanning
2. Decides on M 's move based on symbols being scanned and its current state (remember in the state of the TM M')
3. Visits each $\#$ again, making necessary changes

Simulation of n moves of M requires $O(n^2)$ moves by M' .

Non-deterministic TM (NTM)

Deterministic TM: $\delta: Q \times \Sigma \rightarrow Q \times \Sigma \times \{L, R\}$

Non-deterministic $\delta: \Sigma \rightarrow 2^{Q \times \Sigma \times \{L, R\}}$

$\delta(q, x) = \{(q_1, Y_1, D_1), \dots, (q_k, Y_k, D_k)\}$

- δ defined unambiguously

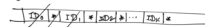
- NTM N accepts w if there is a sequence of moves leading to an accept state:
 $q_0 w \vdash^* w' g$ where $g \in F$

Thm If L is accepted by an NTM N then there is a DTM M st. $L = L(M)$.

Proof (idea)

The DTM M is a multitape TM with two tapes.

Tape 1: Holds a queue of IDs of N , separated by $\#$



Initially: ϵ

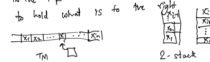
- When an ID reaches the front of the queue M finds all its successor IDs and adds them to the back of the queue using the second tape as scratch
- M accepts if an ID with an accepting state is the current ID
- Because of the queue discipline M reaches every ID eventually

Pushdown Machine

PDAs with more than one stack

- Fact
1. Any PDA can be simulated by a TM
 2. Any PDA with one stack cannot simulate an arbitrary TM.
 3. Any PDA with two stacks can simulate an arbitrary TM.

Thm: Use one stack to hold what is in the tape head and the other to hold what is in the right



Counter Machine

Think of M as

1. Restricted multi-stack machine
 - only two stack symbols: Z_0 and X
 - stack is always of the form $X^n \dots X Z_0$
2. Each stack is a counter
 - holds a nat. no.
 - can only distinguish between zero and non-zero values
 - operations: add 1, subtract 1, and test-if-0

Fact

1. Every language accepted by a k -counter m/c is a CFL accepted by a PDA.
2. 2-counter m/c's have the same power as TMs.

Church-Turing Thesis

"Anything computable via a mechanical procedure can be done on a TM."

- Not a mathematical statement.

— Evidence: (1) Most extensions to the TM model

- Multitape TM
- Nondet. TM
- Multidimensional TM

can be simulated by a TM

- (2) λ -calculus, RAM, General Recursive functions, General grammar, For rewriting, ... are all equivalent to the TMs.