Vectors Assignment-1

Section12thMath- Excercise 12.10.4.1

1. Find $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$,if $\overrightarrow{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\overrightarrow{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

Solution:

The given two vectors are $\overrightarrow{a}=\hat{i} - 7\hat{j} + 7\hat{k}$ and $\overrightarrow{b}=3\hat{i} - 2\hat{j} + 2\hat{k}$

Let

$$\overrightarrow{a} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \overrightarrow{b} = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} \tag{1}$$

The cross product of vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ is given as

$$\left|\overrightarrow{a} \times \overrightarrow{b}\right| = \begin{vmatrix} \mathbf{A_1} & \mathbf{B_1} \\ \mathbf{A_2} & \mathbf{B_2} \\ \mathbf{A_3} & \mathbf{B_3} \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ -7 & -2 \\ 7 & 2 \end{vmatrix} \tag{2}$$

$$\begin{vmatrix} A_1 & B_1 \end{vmatrix} = \begin{vmatrix} -7 & -2 \\ 7 & 2 \end{vmatrix} = -14 + 14 = 0 \tag{3}$$

$$\tag{4}$$

$$\begin{vmatrix} A_2 & B_2 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 7 & 2 \end{vmatrix} = 2 - 21 = -19 \tag{5}$$

$$\tag{6}$$

$$\begin{vmatrix} A_3 & B_3 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ -7 & -2 \end{vmatrix} = -2 + 21 = 19 \tag{7}$$

$$\tag{8}$$

$$\left|\overrightarrow{a} \times \overrightarrow{b}\right| = \sqrt{(-19^2) + (19)^2} \tag{9}$$

$$= \sqrt{2} \times 19 \tag{10}$$

$$= 19\sqrt{2} \tag{11}$$

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