Theoretical Computer Science

Models of Computation

Lecture 2 - Manuel Mazzara

Machines and Grammars

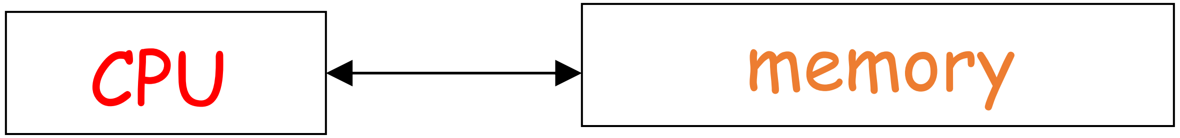
1. Computation is elegantly **<u>modeled</u>** through simple mathematical objects
   - Finite automata, pushdown automata, Turing machines, ...

2. Methods of **<u>generating languages</u>**: regular expressions, grammars…

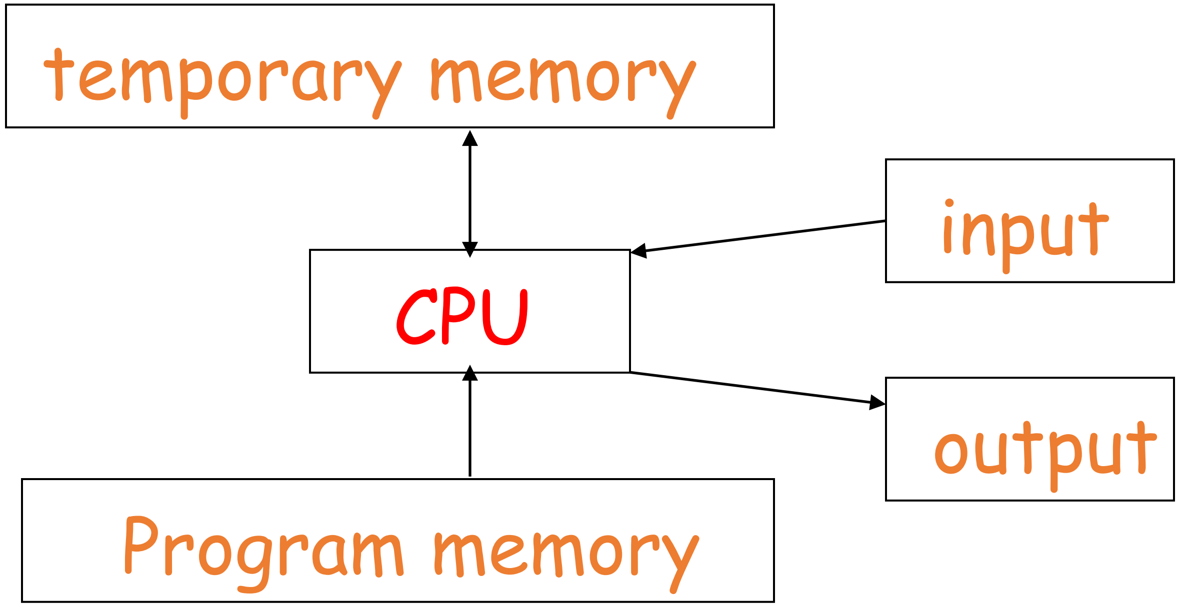
3. <u>Computability</u> theory

Models of Computation

Computation

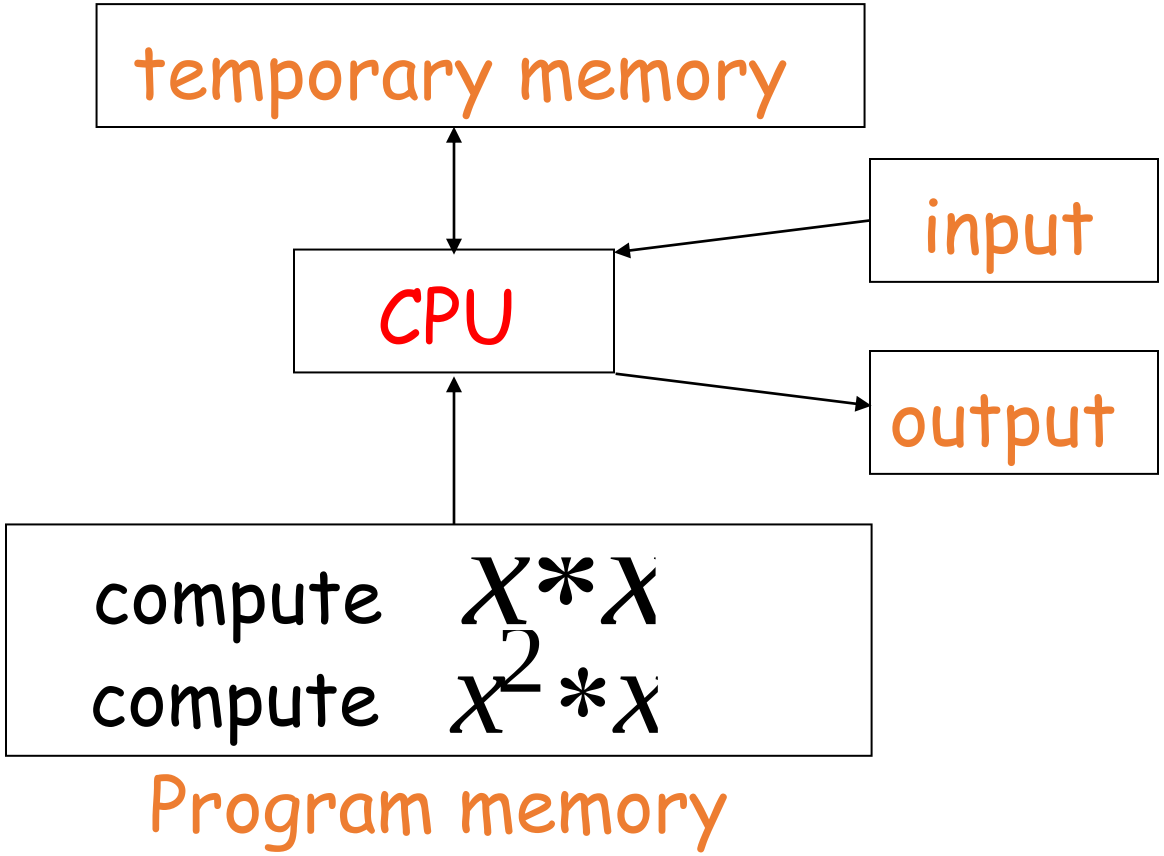


CPU, Memory, I/O



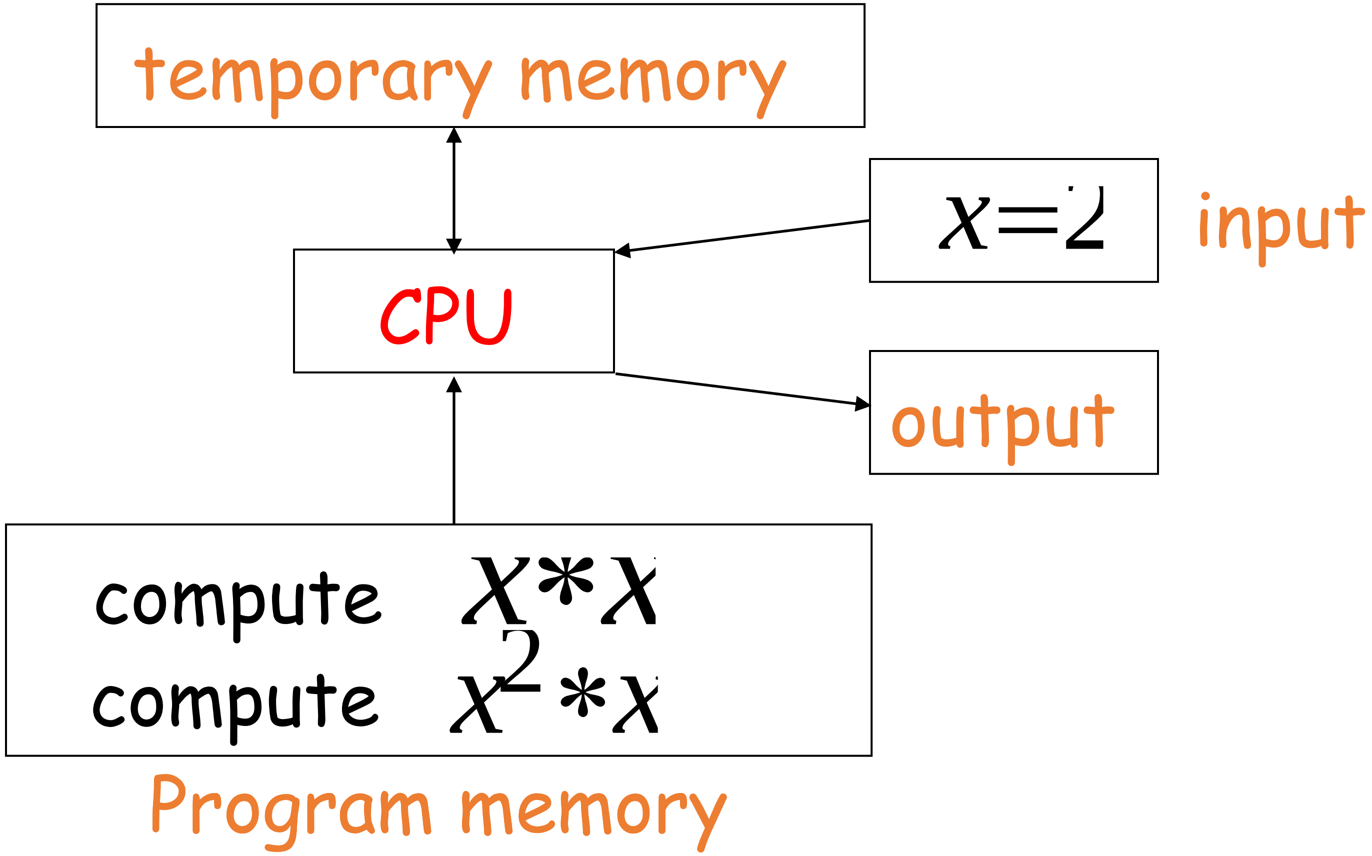
Example (1)

