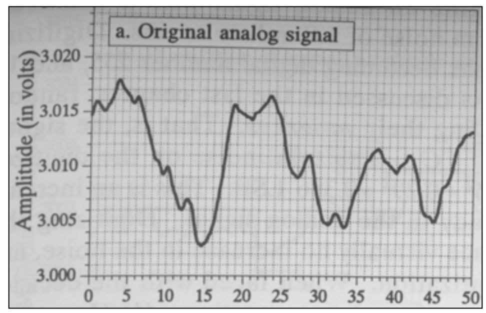
Analog Signals

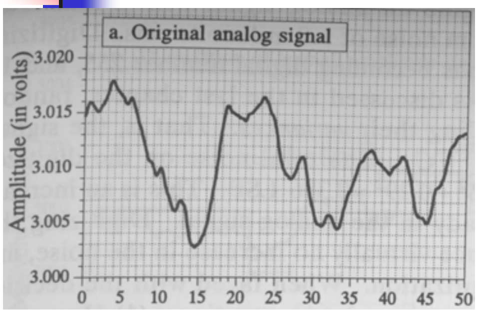
- Both independent and dependent variables can assume a continuous range of values
- Exists in nature

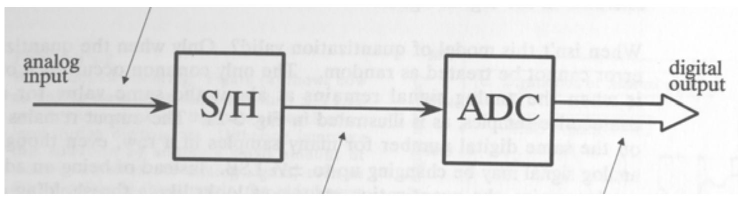


Digital Signals

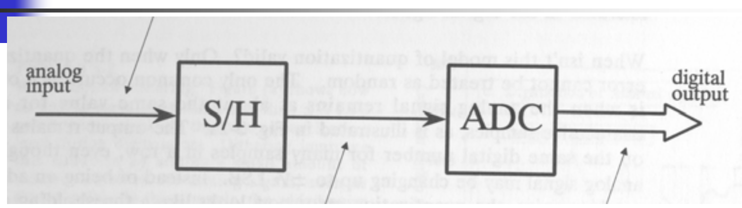
- Both independent and dependent variables are discretized
- Representation in computers
- *Sampling*
  - Discrete independent variable
  - Sample and hold (S/H)
- *Quantization*
  - Discrete dependent variable
  - Analog to Digital Converter (ADC)

