Algorithmic Game Theory

LECTURE 1

Main points covered :

- Need for 'Mechanism Design'
  - Example : 2012 Olympics Women's Badminton
- When is selfish behaviour near optimal ?
  - Braess's Paradox
  - String and Spring
- Can strategic players learn equilibrium?
  - Rock-Paper-Scissors game
  - Nash equilibrium
- Some more examples
- Applications

Example of 2012 Olympics (1/3)
The scandal in women's badminton tournament

- The structure of the game:

  16 teams , 4 groups of 4 teams each

Round-Robin Phase →

Knockout Phase
Quarter-final
Semi-final
Final

Group A
Team 1
Team 2

Group D
Team 1
Team 2

Group B
Team 1
Team 2

Group C
Team 1
Team 2

Example of 2012 Olympics (2/3)
The scandal in women's badminton tournament

- Goals for Players: To win best possible medal
- Goals for game designer: To make every team try to win every game
- What went wrong ?

  1) Last day of RR phase : In group D, team PJ upset favourites TZ | Thus PJ=1^{st} spot, TZ=2^{nd} spot

  2) In group A: Both teams WY & JK had same points | Match to decide 1^{st} & 2^{nd} spots

  3) Knowing that the losing team would face the strong TZ later in tournament, both WY & JK deliberately tried to lose the match

  Contradiction with designer goals !

Example of 2012 Olympics (3/3)
The scandal in women's badminton tournament

- What can be observed :
  - Teams WY & JK tried to lose the match for greater good of their teams
  - Losing would ensure at least a silver medal whereas winning would ensure a maximum of bronze medal (given their opponent TZ are favourites)
  - Player Goals thus satisfied ! | But Designer goal not satisfied !

There is a need to design the game taking into consideration the strategic developments a player can adopt !

~~MECHANISM DESIGN~~

Designing a system such that performance goals are met even in presence of strategic players.

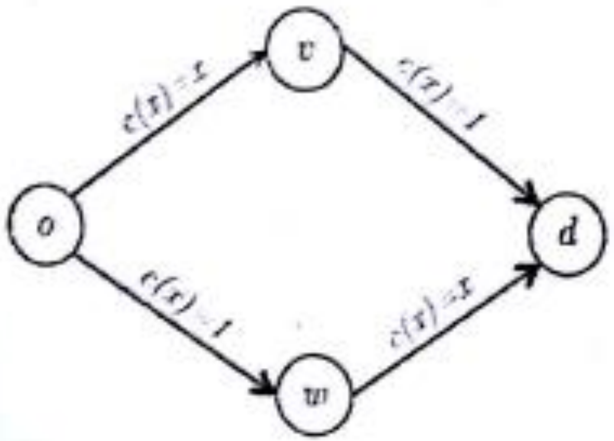
❑ Our suggestions on this particular example:

1)Randomizing the chances of who plays who in the knockout stage can help.

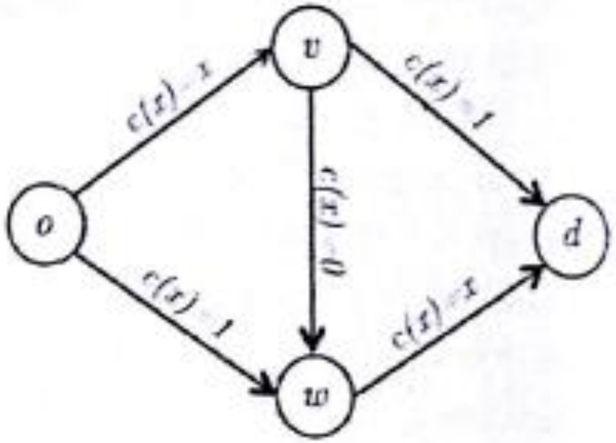
2)Adding an extra criterion like fair-play in matches while deciding upon match points.

