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United International University
School of Science and Engineering
Final Examination Trimester: Fall 2022
Course Title: Coordinate Geometry and Vector Analysis
Course Code: Math 2201 Marks: 40
Total Time: 2 hours

Answer all questions.

1. a) Consider, $F(x,y) = 2e^{2x}\sin y\, i + e^{2x}\cos y\, j$ [6]
   i) Show that F is a conservative vector field on the entire xy −plane.
   ii) Find the potential function $\phi(x,y)$.
   iii) Find $\int_{(0,0)}^{(1,\frac{\pi}{2})} F.dr$ using $\phi(x,y)$

   b) Using Green's theorem find the value of $\oint_C F.dr$ Where [4]
   $F(x,y) = (e^{3x} - y^2)i + (y^3 + 2x^2)j$ and C is the closed circle $x^2 + y^2 = 4$

2. a) Evaluate $\int_C (x-1)dx + (2y-x)dy$ along the rectangle with vertices [5]
   $(0,\ 0), (0,2), (2,2)$ and ~~(0, 2).~~ $(2,0)$

   b) Evaluate the surface integral $\iint_\sigma x^2z^2\, ds$; σ is the part of the cone [5]
   $z = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 3$.

3. a) Find the flux of the vector field $F(x,y,z) = 2xi - yj + 2zk$ across σ, [5]
   where σ is the portion of the surface $z = 5 - x^2 - y^2$ that lies above the xy − plane and suppose that σ is oriented up.

   $-x^2 - y^2 = -5$
   $x^2 + y^2 = 5$
   $r^2 = 5$
   $r = \sqrt{5}$

   b) Use the Divergence Theorem to find the outward flux of the vector field $F(x,y,z) = 2x^3i + 2y^3j + 2z^3k$ across the surface of the region that is enclosed by $z = \sqrt{9 - x^2 - y^2}$ and the plane $z = 0$. [5]

4. a) Use cylindrical coordinate systems to evaluate: [5]

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{9-2(x^2+y^2)} (x^2+y^2)\, dzdydx$$

Or

Using double integral to find the area enclosed by the equations
$-x - 2y = 2, x - y = 1$ and $y = 0$.

b) Use triple integral to find the volume of the solid G enclosed by the sphere $x^2 + y^2 + z^2 = 9$ and the cone $z = \sqrt{x^2 + y^2}$. [5]

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Formula:

1.

$$\int_C f(x,y)\,dx + g(x,y)\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA$$

2.

$$\int_{(x_0,y_0)}^{(x_1,y_1)} \mathbf{F}\cdot d\mathbf{r} = \int_{(x_0,y_0)}^{(x_1,y_1)} \nabla\phi \cdot d\mathbf{r} = \phi(x_1,y_1) - \phi(x_0,y_0)$$

3.

$$\iint_\sigma f(x,y,z)\,dS = \iint_R f(g(y,z),y,z)\sqrt{\left(\frac{\partial x}{\partial y}\right)^2 + \left(\frac{\partial x}{\partial z}\right)^2 + 1}\,dA$$

$$\iint_\sigma f(x,y,z)\,dS = \iint_R f(x,y,g(x,y))\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\,dA$$

$$\iint_\sigma f(x,y,z)\,dS = \iint_R f(x,g(x,z),z)\sqrt{\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2 + 1}\,dA$$

4.

$$\iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R \mathbf{F}\cdot\left(-\frac{\partial z}{\partial x}\mathbf{i} - \frac{\partial z}{\partial y}\mathbf{j} + \mathbf{k}\right) dA$$

σ of the form $z = g(x,y)$ and oriented up

$$\iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R \mathbf{F}\cdot\left(\frac{\partial z}{\partial x}\mathbf{i} + \frac{\partial z}{\partial y}\mathbf{j} - \mathbf{k}\right) dA$$

σ of the form $z = g(x,y)$ and oriented down

5.

$$\iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_G \operatorname{div}\mathbf{F}\,dV$$

6.

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_\sigma (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dS$$