Lecture 05

Simple Harmonic Motion: Sample Problems

A 0.42-kg block is attached to the end of a horizontal ideal spring and rests on a frictionless surface. The block is pulled so that the spring stretches by 2.1 cm relative to its unstrained length. When the block is released, it moves with an acceleration of 9.0 m/s^2. What is the spring constant of the spring?

180 N/m

Ref: google image

Simple Harmonic Motion: Sample Problems

Energy calculations.

For the simple harmonic oscillation where k = 19.6 N/m, A = 0.100 m, x = -(0.100 m) cos 8.08t, and v = (0.808 m/s) sin 8.08t, determine (a) the total energy, (b) the kinetic and potential energies as a function of time, (c) the velocity when the mass is 0.050 m from equilibrium, (d) the kinetic and potential energies at half amplitude ($x = \pm A/2$).

Ref: google image

Simple Harmonic Motion: Sample Problems

a. $E = \frac{1}{2}kA^2 = \frac{1}{2}\cdot 19.6\text{N/m}\cdot(0.100\text{m})^2 = 9.80\times10^{-2}\,\text{J}.$

b. $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2\omega t = (9.80\times10^{-2}\,\text{J})\cos^2 8.08t,$

$K = E - U = (9.80\times10^{-2}\,\text{J})\sin^2 8.08t.$

c. $K = E - U,\quad \frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2,$

$v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \omega\sqrt{A^2 - x^2}$

$= 8.08\text{Hz}\cdot\sqrt{(0.100\text{m})^2 - (0.050\text{m})^2} = 0.70\text{m/s}.$

d. $U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{4}E = 2.5\times10^{-2}\,\text{J},$

$E = K - U = 7.3\times10^{-2}\,\text{J}.$

Ref: google image

Simple Harmonic Motion: Sample Problems

A 500 g block on a spring is pulled a distance of 20 cm and released. The subsequent oscillations are measured to have a period of 0.80 s. At what position (or positions) is the speed of the block 1.0 m/s?

$$T = 0.80\,\text{s} \text{ so } \omega = \frac{2\pi}{T} = \frac{2\pi}{(0.80\,\text{s})} = 7.85\,\text{rad/s}$$

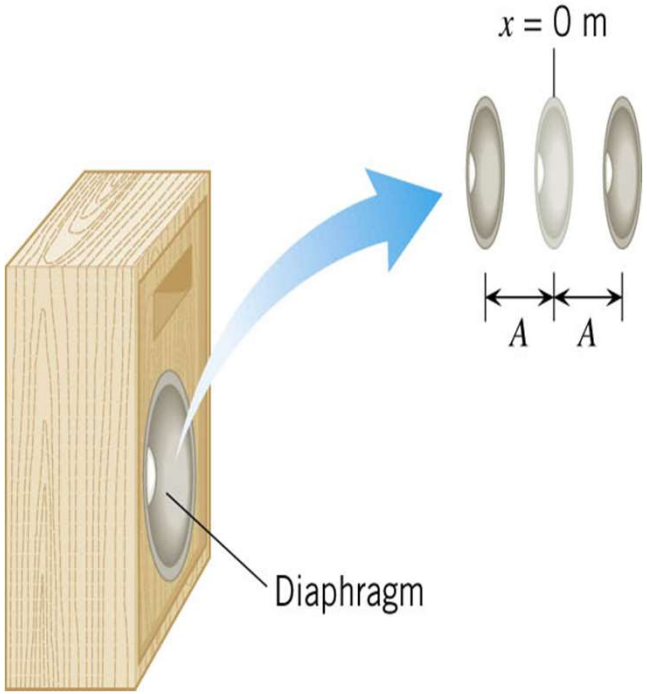
$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \omega\sqrt{A^2 - x^2}$$

$$x = \pm\sqrt{A^2 - \left(\frac{v}{\omega}\right)^2} = \pm\sqrt{(0.20\,\text{m})^2 - \left(\frac{(1.0\,\text{m/s})}{(7.85\,\text{rad/s})}\right)^2} = \pm 0.154\,\text{m} = \pm 15.4\,\text{cm}$$

Ref: google image

Simple Harmonic Motion: Sample Problems

The diaphragm of a loudspeaker moves back and forth in simple harmonic motion to create sound. The frequency of the motion is $f = 1.0$ kHz and the amplitude is $A = 0.20$ mm.



