Floyd-Warshall Algorithm Lecture 09(a)

Ritu Kundu

King's College London

Tue, Nov 28, 2017

1 / 15

- All Pair Shortest Path Distance
 - The Floyd-Warshall Algorithm

- Transitive Closure
 - The adapted Floyd-Warshall Algorithm

Outline

- All Pair Shortest Path Distance
 - The Floyd-Warshall Algorithm

- Transitive Closure
 - The adapted Floyd-Warshall Algorithm

Introduction

Problem

Given a Weighted Directed Graph G=(V,E) with a weight function $w:E\to\Re$ where \Re is the set of real numbers, find the length of the **shortest path** between all pairs of vertices in G.

Input

```
Given: G = (V, E) and w : E \to \Re such that |V| = n. Format: G as adjacency matrix of weights W such that W[i][j] = w_{ij} \begin{cases} 0 & \text{if } (i = j) \\ \text{weight on edge } (i,j) & \text{if } i \neq j \text{ and } (i,j) \in E \\ \infty & \text{if } i \neq j \text{ and } (i,j) \notin E \end{cases}
```

Output

 $n \times n$ matrix D such that $D[i][j] = d_{ij}$ where d_{ij} is the length of the shortest path (or shortest distance) between the vertices i and j.

Assumption: no negative cycle in G.



Notations

Definition: Intermediate Vertices

For a path $P = \langle v_1, v_2, \dots, v_l \rangle$, an **intermediate vertex** is any vertex of P other than v_1 or v_l .

 d_{ij}^k

- Let d^k_{ij} = shortest distance (path-weight) of any path i → j with all intermediate vertices in {1,2,···, k}.
- $d_{ii}^0 = w_{ij}$ as no intermediate vertices.
- d_{ii}^n = shortest distance (path-weight) between i and j.

 D^k

- $D^0 = W$
- O $D^n = D$

Recurrence Relation

Consider a shortest path $i \stackrel{P}{\iff} j$ such that all intermediate vertices are in $\{1, 2, \cdots, k\}$ for $(k \ge 1)$ 2 possibilities:

- k is not an intermediate vertex $\Rightarrow P$ has shortest distance $= d_{ii}^{k-1}$.
- k is an intermediate vertex $\Rightarrow i \xrightarrow{p_1} k \xrightarrow{p_2} j$ such that all intermediate vertices of p_1 and p_2 are in $\{1, 2, \dots, k-1\}$ $\Rightarrow P$ has shortest distance $= d_{ik}^{k-1} + d_{ki}^{k-1}$.

For
$$k \ge 1$$

 $d_{ij}^k = \min(d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1})$



Recurrence Relation

Consider a shortest path $i \stackrel{P}{\iff} j$ such that all intermediate vertices are in $\{1, 2, \cdots, k\}$ for $(k \ge 1)$ 2 possibilities:

- k is not an intermediate vertex $\Rightarrow P$ has shortest distance $= d_{ii}^{k-1}$.
- k is an intermediate vertex $\Rightarrow i \xrightarrow{p_1} k \xrightarrow{p_2} j$ such that all intermediate vertices of p_1 and p_2 are in $\{1, 2, \dots, k-1\}$ $\Rightarrow P$ has shortest distance $= d_{ik}^{k-1} + d_{ki}^{k-1}$.

For
$$k \ge 1$$

 $d_{ij}^k = \min(d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1})$



Pseudocode

Algorithm 1 Floyd-Warshall Algo

```
procedure All Pairs Hortest Distance (W,n) D^0 \leftarrow W *** Initialization *** for k \leftarrow 1 to n do for i \leftarrow 1 to n do for j \leftarrow 1 to n do d_{ij} \leftarrow \min(d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1}) end for end for Return D^n end procedure
```

7 / 15

Pseudocode

Algorithm 2 Floyd-Warshall Algo

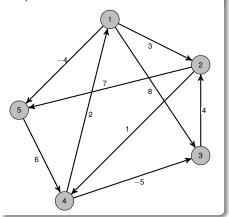
```
procedure AllPairShortestDistance(W, n) D^0 \leftarrow W *** Initialization *** for k \leftarrow 1 to n do for i \leftarrow 1 to n do for j \leftarrow 1 to n do d_{ij} \leftarrow \min(d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1}) end for end for Return D^n end procedure
```

Analysis

```
Time Complexity: \Theta(n^3)
Space Complexity: \Theta(n^3); but reuse to get \Theta(n^2) (Drop Superscript k)
```

