MATH3001: Project in Mathematics

Flood Modelling and Analysis: Modelling large scale river flooding to communicate flood mitigation strategies to local councils and the general public

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Contents

1 Introduction

Flooding in the United Kingdom in recent years has led to major destruction of houses, the environment and government properties. Many people throughout the country have lost homes, money and occasionally even lives. This report will attempt to investigate some of the major floods the UK has experienced over the last two decades and attempt to provide river-mitigation solutions apt to the environment, economy and aesthetics of the surrounding area and council.

This project will be attempting to investigate:

- What types of flood mitigation strategies are available and how they work.
- What FEV (flood excess volume) is and how it is estimated, using rating equations given by the Environmental Agency under the 'Freedom of Information Act', and describe how this model can be used to mitigate flooding.
- The Boxing Day 2015 floods the River Aire, Leeds, the River Calder, Mytholmroyd and the June 2007 flood of the River Don, Hadfields. This will be done as an exercise to familiarize oneself with the coding skills and the data collection methods needed to investigate other river floods. Knowledge of estimated FEV will be applied and examples of potential flood mitigation strategies will be discussed. This will be an attempt to verify and recreate data completed originally by Onno B and Thomas Kent.
- The Boxing Day 2015 flood of the River Wharfe, Addingham. This will be primary research, where FEV will be estimated and flood mitigation strategies will be discussed which are most apt to the requirements of the river and surrounding council.
- What current strategies are being set out by local councils and suggesting alternatives that may be more cost/environmentally effective.

2 Flood Excess Volume

2.1 Return Period

Flood Excess Volume (FEV) is the estimated volume of water that a river has flooded by. Thus, the FEV is the approximate volume of excess water needed to be contained or mitigated in order to prevent a flood of the same size within a given return period. A return period is the inverse probability (given as a percentage) of the estimated time period between similarly large events, in this case flooding (NIWA 2016). Commonly misconstrued, this is not to say that if a return flood period is 100 years, that the next flood of the same magnitude would occur 100 years later, but rather that there is a 0.01 probability that a similar flood could occur any given year succeeding this.

2.2 Calculation

To calculate the FEV of a flood, first data must be collected and analyzed from the Environmental Agency. They analyze the river level (height) against a timestamp typically at 15 minute intervals across hundreds of gauge stations over the country. For each individual gauge station, it is possible to estimate the threshold level of the river H_t, measured in m. This value can be estimated many different ways, from social media time stamped photos, to online live resources such as (www.gaugemap.com). To analyze this data, a rating curve is created to estimate the flow of water, measured in m^3/s at the river's given level and time. The frequent measurements taken at the gauge station can then be plotted onto a graph using the rating equation $Q = Q(\bar{h})$, where Q is the flow of water (m^3/s). We can then apply H_t into the rating curve to give the threshold flow $Q_t = Q(H_t)$. FEV can then be defined as:

$$FEV = \int Q(t) - Q_t,$$

over time T_f, where T_f is the time when $Q(t) - Q_t > 0$, measured in seconds. This gives a volume of water in which the river has flooded by. A way to estimate this value is to evaluate H_m, the mean level of the river given the level is above H_t, evaluated by:

$$\frac{\Sigma_{i=1}^{n} h_i}{n}, \forall h_i \geq h_t.$$

Again, this value can be applied into the rating equation to give a flow value Q_m. Thus,

$$FEV = V_e \approx \; T_f(Q_m - Q_t).$$

This is the formula that will be used to estimate FEV from the quadrant plots that have been created for each river.

2.3 Square Lake Graphs

One way to display the FEV in a simpler and more visual way is to translate it into a hypothetical square lake. Imagining a 2 metre deep square lake, with the total volume equal to the FEV, depicts the exact quantity of water to be mitigated. The lake can then be split into parts for each mitigation strategy to clearly display the methods of flood-mitigation that can be used and the volume of water each method mitigates.

3 River-Mitigation

By definition, to mitigate is to, 'reduce the risk or have a lower occurrence of a specific event.' In the case of rivers this is to reduce the occurrence of flooding (Nebraska DoNR 2018). There are many different types of river-mitigation, falling under two categories; structural and non-structural flood-mitigation.

3.1 Structural Flood-Mitigation

Structural flood-mitigation techniques are methods which actively reduce the flood's destruction in certain areas, including:

- Flood Walls: Permanent walls located around the bank of the river. A durable, long term defence, however walls can be seen as obtrusive and aesthetically displeasing, especially when over 1 meter tall (Flood Control International 2018).
- Removable River Barriers: Generally made from posts and horizontal beams that can be placed into the ground by the river ed in preparation of a flood. These are less effective than permanent flood walls, but are beneficial due to their maneuverability (Flood Control International 2018).
- Reservoirs: A body of water upstream of the urban area that it is protecting, generally at an area of lower economic and environmental cost. Compared to other strategies, reservoirs can be expensive to create and maintain (Hettiarachchi 2011).
- Make Room for the River: A strategy where the area surrounding the river is changed or manipulated to allow a greater volume of water to pass before flooding takes place. Although specific to each river, some examples are; deepening the riverbed, removing obstacles and lowering the flood plane (Ruimte Voor de Rivier 2016).
- Diverting the River: Once the river level reaches a certain height, some of the water is then diverted down an artificial channel to lesser populated areas, thus reducing the level in the town or city downstream.

3.2 Non-Structural Flood-Mitigation

Non-structural flood-mitigation is aimed more at changing the way the population act regarding flooding, and raising awareness of how to reduce damage to property (Nebraska DoNR 2018). These methods include:

- Property Surveys: Giving detailed surveys of properties which have or could be effected by flooding would give homeowners, businesses and building developers more awareness of the risks of flooding (Northern Territory Government 2018).
- Early Warning Systems: Creating earlier warning systems for property owners in high risk areas would give more time for people to defend their property from flood, for example adding sandbags to their property or moving their motor vehicles to areas with less flood risk (Northern Territory Government 2018).
- Improving General Understanding and Awareness of Flooding: Greater knowledge of the risk and history of floods in people's area of home can increase the ability to respond to a flood warning effectively (Northern Territory Government 2018).

4 Analysis of the River Aire

4.1 Background

The River Aire flows through Yorkshire, and is of the the UK's more major rivers, 114km in length. It runs from Malham to Airmyn flowing through 5 conurbations; Skipton, Keighley, Bradford, Leeds and Castleford, with a combined population of 1.5 million (Aire Rivers Trust 2019). Specifically, this project analyses the flood of the River Aire at Armley, Leeds, on Boxing Day 2015. This was a very large flood, caused by Storm Eva, a violent storm that hit Northern England over December 2015. This flood had a return period of 0.5% (Leds.gov 2019), with millions of pounds worth of damage to property and infrastructure.

