

Theoretical Mechanics HW3

Mukhammadrizo Maribjonov

February 2024

MEME

1 Task 1

1.1 Description

You should find an absolute velocity and coriolis acceleration, and absolute acceleration of particle M at the time $t = t_1$.

1.

$$OM = s_r(t) = f_3(t) = 2t^3 + 3t \quad (1)$$

2.

$$\phi(t) = \frac{1}{24}\pi t^2 \quad (2)$$

3. $t_1 = 2$;

4. $R = 15$;

5. Picture is given in Fig. 1.

1.2 Solution

Let's start by finding the position of object D and plane M at given point of time $t = 2$; Using equation (2), we get $\phi(2) = \frac{1}{24}\pi 2^2 = \frac{\pi}{6}$. Absolute velocity of point M is determined as a sum of *relative* and *transport* velocities.

$$\vec{v}_{abs} = \vec{v}_{rel} + \vec{v}_{tr} \quad (3)$$

$$|\vec{v}_{rel}| = \dot{s}_r(t) = 6t^2 + 3; \text{ At } t = 2, |\vec{v}_{rel}| = 27.$$

$$|\vec{v}_{tr}| = R \cdot \omega, \text{ where } \omega(t) = \dot{\phi}(t) = \frac{1}{12}\pi t. \text{ At } t = 2, |\vec{v}_{tr}| = 2.5\pi.$$

Now we have the scalar values of the velocities. Now let's find v_{abs} by OX and OY components:

$$\vec{v}_{abs} = \begin{bmatrix} -v_{rel} - v_{tr} \cos(60^\circ) \\ v_{tr} \sin(60^\circ) \end{bmatrix} = \begin{bmatrix} -30.93 \\ 6.8 \end{bmatrix} \quad (4)$$

As a result we get the value of $v_{abs} = 31.67$

Now let's try to find acceleration. Note, that there is no coriolis acceleration because body D is not rotating. It is making a translatory motion. M experiences coriolis acceleration when body D rotates. Absolute acceleration is found as:

$$\vec{a}_{abs} = \vec{a}_{rel} + \vec{a}_{tr} \quad (5)$$

As M is making rotational transport motion, we can divide it's transport acceleration into tangential and normal transport accelerations.

$$\vec{a}_{abs} = \vec{a}_{rel} + \vec{a}_{tr}^{\tau} + \vec{a}_{tr}^n \quad (6)$$

$$|\vec{a}_{rel}(t=2)| = \ddot{s}_r(t) = 12t, \text{ when } t=2, |\vec{a}_{rel}| = 24.$$

$$|\vec{a}_{tr}^{\tau}(t=2)| = \epsilon R, \epsilon = \ddot{\phi}(t) = \frac{1}{12}\pi.$$

$$\text{So, } |\vec{a}_{tr}^{\tau}(t=2)| = \frac{1}{12}\pi R = \frac{5}{4}\pi;$$

$$|\vec{a}_{tr}^n(t=2)| = \omega(t=2)^2 R = \frac{5}{12}\pi^2$$

$$\vec{a}_{abs} = \begin{bmatrix} -a_{rel} - a_{tr}^n \cos(30^\circ) - a_{tr}^{\tau} \cos(60^\circ) \\ a_{tr}^{\tau} \sin(60^\circ) - a_{tr}^n \sin(30^\circ) \end{bmatrix} \quad (7)$$

As a result we get the value of $a_{abs} = 29.55$

1.3 **Answers:**

1. $v_{abs} = 31.67$
2. There is no coriolis acceleration because body D is not rotating. It is making a translatory motion.
3. $a_{abs} = 29.55$

2 Task 2

2.1 Description

You should find:

1. simulate this mechanism (obtain all positions);
2. find absolute, transport and relative velocities and accelerations for M ;
3. find t , when M reaches O point;

4. draw plots v_{rel} , v_{tr} , a_{tr} , a_{rel} , a respect to time;

Needed variables:

1. $\phi_e(t) = 0.2t^3 + t$;
2. $OM(t) = 5\sqrt{2}(t^2 + t)$;
3. $a = 60$, $\alpha = \frac{\pi}{4}$ rad;
4. Picture is given in Fig. 2;

