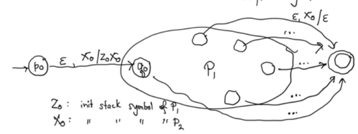
CS 205
Lecture 22
8/2/21

Equivalence of
two notions of acceptance by PDAs

✓(1) Final state to empty stack
(2) Empty stack to final state

Let $L = N(P_1)$ for PDA P_1
↑ accepted by empty stack

Construct PDA P_2 s.t. $L = L(P_2)$ as follows
↑ accepted by final state



Z_0 : init stack symbol of P_1
X_0 : " " " " P_2

CFGs and PDA

Thm : The following are equivalent
(1) L is a CFL.
(2) $L = N(P_1)$ for some PDA P_1.
(3) $L = L(P_2)$ " " " P_2.

already shown

We will show
(1) For every CFG G there is a PDA P s.t. $L(G) = N(P)$.
(2) For every PDA P there is a CFG G s.t. $N(P) = L(G)$.

accept by empty stack

(1) From CFG to PDA

Idea : PDA P simulates the leftmost derivations of G

Ex $S \to aAB$, $A \to C \mid a$, $B \to b \mid A$, $C \to bBd$

$\omega = abbdb$

PDA P has a single state and accepts by empty stack.

Input Left	Stack	Derivation
abbdb	S	S
abbdb	aAB	aAB
bbdb	AB	—
bb db	CB	aCB
bbdb	bBdB	ab BdB
bdb	BdB	—
bdb	bdB	abbdB
db	dB	—
b	B	—
b	b	abbdb
ε	ε	—

Formal Construction

Let $G = (V, T, R, S)$ be a CFG.
The PDA $P = (\{q\}, T, V \cup T, \delta, q, S)$

T ↑ inp. alph.; $V \cup T$ ↑ stack alph.; S ↑ Init stack sym

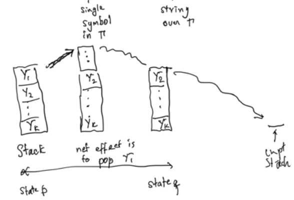
where δ is defined by
1. For each variable A, $\delta(q, \varepsilon, A) = \{(q, \beta) \mid A \to \beta$ is a prodn. in $R\}$
2. For each terminal a, $\delta(q, a, a) = \{(q, \varepsilon)\}$

From PDA to CFG

Observation : In acceptance by empty stack a fundamental event in the execution of the PDA is the "net popping" of a symbol from the stack and the associated change in state.

Single step: $\delta(q, a, Z) = \{(p_1, \gamma_1), \dots\}$

Z ↑ single symbol in Γ; γ_1 ↑ string over Γ



Formal Construction

Let $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0)$. Then $G = (V, \Sigma, R, S)$
where
- S is the start symbol
- the other variables are of the form $[pXq]$ where $p, q \in Q$ and $X \in \Gamma$.

The productions in R are given by
- $S \to [q_0 Z_0 p]$ for every $p \in Q$
- If $(p_1, Y_1 \dots Y_k) \in \delta(p, a, X)$ where a is possibly ε and k is possibly zero, then for every choice of $p_2, \dots, p_{k+1} \in Q$ include the production

$$\underbrace{[p X p_{k+1}]}_{\text{net effect of popping } X} \to a[p_1 Y_1 p_2][p_2 Y_2 p_3] \dots [p_k Y_k p_{k+1}]$$

$\boxed{\text{Prop} \quad [pXq] \overset{*}{\Rightarrow} \omega \text{ iff } (p, \omega, X) \vdash^{*} (q, \varepsilon, \varepsilon)}$