Thoretical Mechanics HW1

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Source

1 Task 1, Coding

1. simulate the move of $\vec{O}$ for $t = [0,\ 10]$;
2. find and draw plots v, a, a_n, a_τ, k (Osculating circle) respect to t;
3. find $y(x)$, $\vec{v}$, $\vec{a}$, $\vec{a_n}$, $\vec{a_\tau}$ and show it on the simulation.

$$\vec{O} = \begin{cases} x = 3\cos(2t)\cos(t) + 0.82 \\ y = 3\cos(2t)\sin(t) + 0.82 \end{cases} . \tag{1}$$

Solution:

1. Link for simulation
2. v, a, a_n, a_τ, k
   (a) $v(t)$ We know that *velocity* is the fist derivative of position vector. Hence we have the following *velocity vector*:

   $$\boxed{\vec{v}(t)} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -6\sin(2t)\cos(t) - 3\cos(2t)\sin(t) \\ -6\sin(2t)\sin(t) + 3\cos(2t)\cos(t) \end{bmatrix} \tag{2}$$

   Equation (2) has the following scalar value:

   $$\boxed{v(t)} = |\vec{v}| = 3\sqrt{1 + 3\sin(2t)^2} \tag{3}$$

   (b) $a(t)$ Acceleration is the first derivative of velocity, or the second derivative of position.

   $$\boxed{\vec{a}(t)} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} -15\cos(2t)\cos(t) + 12\sin(2t)\sin(t) \\ -15\cos(2t)\sin(t) - 12\sin(2t)\cos(t) \end{bmatrix} \tag{4}$$

   And the magnitude of equation (4) is:

   $$\boxed{a(t)} = |\vec{a}| = 3\sqrt{9\cos(2t)^2 + 16} \tag{5}$$

(c) a_τ Tangential acceleration formula $a_\tau(t) = \frac{\vec{v}\cdot\vec{a}}{|\vec{v}|}$

$$a_\tau(t) = \begin{bmatrix} -6\sin(2t)\cos(t) - 3\cos(2t)\sin(t) \\ -6\sin(2t)\sin(t) + 3\cos(2t)\cos(t) \end{bmatrix} \cdot \begin{bmatrix} -15\cos(2t)\cos(t) + 12\sin(2t)\sin(t) \\ -15\cos(2t)\sin(t) - 12\sin(2t)\cos(t) \end{bmatrix} \frac{1}{3\sqrt{1+3\sin(2t)^2}} \tag{6}$$

$$\boxed{a_\tau(t)} = \frac{9\sin 4t}{\sqrt{1+3\sin^2 2t}} \tag{7}$$

Vectored tangential acceleration $\vec{a_\tau}$ can be found by multiplying unit vector of velocity by equation (7) as following:

$$\boxed{\vec{a_\tau}} = a_\tau(t) \cdot \frac{\vec{v(t)}}{|\vec{v(t)}|} = \boxed{\frac{3\sin 4t}{1+3\sin^2 2t} \begin{bmatrix} -6\sin(2t)\cos(t) - 3\cos(2t)\sin(t) \\ -6\sin(2t)\sin(t) + 3\cos(2t)\cos(t) \end{bmatrix}} \tag{8}$$

(d) a_n *Normal acceleration* $\vec{a_n}$ is found as $\vec{a_n} = \vec{a} \times \vec{v}\frac{1}{|\vec{v}|}$.

$$\vec{a_n}(t) = \begin{bmatrix} -15\cos(2t)\cos(t) + 12\sin(2t)\sin(t) \\ -15\cos(2t)\sin(t) - 12\sin(2t)\cos(t) \end{bmatrix} \times \begin{bmatrix} -6\sin(2t)\cos(t) - 3\cos(2t)\sin(t) \\ -6\sin(2t)\sin(t) + 3\cos(2t)\cos(t) \end{bmatrix} \frac{1}{3\sqrt{1+3\sin(2t)^2}} \tag{9}$$

Finally, we get $\boxed{\vec{a_n}(t) = \begin{bmatrix} 0 \\ 0 \\ -27\sin^2 2t - 45 \end{bmatrix} \frac{1}{3\sqrt{1+3\sin(2t)^2}}}$

$$\boxed{a_n(t) = \frac{9\sin^2 2t + 15}{\sqrt{1+3\sin(2t)^2}}} \tag{10}$$

(e) Curvature can be find by

$$\boxed{k(n)} = \frac{a_n}{v(t)^2} = \boxed{\frac{3\sin^2 2t + 5}{3(1+3\sin^2 2t)\sqrt{1+3\sin^2 2t}}} \tag{11}$$

