CS 205
Lecture 10
2/2/21

Recap: Equivalence of REs and FA

✓ DFA $\longrightarrow$ RE $\qquad r_{ij}^{k}$ denoting R_{ij}^{k}

RE $\rightsquigarrow$ ε-NFA

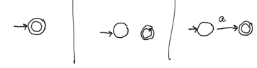
Thm: Let r be a r.e. Then there exists an ε-NFA M that accepts $L(r)$.

Proof: By induction on the structure of r we show that $L(r) = L(M)$ for some ε-NFA M with

(1) exactly one accepting state
(2) no transitions into the initial state
(3) no " out of " accepting state

Basis RE $\quad \varepsilon \quad | \quad \emptyset \quad | \quad a$

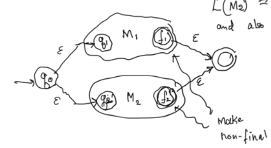
ε-NFA



Inductive Case

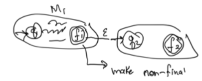
Case 1 : $r = r_1 + r_2$

Assume M_1, M_2 are s.t.
$L(M_1) = L(r_1)$
$L(M_2) = L(r_2)$
and also satisfy (1), (2), (3)

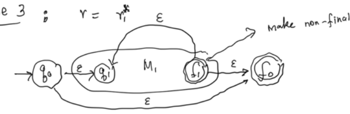


Case 2 : $r = r_1 r_2$

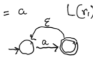
Assume M_1, M_2 are ε-NFA s.t. $L(r_i) = L(M_i)$ and each M_i satisfies (1), (2) & (3)



Case 3 : $r = r_1^*$



Ex $\quad r = a^* \qquad$ Here $r_1 = a \qquad L(r_1) = \{a\}$

$M =$ 

$r = r_1^* \qquad L(r) = \{\varepsilon, a, aa, \ldots\}$

M does not accept ε !

Def. of RE $\quad r = r_1^* \qquad L(r) = (L(r_1))^*$

$$L^* = \bigcup_{i \geq 0} L^i$$

$L^0 = \{\varepsilon\}$
$L^1 = L$
$L^2 = L.L$
...

Algebraic Laws

1. Commutativity $\quad r_1 + r_2 = r_2 + r_1$
2. Associativity for + $\quad (r_1 + r_2) + r_3 = r_1 + (r_2 + r_3)$
3. Associativity for . $\quad (r_1 r_2) r_3 = r_1 (r_2 r_3)$
4. Identity $\quad \emptyset + r = r + \emptyset = r$
   $\varepsilon . r = r . \varepsilon = r$
5. Annihilator $\quad \emptyset . r = r . \emptyset = \emptyset$
6. Distributivity $\quad r_1(r_2 + r_3) = r_1 r_2 + r_1 r_3$
   $(r_1 + r_2) r_3 = r_1 r_3 + r_2 r_3$
7. Idempotence $\quad r + r = r$

Semiring

So far: Regular languages have four different characterizations

(1) DFA
(2) NFA
(3) ε-NFA
(4) RE (regular expressions)

Not every language is regular.

Thm (The Pumping Lemma for Regular Languages)
Let L be a regular language. Then there exists a constant n (which depends on L) s.t. for every string $w \in L$ with $|w| \geq n$, we can break w into three strings as $w = xyz$ s.t.

1. $y \neq \varepsilon$
2. $|xy| \leq n$
3. $xy^k z \in L$ for all $k \geq 0$

Nesting of quantifiers
$\forall L \, \exists n \, \forall w \, \exists x \, \exists y \, \exists z \, [\ldots \, \forall k \ldots]$

That is, we can always find a nonempty string y satisfying (2) that can be "pumped" an arbitrary no. of times or deleted (when $k = 0$) and the resulting string is again in L.

Ex $\quad L = \{0^n 1^n \mid n \geq 0\} \qquad L = \{\varepsilon, 01, 0011, 000111, \ldots\}$

Show that L is not regular.