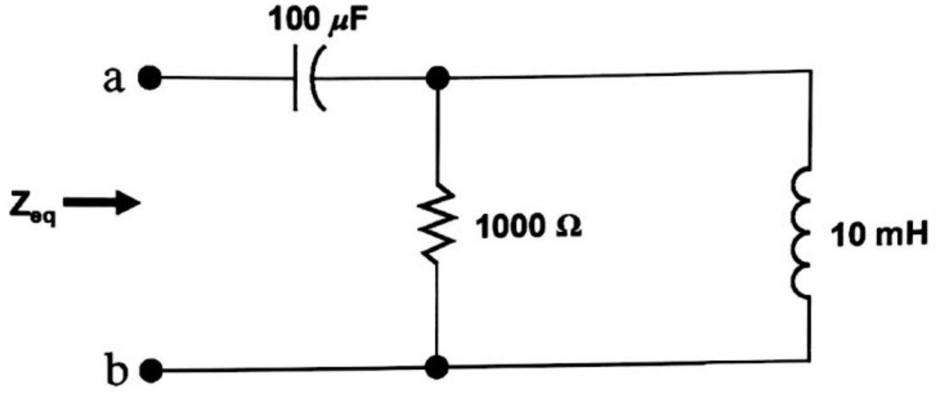
Fall '15 Exam #3 ..
Dhanya Solution.

1. Equivalent Impedance [20]

✓ Find the equivalent impedance, **Zeq**, of the following circuit. Assume that frequency $\boldsymbol{f}$ **= 1 kHz.**



$$Z_{eq} = Z_L \parallel Z_R + Z_C .$$

$$Z_L = j\omega L = j \times 2\pi \times 1000 \times 10 \times 10^{-3} = j\,62.8\,\Omega .$$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{2 \times \pi \times 1000 \times 100 \times 10^{-6}} = -j1.59\,\Omega$$

$$Z_L \parallel Z_R = \frac{Z_L \times Z_R}{Z_L + Z_R} = \frac{(j62.8)(1000)}{\underset{62800}{1000} + j62.8} \frac{[1000 - j62.8]}{[1000 - j62.8]}$$

$$= \frac{394384 + j\,62800000}{1000^2 + 62.8^2}$$

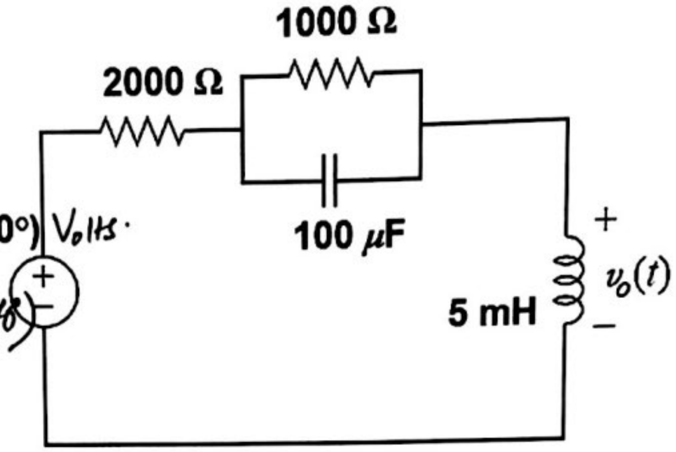
$$= \frac{394384 + j\,62800000}{1003944} = 3.93 + j62.55$$

$$Z_{eq} = 3.93 + j62.55 + -j1.59$$

$$= \underline{3.93 + j\,60.96}\ \Omega$$

2. Voltage Across Impedance [25]

For the circuit shown below, find $v_o(t)$.
Hint: Use voltage divider law after converting to frequency domain circuit.