4.2 Armley Quadrant Plot

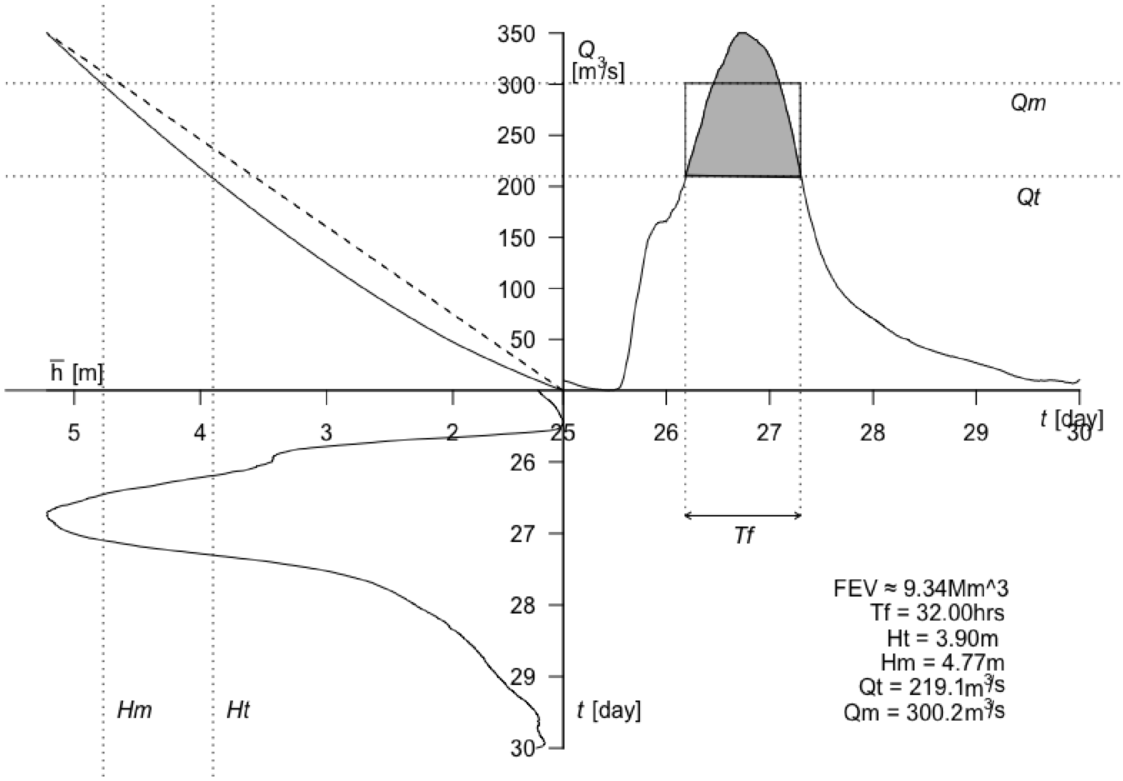

Figure 1: Quadrant plot depicting the Boxing Day 2015 Floods of the River Aire at Armley, Leeds. The data used was compiled and requested for from the Environmental Agency. The bottom left quadrant depicts stage data, $\bar{h}$, against time, t. The upper left quadrant depicts the rating curve and its dotted liner approximation (discussed in more depth below). The upper right quadrant depicts time, t, against flow data, Q, collected from the rating equation. Raw $\bar{h}$ and t data was provided by the Environment Agency, however Q was evaluated from the rating equation, also provided by the Environment Agency.

4.3 Plotting the Graph

To allow all of the individual data sets to be compiled into one graph, first all of the stage and height data had to be scaled to fall between 0 and 1, and then the signs altered to allow the individual plots to fall within the correct axis. All of the data was then programmed into the software 'R' and then plotted.

4.4 The Rating Curve

The rating curve, which is used to estimate the flow data and thus Q_t and Q_m, is based on an equation given by the Environment Agency, specific for each individual gauge station across the county. The general formula is given by:

$$Q(h) = c(h - a)^b,$$

where h is stage, given as arbitrary values between 0 and 6, rather than the actual recorded river heights. Q is the flow, and a, b and c are coefficients, given by the Environment Agency. For Armley, the rating data is given by:

Lower Stage Limit (m)	**Upper Stage Limit (m)**	**c**	**b**	**a**
0.2	0.685	30.69	1.115	0.156
0.685	1.917	27.884	1.462	0.028
1.917	4.17	30.127	1.502	0.153

Table 1: Depicting different values of the coefficients of the rating equation at Armley, at different stages h.

From this data, a programme was created which ran through the different stages h could take, which then plotted a set of points for flow Q at each stage h of the river.

4.5 H_t and Q_t Estimations

H_t and subsequently Q_t were estimated at Armley by Onno B. and Thomas K., by personally viewing the gauge station and by a timestamped photo found on (www.Facebook.com) at the time when the river burst its banks. This information could then be correlated with the stage data given for that specific time by the Environment Agency, giving an approximation of 3.9 meters for H_t. H_t was then applied to the rating curve to give $Q_t = Q(H_t)$.

4.6 FEV Estimation

The square found in the top right quadrant is the visual estimation of FEV, where:

$$V_e \approx T_f(Q_m - Q_t),$$

where T_f is 32 hours, calculated as the total time at which $Q(t) - Q_t > 0$. This square is a good approximation of the shaded area of the graph, equating to $FEV = 9.34Mm^3$.

4.7 Flood-Mitigation

After this flood, Leeds City Council responded by constructing a Leeds Flood Alleviation Scheme (Leeds.gov 2019), which will be undertaken in two phases. The first phase was opened in October 2017, and has been designed to protect a flood with a 1% return period. According to (Leeds.gov 2019), this phase will not fully mitigate a flood with a magnitude of Boxing Day 2015, however there would be a massive reduction to damage to property and infrastructure. Phase two work is scheduled to begin in the summer of 2019, with the aim of protecting the Aire from floods up to a 0.5% return period, enough to match the floods caused by Storm Eva.

4.8 Square Lake Graphs

To visualise the cost-effectiveness of each mitigation technique, first one must determine the cost of each hypothetical scheme and the volume of water it mitigates. Dr Onno B. and Thomas K. estimated this in this example by working with the Leeds City Council. From this, they deduced that possible schemes on the Aire were:

- Calverley Storage: Extra storage created by adding an adjustable weir. Mitigating 8% of the flood. Approximately £10M or £1.25M/1% of flood protection.
- Rodley Storage: Flood plain enhancement. Mitigating 12% of the flood. Approximately £14M or £1.17M/1% of flood protection.
- Floods Walls: Increasing wall height at different areas of the river. Approximately £0.707/1% of flood protection.
- Cononley Washlands and Holden Park Storage: Flood plain enhancement. Mitigating 50.4% of the flood. Approximately £35M or £0.69/1% of flood protection.

From this, Onno B. and Thomas K. created square lake graphs of the different scenarios of possible flood protection schemes, similar to as shown below, with a constant depth of 2 meters. From figure 2, it is clear to see that the cheapest option to mitigate the whole of the FEV is to enhance the flood plain at Cononley Washlands and Holden Park, and add flood wall defences to mitigate the rest of the FEV, costing approximately £70.1M. The most expensive option is to have Calverley and Rodley storage supplement the flood walls[1].

5 Analysis of the River Calder

5.1 Background

The River Calder runs through West Yorkshire, beginning on the Pennines West of Todmorden, and running through the conurbations Halifax, Bridgehouse, Huddersfield, Dewesbury and Wakefield before joining the River Aire at Castleford. The Calder is approximately 87km in length, with its name

[1] The 6% discrepancy between the combined Calverley and Rodley storage and their respective individual storage, as discussed by Onno B. and Thomas K. is noticed however unaccounted for.