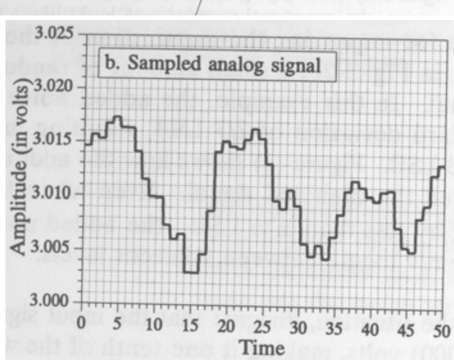
Digital Signal



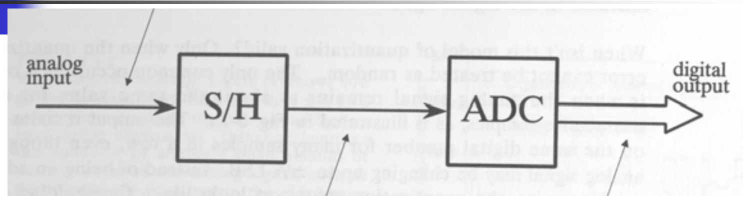


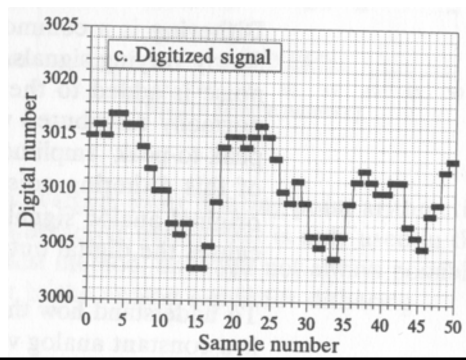
Sampled Signal





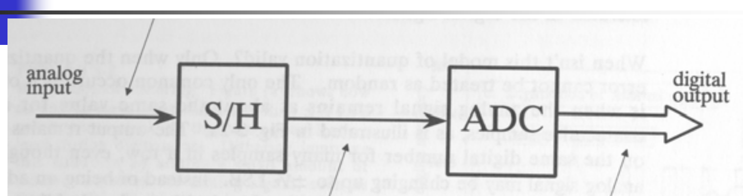
Digitized Signal

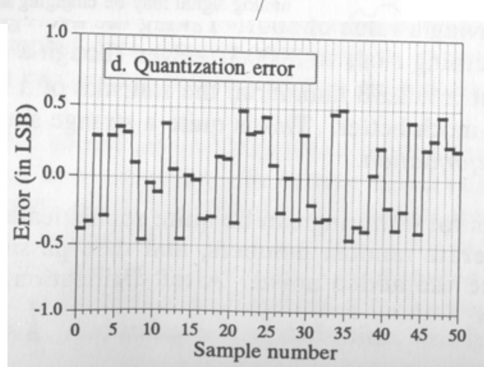




- Depends on number of bits
- 12 bits = 4095 levels
- $0.0 \le$ Voltage ≤ 4.096
- 2.56 and 2.5601 TO 2560
- Each level (LSB) = 0.001
- Error $\le \pm 1/2$ LSB
- Called *Quantization Error*

Digitized Signal





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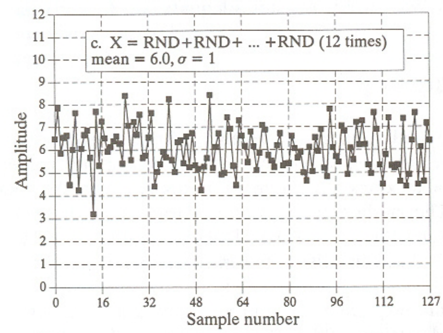
Quantization Error

- Usually like random noise
- Noise is present in most signal acquisition systems
- Random uncorrelated samples added to the original signal



Proper Sampling

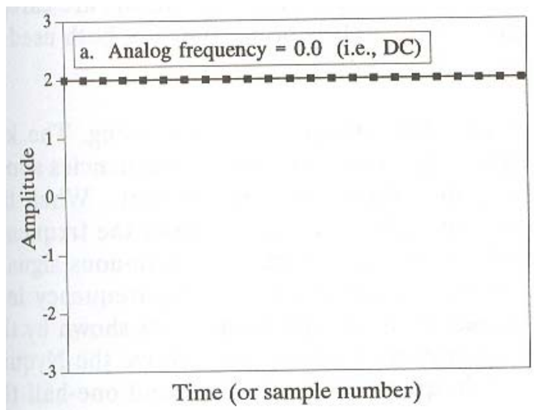
- If the original signal can be reconstructed *unambiguously* from the sampled signal

$$\text{Cycles/ Sample} = \frac{\text{Number of cycles per second}}{\text{Number of samples per second}}$$

$$= \frac{\text{Analog Frequency}}{\text{Sampling Rate}}$$

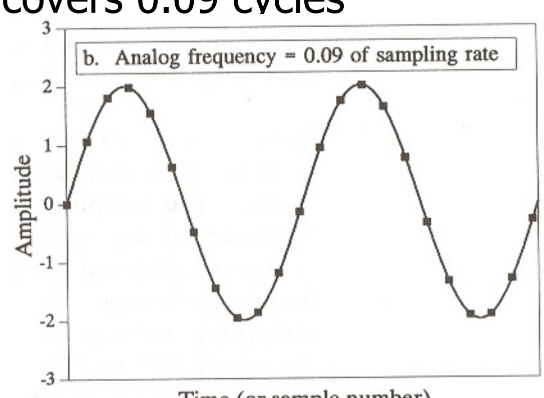
Is it Proper Sampling?

- DC signal
- Freq = 0.0 x Sampling Rate
- Proper



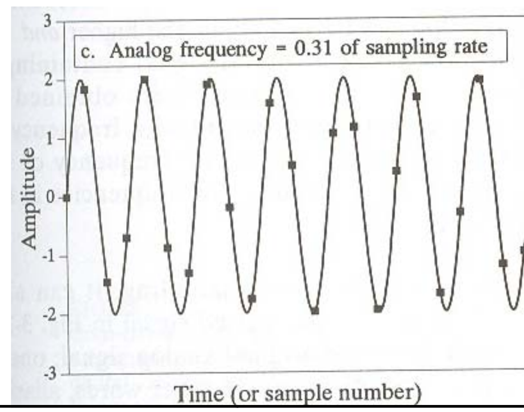
Is it Proper Sampling?

