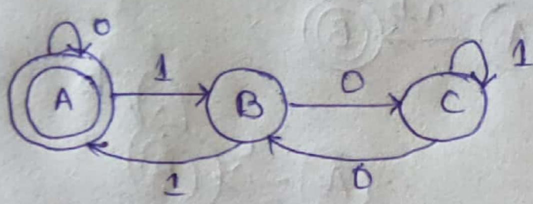
22-12-2021

1) Construct a MDFA which accepts strings over $w = \{0,1\}$ which when interpreted as binary numbers is divisible by 3.

Remainder table

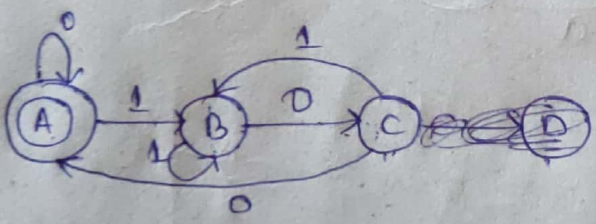
| 0 | 1 | 2 |
|---|---|---|
| 0 | 01 | 10 |
| 11 | 100 | 101 |
| 110 | 111 | 1000 |
| 1001 | 1010 | 1011 |

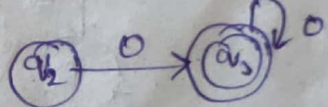


divisible by 4

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 0 | 01 | 10 | 11 |
| 100 | 101 | 110 | 111 |
| 1000 | 1001 | 1010 | 1011 |

10100

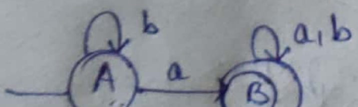


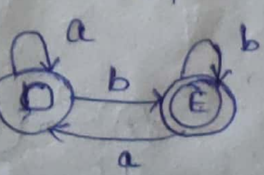


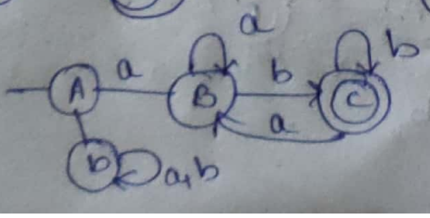
2) $\Sigma = \{a, b\}$ starting with a and ending with b

$L_1 = \{a, ab^n, aa^n, ab^na^n, \ldots$

$L_2 = \{\varepsilon, b, a^nb, b^nb, a^nb^nb, \ldots$

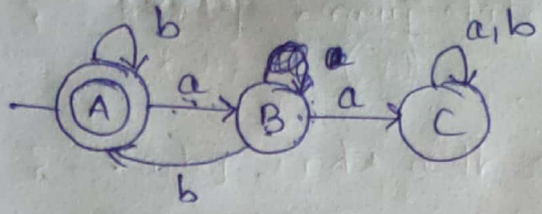






Every a should be followed by b

$L = \{\varepsilon, b, b^x, ab, ab^n, abab, ab^nab$

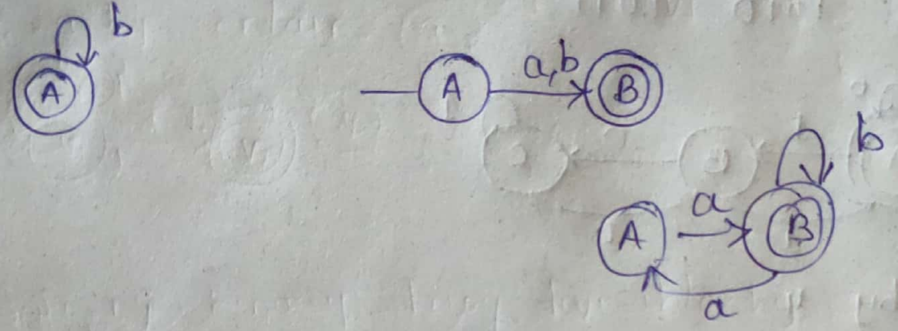


aab

1) start's & ends

a should never follow b

$L = \{\varepsilon, b, b^n, a, a^n, b^n a$

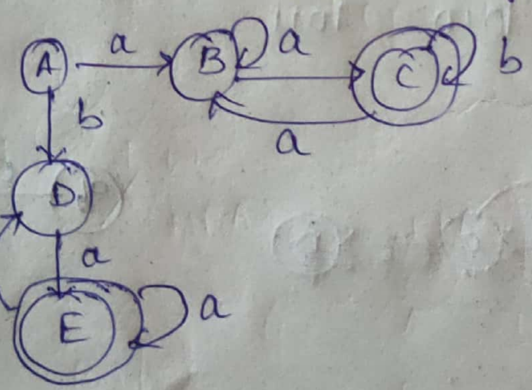


a

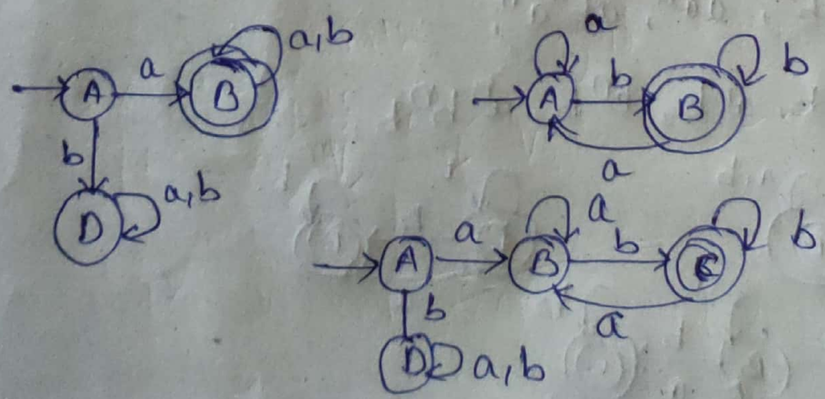
DFA Properties:-

① Union:- (Merge two DFA) Ex:-

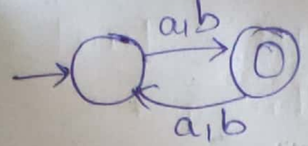
→ start and ends with different symbol

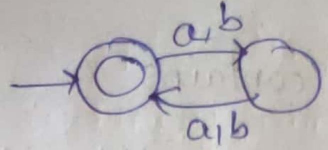


② Concatenation:- starts with a and ends with B

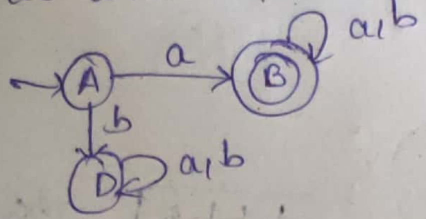


③ Complement

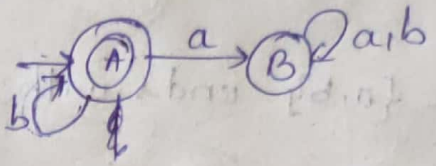



starts with a:-

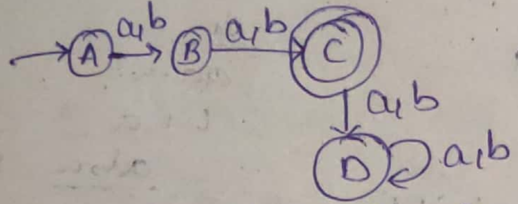



Not starts with a:-




④ Reversal

Accept strings of length two




change initial to final state and final state to initial state.

and change initial arrow directions.

→ we can't solve $a^n b^n \geq 0$ with DFA

(28-12-2021)

NFA:-

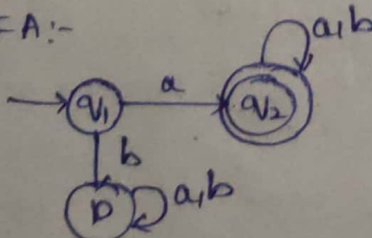
NFA = $\{Q, \Sigma, \delta, q_0, F\}$

$\delta : Q \times \Sigma \rightarrow 2^Q$

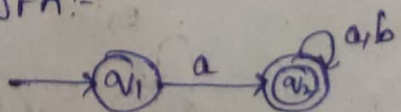
→ Basically all real life examples fall under DFA

① $\Sigma = \{a, b\}$ that starts with a

DFA:-




NFA:-




→ In NFA, complement doesn't work.