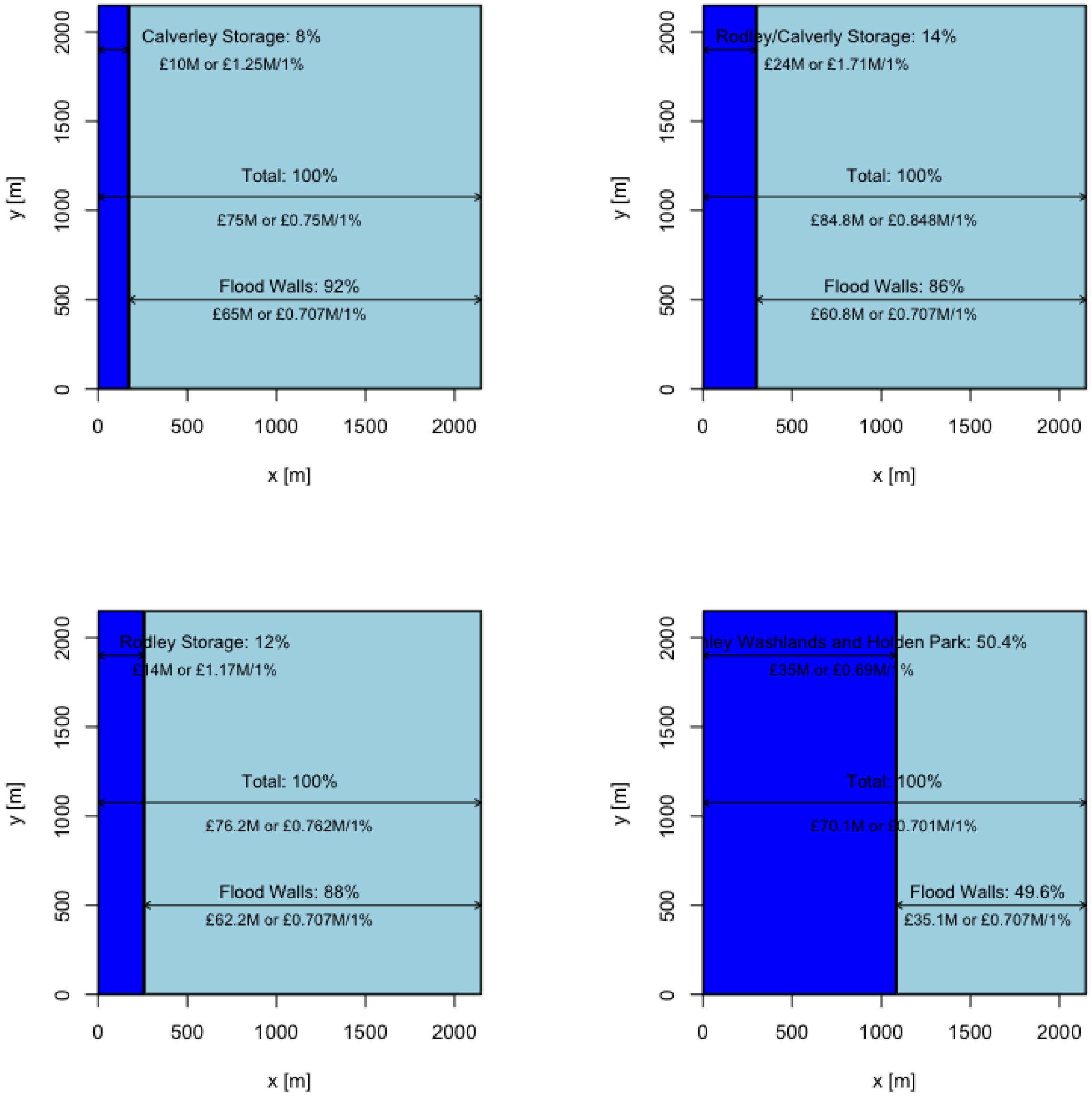

Figure 2: 4 possible mitigation strategies to alleviate 100% of the FEV of the Boxing Day floods. The top left is a combination of Calverley storage and flood walls, the bottom left is Rodley storage and flood walls. The top right is a combination of Rodley and Calverley storage and flood walls, and the bottom right graph depicts a combination of Cononley Washlands and Holden Park storage with flood walls.

originating from the early British, translating to 'violent waters or stream' (AEDA 2019). Storm Eva also hit the Calder in December 2015, forcing the river to burst its banks in multiple areas, causing mass flooding throughout West Yorkshire.

5.2 Mytholmroyd Quadrant Plot

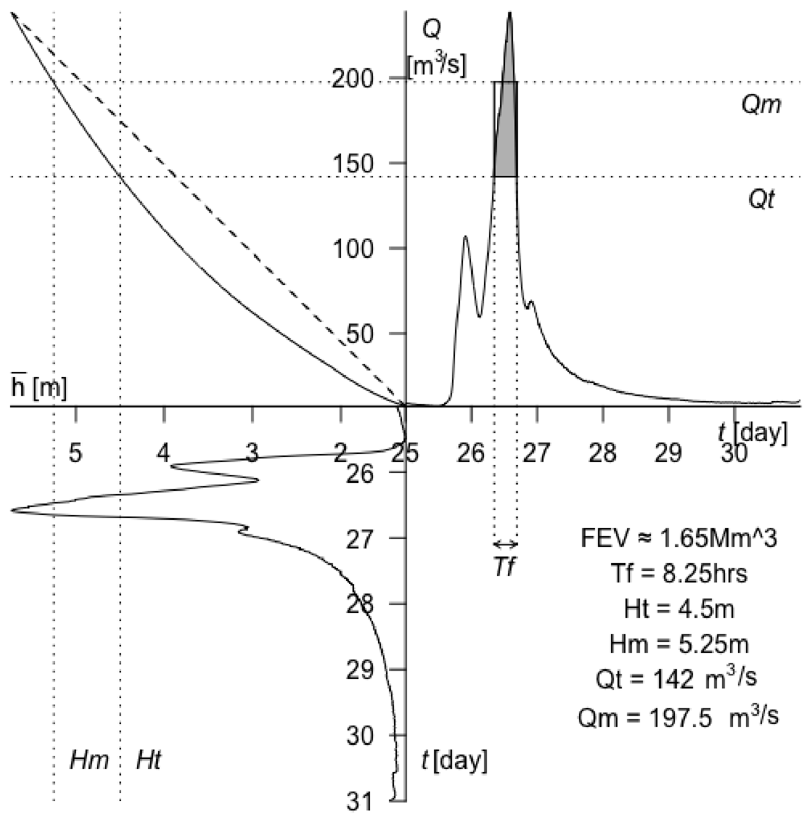

Figure 3: Quadrant plot depicting the Boxing Day 2015 Floods of the River Calder at Mytholmroyd, West Yorkshire. The data used was compiled and requested for from the Environmental Agency. The bottom left quadrant depicts stage data, $\bar{h}$, against time, t. The upper left quadrant depicts the rating curve and its dotted liner approximation (discussed in more depth below). The upper right quadrant depicts time, t, against flow data, Q, collected from the rating equation. Raw $\bar{h}$ and t data was provided by the Environment Agency, however Q was evaluated from the rating equation, also provided by the Environment Agency.

5.3 The Rating Curve

The rating curve's general formula used to plot the above graph was the same as for the River Aire, which is:

$$Q(h) = c(h - a)^b,$$

where h is stage, given as arbitrary values between 0 and 5.8, rather than the actual recorded river heights. Q is the flow, and a, b and c are coefficients, given by the Environment Agency. For the Calder, however, the limits and coefficients of the equation were different, given as:

Lower Stage Limit (m)	**Upper Stage Limit (m)**	**c**	**b**	**a**
0	2.107	8.459	2.239	0.342
2.107	3.088	21.5	1.37	0.826
3.088	5.8	2.086	2.515	-0.856

Table 2: Depicting different values of the coefficients of the rating equation at Mytholmroyd, at different stages h.

