Code:1943 Roll No.: 4158

M.Sc(Informatics),IV-Sem., 2019

Paper IT-43: Modeling Simulation and Performance Evaluation

Time:3hrs Max.Marks:75

Write your Roll.No. on the top right corner immediately on receipt of this question paper

Attempt five questions in all

Question No. 1 is compulsory

Q.1(a) In a coin tossing experiment, if the coin shows head, 1 dice is thrown and the result is recorded. But if the coin shows tail, 2 dice are thrown and their sum is recorded. What is the probability that the recorded number will be 2? (3)

(b)Each of two persons A and B tosses 3 fair coins. What is the probability that they obtain the same number of heads? (3)

(c) A random variable X may assume 4 values with probabilities $(1+3x)/4$, $(1-x)/4$, $(1+2x)/4$ and $(1-4x)/4)$. Find the condition on x so that these values represent the probability function of X? (3)

(d) If

$$p(x) = \begin{cases} xe^{-x^2/2} & ,x \geq 0 \\ 0 & ,x < 0 \end{cases}$$

(*i*) Show that $p(x)$ is a *pdf* (of a continuous random variable X.) (*ii*) Find its distribution function $P(x)$. (3)

(e) If the random variable X follows $N(0,2)$ and $Y = 3X^2$, find the mean and variance of Y. (3)

Q.2(a) Let λ_n be the average arrival rate when there ar n customers in the system (both waiting in the queue and being served) and let μ_n be the average service rate when there are n customers in the system. Show that in case of statistical equilibrium (6)

$$\begin{aligned} \lambda_{n-1}P_{n-1} - (\lambda_n + \mu_n)P_n + \mu_{n+1}P_{n+1} &= 0 \\ \text{and} - \lambda_0 P_0 + \mu_1 P_1 &= 0 \end{aligned}$$

Find the value of P_n and P_0.

(b) In case $\lambda_n = \lambda$ and $\mu_n = \mu$, find the probability that number of customers in the queue exceed k. (4)

(c) At what average rate must a clerk in a supermarket work in order to ensure a probability of 0.90 that the customer will not wait longer than $12min$? It is assumed that there is only one counter at which customers arrive in a Poisson fashion at an average rate of 15 per hour and that the length of the service by the clerk has an exponential distribution. (5)

Q.3(a) For an M/M/c queue, when $\lambda_n = \lambda$ and $\mu_n = \mu$ for all n, find the mean waiting time in the queue for those who actually wait. Also find the average number of customers (in non empty queue), who have to actually wait. (8)

(b) A telephone company is planning to install telephone booths in a new airport. It has established the policy that a person should not have to wait more than 10% of the time he tries to use a phone. The demand for use is estimated to be Poisson with an average of 30 per hour. The average phone call has an exponential distribution with mean time of 5 min. How many phone booths should be installed? (C) (7)

Q.4(a) Fit a binomial distribution for the following data: (6)

x:	0	1	2	3	4	5	6	Total
f:	5	18	28	12	7	6	4	80

(b) Fit a Poisson distribution for the following distribution and hence find the expected frequencies. (6)

x:	0	1	2	3	4	5	6
f:	314	335	204	86	29	9	3

(c) Explain the memoryless property of the exponential distribution. (3)

Q.5(a) The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. What is the probability that the repair time exceeds $2h$? (5)

(b) A program consists of two modules. The number of errors X_1 in the first module has the pmf $P_1(x)$, and the number of errors X_2 in the second module has the pmf $P_2(x)$, independently of X_1, where

x	$P_1(x)$	$P_2(x)$
0	0.5	0.7
1	0.3	0.2
2	0.1	0.1
3	0.1	0

Find the pmf and cdf of $Y = X_1 + X_2$, the total number of errors. (5)

(c) A survey of 320 families with 5 children revealed the following distribution:

No. of boys:	0	1	2	3	4	5
No. of girls:	5	4	3	2	1	0
No. of families:	12	40	88	110	56	14

Is this result consistent with the hypothesis that male and female births are equally probable? Given that $\chi^2_{0.05,\nu=5} = 11.07$. (5)

Q.6(a) Describe the steps involved for the Northwest-corner method of obtaining the starting basic solution of the transport problem. (3)

(b) SunRay Transport Company ships truckloads of grain from three silos to four mills. The supply (in truckloads) and the demand (also in truckloads) together with unit transportation

Table:1

Mills→ / Silo↓	1	2	3	4	Supply
1	10 x_{11}	2 x_{12}	20 x_{13}	11 x_{14}	15
2	12 x_{21}	7 x_{22}	9 x_{23}	20 x_{24}	25
3	4 x_{31}	14 x_{32}	16 x_{33}	18 x_{34}	10
Demand	5	15	15	15	

costs per truckload on the different routes are summarized in the following transportation model in Table:1, where symbols have their usual meanings.

(i) Obtain the starting basic solution for the SunRay Transport Company using the Northwest-corner method. (4)

(ii) Obtain the starting basic solution for the SunRay Transport Company using the Vogel approximation method. (5)

(iii) Describe the steps involved for iterative computations of the transport algorithm. (3)

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