Random Variable

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Unit - 4

Probability and Distribution: Introduction, Addition and multiplication law of probability, Conditional probability, Baye's theorem, Random variables (Discrete and Continuous Random variable) Probability mass function and Probability density function, Expectation and variance, Discrete and Continuous Probability distribution: Binomial, Poission and Normal distributions.

Random Variable

- ① Discrete Random Variable
  - Probability Mass fuction
    - Binomial Distribution
    - Poisson distribution
- ② Continuous Random Variable
  - Probability Density fuction
    - Normal distribution

Random Variable A real valued fuction x, difined on a sample space S of any random experiment, is called a Random variable which assigned to each point of sample space $s \in S$ one only one real number such that

$$X(s) = x$$

where x is any real number

Domain of the random variable x is sample space S and Range is non-empty set of real number.

Random experiment :- Three coins are tossed (Sample space S)

Sample Point	X(S)	X = No. of heads	Probability
HHH	X(HHH)	3	$\frac{1}{8}$
HTT, THT, TTH		1	$\frac{3}{8}$
HHT, HTH, THH		2	$\frac{3}{8}$
TTT		0	$\frac{1}{8}$

Type of Random variable

Random Variable

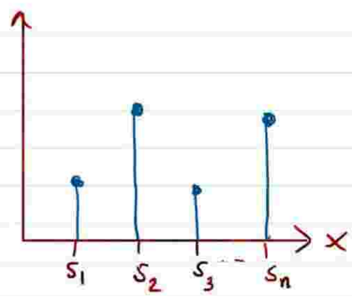
① Discrete Random Variable

② Continuous Random Variable

Discrete Random variable :-

A discrete Random variable which can assume only a discrete set of values or isolated values

For Example. The number of head in 3 tosses of a coin is a discrete Random variable as it assume discrete values 0, 1, 2, 3.



Probability Mass function (PMF)

let $x_1, x_2, \ldots x_n$ be the values of discrete random variable X and let $P_1, P_2, \ldots P_n$ (when $P_i \geq 0$) be a corresponding probability then function $f(x)$ or $P(x)$ is defined as:

$$\boxed{P(X=x) = P(x) = \begin{cases} P(x_i) \text{ or } P_i, & i = 1, 2, 3 \ldots \\ 0 & \text{otherwise} \end{cases}}$$

is called Probability mass function of the discrete random variable X.

Note :- (i) $P(x_i) \geq 0$ (ii) $\sum P(x_i) = 1$.

Probability Distribution of a discrete random variable

Suppose the random variable X such that

X	:	x_1	x_2	x_3	x_n
$P(X=x) = P(x)$	:	P_1	P_2	P_3	P_n

The set of ordered pair $[x_i, P(x_i)]$ is

$P(X=x)=P(x)$:	P_1	P_2	P_3	P_n

pair $[x_i, P(x_i)]$ is called Probability distribution

For Example suppose a coin is tossed three times then the distribution of the number of head is

$X=x$:	0	1	2	3
$P(x)$:	$1/8$	$3/8$	$3/8$	$1/8$

Mean and variance of random variable :-

let X : x_1 x_2 x_3 — ...... x_n
$P(x)$: P_1 P_2 P_3 - ...... P_n

be a Probability distribution function

$$\text{Mean } (\mu) = \frac{\Sigma P_i x_i}{\Sigma P_i} = \Sigma P_i x_i \qquad \{\text{Since } \Sigma P_i = 1$$

$$\text{Variance } \sigma^2 = \Sigma P_i (x_i - \bar{x})^2$$

$$\sigma^2 = \Sigma P_i x_i^2 - \bar{x}$$

$$\text{Standard deviation} = (\sigma) = +\sqrt{\text{Variance}}.$$

Example:- Five defective bulbs are accidentally mixed twenty good ones. It is not possible to just look at a bulb whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

