CS 205
Lecture # 8
25/01/21

Recap : DFA, NFA and ϵ-NFA all accept the same class of languages.

Recall : Concatenation of languages

$L_1, L_2 \subseteq \Sigma^*$

$L_1 L_2 = \{xy \mid x \in L_1,\ y \in L_2\}$

For $L \subseteq \Sigma^*$

define $L^0 = \{\varepsilon\}$

$L^i = L L^{i-1} = \underbrace{LLL\ldots L}_{i \text{ times}}$

$L^* = \bigcup_{i \geq 0} L^i = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \cdots$

Kleene-star (after Stephen Kleene)

$L^+ = \bigcup_{i > 0} L^i = L \cup L^1 \cup L^2 \cup \cdots$

Q: Is it possible that $L^* = L^+$ for some L?

A: Yes, iff $\varepsilon \in L$.

Def : Given an alphabet Σ, the regular expressions over Σ and the sets they denote are defined inductively as follows:

r.e. E ← syntactic object	$L(E)$, set denoted by E ← Semantics
1. $\emptyset$	$\emptyset$ empty set
2. ε	$\{\varepsilon\}$
3. a where $a \in \Sigma$	$\{a\}$

4. If r and s are r.e. denoting sets R and S then

$(r+s)$	$R \cup S$ (union of languages)
(rs)	RS (concatenation of languages)
(r^*)	R^* (Kleene-star)

Convention: $*$ has higher precedence than $\cdot$ or $+$

$\cdot$ " " " $+$

Usually omit parentheses

Ex: $r = 01^* + 0$ $\quad r' = ((0(1^*)) + 0)$ ← with parenthesis

$L(r) = \{0, 01, 011, 0111, \ldots\}$

$L(r) = L(01^*) \cup L(0)$

$= L(0) \cdot L(1^*) \cup \{0\}$

$= \{0\} \cdot (L(1))^* \cup \{0\}$

$= \{0\} \cdot \{1\}^* \cup \{0\}$

Ex: 1. $(0+1)^*$: set of all strings over Σ $\quad \Sigma = \{0,1\}$

2. $(0+1)^* 00 (0+1)^*$: set of all strings with at least two consecutive 0s

3. $(1 + 10)^*$ i.e. $(1+10)(1+10)^*$: set of all strings beginning with a 1 and not containing two consecutive 0s.

choice, iteration

shorthand for

$L = \{1, 11, 10, 111, 110, 101, \ldots\}$

Complement

4. $(0+\varepsilon)(1+10)^*$: set of all strings of 0s and 1s that do not have two consecutive 0s.

5. $(0+1)^* 00$: set of all strings ending with 00

Equivalence of FA and REs

The languages accepted by FA are precisely the languages denoted by regular expressions.

RE $\longrightarrow$ FA

FA $\longrightarrow$ RE ✓✓

Thm (Kleene) : If $L = L(A)$ for some DFA A, then there is a r.e. r s.t. L is denoted by r, i.e., $L = L(r)$.

Proof : (Dynamic Programming)

Let $A = (Q, \Sigma, \delta, q_1, F)$ be a DFA

where $Q = \{q_1, q_2, \ldots, q_n\}$

Let R^k_{ij} denote the set of all strings w s.t. w is the label of a path from q_i to q_j in A and that path has no intermediate node q_l with $l > k$.