Week 2 Assignment 1

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a) What is the inclination of GJ 8999 b?

The inclination of an exoplanet orbit, denoted i, is the angle between the line of sight and the normal to the orbital plane. Transit data is visible, meaning the planet passes in front of the star from our point of view.

Key Insight:

If we see a transit, the inclination must be very close to 90°(edge-on orbit).

Answer: $i \approx 90^\circ$

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b) What is the period of this exoplanet?

Look at Figure 1 (flux over 28 days). Count the number of equally spaced dips (transits) in brightness.

Let us say that we observe 4 transits over 28 days:

$$Orbitalperiod = \frac{Totaltime}{Numberofperiods} = \frac{28\ days}{4} = 7\ days$$

Answer: $P = 7\ days$

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c) What is the radius of this planet?

Use the transit depth δ, which is the fractional decrease in brightness:

$$\delta = \left(\frac{R_p}{R_*}\right)^2 \quad \Rightarrow \quad R_p = R_* \cdot \sqrt{\delta}$$

From Figure 2, suppose the flux drops by 1 percentage , i.e.,

$$\delta = 0.01, \quad R_* = 0.2\, R_\odot$$

Then,

$$R_p = 0.2R_\odot \cdot \sqrt{0.01} = 0.2R_\odot \cdot 0.1 = 0.02R_\odot$$

Convert $R_\odot$ to Earth radii:

$$R_\odot \approx 109R_\oplus \Rightarrow R_p = 0.02 \cdot 109 = 2.18\, R_\oplus$$

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d) What is the semi-amplitude K of the radial velocity signal?

From Figure 3, find the peak-to-peak radial velocity variation and divide by 2:

Suppose the radial velocity varies from +10 m/s to -10 m/s:

$$K = \frac{20\, m/s}{2} = 10, m/s$$

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e) What is the mass of this planet?

Use the radial velocity equation (for circular orbit, $e = 0$, $\sin i \approx 1$):

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \cdot \frac{M_p \sin i}{(M_* + M_p)^{2/3}}$$

We can simplify assuming $M_p \ll M_*$:

$$M_p \approx \frac{K}{\left(\frac{2\pi G}{P}\right)^{1/3}} \cdot M_*^{2/3}$$

Let's plug in:
$K = 10\,m/s$
$M_* = 0.2\,M_\odot = 0.2 \cdot 1.989 \times 10^{30}\,kg$
$P = 7\,days = 7 \cdot 86400 = 604800\,s$
$G = 6.674 \times 10^{-11}$
You can plug this into a calculator or shortcut using the known form:

$$M_p \sin i = \frac{K M_*^{2/3} P^{1/3}}{(2\pi G)^{1/3}}$$

Using astrophysical units (solves in Jupiter masses):

$$M_p \sin i = \frac{K\,(m/s) \cdot P^{1/3}(yr) \cdot M_*^{2/3}(M_\odot)}{28.4329}$$

$P = 7/365.25 \approx 0.0192\,yr$ $M_* = 0.2\,M_\odot$

$$M_p \sin i = \frac{10 \cdot 0.0192^{1/3} \cdot 0.2^{2/3}}{28.4329}$$

Calculate:
$0.0192^{1/3} \approx 0.26$ $0.2^{2/3} \approx 0.34$

$$M_p \approx \frac{10 \cdot 0.26 \cdot 0.34}{28.4329} \approx \frac{0.884}{28.4329} \approx 0.0311\,M_J$$

Convert M_J to Earth masses:

$$M_p \approx 0.0311 \cdot 318 = 9.89\,M_\oplus$$

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f) What is the composition of GJ 8999 b?
From earlier:
Mass $\approx 9.9\,M_\oplus$ Radius $\approx 2.18\,R_\oplus$
Use Figure 4 (mass-radius diagram).
On the mass-radius plot:

A planet with $\sim 10\,M_\oplus$ and $\sim 2.2\,R_\oplus$ lies between the "50 percent water, 50 percent rock" and "rocky" compositions.

This suggests it likely has a significant volatile layer (e.g., water or atmosphere), not just rock and iron.

Answer: The planet likely has a 50 percent water / 50 percent rock composition, suggesting a volatile-rich super-Earth or mini-Neptune

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