Convolutional Neural Nets

Some slides were adated/taken from various sources, including Andrew Ng's Coursera Lectures, CS231n: Convolutional Neural Networks for Visual Recognition lectures, Stanford University CS Waterloo Canada lectures, Aykut Erdem, et.al. tutorial on Deep Learning in Computer Vision, Ismini Lourentzou's lecture slide on "Introduction to Deep Learning", Ramprasaath's lecture slides, and many more. We thankfully acknowledge them. Students are requested to use this material for their study only and NOT to distribute it.

Background: Image Processing

A **convolution matrix** is used in image processing for tasks such as edge detection, blurring, sharpening, etc.

Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Convolution

0	0	0
0	1	1
0	1	0

Convolved Image

1	1	1	1	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	0	0	0	0

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Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Convolution

0	0	0
0	1	1
0	1	0

Convolved Image

3	2	2	3	1
2	0	2	1	0
2	2	1	0	0
3	1	0	0	0
1	0	0	0	0

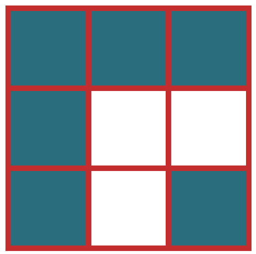
Background: Image Processing

A **convolution matrix** is used in image processing for tasks such as edge detection, blurring, sharpening, etc.

Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Convolution



Convolved Image

3	2	2	3	1
2	0	2	1	0
2	2	1	0	0
3	1	0	0	0
1	0	0	0	0

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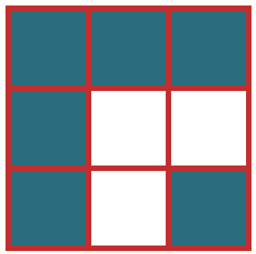
Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Convolution



Convolved Image

3	2	2	3	1
2	0	2	1	0
2	2	1	0	0
3	1	0	0	0
1	0	0	0	0

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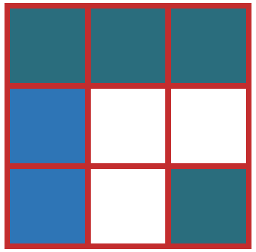
Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Convolution



Convolved Image

3	2	2	3	1
2	0	2	1	0
2	2	1	0	0
3	1	0	0	0
1	0	0	0	0

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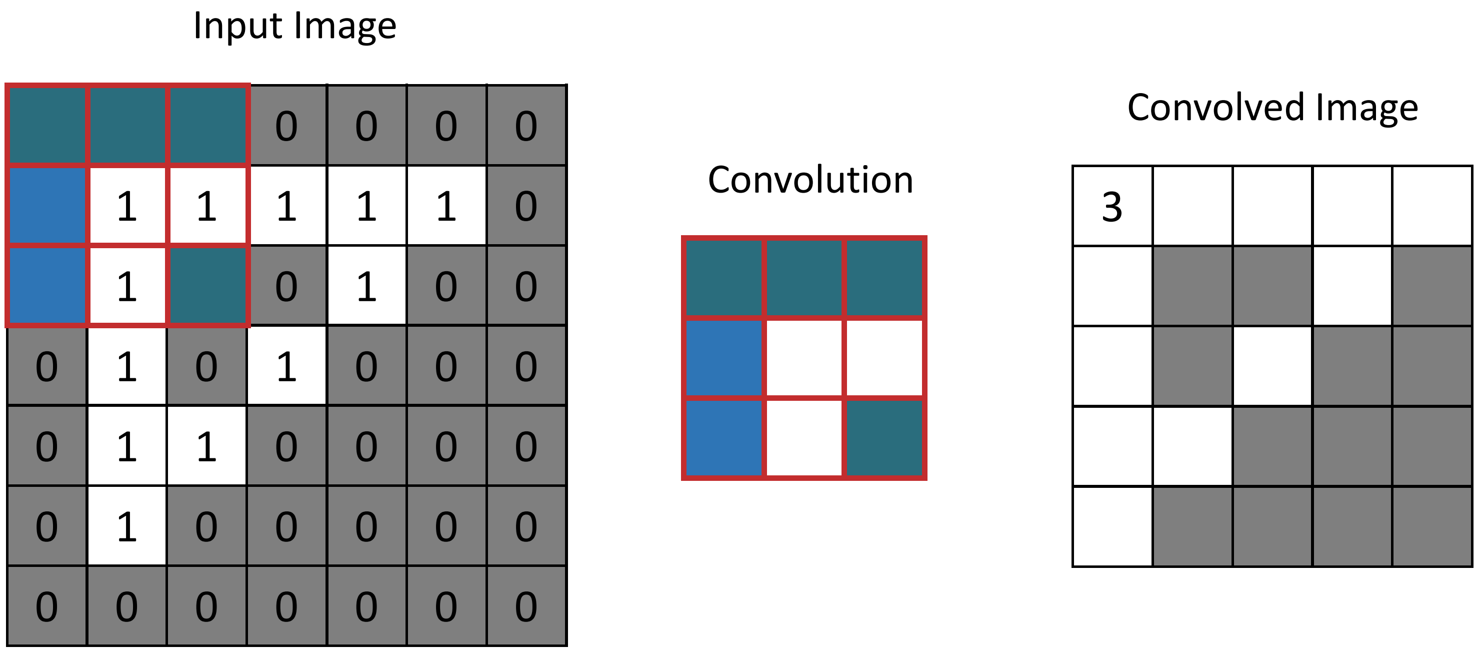

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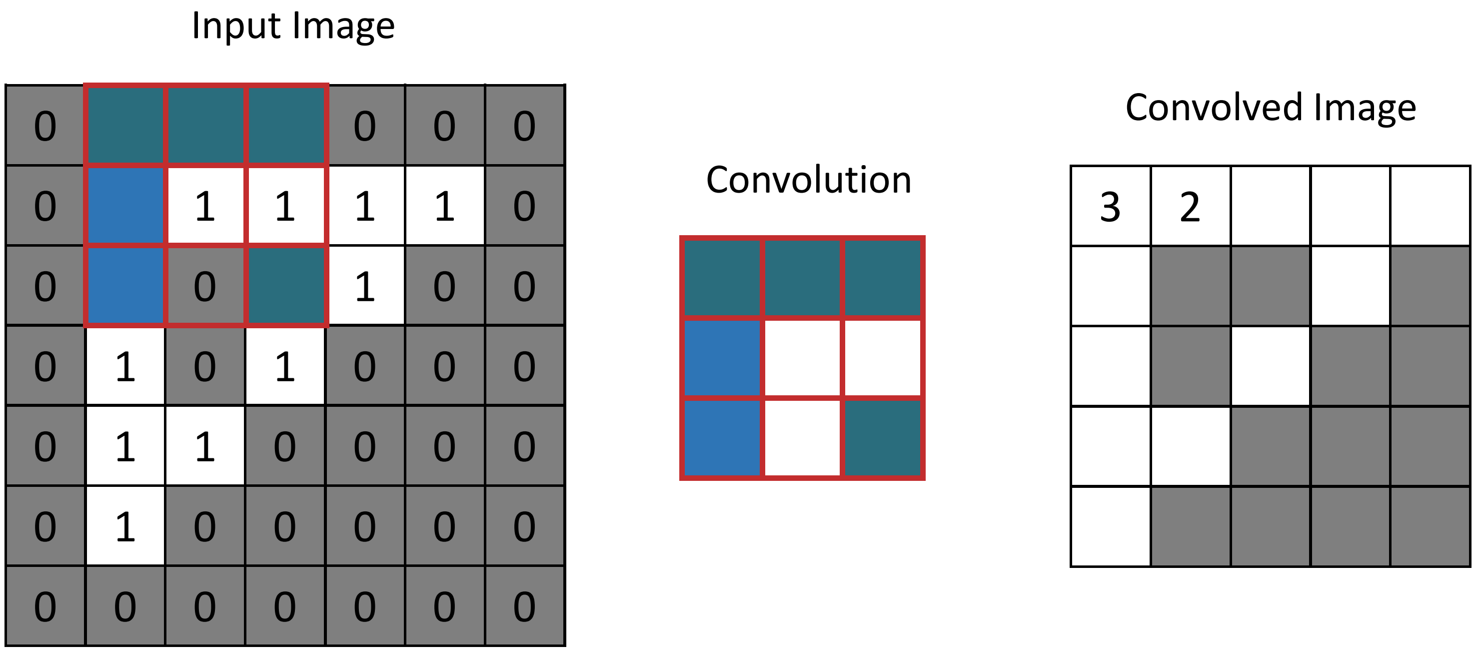

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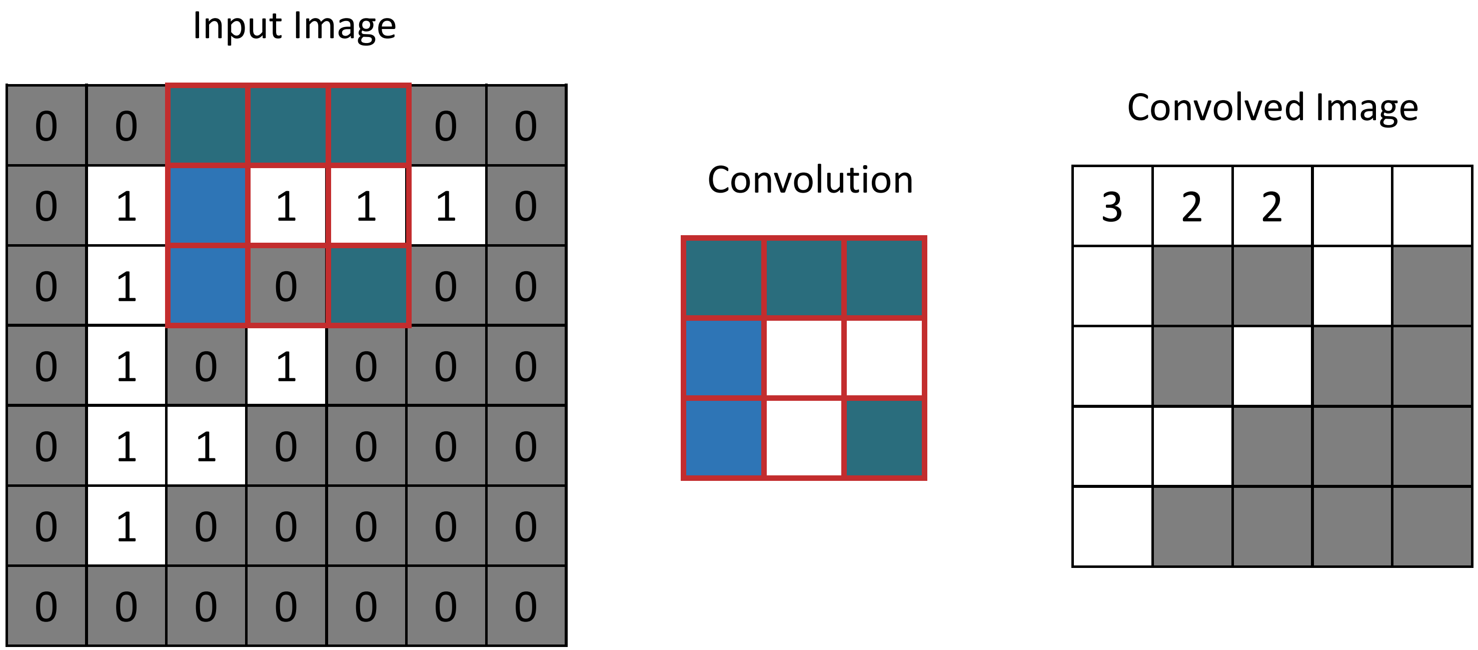

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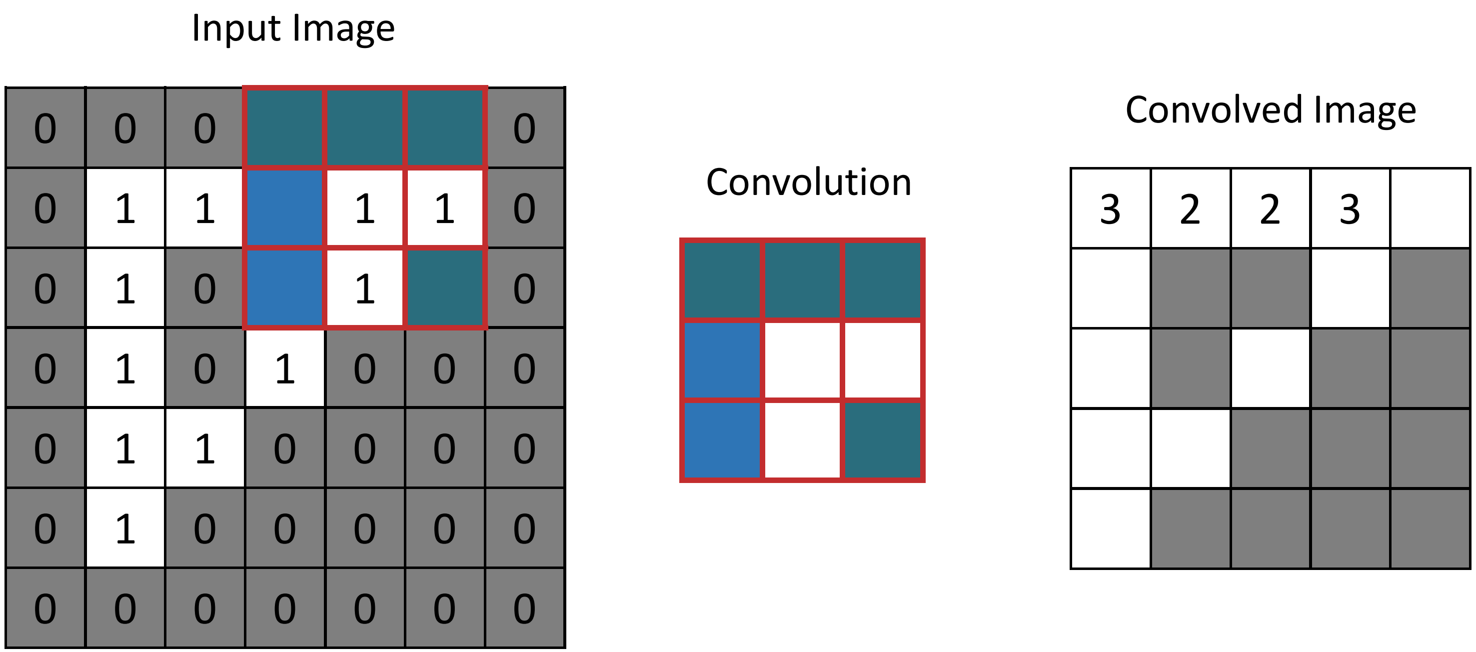

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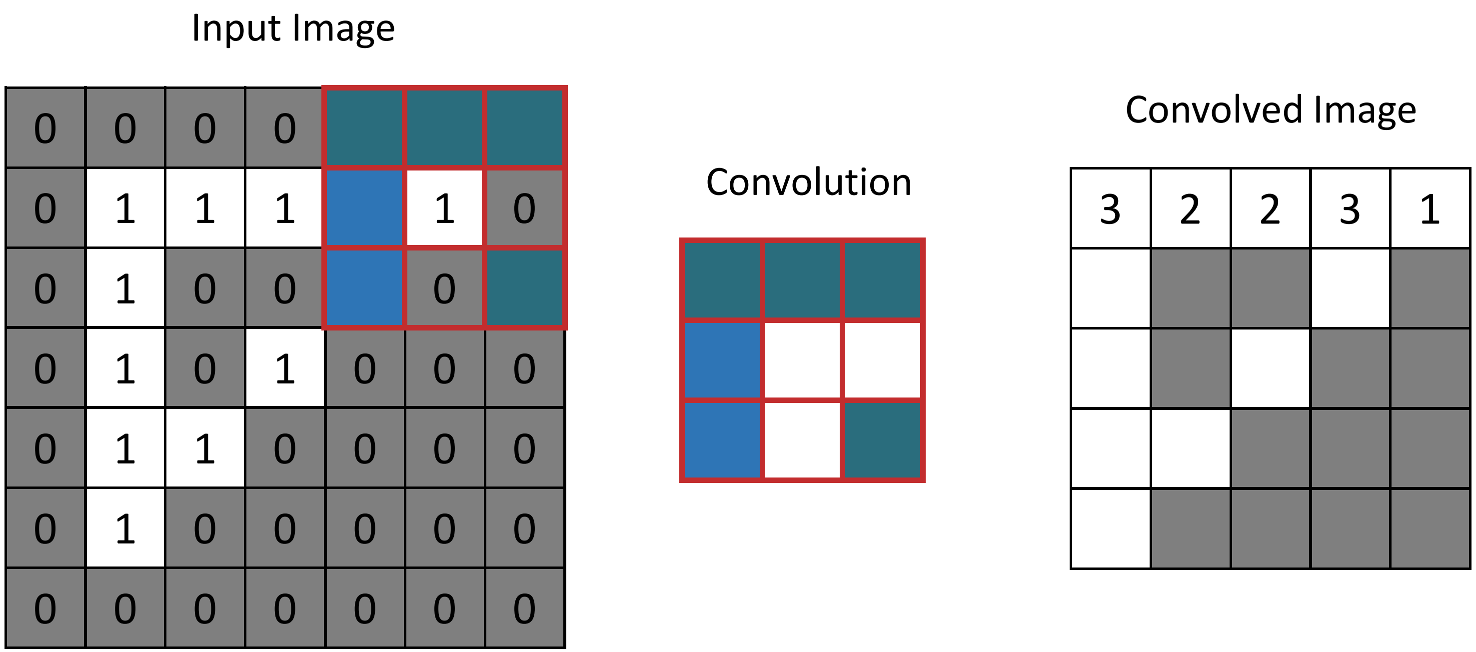

