Theoretical Mechanics HW3

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MEME

1 Task 1

1.1 Description

You should find an absolute velocity and coriolis acceleration, and absolute acceleration of particle M at the time $t = t_1$.

1. $$OM = s_r(t) = f_3(t) = 2t^3 + 3t \tag{1}$$

2. $$\phi(t) = \frac{1}{24}\pi t^2 \tag{2}$$

3. $t_1 = 2$;

4. $R = 15$;

5. Picture is given in Fig. 1.

1.2 Solution

Let's start by finding the position of object D and plane M at given point of time $t = 2$; Using equation (2), we get $\phi(2) = \frac{1}{24}\pi 2^2 = \frac{\pi}{6}$. Absolute velocity of point M is determined as a sum of *relative* and *transport* velocities.

$$\vec{v}_{abs} = \vec{v}_{rel} + \vec{v}_{tr} \tag{3}$$

$|\vec{v}_{rel}| = \dot{s}_r(t) = 6t^2 + 3$; At $t = 2$, $\vec{v}_{rel}| = 27$.

$|\vec{v}_{tr}| = R \cdot \omega$, where $\omega(t) = \dot{\phi}(t) = \frac{1}{12}\pi t$. At $t = 2$, $|\vec{v}_{tr}| = 2.5\pi$.

Now we have the scalar values of the velocities. Now let's find v_{abs} by OX and OY components:

$$\vec{v}_{abs} = \begin{bmatrix} -v_{rel} - v_{tr}\cos(60^\circ) \\ v_{tr}\sin(60^\circ) \end{bmatrix} = \begin{bmatrix} -30.93 \\ 6.8 \end{bmatrix} \tag{4}$$

As a result we get the value of $v_{abs} = 31.67$

Now let's try to find acceleration. Note, that there is no corriolis acceleration because body D is not rotating. It is making a translatory motion. M experiences corriolis acceleration when body D rotates. Absolute acceleration is found as:

$$\vec{a}_{abs} = \vec{a}_{rel} + \vec{a}_{tr} \tag{5}$$

As M is making rotational transport motion, we can divide it's transport acceleration into tangential and normal transport accelerations.

$$\vec{a}_{abs} = \vec{a}_{rel} + \vec{a}^{\tau}_{tr} + \vec{a}^{n}_{tr} \tag{6}$$

$|\vec{a}_{rel}(t = 2)| = \ddot{s}_r(t) = 12t$, when $t = 2$, $|\vec{a}_{rel}| = 24$.

$|\vec{a}^{\tau}_{tr}(t = 2)| = \epsilon R$, $\epsilon = \ddot{\phi}(t) = \frac{1}{12}\pi$.

So, $|\vec{a}^{\tau}_{tr}(t = 2)| = \frac{1}{12}\pi R = \frac{5}{4}\pi$;

$|\vec{a}^{n}_{tr}(t = 2)| = \omega(t = 2)^2 R = \frac{5}{12}\pi^2$

$$\vec{a}_{abs} = \begin{bmatrix} -a_{rel} - a^{n}_{tr}\cos(30°) - a^{\tau}_{tr}\cos(60°) \\ a^{\tau}_{tr}\sin(60°) - a^{n}_{tr}\sin(30°) \end{bmatrix} \tag{7}$$

As a result we get the value of $a_{abs} = 29.55$

1.3 Answers:

1. $v_{abs} = 31.67$
2. There is no corriolis acceleration because body D is not rotating. It is making a translatory motion.
3. $a_{abs} = 29.55$

2 Task 2

2.1 Description

You should find:

1. simulate this mechanism (obtain all positions);
2. find absolute, transport and relative velocities and accelerations for M;
3. find t, when M reaches O point;

4. draw plots v_{rel}, v_{tr}, a_{tr}, a_{rel}, a respect to time;

Needed variables:

1. $\phi_e(t) = 0.2t^3 + t$;
2. $OM(t) = 5\sqrt{2}(t^2 + t)$;
3. $a = 60$, $\alpha = \frac{\pi}{4}\ rad$;
4. Picture is given in Fig. 2;

2.2 Solution

1. Link for simulation
2. Let's start by analyzing the motion of point M in given problem. Triangle O_1OO_2 is rotating about point O_1. As a result point M experiences corriolis acceleration. Point M is has linear relative trajectory along the OO_2. We can split the absolute velocity of M into translatory and relative velocities.

