SYLLABUS

B.Com I Year

Subject – Business Mathematics

UNIT – I	Ratio – Gaining and Sacrificing Ratio, Proportion, Percentage, Commission, Discount and Brokerage.
UNIT – II	Simultaneous Equations – Meaning, Characteristic, Types and Calculations, Preparation of Invoice.
UNIT – III	Elementary Matrices – Definition and Calculations, Types of Matrices.
UNIT – IV	Logarithms and Antilogarithms – Principles and Calculations, Simple and Compound Interest.
UNIT – V	Average – Simple, Weighted and Various Averages Arithmetic mean, Harmonic mean, Geometric mean, Profit and Loss.

UNIT-I
Chapter -1 – RATIO

A ratio can exist only between two quantities of the same type. If x and y are any two numbers and y $\neq 0$ then the fraction $\frac{x}{y}$ is called the ratio of x and y is written as x:y.

Characteristics of Ratio –

The following characteristics are attributed to ratio relationship:

   i)   Ratio is a cross relation found between two or more quantities of same type.
   ii)   It must be expressed in the same units.
   iii)   By the fraction laws a ratio can be expressed as below:

$$\frac{y}{x} = x:y$$

$$\frac{10}{5} = 10:5 \text{ or } 2:1$$

   iv)   A ratio expresses the number of times that one quantity contains another.
   v)   Two or more ratios may be compared by reducing their equivalent fractions to a common denominator.

Different types of Ratio –

Ratio can be divided into following ways –

   1) Unit Ratio – When homogeneous items are same on the basis of unit, it is called unit ratio.
       For example – Ram and Shyam are getting Rs. 5 each.

$$\frac{x}{y} = \frac{5}{5} \text{ or } 5:5 \text{ or } 1:1$$

   2) Duplicate Ratio – When the homogeneous items are shown in unit with square, it is called duplicate ratio.
       For Example, 2:3 square means $2^2:3^2$ or 4:9
   3) Triplicate ratio – When homogenous item is multiplied by 3, it is known as triplicate ratio.
       For example, $2^3:3^3$ = x2x2x2:3x3x3   8:27
   4) Sub triplicate ratio – When ratio is expressed in cube root it is known as sub triplicate ratio.
       For example, $\sqrt[3]{8}: \sqrt[2]{27}$ = 2:3
   5) Ratio of greater in equality – In this type of ratio the first item of given ratio is greater than other items.
       For example,   8:3, 13:8.
   6) Ratio of less in equality – When first item of given ratio is less than the other items of ratio, it is called ratio of less of equality.
       For example,   2:7, 5:12, 1:3
   7) Equality ratio – In this type of ratio first item is equal to other item of ratio.
       For example,   5:5, 8:8, 12:12

Proportion

Relationship between the two ratio's is called proportion. Here, quantity ratio of first two items is equality to rest two terms.
       For example,   2:5::6:15
       Proportion is expressed by four parallel points (::).

In the simple proportion here its not necessary that two items of first ratio and the items of second ratio should be homogeneous. But the items of second set of ratio has the same relationship which is found between the items of first ratio. For example 2:5::6:15. Here 5 is 2.5 times of 2 in case of first ratio. In the same 15 is 2.5 times of 6 in the second set of ratio.

Characteristics of Proportion –
   i)   Proportion is given in four parts. So first number is known as first item, second number is second item, third number is third item and fourth number is known as fourth item.
   ii)   First and fourth items are known as extremes items and second and third items are known as mean items.
   iii) It is not necessary in proportion that all four items should be homogenous. But the ratios of first and second and third and fourth should be the same.

Types of Proportion –

1) **Continued proportion** –
   If ratio of items is going on continuously, e.g., ratio of first and two is equal to two and three and ratio of two and three is equal to three and fourth item and so on, thus, ratio is known as continued ratio.
   For example, $\frac{A}{B} = \frac{B}{C} = \frac{C}{D} = \frac{D}{E} = \frac{E}{F} \ldots$
   Here A, B, C, D, E and F are in continued ratio.
2) **Direct Proportion** –
   In this type of ratio, two different items has the such relation that if the one is increased or decreased,   another will change accordingly in the same ratio.

