

**Short Answer (S.A) :**

- Find the unit vector in the direction of sum of vectors  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = 2\hat{j} + \hat{k}$ .
- If  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$  and  $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$ , find the unit vector in the direction of  
(i)  $6\vec{a}$  (ii)  $2\vec{a} - \vec{b}$
- Find a unit vector in the direction of  $\overrightarrow{PQ}$ , where P and Q have co-ordinates (5,0,8) and (3,3,2), respectively.
- If  $\vec{a}$  and  $\vec{b}$  are the position vectors of A and B, respectively, find the position vector of a point C in BA produced such that  $BC = 1.5BA$ .
- Using vectors, find the value of  $k$  such that the points  $(k, -10, 3)$ ,  $(1, -1, 3)$  and  $(3, 5, 3)$  are collinear.
- A vector  $\vec{r}$  is inclined at equal angles to the three axis. If the magnitude of  $\vec{r}$  is  $2\sqrt{3}$  units, find  $\vec{r}$ .
- A vector  $\vec{r}$  has a magnitude 14 and direction ratios 2,3,-6. Find the direction cosines and components of  $\vec{r}$ , given that  $\vec{r}$  makes an acute angle with x-axis.
- Find a vector of magnitude 6, which is perpendicular to both the vectors  $2\hat{i} - \hat{j} + 2\hat{k}$  and  $4\hat{i} - \hat{j} + 3\hat{k}$ .
- Find the angle between the vectors  $2\hat{i} - \hat{j} + \hat{k}$  and  $3\hat{i} + 4\hat{j} - \hat{k}$ .
- If  $\vec{a} + \vec{b} + \vec{c} = 0$ , show that  $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ . Interpret the result geometrically?
- Find the sine of the angle between the vectors  $\vec{a} = 3\vec{i} + \vec{j} + 2\vec{k}$  and  $\vec{b} = 2\vec{i} - 2\vec{j} + 4\vec{k}$ .
- If A,B,C,D are the points with position vectors  $\hat{i} + \hat{j} - \hat{k}$ ,  $2\hat{i} - \hat{j} + 3\hat{k}$ ,  $2\hat{i} - 3\hat{k}$ ,  $3\hat{i} - 2\hat{j} + \hat{k}$ , respectively, find the projection of  $\overrightarrow{AB}$  along  $\overrightarrow{CD}$ .
- Using vectors, find the area of  $\triangle ABC$  with vertices A(1,2,3), B(2,-1,4) and C(4,5,-1).
- Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.

**Long Answer (L.A)** 15. Prove that in any  $\triangle ABC$ ,  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ , where a,b,c are the magnitudes of the sides opposite to the vertices A,B,C respectively.

16. If  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$ , determine the vertices of a triangle, show that  $\frac{1}{2} [\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}]$  gives the vector area of the triangle. Hence deduce the condition that the three points  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$ , are collinear. Also find the unit vector normal to the plane of the triangle.

17. Show that area of the parallelogram whose diagonals are given by  $\vec{a} \times \vec{b}$  is  $\frac{|\vec{a} \times \vec{b}|}{2}$ . Also find the area of the parallelogram whose diagonals are  $2\hat{i} - \hat{j} + \hat{k}$  and  $\hat{i} + 3\hat{j} - \hat{k}$ .

18. If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{j} - \hat{k}$ , find a the vector  $\vec{c}$  such that  $\vec{a} \times \vec{c} = \vec{b}$  and  $\vec{a} \cdot \vec{c} = 3$ .

**Objective Type Questions** Choose the correct answer from the given four options in each of the Exercise from 19 to 33(M.C.Q)

19. The vector in the direction of the vector  $\hat{i} - 2\hat{j} + 2\hat{k}$  that has magnitude 9 is

- |                                     |  |
|-------------------------------------|--|
| (A) $\hat{i} - 2\hat{j} + 2\hat{k}$ | (C) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ |
| (B) $\hat{i} - 2\hat{j}$            | (D) $9(\hat{i} - 2\hat{j} + 2\hat{k})$ |

20. The position vector of the point which divides the join of points  $2\vec{a}-3\vec{b}$  and  $\vec{a}+\vec{b}$  in the ratio 3:1 is

- (A)  $\frac{3\vec{a}-2\vec{b}}{2}$  (B)  $\frac{7\vec{a}-8\vec{b}}{4}$  (C)  $\frac{3\vec{a}}{4}$  (D)  $\frac{5\vec{a}}{4}$

21. The vector having initial and terminal points as (2,5,0) and (-3,7,4), respectively is

- (A)  $-\hat{i}+12\hat{j}+4\hat{k}$  (C)  $5\hat{i}+2\hat{j}+4\hat{k}$   
(B)  $5\hat{i}+2\hat{j}-4\hat{k}$  (D)  $\hat{i}+\hat{j}+\hat{k}$

22. The angles between two vectors  $\vec{a}$  and  $\vec{b}$  with magnitude  $\sqrt{3}$  and 4, respectively, and  $\vec{a}, \vec{b} = 2\sqrt{3}$  is

- (A)  $\frac{\pi}{6}$  (B)  $\frac{\pi}{3}$  (C)  $\frac{\pi}{2}$  (D)  $\frac{5\pi}{2}$

23. Find the value of  $\lambda$  such that the vectors  $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$  are orthogonal.

