Lecture 03

Lecture: Waves and Oscillations

Ref book: Physics for Engineers - Giasuddin Ahmad (Part-1)
University Physics - Sears, Zemansky, Young & Freedman

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Web ref provided on slides

Simple Harmonic Motion: Equation

We can combine the constants *k* and *m* by makin[g] the substitution:

$\frac{k}{m} = \omega_0^2$, which results

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0.$$

Some solutions of this equ[ation] are:

$x = A \sin(\omega_0 t + \varphi)$
$x = A \cos(\omega_0 t + \varphi)$

This solutions can be prov[ed to] be the solutions of the above

$$x(t) = A\cos(\omega_0 t + \phi)$$

$$v(t) = \frac{dx(t)}{dt} = -A\omega_0 \sin(\omega_0 t + \phi)$$

$$a(t) = \frac{d^2x(t)}{dt^2} = \frac{dv(t)}{dt} = -A\omega_0^2 \cos(\omega_0 t + \phi)$$

$$\ldots \text{ acceleration } = -(\text{constant}) \,.\, (\text{displacement})$$

$$= -A\omega_0^2 \cos(\omega_0 t + \phi)$$

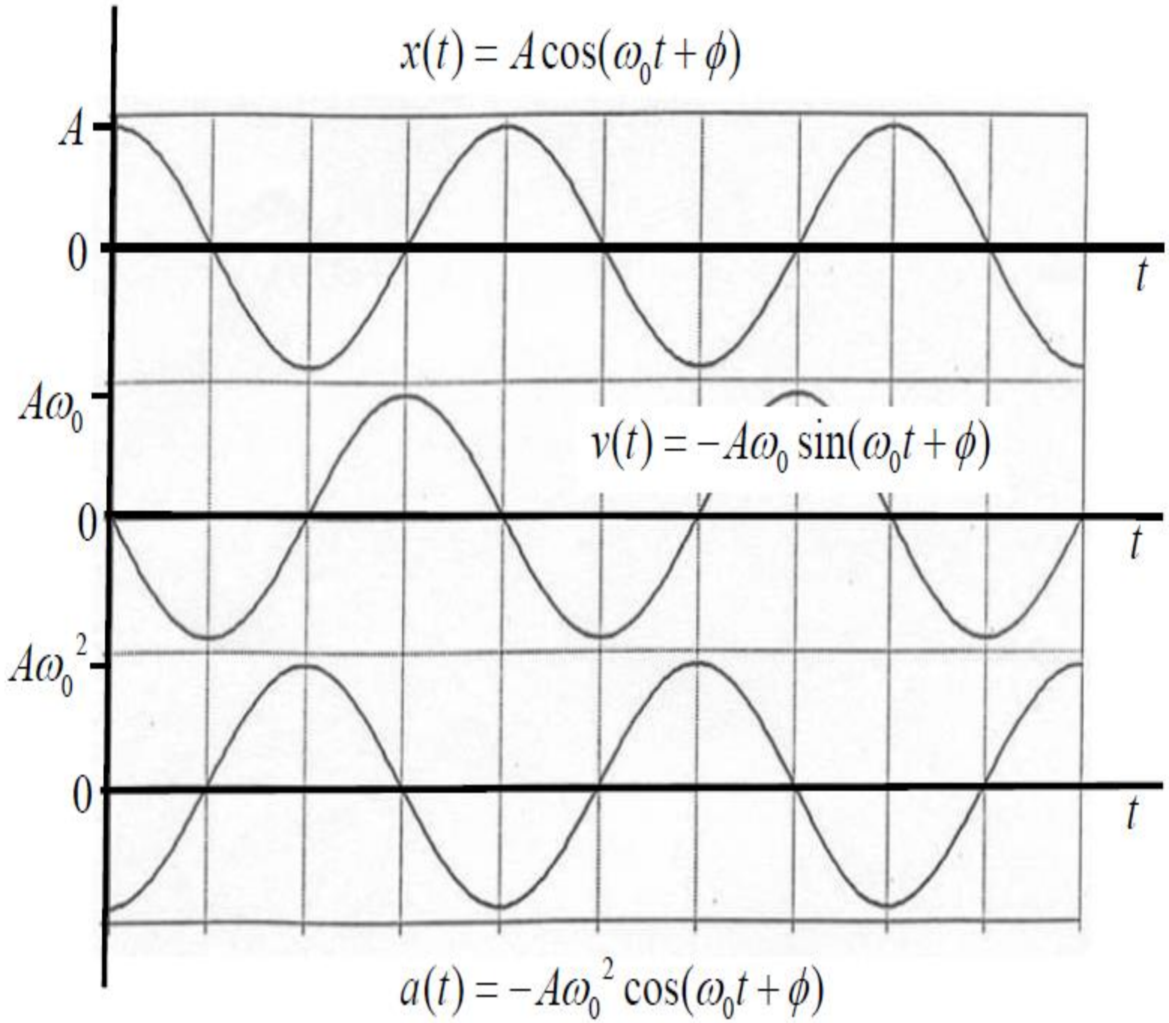
$$= A\omega_0^2 \cos(\omega_0 t + \phi + \pi)$$

