Parameters and Formulas for Spin Response Function

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$$\beta_y,\ \alpha_y,\ \mu_y = (twiss\ data), \qquad \gamma,\ \nu_y = (twiss\ table)\ , \qquad \kappa_y = {d\theta}/{dl}, \qquad \alpha = \frac{\pi}{2}, \qquad \beta_y{}' = -2\alpha_y,$$

$$\left(\mu_y = \underbrace{\int_0^z \frac{dz}{\beta_y}}_{till\ to\ z} = \underbrace{\int_0^l \frac{dl}{\beta_y}}_{till\ to\ l}, \quad dz = dl \right),$$

$$G = megnetic\ moment\ anomaly$$
$$= 1.79284739(proton),\ 0.00115965219(electron)$$

$$\begin{cases} f_y(z) = \sqrt{\beta_y}e^{i\mu_y}\ , \quad f_y{}'(z) = \dfrac{e^{i\mu_y}}{\sqrt{\beta_y}}(i-\alpha_y) \\ f_y^*(z) = \sqrt{\beta_y}e^{-i\mu_y}, \quad f_y^{*\prime}(z) = -\dfrac{e^{-i\mu_y}}{\sqrt{\beta_y}}(i+\alpha_y) \end{cases},$$

$$\psi(z) = \gamma G \underbrace{\int_0^z \kappa_y \zeta\, dz}_{till\ to\ z} = \gamma G \underbrace{\int_0^\theta \zeta\, d\theta}_{till\ to\ \theta}\ , \qquad \psi'(z) = \frac{d\psi}{dz} = \gamma G \kappa_y \zeta = \gamma G \zeta \frac{d\theta}{dl}, \qquad \frac{d}{dz}e^{i\psi} = ie^{i\psi}\psi'(z) = ie^{i\psi}\gamma G\zeta \frac{d\theta}{dl}$$

$$\begin{cases} \Phi(z) = e^{i\alpha(1-\zeta)} \dfrac{d}{dz}e^{i\psi} = ie^{i\alpha(1-\zeta)}e^{i\psi}\psi'(z) = ie^{i\alpha(1-\zeta)}e^{i\psi}\gamma G\zeta \dfrac{d\theta}{dl} \\ \text{or} \quad \Phi(z) = -\cos\left[\alpha(1-\zeta)\right]e^{i\psi}\gamma G\zeta \dfrac{d\theta}{dl} \qquad \because \left(\zeta = \pm 1 \text{ and } \alpha = \dfrac{\pi}{2} \right) \end{cases}$$

Let $\phi(z) = ie^{i\alpha(1-\zeta)}e^{i\psi}\gamma G\zeta$

$$F_1(z) = \underbrace{\int_{-\infty}^z f_y{}'(z)\Phi(z)dz}_{till\ to\ z} = \frac{1}{e^{i2\pi\nu_y}-1}\underbrace{\int_0^L f_y{}'(z)\Phi(z)dz}_{all\ \Delta l(in\ \text{one}\ period)} + \underbrace{\int_0^z f_y{}'(z)\Phi(z)dz}_{till\ to\ z}$$
$$= \frac{1}{e^{i2\pi\nu_y}-1}\underbrace{\int_0^\Theta f_y{}'(\theta)\phi(\theta)d\theta}_{all\ \Delta\Theta(in\ \text{one}\ period)} + \underbrace{\int_0^\theta f_y{}'(\theta)\phi(\theta)d\theta}_{till\ to\ \theta} = f_{11} + f_{12}$$

$$F_2(z) = \underbrace{\int_{-\infty}^z f_y^{*\prime}(z)\Phi(z)dz}_{till\ to\ z} = \frac{1}{e^{-i2\pi\nu_y}-1}\underbrace{\int_0^L f_y^{*\prime}(z)\Phi(z)dz}_{all\ \Delta l(in\ \text{one}\ period)} + \underbrace{\int_0^z f_y^{*\prime}(z)\Phi(z)dz}_{till\ to\ z}$$
$$= \frac{1}{e^{-i2\pi\nu_y}-1}\underbrace{\int_0^\Theta f_y^{*\prime}(\theta)\phi(\theta)d\theta}_{all\ \Delta\Theta(in\ \text{one}\ period)} + \underbrace{\int_0^\theta f_y^{*\prime}(\theta)\phi(\theta)d\theta}_{till\ to\ \theta} = f_{21} + f_{22}$$

$$F_3 = \frac{1}{2i}\left(f_y^*(z)\underbrace{\int_{-\infty}^z f_y{}'(z)\Phi(z)dz}_{till\ to\ z} - f_y(z)\underbrace{\int_{-\infty}^z f_y^{*\prime}(z)\Phi(z)dz}_{till\ to\ z} \right) = -\frac{i}{2}\left[f_y^*(z)F_1(z) - f_y(z)F_2(z) \right]$$