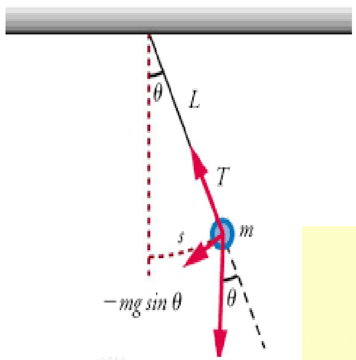
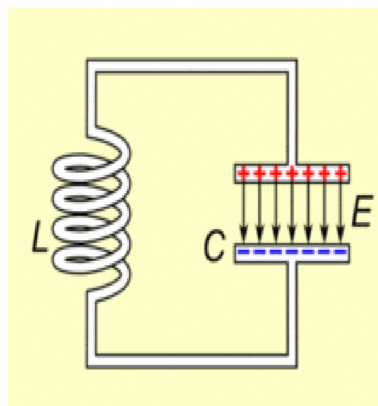
Lecture 2

Harmonic Motion





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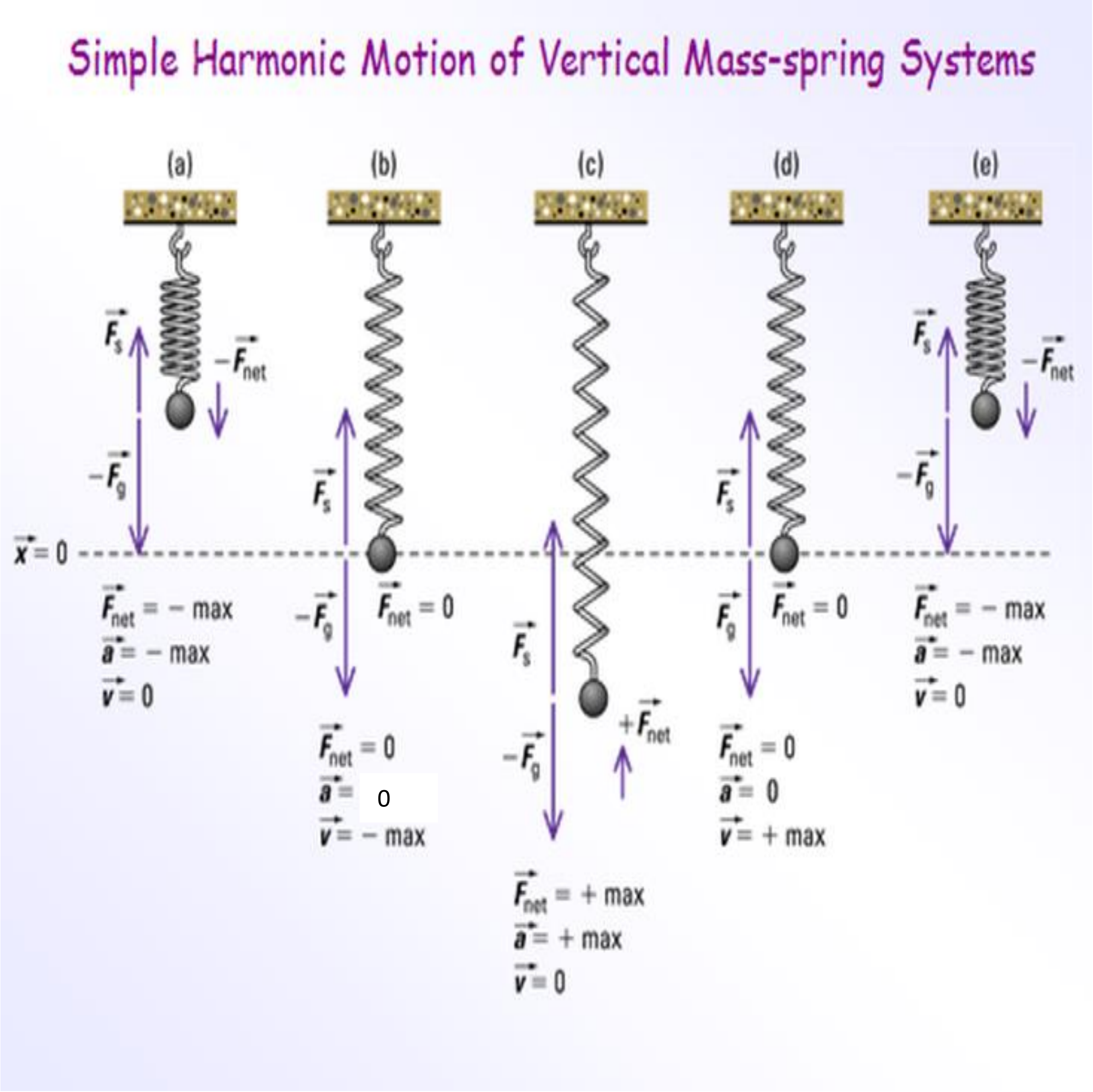
Simple Harmonic Motion



