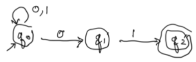
CS 205
Lecture # 7
20/01/21

Ex : Subset construction

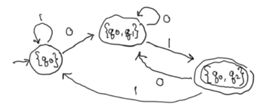
NFA N :



DFA M equivalent to N ; Set of states
$Q_M = 2^{\{q_0, q_1, q_2\}}$

"On-the-fly" construction

DFA M



Unreachable States ϕ, $\{q_1\}$, $\{q_2\}$, $\{q_1, q_2\}$, $\{q_0, q_1, q_2\}$

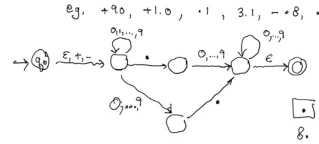
Finite automata with ϵ-transitions

ϵ-NFA : change state w/o reading an input symbol

Ex : ϵ-NFA that accepts decimal nos

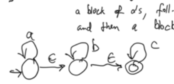
eg. +9.0, +1.0, ·1, 3.1, −·8, ...

(1)



$\boxed{\cdot}$ ×
8. ✓

(2) $\Sigma = \{a, b, c\}$ L = set of strings ~~...~~ of the form a block of a's, foll. by a block of b's and then a block of c's



Def : An NFA with ϵ-transitions is a tuple

$A = (Q, \Sigma, \delta, q_0, F)$ where

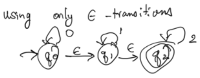
$\delta : Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^Q$

Can extend δ to $\hat{\delta} : Q \times \Sigma^* \rightarrow 2^Q$

Need to define a new concept :

Def : The ϵ-closure of a state q : the set of states that can be reached from q using only ϵ-transitions

Ex



$\delta(q_0, \epsilon) = \{q_1\}$
$\hat{\delta}(q_0, \epsilon) = \{q_0, q_1, q_2\}$

$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$

Def : Inductive defn. of ϵ-closure(q)

(i) $q \in \epsilon\text{-closure}(q)$
(ii) If $p \in \epsilon\text{-closure}(q)$ and $r \in \delta(p, \epsilon)$ then $r \in \epsilon\text{-closure}(q)$

Defn Extended transition fn. $\hat{\delta} : Q \times \Sigma^* \rightarrow 2^Q$ is defined inductively as follows :

(1) $\hat{\delta}(q, \epsilon) = \epsilon\text{-closure}(q)$
(2) $\hat{\delta}(q, wa) = \bigcup_{\substack{p \in \delta(r, a) \\ r \in \hat{\delta}(q, w)}} \epsilon\text{-closure}(p)$ $\quad w \in \Sigma^*$, $a \in \Sigma$

Note In general $\hat{\delta}(q, a) \neq \delta(q, a)$ unlike for DFAs & NFAs
and $\hat{\delta}(q, \epsilon) \neq \delta(q, \epsilon)$

Def : The language accepted by an ϵ-NFA $A = (Q, \Sigma, \delta, q_0, F)$ is

$L(A) = \{w \in \Sigma^* \mid \hat{\delta}(q, w) \cap F \neq \emptyset\}$

Equivalence of ϵ-NFAs and DFAs

Prop : Given any ϵ-NFA E we can find a DFA D s.t. $L(E) = L(D)$.

Proof : Let $E = (Q_E, \Sigma, \delta_E, q_{0E}, F_E)$ be an ϵ-NFA.

Define a DFA $D = (Q_D, \Sigma, \delta_D, q_{0D}, F_D)$ as follows :

$Q_D = 2^{Q_E}$ (but the only reachable/accessible states are ϵ-closed ones)

$q_{0D} = \epsilon\text{-closure}(q_{0E})$

$F_D = \{S \mid S \subseteq Q_E \text{ and } S \cap F_E \neq \emptyset\}$

$\delta_D(S, a) = \bigcup_{\substack{r \in \delta_E(s, a) \\ s \in S}} \epsilon\text{-closure}(r)$

Claim $L(D) = L(E)$

Proof : We show $\hat{\delta}_E(q_{0E}, w) = \hat{\delta}_D(q_{0D}, w)$ for all $w \in \Sigma^*$ by induction on $|w|$.