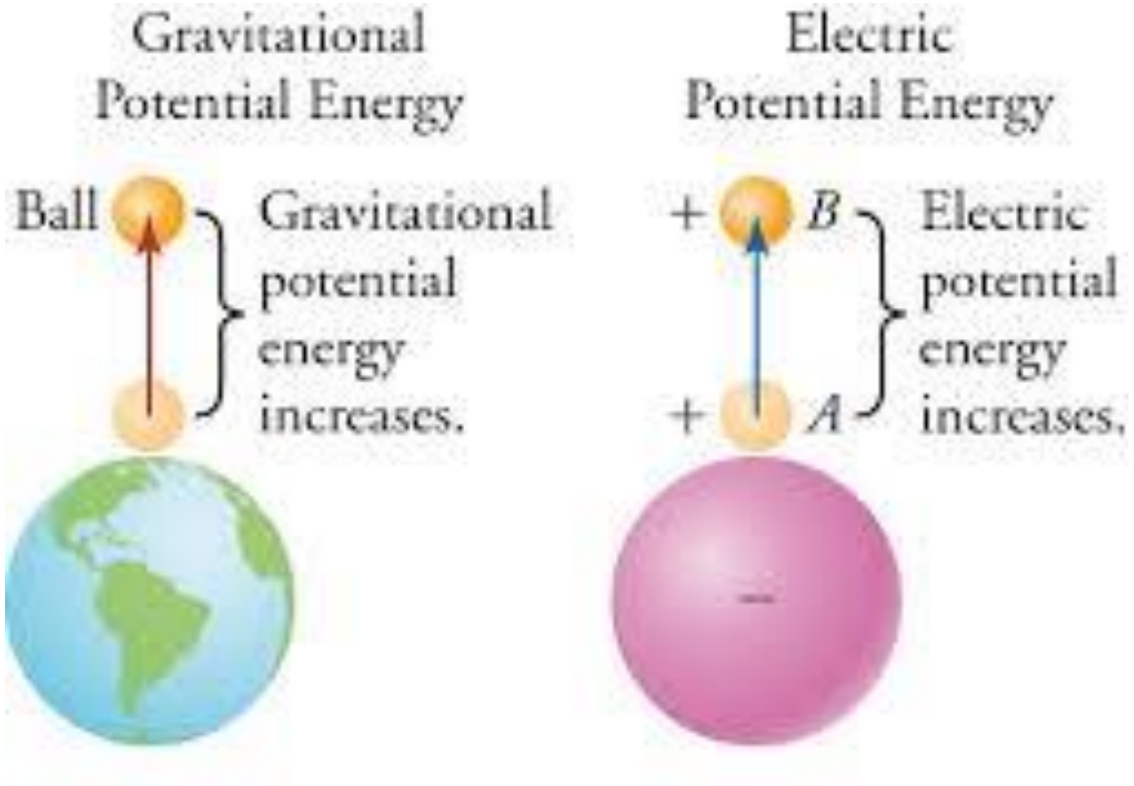
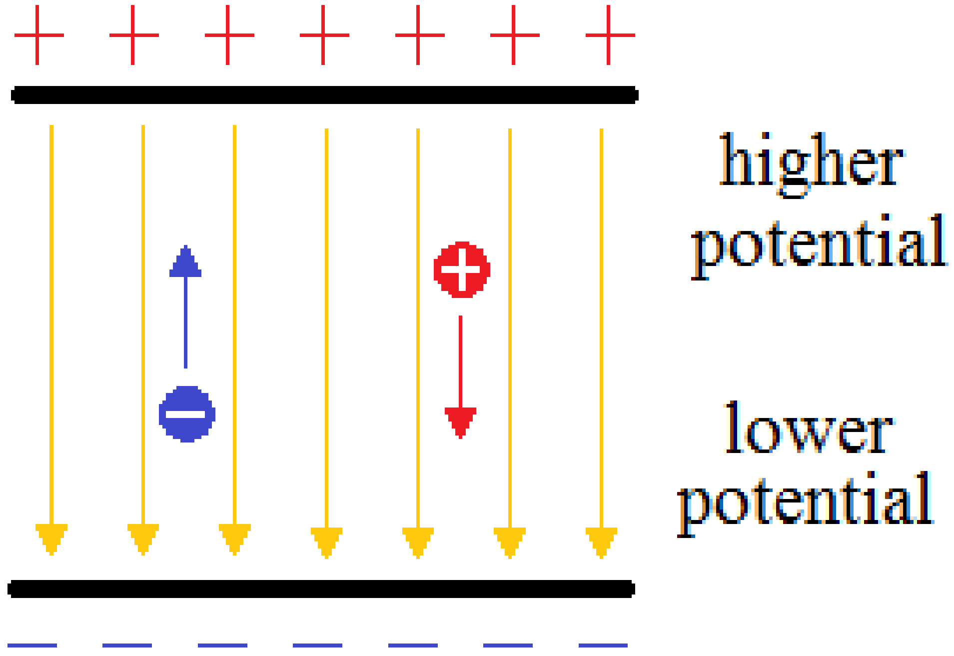
Electric Potential

