Lecture 04

Simple Harmonic Motion: Energy

For the mass-spring system: $x = A\cos(\omega_0 t + \phi)$

Potential energy = $\frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega_0 t + \phi)$

$$E = KE + PE = \frac{1}{2}kA^2$$

k.e. = $\frac{1}{2}mv^2 = \frac{1}{2}m[-A\omega_0\sin(\omega_0 t + \phi)]^2 = \frac{1}{2}mA^2\omega_0^2\sin^2(\omega_0 t + \phi)$

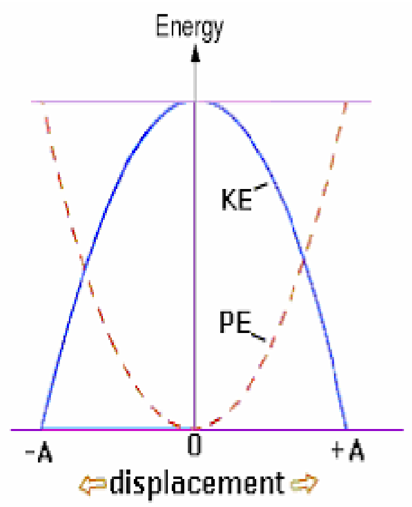
Total energy = p.e. + k.e

$$= \frac{1}{2}kA^2\cos^2(\omega_0 t + \phi) + \frac{1}{2}mA^2\omega_0^2\sin^2(\omega_0 t + \phi)$$

$$= \frac{1}{2}kA^2 \quad (= \frac{1}{2}m\omega_0^2 A^2) \qquad (\therefore E \propto A^2)$$

We can now write: $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$

$$\therefore v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} \quad \text{or} \quad v(x) = \pm\omega_0\sqrt{A^2 - x^2}$$



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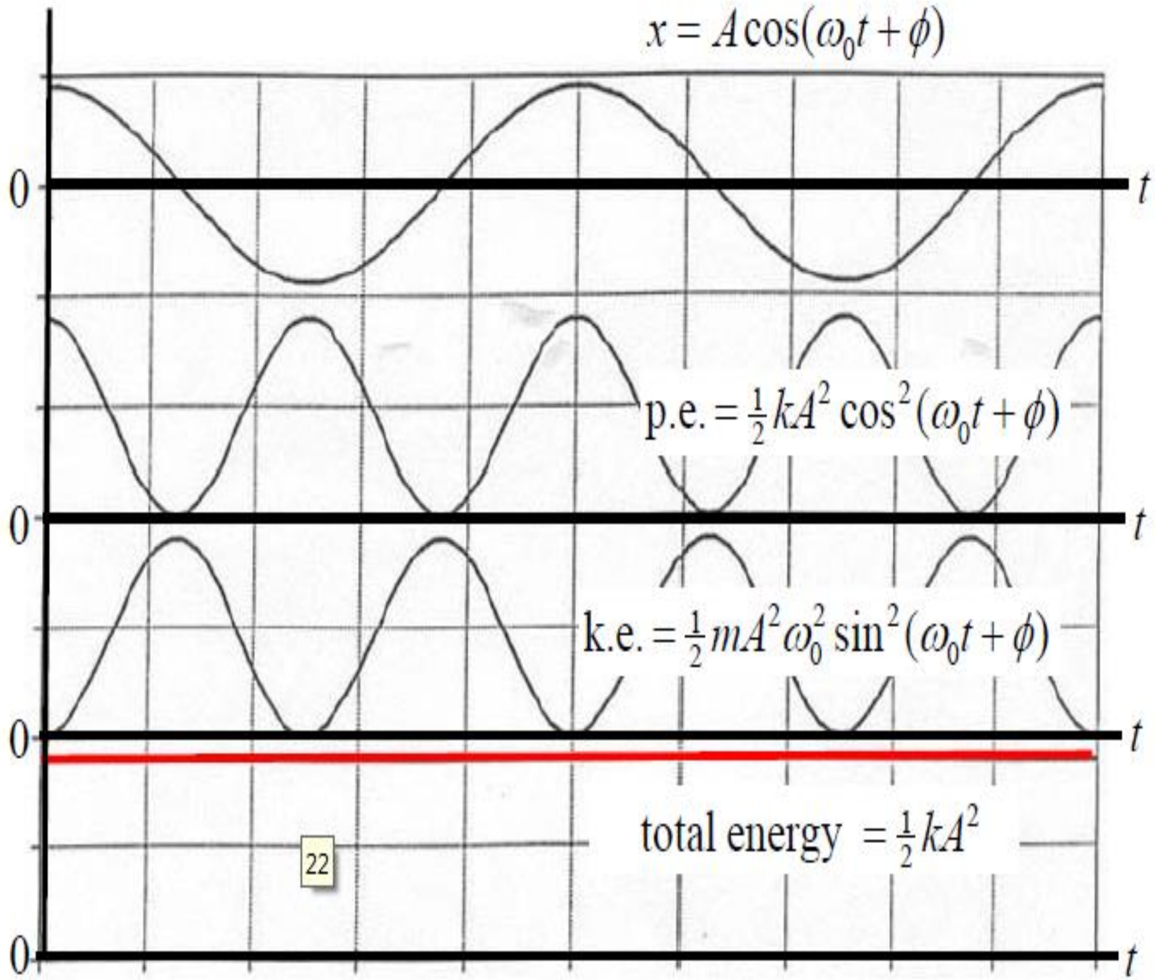
Simple Harmonic Motion: Energy