Difference Between Ratio and Proportion –

S.No.	Ratio	Proportion
1	There are two terms in a ratio.	There are four terms in a proportion.
2	Comparison of two quantities of same type.	Comparison of two ratios.
3	Two quantities must be of same type.	All four quantities are not of same type but the first two are of one type and the last two may be of another type.
4	There is not a product rule	The product of extremes is equal to product of the means.

Chapter -2 - PERCENTAGE
Percent and Percentage
When we take of percentage, we usually refer to "for every one hundred."
Actually percentage can be defined as a fractional expression with 100 as its denominator.

When we talk of 10 percentage of a number, we mean 10 parts put of one hundred parts of the number in consider action the word "percentage" can be denoted by the sign (%).

In the above example 10 percentages can be written as 10% or even$\frac{10}{100}$. When written in the form $\frac{10}{100}$, it is in a fraction form whereby the upper number is the numerator and the bottom the denominator. It can further be simplified as –

$$\frac{10}{100} = \frac{1}{10}$$

From the above discussion we can conclude that when dealing with percentage, a number can be expressed as a fraction of percentage, i.e.,

$\frac{10}{100} = \frac{1}{10}$; or it can be written just in percentage form, i.e., 10 percent = 10%.

Change fractions into percentage –
When changing a fraction into a percentage, we just multiply it by 100 and put the sign %.

Example: Express $\frac{1}{10}$ as a percentage = $\frac{1}{10} X \frac{100}{1} = 10\%$

Change percentage into fraction –
To change a percentage given into a fraction, we divide the fraction by 100.

Example: Express 10% as a fraction = $\frac{10}{10} = \frac{1}{10}$

To find percentage of quantity with another quantity –
Let x and y be two quantities of same type and rate percentage r, such that

        $r \% \ of \ x = y$

                           or

$x . \frac{r}{100} = y$

$r = \frac{y \ X \ 100}{x}$

i.e., Rate percent = $\frac{The \ quantity \ which \ represent \ in \ percent}{Second \ quantity} \ X \ 100$

Example: What percent Rs. 20 of Rs. 350?

Solution: r% $\frac{20 X 100}{350} = 5\frac{5}{7}$

To find the quantity when rate percent and percentage value are known –
If rate percent value are given then

Quantity = $\frac{Percent \ value \ X \ 100}{Rate \ percent}$

Chapter -3 – COMMISSION

The terms commission and discount are commonly applicable in the business world. We should clearly understand the terminologies before solving questions related with them.

Who is an Agent?
Usually businessman may not be directly doing the business transactions themselves because of expanded area of business. They may employ persons to be doing the selling or buying on their behalf. Such person are known as agents. Agents get commission against their works performance.

Commission –
Having transacted the business transactions, the agents will require remuneration from their principal such as remuneration is known as commission. Usually the commission is calculated on the basis of the percentage of total sales done by the agent.

Who is a Broker?
The buyer and seller may not come into contact face to face. Their transaction may be made possible by a middleman. He negotiates the sales and purchase proceeds between the buyer and seller such a negotiator is known as broker.

Brokerage –
This is the remuneration paid to the broker. It is actually a commission paid to the broker. It is calculated on the basis of percentage of the total value of the business transacted by the broker.

Del Credere Agent –
A del-credere agent is a person who guarantees collection of dues for the principal from the customers. They got a special type of commission known as del-credere commission. Usually they deduct the commission on the dues collected and remit the remaining amount to the principal.

Travelling Agent –
This is a person who moves round the trading zone of the principal doing the selling proceeds.

