Theoretical Computer Science

Lab Session 2

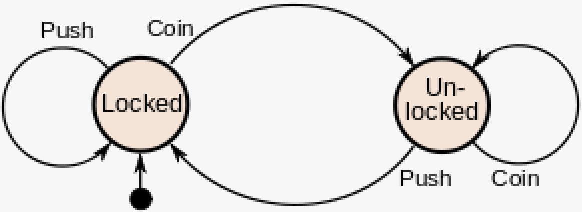
February 8, 2023

Agenda

- Exercises on Finite State Automaton (FSA)

Finite State Automaton

A finite state automaton (sometimes called a finite state machine) is a computation model that can be implemented with hardware or software and can be used to simulate sequential logic and some computer programs. Finite state automata generate regular languages. Finite state machines can be used to model problems in many fields including mathematics, artificial intelligence, games, and linguistics.



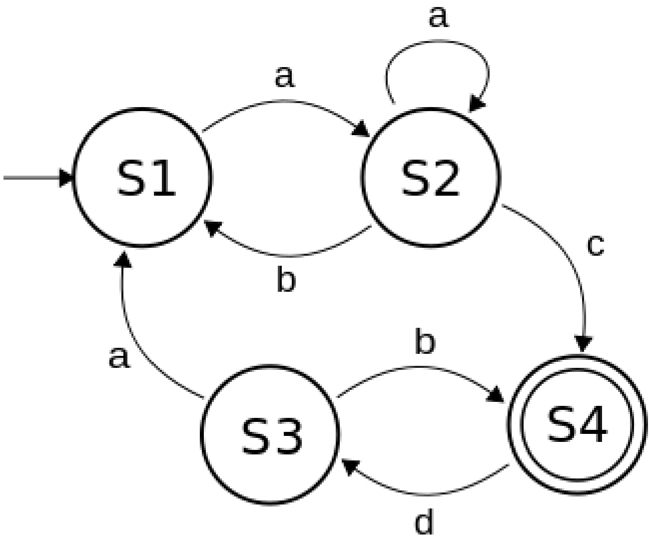
Formal Definition

A deterministic finite automaton M is a 5-tuple, $(Q, \Sigma, \delta, q_0, F)$, consisting of

- a finite set of states Q
- a finite set of input symbols called the alphabet Σ
- a transition function $\delta : Q \times \Sigma \to Q$
- an initial or start state $q_0 \in Q$
- a set of accept states $F \in Q$

Simple Problem

What string cannot be generated by the finite state automaton below?



Exercises

Exercises - Part 1

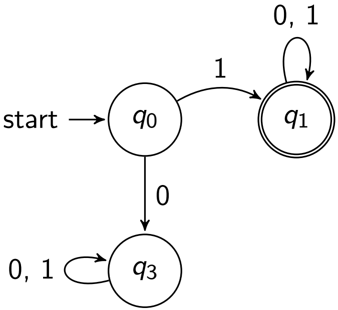
Build complete FSAs that recognize the following languages:
Let Σ be the alphabet $\Sigma = \{0, 1\}$

1. $L_1 = \{x \in \Sigma^* \mid$ x starts with 1$\}$;
2. $L_2 = \{x \in \Sigma^* \mid$ x does not begin with 1$\}$;
3. $L_3 = \{x \in \Sigma^* \mid$ any 0 in x is followed by at least a 1$\}$.
   Strings example: 010111, 1111, 01110111011.
4. $L_4 = \{x \in \Sigma^* \mid$ x ends with 00$\}$;
5. $L_5 = \{x \in \Sigma^* \mid$ x contains exactly 3 zeros$\}$;

Solution (1)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

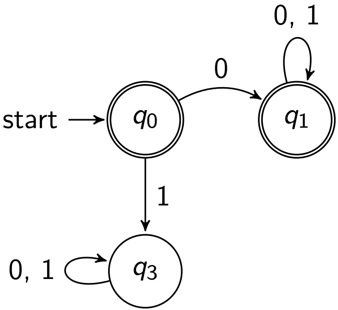
- $L_1 = \{x \in \Sigma^* \mid \ x \text{ starts with } 1\}$;



