Algorithmic Game Theory

LECTURE 2

Main topics covered:

- Mechanism Design
  - Single-item Auctions
  - Sealed-bid Auctions
  - First-price Auctions
  - Second-price Auctions
  - Some useful terms
  - Second-price auctions as Ideal auctions
- Case Study : Sponsored Search Auctions

Course Goal :

To understand how to design systems with strategic participants that have good performance guarantees

Single-item Auctions

- Setup:
  - Seller has a “single” item (e.g., an old-fashioned smartphone)
  - *‘n’* strategic bidders
  - Each bidder *i* has a valuation v_i (max amount that bidder is willing to pay)
  - This valuation is private (unknown to seller and other bidders)
- What does a bidder want ?
  - A bidder wants to acquire the item as cheaply as possible given the maximum selling price is v_i
- Bidder Utility Model
  - Quasilinear Utility Model ~~*(just another complicated name !)*~~
  - If bidder *i* loses, utility=0 !
  - If bidder *i* wins at price *p*, utility=$(v_i - p)$

Sealed-bid Auctions

- Setup:
  - Each bidder *i* privately communicates a bid b_i to the seller
  - The seller decided who wins
  - The seller decides on a selling price
- Selection Rule

  Item is given to the highest bidder !
- Implementation of 3rd step
  - There are many reasonable ways
  - The choice, however, significantly affects bidder behaviour
    - Example:

    If the seller decides to charge nothing to winner, it becomes a game of who names the highest number !

First-price Auctions

- Winner pays her bid
- Disadvantages:
  - Hard for bidder to figure out how to bid
  - Hard for seller/auction designer to predict an outcome

Example:

- Consider a valuation=birth month + birth day
- Minimum=2 (1st Jan) | Maximum=43 (31st Dec)
- Question→ what bid should we submit ?

Turns out, there is no dominant strategy for first-price auctions !

Second-Price Auctions
(Vickery Auctions)

- Winner pays second-highest bid

  If winner bids $100, 2nd highest bid is $90 ➔ winner pays $(90+x) where x is a small increment

- It is a sealed-bid type auction
- Is there a dominant strategy for bidding in this case ?

  YES !

Proposition 1→ *In a second-price auction, every bidder i has a dominant strategy: set the bid b_i equal to her private valuation v_i.*

Proposition 2→ *Non-negative utility: In a second-price auction, every truthful bidder is guaranteed a non-negative utility.*

❑ Advantages:

1. Bidder doesn't have to worry about her competitors' valuations
2. Drastically different from First-price auctions where bidding one's valuation would have guaranteed 0 utility !

Proof of Proposition 1

Let,

- Bidder i has a valuation v_i
- b_{-i} is the vector of of all bids with i^{th} component removed
- $B = \max_{(j\ != i)} b_j$ (i.e., highest bid by some other bidder)

To show: Bidder i's utility is maximised by setting bid $b_i = v_i$

There are only 2 possible outcomes ➔

1. $b_i < B$ → bidder i loses → utility= 0
2. $b_i \geq B$ → bidder i wins → utility= $(v_i - B)$

Therefore, considering these two cases we can conclude:

If $v_i < B$, maximum utility bidder i can obtain= $\max\{0, v_i - B\}$= 0 [which she obtains by bidding truthfully and losing]

If $b_i \geq B$, maximum utility bidder i can obtain= $\max\{0, v_i - B\}$= $(v_i - B)$ [which she obtains by bidding truthfully and winning]

Conclusion: A truthful bidder never regrets participating in a second-price auction !

Proof of Proposition 2

To show: Truthful bidder has non-negative utility

We know, Losers receive utility 0.

Now,

If bidder i is the winner, she receives a utility = $v_i - p$, where p= 2nd highest bid

Since, i is the winner and a truthful bidder, we can say v_i is the highest bid

Therefore, $p < v_i \rightarrow (v_i - p) \geq 0$

$(v_i - p) \geq 0$

This is always true !

Some Useful Terms

- Dominant Strategy Incentive Compatible (DSIC)

  An auction is called DSIC if truthful bidding is the dominant strategy for every bidder and if truthful bidders always obtain non-negative utility

- Social Welfare / Social Surplus

  Social Welfare of an outcome of a single-item auction is defined as

$$\sum_{i=1}^{n} v_i \ . x_i$$

  where, x_i is 1 if i wins and 0 if i loses and v_i is the valuation of i

  An auction is *welfare maximizing* if, when bids are truthful, the auction outcome has the maximum possible social welfare