$$f(x)=x^3$$



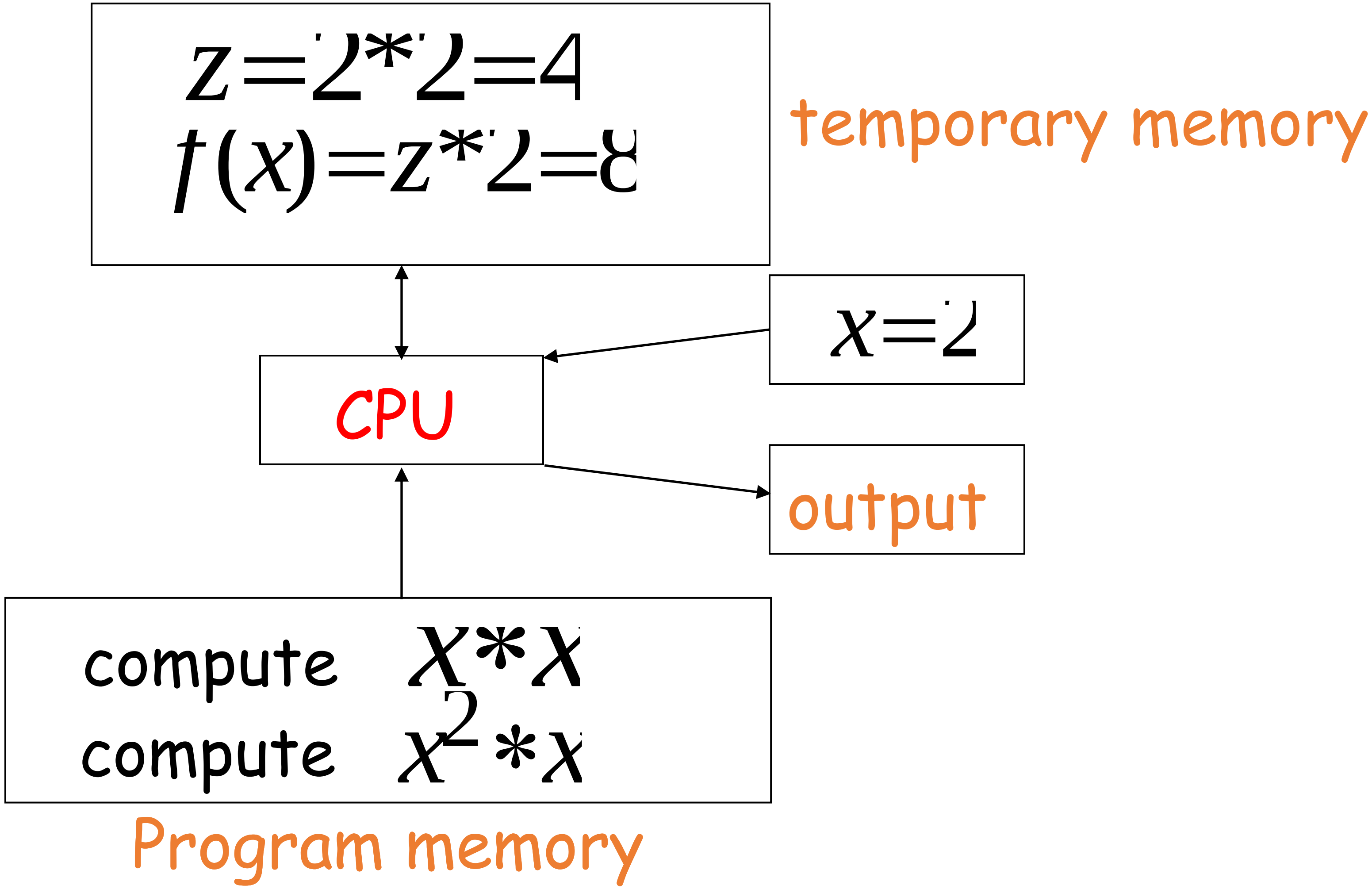
Example (2)

$$f(x) = x^3$$



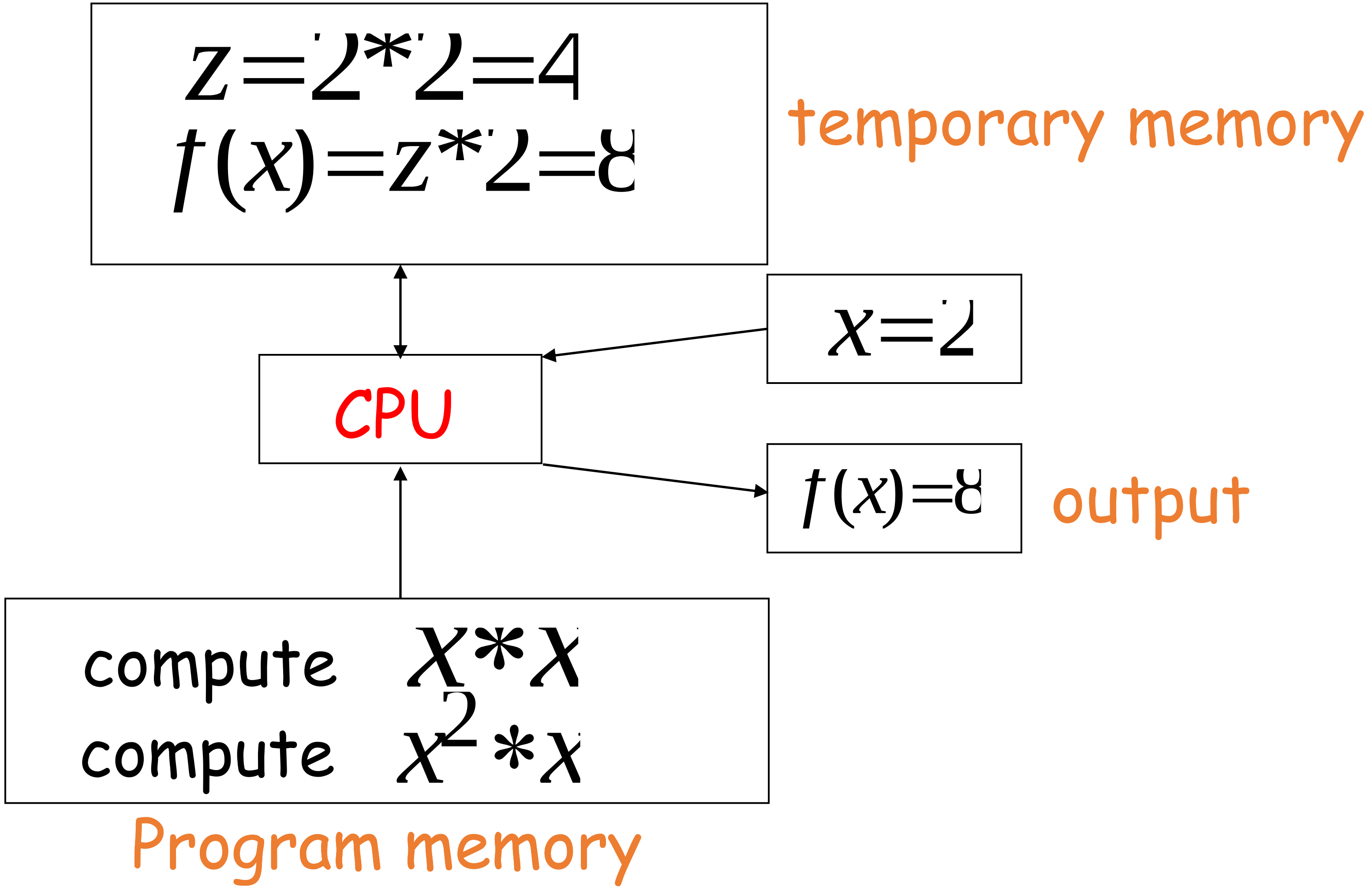
Example (3)

$$f(x)=x^3$$

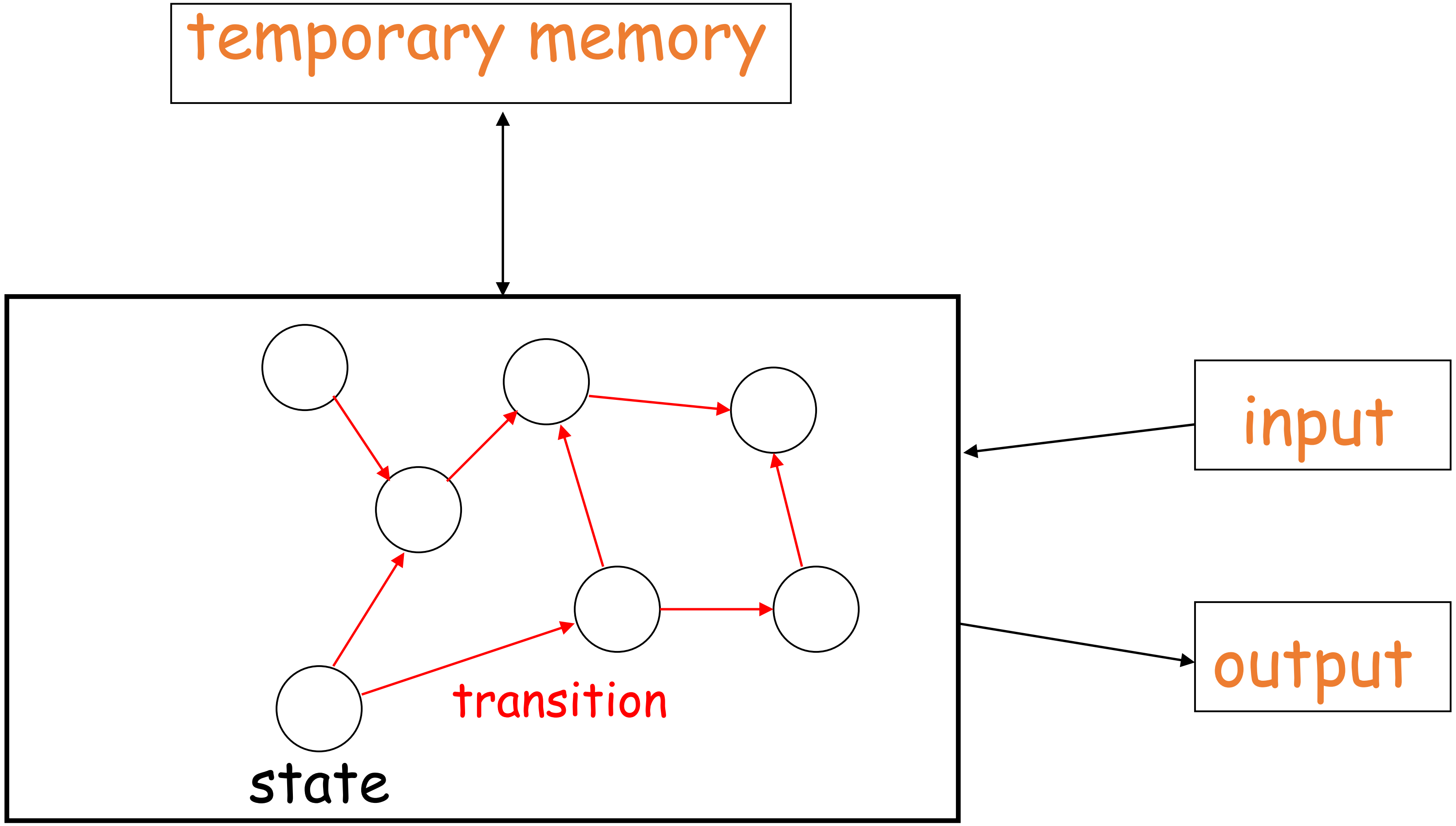


Example (4)