Braess's Paradox (1/2)

- Start point: O & Destination: D
- Two routes each consisting of:
  - A long wide road : Commuting time= 1 hour (constant)
  - A narrow short road : Commuting time directly proportional to fraction of traffic 'x'
  - Augmented network has an extra facility (selfish need): A teleporter



(a) Initial network



(b) Augmented network

Total commuting time = 3/2
[Traffic is 50-50 on both routes]

Total commuting time = 2
[Traffic is 100% on route
o-v-w-d]

Braess's Paradox (2/2)

- Observations from Braess's Paradox

  1) Addition of a 'selfish routing' doesn't necessarily be helpful.

  2) The teleporter in the augmented network in fact increases the total commuting time

- So When is Selfish Routing benign ?

  POA ≈ 1 [POA: Price of Anarchy]

  POA = (system performance with strategic players) / (best possible system performance)

  In this example ;

  POA = 2/(3/2) = 4/3

Strings and Springs
An analogous example

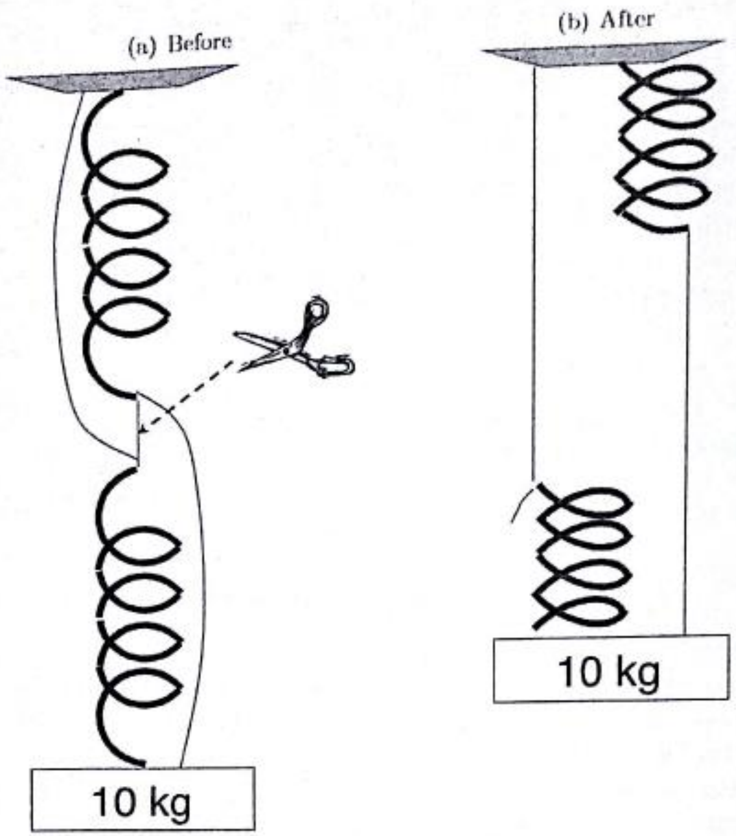
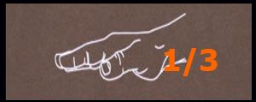


Figure 1.3: Strings and springs. Severing a taut string lifts a heavy weight.

Rock-Paper-Scissors Game

			
	0,0	-1,1	1,-1
	1,-1	0,0	-1 , 1
	-1,1	1 , -1	0,0

- ❖ A Zero-Sum game
- ❖ Can be encoded in the bimatrix form as shown in the figure

Rock-Paper-Scissors & Nash Equilibrium

- What is equilibrium ?

  A steady state of a system where, assuming everything else remains same, the participants want to remain as they are.

- Observations from R-P-S game:

  There is no ‘deterministic equilibrium’. Whatever be the current state, one player will be benefitted and the other would wish to change his/her choice.

- Mixed Strategy :

  When the opponent randomizes over the possible strategies in a system, the resulting probability distribution is called ‘Mixed strategy’

- Nash Equilibrium :

  If both players in R-P-S game randomize uniformly over the 3 possible strategies, neither can increase his/her expected payoff by a unilateral deviation. A pair of probability distributions with this property is a ‘Nash Equilibrium’ !

