DR_CAN 动态系统的建模与分析

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笔记：王崇卫

说明

致敬b站DR_CAN博士。

笔记是个人根据视频仿照DR_CAN老师，极慢的方式把笔记使用drawio软件做了一遍。

这么好的视频，我希望有个还凑合的笔记，供查阅。所以我花了时间来做这件事情。希望对大家有所帮助。

（电子笔记仅供参考翻阅，学习时应当动笔在纸上跟着up主计算）

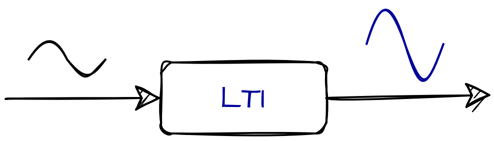
如有错误，欢迎指出，邮箱1084746243@qq.com

动态系统的建模与分析 8

频率响应与滤波器

控制和信号系统至关重要

$G(j\omega)$ ~~玄~~学？ 数学！



$M_i \sin(\omega t + \phi_i)$ $\qquad$ $M_o \sin(\omega t + \phi_o)$

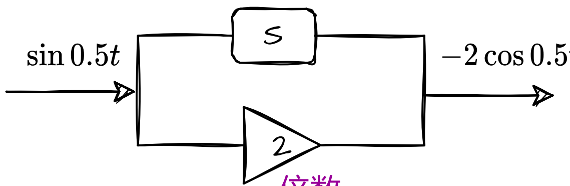
Input Magnitude 输入振幅 $\qquad$ Input Phase 输入相位

$$\frac{M_o}{M_I} = M$$ Magnitude Response 振幅响应

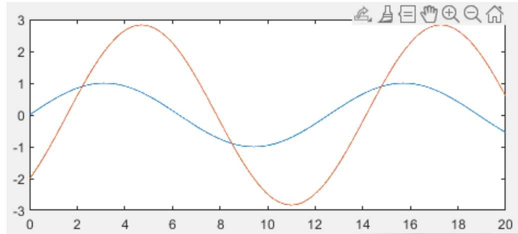
$$\phi_o - \phi_i = \phi$$ Phase Response 幅角响应

频率无变化，幅值变大了，发生了平移

$\sin 0.5t$



$-2\cos 0.5t + 2\sin 0.5t$



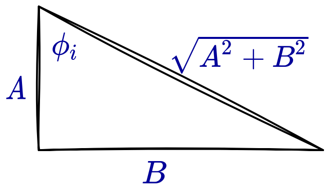
```
t=[0:0.02:20];
f1=sin(0.5*t);
f2=-2*cos(0.5*t)+2*sin(0.5*t);
plot(t,f1);hold on;
plot(t,f2);
```

$$u(t) = A\sin\omega t + B\cos\omega t$$

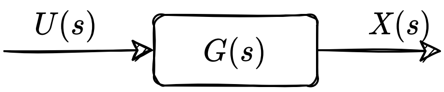
$$= \sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\sin\omega t + \frac{A}{\sqrt{A^2+B^2}}\cos\omega t\right)$$

$$= \sqrt{A^2+B^2}(\cos\phi_i \sin\omega t + \sin\phi_i \cos\omega t)$$

$$= \underbrace{\sqrt{A^2+B^2}}_{M_i}(\sin(\omega t + \phi_i)) \quad = M_i \sin(\omega t + \phi_i)$$



Transfer Function



$$X(s) = U(s)G(s)$$

$$U(s) = \mathscr{L}[U(t)] = \frac{A\omega}{s^2+\omega^2} + \frac{Bs}{s^2+\omega^2} = \frac{A\omega + Bs}{s^2+\omega^2} = \frac{A\omega+Bs}{(s+j\omega)(s-j\omega)} \qquad j = \sqrt{-1}$$

$$G(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s-P_1)(s-P_2)\cdots(s-P_n)}$$

$P_1, P_2 \cdots P_n$ Poles 极点

$$X(s) = U(s)G(s) = \frac{A\omega+Bs}{s^2+\omega^2}\cdot\frac{N(s)}{(s-P_1)(s-P_2)\cdots(s-P_n)}$$

待定系数法

$$= \frac{K_1}{s+j\omega} + \frac{K_2}{s-j\omega} + \frac{C_1}{s-P_1} + \frac{C_2}{s-P_2} + \cdots \frac{C_n}{s-P_n}$$

$(A\omega + Bs)N(s)$

$$= \frac{\boxed{K_1(s-j\omega)D(s) + K_2(s+j\omega)D(s) + C_1(s-j\omega)(s+j\omega)(\cdot\cdot) + C_2(s+j\omega)(s-j\omega)(\cdot\cdot) + \cdot\cdot}}{(s-j\omega)(s+j\omega)(s-P_1)(s-P_2)\cdots(s-P_n)}$$

$$X(t) = \mathscr{L}^{-1}(X(s)) \; = K_1 e^{-j\omega t} + K_2 e^{j\omega t} + C_1 e^{P_1 t} + C_2 e^{P_2 t} + \cdots C_n e^{P_n t}$$

对于稳定系统： $P_1, P_2, \cdots$的实部小于0，例如 $P_1 = -2$, $P_2 = -1+i$ $\Longrightarrow$ $t \to \infty$ $C_1 e^{-2t} + C_2 e^{-1+it} = 0$

$$\boxed{X_{ss}(t) = K_1 e^{-j\omega t} + K_2 e^{j\omega t}}$$

steady state
稳态

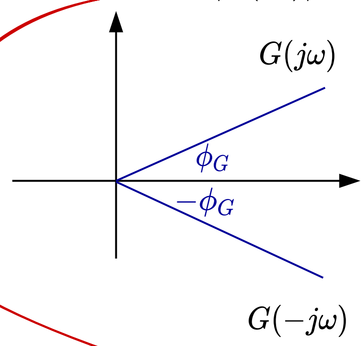
$$K_1(s-j\omega)D(s) + K_2(s+j\omega)D(s) + C_1(s-j\omega)(s+j\omega)(\cdot\cdot)\cdots = (A\omega + Bs)N(s)$$

let $s = -j\omega$

$$K_1(-j\omega - j\omega)D(-j\omega) + 0 + 0 \cdots + 0 = (A\omega - Bj\omega)N(-j\omega)$$

$$K_1 = \frac{A\omega - Bj\omega}{-2j\omega} = \frac{N(-j\omega)}{D(-j\omega)}$$

$|G(j\omega)|e^{j\phi_G}$



$$K_1 = \frac{B+Aj}{2}G(-j\omega)$$

let $s = j\omega$

$$K_2 = \frac{B-Aj}{2}G(j\omega)$$

$|G(j\omega)|e^{-j\phi_G}$

$$X_{ss}(t) = \frac{B+Aj}{2}|G(j\omega)|e^{j\phi_G}e^{-j\omega t} + \frac{B-Aj}{2}e^{j\phi_G}e^{j\omega t}$$

$$= \frac{1}{2}|G(j\omega)|((B+Aj)e^{-(\phi_G+\omega t)j} + (B-Aj)e^{(\phi_G+\omega t)j})$$

欧拉公式

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-(\phi_G+\omega t)j} = \cos(-(\phi_G+\omega t)) + j\sin(\phi_G+\omega t)$$

$$= \cos(\phi_G+\omega t) - j\sin(\phi_G+\omega t)$$

$$e^{(\phi_G+\omega t)j} = \cos((\phi_G+\omega t)) + j\sin(\phi_G+\omega t)$$

$$X_{ss}(t) = \frac{1}{2}|G(j\omega)|(B\cos(\phi_G+\omega t) - Bj\sin(\phi_G+\omega t) + Aj\cos(\phi_G+\omega t) + A\sin(\phi_G+\omega t)$$
$$+ B\cos(\phi_G+\omega t) + Bj\sin(\phi_G+\omega t) - Aj\cos(\phi_G+\omega t) + A\sin(\phi_G+\omega t))$$

$$X_{ss}(t) = \frac{1}{2}|G(j\omega)|(2B\cos(\phi_G+\omega t) + 2A\sin(\phi_G+\omega t))$$

$$= |G(j\omega)|\sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\cos(\phi_G+\omega t) + \frac{A}{\sqrt{A^2+B^2}}\sin(\phi_G+\omega t)\right)$$

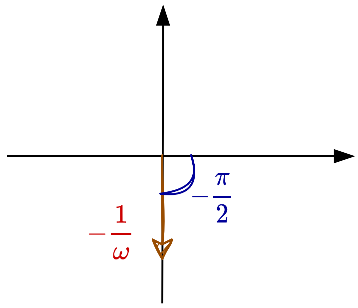
$$\boxed{= |G(j\omega)|M_i\sin(\omega t + \phi_i + \phi_G)}$$

$$\boxed{\begin{aligned} M_G &= |G(j\omega)| \\ \phi_G &= \angle G(j\omega) \end{aligned}}$$