(a) What is the maximum speed of the diaphragm?

(b) Where in the motion does this maximum speed occur?

Ref: google image

Simple Harmonic Motion: Sample Problems

(a)

$$v_{max} = A\omega = A(2\pi f) = (0.20 \times 10^{-3}\,\text{m})(2\pi)(1.0 \times 10^{3}\,\text{Hz}) = \boxed{1.3\,\text{m/s}}$$

(b) The speed of the diaphragm is zero when the diaphragm momentarily comes to rest at either end of its motion: $x = +A$ and $x = -A$. Its maximum speed occurs midway between these two positions, or at $x = 0$ m.

Ref: google image

Simple Harmonic Motion: Sample Problems

What is the oscillation period of an FM radio station that broadcasts at 100 MHz?

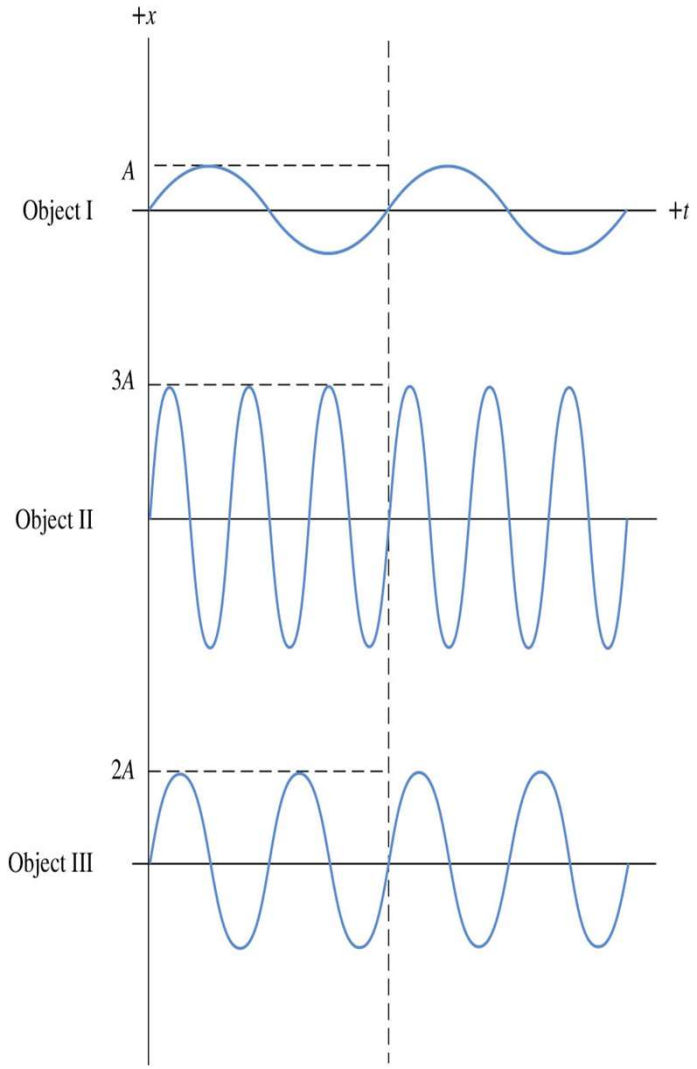
$$f = 100\,\text{MHz} = 1.0\times10^{8}\,\text{Hz}$$

$$T = 1/f = \frac{1}{1.0\times10^{8}\,\text{Hz}} = 1.0\times10^{-8}\,\text{s} = 10\,\text{ns}$$

Note that 1/Hz = s

Ref: google image

Simple Harmonic Motion: Sample Problems



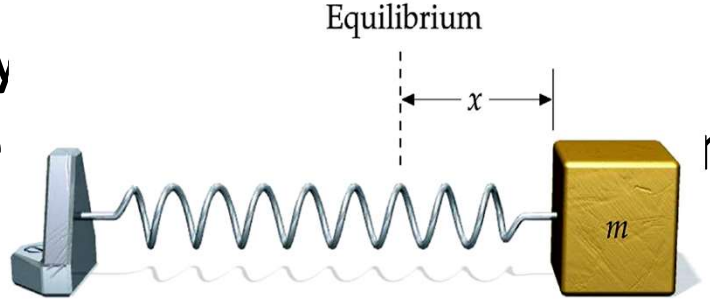
The drawing shows plots of the displacement *x* versus the time *t* for three objects undergoing simple harmonic motion. Which object, I, II, or III, has the greatest maximum velocity? I

Ref: google image

Simple Harmonic Motion: Sample Problems

A 2.00 kg block is attached to a spring as shown.
The force constant of the spring is $k = 196$ N/m.
The block is held a distance of 5.00 cm from
equilibrium and released at $t = 0$.

