Theoretical Computer Science

Tutorial Week 4

Prof. Andrey Frolov

Agenda

Finite State Automaton (FSA)

- **Representations of FSA**
  - Complete
  - Non-complete
- Operations on FSA
- Myhill-Nerode criteria
  - Positive Examples
  - Negative Examples

FSA (Formal definition)

Definition

A (complete) Finite State Automaton is a tuple $\langle Q, \Sigma, q_0, A, \delta \rangle$, where

Q is a finite set of *states*;
Σ is a finite *input alphabet*;
$q_0 \in Q$ is the *initial* state;
$A \subseteq Q$ is the set of *accepting* states;
$\delta : Q \times \Sigma \to Q$ is a (total) *transition* function.

FSA: formally

Example (by formal definition)

$$M = \langle \{q_0, q_1\}, \{0,1\}, \\ \{((q_0,0),q_0), ((q_0,1),q_1), ((q_1,0),q_0), ((q_1,1),q_1)\}, q_0, \{q_1\}\rangle$$

or

Example (by formal definition)

$M = \langle \{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_1\}\rangle$, where
$\delta(q_0,0) = q_0, \delta(q_0,1) = q_1, \delta(q_1,0) = q_0, \delta(q_1,1) = q_1$

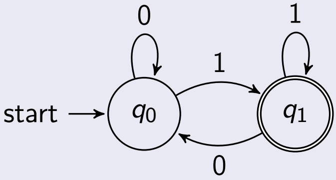
FSA: formally

Example of a FSA (by formal definition)

$M = \langle$	
$\{q_0, q_1\}$,	set of states
$\{0, 1\}$,	input alphabet
$\{((q_0, 0), q_0), ((q_0, 1), q_1),$	
$((q_1, 0), q_0), ((q_1, 1), q_1)\}$,	total transition function
q_0,	initial state
$\{q_1\}$	set of final states
$\rangle$	

FSA: Graphical Representation

State Transition Diagram



Example (by formal definition)

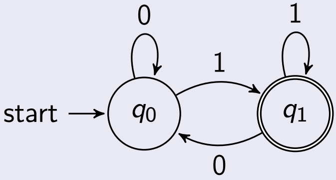
$M = \langle \{q_0, q_1\}, \{0, 1\}, \delta, q_0, \{q_1\} \rangle$, where
$\delta(q_0, 0) = q_0, \delta(q_0, 1) = q_1, \delta(q_1, 0) = q_0, \delta(q_1, 1) = q_1$

FSA: Table Representation

State Transition Table

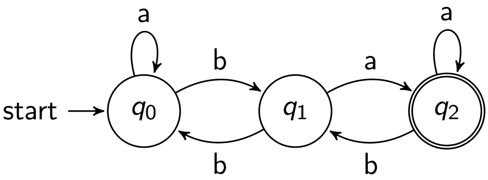
	0	1
$\rightarrow q_0$	q_0	q_1
$^*q_1$	q_0	q_1

State Transition Diagram



State Transition Table: Example

Given an FSA as a State Transition Diagram, build a State Transition Table



State Transition Table

	a	b
$\rightarrow q_0$		
q_1		
$^*q_2$		

State Transition Table: Example

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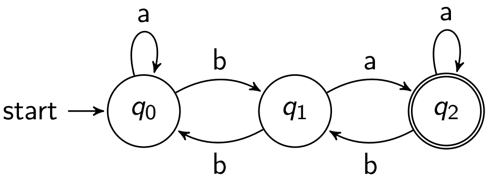

State Transition Table

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State Transition Table: Example

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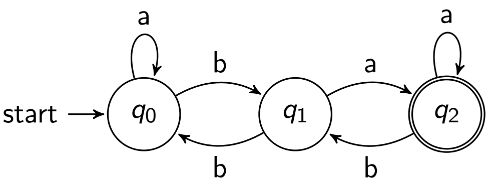

State Transition Table

	a	b
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State Transition Table: Example

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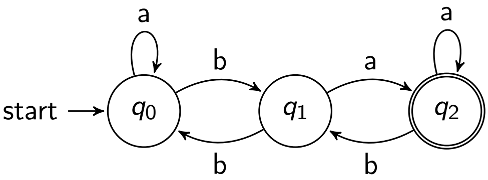

State Transition Table

	a	b
$\rightarrow q_0$	q_0	q_1
q_1	q_2	
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State Transition Table: Example

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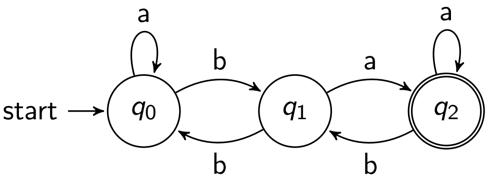
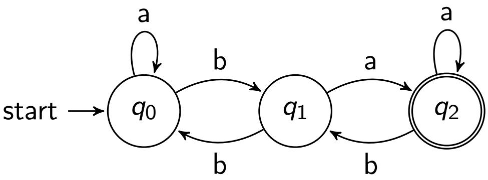


State Transition Table

	a	b
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_0
$^*q_2$		

State Transition Table: Example

Given an FSA as a State Transition Diagram, build a State Transition Table



State Transition Table

	a	b
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_0
$^*q_2$	q_2	

State Transition Table: Example

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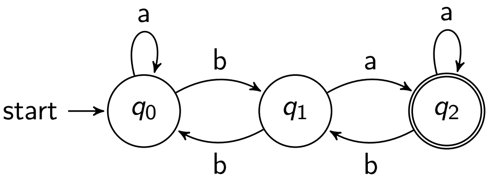

State Transition Table

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q_1	q_2	q_0
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  - Complete
  - **Non-complete**
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  - Positive Examples
  - Negative Examples

FSA (Formal definition)

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- $q_0 \in Q$ is the *initial* state;
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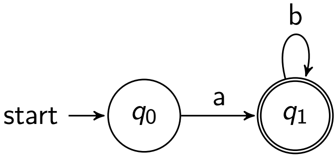
- $\delta : Q \times \Sigma \to Q$ is a partial *transition* function.

Non-complete FSA

If a FSA is not complete?



