UNIVERSITY OF CORDOBA
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PHYSICAL TECHNOLOGY: RESEARCH AND APPLICATIONS

MASTER'S THESIS

Exploring the relationship between Hawkes processes and self-organized criticality in living systems

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Line of research: Modelling of complex systems and their interdisciplinary applications

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Autorización de defensa del Trabajo Fin de Máster

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INFORMAN: Que el presente trabajo titulado *Exploring the relationship between Hawkes processes and self-organized criticality in living systems* que constituye la memoria del Trabajo Fin de Máster ha sido realizado por Antonio Rivas Blanco y AUTORIZA/N su presentación para que pueda ser defendido en la convocatoria 1ª con fecha 23/07/2024.

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“Va por los días en los que quiero un abrazo
y acabo encerrado solo en en cuarto”.
—Piezas

Cambiar encabezado y pie de página en .sty

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This section provides the main results of the investigation. First, the results reproduced from the original paper [1] are presented. Then, the results of the analysis with $n = 2$ are shown. Finally, we have studied the behaviour of an inhibitory and excitatory neuron coupled.

4.1 Results from the original paper

The first result is the percolation phase diagram is shown in Figure 4.1. It displays the percolation strength P_∞ versus the resolution parameter Δ.

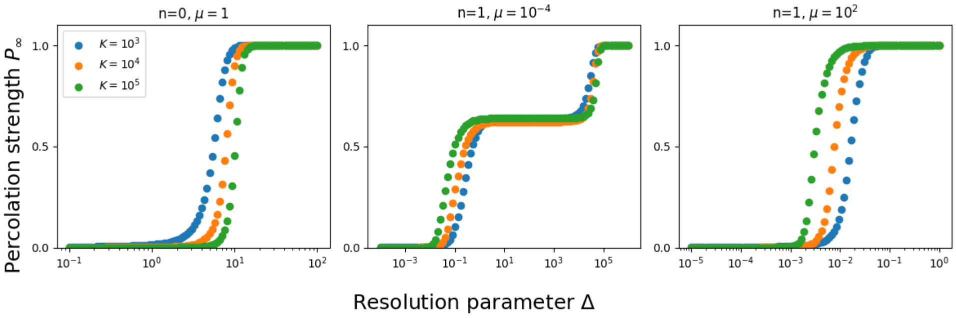

Figure 4.1. Percolation phase diagrams for different event number K taking average values of $R = 1000$ realizations.