(a) Find the angular frequency ω, the frequency
(b) Write me.



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Simple Harmonic Motion: Sample Problems

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{(196\ \mathrm{N/m})}{(2.00\ \mathrm{kg})}} = 9.90\ \mathrm{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{(9.90\ \mathrm{rad/s})}{2\pi} = 1.58\ \mathrm{Hz}$$

$T = 1/f = 0.635\ \mathrm{s}$ $A = 5.00\ \mathrm{cm}$ and $\delta = 0$

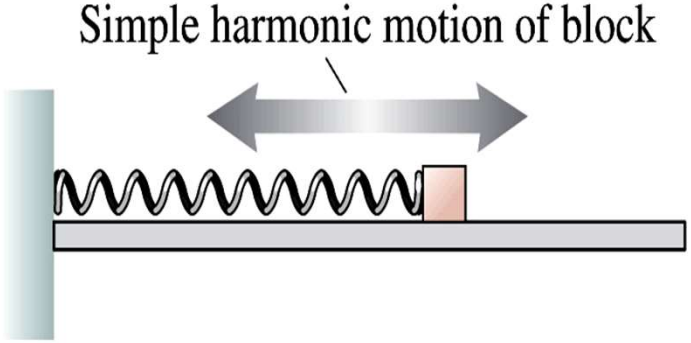
$$x = (5.00\ \mathrm{cm})\cos\left[(9.90\ \mathrm{rad/s})t\right]$$

Ref: google image

Simple Harmonic Motion: Sample Problems

An air-track glider is attached to a spring, pulled 20 cm to the right, and released at t =0. It makes 15 complete oscillations in 10 s.

a. What is the period of oscillation?
b. What is the object's maximum speed?
c. What is its position and velocity at t=0.80 s?



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Simple Harmonic Motion: Sample Problems

$$f = \frac{15\ \text{oscillations}}{10\ \text{s}}$$

$$= 1.5\ \text{oscillations/s} = 1.5\ \text{Hz}$$

$$T = 1/f = 0.667\ \text{s} \qquad v_{\max} = \frac{2\pi A}{T} = \frac{2\pi(0.20\ \text{m})}{(0.667\ \text{s})} = 1.88\ \text{m/s}$$

$$x = A\cos\frac{2\pi t}{T} = (0.20\ \text{m})\cos\frac{2\pi(0.80\ \text{s})}{(0.667\ \text{s})} = 0.062\ \text{m} = 6.2\ \text{cm}$$

$$v = -v_{\max}\sin\frac{2\pi t}{T} = -(1.88\ \text{m/s})\sin\frac{2\pi(0.80\ \text{s})}{(0.667\ \text{s})} = -1.79\ \text{m/s}$$

Ref: google image

Simple Harmonic Motion: Sample Problems

A mass, oscillating in simple harmonic motion, starts at $x = A$ and has period T.

At what time, as a fraction of T, does the mass first pass through $x = ½A$?

$$x = \frac{1}{2}A = A\cos\frac{2\pi t}{T}$$

$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{1}{2}\right) = \frac{T}{2\pi}\frac{\pi}{3} = \frac{1}{6}T$$

Ref: google image

Simple Harmonic Motion: Sample Problems

A particle execute s simple harmonic motion given by the equation

$$y = 12\sin(\frac{2\pi t}{10} + \frac{\pi}{4})$$

Calculate (i) amplitude, (ii) frequency, (iii) displacement at t= 1.25s, (iv) velocity at t= 2.5s (v) acceleration at t= 5s.

Ref: google image

Simple Harmonic Motion: Sample Problems

A particle execute s simple harmonic motion given by the equation

$$y = 10\sin(10t - \frac{\pi}{6})$$

Calculate (i) frequency, (ii) time period (iii) the
maximum displacement (iv)the maximum velocity
(v) the maximum acceleration acceleration.

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