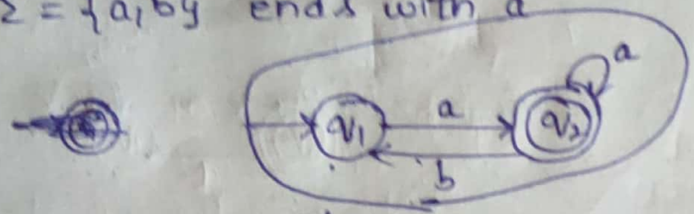
a

② $\Sigma = \{a,b\}$ ends with a

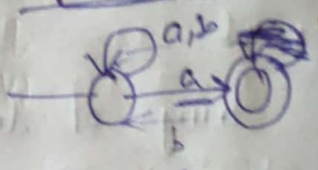
③ $\Sigma = \{a,b\}$ contains a

④ $\Sigma = \{0,1\}$ binary states divisible by 2

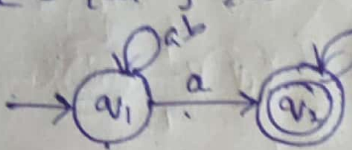
② $\Sigma = \{a,b\}$ ends with a



bbba aba



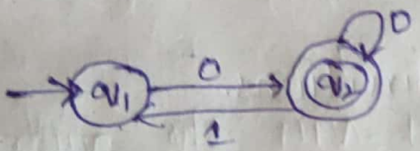
③ $\Sigma = \{a,b\}$ should contains a



a

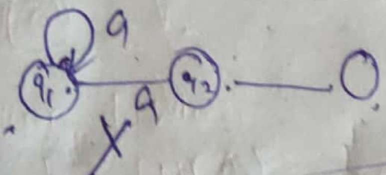
a
aaa
bba
aba

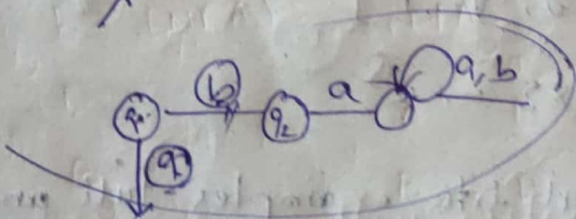
④ $\Sigma = \{0,1\}$ binary string divisible by 2



00 01 10
101

$\{a,b\}$

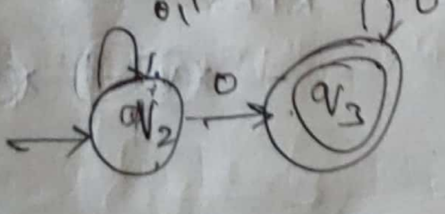




$Q \times \Sigma \to Q$

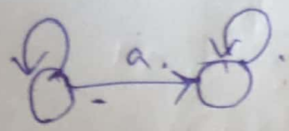
$Q \times \Sigma \to 2^{Q}$

100

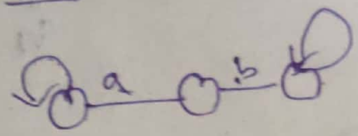


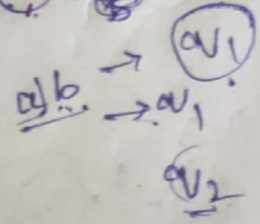
$\{a,b\}$ 

$\underline{L_1} = \{a, ab, \underline{abab}, \underline{a}$

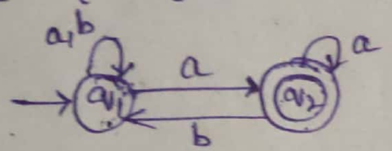


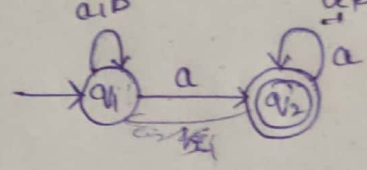
ab





② $\Sigma = \{a,b\}$ ends with a

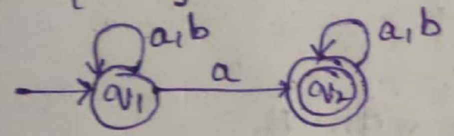




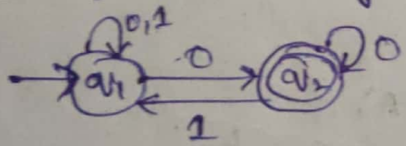
abab

aba

③ $\Sigma = \{a,b\}$ contains a



④ $\Sigma = \{0,1\}$ binary string divisible by 2



⑤ $\Sigma = \{0,1\}$ binary string divisible by 3

| | | |
|---|---|---|
| 0 | 1 | 2 |
| 0 | 01 | 10 |
| 11 | 100 | 101 |
| 110 | 111 | 1000 |
| 1001 | 1010 | 1011 |
| 1100 | 1101 | 1110 |
| 1111 | | |

111111
32 16 8 4 2 1

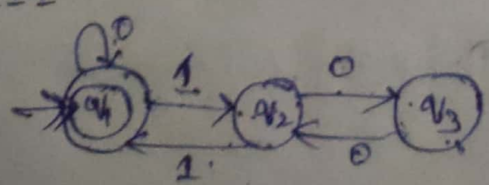
101100

011

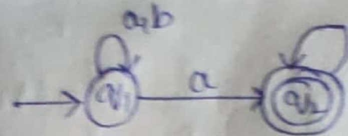
0110

1011 0111

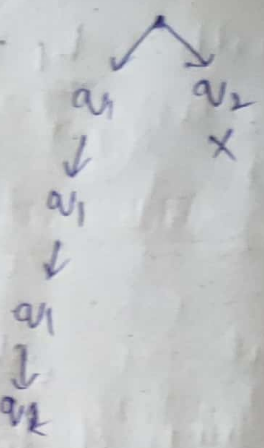
1001 1001

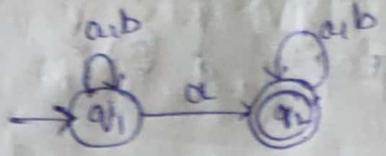


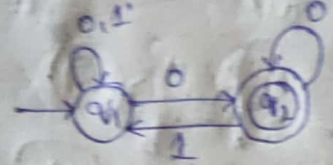
29-12-2021

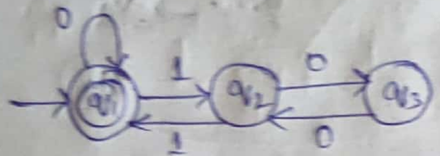
② 


abbb




③ 

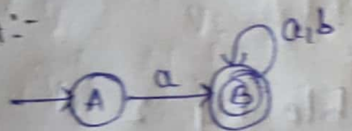

④ 


⑤ 


⑥

Conversion of NFA to DFA:-

NFA:- starting with A




State transition table:-

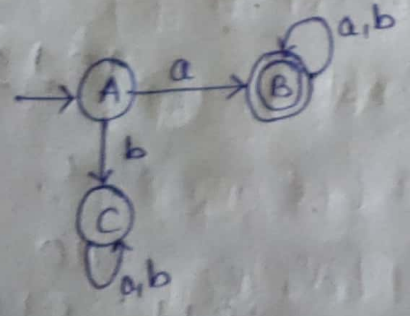
(NFA)

| | a | b |
|---|---|---|
| A | B | D |
| B | B | B |

(DFA):-

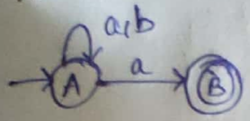
| | a | b |
|---|---|---|
| A | B | ~~B~~ C |
| B | B | B |
| C | C | C |

DFA:-




Scanned with CamScanner

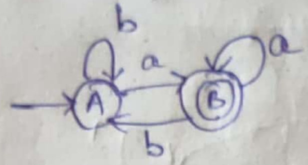
NFA:-



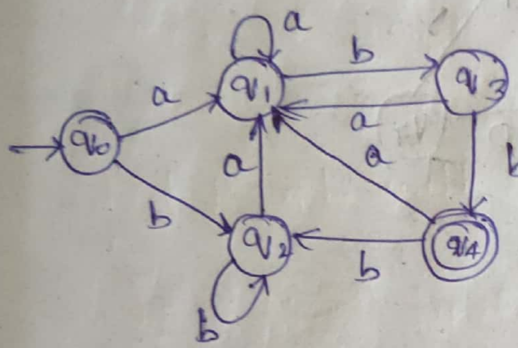

| | a | b |
|---|---|---|
| A | AB | A |
| B | D | D |

DFA:-

| | a | b |
|---|---|---|
| A | {A, B} | A |
| [AB] | [A B] | A |




Minimization of states in DFA:-

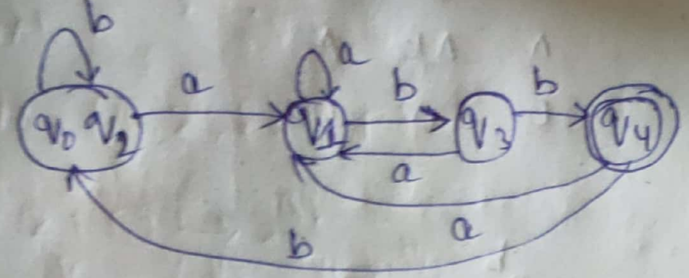


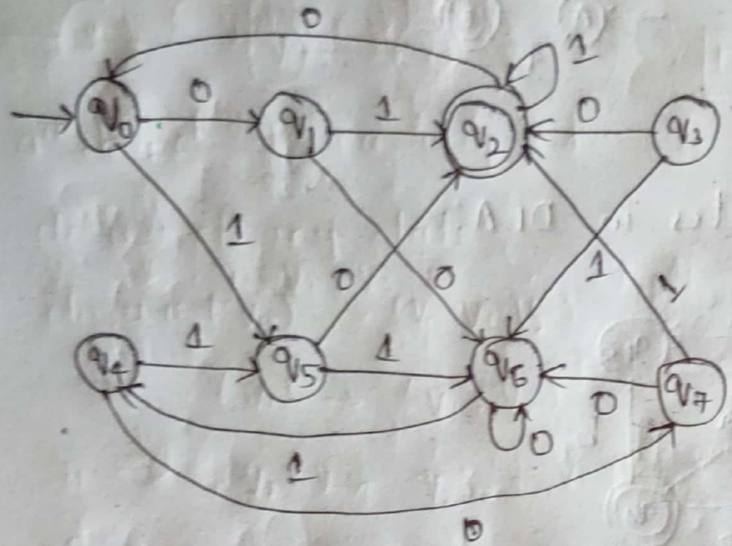

state table

| | a | b |
|---|---|---|
| $q_0$ | $q_1$ | $q_2$ |
| $q_1$ | $q_1$ | $q_3$ |
| $q_2$ | $q_1$ | $q_2$ |
| $q_3$ | $q_1$ | $q_4$ |
| $q_4$ | $q_1$ | $q_2$ |

