

## 2025\_“ShuWei Cup”

### Problem A: Active Vibration Suppression Design Based on Centrifugal Actuators

#### (I) Background

Vibration is an "invisible killer" of actual industrial equipment. It easily causes structural fatigue damage, reduces operational accuracy and personnel comfort, and can even shorten the equipment's service life in severe cases. Therefore, there is an urgent need to develop active suppression technologies. Among them, the active vibration reduction design based on centrifugal actuators is a cutting-edge and hot direction, which has been applied in helicopter rotor systems, heavy-duty vehicle suspensions and other systems.

The basic principle of a centrifugal actuator is to generate controllable centrifugal force through a high-speed rotating eccentric mass. Currently, the widely used centrifugal actuators are of the dual-motor configuration. They use two sets of servo motors to control the rotational speed, which then drives four eccentric masses to rotate through bevel gear transmission, as shown in Figure 1. The dual-motor centrifugal actuator adopts two motors to drive two sets of eccentric masses to rotate in opposite directions respectively (the eccentric masses in the same set have the same angular velocity magnitude but opposite directions, and are installed symmetrically about the y-axis), thereby generating the expected acting force. The specific principle is shown in Figure 2. Ignoring the mass of the connecting rod structure and servo motors, the rotational angular velocity and angular acceleration of the eccentric masses have certain physical constraints. The centrifugal force generated by each rotating eccentric mass satisfies:

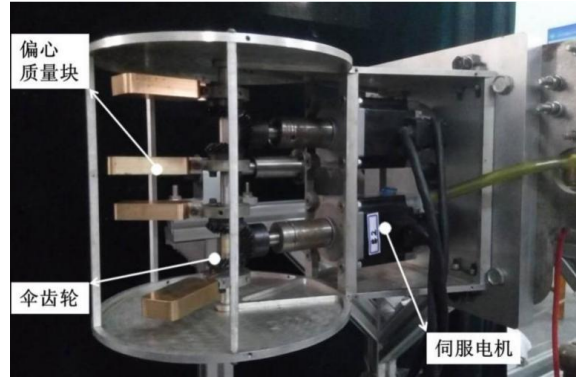
The magnitude of centrifugal force = rotational radius  $\times$  mass of the eccentric mass  $\times$  square of the rotational angular velocity; that is

$$|F_l| = rm\omega^2$$

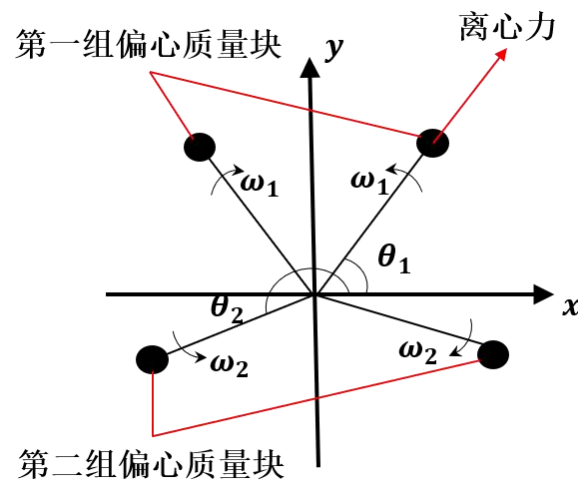
where  $|F_l|$  denotes the magnitude of centrifugal force,  $r$  represents the rotational

radius,  $m$  represents the mass of the eccentric mass, and  $\omega$  represents the rotational angular velocity.

Direction of centrifugal force: pointing outward from the center of rotation to the eccentric mass.



**Figure 1 Schematic Diagram of the Dual-Motor Centrifugal Actuator [1]**



**Figure 2 Schematic Diagram of the Dual-Motor Centrifugal Actuator (Top View)**

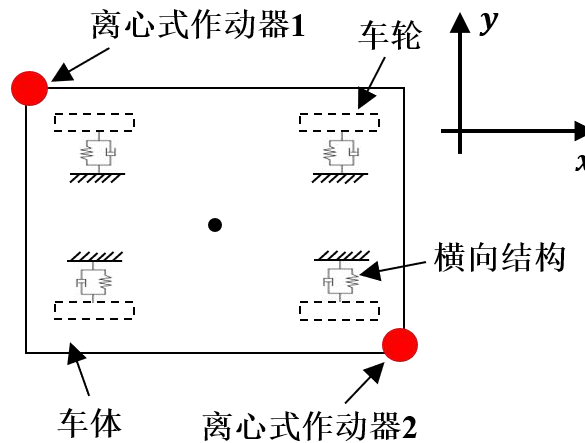
Now, consider the lateral vibration suppression problem of a large vehicle. A design scheme using centrifugal actuators is planned, with 2 dual-motor centrifugal actuators installed at the diagonal positions of the vehicle body respectively, as shown in Figure 3. Neglecting rotations such as vehicle body yaw, let the vehicle's forward direction be the x-axis direction and the lateral movement direction be the y-axis direction. The coordinate system direction in Figure 2 is the same as that in Figure 3. The lateral structure between the vehicle body and the four wheels can be approximated as a spring-damper structure. The lateral force generated under the

relative motion between the vehicle body and the wheels satisfies:

The lateral force on the vehicle body = -equivalent stiffness coefficient  $\times$  relative lateral displacement between the vehicle body and the wheels - equivalent damping coefficient  $\times$  relative lateral velocity between the vehicle body and the wheels; that is

$$F_h = -k \cdot \Delta y - c \cdot \Delta v$$

where  $F_h$  denotes the driving force on the equipment,  $k$  denotes the equivalent spring stiffness,  $\Delta y$  denotes the relative displacement between the equipment and the base,  $c$  denotes the equivalent driving damping,  $\Delta v$  denotes the relative velocity between the equipment and the base. Neglect the relative deformation between parts of the base, i.e.: take a certain base node as the coordinate origin, and other nodes are relatively stationary.



**Figure 3 Vehicle Body Vibration Suppression Design Diagram Based on Centrifugal Actuators (Top View)**

Based on the data in Appendix 1, design a vehicle lateral vibration suppression strategy using centrifugal actuators. In the following problems, the initial lateral displacement of the vehicle body is 0.01 m, and the initial lateral velocity is 0 m/s. Adopt the square of the relative lateral acceleration between the vehicle body and the wheels, integrate it, and average it over time, which is denoted as the vehicle lateral vibration index; i.e.,

$$I_h = \frac{1}{T} \int_0^T a_y^2 dt$$

where  $I_h$  denotes the lateral vibration index,  $T$  denotes the total operation time, and  $a_y$  denotes the relative lateral acceleration between the vehicle body and the wheels.

## (II) Problem Data

Refer to Appendix 1.

## (III) Problems to be Solved

1. According to the structure shown in Figure 3, establish the lateral motion model of the vehicle body (neglecting the lateral motion of wheels). When the vehicle body is subjected to the lateral disturbance force in Scenario 1 of Appendix 1, provide:

1) Without activating the centrifugal actuator, the lateral displacement and lateral acceleration curves of the vehicle body from 0 second to 10 seconds, and the vehicle body lateral vibration index;

2) With the centrifugal actuator activated and the rotation angle of the centrifugal actuator changing according to Scenario 1 of Appendix 1, the lateral displacement and lateral acceleration curves of the vehicle body from 0 second to 10 seconds, and the vehicle body lateral vibration index.

2. During the vehicle's travel, various lateral disturbance forces will be transmitted to the vehicle body through the wheels. When the vehicle body is subjected to the lateral disturbance force in Scenario 2 of Appendix 1, please design the angular displacement curve of the centrifugal actuator based on the information such as lateral disturbance force, vehicle body lateral displacement, lateral velocity, and lateral acceleration, so as to minimize the vehicle body lateral vibration index. Provide the lateral displacement and lateral acceleration curves of the vehicle body from 0 second to 10 seconds without and with the centrifugal actuator activated, as

well as the angular displacement curve of the centrifugal actuator, and provide the vehicle body lateral vibration index under both conditions.

3. In actual scenarios, the disturbance force cannot be measured, and only sensors are installed to collect real-time lateral acceleration information of the vehicle body. Please, under the condition of limited information, provide a design scheme for the angular displacement change of the centrifugal actuator to minimize the vehicle body lateral vibration index, and for the disturbance in Scenario 2 of Appendix 1, provide the lateral displacement and lateral acceleration change curves of the vehicle body from 0 second to 10 seconds, the angular displacement change curve of the centrifugal actuator, and the result of the vehicle body lateral vibration index.

## References

- [1] Song Kuihui. Research on Rotor Hub Vibration Suppression Technology Based on Centrifugal Actuators [D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 2021.
- [2] You Xiaoliang. Research on Key Technologies of Centrifugal Actuators for Active Vibration Control [D]. Nanjing University of Aeronautics and Astronautics, 2013.
- [3] Hu Mingliang, Wang Lingwei, Chen Kui, et al. Research and Implementation of Centrifugal Actuator and Drive System [J]. Science Technology and Engineering, 2023, 23(06): 2649-2655.

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