ELL 311: Communication Engineering	**I Semester 2020-2021**

Lectures 13-14-15: Oct. 19-23

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1 Frequency Division Multiplexing

In telecommunications, frequency-division multiplexing (FDM) is a technique by which the total bandwidth available in a communication medium is divided into a series of non-overlapping frequency bands, each of which is used to carry a separate signal.

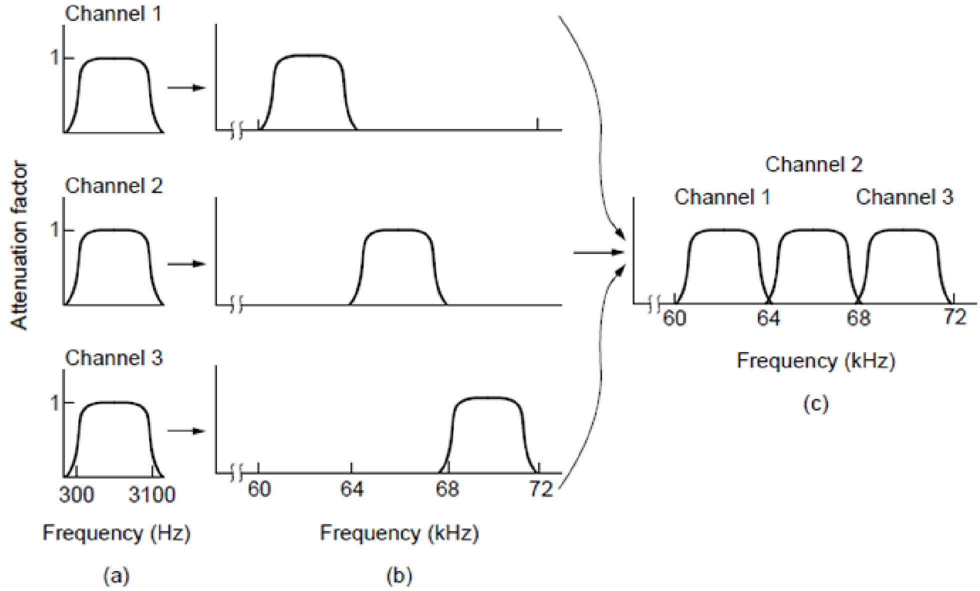


Figure 1: Frequency Division Multiplexing

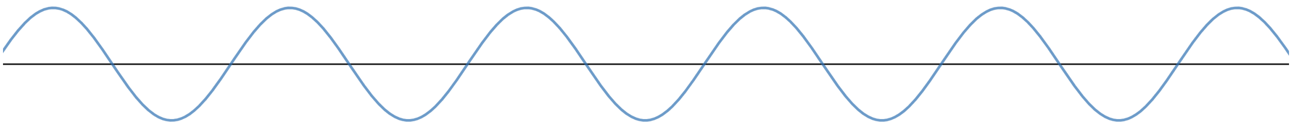
2 Modulation

Modulation is the process that moves the message signal into a specific frequency band that depends on the physical channel such that this operation is invertible. Modulation allows us to send a signal over a bandpass frequency range which allows us to transmit **multiple different signals simultaneously over the same channel** using different frequency ranges (FDM). Another advantage of modulation is that it allows us to use **antennas smaller in size**. A baseband(low frequency) signal would require a large antenna(of the order $\lambda/2$ where λ is the wavelength of the signal) for efficient communication which might not be practically feasible ($\sim$ 30 km for a 10 kHz signal), but with the help of modulation, these baseband signals are shifted to a much higher frequency thus allowing the use of a much smaller antenna. The message signal or the modulating signal, $m(t)$, modulates the carrier signal to form a *Modulated signal*. The signal obtained after demodulating the modulated signal is the *Demodulated signal*. In theory, the demodulated and the original signal should be the same but due to distortion by the channel, they are not exactly the same. There are basically three types of modulations that are widely adopted in communication systems:

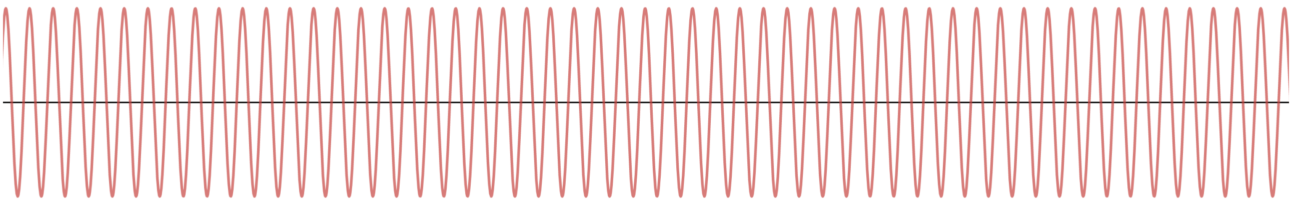
1. Amplitude Modulation
2. Frequency Modulation
3. Phase Modulation