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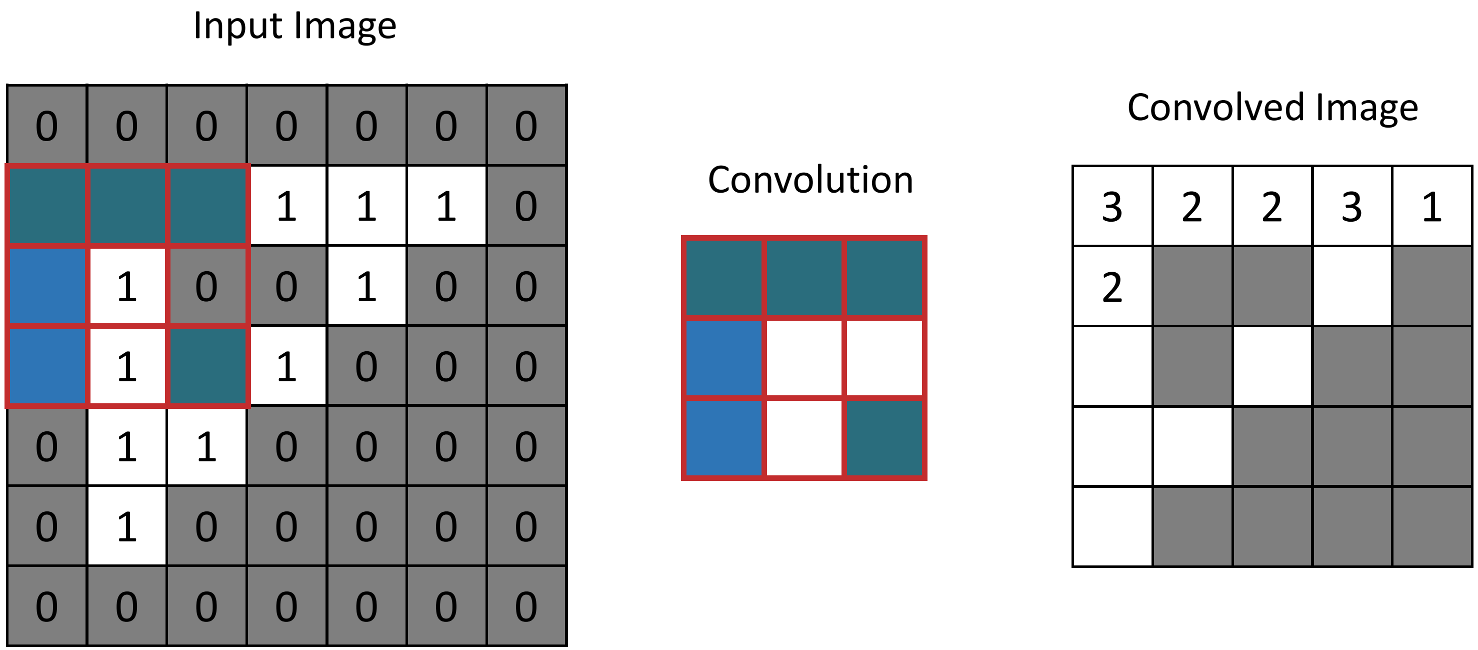

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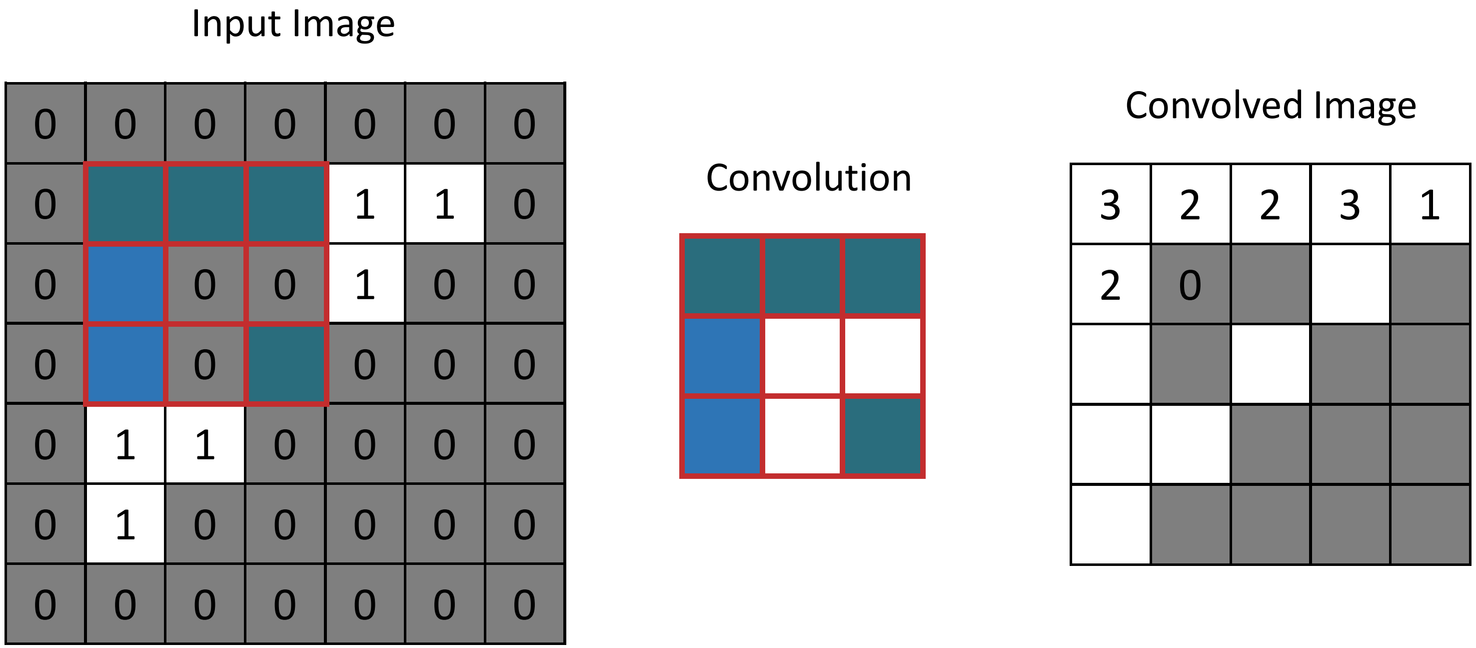


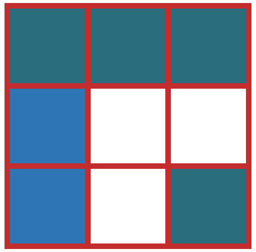
Background: Image Processing

A **convolution matrix** is used in image processing for tasks such as edge detection, blurring, sharpening, etc.

Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Convolution



Convolved Image

3	2	2	3	1
2	0	2	1	0
2	2	1	0	0
3	1	0	0	0
1	0	0	0	0

Background: Image Processing

A **convolution matrix** is used in image processing for tasks such as edge detection, blurring, sharpening, etc.

Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Identity Convolution

0	0	0
0	1	0
0	0	0

Convolved Image

1	1	1	1	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	0	0	0	0

Background: Image Processing

A **convolution matrix** is used in image processing for tasks such as edge detection, blurring, sharpening, etc.

Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Blurring Convolution

.1	.1	.1
.1	.2	.1
.1	.1	.1

Convolved Image

.4	.5	.5	.5	.4
.4	.2	.3	.6	.3
.5	.4	.4	.2	.1
.5	.6	.2	.1	0
.4	.3	.1	0	0

Background: Image Processing

A **convolution matrix** is used in image processing for tasks such as edge detection, blurring, sharpening, etc.

Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Blurring Convolution

.1	.1	.1
.1	.2	.1
.1	.1	.1

Convolved Image

.4	.5	.5	.5	.4
.4	.2	.3	.6	.3
.5	.4	.4	.2	.1
.5	.6	.2	.1	0
.4	.3	.1	0	0

Convolutional Neural Network (CNN)

CNN key idea:
Treat convolution matrix as parameters and learn them!

Input Image

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	1	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0

Learned Convolution

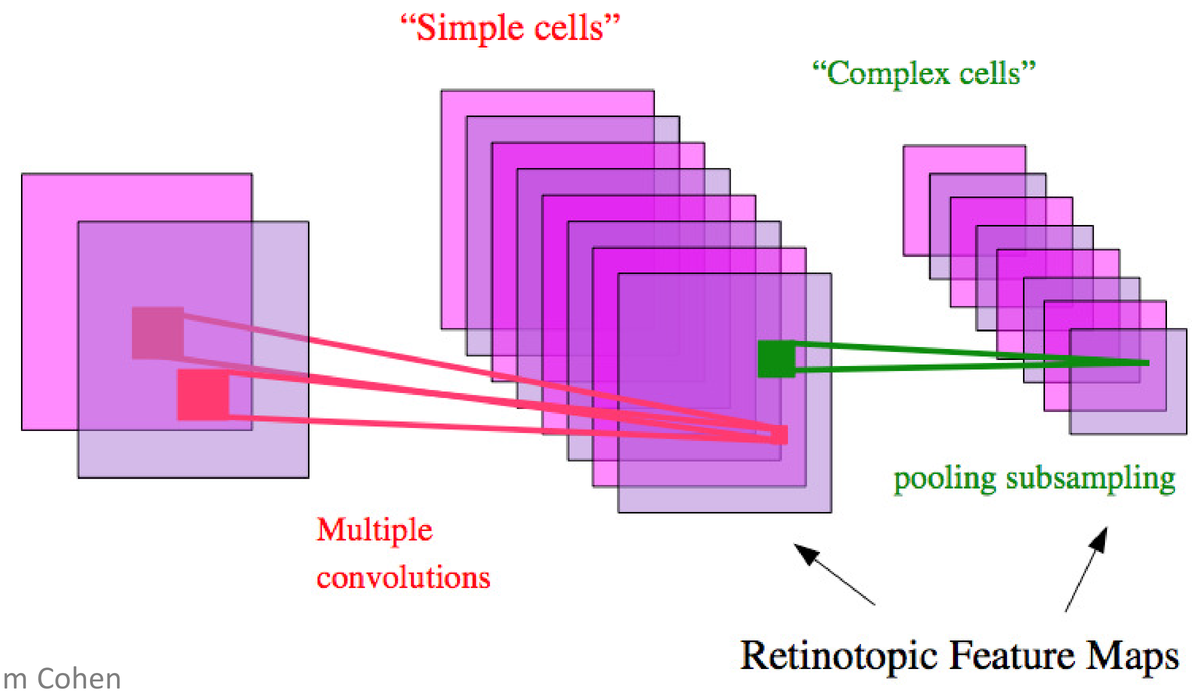
θ_{11}	θ_{12}	θ_{13}
θ_{21}	θ_{22}	θ_{23}
θ_{31}	θ_{32}	θ_{33}

Convolved Image

.4	.5	.5	.5	.4
.4	.2	.3	.6	.3
.5	.4	.4	.2	.1
.5	.6	.2	.1	0
.4	.3	.1	0	0

Model of vision in animals

- **[Hubel & Wiesel 1962]:**
  - simple cells detect local features
  - complex cells "pool" the outputs of simple cells within a retinotopic neighborhood.

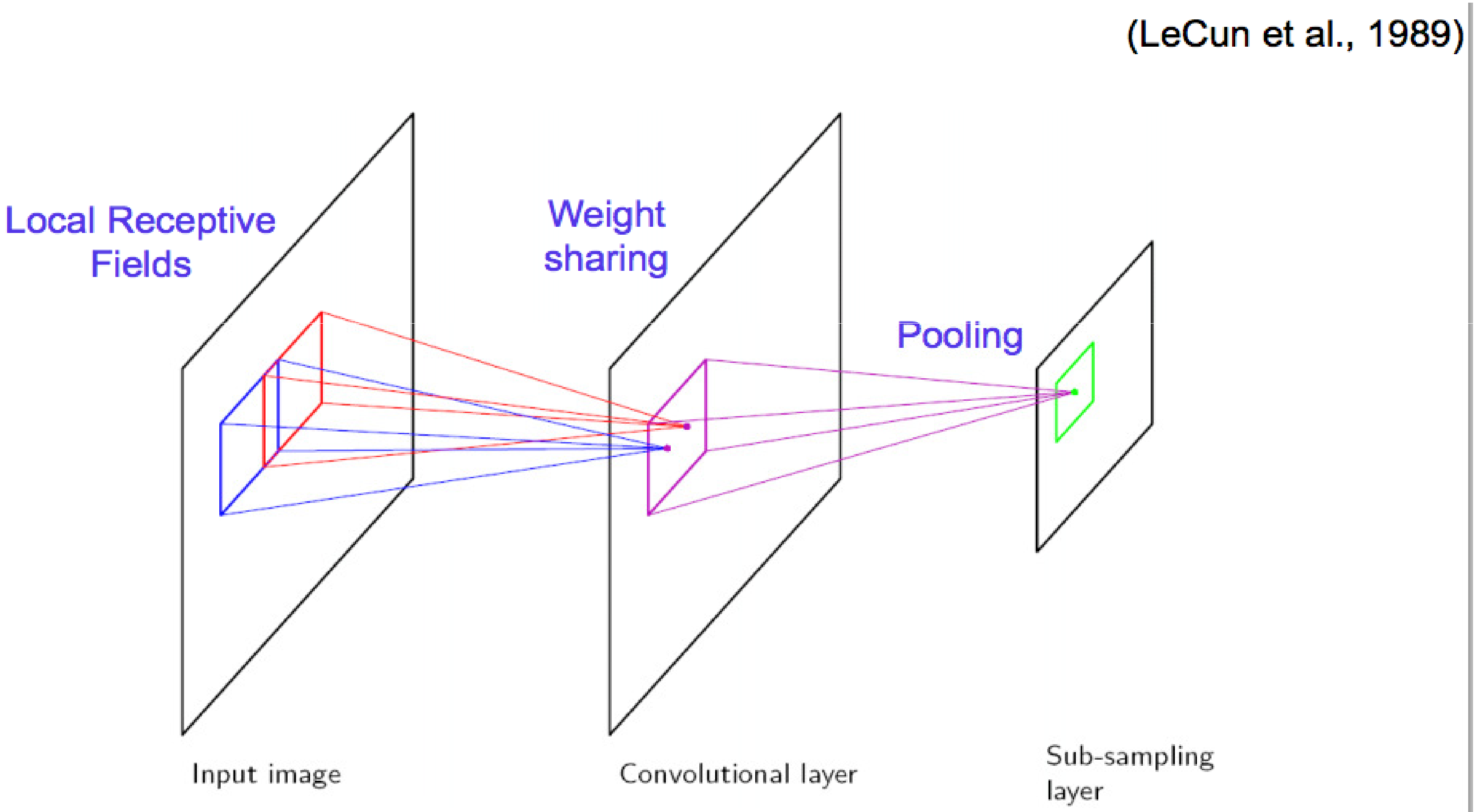


Slide from William Cohen

Huber & Wiesel Video

https://www.youtube.com/watch?v=8VdFf3egwfg

Vision with ANNs



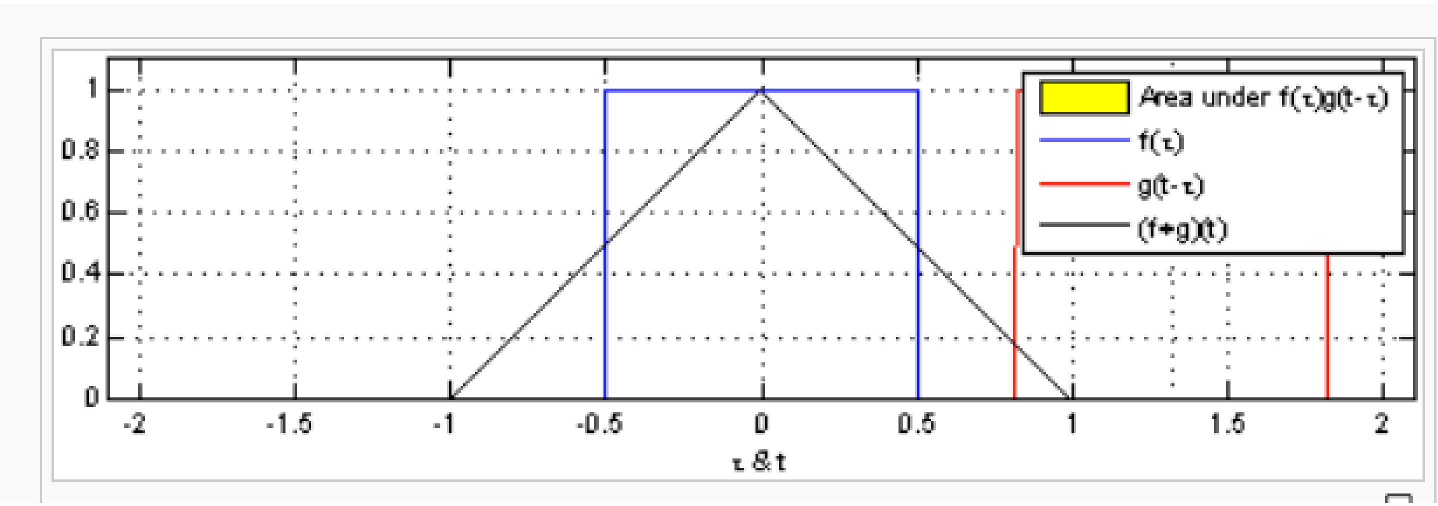
Slide from William Cohen

What’s a convolution?