Steps:-

1) If any state is not reachable from initial state, delete that state and its selected transitions.
2) 0 - equivalences - [$q_0$ $q_1$ $q_2$ $q_3$] [$q_4$]
3) 1 - equivalence - [$q_0$ $q_1$ $q_2$] [$q_3$] [$q_4$]
4) 2 - equivalence - [$q_0$ $q_2$] [$q_1$] [$q_3$] [$q_4$]
5) 3 - equivalence - [$q_0$ $q_2$] [$q_1$] [$q_3$] [$q_4$]





1.) Omit $q_3$

| | 0 | 1 |
|---|---|---|
| $q_0$ | $q_1$ | $q_5$ |
| $q_1$ | $q_6$ | $q_2$ |
| $q_2$ | $q_0$ | $q_2$ |
| $q_4$ | $q_7$ | $q_5$ |
| $q_5$ | $q_2$ | $q_6$ |
| $q_6$ | $q_6$ | $q_4$ |
| $q_7$ | $q_6$ | $q_2$ |

1.) 0 - $[q_0\ q_1\ q_4\ q_5\ q_6\ q_7]$ $[q_2]$

2.) 1 - $[q_0\ q_4\ q_5\ q_6\ q_7]$ $[q_1]$ $[q_2]$

3.) 2 - $[q_0\ q_4\ q_6\ q_7]$ $[q_5]$ $[q_1]$ $[q_2]$

~~4.)~~ $[q_0\ q_4\ q_6\ q_7]$ $[q_5]$ $[q_1]$ $[q_2]$

4.) 3 - $[q_0$ $[q_4$

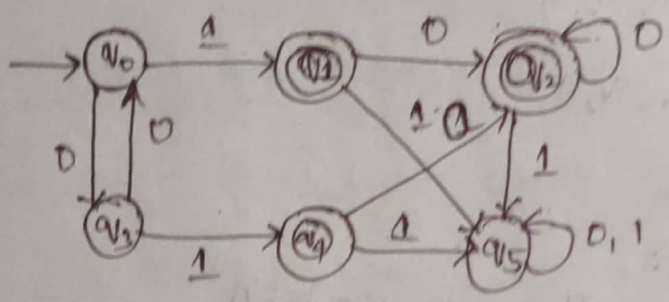
1.) 0 - [$q_0$ $q_1$ $q_4$ $q_5$ $q_6$ $q_7$] [$q_2$]

2.) 1 - [$q_0$, $q_4$, $q_6$] [$q_1$, $q_7$][$q_5$] [$q_2$]

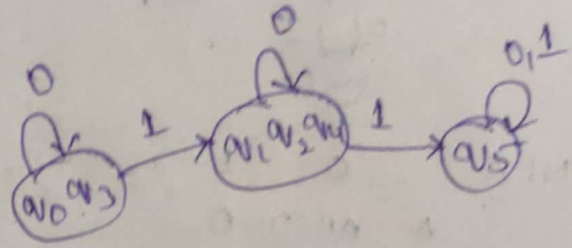
3.) 2 - [$q_0$ $q_4$] [$q_6$] [$q_1$, $q_7$] [$q_5$] [$q_2$]

4.) 3 - [$q_0$ $q_4$] [$q_6$] [$q_1$ $q_7$] [$q_5$] [$q_2$]

[$q_0$      [$q_1$



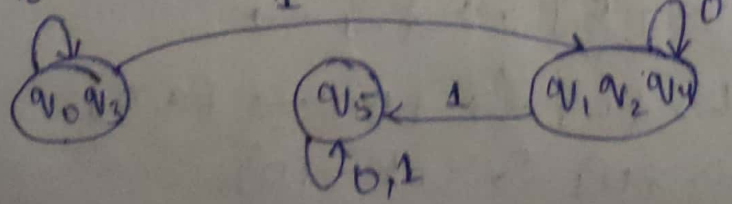

| | 0 | 1 |
|---|---|---|
| $q_0$ | $q_3$ | $q_1$ |
| $q_1$ | $q_2$ | $q_5$ |
| $q_2$ | $q_2$ | $q_5$ |
| $q_3$ | $q_0$ | $q_4$ |
| $q_4$ | $q_2$ | $q_5$ |
| $q_5$ | $q_5$ | $q_5$ |




1) 0 - [$q_0$ $q_3$ $q_5$] [$q_1$ $q_2$ $q_4$]

2) 1 - [$q_0$ $q_3$] [$q_5$] [$q_1$ $q_2$ $q_4$]

3) 2 - [$q_0$ $q_3$] [$q_5$] [$q_1$ $q_2$ $q_4$]

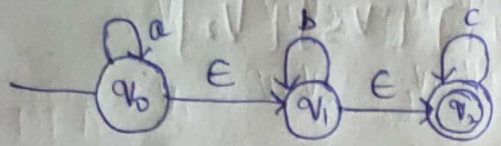


(30-12-2021)

$\epsilon$ - NFA:-

$\epsilon$-NFA = $(Q, \Sigma, \delta, q_0, F)$

$\delta: Q \times (\Sigma \cup \epsilon) \rightarrow 2^Q$




$\boxed{DFA \cong NFA \cong \epsilon\text{-}NFA}$

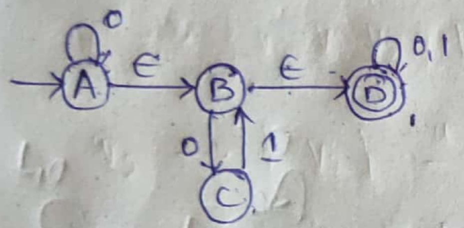
$a^m . b^n . c^p$ all are accepted.

$\epsilon$ - closure of any state:-

$\epsilon$-closure $(q_0) \rightarrow (q_0\, q_1\, q_2)$

$(q_1) \rightarrow (q_1, q_2)$

$(q_2) \rightarrow q_2$




$\rightarrow$ This is $\epsilon$-NFA

| NFA | 0 | 1 |
|---|---|---|
| A | ABCD | D |
| B | CD | D |
| C | $\phi$ | BD |
| D | D | D |

$\epsilon$-closure(A) $\rightarrow$ {A B D} $\xrightarrow{0}$ {A C D} $\rightarrow$ {A B D C}

$\hookrightarrow$ {A B D} $\xrightarrow{1}$ {D} $\rightarrow$ {D}

$\epsilon$-closure (B) $\rightarrow$ {B, D} $\xrightarrow{0}$ {C D} $\rightarrow$ {C D}

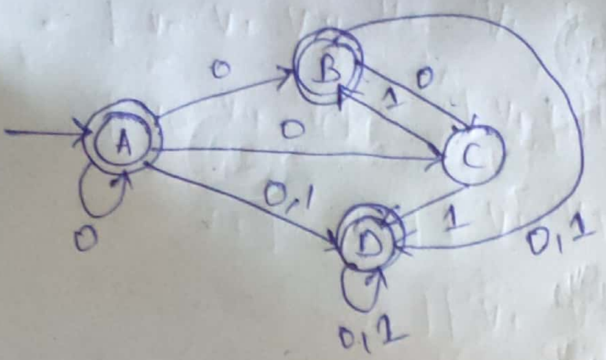
$\hookrightarrow$ {B D} $\xrightarrow{1}$ {D} $\rightarrow$ {D}

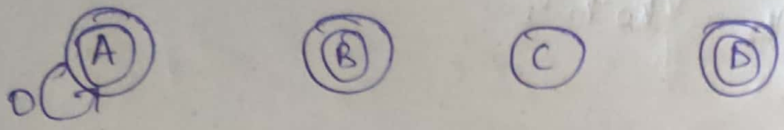
$\epsilon$ closure (C) $\rightarrow$ {C} $\xrightarrow{0}$ { } $\rightarrow$

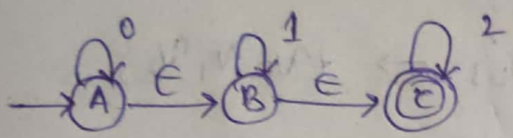
$\hookrightarrow$ {C} $\xrightarrow{1}$ {B} $\rightarrow$ {B, D}

$\epsilon$ closure (D) $\rightarrow$ {D} $\xrightarrow{0}$ {D} $\rightarrow$ {D}

$\hookrightarrow$ {D} $\xrightarrow{1}$ {D} $\rightarrow$ {D}




| NFA | 0 | 1 | 2 |
|---|---|---|---|
| A | ABC | BC | C |
| B | $\phi$ | BC | C |
| C | $\phi$ | $\phi$ | C |

$\epsilon$ closure (A) $\rightarrow \{A B C\} \xrightarrow{0} \{A\} \rightarrow \{A B C\}$

$\xrightarrow{1} \{B\} \rightarrow \{B, C\}$

$\xrightarrow{2} \{C\} \rightarrow \{C\}$

$\epsilon$ closure (B) $\rightarrow \{B C\} \xrightarrow{0} \{\ \}$

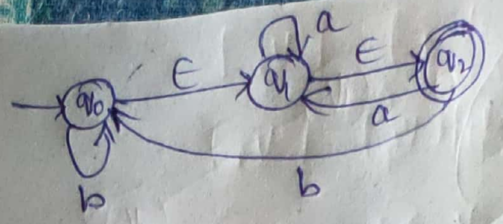
$\xrightarrow{1} \{B\} \rightarrow \{B, C\}$

$\xrightarrow{2} \{C\} \rightarrow \{C\}$

$\epsilon$ closure (C) $\rightarrow \{C\} \xrightarrow{0} \{\ \}$

$\xrightarrow{1} \{\ \}$

$\xrightarrow{2} \{C\} \rightarrow \{C\}$




| NFA | a | b |
|---|---|---|
| $q_0$ | $q_1 q_2$ | $q_0 q_1 q_2$ |
| $q_1$ | $q_1 q_2$ | $q_0 q_1 q_2$ |
| $q_2$ | $q_1 q_2$ | $q_0 q_1 q_2$ |

