Image Reconstruction from Reduced Dimension Space

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***Abstract*—This objective of this assignment is to understand how an image can be reconstructed back after having reduced dimension space. The Yale Face database is utilised in this experiment.**

I. Dimensionality Reduction and Reconstruction

Dimensionality reduction, or dimension reduction, is the transformation of data from a high-dimensional space into a low-dimensional space so that the low-dimensional representation retains some meaningful properties of the original data, ideally close to its intrinsic dimension. Working in high-dimensional spaces can be undesirable for many reasons; raw data are often sparse as a consequence of the curse of dimensionality, and analyzing the data is usually computationally intractable.

However, when dimensions are reduced, there is an inevitable loss of data in the process. This is overlooked since the benefits of reduced dimensions far outweigh the loss of some data points.The loss of information in the form of reconstructed vs original image quality is observed in the following experiment.

II. PCA and Eigenfaces

Principal Component Analysis, PCA, is one of the most famous dimension reduction techniques in computer vision. PCA is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components.In a multi-dimensional data, it finds the dimensions that are most useful and contain the most information. thereby extracting essential information from data by reducing the dimensions.

Eigenfaces are an application of PCA. A set of eigenfaces can be generated by performing PCA on a large set of images depicting different human faces. They are used for applications like Face Recognition and Facial Landmark Detection.

III. Algorithm

A. *Read and Vectorize the Input*

Extract the images from the Yale Face Database and flatten each image into a row vector for easy computation. Here the first image is set aside to act as the test image. The rest are utilised to obtain the eigenfaces.

B. *Principal Component Analysis*

- Compute Covariance Matrix C

$$C = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)(X_i - \mu)^T \tag{1}$$

- Obtain eigenvalue and eigenvector

$$Cv_i = \lambda_i v_i \tag{2}$$

  where
  v_i is the eigenvector and λ_i is the eigenvalue.

- Set principal components for reducing dimensions
  Arrange the eigenvalue and the associated eigenvectors in the decreasing order of eigenvalues and choose an arbitrary number of principal components (dimensions). This is done to obtain the Eigenvectors with the highest eigenvalues i.e. the vectors having the higest information.

C. *Reconstruction*

Given a new image, find it's coefficients with respect to the eigenfaces and sum them to obtain the reconstructed image.

- Find the coefficients of the test image with respect to the eigenfaces using eigenvalues and eigenvectors.
- Multiply the coefficients with the eigenvectors to obtain the reconstructed component values for each pixel in each dimension
- Now add all the dimension components to obtain the final reconstructed image.
- Reshape the image vector to plot the reconstructed image.

IV. Observations

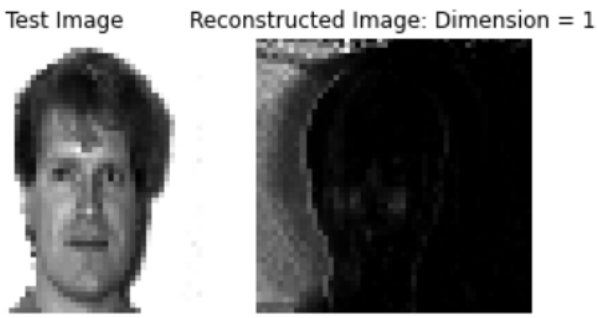

Fig. 1: Image reconstructed from 1 dimension

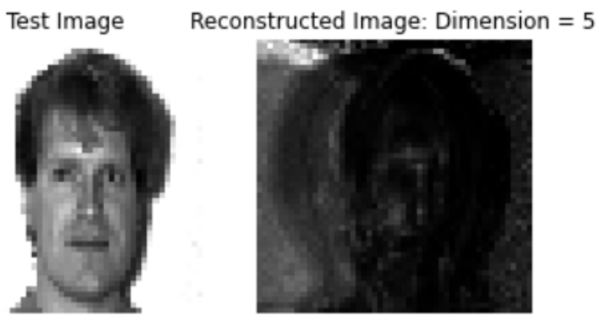

Fig. 2: Image reconstructed from 5 dimensions

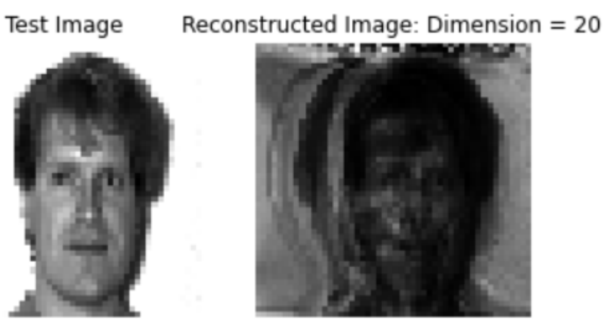

Fig. 3: Image reconstructed from 20 dimensions

Fig. 4: Image reconstructed from 50 dimensions

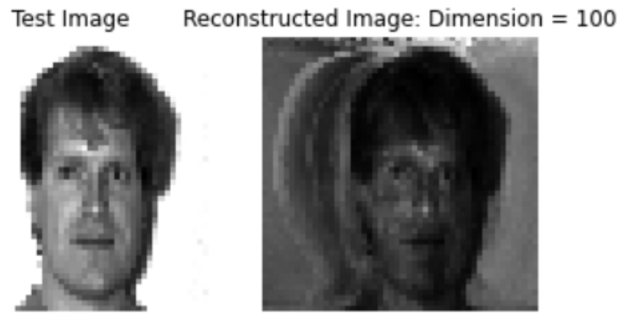

Fig. 5: Image reconstructed from 100 dimensions

Fig. 6: Image reconstructed from 150 dimensions

V. Inference

- The more the number of principal components, more the number of coordinates used to project the images.
- As the dimensions increase, better is the result for reconstruction. This is because more information (from the original image that may be present in different coordinates) from the additional dimensions is utilised to form the reconstructed image.

VI. Result

Reconstruction of images from reduced dimensions is analysed. It is observed that the quality of the reconstructed image depends on the dimensions to which original image is projected.