DR_CAN 傅里叶级数与傅里叶变换

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笔记：粉丝王崇卫

说明

致敬b站DR_CAN博士。

笔记是个人根据视频仿照DR_CAN老师，极慢的方式把笔记使用drawio软件做了一遍。

这么好的视频，我希望有个还凑合的笔记，供查阅。所以我花了时间来做这件事情。希望对大家有所帮助。

（电子笔记仅供参考翻阅，学习时应当动笔在纸上跟着up主计算）

如有错误，欢迎指出，邮箱1084746243@qq.com

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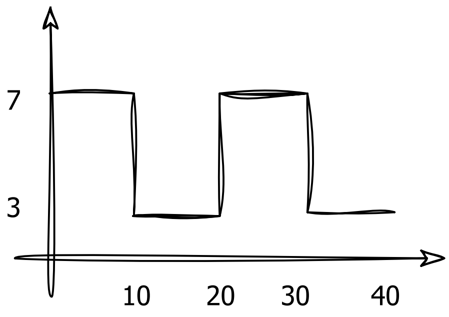
傅里叶级数与变换

PartI 三角函数的正交性

分享知乎文章： [傅里叶分析之掐死教程（完整版）更新于2014.06.06]



$F(t)=?$

周期函数，如何展开为傅里叶级数

展开后的傅里叶级数，每一个参数的意义是怎么来的

三角函数系：集合

取两个不同的，乘积在[-π,π]，定积分为0

$$\{\,0\,,1,\sin x,\cos x,\sin 2x,\cos 2x,\cdots\cdots\sin nx,\cos nx,\cdots\cdots\}$$

$\sin 0x$ $\cos 0x$

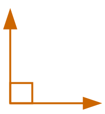
$\sin nx$ $\cos nx$ $\quad n=0,1,2,3,\cdots$

正交： 上面集合任取两个不同的项

$$\int_{-\pi}^{\pi}\sin nx\ \cos mx dx\ =\ 0 \qquad n\neq m$$

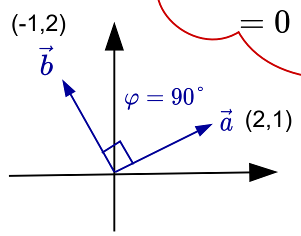
正交

垂直



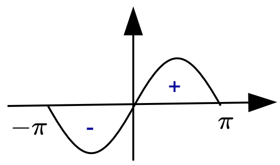
$$\int_{-\pi}^{\pi}\cos nx\ \cos mx dx\ =\ 0 \qquad n\neq m$$

$$\vec{a}\vec{b}=(2,1)\cdot(-1,2)$$
$$=2(-1)+1\times 2$$
$$=0$$



证明：

$$\int_{-\pi}^{\pi}\cos 0x\ \sin x dx\ =\int_{-\pi}^{\pi}\sin x dx=0$$



$$\vec{a}\vec{b}=|\vec{a}||\vec{b}|\cos\underbrace{\varphi}_{0}$$

$\vec{a}\cdot\vec{b}=0$ 正交

积化和差公式

$$\int_{-\pi}^{\pi}\cos nx\ \cos mx dx \rightarrow \frac{1}{2}[\cos(n-m)x+\cos(n+m)x]$$

$$=\frac{1}{2}[\int_{-\pi}^{\pi}\cos(n-m)x dx\ +\int_{-\pi}^{\pi}\cos(n+m)x dx]$$

$$=\frac{1}{2}[\frac{1}{n-m}\overset{0}{\cancel{\sin(n-m)x}}\ |_{-\pi}^{\pi}+\frac{1}{n+m}\overset{0}{\cancel{\sin(n+m)x}}\ |_{-\pi}^{\pi}]$$

$$=0$$

$$\vec{a}=(1,2,5)$$
$$\vec{b}=(1,2,-1)$$
$$\vec{a}\cdot\vec{b}=1\cdot1+2\cdot2-5\cdot1=0$$

$$\vec{a}=(a_1,a_2,a_3,\cdots,a_n)$$
$$\vec{b}=(b_1,b_2,b_3,\cdots,b_n)$$
$$\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+\cdots+a_nb_n$$
$$=\sum_{i=1}^{n}a_ib_i\ =0$$

