M.Sc-I Sem, 2018

Paper: IT-14, Mathematical Foundation for Computer Science

Time: 3hrs.

Max.Marks:75

Attempt **five** *questions in all. Q.1 is compulsary.*

Q.1(a) Find the sets A and B, if (2)

$$A - B = \{1, 3, 7, 11\}, \quad B - A = \{2, 6, 8\}$$

$$\text{and} \quad A \cap B = \{4, 9\}$$

(b) If a set S has n elements then show that the power set has 2^n elements. (2)

(c) if R is an equivalence relation on the set $A = \{1, 2, 3, 4, 5, 6\}$ given below , find the partition of A induced by R: (2)

$$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4), (4,5), (5,4), (5,5), (6,6)\}$$

(d) Draw the directed graph representing the relation on $\{1, 2, 3, 4\}$ whose matrix representation is (2)

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

(e) The truth table for the half adder is given as follows: (5)

Table 1

Inputs		Outputs	
x	y	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Draw the half circuit and express the functions (s, c) in terms the inputs (x, y). Draw a simplified circuit.

(f) Draw a complete graph on 5 vertices. (2)

Q.2(a) Let $A = \{a, b, c, d\}$, and let R be the relation on A that has the matrix (3)

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

Construct the digraph of R , and list in-degrees and ou-degrees of all vertices.

(b) The relation R and S have the following matrix representations: (3)

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad M_S = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Find $S \circ R$?

(c) Let $A = \{a, b, c, d, e\}$ and let the equivalence relations R and S on A be given by (4)

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad M_S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Compute the partition of A corresponding to $R \cap S$.

(d)Let $A = \{a, b, c, d, e\}$and let R and S be the relation on A described by (5)

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad M_S = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Use Warshall's algorithm to compute the transitive closure of $R \cup S$.

Q.3(a)The state table of a finite state machine M is given in Table:2 (5)

(i) Find the input set I, the state set S, the output set O and the initial state of M.

(ii) Draw the diagram of M.

(iii) Find the output of the word $w = a^2bab^2a$.

Table 2

f, g	a	b
s_0	s_0, b	s_4, b
s_1	s_0, a	s_3, b
s_2	s_0, a	s_2, b
s_3	s_1, b	s_1, b
s_4	s_1, b	s_0, a

(b) Design an FSM that outputs 1 whenever it sees 101 as consecutive input bits and outputs 0 otherwise. (5)

(c) Draw the state diagram for the NFA for which the state table is given in Table:3 and accepting states are s_1 and s_3. Find (i) the language accepted by this NFA, and (ii) the equivalent DFA. (5)

Table 3

PS	δ	
	a	b
s_0	s_2	s_1
s_1	s_1, s_2	s_3
s_2	ϕ	ϕ
s_3	s_2, s_3	s_2

Q.4(a)Show that the string $w = baab$ is accepted by the PDA which accepts the language $L = \{ww^R : w\epsilon\{a,b\}^+\}$ in general following the rules given below. (6)

$$\delta(s_0, a, z_0) = (s_0, az_0), \quad \delta(s_0, b, z_0) = (s_0, bz_0 \quad \delta(s_0, a, a) = (s_0, aa)$$

$$\delta(s_0, a, b) = (s_0, ab), \quad \delta(s_0, b, a) = (s_0, ba), \quad \delta(s_0, b, b) = (s_0, bb)$$

$$\delta(s_0, \lambda, a) = (s_1, a), \quad \delta(s_0, \lambda, b) = (s_1, b)$$

$$\delta(s_1, a, a) = (s_1, \lambda), \quad \delta(s_1, b, b) = (s_1, \lambda)$$

$$\delta(s_1, \lambda, z_0) = (s_2, z_0)$$

(b)Examine whether the following grammar G is ambiguous or not: (4)

