Assignment 1

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1

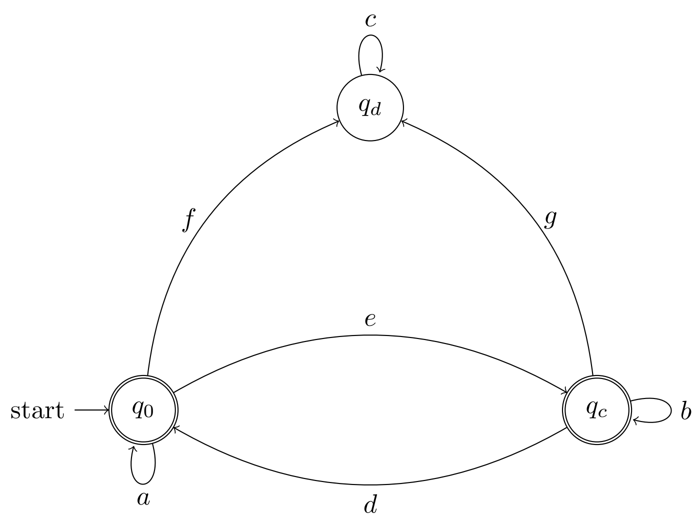
So, in our language L we will be given 3 X N matrix as input string where N>=0. As we process the input matrix we will be having 2 possibilities either carry is there or not. So, based on this fact I will be having 3 states one for carry(q_c), one without carry(q_0) and one dead state(q_d) for treating that this can never be in our language L.

We will be starting at q_0 as there is no initial carry. Taking $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ as input if (a+b)mod2 = (c)mod2 where mod denotes remainder as divided by 2 and greater than equal to 0. So, if we are at q_0 only $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ these possibilities are possible and all other will be gone to dead states. Here $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ will remain at q_0 and other value $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ will lead to transition to q_c as it will provide carry.

Next if we are at q_c then taking $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ as input if (a+b+1)mod2=(c)mod2 where mod denotes remainder as divided by 2 and greater than equal to 0.

So, if we are at q_c only $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ these possibilities are possible and all other will be gone to dead state. Here $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ these will remain at q_c as they generate carry and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ will lead to q_0.

This will be resulting DFA and q_0, q_c will be accepted states.



$$\Sigma = \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\text{a=} \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix} \text{ b=} \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \text{ c=}\Sigma \text{ d=} \begin{bmatrix}0\\0\\1\end{bmatrix}$$

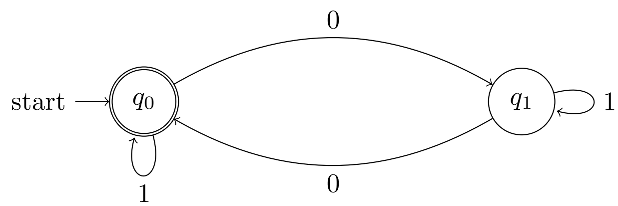
$$\text{e=} \begin{bmatrix}1\\1\\0\end{bmatrix} \text{ f=} \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \text{ g=} \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}$$

And we have to ignore carry bit of MSB if it is there. So, q_c also becomes accepting state. As we can draw DFA for this language and proved this DFA is accepted by required condition. So, Language L is regular.

2

L is a regular language and we need to show L' is also regular.
L'= {w |w $\in$ L and have even number of zeroes}. L and L' both are defined on set {0,1}. So, from question we can see that L' is intersection of 2 regular languages i.e. L' = L $\cap$ M where M is regular language having even number of zeroes. Now, we just need to prove that intersection of two regular languages is regular.
Proof –
Let's say there are two regular languages as A and B
Where A =(QA,Σ,δA,qA,FA) and B= (QB,Σ,δB,qB,FB)
For intersection we need to make pairs from QA and QB that will simulate both DFA states from A and B

For example Lets say T is final DFA and where we are in a state (p,q) in T where p is a state in A and q is a state in B. Here we get input as a and p goes to s in A for input a and q goes to t in B for input a So, our DFA T must move to state (s,t).
T =(QA X QB,Σ,δT,(qA,qB),FA X FB).
Here start state of T is pair of start state of A and B and similarly final states are pairs of final states from both A and B.
We can compute δT ((p,q),a) = (δA (p,a), δB (q,a)) and similarly extend this to δ'.
We can see that L(T)= L(A) $\cap$ L(B). As from δ' and induction on any string will be accepted in T if and only if it exists both in A and B.
So, this implies T is intersection of A and B and T is regular as its DFA exists.
So, we are given that L is regular and we can show M is regular by its DFA as



And As L' is intersection of L and M. and we have proved that intersection of two regular languages is regular. So, L' is regular.
Hence proved.