Important formulae –

i)   Amount of commission $= \dfrac{Rate\ of\ commission\ \ X\ Amount\ \ of\ sales}{100}$

ii)   Rate of commission $= \dfrac{Rate\ of\ commission\ \ X\ 100}{Amount\ \ of\ Sales}$

iii)   Amount of Sales $= \dfrac{Rate\ of\ commission\ \ X\ 100}{Rate\ of\ commission}$

iv)   Amount of Del-cradere commission $= \dfrac{Credit\ Sales\ X\ Rate\ of\ del-credare\ \ commission}{100}$

DISCOUNT

The allowance or deduction from the market price of goods sold given by the vendor (Seller) to the purchaser (Buyer) is called discount. Discount is also known as allowance. The objective of allowing discount are –
- To increase the sales
- To retain the customership
- To encourage the customers to make the payment early

Kinds of Discount –
General there are two types of discounts are allowed to the customers –
Trade Discount and Cash Discount

1) **Trade Discount –** The Discount which is allowed by the seller according to the customs and traditions of the Business and which is allowed to all the customers irrespective of the payments conditions is called Trade Discount. The objective allowing Trade Discount is to increase the sales.

2) **Cash Discount –** The deduction on the marked price or invoice price or the selling price to the customer to encourage them to pay in cash or to make earlier cash payments is called cash discount.

In general Trade discount is given on marked price and cash discount is given on the remaining amount after deducting trade discount. In this way the purchaser in cash is entitled to get both type of discount.

Apart from these two discounts, there are some more types of discount.

Bulk discount or Quantity discount – It is allowed to the customers on purchasing on good in big quantity or bulk quantity.

Successive discount – When another discount is given after a discount, then the combination of these two discounts are known as successive discounts.

Equivalent Rate of Discount – The discount for which the amount due is equal o the amount due for successive discount is called their equivalent discount. Equivalent discount rate is also called single rate of equivalent discount.

It is to be noted that the total amount of successive discount is equal to the among of equivalent discount.

For example:
If a trader allows successive discount of 20% and 5% then the single rate/equivalent rate of discount will be –
D = 20+5 – 20x5 / 100 = 24%

NINE-VALUE TABLE
It is a method of calculating discount on a certain sum of list price/marked price. In this method on the basis of rate given first of all we have to calculate the discount for Rupee 1 and accordingly for Rupees 2, 3, 4, 5, 6, 7, 8 and 9.

With the help of this table we can calculate the commission or discount on any quantity.

Questions to be prepared on Unit-I :
1)   Give the definition and characteristics of Ratio and also explain its types.
2)   Describe the various type of Proportion.
3)   Distinguish between Ratio and Proportion.
4)   Explain the importance/significance of Percentage.
5)   Explain the terms Commission, Discount & Brokerage.
6)   What is successive discount?
7)   Explain equivalent rate of discount with example.
8)   Explain Nine-Values Table with example.

UNIT-II
Chapter – 4 - Simultaneous Equations

Equation – Equations signify relation of equality between two algebraic expressions symbolized by the sign of equality '='. In other words, an equation is statement which says that the two algebraic expressions are equal and is satisfied only for certain values of the variables.

Identify – When equality of two algebraic expressions hold true for all values of variables then it is called an identity.

Root of an Equation – The value of unknown or variable for which the equation is true is known as the root of the equation. To find the roots of an equation means to solve the equations.

Degree of an Equation – The degree of an equation is the highest exponent of the variable x or variables (x, y, ...) present in the equation is called the degree of an equation.

Linear Equation – An equation which involves power of an unknown quantity not higher than unity (one) is called a linear equation.

One variable Linear Equation – A linear equation in one variable (x, say) in which the highest degree of the variable x is 1. A linear equation in one variable is, in general, written as ax+by = c or ax = c. This equation is also called, "First degree equation in x" or simple equation.

Two variable equation – A linear equation in two variables (x, y, say) in which the highest degree of the variables x and y each is 1. A linear equation in two variables, is general, is written as ax+by+c = 0 or ax+by=d.

Three variable equation – A linear equation in three variables (x, y, z, say) in which the highest degree of the variables x, y and z each is 1. A linear equation in three variables, in general, is written as $a_1x+b_1y+c_1z=d$.

