Latex Math Assignment ncert-exemplar/12/leep210.pdf-Excercise 10.3

Short Answer (S.A)

1. Find the unit vector in the direction of sum of vectors $\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{b} = 2\hat{j} + \hat{k}$.

2. If $\overrightarrow{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, find the unit vector in the direction of

(i) $6\overrightarrow{a}$ (ii) $2\overrightarrow{a} - \overrightarrow{b}$

3. Find a unit vector in the direction of $\overline{PQ}$, where P and Q have co-ordinates(5,0,8)and (3,3,2),respectively.

4. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are the postion vectors of A and B, respectively, find the position vector of a point C in BA produced such that BC=1.5 BA.

5. Using vectors, find the value of k such that the points (k,-10,3),(1,-1,3)and (3,5,3) are colinear

6. A vector $\overrightarrow{r}$ is inclined at equal angles to the three axes. If the magnitude of $\overrightarrow{r}$ is $2\sqrt{3}$ units, find $\overrightarrow{r}$.

7. A vector $\overrightarrow{r}$ has a magnitude 14 and direction ratios 2,3,-6. Find the direction cosines and components of $\overrightarrow{r}$,given that $\overrightarrow{r}$ makes an acute angle with x-axis.

8. Find a vector of magntude6, which is perpendicular to both the vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$.

9. Find the angle between the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$.

10. If $\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = 0$, show that $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{b} \times \overrightarrow{c} = \overrightarrow{c} \times \overrightarrow{a}$. Interpret the result geometrically?

11. Find the sine of the angle between the vectors $\overrightarrow{a} = 3\overrightarrow{i} + \overrightarrow{j} + 2\overrightarrow{k}$ and $b = 2\overrightarrow{i} - 2\overrightarrow{j} + 4\overrightarrow{k}$.

12. If A,B,C,D are the points with position vectors $\hat{i} + \hat{j} - \hat{k}, 2\hat{i} - \hat{j} + 3\hat{k}, 2\hat{i} - 3\hat{k}, 3\hat{i} - 2\hat{j} + \hat{k}$, respectively, find the projection of $\overline{AB}$ along $\overline{CD}$.

13. Using vectors, find the area of trianlge ABC with vertices A(1,2,3), B(2,-1,4) and C(4,5,-1).

14. Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.

Long Answer Questions(L.A)

15. Prove that in any triangle ABC, cosA=$\frac{b^2+c^2-a^2}{2bc}$, where a,b,c are the magnitudes of the sides opposite to the vertices A,B,C respectively.

16. If $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ determine the vertices of a triangle, show that $\frac{1}{2}\left[\overrightarrow{b} \times \overrightarrow{c} + \overrightarrow{c} \times \overrightarrow{a} + \overrightarrow{a} \times \overrightarrow{b}\right]$ gives the vector area of the triangle. Hence deduce the condition that the three points $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ are collinear. Also find the unit vector of normal to the plane of triangle.

17. Show that area of the parallelogram whose diagonals are given by $\overrightarrow{a} \times \overrightarrow{b}$ is $\frac{|a \times \ b|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

18. If $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$ and $\overrightarrow{b} = \hat{j} - \hat{k}$, find a the vector $\overrightarrow{c}$ such that $\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b}$ and $\overrightarrow{a}.\overrightarrow{c} = 3$.

Objective Type Questions

Choose the correct answer from the given four options in each of the Excercise from 19 to 33(M.C.Q)

19. The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude9 is

(A) $\hat{i} - 2\hat{j} + 2\hat{k}$

(B) $\dfrac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

(C) $3(\hat{i} - 2\hat{j} + 2\hat{k})$

(D) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

20. The position vector of the point which divides the join of points $2\overrightarrow{a} - 3\overrightarrow{b}$ and $\overrightarrow{a} + \overrightarrow{b}$ in the ratio 3:1 is

(A) $\dfrac{3\overrightarrow{a} - 2\overrightarrow{b}}{2}$ (B) $\dfrac{7\overrightarrow{a} - 8\overrightarrow{b}}{4}$ (C) $\dfrac{\overrightarrow{3a}}{4}$ (D) $\dfrac{\overrightarrow{5a}}{4}$

21. The vector having intial and terminal points as (2,5,0)and (-3,7,4),respectively is

(A) $-\hat{i} + 12\hat{j} + 4\hat{k}$

(B) $-5\hat{i} + 2\hat{j} + 4\hat{k}$

(C) $5\hat{i} + 2\hat{j} - 4\hat{k}$

(D) $\hat{i} + \hat{j} + \hat{k}$

22. The angles between two vectors $\overrightarrow{a} and \overrightarrow{b}$ with magnitude $\sqrt{3}$ and 4, respectively, and $\overrightarrow{a}, \overrightarrow{b} = 2\sqrt{3}$ is

(A) $\dfrac{\pi}{6}$ (B) $\dfrac{\pi}{3}$ (C) $\dfrac{\pi}{2}$ (D) $\dfrac{5\pi}{2}$

23. Find the value of λ such that the vectors $\overrightarrow{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\overrightarrow{b} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k}$ are orthogonal.

