Theoretical Mechanics HW4

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MEME

1 Task

1.1 Description

Determine the reaction forces and the forces in the interim pins of the composite stud. The studs and acting forces are shown. Needed variables:

- $P_1 = 6$;

- $P_2 = 10$;

- $M_1 = 30$;

- $q = 1.5$;

1.2 Solution

Research Object: rod OC, rod CD, rod DF, all of them are fixed. Reaction forces are shown in Fig. 1.

Force Analysis:

- $P_1 = 6$;

- $P_2 = 10$;

- $M_1 = 30$;

- $F_1 = 2q = 3$;

- $F_2 = 3q = 4.5$;

- $F_3 = 1.5q = 2.25$;

- R_{ax}, R_{ay}, $\vec{R}_b$, $\vec{R}_e$, $\vec{R}_f$, R_{cx}, R_{cy}, R_{dx}, R_{dy} - unknowns.

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Solution: Equation of rod OC:

$$\begin{cases} OX : -P_1\cos(60°) + R_{ax} + R_{cx}^{ac} = 0 \\ OY : -P_1\sin(60°) + R_{ay} + R_b - F_1 + R_{cy}^{ac} = 0 \\ M_a : 2P_1\sin(60°) + M_1 + 4R_b - 5F_1 + 6R_{cy}^{ac} = 0 \end{cases} \quad (1)$$

Equation of rod CD:

$$\begin{cases} OX : -R_{cx}^{cd} + R_{dx}^{cd} = 0 \\ OY : -R_{cy}^{cd} + R_{dy}^{cd} - F_2 = 0 \\ M_c : -1,5F_2 + 3R_{dy}^{dc} = 0 \end{cases} \quad (2)$$

Equation of rod DF:

$$\begin{cases} OX : -R_{dx}^{df} - R_f\cos(30°) = 0 \\ OY : -R_{dy}^{df} - F_3 + R_e - P_2 + R_f\sin(30°) = 0 \\ M_d : -0.75F_3 + 1.5R_E - 3P_2 + 5R_f\sin(30°) = 0 \end{cases} \quad (3)$$

Solving the system of linear equations in $sympy$, we get the following results:

- $Rax = 7.91778711434735$

- $Ray = 13.4192286340600$

- $Rb = -2.97307621135332$

- $Re = 11.6607142857143$

- $Rf = 5.67857142857143$

- $Rcx = -4.91778711434735$

- $Rcy = -2.25000000000000$

- $Rdx = -4.91778711434735$

- $Rdy = 2.25000000000000$

;

1.3 Answer:

- $Rax = 7.91778711434735$

- $Ray = 13.4192286340600$

- $Rb = -2.97307621135332$

- $Re = \quad 11.6607142857143$

- $Rf = \quad 5.67857142857143$

- $Rcx = -4.91778711434735$

- $Rcy = -2.25000000000000$

- $Rdx = -4.91778711434735$

- $Rdy = \quad 2.25000000000000$

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2 Task

2.1 Description

Determine the reaction forces in rods supporting a thin horizontal rectangular plate of weight G under action of force P applied along the side AB. The constructions and the acting forces are shown Fig. 3. Needed variables:

- $G = 10$;

- $P = 20$;

- $a = 8, 5$;

- $b = 2, 5$;

- $c = 3, 5$;

- $d = 2$;

2.2 Solution

Research Object Let's start by putting $Oxyz$ on a point with a most reaction intersections. One of such points is point A, and it will be origin of $Oxyz$. AC is positive x direction. AB is positive y direction. And up from A is positive z direction. Reaction forces are shown in Fig. 2.

- Rod 1 - fixed, with pin support;

- Rod 2 - fixed, with pin support;

- Rod 3 - fixed, with pin support;

- Rod 4 - fixed, with pin support;

- Rod 5 - fixed, with pin support;

- Rod 6 - fixed, with pin support;

- point A, with reaction forces $\vec{s}_5$, $\vec{s}_4$;

- point B, with reaction force $\vec{s}_3$;

- point C, with reaction force $\vec{s}_1$, $\vec{s}_2$;

- point D, with reaction force $\vec{s}_6$;

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Force Analysis

- $\vec{G}$ gravitation;

- $\vec{P}$ along AB;

- $\vec{s}_1$, $\vec{s}_2$, $\vec{s}_3$, $\vec{s}_4$, $\vec{s}_5$, $\vec{s}_6$ - unknowns;

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Solution We have six unknowns, and we can make six linear equations in the form of XYM_{point}.