2.1 Amplitude Modulation

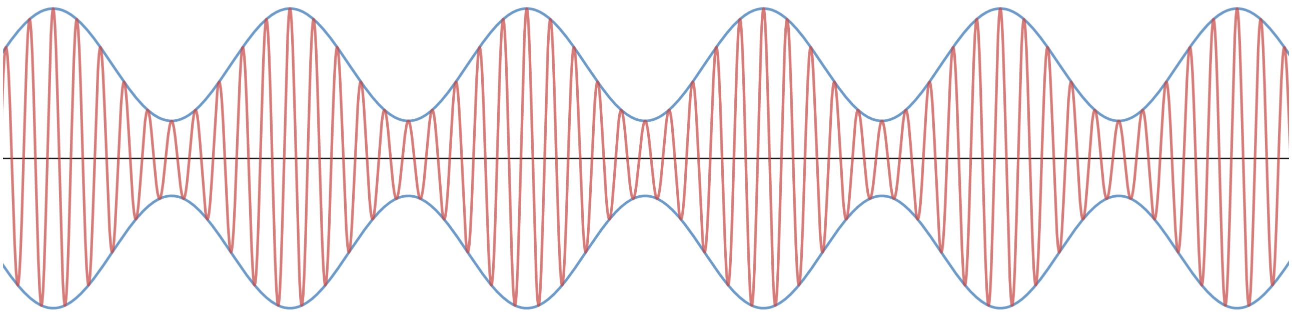
Amplitude modulation is a technique widely used in communication systems most commonly for transmitting a piece of information over radio carrier waves. In this type of modulation, the amplitude(signal strength) of the carrier wave is varied in proportion to that of the baseband (message) signal, $m(t)$. Amplitude $a(t)$ of the carrier wave $a(t)\, cos(\omega_c t + \theta_c)$ is varied in this process. An important observation is that $a(t) \neq m(t)$ (as $m(t)$ can take both negative and positive values but $a(t)$, that is the amplitude of the signal cannot be negative). The frequency ω_c and phase θ_c are constant. The frequency of the carrier wave, $\omega_c >> B$ (where B is the message bandwidth or the maximum frequency content of the signal $m(t)$) due to reasons stated in the above section. Figure 2 shows an illustration of an amplitude modulated (AM) signal.



(a) Message Signal



(b) Carrier Signal



(c) Amplitude Modulated Signal ($\mu < 1$)

Figure 2: An illustration of Amplitude Modulation

We add a constant, A, which can be considered as a DC shift to the message signal to make amplitude of the carrier wave positive. Hence,

$$a(t) = (A + m(t)) \tag{1}$$

The modulated signal, x_{AM}, now becomes

$$x_{AM}(t) = (A + m(t))\, cos(\omega_c t) = A(1 + k_a m(t))\, cos(\omega_c t) \tag{2}$$

Since we need to choose constants in order to satisfy a positive amplitude (by giving a DC shift A),

$$|k_a m(t)| < 1 \tag{3}$$

$max(|k_a m(t)|)$ gives us the ***Depth of Modulation*** or the ***Modulation Index,*** μ. In our analysis, we consider a message signal which is a single tone sinusoidal signal (single frequency). Since any complex/random signal can be expressed in the form of exponentials, the analysis of a sinusoidal signal can hence be extended to the domain of complex signals. The single tone sinusoidal message signal is represented as:

$$m(t) = A_m \, cos(\omega_m t) \tag{4}$$

The final signal after amplitude modulation becomes,

$$x_{AM}(t) = (A + A \cdot k_a \cdot A_m \, cos(\omega_m t))cos(\omega_c t) = A(1 + k_a \cdot A_m \, cos(\omega_m t))cos(\omega_c t) \tag{5}$$