5.4 FEV Estimation

H_t was estimated by Onno B. and Thomas K. to be $4.5m$ for the Calder at Mytholmroyd. With this estimation, it can be evaluated that FEV is approximately:

$$V_e \approx \; T_f(Q_m - Q_t) \; = 8.25(197.5 - 142) \; = 1.65Mm^3.$$

This approximation can be seen in figure 3 as the area of the rectangle in the top right quadrant, an approximation of the shaded area of the graph.

5.5 Flood-Mitigation

Some of the schemes addressed for the River Calder, explored by Onno B. and Thomas K., include:

- Reservoir Storage: Using reservoirs to temporarily store potential flood water upstream of the flood risk area. Mitigating between 26.7% and 53.3% of the total FEV, approximately £$[0.56, 1.13]M/1\%$ of FEV.
- Natural Flood Management (NFM): A method of flood defence using the environment and natural processes to increase the volume of water the surrounding area can uptake, reducing run off, for the Calder this includes debris damns[2](Eye on Calderdale 2019). Mitigating between 4.2% and 8.5% of the total FEV, approximately £$[0.63, 1.27]M/1\%$ of FEV.
- Tree Planting: Involves planting trees or vegetation in areas with close proximity to the river to increase uptake of water by the soil, decreasing run off volume. Mitigating between 2.5% and 5% of the total FEV, approximately £$[1, 2]M/1\%$ of FEV.

[2]NFM has been up-scaled for the square lake graph. As proposed by Onno B. and Thomas K., whilst NFM has proven to mitigate local floods with very small return periods, it has been shown to be fairly ineffective when dealing with large scale flooding.

5.6 Square Lake Graphs

From this data it is possible to create a square lake graph of depth 2 meters. The % of total FEV alleviated is depicted as the amount of the graph that is coloured, using the upper and lower approximations of the effectiveness of each mitigating measure.

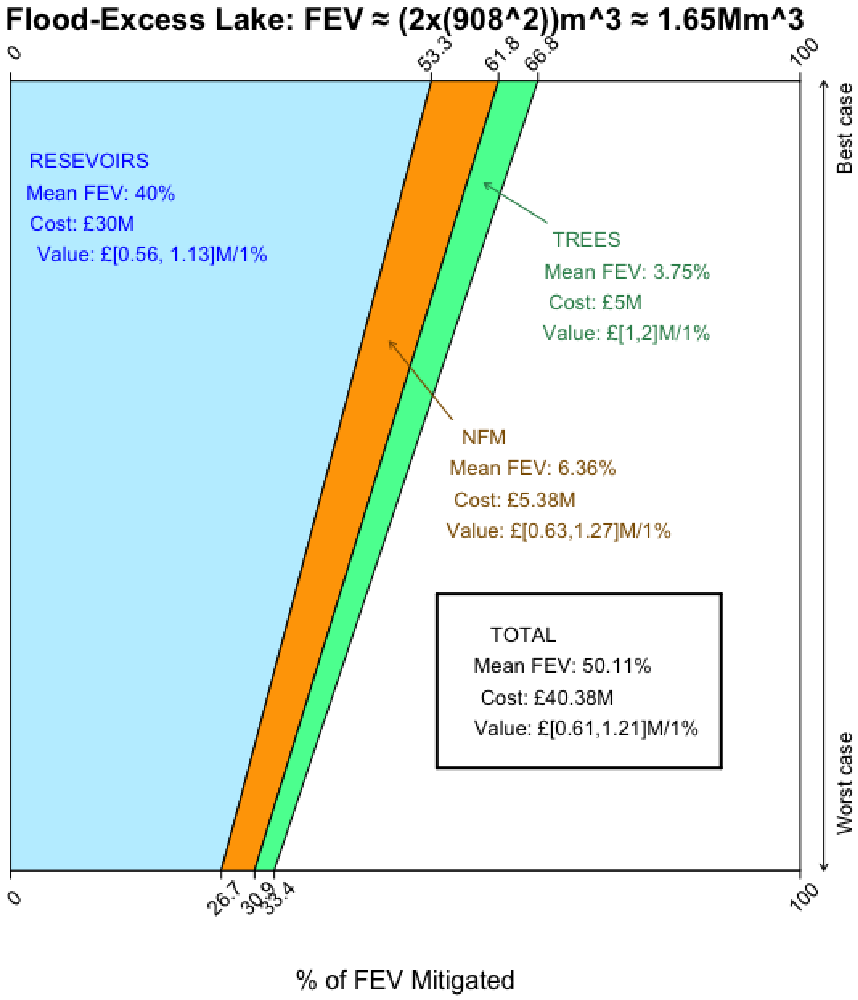

Figure 4: Flood-Excess Lake:

$$V_e \approx \ 2 \cdot (908)^2 \approx 1.65Mm^3.$$

Square lake graph of potential flood alleviation schemes to reduce FEV at Mytholmroyd, River Calder.

As the Boxing Day 2015 flood of the River Calder had approximately a 1% return period, it is clear to see these flood alleviation methods shown in figure 4 mitigate between 33.4% and 66.8% of that flood. This would suggest that these strategies would not fully mitigate another flood of this magnitude, and would need to be supplemented with other strategies, such as flood walls.

6 Analysis of the River Don

6.1 Background

The River Don is a primarily South Yorkshire river, originating in the Pennine Hills, at around 450m elevation (Britainnica 2019). It is approximately 110km in length before it joins the River Ouse at Goole. It flows through the cornubations of Sheffield, Doncaster, Rotherham, Barnsley and Chesterfield, with a combined population of approximatly of 1.3 million (Sheffield.ac 2019). The River Don has been analysed at Hadfields, Sheffield, focusing on the devastating June 2007 flooding. The flood reached its peak at 11pm June 25th, and caused more than 1000 people in the area to evacuate their homes (Wiliams and Glendinning 2007). This flood had approximately a 0.5% return period.