https://en.wikipedia.org/wiki/Convolution

1-D

$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} f(t-\tau)\, g(\tau)\, d\tau.$$



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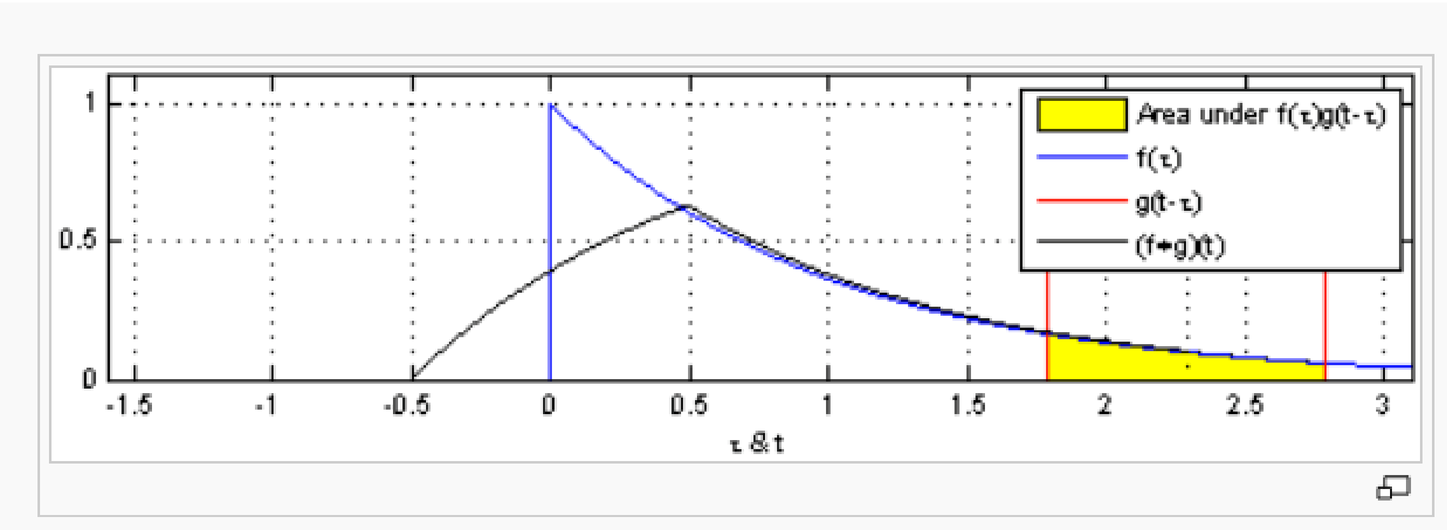


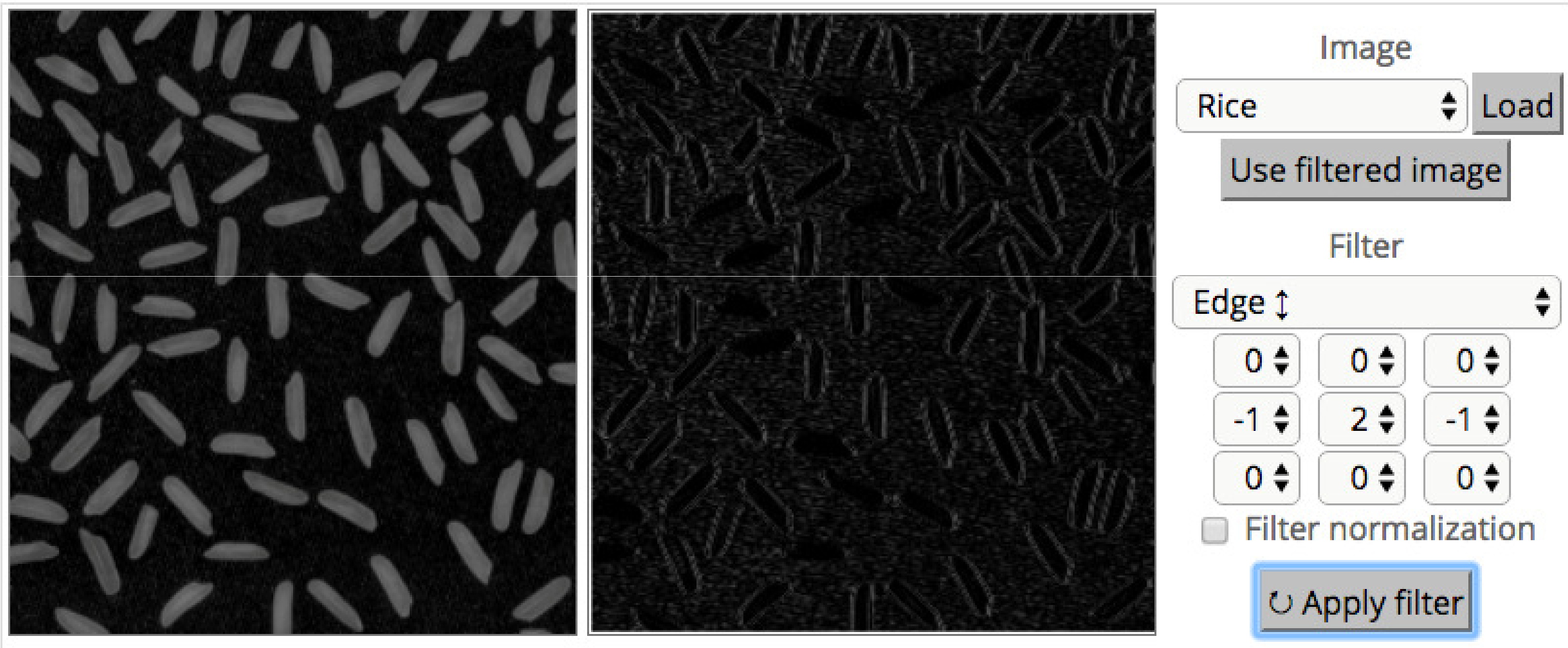
Slide from William Cohen

Convolution

Visual Interpretation

What’s a convolution?

http://matlabtricks.com/post-5/3x3-convolution-kernels-with-online-demo



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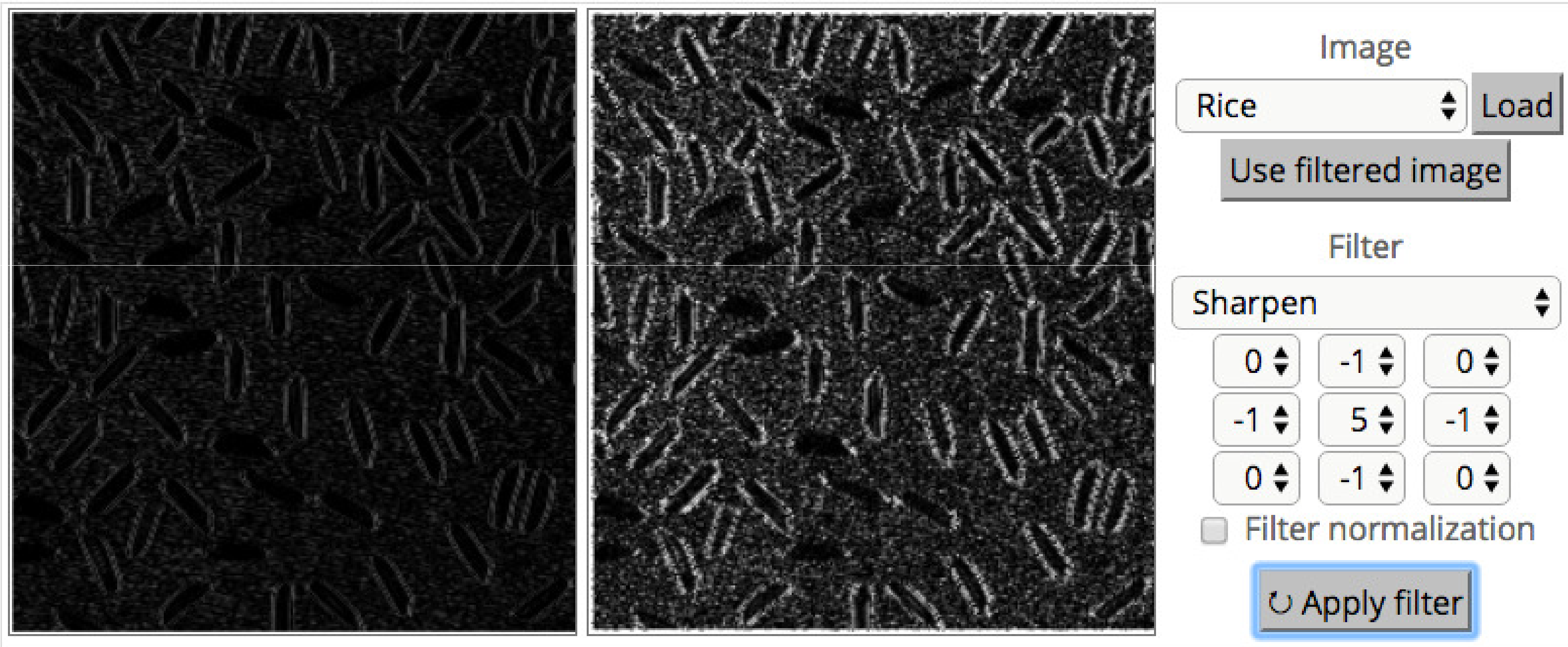

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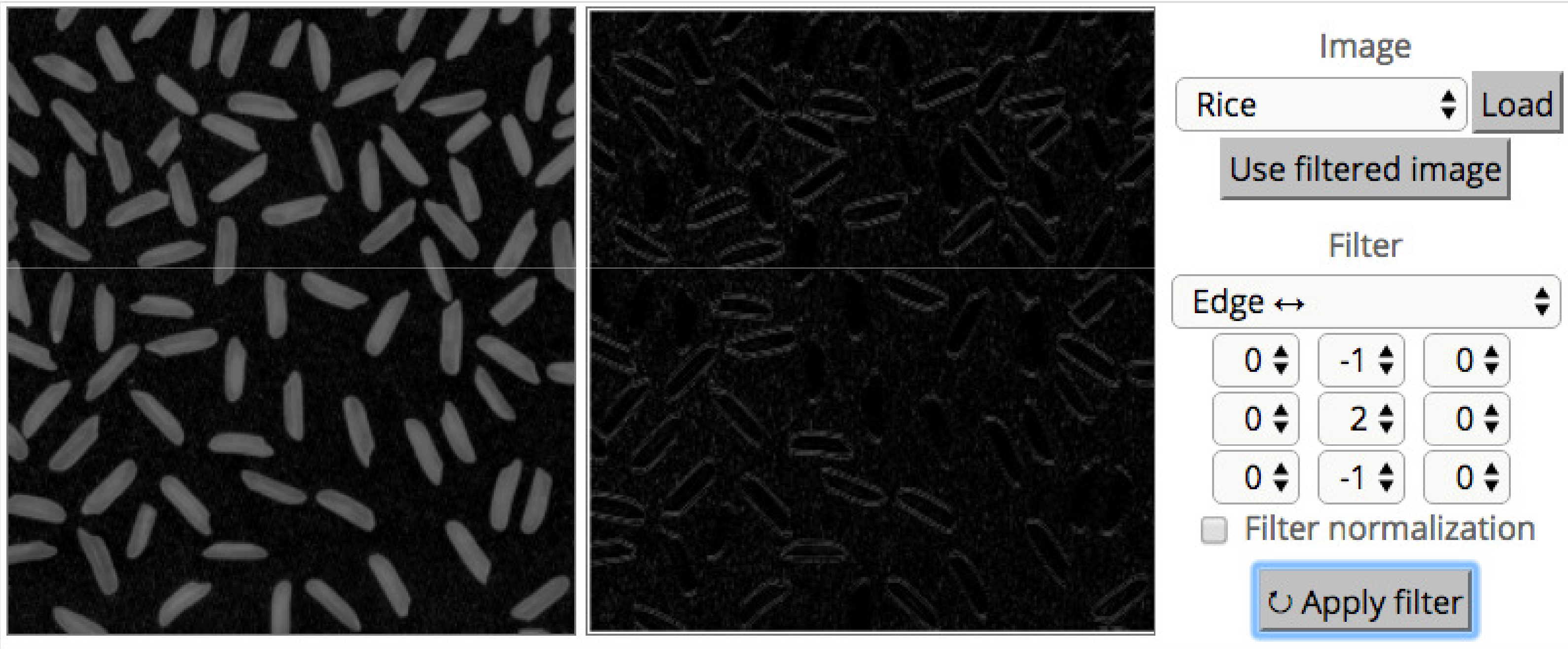

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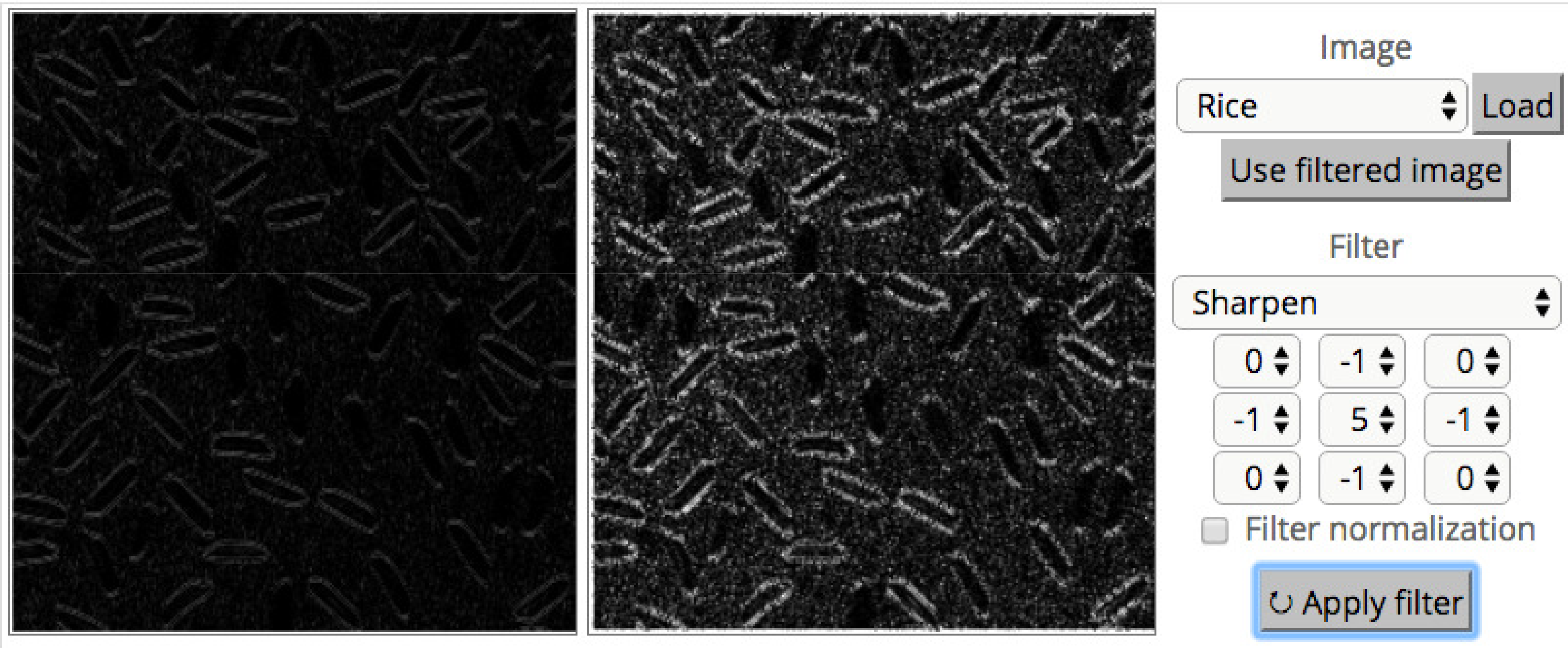

