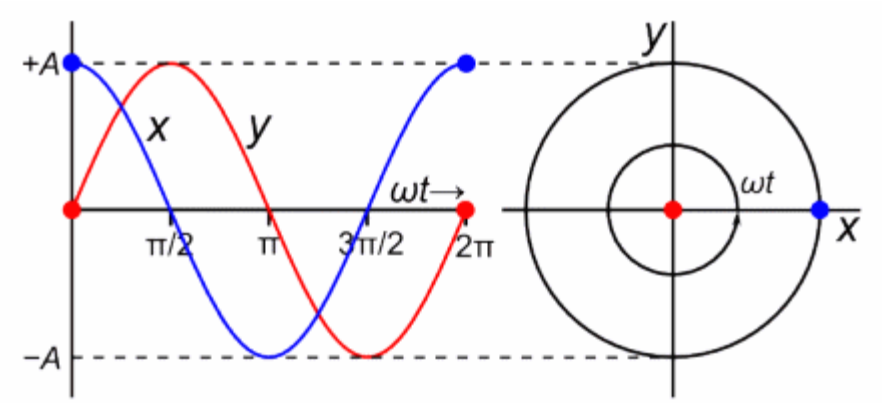
United International **University**

A Complete Guideline for Graphical Phase Change analysis in Simple Harmonic Motion



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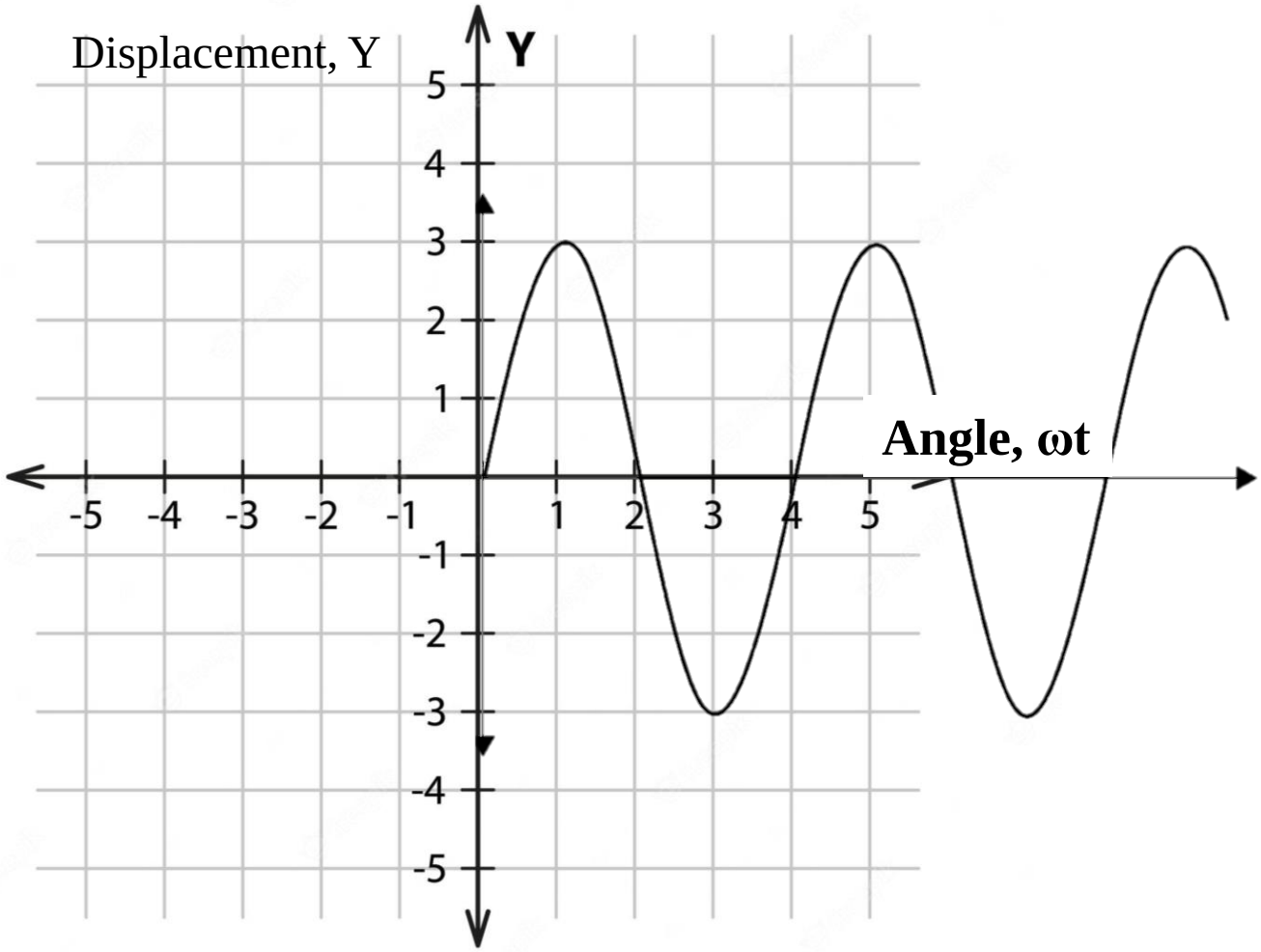
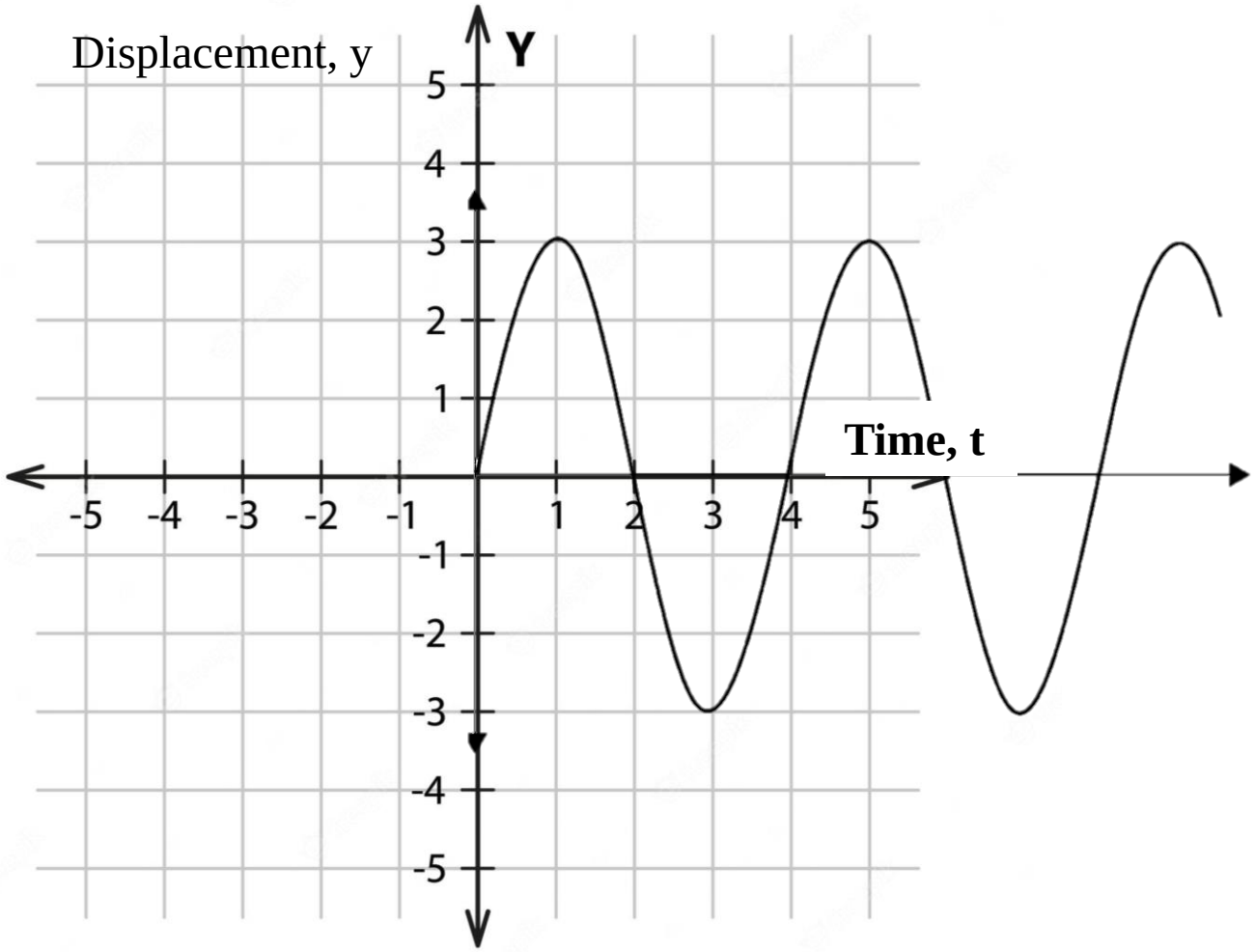
Let us consider the displacement of a simple harmonic oscillator is, **$y = A \sin\omega t$**
Let's see the graph can be represented as