6.2 Hadfields Quadrant Plot

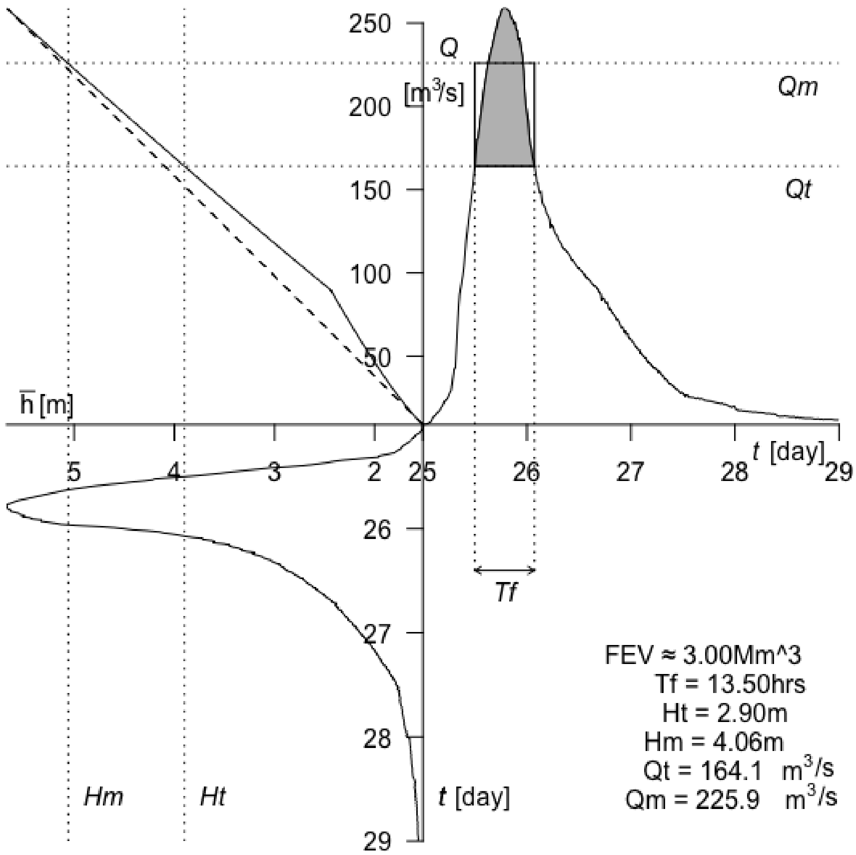

Figure 5: Quadrant plot depicting the June 2007 Floods of the River Don at Hadfields, Sheffield. The data used was compiled and requested for from the Environmental Agency. The bottom left quadrant depicts stage data, $\bar{h}$, against time, t. The upper left quadrant depicts the rating curve and its dotted liner approximation (discussed in more depth below). The upper right quadrant depicts time, t, against flow data, Q, collected from the rating equation. Raw $\bar{h}$ and t data was provided by the Environment Agency, however Q was evaluated from the rating equation, also provided by the Environment Agency.

6.3 The Rating Curve

Again, as before for the other modelled rivers, the general rating equation is:

$$Q(h) = c(h - a)^b,$$

where h is stage, given as arbitrary values between 0 and 4.7, not the precise recorded river stages. Q is the flow, and a, b, and c are the coefficients, given by the Environment Agency. For the case of the River Don, the limits and coefficients are:

Lower Stage Limit (m)	Upper Stage Limit (m)	c	b	a
0	0.52	78.4407	1.7742	0.223
0.52	0.931	77.2829	1.3803	0.3077
0.931	1.436	79.5956	1.2967	0.34
1.436	3.58	41.3367	1.1066	-0.5767

Table 3: Depicting different values of the coefficients of the rating equation at Hadfields, at different stages h.

These values applied into the general rating equation lead to the non-dotted line in the top left quadrant of figure 5, with the dotted line showing the rating curve's linear approximation.

6.4 FEV Estimation

H_t was again estimated by Onno B. and Thomas K. to be 2.90m for the Don at this point. Gaugemap estimates that flooding can occur at 2.63m, however this is seen as a slight underestimate, thus $H_t = 2.9m$ is the value chosen. Using this value, H_m, Q_t and Q_m can be evaluated from the rating equation, to give FEV equalling approximately:

$$V_e \approx \ T_f(Q_m - Q_t) \ = 13.5(225.9 - 164.1) \ = 3.00Mm^3.$$

This estimation can be visualised in figure 5 as the rectangle in the top right quadrant of the plot, itself a good approximation of the shaded area, also in the top right quadrant.

6.5 Flood-Mitigation

There is a large scale flood-alleviation scheme the had just been completed in Sheffield, costing £21 million. This has focused on an 8km stretch of the Don, using structural flood-mitigation strategies to reduce the risk of flooding. These include:

- Flood Walls: Permanent flood walls have been implemented along this stretch of river, up to 1.1m high (Gov.uk 2018)
- Flood Gates: Flood gates have been installed along the river to divert and reduce flooding.
- Flap Valves and Pipes: Methods of diverting the river to reduce the volume and flow rate of water to certain areas at times when flooding may be possible.

7 Analysis of the River Wharfe

7.1 Background

The river I have chosen to investigate is the River Wharfe, in Yorkshire. It originates in the Pennines, and flows 97km southeast where it joins the Ouse near York (Britainnica 2019). Historically it is considered the border between West and North Yorkshire. From its origin to around Addingham (the chosen investigating area) the river is very twisting and fast flowing, however has a much more timid nature further downstream (Wikipedia 2019). Its banks are a popular rambling site but are known to be dangerous in areas due to common flooding and fast flowing waters. The Strid, an area of the Wharfe just a few km upstream of Addingham, is renowned for its deadly rapids and undercut banks, with a married couple famously falling victim to the river. The Boxing Day floods of 2015 caused by Storm Eva that affected much of the north of England hit the River Wharfe harshly, causing a flood with approximately a 4% return period (Leeds.gov 2019).

7.2 Addingham Quadrant Plot

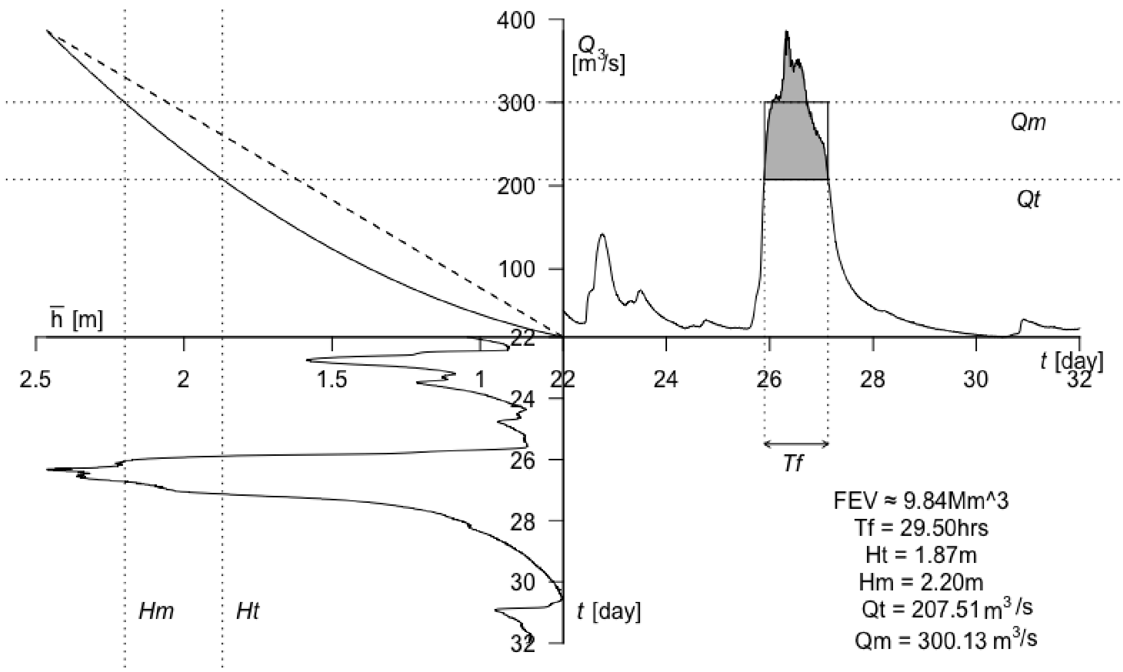

Figure 6: Quadrant plot depicting the Boxing Day 2015 Floods of the River Wharfe at Addingham, Yorkshire. The data used was compiled and requested for from the Environmental Agency. The bottom left quadrant depicts stage data, $\bar{h}$, against time, t. The upper left quadrant depicts the rating curve and its dotted liner approximation (discussed in more depth below). The upper right quadrant depicts time, t, against flow data, Q, collected from the rating equation. Raw $\bar{h}$ and t data was provided by the Environment Agency, however Q was evaluated from the rating equation, also provided by the Environment Agency.