Q.2. A Random variable x has the following probability fun^n value of x,

x:	0	1	2	3	4	5	6	7
$P(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

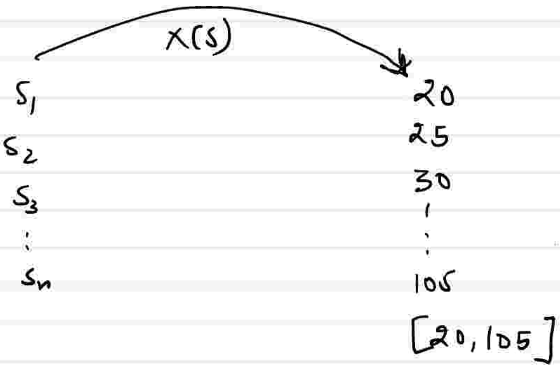
(i) Find k,

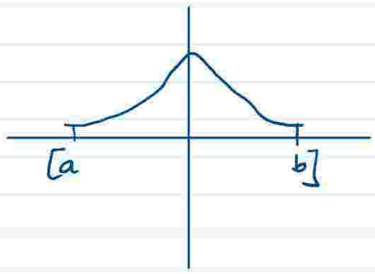
(ii) Evaluate $P(x<6)$, $P(x \geq 6)$, $P(3<x<6)$

(iii) Find the minimum value of x so that $P(X \le x) > \frac{1}{2}$.

Continuous Random Variable: A continuous Random variables is one which can assume any value within an interval

① Age of persons of a group
② Hight of boys in a class
③ Weight of student in a class





Note:- The interval may be finite or infinite

Probability Density function :- let x be Continuous random variable the PDF is defined as:

$$P(a \le x \le b) = f(x) \text{ or } f(x) = \begin{cases} 0 & x < a \\ \phi(x) & a \le x \le b \\ 0 & x > b \end{cases}$$

Such that

(i) $f(x) \geq 0$, (ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

Note :- $\boxed{P(a \le x \le b) = \int_a^b f(x)\, dx}$

Q.1. If the function $f(x)$ is defined by $f(x) = ce^{-x}$ $0 \le x \le \infty$, find the value of c which changes $f(x)$ to a probability density function.

Q.2. If $f(x)$ has probability density cx^2, $0 \le x \le 1$ determine c and find the probability that ...

c and find the probability that $\frac{1}{3} < x < \frac{1}{2}$ i.e. $P\left(\frac{1}{3} < x < \frac{1}{2}\right)$

Mathematical Expectation for Descrete Random Variable :-

#① Relation b/w expectation of a random veriable x with moments.

If x is descrete random veriable with probability distribution

$$\begin{array}{lllll} x: & x_1 & x_2 & \cdots & x_n \\ p(x): & p_1 & p_2 & \cdots & p_n \end{array} \quad \text{where } \sum_{i=1}^{n} p_i = 1.$$

Then the expectation of x is denoted by $E(x)$ and defined as:-

$$E(x) = x_1p_1 + x_2p_2 + x_3p_3 + \cdots + x_np_n$$

$$E(x) = \sum_{i=1}^{n} x_ip_i = \sum xp = \bar{x} = \mu_1 = v_1$$

where v_1 is 1^{st} moment about origin

It is also known as "Mean value or Avg value"

or

$$E(x) = \sum x \cdot p(x)$$

Remarks: ①

$$E(x^2) = \sum x^2 p(x)$$

$$E(x^3) = \sum x^3 p(x)$$

$$\vdots$$

$$E(x^r) = \sum x^r p(x) = \sum_{i=1}^{n} x_i^r p_i = v_r$$

r^{th} moment of discrete probability distribution about origin (i.e. $x \to 0$)

$$v_r = \frac{\sum p_i x_i^r}{\sum p_i} = \sum p_i x_i^r = E(x^r) \quad \left\{ \because \sum p_i = 1 \right.$$

$$v_r = \frac{\sum f x^r}{\sum f}$$

याद ?

#

Some imp formulas:- ① $E(c) = c$ (where c is any Constant)

② $E(ax) = aE(x)$

③ $E(x \pm y) = E(x) \pm E(y)$

④ $E(xy) = E(x) \cdot E(y)$

r^{th} moment of discrete probability distribution about mean

$$\mu_r = \frac{\sum p_i (x_i - \bar{x})^r}{\sum p_i} = \sum p_i (x_i - \bar{x})^r \quad \text{where } \sum p_i = 1$$

$$\boxed{\mu_r = E\{(x - \bar{x})^r\}}$$

$$\text{Variance} = \mu_2 = E(x-\bar{x})^2 = E(x^2) - 2\bar{x}E(x) + \bar{x}^2 E(1)$$

$$= E(x^2) - 2\bar{x}^2 + \bar{x}^2$$

$$\mu_2 = E(x^2) - \bar{x}^2 \quad \text{or} \quad \boxed{\mu_2 = E(x^2) - (E(x))^2}$$

and $S.D = \sqrt{\text{Variance}}$

Q.1. What is the expected value of the number of points that will be obtained a single throw with an ordinary die? Find variance also.