$$\mu = max\{k_a m(t)\} = k_a A_m \tag{6}$$

Hence,

$$x_{AM}(t) = A(1 + \mu \, cos(\omega_m(t)))cos(\omega_c(t)) \tag{7}$$

If we increase μ to 1 (100% modulation), the message signal becomes

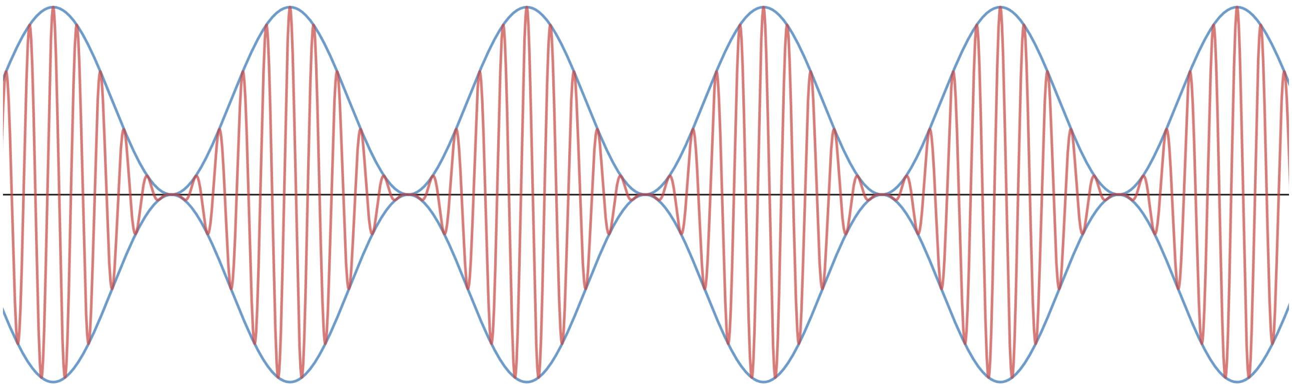


Figure 3: Amplitude Modulation at $\mu = 1$

For $\mu > 1$, envelope distortions occur. Coherent demodulation (3.2) cascaded with a low pass filter can still be used to obtain the original signal but Envelope Detection (6.2) fails in this case.

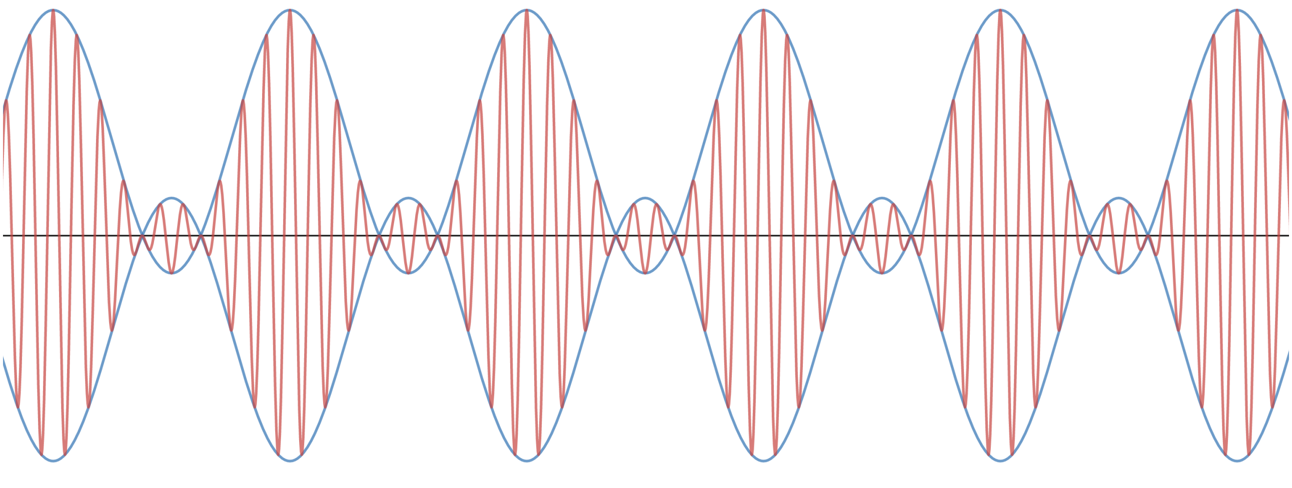


Figure 4: Amplitude Modulation at $\mu > 1$

Sideband and Carrier Power

For a single tone message signal,

$$x_{AM}(t) = \underbrace{A\,cos(\omega_c t)}_{\text{carrier}} + \underbrace{m(t)\,cos(\omega_c t)}_{\text{message signal}} \tag{8}$$

The carrier power P_c is the mean square value of $A\,cos(\omega_c t)$, which is $A^2/2$. The sideband power, P_s is the power of $m(t)\,cos(\omega_c t)$, which is $k_a^2 A^2 A_m^2/4$. Hence,

$$P_c = \frac{A^2}{2} \qquad\qquad P_s = \frac{k_a^2 A^2 A_m^2}{4}$$

The sideband power is the useful power whereas the carrier power is the power wasted for convenience and does not contain any information. Total power of the signal is the sum of both the powers. Therefore, the efficiency, η

$$\eta = efficiency = \frac{useful\ \ power}{total\ \ power} = \frac{P_s}{(P_s + P_c)} = \frac{\mu^2}{\mu^2 + 2} \tag{9}$$