We can also plot this **y = A sinωt** graph for y vs. t and the graph will be like the figure below



Now if we consider 2nd particle with $\mathbf{y = A \sin(\omega t+\varphi)}$

Displacement, $\mathbf{y = A \sin(\omega t+\varphi)}$

Now if we want to calculate the value of wt for which the value of **y= 0**

For $\mathbf{y = 0}$, $\mathbf{\sin(\omega t+\varphi) = 0}$

$$\mathbf{(\omega t+\varphi) = 0}$$

$$\mathbf{\omega t = - \varphi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)}$$

So the wave pattern of the particle will originates from an angle, $\mathbf{\omega t = - \varphi}$ if we consider four cases of displacements for graphical representation

(i) $\mathbf{y = A \sin(\omega t + \frac{\pi}{4})}$

(ii) $\mathbf{y = A \sin(\omega t - \frac{\pi}{4})}$

(iii) $\mathbf{y = A \sin(\omega t + \frac{\pi}{2})}$

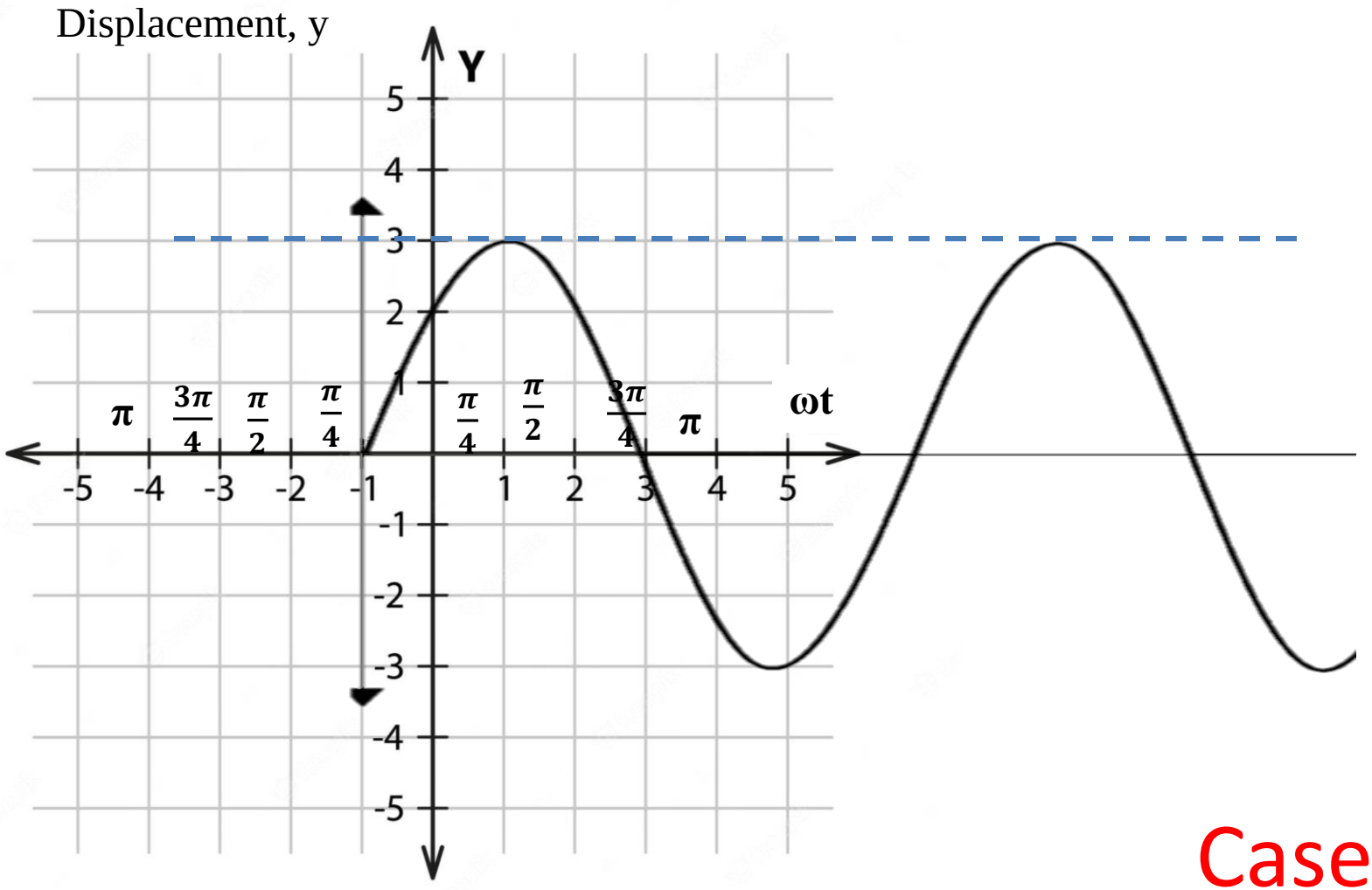
(iv) $\mathbf{y = A \sin(\omega t + \frac{\pi}{3})}$

Please note that, you must plot the graph from $\omega t = 0$ position. We have only provided the negative value of ωt for primary understanding and practice purpose. But you must start the graph from wt = 0 when you understand the patterns.