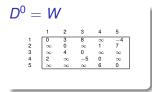
Example

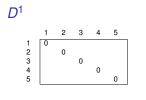




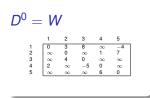
Weight Matrix

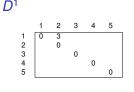
	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	$8 \\ \infty \\ 0 \\ -5$	1	7
3	∞	4	0	∞	∞
4	2	∞	-5	0	∞
5	∞	∞	∞	6	0





$$d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3$$



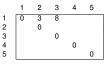


$$D^0 = W$$

$$\begin{smallmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 0 & 3 & 8 & \infty & -4 \\ 2 & \infty & 0 & \infty & 1 & 7 \\ 3 & \infty & 4 & 0 & \infty & \infty \\ 4 & 2 & \infty & -5 & 0 & \infty \\ 5 & \infty & \infty & \infty & 6 & 0 \end{smallmatrix}$$

$$\begin{array}{l} \textit{d}_{12}^{1} = \min(\textit{d}_{12}^{0}, \textit{d}_{11}^{0} + \textit{d}_{12}^{0}) = \min(3, 0 + 3) = \min(3, 3) = 3 \\ \textit{d}_{13}^{1} = \min(\textit{d}_{13}^{0}, \textit{d}_{11}^{0} + \textit{d}_{13}^{0}) = \min(8, 0 + 8) = \min(8, 8) = 8 \end{array}$$

2



$$D^0 = W$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 3 & 8 & \infty & -4 \\ 2 & \infty & 0 & \infty & 1 & 7 \\ 3 & 0 & 4 & 0 & \infty & \infty \\ 4 & 2 & \infty & -5 & 0 & \infty \\ 5 & \infty & \infty & \infty & 6 & 0 \end{bmatrix}$$

$$\begin{split} & d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3 \\ & d_{13}^1 = \min(d_{13}^0, d_{11}^0 + d_{13}^0) = \min(8, 0 + 8) = \min(8, 8) = 8 \\ & d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty \end{split}$$

1 2 3 4 5
1 0 3 8 ∞ 2 0 0 3
4 0 0 5

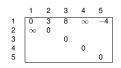
$$\begin{split} & d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3 \\ & d_{13}^1 = \min(d_{13}^0, d_{11}^0 + d_{13}^0) = \min(8, 0 + 8) = \min(8, 8) = 8 \\ & d_{14}^1 = \min(d_{14}^0, d_{11}^0, d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty \\ & d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4 \end{split}$$



$$D^{0} = W$$

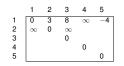
$$\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
0 & 3 & 8 & \infty & -4 \\
2 & \infty & 0 & \infty & 1 & 7 \\
3 & \infty & 4 & 0 & \infty & \infty \\
4 & 2 & \infty & -5 & 0 & \infty \\
5 & \infty & \infty & \infty & 6 & 0
\end{bmatrix}$$

$$\begin{array}{l} d_{12}^1=\min(d_{12}^0,d_{11}^0+d_{12}^0)=\min(3,0+3)=\min(3,3)=3\\ d_{13}^1=\min(d_{13}^0,d_{11}^0+d_{13}^0)=\min(8,0+8)=\min(8,8)=8\\ d_{14}^1=\min(d_{14}^0,d_{11}^0+d_{14}^0)=\min(\infty,0+\infty)=\min(\infty,\infty)=\infty\\ d_{15}^1=\min(d_{15}^0,d_{11}^0+d_{15}^0)=\min(-4,0+-4)=\min(-4,-4)=-4\\ d_{21}^1=\min(d_{21}^0,d_{21}^0+d_{11}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty \end{array}$$





$$\begin{array}{l} d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3 \\ d_{13}^1 = \min(d_{13}^0, d_{11}^0 + d_{13}^0) = \min(8, 0 + 8) = \min(8, 8) = 8 \\ d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty \\ d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4 \\ d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty \\ d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty \end{array}$$





$$\begin{array}{l} d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3 \\ d_{13}^1 = \min(d_{13}^0, d_{11}^0 + d_{13}^0) = \min(8, 0 + 8) = \min(8, 8) = 8 \\ d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^1) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty \\ d_{15}^1 = \min(d_{16}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4 \\ d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty \\ d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty \\ d_{24}^1 = \min(d_{24}^0, d_{21}^0 + d_{14}^0) = \min(1, \infty + \infty) = \min(1, \infty) = 1 \\ \end{array}$$

$$\begin{array}{l} d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3 \\ d_{13}^1 = \min(d_{13}^0, d_{11}^0 + d_{13}^0) = \min(8, 0 + 8) = \min(8, 8) = 8 \\ d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty \\ d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4 \\ d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty \\ d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty \\ d_{24}^1 = \min(d_{24}^0, d_{21}^0 + d_{14}^0) = \min(1, \infty + \infty) = \min(1, \infty) = 1 \\ d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(7, \infty + -4) = \min(7, \infty) = 7 \end{array}$$