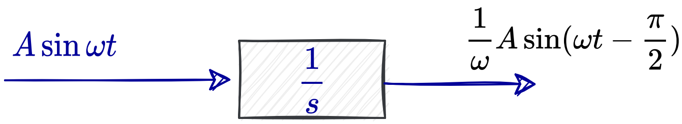
积分例子

$$\int U(t)\mathrm{d}t \qquad\qquad G(s) = \frac{1}{s}$$

$$G(j\omega) = \frac{1}{j\omega} = -\frac{1}{\omega}j$$



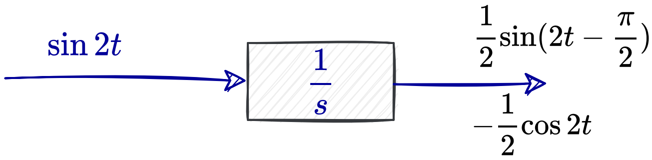
$$|G(j\omega)| = \frac{1}{\omega}$$



$\omega \uparrow$

$|G(j\omega)| \downarrow$

低通



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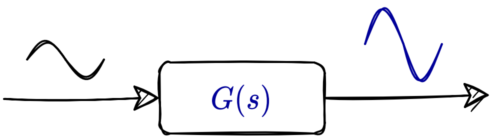
一阶系统的频率响应_低通滤波器



$M_i \sin(\omega t + \phi_i)$ $\qquad$ $M_o \sin(\omega t + \phi_o)$

Input Magnitude 输入振幅 $\qquad$ Input Phase 输入相位

$$\boxed{\begin{aligned} \frac{M_o}{M_I} &= |G(j\omega)| \\ \phi_o - \phi_i &= \angle G(j\omega) \end{aligned}}$$

Magnitude Response 振幅响应

Phase Response 幅角响应

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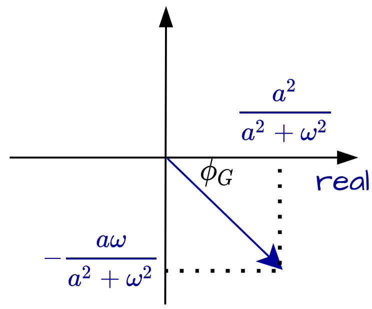
1 st Order System 一阶系统

$$G(s) = \frac{a}{s+a}$$

$s = j\omega$ $\Longrightarrow$

$$G(j\omega) = \frac{a}{a+j\omega} = \frac{a(a-j\omega)}{(a+j\omega)(a-j\omega)} = \frac{a^2 - aj\omega}{a^2+\omega^2}$$

$$= \underbrace{\frac{a^2}{a^2+\omega^2}}_{\text{Real}} + \underbrace{\left(-\frac{a\omega}{a^2+\omega^2}\right)}_{\text{Imaginary}} j$$



$$|G(j\omega)| = \sqrt{\left(\frac{a^2}{a^2+\omega^2}\right)^2 + \left(\frac{a\omega}{a^2+\omega^2}\right)^2}$$

$$= \sqrt{\frac{a^2(a^2+\omega^2)}{a^2+\omega^2}}$$

$$= \sqrt{\frac{1}{1+(\frac{\omega}{a})^2}}$$

高通

$$\sqrt{\frac{1}{1+(\frac{a}{\omega})^2}}$$

$$G(s) = \frac{s}{s+a}$$

$\omega << a \Longrightarrow (\frac{\omega}{a})^2 \to 0 \Longrightarrow |G(j\omega)| \to 1$

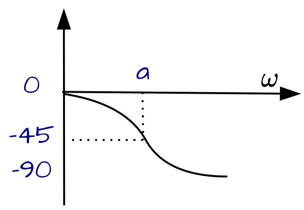
$\omega \uparrow \Longrightarrow (\frac{\omega}{a})^2 \uparrow \Longrightarrow |G(j\omega)| \downarrow$

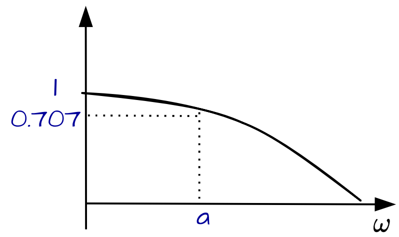
$\omega = a \Longrightarrow |G(j\omega)| = \sqrt{\frac{1}{2}} = 0.707$

$\omega >> a \Longrightarrow (\frac{\omega}{a})^2 \to \infty \Longrightarrow |G(j\omega)| \to 0$

$$\phi_G = \arctan\left(-\frac{a\omega}{a^2}\right)$$

$$= -\arctan\left(\frac{\omega}{a}\right)$$

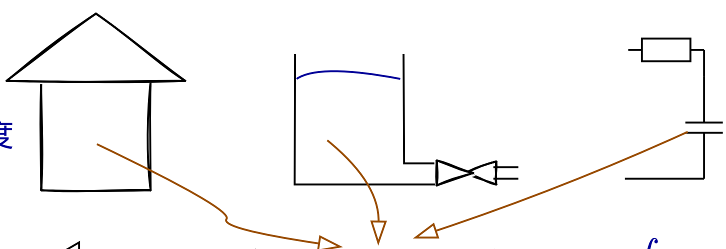




cut-off frequency 截止频率

低通滤波器 Low Pass Filter

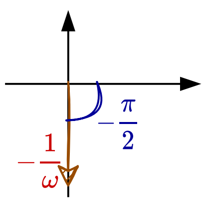
空调温度



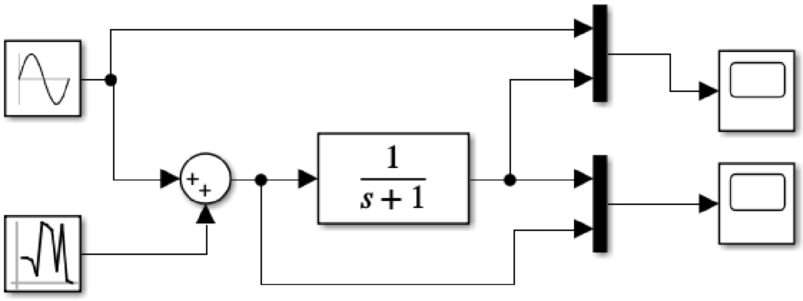
延迟 $\Longleftarrow$ 缓冲 $\Longleftarrow$ "容器" $\Longrightarrow$ 积分 $\int$ $\Longrightarrow$ $\frac{1}{s}$ $\quad s = j\omega$

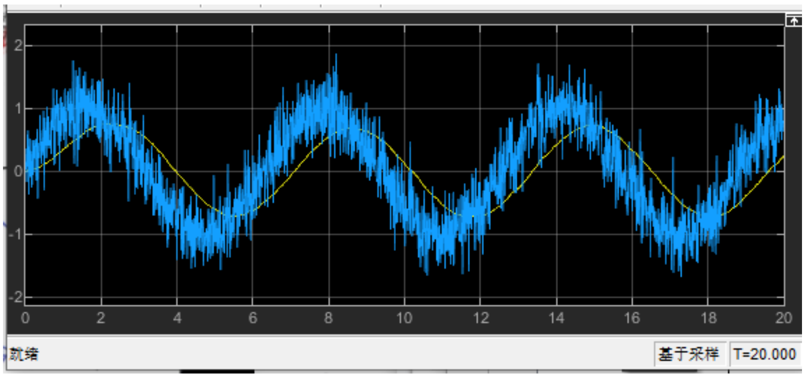
$$\frac{1}{j\omega} = -\frac{1}{\omega} j$$

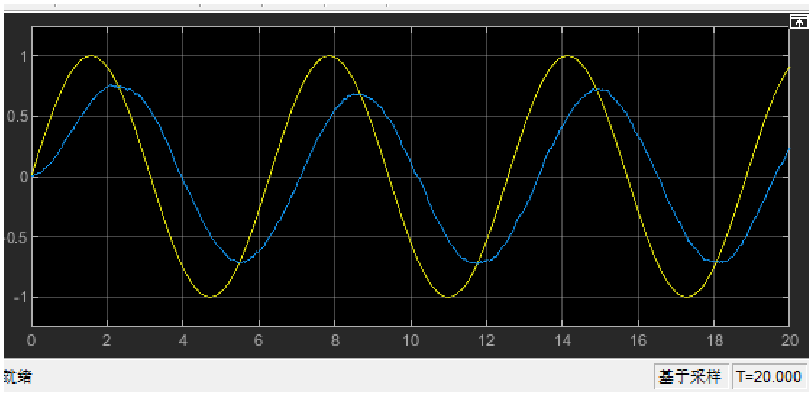
$$\left|G\left(\frac{1}{j\omega}\right)\right| = \frac{1}{\omega}$$



Simulink 仿真


$\frac{1}{s+1}$


2
1
0
-1
-2
0 2 4 6 8 10 12 14 16 18 20
就绪
基于采样
T=20.000


1
0.5
0
-0.5
-1
0 2 4 6 8 10 12 14 16 18 20
就绪
基于采样
T=20.000

动态系统的建模与分析

笔记:王崇卫

二阶系统动态响应 2nd order System

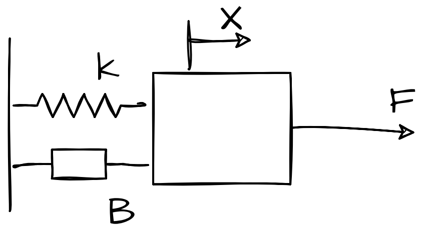
Newton's 2nd Law

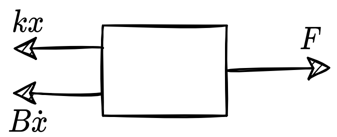
位移对时间的二次导

$$F = ma \qquad a: \text{加速度} \qquad \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \qquad \ddot{x} \quad \ddot{\theta}$$