Phase difference between acceleration and displacement is π

Phase difference between v and x (and v & a) is $\frac{\pi}{2}$

Ref: google image

Simple Harmonic Motion: Graphs



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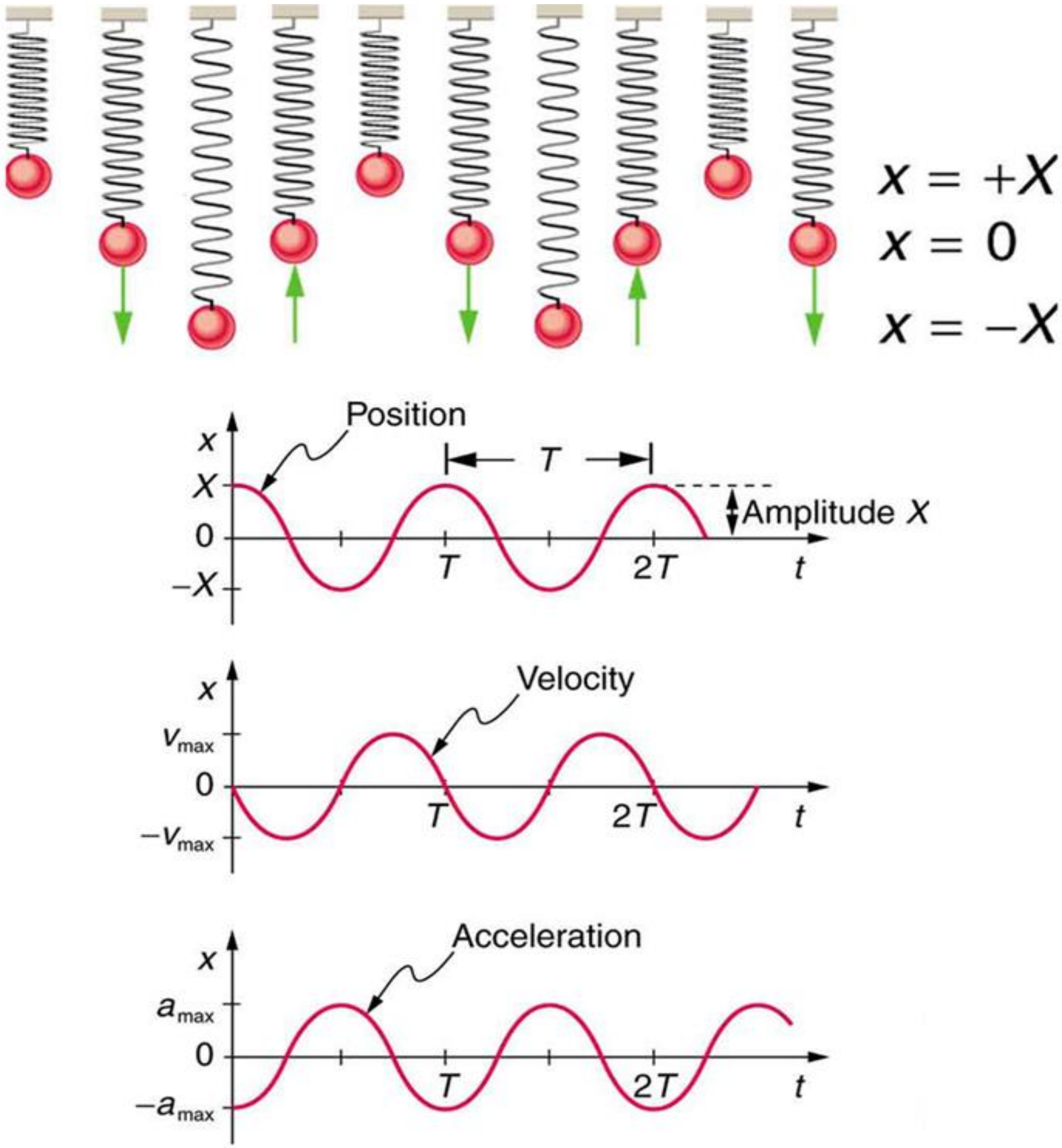
Simple Harmonic Motion: Graphs