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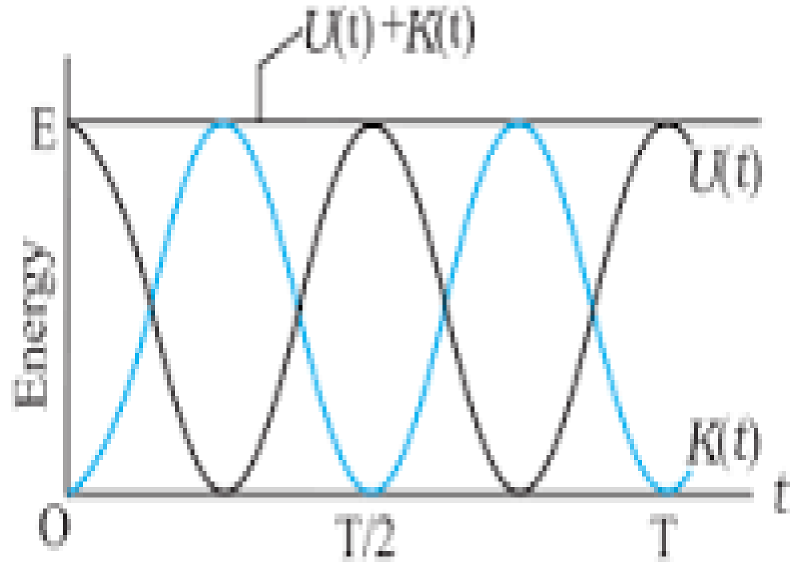
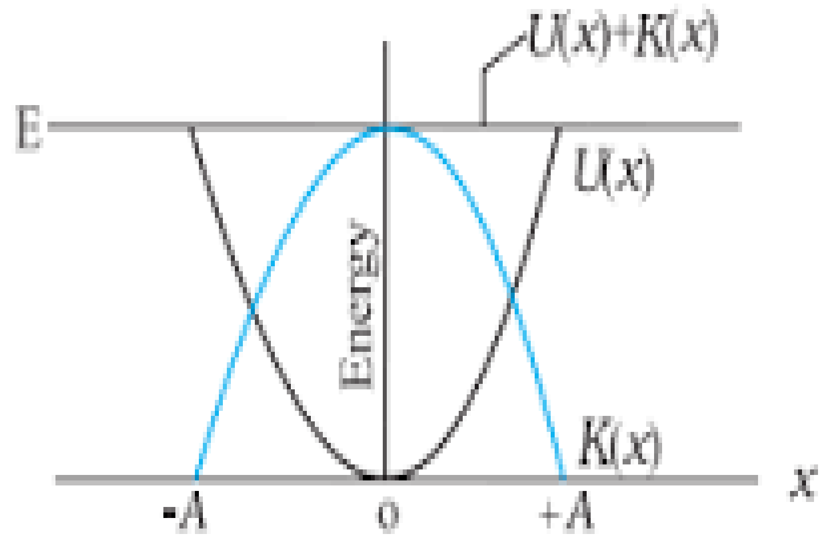
Simple Harmonic Motion: Energy





