《算法复杂性理论》
第3讲 穷举法

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穷举法定义

- In computer science, brute-force search or exhaustive search, also known as generate and test, is a very general problem-solving technique that consists of systematically enumerating all possible candidates for the solution and checking whether each candidate satisfies the problem's statement.

Wikipedia

穷举法定义

- In order to apply brute-force search, one must implement four procedures, first, next, valid, and output. They should take the problem instance P a parameter, and do the following:
  1. first (P): generate a first candidate solution for P.
  2. next (P, c): generate the next candidate for P after the current one c.
  3. valid (P, c): check whether candidate c is a solution for P.
  4. output (P, c): use the solution c of P as appropriate to the application.
- The next procedure must also tell when there are no more candidates for the instance P, usually by returning a "null candidate", Λ. Likewise the first procedure should return Λ if there are no candidates at all for the instance P.

Wikipedia

穷举法的通用算法

- 算法名称：通用穷举法(ExhaustiveSearch)
- 输入：问题实例P
- 输出：问题的解
- 1: c ← first(P)
- 2: while c ≠ Λ
- 3: if valid(P,c) then output(P, c)
- 4: c ← next(P)
- 5: end while

百元买百鸡问题

- 鸡翁一值钱5，鸡母一值钱3，鸡雏三值钱1。百钱买百鸡，问鸡翁、母、雏各几何？”
- 算法问题：n元买n鸡问题
- 数学模型
  - $x + y + z = n$
  - $5x + 3y + \frac{z}{3} = n$

百元买百鸡问题

```
1: for x=1 to n
2:     for y=1 to n
3:         for z=1 to n
4:             if x+y+z=n then
5:                 if z mod 3 = 0 then
6:                     if 5x+3y+z/3 = n then
7:                         print x,y,z
```


百元买百鸡问题

```
1: for x=1 to n/5
2:     for y=1 to n/3
3:         z=n-x-y
5:         if z mod 3 = 0 then
6:             if 5x+3y+z/3 = n then
7:                 print x,y,z
```


素数测试—试除法(trial division)

- 试除法是测试一个数N是否为素数的蛮力方法
  - 由于如果N有大于$\sqrt{N}$的因子p，则一定有一个小于$\sqrt{N}$因子q，因此只要用小于$\sqrt{N}$每个素数去试除N，如果找到一个数能够除尽N，则N就不是素数，如果所有的素数都除不尽N，则N必是素数
  - 上述方法未考虑获得所有小于$\sqrt{N}$的素数的代价
  - 也未考虑计算$\sqrt{N}$的代价

素数测试—朴素(na ïve)试除法伪代码

```
1: ret = true
2: for i=2 to sqrt(N)
3:     if N MOD i =0
4:         ret = false
5:         break
6:     end if
7: return ret
```

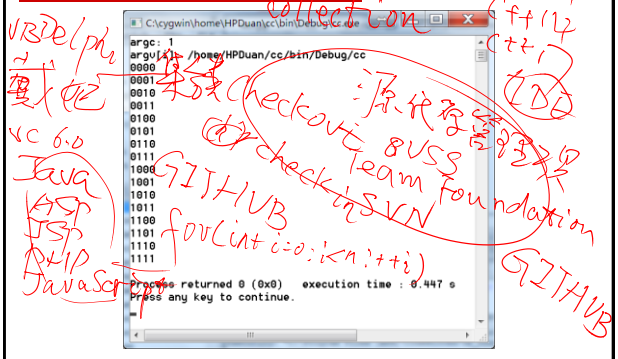

素数测试—朴素(na ïve)试除法伪代码

```
1: ret = true
2: i = 2
3: do
4:     if N MOD i =0
5:         ret = false
6:         break
7:     end if
8:     i = i+1
9: while i*i<=N
```

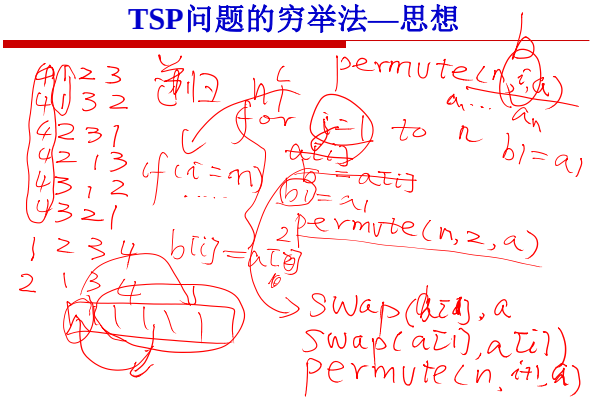

0-1背包问题的穷举法

```
vector<int> KSv;
void Knapsack(int n) {
  if (n==0) {
    for (auto x:KSv)
      cout << x;
    cout << endl;
    return; }
  KSv.push_back(0);
  Knapsack(n-1);
  KSv.pop_back();
  KSv.push_back(1);
  Knapsack(n-1);
  KSv.pop_back();
}
```

0-1背包问题的穷举法



TSP问题的穷举法—思想



TSP问题的穷举法—算法

```
vector<int> TSPv;
void TSP(int i, int n) {
  if (i==n-1) {
    for (auto x:TSPv)
      cout << x+1;
    cout << endl;
    return;
  }
  for (int j=i; j<n; ++j) {
    swap(TSPv[i],TSPv[j]);
    TSP(i+1, n);
    swap(TSPv[i],TSPv[j]);
  }
}
```

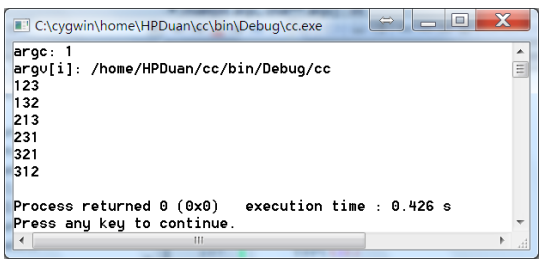
```
void TSPt(int n)
{
  for (int i=0; i<n; ++i)
    TSPv.push_back(i);
  TSP(0,n);
}
```


TSP问题的穷举法—算法

```
vector<int> TSPv(10);
void TSP(int i, int n){
  if (i==n-1) {
    for_each (TSPv.begin(),
      TSPv.begin()+n,
      [](int x){cout << x;});
    cout << endl;
    return;  }
  for (int j=i; j<n; ++j) {
    swap(TSPv[i],TSPv[j]);
    TSP(i+1, n);
    swap(TSPv[i],TSPv[j]);
  }
}
```

```
void TSPt(int n)
{
  iota(TSPv.begin(),
    TSPv.end(), 1);
  TSP(0,n);
}
```


TSP问题的穷举法



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