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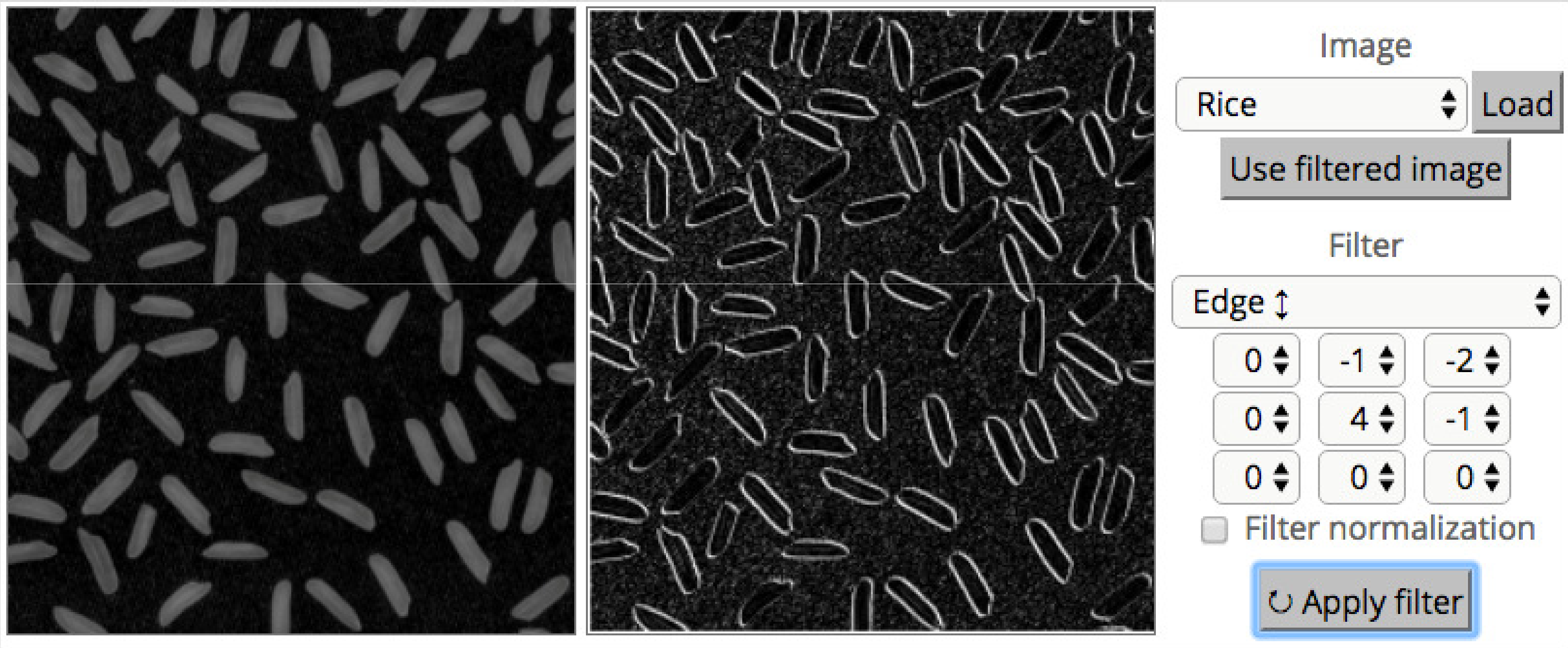

Slide from William Cohen

What's a convolution?

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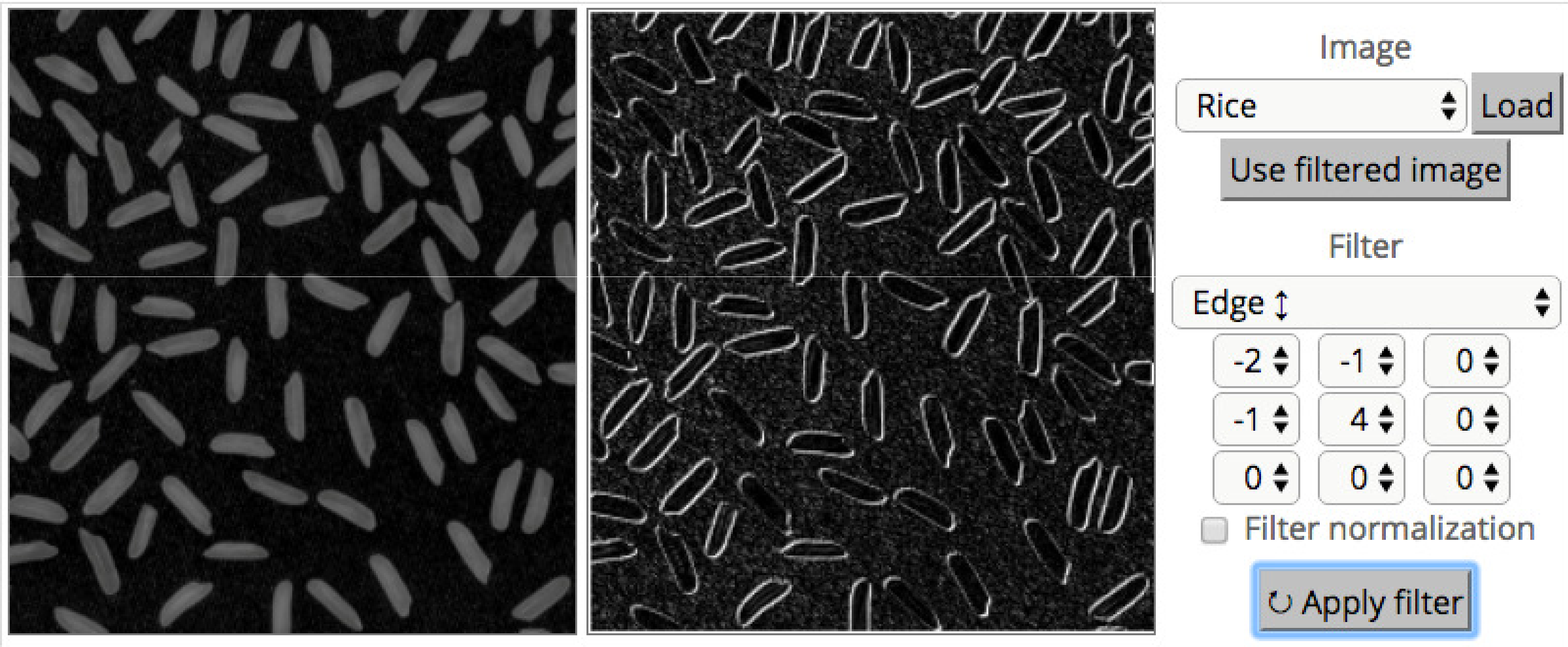


Slide from William Cohen

What’s a convolution?

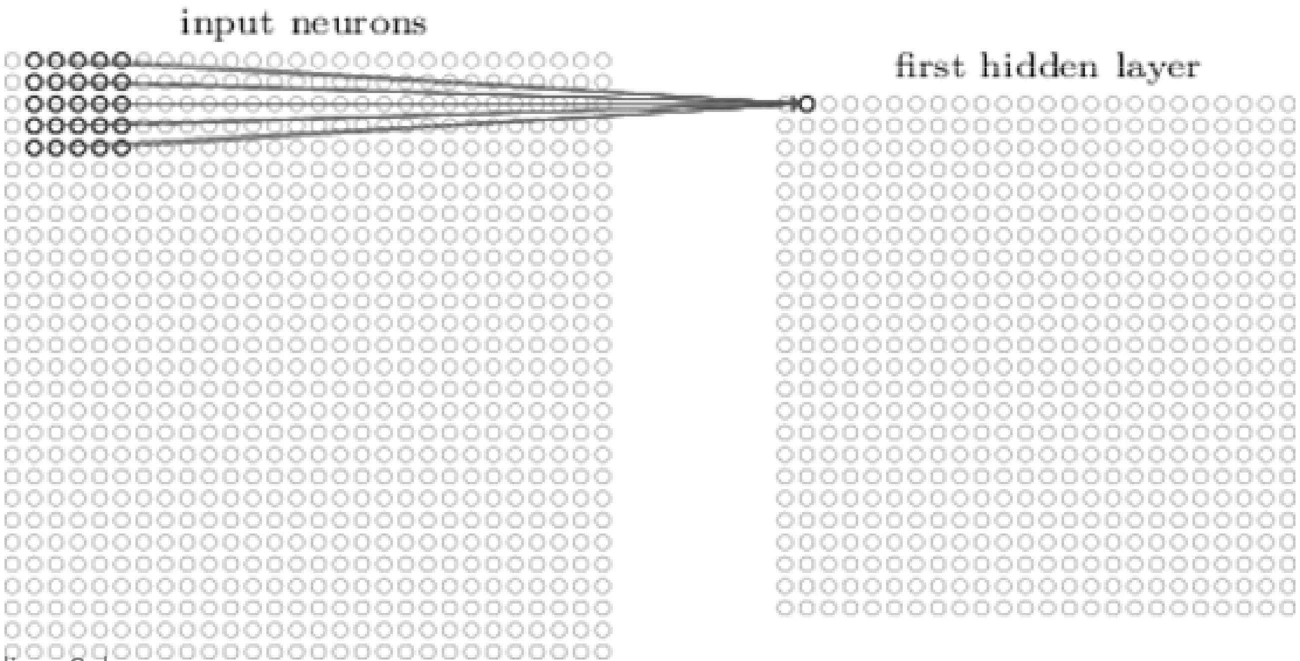
- Basic idea:
  - Pick a 3x3 matrix F of weights
  - Slide this over an image and compute the “inner product” (similarity) of F and the corresponding field of the image, and replace the pixel in the center of the field with the output of the inner product operation

- Key point:
  - Different convolutions extract different types of low-level “features” from an image
  - All that we need to vary to generate these different features is the weights of F

Slide from William Cohen

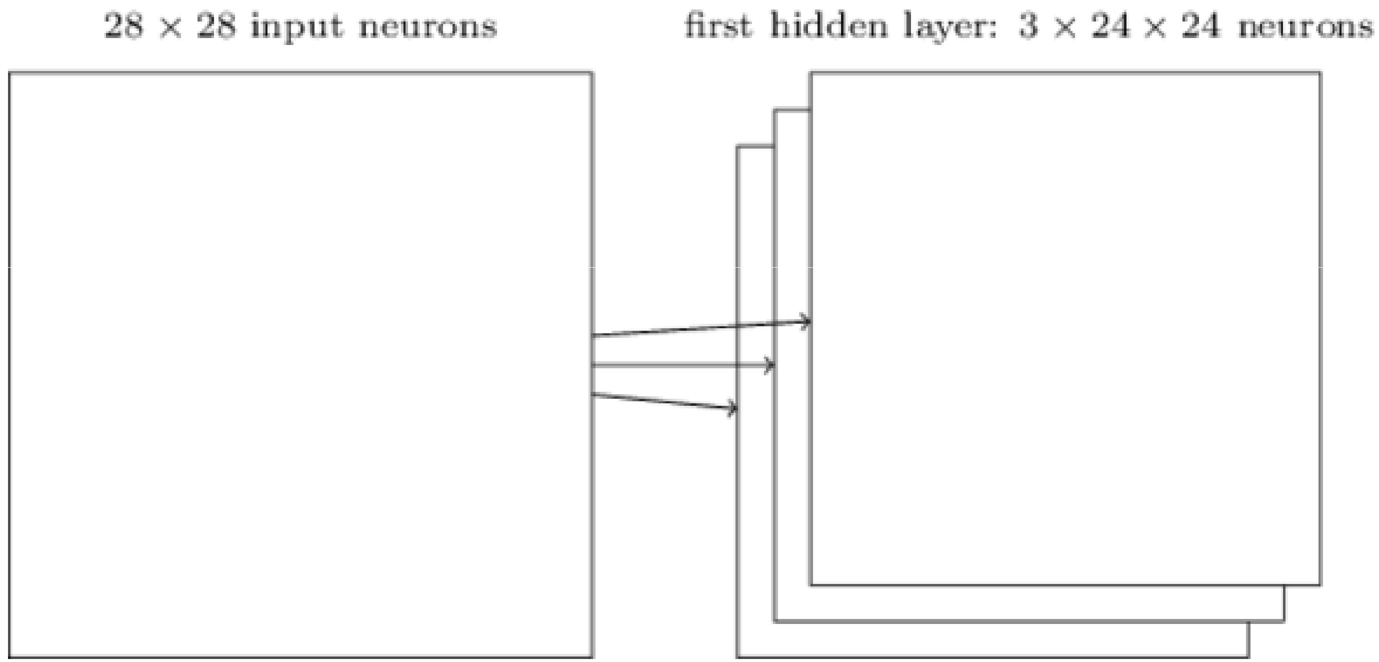
How do we convolve an image with an ANN?

Note that the parameters in the matrix defining the convolution are **tied** across all places that it is used



Slide from William Cohen

How do we do many convolutions of an image with an ANN?



Slide from William Cohen

Convolutional Neural Network (CNN)

Typical layers include:

- Convolutional layer
- Max-pooling layer
- Fully connected layer
- (Nonlinear) Normalization layer
- Softmax

These can be arranged into arbitrarily deep topologies

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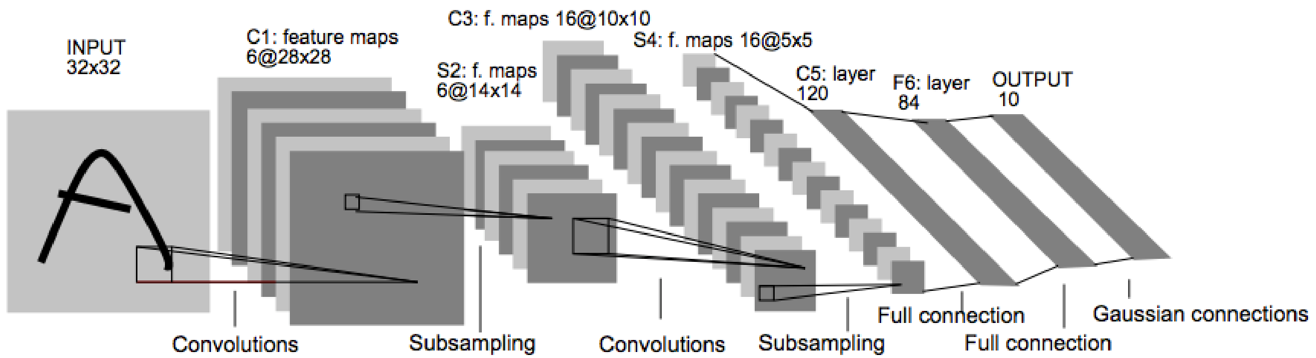


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

Convolution of a Color Image

- Color images consist of 3 floats per pixel for RGB (red, green blue) color values
- Convolution must also be 3-dimensional