Case (i)

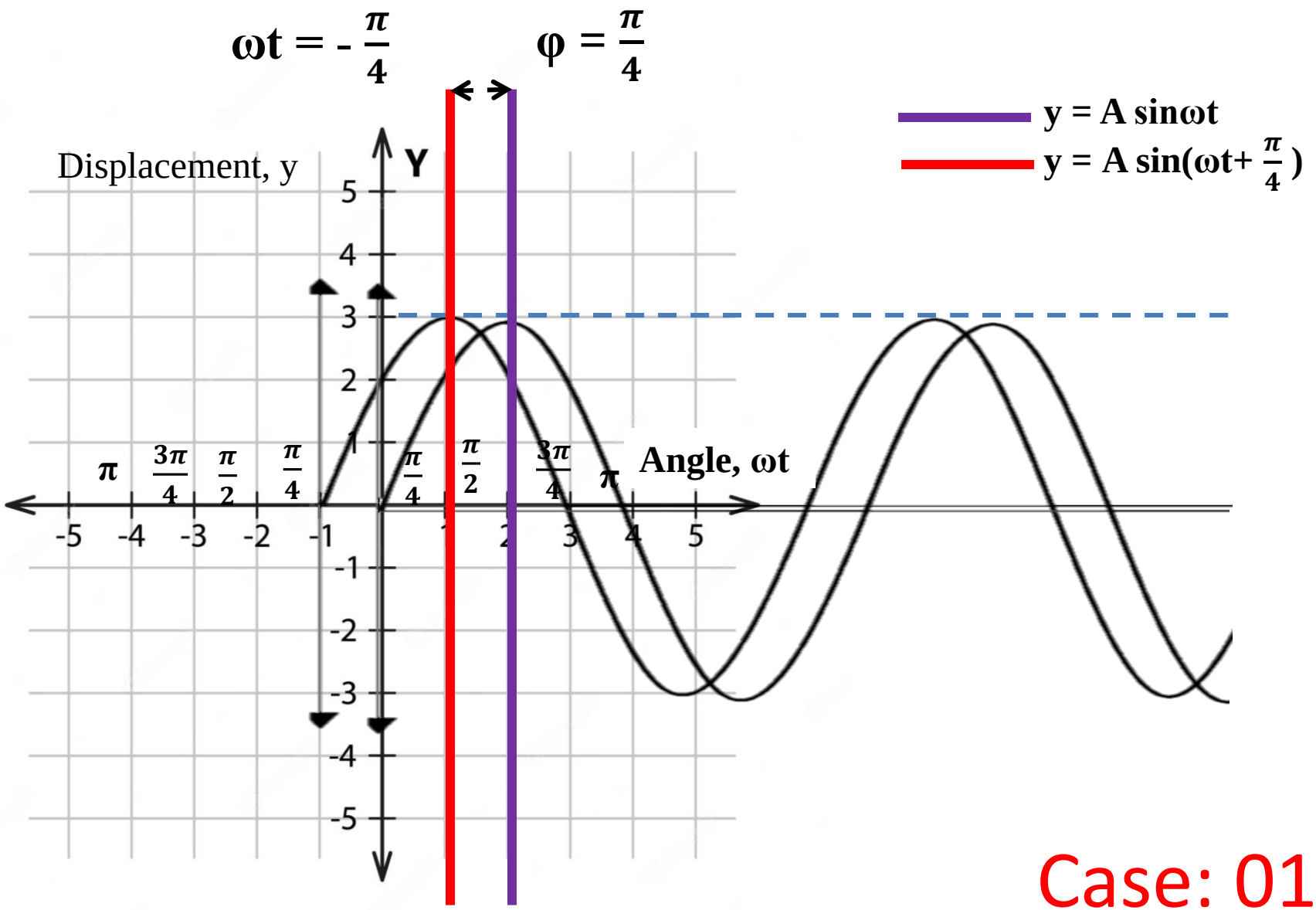
$y = A \sin(\omega t + \frac{\pi}{4})$

From Eq^n (1), the graph will start from $\omega t = -\frac{\pi}{4}$



Case: 01

Displacement y vs. Angle ωt

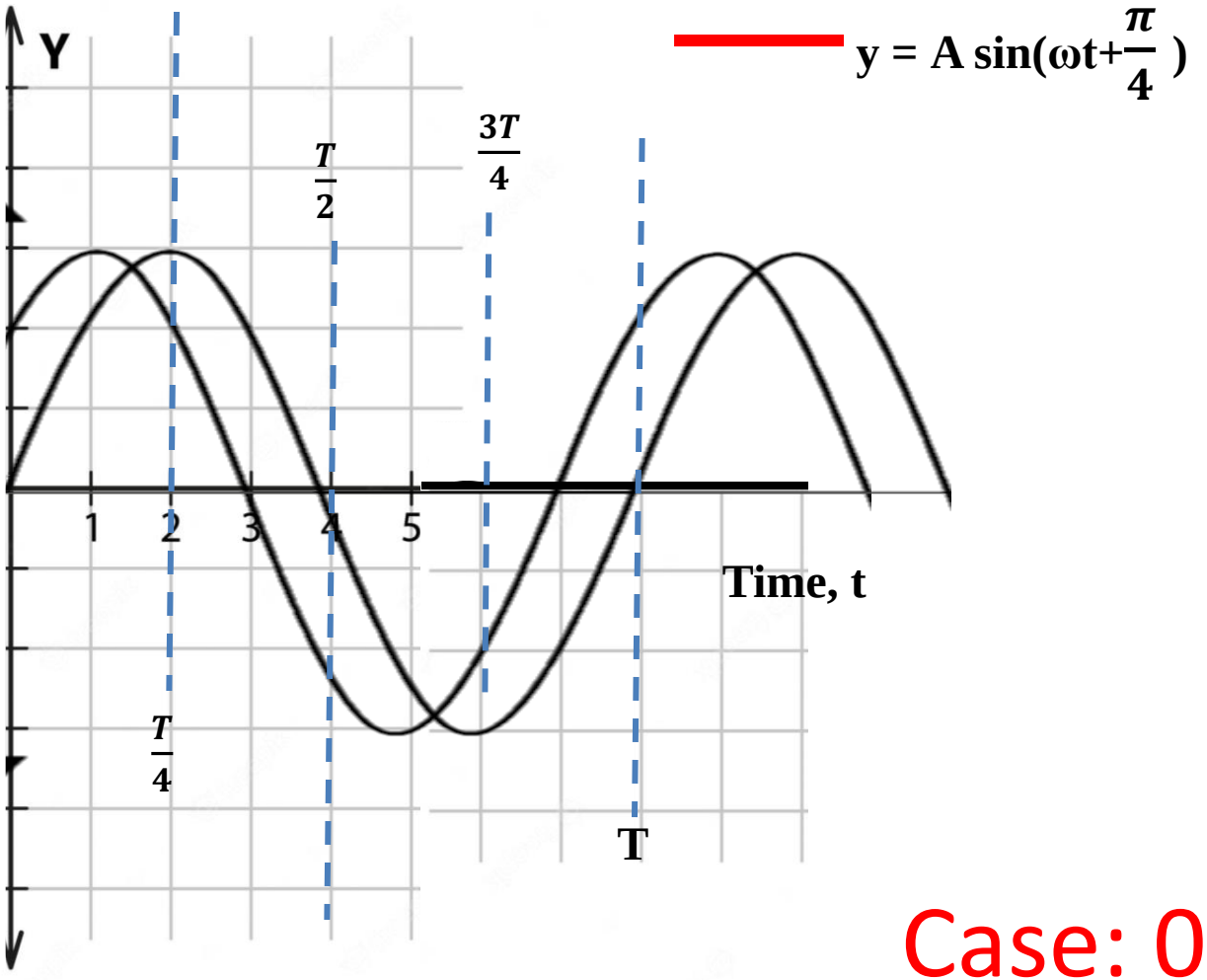


Case: 01

Displacement y vs. Time t