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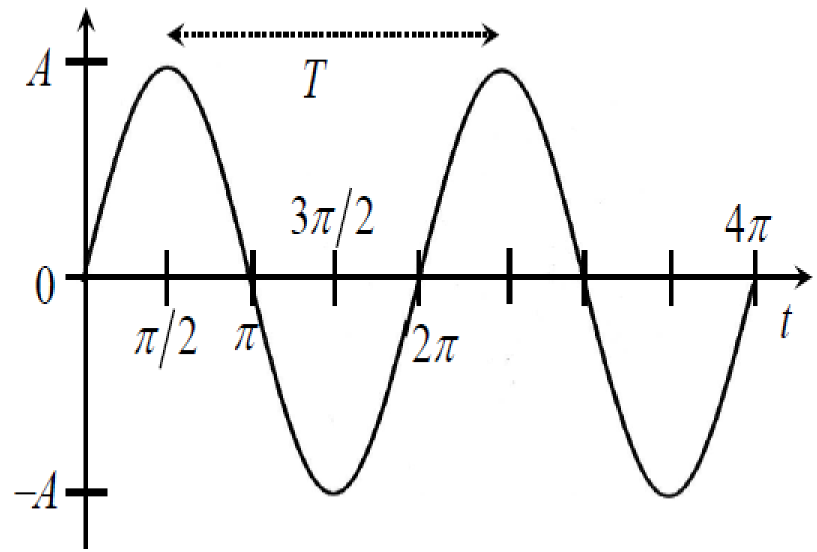
Simple Harmonic Motion: Definition

Definitions of some related quantities for $y = A \sin(\omega t+\varphi)$



$\omega T = 2\pi$ where T : period (s)

ω : angular frequency (rad s^{-1})

some books use υ → $f = \dfrac{1}{T}$ where f : frequency (Hz)

A : Amplitude

ϕ : phase angle, initial phase or phase constant

Amplitude: The amplitude of the motion, denoted by A, is the maximum magnitude of displacement from the equilibrium position. It is always positive

Period: The period T, is the time required for one oscillation

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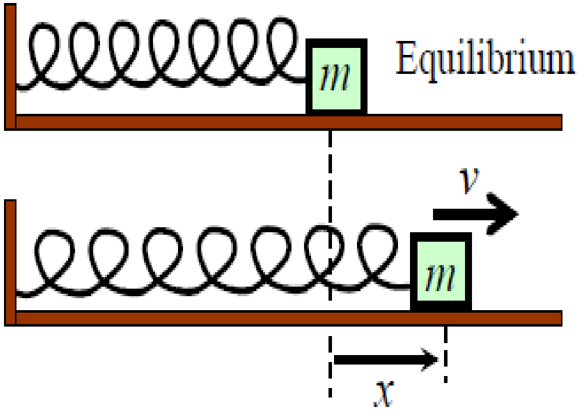
Simple Harmonic Motion: Energy



Suppose that the mass has a speed v when it has displacement x

Kinetic energy of mass = $\frac{1}{2}mv^2$

Potential energy of spring = $\int_0^x F dx' = \int_0^x kx' dx' = \frac{1}{2}kx^2$

There are no dissipative mechanisms in our model (no friction).
… the total energy of the mass-spring system is conserved.