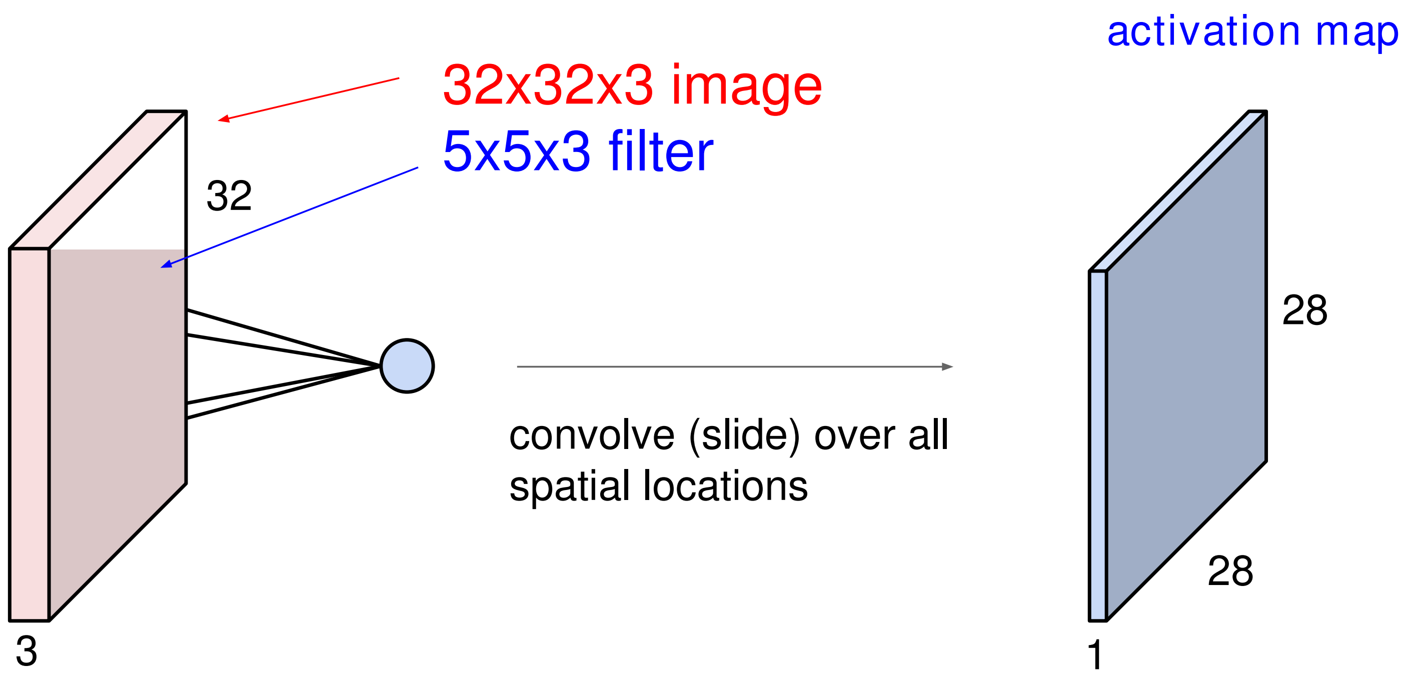
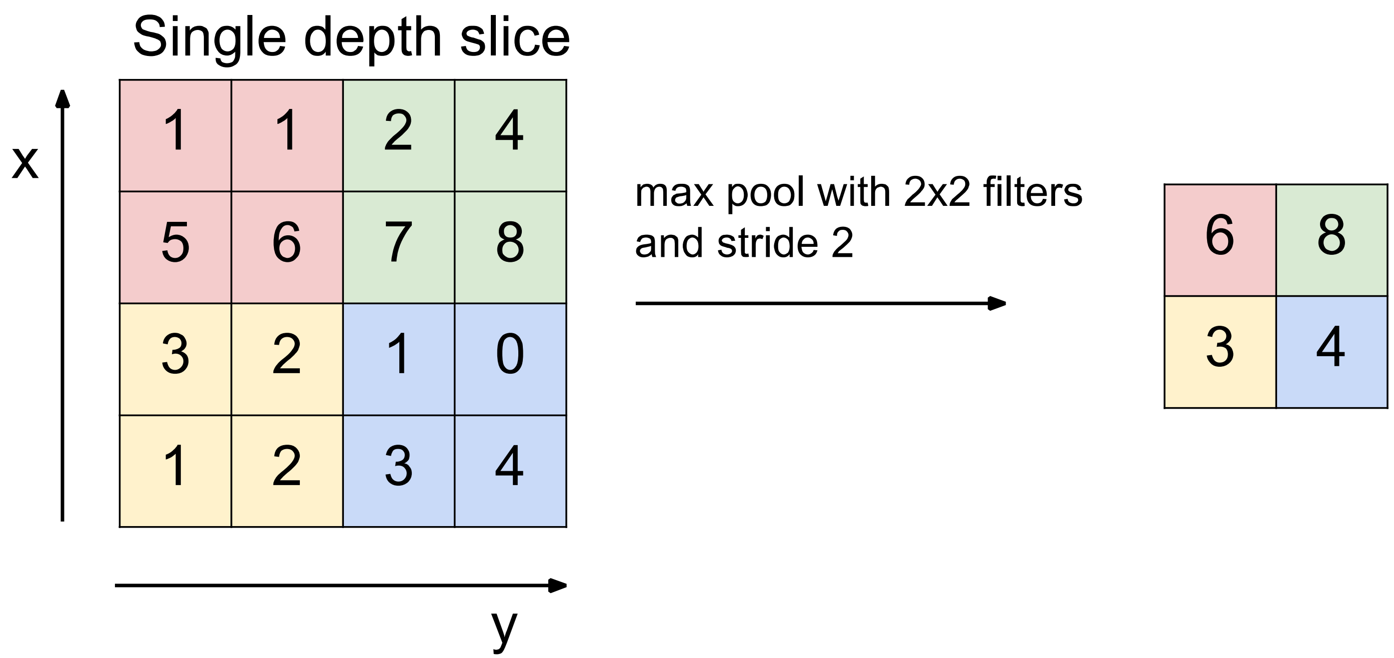


Figure from Fei-Fei Li & Andrej Karpathy & Justin Johnson (CS231N)

Max-pooling



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Max-pooling

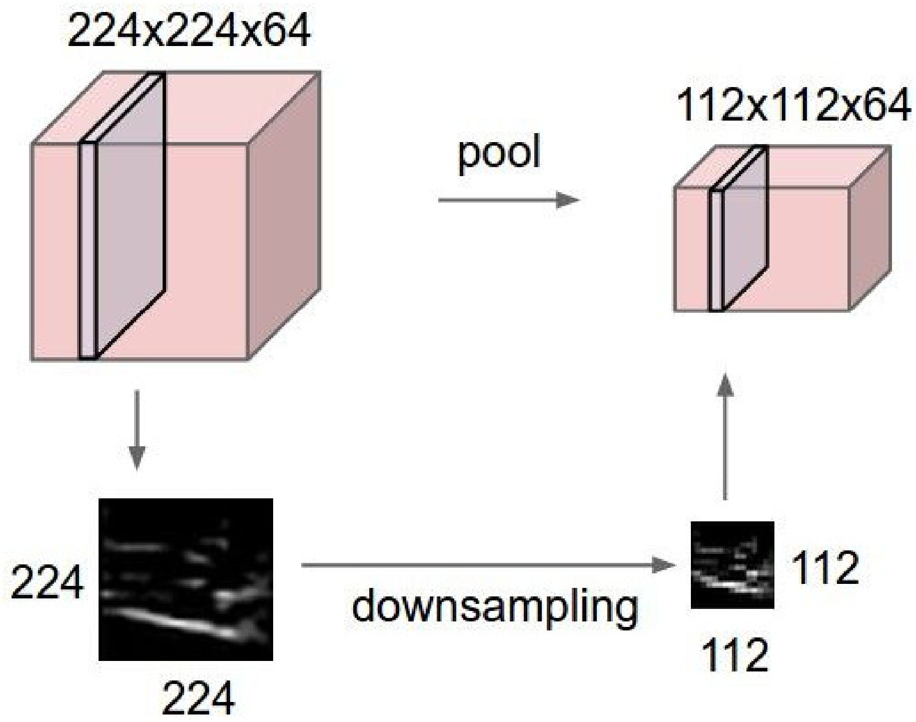

Figure from Fei-Fei Li & Andrej Karpathy & Justin Johnson (CS231N)

Why do max-pooling?

- Saves space
- Reduces overfitting?
- Because I'm going to add *more* convolutions after it!
  - Allows the short-range convolutions to extend over larger subfields of the images
    - So we can spot larger objects
    - Eg, a long horizontal line, or a corner, or …

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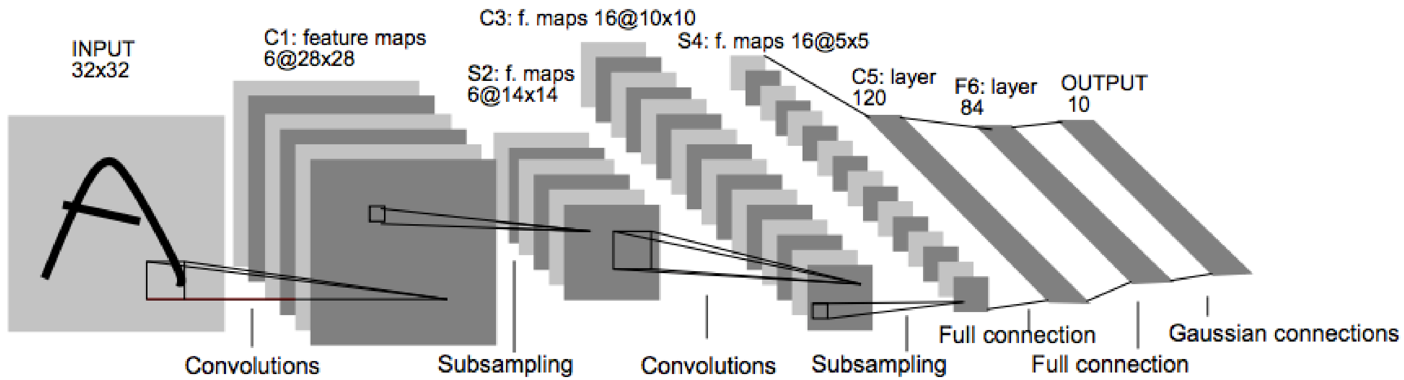

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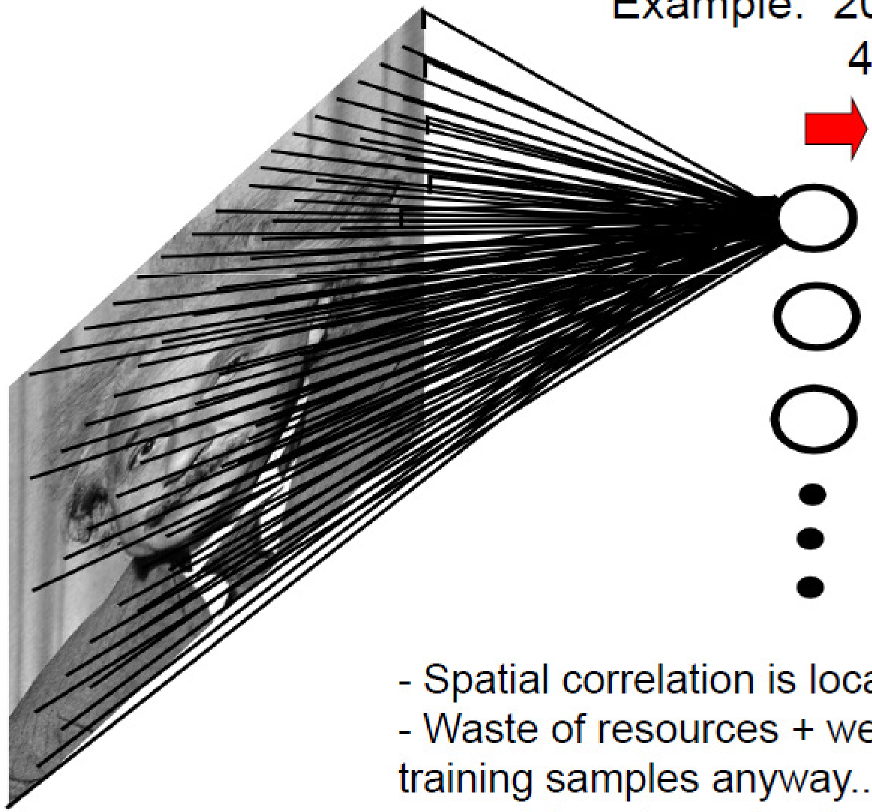
Slide from William Cohen

Naïve CNN Architecture

Fully Connected Layer

Example: 200x200 image
40K hidden units
~2B parameters!!!

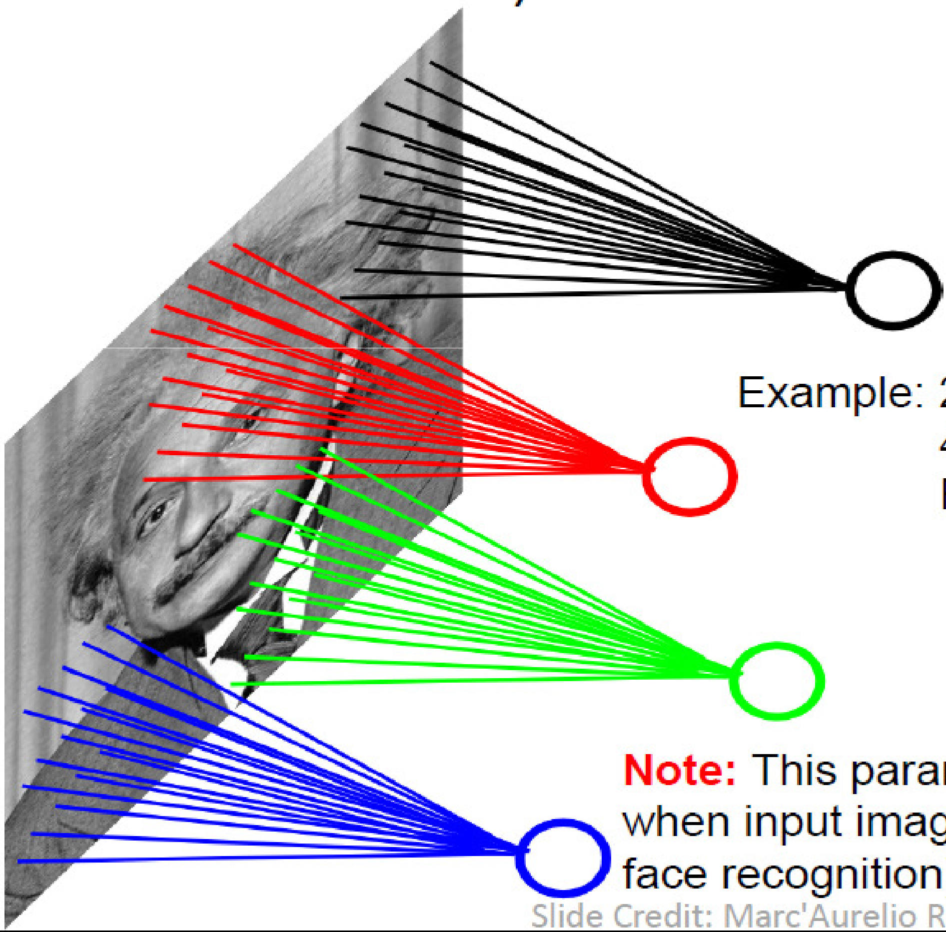


- Spatial correlation is local
- Waste of resources + we have not enough training samples anyway..

Slide Credit: Marc'Aurelio Ranzato

Naïve CNN Architecture

Locally Connected Layer



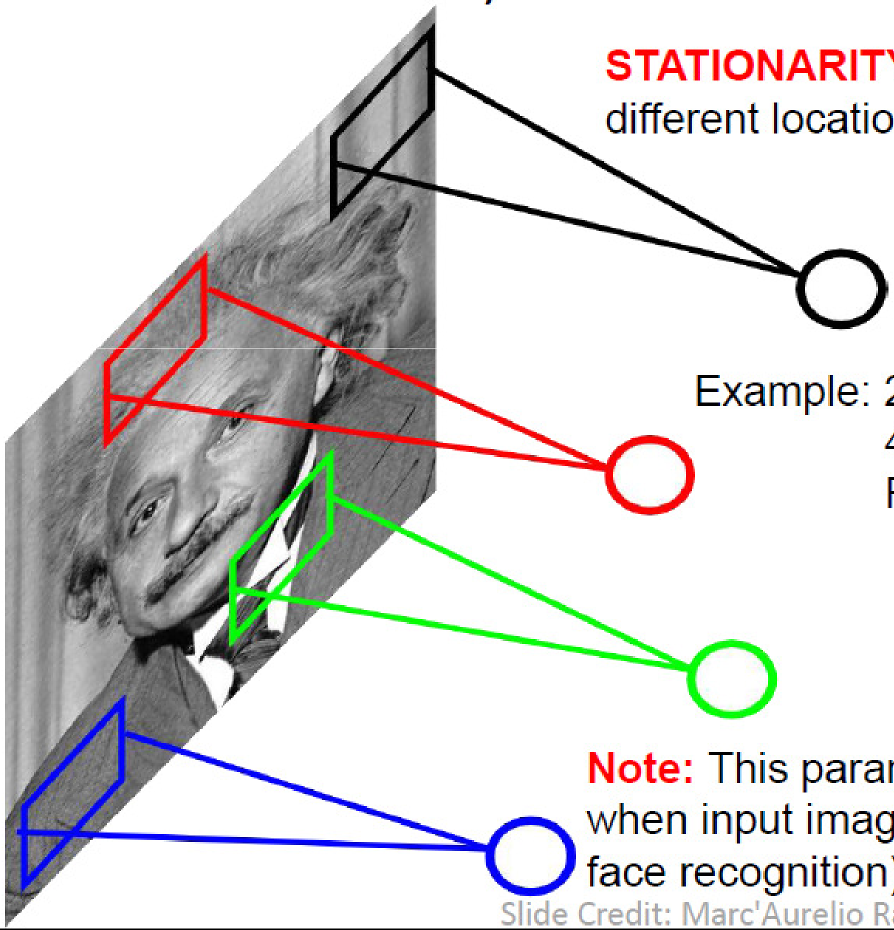
Example: 200x200 image
40K hidden units
Filter size: 10x10
4M parameters

Note: This parameterization is good when input image is registered (e.g., face recognition).