$$f(x)=x^3$$



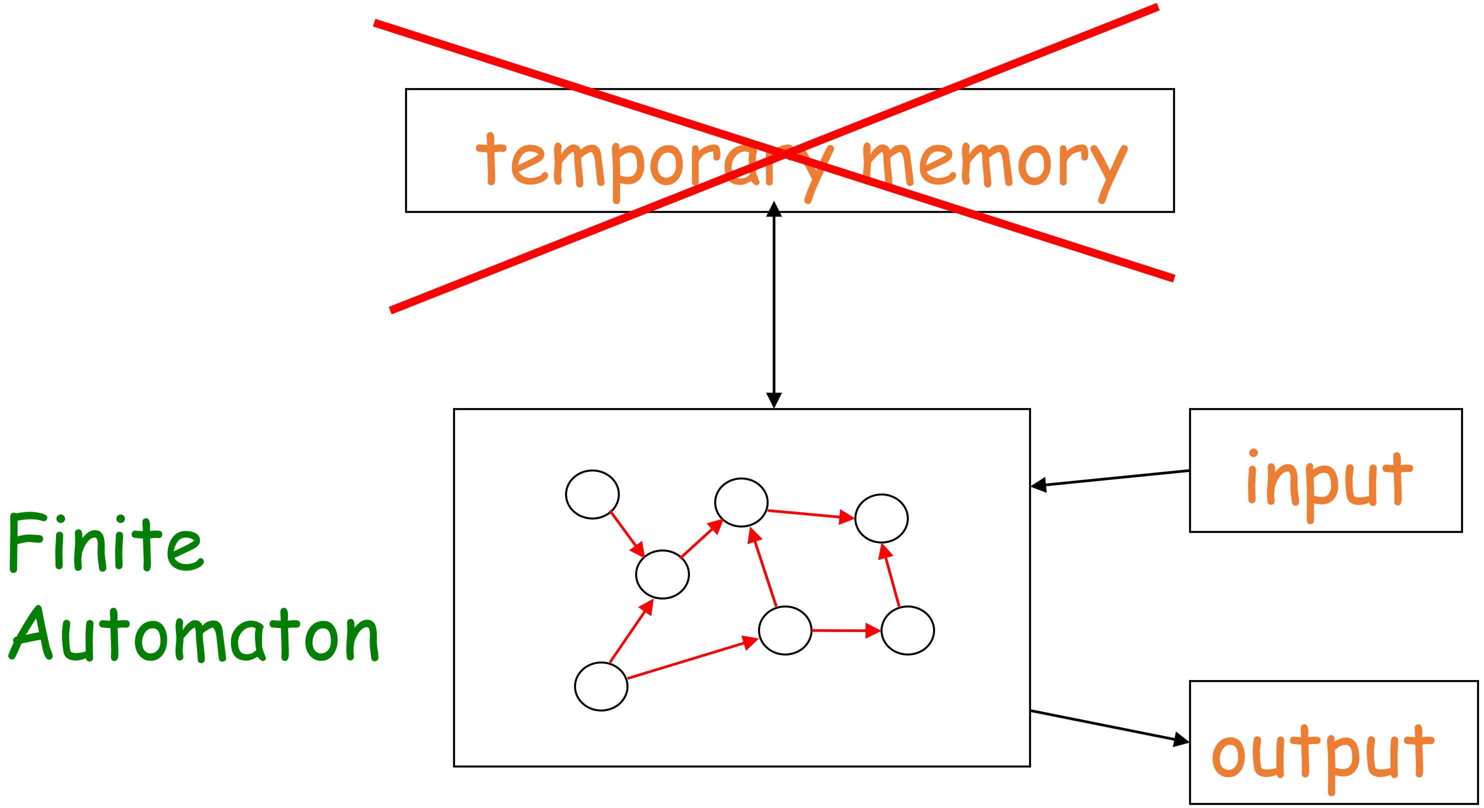
Automaton



Different kind of Automata

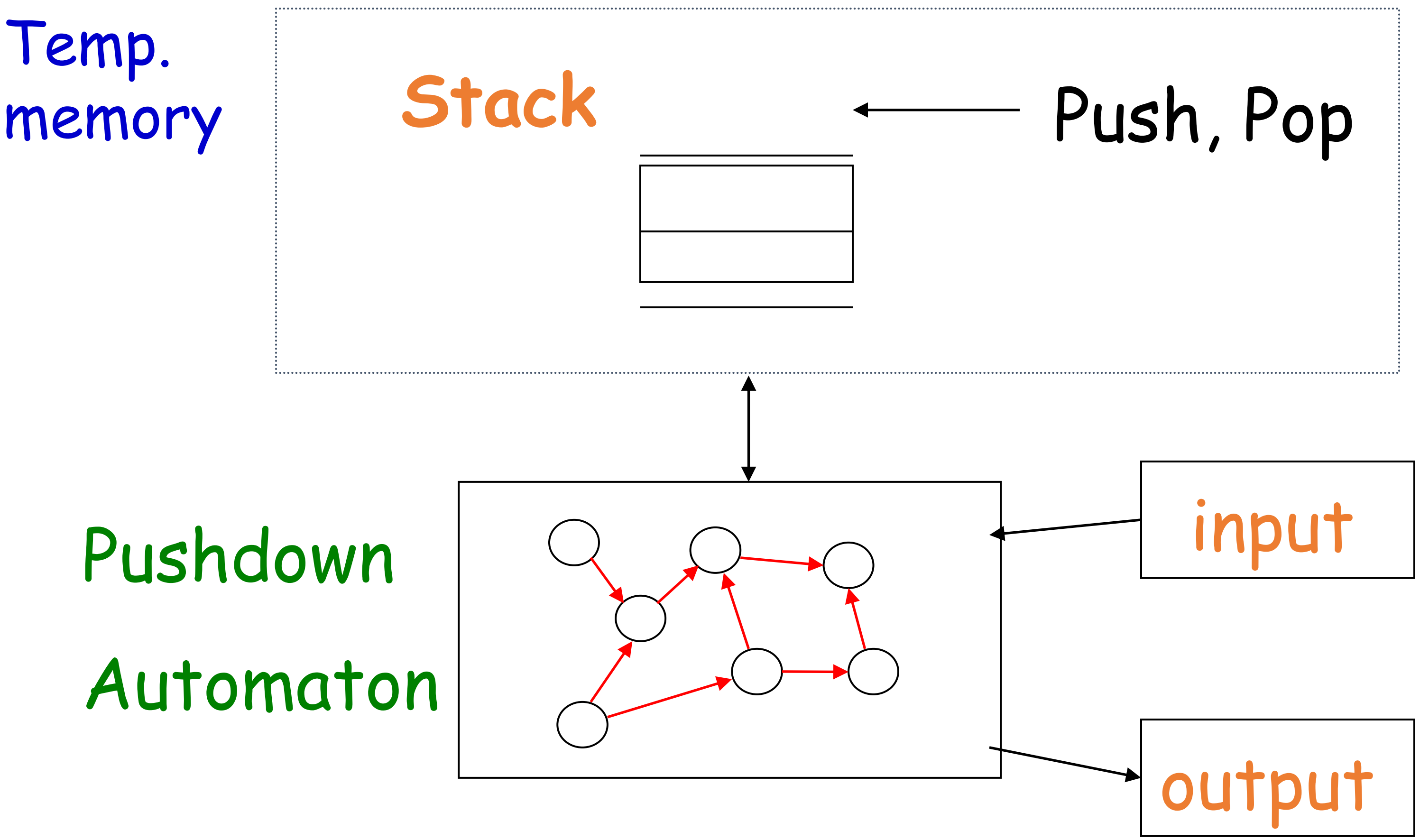
- Finite State Automata (**FSA**)
  - **no temporary memory, just states are used to memorize**
- Pushdown Automata (**PDA**)
  - **stack (destructive memory), need to destroy while reading**
- Turing Machines (**TMs**)
  - **equivalent to random (non-sequential) access memory**
  - In fact, it is sequential, but does not change the computational power

FSA



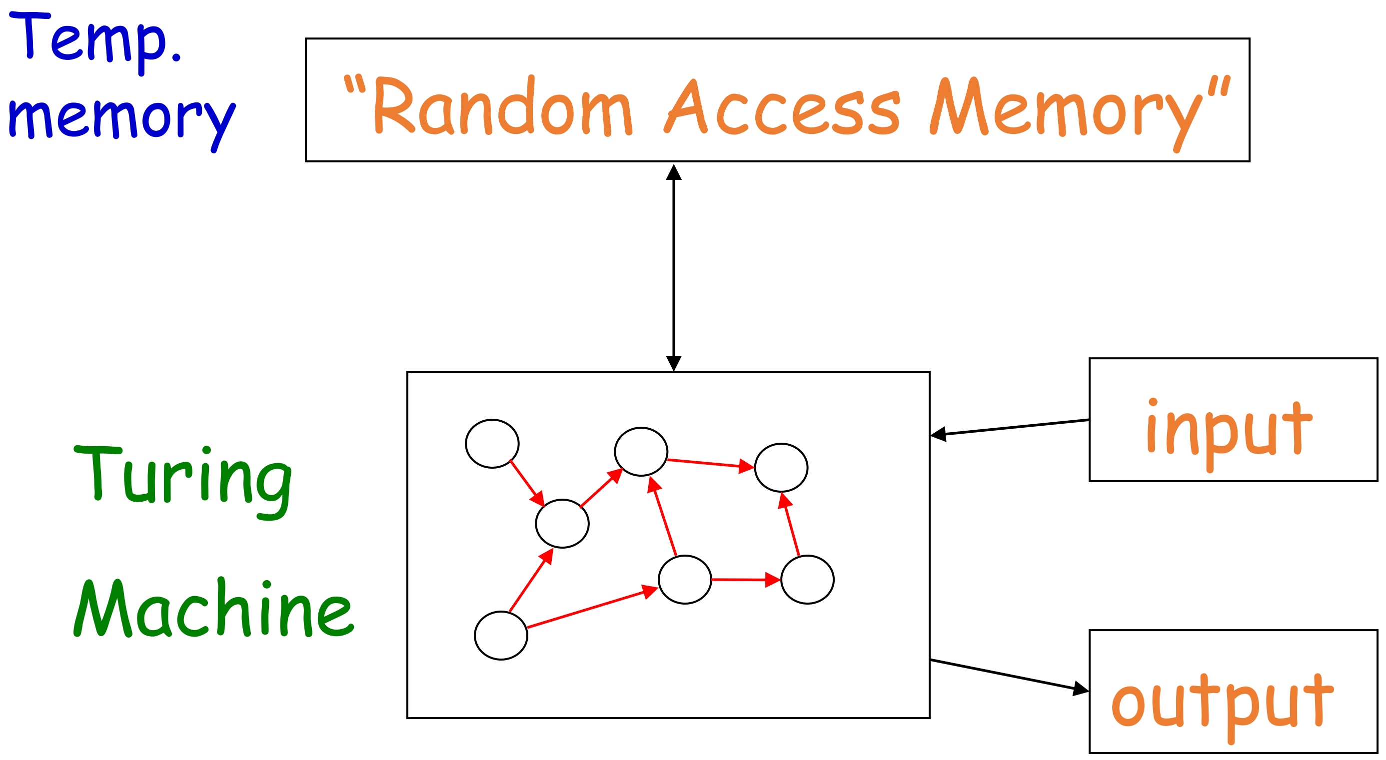
Example: Elevators, Vending Machines ("small" computing power)

PDA



Example: Compilers for Programming Languages (“medium” computing power)

TM



Any Algorithm ("highest" known computing power)

Power of Automata

Simple problems	More complex problems	Hardest problems
Finite Automata <	Pushdown Automata <	Turing Machine

Less power → More power

Solve more computational problems

A course-long question

- **Turing Machine** is the most powerful computational model known
- Are there computational problems that a Turing Machine cannot solve?
- The Answer is “yes” (unsolvable problems)
- **There are indeed unsolvable problems**, and we will see in detail what it means

