FLAPS
the FLexible Axisymmetric Planet Solver

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1 Introduction

2 The physics

2.1 Axisymmetric formulation

2.2 Dynamic topography

3 Numerical methods

3.1 Finite elements

3.2 Mapping

3.3 Quadrature

3.4 Computing normals

3.5 Free slip

3.6 Pressure normalisation

4 The data

4.1 Earth

We assume that viscosity is purely a function of depth[1]..

Five radial viscosity profiles are available:

- The first viscosity profile is a constant viscosity for all depths of 10^{22} Pa s. This value is an estimated value of what is normally found in the literature.
- The second viscosity profile comes from Yoshida et al (2001) [8]. It uses three different regions: lithosphere (0 km to 150 km), upper mantle (150 km to 670 km) and lower mantle (670 km to 2900 km).
- The third viscosity profile comes from Steinberger & Holmes (2008) [7] which is comparable to [6], but of the latter no available data was available. Data is read from the file `DATA/EARTH/eta_stho08.ascii`.
- The fourth and fifth profile come from Ciskova et al (2012) [1]. Data is read from the file `DATA/EARTH/eta_civs.ascii`. The paper showcases two main families of radial viscosity profiles in literature. Family A, which has a sharp increase below the 660 km transition zone and remains constant for most of the lower mantle and family B which is much smoother over the transition zone and increases with depth in the lower mantle.

Three radial density profiles are available:

- PREM [2]
- ak135f [3] `http://rses.anu.edu.au/seismology/ak135/ak135f.html` Data is read from the file `DATA/EARTH/rho_ak135f`
- stw105 [5] `http://ds.iris.edu/ds/products/emc-stw105/` Data is read from the file `DATA/EARTH/rho_stw105.ascii`.

[1] This is borrowed from stone 71

4.2 Mars

5 Benchmarking

5.1 Computing volume/mass

5.2 Annulus convection manufactured solution

6 The '4D dynamic earth' inter-code benchmark

[4]

A Misc

A.1 Notes to self

What I have tried to cure the pb of the weird anomalies at the poles.

- turning elements into real trapezes. Made things worse
- different mappings. not much difference
- when using blob, reduced densities. no difference
- nb of quad points, no real difference
- nb of elements in tangential direction, some difference but no cure
- when using blob, drho/rho, no diff
- type of b.c. at point corner below poles, no real diff
- scaling of G matrix
- different rotations/bc for free slip, no difference
- using cmat matrix for dev strain rate, helped a little bit, no cure

A.2 To do list

- visc profiles
- rho profiles
- time stepping
- gravity calculations. import from f96, re-benchmark
- CBF?
- compute self gravity for reduced density case
- export exx1 and exx3 to outside function, clean their code too?
- remove call to math
- bottom free slip
- change y for z in stone
- use PREM gravity value
- aspect with GMG ?
- compute moment of inertia
- by default code now uses elemental rho and eta. it changes things wrt exp0 benchmark results!

References

[1] H. Čížková, A.P. van den Berg, W. Spakman, and Ctirad Matyska. The viscosity of the earth's lower mantle inferred from sinking speed of subducted lithosphere. 200–201:56–62, 2012.

[2] A.M. Dziewonski and D.L. Anderson. Preliminary reference Earth model. 25:297–356, 1981.

[3] B.L.N. Kennett, E.R. Engdahl, and R. Buland. Travel times for global earthquake location and phase association. 122:108–124, 1995.

[4] M. Kronbichler, T. Heister, and W. Bangerth. High accuracy mantle convection simulation through modern numerical methods. 191:12–29, 2012.

[5] B Kustowski, G Ekström, and AM Dziewoński. Anisotropic shear-wave velocity structure of the earth's mantle: A global model. *Journal of Geophysical Research: Solid Earth*, 113(B6), 2008.

[6] B. Steinberger and A.R. Calderwood. Models of large-scale viscous flow in the Earth's mantle with constraints from mineral physics and surface observations. 167:1461–1481, 2006.

[7] B Steinberger and R Holme. Mantle flow models with core-mantle boundary constraints and chemical heterogeneities in the lowermost mantle. *Journal of Geophysical Research: Solid Earth*, 113(B5), 2008.

[8] Masaki Yoshida, Satoru Honda, Motoyuki Kido, and Yasuyuki Iwase. Numerical simulation for the prediction of the plate motions. *Earth, planets and space*, 53(7):709–721, 2001.