Q.2. Find $E(x)$, $E(x^2)$, $E\{(x-\bar{x})^2\}$ for the following probability distribution:

x:	8	12	16	20	24
$p(x)$:	1/8	1/4	3/8	1/4	1/16

Q.3. In four tosses of a coin, let x be the number of head. Calculate $E(x)$.

Expectation for Continuous Random Variable :-

If x is a continuous random variable, then the expectation of x is:

$$E(x) = \int_{-\infty}^{\infty} x\, p(x)\, dx \qquad \text{and} \qquad \int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2\, p(x)\, dx$$

$$\vdots$$

$$E(x^r) = \int_{-\infty}^{\infty} x^r\, p(x)\, dx$$

Note ① $E(x) = \bar{x} =$ mean value / Avg value $= \mu$

② $E(x^r) = \nu_r = r^{th}$ moment about origin

③ $E(x-\bar{x})^r = \mu_r =$ moment about mean

④ $\mu_1 = 0$,

⑤ $\mu_2 = E(x^2) - (E(x))^2 =$ Variance

⑥ $S.D = +\sqrt{\text{variance}} = \sqrt{\mu_2}$

Q. 1. A continuous random variable x has

$$f(x) = \begin{cases} \frac{1}{2}(x+1) & \text{for } -1<x<1 \\ 0 & \text{elsewhere} \end{cases}$$

represent the density, find the mean and variance, S.D.

Binomial Probability Distribution

(Binomial Distribution)

Let there be n independent trail in an experiment. Let a random variable x denote the number of successes in these n trails. Let p be the probability of a success and q be the probability of failure in a single trail so that $p+q=1$

"Let the trail be endependent and p be constant for every trail."

r succers can be obtained in n trails with ${}^nC_r$ ways:

Now the probability of r successes in n trails is

$$p(r) = p(x=r) = {}^nC_r\, P(\underbrace{S\cdot S\cdot S\cdot S \cdots S}_{r^{th}\text{ times}}\cdot \underbrace{F\cdot F\cdot F \cdots F}_{(n-r)^{th}\text{ times}})$$

$$= {}^nC_r\, P(S)\cdot P(S)\cdot P(S)\cdots P(S)\cdot P(F)\cdot P(F)\cdots P(F)$$

$$= {}^nC_r\, \underbrace{p\cdot p\cdot p \cdots p}_{r^{th}}\cdot \underbrace{q\cdot q\cdot q \cdots q}_{(n-r)^{th}\text{ times}} \qquad \left\{ \begin{array}{l} P(S)=p \\ P(F)=q \end{array} \right.$$

$$= {}^nC_r\, p^r q^{n-r}$$

Thus $\boxed{P(r) = P(x=r) = {}^nC_r\, p^r q^{n-r}}$, $r = 0,1,2,3 \dots n$

It is called Binomial Distribution

$\left\{ (a+x)^n = \sum_{r=1}^{n} {}^nC_r\, a^r x^{n-r} \right.$

Remarks:- ① If n independent trails consistute one expriment and this experiment is repeated N times the frequency of r successes is $N\cdot {}^nC_r\, p^r q^{n-r}$ or $N(p+q)^n$ is called B.D.

imp Mean, Variance and Standard Deviation of Binomial distribution:-

The Binomial distribution is $P(r) = {}^nC_r\, q^{n-r} p^r$ $r = 0,1,2,\dots$

①

$$\text{Mean } (\mu_1) = E(r) = \sum_{r=0}^{n} r\, p(r)$$

$$= \sum_{r=0}^{n} r\, {}^nC_r\, p^r q^{n-r}$$

$$= \sum_{r=0}^{n} r \frac{n!}{r!(n-r)!} p^r q^{n-r}$$

$$= \sum_{r=0}^{n} \cancel{r} \frac{n(n-1)!}{\cancel{r}(r-1)!(n-r)!} p^r q^{n-r}$$

$$= np \sum_{r=1}^{n} \frac{(n-1)!}{(r-1)!(n-r)!} p^{r-1} q^{n-r}$$

$$= np\,(p+q)^{n-1}$$

$$\text{Mean}(\mu_1) = np \quad \text{ans}$$

Hence, mean of the binomial distribution is np.