State Transition Table

	a	b
$\rightarrow q_0$	q_1	
$^*q_1$		q_1

Non-complete FSA

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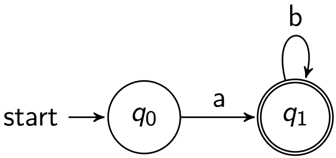

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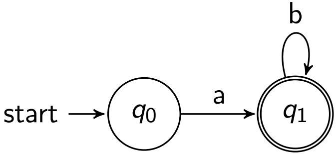

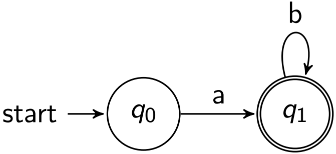

Non-complete FSA

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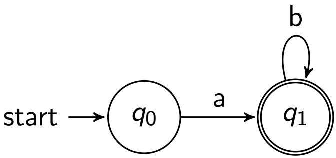

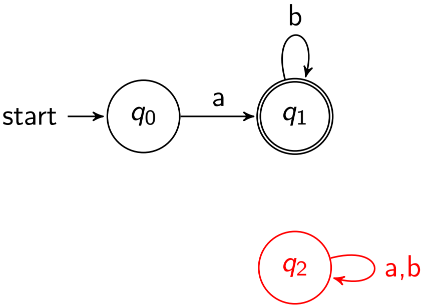

Non-complete FSA

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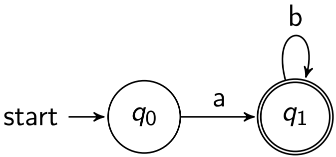

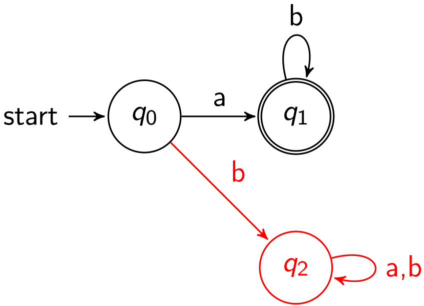

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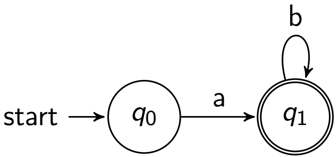

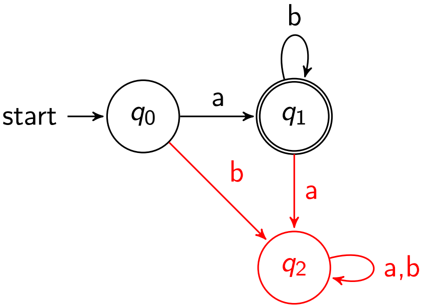

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Finite State Automaton (FSA)

- Representations of FSA
  - Complete
  - Non-complete
- **Operations on FSA**
- Myhill-Nerode criteria
  - Positive Examples
  - Negative Examples

Regular Languages

Definition

A languages is called regular, if it recognized by a FSA.

Operations

Problem

If we have an algorithm to accept L, how can we formulate an algorithm to accept L^c?

Problem

Suppose L_1 and L_2 are both languages over the alphabet A.

If we have one algorithm to accept L_1 and another to accept L_2, how can we formulate an algorithm to accept $L_1 \cap L_2$? (similarly, $L_1 \cup L_2$ or $L_1 \backslash L_2$).

Operations

Problem

Suppose $M = (Q^1, A, \delta^1, q_0^1, F^1)$ is a finite automaton accepting L.

What is an automaton which accepts L^c?

Problem

Suppose $M^1 = (Q^1, A, \delta^1, q_0^1, F^1)$ and $M^2 = (Q^2, A, \delta^2, q_0^2, F^2)$ are finite automata accepting L_1 and L_2, respectively.

What is an automaton which accepts $L_1 \cap L_2$?
(similarly, $L_1 \cup L_2$, $L_1 \backslash L_2$)?

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Finite State Automaton (FSA)

- Representations of FSA
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  - **Complement**
  - Intersection
  - Union
  - Difference
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Complement

Suppose $M = (Q, A, \delta, q_0, F)$ is a complete FSA accepting L.

The automaton $M^c = (Q, A, \delta, q_0, F^c)$ accepts the language L^c.

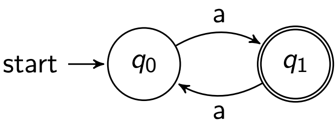
Recall that

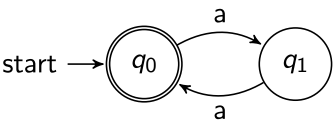
$$F^c = Q \setminus F$$

Complement: Example

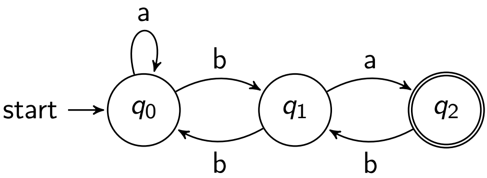
$$M = \langle \{q_0, q_1\}, \{a\}, \{((q_0, a), q_1), ((q_1, a), q_0)\}, q_0, \{q_1\} \rangle$$

$$M^c = \langle \{q_0, q_1\}, \{a\}, \{((q_0, a), q_1), ((q_1, a), q_0)\}, q_0, \{q_0\} \rangle$$



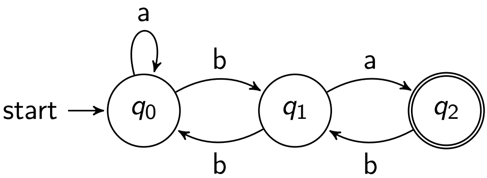


Complement: Example

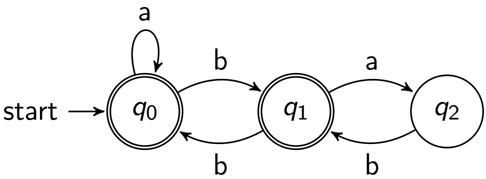


What would be the complement M^c?

Complement: Example

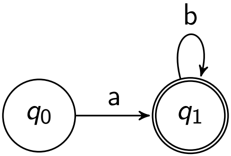


What would be the complement M^c?



Complement: Example

If a FSA is not complete?



Complement: Example

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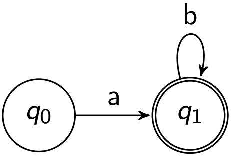

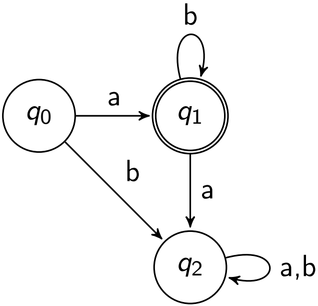

Complement: Example

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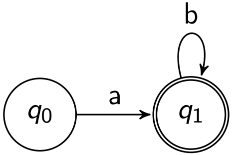

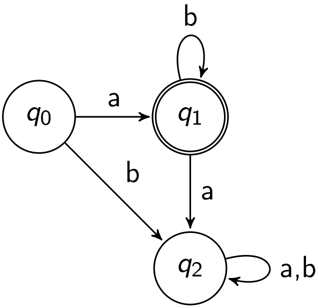

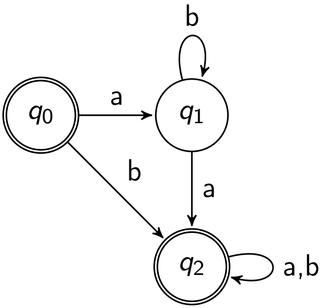

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  - **Intersection**
  - Union
  - Difference
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Intersection