$q_0 \rightarrow \{q_0 q_1 q_2\} \xrightarrow{a} \{q_1\} \rightarrow \{q_1 q_2\}$

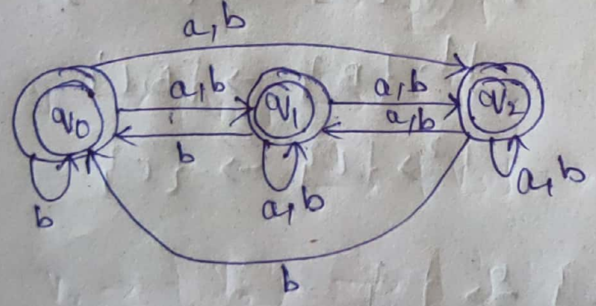
$\quad \xrightarrow{b} \{q_0\} \rightarrow \{q_0 q_1 q_2\}$

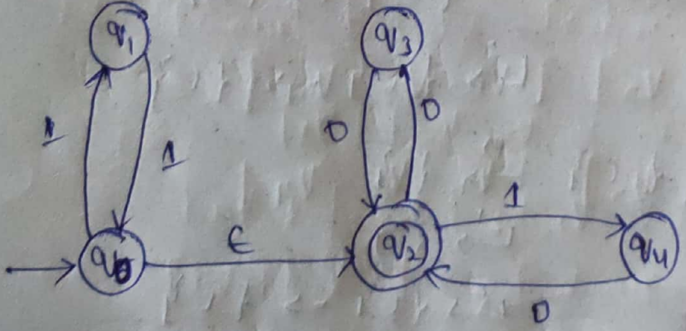
$q_1 \rightarrow \{q_1 q_2\} \xrightarrow{a} \{q_1\} \rightarrow \{q_1 q_2\}$

$\quad \xrightarrow{b} \{q_0\} \rightarrow \{q_0 q_1 q_2\}$

$q_2 \rightarrow \{q_2\} \xrightarrow{a} \{q_1\} \rightarrow \{q_1 q_2\}$

$\quad \xrightarrow{b} \{q_0\} \rightarrow \{q_0 q_1 q_2\}$



| | | 0 | 1 |
|---|---|---|---|
| NFA | $q_0$ | $q_3$ | $q_1 q_4$ |
| | $q_1$ | $\phi$ | $q_0 q_2$ |
| | $q_2$ | $q_3$ | $q_4$ |
| | $q_3$ | $q_2$ | $\phi$ |
| | $q_4$ | $q_2$ | $\phi$ |

$q_0 \rightarrow \{q_0\, q_2\} \xrightarrow{0} \{q_3\} \rightarrow \{q_3\}$

$\xrightarrow{1} \{q_1, q_4\} \rightarrow \{q_1, q_4\}$

$q_1 \rightarrow \{q_1\} \xrightarrow{0} \{\}$

$\xrightarrow{1} \{q_0\} \longrightarrow \{q_0, q_2\}$

$q_2 \longrightarrow \{q_2\} \xrightarrow{0} \{q_3\} \rightarrow \{q_3\}$

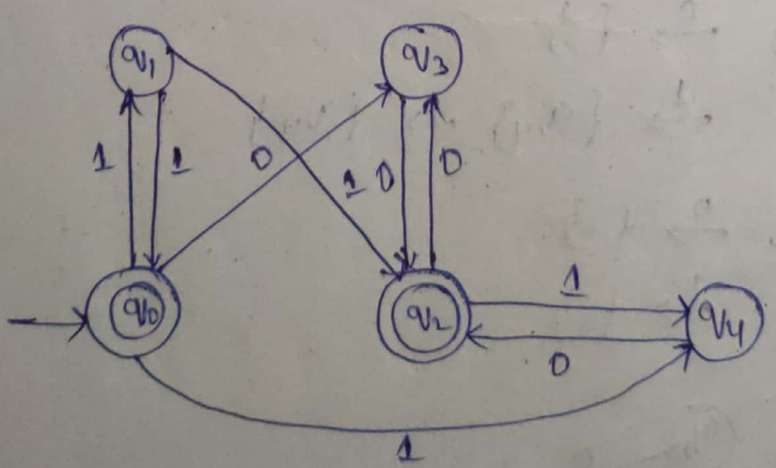
$\xrightarrow{1} \{q_4\} \rightarrow \{q_4\}$

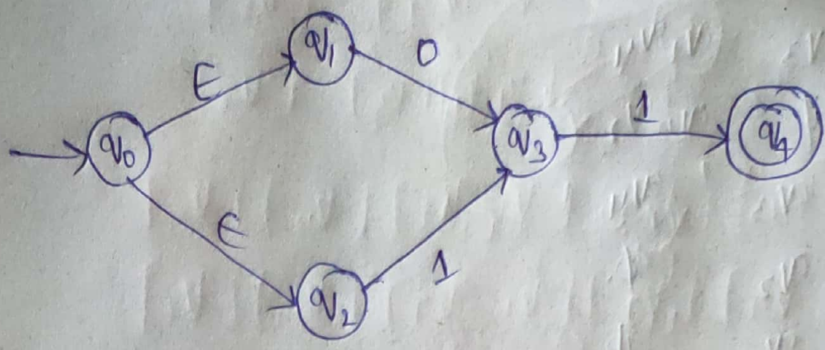
$q_3 \rightarrow \{q_3\} \xrightarrow{0} \{q_2\} \rightarrow \{q_2\}$

$\xrightarrow{1} \{\}$

$q_4 \rightarrow \{q_4\} \xrightarrow{0} \{q_2\} \rightarrow \{q_2\}$

$\xrightarrow{1} \{\}$






| NFA | 0 | 1 |
|---|---|---|
| $q_0$ | $q_3$ | $q_3$ |
| $q_1$ | $q_3$ | $\phi$ |
| $q_2$ | $\phi$ | $q_3$ |
| $q_3$ | $\phi$ | $q_4$ |
| $q_4$ | $\phi$ | $\phi$ |

$q_0 \rightarrow \{q_0 q_1 q_2\} \xrightarrow{0} \{q_3\} \rightarrow \{q_3\}$

$\xrightarrow{1} \{q_3\} \rightarrow \{q_3\}$

$q_1 \rightarrow \{q_1\} \xrightarrow{0} \{q_3\} \rightarrow \{q_3\}$

$\xrightarrow{1} \{\ \}$

$q_2 \rightarrow \{q_2\} \xrightarrow{0} \{\}$

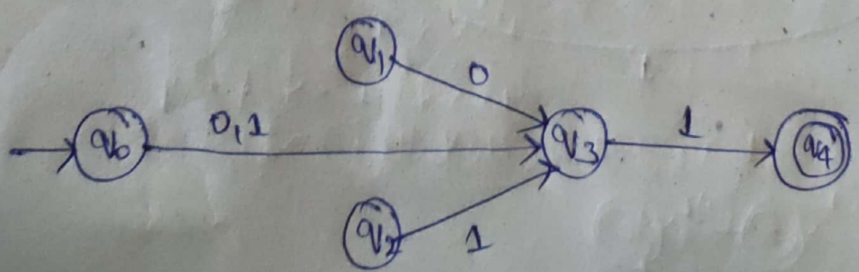
$\xrightarrow{1} \{q_3\} \rightarrow \{q_3\}$

$q_3 \rightarrow \{q_3\} \xrightarrow{0} \{\}$

$\xrightarrow{1} \{q_4\} \rightarrow \{q_4\}$

$q_4 \rightarrow \{q_4\} \xrightarrow{0} \{\ \}$

$\xrightarrow{1} \{\ \}$



18-01-2022

FSM

- with o/p
  - Moore
  - Meele
- without o/p
  - DFA
  - NFA
  - NFA-$\epsilon$

Regular langauge → Langauge accepted by FSM.

→ Representing regular language using mathematical terms is called regular expression.

Regular Expression:-

1.) Union (+)

2.) Concatenation (•)

3.) Kleene closure (*)

Ex:- $\Sigma = \{a, b\}$

$L = \{ab, ba, bb, aa\}$

$= (a+b) \cdot (a+b)$

$a^* \rightarrow \epsilon, a, aa, aaa, \ldots$ any number

$a^+ \rightarrow a, aa, aaa, \ldots$ one or more

$(a+b)^* = \{\epsilon, a, ab, b, bb, \ldots$

① All strings of length 2 $\Sigma = \{a, b\}$

$L_1 = \{aa, ab, ba, bb\}$

$R.E(L_1) = (a+b) \cdot (a+b)$

② L = all strings $\Sigma = \{a, b\}$ having length atleast two

$L = \{aa, ab, ba, bb, aaa, \ldots bbb, \ldots$

$R.E(L) = (a+b)(a+b)(a+b)^*$

$= (a+b)(a+b)^+$

③ $\Sigma = \{a, b\}$ L = length atmost 2

$L = \{\epsilon, a, b, aa, ab, ba, bb\}$

$R.E(L) = (\epsilon + a + b)(\epsilon + a + b)$

④ Find even length string over $\Sigma = \{a, b\}$

$L = \{\epsilon, aa, ab, ba, bb, aaaa, \ldots$ infinite.

$[(\epsilon + a + b)(\epsilon + a + b)]^*$

$R.E(L) = [(a+b)(a+b)]^*$

⑤ odd length string.