动力学和运动学

Vibration 振动　　分析二阶系统　　matllab/simulink





mass-spring-damping

定义一些参数

$$\omega_n = \sqrt{\frac{k}{m}}$$ Natural Frequency 固有频率

$$\zeta = \frac{B}{2\sqrt{km}}$$ Damping Ratio 阻尼比

Response to Inital Conditions 初始条件

$$F = 0 \quad x_{(0)} = x_0 \quad \dot{x}_{(0)} = \dot{x}_0$$

$$m\ddot{x} = F - B\dot{x} - kx$$

$$\boxed{\ddot{x} + \frac{B}{m}\dot{x} + \frac{k}{m}x = F}$$

$$\boxed{\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0}$$

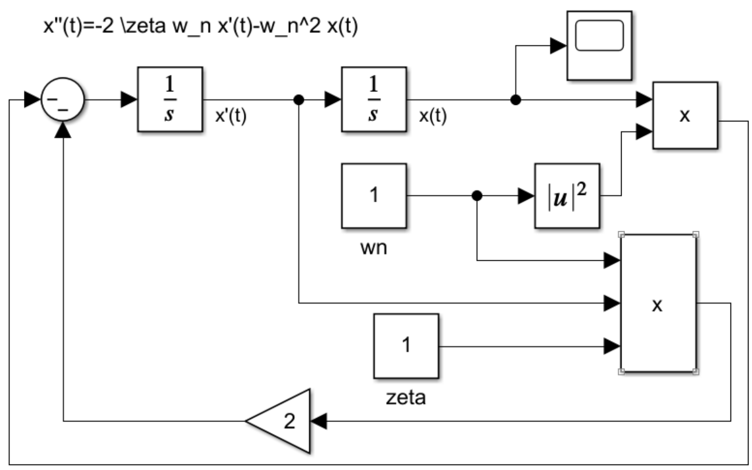
求解微分方程

simulink

$$\ddot{x} = -2\zeta\omega_n\dot{x} - \omega_n^2 x$$

$$x_{(0)} = 5$$

$$\dot{x}_{(0)} = 0$$



$$x_{(t)} = e^{\lambda t} \qquad \dot{x} = \lambda e^{\lambda t} \qquad \ddot{x} = \lambda^2 e^{\lambda t}$$

$$\lambda^2 e^{\lambda t} + 2\zeta\lambda e^{\lambda t} + \omega_n^2 e^{\lambda t} = 0$$

$$(\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2)e^{\lambda t} = 0 \qquad e^{\lambda t} \neq 0$$

$$\boxed{\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0}$$

characteristic equation
特征方程

$$\lambda = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$\lambda_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

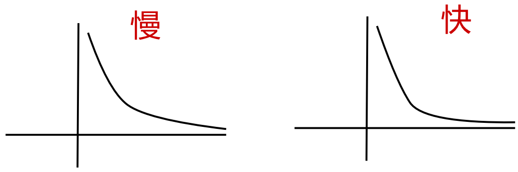
$$\lambda_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$

分情况讨论

① $\zeta > 1$ $\quad \frac{B}{2\sqrt{km}} > 1$ Over damped 过阻尼

$$\lambda_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} \quad < 0$$

$< \zeta$



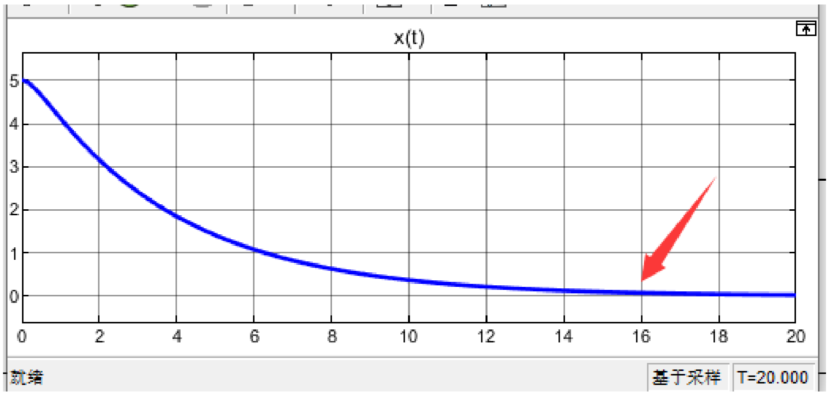
$$\lambda_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1} \quad < 0$$

< 0 $\quad < 0$

$$x_{(t)} = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

$|\lambda_1| < |\lambda_2|$

simulink中 $\zeta = 2$



② $\zeta = 1$ $\quad \frac{B}{2\sqrt{km}} = 1$ critical damped 临界阻尼

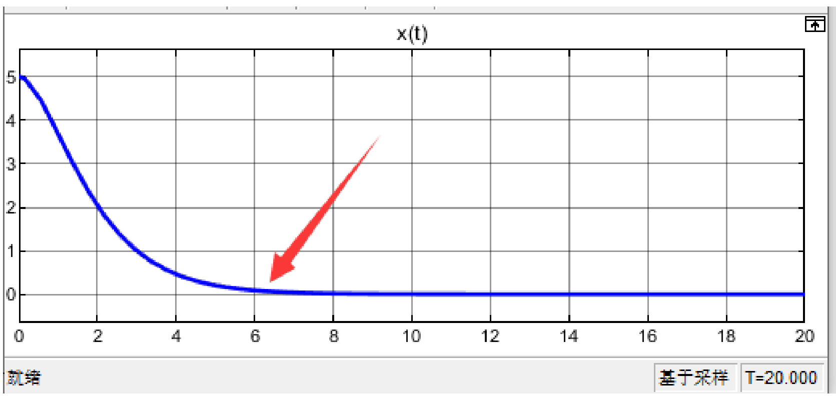
$$x_{(t)} = (C_1 + C_2 t)e^{\lambda t}$$

不存在一个慢的收敛项，收敛比过阻尼快

$$\lambda_1 = \lambda_2 = -\omega_n$$

$|\lambda_1| < \lambda < |\lambda_2|$

simulink中 $\zeta = 1$



③ $0 < \zeta < 1$ $\quad 0 < \frac{B}{2\sqrt{km}} < 1$ underdamped 欠阻尼

ω_d Damped natural frequency 阻尼固有频率

$$\lambda = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

< 0

$$x_{(t)} = e^{-\zeta\omega_n t}(C_1 \cos \underbrace{\omega_n\sqrt{1 - \zeta^2}}_{\omega_d} t + C_2 \sin \omega_n\sqrt{1 - \zeta^2} t)$$

$$\lambda_1 = -\zeta\omega_n + i\omega_n\sqrt{1 - \zeta^2}$$

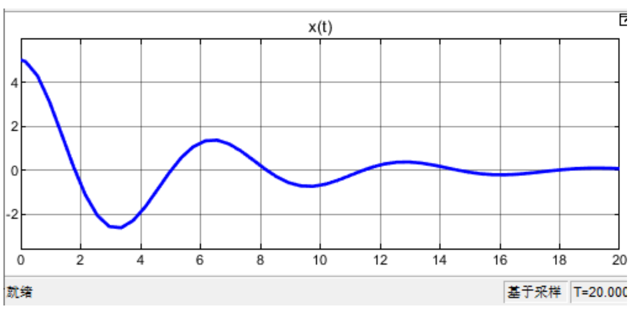
共轭

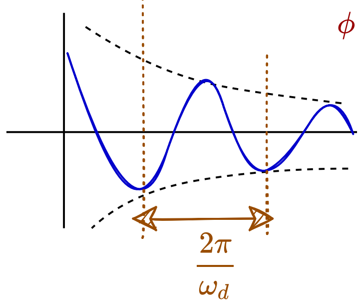
$$\lambda_2 = -\zeta\omega_n - i\omega_n\sqrt{1 - \zeta^2}$$

$$\boxed{x_{(t)} = \sqrt{C_1^2 + C_2^2}\, e^{-\zeta\omega_n t}(\sin(\omega_d t + \phi))}$$