Theoretical Computer Science

About Theoretical Computer Science

Lecture 2 - Manuel Mazzara

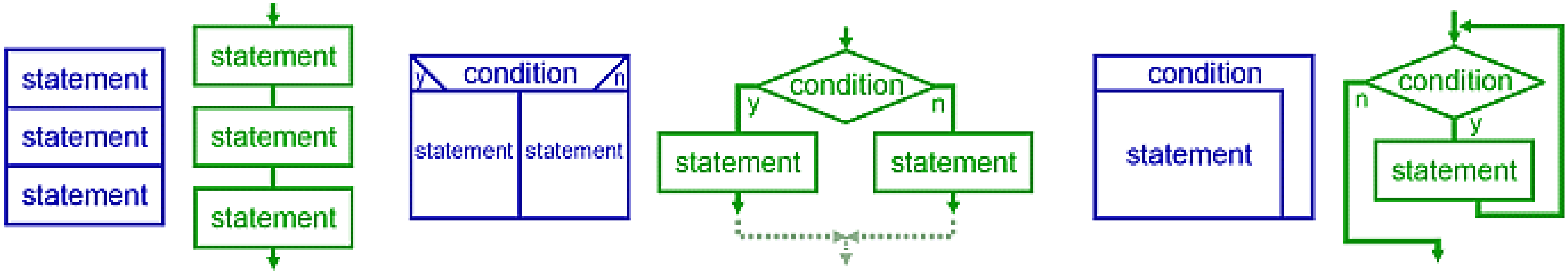
Edsger Wybe Dijkstra
11 May 1930 – 6 August 2002

- **Structured Programming**
- Software Engineering
- Concurrent and Distributed Computing
    - Semaphores
    - Mutual exclusion
    - Deadlock

- *Solution of a Problem in Concurrent Programming Control* - E.W. Dijkstra, Communications of the ACM, Vol. 8 , No. 9, p. 569, **1965**

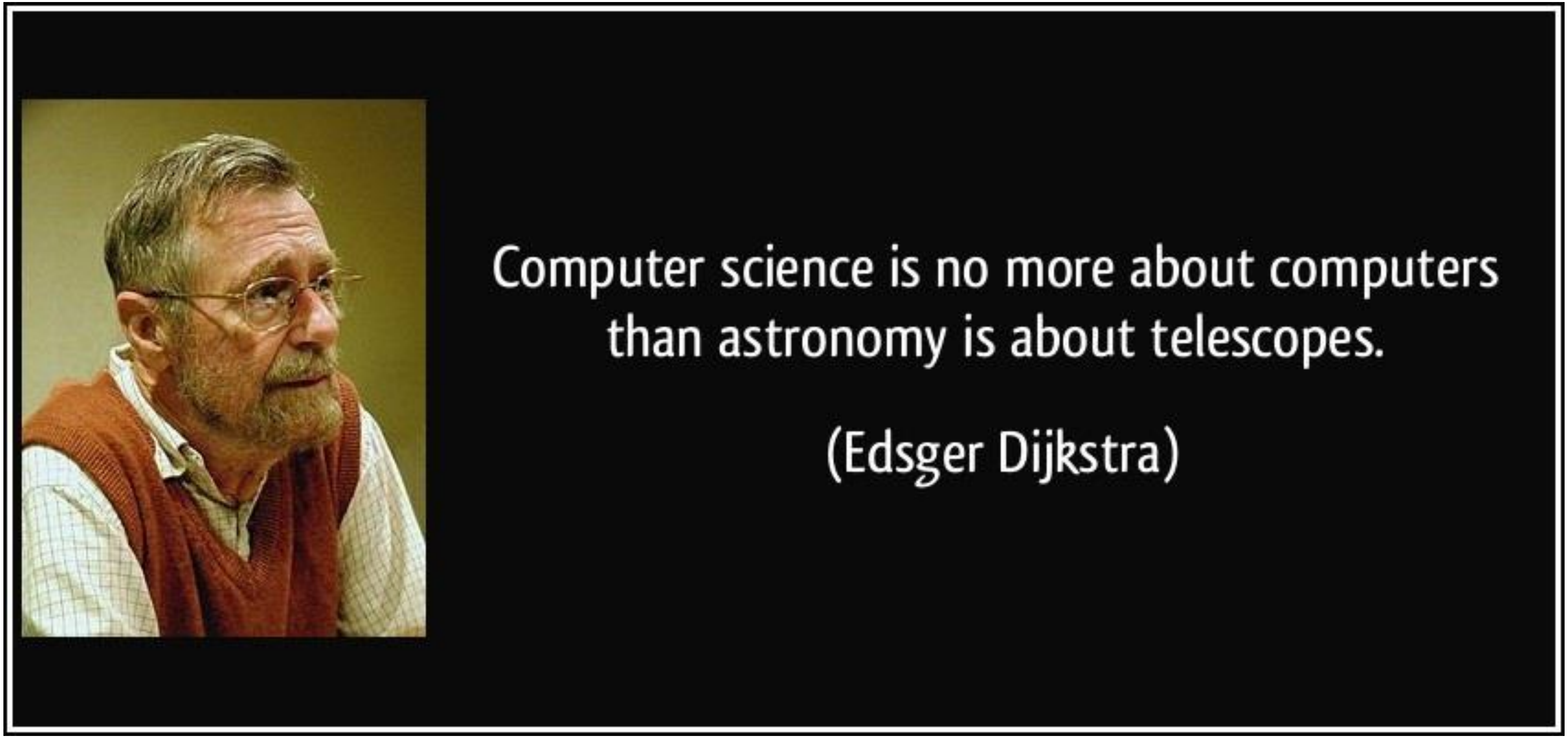
*"The **revolution** in views of programming started by Dijkstra's iconoclasm led to a movement known as **structured programming**, which advocated a **systematic, rational approach to program construction**. Structured programming is **the basis for all that has been done since in programming methodology**, including object-oriented programming."*

Bertrand Meyer - Touch of Class (page 188)

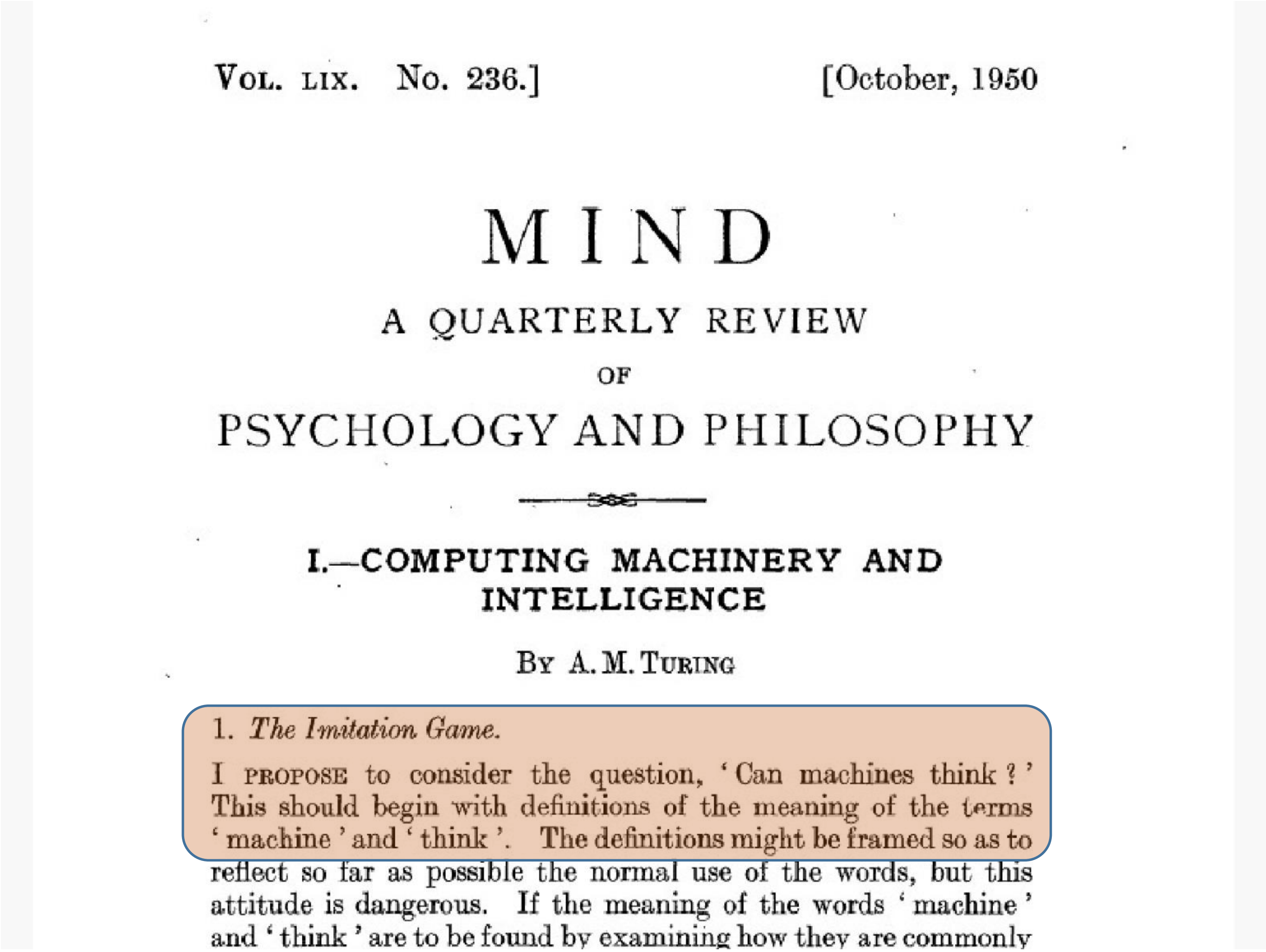


Structured programming

- **Structured programming** is a programming paradigm aimed at improving the **clarity**, **quality**, **development and maintenance time** of a computer program by making use of the **structured control flow constructs** of selection (if/then/else) and repetition (while and for), block structures, and subroutines.


Computer science is no more about computers
than astronomy is about telescopes.
(Edsger Dijkstra)

Seminal Turing's article on AI



VOL. LIX. No. 236.] [October, 1950

MIND

A QUARTERLY REVIEW

OF

PSYCHOLOGY AND PHILOSOPHY

I.—COMPUTING MACHINERY AND INTELLIGENCE

BY A. M. TURING

1. *The Imitation Game.*

I PROPOSE to consider the question, 'Can machines think ?' This should begin with definitions of the meaning of the terms 'machine' and 'think'. The definitions might be framed so as to reflect so far as possible the normal use of the words, but this attitude is dangerous. If the meaning of the words 'machine' and 'think' are to be found by examining how they are commonly

“The question of whether a computer can think is no more interesting than the question of whether a submarine can swim.”