7.3 Plotting the Graph

The graph was plotted using the programming system R. First all of the data was collected into R that was provided upon request from the Environment Agency. Then it all had to be scaled to allow all of the different plots to fit on one set of axis. Next, the axis and segment lines needed to be scaled by the same scale-vector to keep all of the data in proportion.

7.4 The Rating Curve

Upon request from the Environmental Agency, the rating equation values were provided as shown below:

Lower Stage Limit (m)	Upper Stage Limit (m)	c	b	a
0	0.106	24.12	1.5042	0
0.106	0.27	25.935	1.5615	-0.004
0.27	0.462	26.489	1.6314	-0.016
0.462	0.667	26.18	1.6811	-0.03
0.667	2.4631	77.926	2.001	0.239

Table 4: Depicting different values of the coefficients of the rating equation at Addingham, at different stages h.

These limits[3] and coefficients were then inputted into a programme based on the rating equation:

$$Q(h) = c(h - a)^b,$$

forming the rating curve seen in the top left quadrant of figure 6. The Environmental Agency tested the error in the rating curve values against the gauged river heights over time as seen below in figure 7:

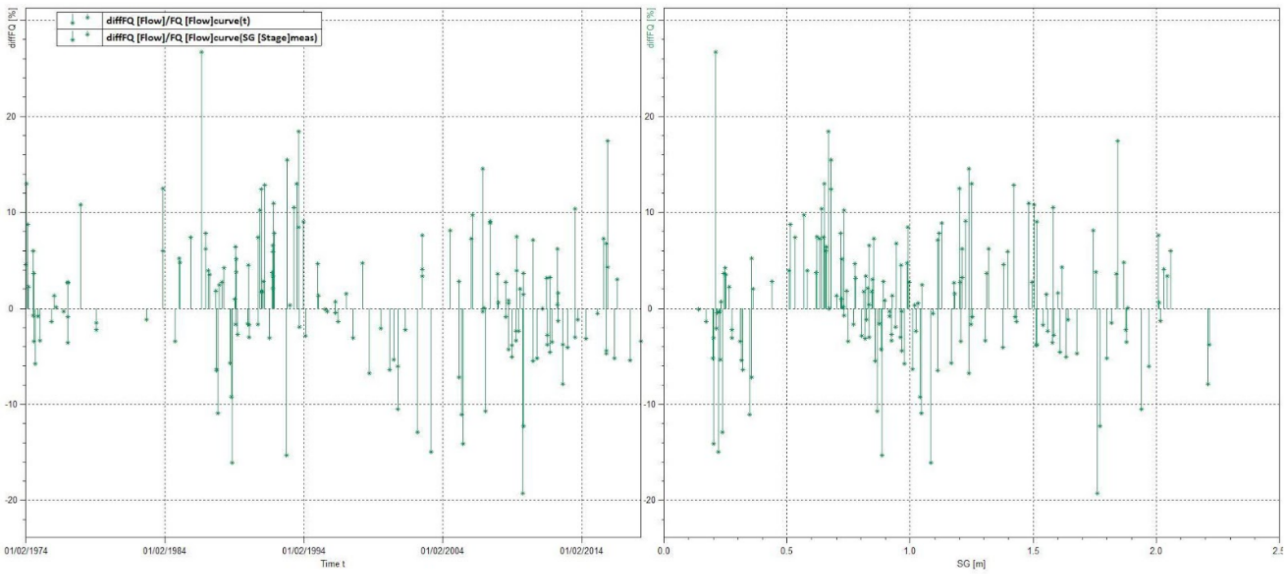

Figure 7: Plot of the deviation of gaugings compared to the rating. The left plot depicts deviation over time and the right plot depicts deviation with increasing stage. Graph provided by the Environmental Agency under the 'Freedom of Information Act'.

[3]The upper limit 2.4631 value has been extrapolated to increase the upper limit. Originally 1.9, the Environmental Agency review suggested it could be increased to 2.4631 meters without changing the standard error.

The graph shows that only one gauging has larger than a 20% error, with most values falling within $+/-10\%$. The standard eror for each of the individual limbs of the equation are shown below:

Limb Number	**Upper Stage Limit (m)**	**Standard Error (%)**
1	0.106	No gaugings
2	0.270	9.8
3	0.462	6.4
4	0.667	8.5
5	2.4631	6.7

Table 5: Depicting the standard error of each individual limb of the rating equation compared to the observed gaugings.

From figure 7, it can be seen that the deviations shown in table 5 are both positive and negative and don't shown any particular trends, either with increasing river level or over time. Due to this, the current rating is a good estimation of flow.

7.5 H_t and Q_t Estimations

(Gaugemap 2019) approximated H_t to be 1.87m at Addingham. It was difficult to see the accuracy of this as a stand alone measure so I viewed the gauge station personally, indicating a threshold value of somewhere in the region of 1.8-2.0m. Two other live online river level sources, (River Levels 2019) and (Flood Information Service 2019), the latter being a government run site, both also estimated flooding to occur above 1.87m. From this, The estimate from (Gaugemap 2019) seemed reasonable so I used this value for H_t.

7.6 FEV Estimation

The square found in the top right quadrant of figure 6 is the visual estimation of FEV, where:

$$V_e \approx T_f(Q_m - Q_t),$$

where T_f is 29.5 hours, calculated as the total time at which $Q(t) - Q_t > 0$. This square is a good approximation of the shaded area of the graph, equating to FEV equalling approximately:

$$V_e \approx T_f(Q_m - Q_t) = 106200(300.13 - 207.51) = 9.84Mm^3.$$

Since H_t was evaluated without complete certainty, I have also evaluated FEV at the upper and lower bounds of my previous approximation of H_t in order to compare the different FEVs. Using the upper bound of my approximation $H_{tu} = 2.0m$, H_{mu} can be evaluated by:

$$H_{mu} = \frac{\Sigma_{i=1}^{n} h_i}{n}, \forall h_i \geq h_{tu},$$

meaning $H_{mu} = 2.22m$. From these values, Q_{tu} and Q_{mu} can be evaluated by applying to the rating equation using the fifth limb, giving:

$$Q(H_{tu}) = Q_{tu} = c(H_{tu} - a)^b = 241.79m^3/s,$$

and

$$Q(H_{mu}) = Q_{mu} = c(H_{mu} - a)^b = 306.02m^3/s$$

respectively. From this, T_{fu} (the total time which $Q(t) - Q_{tu} > 0$) can be evaluated as $T_{fu} = 27hrs = 97200s$, meaning the lower approximation of FEV would be:

$$V_{eu} \approx \ T_{fu}(Q_{mu} - Q_{tu}) \ = 97200(306.02 - 241.79) \ = 6.24Mm^3.$$

Similar approximations can be made using the lower bound of $H_{tl} = 1.80m$. Using the same process as above, one can evaluate: $H_{ml} = 2.18m$, $Q_{tl} = 189.97m^3/s$, $Q_{ml} = 293.78m^3/s$ and $T_{fl} = 30.75hrs = 110700s$. These values can again be applied to FEV approximation equation, to give an upper approximation of FEV:

$$V_{el} \approx \ T_{fl}(Q_{ml} - Q_{tl}) \ = 110700(293.78 - 189.97) \ = 11.49Mm^3.$$

$V_{eu} \approx$ 27% less than V_e, and $V_{el} \approx$ 17% greater than V_e. Thus, the estimation of H_t has a massive impact on the resultant FEV, emphasising the importance of using the most accurate river threshold value possible. Below in figure 8, approximations of V_{el} and V_{eu} can be seen as area of the blue and red rectangles in the top right quadrant of the graph.