$\omega t = -\frac{\pi}{4}$ $\varphi = \frac{\pi}{4}$

Displacement, y

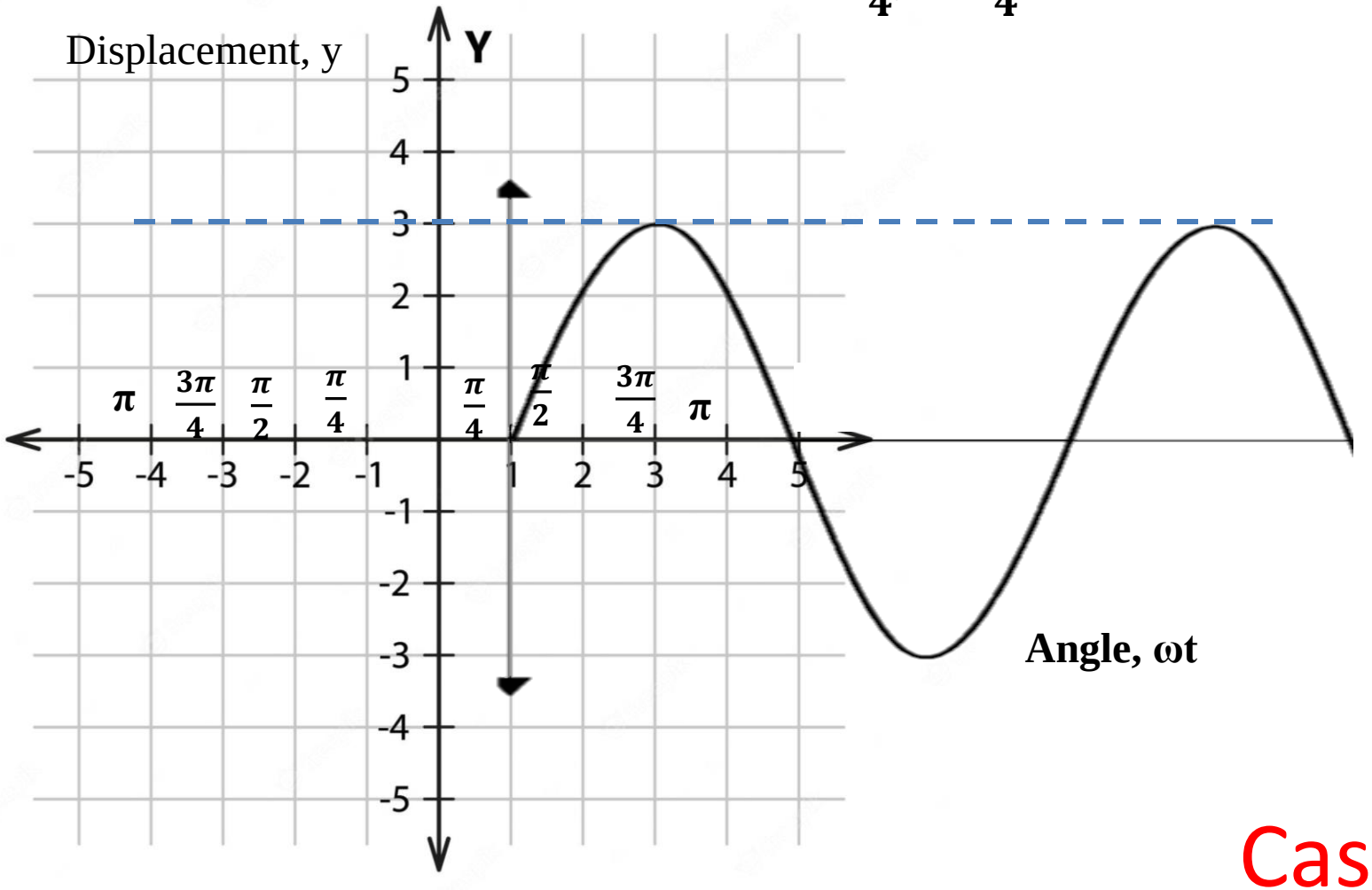


Case: 01

Case (ii)

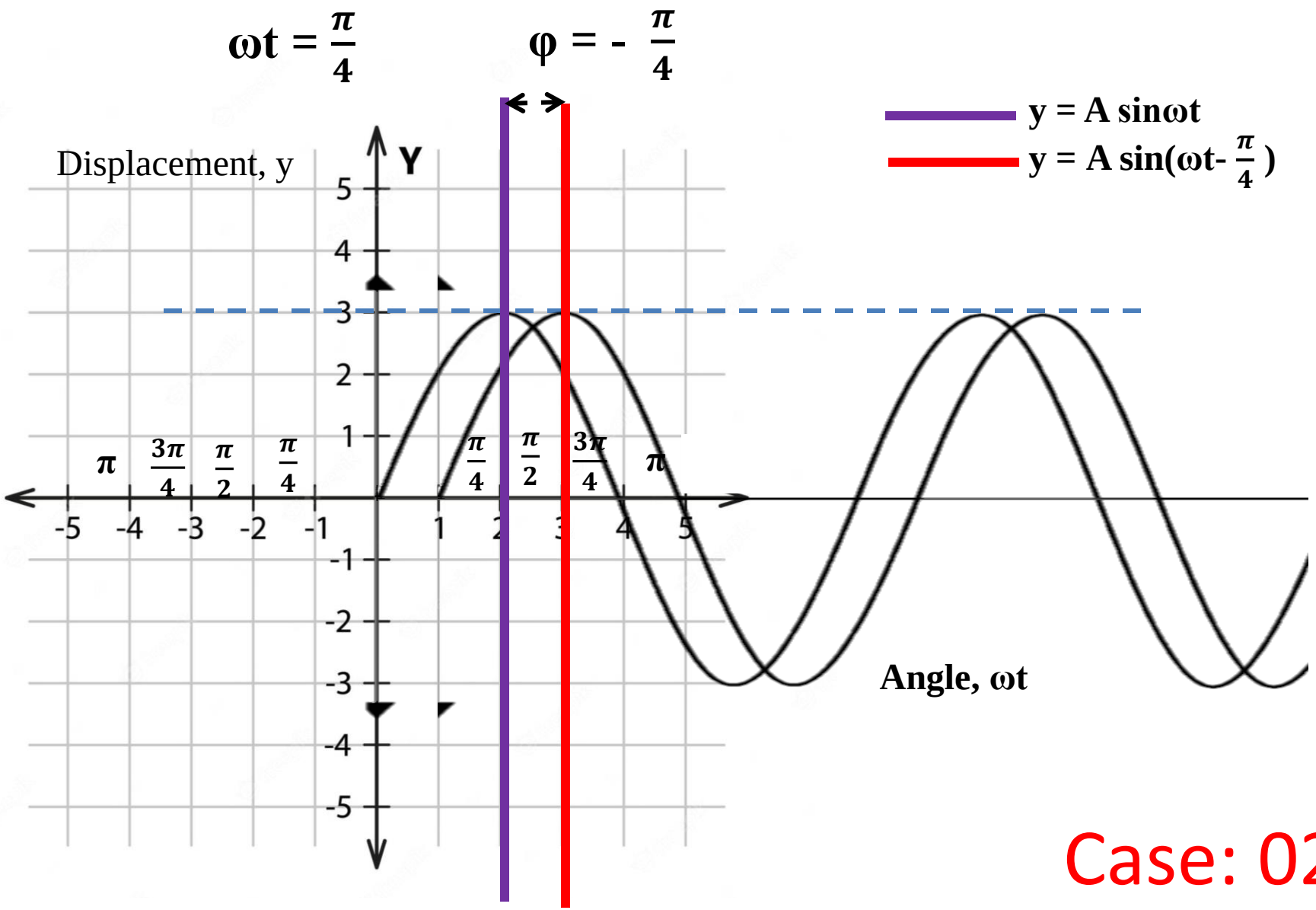
$y = A \sin(\omega t - \frac{\pi}{4})$

From Eq^{n} (1), the graph will start from $\omega t = -(-\frac{\pi}{4}) = \frac{\pi}{4}$