$$\text{Variance } \sigma^2 = E(r^2) - (E(r))^2 = \sum_{r=0}^{n} r^2 p(r) - (np)^2 \quad \text{— ①}$$

$$= \sum_{r=0}^{n} (r + r^2 - r)\, p(r) - (np)^2$$

$$= \sum_{r=0}^{n} r p(r) + \sum_{r=0}^{n} r(r-1)\, p(r) - (np)^2$$

$$= (np) + \sum_{r=0}^{n} r(r-1)\, p(r) - (np)^2 \quad \text{— ②}$$

Now, $\sum_{r=0}^{n} r(r-1)\, p(r) = \sum_{r=0}^{n} r(r-1)\, {}^nC_r\, p^r q^{n-r}$

$$= \sum_{r=0}^{n} \cancel{r(r-1)} \frac{n(n-1)(n-2)!}{\cancel{r(r-1)}(r-2)!(n-r)!} \times p^r q^{n-r}$$

$$= n(n-1)p^2 \sum_{r=2}^{n} \frac{(n-2)!}{(r-2)!(n-r)!} p^{r-2} q^{n-r}$$

$$= n(n-1)p^2\,(p+q)^{n-2}$$

$$= n(n-1)p^2$$

from ②

$$\text{Variance } (\sigma^2) = np + n(n-1)p^2 - n^2p^2$$

$$= np - np^2 = np(1-p)$$

$$\text{Variance}(\sigma^2) = npq \quad \text{and} \quad \text{S.D. } (\sigma) = \sqrt{npq}$$

imp

Moment Generating function of Binomial Distribution :-

① About Origin :-

$$M_r(t) = E(e^{tr}) = \sum_{r=1}^{n} e^{tr} p(r)$$

$$= \sum_{r=0}^{n} e^{tr} \, {}^nC_r \, p^r q^{n-r}$$

$$= \sum_{r=0}^{n} {}^nC_r (pe^t)^r q^{n-r}$$

$$M_r(t) = (q + pe^t)^n$$

② About Mean :-

$$M_{r-np}(t) = E\left\{e^{t(r-np)}\right\}$$

$$= E\left\{e^{tr} \cdot e^{-npt}\right\}$$

$$= e^{-npt} E(e^{tr})$$

$$\boxed{M_{r-np}(t) = e^{-npt} M_r(t)}$$

Recurrence or Recursion formula for the Binomial Distribution

$$P(r+1) = \frac{n-r}{r+1} \cdot \frac{p}{q} P(r)$$

Moment about mean af binomial distribution

$$\boxed{\mu_2 = npq, \quad \mu_3 = npq(q-p), \quad \mu_4 =}$$

Note:-

$$\text{Skewness} = \gamma_1 = \sqrt{\beta_1} = \frac{\mu_3^2}{\mu_2^3} = \frac{1-2p}{\sqrt{npq}}$$

$p < \frac{1}{2}$ skewness is positive

$p > \frac{1}{2}$ skewness is Negative

$p = \frac{1}{2}$ it is zero (Symmetrical)

Kurtosis:-

$$\gamma_2 = 3 + \beta_2 = 3 + \frac{\mu_4}{\mu_2^2}$$

$$= 3 + 3 + \frac{1-6pq}{npq}$$

$$\gamma_2 = 6 + \frac{1-6pq}{npq}$$

which is called Measure of kurtosis.

Q.1. (I) Comment on the following statement:- For a Binomial distribution, mean is 6 and variance is 9.

(ii) A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the number of success.

Q.2 A binomial variable X satisfies the relation $9P(X=4) = P(X=2)$ when $n=6$. Find the value of the parameter p and $P(X=1)$.

Q.3. If the probability of hitting a target is 10% and 10 shots are fired independently. What is the probability that the target will be hit at least once.

the target will be hit at least once.

Q.4. Out of 800 families with 4 children each, how many families would be expected to have

(i) 2 boys and girls (ii) at least one boy

(ii) no girl (iv) at most two girls

Assume equal probability for boys and girls.

Q.5. If 10% of the bolts produced by a machine are defective, determine the probability that out of 10 bolts choosen at random.

(i) 1 (ii) None (iii) at most 2 bolts will be defective.

Q.6. Six dice are throw, 729 times. How many times do you expect at least three dice to show a four or six.