— Edsger W. Dijkstra

Theoretical Computer Science

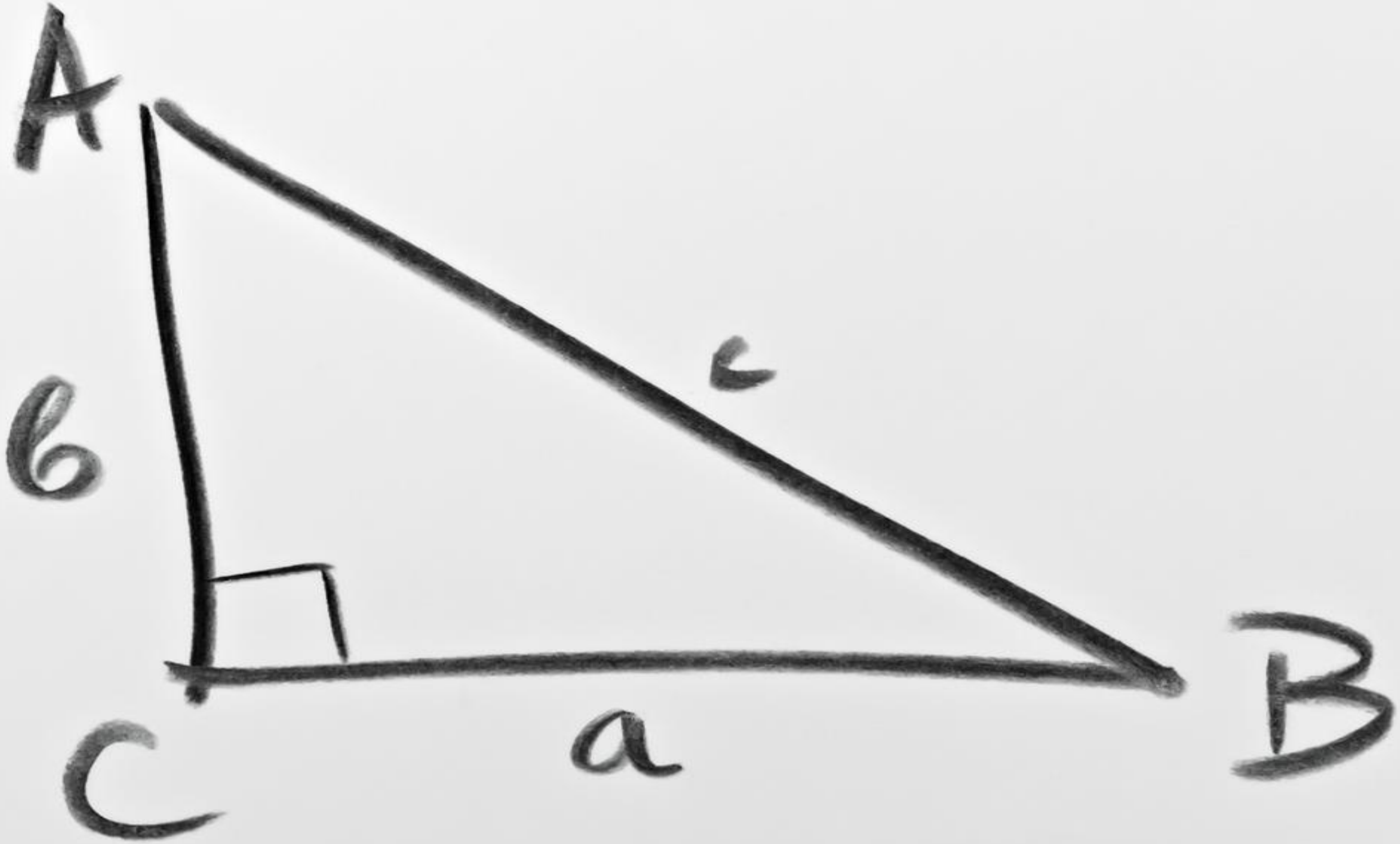
Models and Abstractions

Lecture 2 - Manuel Mazzara

WHAT IS A SCIENTIFIC MODEL?

scientific model

a representation of a particular phenomena in the world



$$c^2 = a^2 + b^2$$

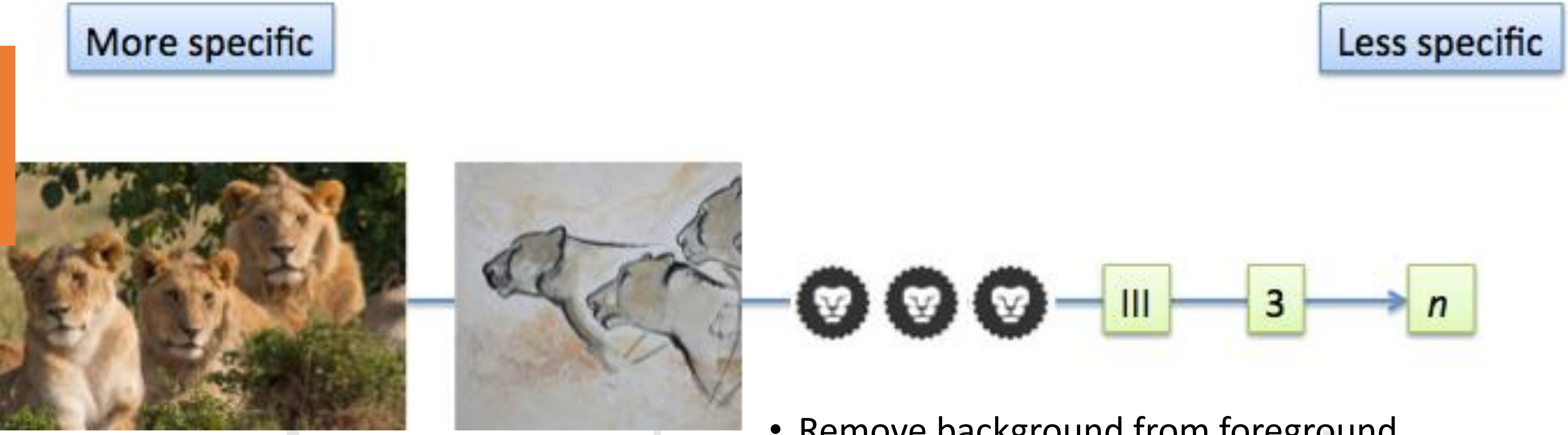
Rickmansworth
Moor Park
Northwood
Northwood Hills
Pinner
North Harrow
Harrow & Wealdstone
Kenton
Northwick Park
West Ruislip
Hillingdon
Ruislip Manor
Rayners Lane
Uxbridge
Ickenham
Ruislip
Eastcote
West Harrow
Harrow on-the-Hill
Preston Road
Ruislip Gardens
South Ruislip
South Harrow
Sudbury Hill
Sudbury Town
Alperton
Northolt
Greenford
Perivale
Hanger Lane
Park Royal
North Ealing
Ealing Broadway
West Acton
North Acton
East Acton
Ealing Common
South Ealing
Northfields
Boston Manor
Osterley
Hounslow East
Hounslow Central
Hounslow West
Hatton Cross
Heathrow Terminals 1, 2, 3
Heathrow Terminal 4
Acton Town
Chiswick Park
Turnham Green
Stamford Brook
Ravenscourt Park
Gunnersbury
Kew Gardens
Richmond
South Kenton
North Wembley
Wembley Central
Stonebridge Park
Harlesden
Willesden Junction
Kensal Green
Queen's Park
Kilburn Park
Maida Vale
Warwick Avenue
Royal Oak
Westbourne Park
Ladbroke Grove
Latimer Road
White City
Holland Park
Shepherd's Bush
Goldhawk Road
Hammersmith
Barons Court
West Kensington
Kensington (Olympia)
Earl's Court
West Brompton
Fulham Broadway
Parsons Green
Putney Bridge
East Putney
Southfields
Wimbledon Park
Canons Park
Queensbury
Kingsbury
Wembley Park
Neasden
Dollis Hill
Willesden Green
Kilburn
West Hampstead
Finchley Road
Swiss Cottage
St. John's Wood
Burnt Oak
Colindale
Hendon Central
Brent Cross
Golders Green
Hampstead
Belsize Park
Chalk Farm
Camden Town
Mornington Crescent
Euston
Edgware Road
Marylebone
Great Portland Street
Paddington
Edgware Road
Baker Street
Bayswater
Queensway
Marble Arch
Notting Hill Gate
Lancaster Gate
Bond Street
High Street Kensington
Hyde Park Corner
Knightsbridge
Green Park
Gloucester Road
South Kensington
Sloane Square
Victoria
Pimlico
Vauxhall
St. James's Park
Westminster
Regent's Park
Oxford Circus
Euston Square
Warren Street
Goodge Street
Tottenham Court Road
Piccadilly Circus
Leicester Square
Covent Garden
Charing Cross
Embankment
Charing Cross
Waterloo
Lambeth North
Kennington
Oval
Stockwell
Brixton
Clapham North
Clapham Common
Clapham South
Balham
Tooting Bec
Tooting Broadway
West Finchley
Finchley Central
East Finchley
Highgate
Archway
Tufnell Park
Kentish Town
King's Cross St. Pancras
Caledonian Road
Holloway Road
Arsenal
Russell Square
Holborn
Aldwych
Angel
Farringdon
Barbican
St. Paul's
Chancery Lane
Blackfriars
Temple
Mansion House
Cannon Street
Monument for Bank
Bank
Moorgate
Liverpool Street
Aldgate
Tower Hill
Fenchurch Street
London Bridge
Borough
Elephant & Castle
Arnos Grove
Bounds Green
Wood Green
Turnpike Lane
Manor House
Finsbury Park
Seven Sisters
Tottenham Hale
Walthamstow Central
Blackhorse Road
Drayton Park
Highbury & Islington
Essex Road
Old Street
G.N. Electrics
Docklands Light Railway
Waterloo & City Line
Buckhurst Hill
Roding Valley
Chigwell
Grange Hill
Woodford
No Service after 20 00
Hainault
Fairlop
Barkingside
Newbury Park
South Woodford
Snaresbrook
Redbridge
Wanstead
Gants Hill
Leytonstone
Leyton
Stratford
Upminster
Upminster Bridge
Hornchurch
Elm Park
Dagenham East
Dagenham Heathway
Becontree
Upney
Barking
East Ham
Upton Park
Plaistow
West Ham
Bromley-by-Bow
Bow Road
Mile End
Bethnal Green
Shoreditch
Stepney Green
Whitechapel
Aldgate East
Shadwell
Wapping
Rotherhithe
Surrey Docks
New Cross Gate
New Cross
River Thames
UNDERGROUND
Daily Services
Peak Hour and Special Services
Bakerloo Line
peak hours
Central Line
peak hours
Circle Line
Jubilee Line
Metropolitan Line
peak hours
East London Line
Piccadilly Line
peak hours
Victoria Line
British Rail
Interchange with other lines
Interchange with British Rail
Interchange with British Rail within walking distance
Closed Sundays
Closed Saturdays and Sundays
Served by Piccadilly Line early mornings and late evenings Mondays to Saturdays and all

Mathematical abstractions

- Representing real systems
  - **Abstraction allows you to focus on the important aspects of a problem**

- **Formal reasoning** can improve our ability to design and build systems
  - **Uncover design flaws**
  - Precisely **define requirements**
  - Mathematics allows you to **reason about solutions** to the problem