$L = \{a, b, aaa, bbb, \ldots\}$

$R.E(L) = (a+b)[(a+b)(a+b)]^*$

⑥ set of all strings whose length is divisible by 3

$L = \{0, 3, 6, \ldots\}$

$RE = [(a+b)(a+b)(a+b)]^*$

⑦ set of all strings of length 2 when modulus with 3 gives 2

$L = \{2, 5, 8, 11, \ldots$

$R.E = [(a+b)(a+b)]^+ (a+b)^*$

$= (a+b)(a+b)[(a+b)(a+b)(a+b)]^*$

(8) $\Sigma = \{a, b\}$ where a's are atleast 2

$L = \{aa, baa, baab, \ldots$

$b^* a b^* a$

$(a+b)^*$

baabab

$R.E = (aa)^+ (a+b)^*$

$= (a+b)^* a^+ (a+b)^* a^+ (a+b)^*$

(9) $\Sigma = \{a, b\}$ ends with a

$L = \{a, ba, aa, aaa, \ldots\}$

$R(L) = (a+b)^* a$

(10) $\Sigma = \{a, b\}$ contains a

$R.E = (a+b)^* a (a+b)^*$

(11) $\Sigma = \{a, b\}$ start and end differently.

$L = \{ab, ba, , \ldots a \ldots b, b \ldots a \ldots\}$

$R.E(L) = a^* (a+b)^* b^+ + b^+ (a+b)^* a^+$

Scanned with CamScanner

19-01-2022

Applications of Regular Expressions:-

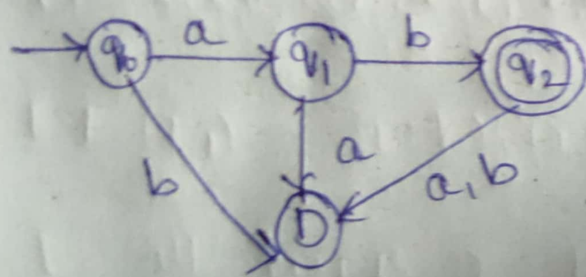
1) Search patterns

a.) UNIX - grep

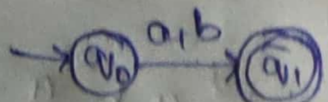
b.) Google search suggestions/ auto complete.

2) compiler → lexical analyzer

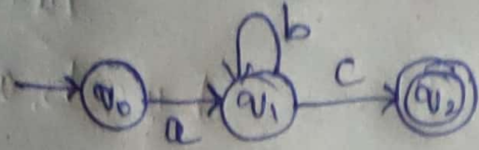
Conversion of DFA to Regular Expressions.



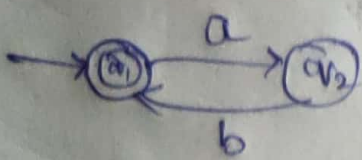

R.E = ab

(2) 

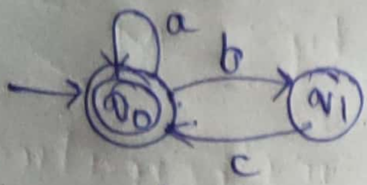

R.E = $a+b$

(3) 

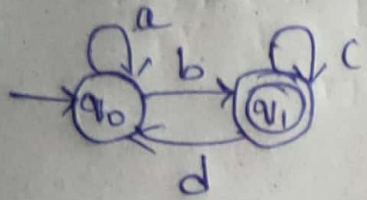

R.E = $ab^*c$

(4) 


R.E = $(ab)^*$

(5) 


R.E = $(a+bc)^*$

(6) 


R.E = $a^*b + a^*bc^* + a^*bc^*da^*bc^*$

$= a^*b(c + da^*b)^*$

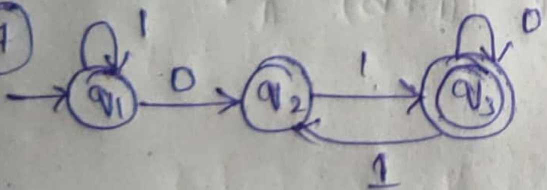
Precedence of operators:-

1.) Union :- (+)

2.) Concat :- (.)

3.) Klene closure :- (*)

$(*) > (.) > (+)$

(7) 


R.E = $1^*010^*$

$+ 1^*010^*110^*$

$= 1^*010^*(110^*)^*$

$= 1^*01(0^*+11)^*$

20-01-2022

Theorem 3.4:- In book - reference

① 


$R.E = 1^* 01(0+11)^*$

$$R_{ij}^{(k)} = R_{ij}^{k-1} + R_{ik}^{(k-1)} \cdot \left(R_{kk}^{(k-1)}\right)^* \cdot R_{kj}^{(k-1)}$$

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $R_{11}$ | $1+\epsilon$ | $1^*$ | | |
| $R_{12}$ | $0$ | $1^*0$ | | |
| $R_{13}$ | $\phi$ | | | |
| $R_{21}$ | $\phi$ | | | |
| $R_{22}$ | $\phi+\epsilon$ | | | |
| $R_{23}$ | $1$ | | | |
| $R_{31}$ | $\phi$ | | | |
| $R_{32}$ | $1$ | | | |
| $R_{33}$ | $\epsilon+0$ | | | |

col-0 :- single step

$R_{11} \Rightarrow q_1 \xrightarrow{\epsilon,1} q_2$

$R_{12} \Rightarrow q_1 \xrightarrow{0} q_2$

$R_{13} \Rightarrow q_1 \xrightarrow{\phi} q_3$

$R_{21} \Rightarrow q_2 \xrightarrow{\phi} q_1$

consider

$R^{(\text{No: of states})}_{[\text{ini}] \to [\text{final}]}$

col-1 :- use formula above

$$R_{11}^{1} = R_{11}^{0} + R_{11}^{0} \cdot (R_{11}^{0})^* \cdot R_{11}^{0}$$

$$= (1+\epsilon) + (1+\epsilon)(1+\epsilon)^*(1+\epsilon)$$

$$= (1+\epsilon) + (1+\epsilon)^*$$

$$= (1+\epsilon)^*$$

$$R_{12}^{1} = R_{12}^{0} + R_{11}^{0} \, (R_{11}^{0})^* \, R_{12}^{0}$$

$$= \cancel{(1+\epsilon)}\,0) + (1+\epsilon)(1+\epsilon)^* 0$$

$$= 0 + (1+\epsilon)^* 0$$

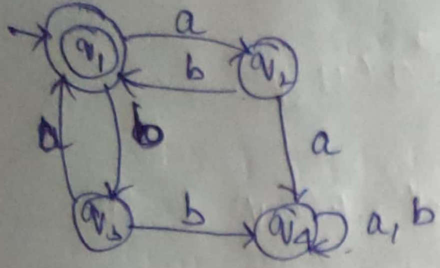
$$= 0 + 1^* 0$$

$$= 1^* 0$$

Adem's theorem:-

if $R = P + RQ$

$R = PQ^*$



$q_1 \rightarrow \epsilon + q_2(b) + q_3(a)$

$q_2 - q_1(a)$

$q_3 - q_1(b)$

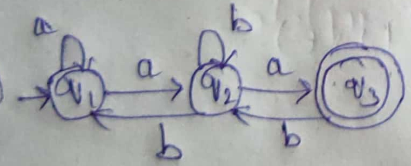
$q_4 - q_3(b) + q_2(a) + q_4(a) + q_4(b)$

$q_1 = \epsilon + q_1(a)(b) + q_1(b)(a)$

$= \epsilon + q_1(ab + ba)$

$= \epsilon(ab + ba)^*$

$= (ab + ba)^*$

② 

$q_1 \rightarrow \epsilon + q_1(a) + q_2(b)$

$q_2 \rightarrow q_1(a) + q_2(b) + q_3(b)$

$q_3 \rightarrow q_2(a)$

$= q_1(aa) + q_2(ba) + q_3 ba$

$q_2 = q_1(a) + q_2(b) + q_2(a)(b)$

$$= q_1(a) + q_2(b+ab)$$

$$\therefore q_2 = q_1 a \cdot (b+ab)^*$$

$$q_1 = \epsilon + q_1(a) + q_1 a \cdot (b+ab)^* b$$

$$= \epsilon + q_1[a + a(b+ab)^* b]$$

$$q_1 = \epsilon[a + a(b+ab)^* b]^*$$

$$q_2 = (a + a(b+ab)^* b)^* a (b+ab)^*$$

$$q_3 = (a + a(b+ab)^* b)^* a (b+ab)^* a$$

(3)

| | 0 | 1 |
|---|---|---|
| $q_1$ | $q_2$ | $q_1$ |
| $q_2$ | $q_2$ | $q_3$ |
| $q_3$ | $q_3$ | $q_2$ |


$q_1 - \epsilon + q_1(1)$

$q_2 - q_2(0) + q_3(1) + q_1(0)$

$q_3 - q_2(1) + q_3(0)$

$q_3 = q_2(1)0^*$

$q_1 = \epsilon + q_1(1) \Rightarrow q_1 = \epsilon 1^* = 1^*$

$q_2 = q_2 0 + q_3 1 + q_1 0 = q_2 0 + q_3 1 + 1^* 0$

$q_3 = q_2 1 + q_3 0$

$\quad = q_2 (1) 0^*$

$q_2 = q_2 0 + q_2 1 0^* 1 + 1^* 0$

$\quad = 1^* 0 + q_2 (0 + 10^* 1)$

$q_2 = 1^* 0 (0 + 10^* 1)^*$

$q_3 = 1^* 0 (0 + 10^* 1)^* 1 0^*$

1. 3.2.4
2. Applications of Regular E
3. Algebric laws for Regular E
   (i) Associative & commutative
4. Identities & Annihilators

Conversion of PDA to CFG (14-02-2022)

$M = \{\{q_0, q_1\}, \{a, b\}, \delta, \{x, z_0\}, z_0, q_0, \phi\}$

1.) $\delta(q_0, a, z_0) = (q_0, xz_0)$

2.) $\delta(q_0, a, z) = (q_0, xx)$

3.) $\delta(q_0, b, x) = (q_1, \epsilon)$

4.) $\delta(q_1, b, x) = (q_1, \epsilon)$

5.) $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$\delta(q_0, a, z) = (q_0, xx)$