Poisson Distribution

Poisson Distribution as a limiting case of Binomial Distribution

- If the parameters n and p of a binomial distribution are known, we can find distribution.
- But in situation where n is very large and p is very small application of binomial distribution is very labourious.
- However, if we assume that as $n \to \infty$ and $p \to 0$ such that np always remain finite (say λ), we get the poisson approximation to the Binomial Distribution, thus $\lambda = np$

Now for the Binomial distribution

$$P(X=r) = {}^nC_r \, p^r q^{n-r}$$

$$= \frac{n!}{r!\,(n-r)!} \cdot p^r (1-p)^{n-r} \qquad (\because p+q=1)$$

$$= \frac{n(n-1)(n-2)\cdots(n-(r-1))\cancel{(n-r)!}}{r!\,\cancel{(n-r)!}} \left(\frac{\lambda}{n}\right)^r \left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{\lambda^r}{r!} \frac{n(n-1)(n-2)\cdots(n-r+1)}{n^r} \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$= \frac{\lambda^r}{r!}\left(\frac{n}{n}\right)\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{r-1}{n}\right) \times \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r} \quad \text{--- (1)}$$

As $n \to \infty$ then,

$$\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{n-1}{n}\right) \to 1 \text{ and } \left(1-\frac{\lambda}{n}\right)^r \to 1$$

and $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^{-x}$ thus, $\lim_{n\to\infty}\left[\left(1-\frac{\lambda}{n}\right)^{\frac{-n}{\lambda}}\right]^{\lambda} = e^{-\lambda}$

from equation ①.

$$P(X=r) = \frac{e^{-\lambda}\cdot\lambda^r}{r!} \quad (r=0,1,2,\dots)$$

where λ is finite number $= np$
which is called a probability distribution which is called the Poisson probability distribution.

Note:- ① λ is called the parameter of distribution
② The sum of probabilities $P(r)$ for $r=0,1,2,3,\dots$ is 1.

$$\Sigma P(r) = 1$$

Mean and variance of the poisson distribution :-

The poisson distribution $P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$

$$\text{Mean} = E(r) = \sum_{r=0}^{\infty} r\,p(r) = \sum_{r=0}^{\infty} r\frac{e^{-\lambda}\lambda^r}{r!}$$

$$= \sum_{r=0}^{\infty} \cancel{r}\frac{e^{-\lambda}\lambda^r}{\cancel{r}(r-1)!}$$

$$= \lambda e^{-\lambda}\sum_{r=1}^{\infty}\frac{\lambda^{r-1}}{(r-1)!}$$

$$= \lambda e^{-\lambda}\cdot e^{\lambda} = \lambda = np$$

Hence mean of the poisson distribution is $\lambda = np$

$$\text{Variance } \sigma^2 = E(r^2) - \{E(r)\}^2$$

$$= \sum_{r=0}^{n} r^2 P(r) - \lambda^2$$

$$= \sum_{r=0}^{n}(r+r^2-r)P(r) - \lambda^2$$

$$= \sum_{r=0}^{n} r\,p(r) + \sum_{r=0}^{n} r(r-1)P(r) - \lambda^2$$

$$= \lambda + \sum_{r=0}^{n} r(r-1)P(r) - \lambda^2 \quad \text{--- (12)}$$

Now,

$$\sum_{r=0}^{n} r(r-1)P(r) = \sum_{r=0}^{n} r(r-1)\cdot\frac{e^{-\lambda}\lambda^r}{r!} = \sum_{r=0}^{n} \cancel{r(r-1)}\frac{e^{-\lambda}\lambda^r}{\cancel{r(r-1)}(r-2)!}$$

$$= \sum_{r=2}^{n} \frac{e^{-\lambda}\lambda^r}{(r-2)!} = e^{-\lambda}\cdot\lambda^2 \sum_{r=2}^{n}\frac{\lambda^{r-2}}{(r-2)!}$$

$$= e^{-\lambda}\cdot\lambda^2 e^{\lambda} = \lambda^2$$

from equation (12)

$$\text{Variance } \sigma^2 = \lambda + \cancel{\lambda^2} - \cancel{\lambda^2} = \lambda$$

Hence Mean $\lambda = np =$ Variance σ^2

Q.1. Using Poisson distribution, find the probability that the ace of spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trials.

Q.2. Fit a poisson distribution to the following data and calculate theoretical frequencies.