- Freq = 0.09 x Sampling Rate
- Each sample covers 0.09 cycles
- Proper



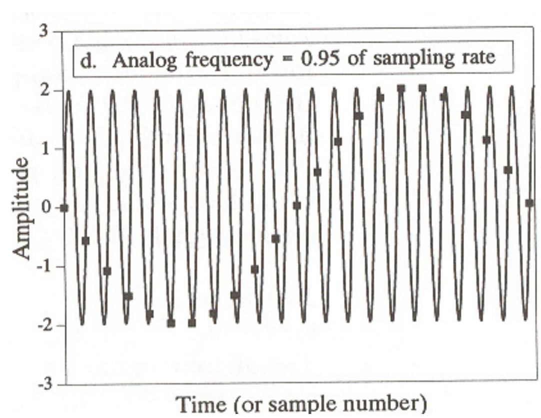
Is it Proper Sampling?

- Freq = 0.31 x Sampling Rate
- Larger fraction of cycles per sample
- Proper



Is it Proper Sampling?

- Freq = 0.95 x Sampling Rate
- Much larger parts of cycles per sample
- Not Proper
- Aliasing
- Changes frequency and phase

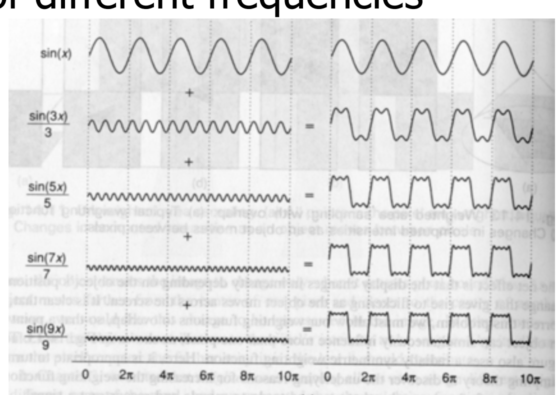


Sampling Theorem

- Proper Sampling: At least one sample per half cycle
- Freq ≤ 0.5 x Sampling Rate
- Sampling Rate ≥ 2 x Frequency
- Nyquist Rate

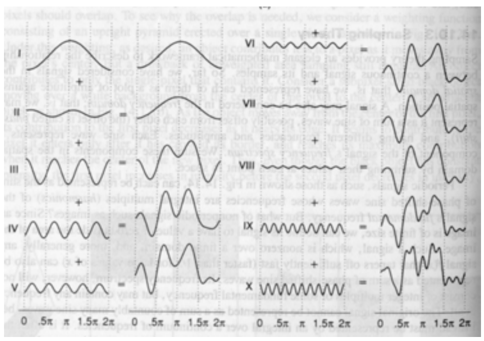
Time (Spatial) Domain vs. Frequency Domain

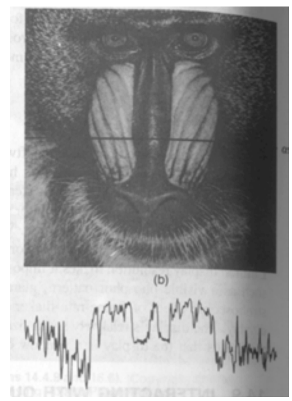
- Any one dimensional *analog signal* can be represented as a linear combination of sine waves of different frequencies



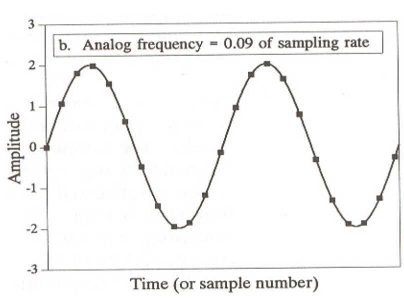
1D Signal

- Example: Once scan line of an image
- Amount of each sine wave defined by its amplitude and phase



