Luminosity Calculation -- River Huang, Vasiliy Morozov

Define the following parameters:

• φ_x is the angle between the projection of the beam1 trajectory on the (*x*, *s*) plane and the s-axis.

• φ_y is the angle between the projection of the trajectory on the (*y*, *s*) plane and the s-axis.

• offset $\delta x = \Delta x_2 - \Delta x_1$ and offset $\delta y = \Delta y_2 - \Delta y_1$.

and also define $\sigma_x^* = \frac{1}{\sqrt{2}}\sqrt{{\sigma_{1x}^*}^2 + {\sigma_{2x}^*}^2}$, $\sigma_y^* = \frac{1}{\sqrt{2}}\sqrt{{\sigma_{1y}^*}^2 + {\sigma_{2y}^*}^2}$, $\sigma_s = \frac{1}{\sqrt{2}}\sqrt{\sigma_{1s}^2 + \sigma_{2s}^2}$

1) Analytic solution 1

Condition: $\varphi_y = 0$, $\delta x = 0$, $\delta y = 0$, $\beta_{1x}^* \gg (\sigma_{1s} + \sigma_{2s})$, $\sigma_{1x}^* \gg \sigma_{2x}^*$

let $a = \frac{\sin^2\varphi_x}{{\sigma_x^*}^2} + \frac{\sec^2\varphi_x}{\sigma_s^2}$, $B = \left(\frac{{\sigma_{1y}^*}^2}{{\beta_{1y}^*}^2} + \frac{{\sigma_{2y}^*}^2}{{\beta_{2y}^*}^2}\right)\frac{1}{2{\sigma_y^*}^2}$

the luminosity is

$$L = L_0\sqrt{2-\cos^2\varphi_x}\,\frac{1}{\sqrt{\pi}\sigma_s\sqrt{B}}\,e^{\frac{a}{2B}}K_0\left(\frac{a}{2B}\right)$$

where K_0 is the modified Bessel function.

2) Analytic solution 2

Condition: $\delta x < \sigma_{1x}^*$ or $\delta x < \sigma_{2x}^*$, $\beta_{1x}^* \gg (\sigma_{1s} + \sigma_{2s})$, $\beta_{1y}^* \gg (\sigma_{1s} + \sigma_{2s})$, $\sigma_{1x}^* \gg \sigma_{2x}^*$, $\sigma_{1y}^* \gg \sigma_{2y}^*$

Let $a = \frac{\sin^2\varphi_x}{{\sigma_x^*}^2} + \frac{\sin^2\varphi_y}{{\sigma_y^*}^2} + \frac{1+\tan^2\varphi_x+\tan^2\varphi_y}{\sigma_s^2}$, $b = \frac{(\delta x)\sin\varphi_x}{{\sigma_x^*}^2} + \frac{(\delta y)\sin\varphi_y}{{\sigma_y^*}^2}$, $c = \frac{(\delta x)^2}{4{\sigma_x^*}^2} + \frac{(\delta y)^2}{4{\sigma_y^*}^2}$,

the luminosity is

$$L = L_0\sqrt{2 - cos^2\,\varphi_x}\,\frac{1}{\sqrt{a}\sigma_s}\,e^{\frac{b^2}{4a}-c}$$

3) Analytic solution 3

Condition: $\varphi_y = 0$, $\delta x < \sigma_{1x}^*$ or $\delta x < \sigma_{2x}^*$, $\beta_{1x}^* \gg (\sigma_{1s} + \sigma_{2s})$, $\beta_{1y}^* \gg (\sigma_{1s} + \sigma_{2s})$, $\sigma_{1x}^* \gg \sigma_{2x}^*$, $\sigma_{1y}^* \gg \sigma_{2y}^*$

Let $a = \frac{\sin^2\varphi_x}{{\sigma_x^*}^2} + \frac{\sec^2\varphi_x}{\sigma_s^2}$, $b = \frac{(\delta x)\sin\varphi_x}{{\sigma_x^*}^2}$, $c = \frac{(\delta x)^2}{4{\sigma_x^*}^2} + \frac{(\delta y)^2}{4{\sigma_y^*}^2}$,

the luminosity is

$$L = L_0\sqrt{2 - cos^2\,\varphi_x}\,\frac{1}{\sqrt{a}\sigma_s}\,e^{\frac{b^2}{4a}-c}$$

If φ_x is small enough, the luminosity is

$$L = L_0\left(1 + \frac{\sigma_s^2}{{\sigma_x^*}^2}\tan^2\varphi_x\right)^{-1/2}\exp\left[-\left(\frac{\delta x}{2}\right)^2\left(\frac{1}{{\sigma_x^*}^2\cos^2\varphi_x + \sigma_s^2\sin^2\varphi_x}\right) - \left(\frac{\delta y}{2\sigma_y^*}\right)^2\right]$$