$$\mu := k_a A_m \tag{10}$$

As evident from the above equations, lower modulation indices lead to less power across the message signal and more power across the carrier which would result in a lower SNR (Signal to Noise Ratio) which is not desirable. Hence **higher modulation indices are generally preferred**. Maximum efficiency for a single tone signal occurs at $\mu = 1\ \ or\ \ 100\%$, and is given by

$$maximum\ \ efficiency\ \ (e) = \frac{1}{1+2} = 0.33 \tag{11}$$

We have,

$$x_{AM}(t) = A(1 + k_a m(t))\,cos(\omega_c t) \tag{12}$$

$$x_{AM}(t) = A(1 + k_a \cdot A_m\,cos(\omega_m t))cos(\omega_c t) = A\,cos\omega_c t + \frac{k_a A A_m}{2}(cos(\omega_c + \omega_m)t + cos(\omega_c - \omega_m)t) \tag{13}$$

Taking fourier transform of the above equation on both sides,

$$x_{AM}(\omega) = A(\pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c)) + \frac{k_a A}{2}(M(\omega - \omega_c) + M(\omega - \omega_c)) \tag{14}$$

Consider the following example where $M(\omega)$ (Fourier transform of the message signal) is as shown:

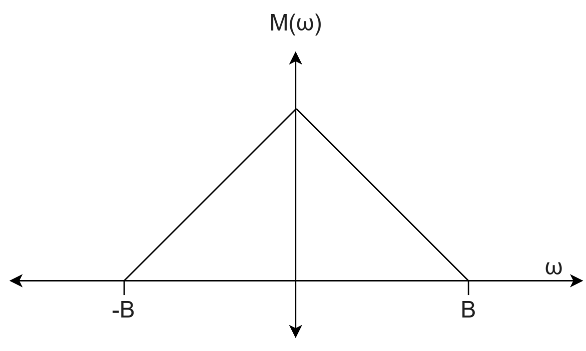


Figure 5:

$x_{AM}(\omega)$ for this signal would be

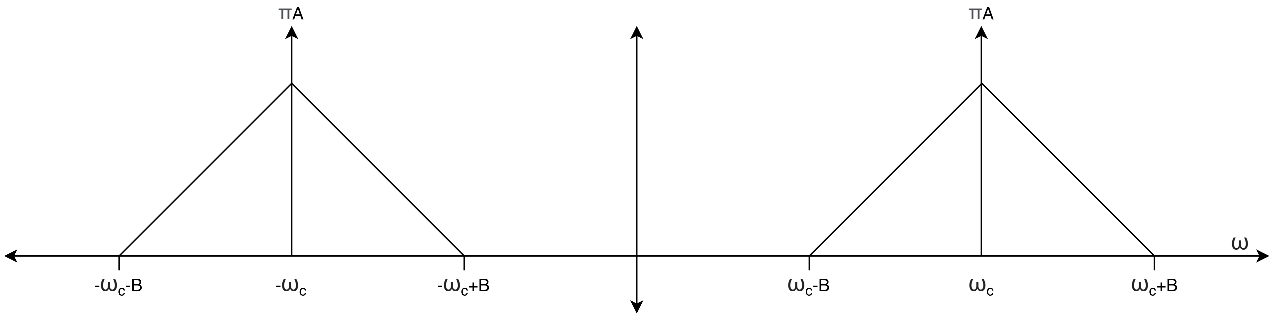


Figure 6:

Note that the bandwidth of the message signal is 'B' Hz (Positive half in the graph of $M(\omega)$, $0\ \ to\ \ B$), whereas the bandwidth of the transmitted AM signal is '$2B$' ($\omega_c - B\ \ to\ \ \omega_c + B$). This is one of the disadvantages of such a modulation scheme as more bandwidth is consumed. Another disadvantage of such a scheme is that the carrier frequency, despite of carrying no information, consumes a lot of power.

3 Synthesizing and Recovering Signal

In this section we will discuss how to synthesize Amplitude modulated signal from message signal and how to recover message signal from the received signal.

3.1 Synthesizing

For this purpose we'll use a square law device. We are using a square law device because it will be able to provide a good estimation of any non linear function like exponential functions. We will have to keep in mind that the amplitude of the message signal is not too large because in that case square law device won't work and we would require higher order polynomial. The equation of the square law device is given below:

$$y(t) = ax(t) + b(x(t))^2 \tag{15}$$

Before passing the message signal $m(t)$ we will add the carrier signal $A\ cos(\omega_c t)$ to the signal to get $x(t) = m(t) + A\ cos(\omega_c t)$. Which gives us:

$$\begin{aligned} y(t) &= a(m(t) + A\,cos(\omega_c t)) + b(m(t) + A\,cos(\omega_c t))^2 \\ &= am(t) + aA\,cos(\omega_c t) + bm^2(t) + 2bm(t)\cdot A\,cos(\omega_c t) + bA^2\,cos^2(\omega_c t) \\ &= am(t) + aA\,cos(\omega_c t) + bm^2(t) + 2bm(t)\cdot A\,cos(\omega_c t) + bA^2\left(\frac{1+cos(2\omega_c t)}{2}\right) \end{aligned} \tag{16}$$

The message signal in the Fourier domain is shown below, 2B is the Band width of the signal.

