CS 205, Formal Languages, Automata Theory and Computation
Quiz 2 Solutions, Winter 2020-21
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1. Consider the regular language L denoted by the regular expression $0^*10^*1(0+1)^*$. Enumerate and describe the equivalence classes of the relation R_L, where R_L is the relation used in the statement of the Myhill-Nerode theorem.

Solution: The language L is the set of all strings with at least two 1's. The equivalence classes of R_L are described by the following regular expressions.

$$C_1 = 0^* \text{ } i.e., \text{ all strings with no 1's}$$
$$C_2 = 0^*10^* \text{ } i.e., \text{ all strings with exactly one 1}$$
$$C_2 = 0^*10^*1(0+1)^* \text{ } i.e., \text{ all strings with two or more 1's}$$

2. State whether the following statement is true or false with proper justification: If A and B are two regular languages over the alphabet $\{0,1\}$ such that R_A is a refinement of R_B then A must be a subset of B. You must either give a proof if the answer is true or a counterexample with two concrete languages A and B if it is false. Here R_L is the equivalence relation in the statement of the Myhill-Nerode theorem.

Solution: False. Consider the following languages

$$A = \text{ set of all strings with an even number of 0's, and}$$
$$B = \text{ set of all strings with an odd number of 0's.}$$

Then R_A and R_B are identical equivalence relations and hence refine each other. But neither language is contained in the other.

3. Prove that the following language over the alphabet $\{0,1\}$

$$L = \{w \mid w \text{ can be written as } w = x1x \text{ for some } x \in \{0,1\}^*\}$$

is not regular by using the Myhill-Nerode theorem.

Solution: We show that the relation R_L has infinite index. For all pairs i, j where $i \neq j$, the strings 0^i1 and 0^j1 are in distinct equivalence classes of R_L because $0^i10^i \in L$ whereas $0^j10^i \notin L$. Hence the number of equivalence classes of R_L is infinite. By the Myhill-Nerode theorem L is not regular.

4. Write a CFG for the following language over the alphabet $0,1$: The set of all palindromes with an even number of 0's.

Solution:

$$S \to \epsilon \mid 1 \mid 0S0 \mid 1S1$$