

## United International University School of Science and Engineering

Final Examination Trimester: Fall 2022

Course Title: Coordinate Geometry and Vector Analysis

Course Code: Math 2201 Marks: 40

**Total Time: 2 hours** 



Answer all questions.

[6]

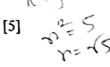
- a) Consider,  $F(x, y) = 2e^{2x} \sin y \, i + e^{2x} \cos y \, j$ i) Show that F is a conservative vector field on the entire xy -plane.
  - Find the potential function  $\phi(x, y)$ . ii)
  - Find  $\int_{(0,0)}^{(1,\frac{\pi}{2})} F. dr$  using  $\phi(x,y)$ iii)
- b) Using Green's theorem find the value of  $\oint_c F \cdot dr$  Where [4]  $F(x,y) = (e^{3x} - y^2)i + (y^3 + 2x^2)j$  and C is the closed circle  $x^2 + y^2 = 4$
- a) Evaluate  $\int_{c} (x-1)dx + (2y-x)dy$  along the rectangle with vertices [5] (0, 0), (0, 2), (2,2) and (9,2)-(2,0)b) Evaluate the surface integral  $\iint_{\sigma} x^2 z^2 ds$ ;  $\sigma$  is the part of the cone
  - [5]

 $z = \sqrt{x^2 + y^2}$  that lies between the planes z = 1 and z = 3.



a) Find the flux of the vector field F(x, y, z) = 2xi - yj + 2zk across  $\sigma$ , [5] where  $\sigma$  is the portion of the surface  $z = 5 - x^2 - y^2$  that lies above the xyplane and suppose that  $\sigma$  is oriented up.

b) Use the Divergence Theorem to find the outward flux of the vector field  $F(x, y, z) = 2x^3i + 2y^3j + 2z^3k$  across the surface of the region that is emplosed by  $z = \sqrt{9 - x^2 - y^2}$  and the plane z = 0.



a) Use cylindrical coordinate systems to evaluate:

[5]

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{9-2(x^2+y^2)} (x^2+y^2) \, dz dy dx$$

Using double integral to find the area enclosed by the equations

$$-x - 2y = 2$$
,  $x - y = 1$  and  $y = 0$ .

b) Use triple integral to find the volume of the solid G enclosed by the sphere [5]  $x^2 + y^2 + z^2 = 9$  and the cone  $z = \sqrt{x^2 + y^2}$ .

## United International University

## School of Science and Engineering

Final Examination

Trimester: Summer: 2022

Course Title: Coordinate Geometry and Vector Analysis

Course Code: MAT 2109 / Math 201 Marks: 40 Time: 2 Hours.

Formula:

1.

$$\int_{C} f(x, y) dx + g(x, y) dy = \iint_{R} \left( \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA$$

$$\int_{(x_0, y_0)}^{(x_1, y_1)} \mathbf{F} \cdot d\mathbf{r} = \int_{(x_0, y_0)}^{(x_1, y_1)} \nabla \phi \cdot d\mathbf{r} = \phi(x_1, y_1) - \phi(x_0, y_0)$$

3.

$$\iint\limits_{\mathcal{S}} f(x, y, z) dS = \iint\limits_{\mathcal{R}} f(g(y, z), y, z) \sqrt{\left(\frac{\partial x}{\partial y}\right)^2 + \left(\frac{\partial x}{\partial z}\right)^2 + 1} dA$$

$$\iint f(x, y, z) dS = \iint_{\mathcal{P}} f(x, y, g(x, y)) \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1} dA$$

$$\iint f(x, y, z) dS = \iint_{R} f(x, g(x, z), z) \sqrt{\left(\frac{\partial y}{\partial x}\right)^{2} + \left(\frac{\partial y}{\partial z}\right)^{2} + 1} dA$$

4.

$$\iint_{\mathcal{D}} \mathbf{F} \cdot \mathbf{n} \, dS = \iint_{\mathcal{D}} \mathbf{F} \cdot \left( -\frac{\partial z}{\partial x} \mathbf{i} - \frac{\partial z}{\partial y} \mathbf{j} + \mathbf{k} \right) \, dA$$

 $\sigma$  of the form z = g(x, y)and oriented up

$$\iint_{R} \mathbf{F} \cdot \mathbf{n} \, dS = \iint_{R} \mathbf{F} \cdot \left( \frac{\partial z}{\partial x} \mathbf{i} + \frac{\partial z}{\partial y} \mathbf{j} - \mathbf{k} \right) \, dA$$

 $\sigma$  of the form z = g(x, y)and oriented down

5.

$$\iint_{G} \mathbf{F} \cdot \mathbf{n} \, dS = \iiint_{G} \operatorname{div} \mathbf{F} \, dV$$

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint (\text{curl } \mathbf{F}) \cdot \mathbf{n} \, dS$$