Deaths :	0	1	2	3	4
Frequency :	122	60	15	2	1

Q.3.(i) If the probability of a bad reaction from a certain injection is 0.0002, determine the chance that out of 1000 individual more than two will get a bad reactions.

(ii) The probability that a man aged 50 years will within a

(ii) The probability that a man aged 50 years will [illegible] a years is 0.01125. what probability that of 12 such men, at least 11 will reach their 51 st birthday. ($e^{-0.135} = 0.87366$)

Q.4. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective The blads are supplied in packets of 10. Calculate the approximate number of packets containing no defective; one defective and two defective blads in a consignment of 10000 packets

(Given: $e^{-0.02} = 0.9802$)

Q: (i) Six coins are tossed 6400 times. Using the poisson distribution, determine the approximate probability of getting six heads x times

(ii) A poisson distribution has a double mode at $x=3$ and $x=4$. what is the probability that x will have one or the other of these two values.

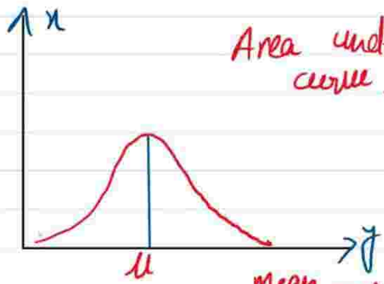
Normal distribution

Definition A continuous random variable x is said to have a normal distribution with parameter $\bar{x} = \mu$ (called mean) and σ^2 (called variance) if probability denisty function is given by

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

(i) $f(x) \geq 0 \quad \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$



Area under normal curve 1.

Mean = mode = median

i.e ① the total area under the normal curve above the x-axis is 1.

② The normal distribution is symmetrical about the mean

③ The mean, mode & median of the distribution coincide.

Mean of the normal distribution: (2015, 2018).

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\text{Mean} = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} x \times \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \quad \text{— ①}$$

Let $\frac{x-\mu}{\sigma} = z$, $x = \mu + \sigma z$

$dx = \sigma\,dz$ when $x = -\infty$, $z = -\infty$
$x = \infty$, $z = \infty$

from ①.

$$\text{Mean} = \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z)\, e^{-\frac{1}{2}z^2}\, \cancel{\sigma}\,dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int^{\infty} \mu e^{-\frac{1}{2}z^2} dz + \sigma \int^{\infty} z\, e^{-\frac{1}{2}z^2} dz \right]$$

$$= \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{\infty} \mu e^{-\frac{1}{2}z^2} dz + \sigma \int_{-\infty}^{\infty} \underbrace{z e^{-\frac{1}{2}z^2} dz}_{\text{odd fution}}\right]$$

$$= \frac{\mu}{\sqrt{2\pi}} \times 2 \int_{0}^{\infty} e^{-\frac{1}{2}z^2} dz$$

Let $\frac{z^2}{2} = t$, then $z^2 = 2t \Rightarrow z = \sqrt{2}\sqrt{t}$

$dz = \frac{\sqrt{2}}{2} t^{\frac{1}{2}-1} dt$

when $z=0$, $t=0$

$z=\infty$, $t=\infty$

$$\text{Mean} = \frac{\mu\cancel{\sqrt{2}}}{\sqrt{\pi}} \int_0^{\infty} e^{-t} \frac{1}{\cancel{\sqrt{2}}} t^{\frac{1}{2}-1} dt$$

$$= \frac{\mu}{\sqrt{\pi}} \int_0^{\infty} e^{-t} t^{\frac{1}{2}-1} dt \qquad \left\{ \text{By } \Gamma n = \int_0^{\infty} e^{-t} t^{n-1} dt \right.$$

$$= \frac{\mu}{\sqrt{\pi}} \times \Gamma\tfrac{1}{2}$$

$$= \frac{\mu}{\cancel{\sqrt{\pi}}} \times \cancel{\sqrt{\pi}}$$

$$\boxed{\text{Mean } (\bar{x}) = \mu}$$

Moments about mean :- $\quad \mu_r = E(x-\bar{x})^r$

(i) first moment about mean $\mu_1 = 0$ (always)