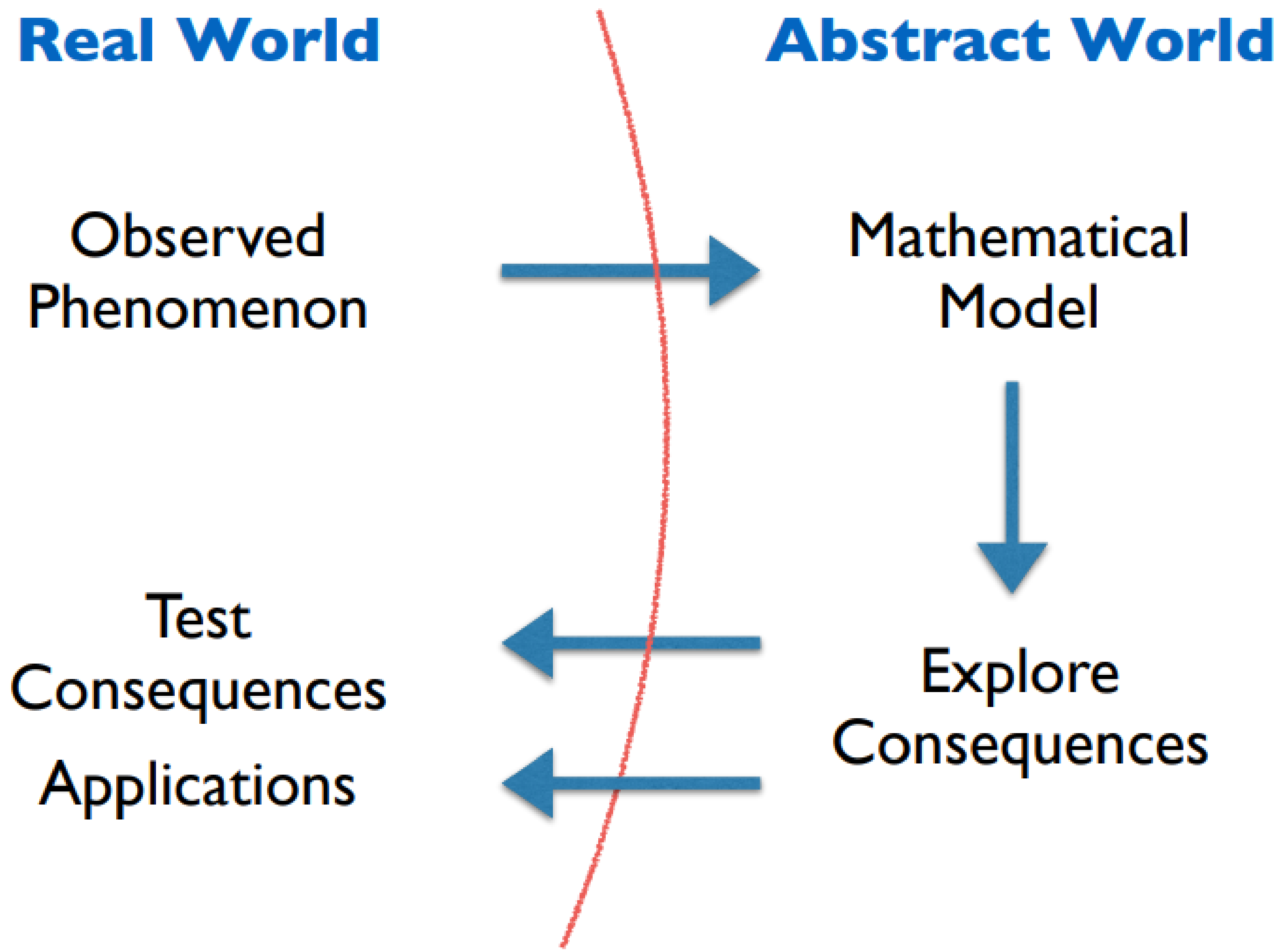
- Different models have different strengths and weaknesses



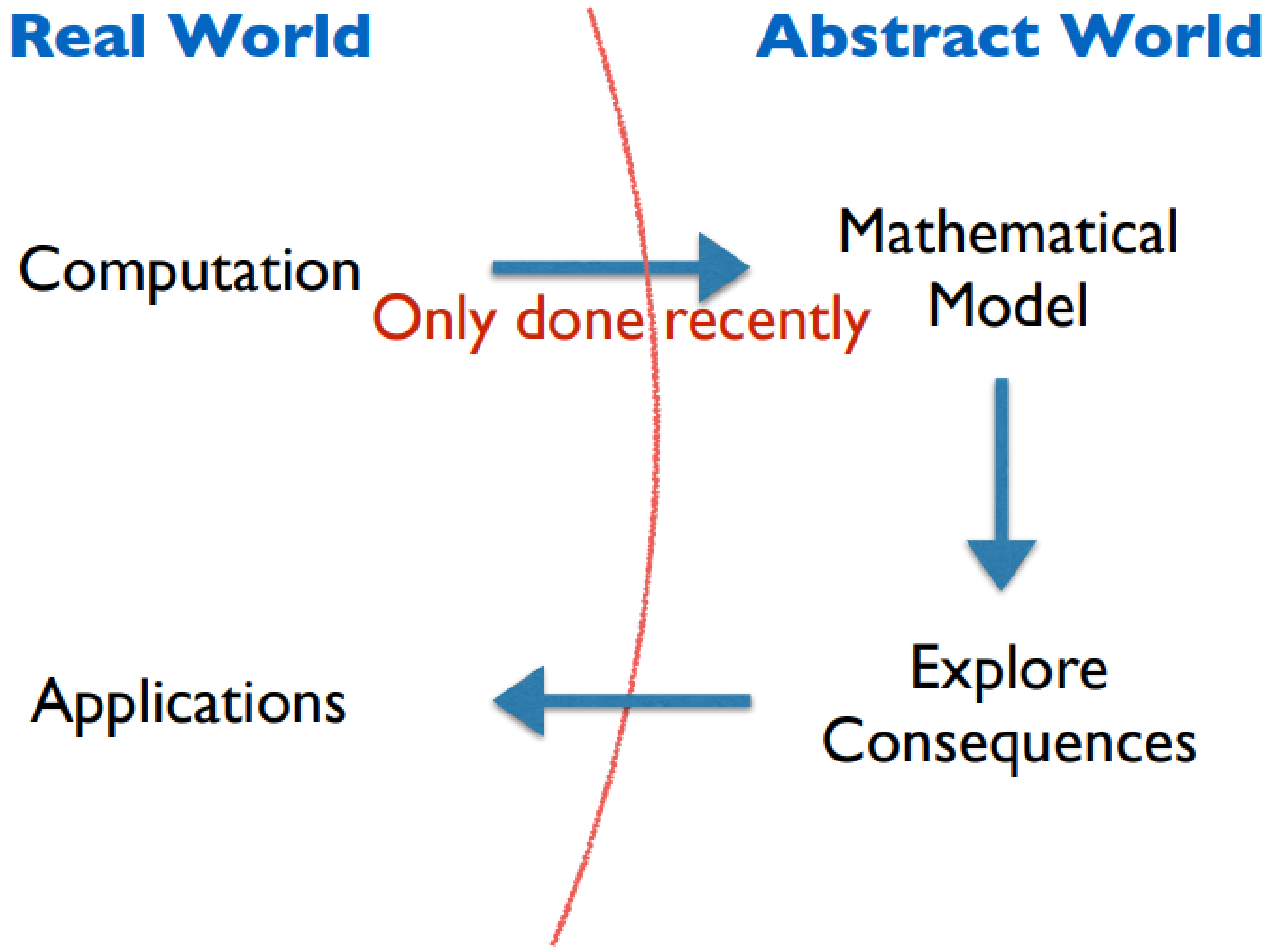
Abstraction

- Remove background from foreground
- Remove differences between each animal
- Remove "animal-ness" (treat lions as generic "lines")
- Remove need to count objects with literal lines
- Remove need to specify a fixed number

The role of theoretical physics

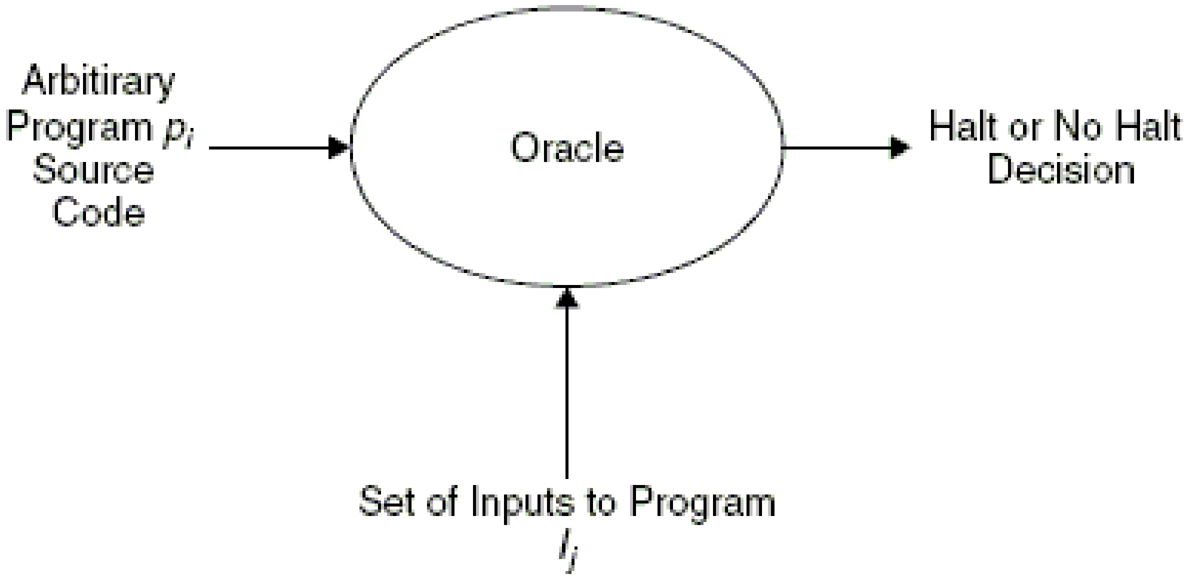


The role of theoretical computer science



Our big question

- Is every function computable?
  - Can I write an algorithm for any function N→N?
  - **Halting Problem**



Theoretical Computer Science

Introduction to Formal Languages

Lecture 2 - Manuel Mazzara

Big “existential” question (1)

- Do you know anything in nature with an infinite set of “bricks” non determined a-priori?
  - Think about chemistry, physics and and biology!

- Everything, including life seems to be expressed by **building bricks**, finite in nature and **pre-determined**, that do not change over time, and express complexity trough *“combinatorial explosion”*

- ***Can you falsify this statement?***

Periodic Table of the Elements

Atomic Number	Boiling Point
Symbol	
Name	
Atomic Mass	

Normal boiling points are in °C.
SP = Triple Point
Pressure is listed if not 1 atm.
Allotrope is listed if more than one allotrope.

1 IA 1A	2 IIA 2A	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8	9 VIII 8	10	11 IB 1B	12 IIB 2B	13 IIIA 3A	14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	18 VIIIA 8A
1 -252.762 H Hydrogen 1.008																	2 -268.93 He Helium 4.003
3 1342 Li Lithium 6.941	4 2471 Be Beryllium 9.012											5 4000 B Boron 10.811	6 graphite 3825 SP C Carbon 12.011	7 -195.798 N Nitrogen 14.007	8 -182.953 O Oxygen 15.999	9 -188.12 F Fluorine 18.998	10 -246.053 Ne Neon 20.180
11 882.940 Na Sodium 22.990	12 1090 Mg Magnesium 24.305											13 2519 Al Aluminum 26.982	14 3265 Si Silicon 28.086	15 white 280.5 P Phosphorus 30.974	16 444.61 S Sulfur 32.066	17 -101.5 Cl Chlorine 35.453	18 -185.847 Ar Argon 39.948
19 759 K Potassium 39.098	20 1484 Ca Calcium 40.078	21 2836 Sc Scandium 44.956	22 3287 Ti Titanium 47.88	23 3407 V Vanadium 50.942	24 2671 Cr Chromium 51.996	25 2061 Mn Manganese 54.938	26 2861 Fe Iron 55.933	27 2927 Co Cobalt 58.933	28 2913 Ni Nickel 58.693	29 2562 Cu Copper 63.546	30 907 Zn Zinc 65.39	31 2204 Ga Gallium 69.732	32 2833 Ge Germanium 72.61	33 616 SP As Arsenic 74.922	34 685 Se Selenium 78.972	35 58.8 Br Bromine 79.904	36 -153.34 Kr Krypton 84.80
37 688 Rb Rubidium 84.468	38 1382 Sr Strontium 87.62	39 3345 Y Yttrium 88.906	40 4409 Zr Zirconium 91.224	41 4744 Nb Niobium 92.906	42 4639 Mo Molybdenum 95.95	43 4265 Tc Technetium 98.907	44 4150 Ru Ruthenium 101.07	45 3695 Rh Rhodium 102.906	46 2963 Pd Palladium 106.42	47 2162 Ag Silver 107.868	48 767 Cd Cadmium 112.411	49 2072 In Indium 114.818	50 2602 Sn Tin 118.71	51 1587 Sb Antimony 121.760	52 988 Te Tellurium 127.6	53 184.4 I Iodine 126.904	54 -108.09 Xe Xenon 131.29
55 671 Cs Cesium 132.905	56 1897 Ba Barium 137.327	57-71	72 4603 Hf Hafnium 178.49	73 5458 Ta Tantalum 180.948	74 5555 W Tungsten 183.85	75 5596 Re Rhenium 186.207	76 5012 Os Osmium 190.23	77 4428 Ir Iridium 192.22	78 3825 Pt Platinum 195.08	79 2856 Au Gold 196.967	80 356.62 Hg Mercury 200.59	81 1473 Tl Thallium 204.383	82 1749 Pb Lead 207.2	83 1564 Bi Bismuth 208.980	84 962 Po Polonium [208.982]	85 337 At Astatine 209.987	86 -61.7 Rn Radon 222.018
87 677 Fr Francium 223.020	88 1737 Ra Radium 226.025	89-103	104 unknown Rf Rutherfordium [261]	105 unknown Db Dubnium [262]	106 unknown Sg Seaborgium [266]	107 unknown Bh Bohrium [264]	108 unknown Hs Hassium [269]	109 unknown Mt Meitnerium [268]	110 unknown Ds Darmstadtium [269]	111 unknown Rg Roentgenium [272]	112 unknown Cn Copernicium [277]	113 unknown Uut Ununtrium unknown	114 unknown Fl Flerovium [289]	115 unknown Uup Ununpentium unknown	116 unknown Lv Livermorium [298]	117 unknown Uus Ununseptium unknown	118 unknown Uuo Ununoctium unknown