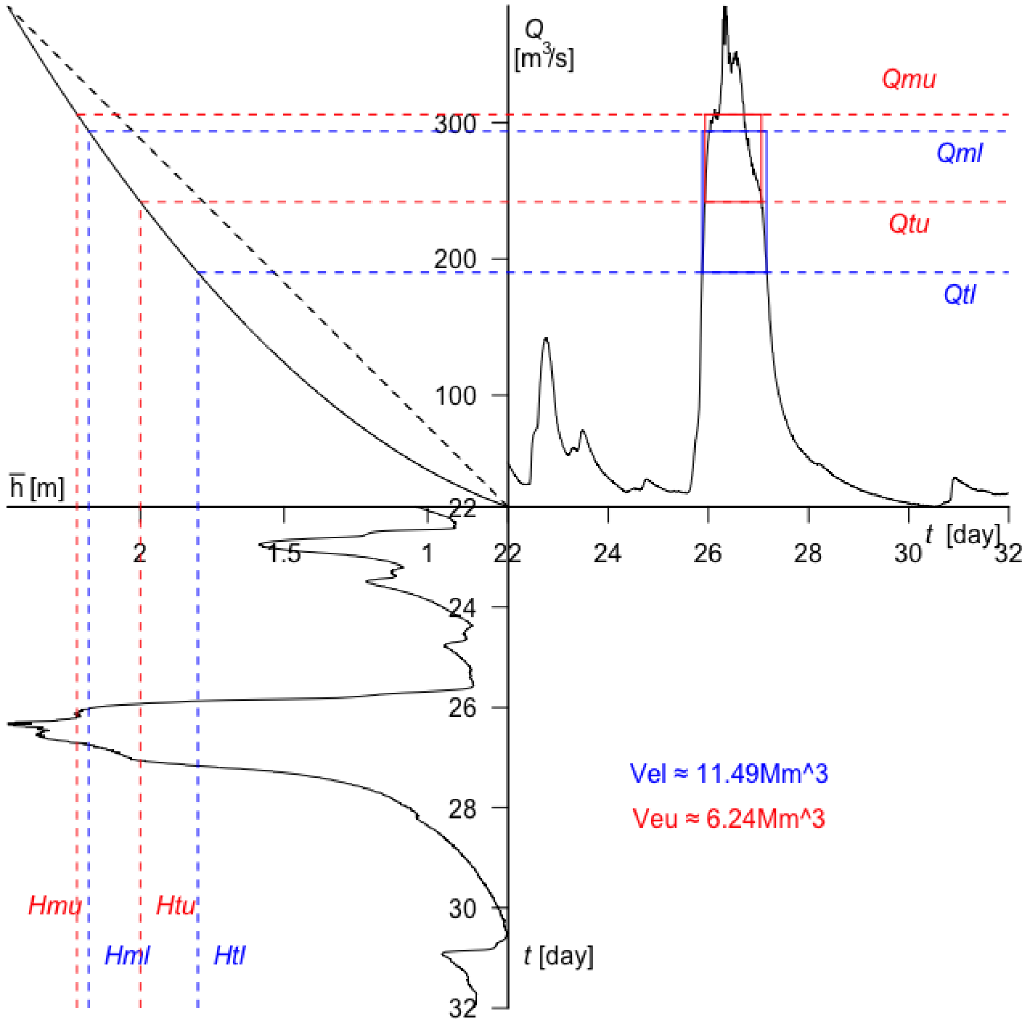

Figure 8: Quadrant plot depicting the Boxing Day 2015 Floods of the River Wharfe at Addingham, Yorkshire, using varying estimations of H_t.

7.7 Flood-Mitigation

Leeds City Council have began a large scale flood alleviation scheme on the River Wharfe focused on the town of Otley, with plans to start construction in Autumn 2019 (Leeds Flood Scheme 2019). A number of different methods of flood mitigation have been proposed, with the council proposing a £2 million budget for the scheme. Leeds Council and the Environmental Agency estimated the effect of these schemes in Otley based on the flow rates of the river upstream (Leeds Flood Scheme 2019). The Addingham gauging station is the closest upstream station from Otley, thus it is appropriate to base the Otley mitigation schemes of the flow data compiled from Addingham. All of the scheme costs have been estimated by the Leeds City Council based on design, construction, and 50 years of maintenance.

These methods include:

- Scheme 1. Flood Walls in Wharfe Meadow up to 1.8m: Increased river embankments on the Wharfe at Wharfe Meadow (blue area in fig. 9), can be placed to reduce the FEV to 0 in a flood similar to the Boxing Day floods. This in total would cost between £$[5, 8]M$, or £$[0.05, 0.08]M/1\%$ of FEV (Leeds Flood Scheme 2019).
- Scheme 2. Flood Walls on the western extents of Billams Hill up to 1.1m: Raising the flood embankments on the west of Billams Hill (yellow area in fig. 9) can be systematically placed to reduce the FEV, to mitigate a flood with a 4% return period, up to the magnitude of the Boxing Day floods. This in total would cost between £$[3.5, 3.9]M$, or £$[0.035, 0.039]M/1\%$[4] of FEV (Leeds Flood Scheme 2019).
- Scheme 3. Removal of Vegetation on the gravel bar in Otley: Removal of vegetation and trees blocking the flow of the river along the gravel bar, in Otley (green area fig. 9). Would reduce FEV locally by $25 - 35\%$, costing approximately £$100 - 350k$, or £$[0.003, 0.014]M/1\%$. This solution would only add localised flood protection to the town of Otley, and would have no effect on flood levels a few 100 meters downstream at Poole or further settlements.
- Scheme 4. Realigning 50-100m of the riverbank upstream of Otley Bridge: This would reduce the FEV of the Boxing Day floods by $15 - 20\%$, costing approximately £$200 - 300k$, or £$[0.01, 0.02]M/1\%$ or FEV mitigated (Otley Flood Scheme 2019).

[4] Based off flood wall expenditure $+/-5\%$ at Billams Hill proposed by Leeds Flood Alleviation Scheme 2019.