Second-price auctions as Ideal auctions

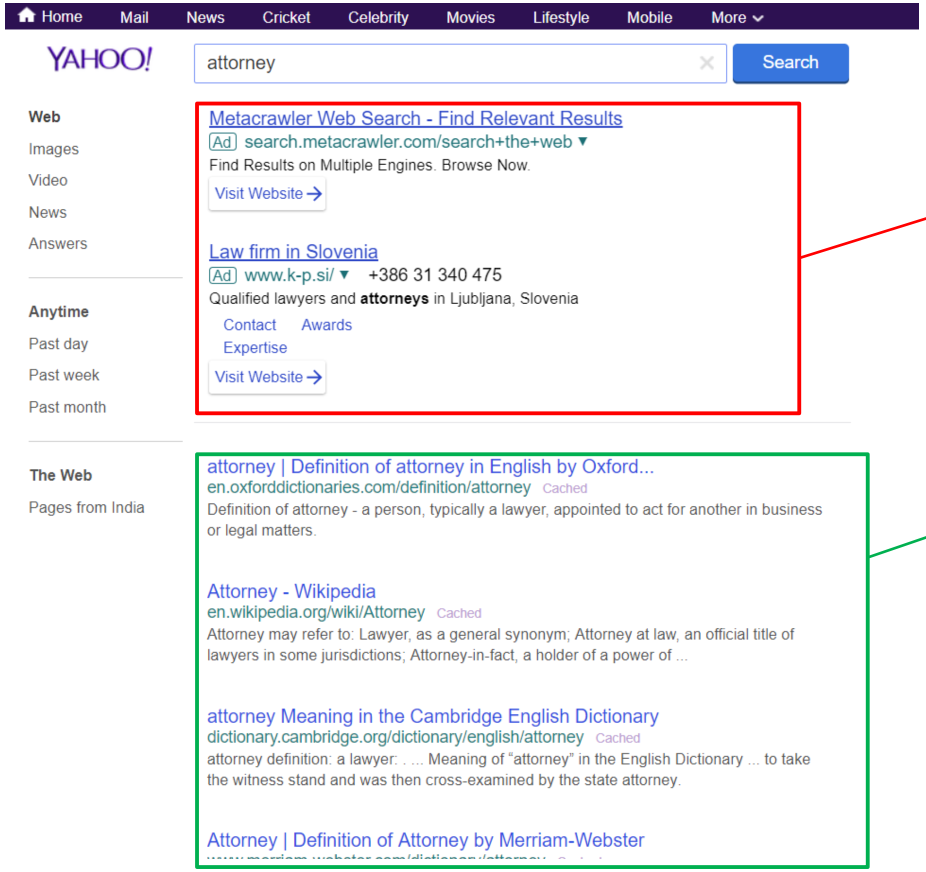
A second-price single-item auction satisfies the following:

- [strong incentive guarantees] It is a DSIC auction
- [strong performance guarantees] It is welfare maximising
- [computational efficiency] It can be implemented in time polynomial (indeed, linear) in the size of the inputs, meaning the number of bits necessary to represent the numbers $v_{1,\dots,}v_n$

Case Study: Sponsored Search Auctions (1/3)

BACKGROUND :

When we query a web search, we get a results page like this



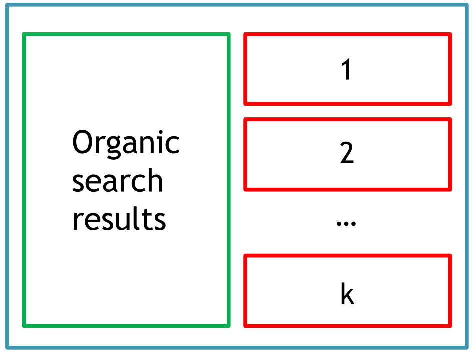
We get a list of advertisements or 'sponsored links' which have been paid for by the advertisers

We get a list of organic search results that concern the keyword that we had queried for

Quick Fact: Sponsored search auctions generated about 98% of Google's revenue around 2006 !!!

Case Study: Sponsored Search Auctions (2/3)

The basic model of sponsored search auctions:



- There are ‘k’ slots for sale to advertisers
- Each slot will contain a unique Ad i.e., not identical to any other Ad
- Each slot will be assigned to only 1 advertiser
- The Ads will be in an ordered list i.e., higher slots are more valuable than lower slots

- The difference between different slots is quantified using the click-through-rates (CTRs)
  CTR (α_j) of a slot j = Probability that the user clicks on this slot
  $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_j \geq \ldots \geq \alpha_k$
- Each advertiser i has a quality score β_i (the higher the better)
- CTR of advertiser i in slot j = $\beta_i.\alpha_j$
- The advertiser also has a private valuation v_i on for each click on her link
- Expected value derived by advertiser = $v_i.\alpha_j$

Case Study: Sponsored Search Auctions (3/3)

- What do we want for an ideal sponsored search auction ?
  1. DSIC: Truthful bidding is dominant strategy
  2. Social Welfare Maximization: Assignment of bidders should maximize $\sum_{i=1}^{n} v_i \,.\, x_i$, where x_i is now the CTR of the slot
  3. Computational Efficiency: Polynomial (or even near-linear) running time

- Our Design Approach ➔
  - STEP 1: Assume, without justification that bidders bid truthfully. Then how do we assign bidders to slots so that (2) and (3) hold?

    Using the natural greedy algorithm could be a possible solution
  - STEP 2: Given our answer to step 1, how should we set selling price so that (1) holds?

    Something analogous to second-price rule.

    Myerson's lemma comes to our rescue here !

References

- Twenty Lectures on Algorithmic Game Theory by Tim Roughgarden, 2016, Cambridge University Press
- https://www.youtube.com/playlist?list=PLEGCF-WLh2RJBqmxvZ0_ie-mleCFhi2N4

THANK YOU