

Assignment 5

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Question

Find the probability P_s that in a men's tennis tournament the final match will last five sets.

- (a) Assume that the probability p that a player wins a set equals 0.5.
- (b) Use bayesian statistic with uniform prior (see law of succession).

Solution I

(a) Suppose that the probability $P(A)$ that player A wins a set equals $p=1-q$. He wins the match in five sets if he wins two of the first four sets and the fifth set. Hence, the probability $P_5(A)$ that he wins in five equals $6p^3q^2$. Similarly, the probability $p_5(B)$ that player B wins in five equals $6p^2q^3$. Hence,

$$p_5 = p_5(A) + p_5(B) = 6p^3q^2 + 6p^2q^3 = 6p^2q^2$$

is the probability that the match last five sets.

If $p=q=1/2$, then $p_5 = 3/8$

(b) Suppose now that $P(A) = \tilde{p}$ is an RV with density $f(p)$. In this case, $\tilde{p}_5 = 6\tilde{p}^2(1 - \tilde{p}^2)$ is an RV. We wish to find its best bayesian estimate .

Using the MS criterion, we obtain

$$\hat{p}_5 = E(p_5) = \int_0^1 6p^2(1 - p^2)f(p) dp$$

If $f(p)=1$, then $\hat{p}_5 = 1/5$