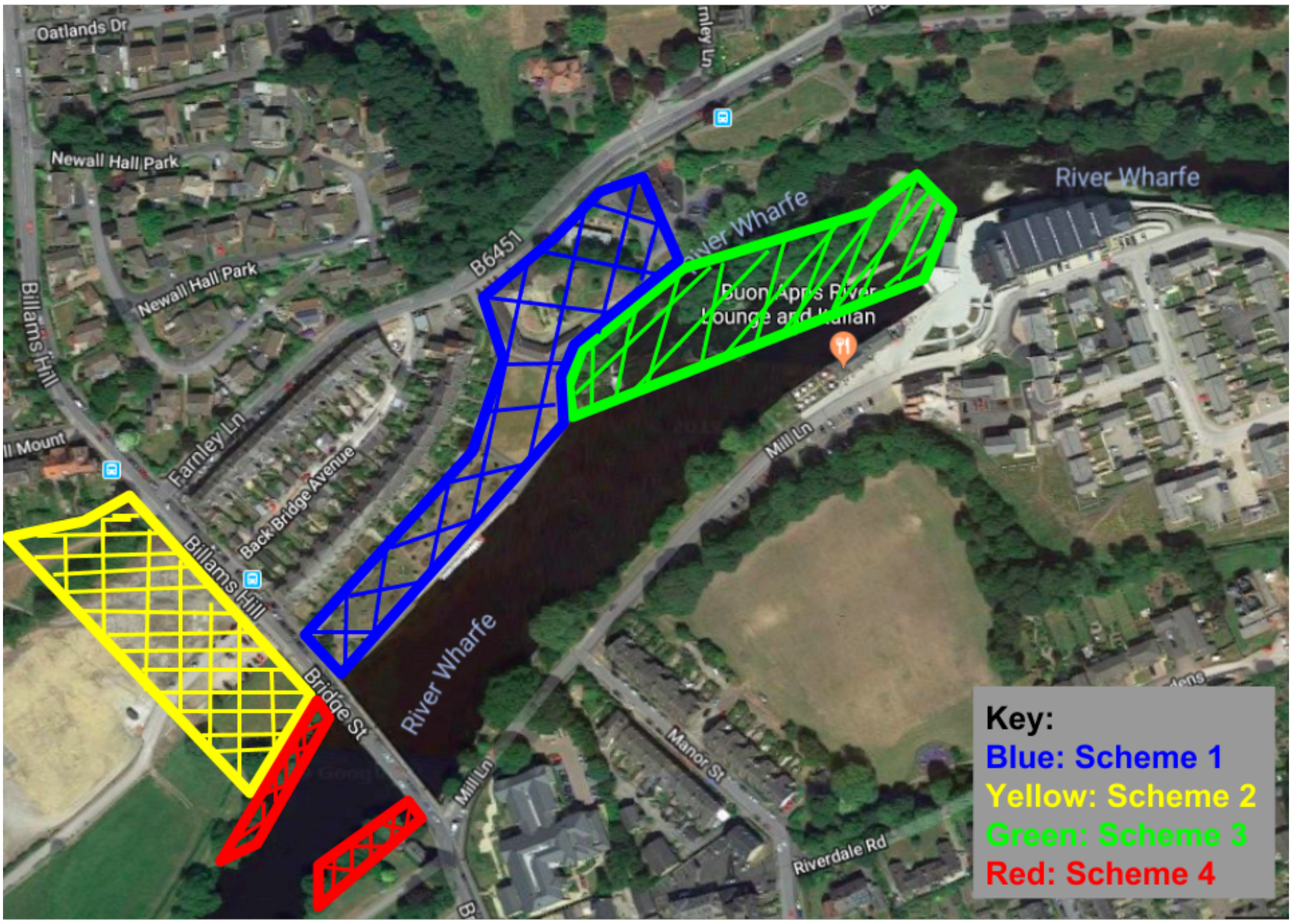

Figure 9: Areas for possible flood-mitigation schemes in Otley. Areas chosen to target have been adapted from plans from the Environmental Agency (Google Maps 2019).

7.8 Square Lake Graphs

As stated above, flood walls in different areas of Otley can in themselves mitigate a flood of the same magnitude as the Boxing Day floods, however these solutions have their drawbacks as stand alone mitigation. Firstly, they are expensive, and to mitigate the entire FEV would be substantially over Leeds Council's budget for the project. The flood wall schemes also have a large footprint, potentially disrupting the aesthetics and environment, not in keeping of the historic town's atmosphere. Finally, for a project this large, there would be large scale traffic and recreational disruption whilst the development of the project is taking place. Due to this, a collaboration of the schemes is a more viable solution economically and aesthetically. Below square lake graphs have been created to better demonstrate the information shown in §7.7, created whilst taking the median assumption of $V_e \approx 9.84 Mm^3/s$. Option 1: Schemes 1, 3 and 4 for mitigating floods with a 4% return period:

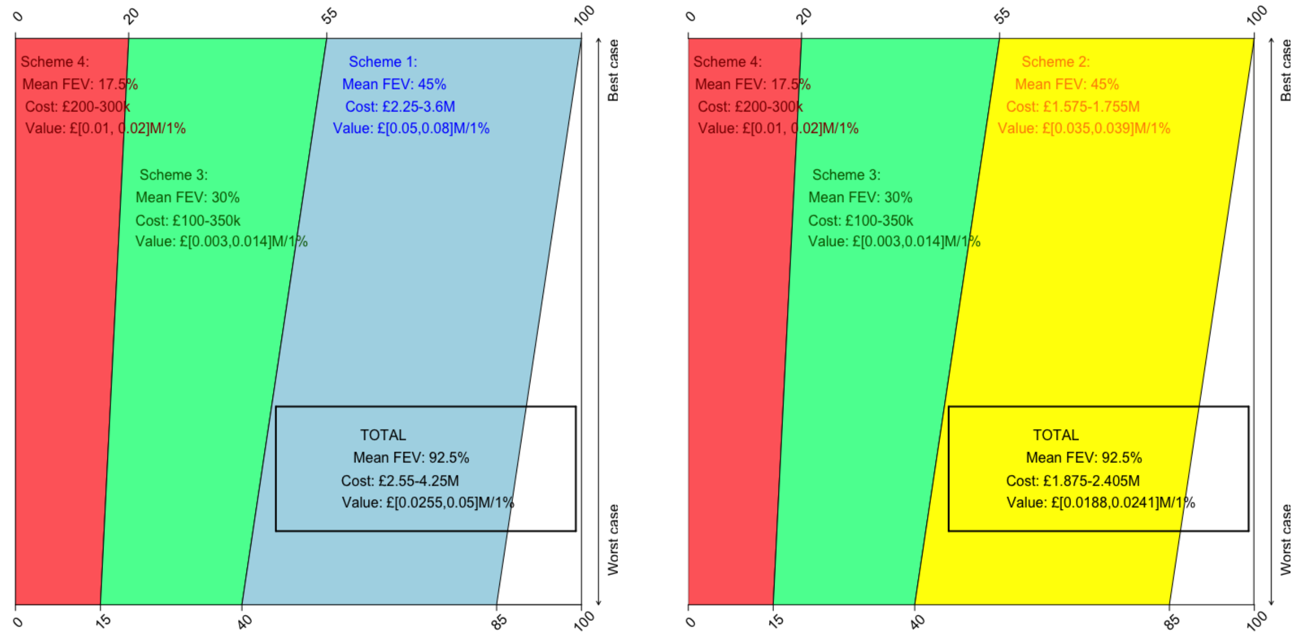

Figure 10: Two possible square lake graph A combin The x axis is the percent of total FEV mitigated, with the y axis depicting the worst to best cases.aLeft square lake (option 1): tion of schemes 1,options. 3 and 4 to mitigate all of the FEV of a flood with a 4% return period. Right square lake (option 2): A combination of schemes 2, 3 and 4 to mitigate all of the FEV of a flood with a 4% return period.

7.9 Comparison of Solutions

7.10 Future Considerations

8 Analysis of the Extra River

8.1 Background and Reasoning

8.2 Quadrant Plot

9 Conclusion

10 References