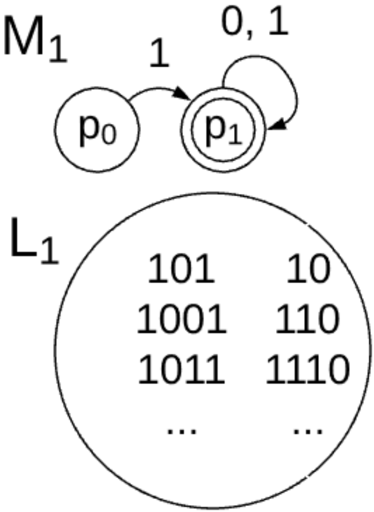
Suppose $M^1 = (Q^1, A, \delta^1, q_0^1, F^1)$ and $M^2 = (Q^2, A, \delta^2, q_0^2, F^2)$ are finite automata accepting L_1 and L_2, respectively.

$$Q = Q^1 \times Q^2$$
$$A$$
$$q_0 = (q_0^1, q_0^2)$$
$$\delta((q,p),a) = (\delta^1(q,a), \delta^2(p,a))$$
$$F = \{(q,p) \in Q^1 \times Q^2 \mid q \in F^1 \; \& \; p \in F^2\}$$

The automaton $M = (Q, A, \delta, q_0, F)$ accepts the language $L_1 \cap L_2$.

$$M = M_1 \cap M_2$$

Intersection: Example 1



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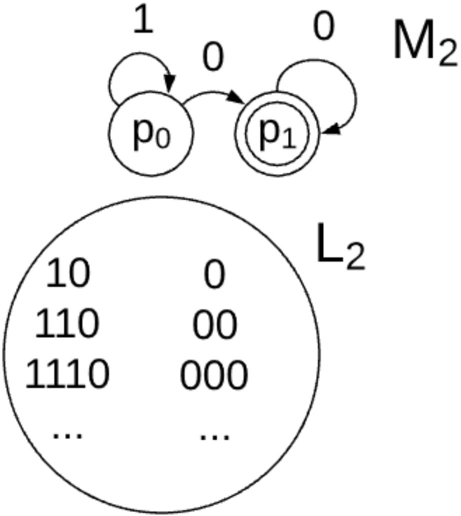

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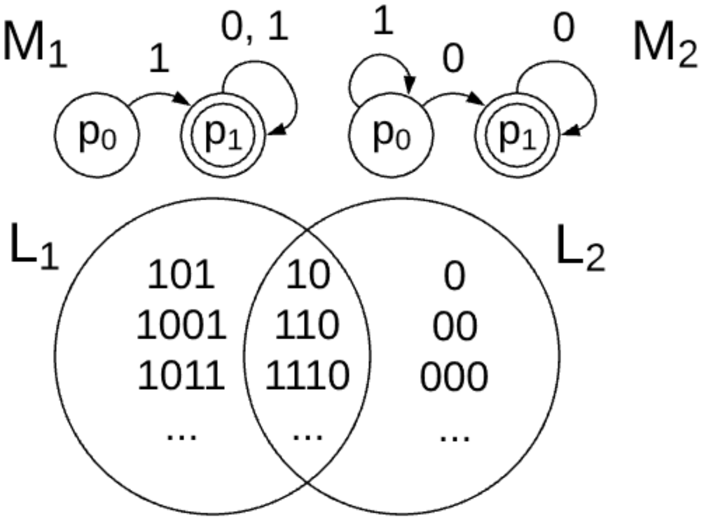

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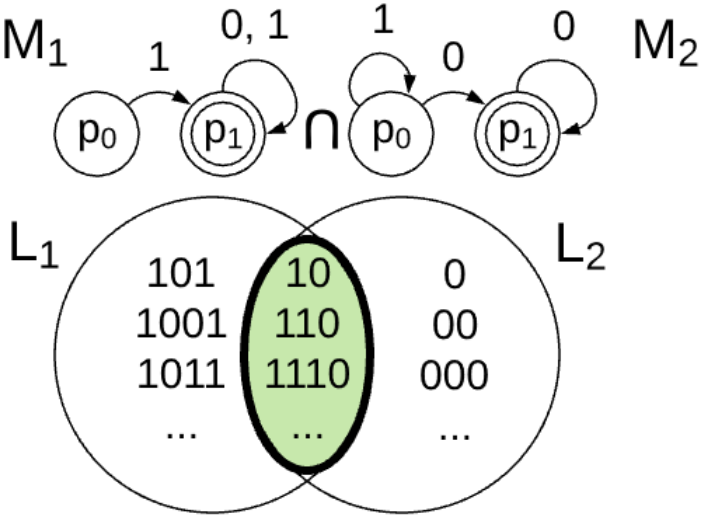
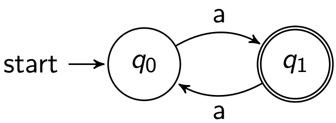


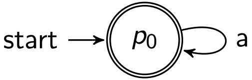
Intersection: Example 2

$$M^1 = \langle \{q_0, q_1\}, \{a\},\\ \{((q_0, a), q_1), ((q_1, a), q_0)\},\\ q_0, \{q_1\}\rangle$$



Intersection: Example 2

$$M^2 = \langle \{p_0\}, \{a\},$$
$$\{((p_0, a), p_0)\},$$
$$p_0, \{p_0\}\rangle$$



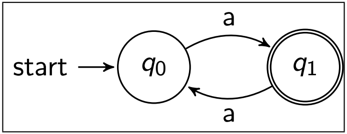
Intersection: Example 2

$$\begin{aligned} M^1 = \langle &\{q_0, q_1\}, \{a\}, \\ &\{((q_0, a), q_1), ((q_1, a), q_0)\}, \\ &q_0, \{q_1\}\rangle \end{aligned}$$

$$\begin{aligned} M^2 = \langle &\{p_0\}, \{a\}, \\ &\{((p_0, a), p_0)\}, \\ &p_0, \{p_0\}\rangle \end{aligned}$$

$$\begin{aligned} (M^1 \cap M^2) = \langle &\{(q_0, p_0), (q_1, p_0)\}, \{a\}, \\ &\Big\{\Big(((q_0, p_0), a), (q_1, p_0)\Big), \Big(((q_1, p_0), a), (q_0, p_0)\Big)\Big\}, \\ &(q_0, p_0), \{(q_1, p_0)\}\rangle \end{aligned}$$

Intersection: Example 2



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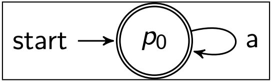

