Algorithmic Game Theory

LECTURE 7

Main Topics Covered

- General Mechanism Design Environments
- VCG Mechanism
- Practical Considerations

Introduction

- Till now, we have considered only “single-parameter” environments
- Multiple-parameter environments are also possible
- The “Vickery-Clarke-Groves” mechanism is useful in such cases
- VCG mechanism says that DSIC welfare maximization is possible *in principle* in every multiple-parameter environment

General Mechanism Design Environments

- The basic ingredients of a general mechanism design environment are as follows:

  - *"n" strategic participants/agents*

  - *A finite set Ω of outcomes*
    [Outcome set Ω is abstract and could be very large]

  - *Each agent "i" has a non-negative valuation $v_i(w)$ for each outcome $w \in \Omega$*

Here, the social welfare of an outcome $w \in \Omega$ is defined as $\sum_{i=1}^{n} v_i(w)$

Examples – General Mechanism Design Environments

- Single-Item Auction revisited

  - Ω has n+1 elements corresponding to the winner of the item (if any)

  - In single-parameter case, every loser bidder had only one valuation = 0

  - In this multi-parameter case, a bidder might have different valuations for each possible winner of the auction

  - For e.g., →
    In a bidding war over a hot start-up, if a bidder loses he/she might prefer that the start-up be bought (won) by a bidder from a different market rather than a direct competitor !

Examples – General Mechanism Design Environments

- Combinatorial Auctions
  - Multiple indivisible items for sale
  - Each bidder can have complex preferences between different subsets of items called “bundles”
  - With *n* bidders and a set *M* of *m* items, the outcome Ω corresponds to n-vectors $(S_1, S_2, \dots, S_n)$ where $S_i \subseteq M$ denotes the bundle allocated to i’th bidder
  - No item is allocated twice
  - There are $(n+1)^m$ different outcomes
  - Each bidder i has private valuation $v_i(S)$ for each bundle $S \subseteq M$ he/she might get

    ∴ Each bidder has 2^m private parameters

Applications: Government spectrum auctions, bidders in telecommunication companies like Verizon or AT&T

VCG Mechanism (1/3)

Theorem 1: In every general mechanism design environment, there is a DSIC welfare-maximizing mechanism

NOTE: Theorem 1 asserts the 1^{st} two properties (DSIC and welfare maximizing) of an ideal auction [Lecture 2]. But, it does not assert the 3^{rd} (computational efficiency) property.

How can we design this mechanism ?

We will use the same 2-step approach used in previous lectures [Lecture 2]

1. Assume without justification that bids are truthful. Predict allocation rule.
2. Predict a suitable payment rule.

VCG Mechanism (2/3)

- Step 1 →
  - Assume, without justification, that agents truthfully report their private information
  - Bids $b_1, b_2, \ldots, b_n$ are used as proxies for the unknown valuations, i.e., bid b_i is now a vector indexed by Ω
  - The correct choice, now, would be to choose a bid that maximizes welfare as follows

$$x(b) = argmax_{w \in \Omega} \sum_{i=1}^{n} b_i(w) \qquad \ldots (1)$$

- Step 2→
  - For deciding the payment rule, we cannot use the Myerson's lemma anymore as the bids are multidimensional now !
  - Instead, for characterizing agent i's payment we will the "*externalities*" caused by i
  - That means, we will use the "welfare loss" inflicted on the other n-1 agents by agent i

VCG Mechanism (3/3)

- Step 2 continuation →
  - Using the externalities, we can formulate a payment rule as follows

$$p_i(b) = \left(max_{w\in\Omega} \sum_{j\neq i} b_j(w) \right) - \sum_{j\neq i} b_j(w^*) \qquad \ldots (2)$$

  - Here, $w^* = x(b)$ is the outcome chosen in (1)
  - $p_i(b)$ is always at least 0 !

We can now define a VCG Mechanism as follows:

A mechanism (x,p) with allocation and payment rules as in (1) and (2) respectively is a VCG Mechanism

NOTE: We can rewrite (2) as the difference of i's bid and a "rebate" where rebate equals to the increase in welfare attributable to i's presence

$$p_i(b) = \underbrace{b_i(w^*)}_{\text{Bid}} - \underbrace{\left[\sum_{j=1}^{n} b_j(w^*) - max_{w\in\Omega} \sum_{j\neq i} b_j(w)\right]}_{\text{Rebate}}$$

Practical Considerations

1. Preference Elicitation:

The challenge of getting the reports/bids $b_1, b_2, \ldots, b_n$ from the agents
For e.g., in a combinatorial auction with m items, there are 2^m private parameters.
The value becomes huge for even a small m like 10 or 20 !

2. Computational Intractability of welfare maximization problems

3. Bad Revenue and Incentive Properties (despite being DSIC)

For e.g., in a combinatorial auction with two agents (P & Q) and 2 items(A & B):
P only wants both items (i.e., $v_1(\mathrm{AB})$=1 and is 0 otherwise)
Q only wants item A (i.e., $v_2(AB) = v_2(A)$=1 and is 0 otherwise)

The revenue of VCG mechanism is 1 in the above example.
But if a 3rd agent R, who only wants item B, is added then revenue will drop to 0 !

References

- Twenty Lectures on Algorithmic Game Theory by Tim Roughgarden, 2016, Cambridge University Press
- https://www.youtube.com/playlist?list=PLEGCF-WLh2RJBqmxvZ0_ie-mleCFhi2N4

THANK YOU