(ii) $\mu_2 = E(x-\bar{x})^2 = 2^{nd}$ moment about

$$\text{Variance} = E(x-\mu)^2$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$= \frac{1}{} \int^{\infty} (\sigma z)^2 e^{-\frac{1}{2}z^2} \sigma dz \qquad \left\{ \begin{array}{l} \frac{x-\mu}{\sigma} = z \\ x = \mu + \sigma z \end{array} \right.$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z)^2 \, e^{-\frac{1}{2}z^2} \, \sigma \, dz \qquad \left\} \begin{array}{l} x = \mu + \sigma z \\ dx = \sigma dz \end{array} \right.$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 \, e^{-\frac{1}{2}z^2} \, dz$$

$$= \frac{\sigma^2 \, 2}{\sqrt{2\pi}} \int_{0}^{\infty} z^2 \, e^{-\frac{1}{2}z^2} \, dz$$

$$= \frac{\sigma^2 \sqrt{2}}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t} \cdot 2t \, \frac{1}{\sqrt{2}} \, t^{\frac{1}{2}-1} \, dt \qquad \left\{ \begin{array}{l} \frac{1}{2}z^2 = t \\ dz = \frac{1}{\sqrt{2}} t^{\frac{1}{2}-1} dt \end{array} \right.$$

$$= \frac{\sigma^2}{\sqrt{\pi}} \cdot 2 \int_{0}^{\infty} e^{-t} \, t^{\frac{3}{2}-1} \, dt$$

$$= \frac{\sigma^2}{\sqrt{\pi}} \, 2 \times \Gamma\left(\frac{3}{2}\right)$$

$$= \frac{\sigma^2}{\cancel{\sqrt{\pi}}} \times \cancel{2} \, \frac{1}{\cancel{2}} \, \cancel{\sqrt{\pi}}$$

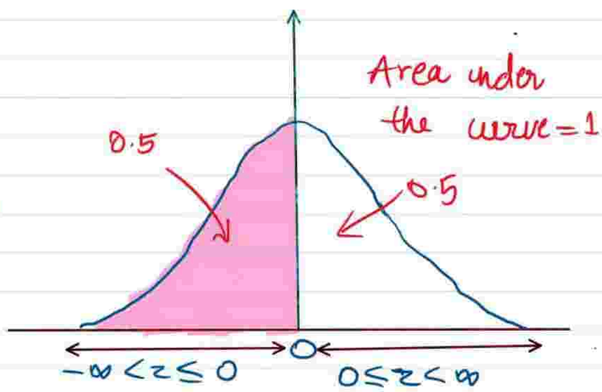
$$= \sigma^2$$

The standard deviation of the normal distribution is σ.

Standard form of the normal distribution:- If x is a normal random with mean μ and S.D. σ, then the random variable

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

has the normal distribution with mean 0 and S.D. 1.



The random variable z is called standard normal variable z and the standard form of the normal distribution is

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}}$$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-}$$

Q.1. The life of army shoes is normally distributed with mean 8 months standard deviation 2 months. If 5000 pairs are injured has many pairs would be expected to need replacement after 12 months?

Q.2. Assume mean height of soldiers to be 68.22 inches with a variance of 10.8 inches square. How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall, given that the are under that standard normal curve b/w $z=0$ and $z=0.35$ i.e; 0.1368 and b/w $z=0$ and $z=1.15$ i.e; 0.3746

Q.3. A sample of 100 dry battery cells tested to find the length of life produced the following results

$$\bar{x} = 12 \text{ hours}, \quad \sigma = 3 \text{ hours}$$

Assuming the data to be normally distributed, what percentage of battery celles are expected to have life

(i) more than 15 hours, (ii) less than 6 hours

(iii) between 10 and 14 hours.

Q. In a sample of 1000 cases, the mean of certain test is 14 and S.D. is 2.5. Assuming the distribution to be normal find

(i) how many students score between 12 and 15

(ii) how many score above 18

(iii) how many score below 8

(iv) how many score 16.