$$\begin{array}{ll} d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3 \\ d_{13}^1 = \min(d_{13}^0, d_{11}^0 + d_{13}^0) = \min(8, 0 + 8) = \min(8, 8) = 8 \\ d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty \\ d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4 \\ d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty \\ d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty \\ d_{24}^1 = \min(d_{24}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = 1 \\ d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(1, \infty + \infty) = \min(1, \infty) = 1 \\ d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(7, \infty + -4) = \min(7, \infty) = 7 \\ d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{12}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty \end{array}$$

$$\begin{array}{l} d_{12}^1=\min(d_{12}^0,d_{11}^0+d_{12}^0)=\min(3,0+3)=\min(3,3)=3\\ d_{13}^1=\min(d_{13}^0,d_{11}^0+d_{13}^0)=\min(8,0+8)=\min(8,8)=8\\ d_{14}^1=\min(d_{14}^0,d_{11}^0+d_{14}^0)=\min(\infty,0+\infty)=\min(\infty,\infty)=\infty\\ d_{15}^1=\min(d_{15}^0,d_{11}^1+d_{15}^0)=\min(-4,0+0)=\min(-4,-4)=-4\\ d_{21}^1=\min(d_{21}^0,d_{21}^0+d_{11}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{23}^1=\min(d_{23}^0,d_{21}^0+d_{13}^0)=\min(\infty,\infty+8)=\min(\infty,\infty)=\infty\\ d_{24}^1=\min(d_{24}^0,d_{21}^0+d_{14}^0)=\min(1,\infty+\infty)=\min(1,\infty)=1\\ d_{25}^1=\min(d_{25}^0,d_{21}^0+d_{15}^0)=\min(1,\infty+4)=\min(1,\infty)=7\\ d_{31}^1=\min(d_{31}^0,d_{31}^0+d_{11}^1)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{29}^1=\min(d_{39}^0,d_{31}^0+d_{11}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{29}^1=\min(d_{29}^0,d_{31}^0+d_{12}^0)=\min(4,\infty+3)=\min(4,\infty)=4\\ \end{array}$$

$$D^{\prime}$$

$$\begin{array}{l} d_{12}^1=\min(d_{12}^0,d_{11}^0+d_{12}^0)=\min(3,0+3)=\min(3,3)=3\\ d_{13}^1=\min(d_{13}^0,d_{11}^0+d_{13}^0)=\min(8,0+8)=\min(8,8)=8\\ d_{14}^1=\min(d_{14}^0,d_{11}^0+d_{14}^0)=\min(\infty,0+\infty)=\min(\infty,\infty)=\infty\\ d_{15}^1=\min(d_{15}^0,d_{11}^0+d_{14}^0)=\min(-4,0+-4)=\min(-4,-4)=-4\\ d_{21}^1=\min(d_{21}^0,d_{21}^0+d_{13}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{23}^1=\min(d_{23}^0,d_{21}^0+d_{13}^0)=\min(\infty,\infty+8)=\min(\infty,\infty)=\infty\\ d_{24}^1=\min(d_{24}^0,d_{21}^0+d_{14}^0)=\min(1,\infty+\infty)=\min(1,\infty)=1\\ d_{25}^1=\min(d_{22}^0,d_{21}^0+d_{14}^0)=\min(1,\infty+\infty)=\min(1,\infty)=1\\ d_{25}^1=\min(d_{22}^0,d_{21}^0+d_{14}^0)=\min(1,\infty+0)=\min(1,\infty)=7\\ d_{31}^1=\min(d_{31}^0,d_{31}^0+d_{11}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{32}^1=\min(d_{32}^0,d_{31}^0+d_{12}^0)=\min(4,\infty+3)=\min(4,\infty)=4\\ d_{34}^1=\min(d_{34}^0,d_{31}^0+d_{14}^0)=\min(\infty,\infty+\infty)=\min(\infty,\infty)=\infty\\ d_{34}^1=\min(d_{34}^0,d_{31}^0+d_{14}^0)=\min(\infty,\infty+\infty)=\min(\infty,\infty)=\infty\\ \end{array}$$