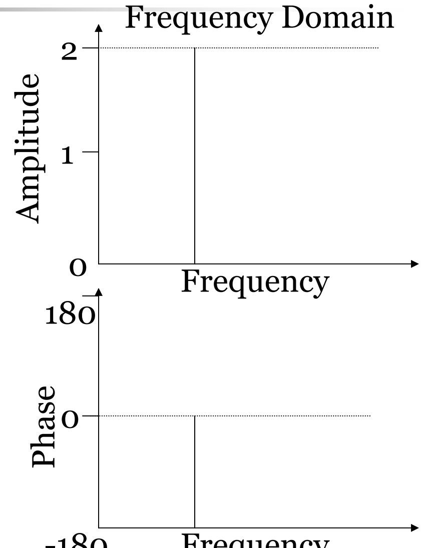


Representation in Both Domains

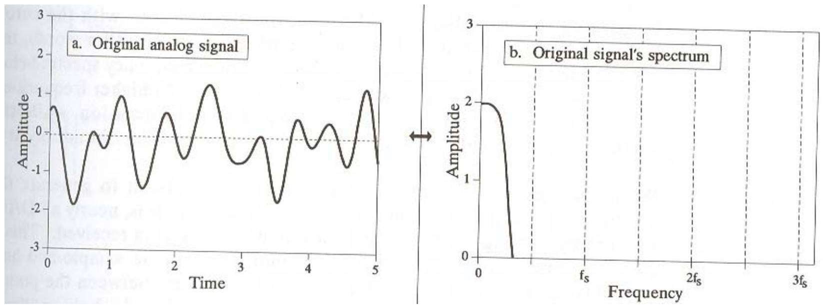


Time Domain

Frequency Domain



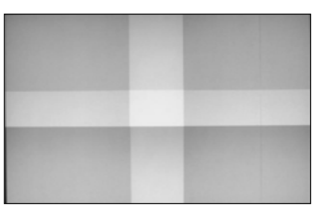
Representation in Both Domains

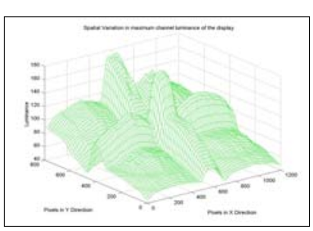


Time Domain

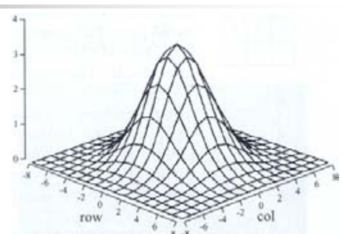
Frequency Domain

Extending it to 2D

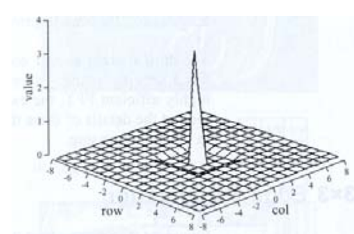




Amplitude



Phase



Amplitude

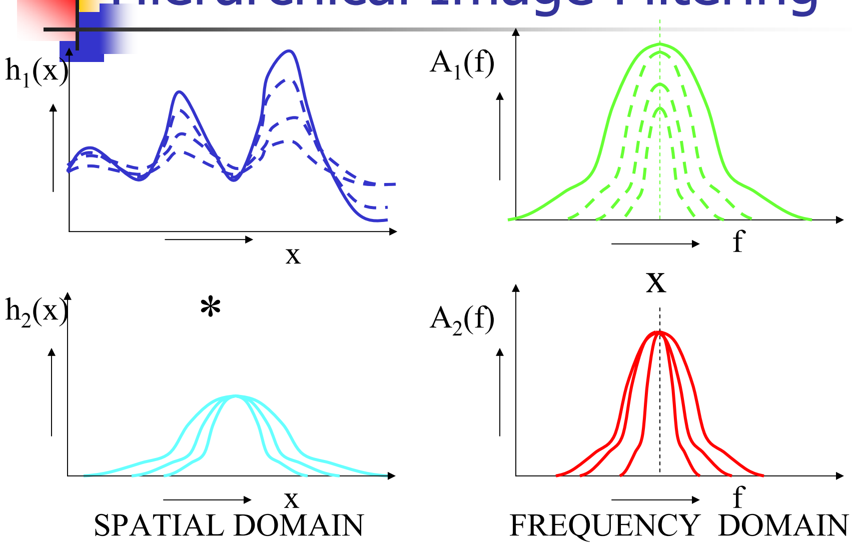
- Amplitude
  - **How** much details?
  - Sharper details signify higher frequencies
  - Will deal with this mostly