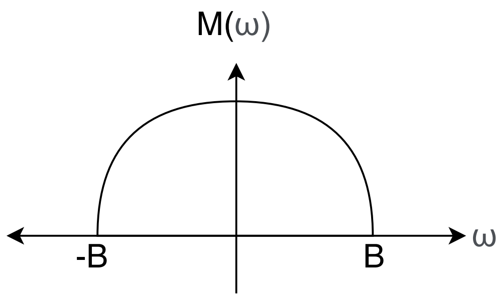


Figure 7: Fourier transform of Message Signal

Now we will take Fourier transform of the output y(t) and plot each term. Note that while taking Fourier transform of $m^2(t)$ we will perform convolution of $M(\omega)$ with itself in frequency domain to get the output in -2B to 2B range. In most of the cases $\omega_c >> B$

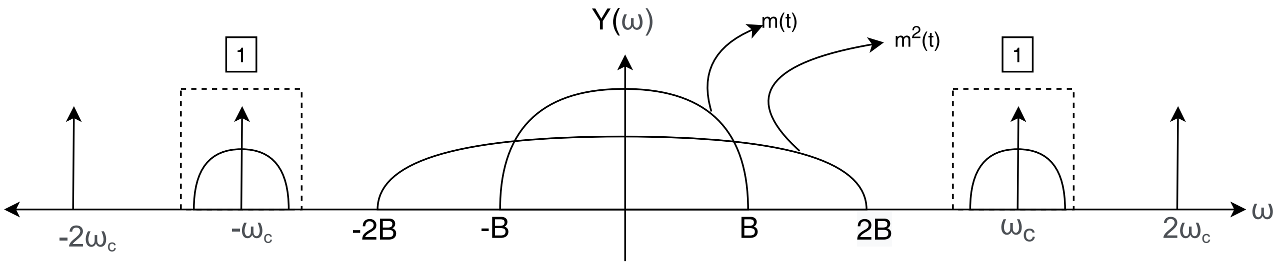


Figure 8: Fourier transform of output signal y(t)

Before finally passing the signal we put a band pass filter 1 as shown in the Figure 8 which allows only bands near ω_c to pass. $\omega_c >> B$ results in no overlap of the components, which in-turn helps in easy retrieval. Let $Y^{'}(\omega)$ be the final signal in the Fourier domain. Taking inverse Fourier transform we get the final signal in time domain as:

$$x_{AM}(t) = \mathscr{F}^{-1}(Y^{'}(\omega)) = aA\, cos(\omega_c t) + 2bAm(t)\, cos(\omega_c t) \tag{17}$$

$$= aA\left(1 + \underbrace{\frac{2bA}{a}}_{k_a} m(t)\right) cos(\omega_c t) \tag{18}$$

3.2 Retrieval of original signal

To retrieve the original signal at the receiving end we will first multiply the received signal with $cos(\omega_c t)$ and then pass it through a low pass filter. We get:

$$x_{AM}(t)\, cos(\omega_c t) = A(1 + k_a m(t))\left(\frac{1 + cos(2\omega_c t)}{2}\right) \tag{19}$$

Plotting this signal in Frequency domain we get:

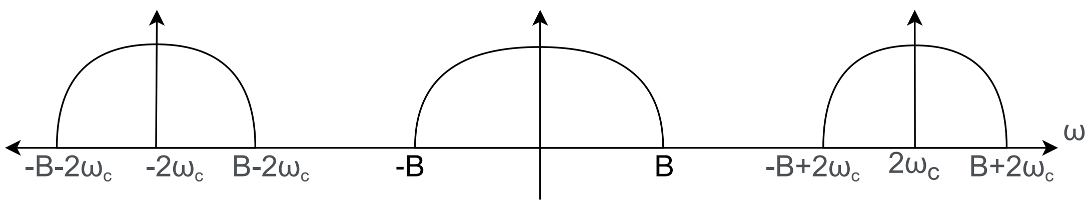


Figure 9: Signal after multiplying it with $\cos(\omega_c t)$

Now, we can pass this signal through a low pass filter which allows only frequencies between -B and B. By doing this, we will be able to regenerate the original signal $m(t)$.

