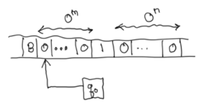
CS 205
Lecture 29
24/3/21

Computing partial functions $f: \mathbb{N} \to \mathbb{N}$ with TM M

- Number i represented as 0^i
- If $q_0 0^i \vdash^* q_h 0^j$ then $M(i) = j$.
  q_h: special halting state
- Multiple arguments and outputs:
  - $\langle i_1, i_2, \ldots, i_n \rangle$ represented by $0^{i_1} 1 0^{i_2} 1 \ldots 1 0^{i_n}$ (the 1s are separators)

Addition



Want to reach the ID $q_h 0^{m+n}$

- Change the 1 to a 0.
- Erase the last 0. (If the input is not of the right form then halt)
- Move to the leftmost symbol, change state to q_h, and halt.

Q: How to do subtraction, multiplication

Programming Techniques for TMs

(1) Storing in Finite Control

Ex: $L = 01^* + 10^*$

TM M: remembers the first symbol read in its state

$Q = \{q_0, q_1\} \times \{0, 1, B\}$ → remembers the 1st symbol

q_0: first symbol has not been read
q_1: 1st sym has been read

Initial state: $\langle q_0, B \rangle$

Single Final state: $\langle q_1, B \rangle$

$\boxed{\delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\} \text{ is a partial fn.}}$

$\delta(\langle q_0, B\rangle, a) = (\langle q_1, a\rangle, a, R)$ $\quad a \in \{0,1\}$

$\delta(\langle q_1, a\rangle, a') = (\langle q_1, a\rangle, a', R)$ $\quad$ where $a, a' \in \{0,1\}$, $a' \neq a$

$\delta(\langle q_1, a\rangle, B) = (\langle q_1, B\rangle, B, R)$

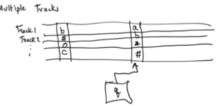
(2) Shift Symbols : Move the entire tape contents K cells to the right

Solution: Remember the last K symbols in finite control

- States: $\{q_0\} \times \Gamma^K$
- Initial state: $(q_0, \langle B, B, \ldots, B\rangle)$
- $\delta((q_0, \langle a_1, \ldots, a_i, B, B, \ldots, B\rangle), a) = ((q_0, \langle a_1, a_2, \ldots, a_i, a, B, \ldots B\rangle), B, R)$

  where $a_j \neq B$ and $a \neq B$
- $\delta((q_0, \langle a_1, \ldots, a_K\rangle), b) = ((q_0, \langle a_2, \ldots a_K, b\rangle), a_1, R)$
- When M finishes reading the tape, it writes everything in the finite control and accepts.

(3) Multiple Tracks



Problem: Marking tape cells

Ex $L = \{ wcw \mid w \in \{0,1\}^* \}$.

TM M with two tracks – for the input date and for the mark

Idea:
- Read a symbol in the left portion
- Remember what it is
- Mark that it has been read
- Move to the corresponding position in the right portion
- Check if the symbol is the same
- Mark this symbol as well

States: $Q \times \{0, 1, B\}$

Tape Alphabet: $\Gamma = \Sigma \cup (\{0,1,c,B\} \times \{B, *\}) \cup \{B\}$ ($*$: mark)

Blank symbol $\langle B, B\rangle$

- First rewrite the input $a_1 a_2 \ldots a_K c\, b_1 b_2 \ldots b_\ell$ as $\langle a_1, B\rangle \langle a_2, B\rangle \ldots \langle a_K, B\rangle \langle c, B\rangle \langle b_1, B\rangle \ldots \langle b_\ell, B\rangle$
- ...

(4) Subroutines

- Useful to think of TMs as a collection of interacting components / subroutines
- M_1 calls M_2 by passing control to the initial state of M_2

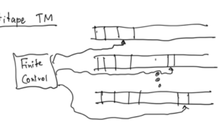
Example: Multiplication

TM M starts with $0^m 1 0^n 1$ and must end with 0^{mn} on its tape.

Idea
Repeatedly copy the block 0^n to the right and erase one zero from the first block

- During the computation the tape will have $0^i 1 0^n 1 0^K$ where 0^K represents the partial answer
- Subroutine → $\boxed{\text{Basic step: M erases the left most 0, copies the middle sequence of 0's to the right end.}}$
- New tape: $0^{i-1} 1 0^n 1 0^{K+n}$
- When all the 0's in the first group are erased, the tape has $1 0^n 1 0^{mn}$.
- Finally, M erases 10^n1.

(5) Multitape TM



- Input is on Tape 1
- Cells in Tapes 2, ..., K are initially blank
- One step of computation:
  - read sym under each of the K heads
  - write new symbols on each tape
  - move the tape heads
  - change state