Web ref provided on slides

Electric potential

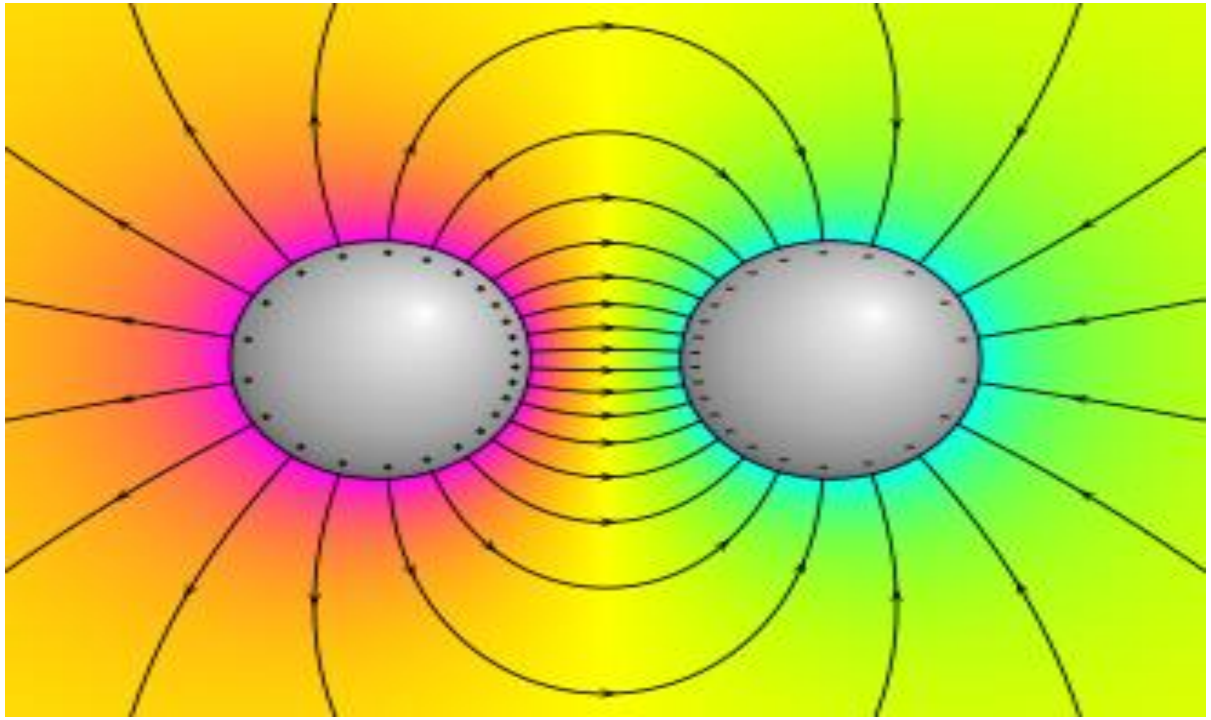




Ref: google image

Electric potential

Electric Potential: An **electric potential** (also called the *electric field potential*, potential drop or the **electrostatic potential**) is the amount of work needed to move a unit of positive charge from a reference point to a specific point inside the field without producing an acceleration.



Electric potential around two spheres at opposite potential. The color coding runs from cyan (negative) through yellow (neutral) to pink (positive).

Ref: wikipedia

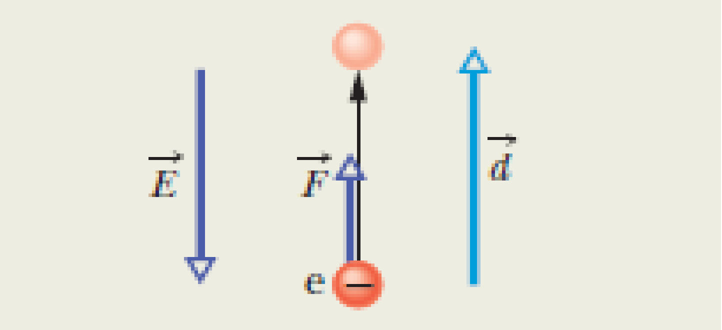
Electric potential energy

Electric Potential Energy: Electric potential energy, or electrostatic potential energy, is a potential energy (measured in joules) that results from conservative Coulomb forces and is **associated** with the configuration of a particular set of point charges within a defined system.

When an electrostatic force acts between two or more charged particles within a system of particles, we can assign an **electric potential energy** *U* to the system. If the system changes its configuration from an initial state *i* to a different final state *f*, the electrostatic force does work *W* on the particles.

$$\Delta U = U_f - Ui = -W$$

- **Problem-1:** Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electric force $\vec{F}$ causes it to move vertically upward due to an electric field $\vec{E}$ with field strength 150 N/C which is directed downward and the displacement of electron is 520 m. Calculate the change in electric potential energy and change in electric potential for which the electron moved.
- Soln: Steps to be followed:



(1) The change ΔU in the electric potential energy of the electron is related to the work W done on the electron by the electric field. Equation 24-5 ($W = -\Delta U$) gives the relation. (2) The work done by a constant force $\vec{F}$ on a particle undergoing a displacement $\vec{d}$ is

$$W = \vec{F} \cdot \vec{d}.$$

(3) The electric force and the electric field are related by the force equation $\vec{F} = q\vec{E}$, where here q is the charge of an electron ($= -1.6 \times 10^{-19}$ C).

Calculations: Substituting the force equation into the work equation and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos \theta,$$

where θ is the angle between the directions of $\vec{E}$ and $\vec{d}$. The field $\vec{E}$ is directed downward and the displacement $\vec{d}$ is directed upward; so $\theta = 180°$. We can now evaluate the work as

$$\begin{aligned} W &= (-1.6 \times 10^{-19}\ \text{C})(150\ \text{N/C})(520\ \text{m}) \cos 180° \\ &= 1.2 \times 10^{-14}\ \text{J}. \end{aligned}$$

Equation 24-5 then yields

$$\Delta U = -W = -1.2 \times 10^{-14}\ \text{J}. \qquad \text{(Answer)}$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by 1.2×10^{-14} J. To find the change in electric potential, we apply Eq. 24-4:

$$\begin{aligned} \Delta V &= \frac{\Delta U}{-q} = \frac{-1.2 \times 10^{-14}\ \text{J}}{-1.6 \times 10^{-19}\ \text{C}} \\ &= 4.5 \times 10^{4}\ \text{V} = 45\ \text{kV}. \qquad \text{(Answer)} \end{aligned}$$

This tells us that the electric force does work to move the electron to a *higher* potential.