Slide Credit: Marc'Aurelio Ranzato

Naïve CNN Architecture

Locally Connected Layer



STATIONARITY? Statistics is similar at different locations

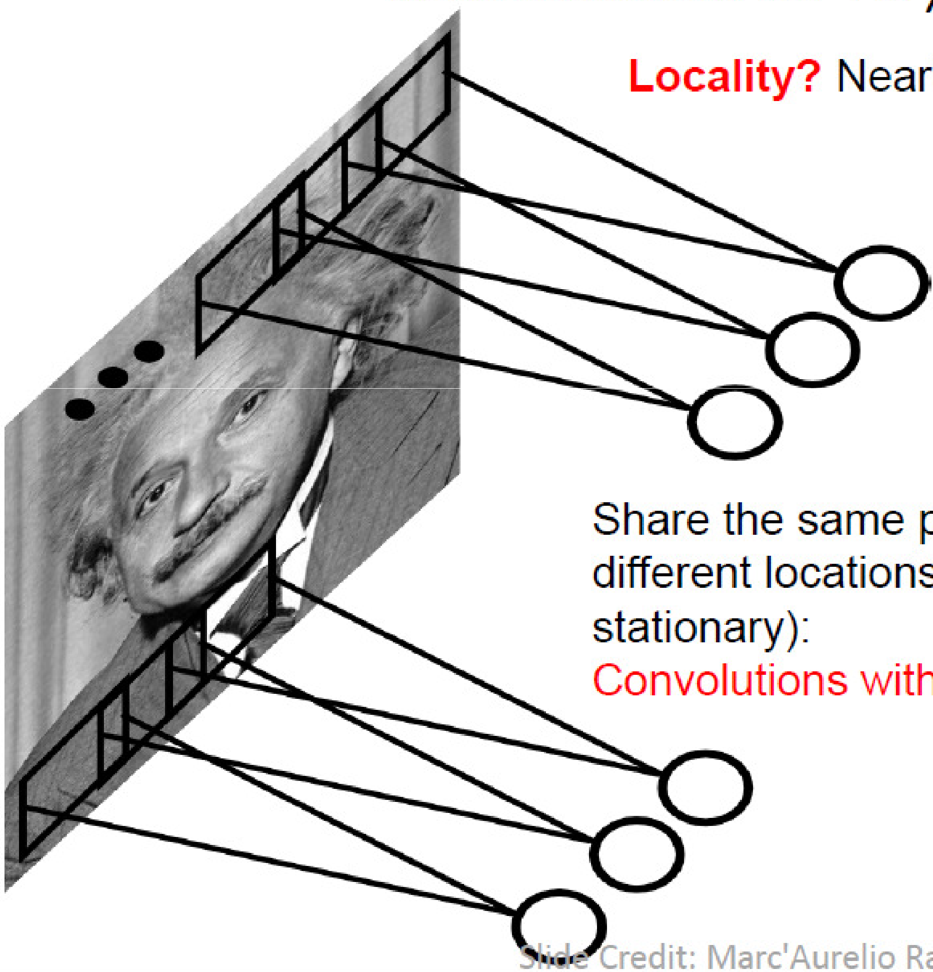
Example: 200x200 image
40K hidden units
Filter size: 10x10
4M parameters

Note: This parameterization is good when input image is registered (e.g., face recognition).

Slide Credit: Marc'Aurelio Ranzato

Naïve CNN Architecture

Convolutional Layer



Locality? Nearby pixels are correlated

Share the same parameters across different locations (assuming input is stationary):
Convolutions with learned kernels

Slide Credit: Marc'Aurelio Ranzato

Convolution: Example

$$* \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} =$$

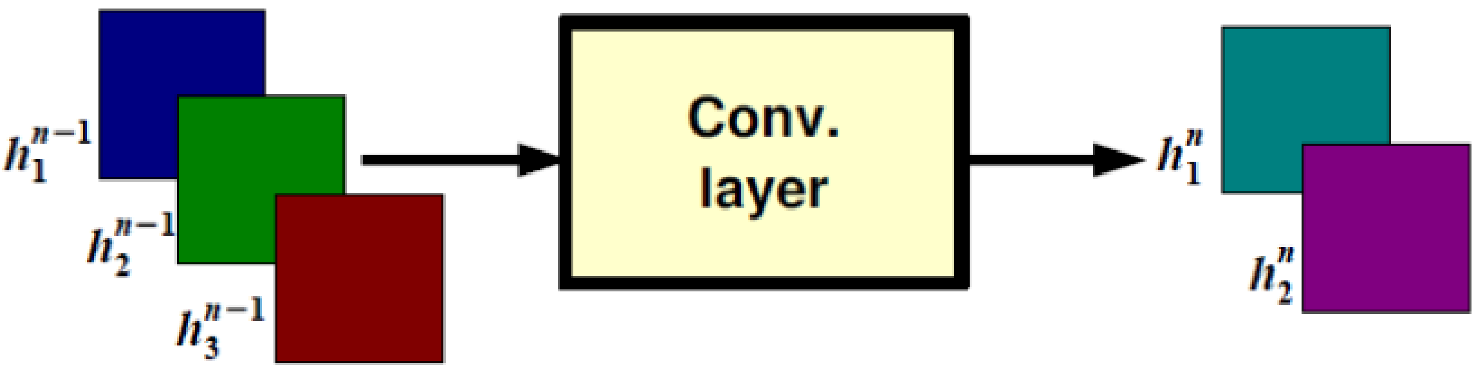
Convolution Layer

$$h_j^n = max\left(0, \sum_{k=1}^{K} h_k^{n-1} * w_{kj}^n\right)$$

output feature map

input feature map

kernel



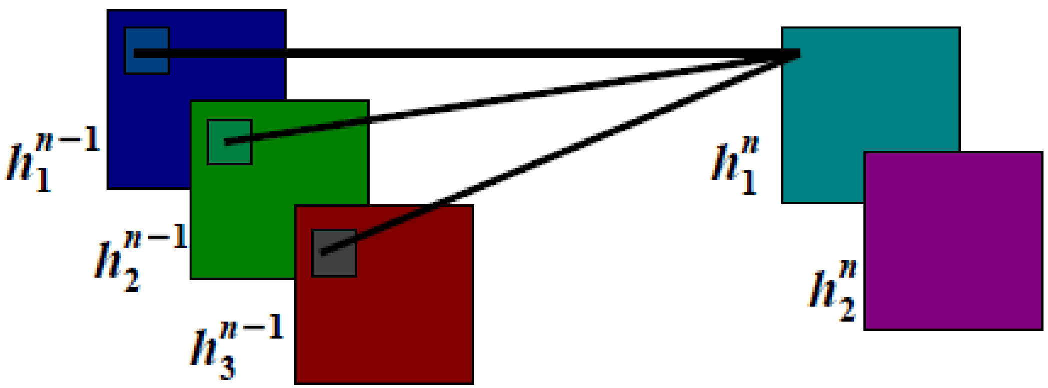
Convolution Layer

$$h_j^n = max\left(0, \sum_{k=1}^{K} h_k^{n-1} * w_{kj}^n\right)$$

output feature map

input feature map

kernel



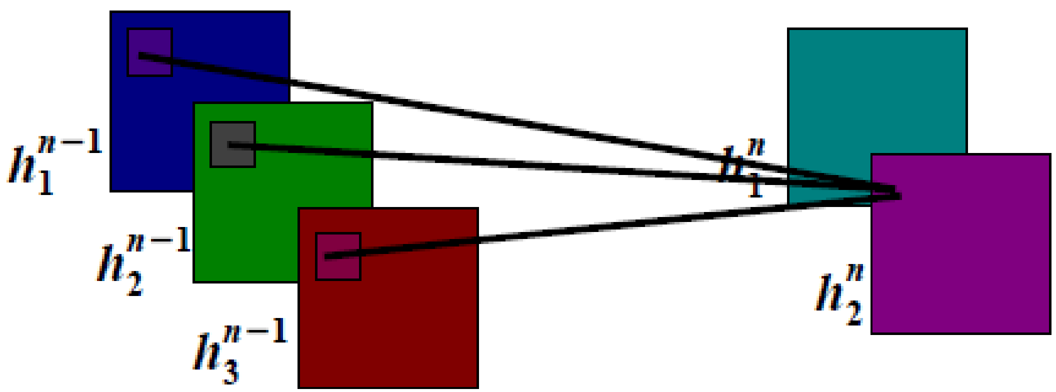
Convolution Layer

$$h_j^n = max\left(0, \sum_{k=1}^{K} h_k^{n-1} * w_{kj}^n\right)$$

output feature map

input feature map

kernel



CNN: Key Ideas

A standard neural net applied to images:

- scales quadratically with the size of the input
- does not leverage stationarity

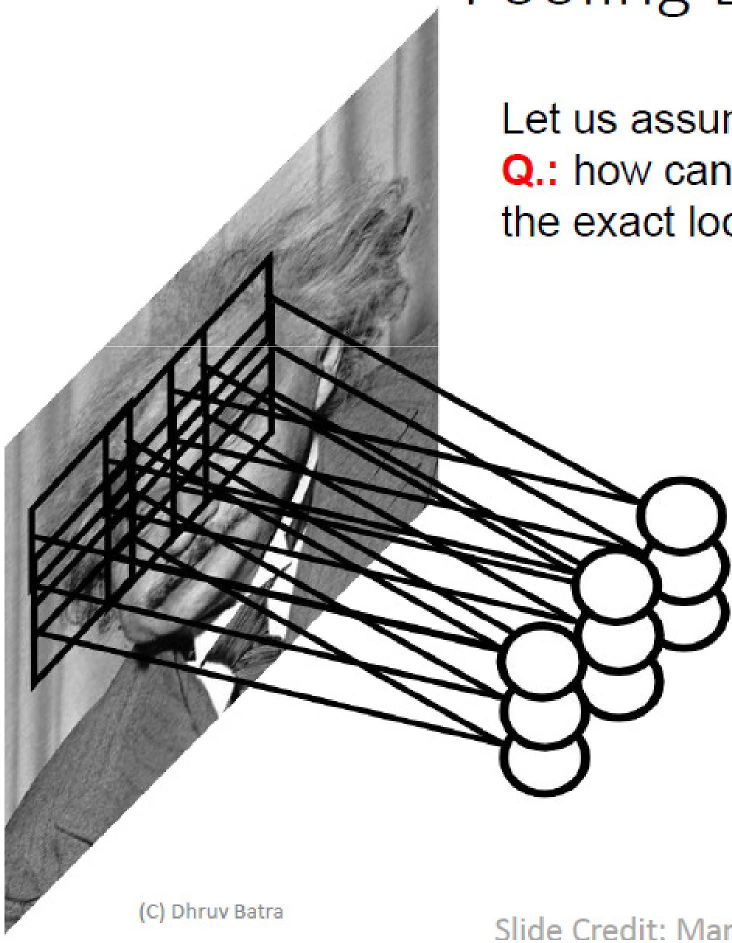
Solution:

- connect each hidden unit to a small patch of the input
- share the weight across space

This is called: **convolutional layer.**
A network with convolutional layers is called **convolutional network.**

Pooling

Pooling Layer

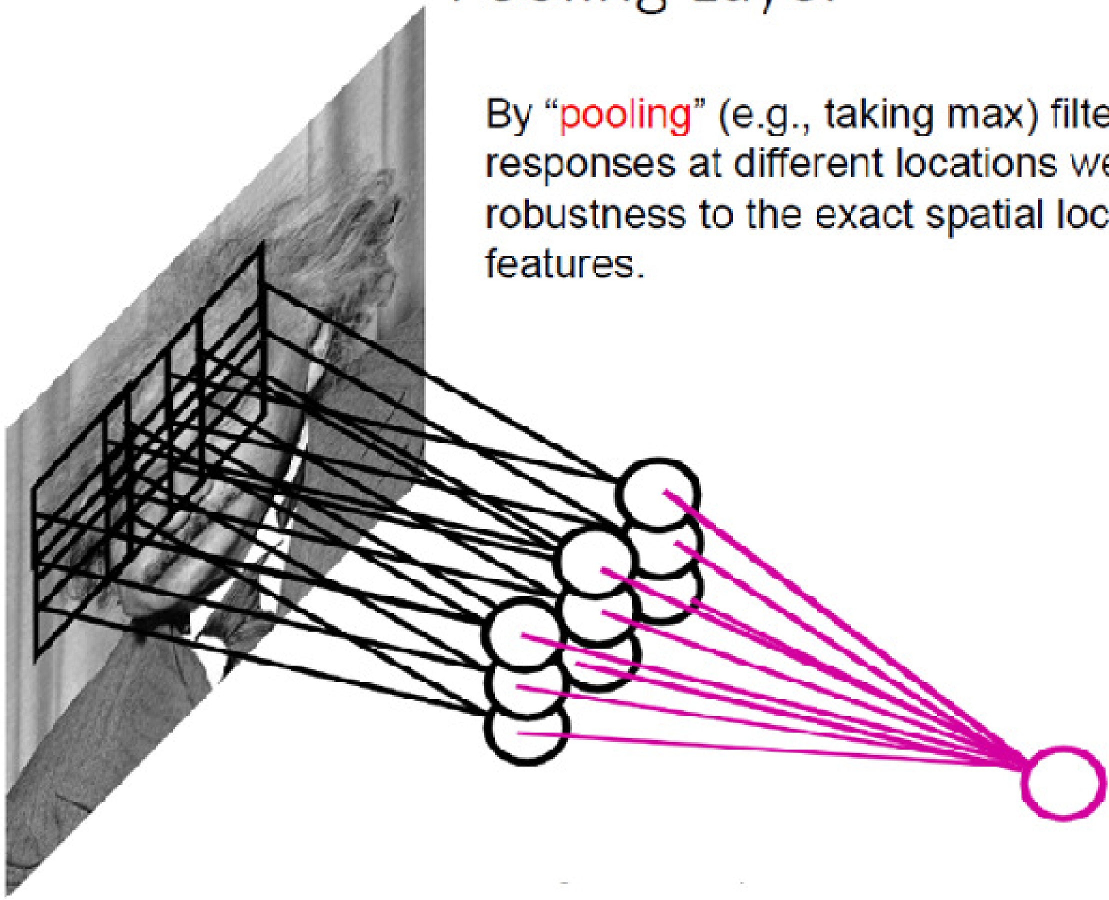


Let us assume filter is an "eye" detector.
Q.: how can we make the detection robust to the exact location of the eye?

Pooling

Pooling Layer

By "pooling" (e.g., taking max) filter responses at different locations we gain robustness to the exact spatial location of features.