$$D^1$$

$$\begin{array}{l} d_{12}^1=\min(d_{12}^0,d_{11}^0+d_{12}^0)=\min(3,0+3)=\min(3,3)=3\\ d_{13}^1=\min(d_{13}^0,d_{11}^0+d_{13}^0)=\min(8,0+8)=\min(8,8)=8\\ d_{14}^1=\min(d_{14}^0,d_{11}^0+d_{14}^0)=\min(\infty,0+\infty)=\min(\infty,\infty)=\infty\\ d_{15}^1=\min(d_{15}^0,d_{11}^0+d_{15}^0)=\min(-4,0+-4)=\min(-4,-4)=-4\\ d_{21}^1=\min(d_{21}^0,d_{21}^0+d_{11}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{23}^1=\min(d_{23}^0,d_{21}^0+d_{13}^0)=\min(\infty,\infty+8)=\min(\infty,\infty)=\infty\\ d_{24}^1=\min(d_{24}^0,d_{21}^0+d_{14}^0)=\min(1,\infty,\infty+8)=\min(1,\infty)=1\\ d_{25}^1=\min(d_{25}^0,d_{21}^0+d_{14}^0)=\min(1,\infty+\infty)=\min(1,\infty)=1\\ d_{25}^1=\min(d_{25}^0,d_{21}^0+d_{15}^0)=\min(1,\infty+4)=\min(1,\infty)=1\\ d_{31}^1=\min(d_{31}^0,d_{31}^0+d_{11}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{32}^1=\min(d_{32}^0,d_{31}^0+d_{12}^0)=\min(4,\infty+3)=\min(4,\infty)=4\\ d_{34}^1=\min(d_{34}^0,d_{31}^0+d_{13}^0)=\min(\infty,\infty+\infty)=\min(\infty,\infty)=\infty\\ d_{35}^1=\min(d_{34}^0,d_{31}^0+d_{14}^0)=\min(\infty,\infty+\infty)=\min(\infty,\infty)=\infty\\ d_{36}^1=\min(d_{36}^0,d_{31}^0+d_{15}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{36}^1=\min(d_{36}^0,d_{36}^0)=\lim_{\alpha}^0=\lim_{\alpha}^$$

$$D^0 = W$$

$${\begin{smallmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 3 & 8 & \infty & -4 \\ 2 & \infty & 0 & \infty & 1 & 7 \\ 3 & \infty & 4 & 0 & \infty & \infty \\ 4 & 2 & \infty & -5 & 0 & \infty \\ 5 & \infty & \infty & \infty & 6 & 0 \\ \end{smallmatrix}$$

$$D^1$$

$$\begin{array}{l} d_{12}^1=\min(d_{12}^0,d_{11}^0+d_{12}^0)=\min(3,0+3)=\min(3,3)=3\\ d_{13}^1=\min(d_{13}^0,d_{11}^0+d_{13}^0)=\min(8,0+8)=\min(8,8)=8\\ d_{14}^1=\min(d_{14}^0,d_{11}^0+d_{14}^0)=\min(\infty,0+\infty)=\min(\infty,\infty)=\infty\\ d_{15}^1=\min(d_{15}^0,d_{11}^0+d_{14}^0)=\min(-4,0+-4)=\min(-4,-4)=-4\\ d_{21}^1=\min(d_{21}^0,d_{21}^0+d_{11}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{23}^1=\min(d_{23}^0,d_{21}^0+d_{11}^0)=\min(\infty,\infty+8)=\min(\infty,\infty)=\infty\\ d_{24}^1=\min(d_{24}^0,d_{21}^0+d_{14}^0)=\min(1,\infty+\infty)=\min(1,\infty)=1\\ d_{25}^1=\min(d_{25}^0,d_{21}^0+d_{15}^0)=\min(7,\infty+4)=\min(7,\infty)=7\\ d_{31}^1=\min(d_{25}^0,d_{21}^0+d_{15}^0)=\min(7,\infty+4)=\min(7,\infty)=7\\ d_{31}^1=\min(d_{31}^0,d_{31}^0+d_{11}^0)=\min(\infty,\infty+0)=\min(\infty,\infty)=\infty\\ d_{32}^1=\min(d_{32}^0,d_{31}^0+d_{12}^0)=\min(4,\infty+3)=\min(4,\infty)=4\\ d_{34}^1=\min(d_{34}^0,d_{31}^0+d_{14}^0)=\min(\infty,\infty+\infty)=\min(\infty,\infty)=\infty\\ d_{35}^1=\min(d_{35}^0,d_{31}^0+d_{15}^0)=\min(\infty,\infty+4)=\min(\infty,\infty)=\infty\\ d_{41}^1=\min(d_{31}^0,d_{31}^0+d_{15}^0)=\min(\infty,\infty+4)=\min(\infty,\infty)=\infty\\ d_{41}^1=\min(d_{31}^0,d_{31}^0+d_{15}^0)=\min(\infty,\infty+4)=\min(\infty,\infty)=\infty\\ d_{41}^1=\min(d_{31}^0,d_{31}^0+d_{15}^0)=\min(\infty,\infty+4)=\min(\infty,\infty)=\infty\\ d_{41}^1=\min(d_{31}^0,d_{31}^0+d_{15}^0)=\min(\infty,\infty+4)=\min(\infty,\infty)=\infty\\ d_{41}^1=\min(d_{31}^0,d_{31}^0+d_{15}^0)=\min(\infty,\infty+4)=\min(2,2)=2\\ \end{array}$$