Solution (2)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

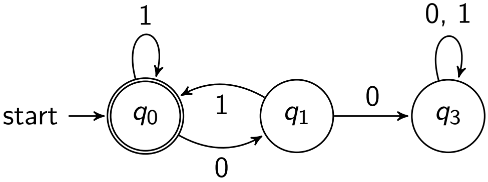
- $L_2 = \{x \in \Sigma^* \mid \; x \text{ does not begin with } 1\}$;



Solution (3)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

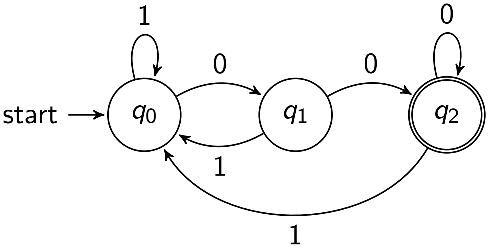
- $L_3 = \{x \in \Sigma^* \mid$ any 0 in x is followed by at least a 1$\}$.
  Strings example: 010111, 1111, 01110111011.



Solution (4)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

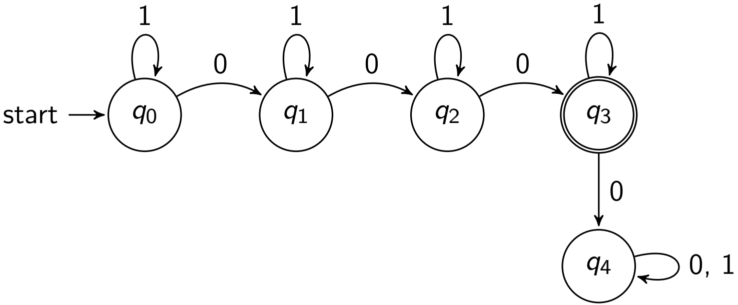
- $L_4 = \{x \in \Sigma^* \mid \ x \text{ ends with } 00\}$;



Solution (5)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

- $L_5 = \{x \in \Sigma^* \mid \ x \text{ contains exactly 3 zeros}\}$;



Exercises - Part 2

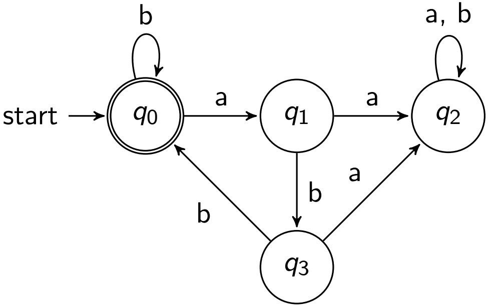
Build complete FSAs that recognize the following languages:
Let Σ be the alphabet $\Sigma = \{a, b\}$

6. $L_6 = \{x \in \Sigma^* \mid$ every a in x (if there are any) is followed immediately by $bb\}$.
7. $L_7 = \{x \in \Sigma^* \mid$ x ends with b and does not contain the substring $aa\}$.
8. $L_8 = \{x \in \Sigma^* \mid x$ contains the substring $abbaab\}$;
9. $L_9 = \{x \in \Sigma^* \mid$ x has an even number of a's and an even number of b's$\}$;

Solution (6)

Let Σ be the alphabet $\Sigma = \{a, b\}$

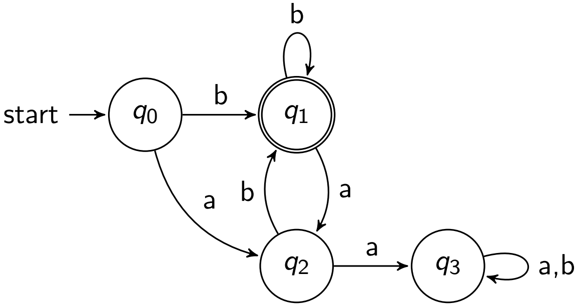
- $L_6 = \{x \in \Sigma^* \mid$
  every a in x (if there are any) is followed immediately by $bb\}$.