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Ref: google

Simple Harmonic Motion: Energy

For our mass-spring system: $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$

$$\therefore \frac{d}{dt}\left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2\right) = 0$$

$$\therefore mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

$$\therefore mv\frac{dv}{dt} + kxv = 0$$

$$\therefore m\frac{dv}{dt} + kx = 0$$

$$\therefore \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

… as before

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Simple Harmonic Motion: Energy

For the mass-spring system: $x = A\cos(\omega_0 t + \phi)$

Potential energy = $\frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega_0 t + \phi)$

k.e. = $\frac{1}{2}mv^2 = \frac{1}{2}m[-A\omega_0\sin(\omega_0 t + \phi)]^2 = \frac{1}{2}mA^2\omega_0^2\sin^2(\omega_0 t + \phi)$

Total energy = p.e. + k.e

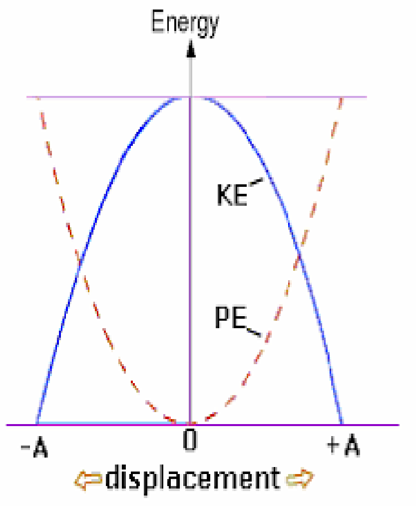
$$= \frac{1}{2}kA^2\cos^2(\omega_0 t + \phi) + \frac{1}{2}mA^2\omega_0^2\sin^2(\omega_0 t + \phi)$$

$$= \frac{1}{2}kA^2 \quad (= \frac{1}{2}m\omega_0{}^2A^2) \qquad (\therefore E \propto A^2)$$

We can now write: $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$

$$\therefore v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} \quad \text{or} \quad v(x) = \pm\omega_0\sqrt{A^2 - x^2}$$