Electric potential and energy

Differences between Electric Potential Electric Potential Energy:

The potential energy per unit charge at a point in an electric field is called the **electric potential** V (or simply the **potential**) at that point. Thus,

$$V = \frac{U}{q}.$$

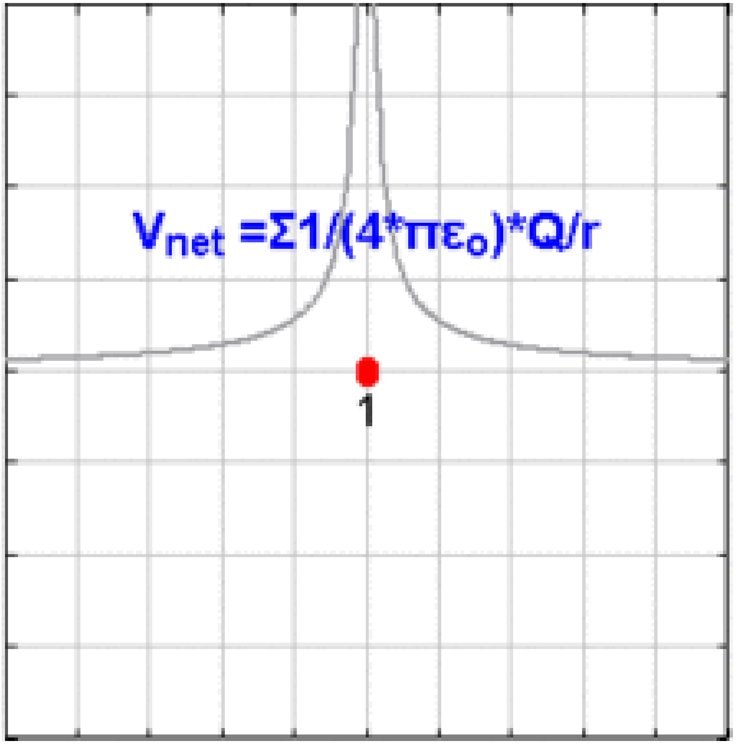
An **electric potential** (also called the electric field potential or the electrostatic potential) is the amount of work needed to move a unit positive charge from a reference point to a specific point inside the field without producing any acceleration. Typically, the reference point is Earth or a point at Infinity, although any point beyond the influence of the electric field charge can be used.

According to theoretical electromagnetics, electric potential is a scalar quantity denoted by V, equal to the electric potential energy of any charged particle at any location (measured in joules) divided by the charge of that particle (measured in coulombs). By dividing out the charge on the particle a remainder is obtained that is a property of the electric field itself.

Electric potential energy, or electrostatic potential energy, is a potential energy (measured in joules) that results from conservative Coulomb forces and is associated with the configuration of a particular set of point charges within a defined system. An object may have electric potential energy by virtue of two key elements: its own electric charge and its relative position to other electrically charged objects.

The term "electric potential energy" is used to describe the potential energy in systems with time-variant electric fields, while the term "electrostatic potential energy" is used to describe the potential energy in systems with time-invariant electric fields.

Electric potential: Equation



The electric potential created by a charge Q is $V=Q/(4\pi\varepsilon_o r)$. Different values of Q will make different values of electric potential V (shown in the image).

Ref: google, Ref book

Electric potential: Due to a point charge

The expression for the electric potential *V* relative to the zero potential at infinity. In an arbitrary electric field $\vec{E}$ a positive test charge q_0 that moves from point *i* to a point *f* has an electrostatic force $q_0\,\vec{E}$. The potential difference between these two points along surface $\vec{ds}$

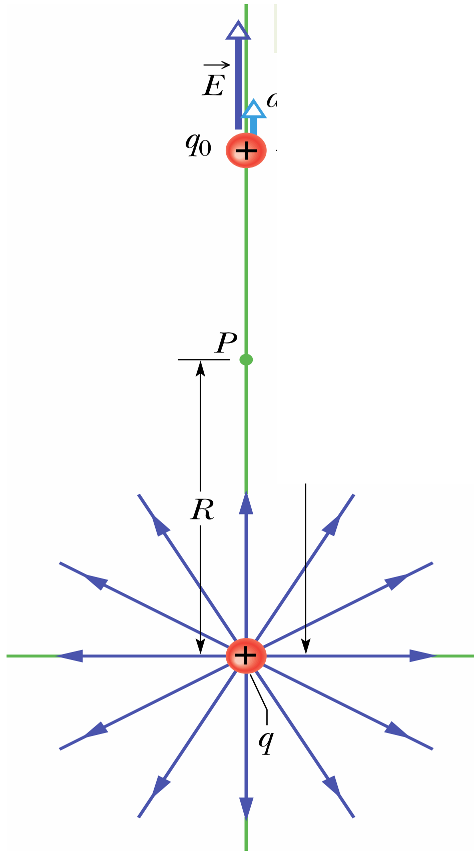
$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}.$$

Consider a point *P* at distance *R* from a fixed particle of positive charge *q*. To move a positive test charge *q*0 from point *P* to infinity the dot product becomes

$$\vec{E}\cdot d\vec{s} = E\cos\theta\, ds.$$

Hence,

$$V_f - V_i = -\int_R^\infty E\,dr.$$



Ref: Ref book

Electric potential: Due to a point charge

The the magnitude of the electric field at the site of the test charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