Phase

- **Where** are the details?
- Though we do not use it much, it is important, especially for perception

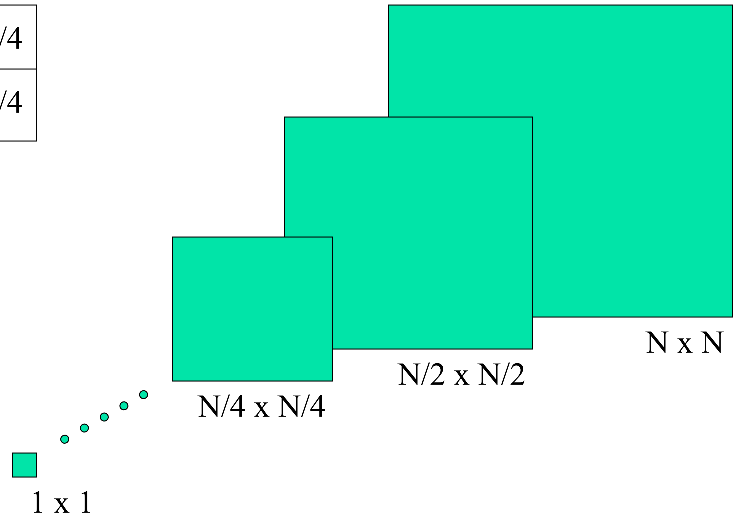
Low Pass Filtering

Hierarchical Image Filtering

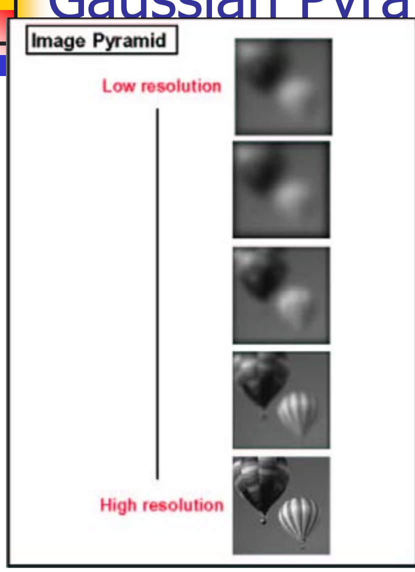


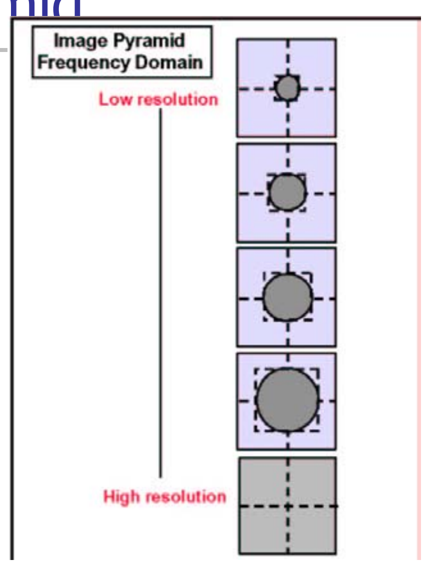
Hierarchical Filtering

1/4	1/4
1/4	1/4

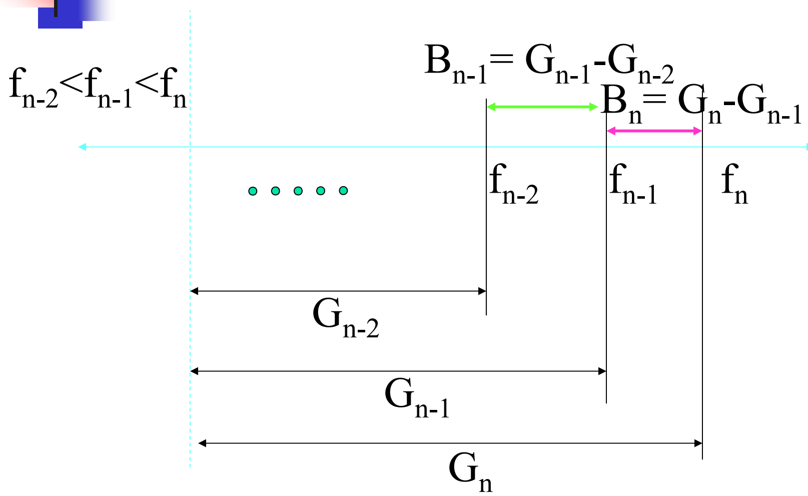


Gaussian Pyramid

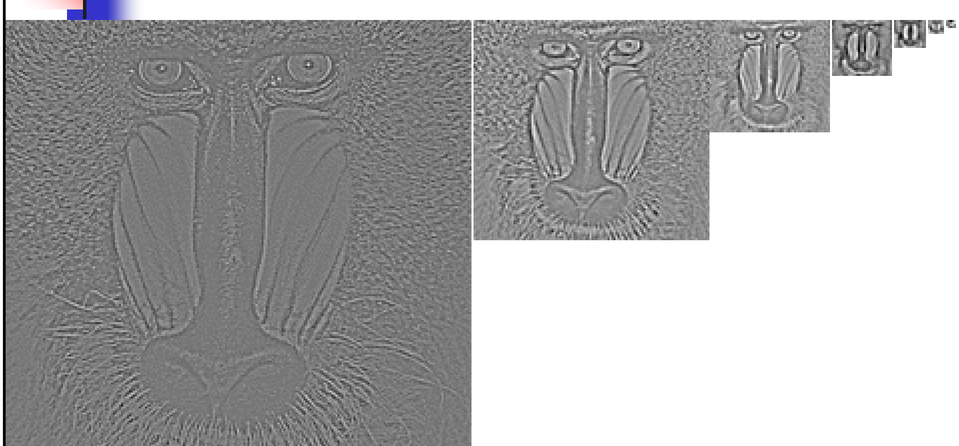




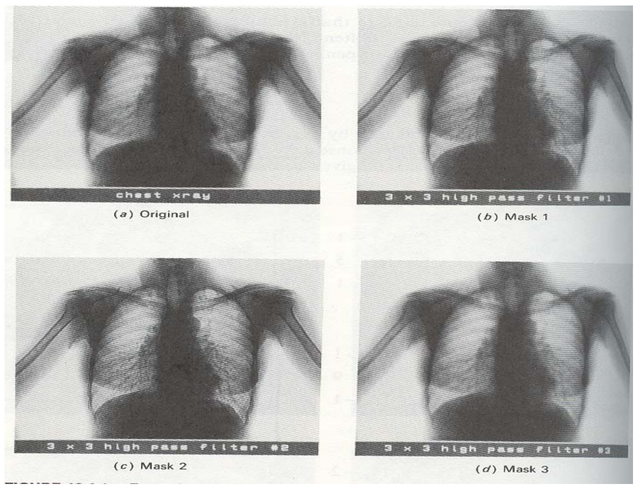