Solution (7)

Let Σ be the alphabet $\Sigma = \{a, b\}$

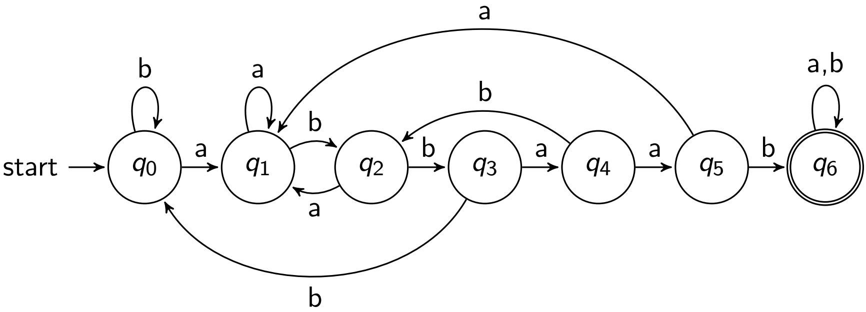
- $L_7 = \{x \in \Sigma^* \mid$
  x ends with b and does not contain the substring $aa\}$.



Solution (8)

Let Σ be the alphabet $\Sigma = \{a, b\}$

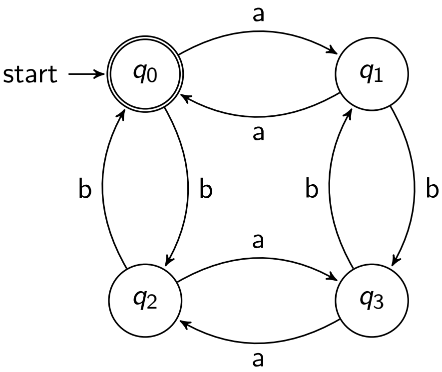
- $L_8 = \{x \in \Sigma^* \mid x \text{ contains the substring } abbaab\}$;



Solution (9)

Let Σ be the alphabet $\Sigma = \{a, b\}$

- $L_9 = \{x \in \Sigma^* \mid$
  x has an even number of a's and an even number of b's$\}$;



Exercises - Part 3

Build complete FSAs accepting the following languages over the alphabet $\Sigma = \{0, 1\}$

10. $L_a = \{x \in \Sigma^* \mid$ x is a binary representation of an integer divisible by 5 and it begins with 1$\}$;
11. $L_b = \{x \in \Sigma^* \mid |x| \geq 2 \wedge$ final two symbols are the same$\}$;

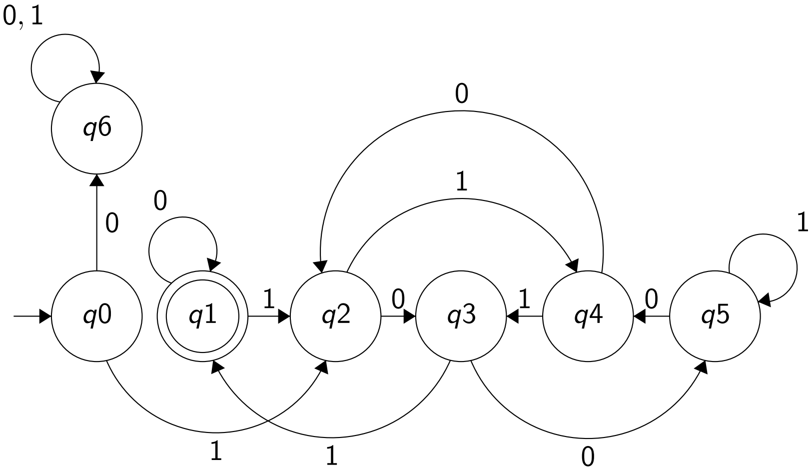
Build a complete FSA accepting the following language over the alphabet $\Sigma = \{a, b, c\}$

12. $L_c = \{x \in \Sigma^* \mid$ the substring abc in x occurs an odd number of times$\}$.

Solution (10)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

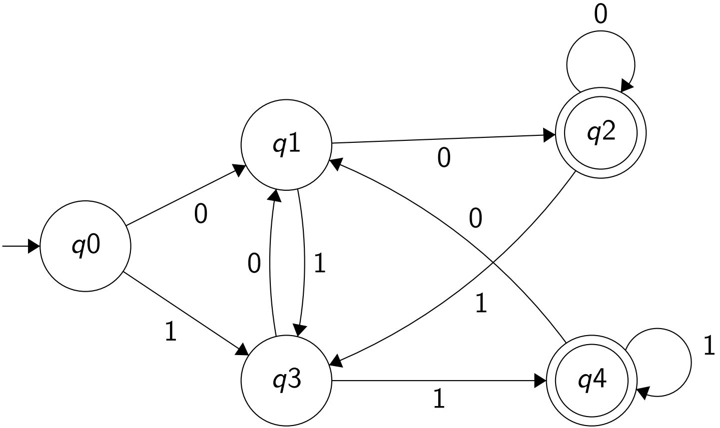
- $L_a = \{x \in \Sigma^* \mid \ x$ is a binary representation of an integer divisible by 5 and it begins with $1\}$;