Different Pooling

Max-pooling:

$$h_j^n(x, y) = max_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

Average-pooling:

$$h_j^n(x, y) = 1/K \sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

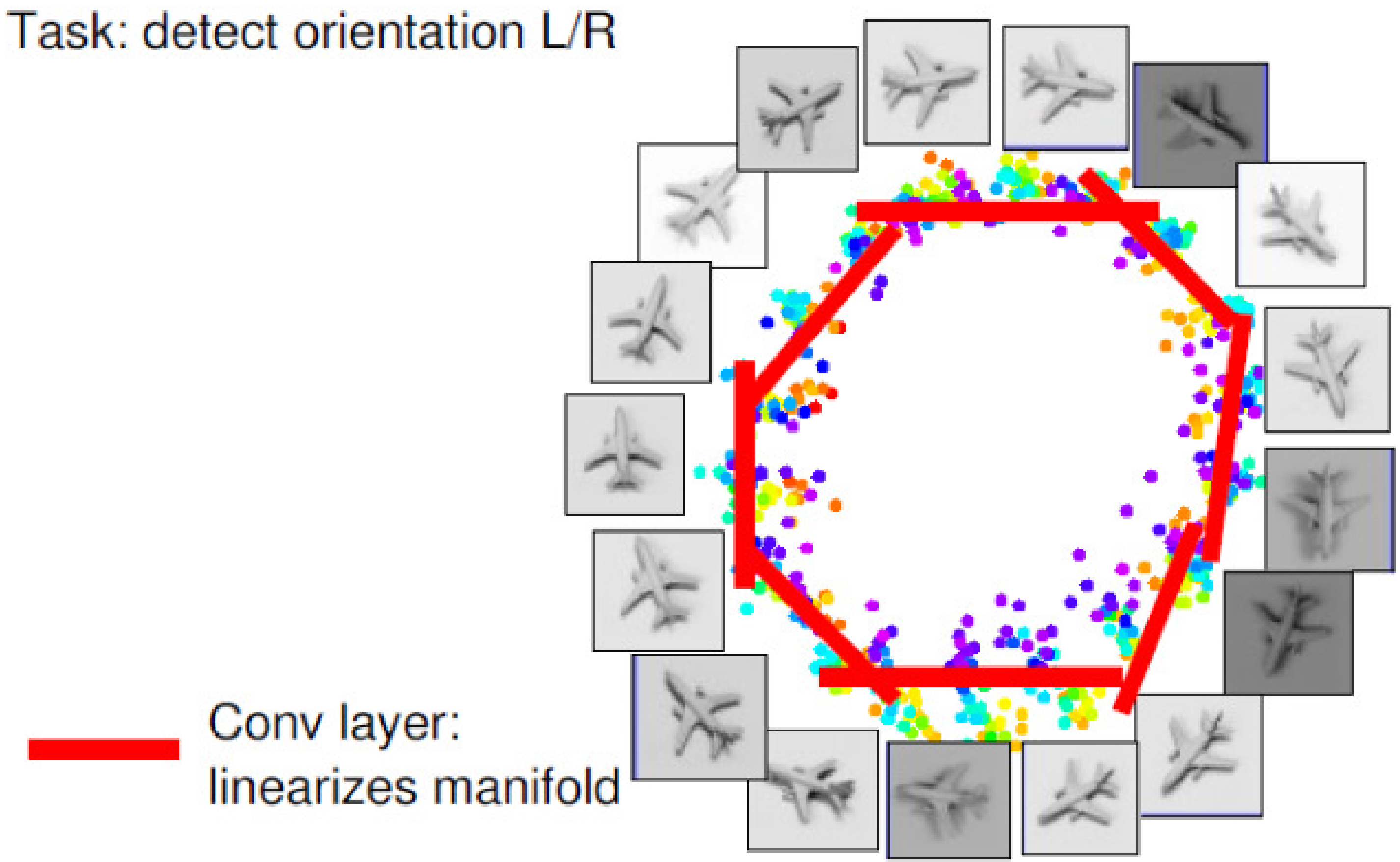
L2-pooling:

$$h_j^n(x, y) = \sqrt{\sum_{\bar{x} \in N(x), \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})^2}$$

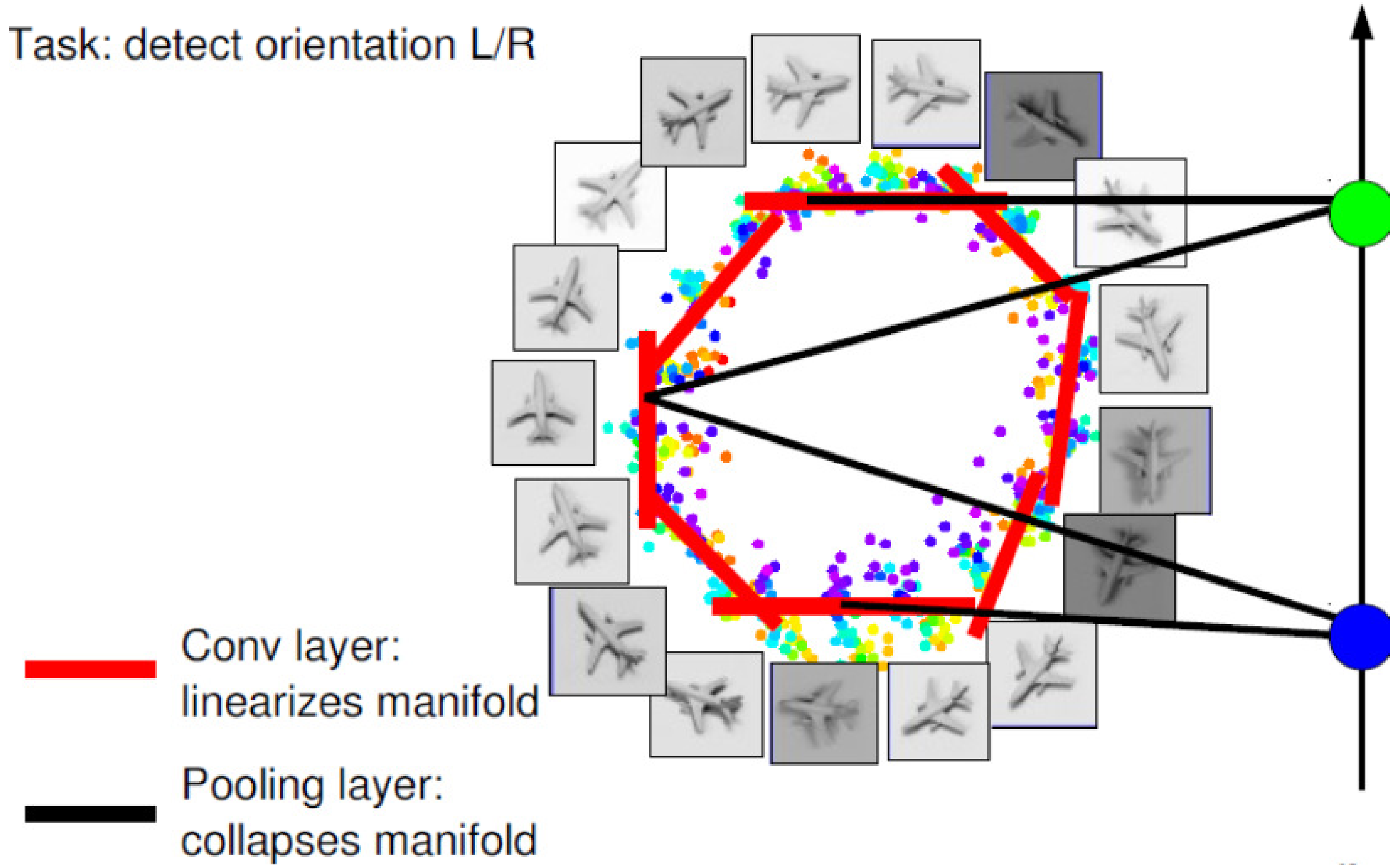
L2-pooling over features:

$$h_j^n(x, y) = \sqrt{\sum_{k \in N(j)} h_k^{n-1}(x, y)^2}$$

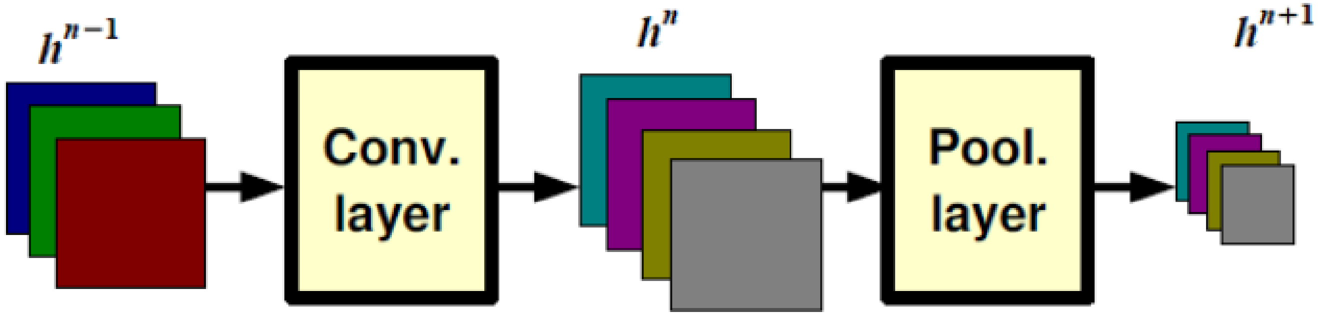
Pooling: Visual Interpretation



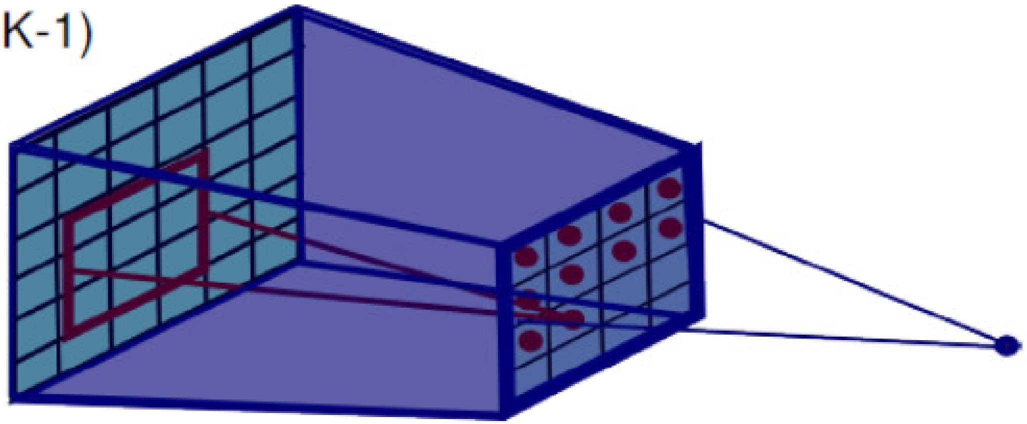
Pooling: Visual Interpretation



Pooling Layer: Receptive Field Size

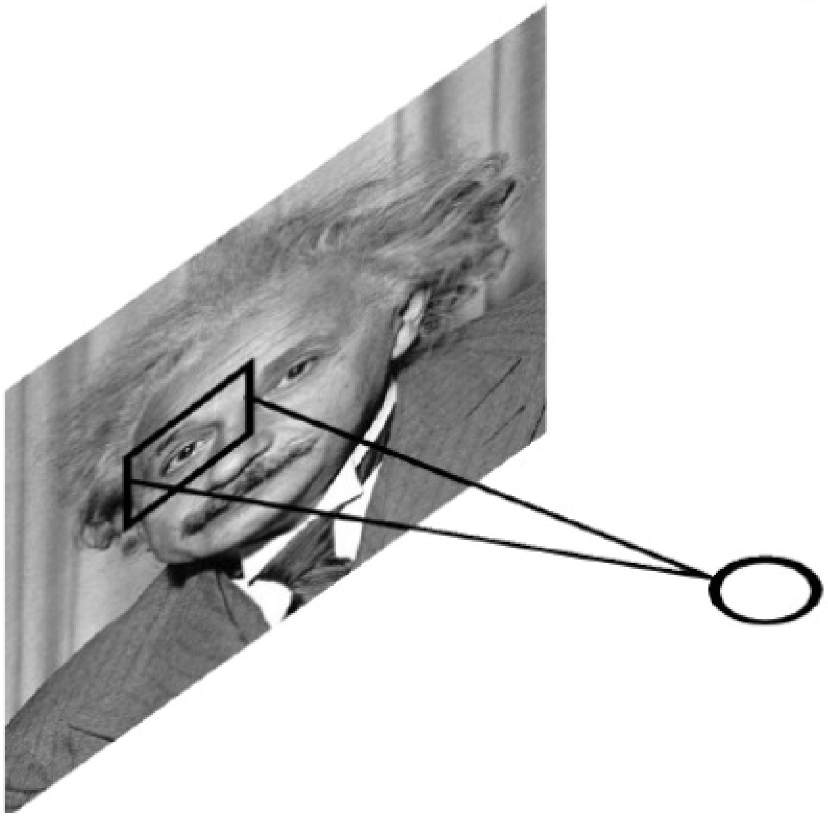


If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size: (P+K-1)x(P+K-1)



Local Contrast Normalization

$$h^{i+1}(x,y)=\frac{h^i(x,y)-m^i(N(x,y))}{\sigma^i(N(x,y))}$$



Local Contrast Normalization

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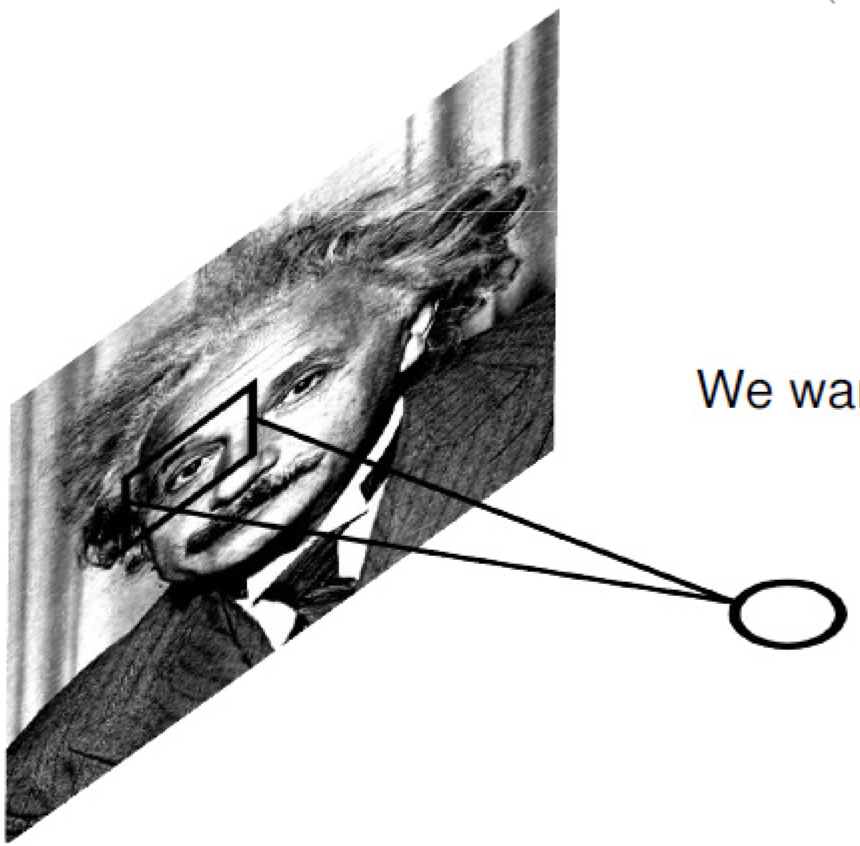
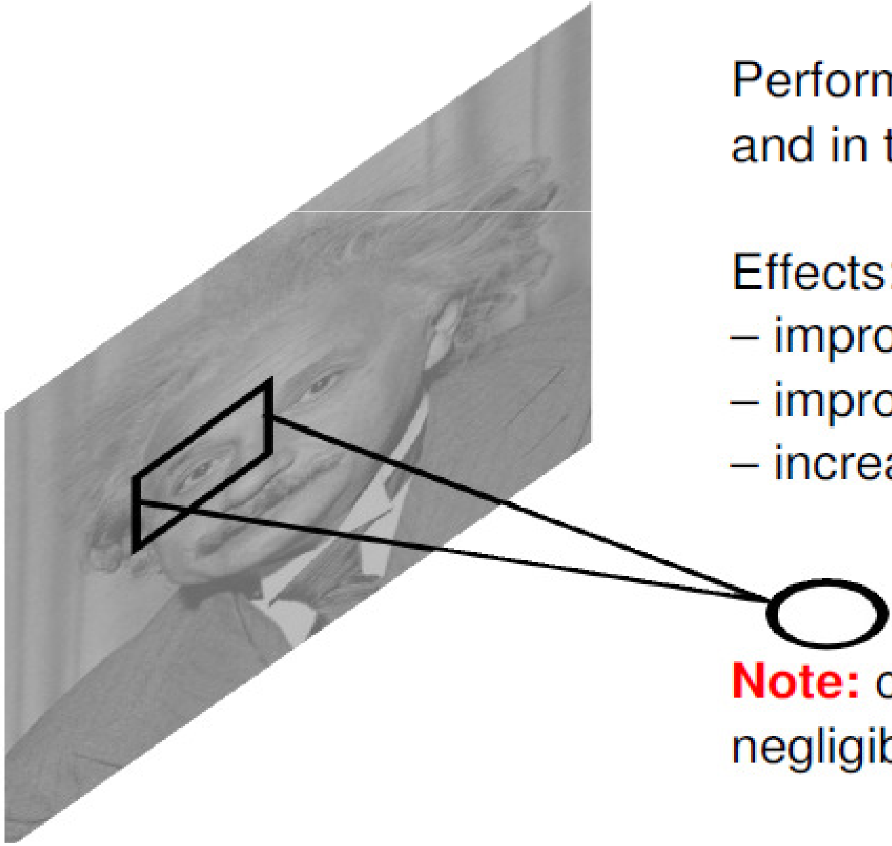


We want the same response.