$(q_0, z, q_0) = a(q_0, x, q_0)(q_0, x, q_0)$

$(q_0, z, q_0) = a(q_0, x, q_1)(q_1, x, q_0)$

$(q_0, z, q_1) = a(q_0, x, q_0), (q_0, x, q_1)$

$(q_0, z, q_1) = a(q_0, x, q_1)(q_1, z, q_1)$

$\delta(q_0, a, z) = (q_1, z)$

$(q_0, z, q_0) = a(q_1, z, q_0)$ ~~$(q_0, z, q$~~

$(q_0, z, q_1) = a(q_1, z, q_1)$

$\delta(q_1, x, A) = (q_1, \epsilon)$

$(q_1, A, P) = x$

$S \rightarrow (q_0, z_0, q_0)$

$S \rightarrow (q_0, z_0, q_1)$

$[q_0, z_0, q_0] = a(q_0, x, q_0)(q_0, z_0, q_0]$

$q_0 z_0 q_0 = a[q_0 x q_1][q_1 z_0 q_0]$

$q_0 z_0 q_1 = a[q_0 x q_0][q_0 z_0 q_1]$

$q_0 z_0 q_1 = a[q_0 x q_1][q_1 z_0 q_1]$

$[q_0 \times q_0] = a\,[q_0 \times q_0][q_0 \times q_0]$

$[q_0 \times q_0] = a\,[q_0 \times q_1][q_1 \times q_0]$

$[q_0 \times q_1] = a\,[q_0 \times q_0][q_0 \times q_1]$

$[q_0 \times q_1] = a\,[q_0 \times q_1][q_1 \times q_1]$

$\delta(q_0, b, x) = (q_1, \epsilon)$

$(q_0\ x\ q_1) \rightarrow b$

$\delta(q_1, b, x) = (q_1\ \epsilon)$

$q_1 \times q_1 = b$

$q_1\ Z_0\ q_1 = \epsilon$

1.) $(q_1, Z_0, q_0)$ is not present in LHS

$(q_1, x, q_0)$ is not present in LHS

2) $A = \{(q_0, q_1), (a, b), (Z_0, Z), q_0, Z_0, \phi\}$

$\delta(q_0, b, Z_0) = (q_0, ZZ_0)$

$\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$

$\delta(q_0, b, Z) = (q_0, ZZ)$

$\delta(q_0, a, Z) = (q_1, Z)$

$\delta(q_1, b, Z) = (q_1, \epsilon)$

$\delta(q_1, a, Z_0) = (q_0, Z_0)$

$$\left.\begin{aligned}
q_0\ Z_0\ q_0 &= b(q_0\ Z\ q_0)(q_0\ Z_0\ q_0)\\
q_0\ Z_0\ q_0 &= b(q_0\ Z\ q_1)(q_1\ Z_0\ q_0)\\
q_0\ Z_0\ q_1 &= b(q_0\ Z\ q_0)(q_0\ Z_0\ q_1)
\end{aligned}\right\}\ \text{1st}$$

$q_0\ Z_0\ q_1 = b(q_0\ Z\ q_1)(q_1\ Z_0\ q_1)$

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$(q_0, z, q_0) = b(q_0\ z\ q_0)(q_0\ z\ q_0)$
$(q_0\ z, q_0) = b(q_0\ z\ q_1)(q_1\ z\ q_0)$
$(q_0\ z\ q_1) = b(q_0\ z\ q_0)(q_0\ z\ q_1)$
$(q_0\ z\ q_1) = b(q_0\ z\ q_1)(q_1\ z\ q_1)$ $\rightarrow$ 3rd

$(q_1\ z\ q_1 = b) \rightarrow 5^{th}$ transition

$(q_0\ z\ q_0) = a(q_1\ z\ q_0)$
$(q_0\ z\ q_1) = a(q_1\ z\ q_1)$ $\rightarrow$ 4th

$(q_1\ z_0\ q_0) = a(q_0\ z_0\ q_0)$
$(q_1\ z_0\ q_1) = a(q_0\ z_0\ q_1)$ $\rightarrow$ 6th

$(q_0\ z_0\ q_0) = \epsilon \rightarrow 2^{nd}$

let $q_0 z_0 q_0 = A$
$q_0 z_0 q_1 = B$
$q_0\ z\ q_0 = C$
$q_0\ z\ q_1 = D$
$q_1 z_0 q_0 = E$
$q_1\ z_0\ q_1 = F$
$q_1\ z\ q_1 = G$

For $\delta(q_0, b, z_0) = (q_0, zz_0)$
$A \rightarrow bCA$
$A \rightarrow bDE$
$B \rightarrow bCB$
$B \rightarrow bDF$

For $\delta(q_0, b, z) = (q_0, zz)$
$C \rightarrow bCC$
$C \rightarrow bD(q_1\ z\ q_0)$
$D \rightarrow bCD$
$D \rightarrow bDG$

For $\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$

$(q_0 Z_0 q_0) = \epsilon$

$A \to \epsilon$

For $\delta(q_0, a, Z) = (q_1, Z)$

$C \to a(q_1 Z q_0)$

$D \to aG$

For $\delta(q_1, b, Z) = (q_1, \epsilon)$

$G \to b$

For $\delta(q_1, a, Z_0) = (q_0, Z_0)$

$E \to aA$

$F \to aB$

$A \to bCA \mid bDE \mid \epsilon$

$B \to bCB \mid bDF$

$C \to bCC$

$D \to bCD \mid bDG \mid aG$

$E \to aA$

$F \to aB$

$G \to b$

Conversion of CFG to PDA

① $S \rightarrow aSb \mid ab$ → This is of non GNF form

Machine, $M = (Q, \Sigma, \delta, \Gamma, Z_0, q_0, F)$

$= q, \Sigma, \delta, (V \cup T), S, q, \phi$

↳ for PDA with accepting empty stack.

① $\delta(q, \epsilon, S) = (q, aSb)$

② $\delta(q, \epsilon, S) = (q, ab)$

③ $\delta(q, a, a) = (q, \epsilon)$

④ $\delta(q, b, b) = (q, \epsilon)$

aaabbbb

$\vdash (q, aaabbbb, S)$

$\vdash (q, aaabbbb, aSb)$

$\vdash (q, aabbbb, Sb)$

$\vdash (q, aabbbb, aSbb)$

$\vdash (q, abbbb, Sbbb)$

$\vdash (q, abbbb, abbb)$

$\vdash (q, bbbb, bbb)$

$\vdash (q, b, S)$

$S \to aA \mid aAA \mid aAAA \mid a$ $(Q, \Sigma, \delta, V, S, q, \phi)$

$S \to 0BB$

$B \to 0S \mid 1S \mid 0$

1.) $\delta(q, 0, S) = (q, BB)$ $\quad qSq = 0(qBq)(qBq)$

2.) $\delta(q, 0, B) = (q, S)$ $\quad qBq = 0(qSq)$

3.) $\delta(q, 1, B) = (q, S)$ → two transitions Non deterministic

4.) $\delta(q, 0, B) = (q, \epsilon)$

$\vdash (q, 010000, S) \vdash (q, 10000, BB)$

$\vdash (q, 0000, SB)$

$\vdash (q, 000, BBB)$

$\vdash (q, 00, BB)$

$\vdash (q, \epsilon, \epsilon)$

CFG → GNF

$S \to absb \mid aa$

if $A \to a$

$B \to b$

$S \to aBSB \mid aA$

$S \to AB$

$A \to aA \mid bB \mid b$

$B \to b$

$S \to aAB \mid bBB \mid bB$

$A \to aA \mid bB \mid b$

$B \to b$

$S \to CA \mid BB$

$B \to b \mid SB$

$C \to b$

$A \to a$

$S \to bA \mid BB$

$B \to b \mid SB$

$C \to b$

$A \to a$

(11-03-2023)

Turing Machines

- Unrestricted grammar
- Most inclusive

Turing machine, $T = \{\Sigma, Q, q_0, F, \Gamma, B, \delta\}$

$\Sigma$ - input symbols

$Q$ - set of all states

$q_0$ - initial state

F - final states.

$\Gamma$ - contains all symbols $(\Sigma + B + Q)$

B → like blank space.

$\delta : q \times \Gamma \rightarrow q \times \Gamma \times (L/R)$

Instantaneous description:-

① $a\underset{q_1}{\underset{\uparrow}{a}}aa$ ② $a\overset{\downarrow}{\underset{q_1}{a}}aa$ ③ $aaaq_1a$

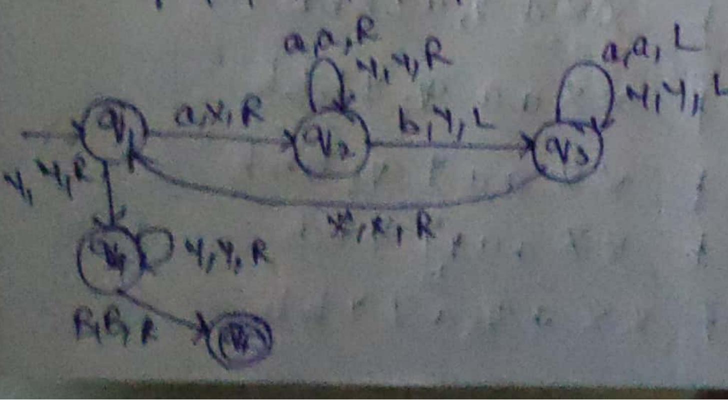
→ Any of the representation can be used.