Case: 02

Displacement y vs. Angle ωt

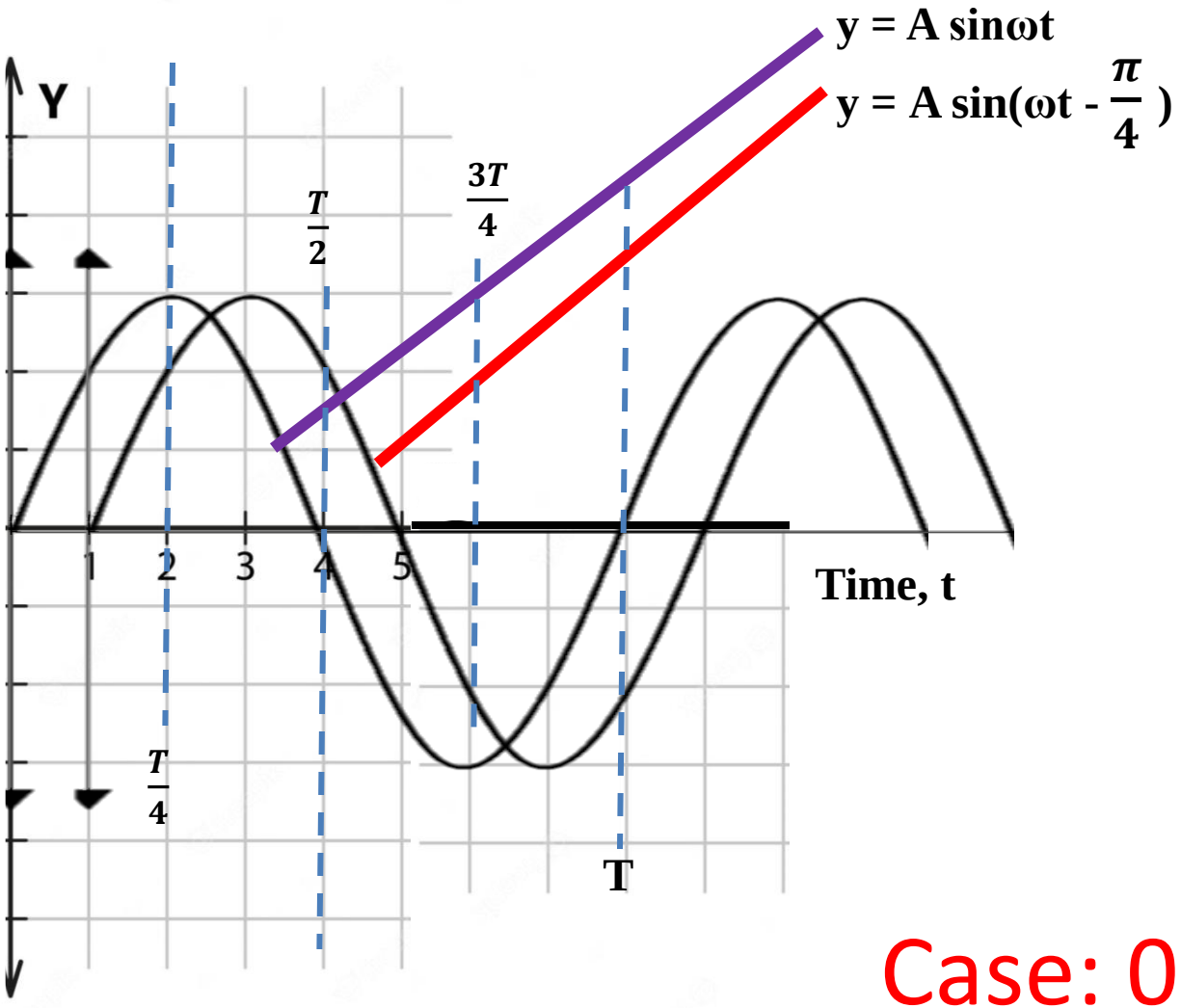


Case: 02

Displacement y vs. Time t

$\omega t = \frac{\pi}{4}$ $\varphi = -\frac{\pi}{4}$

Displacement, y

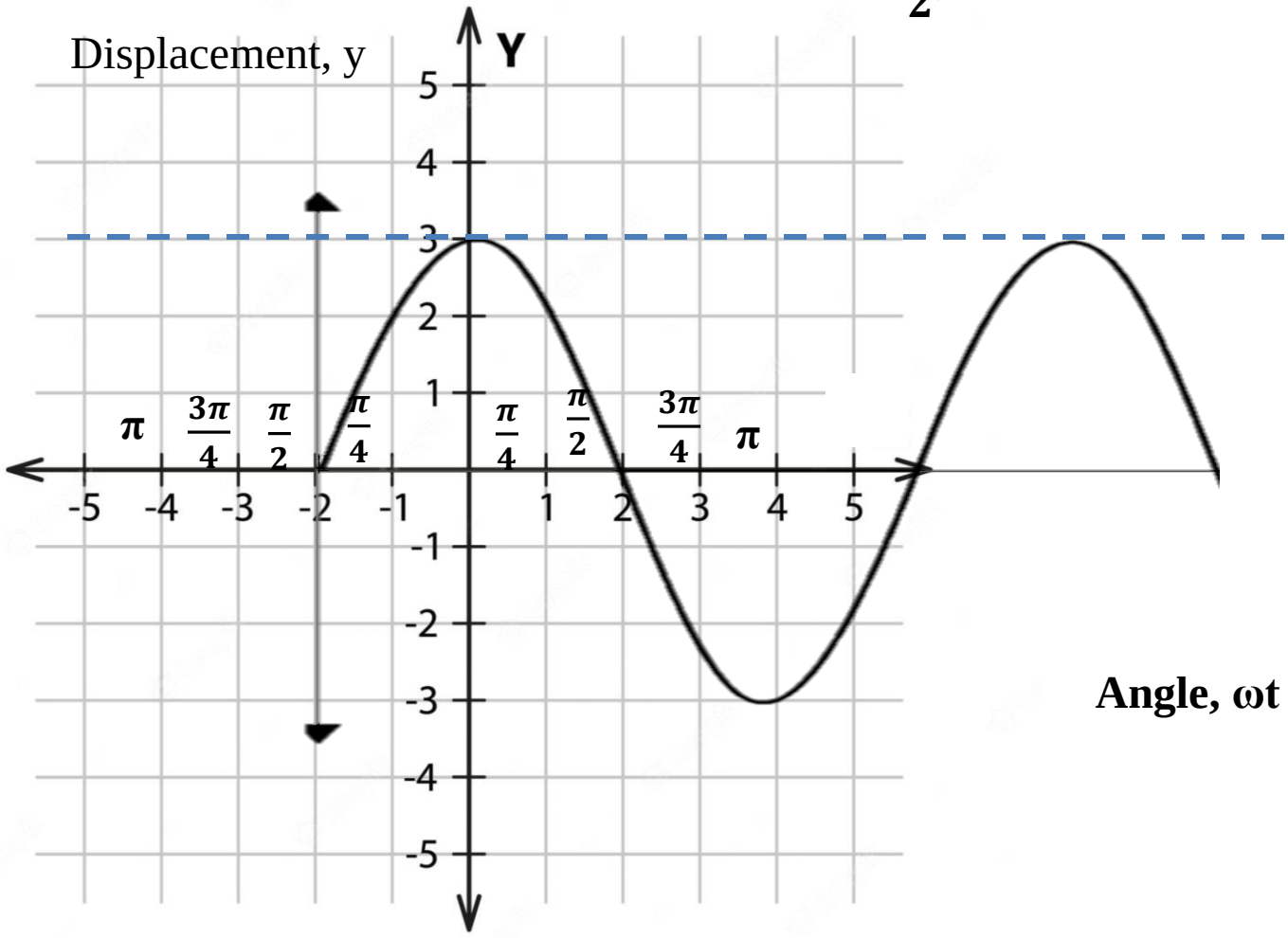


Case: 02

Case (iii)