$$G = \{N, T, S, P\}, \quad \text{where} N = \{S, A\}, \quad T = \{a, b\}$$

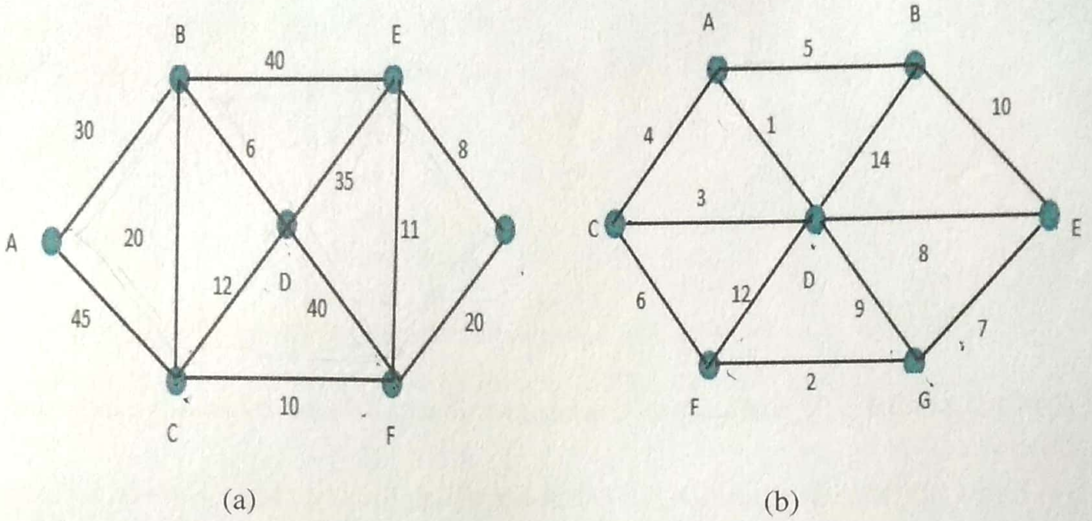

Figure 1: Graphs (a) G_1, (b) G_2

and P consists of the rules $S \to aAb, \quad S \to abSb, \quad S \to a, \quad A \to bS, \quad A \to aAAB.$

(c) Construct derivation trees for the words (i) $ababbbba$, (ii) $bbbcbba$ using the grammars G_1 and G_2 respectively, where G_1 consists of the productions $\{S \to AbS, \quad A \to aS, \quad S \to ba, \quad \text{and} \quad A \to b\}$ and G_2 consists of the production $\{S \to bcS, \quad S \to bbS, \quad S \to cb, \quad S \to a\}$. (5)

Q.5(a) Define (i) simple graph, (ii) Regular graph, and (iii) Bipartite graph. (3)

(b) Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are such that, (6)

$$V_1 = \{A_1, A_2, A_3, A_4\}, \quad V_2 = \{B_1, B_2, B_3, B_4\}$$

$$E_1 = \{A_1A_2, A_1A_3, A_1A_4, A_2A_4, A_3A_4\}, \quad E_2 = \{B_1B_2, B_1B_3, B_1B_4, B_2B_3, B_3B_4\}$$

Find the adjacency matrix of the two graphs and also find a permutation matrix P such that the two graphs are isomorphic i.e., $PAP^T = B$, where A is the adjacency matrix of the graph G_1 while B is the adjacency matrix of graph G_2.

(c) Find the Eulerian circuit or a path in graph G_1, if any. Give reason for your answer. (2)

(d)Find the number of paths of length 4 between the vertices A_2 and A_3 in graph G_1. (4)

Q.6(a) Describe the Dijkstra's method of finding the shortest path in a network. Find the shortest path between the vertex A and G in the graph G_1 (Fig.1(a)). (8)

(b) Use Prim's algorithm to find the minimum spanning tree in graph G_2 (Fig.1(b)). (4)

(c) Represent the postfix expression $ab + cd * ef / - -a*$ as a binary tree and write also the corresponding infix and prefix forms. (3)