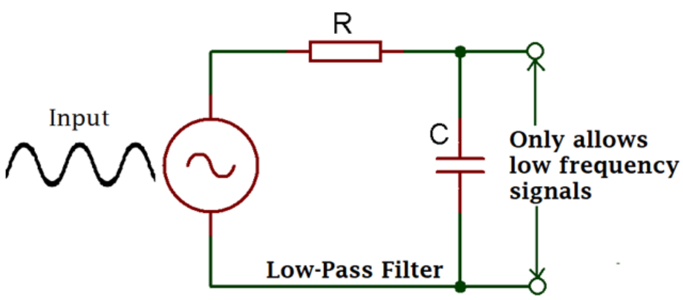


Figure 10: Low Pass Filter Circuit Diagram

The Figure 11 shows the final flow for demodulating the signal. In the above demodulation scheme we need to make sure that the $cos(\omega_c)$ signal that we multiplied with the received signal is in perfect sync with the $cos(\omega_c)$ term of the received signal i.e their is no phase delay and their frequency matches. Such type of process are called **Coherent Demodulation** or **Synchronous Demodulation**.

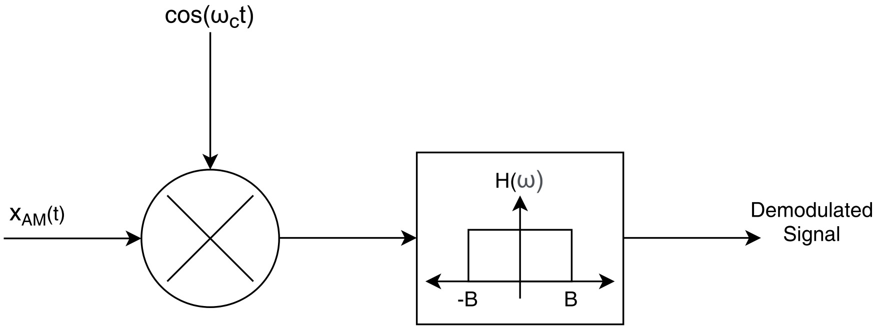


Figure 11: Flow diagram of Demodulation system

3.3 Drawbacks of Amplitude Modulation

Following are a few drawbacks of Amplitude Modulation:

When we frequency translate our signal, the left half and right half of the signal, in terms of information content or any other aspect, are identical. Hence, the result of frequency translation is duplication of information. This results in a signal occupying double of the required bandwidth for transferring information. For example, if B is the maximum frequency content of the signal, the bandwidth required for transmission is 2B.

Secondly, the maximum power that can be transmitted in an AM signal occurs at 100% modulation, and is equal to only 33.33% of the entire signal power. Rest of the power is used up by the carrier signal. There is a scope for significant improvement in terms of efficiency of message signal content. This can be achieved by different techniques involving suppressing of the carrier signal and only including one side band of the signal.

4 Introduction to DSBSC and SSBSC Modulation Schemes

In the process of Amplitude Modulation, the modulated wave consists of the carrier wave and two sidebands. The modulated wave has the information only in the sidebands. Sideband is nothing but a band of frequencies, containing power, which are the lower and higher frequencies of the carrier frequency.

The transmission of a signal, which contains a carrier along with two sidebands can be termed as Double Sideband Full Carrier system or simply DSBFC. It is plotted as shown in the following figure.

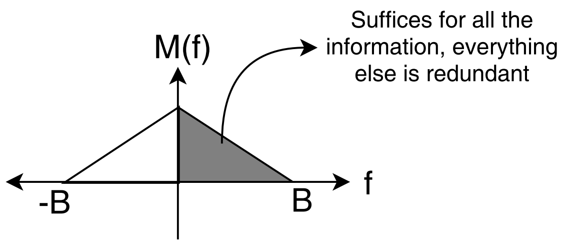


Figure 12: DSBFC with shaded Upper Side Band

A more effective way of transferring the signal is by suppressing the carrier. This results in a Double Sideband Suppressed Carrier system or DSBSC. The DSBSC signal can be formulated as:

$$x_{DSBSC}(t) = A(t))cos(\omega_c(t) \tag{20}$$
$$\tag{21}$$

and the power associated with the signal can be written as $A^2\frac{P_m}{2}$. This clearly doesn't involve any energy being occupied by the carrier wave, and is thus more efficient than Synchronous Demodulation.

We can improve the quality of modulation even further by reducing the required channel bandwidth of the signal, and including only one side band, as both the side bands are identical for real signals. This gives us a very effective and widely used modulation technique known as Single Sideband Suppressed Carrier system or SSBSC.