$$\phi = \arctan(\frac{C_1}{C_2})$$

simulink中 $\zeta = 0.2$

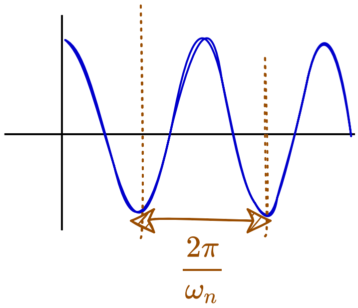




④ $\zeta = 0 \qquad B = 0$

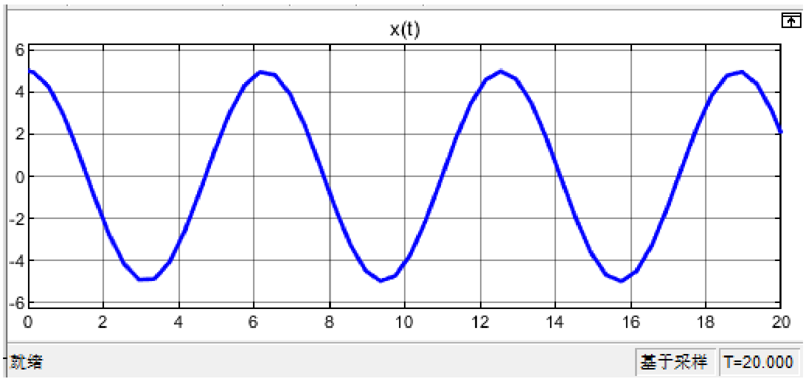
$$x_{(t)} = e^0 (C_1 \cos \omega_n t + C_2 \sin \omega_n t)$$

$$x_{(t)} = \sqrt{C_1^2 + C_2^2} \sin(\omega_d t + \phi)$$

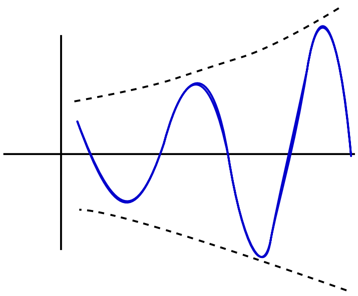


simulink中

$\zeta = 0$



⑤ $-1 < \zeta < 0$



不稳定

$$\lambda = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

< 0

$$\lambda_1 = -\zeta\omega_n + i\omega_n \sqrt{1 - \zeta^2}$$

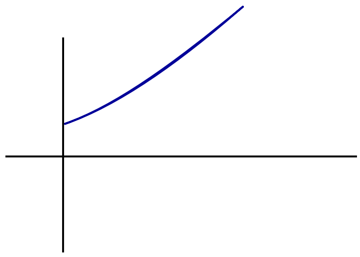
$$\lambda_2 = -\zeta\omega_n - i\omega_n \sqrt{1 - \zeta^2}$$

$$x_{(t)} = e^{-\zeta\omega_n t} (C_1 \cos \omega_n \sqrt{1 - \zeta^2} t + C_2 \sin \omega_n \sqrt{1 - \zeta^2} t)$$

$$x_{(t)} = \sqrt{C_1^2 + C_2^2} \underline{e^{-\zeta\omega_n t}} (\sin(\omega_d t + \phi))$$

$-\zeta\omega_n > 0$

⑥ $\zeta < -1$



$$\lambda = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

$< \zeta$

$$\lambda_1 = -\zeta\omega_n + \omega_n \sqrt{\zeta^2 - 1} \quad > 0$$

$$\lambda_2 = -\zeta\omega_n - \omega_n \sqrt{\zeta^2 - 1} \quad > 0$$

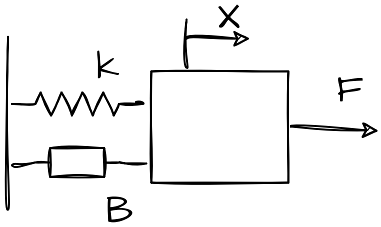
$$x_{(t)} = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

> 0

动态系统的建模与分析

2nd order System Unit Step Response 二阶系统单位阶跃

mass-spring-damping



$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = F$$

$$\zeta = \frac{B}{2\sqrt{km}}$$ Damping Ratio 阻尼比

$$\omega_n = \sqrt{\frac{k}{m}}$$ Natural Frequency 固有频率

输入 Input $u_{(t)} = \dfrac{F}{\omega_n^2}$ → 单位化

输出 Input $x_{(t)}$

0初始条件

$x_{(0)} = x_0 \quad \dot{x}_{(0)} = \dot{x}_0$

1.微分方程求解

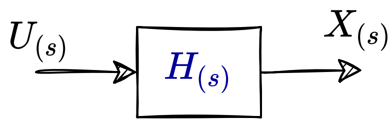
$x_{(t)} = x_n + x_p$

2. 传递函数

$$\mathscr{L}\ \ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \mathscr{L}\ \omega_n^2 u_{(t)}$$

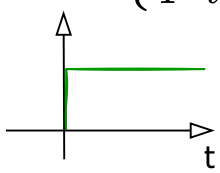
$$s^2 X_{(s)} + 2\zeta\omega_n s X_{(s)} + \omega_n^2 X_{(s)} = \omega_n^2 U_{(s)}$$

$$H_{(s)} = \frac{X_{(s)}}{U_{(s)}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



Unit step Input
单位阶跃

$$u_{(t)} = \begin{cases} 0 & t = 0 \\ 1 & t > 0 \end{cases}$$



$$\mathscr{L}[u_{(t)}] = \frac{1}{s}$$

$$X_{(s)} = U_{(s)} H_{(s)}$$

①求$X_{(s)}$

②$\mathscr{L}^{-1}[X_{(s)}] = X_{(t)}$

$$\boxed{X_{(s)} = \frac{1}{s}\,\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

Find Poles
极点

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$p_1 = 0$$
$$p_2 = -\zeta\omega_n + i\omega_n\sqrt{1-\zeta^2}$$
$$p_3 = -\zeta\omega_n - i\omega_n\sqrt{1-\zeta^2}$$

$$s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Under Damped
欠阻尼

$0 < \zeta < 1$

$$s = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$$

$$X_{(s)} = \frac{A}{s-p_1} + \frac{B}{s-p_2} + \frac{C}{s-p_3} = \frac{A(s-p_2)(s-p_3) + B(s-p_1)(s-p_3) + C(s-p_1)(s-p_2)}{(s-p_1)(s-p_2)(s-p_3)}$$

$= \omega_n^2$

$$A(s-p_2)(s-p_3) + B(s-p_1)(s-p_3) + C(s-p_1)(s-p_2) = \omega_n^2$$

$s = p_1 = 0$

$$A(-p_2)(-p_3) = \omega_n^2$$

$$A(\zeta\omega_n - i\omega_n\sqrt{1-\zeta^2})(\zeta\omega_n + i\omega_n\sqrt{1-\zeta^2}) = \omega_n^2$$

$$A(\cancel{\zeta^2\omega_n^2} + \omega_n^2 - \cancel{\omega_n^2\zeta^2}) = \omega_n^2$$

$$\boxed{A = 1}$$

$s = p_2 = -\zeta\omega_n + i\omega_n\sqrt{1-\zeta^2}$

$$B(-\zeta\omega_n + i\omega_n\sqrt{1-\zeta^2} - 0)(\cancel{-\zeta\omega_n} + i\omega_n\sqrt{1-\zeta^2} + \cancel{\zeta\omega_n} + i\omega_n\sqrt{1-\zeta^2}) = \omega_n^2$$

$$B(-\zeta\omega_n + i\omega_n\sqrt{1-\zeta^2})(2i\omega_n\sqrt{1-\zeta^2}) = \omega_n^2$$

$$B(-2i\zeta\omega_n^2\sqrt{1-\zeta^2} - 2\omega_n^2(1-\zeta^2)) = \omega_n^2$$

$$B(-2\cancel{\omega_n^2})(i\zeta\sqrt{1-\zeta^2} + (1-\zeta^2)) = \cancel{\omega_n^2}$$

$$B = -\frac{1}{2}\frac{1}{(1-\zeta^2) + i\zeta\sqrt{1-\zeta^2}} \cdot \frac{(1-\zeta^2) - i\zeta\sqrt{1-\zeta^2}}{(1-\zeta^2) - i\zeta\sqrt{1-\zeta^2}}$$

$$= -\frac{1}{2}\frac{(1-\zeta^2) - i\zeta\sqrt{1-\zeta^2}}{(1-\zeta^2)^2 + \zeta^2(1-\zeta^2)} = -\frac{1}{2}\frac{(1-\zeta^2) - i\zeta\sqrt{1-\zeta^2}}{1 - 2\zeta^2 + \cancel{\zeta^4} + \zeta^2 - \cancel{\zeta^4}}$$

$$= -\frac{1}{2}\frac{(1-\zeta^2) - i\zeta\sqrt{1-\zeta^2}}{1-\zeta^2} = -\frac{1}{2}[1 - \frac{\zeta\sqrt{1-\zeta^2}}{1-\zeta^2}i]$$

$$B = -\frac{1}{2}[1 - \frac{\zeta}{\sqrt{1-\zeta^2}}i]$$

$s = p_3 \quad C = B^*$

$$C = -\frac{1}{2}[1 + \frac{\zeta}{\sqrt{1-\zeta^2}}i]$$

$$X_{(s)} = \frac{\overset{A}{1}}{s} \boxed{-\frac{1}{2}[1 - \frac{\zeta}{\sqrt{1-\zeta^2}}i]}^{B}\frac{1}{s-p_2} \boxed{-\frac{1}{2}[1 + \frac{\zeta}{\sqrt{1-\zeta^2}}i]}^{C}\frac{1}{s-p_3}$$