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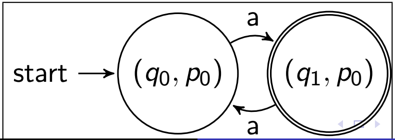
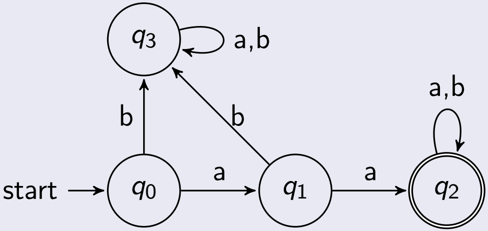
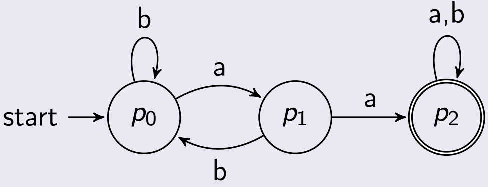


Intersection: Example 3

M_1

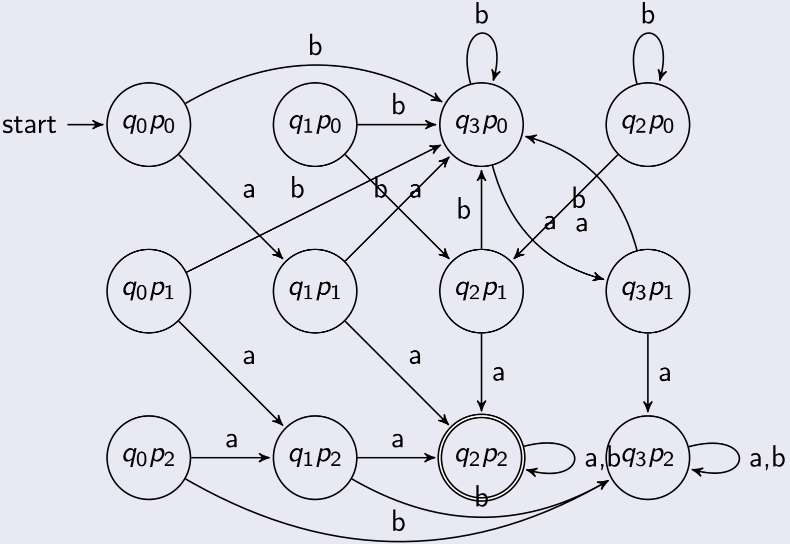


M_2



Intersection: Example 3

$M_1 \cap M_2$



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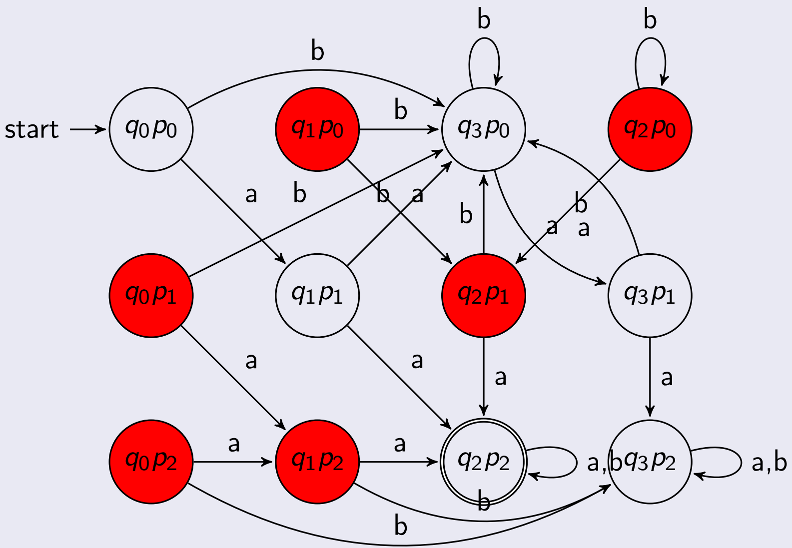

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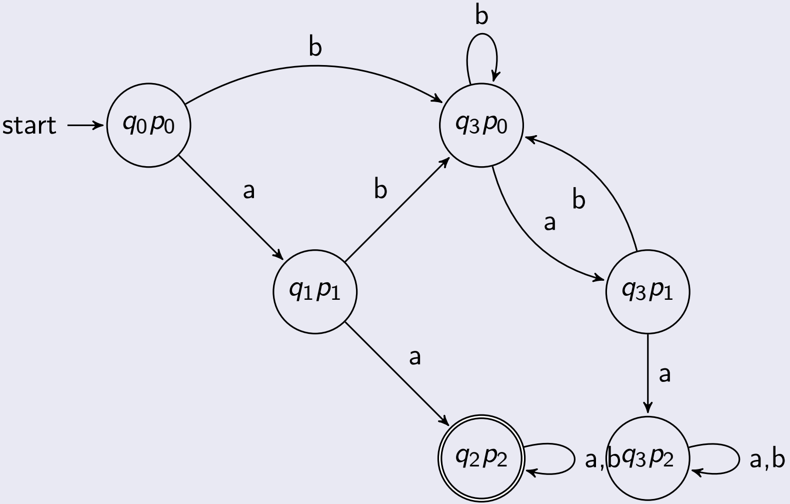

Intersection: Example 3

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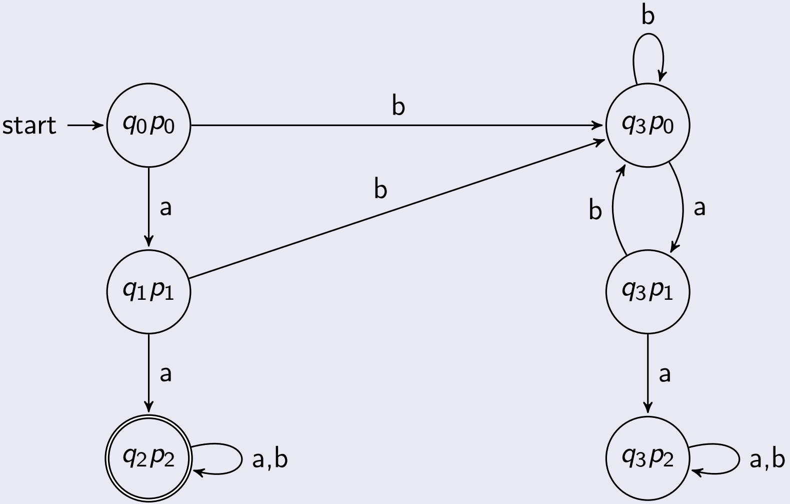