We use

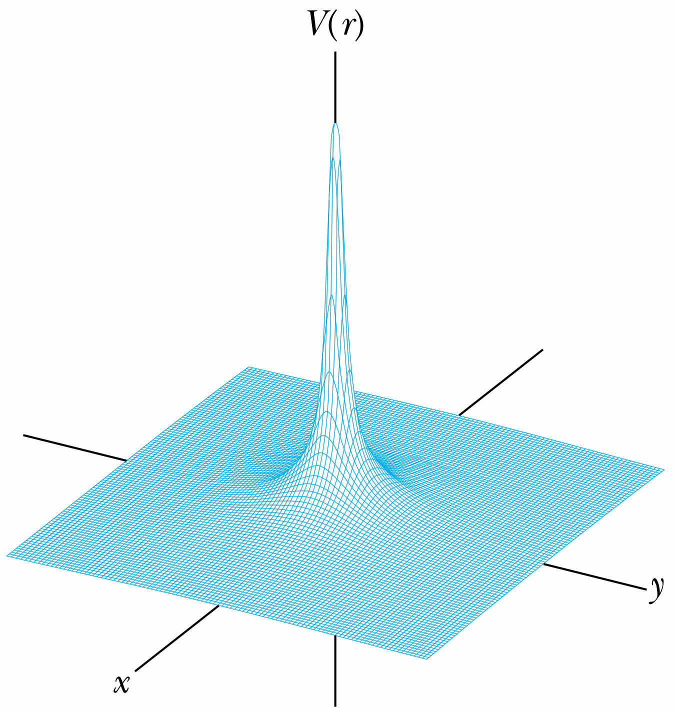
$$V_f = 0 \text{ (at } \infty\text{) and } V_i = V \text{ (at } R\text{)}$$

to have

$$-V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$

that gives

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$



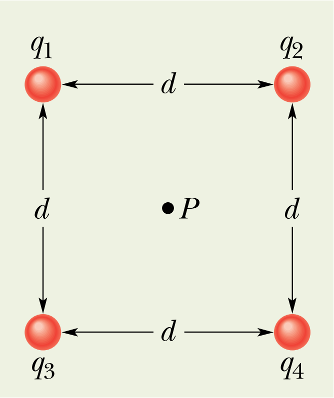
A computer-generated plot of the electric potential $V(r)$ due to a positive point charge located at the origin of an xy plane.

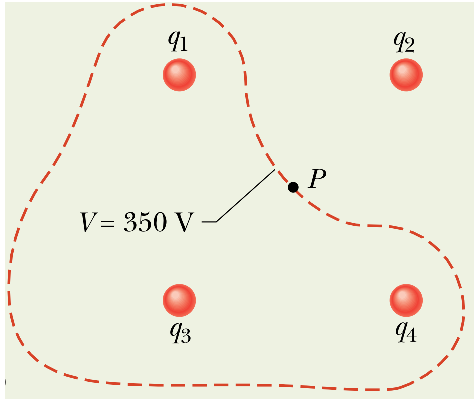
Electric potential: Due to a group of point charges

The potential due to 'n' charges

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \qquad (n \text{ point charges}).$$

Problem 2: What is the electric potential at point *P*, located at the center of the square of point charges shown in figure below? The distance *d* is 1.3 m, and the charges are q_1=12nC, q_2=-24nC, q_3=31nC, and q_4=17nC.





Ans: 350V

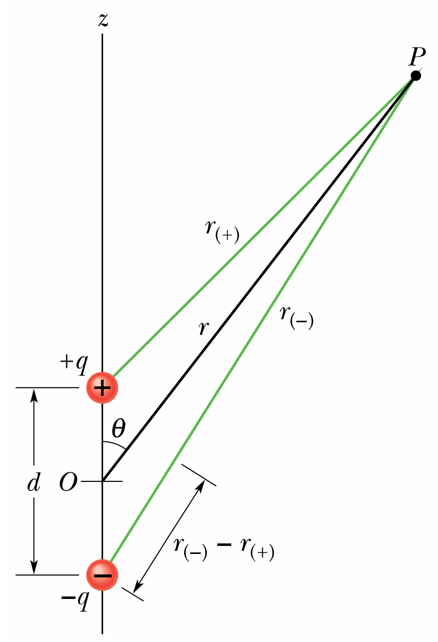
Ref: Ref book

Electric potential: Due to an electric dipole

The potential due to an electric dipole at an arbitrary point *P* in the can be calculated using

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

for *n=2*. That gives for the figures

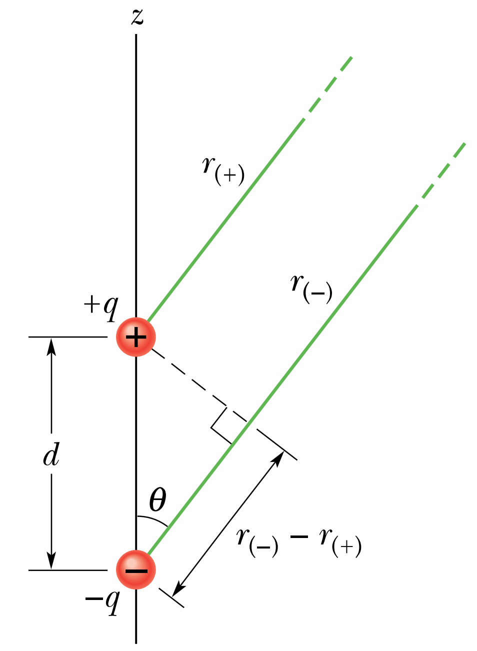


$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{r_{(-)} - r_{(+)}}{r_{(-)} r_{(+)}}.$$

Here, $r_{(-)} - r_{(+)} \approx d\cos\theta$ and $r_{(-)} r_{(+)} \approx r^2$.

