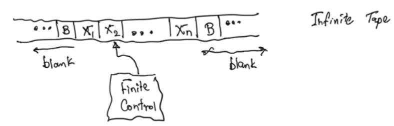
CS 205
Lecture 27
22/3/21

Turing Machines & Computability

Machines encountered so far : DFAs, NFAs, PDAs
Limited capability
$L = \{a^i b^i c^i \mid i \geq 0\}$

Q : What is the most general computing model ?
A : Turing Machine



Depending on the current state & current scanned symbol
the m/c does the following :
- changes its state
- overwrites the current tape cell
- move the tape head left or right

Formal Definition

A TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

- Q — finite set of states
- Σ — input alphabet
- Γ — tape alphabet
- δ — transition fn.
- q_0 — initial state
- B — blank symbol
- F — accepting states $F \subseteq Q$

$\Sigma \subseteq \Gamma$
$B \in \Gamma$, $B \notin \Sigma$

δ is a partial function

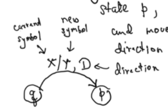
Deterministic TM

$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$

- Q (left) — current state
- Γ (left) — tape symbol being read
- Q (right) — next state
- Γ (right) — symbol to be written
- $\{L, R\}$ — direction in which the tape head moves

$\delta(q, X) = (p, Y, D)$: when in state q and reading symbol X the m/c will move to state p, overwrite X with Y and move its tape head in direction $D \in \{L, R\}$.

Depicted

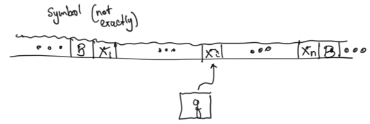


Instantaneous Description (ID)

Captures the "current state of the computation"

ID : $\boxed{X_1 X_2 \ldots X_{i-1}\, q\, X_i \ldots X_n}$

- X_1 — leftmost non-blank symbol (not exactly)
- q — current state
- X_i — scanning the i^{th} symbol X_i
- X_n — rightmost non-blank symbol (not exactly)



Moves of TMs

Left move (1) If $\delta(q, X_i) = (p, Y, L)$ then
(new state p, new symbol Y, move left L)

$X_1 X_2 \ldots X_{i-1}\, q\, X_i\, X_{i+1} \ldots X_n$
$\vdash$
$X_1 X_2 \ldots X_{i-2}\, p\, X_{i-1}\, Y\, X_{i+1} \ldots X_n$

$ID_1 \vdash ID_2$
one move of the TM

Boundary Cases

- If $i = 1$: $q X_1 X_2 \ldots X_n \overset{?}{\vdash} p B Y X_2 \ldots X_n$ — Symbol being scanned is B.
- If $i = n$ and $Y = B$: $X_1 X_2 \ldots X_{n-1}\, q\, X_n \vdash X_1 X_2 \ldots X_{n-2}\, p\, X_{n-1}$

Right move (2) If $\delta(q, X_i) = (p, Y, R)$ then
...

Acceptance

Def : A TM M accepts w if $q_0 w \vdash^{*} \alpha_1 q \alpha_2$ where $q \in F$, $\alpha_1, \alpha_2 \in \Gamma^*$
(for $w \in \Sigma^*$)

In words : "M accept w if when started in q_0 with w as input, M reaches an accept state".

Note : M may not read the entire input w.
It may pass back and forth over symbols of w many times.

Language accepted by TM M : $L(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}$

TM "halts" : no transitions for current state & tape symbol

Convention : All final states are halting.