$$D^1$$

```
d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3
d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(8, 0 + 8) = \min(8, 8) = 8
d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty
d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty
d_{24}^1 = \min(d_{24}^0, d_{21}^0 + d_{14}^0) = \min(1, \infty + \infty) = \min(1, \infty) = 1
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(7, \infty + -4) = \min(7, \infty) = 7
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{32}^1 = \min(d_{32}^0, d_{31}^0 + d_{12}^0) = \min(4, \infty + 3) = \min(4, \infty) = 4
d_{34}^1 = \min(d_{34}^0, d_{31}^0 + d_{14}^0) = \min(\infty, \infty + \infty) = \min(\infty, \infty) = \infty
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(\infty, \infty + -4) = \min(\infty, \infty) = \infty
d_{41}^{1} = \min(d_{41}^{0}, d_{41}^{0} + d_{11}^{0}) = \min(2, 2 + 0) = \min(2, 2) = 2
d_{42}^1 = \min(d_{42}^0, d_{41}^0 + d_{12}^0) = \min(\infty, 2+3) = \min(\infty, 5) = 5
```

$$D^1$$

```
d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3
d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(8, 0 + 8) = \min(8, 8) = 8
d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty
d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty
d_{24}^1 = \min(d_{24}^0, d_{21}^0 + d_{14}^0) = \min(1, \infty + \infty) = \min(1, \infty) = 1
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(7, \infty + -4) = \min(7, \infty) = 7
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{32}^1 = \min(d_{32}^0, d_{31}^0 + d_{12}^0) = \min(4, \infty + 3) = \min(4, \infty) = 4
d_{34}^1 = \min(d_{34}^0, d_{31}^0 + d_{14}^0) = \min(\infty, \infty + \infty) = \min(\infty, \infty) = \infty
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(\infty, \infty + -4) = \min(\infty, \infty) = \infty
d_{41}^{1} = \min(d_{41}^{0}, d_{41}^{0} + d_{11}^{0}) = \min(2, 2 + 0) = \min(2, 2) = 2
d_{42}^1 = \min(d_{42}^0, d_{41}^0 + d_{12}^0) = \min(\infty, 2+3) = \min(\infty, 5) = 5
d_{43}^1 = \min(d_{43}^0, d_{41}^0 + d_{13}^0) = \min(-5, 2 + 8) = \min(-5, 10) = -5
```

$$D^1$$

$$\begin{array}{l} d_{12}^1=\min(d_{12}^0,\,d_{11}^0+\,d_{12}^0)=\min(3,\,0+3)=\min(3,\,3)=3\\ d_{13}^1=\min(d_{13}^0,\,d_{11}^0+\,d_{13}^0)=\min(8,\,0+8)=\min(8,\,8)=8\\ d_{14}^1=\min(d_{14}^0,\,d_{11}^0+\,d_{14}^0)=\min(\infty,\,0+\infty)=\min(\infty,\,\infty)=\infty\\ d_{15}^1=\min(d_{15}^0,\,d_{11}^0+\,d_{15}^0)=\min(-4,\,0+-4)=\min(-4,\,-4)=-4\\ d_{21}^1=\min(d_{21}^0,\,d_{21}^0+\,d_{11}^0)=\min(\infty,\,\infty+0)=\min(\infty,\,\infty)=\infty\\ d_{23}^1=\min(d_{23}^0,\,d_{21}^0+\,d_{13}^0)=\min(\infty,\,\infty+8)=\min(\infty,\,\infty)=\infty\\ d_{24}^1=\min(d_{24}^0,\,d_{21}^0+\,d_{13}^0)=\min(\infty,\,\infty+8)=\min(1,\,\infty)=1\\ d_{25}^1=\min(d_{25}^0,\,d_{21}^0+\,d_{15}^0)=\min(7,\,\infty+4)=\min(7,\,\infty)=7\\ d_{31}^1=\min(d_{31}^0,\,d_{31}^0+\,d_{11}^0)=\min(\infty,\,\infty+0)=\min(\infty,\,\infty)=\infty\\ d_{32}^1=\min(d_{32}^0,\,d_{31}^0+\,d_{11}^0)=\min(\infty,\,\infty+0)=\min(\infty,\,\infty)=\infty\\ d_{31}^1=\min(d_{32}^0,\,d_{31}^0+\,d_{11}^0)=\min(\infty,\,\infty+0)=\min(\infty,\,\infty)=\infty\\ d_{32}^1=\min(d_{32}^0,\,d_{31}^0+\,d_{12}^0)=\min(\infty,\,\infty+0)=\min(\infty,\,\infty)=\infty\\ d_{34}^1=\min(d_{33}^0,\,d_{31}^0+\,d_{14}^0)=\min(\infty,\,\infty+0)=\min(\infty,\,\infty)=\infty\\ d_{41}^1=\min(d_{41}^0,\,d_{41}^0+\,d_{11}^0)=\min(\infty,\,\infty+0)=\min(\infty,\,\infty)=\infty\\ d_{41}^1=\min(d_{41}^0,\,d_{41}^0+\,d_{11}^0)=\min(\infty,\,2+0)=\min(\infty,\,2)=2\\ d_{42}^1=\min(d_{42}^0,\,d_{41}^1+\,d_{12}^0)=\min(\infty,\,2+3)=\min(\infty,\,5)=5\\ d_{43}^1=\min(d_{43}^0,\,d_{41}^0+\,d_{13}^0)=\min(-5,\,2+8)=\min(-5,\,10)=-5\\ d_{45}^1=\min(d_{45}^0,\,d_{41}^0+\,d_{15}^0)=\min(\infty,\,2+4)=\min(\infty,\,-2)=-2\\ \end{array}$$