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Union

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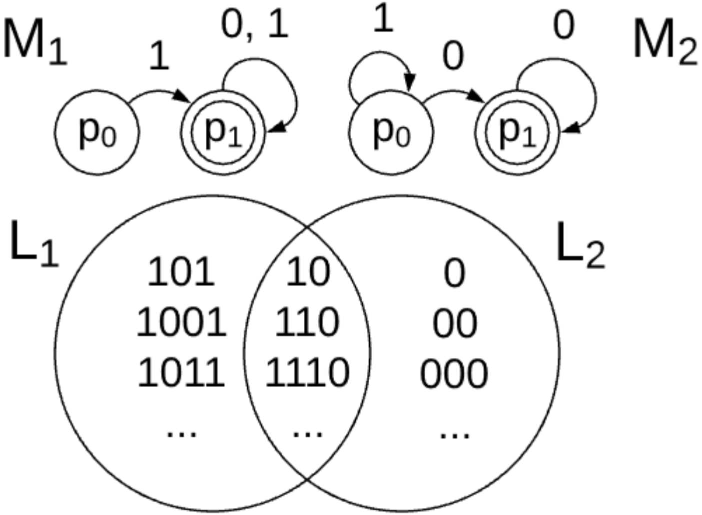
$$\begin{aligned}
&Q = Q^1 \times Q^2 \\
&A \\
&q_0 = (q_0^1, q_0^2) \\
&\delta((q,p),a) = (\delta^1(q,a), \delta^2(p,a)) \\
&F = \{(q,p) \in Q^1 \times Q^2 \mid q \in F^1 \vee p \in F^2\}
\end{aligned}$$

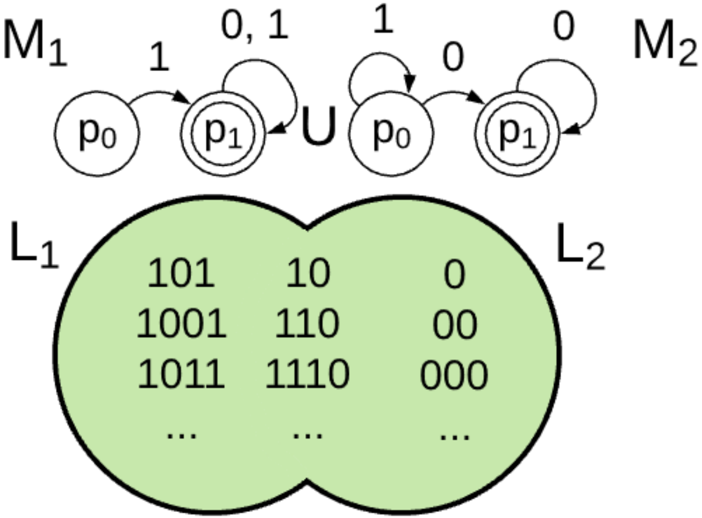
The automaton $M = (Q, A, \delta, q_0, F)$ accepts the language $L_1 \cup L_2$.