Band-limited Images (Laplacian Pyramid)



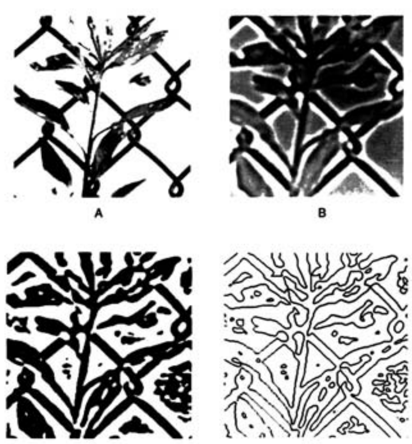
Band-limited Images (Laplacian Pyramid)



Edge Crispening

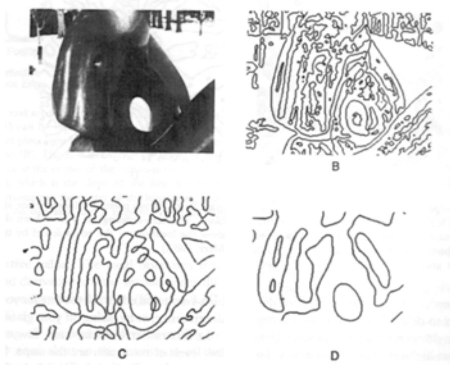


Second Order Edge Detection



A. The image
B. Image after convolution
C. Segmented convolved image
D. Edge detected image

Scaling Problem



- Edges in coarser level do not disappear in finer levels
- New edges are added
- Coarser level edges are most important
- Advances like a hierarchy