$$D^1$$

```
d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3
d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(8, 0 + 8) = \min(8, 8) = 8
d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty
d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty
d_{24}^1 = \min(d_{24}^0, d_{21}^0 + d_{14}^0) = \min(1, \infty + \infty) = \min(1, \infty) = 1
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(7, \infty + -4) = \min(7, \infty) = 7
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{32}^1 = \min(d_{32}^0, d_{31}^0 + d_{12}^0) = \min(4, \infty + 3) = \min(4, \infty) = 4
d_{34}^1 = \min(d_{34}^0, d_{31}^0 + d_{14}^0) = \min(\infty, \infty + \infty) = \min(\infty, \infty) = \infty
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(\infty, \infty + -4) = \min(\infty, \infty) = \infty
d_{41}^{1} = \min(d_{41}^{0}, d_{41}^{0} + d_{11}^{0}) = \min(2, 2 + 0) = \min(2, 2) = 2
d_{42}^1 = \min(d_{42}^0, d_{41}^0 + d_{12}^0) = \min(\infty, 2+3) = \min(\infty, 5) = 5
d_{43}^1 = \min(d_{43}^0, d_{41}^0 + d_{13}^0) = \min(-5, 2 + 8) = \min(-5, 10) = -5
d_{45}^1 = \min(d_{45}^0, d_{41}^0 + d_{15}^0) = \min(\infty, 2 + -4) = \min(\infty, -2) = -2
d_{51}^1 = \min(d_{51}^0, d_{51}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
```

$$D^1$$

```
d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3
d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(8, 0 + 8) = \min(8, 8) = 8
d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty
d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty
d_{24}^1 = \min(d_{24}^0, d_{21}^0 + d_{14}^0) = \min(1, \infty + \infty) = \min(1, \infty) = 1
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(7, \infty + -4) = \min(7, \infty) = 7
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{32}^1 = \min(d_{32}^0, d_{31}^0 + d_{12}^0) = \min(4, \infty + 3) = \min(4, \infty) = 4
d_{34}^1 = \min(d_{34}^0, d_{31}^0 + d_{14}^0) = \min(\infty, \infty + \infty) = \min(\infty, \infty) = \infty
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(\infty, \infty + -4) = \min(\infty, \infty) = \infty
d_{41}^{1} = \min(d_{41}^{0}, d_{41}^{0} + d_{11}^{0}) = \min(2, 2 + 0) = \min(2, 2) = 2
d_{42}^1 = \min(d_{42}^0, d_{41}^0 + d_{12}^0) = \min(\infty, 2+3) = \min(\infty, 5) = 5
d_{43}^1 = \min(d_{43}^0, d_{41}^0 + d_{13}^0) = \min(-5, 2 + 8) = \min(-5, 10) = -5
d_{45}^1 = \min(d_{45}^0, d_{41}^0 + d_{15}^0) = \min(\infty, 2 + -4) = \min(\infty, -2) = -2
d_{51}^1 = \min(d_{51}^0, d_{51}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{52}^1 = \min(d_{52}^0, d_{51}^0 + d_{12}^0) = \min(\infty, \infty + 3) = \min(\infty, \infty) = \infty
```

$$D^{0} = W$$

$$\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
0 & 3 & 8 & \infty & -4 \\
0 & 0 & \infty & 1 & 7 \\
0 & 0 & \infty & 1 & 7 \\
0 & 0 & \infty & 0 & \infty \\
0 & 0 & 0 & \infty & 0 \\
0 & 0 & 0 & 0 & \infty \\
0 & 0 & 0 & 0 & \infty \\
0 & 0 & 0 & 0 & 0 & \infty
\end{bmatrix}$$

