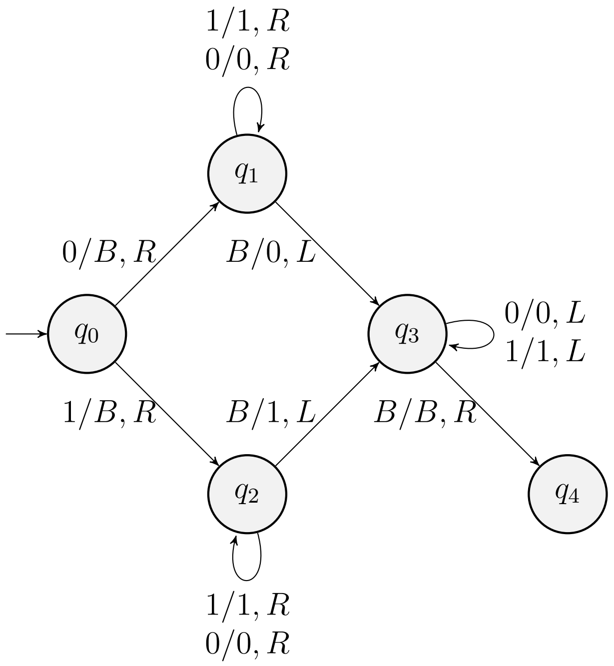
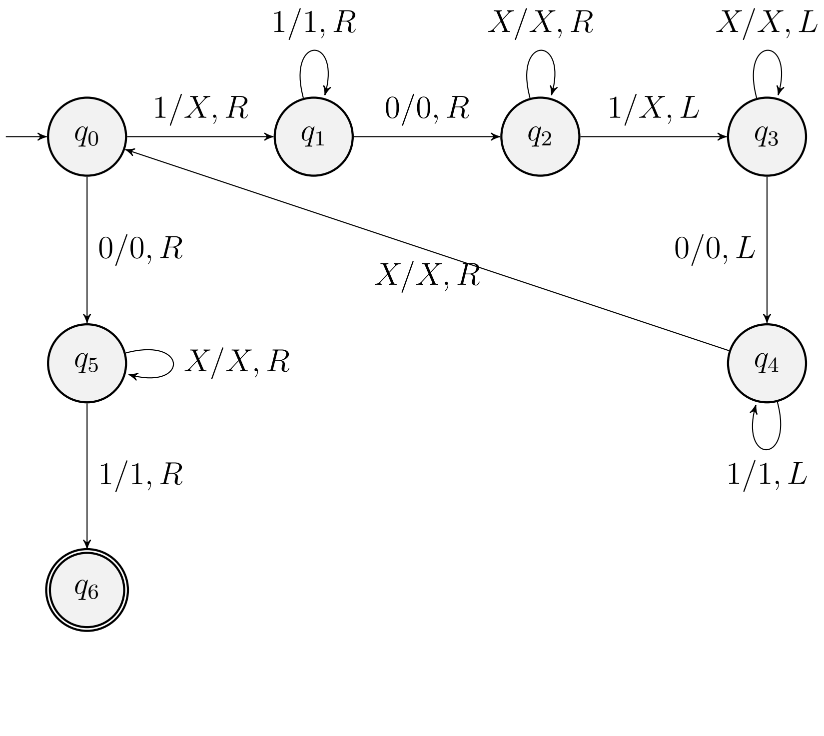
CS 205, Formal Languages, Automata Theory and Computation
Quiz 6 Solutions, Winter 2020-21
Department of CSE, IIT Guwahati

1. The following Turing machine takes as input a binary string and produces as output another binary string and halts on the leftmost symbol. Describe the function computed by the machine. Here the tape alphabet is $\{0, 1, B\}$.



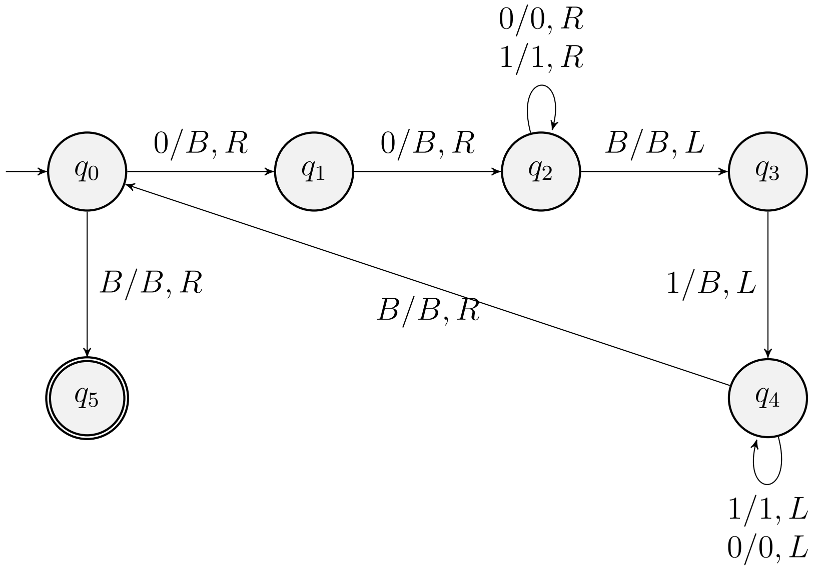
Solution: The TM does a cyclic left shift of the input by one bit, *i.e.*, $b_1 b_2 \ldots b_n$ is mapped to $b_2 b_3 \ldots b_n b_1$.

2. Describe the language accepted by the following Turing machine over the binary alphabet. Here the tape alphabet is $\{0, 1, B, X\}$.



Solution: $L = \{1^m 0 1^n | m < n\}$

3. Describe the language accepted by the following Turing machine over the binary alphabet. Here the tape alphabet is $\{0, 1, B\}$.



Solution: $L = \{0^{2n}1^n \mid n >= 0\}$

4. Show that the following problem is undecidable: Given the code $\langle M \rangle$ of a Turing machine M decide whether M halts on all input strings in 0^*. Assume that the input alphabet of M is $\{0, 1\}$. Do not use Rice's theorem in your proof. You can use any other result proved in the class.

Solution: Let $L = \{\langle M \rangle \mid \text{ TM } M \text{ halts on all inputs in } 0^*\}$. We give a many-one reduction f from $L_{\text{halt}} = \{\langle M \rangle \# w \mid M \text{ halts on } w\}$ to L as follows. The reduction f takes a string $\langle M \rangle \# w$ as input and produces as output the code $\langle M' \rangle$ of a TM M' described below.

M' : On input u
Run M on w
if M halts on w then
M' accepts and halts on u

Clearly $\langle M \rangle \# w \in L_{\text{halt}}$ iff M' halts on any string in 0^* iff $f(\langle M \rangle \# w) \in L$. Since L_{halt} is uncomputable, so is L.