$$M = M_1 \cup M_2$$

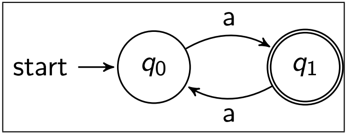
Union: Example 1



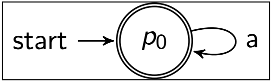
Union: Example 1



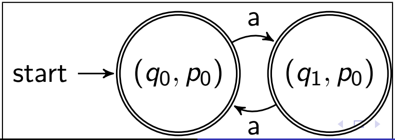
Union: Example 2



$$\cup$$

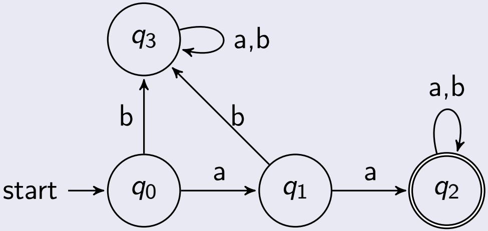


$$=$$

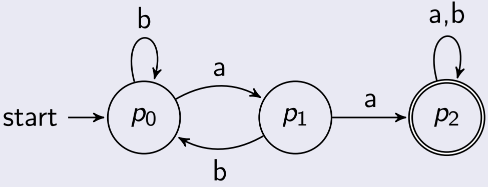


Union: Example 3

M_1

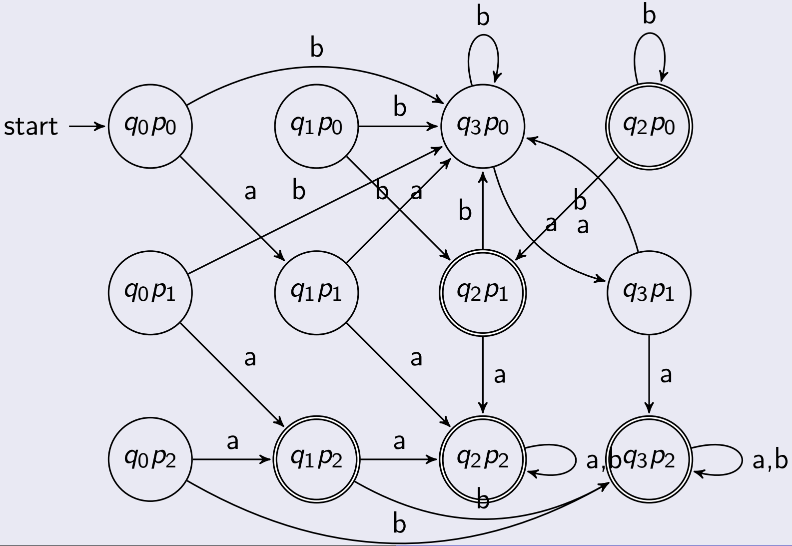


M_2



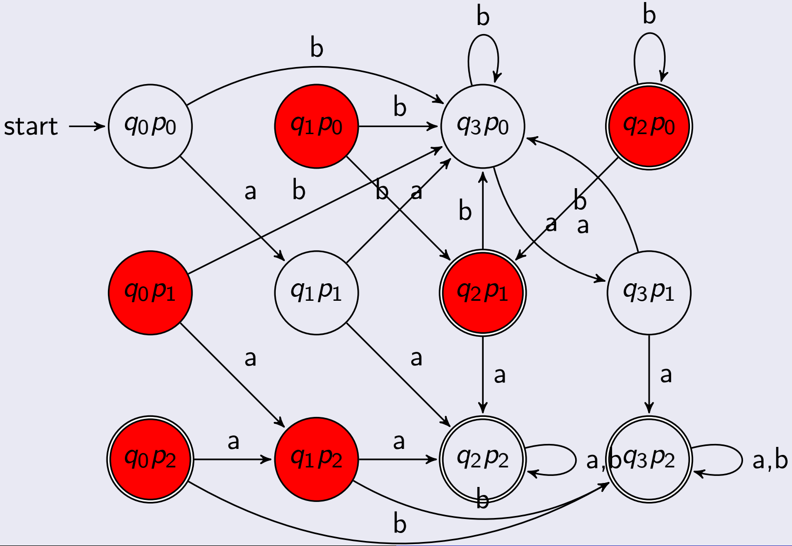
Union: Example 3

$M_1 \cup M_2$



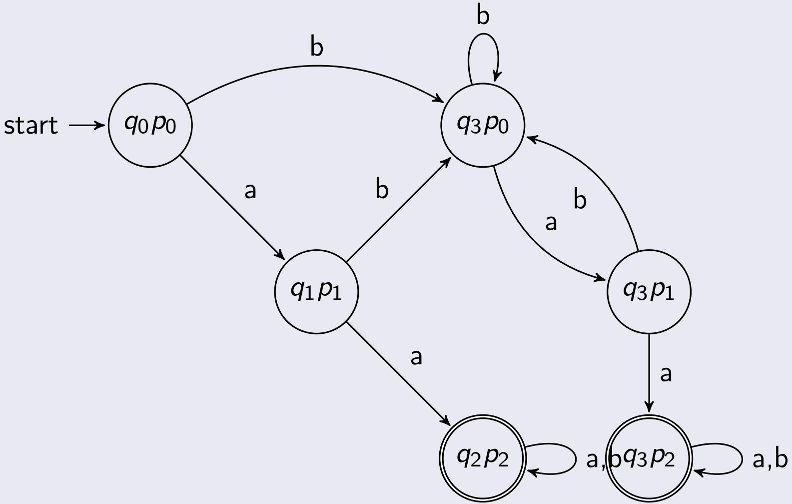
Union: Example 3

$M_1 \cup M_2$



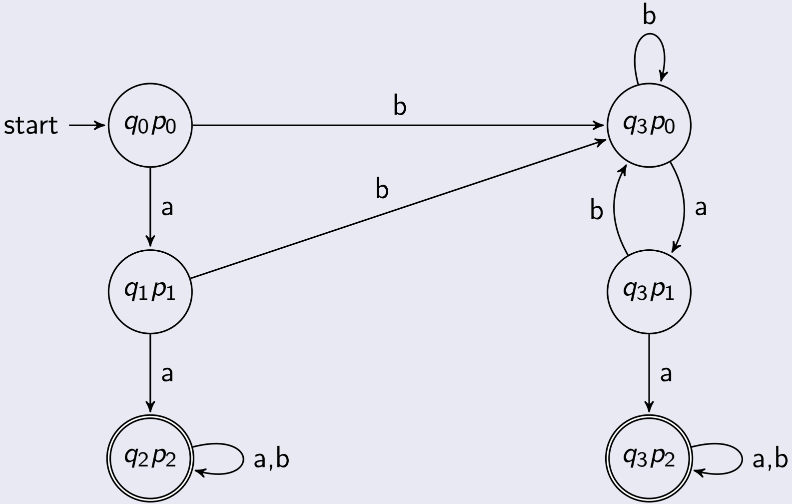
Union: Example 3

$M_1 \cup M_2$



Union: Example 3

$M_1 \cup M_2$



Agenda

Finite State Automaton (FSA)

- Representations of FSA
  - Complete
  - Non-complete
- Operations on FSA
  - Complement
  - Intersection
  - Union
  - **Difference**
- Myhill-Nerode criteria
  - Positive Examples
  - Negative Examples

Difference

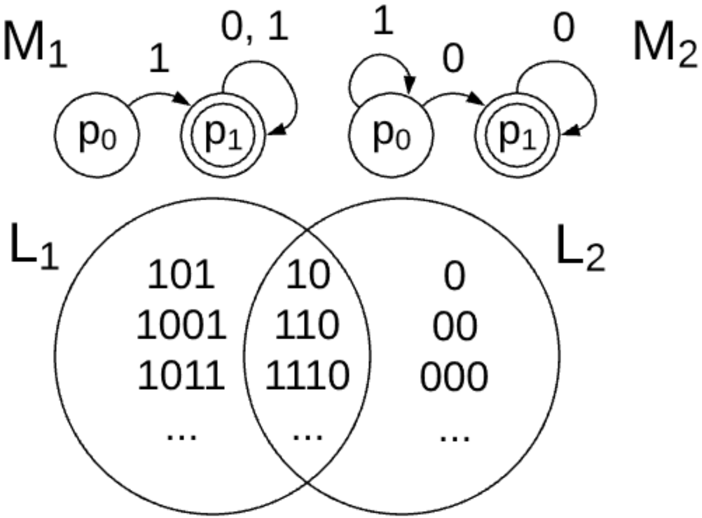
Suppose $M^1 = (Q^1, A, \delta^1, q_0^1, F^1)$ and $M^2 = (Q^2, A, \delta^2, q_0^2, F^2)$ are finite automata accepting L_1 and L_2, respectively.

$$
\begin{aligned}
&Q = Q^1 \times Q^2 \\
&A \\
&q_0 = (q_0^1, q_0^2) \\
&\delta((q,p),a) = (\delta^1(q,a), \delta^2(p,a)) \\
&F = \{(q,p) \in Q^1 \times Q^2 \mid q \in F^1 \ \& \ p \notin F^2\}
\end{aligned}
$$

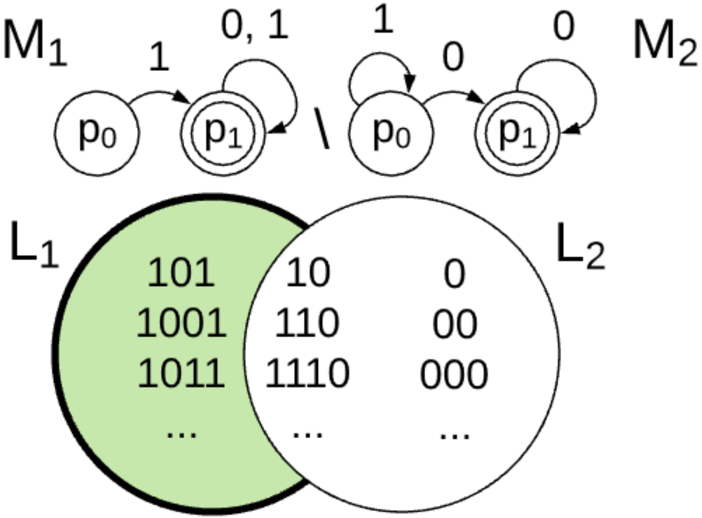
The automaton $M = (Q, A, \delta, q_0, F)$ accepts the language $L_1 \setminus L_2$.

$$M = M_1 \setminus M_2$$

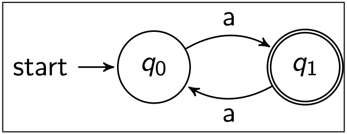
Difference (Example 1)



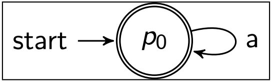
Difference (Example 1)