$$\begin{array}{ll} d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3 \\ d_{13}^1 = \min(d_{13}^0, d_{11}^0 + d_{13}^0) = \min(8, 0 + 8) = \min(8, 8) = 8 \\ d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty \\ d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4 \\ d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty \\ d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty \\ d_{24}^1 = \min(d_{24}^0, d_{21}^0 + d_{14}^0) = \min(1, \infty, \infty + 8) = \min(1, \infty) = 1 \\ d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(7, \infty + -4) = \min(7, \infty) = 7 \\ d_{31}^1 = \min(d_{31}^0, d_{31}^0 + d_{15}^0) = \min(7, \infty + 0) = \min(\infty, \infty) = \infty \\ d_{32}^1 = \min(d_{32}^0, d_{31}^0 + d_{15}^0) = \min(0, \infty, \infty + 0) = \min(0, \infty) = \infty \\ d_{34}^1 = \min(d_{32}^0, d_{31}^0 + d_{12}^0) = \min(0, \infty, \infty + 0) = \min(0, \infty) = \infty \\ d_{34}^1 = \min(d_{34}^0, d_{31}^0 + d_{12}^0) = \min(0, \infty, \infty + \infty) = \min(0, \infty) = \infty \\ d_{35}^1 = \min(d_{35}^0, d_{31}^0 + d_{14}^0) = \min(0, \infty, \infty + \infty) = \min(0, \infty) = \infty \\ d_{41}^1 = \min(d_{34}^0, d_{31}^0 + d_{14}^0) = \min(0, \infty, \infty + \infty) = \min(0, \infty) = \infty \\ d_{41}^1 = \min(d_{34}^0, d_{31}^0 + d_{14}^0) = \min(0, \infty, \infty + \infty) = \min(0, \infty) = \infty \\ d_{41}^1 = \min(d_{34}^0, d_{31}^0 + d_{12}^0) = \min(0, \infty, \infty + \infty) = \min(0, \infty) = \infty \\ d_{41}^1 = \min(d_{34}^0, d_{31}^0 + d_{14}^0) = \min(0, \infty, \infty + \infty) = \min(0, \infty) = \infty \\ d_{41}^1 = \min(d_{34}^0, d_{31}^0 + d_{13}^0) = \min(0, \infty, \infty + \infty) = \min(0, \infty) = \infty \\ d_{41}^1 = \min(d_{34}^0, d_{31}^0 + d_{13}^0) = \min(0, \infty, \infty + \infty) = \min(\infty, \infty) = \infty \\ d_{42}^1 = \min(d_{35}^0, d_{31}^0 + d_{13}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty \\ d_{43}^1 = \min(d_{35}^0, d_{31}^0 + d_{13}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty \\ d_{45}^1 = \min(d_{35}^0, d_{31}^0 + d_{13}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty \\ d_{52}^1 = \min(d_{52}^0, d_{51}^0 + d_{12}^0) = \min(0, \infty, \infty + 3) = \min(\infty, \infty) = \infty \\ d_{52}^1 = \min(d_{52}^0, d_{51}^0 + d_{12}^0) = \min(0, \infty, \infty + 3) = \min(\infty, \infty) = \infty \\ d_{52}^1 = \min(d_{52}^0, d_{51}^0 + d_{12}^0) = \min(0, \infty, \infty + 3) = \min(\infty, \infty) = \infty \\ d_{52}^1 = \min(d_{52}^0, d_{51}^0 + d_{12}^0) = \min(0, \infty, \infty + 3) = \min(\infty, \infty) = \infty \\ d_{52}^1 = \min(d_{52}^0, d_{51}^0 + d_{12}^0) = \min(0, \infty, \infty + 3) =$$

$$D^{0} = W$$

$$\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
0 & 3 & 8 & \infty & -4 \\
0 & 0 & \infty & 1 & 7 \\
0 & 0 & \infty & 0 & \infty \\
0 & 0 & 0 & \infty & \infty \\
0 & 0 & 0 & 0 & \infty \\
0 & 0 & 0 & 0 & \infty \\
0 & 0 & 0 & 0 & \infty \\
0 & 0 & 0 & 0 & 0 & \infty
\end{bmatrix}$$

```
d_{12}^1 = \min(d_{12}^0, d_{11}^0 + d_{12}^0) = \min(3, 0 + 3) = \min(3, 3) = 3
d_{13}^1 = \min(d_{13}^0, d_{11}^0 + d_{13}^0) = \min(8, 0 + 8) = \min(8, 8) = 8
d_{14}^1 = \min(d_{14}^0, d_{11}^0 + d_{14}^0) = \min(\infty, 0 + \infty) = \min(\infty, \infty) = \infty
d_{15}^1 = \min(d_{15}^0, d_{11}^0 + d_{15}^0) = \min(-4, 0 + -4) = \min(-4, -4) = -4
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{23}^1 = \min(d_{23}^0, d_{21}^0 + d_{13}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty
d_{24}^1 = \min(d_{24}^0, d_{21}^0 + d_{14}^0) = \min(1, \infty + \infty) = \min(1, \infty) = 1
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(7, \infty + -4) = \min(7, \infty) = 7
d_{21}^1 = \min(d_{21}^0, d_{21}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{32}^1 = \min(d_{32}^0, d_{31}^0 + d_{12}^0) = \min(4, \infty + 3) = \min(4, \infty) = 4
d_{34}^1 = \min(d_{34}^0, d_{31}^0 + d_{14}^0) = \min(\infty, \infty + \infty) = \min(\infty, \infty) = \infty
d_{25}^1 = \min(d_{25}^0, d_{21}^0 + d_{15}^0) = \min(\infty, \infty + -4) = \min(\infty, \infty) = \infty
d_{41}^{1} = \min(d_{41}^{0}, d_{41}^{0} + d_{11}^{0}) = \min(2, 2 + 0) = \min(2, 2) = 2
d_{42}^1 = \min(d_{42}^0, d_{41}^0 + d_{12}^0) = \min(\infty, 2+3) = \min(\infty, 5) = 5
d_{43}^1 = \min(d_{43}^0, d_{41}^0 + d_{13}^0) = \min(-5, 2 + 8) = \min(-5, 10) = -5
d_{45}^1 = \min(d_{45}^0, d_{41}^0 + d_{15}^0) = \min(\infty, 2 + -4) = \min(\infty, -2) = -2
d_{51}^1 = \min(d_{51}^0, d_{51}^0 + d_{11}^0) = \min(\infty, \infty + 0) = \min(\infty, \infty) = \infty
d_{52}^1 = \min(d_{52}^0, d_{51}^0 + d_{12}^0) = \min(\infty, \infty + 3) = \min(\infty, \infty) = \infty
d_{52}^1 = \min(d_{52}^0, d_{51}^0 + d_{12}^0) = \min(\infty, \infty + 8) = \min(\infty, \infty) = \infty
d_{54}^{1} = \min(d_{54}^{0}, d_{51}^{0} + d_{14}^{0}) = \min(6, \infty + \infty) = \min(6, \infty) = 6
```