可以试着证明：

$$\int_{-\pi}^{\pi}\cos nx\ \sin mx dx\ =\ 0 \qquad n\neq m$$

$$\int_{-\pi}^{\pi}\sin nx\ \sin mx dx\ =\ 0 \qquad n\neq m$$

$IF\ m = n?$

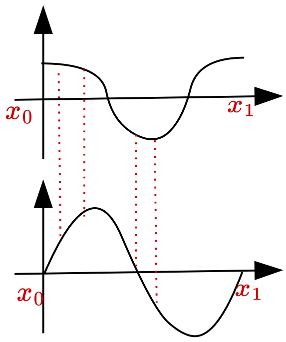
$$\int_{-\pi}^{\pi} \cos mx\ \cos mx dx \quad \frac{1}{2}[1+\cos 2mx]$$

$$\cos 0x \ \cdot \cos 2mx = 0$$

$$= \frac{1}{2}[\int_{-\pi}^{\pi} 1dx \ + \cancel{\int_{-\pi}^{\pi} \cos 2mx dx}]$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} 1dx \ = \frac{1}{2}x\ |_{-\pi}^{\pi} = \pi$$

$$a = f(x)$$



$$b = g(x)$$

$$a \cdot b = \int_{x_0}^{x_1} f(x)g(x)dx = 0$$

这两个函数正交

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笔记：王崇卫

傅里叶级数与变换

Part2 周期为"2π"的函数展开为傅里级数

复习：三角函数的正交性

取两个不同的，乘积在[-π,π]，定积分为0

$$\{\ 0\ , 1, \sin x, \cos x, \sin 2x, \cos 2x, \cdots \sin nx, \cos nx, \cdots \sin mx, \cos mx, \cdots\}$$

$\sin 0x$ $\cos 0x$

上面集合任取两个不同的项 $n \neq m$

$$\int_{-\pi}^{\pi} \sin x\ \sin nx dx = 0$$

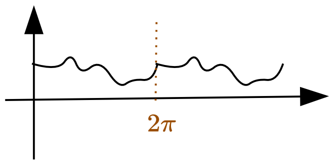
$$\int_{-\pi}^{\pi} \cos 2x dx = 0$$

$$\int_{-\pi}^{\pi} \cos nx\ \sin mx dx = 0$$

$n = m$

$$\int_{-\pi}^{\pi} \cos nx\ \cos nx dx = \pi$$

周期 $T = 2\pi$ $f(x) = f(x + 2\pi)$



像这样的一个函数
可以展开三角级数

$$f(x) = \sum_{n=0}^{\infty} a_n \cos nx + \sum_{n=0}^{\infty} b_n \sin nx$$

教科书的形式:

$$f(x) = \boxed{\frac{a_0}{2}} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

区别:n的起始值，多了一项

$$= a_0 \overset{1}{\cos 0x} + \sum_{n=1}^{\infty} a_n \cos nx + b_0 \overset{0}{\sin 0x} + \sum_{n=1}^{\infty} b_n \sin nx$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

1.找a_0 $\int_{-\pi}^{\pi} dx$

$$\int_{-\pi}^{\pi} f(x) dx = \int_{-\pi}^{\pi} a_0 dx + \overset{=0}{\cancel{\int_{-\pi}^{\pi} \sum_{n=1}^{\infty} a_n \overset{1 \cdot \cos nx}{\cos nx dx}}} + \overset{=0}{\cancel{\int_{-\pi}^{\pi} \sum_{n=1}^{\infty} b_n \sin nx dx}}$$

$$\int_{-\pi}^{\pi} f(x) dx = a_0 \int_{-\pi}^{\pi} dx = a_0 x \mid_{-\pi}^{\pi} = 2\pi a_0 \Longrightarrow a_0 = \frac{1}{\cancel{2}\pi} \int_{-\pi}^{\pi} f(x) dx$$

2.找a_n　　①等式两边乘 $\cos mx$　　② $\int_{-\pi}^{\pi} dx$

$$f(x)\cos mx=\frac{a_0}{2}\cos mx+\sum_{n=1}^{\infty}a_n\cos nx\cos mx+\sum_{n=1}^{\infty}b_n\sin nx\cos mx$$

$$\int_{-\pi}^{\pi}f(x)\cos mxdx=\int_{-\pi}^{\pi}\frac{a_0}{2}\cos mxdx+\int_{-\pi}^{\pi}\sum_{n=1}^{\infty}a_n\cos nx\cos mxdx+\int_{-\pi}^{\pi}\sum_{n=1}^{\infty}b_n\sin nx\cos mxdx$$

$$\int_{-\pi}^{\pi}f(x)\cos mxdx=\int_{-\pi}^{\pi}\sum_{n=1}^{\infty}a_n\cos nx\cos mxdx$$