Finally, $$V = \frac{q}{4\pi\varepsilon_0} \frac{d\cos\theta}{r^2}$$



Electric potential: Due to an electric dipole

For a dipole moment *p=qd*, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

For continuous charge distribution:

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}.$$

Here *r* is the distance between *P* and a differential element of charge *dq.*

Line of Charge

In Fig. 24-12*a*, a thin nonconducting rod of length *L* has a positive charge of uniform linear density l. Let us determine the electric potential *V* due to the rod at point *P*, a perpendicular distance *d* from the left end of the rod. We consider a differential element *dx* of the rod as shown in Fig. 24-12*b*.This (or any other) element of the rod has a differential charge of

$$dq = \lambda\, dx. \tag{24-33}$$

This element produces an electric potential dV at point P, which is a distance $r = (x^2 + d^2)^{1/2}$ from the element (Fig. 24-12*c*). Treating the element as a point

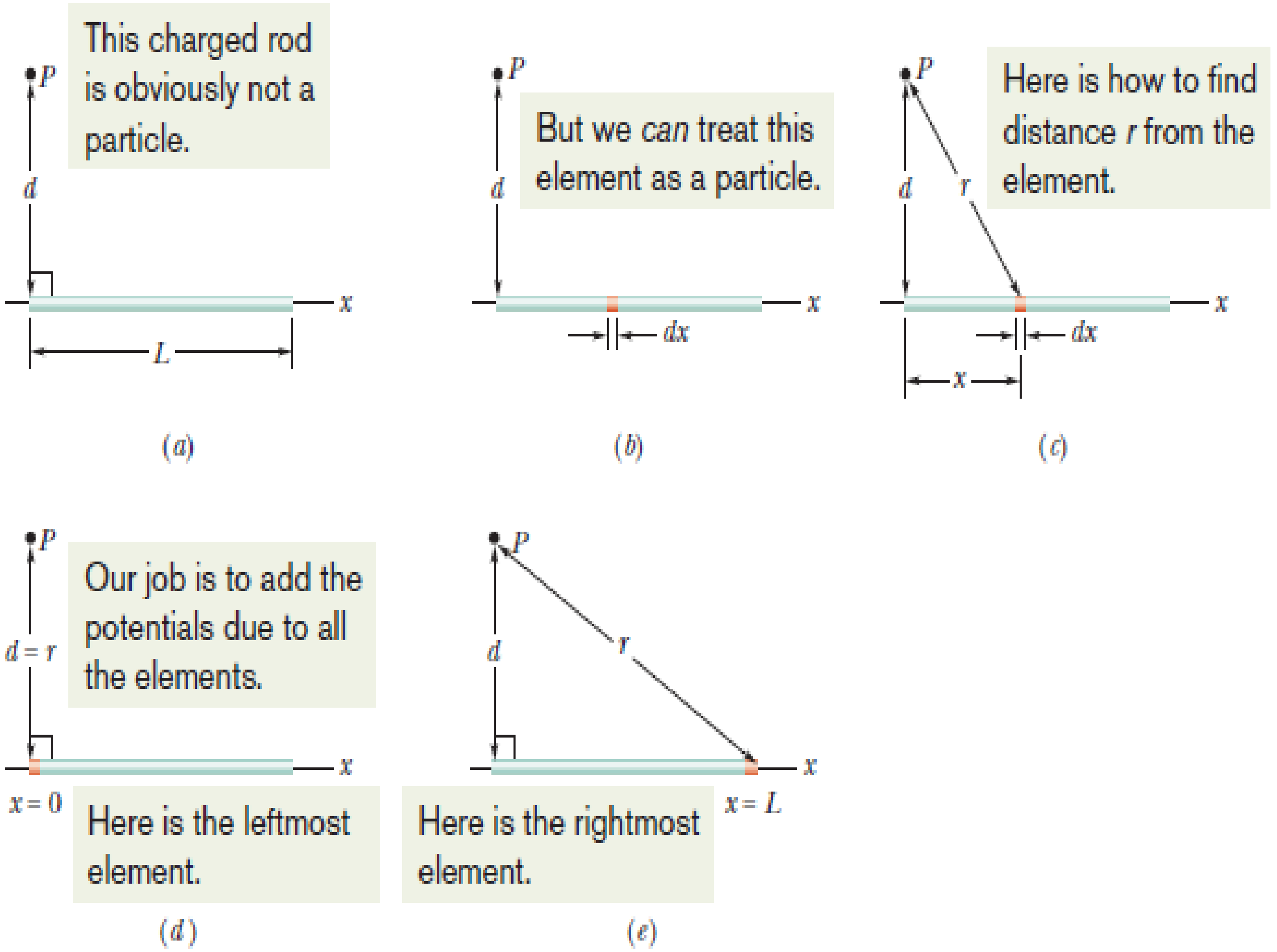


Fig. 24-12 (*a*) A thin, uniformly charged rod produces an electric potential V at point P. (*b*) An element can be treated as a particle. (*c*) The potential at P due to the element depends on the distance r. We need to sum the potentials due to all the elements, from the left side (*d*) to the right side (*e*).

charge, we can use Eq. 24-31 to write the potential dV as

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2 + d^2)^{1/2}}. \qquad (24\text{-}34)$$

Since the charge on the rod is positive and we have taken $V = 0$ at infinity, we know from Section 24-6 that dV in Eq. 24-34 must be positive.

We now find the total potential V produced by the rod at point P by integrating Eq. 24-34 along the length of the rod, from $x = 0$ to $x = L$ (Figs. 24-12d and e), using integral 17 in Appendix E. We find

$$\begin{aligned}
V = \int dV &= \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{(x^2 + d^2)^{1/2}}\, dx \\
&= \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}} \\
&= \frac{\lambda}{4\pi\varepsilon_0} \left[\ln\left(x + (x^2 + d^2)^{1/2} \right) \right]_0^L \\
&= \frac{\lambda}{4\pi\varepsilon_0} \left[\ln\left(L + (L^2 + d^2)^{1/2} \right) - \ln d \right].
\end{aligned}$$

We can simplify this result by using the general relation $\ln A - \ln B = \ln(A/B)$. We then find

