Theoretical Mechanics HW4

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February 2024

MEME

1 Task

1.1 Description

Determine the reaction forces and the forces in the interim pins of the composite stud. The studs and acting forces are shown. Needed variables:

- $P_1 = 6$;
- $P_2 = 10$;
- $M_1 = 30$;
- $q = 1.5$;

1.2 Solution

Research Object: rod OC, rod CD, rod DF, all of them are fixed. Reaction forces are shown in Fig. 1.

Force Analysis:

- $P_1 = 6$;
- $P_2 = 10$;
- $M_1 = 30$;
- $F_1 = 2q = 3$;
- $F_2 = 3q = 4.5$;
- $F_3 = 1.5q = 2.25$;
- R_{ax}, R_{ay}, $\vec{R}_b$, $\vec{R}_e$, $\vec{R}_f$, R_{cx}, R_{cy}, R_{dx}, R_{dy} - unknowns.

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Solution: Equation of rod OC:

$$\begin{cases} OX: -P_1\cos(60^\circ) + R_{ax} + R_{cx}^{ac} = 0 \\ OY: -P_1\sin(60^\circ) + R_{ay} + R_b - F_1 + R_{cy}^{ac} = 0 \\ M_a: 2P_1\sin(60^\circ) + M_1 + 4R_b - 5F_1 + 6R_{cy}^{ac} = 0 \end{cases} \quad (1)$$

Equation of rod CD:

$$\begin{cases} OX: -R_{cx}^{cd} + R_{dx}^{cd} = 0 \\ OY: -R_{cy}^{cd} + R_{dy}^{cd} - F_2 = 0 \\ M_c: -1,5F_2 + 3R_{dy}^{dc} = 0 \end{cases} \quad (2)$$

Equation of rod DF:

$$\begin{cases} OX: -R_{dx}^{df} - R_f\sin(30^\circ) = 0 \\ OY: -R_{dy}^{df} - F_3 + R_e - P_2 + R_f\cos(30^\circ) = 0 \\ M_d: -0.75F_3 + 1.5R_E - 3P_2 + 5R_f\cos(30^\circ) = 0 \end{cases} \quad (3)$$

Solving the system of linear equations in *sympy*, we get the following results:

- $Rax = 4.64$
- $Ray = 13.42$
- $Rb = -2.973$
- $Re = 11.66$
- $Rf = 3.28$
- $Rcx = -1.64$
- $Rcy = -2.25$
- $Rdx = -1.64$
- $Rdy = 2.25$

;

1.3 Answer:

- $Rax = 4.64$
- $Ray = 13.42$
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2 Task

2.1 Description

Determine the reaction forces in rods supporting a thin horizontal rectangular plate of weight G under action of force P applied along the side AB. The constructions and the acting forces are shown Fig. 3. Needed variables:

- $G = 10$;
- $P = 20$;
- $a = 8,5$;
- $b = 2,5$;
- $c = 3,5$;
- $d = 2$;

2.2 Solution

Research Object Let's start by putting $Oxyz$ on a point with a most reaction intersections. One of such points is point A, and it will be origin of $Oxyz$. AC is positive x direction. AB is positive y direction. And up from A is positive z direction. Reaction forces are shown in Fig. 2.

- Rod 1 - fixed, with pin support;
- Rod 2 - fixed, with pin support;
- Rod 3 - fixed, with pin support;
- Rod 4 - fixed, with pin support;
- Rod 5 - fixed, with pin support;
- Rod 6 - fixed, with pin support;

- point A, with reaction forces $\vec{s}_5$, $\vec{s}_4$;
- point B, with reaction force $\vec{s}_3$;
- point C, with reaction force $\vec{s}_1$, $\vec{s}_2$;
- point D, with reaction force $\vec{s}_6$;

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Force Analysis

- $\vec{G}$ gravitation;
- $\vec{P}$ along AB;
- $\vec{s}_1$, $\vec{s}_2$, $\vec{s}_3$, $\vec{s}_4$, $\vec{s}_5$, $\vec{s}_6$ - unknowns;

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Solution We have six unknowns, and we can make six linear equations in the form of XYM_{point}.

