CS 205
Lecture 16
17/02/2021

Example of R_L and M_L

$\boxed{\text{Recall : } xR_Ly \text{ iff } xz \in L \Leftrightarrow yz \in L \text{ for all } z \in \Sigma^*}$

Let $L = 0^*10^*$

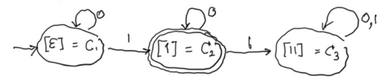
Then xR_Ly iff

(i) x, y have no 1's : $C_1 = 0^*$

(ii) x, y " one 1 $C_2 = 0^*10^*$

(iii) x, y " more than one 1 $C_3 = 0^*10^*1(0+1)^*$

DFA M_L



Fact : If L is regular then the DFA M_L is a minimum state DFA that accepts L.

Why? Because R_M refines R_L.

States of DFA $M \Leftrightarrow$ Equiv. classes of R_M

" " " $M_L \Leftrightarrow$ " " " R_L

Isomorphism between DFA $M_i = (Q_i, \Sigma, \delta_i, q_{0i}, F_i)$ $i = 1,2$

: a bijection $f : Q_1 \to Q_2$ s.t.

$$f(q_{01}) = q_{02}$$

$$f(\delta_1(q, a)) = \delta_2(f(q), a) \quad \text{for all } q, a$$

$$q \in F_1 \Leftrightarrow f(q) \in F_2$$

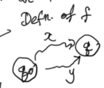
Thm The minimum state DFA accepting a set L is unique up to an isomorphism and is given by M_L in the proof of the Myhill Nerode theorem.

Proof : Since R_M refines R_L for any DFA $M = (Q, \Sigma, \delta, q_0, F)$ that accepts L, the no. of states of M is greater than or equal to the no. of states of M_L.

If equality holds, consider the following function $f : Q \to Q_L$.

Let q be a state in M. Then q must be reachable by a string x, i.e., $\delta(q_0, x) = q$.

Defn. of f



$\boxed{\text{Define } f(q) = \delta_L(q_{0L}, x).}$

$\boxed{\begin{array}{l} \text{Now if } \delta(q_0, x) = \delta(q_0, y) \longrightarrow xR_My \\ \text{then } \delta_L(q_{0L}, x) = \delta_L(q_0, y) \longrightarrow xR_Ly \end{array}}$

Why? Since R_M refines R_L.

Hence f is well defined.

Also, f is injective and surjective. (Easy)

(i) $f(q_0) = [\varepsilon] = q_{0L}$ since $\delta(q_0, \varepsilon) = q_0$.

(ii) $f(\delta(q, a)) = f(\delta(\delta(q_0, x), a))$ by assump.

$= f(\delta(q_0, xa))$ property of δ

$= \delta_L(q_{0L}, xa)$ by defn. of f

$= \delta_L(\delta_L(q_{0L}, x), a)$ prop. of δ_L

$= \delta_L(f(q), a)$

DFA Minimization : Find the minimum state DFA M' equivalent to a given DFA $M = (Q, \Sigma, \delta, q_0, F)$

$\boxed{\begin{array}{l} \text{Define the equivalence relation } \equiv \subseteq Q \times Q \text{ as} \\ p \equiv q \text{ iff for each } x \in \Sigma^*, \ \delta(p, x) \in F \\ \qquad \text{iff} \\ \qquad \delta(q, x) \in F. \end{array}}$

Note : There is an isomorphism between

(1) Equivalence classes of $\equiv$ that contain a state reachable from q_0 by some input string, and

(2) Q_L, the states of the minimum state DFA M_L

Def

Say that

(1) p is equivalent to q if $p \equiv q$

(2) p is distinguishable from q if there exists an x s.t. $\delta(p, x) \in F$ and $\delta(q, x) \notin F$

or

$\delta(p, x) \notin F$ and $\delta(q, x) \in F$.

Algorithm for computing the equivalence classes of $\equiv$