

Short Answer (S.A) :

- Find the unit vector in the direction of sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{j} + \hat{k}$.
- If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, find the unit vector in the direction of
 - $6\vec{a}$
 - $2\vec{a} - \vec{b}$
- Find a unit vector in the direction of \overline{PQ} , where P and Q have co-ordinates (5,0,8) and (3,3,2), respectively.
- If \vec{a} and \vec{b} are the position vectors of A and B, respectively, find the position vector of a point C in BA produced such that $BC = 1.5BA$.
- Using vectors, find the value of k such that the points $(k, -10, 3)$, $(1, -1, 3)$ and $(3, 5, 3)$ are collinear.
- A vector \vec{r} is inclined at equal angles to the three axis. If the magnitude of \vec{r} is $2\sqrt{3}$ units, find \vec{r} .
- A vector \vec{r} has a magnitude 14 and direction ratios 2,3,-6. Find the direction cosines and components of \vec{r} , given that \vec{r} makes an acute angle with x-axis.
- Find a vector of magnitude 6, which is perpendicular to both the vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$.
- Find the angle between the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$.
- If $\vec{a} + \vec{b} + \vec{c} = 0$, show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. Interpret the result geometrically?
- Find the sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$.
- If A,B,C,D are the points with position vectors $\hat{i} + \hat{j} - \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{k}$, $3\hat{i} - 2\hat{j} + \hat{k}$, respectively, find the projection of \overline{AB} along \overline{CD} .
- Using vectors, find the area of $\triangle ABC$ with vertices A(1,2,3), B(2,-1,4) and C(4,5,-1).
- Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.

Long Answer (L.A)

- Prove that in any $\triangle ABC$, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, where a,b,c are the magnitudes of the sides opposite to the vertices A,B,C respectively.
- If \vec{a} , \vec{b} , \vec{c} , determine the vertices of a triangle, show that $\frac{1}{2} [\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}]$ gives the vector area of the triangle. Hence deduce the condition that the three points \vec{a} , \vec{b} , \vec{c} , are collinear. Also find the unit vector normal to the plane of the triangle.
- Show that area of the parallelogram whose diagonals are given by $\vec{a} \times \vec{b}$ is $\frac{|\vec{a} \times \vec{b}|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.
- If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a the vector \vec{c} such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

Objective Type Questions

Choose the correct answer from the given four options in each of the Exercise from 19 to 33 (M.C.Q)

- The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is
 - $\hat{i} - 2\hat{j} + 2\hat{k}$
 - $\hat{i} - 2\hat{j}$
 - $3(\hat{i} - 2\hat{j} + 2\hat{k})$
 - $9(\hat{i} - 2\hat{j} + 2\hat{k})$

20. The position vector of the point which divides the join of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3:1 is
- $\frac{3\vec{a}-2\vec{b}}{2}$
 - $\frac{7\vec{a}-8\vec{b}}{4}$
 - $\frac{3\vec{a}}{4}$
 - $\frac{5\vec{a}}{4}$
21. The vector having initial and terminal points as (2,5,0) and (-3,7,4), respectively is
- $-\hat{i} + 12\hat{j} + 4\hat{k}$
 - $5\hat{i} + 2\hat{j} - 4\hat{k}$
 - $5\hat{i} + 2\hat{j} + 4\hat{k}$
 - $\hat{i} + \hat{j} + \hat{k}$
22. The angles between two vectors \vec{a} and \vec{b} with magnitude $\sqrt{3}$ and 4, respectively, and $\vec{a} \cdot \vec{b} = 2\sqrt{3}$ is
- $\frac{\pi}{6}$
 - $\frac{\pi}{3}$
 - $\frac{\pi}{2}$
 - $\frac{5\pi}{2}$
23. Find the value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal.
- 0
 - 1
 - $\frac{3}{2}$
 - $-\frac{5}{2}$
24. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is
- $\frac{2}{3}$
 - $\frac{3}{2}$
 - $\frac{5}{2}$
 - $\frac{2}{5}$
25. The vector from origin to the points A and B are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$, respectively, then the area of $\triangle OAB$ is
- 340
 - $\sqrt{25}$
 - $\sqrt{229}$
 - $\frac{1}{2}\sqrt{229}$
26. For any vector \vec{a} , the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to
- a
 - 3a
 - 4a
 - 2a
27. If $|\vec{a}|=10$, $|\vec{b}|=2$ and $\vec{a} \cdot \vec{b}=12$, then value of $|\vec{a} \times \vec{b}|$ is
- 5
 - 10
 - 14

(d) 16

28. The vectors $\lambda\hat{i} + \lambda\hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if

(a) $\lambda = -2$

(b) $\lambda = 0$

(c) $\lambda = 1$

(d) $\lambda = -1$

29. If \vec{a} , \vec{b} , \vec{c} are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is

(a) 1

(b) 3

(c) $-\frac{3}{2}$

(d) None of these

30. Projection vector of \vec{a} on \vec{b} is

(a) $\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right)$

(b) $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

(c) $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

(d) $\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \right)$

31. If \vec{a} , \vec{b} , \vec{c} are the three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 5$, the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is

(a) 0

(b) 1

(c) -19

(d) 38

32. If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 2$, then the range of $|\lambda\vec{a}|$ is

(a) $[0, 8]$

(b) $[-12, 8]$

(c) $[0, 12]$

(d) $[8, 12]$

33. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(a) one

(b) two

(c) three

(d) infinite

Fill in the blanks in each of the Exercises from 34 to 40.

34. The vector $\vec{a} + \vec{b}$ bisects the angle between the non-collinear vectors \vec{a} and \vec{b} if _____.

35. If $\vec{r} \cdot \vec{a} = 0$, $\vec{r} \cdot \vec{b} = 0$ and $\vec{r} \cdot \vec{c} = 0$ for some non-zero vector \vec{r} , then the value of $\vec{a} \cdot (\vec{b} \times \vec{c})$ is _____.

36. The vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{k}$ are the adjacent sides of a parallelogram. The acute angle between its diagonals is _____.

37. The values of k for which $\left|k\vec{a}\right| < |\vec{a}|$ and $k\vec{a} + \frac{1}{2}\vec{a}$ is parallel to \vec{a} holds true are _____.
38. The value of the expression $\left|\vec{a} \times \vec{b}\right| + (\vec{a} \cdot \vec{b})$ is _____.
39. If $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a} \cdot \vec{b}\right|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to _____.
40. If \vec{a} is any non-zero vector, then $(\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$ equals _____.
State **True** or **False** in each of the following Exercises.
41. If $|\vec{a}| = |\vec{b}|$, then necessarily it implies $\vec{a} = \pm\vec{b}$.
42. Position vector of point P is a vector whose initial point is origin.
43. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the vectors \vec{a} and \vec{b} are orthogonal.
44. The formula $(\vec{a} + \vec{b}) = \vec{a} + \vec{b} + 2\vec{a} \times \vec{b}$ is valid for non-zero vectors \vec{a} and \vec{b} .
45. If \vec{a} and \vec{b} are adjacent sides of a rhombus, then $\vec{a} \cdot \vec{b} = 0$.