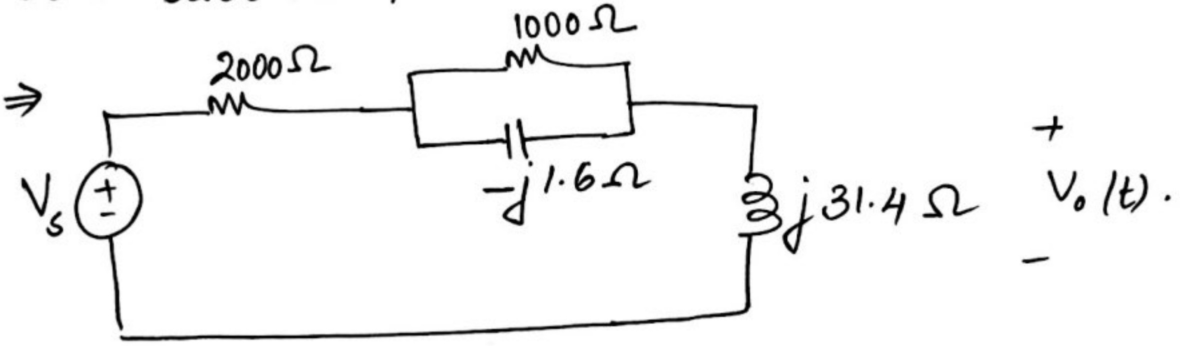


$V_s = 2\sin(6280t + 20°)$ Volts.

$= 2\cos(6280t - 70)$

$\omega = 6280$ rad/sec.

$\Rightarrow$



$\Rightarrow 1000\,\Omega \parallel -j1.6\,\Omega$

$\Rightarrow Z_{eq}' = \dfrac{1000 \cdot (-j1.6)}{1000 - j1.6}$

$= \dfrac{-j1600\,(1000 + j1.6)}{1000^2 + 1.6^2}$

$= -j\,0.0016\,(1000 + j1.6)$

$= 0.00256 - j1.6$

$Z_{total} \Rightarrow 2000 + Z_{eq}' + j31.4 = 2000.00256 - j1.6 + j31.4$

$\Rightarrow V_o(t) = \dfrac{2\angle -70°\,(j31.4)}{2000.00256 + j29.8}$

$= \dfrac{(2\angle -70°)(31.4\angle 90°)}{2000.22\angle +0.85°}$

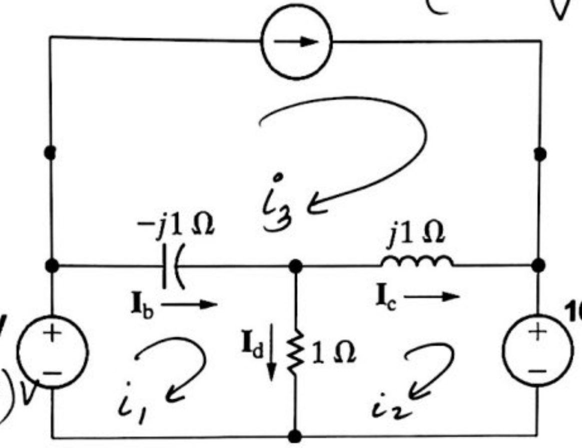
$= 0.0314\angle 19.15°$

$= 0.0314\cos(6280t + 19°)$

3. Mesh Current Method [25]

Find the branch currents, **Ib**, **Ic**, and **Id** using mesh current method in the circuit below [20]. Write the final answers in the phasor form [5].

$1.5\angle{-45^\circ}$ A $= (1.06 - j1.06)$ A.



$= (14.14 + j14.14)$ V

mesh ① $\longrightarrow -(14.14 + j14.14) - j1(i_1 - i_3) + 1(i_1 - i_2) = 0$.

mesh ③ $\Rightarrow i_3 = (1.06 - j1.06)$ A.

$\Rightarrow -14.14 - j14.14 - ji_1 + j(1.06 - j1.06) + i_1 - i_2 = 0$

$\Rightarrow -13.08 - j13.08 + i_1(1-j) + i_2(-1) = 0$ —— ①

mesh ② $\longrightarrow 1(i_2 - i_1) + j1(i_2 - i_3) + 10 = 0$

$\Rightarrow i_1(-1) + i_2(1+j) - 1.06 - j1.06 + 10 = 0$

$\Rightarrow i_1(-1) + i_2(1+j) = -8.94 + j1.06$ —— ②.

Solving ① & ② $\longrightarrow i_1 = -8.94 + j27.22$ A ; $i_2 = 5.2 + j23.08$ A

$\Rightarrow I_b = i_1 - i_3 = -10 + j28.28 = 30.5\angle 109.5^\circ$ A

$I_c = i_2 - i_3 = 4.14 + j24.14 = 24.5\angle 80.3^\circ$ A.

