CS 205

Lecture 9

27/01/21

Recap : Regular expressions over an alphabet Σ

RE r	$L(r)$
$\emptyset$	$\emptyset$
ε	$\{\varepsilon\}$
$a \quad (a \in \Sigma)$	$\{a\}$

If r_1, r_2 are REs denoting sets R_1 and R_2 then

$(r+s)$	$R_1 \cup R_2$
(rs)	$R_1 R_2$
(r_1^*)	R_1^*

Thm If $L = L(A)$ for some DFA, then there is a r.e. r s.t. $L = L(r)$.

Proof : Let $A = (Q, \Sigma, \delta, q_1, F)$

Where $Q = \{q_1, \dots, q_n\}$. $\quad F \subseteq Q$

states have indices $\leq K$.

$\boxed{\text{Let } R_{ij}^{K} \text{ denote the set of all strings } w \text{ which take the DFA } A \text{ from } q_i \text{ to } q_j \text{ w/o passing through any state } q_l \text{ whose index } l > K.}$

$$L(A) = \bigcup_{q_j \in F} R_{1j}^{n}$$

If $q_j \in F$

We define R_{ij}^{K} inductively as follows:

Base Case

$$R_{ij}^{0} = \begin{cases} \{a \mid \delta(q_i, a) = q_j\} & \text{if } i \neq j \\ \{a \mid \delta(q_i, a) = q_j\} \cup \{\varepsilon\} & \text{if } i = j \end{cases}$$

Inductive Case

$$R_{ij}^{K} = R_{ij}^{K-1} \cup R_{iK}^{K-1} \cdot (R_{KK}^{K-1})^* \cdot R_{Kj}^{K-1}$$

R_{ij}^{K-1}

We show that for each i, j and K there exists a r.e. r_{ij}^{K} which denotes the language R_{ij}^{K} by induction on K.

Basis $(K=0)$ R_{ij}^{0} is a finite set of strings each of which is either ε or a single symbol.

Then $r_{ij}^{0} = a_1 + a_2 + \dots + a_p$ when $i \neq j$, $R_{ij}^{0} = \{a_1, \dots, a_p\}$

Or

$a_1 + a_2 + \dots + a_p + \varepsilon$ when $i = j$, $R_{ij}^{0} = \{a_1, \dots a_p, \varepsilon\}$

Ex 

no transition from q_i to q_j

i.e. for no $a \in \Sigma$, $q_j = \delta(q_i, a)$

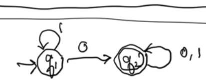
$R_{ij}^{0} \overset{?}{=} \emptyset$

$r_{ij} \overset{?}{=} \emptyset$

phi	$\phi \quad \varphi$
empty set	$\emptyset$

Inductive Step

$$\boxed{r_{ij}^{K} = r_{ij}^{K-1} + r_{iK}^{K-1} \cdot (r_{KK}^{K-1})^* \cdot r_{Kj}^{K-1}}$$

Ex DFA A 

$K = 0$

$r_{11}^{0} = 1 + \varepsilon$

$r_{12}^{0} = 0$

$r_{21}^{0} = \emptyset$

$r_{22}^{0} = 0 + 1 + \varepsilon$

$K = 1$

$r_{11}^{1} = r_{11}^{0} + r_{11}^{0} \cdot (r_{11}^{0})^* \cdot r_{11}^{0} = (1+\varepsilon) + (1+\varepsilon)(1+\varepsilon)^*(1+\varepsilon)$

$r_{12}^{1} = r_{12}^{0} + r_{11}^{0} \cdot (r_{11}^{0})^* \cdot r_{12}^{0}$

$r_{21}^{1} = \dots$

$r_{22}^{1} = \dots$

RE for $L(A)$? $\quad r = r_{12}^{2}$