$x_1 x_2 x_3 \ldots x_{i-1} q x_i \; x_{i+1}$

$\delta(q, x_i) \;(P, Y, L)$

$x_1 x_2 x_3 \ldots P x_{i-1} Y x_{i+1}$

B | a | a | a | b | b | b | B



B | a | a | a | a | b | b | b | b | B

T table:

| | a | b | x | y | B |
|---|---|---|---|---|---|
| $q_1$ | $q_2 X R$ | - | - | $q_4 Y R$ | - |
| $q_2$ | $q_2 a R$ | $q_3 Y L$ | - | $q_2 Y R$ | - |
| $q_3$ | $q_3 a L$ | - | $q_1 X R$ | $q_3 Y L$ | - |
| $q_4$ | - | - | - | $q_4 Y R$ | $q_5 B R$ |
| $q_5$ | - | - | - | - | - |

$q_1 aaaa\,bbbb \vdash X q_2 aaa\,bbbb$

$\vdash X a q_2 aa\,bbbb$

$\vdash X aa q_2 a\,bbbb$

$\vdash X aaa q_2 bbbb$

$\vdash X aa q_3 a Y bbb$

$\vdash X a q_3 aa Y bbb$

$\vdash X q_3 aaa Y bbb$

$\vdash q_3 X aaa Y bbb$

$\vdash X q_1 aaa Y bbb$

$\vdash X X q_2 aa Y bbb$

$\vdash X X a q_2 a Y bbb$

$\vdash X X aa q_2 Y bbb$

$\vdash X X aa Y q_2 bbb$

$\vdash X X aa q_3 Y Y bb$

$\vdash X X a q_3 a Y Y bb$

$\vdash x\,x\,q_3\,a\,a\,y\,y\,b\,b$

$\vdash x\,q_3\,x\,a\,a\,y\,y\,b\,b$

$\vdash x\,x\,q_1\,a\,a\,y\,y\,b\,b$

$\vdash x\,x\,x\,q_2\,a\,y\,y\,b\,b$

$\vdash x\,x\,x\,a\,q_2\,y\,y\,b\,b$

$\vdash x\,x\,x\,a\,y\,q_2\,y\,b\,b$

$\vdash x\,x\,x\,a\,y\,y\,q_2\,b\,b$

$\vdash x\,x\,x\,a\,y\,q_3\,y\,y\,b$

$\vdash x\,x\,x\,a\,q_3\,y\,y\,y\,b$

$\vdash x\,x\,x\,q_3\,a\,y\,y\,y\,b$

$\vdash x\,x\,q_3\,x\,a\,y\,y\,y\,b$

$\vdash x\,x\,x\,q_1\,a\,y\,y\,y\,b$

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$\vdash x\,x\,x\,x\,y\,y\,y\,q_2\,b$

$\vdash x\,x\,x\,x\,y\,y\,q_3\,y\,y$

$\vdash x\,x\,x\,x\,y\,q_3\,y\,y\,y$

$\vdash x\,x\,x\,x\,q_3\,y\,y\,y\,y$

$\vdash x\,x\,x\,q_3\,x\,y\,y\,y\,y$

$\vdash x\,x\,x\,x\,q_1\,y\,y\,y\,y$

$\vdash x\,x\,x\,x\,y\,q_4\,y\,y\,y$

$\vdash x\,x\,x\,x\,y\,y\,q_4\,y\,y$

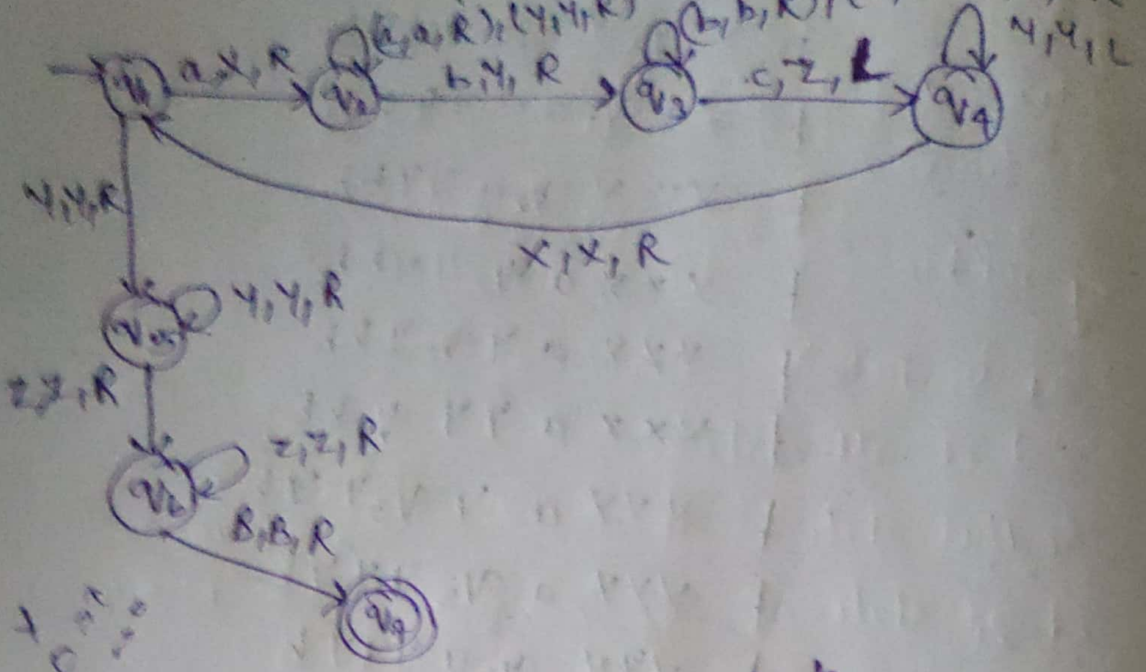
$\vdash x\,x\,x\,x\,y\,y\,y\,q_4\,y$

$\vdash x\,x\,x\,x\,y\,y\,y\,y\,q_4$

$\vdash x\,x\,x\,x\,y\,y\,y\,y\,q_5$

Accepted.