$$\vec{v}_{abs}^{M} = \vec{v}_{rel} + \vec{v}_{tr} \tag{8}$$

$$v_{rel}(t) = \dot{O}M(t) = 5\sqrt{2}(2t + 1) \tag{9}$$

$$v_{tr} = R\omega \tag{10}$$

where R is the shortest distance from O_1 to M, and ω is angular velocity of triangle O_1OO_2.

$$\omega = \dot{\phi}_e(t) = 0.6t^2 + 1 \tag{11}$$

As M moves distance MO_1 is some function of time, let's say $R(t)$. We find $R(t)$ by analyzing triangle O_1OM. In triangle O_1OM, we know angle O, length OM and O_1O. We need to find O_1M (aka R). Using *cosine law*, we get:

$$\begin{aligned} R(t) &= \sqrt{O_1O^2 + OM(t)^2 - 2 \cdot O_1O \cdot OM(t) \cdot \cos(45^\circ)} \\ &= \sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)} \end{aligned} \tag{12}$$

Now, we have all the variables to calculate $\vec{v}_{tr}$ from Eq. (10).

$$v_{tr}(t) = R(t)\omega(t) = (0.6t^2 + 1) \cdot \sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)} \tag{13}$$

By adding $\vec{v}_{rel}$(Eq. 9) and $\vec{v}_{tr}$(Eq. 13) we get absolute acceleration:

$$\vec{v}_{abs} = \begin{bmatrix} -v_{tr}\sin(\alpha) - v_{rel}\cos(45^\circ - \phi) \\ v_{tr}\cos(\alpha) + v_{rel}\sin(45^\circ - \phi) \end{bmatrix} \tag{14}$$

where

$$\alpha = \phi + \theta \tag{15}$$

θ is angle MO_1O. We find θ by analysing triangle O_1OM and using *sine laws.*

$$\frac{OM(t)}{sin(\theta)} = \frac{R(t)}{sin(45°)} \tag{16}$$

As a result:

$$\theta = \arcsin(\frac{OM(t) \cdot sin(45°)}{R(t)}) \tag{17}$$

Now, we analyze acceleration of point M.

$$\vec{a}_{abs} = \vec{a}_{rel} + \vec{a}_{tr} + \vec{a}_{corr} \tag{18}$$

$$a_{rel} = 10\sqrt{2} \tag{19}$$

$$\vec{a}_{tr} = \vec{a}_{tr}^{\tau} + \vec{a}_{tr}^{n} \tag{20}$$

$a_{tr}^{\tau} = \epsilon \cdot R(t)$, where $R(t)$ is in Eq. 12, and $\epsilon = \dot{\omega}(t) = 1.2t$.

$$a_{tr}^{\tau} = 1.2t \cdot \sqrt{3600 + 50(t^2+t)^2 - 600(t^2+t)} \tag{21}$$

$a_{tr}^{n} = \frac{v_{tr}^2}{R(t)}$

$$a_{tr}^{n} = \frac{(0.6t^2+1)\sqrt{50t^4 + 100t^3 - 550t^2 - 600t + 3600}}{\sqrt{3600 + 50(t^2+t)^2 - 600(t^2+t)}} \tag{22}$$

$$\vec{a}_{tr} = \begin{bmatrix} -a_{tr}^{\tau}\sin(\alpha) - a_{tr}^{n}\cos(\alpha) \\ a_{tr}^{\tau}\cos(\alpha) - a_{tr}^{n}\sin(\alpha)] \end{bmatrix} \tag{23}$$

$$\vec{a}_{corr} = 2 \cdot \vec{\omega} \times \vec{v}_{rel} \tag{24}$$

$a_{corr} = 2\omega \cdot v_{rel}\sin(\gamma) = 2\omega \cdot v_{rel}$, where γ is angle between $\vec{\omega}$ and $\vec{v}_{rel}$, and it is $\gamma = 90°$. Using Eq. 9 and Eq. 11 we get:

$$a_{corr} = 10\sqrt{2}(0.6 \cdot t^2 + 1)(2t+1) \tag{25}$$

Now, the absolute acceleration is:

$$\vec{a}_{abs} = \begin{bmatrix} -a_{tr}^{\tau}\sin(\alpha) - a_{tr}^{n}\cos(\alpha) - a_{rel}\cos(45° - \phi) - a_{corr}\cos(\alpha) \\ a_{tr}^{\tau}\cos(\alpha) - a_{tr}^{n}\sin(\alpha) + a_{rel}\cos(45° - \phi) - a_{corr}\cos(\alpha) \end{bmatrix} \tag{26}$$