Types of Simultaneous Equations –
  i)   Linear Simultaneous Equations in two Variables – Two linear equations in two variables together are linear simultaneous equations in two variables, e.g.:
$$4x+y = 2$$
$$3x-5y = 18$$
  ii)   Linear Simultaneous Equations in three Variables – Three linear equations in three variables together are linear simultaneous equations in three variables, e.g.:
$$3x+5y-7z = 13$$
$$4x+y-12z = 6$$
$$2x+9y-3z = 20$$
  iii)   Specific type of Simultaneous Equations – The equations in other than linear form are called specific type equations, e.g.:
  i)   quadratic equation : $ax^2 + bx + c = 0$
  ii) reciprocal equation : $\frac{a}{x} + \frac{b}{y} = c$
  iii) $a\left(\frac{y}{x}\right) + c = by, etc.$

Characteristics of Simultaneous Equations –
  1) A system of linear equations in one variable is not taken under simultaneous equations.

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2) The set of values of two variables x and y which satisfy each equation in the system of equations is called the solution of simultaneous equations.

The solutions of two variable linear simultaneous equations may be –

i)   Infinitely many,

ii)   An unique solution, or

iii)   No solution.

3) For simultaneous equations –

$a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

     a.   If $\frac{a1}{a2} = \frac{b1}{b2} = k$ and $c1 = k$ C_2 then there are infinitely many solutions.

     b.   If $\frac{a1}{a2} = \frac{b1}{b2} = c1$ $\neq$ kc_2, then there is no solution.

     c.   If $c_1 \neq 0$, then $c_1 = kc2$ $\rightarrow$ $\frac{c1}{c2} = k, hence$

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2} \rightarrow infinitely\ many\ solutions$$

$$and\ \frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2} \rightarrow no\ solution$$

     d.   If c_1 and c_2 both are zero (i.e., $c_1=0=c_2$)

Chapter – 5 - Preparation of Invoice

After the dispatch of goods, the seller prepares an invoice of the goods sold in which the quantity and quality of goods and their price is mentioned. Discount, if any, is deducted from the total amount, to this are added the seller's other expenses. Railway receipt number is also mentioned in the invoice if the goods have been sent by train.

Types of Invoice –

1) *Local invoices* – In these invoices only the cost of the goods less any trade discount is borne by the seller. All expenses of packing, cartage, loading and freight for carrying goods to the place of buyer are shown extra and charged to the buyer. It means that the cost includes cost of the goods only and all other expenses are extra and are recovered from the buyer.

2) *At station invoice* – This implies that all costs upto the stage of putting the goods at railway station will be borne by the seller and are included in the cost of goods but expenses beyond that, i.e., the railway fare, insurance, etc., will be borne by the buyer.

3) *Free on Rails (FOR) invoices* – Under this all the costs, i.e., cost of goods, cost of packing, carrying the goods to railway station, loading them in wagons, are borne by the seller and further expenses are borne by the buyer, i.e., they are charged over and above the cost of goods.

4) *Cost and Freight (C and F) invoice* – In such type of invoices it is presumed that cost which the seller is charging includes cost of goods, cost of packing, freight for carrying goods to the buyer and all other incidental expenses. All expenses other than above, i.e., insurance will be charged extra.

5) *Cost, Insurance and freight, (CI & F) invoice* – In such invoices cost charged by the seller includes cost of goods, cost of packing, freight and insurance. Other expenses are charged extra.

6) *Franco invoice* – In such invoices all costs upto putting the goods at the door of the buyer are borne by the seller, i.e., the price which he has quoted includes all expenses incurred in carrying the goods to the buyer's place. For example, Franco cost invoice will mean that cost charged by the seller includes cost of the goods, cost of packing, freight, insurance and local transportation charges.

Preparation of Invoice –
Invoice is prepared in duplicate. The original copy is sent to the buyer and the duplicate is kept for future reference. The usual contents of the invoice are –
- i)   Name and address of the seller
- ii)   No. and date of the invoice
- iii)   No. of the purchase order
- iv)   Name and address of the buyer
- v)   Place where it has been made
- vi)   Terms of trade
- vii)   Details about quantity of goods like weight or length etc.
- viii)   Date
- ix)   Separate price of each item and total price.
- x)   Trade discount, if any.
- xi)   Expenses incurred on sending the goods.
- xii)   Advance payment received (if paid by the buyer)
- xiii)   Net amount payable.
- xiv)   Details about mode of sending the goods
- xv)   Errors and omissions excepted
- xvi)   Special information, if any
- xvii)   Signature of the seller.