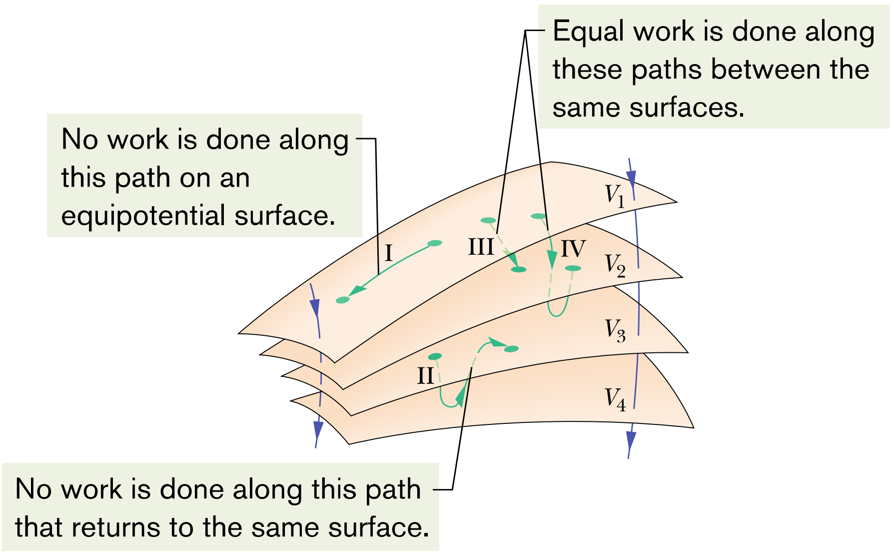
$$V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right]. \tag{24-35}$$

Because V is the sum of positive values of dV, it too is positive, consistent with the logarithm being positive for an argument greater than 1.

Electric potential: Equipotential Surfaces

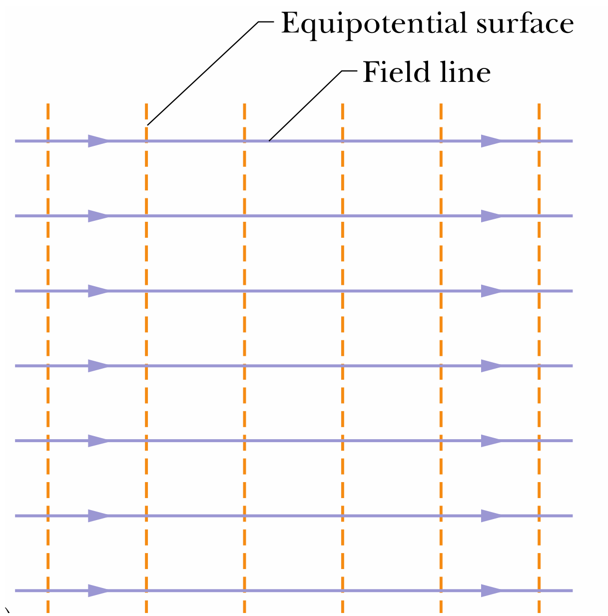
Adjacent points that have the same electric potential form an **equipotential surface,** which can be either an imaginary surface or a real, physical surface.

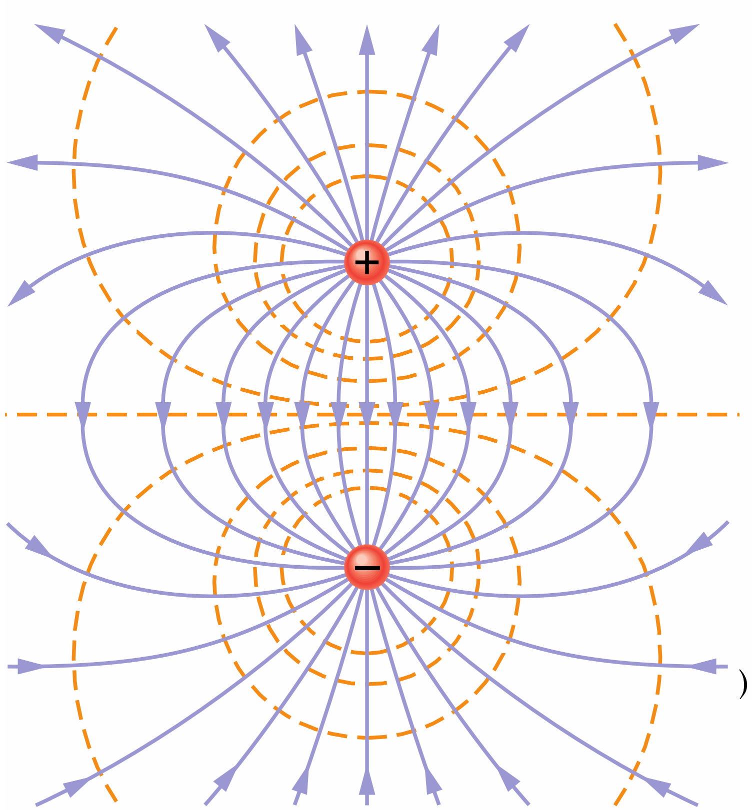
No net work *W* is done on a charged particle by an electric field.

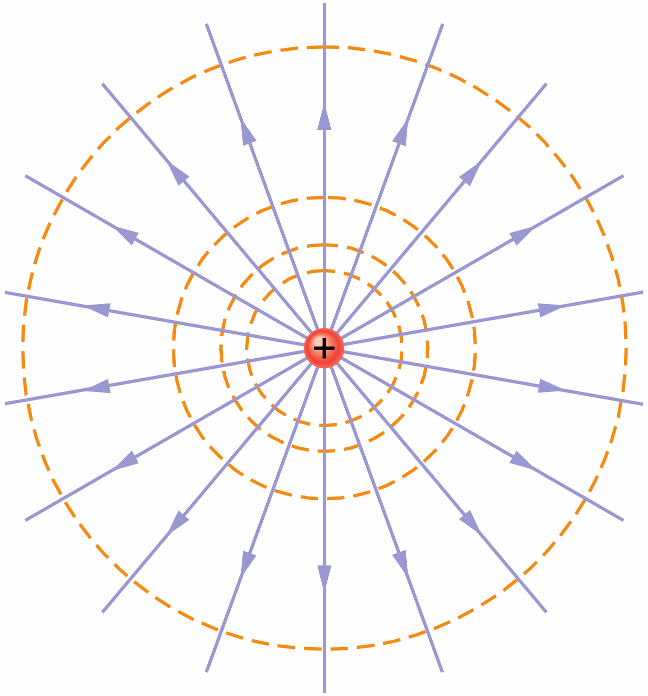


Electric potential: Equipotential Surfaces

Examples:







Electric field lines (purple) and cross sections of equipotential surfaces (gold) for (*a*) a uniform electric field, (*b*) the field due to an electric dipole, and (*c*) the field due to a point charge

Ref: google, Ref book

Electric potential: From electric field

We consider an arbitrary electric field vector $\vec{E}$, represented by the field lines, and the potential difference between any two points *i* and *f* can be calculated from the work done on a positive test charge q_0 by the field as the charge moves from *i* to *f*.