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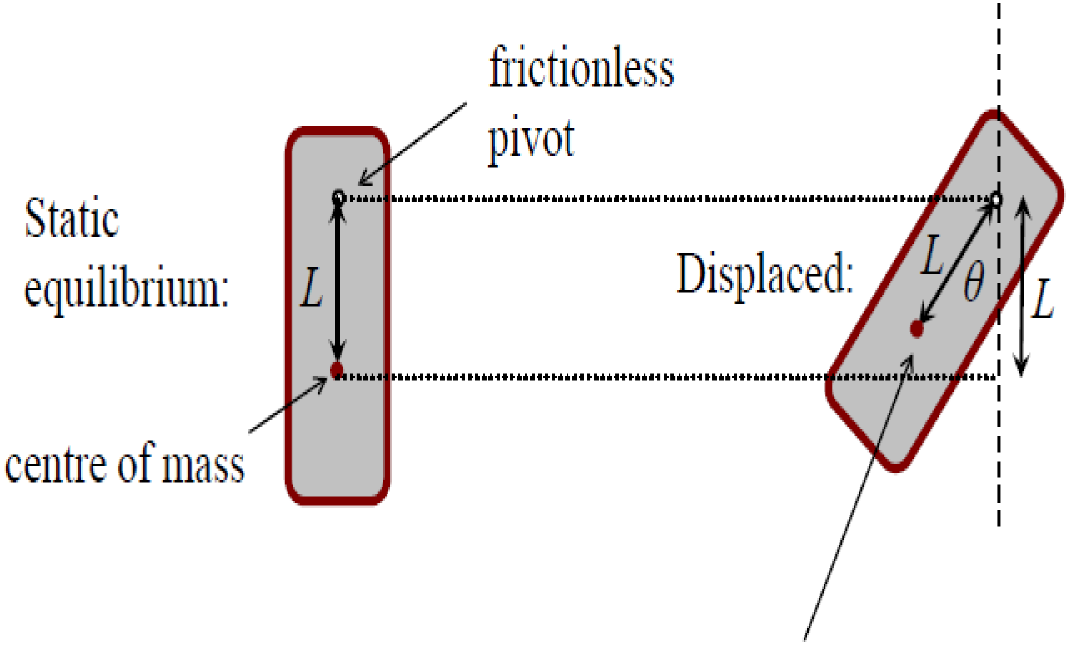
Simple Harmonic Motion: Pendulum

The pendulum: general case



In displaced position, centre of mass is $L - L\cos\theta$ above the equilibrium position.

Recall $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots$ For small angles, $\cos\theta \approx 1 - \frac{\theta^2}{2}$

Gravitational potential energy = $mgL(1 - \cos\theta) = mgL\frac{\theta^2}{2}$

Simple Harmonic Motion: Pendulum

Gravitational potential energy = $\frac{1}{2}mgL\theta^2$

Kinetic energy = $\frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2$

Total energy = $\frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}mgL\theta^2$ = constant

$$\therefore I\frac{d\theta}{dt}\frac{d^2\theta}{dt^2} + mgL\theta\frac{d\theta}{dt} = 0$$ … true for all $\frac{d\theta}{dt}$

$$\therefore \frac{d^2\theta}{dt^2} = -\frac{mgL}{I}\theta = -\omega_0^2\theta$$ where $\omega_0 = \sqrt{\frac{mgL}{I}}$

Equation of SHM

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Simple Harmonic Motion: Pendulum