$$E = KE + PE = \frac{1}{2}kA^2$$



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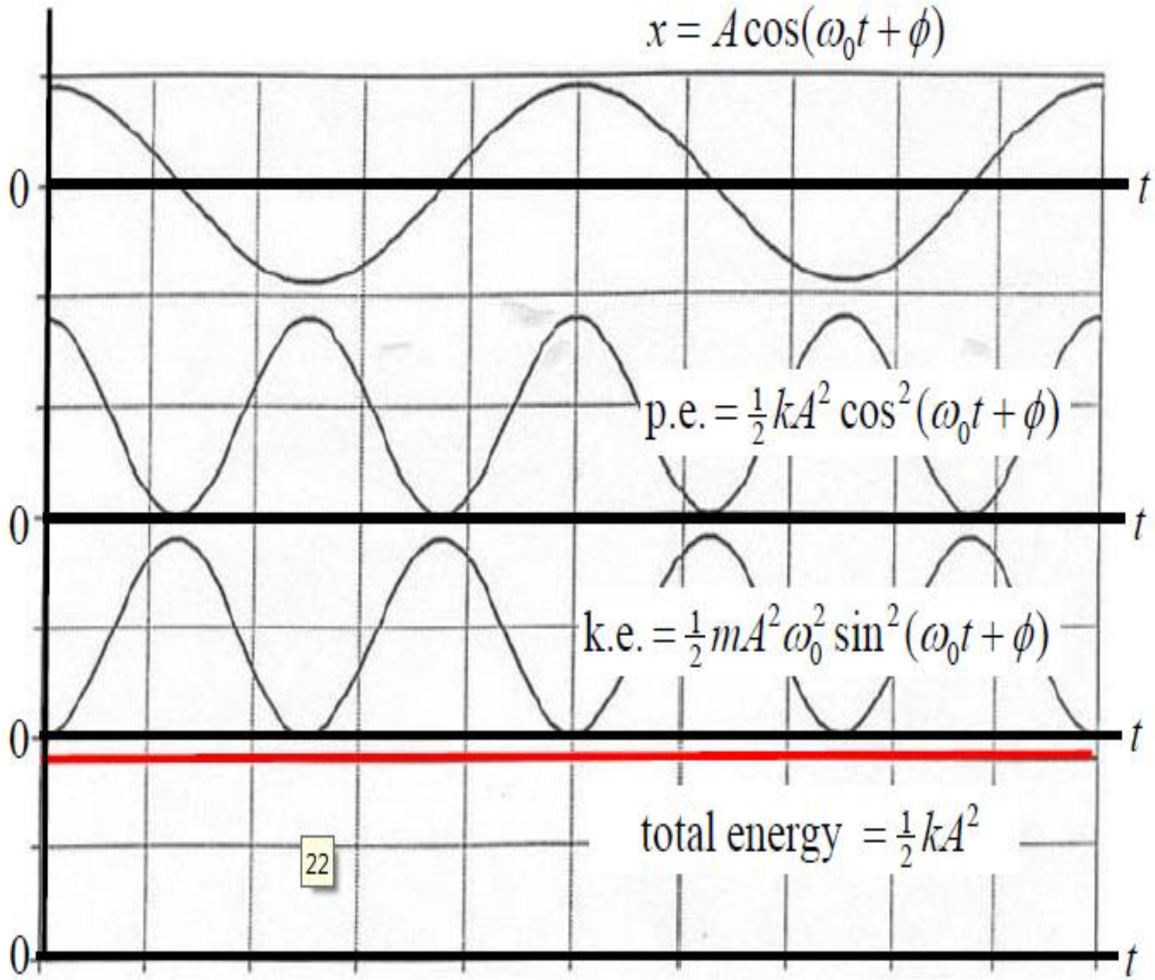
Simple Harmonic Motion: Energy



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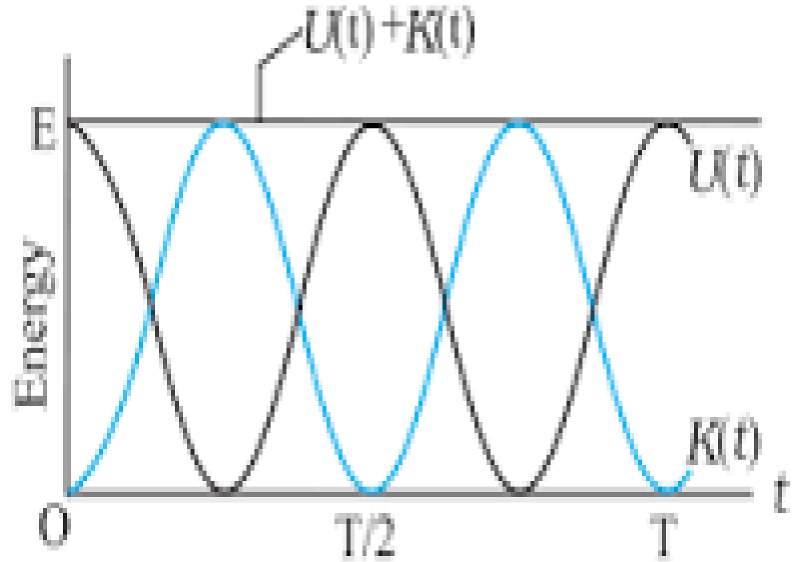
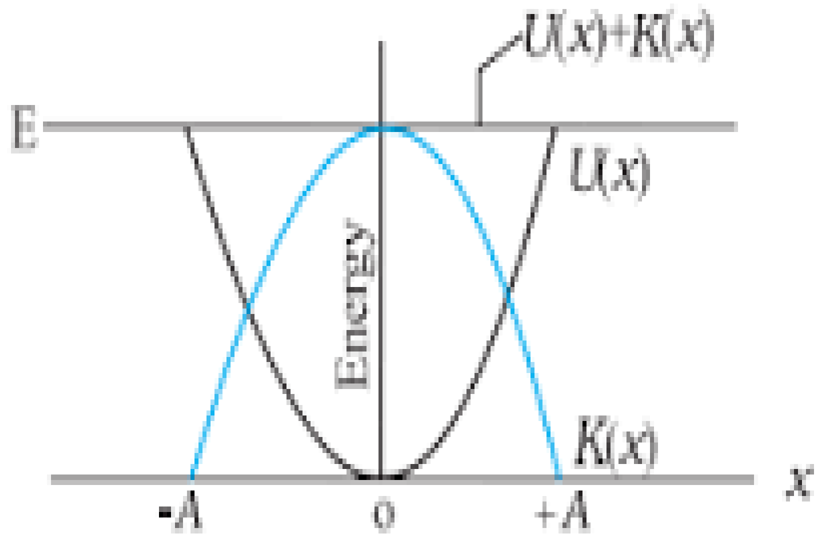
Simple Harmonic Motion: Energy