- (A) 0 (B) 1 (C)  $\frac{3}{2}$  (D) -52

24. The value of  $\lambda$  for which the vectors  $3\hat{i} - 6\hat{j} + \hat{k}$  and,  $2\hat{i} - 4\hat{j} + \lambda\hat{k}$  are parallel is

- (A)  $\frac{2}{3}$  (B)  $\frac{3}{2}$  (C) 52 (D) 25

25. The vector from origin to the points A and B are  $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$  and  $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ , respectively, then the area of  $\triangle OAB$  is

- (A) 340 (B)  $\sqrt{25}$  (C)  $\sqrt{229}$  (D)  $12\sqrt{229}$

26. For any vector  $\vec{a}$ , the value of  $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$  is equal to

- (A) a (B) 3a (C) 4a (D) 2a

27. If  $|\vec{a}| = 10$ ,  $|\vec{b}| = 2$  and  $\vec{a}, \vec{b} = 12$ , then value of  $|\vec{a} \times \vec{b}|$  is

- (A) 5 (B) 10 (C) 14 (D) 16

28. The vectors  $\lambda\hat{i} + \lambda\hat{j} + 2\hat{k}$ ,  $\hat{i} + \lambda\hat{j} - \hat{k}$  and  $2\hat{i} - \hat{j} + \lambda\hat{k}$  are coplanar if

- (A)  $\lambda = -2$  (B)  $\lambda = 0$  (C)  $\lambda = 1$  (D)  $\lambda = -1$

29. If  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = 0$ , then the value of  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  is

- (A) 1 (B) 3 (C)  $-\frac{3}{2}$  (D) None of these

30. Projection vector of  $\vec{a}$  on  $\vec{b}$  is

- (A)  $\left( \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right)$  (B)  $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$  (C)  $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$  (D)  $\left( \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \right)$

31. If  $\vec{a}, \vec{b}, \vec{c}$  are the three vectors such that  $\vec{a} + \vec{b} + \vec{c} = 0$  and  $|\vec{a}| = 2$ ,  $|\vec{b}| = 3$ ,  $|\vec{c}| = 5$ , the value of  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  is

- (A) 0 (B) 1 (C) -19 (D) 38

32. If  $|\vec{a}| = 4$  and  $-3 \leq \lambda \leq 2$ , then the range of  $|\lambda\vec{a}|$  is

- (A)  $[0, 8]$  (B)  $[-12, 8]$  (C)  $[0, 12]$  (D)  $[8, 12]$

33. The number of vectors of unit length perpendicular to the vectors  $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$  and  $\vec{b} = \hat{j} + \hat{k}$  is

(A) one

(B) two

(C) three

(D) infinite

Fill in the blanks in each of the Exercises from 34 to 40.

34. The vector  $\vec{a} + \vec{b}$  bisects the angle between the non-collinear vectors  $\vec{a}$  and  $\vec{b}$  if \_\_\_\_\_

35. If  $\vec{r} \cdot \vec{a} = 0$ ,  $\vec{r} \cdot \vec{b} = 0$  and  $\vec{r} \cdot \vec{c} = 0$  for some non-zero vector  $\vec{r}$ , then the value of  $\vec{a} \cdot (\vec{b} \times \vec{c})$  is \_\_\_\_\_

36. The vectors  $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{k}$  are the adjacent sides of a parallelogram. The acute angle between its diagonals is \_\_\_\_\_

37. The values of  $k$  for which  $|k\vec{a}| < |\vec{a}|$  and  $k\vec{a} + \frac{1}{2}\vec{a}$  is parallel to  $\vec{a}$  holds true are \_\_\_\_\_

38. The value of the expression  $|\vec{a} \times \vec{b}| + (\vec{a} \cdot \vec{b})$  is \_\_\_\_\_

39. If  $|\vec{a} \times \vec{b}| + |\vec{a} \cdot \vec{b}| = 144$  and  $|\vec{a}| = 4$ , then  $|\vec{b}|$  is equal to \_\_\_\_\_

40. If  $\vec{a}$  is any non-zero vector, then  $(\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$  equals \_\_\_\_\_

State **True** or **False** in each of the following Exercises.

41. If  $|\vec{a}| = |\vec{b}|$ , then necessarily it implies  $\vec{a} = \pm\vec{b}$ .

42. Position vector of point P is a vector whose initial point is origin.

43. If  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ , then the vectors  $\vec{a}$  and  $\vec{b}$  are orthogonal.

44. The formula  $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + 2\vec{a} \cdot \vec{b}$  is valid for non-zero vectors  $\vec{a}$  and  $\vec{b}$ .

45. If  $\vec{a}$  and  $\vec{b}$  are adjacent sides of a rhombus, then  $\vec{a} \cdot \vec{b} = 0$ .