[Question 1:]

(i) Find the directional derivative of a 3-D function: $f(x, y, z) = 5x^2 - 2xy + y^2 - 4yz + z^2 + 3xz$ along the direction $v = (\frac{2}{3}, \frac{1}{3}, \frac{2}{3})$ at (1,-2,3).

(ii) Find the directional derivative of the function: $f(x, y) = y^2 + xy$ along the direction $v = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$ at (1,3).

[Question 2:]

(i) In $\mathbb{R}^3$ compute the inner product of $v_1 = (1, 2, -1)$ and $v_2 = (2, 1, 4)$. What is the length of the two vectors? What is the angle between the two vectors?

(ii) In $\mathbb{R}^3$ compute the inner product of $v_1 = (1, 2, 4)$ and $v_2 = (2, 5, 1)$. What is the length of the two vectors? What is the angle between the two vectors?

[Question 3:]

Consider the set of vectors $V \in \mathbb{R}^+$, and the two operations defined "$\oplus$" and "$\circ$" are defined as $x \oplus y = xy$ and $\alpha \circ x = x^\alpha$, $\alpha \in \mathbb{R}$. Determine whether the given set is a vector space. If not, give at least one axiom that is not satisfied.

[Question 4:]

Find extremals for

$$\int_0^1 \{[\dot{x}(t)]^2 + 10\, t\, x(t)\}\, dt \quad \text{subject to } x(0) = 1, \;\; x(1) = 2.$$