Uses of Invoice –
Following are the uses of invoice –
- a)   It informs the buyer about the price of the goods and other expenses he has to pay.
- b)   If the invoice reaches buyer before the goods, he can make arrangement for their resale.
- c)   The buyer can compare the invoice with his order.
- d)   After taking delivery of the goods he can compare the contents of the packages with the invoice and point discrepancy, if any to the seller.
- e)   Pay Octroi etc. on the basis of the invoice.

Necessary entries can be made in the books of accounts on the basis of invoice.

UNIT-III

Definition of Matrix –
A matrix (plural of matrices) is an array of real numbers (or other suitable elements) arranged in row and columns is called as a matrix. Consider a set of real numbers m and n when multiplied together we get $m \times n$ or mn. These can be used to define a matrix.

1) **Row Matrix or Row Vector –**
   A matrix having only one row is known as a row matrix or a row vector. It is in the form $(1 \times n)$.
   Example –

   $$(3), (7 - 10), (\tfrac{1}{2} \; 3 - 6), (a \; b \; c \; d)$$

2) **Column Matrix or Column Vector –**
   This is a type of Matrix which has only one column. It is in the form (m x 1).
   Example –

   $$(5), \begin{bmatrix} 8 \\ 4 \\ -2 \end{bmatrix}, \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

3) **Zero or Null Matrix –**
   This is a type of Matrix whose every element is zero. It is usually denoted by bold face zero **(0)**.
   Example –

   Examples :
   $$(0), \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, [0 \; 0 \; 0 \; 0 \; 0]$$

4) **Diagonal Matrix –**
   Some matrix are such that all their elements are zero apart from the diagonal extending from the upper left hand corner to the lower right hand corner. These are known as diagonal matrix.
   Examples are -

   $$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

5) **Square Matrix –**
   In this matrix, the number of rows and columns are the same.
   Examples –

   $$(3), \begin{bmatrix} 6 & -7 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 5 & 1/2 \\ 7 & 6 & 3 \\ 2 & 7 & -1 \end{bmatrix}, \begin{bmatrix} 1/2 & 2 & 7 & -5 \\ 3 & 6 & 8 & 2 \\ 5 & 3/4 & -6 & 5 \\ 7 & 8 & -1 & 4 \end{bmatrix}$$

6) Unit or Identity Matrix –
This is a type of matrix where diagonal elements have values of 1. A unit matrix is usually denoted by bold face **(I)**. Examples of unit matrix are as follows –

$$I_1 = [\,1\,], \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7) Scalar Matrix –
This is a diagonal matrix whose diagonal elements are all equal. See examples given below –

$$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \quad \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \quad \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{bmatrix}$$

8) Upper Triangular Matrix –
A square matrix in which every element below the principal diagonal are zero is known as an upper triangular matrix. Examples –

$$\begin{bmatrix} 6 & 2 & 4 \\ 0 & 5 & 7 \\ 0 & 0 & 3 \end{bmatrix}, \quad \begin{bmatrix} 7 & 5 & 4 & 6 \\ 0 & 8 & 3 & 7 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9) Lower Triangular Matrix –
A square matrix in which every element above the principal diagonal are zero is known as the lower triangular matrix. Examples –

$$\begin{bmatrix} 6 & 0 & 0 \\ 2 & 4 & 0 \\ 5 & 3 & 5 \end{bmatrix}, \quad \begin{bmatrix} 8 & 0 & 0 & 0 \\ 4 & 7 & 0 & 0 \\ 3 & 4 & 2 & 0 \\ 9 & 3 & 6 & 5 \end{bmatrix}$$

10) Transpose Matrix –
A matrix obtained by interchanging the row and columns of a matrix is called transpose of A and is denoted by A^T or A'. Example given below –