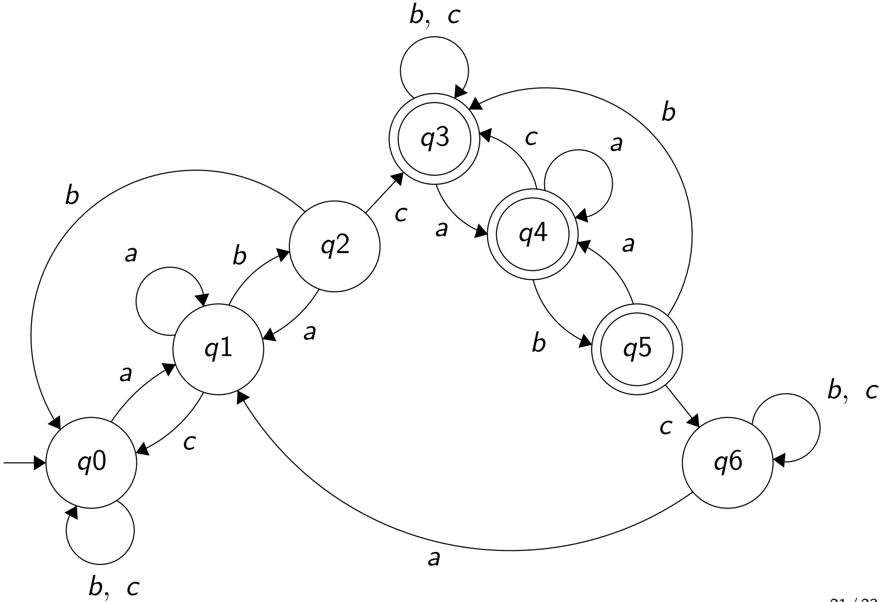
Solution (11)

Let Σ be the alphabet $\Sigma = \{0, 1\}$

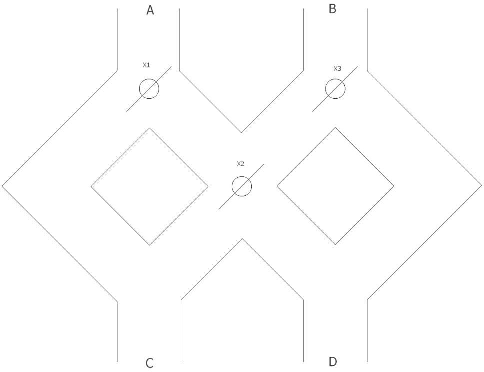
- $L_b = \{x \in \Sigma^* \mid |x| \geq 2 \wedge \text{final two symbols are the same}\}$;



Solution (12): L_c - *abc* occurs odd number of times

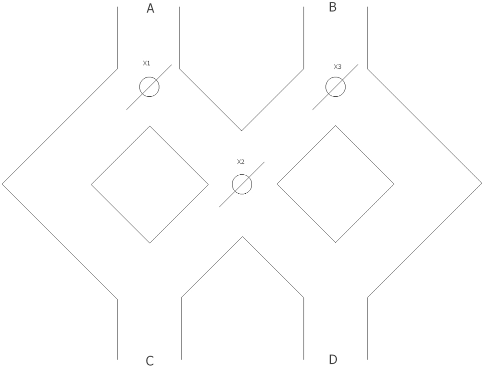


Homework



The figure is a marble toy. A marble is dropped at A or B. Levers $x1$, $x2$, and $x3$ cause the marble to fall either to the left or to the right. Whenever a marble encounters a lever, it causes the lever to reverse after the marble passes, so the next marble will take the opposite branch.

Homework



Model this toy by a complete FSA. Let the inputs A and B represent the input into which the marble is dropped. Let acceptance correspond to the marble exiting at D; nonacceptance represents a marble exiting at C.