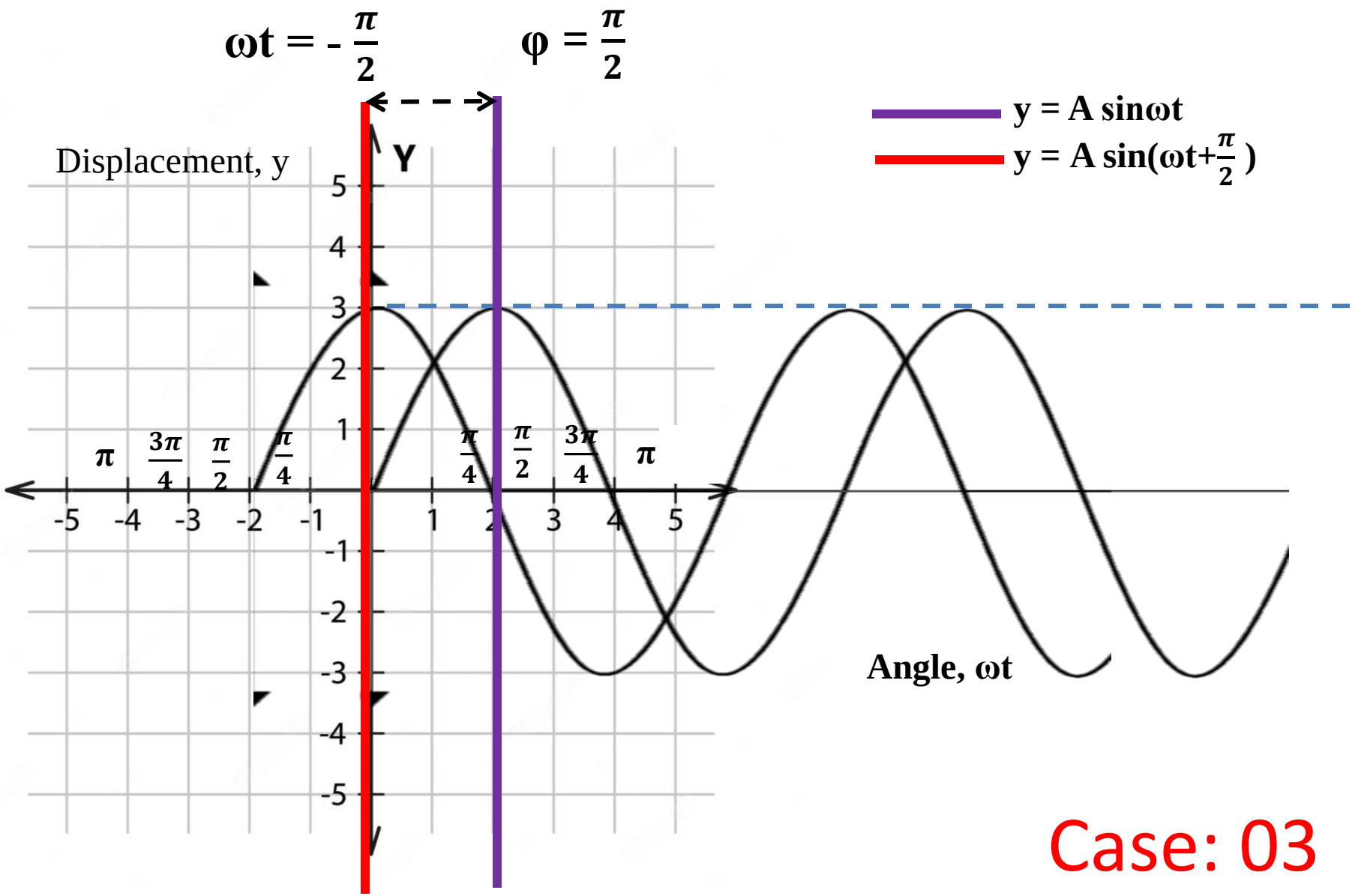
$y = A \sin(\omega t + \frac{\pi}{2})$

From Eq^{n} (1), the graph will start from $\omega t = -(\frac{\pi}{2})$



Case: 03

Displacement y vs. Angle ωt