(f) Plots are shown in Fig. 1

3. Cartesian form of parametric equation (1) can be formed as:

$$\boxed{((x-0.82)^2 + (y-0.82)^2)^3 = 9((x-0.82)^2 - (y-0.82)^2)^2} \tag{12}$$

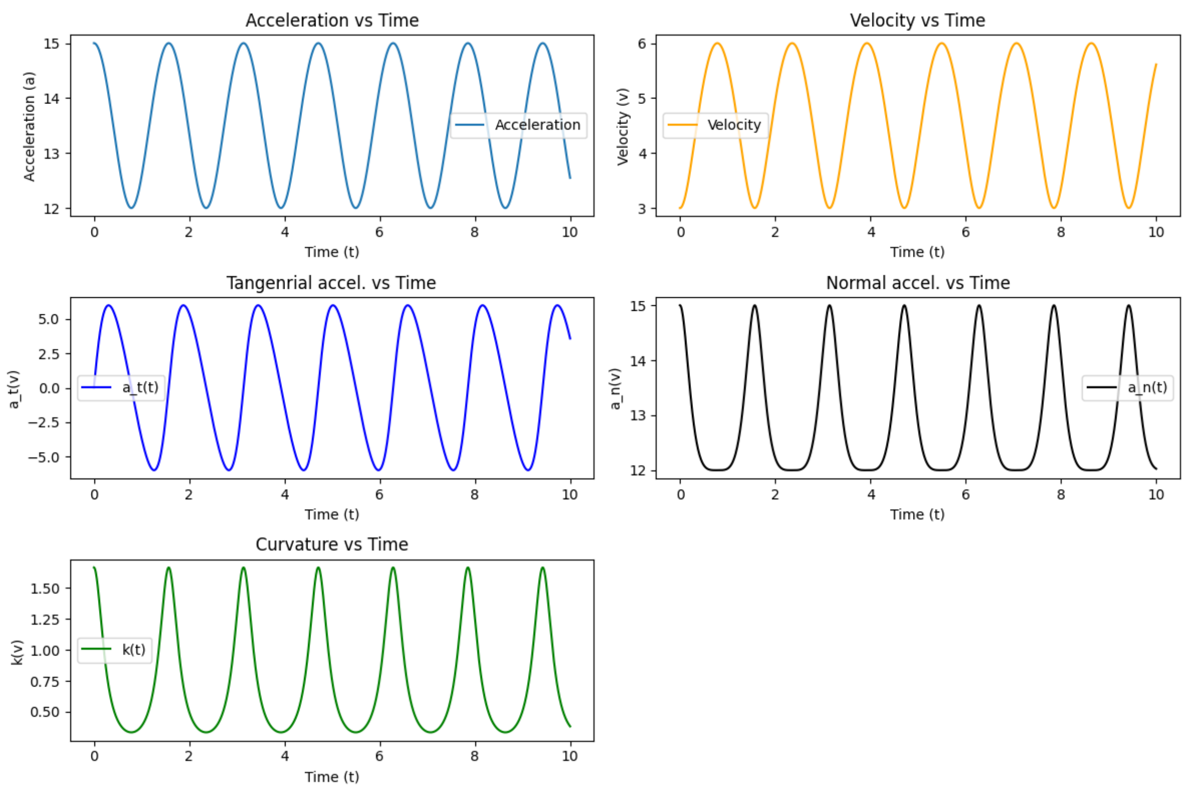

Figure 1: Plots of v, a, a_n, a_τ, k for $t = [0,\ 10]$;

2 Task 2, Coding

You should solve the task, till the M point travels s:

1. simulate this mechanism (obtain all positions of bodies 1, 2, 3)
2. velocity for M(draw plots for magnitudes and show vectors on simulation);
3. accelerations (tangent, normal, overall) for M(draw plots for magnitudes and show vectors on simulation);
4. draw plots of angular velocities for 2, 3 bodies.
5. If $R_2 = 40$, $r_2 = 30$, $R_3 = 15$, $x = x(t) = 3 + 80t^2$, $s_M = [0, 5]$.