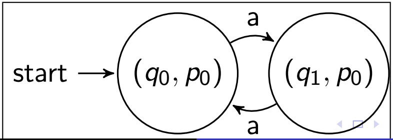
Difference (Example 2 $L_1 \setminus L_2$)



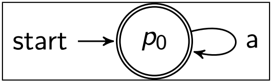
$$\setminus$$



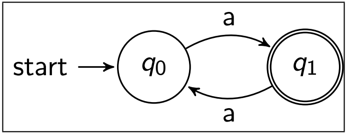
$$=$$



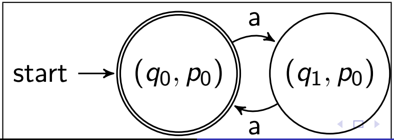
Difference (Example 3 $L_2 \setminus L_1$)



$\setminus$



$=$



Agenda

Finite State Automaton (FSA)

- Representations of FSA
  - Complete
  - Non-complete
- Operations on FSA
- **Myhill-Nerode criteria**
  - Positive Examples
  - Negative Examples

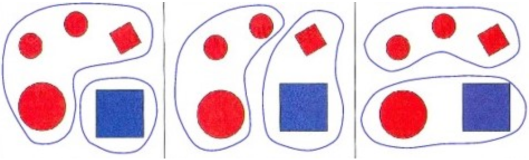
Myhill-Nerode criteria

For a language L over an alphabet A,

$$s_1 \equiv_L s_2 \Leftrightarrow (\forall t \in A^*)\,(s_1 t \in L \leftrightarrow s_2 t \in L)$$

$\equiv_L$ is an equivalence relation

What are equivalence relations in general?



Myhill-Nerode criteria

For a language L over an alphabet A,

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*)\,[(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

t is called a **distinguishing extension**.

Myhill-Nerode criteria

Myhill-Nerode theorem

A language L is regular iff $\equiv_L$ has a finite number of equivalent classes.

Agenda

Finite State Automaton (FSA)

- Representations of FSA
  - Complete
  - Non-complete
- Operations on FSA
- Myhill-Nerode criteria
  - **Positive Examples**
  - Negative Examples

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*) \left[(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L) \right]$$

Example 1: $L_1 = \{0x \mid x \in \Sigma^*\}$, where $\Sigma = \{0, 1\}$

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*) \, [(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

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ϵ,

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ϵ, 0, 1,

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ϵ, 0, 1, 00,

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ϵ, 0, 1, 00, 01,

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$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*)\,[(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

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ϵ, 0, 1, 00, 01,10, 11, ...

1. ϵ

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ϵ, 0, 1, 00, 01,10, 11, ...

1. ϵ
2. 0: $0 \not\equiv_{L_1} \epsilon$, since $0 \cdot \epsilon \in L_1 \,\&\, \epsilon \cdot \epsilon \notin L_1$ (a disting. ext. is ϵ)

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*) \, [(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

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3. 1:

Myhill-Nerode method. Examples

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Example 1: $L_1 = \{0x \mid x \in \Sigma^*\}$, where $\Sigma = \{0, 1\}$
ϵ, 0, 1, 00, 01,10, 11, . . .

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3. 1:

   $1 \not\equiv_{L_1} 0$, since $1 \notin L_1, 0 \in L_1$

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*) \, [(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

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   $1 \not\equiv_{L_1} \epsilon$, since $1 \cdot 0 \notin L_1, \epsilon \cdot 0 \in L_1$ (a distinguishing ext. is 0)

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*)\,[(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

Example 1: $L_1 = \{0x \mid x \in \Sigma^*\}$, where $\Sigma = \{0, 1\}$
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   $1 \not\equiv_{L_1} \epsilon$, since $1 \cdot 0 \notin L_1, \epsilon \cdot 0 \in L_1$ (a distinguishing ext. is 0)
4. $0t \equiv_{L_1} 0$

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*) \, [(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

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ϵ, 0, 1, 00, 01,10, 11, . . .

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4. $0t \equiv_{L_1} 0$
5. $1t \equiv_{L_1} 1$

Myhill-Nerode method. Examples

Example 1: $L_1 = \{0x \mid x \in \Sigma^*\}$

$[\epsilon] = \{\epsilon\}$, $[0] = \{0x \mid x \in \Sigma^*\} = L_1$, $[1] = \{1x \mid x \in \Sigma^*\}$

δ	0	1
$\rightarrow [\epsilon]$		
$*[0]$		
$[1]$		

Myhill-Nerode method. Examples

Example 1: $L_1 = \{0x \mid x \in \Sigma^*\}$

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δ	0	1
$\rightarrow [\epsilon]$	$[0]$	
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δ	0	1
$\rightarrow [\epsilon]$	$[0]$	$[1]$
$*[0]$	$[0]$	
$[1]$		

Myhill-Nerode method. Examples

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$*[0]$	$[0]$	$[0]$
$[1]$		

Myhill-Nerode method. Examples

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δ	0	1
$\rightarrow [\epsilon]$	$[0]$	$[1]$
$*[0]$	$[0]$	$[0]$
$[1]$	$[1]$	

Myhill-Nerode method. Examples

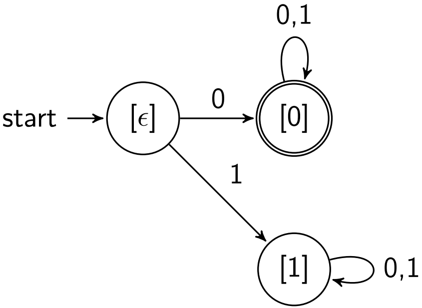
Example 1: $L_1 = \{0x \mid x \in \Sigma^*\}$

$[\epsilon] = \{\epsilon\}$, $[0] = \{0x \mid x \in \Sigma^*\} = L_1$, $[1] = \{1x \mid x \in \Sigma^*\}$

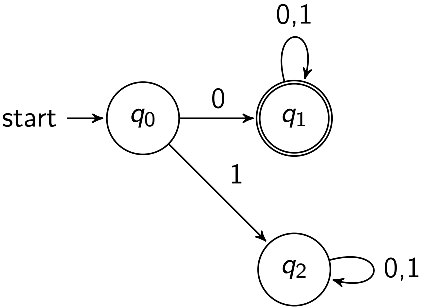
δ	0	1
$\rightarrow [\epsilon]$	$[0]$	$[1]$
$*[0]$	$[0]$	$[0]$
$[1]$	$[1]$	$[1]$

Myhill-Nerode method. Examples

Example 1: $L_1 = \{0x \mid x \in \Sigma^*\}$



Example 1: $L_1 = \{0x \mid x \in \Sigma^*\}$



Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*)\,[(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

Example 2: $L_2 = \{x00 \mid x \in \Sigma^*\}$

1. ϵ

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*)\,[(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