The moment of inertia of the pendulum about an passing through the point of suspension is

$$= mK^2 + mL^2$$

Therefore $\omega_0 = \sqrt{\dfrac{gL}{K^2 + L^2}}$

Time Period

$$T = 2\pi \sqrt{\frac{K^2 + L^2}{Lg}}$$

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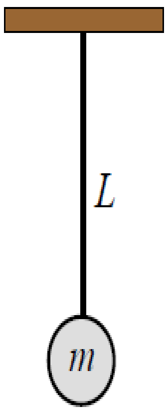
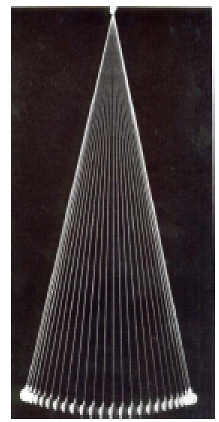
Simple Harmonic Motion: Simple Pendulum

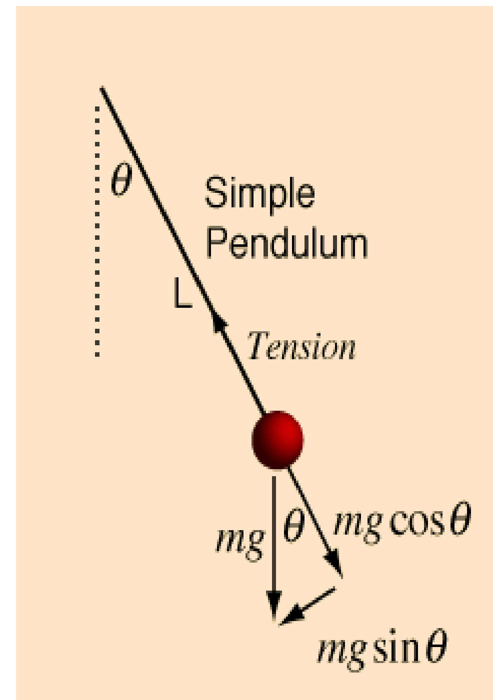


The simple pendulum

$$I = mL^2$$

$$\omega_0 = \sqrt{\frac{mgL}{mL^2}} = \sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$





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Simple Harmonic Motion: Simple Pendulum

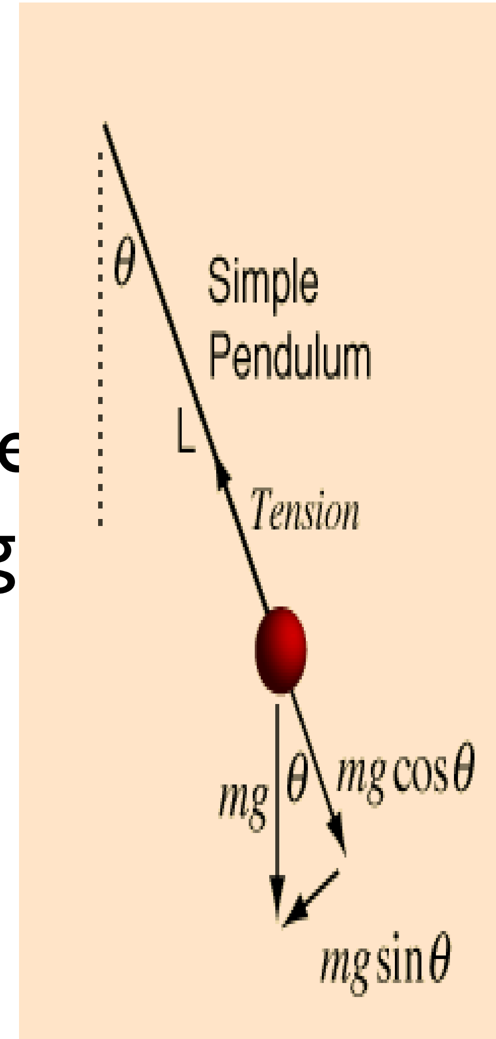
Restoring force

$$F = -mg\sin\theta$$

If the angle θ is very small sinθ is ve
equal to θ. The displacement along

$$x = L\theta$$

Therefore, $F = -mg\theta$



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Simple Harmonic Motion: Simple Pendulum

Acceleration $\dfrac{d^2x}{dt^2} = L\dfrac{d^2\theta}{dt^2}$

$$mL\frac{d^2\theta}{dt^2} = -mg\theta$$

$$Force = mL\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

$$\omega^2 = \frac{g}{L}$$

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

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