② $\mathscr{L}^{-1}[X_{(s)}] = X_{(t)}$

$$X_{(t)} = 1 \cdot e^{0t} - \frac{1}{2}[1 - \frac{\zeta}{\sqrt{1-\zeta^2}}i]e^{p_2 t} - \frac{1}{2}[1 + \frac{\zeta}{\sqrt{1-\zeta^2}}i]e^{p_3 t}$$

Damped natural frequency
阻尼固有频率
ω_d
$||$

$$= 1 - \frac{1}{2}[1 - \frac{\zeta}{\sqrt{1-\zeta^2}}i]e^{(-\zeta\omega_n + i\omega_n\sqrt{1-\zeta^2})t} - \frac{1}{2}[1 + \frac{\zeta}{\sqrt{1-\zeta^2}}i]e^{(-\zeta\omega_n - i\omega_n\underline{\sqrt{1-\zeta^2}})t}$$

$$= 1 - e^{-\zeta\omega_n t}\left[\frac{1}{2}\left(1 - \frac{\zeta}{\sqrt{1-\zeta^2}}i\right)e^{i\omega_d t} + \frac{1}{2}\left(1 + \frac{\zeta}{\sqrt{1-\zeta^2}}i\right)e^{-i\omega_d t}\right]$$

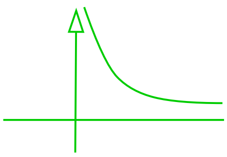
$$= 1 - e^{-\zeta\omega_n t}\left[\frac{1}{2}\overbrace{\left(e^{i\omega_d t} + e^{-i\omega_d t}\right)}^{= \cos\omega_d t} + \frac{1}{2}\frac{\zeta}{\sqrt{1-\zeta^2}}\underbrace{i\left(-e^{i\omega_d t} + e^{-i\omega_d t}\right)}_{= \sin\omega_d t}\right]$$

$$= 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$

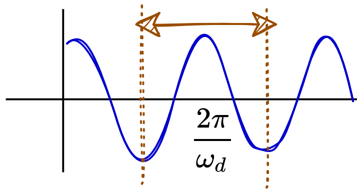
$$A\sin\theta + B\cos\theta = \sqrt{A^2+B^2}\,sin(\theta+\phi)$$

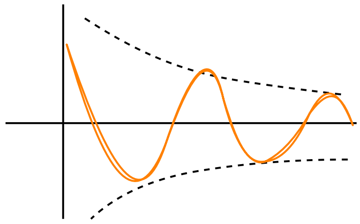
$$\phi = \arctan\frac{B}{A}$$

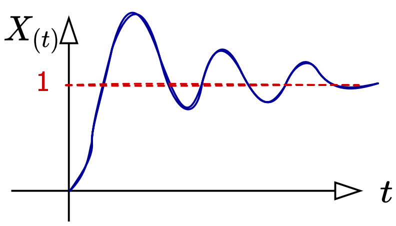
$$= 1 - e^{-\zeta\omega_n t}\sqrt{1 + \frac{\zeta^2}{1-\zeta^2}}\left(\sin(\omega_d t + \phi)\right)$$



$$= \boxed{1 - e^{-\zeta\omega_n t}\sqrt{\frac{1}{1-\zeta^2}}\sin(\omega_d t + \phi)}$$





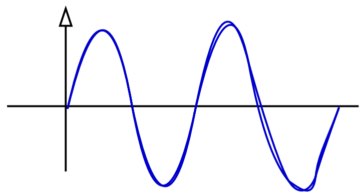


$t = 0 \quad X_{(t)} = 0$

$t \to \infty \quad X_{(t)} = 1$

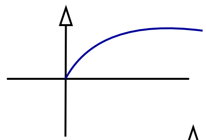
$\zeta = 0$ no damping

$X_{(t)} = 1 - \cos\omega_n t$

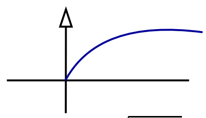


$\zeta = 1$ critical damping

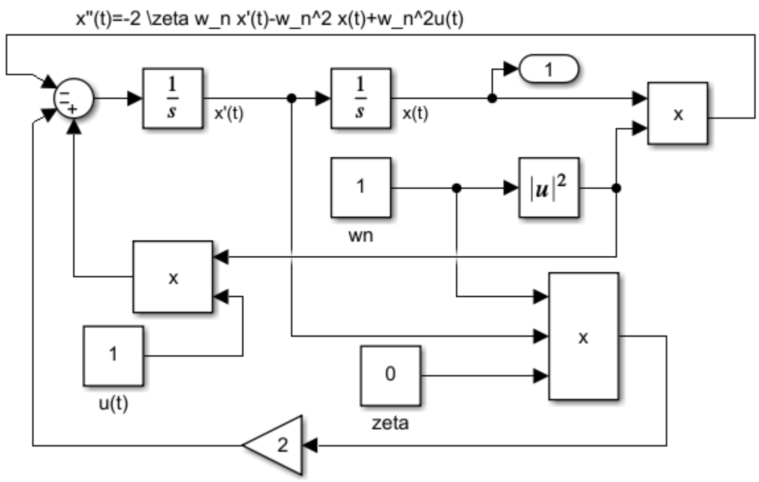
$X_{(t)} = 1 - e^{-\omega_n t}[1 + \omega_n t]$

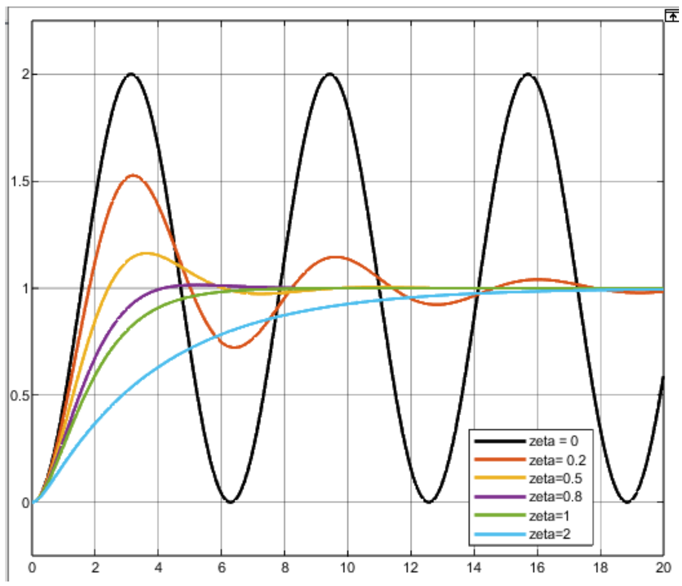


$\zeta > 1$ Over damped



$$X_{(t)} = 1 + \frac{\omega_n}{2\sqrt{\zeta^2-1}}e^{(-\zeta+\sqrt{\zeta^2-1})\omega_n t} + e^{-\zeta-\sqrt{\zeta^2-1}\omega_n t}$$

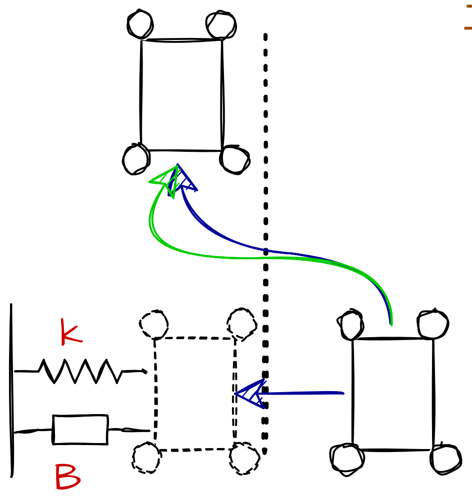




笔记：王崇卫

动态系统的建模与分析

2nd order System Unit Step Response
二阶系统　　　单位阶跃



平衡点

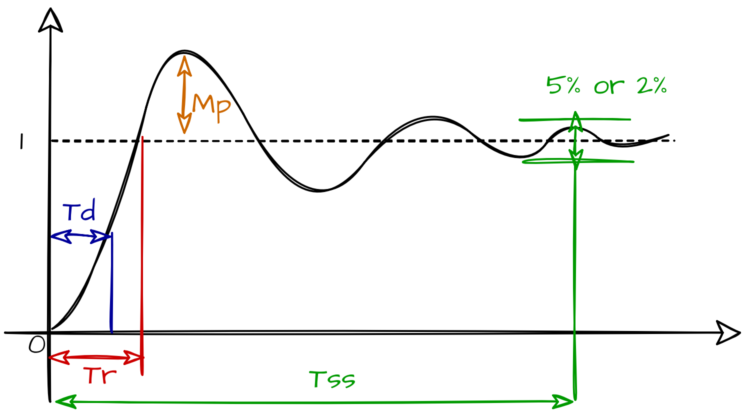
自动驾驶课题：自动变道

这个二阶系统，抽象成一个弹簧阻尼系统

如何判断这两个轨迹的算法好呢？

欠阻尼系统为例

$$X_{(t)} = 1 - e^{-\zeta\omega_n t}\sqrt{\frac{1}{1-\zeta^2}}\sin(\omega_d t + \phi)$$