$$\begin{cases} M_A^X : -G \cdot b\frac{1}{2} - s_3 \sin(\alpha)b + s_6 \sin(\gamma)b = 0 \\ M_A^Y : G\frac{a}{2} - s_6 \sin(\gamma)a + s_2 \sin(\beta)a + s_1 a = 0 \\ M_A^Z : s_2 \cos(\beta)a - s_3 \cos(\alpha)b - s_6 \cos(\gamma)a = 0 \\ OX : s_3 \cos(\alpha) = 0 \\ OY : P + s_2 \cos(\beta) + s_4 \cos(\beta) - s_6 \cos(\gamma) = 0 \\ OZ : -s_1 - s_2 \sin(\beta) - s_3 \sin(\alpha) - s_4 \sin(\beta) - s_5 + s_6 \sin(\gamma) - G = 0 \end{cases} \tag{4}$$

where α, β, γ are angles as shown in Fig. 2. By solving the Eq. 4, we get the following values:

- $s_1 = -8.75$;
- $s_2 = 10.75$;
- $s_3 = 0$;
- $s_4 = -34.41$;
- $s_5 = 23$;
- $s_6 = 8$;

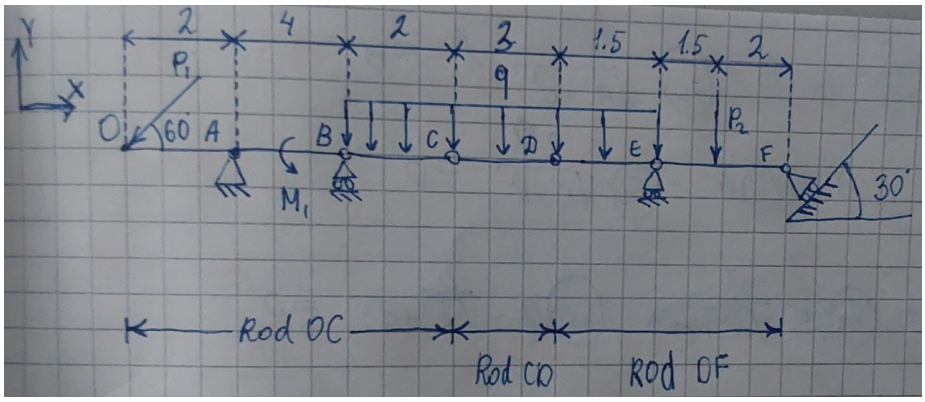

Figure 1: Reaction Forces

2.3 Answer:

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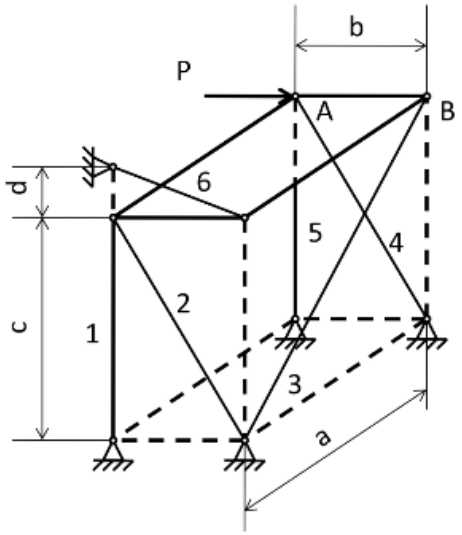

Figure 2: Description

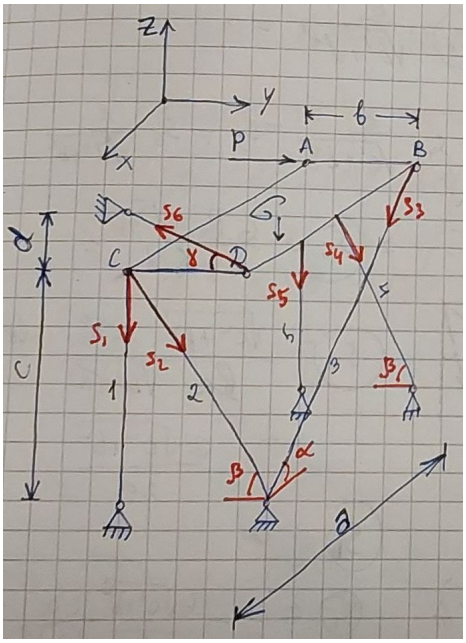

Figure 3: Reaction Forces