$$
\begin{cases}
M_A^X : -G \cdot b\frac{1}{2} - s_3 \sin(\alpha)b + s_6 \sin(\gamma)b = 0 \\
M_A^Y : G\frac{a}{2} - s_6 \sin(\gamma)a + s_2 \sin(\beta)a + s_1 a = 0 \\
M_A^Z : s_2 \cos(\beta)a - s_3 \cos(\alpha)b - s_6 \cos(\gamma)a = 0 \\
OX : s_3 \cos(\alpha) = 0 \\
OY : P + s_2 \cos(\beta) + s_4 \cos(\beta) - s_6 \cos(\gamma) = 0 \\
OZ : -s_1 - s_2 \sin(\beta) - s_3 \sin(\alpha) - s_4 \sin(\beta) - s_5 + s_6 \sin(\gamma) - G = 0
\end{cases}
\tag{4}
$$

where α, β, γ are angles as shown in Fig. 2. By solving the Eq. 4, we get the following values:

- $s_1 = -8.75$;

- $s_2 = 10.75$;

- $s_3 = 0$;

- $s_4 = -34.41$;
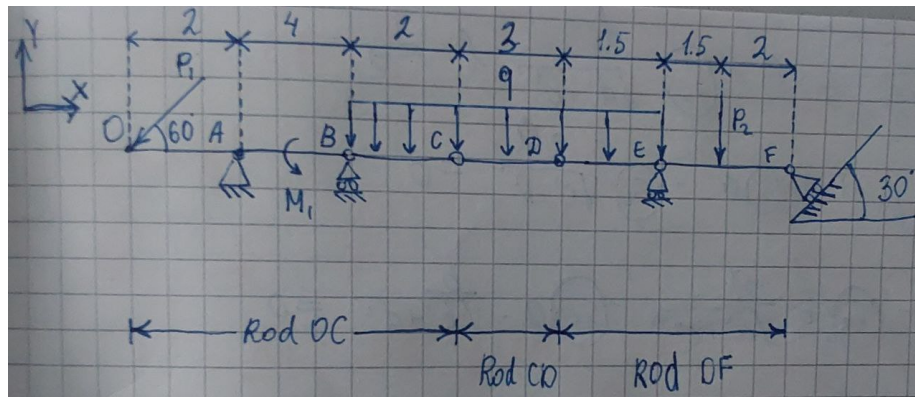
- $s_5 = 23$;

- $s_6 = 8$;

4

Figure 1: Reaction Forces

2.3 Answer:

- $s_1 = -8.75$;
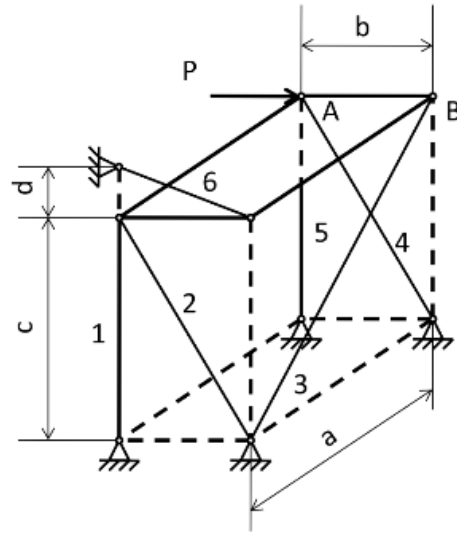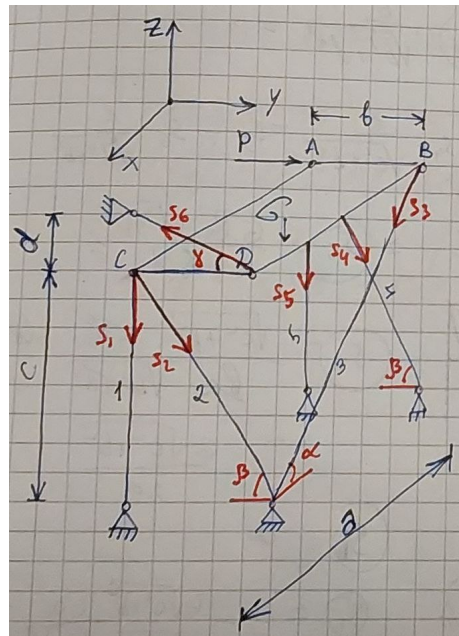- $s_2 = 10.75$;
- $s_3 = 0$;
- $s_4 = -34.41$;
- $s_5 = 23$;
- $s_6 = 8$;

Figure 2: Description

Figure 3: Reaction Forces