The differential work dW done on a particle by a force $\vec{F}$ during a displacement $d\vec{s}$ is given by

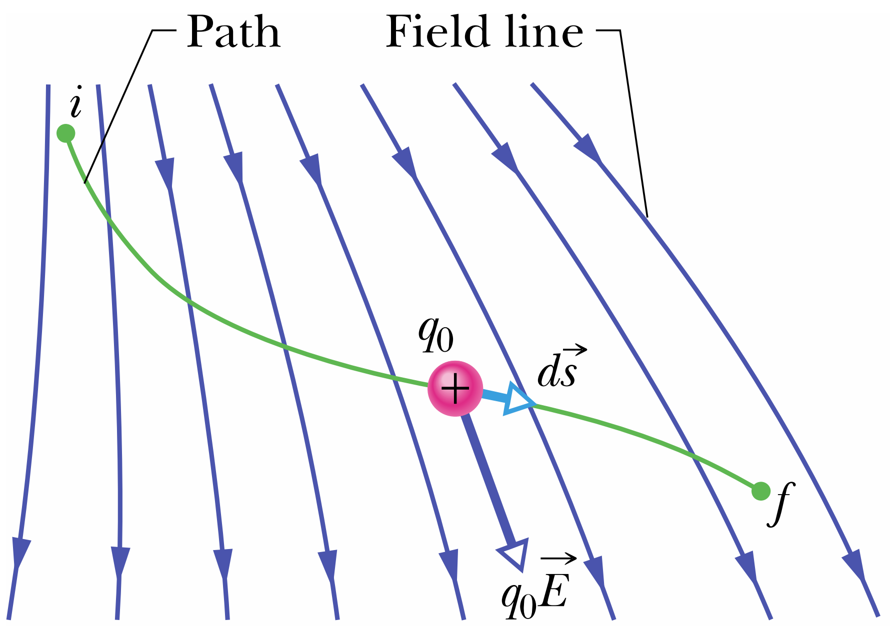
$$dW = \vec{F} \cdot d\vec{s}.$$

or,

$$dW = q_0 \vec{E} \cdot d\vec{s}.$$

To find the total work W done on the particle by the field as the particle moves from point *i* to point *f*, we integrate the works done on the charge

$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}.$$



Ref: google, Ref book

Electric potential: From electric field

Hence, the potential becomes $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$

If we set $V_i=0$, then we have $V = -\int_i^f \vec{E} \cdot d\vec{s}$

in which we have dropped the subscript *f*. And, moving the test charge along the displacement gives $\vec{E} \cdot d\vec{s} = E\, ds \cos\theta = E\, ds.$

This results: $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_i^f E\, ds.$

Or,

$$V_f - V_i = -E \int_i^f ds = -Ed,$$

Where the integral is simply the length *d* of the path.

Electric potential: Gradient

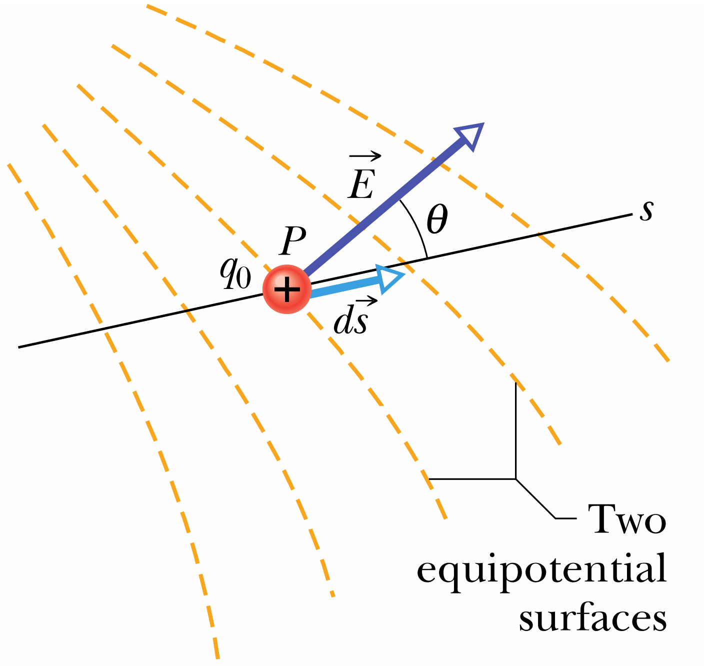
We start from $dW = q_0\vec{E}\cdot d\vec{s}$ for the equipotential surface.

The work the electric field does on the test charge during the move is $q_0 dV$. Equating the two works:

$$-q_0\, dV = q_0 E(\cos\theta)\, ds,$$

which results

$$E\cos\theta = -\frac{dV}{ds}.$$



Since E cosθ is the component of E *vector*, along the displacement vector, the above equation becomes

$$\boxed{E_s = -\frac{\partial V}{\partial s}.}$$

Electric potential: Problems

- **Problem 3:** A charge of -1.0 μC is located on the *y*-axis 1.0 m from the origin at the coordinates (0,1) while a second charge of +1.0 μC is located on the *x*-axis 1.0 m from the origin at the coordinates (1,0). Determine the value of the following quantities at the origin…the magnitude of the electric field,
- the direction of the electric field,
- the electric potential (assuming the potential is zero at infinite distance), and
- the energy needed to bring a +1.0 μC charge to this position from infinitely far away.

https://physics.info/electric-potential/practice.shtml

Electric potential: Problems

- Soln 3: Since the charges are identical in magnitude and equally far from the origin, we can do one computation for both charges.

$$E = \frac{kq}{r^2}$$

E=9000 N/C

Electric field lines come out of positive charges and go into negative charges. At the origin, this results in an electric field that points "left" (away from the positive change) and "up" (toward the negative charge). These two vectors form the legs of a 45°–45°–90° triangle whose sides are in the ratio 1:1:√2.