2.2 Solution

1. Link for simulation
2. Let's start by analyzing the motion of point M in given problem. Triangle O_1OO_2 is rotating about point O_1 . As a result point M experiences coriolis acceleration. Point M is has linear relative trajectory along the OO_2 . We can split the absolute velocity of M into translatory and relative velocities.

$$\vec{v}_{abs}^M = \vec{v}_{rel} + \vec{v}_{tr} \quad (8)$$

$$v_{rel}(t) = \dot{OM}(t) = 5\sqrt{2}(2t + 1) \quad (9)$$

$$v_{tr} = R\omega \quad (10)$$

where R is the shortest distance from O_1 to M , and ω is angular velocity of triangle O_1OO_2 .

$$\omega = \dot{\phi}_e(t) = 0.6t^2 + 1 \quad (11)$$

As M moves distance MO_1 is some function of time, let's say $R(t)$. We find $R(t)$ by analyzing triangle O_1OM . In triangle O_1OM , we know angle O , length OM and O_1O . We need to find O_1M (aka R). Using *cosine law*, we get:

$$\begin{aligned} R(t) &= \sqrt{O_1O^2 + OM(t)^2 - 2 \cdot O_1O \cdot OM(t) \cdot \cos(45^\circ)} \\ &= \sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)} \end{aligned} \quad (12)$$

Now, we have all the variables to calculate \vec{v}_{tr} from Eq. (10).

$$v_{tr}(t) = R(t)\omega(t) = (0.6t^2 + 1) \cdot \sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)} \quad (13)$$

By adding \vec{v}_{rel} (Eq. 9) and \vec{v}_{tr} (Eq. 13) we get absolute acceleration:

$$\vec{v}_{abs} = \begin{bmatrix} -v_{tr} \sin(\alpha) - v_{rel} \cos(45^\circ - \phi) \\ v_{tr} \cos(\alpha) + v_{rel} \sin(45^\circ - \phi) \end{bmatrix} \quad (14)$$

where

$$\alpha = \phi + \theta \quad (15)$$

θ is angle MO_1O . We find θ by analysing triangle O_1OM and using *sine laws*.

$$\frac{OM(t)}{\sin(\theta)} = \frac{R(t)}{\sin(45^\circ)} \quad (16)$$

As a result:

$$\theta = \arcsin\left(\frac{OM(t) \cdot \sin(45^\circ)}{R(t)}\right) \quad (17)$$

Now, we analyze acceleration of point M.

$$\vec{a}_{abs} = \vec{a}_{rel} + \vec{a}_{tr} + \vec{a}_{corr} \quad (18)$$

$$a_{rel} = 10\sqrt{2} \quad (19)$$

$$\vec{a}_{tr} = \vec{a}_{tr}^\tau + \vec{a}_{tr}^n \quad (20)$$

$a_{tr}^\tau = \epsilon \cdot R(t)$, where $R(t)$ is in Eq. 12, and $\epsilon = \dot{\omega}(t) = 1.2t$.

$$a_{tr}^\tau = 1.2t \cdot \sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)} \quad (21)$$

$$a_{tr}^n = \frac{v_{tr}^2}{R(t)}$$

$$a_{tr}^n = \frac{(0.6t^2 + 1)\sqrt{50t^4 + 100t^3 - 550t^2 - 600t + 3600}}{\sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)}} \quad (22)$$

$$\vec{a}_{tr} = \begin{bmatrix} -a_{tr}^\tau \sin(\alpha) - a_{tr}^n \cos(\alpha) \\ a_{tr}^\tau \cos(\alpha) - a_{tr}^n \sin(\alpha) \end{bmatrix} \quad (23)$$

$$\vec{a}_{corr} = 2 \cdot \vec{\omega} \times \vec{v}_{rel} \quad (24)$$

$a_{corr} = 2\omega \cdot v_{rel} \sin(\gamma) = 2\omega \cdot v_{rel}$, where γ is angle between $\vec{\omega}$ and \vec{v}_{rel} , and it is $\gamma = 90^\circ$. Using Eq. 9 and Eq. 11 we get:

$$a_{corr} = 10\sqrt{2}(0.6 \cdot t^2 + 1)(2t + 1) \quad (25)$$

Now, the absolute acceleration is:

$$\vec{a}_{abs} = \begin{bmatrix} -a_{tr}^\tau \sin(\alpha) - a_{tr}^n \cos(\alpha) - a_{rel} \cos(45^\circ - \phi) - a_{corr} \cos(\alpha) \\ a_{tr}^\tau \cos(\alpha) - a_{tr}^n \sin(\alpha) + a_{rel} \cos(45^\circ - \phi) - a_{corr} \cos(\alpha) \end{bmatrix} \quad (26)$$

