Latex Math Assignment ncert-exemplar/12/leep210.pdf- Excercise 10.3

Short Answer (S.A):

1. Find the unit vector in the direction of sum of vectors $\vec{a}= 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{j} + \hat{k}$.
2. If $\vec{a}=\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b}=2\hat{i} + \hat{j} - 2\hat{k}$, find the unit vector in the direction of
   (a) $6\vec{a}$
   (b) $2\vec{a}$-$\vec{b}$
3. Find a unit vector in the direction of $\overline{PQ}$, where P and Q have co-ordinates(5,0,8) and (3,3,2),respectively.
4. If $\vec{a}$ and $\vec{b}$ are the postion vectors of A and B, respectively, find the position vector of a point C in BA produced such that BC=1.5BA.
5. Using vectors, find the value of k such that the points $(k, -10, 3)$, $(1, -1, 3)$ and $(3, 5, 3)$ are collinear.
6. A vector $\vec{r}$ is inclined at equal angles to the three axis. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, find $\vec{r}$.
7. A vector $\vec{r}$ has a magnitude 14 and direction ratios 2,3,-6. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with x-axis.
8. Find a vector of magnitude 6, which is perpendicular to both the vectors $2\hat{i} - \hat{j}$+$2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$.
9. Find the angle between the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$.
10. If $\vec{a} + \vec{b} + \vec{c}$=0, show that $\vec{a} \times \vec{b}$=$\vec{b} \times \vec{c}$=$\vec{c} \times \vec{a}$. Interpret the result geometrically?
11. Find the sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$.
12. If A,B,C,D are the points with position vectors $\hat{i} + \hat{j} - \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{k}$, $3\hat{i}$-$2\hat{j}$+$\hat{k}$, respectively, find the projection of $\overline{AB}$ along $\overline{CD}$.
13. Using vectors, find the area of $\triangle ABC$ with vertices A(1,2,3), B(2,-1,4) and C(4,5,-1).
14. Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.

Long Answer(L.A)

15. Prove that in any $\triangle ABC$, cos A=$\frac{b^2+c^2-a^2}{2bc}$, where a,b,c are the magnitudes of the sides opposite to the vertices A,B,C respectively.
16. If $\vec{a}$, $\vec{b}$, $\vec{c}$,determine the vertices of a triangle, show that $\frac{1}{2}\left[\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}\right]$ gives the vector area of the trianlge. Hence deduce the condition that the three points $\vec{a}, \vec{b}, \vec{c}$, are collinear. Also find the unit vector normal to the plane of the triangle.
17. Show that area of the parallelogram whose diagonals are given by $\vec{a} \times \vec{b}$ is $\frac{\left|\vec{a} \times \vec{b}\right|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.
18. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a the vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

Objective Type Questions

Choose the correct answer from the given four options in each of the Excercise from 19 to 33(M.C.Q)

19. The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is
    (a) $\hat{i} - 2\hat{j} + 2\hat{k}$
    (b) $\hat{i} - 2\hat{j}$
    (c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$
    (d) $9(\hat{i} - 2\hat{j} + 2\hat{k})$
20. The position vector of the point which divides the join of points $2\vec{a}$-$3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3:1 is
    (a) $\frac{3\vec{a}-2\vec{b}}{2}$
    (b) $\frac{7\vec{a}-8\vec{b}}{4}$

(c) $\frac{3\vec{a}}{4}$

(d) $\frac{5\vec{a}}{4}$

21. The vector having intial and terminal points as (2,5,0)and (-3,7,4),respectively is

(a) -$\hat{i} + 12\hat{j} + 4\hat{k}$

(b) $5\hat{i} + 2\hat{j} - 4\hat{k}$

(c) $5\hat{i} + 2\hat{j} + 4\hat{k}$

(d) $\hat{i} + \hat{j} + \hat{k}$

22. The angles between two vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 4, respectively, and $\vec{a}$, $\vec{b}$= $2\sqrt{3}$ is

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) $\frac{5\pi}{2}$

23. Find the value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal.

(a) 0

(b) 1

(c) $\frac{3}{2}$

(d) $-\frac{5}{2}$

24. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and, $2\hat{i} - 4\hat{j}$+$\lambda\hat{k}$ are parallel is