3. Point M reaches point O_2 when it travels distance of $60\sqrt{2}$ because the distance $OO_2 = 60\sqrt{2}$. By solving $OM(t) = 60\sqrt{2}$ for t, we get our t:

$$\begin{aligned} 5\sqrt{2}(t^2 + t) &= 60\sqrt{2} \\ t &= 3 \end{aligned} \tag{27}$$

4. Plots are given in answers sub-section.

2.3 Answers:

1. •

$$\vec{v}_{rel} = \begin{bmatrix} -v_{rel}\cos(45^\circ - \phi) \\ v_{rel}\sin(45^\circ - \phi) \end{bmatrix}, v_{rel}(t) = 5\sqrt{2}(2t+1) \tag{28}$$

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$$\begin{aligned} \vec{v}_{tr} &= \begin{bmatrix} -v_{tr}\cos(90^\circ - \alpha) \\ v_{tr}\sin(90^\circ - \alpha) \end{bmatrix}, v_{tr}(t) = (0.6t^2 + 1) \cdot \sqrt{3600 + 50(t^2+t)^2 - 600(t^2+t)} \\ \alpha &= \theta + \phi, \theta = \arcsin(\frac{OM(t) \cdot sin(45^\circ)}{R(t)}), R(t) = \sqrt{3600 + 50(t^2+t)^2 - 600(t^2+t)} \end{aligned} \tag{29}$$

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$$\vec{v}_{abs} = \begin{bmatrix} -v_{tr}\sin(\alpha) - v_{rel}\cos(45^\circ - \phi) \\ v_{tr}\cos(\alpha) + v_{rel}\sin(45^\circ - \phi) \end{bmatrix} \tag{30}$$

•

$$a_{rel} = 10\sqrt{2} \tag{31}$$

•

$$\begin{aligned} \vec{a}_{tr} &= \begin{bmatrix} -a_{tr}^{\tau}\sin(\alpha) - a_{tr}^{n}\cos(\alpha) \\ a_{tr}^{\tau}\cos(\alpha) - a_{tr}^{n}\sin(\alpha)] \end{bmatrix}, \\ a_{tr}^{\tau} &= 1.2t \cdot \sqrt{3600 + 50(t^2+t)^2 - 600(t^2+t)}, \\ a_{tr}^{n} &= \frac{(0.6t^2+1)\sqrt{50t^4 + 100t^3 - 550t^2 - 600t + 3600}}{\sqrt{3600 + 50(t^2+t)^2 - 600(t^2+t)}} \end{aligned} \tag{32}$$

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$$\begin{aligned} \vec{a}_{abs} &= \begin{bmatrix} -a_{tr}^{\tau}\sin(\alpha) - a_{tr}^{n}\cos(\alpha) - a_{rel}\cos(45^\circ - \phi) - a_{corr}\cos(\alpha) \\ a_{tr}^{\tau}\cos(\alpha) - a_{tr}^{n}\sin(\alpha) + a_{rel}\sin(45^\circ - \phi) - a_{corr}\sin(\alpha) \end{bmatrix}, \\ a_{corr} &= 10\sqrt{2}(0.6 \cdot t^2 + 1)(2t+1) \end{aligned} \tag{33}$$