5 AM for Non-Zero Mean Signals

As we know, the modulation index, or the depth of modulation μ, can be written as

$$\begin{gathered} \mu = max\{k_a m(t)\} \\ \mu : Modulation\ \ Index\ \ or\ \ Depth\ \ of\ \ Modulation \end{gathered} \tag{22}$$

An important observation regarding this is that we assume our signal to have equal excursions on both positive and negative sides, that is $m(t)$ is a zero mean signal, which is not always the case. Let us assume a signal $m(t)$ such that it is a non-zero mean signal. Therefore,

$$\frac{m_{max} + m_{min}}{2} = c\ \ (some\ \ constant) \tag{23}$$

Rewriting,

$$m(t) = \tilde{m}(t) + c \tag{24}$$

where $\tilde{m}(t)$ is a zero-mean signal. Now,

$$\begin{gathered} x_{AM}(t) = (A + c + \tilde{m}(t))cos(\omega_c(t)) \\ x_{AM}(t) = (A + c)\left(1 + \left(\frac{1}{A + c}\right)\tilde{m}(t)\right)cos(\omega_c(t)) \end{gathered} \tag{25}$$

Let the peak value of $\tilde{m}(t)$ be m_p s.t. $\tilde{m}(t) = m_p m^*(t)$. Then,

$$\begin{gathered} m_p = \frac{m_{max} - m_{min}}{2} \\ x_{AM}(t) = (A + c)\left(1 + \left(\frac{1}{A + c}\right)m_p m^*(t)\right)cos(\omega_c(t)) \end{gathered} \tag{26}$$

Therefore,

$$\begin{aligned} \mu &= \left(\frac{1}{A + c}\right)m_p \\ &= \left(\frac{1}{A + \frac{m_{max} + m_{min}}{2}}\right)\left(\frac{m_{max} - m_{min}}{2}\right) \\ \mu &= \left(\frac{1}{2A + m_{max} + m_{min}}\right)(m_{max} - m_{min}) \end{aligned} \tag{27}$$

This way, the Amplitude Modulation can be generalized to non-zero mean message signals.

6 Envelope Detection

An envelope detector is an electronic circuit that takes a (relatively) high-frequency amplitude modulated signal as input and provides an output which is the envelope of the original signal. An envelope detector is sometimes called as a peak detector.

6.1 Importance of Envelope Detection over Coherent Demodulation

After studying Amplitude Modulation schemes, we understand that the demodulation of the signal involved Coherent Demodulation or Synchronous Demodulation, where the knowledge of both the frequency and phase of the carrier signal is required at the receiver end. This is not very feasible and we try to use envelope detection technique where the knowledge of the carrier is not required for demodulation. The only catch is that envelope detection only works if the following condition holds.

$$\mu < 1\ \ for\ \ proper\ \ envelope\ \ detection. \tag{28}$$

6.2 Envelop Detection Modelling

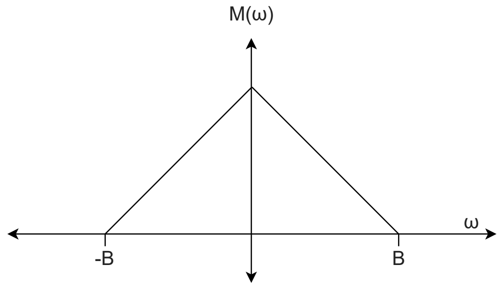
The Amplitude Modulated signal can be written as follows:

$$x_{am}(t) = A(1 + k_a m(t))\, cos(\omega_c t) \tag{29}$$

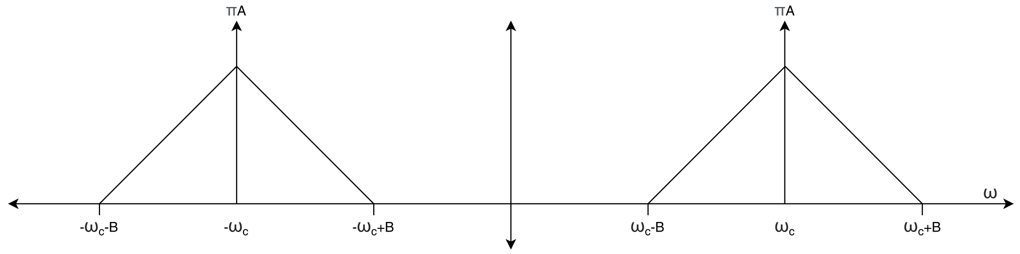
And the corresponding Fourier transform is written as:

$$X_{AM}(f) = \frac{A}{2}(\delta(f - f_c) + \delta(f + f_c)) + \frac{k_a A}{2}(M(f - f_c) + M(f + f_c)) \tag{30}$$

The original message signal in the frequency domain and the corresponding Amplitude Modulated signal can be represented as:



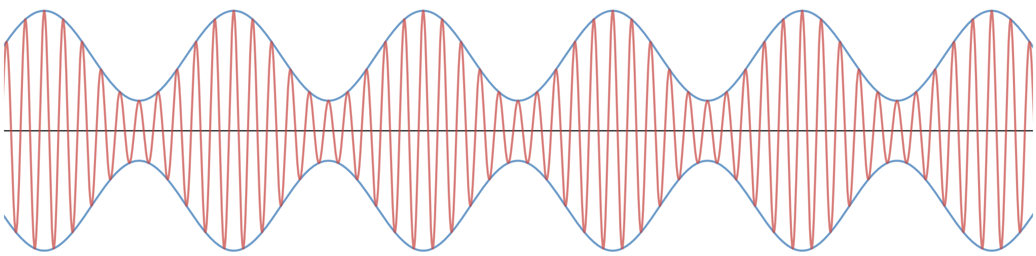
(a) Frequency domain message Signal



(b) Amplitude Modulated signal in frequency domain

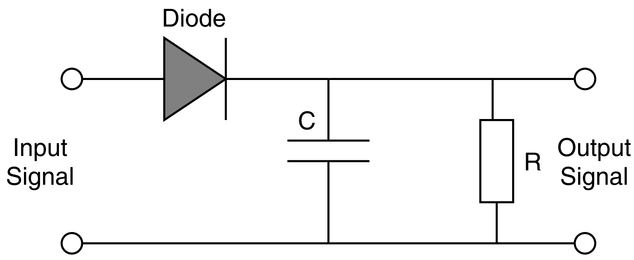
Figure 13:

The Amplitude Modulated Signal in the time domain looks something like this (for $\mu < 1$):



6.3 Retrieval of original signal using Envelop Detection

Assuming that our carrier frequency f_c is very large as compared to our message signal frequency f_m, the excursions of the signal would be very closely packed. Hence, we need to follow the envelop in order to retrieve our signal. For this we use the following circuit.



After passing through the above filter, our input signal becomes somewhat similar to the trace of the envelop. This can be clearly seen in the following figure:

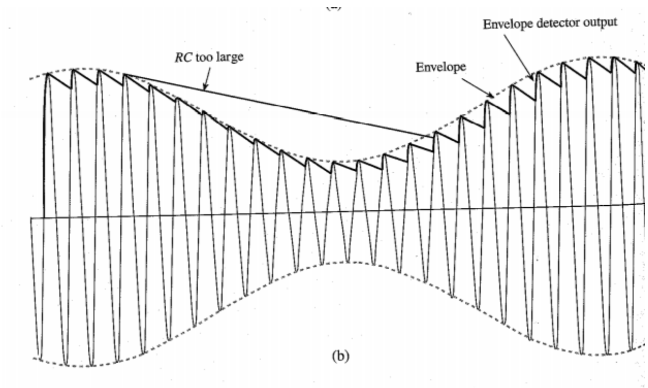


Figure 14: Tracing the envelope of input signal

This process involves charging of the capacitor in the positive cycle as the input voltage keeps increasing. Once it starts falling, the diode gets switched off and the capacitor starts getting discharged across the resistor on the other side.
The importance of this methodology is that we don't need any knowledge of the carrier signal. We can just use an appropriate RC time constant and retrieve our original signal.

The time constant should be chosen in such a way that it satisfies the following properties:

- The time constant should be very high as compared to the carrier signal time period. If this is not the case, the output signal would follow the carrier frequency instead of following the envelop.
- The second important thing is that the time constant should be very low when compared to the message signal time period, or else, it would lead to **Diagonal Clipping**.
- Diagonal Clipping is the distortion that occurs in an AM demodulator (usually associated with diode detection), where the capacitor discharge time constant is set too long for the detector to accurately follow fast changes in the AM signal envelope. Sometimes referred to as "failure to follow distortion," diagonal clipping can also occur in AM modulators when the intelligence bandwidth exceeds that of the modulator.

Hence the time period is as follows:

$$\frac{1}{\omega_c} << RC << \frac{1}{\omega_m} \tag{31}$$

Having $\omega_c >> \omega_m$ helps us in choosing a suitable time constant RC such that we are able to successfully retrieve our signal.

7 Further help

You may take a look at the following sites for further help:

1. Frequency Division Multiplexing: https://www.sciencedirect.com/book/9780128142042/signals-and-systems-us
2. Amplitude Modulation: https://nptel.ac.in/content/storage2/courses/106105080/pdf/M2L5.pdf

3. Demodulation: https://www.electronics-notes.com/articles/radio/modulation/am-synchronous-demodulati
php

4. Envolope Detector: Demodulation: https://www.electronics-notes.com/articles/radio/modulation/am-synchronous-demodulation-detection-detector.php

References

[1] B.P.Lathi Modern Digital and Analog Communication Book,
https://ict.iitk.ac.in/wp-content/uploads/EE320A-Principles-Of-Communication-modern-digital-and-anal
pdf

[2] https://www.wirebiters.com/communication-multiplexing/