3. Point M reaches point O_2 when it travels distance of $60\sqrt{2}$ because the distance $OO_2 = 60\sqrt{2}$. By solving $OM(t) = 60\sqrt{2}$ for t , we get our t :

$$\begin{aligned} 5\sqrt{2}(t^2 + t) &= 60\sqrt{2} \\ t &= 3 \end{aligned} \quad (27)$$

4. Plots are given in answers sub-section.

2.3 Answers:

1. Link for simulation

2. •

$$\vec{v}_{rel} = \begin{bmatrix} -v_{rel} \cos(45^\circ - \phi) \\ v_{rel} \sin(45^\circ - \phi) \end{bmatrix}, v_{rel}(t) = 5\sqrt{2}(2t + 1) \quad (28)$$

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$$\vec{v}_{tr} = \begin{bmatrix} -v_{tr} \cos(90^\circ - \alpha) \\ v_{tr} \sin(90^\circ - \alpha) \end{bmatrix}, v_{tr}(t) = (0.6t^2 + 1) \cdot \sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)}$$

$$\alpha = \theta + \phi, \theta = \arcsin\left(\frac{OM(t) \cdot \sin(45^\circ)}{R(t)}\right), R(t) = \sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)} \quad (29)$$

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$$\vec{v}_{abs} = \begin{bmatrix} -v_{tr} \sin(\alpha) - v_{rel} \cos(45^\circ - \phi) \\ v_{tr} \cos(\alpha) + v_{rel} \sin(45^\circ - \phi) \end{bmatrix} \quad (30)$$

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$$a_{rel} = 10\sqrt{2} \quad (31)$$

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$$\begin{aligned} \vec{a}_{tr} &= \begin{bmatrix} -a_{tr}^\tau \sin(\alpha) - a_{tr}^n \cos(\alpha) \\ a_{tr}^\tau \cos(\alpha) - a_{tr}^n \sin(\alpha) \end{bmatrix}, \\ a_{tr}^\tau &= 1.2t \cdot \sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)}, \\ a_{tr}^n &= \frac{(0.6t^2 + 1)\sqrt{50t^4 + 100t^3 - 550t^2 - 600t + 3600}}{\sqrt{3600 + 50(t^2 + t)^2 - 600(t^2 + t)}} \end{aligned} \quad (32)$$

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$$\begin{aligned} \vec{a}_{abs} &= \begin{bmatrix} -a_{tr}^\tau \sin(\alpha) - a_{tr}^n \cos(\alpha) - a_{rel} \cos(45^\circ - \phi) - a_{corr} \cos(\alpha) \\ a_{tr}^\tau \cos(\alpha) - a_{tr}^n \sin(\alpha) + a_{rel} \sin(45^\circ - \phi) - a_{corr} \sin(\alpha) \end{bmatrix}, \\ a_{corr} &= 10\sqrt{2}(0.6 \cdot t^2 + 1)(2t + 1) \end{aligned} \quad (33)$$