Solution

1. Link for simulation
2. velocity of point M. Let's start by finding velocity $v_x(t)$ of object 1 as derivative of position function.

$$v_x(t) = \dot{x}(t) = 160t \tag{13}$$

Now, let's remember the connection between velocity of object 1 with angular velocity of disk 2

$$v_x(t) = w_2 R_2 \tag{14}$$

which helps us to find angular velocity of *disk 2*

$$w_2(t) = \frac{v_x(t)}{R_2} = \frac{160t}{40} = 4t \tag{15}$$

We know that 2 circles in disk 2 have the same angular velocity w_2, but different linear velocities. Linear velocity of point of outer circle is same as $v_x(t)$. Linear velocity of inner circle of disk 2 is:

$$v_{2inner} = w_2(t) r_2 = 120t \tag{16}$$

Disk 3 and inner circle of disk 2 have the same linear velocity at the point of intersection, but different angular velocities (w_2, w_3). So, points of disk 3 (including point M) have velocity as show in equation (16). Plot of $v_m(t)$ is shown in Fig. 2

$$v_m(t) = v_{2inner} = 120t \tag{17}$$

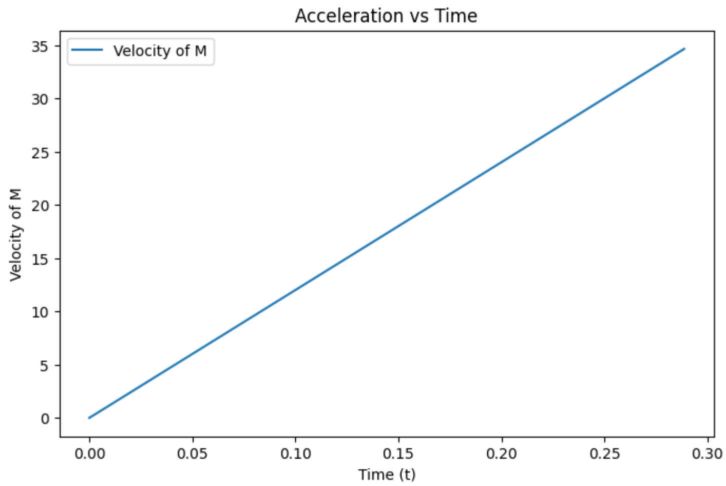

Figure 2: Velocity of M

3. Normal acceleration (a_{nm}), tangential acceleration ($a_{\tau m}$), acceleration (a_m) of point M For normal acceleration (a_{nm}) let's use the following formula:

$$\boxed{a_{nm}} = \frac{v_m^2}{R_3} = \frac{(120t)^2}{15} = \boxed{960t^2} \tag{18}$$

For $a_{\tau m}$ we use the following formula:

$$a_{\tau m} = R_3 \epsilon_m \tag{19}$$

, where ϵ_m is angular acceleration of point M. We find $\epsilon_m = \dot{w}_m$. And w_m is found from:

$$\begin{gathered} v_m = w_m R_3 \\ \boxed{w_m} = \frac{v_m}{R_3} = \frac{120t}{15} = \boxed{8t} \end{gathered} \tag{20}$$

Hence, we have ϵ_m:

$$\epsilon_m = \dot{w}_m(t) = 8 \tag{21}$$

Putting equation (21) into (19), we get:

$$\boxed{a_{\tau m}} = R_3 8 = \boxed{120} \tag{22}$$

Total acceleration of point M is:

$$\boxed{a_m} = \sqrt{a_{\tau m}^2 + a_{nm}^2} = \boxed{\sqrt{14400 + 960^2 t^4}} \tag{23}$$