$\sum E = \sqrt{2} \times 9{,}000$ N/C

- Moving "up" and to the "left" in equal amounts results in a 135° standard angle.
- Once again, since the charges are identical in magnitude and equally far from the origin, we only need to compute one number.

$$V = \frac{kq}{r}$$

$V = 9{,}000$ V

Electric potential: Problems

Electric potential is a scalar quantity. It doesn't have direction, but it does have sign. The positive charge contributes a positive potential and the negative charge contributes a negative potential. Add them up and watch them cancel.

$$\sum V = 9{,}000 \text{ V} - 9{,}000 \text{ V}$$

- The electric potential at a point in space is defined as the work per unit charge required to move a test charge to that location from infinitely far away.

$$\Delta V = \frac{\Delta U_E}{q}$$

Algebra shows that work is charge times potential difference. Since the potential at the origin is zero, no work is required to move a charge to this point. $\Delta U_E = q\Delta V = 0\ J$

https://physics.info/electric-potential/practice.shtml

Electric potential: Problems

- **Prob. 4**: A proton (mass *m*, charge +*e*) and an alpha particle (mass 4*m*, charge +2*e*) approach one another with the same initial speed *v* from an initially large distance. How close will these two particles get to one another before turning around?
- Soln 4: The kinetic energy of the moving particles is completely transformed into electric potential energy at the point of closest approach.

$$U_e = K$$

$$\frac{k(e)(2e)}{r} = \frac{1}{2}(m)v^2 + \frac{1}{2}(4m)v^2$$

$$r = \frac{4ke^2}{5mv^2}$$

Electric potential: Problems

- **Prob. 5:** An electron in the atmosphere is moved upward through the displacement by an electrostatic force due to a downward electric field *E*. Find the potential energy of the electron.

Soln: Sample problem in Fig. 24-1

- **Prob.6:** Figure 24-19 shows three charged particles held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

Soln: P.T.O.

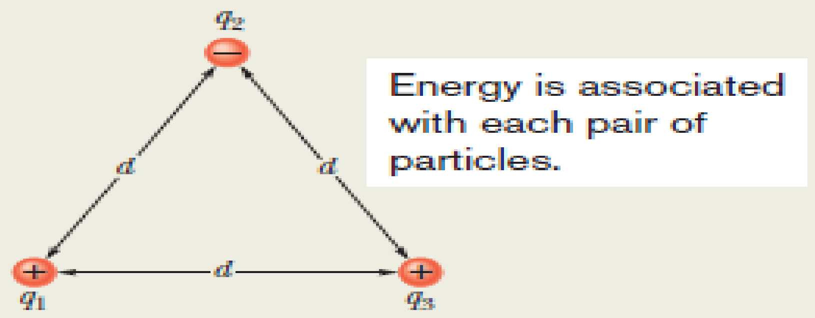


Figure 24-19 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

Electric potential: Problems

Soln:

Figure 24-19 shows three charged particles held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

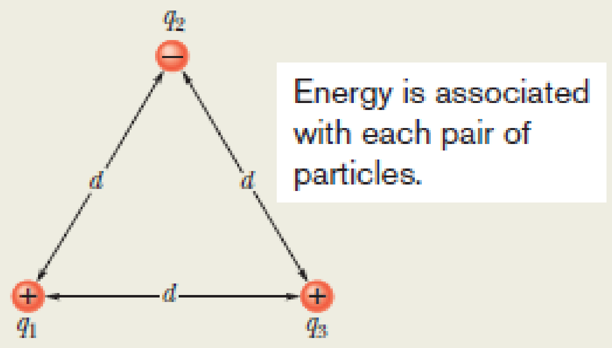


Figure 24-19 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

KEY IDEA

The potential energy U of the system is equal to the work we must do to assemble the system, bringing in each charge from an infinite distance.

Calculations: Let's mentally build the system of Fig. 24-19, starting with one of the charges, say q_1, in place and the others at infinity. Then we bring another one, say q_2, in from infinity and put it in place. From Eq. 24-46 with d substituted for r, the potential energy U_{12} associated with the pair of charges q_1 and q_2 is

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}.$$

We then bring the last charge q_3 in from infinity and put it in place. The work that we must do in this last step is equal to the sum of the work we must do to bring q_3 near q_1 and the work we must do to bring it near q_2. From Eq. 24-46, with d substituted for r, that sum is

$$W_{13} + W_{23} = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{d} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{d}.$$

The total potential energy U of the three-charge system is the sum of the potential energies associated with the three pairs of charges. This sum (which is actually independent of the order in which the charges are brought together) is

Electric potential: Problems

$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right)$$

$$= -\frac{10q^2}{4\pi\varepsilon_0 d}$$

$$= -\frac{(8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})(10)(150 \times 10^{-9}\ \mathrm{C})^2}{0.12\ \mathrm{m}}$$

$$= -1.7 \times 10^{-2}\ \mathrm{J} = -17\ \mathrm{mJ}. \qquad \text{(Answer)}$$

The negative potential energy means that negative work would have to be done to assemble this structure, starting with the three charges infinitely separated and at rest. Put another way, an external agent would have to do 17 mJ of positive work to disassemble the structure completely, ending with the three charges infinitely far apart.

The lesson here is this: If you are given an assembly of charged particles, you can find the potential energy of the assembly by finding the potential of every possible pair of the particles and then summing the results.