$n=m$　$$\int_{-\pi}^{\pi}f(x)\cos mxdx=\int_{-\pi}^{\pi}a_n\cos nx\cos mxdx=a_n\underbrace{\int_{-\pi}^{\pi}(\cos nx)^2dx}_{\pi}=a_n\pi$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx$$

3.找b_n　　①等式两边乘 $\sin mx$　　② $\int_{-\pi}^{\pi} dx$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx$$

$$f(x)=f(x+2\pi)\qquad T=2\pi$$

$$\boxed{\begin{aligned}f(x)&=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx\\a_0&=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx\\a_n&=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx\\b_n&=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx\end{aligned}}$$

$T=2L$?

$e^{i\theta}=\cos\theta+i\sin\theta$?　通过欧拉公式用复数的形式表示傅里叶的展开

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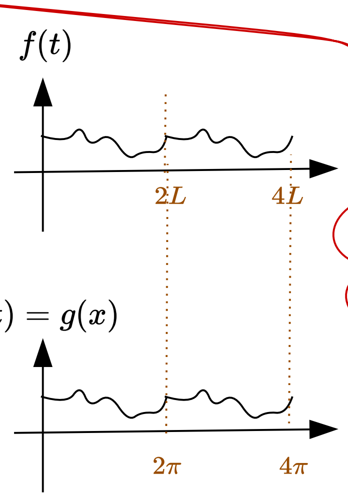
傅里叶级数与变换

Part3 周期为"2L"的函数展开为傅里级数

$$f(t) = f(t+2L)$$

$T = 2\pi$

$g(x) = g(x+2\pi)$



$f(t) = g(x)$

换元的方法

$$x = \frac{\pi}{L}t$$

$$t = \frac{L}{\pi}x$$

t	x
$2L$	2π
$4L$	4π
0	0

$$f(t) = f(\frac{L}{\pi}x) \triangleq g(x)$$

$$g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

① $$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos \frac{n\pi}{L}t + b_n \sin \frac{n\pi}{L}t)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} g(x)dx$$

② $$a_0 = \frac{1}{L}\int_{-L}^{L} f(t)dt$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(x)\cos nx dx$$

③ $$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos \frac{n\pi}{L}t dt$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(x)\sin nx dx$$

④ $$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin \frac{n\pi}{L}t dt$$

$$x = \frac{\pi}{L}t$$

$$\cos nx = \cos \frac{n\pi}{L}t$$

$$\sin nx = \sin \frac{n\pi}{L}t$$

$$g(x) = f(t)$$

$$\int_{-\pi}^{\pi} dx = \int_{-L}^{L} d\frac{\pi}{L}t$$

x	t
$-\pi$	$-L$
π	L

$$\frac{1}{\pi}\int_{-\pi}^{\pi} dx = \frac{1}{\pi}\frac{\pi}{L}\int_{-L}^{L} dt$$

$$= \frac{1}{L}\int_{-L}^{L} dt$$

工程中: 时间没有负数， t从0开始， 周期为 T=2L $\omega = \frac{\pi}{L} = \frac{2\pi}{T}$

$$\int_{-L}^{L} dt \implies \int_{0}^{2L} dt \implies \int_{0}^{T} dt$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_0 = \frac{2}{T}\int_0^T f(t)dt$$

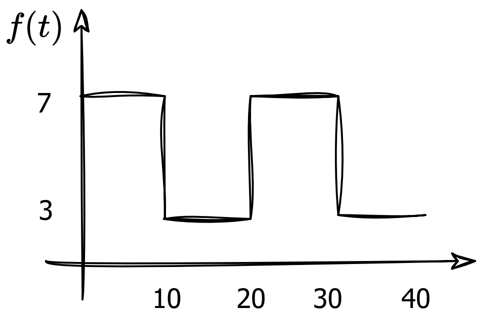
$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt$$

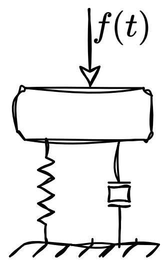
$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