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

 D^2

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

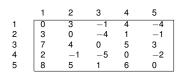
	1	2	3	4	5
1	0	3	8	4	-4
1 2 3 4 5	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	-1	-5	0	-2
5	∞	3 0 4 −1 ∞	∞	6	0

 D^2

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

 D^3

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	3 0 4 _1	∞	1	7
3	∞	4	0	5	11
2 3 4 5	2	-1	-5	0	-2
5	∞	3 0 4 −1 ∞	∞	6	0



 D^2

	1	2	3	4	5
1	0	3	8	4	-4
2	∞	0	∞	1	7
3	∞	4	0	5	11
4	2	5	-5	0	-2
5	∞	∞	∞	6	0

 D^3

	1	2	3	4	5
1	0	3	8	4	-4
2	0 ∞	0	∞	1	7
3	∞	4	0	5	11
4	∞ 2	-1	-5	0	-2
4 5	∞	∞	∞	6	0

 D^2

	1	2	3	4	5
1	0	3	-1	4	-4
2	3	0	-4	1	-1
2	7	4	0	5	3
4 5	2	-1	-5	0	-2
5	8	5	1	6	0

	1	2	3	4	5
1	0	1	-3	2	-4
2	3	0	-4	-	-1
1 2 3 4 5	7	4	0	5	3
4	2	-1	-5	0	-2
5	0 3 7 2 8	-1 5	1	6	0

Outline

- All Pair Shortest Path Distance
 - The Floyd-Warshall Algorithm

- Transitive Closure
 - The adapted Floyd-Warshall Algorithm

Introduction

Problem

Given a Graph G = (V, E), find its **transitive closure** defined as:

 $G = (V, E^*)$ where $E^* = \{(i, j) : \text{there is path } i \iff j \text{ in } G\}.$

Input

Given: G = (V, E) such that |V| = n.

Output

 $n \times n$ matrix D such that $D[i][j] = d_{ij}$ such that $d_{ij} \begin{cases} 1 & \text{if there is path } i \text{ www.} \\ 0 & \text{otherwise} \end{cases}$



Changes

New Weight Matrix

Assign weight of 1 or 0 to each edge. In other words, modify W as follows:

$$W[i][j] = w_{ij} \begin{cases} 1 & \text{if } (i = j) \text{ or } (i, j) \in E \\ 0 & \text{otherwise} \end{cases}$$

In effect, it is unweighted adjacency matrix.

New Operators in Recurrence Relation

Replace the operators in the recurrence relation by following boolean operators:

- min → ∨ (OR)
- $+ \rightarrow \wedge$ (AND)

Modified Recurrence Relation

For
$$k \ge 1$$

 $d_{ij}^k = \overline{d}_{ij}^{k-1} \lor (d_{ik}^{k-1} \land d_{kj}^{k-1})$

 d_{ii}^k

- $d_{ii}^k = 1$ if there is a path $i \iff j$ with all intermediate vertices in $\{1, 2, \dots, k\}$, 0 otherwise.
- $d_{ii}^0 = 1$ if i = j or $(i, j) \in E$, 0 otherwise.
- $d_{ii}^n = 1$ if there is a path $i \iff j$, 0 otherwise.

References

• Cormen, Leiserson, Rivest: Introduction to Algorithms