Td: Delay time 延迟时间　50%

Tr: Rise time 上升时间　100%

Mp: Max Overshoot　最大超调量

$(X_p - 1) \times 100\%$

Tss:settling time 调节时间，稳态

Tr: $\cancel{1} = \cancel{1} - e^{-\zeta\omega_n t}\sqrt{\frac{1}{1-\zeta^2}}\underline{\sin(\omega_d t + \phi)} = 0$

$$\sin(\omega_d t + \phi) = 0$$

$$\omega_d t + \phi = \pi$$

$$tr = \frac{\pi - \phi}{\omega_d}$$

Mp: $X_{(tp)}$ peak time $\dot{X}_{(t)} = 0 \ 1st$

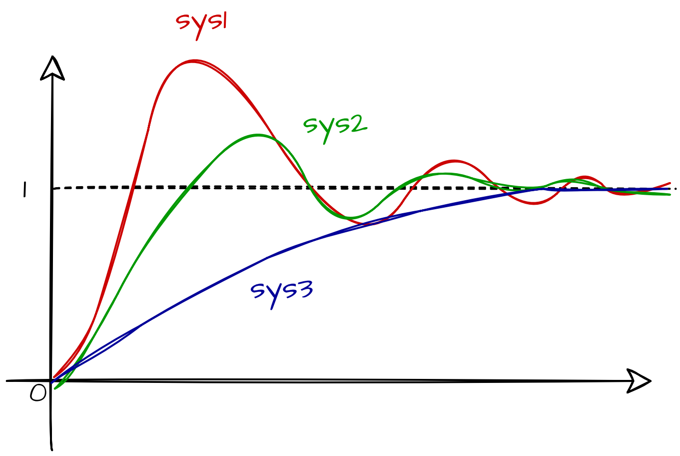
$$tp = \frac{\pi}{\omega_d}$$

$$Mp = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$$

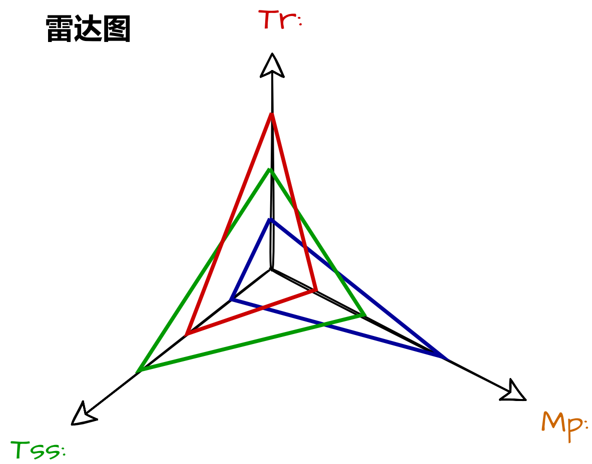
Tss: 2% $Tss = \frac{4}{\zeta\omega_n}$

5% $Tss = \frac{3}{\zeta\omega_n}$

分析



雷达图



	Tr	Mp	Tss
Sys1	3	1	2
Sys2	2	2	3
Sys3	1	3	1

计分规则，1，2，3

Tr越小，分数高

Mp越小，分数高

Tss越小，分数高

这个圈越大越好，但是要针对不同的系统做出考虑

比如转换车道，我们从超调量入手。
选择3号系统，尽管反应慢一些，但不会驶入其他车道

比如紧急避障，我们应该选择1号系统。
尽快的避开危险区。

当然障碍还远，应该选择2号系统。
既能避开，也不会影响乘车体验。

实际上很多的内容不是对错来判断的。
只是在某一种情况下，哪一种更加适合。

笔记：王崇卫

动态系统的建模与分析

2nd order System Frequency Response

二阶系统 频率响应

线性时不变系统

$$M_i sin(\omega+\phi i) \longrightarrow \boxed{\begin{matrix} LTI \\ G(s) \end{matrix}} \longrightarrow M_o sin(\omega+\phi i)$$

steady state

$$\boxed{\begin{aligned} M &= \frac{M_o}{M_i} = |G(j\omega)| \\ \phi &= \phi_o - \phi_i = \angle G(jw) \end{aligned}}$$

振幅响应 Amplitude response

幅角响应 Phase response

$$G_{(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

ω_n Natural Frequency 固有频率

ζ Damping Ratio 阻尼比

$$G_{(j\omega)} = \frac{\omega_n^2}{-\omega^2+2\zeta\omega_n\omega j+\omega_n^2}$$

$$= \frac{1}{-\frac{\omega^2}{\omega_n^2}+2\zeta\frac{\omega}{\omega_n}j+1}$$

输入频率

$$\frac{\omega}{\omega_n} = \Omega$$

$$= \frac{1}{-\Omega^2+2\zeta\Omega j+1} = \frac{1-\Omega^2-2\zeta\Omega j}{(1-\Omega^2+2\zeta\Omega j)(1-\Omega^2-2\zeta\Omega j)}$$

$$= \underbrace{\frac{1-\Omega^2}{(1-\Omega^2)^2+4\zeta^2\Omega^2}}_{\text{Real}} - \underbrace{\frac{2\zeta\Omega}{(1-\Omega^2)^2+4\zeta^2\Omega^2}j}_{\text{Im}}$$

$$|G(j\omega)| = (Real(G_{(j\omega)})^2+Im(G_{(j\omega)})^2)^{\frac{1}{2}}$$

$$= \sqrt{\frac{(1-\Omega^2)^2+4\zeta^2\Omega^2}{((1-\Omega^2)^2+4\zeta^2\Omega^2)^2}}$$

$$\boxed{|G_{(j\omega)}| = \sqrt{\frac{1}{(1-\Omega^2)^2+4\zeta^2\Omega^2}}}$$

$$\frac{\omega}{\omega_n} = \Omega$$

$$\boxed{|G_{(j\omega)}|_{\omega=\omega_n\sqrt{1-2\zeta^2}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$

分析

$\Omega = 0 \quad \omega = 0$

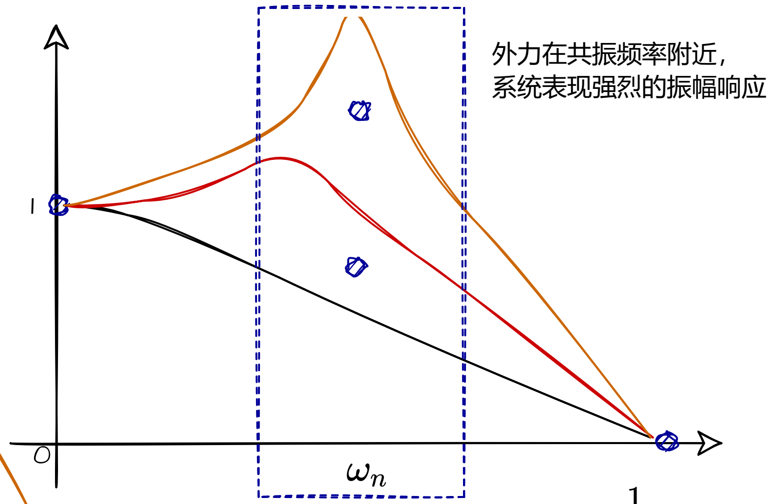
$$|G_{(j\omega)}| = 1$$

$\Omega \to \infty \quad \omega \gg \omega_n \quad \omega \to \infty$

$$|G_{(j\omega)}| \to 0$$

$\Omega = 1 \quad \omega = \omega_n$

$$|G_{(j\omega)}| = \sqrt{\frac{1}{0+4\zeta^2}} = \frac{1}{2\zeta} \qquad \zeta < 0.5 \quad |G_{(j\omega)}| > 1 \qquad \zeta > 0.5 \quad |G_{(j\omega)}| < 1$$



① $\zeta = 1 \qquad \omega = \omega_n \qquad |G_{(j\omega)}| = \frac{1}{2}$

② $\zeta = 0.5 \qquad \omega = \omega_n \qquad |G_{(j\omega)}| = 1$

$\omega = \omega_n\sqrt{1-2\zeta^2} \qquad |G_{(j\omega)}| = 1.16$

③ $\zeta = 0 \qquad \omega = \omega_n$

$|G_{(j\omega)}| = \infty$

pole: $\quad f(\Omega) = (1-\Omega^2)^2 + 4\zeta^2\Omega^2$

$$\frac{df(\Omega)}{d\Omega} = 2(1-\Omega^2)(-2\Omega) + 8\zeta^2\Omega^2 = 0$$

$$\Omega\underbrace{(-1+\Omega^2+2\zeta^2)}_{=0} = 0$$

$$\Omega = \sqrt{1-2\zeta^2} \qquad \Omega = \sqrt{1-2\zeta^2} = \frac{\omega}{\omega_n}$$

$$\boxed{\omega = \omega_n\sqrt{1-2\zeta^2}}$$

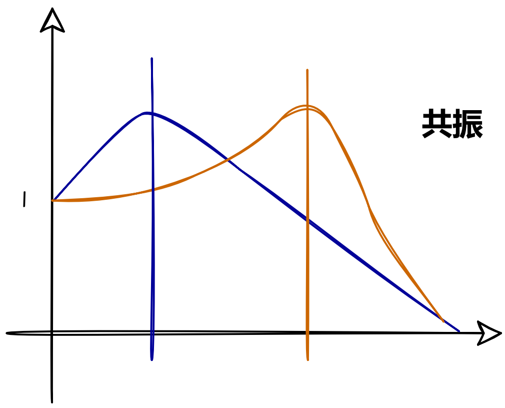
共振频率

Resonance frequency

$1-2\zeta^2 > 0$时存在极值 $\qquad \zeta < \sqrt{\frac{1}{2}}$

$$|G_{(j\omega)}|_{\omega=\omega_n\sqrt{1-2\zeta^2}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$\zeta \to 0, \omega \approx \omega_n$