Simple Harmonic Motion

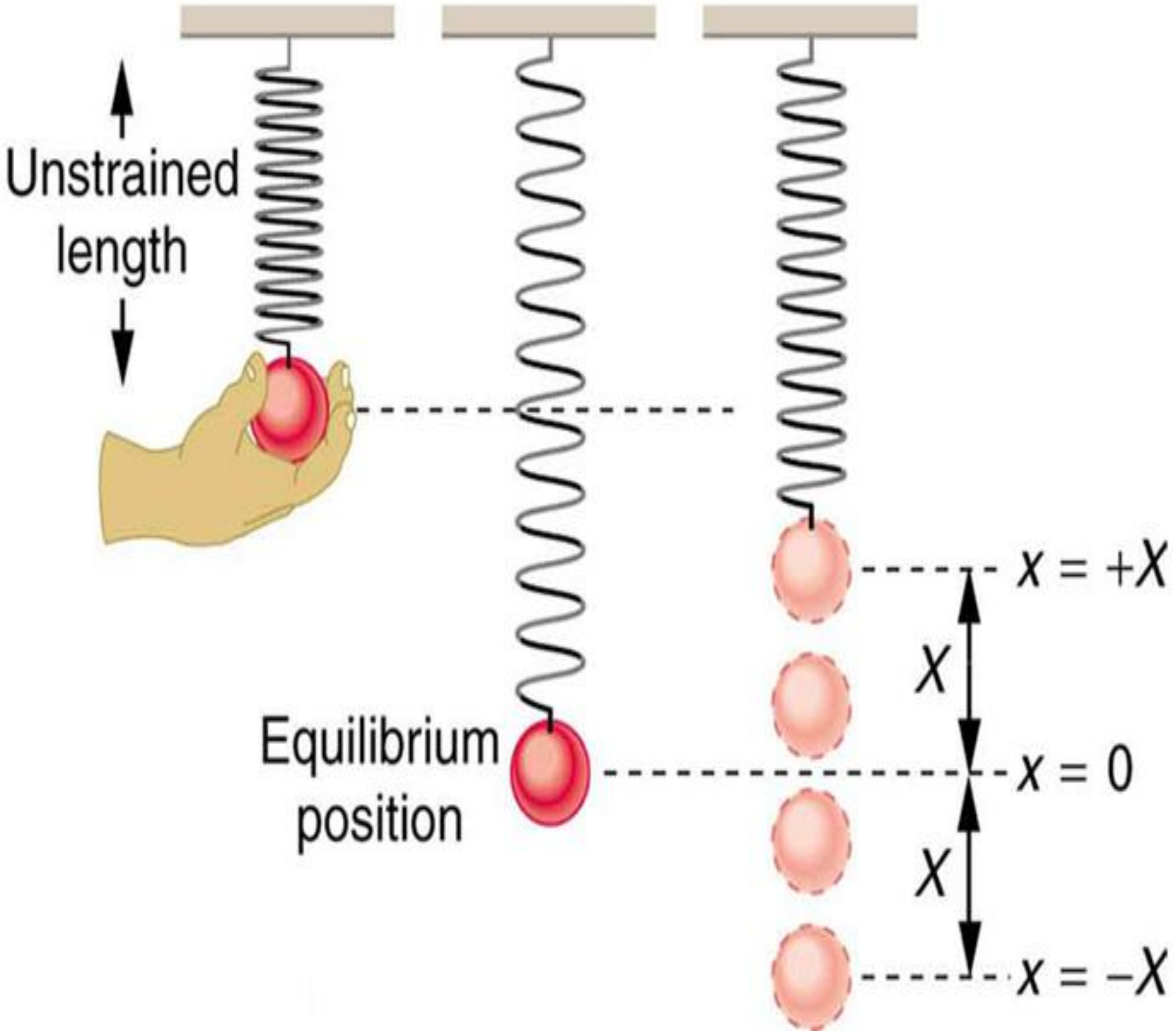
Periodic Motion: A motion which repeats itself in equal intervals of time is periodic motion. For example, the motion of the hands of a clock, the motion of the wheels of a car and the motion of a merry-go-round.