Lanthanide Series	57 3464 La Lanthanum 138.906	58 3443 Ce Cerium 140.115	59 3520 Pr Praseodymium 140.908	60 3074 Nd Neodymium 144.24	61 3000 Pm Promethium 144.913	62 1794 Sm Samarium 150.36	63 1529 Eu Europium 151.966	64 3273 Gd Gadolinium 157.25	65 3230 Tb Terbium 158.925	66 2567 Dy Dysprosium 162.50	67 2700 Ho Holmium 164.930	68 2868 Er Erbium 167.26	69 1950 Tm Thulium 168.934	70 1196 Yb Ytterbium 173.04	71 3402 Lu Lutetium 174.967
Actinide Series	89 3198 Ac Actinium 227.028	90 4788 Th Thorium 232.038	91 4027 Pa Protactinium 231.036	92 4131 U Uranium 238.029	93 4174 Np Neptunium 237.048	94 3228 Pu Plutonium 244.064	95 2011 Am Americium 243.061	96 3100 Cm Curium 247.070	97 2627 Bk Berkelium 247.070	98 unknown Cf Californium 251.080	99 unknown Es Einsteinium [254]	100 unknown Fm Fermium 257.095	101 unknown Md Mendelevium 258.1	102 unknown No Nobelium 259.101	103 unknown Lr Lawrencium [262]

Alkali Metal | **Alkaline Earth** | **Transition Metal** | **Basic Metal** | **Semimetal** | **Nonmetal** | **Halogen** | **Noble Gas** | **Lanthanide** | **Actinide**

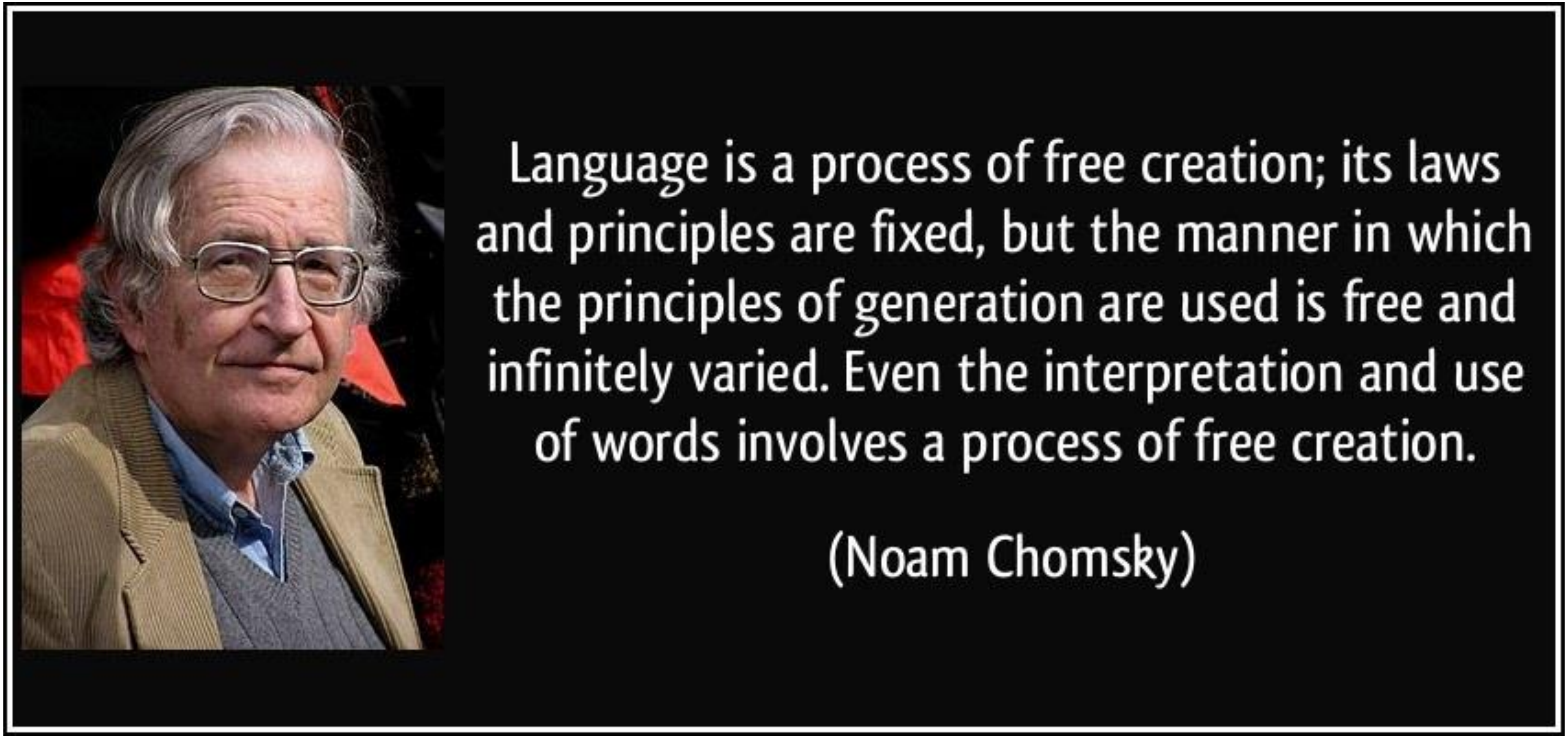
Big “existential” question (2)

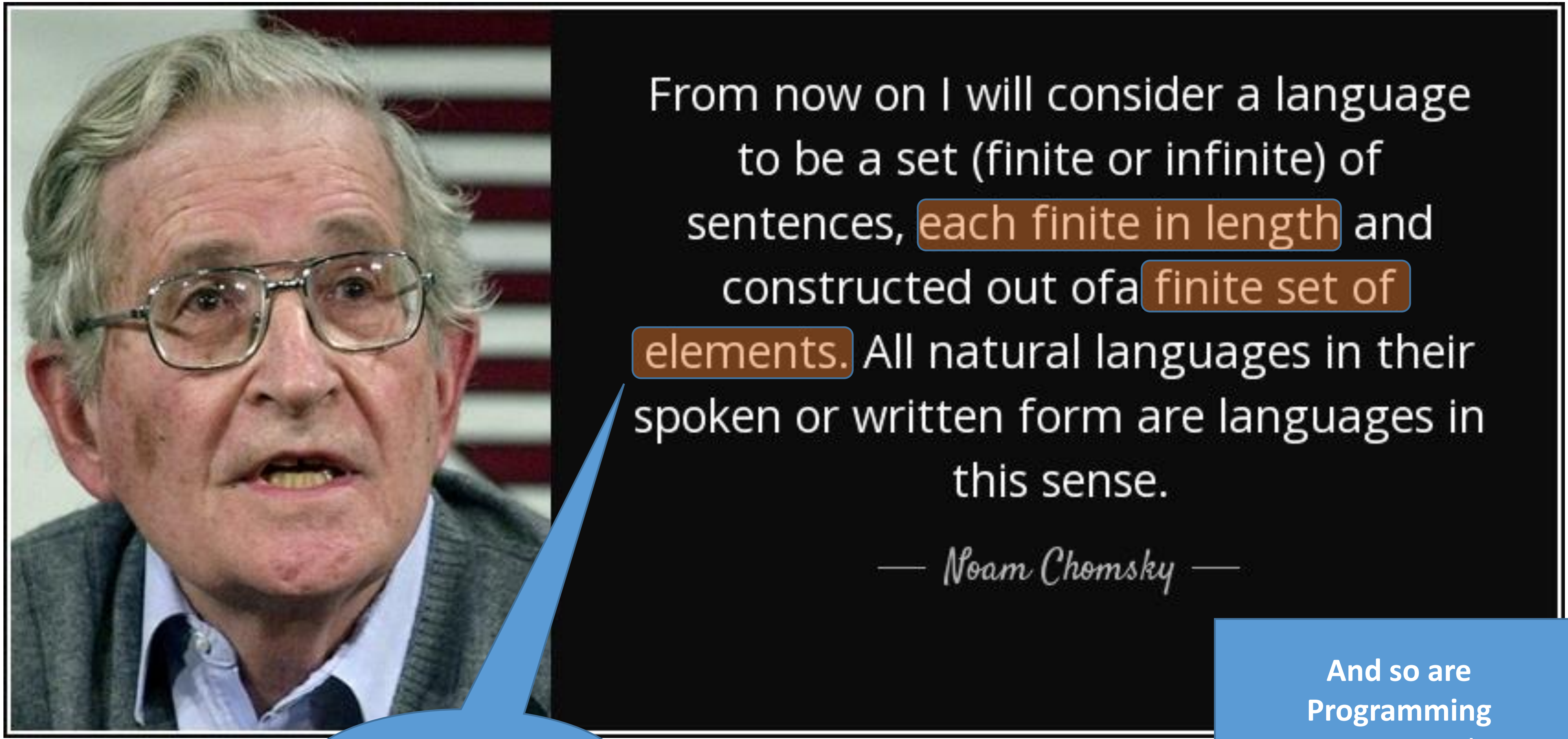
- Everything, including life seems to be expressed by **building bricks, finite in nature and pre-determined**, that do not change over time, and **express complexity trough combinatorial explosion**

- Is brain functioning any different?

- **Language** is not different!

- **Computation** is not different!