3. $t = 3$

4. Plots are in Fig. 3-4;

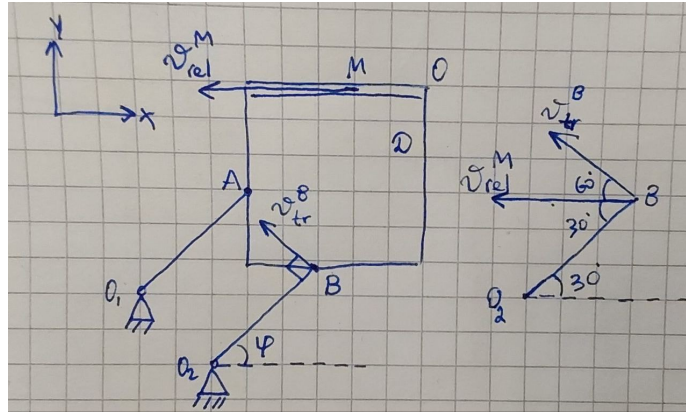


Figure 1: Task 1

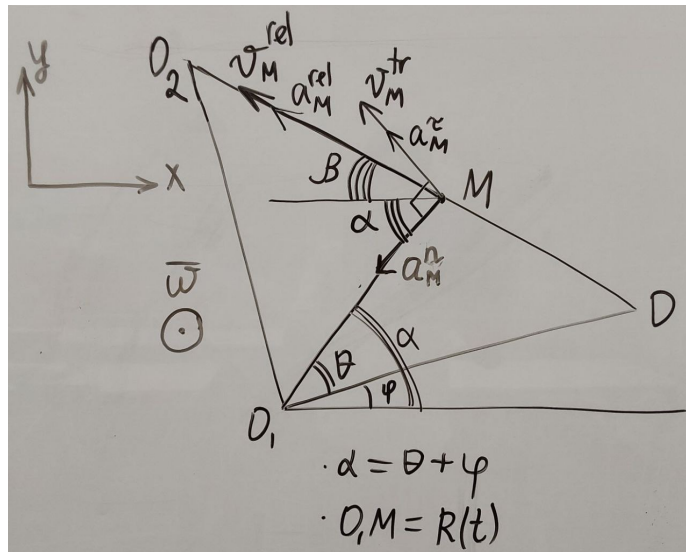


Figure 2: Task 2

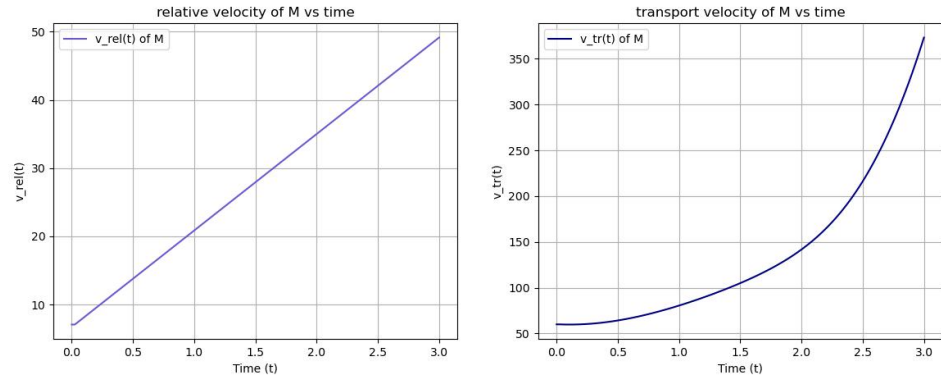


Figure 3: velocities of M

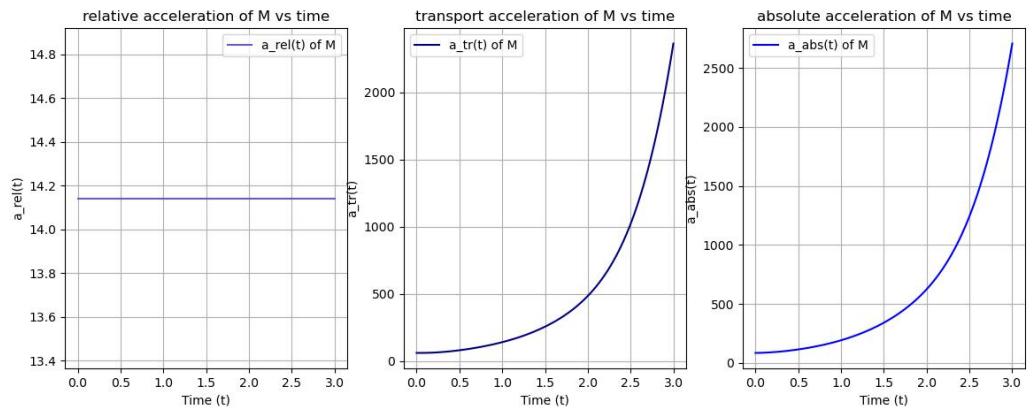


Figure 4: accelerations of M