

# Algorithmic Game Theory

## LECTURE 2

# Main topics covered:

- ▶ Mechanism Design
  - ▶ Single-item Auctions
  - ▶ Sealed-bid Auctions
  - ▶ First-price Auctions
  - ▶ Second-price Auctions
  - ▶ Some useful terms
  - ▶ Second-price auctions as Ideal auctions
- ▶ Case Study : Sponsored Search Auctions

# Course Goal :

To understand how to design systems with strategic participants that have good performance guarantees

# Single-item Auctions

- ▶ Setup:
  - ▶ Seller has a “single” item (e.g., an old-fashioned smartphone)
  - ▶ ‘ $n$ ’ strategic bidders
  - ▶ Each bidder  $i$  has a valuation  $v_i$  (max amount that bidder is willing to pay)
  - ▶ This valuation is private (unknown to seller and other bidders)
- ▶ What does a bidder want ?
  - ▶ A bidder wants to acquire the item as cheaply as possible given the maximum selling price is  $v_i$
- ▶ Bidder Utility Model
  - ▶ Quasilinear Utility Model *~~(just another complicated name !)*~~
  - ▶ If bidder  $i$  loses, utility=0 !
  - ▶ If bidder  $i$  wins at price  $p$ , utility= $(v_i - p)$

# Sealed-bid Auctions

## ▶ Setup:

- ▶ Each bidder  $i$  privately communicates a bid  $b_i$  to the seller
- ▶ The seller decided who wins
- ▶ The seller decides on a selling price

## ▶ Selection Rule

Item is given to the highest bidder !

## ▶ Implementation of 3<sup>rd</sup> step

- ▶ There are many reasonable ways
- ▶ The choice, however, significantly affects bidder behaviour

### ▶ Example:

If the seller decides to charge nothing to winner, it becomes a game of who names the highest number !

# First-price Auctions

- ▶ Winner pays her bid
- ▶ Disadvantages:
  - ▶ Hard for bidder to figure out how to bid
  - ▶ Hard for seller/auction designer to predict an outcome

## Example:

- Consider a valuation=birth month + birth day
- Minimum=2 (1<sup>st</sup> Jan) | Maximum=43 (31<sup>st</sup> Dec)
- Question→ what bid should we submit ?

Turns out, there is **no dominant strategy** for first-price auctions !

# Second-Price Auctions (Vickery Auctions)

- ▶ Winner pays second-highest bid

If winner bids \$100, 2<sup>nd</sup> highest bid is \$90 → winner pays  $$(90+x)$  where  $x$  is a small increment

- ▶ It is a sealed-bid type auction
- ▶ Is there a dominant strategy for bidding in this case ?

YES !

Proposition 1 → *In a second-price auction, every bidder  $i$  has a dominant strategy: set the bid  $b_i$  equal to her private valuation  $v_i$ .*

Proposition 2 → *Non-negative utility: In a second-price auction, every truthful bidder is guaranteed a non-negative utility.*

## ❑ Advantages:

1. Bidder doesn't have to worry about her competitors' valuations
2. Drastically different from First-price auctions where bidding one's valuation would have guaranteed 0 utility !

# Proof of Proposition 1

Let,

- Bidder  $i$  has a valuation  $v_i$
- $b_{-i}$  is the vector of all bids with  $i^{\text{th}}$  component removed
- $B = \max_{(j \neq i)} b_j$  (i.e., highest bid by some other bidder)

**To show:** Bidder  $i$ 's utility is maximised by setting bid  $b_i = v_i$

There are only 2 possible outcomes  $\rightarrow$

1.  $b_i < B \rightarrow$  bidder  $i$  loses  $\rightarrow$  utility = 0
2.  $b_i \geq B \rightarrow$  bidder  $i$  wins  $\rightarrow$  utility =  $(v_i - B)$

Therefore, considering these two cases we can conclude:

If  $v_i < B$ , maximum utility bidder  $i$  can obtain =  $\max\{0, v_i - B\} = 0$  [which she obtains by bidding truthfully and losing]

If  $b_i \geq B$ , maximum utility bidder  $i$  can obtain =  $\max\{0, v_i - B\} = (v_i - B)$  [which she obtains by bidding truthfully and winning]

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**Conclusion:** A truthful bidder never regrets participating in a second-price auction !

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# Proof of Proposition 2

**To show:** Truthful bidder has non-negative utility

We know, Losers receive utility 0.

Now,

If bidder  $i$  is the winner, she receives a utility  $= v_i - p$ , where  $p = 2^{\text{nd}}$  highest bid

Since,  $i$  is the winner and a truthful bidder, we can say  $v_i$  is the highest bid

Therefore,  $p < v_i \rightarrow (v_i - p) \geq 0$

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$$(v_i - p) \geq 0$$

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This is always true !