$T \to \infty$

$f(t)$就不再为周期函数

Fourier transform





力施加在振动系统上
1. 分析系统的响应
2. 找到固有频率
3. 分析周期函数的展开

如果正好固有频率，振幅过大
系统可能出问题

$T = 20 \qquad \omega = \frac{2\pi}{T} = \frac{1}{10}\pi$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_0 = \frac{2}{T}\int_0^T f(t)dt = \frac{1}{10}(\int_0^{10} 7dt + \int_{10}^{20} 3dt) = 10$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt = \frac{1}{10}(\int_0^{10} 7\cos\frac{n}{10}\pi t dt + \int_{10}^{20} 3\cos\frac{n}{10}\pi t dt)$$

$$= \frac{1}{10}(\frac{70}{n\pi}\sin\frac{n\pi t}{10}\ |_0^{10} + \frac{30}{n\pi}\sin\frac{n\pi t}{10}\ |_{10}^{20}) = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt = \frac{1}{10}(\int_0^{10} 7\sin\frac{n}{10}\pi t dt + \int_{10}^{20} 3\sin\frac{n}{10}\pi t dt)$$

$$= \frac{1}{10}(-\frac{70}{n\pi}\cos\frac{n\pi t}{10}\ |_0^{10} - \frac{30}{n\pi}\cos\frac{n\pi t}{10}\ |_{10}^{20})$$

n 为偶数时 $\cos\frac{n\pi t}{10}\Big|_0^{10}=\cos\frac{n\pi t}{10}\Big|_{10}^{20}=0$

n 为奇数时 $b_n=\frac{1}{10}\left(\frac{140}{n\pi}-\frac{60}{n\pi}\right)=\frac{8}{n\pi}$

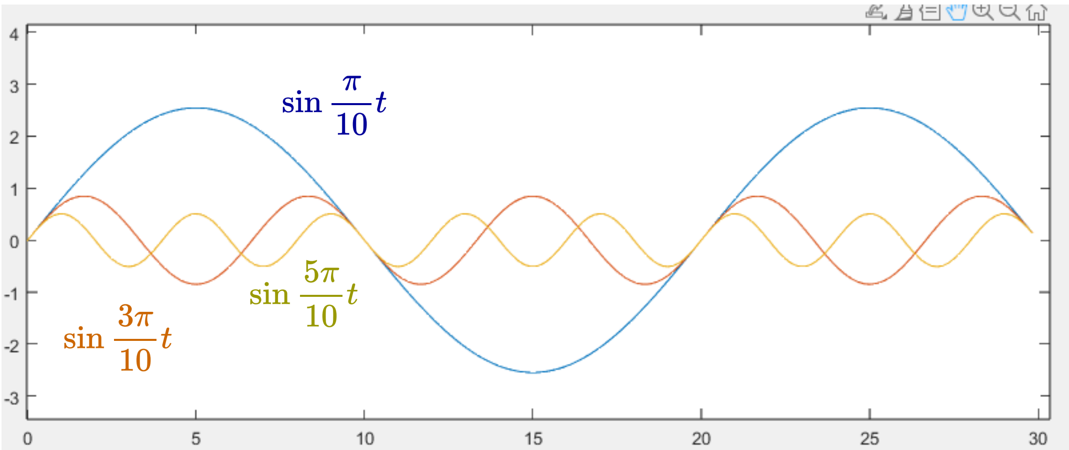
$$f(t)=5+\sum_{n=1}^{\infty}\frac{8}{n\pi}\sin\frac{n\pi}{10}t \qquad n=1,3,5,7,\cdots$$

$n\uparrow \quad \frac{8}{n\pi}\to 0$ 低频率的占了主要的部分，高频率的小的多

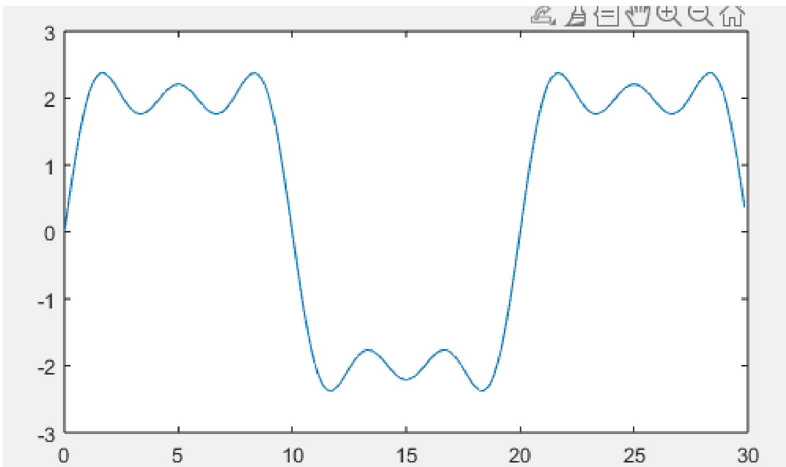
$n=1 \quad \frac{8}{\pi}\sin\frac{\pi}{10}t$