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad A_T = A' = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & 6 & 4 \\ 8 & 5 & 3 \\ 9 & 2 & 1 \end{bmatrix} \qquad A_T = A' = \begin{bmatrix} 7 & 8 & 9 \\ 6 & 5 & 2 \\ 4 & 3 & 1 \end{bmatrix}$$

UNIT-IV

FUNDAMENTAL LAWS OF LOGARITHM

The laws of logarithm which are valid for any base a (> 0 but $\neq 1$) are as follows:

Law 1 : The logarithm of the product of two numbers is equal to the sum of the logarithms of these numbers to the same base, *i.e.,*

$$\log_a(m \times n) = \log_a m + \log_a n$$

Proof : Let

$$\log_a m = x \quad \Rightarrow \quad a^x = m$$

and

$$\log_a n = y \quad \Rightarrow \quad a^y = n$$

Hence

$$a^x \times a^y = m \times n$$

or

$$a^{x+y} = m \times n$$

$\therefore$

$$\log_a(m \times n) = x + y, \qquad \text{by definition}$$

$\Rightarrow$

$$\log_a(m \times n) = \log_a m + \log_a n.$$

Sub-law : The logarithm of the product of two or more numbers is equal to the sum of the logarithms of these numbers to the same base, *i.e.,*

$$\log_a(m \times n \times p \times .....) = \log_a m + \log_a n + \log_a p + ...$$

Law 2 : The logarithm of the quotient of two numbers is equal to the difference of the logarithm of the numerator and the logarithm of the denominator to the same base, *i.e.,*

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Proof : Let

$$\log_a m = x \quad \Rightarrow \quad a^x = m$$

and

$$\log_a n = y \quad \Rightarrow \quad a^y = n$$

Hence

$$\frac{m}{n} = \frac{a^x}{a^y} = a^{x-y}, \qquad \text{by law of indices}$$

$\Rightarrow$

$$\log_a\left(\frac{m}{n}\right) = x - y$$

$\Rightarrow$

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Law 3 : The logarithm of a number raised to a power is equal to the product of the index of the power and the logarithm of the number to the same base, *i.e.,*

$$\log_a(m^n) = n \log_a m$$

Proof : Let

$$\log_a m = x \quad \Rightarrow \quad a^x = m$$

Hence,

$$(a^x)^n = m^n, \quad \text{by raising to the power } n \text{ on both sides.}$$

$\Rightarrow$

$$a^{nx} = m^n$$

$\Rightarrow$

$$\log_a(m^n) = nx = n \log_a m.$$

Remark : If $n = \frac{1}{p}$, where p is a positive integer, then logarithm $\sqrt[p]{m}$ or $m^{1/p}$ is obtained by dividing the logarithm of m by p, *i.e.,*

$$\log_a(\sqrt[p]{m}) = \log_a(m^{1/p}) = \frac{1}{p}\log_a m \quad \text{or} \quad \frac{\log_a m}{p}$$

Change of Base

If the logarithm of numbers are given to the base a, then the logarithms of those numbers can be obtained to another base b.

Let

$$\log_b n = x$$

$\Rightarrow$

$$b^x = n$$

$\Rightarrow$

$$\log_a b^x = \log_a n, \qquad \{\text{taking log to the base } a\}$$

$\Rightarrow$

$$x \log_a b = \log_a n$$

$\Rightarrow$

$$x = \frac{\log_a n}{\log_a b} \quad \text{or} \quad \log_b n = \frac{\log_a n}{\log_a b} \qquad ...(1)$$

Writing $n = a$ in (1),

$$\log_b a = \frac{\log_a a}{\log_a b} = \frac{1}{\log_a b}$$

$\Rightarrow \qquad \log_b a . \log_a b = 1 \qquad\qquad\qquad ...(2)$

Similarly $\qquad\qquad \log_b m = n$

$\Rightarrow \qquad\qquad b^n = m$

$\Rightarrow \qquad\qquad \left(\frac{1}{b}\right)^{-n} = m$

$\Rightarrow \qquad\qquad \log_{1/b} m = -n \qquad\qquad\qquad ...(3)$

Common Logarithm*

The logarithm of a number to the base 10 is called 'common logarithm'. In numerical calculations common logarithms are used when no base is mentioned it is understood to be 10. For example,

$$\log_{10} n = x \text{ is simply written as } \log n = x.$$

The common logarithm of a number consists of two parts : the whole part or the integral part and the decimal part. The integral part is called *characteristic* and the decimal part is called *mantissa*.

$$\text{In } \log_{10} 14 = 1.1461 = 1 + .1461$$

characteristic is 1 and mantissa is .1416.