2. $t = 3$

3. Plots are in Fig. 3-4;

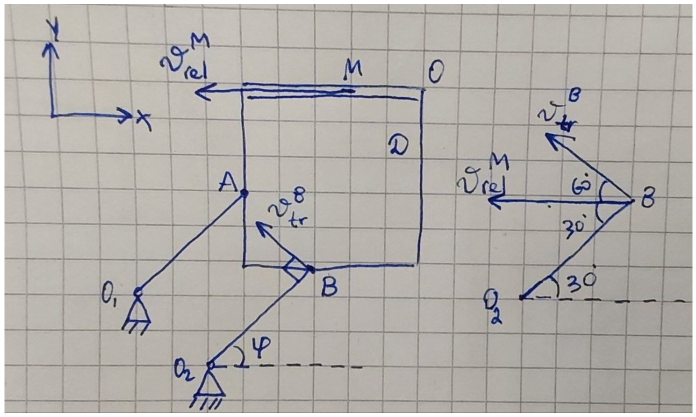

Figure 1: Task 1

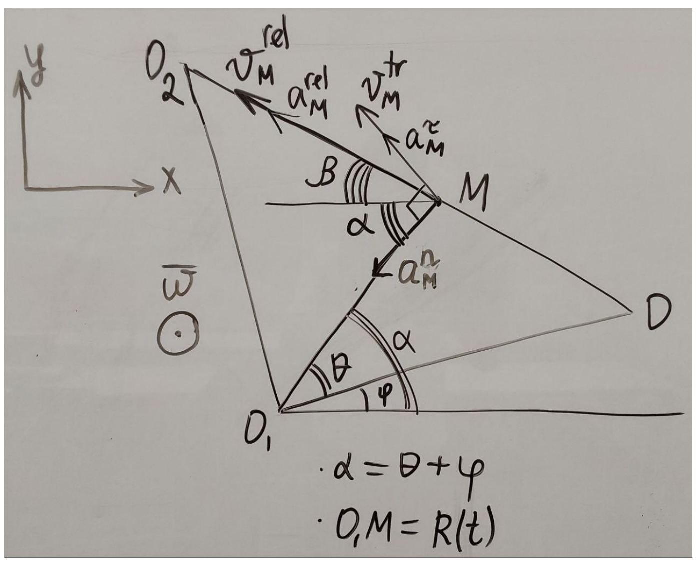

Figure 2: Task 2

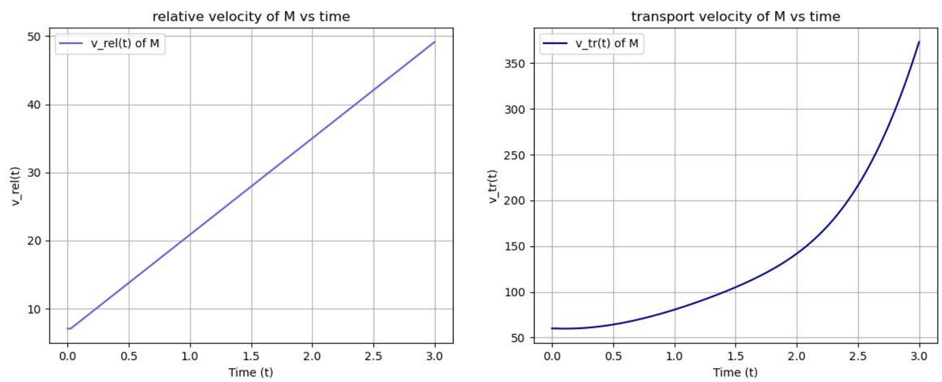

Figure 3: velocities of M

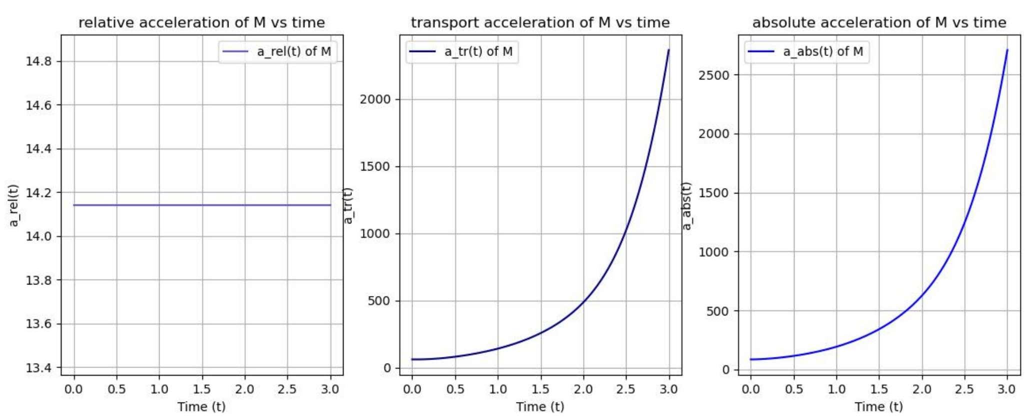

Figure 4: accelerations of M