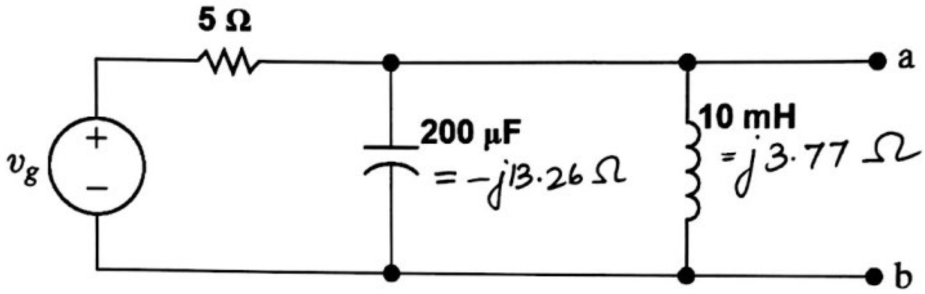
$I_d = i_1 - i_2 = -14.14 + j4.14 = 14.7\angle 163.7^\circ$ A

4. Thevenin Equivalent Source [25]

✓ (a) Find the Thevenin equivalent source for the circuit shown below [15].
Assume that $\boldsymbol{v_g(t)=50\cos(377t-90°)}$. ✓
Note: Thevenin equivalent source=equivalent voltage source in series with an equivalent impedance. It is best to represent the voltage in phaser form. Leave the impedance in rectangular form.

(b) Find the load required for the maximum power transfer [5]. Convert the load impedance into a resistor in series with a capacitor or an inductor [5]. Round the capacitor or inductor value to 1 decimal point.

(c) Find the maximum average power (not max peak power) delivered to the load you found in (b) [5].



$$Z_{Th} = Z_R \,||\, Z_C \,||\, Z_L \Rightarrow Z_{Th}^{-1} = \frac{1}{5} - \frac{1}{j13.26} + \frac{1}{j3.77}$$

$$= \frac{-49.99 - j18.85 + j66.3}{-249.95}$$

$$= \frac{-50 + j47.45}{-250}$$

$$\Rightarrow Z_{Th} = \frac{250}{50 - j47.45}\ \Omega.$$

$$= \frac{12500 + j11862.5}{2500 + 2251.5}$$

$$= \underline{\underline{2.63 + j2.49}}\ \Omega$$

4: Thevenin Equivalent

$$V_{Th} = V_{AB} = V_g \times \left(\frac{Z_L \| Z_C}{Z_L \| Z_C + Z_R} \right)$$

$$Z_L \| Z_C \Rightarrow Z_{eq}' = \frac{j3.77 \times -j13.26}{-j9.49}$$

$$= \frac{49.99}{-j9.49}$$

$$= j5.27 \ \Omega .$$

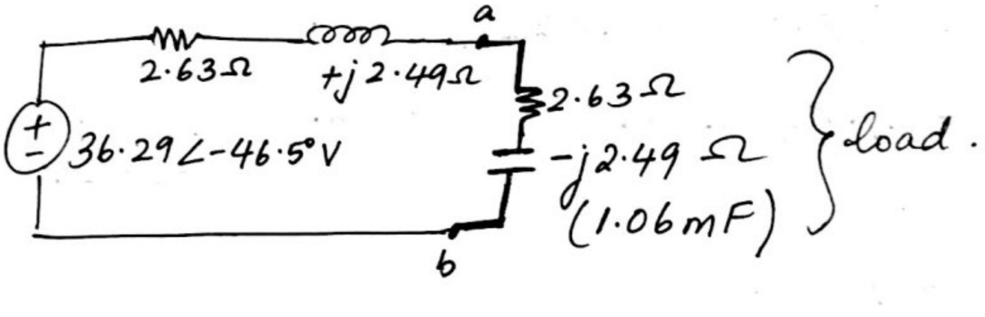
$$\Rightarrow V_{Th} = (50 \angle -90^\circ) \left(\frac{j5.27}{5 + j5.27} \right)$$

$$= (50 \angle -90^\circ) \left(\frac{5.27 \angle +90^\circ}{7.26 \angle 46.5^\circ} \right)$$

$$= \underline{\underline{36.29 \angle -46.5^\circ}}$$

Thevenin equivalent:

$\Rightarrow$



$$P_{max} = \frac{V_{rms}^2}{4R_{Th}} = \frac{(36.29/\sqrt{2})^2}{4 \times 2.63} = \underline{\underline{62.59}} \text{ Watts}$$