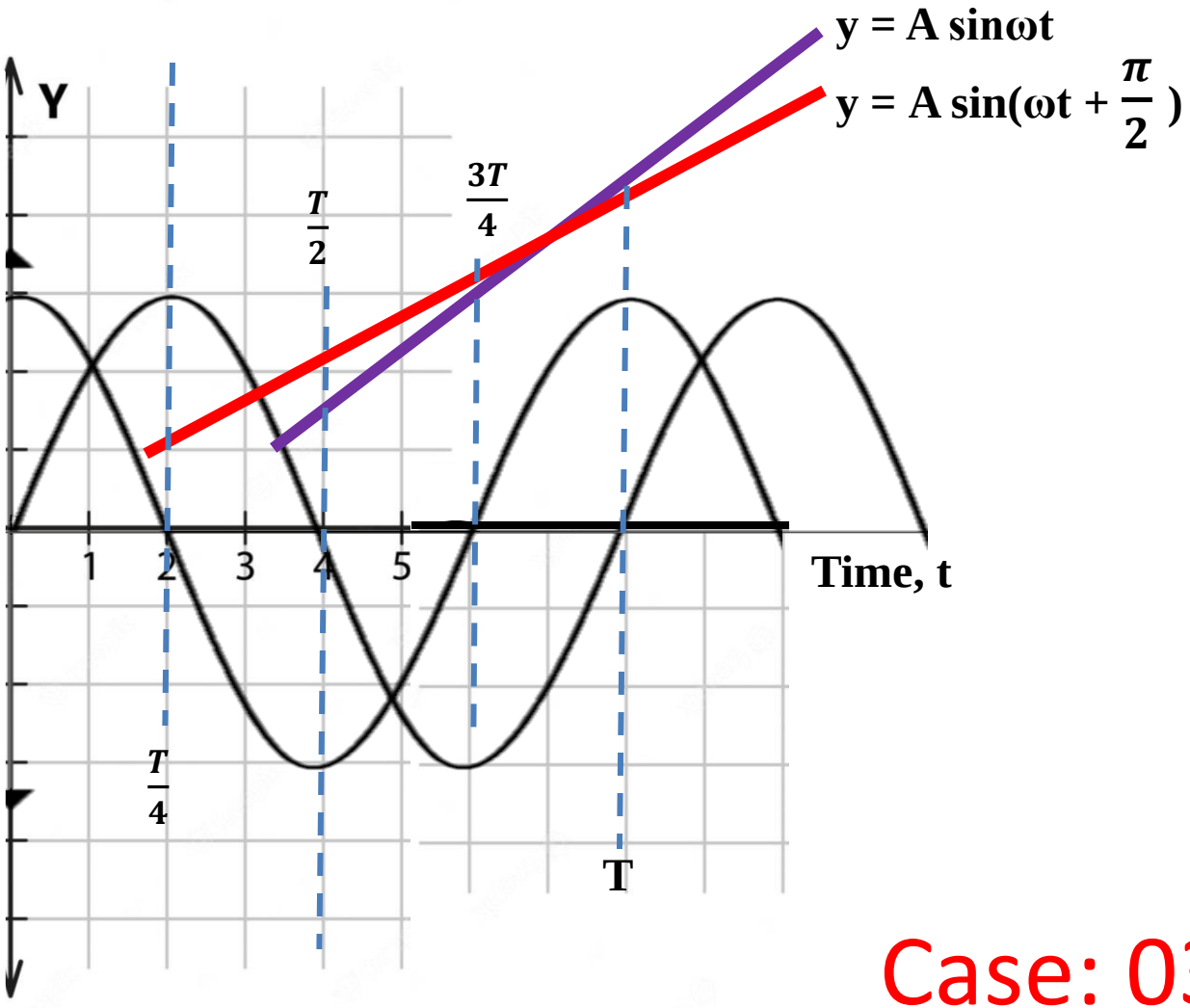


Case: 03

Displacement y vs. Time t

$\omega t = -\frac{\pi}{2}$ $\varphi = \frac{\pi}{2}$

Displacement, y

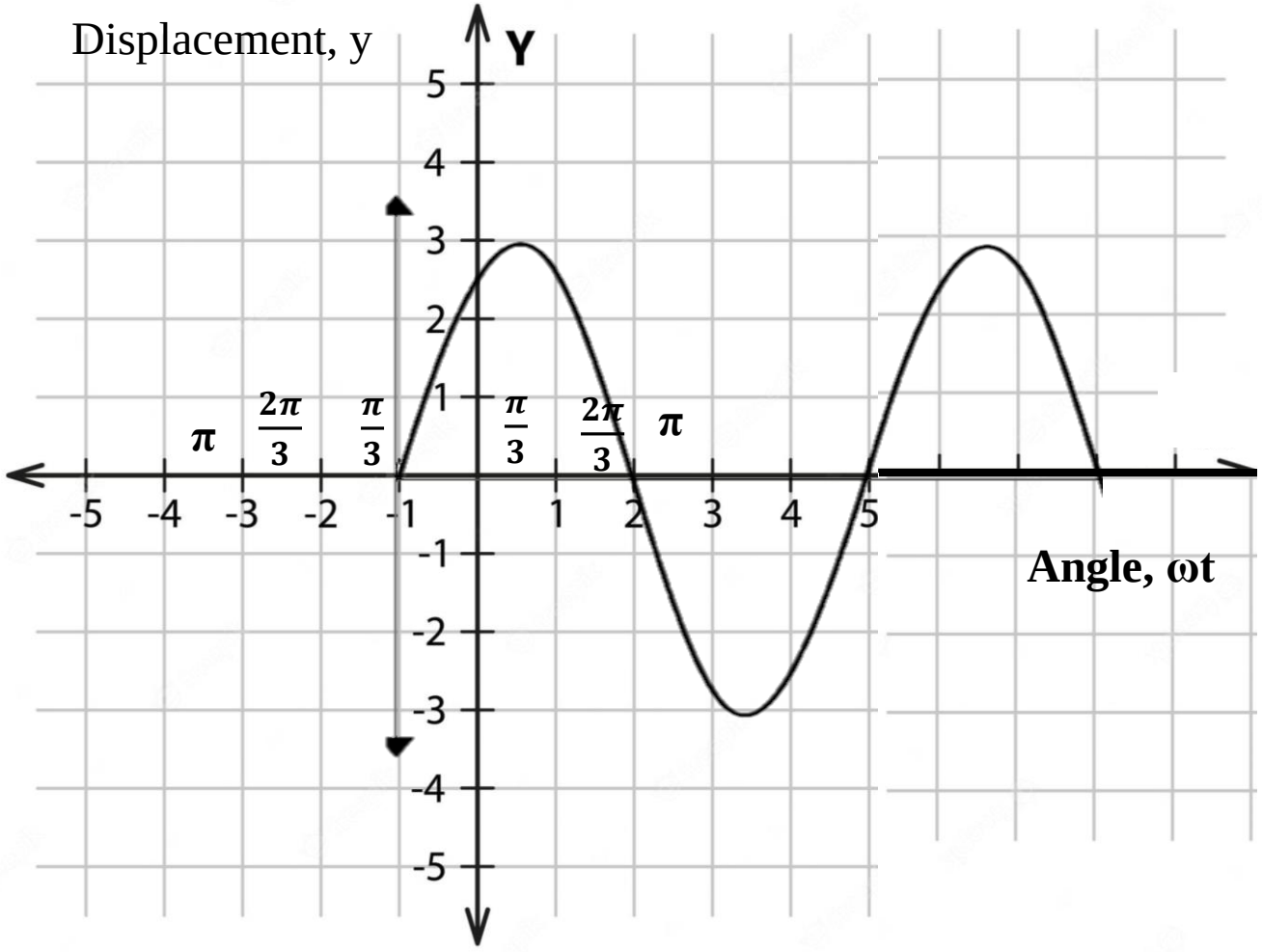


Case: 03

Case (iv):