共振

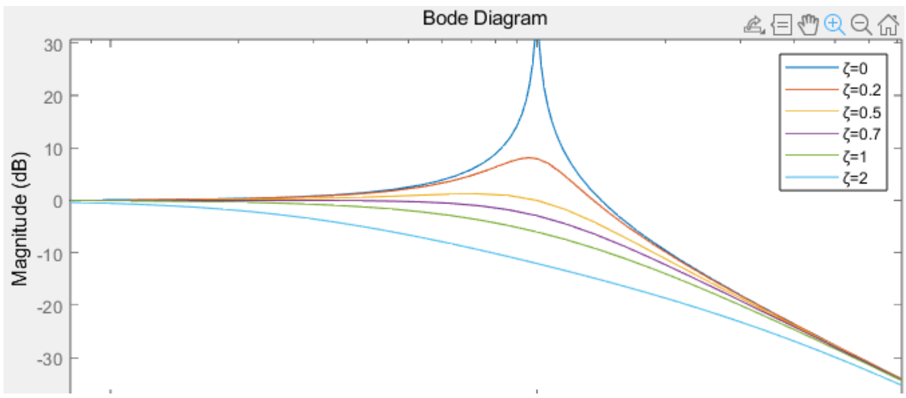
生活中：劝人说话，感同身受是很难的事情

有的人会被**物质**刺激，
有的人会被**颜值**刺激，
有的人会被**精神**刺激
还有的人是**佛系**的

找到对方的共振频率，
才有可能找到共鸣

"频率的作用"：
和有的人相处舒服
和有的人相处尴尬

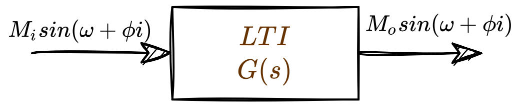
matlab bode函数



动态系统的建模与分析

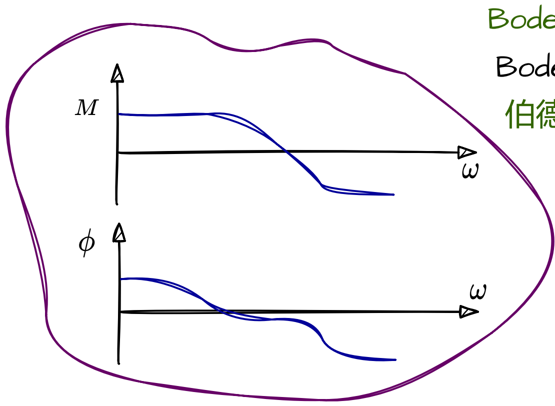
Bode Plot 伯德图 (1)

对于一个线性时不变系统



振幅的变化 $M=\frac{M_o}{M_i}=|G(j\omega)|$

相位的变化 $\phi=\phi_o-\phi_i=\angle G(jw)$



Bode, H.W

Bode Plot

伯德图

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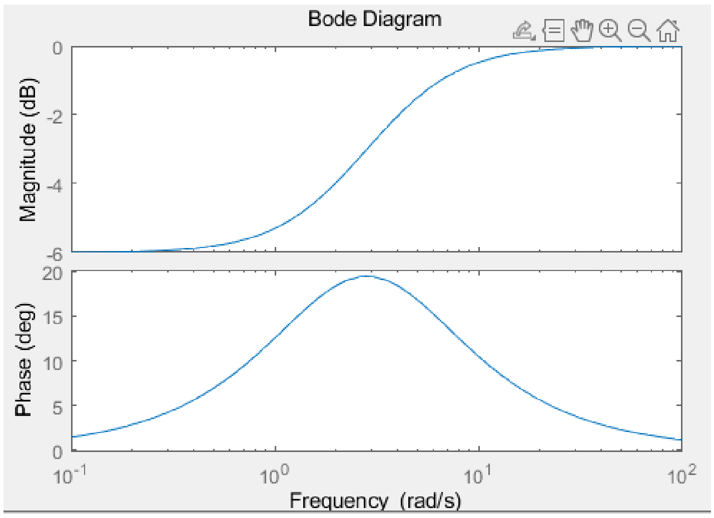
matlab bode()

$G(s)=\frac{s+2}{s+4}$

```
>>bode([1 2],[1 4])
```



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dB decibel 分贝

$\frac{1}{10}$ dm

alexander bell

电话电报的信号损失

60 dB 交流、交谈

80dB 闹市

强度差 100倍

$dB=10log_{10}\frac{P_M}{P_R}$

Measurement 测量

Reference 参考

能量的比值取对数

取对数的原因是把一个很大数缩小

声音强度

可听见的最低音量 $1\times10^{-12}w/m^2$ $10log_{10}1=0dB$

电锯 $1w/m^2$ $10log_{10}10^{12}=120dB$

在bode图中 $20\log_{10}$

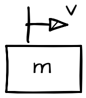
$$M=\frac{M_o}{M_i}$$

一般来说，振幅和功率是平方的关系

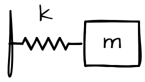
功率 能量

$$P=f(M^2)$$

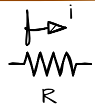
振幅



$\frac{1}{2}mv^2$



$\frac{1}{2}kx^2$



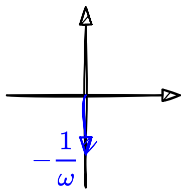
i^2R

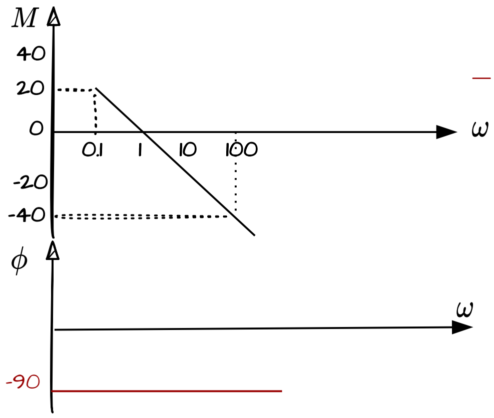
$$dB=10log_{10}\frac{P_M}{P_R}=10log_{10}(\frac{M_o}{M_i})^2=20log_{10}M$$

积分 Integrator $G(s)=\frac{1}{s}$

$$G(jw)=\frac{1}{jw}=-\frac{1}{\omega}j \qquad |G(jw)|=\frac{1}{\omega} \qquad 20log\frac{1}{\omega}=20log\omega^{-1}=-20log\omega$$

复平面





$-20dB/decade$

$$G(s)=\frac{a}{s+a}$$

注：推导见一阶系统的频率响应

$$|G(j\omega)|=\sqrt{\frac{1}{1+(\frac{\omega}{a})^2}} \qquad \angle G(jw)=-arctan(\frac{\omega}{a})$$

低频$\omega<<a$ $\quad |G(j\omega)|=1 \quad 20log|G(jw)|=0 \quad \angle G(jw)=0$

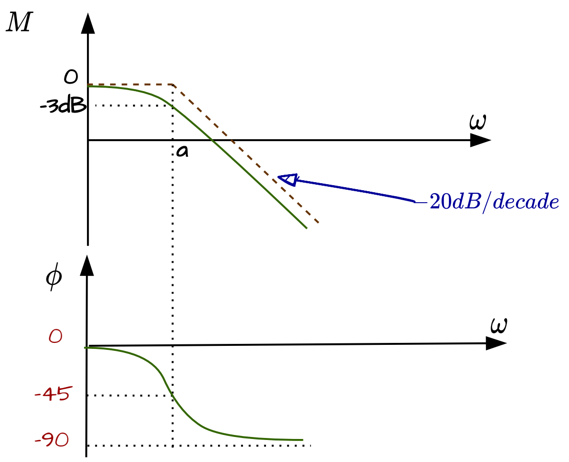
这个 $-3dB$, 输入的振幅是输出的振幅的$\sqrt{\frac{1}{2}}$, 从能量的角度是输出的能量是输出的$\frac{1}{2}$