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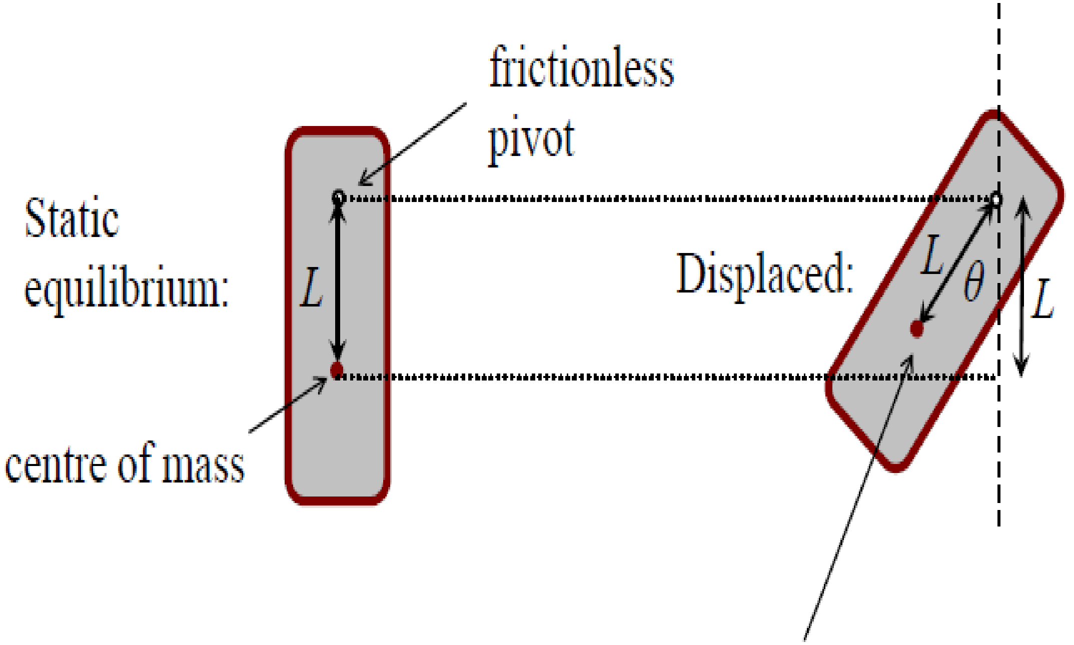
Simple Harmonic Motion: Pendulum

The pendulum: general case



In displaced position, centre of mass is $L - L\cos\theta$ above the equilibrium position.

Recall $\cos\theta = 1 - \dfrac{\theta^2}{2!} + \dfrac{\theta^4}{4!} - \ldots$ For small angles, $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$

Gravitational potential energy = $mgL(1-\cos\theta) = mgL\dfrac{\theta^2}{2}$