Oscillatory Motion: An oscillatory motion is a periodic motion in which an object moves to and fro about its equilibrium position. The object performs the same set of movements again and again after a fixed time. One such set of movements is an Oscillation. The motion of a simple pendulum, the motion of leaves vibrating in a breeze and the motion of a cradle are all examples of oscillatory motion.

SHM: To-and-fro motion under the action of a restoring force. Simple harmonic motion is the simplest example of oscillatory motion.

Ref: toppr.com

Simple Harmonic Motion: Graphs



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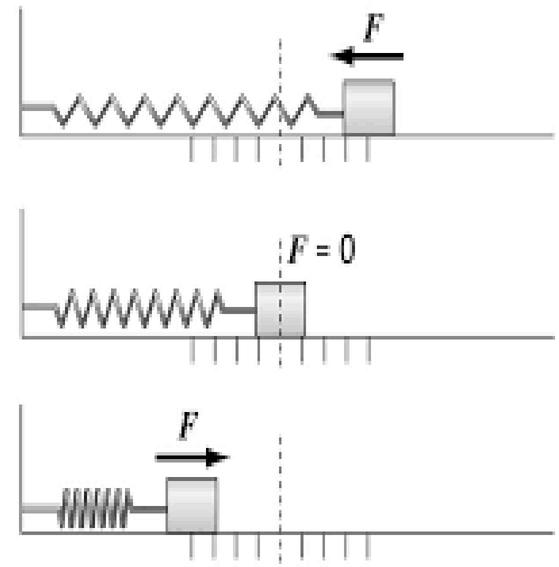
Simple Harmonic Motion: Equation

Hooke's Law: The extension of an elastic object is directly proportional to the force applied to it. Or,

The restoring force applied to an elastic object (such as a spring) is proportional to the displacement (or extension) and in the

hat displacement.



Hooke's Law:

Restoring force,

$$\vec{\mathbf{F}}_{restore} = -k\Delta\vec{\mathbf{x}}$$

where $\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_0$

and k is the "spring constant" [N m^{-1}]

displacement caused

tart with the momentum principle: $\dfrac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{\text{net}}$

For horizontal forces on the mass: $\dfrac{dp_x}{dt} = -kx$

$$\therefore \frac{d(mv_x)}{dt} = -kx \quad \text{or} \quad \frac{d}{dt}\left(m\frac{dx}{dt}\right) = -kx$$

$$\therefore \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Ref: google

Simple Harmonic Motion: Equation

We can combine the constants *k* and *m* by makin[g] the substitution:

$\frac{k}{m} = \omega_0^2$, which results

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0.$$

Some solutions of this equ[ation] are:

$x = A \sin(\omega_0 t + \varphi)$
$x = A \cos(\omega_0 t + \varphi)$

This solutions can be prov[ed to] be the solutions of the above

$$x(t) = A\cos(\omega_0 t + \phi)$$

$$v(t) = \frac{dx(t)}{dt} = -A\omega_0 \sin(\omega_0 t + \phi)$$

$$a(t) = \frac{d^2x(t)}{dt^2} = \frac{dv(t)}{dt} = -A\omega_0^2 \cos(\omega_0 t + \phi)$$

$$\ldots \text{ acceleration } = -(\text{constant}).(\text{displacement})$$

$$= -A\omega_0^2 \cos(\omega_0 t + \phi)$$

$$= A\omega_0^2 \cos(\omega_0 t + \phi + \pi)$$

Phase difference between acceleration and displacement is π

Phase difference between v and x (and v & a) is $\frac{\pi}{2}$

Ref: google image