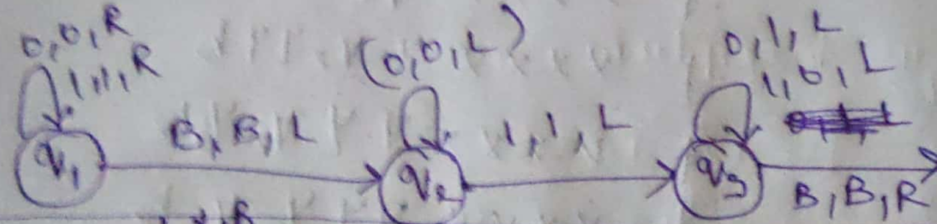
B|a|a|b|b|c|c|B $a^N b^N c^N$

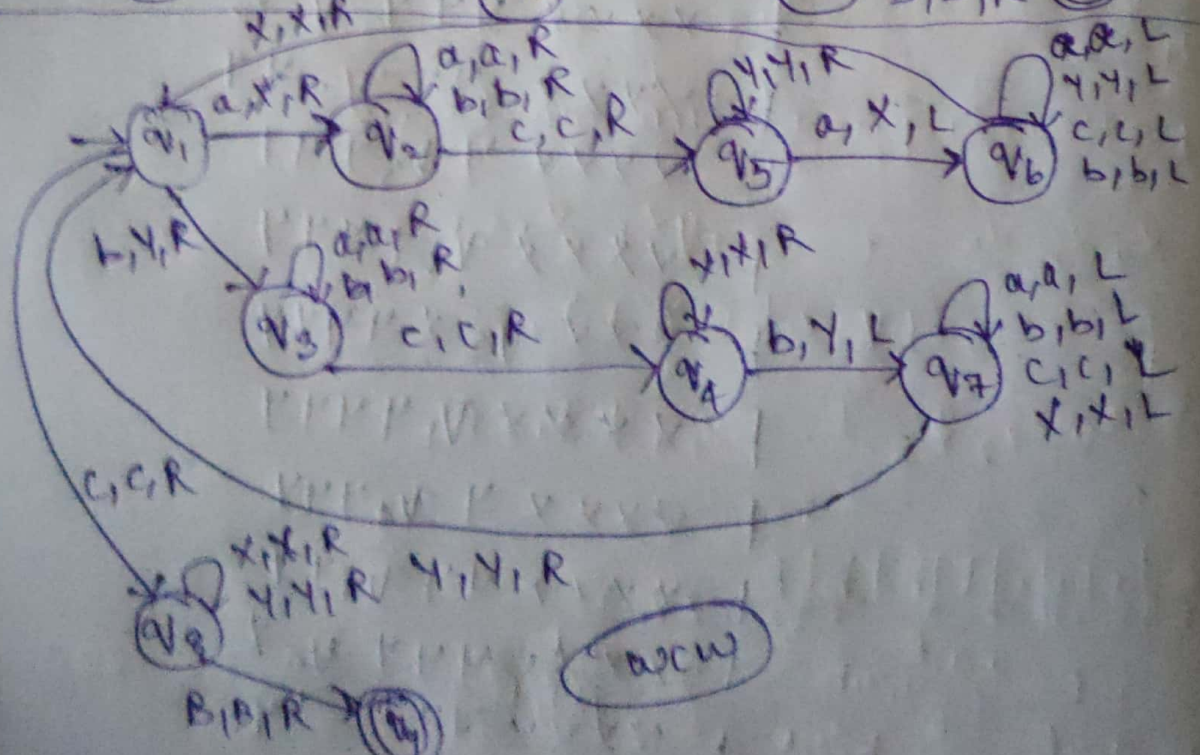



a, b

H.W :- wcw

1's complement :-



$abcab \vdash x q_2 b c a b$

$\vdash x b q_2 c a b$

$\vdash x b c q_5 a b$

$\vdash x b q_6 c x b$

$\vdash x q_6 b c x b$

$\vdash q_6 x b c x b$

$\vdash x q_1 b c x b$

$\vdash x y q_3 c x b$

$\vdash x y c q_4 x b$

$\vdash x y c x q_4 b$

$\vdash x y c q_7 x y$

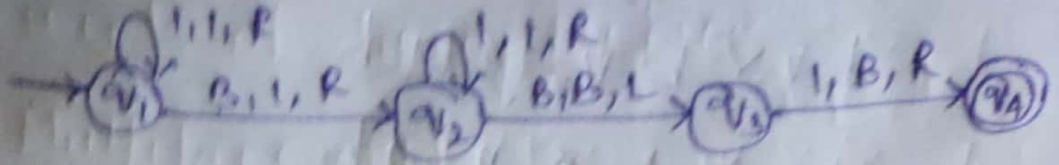
$\vdash x y q_7 c x y$

$\vdash x q_7 y c x y$

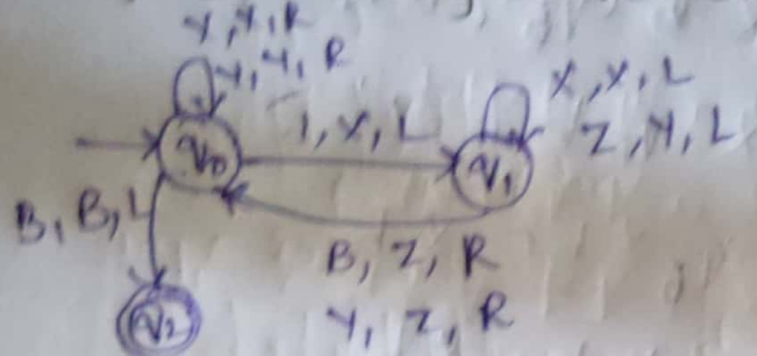
$\vdash x y q_1 c x y$

$\vdash x y c q_8 x y$

Unary Addition in Turing Machines:-




Unary to Binary conversions:-




Z = 1

Y = 0

Unary :- 111 Binary :- 11

BB$q_0$111B ⊢ $q_1$BX11B

⊢ Z$q_0$X11B

⊢ ZX$q_0$11B

⊢ Z$q_1$XX1B

⊢ $q_1$ZXX1B

⊢ $q_1$BYXX1B

⊢ Z$q_0$YXX1B

⊢ ZYXX$q_0$1B

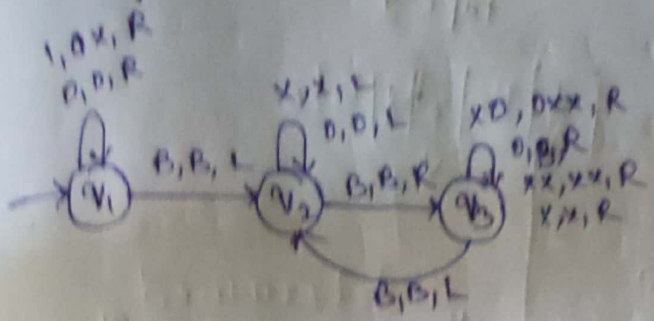
⊢ ZYX$q_1$XXB

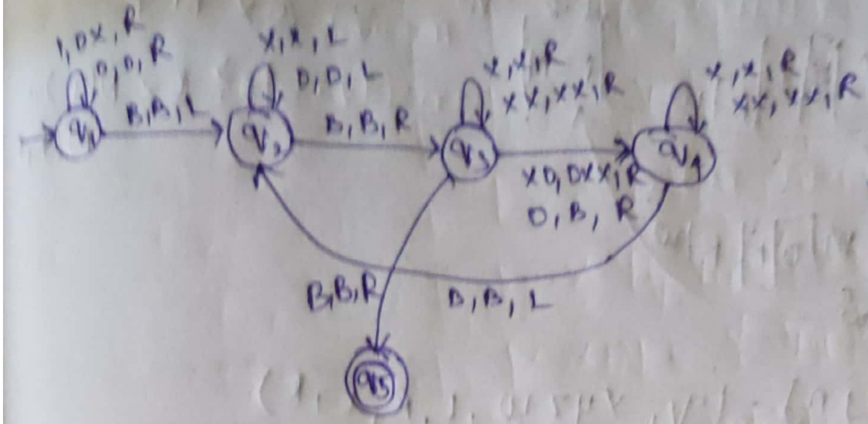
⊢ Z$q_1$YXXXB

⊢ ZZ$q_0$XXXB

Binary to unary conversion:

Binary = 11 Unary = 111 0x0x




11 ⊢ 0x $q_1$1

⊢ 0x0x $q_1$B

⊢ 0x0 $q_2$ xB

⊢ $q_2$ B 0x0xB

⊢ B $q_3$ 0x0xB

⊢ B0 $q_3$ x0xB

⊢ B0 0xx $q_3$ xB

⊢ B00xxx $q_3$ B

⊢ B00xx $q_2$ x

⊢ BB $q_3$ x0xB

⊢ BB 0xx $q_3$ xB

⊢ BB 0xxx $q_3$ B

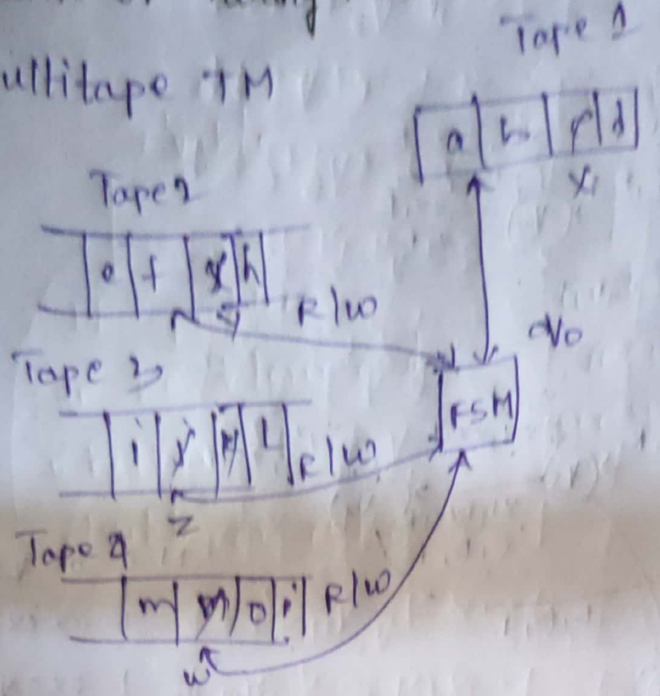
⊢ BB 0xx $q_2$ xB

⊢ BBB xxxB

(09-03-2022)

# Variants of Turing Machine

1) Multitape TM



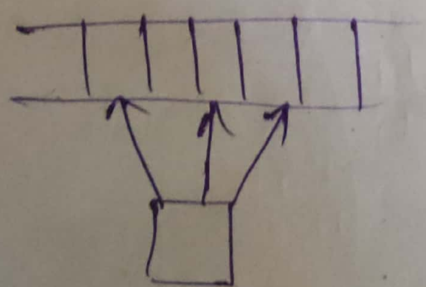

$\delta: Q \times N^{\gamma} \to Q \times N(\gamma) \times N(R,L)$

$\delta(q_0, cgjn) = (q, xyzw, L, R, L, R)$

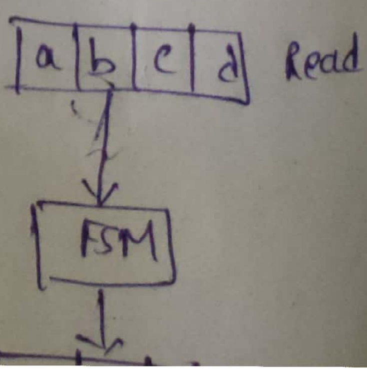
2) Non-deterministic turing Machine :-

$\delta: Q \times \Gamma \to 2^{Q \times \Gamma \times (R|L)}$

3) Multi-head Turing Machine :-



Offline Turing Machine



4) Multi-dimensional TM

$\delta: Q \times \Gamma \rightarrow (Q \times \Gamma \times (L, R, U, D))$

6) TM with stay option:-

$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times (L, R, S)$

5) TM with semi-infinite tape:-

→ one side infinity.

8) Jumping TM

→ Moves left / Right over multiple locations

$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times (L, R, n)$

$n \rightarrow$ number of jumps.

Exp of the form

$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$

where $a_n, a_{n-1}, \ldots$ are constants

Problem

Solvable

If aproblem can be determined whether its solvable or not.

Deterministic soln

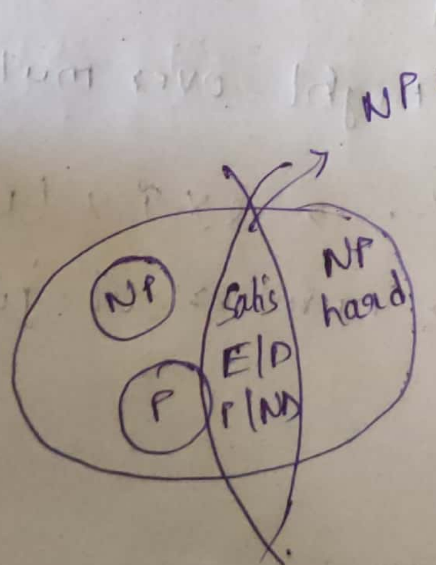
Poly time

exp time

Non deterministic sol^n

Unsolvable

undetermined means unsolvable.




P → Deter, Poly time

NP → Non-Deter Poly time