Language is a process of free creation; its laws and principles are fixed, but the manner in which the principles of generation are used is free and infinitely varied. Even the interpretation and use of words involves a process of free creation.
(Noam Chomsky)

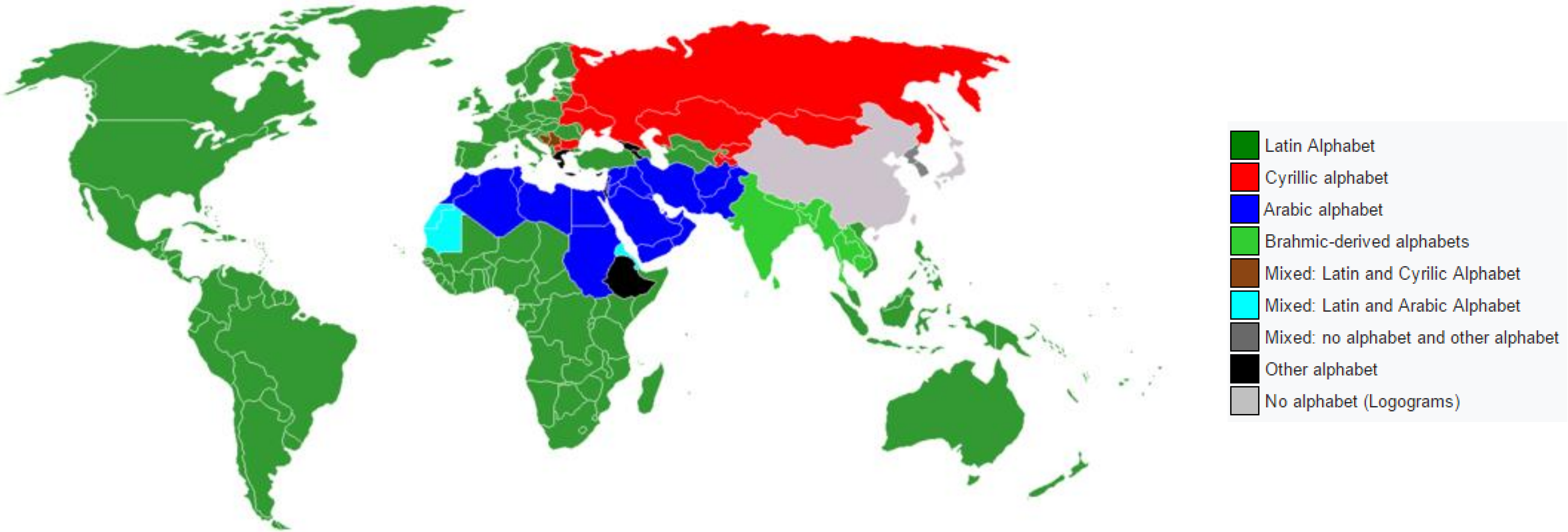


An Alphabet!

And so are Programming Languages!

What is an alphabet?

Natural languages alphabets



Elements of languages

- **Alphabet** or vocabulary
  - Finite set of **basic symbols**
  - Examples:
    - Roman alphabet $\{$a, b, …, z$\}$
    - Digits $\{$0, 1, …, 9$\}$
    - Binary alphabet $\{$0, 1$\}$
- **String** over an alphabet **A**
  - **Finite sequence of symbols** of the alphabet **A**
  - Repetitions are allowed

Examples

- Roman alphabet **A**=$\{$a, b, …, z$\}$
  - **a** is a string on **A**
  - **aa** is a string on **A**
  - **aba**, **add**, **aza**, … are strings on **A**
- Alphabet of digits **D**=$\{$0, 1, …, 9$\}$
  - **0**, **1**, **2**, …,**9** are strings over **D**
  - **012**, **999**, **923456**, … are strings over **D**

Length of a string

- The **length** of a string is the **number of symbols** contained in the string
  - We denote the length of a string **x** as **|x|**
- Examples:
  - $|a|= 1$
  - $|991346|=6$

- The **empty string** is a string that has zero symbols
  - We denote it as ε
  - $|\varepsilon|=0$

Comparing strings

- Two strings
  - $\mathbf{x}=x_1x_2\ldots x_n$
  - $\mathbf{y}=y_1y_2\ldots y_m$

  are **equal** if and only if
  - $|x|=|y|$ $(n=m)$ Same length
  - $x_i=y_i, \forall i\ (1\leq i\leq n)$ Corresponding elements are the same
- Examples
  - aabb and aabba are not equal
  - ababs and baasb are not equal

Concatenation

- Given two strings **x** and **y**, the **concatenation** (or product) of **x** and **y** is a string **xy** (or **x·y**), where **x** is followed by **y**
  - Example: strings on **A**={a, b, c, d}
    - x=abadd
    - y=dcc
    - xy=abadddcc
    - yx=dcc abadd
- Remarks
  - A string **x** concatenated with **ε** is still **x**
  - We abbreviate **xx** as $\mathbf{x}^2$, **xxx** as $\mathbf{x}^3$, …
  - Concatenation is associative and **non-commutative**

Substrings

- A string **x** is a **<u>substring</u>** (or a factor) of a string **s** if there exist two strings **y** and **z** such that **s**=**yxz**
  - y or z can be ε
    - If y= ε, x is called prefix
    - If z= ε, x is called suffix
  - If both y and z are ε, x is equal to s
- Example: s=aadabbc
  - aad is a prefix of s
  - abbc is a suffix of s
  - ada is a substring of s

Kleene Star

- The **Kleene star** is a **unary operator** that applies to a set of symbols or a set of strings
  - It is denoted as *
  - In algebra it is called the **free monoid** on a set
- If **A** is an alphabet, then **A*** is the **set of all strings over symbols in A**, including the **empty string**.
- Examples:
  - If A={a, b, c} then A*={ε, a, b, c, aa, ab, ac, ba, bb, bc, ca, …}
  - If B={0, 1} then B*={ε, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, …}

Do you remember what is a free monoid in abstract algebra (the study of algebraic structures)?

Free monoid

- A **monoid** is a set equipped with an **associative binary operation** and an **identity element**

- The **free monoid** on a set is the monoid whose elements are **all the finite sequences (or strings) of zero or more elements from that set**
  - **String concatenation** is the monoid operation
  - The unique sequence of zero elements, the empty string (denoted by ε or λ) is the **identity element** (it leaves any element of the set unchanged when combined with it)

- **The free monoid on a set A is usually denoted A^***

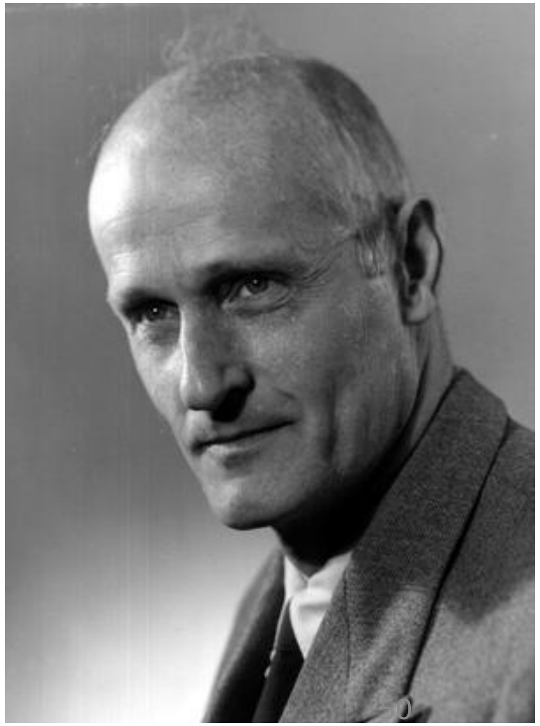
Stephen Kleene

- Kleene star is widely used for **regular expressions**

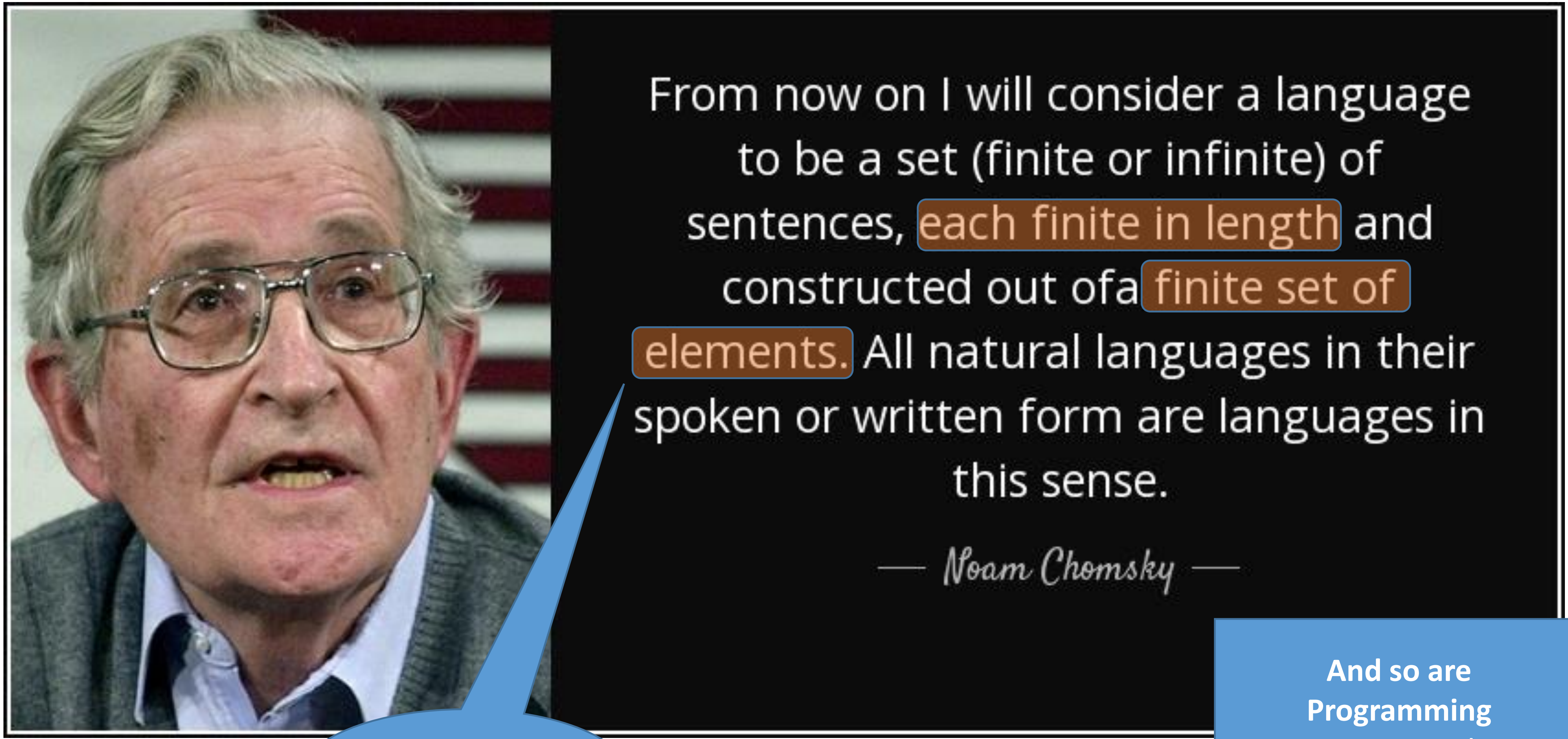
- It was introduced by **Stephen Kleene** in this context

- **Stephen Kleene** (1909-1994)
  - American mathematician
  - Student of **<u>Alonzo Church</u>**

Lambda calculus
Church-Turing thesis



What is a language?



An Alphabet!

And so are Programming Languages!

What map is this?

Languages

- A **language is a set of strings over an alphabet**
- Languages:
  - Russian, Italian, English, French
  - C, Java, Pascal, Eiffel

  but also
  - Graphical languages
  - Music
  - Multimedia

Formally

- A language **L** over an alphabet **A** is a **<u>subset</u>** of **A***
- Examples
  - $A=\{a, b, c\}$

    $A^*=\{\varepsilon, a, b, c, aa, ab, ac, ba, bb, bc, ca, \ldots\}$

    $L_1= \{\varepsilon, a, b, c, bc, ca\}$

    $L_2= \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$