Chapter 5

Conclusions

Anexo

REVISAR LOS CÓDIGOS PARA QUE ESTÉN ACTUALIZADOS

Script 5.1. *Script* Python con todas las funciones.

```python
import numpy as np
import matplotlib.pyplot as plt

def algorithm(rate, mu, n):
    """
    Algorithm that computes interevent times and Hawkes intensity

    #Output: rate x_k, x_k
    """
    # Paso 1
    u1 = np.random.uniform()
    if mu == 0:
        F1 = np.inf
    else:
        F1 = -np.log(u1) / mu

    # Paso 2
    u2 = np.random.uniform()
    if (rate - mu) == 0:
        G2 = 0
    else:
        G2 = 1 + np.log(u2) / (rate - mu)


    # Paso 3
    if G2 <= 0:
        F2 = np.inf
    else:
        F2 = -np.log(G2)

    # Paso 4
    xk = min(F1, F2)

    # Paso 5
    rate_tk = (rate - mu) * np.exp(-xk) + n + mu
    return rate_tk, xk

def generate_series(K, n, mu):
    """
    Generates temporal series for K Hawkes processes

    ##Inputs:
```

```
    K: Number of events
    n: Strength of the Hawkes process
    mu: Background intensity

    ##Output:
    times: time series the events
    rate: time series for the intensity
    """
    times_between_events = [0]
    rate = [mu]
    for _ in range(K):
        rate_tk, xk = algorithm(rate[-1], mu, n)
        rate.append(rate_tk)
        times_between_events.append(xk)
    times = np.cumsum(times_between_events)
    return times, rate

def identify_clusters(times, delta):
    """
    Identifies clusters in a temporal series given a resolution parameter delta

    ## Inputs:
    times: temporal series
    delta: resolution parameter

    ## Output:
    clusters: list of clusters
    """
    clusters = []
    current_cluster = []
    for i in range(len(times) - 1):
        if times[i + 1] - times[i] <= delta:
            if not current_cluster:
                current_cluster.append(times[i])
            current_cluster.append(times[i + 1])
        else:
            if current_cluster:
                clusters.append(current_cluster)
                current_cluster = []
    return clusters

def generate_series_perc(K, n, mu):
    """
    Generates temporal series for K Hawkes processes

    ##Inputs:
    K: Number of events
    n: Strength of the Hawkes process
    mu: Background intensity

    ##Output:
    times_between_events: time series the interevent times
    times: time series the events
    rate: time series for the intensity
    """
    times_between_events = [0]
    rate = [mu]
    for _ in range(K):
        rate_tk, xk = algorithm(rate[-1], mu, n)
        rate.append(rate_tk)
        times_between_events.append(xk)
    times = np.cumsum(times_between_events)
```

```python
    return times_between_events, times, rate

def calculate_percolation_strength(times_between_events, deltas):
    percolation_strengths = []

    for delta in deltas:
        cluster_sizes = []
        # Initialize the size of the current cluster
        current_cluster_size = 1 # The first event is always a cluster

        for i in range(len(times_between_events)):
            if times_between_events[i] <= delta:
                current_cluster_size += 1
            else:
                if current_cluster_size > 1: # Only consider clusters with more than one
    event
                    cluster_sizes.append(current_cluster_size)
                # Reset the size of the current cluster
                current_cluster_size = 1 # The next event is always a cluster

        # Add the size of the last cluster
        if current_cluster_size > 1: # Only consider clusters with more than one event
            cluster_sizes.append(current_cluster_size)

        max_cluster_size = max(cluster_sizes)

        percolation_strengths.append(max_cluster_size / len(times_between_events))
    return percolation_strengths

def model(n_max, mu_E, mu_I, tau, n_EE, n_IE, n_EI, n_II, dt):
    """
    Solve the equations of the mena field model for a given number of iterations n_max

    Inputs:
    n_max: number of iterations
    mu_E: Poisson rate of excitatory neurons
    mu_I: Poisson rate of inhibitory neurons
    tau: characteristic time of the system
    n_EE: influence of excitatory neurons on excitatory neurons
    n_IE: influence of excitatory neurons on inhibitory neurons
    n_EI: influence of inhibitory neurons on excitatory neurons
    n_II: influence of inhibitory neurons on inhibitory neurons
    dt: time step

    Outputs:
    time: time series
    t_events_E: times of events of excitatory neurons
    t_events_I: times of events of inhibitory neurons
    rates_E: rates of excitatory neurons
    rates_I: rates of inhibitory neurons
    """
    n_E = n_I = n = 0
    t_events_E = [0]
    t_events_I = [0]
    rates_E = [mu_E]
    rates_I = [mu_I]
    time = [0]
    while n <= n_max:
        # Excitation neurons
        l_Enew = rates_E[-1]  + dt * (mu_E- rates_E[-1])/tau
        if np.random.uniform() < rates_E[-1]*dt:
            l_Enew += n_EE
```

```
                t_events_E.append(time[-1]+dt*np.random.uniform())
                n_E += 1
            if np.random.uniform() < rates_I[-1]*dt:
                l_Enew -= n_IE
                t_events_E.append(time[-1]+dt*np.random.uniform())
                n_E += 1

            # Inhibition neurons
            l_Inew = rates_I[-1] + dt * (mu_I- rates_I[-1])/tau
            if np.random.uniform() < rates_E[-1]*dt:
                l_Inew += n_EI
                t_events_I.append(time[-1]+dt*np.random.uniform())
                n_I += 1
            if np.random.uniform() < rates_I[-1]*dt:
                l_Inew -= n_II
                t_events_I.append(time[-1]+dt*np.random.uniform())
                n_I += 1
            rates_E.append(l_Enew)
            rates_I.append(l_Inew)
            time.append(time[-1]+dt)

            n = n_E + n_I
        return time, t_events_E, t_events_I, rates_E, rates_I
```