Nash's Theorem

- ***Every finite 2 player game has a Nash Equilibrium***

- Can a Nash Equilibrium be computed efficiently ?
  - In zero-sum games (e.g.; R-P-S) it can be done via linear programming in P time
  - In non-zero-sum games, however, recent results say there is no computationally efficient algorithm
  - But, the problem is NOT NP-Hard !
  - It has intermediate complexity !
  - It belongs to a class called PPAD which we will learn about in the coming lectures

Some more examples

- Battle of the sexes
- Guess Two-Thirds of the Average

Battle of the Sexes (1/3)

PLAYER 1 \ PLAYER 2	Theater!	Football fine
Theater fine	1, 5	0, 0
Football!	0, 0	5, 1

- Structure of the game :
  - Player 1 wants to go to a Football match
  - Player 2 wants to go to the Theatre
  - But, both want to stay together !
  - They need to come up with a solution such that none of them would wish to change their choice and remain satisfied too

Battle of the Sexes (2/3)

PLAYER 1 \ PLAYER 2	Theater!	Football fine
Theater fine	1, 5	0, 0
Football!	0, 0	5, 1

There are 2 cases of 'pure-strategy' Nash equilibrium in this case :

1) (Theatre fine ,Theatre) :

(i) Player 2's wish to go to the theatre is satisfied, She does not wish to change her choice, moreover she already has a payoff of 5

(ii) Although Player 1's wish to go to a Football match is not satisfied he prefers to stick with his choice because if he changes, his payoff would reduce to 0 from 1

(iii) Both get to stay together as well !

2) (Football ,Football fine) :

(i) Player 1's wish is satisfied , Payoff of 5

(ii) Player 2 prefers to stick with the choice as changing would reduce her payoff from 1 to 0

(iii) Both get to stay together !

Battle of the Sexes (3/3)

- Since there are 2 equilibriums , how can Players 1 & 2 take a decision ?
- One of them has to change the strategy !

Player 1 changes his 2nd strategy to 'Theatre great,I'll invite my mom' !!

- Now Player 2 doesn't want to go to the theatre as in that case there would be a 3rd person (an intruder) with them !
- Also, in (Theatre great I'll…. , Theatre) Player 2 has a payoff of -1 which she would definitely want to increase
- Only possible case of a Nash equilibrium is thus unique i.e. (Football , Football fine)
- Player 1 wins !

	Theater!	Football fine
~~Theater fine~~	~~1, 5~~	~~0, 0~~
Football!	0, 0	5, 1
Theater great, I'll invite my mom	2, -1	0, 0

unique Equilibrium

(Football!, Football fine)

Observation : The player with knowledge of game theory was able to remove an unwanted Nash equilibrium and tilt the result in his favour !

Guessing 2/3rd of the Average (1/2)

- Structure of the game :
  - K players viz p1, p2, p3, …., pk
  - Each player submits a number x in the range [0 100]
  - Compute the average x_avg : $\bar{x} := \frac{1}{k}\sum_{i=1}^{k} x_i$
  - Find Xj closest to (2/3)*x_avg
  - Player Pj wins and gets $100 , all other players don't get any reward

Guessing 2/3rd of the Average (2/2)

- Is it rational to play above (2/3)*100 = 66.67 ≈ 67 ?

  NO !

  All strategies above 67 are weakly dominated i.e. if you win with >67 then you will also be able to win with 67 , so they can be eliminated

- Therefore, all strategies above 67 can be eliminated
- Similarly all strategies above (2/3)*67 = (2/3)*(2/3)*100 ≈ 45 can also be eliminated
- ....and so on until all strategies above 0 have been eliminated
- Thus, the rational strategy would be to play 0 !

The all-zero strategy is the only Nash equilibrium of this game

historical facts:	21.6 was the winning value in a large internet-based competition organized by the Danish newspaper Politiken. This included 19,196 people and with a prize of 5000 Danish kroner.

Applications

- GAME →
  - Market → Price Equilibrium
  - Internet → Packet Routing
  - Roads → Traffic Pattern
  - Facebook, other social media → Structure of social network

References

- Twenty Lectures on Algorithmic Game Theory by Tim Roughgarden, 2016, Cambridge University Press
- https://www.youtube.com/playlist?list=PLEGCF-WLh2RJBqmxvZ0_ie-mleCFhi2N4
- http://people.csail.mit.edu/costis/6853fa2011/

Thank You