United International University

School of Science and Engineering

Final Examination Trimester: Spring 2022

Course title: Coordinate Geometry and Vector Analysis

Course Code: Math 2201 Marks: 40 Time: 2 hours

Answer all the questions

1. a) Find parametric equations of the tangent line to the curve of intersection of the paraboloid $z = \sqrt{x^2 + y^2}$ and the plane $2x - 5y + 3z = 9$ at the point $(-1, -2, 3)$. [3]

   b) Evaluate $\int_C x^2 dy + y dx$; where C is the triangle with vertices $(0,0), (3,0) and (3,9)$, oriented counterclockwise. [3]

   c) Find the value of the integral $\int_1^5 \int_2^4 \frac{xy}{\sqrt{x^2+y^2+1}}\, dydx$ [4]

2. a) Find the volume of the tetrahedron bounded by the coordinate planes and the plane $z = 6 - 8x - 2y$. [5]

   b) Evaluate [5]

$$\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{0}^{\sqrt{16-x^2-y^2}} \sqrt{x^2 + y^2}\, dz\, dy\, dx$$

3. a) Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 5$ and between the planes $z = 2$ and $x + z = 8$. [5]

   b) Use Green's theorem to evaluate $\oint \frac{xy}{1+x} dx - \ln(1+x)\, dy$, where C is the rectangle with vertices $(0,0), (4,0), (4,2) and (0,2)$, oriented counterclockwise. [5]

4. a) Evaluate $\int_{(0,0)}^{(1,2)} (3x^2y^2 + 2xcosy)dx + (2x^3y - x^2siny)dy$ using the fundamental theorem of line integrals. [5]

   b) Find the directional derivative of $f(x,y) = \sqrt[3]{x+y}$ at $P(1,0)$ in the direction of $b = 2i + j$. [3]

   c) Prove that $div\ (curl\ F) = 0$ [2]

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