Plots are shown in Fig. 3

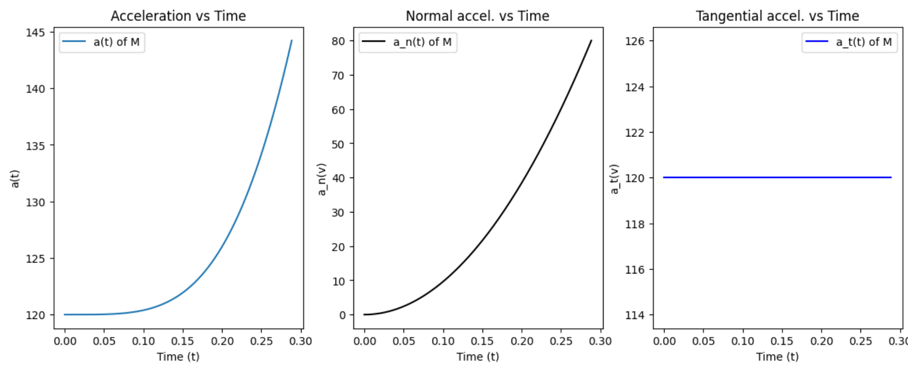

Figure 3: Plots of $a(t)$, a_τ, a_n of M

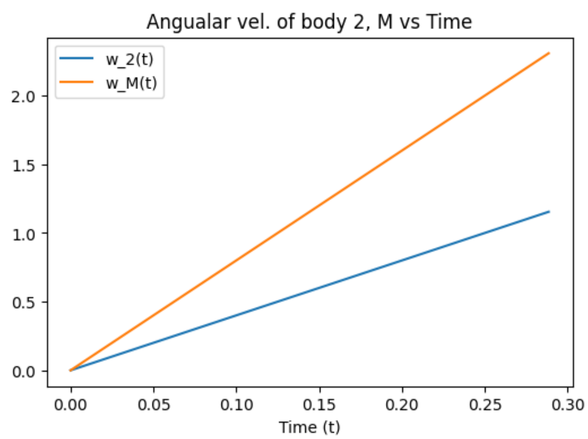

Figure 4: w_2 and w_m vs Time

4. Angular velocity of body 2 is $w_2(t) = 4t$ as show in equation (15). Angular velocity of body 3 is $w_m = 8t$ as shown in equation (20). Plots are shown in Fig. 4.

3 Task 3, Coding

1. simulate this mechanism (obtain all positions.) ($x_i(t)$, $y_i(t)$, where i is A, B, C point)
2. velocities for B, C (draw plots for magnitudes and show vectors on simulation);
3. accelerations for B and C (draw plots for magnitudes and show vectors

on simulation);

4. draw a plot of angular velocity of body BA.
5. If $y_A(t) = 22.5 + 10\sin(\frac{\pi}{5}t)$; $t = [0..10]$ sec.; $AB = 45$, $BC = 30$.

Solution

1. Link for simulation
2. Let's start by finding ICV (Instantaneous center of zero velocity) and name it at point P. ICV is perpendicular to acceleration of points A, B. Thus, AP and BP form a right angle with axes OY, OX respectively. At any point of time t, $AP(t) = OB(t) = \sqrt{45^2 - y_A^2(t)}$, $BP(t) = OA(t) = y_A(t)$. For later use, let's find angle $\angle OBA(t)$:

$$\angle OBA(t) = \arcsin\left(\frac{y_A(t)}{45}\right) \tag{24}$$

Now, $\angle ABP(t) = 90^\circ - OBA(t) = \beta(t)$. Next, we can find $CP(t)$ using *cosine law* as

$$\begin{aligned} CP(t) = \sqrt{CB^2 + BP^2(t) - 2CB \cdot BP(t)\cos(\beta(t))} = \\ \sqrt{900 + y_A^2(t) - 60y_A(t)\cos(\beta(t))} \end{aligned} \tag{25}$$

On segment AB we have point At any point of time t length of OA is $y_A(t)$. Then OB can be found as:

$$OB(t) = \sqrt{45^2 - y_A^2(t)} = \frac{5\sqrt{243 - 72\sin(\frac{\pi x}{5}) - 16\sin^2(\frac{\pi x}{5})}}{2} \tag{26}$$

Now, we can find $v_a(t)$, $v_b(t)$

$$\boxed{v_a(t)} = \dot{y}_A(t) = \boxed{2\pi\cos(\frac{\pi t}{5})} \tag{27}$$

$$\boxed{v_b(t)} = \dot{OB}(t) = \boxed{-\frac{18\pi\cos(\frac{\pi x}{5}) + 4\pi\sin(\frac{2\pi x}{5})}{\sqrt{243 - 72\sin(\frac{\pi x}{5}) - 16\sin^2(\frac{\pi x}{5})}}} \tag{28}$$

3. Accelerations $a_A(t)$, $a_B(t)$

$$\boxed{a_A(t)} = \dot{v}_a(t) = \boxed{-\frac{2\pi^2\sin(\frac{\pi x}{5})}{5}} \tag{29}$$