# Some Useful Terms

- ▶ Dominant Strategy Incentive Compatible (DSIC)

An auction is called DSIC if truthful bidding is the dominant strategy for every bidder and if truthful bidders always obtain non-negative utility

- ▶ Social Welfare / Social Surplus

Social Welfare of an outcome of a single-item auction is defined as

$$\sum_{i=1}^n v_i \cdot x_i$$

where,  $x_i$  is 1 if  $i$  wins and 0 if  $i$  loses and  $v_i$  is the valuation of  $i$

An auction is *welfare maximizing* if, when bids are truthful, the auction outcome has the maximum possible social welfare

# Second-price auctions as Ideal auctions

A second-price single-item auction satisfies the following:

- ▶ **[strong incentive guarantees]** It is a DSIC auction
- ▶ **[strong performance guarantees]** It is welfare maximising
- ▶ **[computational efficiency]** It can be implemented in time polynomial (indeed, linear) in the size of the inputs, meaning the number of bits necessary to represent the numbers  $v_1, \dots, v_n$

# Case Study: Sponsored Search Auctions (1/3)

## BACKGROUND :

When we query a web search, we get a results page like this

The screenshot shows a Yahoo! search results page for the keyword "attorney". The page has a navigation bar at the top with links to Home, Mail, News, Cricket, Celebrity, Movies, Lifestyle, Mobile, and More. Below the navigation bar is the Yahoo! logo and a search bar containing the text "attorney". To the left of the search results is a sidebar with links to Web, Images, Video, News, Answers, Anytime, Past day, Past week, Past month, and The Web. The search results are displayed in a list format. The first result is a sponsored link from Metacrawler, which is highlighted with a red box. The second result is a sponsored link from a law firm in Slovenia, also highlighted with a red box. The third result is an organic search result from Oxford Dictionaries, highlighted with a green box. The fourth result is an organic search result from Wikipedia, highlighted with a green box. The fifth result is an organic search result from the Cambridge English Dictionary, highlighted with a green box. The sixth result is an organic search result from Merriam-Webster, highlighted with a green box.

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en.oxforddictionaries.com/definition/attorney **Cached**  
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[attorney Meaning in the Cambridge English Dictionary](#)  
dictionary.cambridge.org/dictionary/english/attorney **Cached**  
attorney definition: a lawyer: ... Meaning of "attorney" in the English Dictionary ... to take the witness stand and was then cross-examined by the state attorney.

[Attorney | Definition of Attorney by Merriam-Webster](#)  
www.merriam-webster.com/dictionary/attorney

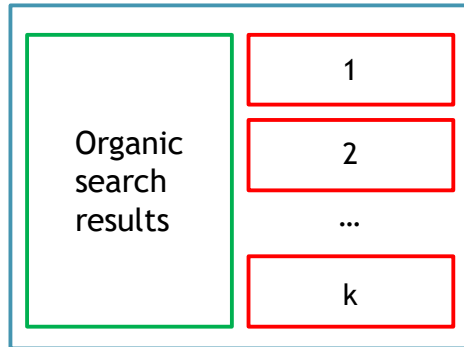
We get a list of advertisements or 'sponsored links' which have been paid for by the advertisers

We get a list of organic search results that concern the keyword that we had queried for

Quick Fact: Sponsored search auctions generated about 98% of Google's revenue around 2006 !!!

# Case Study: Sponsored Search Auctions (2/3)

The basic model of sponsored search auctions:



- There are 'k' slots for sale to advertisers
- Each slot will contain a unique Ad i.e., not identical to any other Ad
- Each slot will be assigned to only 1 advertiser
- The Ads will be in an ordered list i.e., higher slots are more valuable than lower slots

- The difference between different slots is quantified using the click-through-rates (CTRs)  
CTR ( $\alpha_j$ ) of a slot  $j$  = Probability that the user clicks on this slot  
 $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_j \geq \dots \geq \alpha_k$
- Each advertiser  $i$  has a quality score  $\beta_i$  (the higher the better)
- CTR of advertiser  $i$  in slot  $j$  =  $\beta_i \cdot \alpha_j$
- The advertiser also has a private valuation  $v_i$  on for each click on her link
- Expected value derived by advertiser =  $v_i \cdot \alpha_j$

# Case Study: Sponsored Search Auctions (3/3)

- ▶ What do we want for an ideal sponsored search auction ?
  1. DSIC: Truthful bidding is dominant strategy
  2. Social Welfare Maximization: Assignment of bidders should maximize  $\sum_{i=1}^n v_i \cdot x_i$ , where  $x_i$  is now the CTR of the slot
  3. Computational Efficiency: Polynomial (or even near-linear) running time
  
- ▶ Our Design Approach →
  - ▶ **STEP 1:** Assume, without justification that bidders bid truthfully. Then how do we assign bidders to slots so that (2) and (3) hold?

Using the natural greedy algorithm could be a possible solution
  - ▶ **STEP 2:** Given our answer to step 1, how should we set selling price so that (1) holds?

Something analogous to second-price rule.  
Myerson's lemma comes to our rescue here !

# References

- ▶ Twenty Lectures on Algorithmic Game Theory by Tim Roughgarden, 2016, Cambridge University Press
- ▶ [https://www.youtube.com/playlist?list=PLEGCF-WLh2RJBqmxvZ0\\_ie-mleCFhi2N4](https://www.youtube.com/playlist?list=PLEGCF-WLh2RJBqmxvZ0_ie-mleCFhi2N4)

# THANK YOU