Now if we consider the particle with $y = A \sin(\omega t + \frac{\pi}{3})$

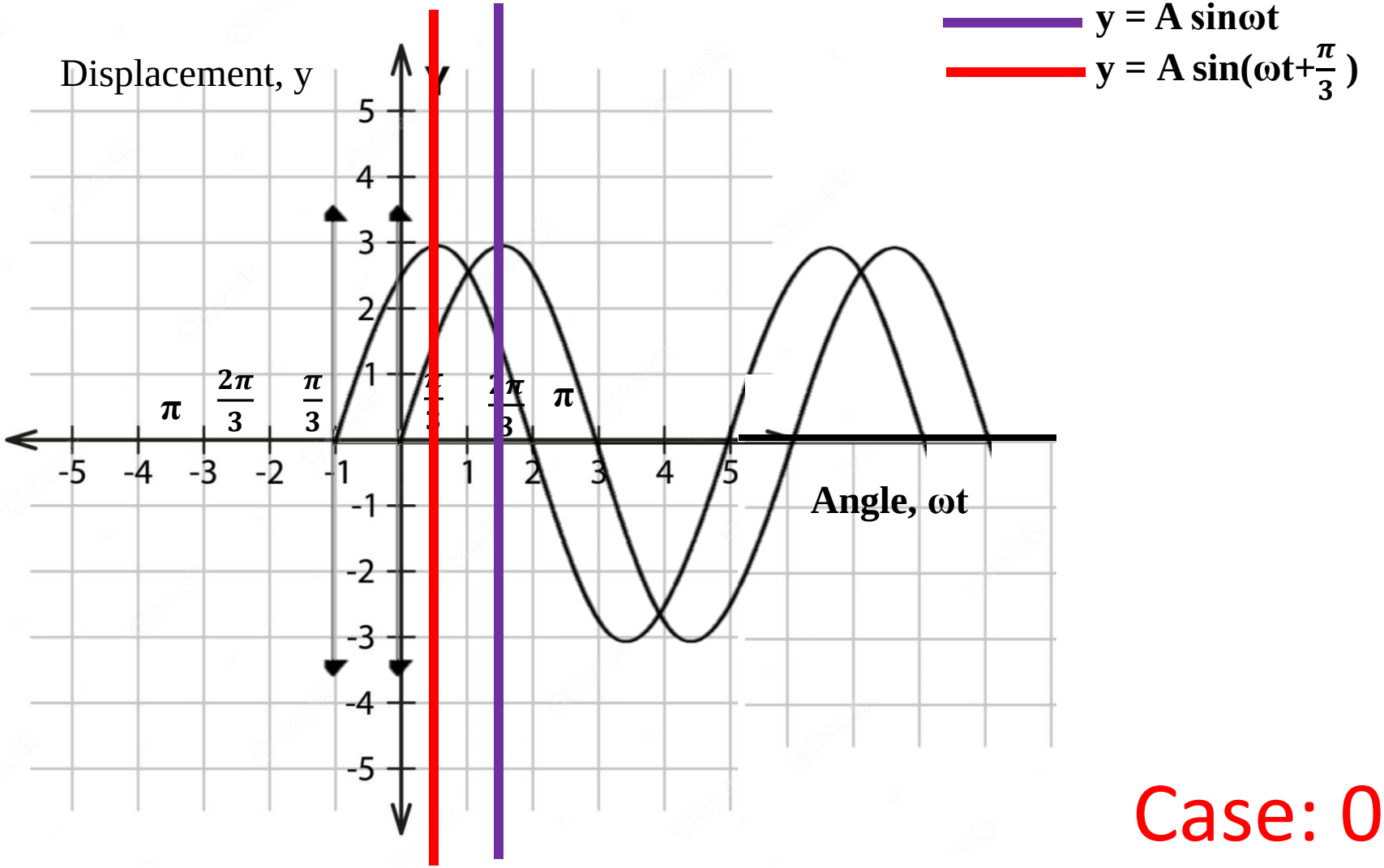
graph for **y** vs. **ωt** and the graph will be like the figure below



Case: 04

Displacement y vs. Angle ωt

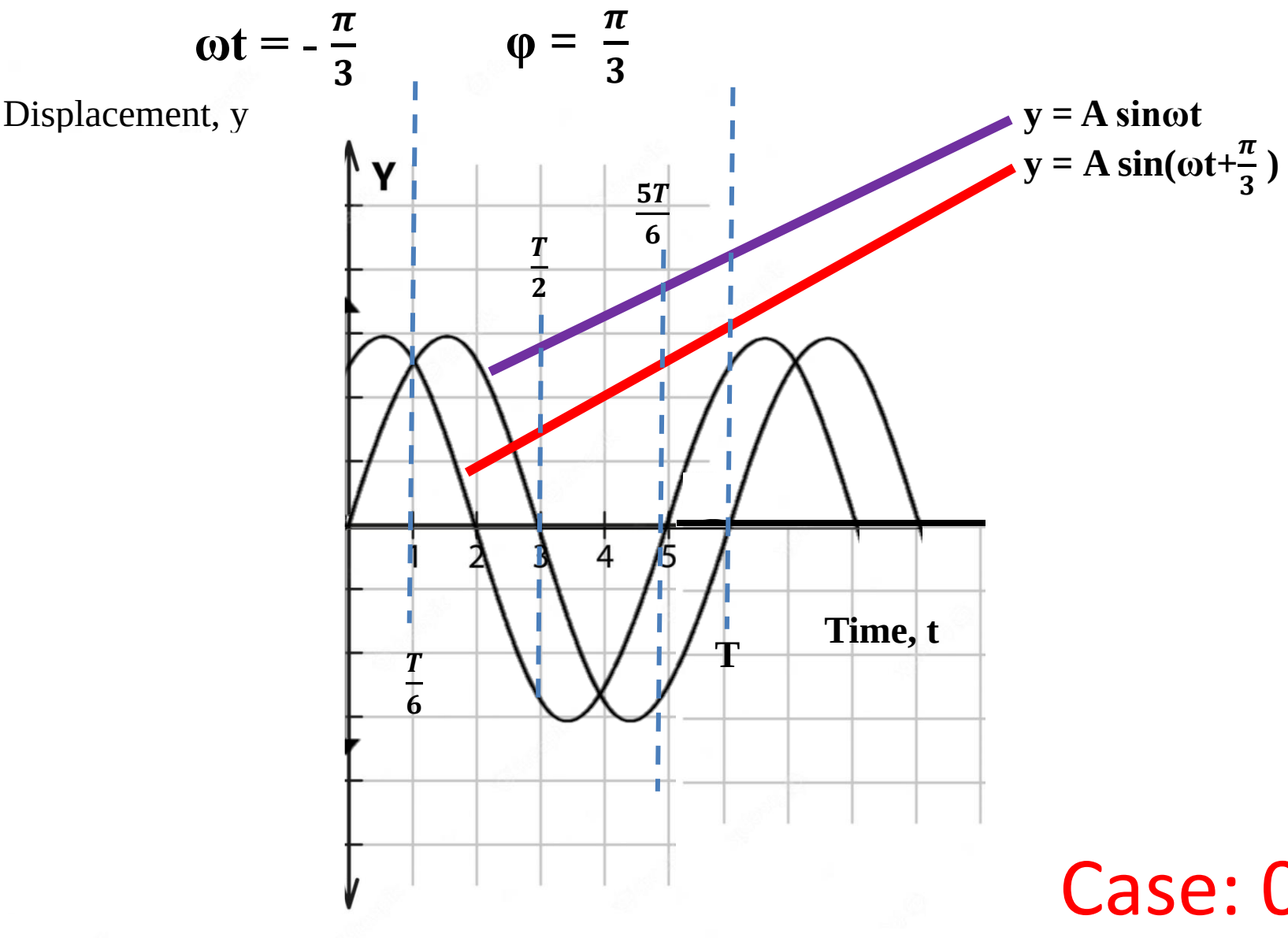
$\omega t = -\frac{\pi}{3}$ $\varphi = \frac{\pi}{3}$



Case: 04

Displacement y vs. time t

$\omega t = -\frac{\pi}{3}$ $\varphi = \frac{\pi}{3}$



Case: 04

Some Practice Problems

(i) $\mathbf{y = A \sin(\omega t - \frac{\pi}{3})}$

(ii) $\mathbf{y = A \sin(\omega t - \frac{\pi}{2})}$

(iii) $\mathbf{y = A \sin(\omega t + \pi)}$

(iv) $\mathbf{y = A \sin(\omega t - \pi)}$

(v) $\mathbf{y = A \sin(\omega t + \frac{3\pi}{4})}$

(vi) $\mathbf{y = A \sin(\omega t - \frac{3\pi}{4})}$

(vii) $\mathbf{y = A \sin(\omega t + \frac{3\pi}{2})}$

(viii) $\mathbf{y = A \sin(\omega t + 2\pi)}$

Please try these figures. If you face any problem please contact me in the counseling hours