To Determine Characteristic of a Common Logarithm

Consider the logarithm of those numbers which are integral powers of 10.

$10^3 = 1,000$	$\Rightarrow$	$\log_{10} 1,000 = 3$
$10^2 = 100$	$\Rightarrow$	$\log_{10} 100 = 2$
$10^1 = 10$	$\Rightarrow$	$\log_{10} 10 = 1$
$10^0 = 1$	$\Rightarrow$	$\log_{10} 1 = 0$
$10^{-\infty} = 0$	$\Rightarrow$	$\log_{10} 0 = -\infty$
$10^{-1} = 0.1$	$\Rightarrow$	$\log_{10} 0.1 = -1$
$10^{-2} = 0.01$	$\Rightarrow$	$\log_{10} 0.01 = -2$
$10^{-3} = 0.001$	$\Rightarrow$	$\log_{10} 0.001 = -3$

From the above results we observe that :

(1) The numbers which are integral powers of 10 are straight forward and it contains characteristic only.

(2) The common logarithm of a number between 1 and 10 lies between 0 and 1.

(3) The common logarithm of a number between 10 and 100 lies between 1 and 2.

(4) The common logarithm of a number between 100 and 1,000 lies between 2 and 3.

(5) The common logarithm of a number between .1 and 1 is between −1 and 0.

(6) The common logarithm of a number between .01 and 0.1 is between (− 2) and (− 1).

(7) The characteristic of a common logarithm is found out by mere inspection. In this connection the following rules should be kept in mind.

Definition of The Derivative

The derivative of a function represents an infinitesimal change in the function with respect to one of its variables.

Mathematical form of derivative: The derivative of the function $f(x)$ at the point x_0 is given and denoted by

$$\frac{df}{dx}(x_0) = f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Some Basic Differentiation formulae

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d}{dx}(c\,u) = c\,\frac{du}{dx}$$

$$\frac{d}{dx}(u\,v) = u\,\frac{dv}{dx} + v\,\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,\dfrac{du}{dx} - u\,\dfrac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}(u^n) = n u^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}(u \circ v) = \frac{dv}{dx}\left(\frac{du}{dx} \circ v\right)$$

Chain Rule

$$\frac{d}{dx}(u \circ v) = \frac{dv}{dx}\left(\frac{du}{dx} \circ v\right)$$

is known as the Chain Rule formula. It may be rewritten as

$$\frac{du}{dx} = \frac{du}{dv}\frac{dv}{dx}$$

Another similar formula is given by

$$\frac{du}{dx} = \frac{\dfrac{du}{dv}}{\dfrac{dx}{dv}}$$

Partial differentiation or derivative

Partial differentiation or derivative of a function of several variables is its derivative with respect to one of those variables, with the others held constant (as opposed to the total derivative, in which all variables are allowed to vary). Partial derivatives are used in vector calculus and differential geometry.

The partial derivative of a function $f(x, y, ...)$ with respect to the variable x is variously denoted by

$$f'_x, \ f_x, \ \partial_x f, \frac{\partial}{\partial x}f, \ \text{or} \ \frac{\partial f}{\partial x}.$$

Partial derivative formula

If f is a function of two variables, its partial derivatives fx and fy are also function of two variables; their partial derivatives (fx)x, (fx)y, (fy)x, and (fy)y are second order partial derivatives. If z=f(x,y), then

$$(fx)x = fxx = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 z}{\partial x^2}$$
$$(fx)y = fxy = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 z}{\partial y \partial x}$$

$$(fy)x = fxy = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 z}{\partial x \partial y}$$

$$(fy)y = fxx = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 z}{\partial y^2}$$