$n=3 \quad \frac{8}{3\pi}\sin\frac{3\pi}{10}t$

$n=5 \quad \frac{8}{5\pi}\sin\frac{5\pi}{10}t$



加起来



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傅里叶级数与变换

Part4 傅里叶级数的复数形式

$f(t) = f(t+T)$ T 周期

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t) \qquad \omega = \frac{2\pi}{T}$$

$$a_0 = \frac{2}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

欧拉公式的证明

【工程数学基础】5_如何证明宇宙第一美公式？？—欧拉公式证明

【工程数学基础】6_SinX=2? 复变函数 欧拉公式

Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

$$\sin\theta = -\frac{1}{2}i(e^{i\theta} - e^{-i\theta})$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n\frac{1}{2}(e^{in\omega t} + e^{-in\omega t}) - \frac{1}{2}ib_n(e^{in\omega t} - e^{-in\omega t})]$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty}[\frac{a_n - ib_n}{2}e^{in\omega t} + \frac{a_n + ib_n}{2}e^{-in\omega t}]$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty}\frac{a_n - ib_n}{2}e^{in\omega t} + \boxed{\sum_{n=1}^{\infty}\frac{a_n + ib_n}{2}e^{-in\omega t}} \qquad n \to (-n)$$

$n = 0, e^{in\omega t} = 1$

$$= \sum_{n=0}^{0}\frac{a_0}{2}e^{in\omega t} + \sum_{n=1}^{\infty}\frac{a_n - ib_n}{2}e^{in\omega t} + \sum_{n=-\infty}^{-1}\frac{a_{-n} + ib_{-n}}{2}e^{in\omega t} \qquad n \to (-\infty, \infty)$$

$$= \sum_{n=-\infty}^{\infty}\boxed{C_n}e^{in\omega t}$$

$$C_n = \begin{cases} \frac{a_0}{2}, & n = 0 \\ \frac{a_n - ib_n}{2}, & n = 1, 2, 3, 4 \cdots \\ \frac{a_n + ib_n}{2}, & n = -1, -2, -3, -4 \cdots \end{cases}$$

$$a_0 = \frac{2}{T} \int_0^T f(t) dt$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega t dt$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega t dt$$

$$\underset{n=0}{C_n} = \frac{a_0}{2} = \frac{1}{2} \frac{2}{T} \int_0^T f(t) dt = \boxed{\frac{1}{T} \int_0^T f(t) dt}$$

$$\underset{n=1,2,3\cdots}{C_n} = \frac{1}{2} \left(\frac{2}{T} \int_0^T f(t) \cos n\omega t dt - i \frac{2}{T} \int_0^T f(t) \sin n\omega t dt \right)$$

$$= \frac{1}{T} \int_0^T f(t)(\underline{\cos n\omega t - i \sin n\omega t}) dt$$

$$\cos n\omega t - i \sin n\omega t$$

$$= \cos(-n\omega t) + i \sin(-n\omega t)$$

$$= e^{-in\omega t}$$

$$= \boxed{\frac{1}{T} \int_0^T f(t) e^{-in\omega t} dt}$$

$$\underset{n=-1,-2,-3\cdots}{C_n} = \frac{1}{2} \left(\frac{2}{T} \int_0^T f(t) \cos(-n)\omega t dt + i \frac{2}{T} \int_0^T f(t) \sin(-n)\omega t dt \right)$$

$$= \frac{1}{T} \int_0^T f(t)(\cos n\omega t - i \sin n\omega t) dt = \boxed{\frac{1}{T} \int_0^T f(t) e^{-in\omega t} dt}$$

$$n = 0: \quad \frac{1}{T} \int_0^T f(t) e^0 dt = \boxed{\frac{1}{T} \int_0^T f(t) dt}$$

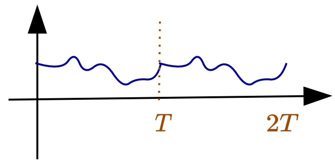
$$f(t) = f(t + T)$$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega t}$$

$$C_n = \frac{1}{T} \int_0^T f(t) e^{-in\omega t} dt$$

傅里叶级数与变换

Part5 傅里叶变换 FT



时域表达

$$f_T(t) = f(t+T)$$

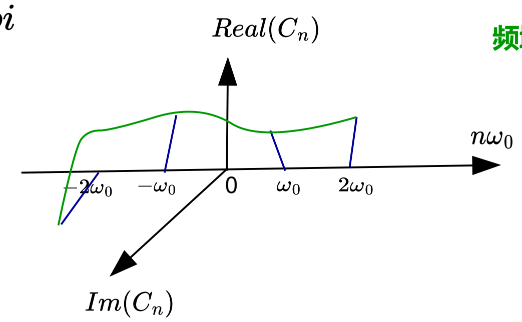
定义了函数

$$f_T(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega_0 t} \quad (1) \qquad \omega_0 = \frac{2\pi}{T} \text{ 基频率}$$