Example 2: $L_2 = \{x00 \mid x \in \Sigma^*\}$

1. ϵ
2. 0: $0 \not\equiv_{L_2} \epsilon$, since $00 \in L_2 \,\&\, \epsilon 0 = 0 \notin L_2$ (a disting. ext. is 0)

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*)\,[(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

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2. 0: $0 \not\equiv_{L_2} \epsilon$, since $00 \in L_2 \,\&\, \epsilon 0 = 0 \notin L_2$ (a disting. ext. is 0)
3. 1: $1 \equiv_{L_2} \epsilon$, $t1 \equiv_{L_2} \epsilon$

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*) \, [(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

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2. 0: $0 \not\equiv_{L_2} \epsilon$, since $00 \in L_2 \,\&\, \epsilon 0 = 0 \notin L_2$ (a disting. ext. is 0)
3. 1: $1 \equiv_{L_2} \epsilon$, $t1 \equiv_{L_2} \epsilon$
4. 00:

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*)\,[(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

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1. ϵ
2. 0: $0 \not\equiv_{L_2} \epsilon$, since $00 \in L_2 \,\&\, \epsilon 0 = 0 \notin L_2$ (a disting. ext. is 0)
3. 1: $1 \equiv_{L_2} \epsilon$, $t1 \equiv_{L_2} \epsilon$
4. 00:

$$00 \not\equiv_{L_2} \epsilon, \text{ since } 00 \in L_2 \,\&\, \epsilon \notin L_2 \text{ (a distinguishing ext. is } \epsilon)$$

Myhill-Nerode method. Examples

$$s_1 \not\equiv_L s_2 \Leftrightarrow (\exists t \in A^*)\,[(s_1 t \notin L \,\&\, s_2 t \in L) \vee (s_1 t \in L \,\&\, s_2 t \notin L)]$$

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Example 2: $L_2 = \{x00 \mid x \in \Sigma^*\}$

$[\epsilon] = \{\epsilon\} \cup \{x1 \mid x \in \Sigma^*\}$, $[0] = \{x10 \mid x \in \Sigma^*\}$,
$[00] = \{x00 \mid x \in \Sigma^*\} = L_2$

δ	0	1
$\rightarrow [\epsilon]$		
$[0]$		
$*[00]$		

Myhill-Nerode method. Examples

Example 2: $L_2 = \{x00 \mid x \in \Sigma^*\}$

$[\epsilon] = \{\epsilon\} \cup \{x1 \mid x \in \Sigma^*\}$, $[0] = \{x10 \mid x \in \Sigma^*\}$,
$[00] = \{x00 \mid x \in \Sigma^*\} = L_2$

δ	0	1
$\rightarrow [\epsilon]$	$[0]$	
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Myhill-Nerode method. Examples

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$[00] = \{x00 \mid x \in \Sigma^*\} = L_2$

δ	0	1
$\rightarrow [\epsilon]$	$[0]$	$[\epsilon]$
$[0]$		
$*[00]$		

Myhill-Nerode method. Examples

Example 2: $L_2 = \{x00 \mid x \in \Sigma^*\}$

$[\epsilon] = \{\epsilon\} \cup \{x1 \mid x \in \Sigma^*\}$, $[0] = \{x10 \mid x \in \Sigma^*\}$,
$[00] = \{x00 \mid x \in \Sigma^*\} = L_2$

δ	0	1
$\rightarrow [\epsilon]$	$[0]$	$[\epsilon]$
$[0]$	$[00]$	
$*[00]$		

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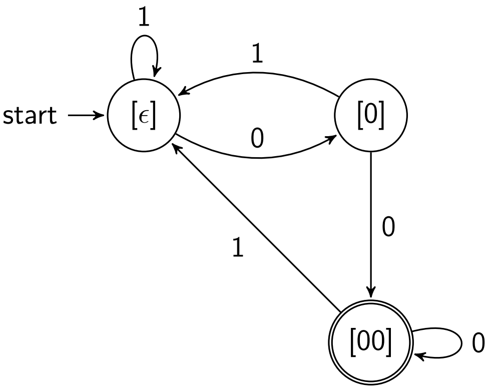

Example 3: $L_3 = \{x \in \Sigma^* \mid x$ is a binary representation of an integer divisible by 5 and it begins with $1\}$, where $\Sigma = \{0, 1\}$

1. $[\epsilon]$
2. $[0] = \{0x \mid x \in \Sigma^*\}$
3. $[1] = \{x \mid$ the remainder after dividing x by 5 is $1\}$
4. $[10] = \{x \mid$ the remainder after dividing x by 5 is $2\}$
5. $[11] = \{x \mid$ the remainder after dividing x by 5 is $3\}$
6. $[100] = \{x \mid$ the remainder after dividing x by 5 is $4\}$
7. $[101] = \{x \mid x$ is divisible by $5\}$

Myhill-Nerode method. Examples

Example 3: $L_3 = \{x \in \Sigma^* \mid x$ is a binary representation of an integer divisible by 5 and it begins with $1\}$

δ	0	1
$\rightarrow [\epsilon]$	[0]	[1]
[0]	[0]	[0]
[1]		
[10]		
[11]		
[100]		
*[101]		

Myhill-Nerode method. Examples

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[0]	[0]	[0]
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δ	0	1
$\to [\epsilon]$	[0]	[1]
[0]	[0]	[0]
[1]	[10]	[11]
[10]	[100]	[101]
[11]	[1]	[10]
[100]	[11]	[100]
*[101]	[101]	

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$\rightarrow [\epsilon]$	[0]	[1]
[0]	[0]	[0]
[1]	[10]	[11]
[10]	[100]	[101]
[11]	[1]	[10]
[100]	[11]	[100]
*[101]	[101]	[1]

Agenda

Finite State Automaton (FSA)

- Representations of FSA
  - Complete
  - Non-complete
- Operations on FSA
- Myhill-Nerode criteria
  - Positive Examples
  - **Negative Examples**

Myhill-Nerode method. Examples

Negative Example 1

$L_1 = \{a^n b^n \mid n \in \mathbb{N}\}$ is not regular.

Proof

For $m \neq k$,

$$a^m \not\equiv_{L_1} a^k,$$

since $a^m b^k \notin L_1, a^k b^k \in L_1$ (a distinguishing ext. is b^k).
Therefore, there are infinity many equivalence classes!
So, L_1 is not regular.

Myhill-Nerode method. Examples

Negative Example 2

$L_2 = \{a^n b a^n \mid n \in \mathbb{N}\}$ is not regular.

Proof

For $m \neq k$,

$$a^m b \not\equiv_{L_2} a^k b,$$

since $a^m b a^k \notin L_2, a^k b a^k \in L_2$ (a distinguishing ext. is a^k).
Therefore, there are infinity many equivalence classes!
So, L_2 is not regular.

Thank you for your attention!