Local Contrast Normalization

$$h^{i+1}(x,y) = \frac{h^i(x,y) - m^i(N(x,y))}{\sigma^i(N(x,y))}$$

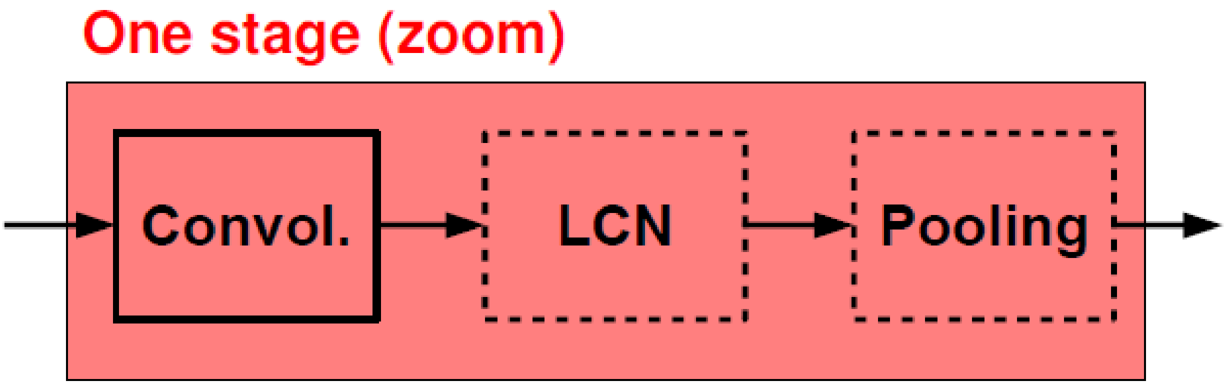


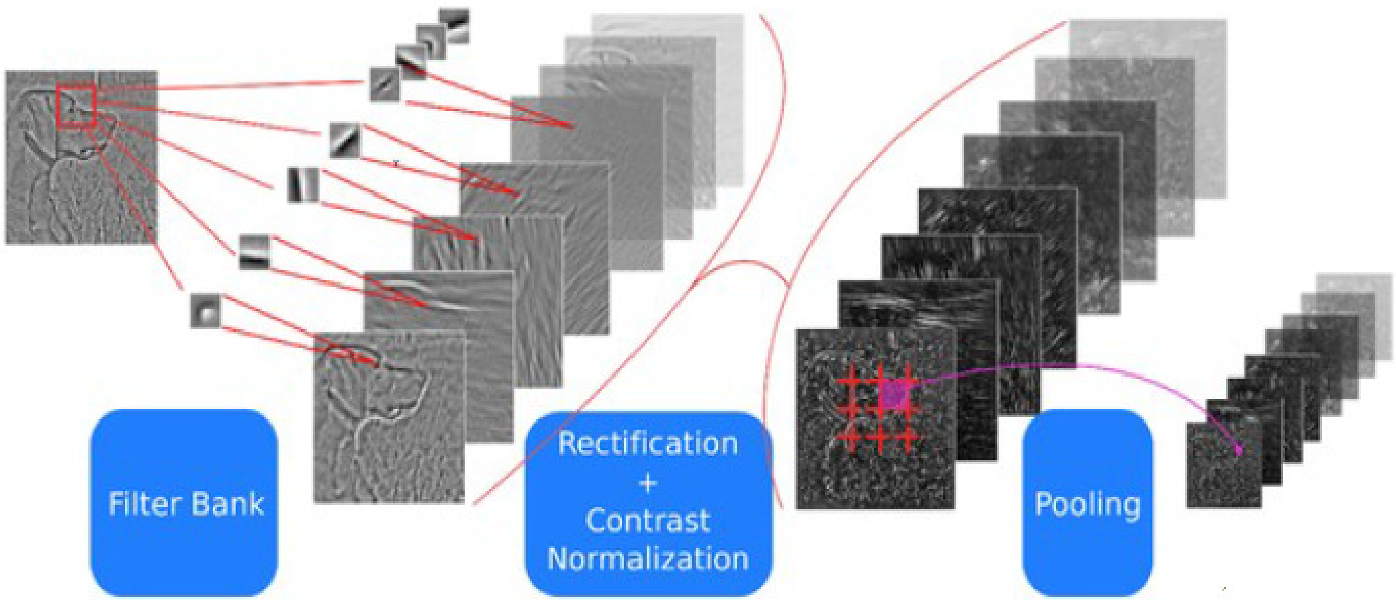
Performed also across features and in the higher layers..

Effects:
– improves invariance
– improves optimization
– increases sparsity

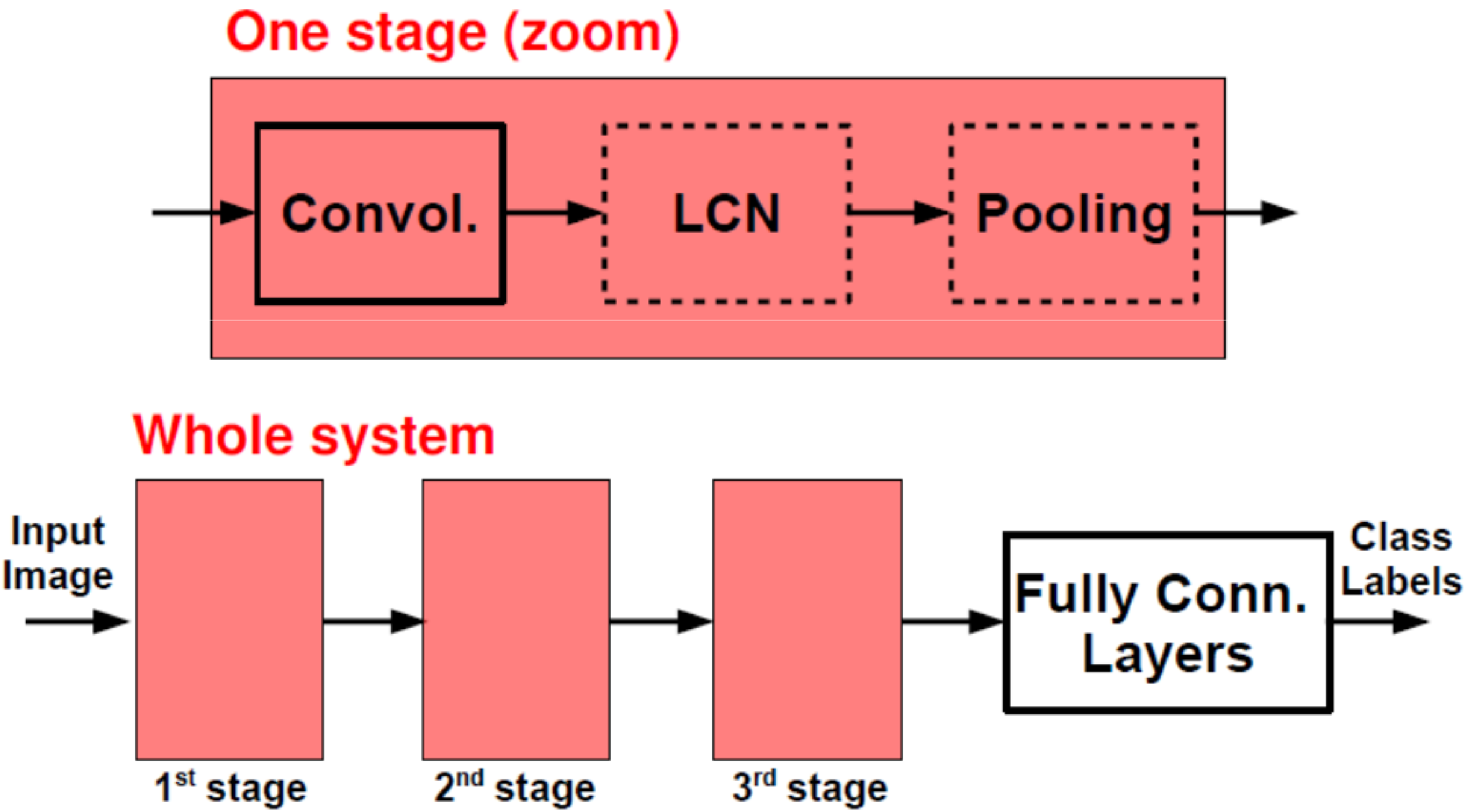
Note: computational cost is negligible w.r.t. conv. layer.

Stages of CNN

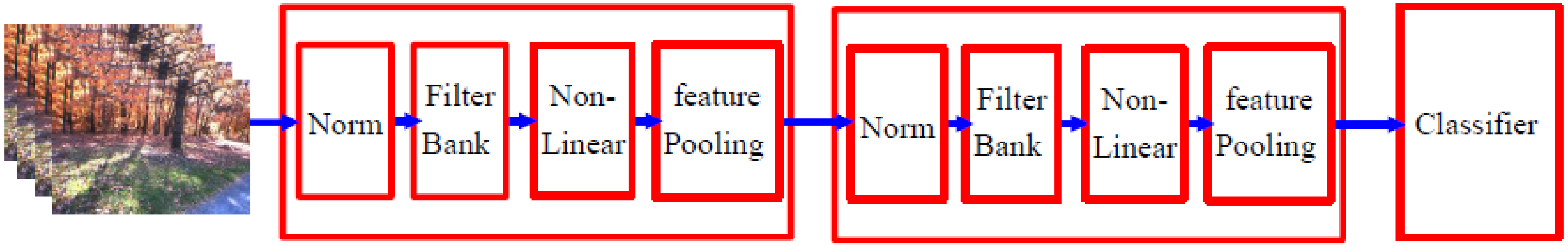




Typical CNN Architecture

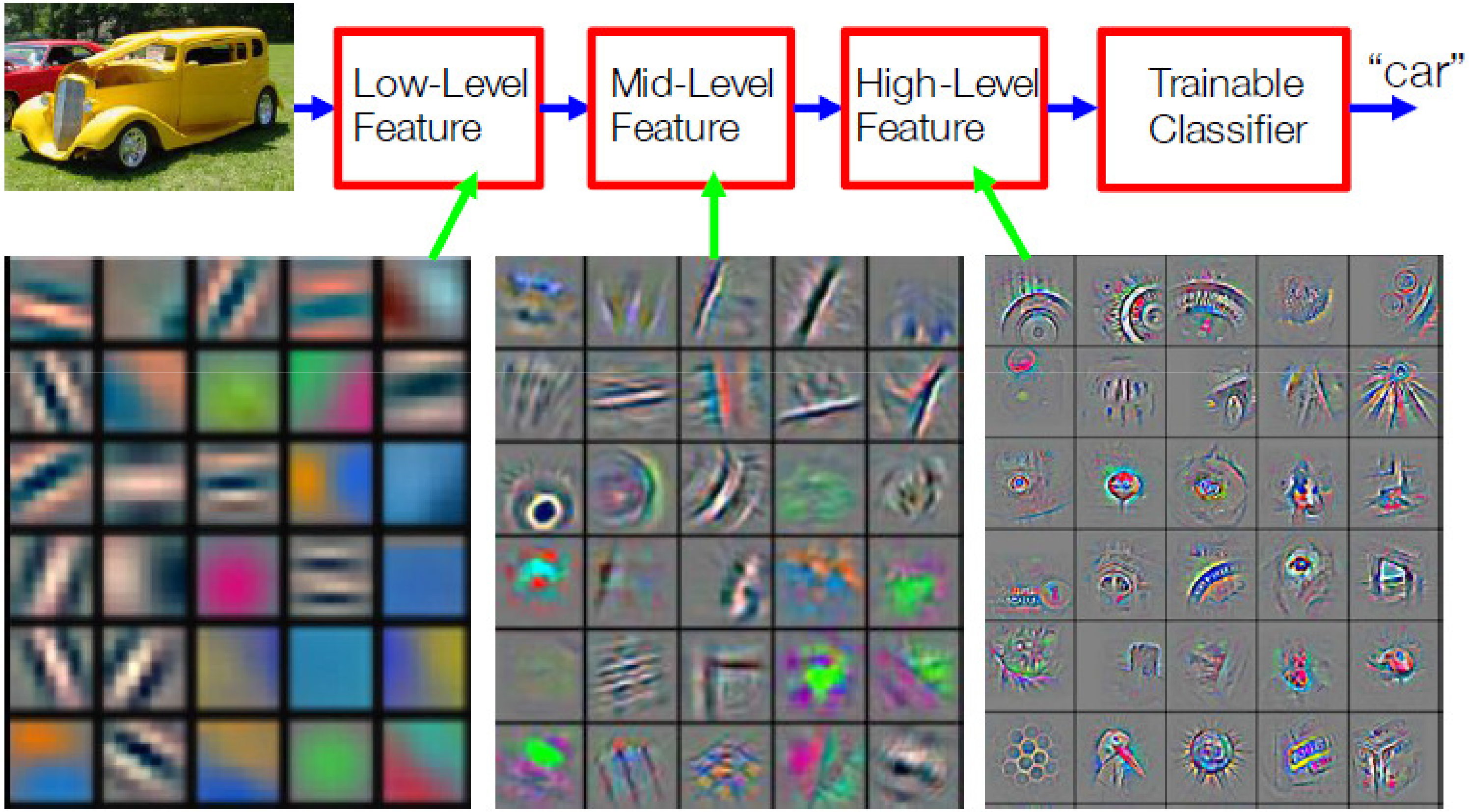


Typical CNN Architecture



- Normalization: eg. Contrast Normalization
- Filter Bank: Matrix Multiplication
- Non-Linearity: eg. ReLU
- Pooling: aggregation over space or feature type

Deep Learning = Hierarchical Compositionality



M.D. Zeiler and R. Fergus, **"Visualizing and Understanding Convolutional Networks"**, In ECCV 2014 Slide credit: Yann LeCun

Three key ideas of deep learning

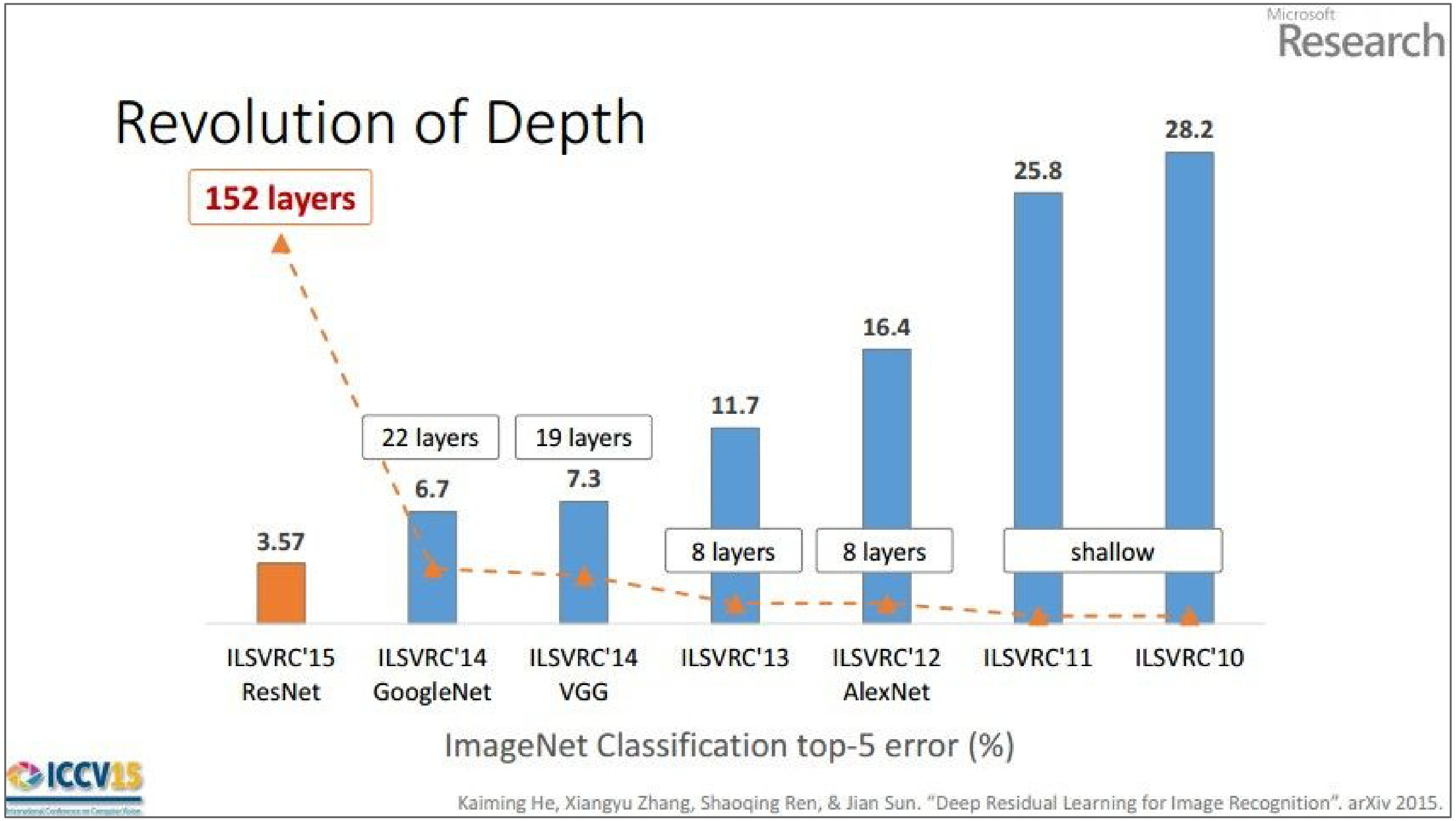
- **(Hierarchical) Compositionality**
  - Cascade of non-linear transformations
  - Multiple layers of representations

- **End-to-End Learning**
  - Learning (goal-driven) representations
  - Learning to feature extract

- **Distributed Representations**
  - No single neuron “encodes” everything
  - Groups of neurons work together

Slide credit: Dhruv Batra

CNNs for Image Recognition



Slide from Kaiming He

to continue…