频谱 $|C_n|$幅度

$$C_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f_T(t) e^{-in\omega_0 t} dt \quad (2) \qquad C_n = a + bi$$

频域表达



非周期函数的一般形式

无限久后重复 $T \to \infty$

$$\lim_{T\to\infty} f_T(t) = f(t)$$

$$\cdots + C_{-1}e^{-i\omega_0 t} + C_0 e^0 + C_1 e^{i\omega_0 t} + C_2 e^{i2\omega_0 t} + \cdots$$

两个频率之间的距离定义为

离散 $\to$ 连续

$$\Delta\omega = (n+1)\omega_0 - n\omega_0 = \omega_0 = \frac{2\pi}{T}$$

$T\uparrow, \Delta\omega\downarrow$ 距离就没有了，连续了

$$\frac{1}{T} = \frac{\Delta\omega}{2\pi}$$

把(2)代入(1)

$$f_T(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f_T(t) e^{-in\omega_0 t} dt \;\; e^{in\omega_0 t}$$

$$f_T(t) = \sum_{n=-\infty}^{\infty} \frac{\Delta\omega}{2\pi}\int_{-\frac{T}{2}}^{\frac{T}{2}} f_T(t) e^{-in\omega_0 t} dt \;\; e^{in\omega_0 t}$$

$T \to \infty$

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} dt \to \int_{+\infty}^{-\infty} dt$$

$$n\omega_0 \to \omega$$

$$\sum_{n=-\infty}^{\infty} \Delta\omega \to \int_{-\infty}^{+\infty} d\omega$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt \;\; e^{i\omega t} d\omega$$

FT 傅里叶变换

$$\boxed{F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt}$$

逆变换

$$\boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega}$$

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https://www.bilibili.com/video/BV1St41117fH/?spm_id_from=333.999.0.0&vd_source=1f88f15c4a8c95c1d720fa4c6218bc54
笔记：王崇卫

傅里叶级数与变换

知识点

① 三角函数的正交性

取两个不同的，乘积在[-π,π]，定积分为0

$$\{\ 0\ , 1, \sin x, \cos x, \sin 2x, \cos 2x, \cdots\cdots \sin nx, \cos nx, \cdots\cdots\}$$

$\sin 0x$ $\cos 0x$

$\sin nx$
$\cos nx$ $n = 0, 1, 2, 3, \cdots$

正交： 上面集合任取两个不同的项

$$\int_{-\pi}^{\pi} \cos x\ \cos 2x dx = 0$$

$$\int_{-\pi}^{\pi} \sin nx\ dx = 0$$

② 周期为"2π"

$$f(x) = f(x + 2\pi)$$

$$f(x) = \sum_{n=0}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

左边和右边同时

$$\int_{-\pi}^{\pi} dx$$

$$\int_{-\pi}^{\pi} \cos nx dx$$

$$\int_{-\pi}^{\pi} \sin nx dx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx dx$$

③ 周期为"2L"

$$T = 2L \quad f(t) = f(t + 2L)$$

$$x = \frac{\pi}{L}t$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos \frac{n\pi}{L}t + b_n \sin \frac{n\pi}{L}t)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(t) dt$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t) \cos \frac{n\pi}{L} t dt$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(t) \sin \frac{n\pi}{L} t dt$$

④ 复指数

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$f(t) = f(t+T) \qquad \omega = \frac{2\pi}{T}$$

$$f(t) = \sum_{-\infty}^{\infty} C_n e^{in\omega t}$$

$$C_n = \frac{1}{T}\int_0^T f(t)e^{-in\omega t}dt$$

⑤ TF

$$f(t) = f(t+T) \qquad T \to \infty$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt \; e^{i\omega t}d\omega$$

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt \qquad \text{FT}$$

$$s = i\omega$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{i\omega t}d\omega \qquad \text{IFT}$$

$$F(s) = \int_{-\infty}^{+\infty} f(t)e^{-st}dt \qquad \text{Laplace}$$

desmos https://www.desmos.com/calculator?lang=zh-CN