截至频率$\omega=a$ $\quad |G(jw)|=\sqrt{\frac{1}{2}} \quad 20logG(jw)=20log(\sqrt{2})^{-1}=-20log\sqrt{2}=-3dB$

$$\angle G(jw)=-arctan1=-45^\circ$$

高频$\omega>>a$ $\quad |G(jw)|=\frac{1}{\omega} \quad 20log\frac{1}{\omega}=20log\omega^{-1}=-20log\omega \quad -20dB/decade$

$$\angle G(jw)=-90^\circ$$



试一试

$$G(s)=s$$

$$G(s)=as+a$$

bode图的另一个好处：

把乘积变成加和

$$\boxed{log_{10}AB=log_{10}A+log_{10}B}$$

把复杂的传递函数分解

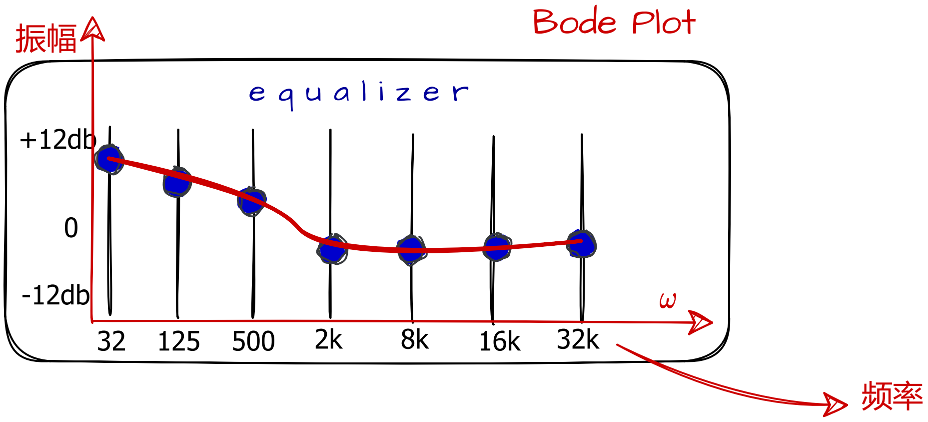
动态系统的建模与分析

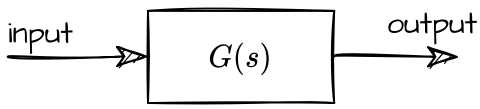
Bode Plot 伯德图 (2)

Year of 9102 **数学规则** **应用**

调音例子

音乐





复数

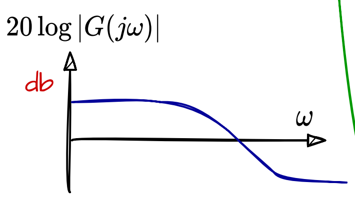
振幅的变化 $M=\dfrac{M_o}{M_i}=|G(j\omega)|$ $G(j\omega)=re^{j\theta}$ $r=|G(j\omega)|$

相位的变化 $\phi=\phi_o-\phi_i=\angle G(jw)$ $\theta=\angle G(jw)$

相加

$$G(j\omega)=G_1(j\omega)\cdot G_2(j\omega)=r_1e^{j\theta_1}\cdot r_2e^{j\theta_2}=r_1r_2e^{j(\theta_1+\theta_2)}$$

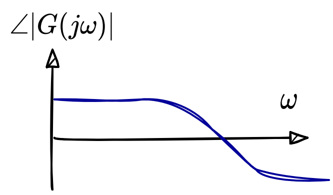
相乘



$$\boxed{\log AB=\log A+\log B}$$

$$20\log|G(j\omega)|=20\log|G_1(j\omega)|+20\log|G_2(j\omega)|$$

非常重要的一个性质，也是为什么bode图这么好用的一个原因

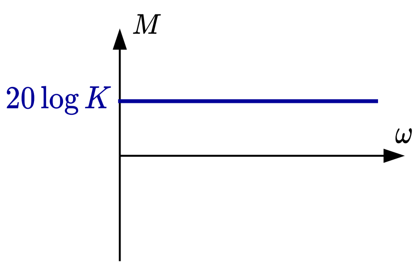


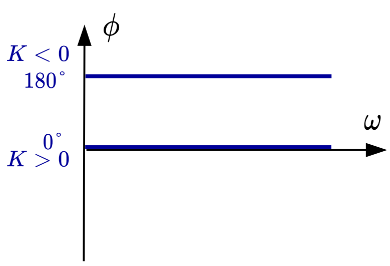
相位相加的关系

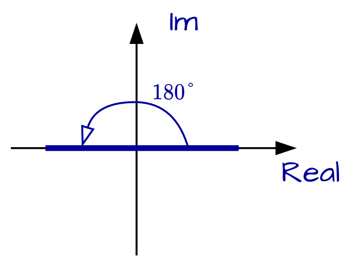
当我们掌握一些基本的，常用的传递函数的bode的绘制
对于复杂系统来说，只是**拆分组合**

常用：

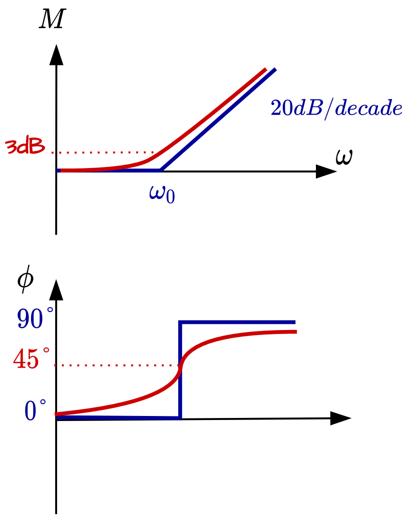
① K



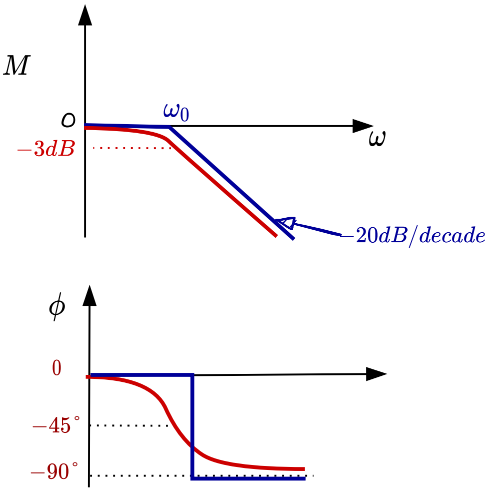




② $1+\frac{1}{\omega_0}s$



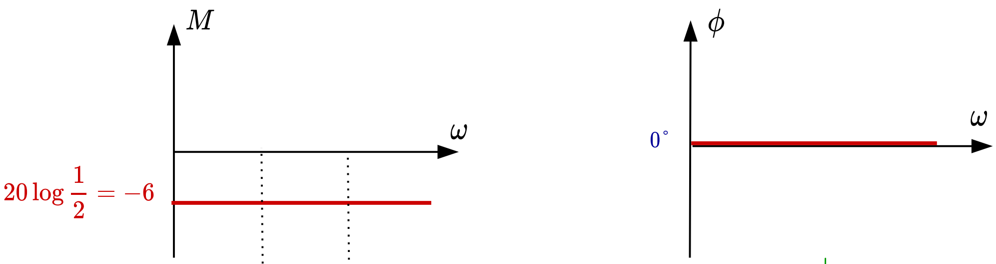
③ $\dfrac{1}{1+\frac{1}{\omega_0}s}$



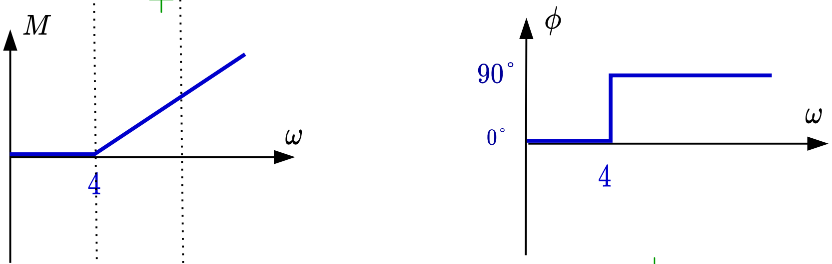
上面三个作为基础，就可以做组合了

例 $$G(s)=\frac{s+4}{s+8}=\frac{4(\frac{1}{4}s+1)}{8(\frac{1}{8}s+1)}=\underbrace{\frac{1}{2}}_{①}\cdot\underbrace{(\frac{1}{4}s+1)}_{②}\underbrace{\frac{1}{\frac{1}{8}s+1}}_{③}$$

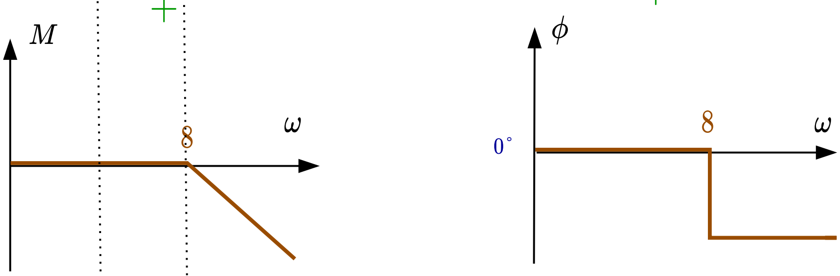
① $\frac{1}{2}$



① $\frac{1}{4}s+1$



③ $\dfrac{1}{\frac{1}{8}s+1}$



$G(s)=\dfrac{s+4}{s+8}$