CS 205

Lecture 11

3/2/21

Pumping Lemma : If L is regular then there is a constant n s.t. $\forall w \in L$ with $|w| \geq n$, $\exists x, y, z \in \Sigma^*$ s.t.

1. $w = xyz$
2. $y \neq \varepsilon$
3. $|xy| \leq n$
4. $\forall k \geq 0.\ xy^kz \in L$

Proof (Informal)

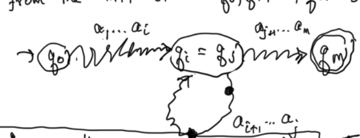
Suppose L is regular and M is a DFA with n states s.t. $L(M) = L$.

Let $w = a_1 a_2 \dots a_m$ with $m \geq n$. and $w \in L$.

Consider M's run on w:

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \cdots q_{m-1} \xrightarrow{a_m} q_m$$

Since $m \geq n$, there must be two states q_i and q_j $(i \neq j)$ from the $m+1$ states $q_0, q_1, \dots, q_m$ s.t. $q_i = q_j$. (Say $i < j$)



$\boxed{\text{Suppose } j \text{ is the smallest such index.}}$ Then

1. $|a_1 \dots a_i \dots a_j| \leq n$ (Since at most $n+1$ states are visited when processing $a_1 \dots a_j$.)
2. $|a_{i+1} \dots a_j| \neq \varepsilon$

Let $x = a_1 \dots a_i$
$y = a_{i+1} \dots a_j$
$z = a_{j+1} \dots a_m$

Then for all $k \geq 0$. xy^kz is also in L. (why?)

Ex 1 $L = \{0^i 1^i \mid i \geq 0\}$ is not regular.

Proof: Suppose L is regular. Let n be the constant in the PL.

Consider $\boxed{w = 0^n 1^n}$ Then by the PL there exist $x, y, z \in \Sigma^*$ s.t. $w = xyz$, $\boxed{y \neq \varepsilon}$, $|xy| \leq n$ and $xy^kz \in L$ for all $k \geq 0$.

since

Consider any such x, y, z. Since $|xy| \leq n$,

$x = 0^r$, $\boxed{y = 0^s}$ and $z = 0^t 1^n$ where $r, t \geq 0$ and $s \geq 1$.

Now, $xy^0z = xz = 0^r 0^t 1^n = 0^{r+t} 1^n \notin L$

since $r + t < n$ (Why?)
(Since $r+s+t = n$ and $s \geq 1$)

Insight : 1. Choice of w s.t. $w \in L$ and $|w| \geq n$
2. Choice of k s.t. $xy^kz \notin L$

Ex 2 : $L = \{w \mid w$ has an equal no. of 0s and 1s$\}$ is not regular.

Proof : Same as above.

Ex 3 : $L = \{0^i \mid i \text{ is prime}\}$ is not regular $\Sigma = \{0\}$