(A) 0 (B) 1 (C) $\dfrac{3}{2}$ (D) $-\dfrac{5}{2}$

24. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j}+\lambda\hat{k}$ are parallel is

(A) $\dfrac{2}{3}$ (B) $\dfrac{3}{2}$ (C) $\dfrac{5}{2}$ (D) $\dfrac{2}{5}$

25. The vector from origin to the points A and B are $\overrightarrow{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\overrightarrow{b} = 2\hat{i} + 3\hat{j} + \hat{k}$, respectively, then the area of trianlge OAB is

(A) 340 (B) $\sqrt{25}$ (C) $\sqrt{229}$ (D) $\dfrac{1}{2}\sqrt{229}$

26. For any vector $\overrightarrow{a}$, the value of $(\overrightarrow{a} \times \overrightarrow{i})^2 + (\overrightarrow{a} \times \overrightarrow{j})^2 + (\overrightarrow{a} \times \overrightarrow{k})^2$ is equal to

(A) a (B) 3a (C) 4a (D) 2a

27. If $|\overrightarrow{a}| = 10, |\overrightarrow{b}| = 2$ and $\overrightarrow{a}, \overrightarrow{b} = 12$, then value of $|\overrightarrow{a} \times \overrightarrow{b}|$ is

(A) 5 (B) 10 (C) 14 (D) 16

28. The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k},\ \hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if

(A) $\lambda = -2$ (B) $\lambda = 0$ (C) $\lambda = 1$ (D) $\lambda = 1$

29. If $\hat{a}, \hat{b}, \hat{c}$ are unit vectors such that $\hat{a} + \hat{b} + \hat{c} = 0$, then the value of $\hat{a}.\hat{b} + \hat{b}.\hat{c} + \hat{c}.\hat{a}$ is

(A) 1 (B) 3 (C) $-\dfrac{3}{2}$ (D) None of these

30. Projection vector of $\overrightarrow{a}$ on $\overrightarrow{b}$ is

(A) $\left(\dfrac{\overrightarrow{a}.\overrightarrow{b}}{|\overrightarrow{b}|^2}\right)$ (B) $\dfrac{\overrightarrow{a}.\overrightarrow{b}}{|\overrightarrow{b}|}$ (C) $\dfrac{\overrightarrow{a}.\overrightarrow{b}}{|\overrightarrow{a}|}$ (D) $\left(\dfrac{\overrightarrow{a}.\overrightarrow{b}}{|\overrightarrow{a}|^2}\right)$

31. $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ are the three vectors such that $\overrightarrow{a}+\overrightarrow{b}+\overrightarrow{c} = 0$ and $|\overrightarrow{a}| = 2, |\overrightarrow{b}| = 3, |\overrightarrow{c}| = 5$, the value of $\overrightarrow{a}.\overrightarrow{b} + \overrightarrow{b}.\overrightarrow{c} + \overrightarrow{c}.\overrightarrow{a}$ is

(A) 0 (B) 1 (C) -19 (D) 38

32. If $|\overrightarrow{a}| = 4$ and $-3 \leq \lambda \leq 2$, then the range of $|\lambda \overrightarrow{a}|$ is

(A) $[0, 8]$ (B) $[-12, 8]$ (C) $[0, 12]$ (D) $[8, 12]$

33. The number of vectors of unit length perpendicular to the vectors $\overrightarrow{a} = 2\overrightarrow{i} + \overrightarrow{j} + 2\overrightarrow{k}$ and $\overrightarrow{b} = \overrightarrow{j} + \overrightarrow{k}$ is

(A) one (B) two (C) three (D) infinite

Fill in the blanks in each of Excersices from 34 to 40.

34. The vector $\overrightarrow{a} + \overrightarrow{b}$ bisects the angle between the non-collinear vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ if ______

35. If $\overrightarrow{r}.\overrightarrow{a} = 0, \overrightarrow{r}.\overrightarrow{b} = 0$ and $\overrightarrow{r}.\overrightarrow{c} = 0$ for some non-zero vector $\overrightarrow{r}$, then the value of $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$ is ______

36. The vectors $\overrightarrow{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{b} = -\hat{i} - 2\hat{k}$ are the adjancent sides of a parallelogram. The acute angle between its diagonals is ______

37. The values of k for which $|k\overrightarrow{a}| < |\overrightarrow{a}|$and k$\overrightarrow{a} + \frac{1}{2}\overrightarrow{a}$ is parallel to $\overrightarrow{a}$ holds true are ______

38. The value of the expression $|\overrightarrow{a} \times \overrightarrow{b}|^2 + (\overrightarrow{a}.\overrightarrow{b})^2$ is ______

39. If $|\overrightarrow{a} \times \overrightarrow{b}|+|\overrightarrow{a}.\overrightarrow{b}|$=144 and $|\overrightarrow{a}| = 4$, then $|\overrightarrow{b}|$ is equal to ______

40. If $\overrightarrow{a}$ is any non-zero vector, then $(\overrightarrow{a}.\hat{i})\hat{i}$+$(\overrightarrow{a}.\hat{j})\hat{j}$+ $(\overrightarrow{a}.\hat{k})\hat{k}$ equals ______

State **True** or **False** in each of the Excercises.

41. If $|\overrightarrow{a}| = |\overrightarrow{b}|$, then necessarily it implies $\overrightarrow{a} = \pm\overrightarrow{b}$.

42. Position vector of point P is a vector whose intial point is origin.

43. If $|\overrightarrow{a} + \overrightarrow{b}|$=$|\overrightarrow{a} - \overrightarrow{b}|$, then the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ are orthogonal.

44. The formula $(\overrightarrow{a} + \overrightarrow{b})$= $\overrightarrow{a}^2 + \overrightarrow{b}^2 + 2\overrightarrow{a} \times \overrightarrow{b}$ is valid for non-zero vectors $\overrightarrow{a}$ and $\overrightarrow{b}$.

45. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are adjacent sides of a rhombus, then $\overrightarrow{a}.\overrightarrow{b}$.=0.