(a) $\frac{2}{3}$

(b) $\frac{3}{2}$

(c) $\frac{5}{2}$

(d) $\frac{2}{5}$

25. The vector from origin to the points A and B are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$, respectively, then the area of $\triangle OAB$ is

(a) 340

(b) $\sqrt{25}$

(c) $\sqrt{229}$

(d) $\frac{1}{2}\sqrt{229}$

26. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$is equal to

(a) a

(b) 3a

(c) 4a

(d) 2a

27. If $|\vec{a}|$=10, $\left|\vec{b}\right| = 2$ and $\vec{a}$, $\vec{b}$=12, then value of $\left|\vec{a} \times \vec{b}\right|$ is

(a) 5

(b) 10

(c) 14

(d) 16

28. The vectors $\lambda\hat{i} + \lambda\hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if

(a) $\lambda = -2$

(b) $\lambda = 0$

(c) $\lambda = 1$
(d) $\lambda = -1$

29. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is
   (a) 1
   (b) 3
   (c) $\frac{-3}{2}$
   (d) None of these

30. Projection vector of $\vec{a}$ on $\vec{b}$ is
   (a) $\left(\dfrac{\vec{a}.\vec{b}}{\left|\vec{b}\right|^2}\right)$
   (b) $\frac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}$
   (c) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$
   (d) $\left(\dfrac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)$

31. If $\vec{a}, \vec{b}, \vec{c}$ are the three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=2$, $|\vec{b}|$=3, $|\vec{c}|$=5, the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is
   (a) 0
   (b) 1
   (c) -19
   (d) 38

32. If $|\vec{a}|=4$ and $-3 \leq \lambda \leq 2$, then the range of $|\lambda\vec{a}|$ is
   (a) $[0, 8]$
   (b) $[-12, 8]$
   (c) $[0, 12]$
   (d) $[8, 12]$

33. The number of vectors of unit length perpendicular to the vectors $\vec{a}=2\hat{\imath}+\hat{\jmath}+2\hat{k}$ and $\vec{b}=\hat{\jmath}+\hat{k}$ is
   (a) one
   (b) two
   (c) three
   (d) infinite

Fill in the blanks in each of the Excercises from 34 to 40.

34. The vector $\vec{a}+\vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$ if ______.

35. If $\vec{r}.\vec{a}=0$, $\vec{r}.\vec{b}=0$ and $\vec{r}.\vec{c}=0$ for some non-zero vector $\vec{r}$, then the value of $\vec{a}.(\vec{b}\times\vec{c})$ is ______.

36. The vectors $\vec{a}=3\hat{\imath}-2\hat{\jmath}+2\hat{k}$ and $\vec{b}=\hat{\imath}-2\hat{k}$ are the adjancent sides of a parallelogram. The acute angle between its diagonals is ______.

37. The values of k for which $\left|k\vec{a}\right| < |\vec{a}|$ and $k\vec{a}+\dfrac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true are ______.

38. The value of the expression $\left|\vec{a}\times\vec{b}\right| + (\vec{a}.\vec{b})$ is ______.

39. If $\left|\vec{a}\times\vec{b}\right|^2 + \left|\vec{a}.\vec{b}\right|^2$=144 and $|\vec{a}|$=4, then $\left|\vec{b}\right|$ is equal to ______.

40. If $\vec{a}$ is any non-zero vector, then $(\vec{a}.\hat{i})\hat{i}+(\vec{a}.\hat{j})\hat{j}+(\vec{a}.\hat{k})\ \hat{k}$ equals ______.

State **True** or **False** in each of the following Exercises.

41. If $|\vec{a}| = \left|\vec{b}\right|$, then necessarily it implies $\vec{a} = \pm\vec{b}$.

42. Position vector of point P is a vector whose intial point is origin.

43. If $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$, then the vectors $\vec{a}$ and $\vec{b}$ are orthogonal.

44. The formula $(\vec{a}+\vec{b}) = \vec{a}+\vec{b} + 2\vec{a}\times\vec{b}$ is valid for non-zero vectors $\vec{a}$ and $\vec{b}$.

45. If $\vec{a}$ and $\vec{b}$ are adjacent sides of a rhombus, then $\vec{a}.\vec{b}.=0$.