Standard normal distribution table $P(0 \leq z < \infty) = 0\cdot 5$

	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	0·0000	0·0040	0·0080	0·0120	0·0160	0·0199	0·0239	0·0279	0·0319	0·0359
0·1	0·0398	0·0438	0·0478	0·0517	0·0557	0·0596	0·0636	0·0675	0·0714	0·0753
0·2	0·0793	0·0832	0·0871	0·0910	0·0948	0·0987	0·1026	0·1064	0·1103	0·1141
0·3	0·1179	0·1217	0·1255	0·1293	0·1331	0·1368	0·1406	0·1443	0·1480	0·1517
0·4	0·1554	0·1591	0·1628	0·1664	0·1700	0·1736	0·1772	0·1808	0·1844	0·1879
0·5	0·1915	0·1950	0·1985	0·2019	0·2054	0·2088	0·2123	0·2157	0·2190	0·2224
0·6	0·2257	0·2291	0·2324	0·2357	0·2389	0·2422	0·2454	0·2486	0·2517	0·2549
0·7	0·2580	0·2611	0·2642	0·2673	0·2704	0·2734	0·2764	0·2794	0·2823	0·2852
0·8	0·2881	0·2910	0·2939	0·2967	0·2995	0·3023	0·3051	0·3078	0·3106	0·3133
0·9	0·3159	0·3186	0·3212	0·3238	0·3264	0·3289	0·3315	0·3340	0·3365	0·3389
1·0	0·3413	0·3438	0·3461	0·3485	0·3508	0·3531	0·3554	0·3577	0·3599	0·3621
1·1	0·3643	0·3665	0·3686	0·3708	0·3729	0·3749	0·3770	0·3790	0·3810	0·3830
1·2	0·3849	0·3869	0·3888	0·3907	0·3925	0·3944	0·3962	0·3980	0·3997	0·4015
1·3	0·4032	0·4049	0·4066	0·4082	0·4099	0·4115	0·4131	0·4147	0·4162	0·4177
1·4	0·4192	0·4207	0·4222	0·4236	0·4251	0·4265	0·4279	0·4292	0·4306	0·4319
1·5	0·4332	0·4345	0·4357	0·4370	0·4382	0·4394	0·4406	0·4418	0·4429	0·4441
1·6	0·4452	0·4463	0·4474	0·4484	0·4495	0·4505	0·4515	0·4525	0·4535	0·4545
1·7	0·4554	0·4564	0·4573	0·4582	0·4591	0·4599	0·4608	0·4616	0·4625	0·4633
1·8	0·4641	0·4649	0·4656	0·4664	0·4671	0·4678	0·4686	0·4693	0·4699	0·4706
1·9	0·4713	0·4719	0·4726	0·4732	0·4738	0·4744	0·4750	0·4756	0·4761	0·4767
2·0	0·4772	0·4778	0·4783	0·4788	0·4793	0·4798	0·4803	0·4808	0·4812	0·4817
2·1	0·4821	0·4826	0·4830	0·4834	0·4838	0·4842	0·4846	0·4850	0·4854	0·4857
2·2	0·4861	0·4864	0·4868	0·4871	0·4875	0·4878	0·4881	0·4884	0·4887	0·4890
2·3	0·4893	0·4896	0·4898	0·4901	0·4904	0·4906	0·4909	0·4911	0·4913	0·4916
2·4	0·4918	0·4920	0·4922	0·4925	0·4927	0·4929	0·4931	0·4932	0·4934	0·4936
2·5	0·4938	0·4940	0·4941	0·4943	0·4945	0·4946	0·4948	0·4949	0·4951	0·4952
2·6	0·4953	0·4955	0·4956	0·4957	0·4959	0·4960	0·4961	0·4962	0·4963	0·4964
2·7	0·4965	0·4966	0·4967	0·4968	0·4969	0·4970	0·4971	0·4972	0·4973	0·4974
2·8	0·4974	0·4975	0·4976	0·4977	0·4977	0·4978	0·4979	0·4979	0·4980	0·4981
2·9	0·4981	0·4982	0·4982	0·4983	0·4984	0·4984	0·4985	0·4985	0·4986	0·4986
3·0	0·4987	0·4987	0·4987	0·4988	0·4988	0·4989	0·4989	0·4989	0·4990	0·4990

Q. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given that

if $\boxed{f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2}\, dx}$

Q. Assuming that the diameter of 1000 brass plugs taken consecutively from a machine, from a normal distribution with mean 0.7515 cm and S.D. 0.002cm. how many of with mean 0.7515 cm and S.D. 0.002 cm. how many of the plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm.

Q. If the heights of 300 students are normally distributed with 64.5 inches and S.D. 3.3 inches. Find the hight below which 99% of the students lie.

Q. The income of the group of 10000 persons was found to be normally distributed with mean ₹ 750 p.m. and S.D of ₹ 50. Show that, of this group, about 95% had income exceeding ₹ 668 and only 5% had income ₹832 Also find the lowest income among the richest 100.