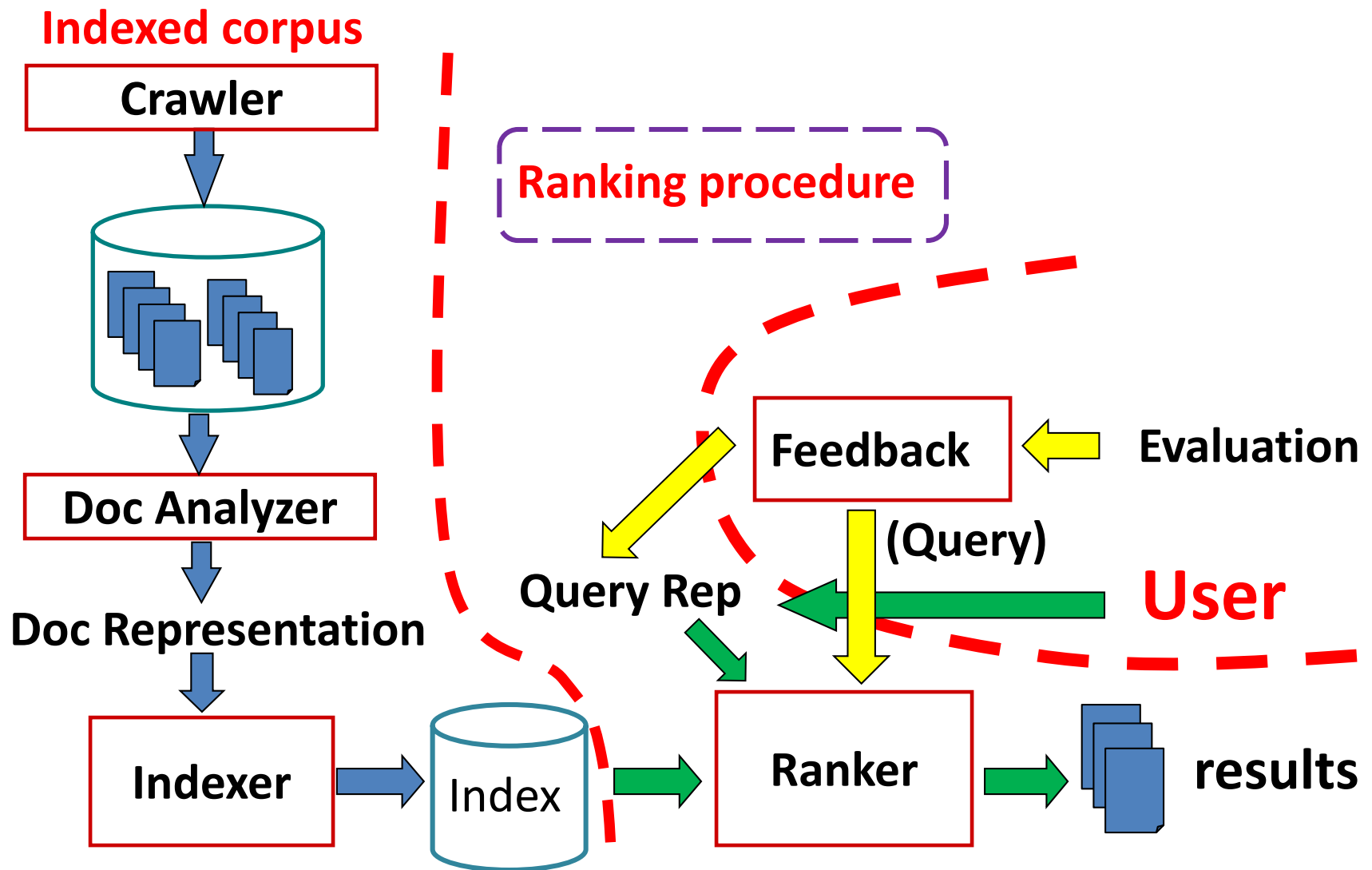


# Boolean Model

Hongning Wang

CS@UVa

# Abstraction of search engine architecture

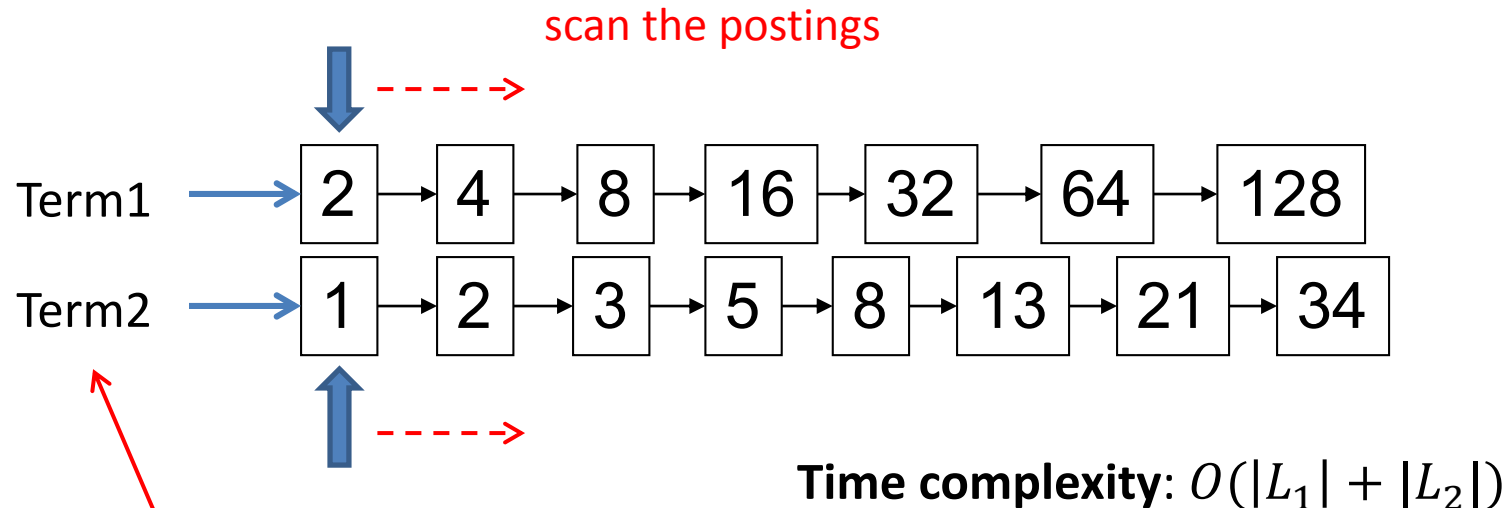


# Search with Boolean query

- Boolean query
  - E.g., “obama” AND “healthcare” NOT “news”
- Procedures
  - Lookup query term in the dictionary
  - Retrieve the posting lists
  - Operation
    - AND: intersect the posting lists
    - OR: union the posting list
    - NOT: diff the posting list

# Search with Boolean query

- Example: AND operation

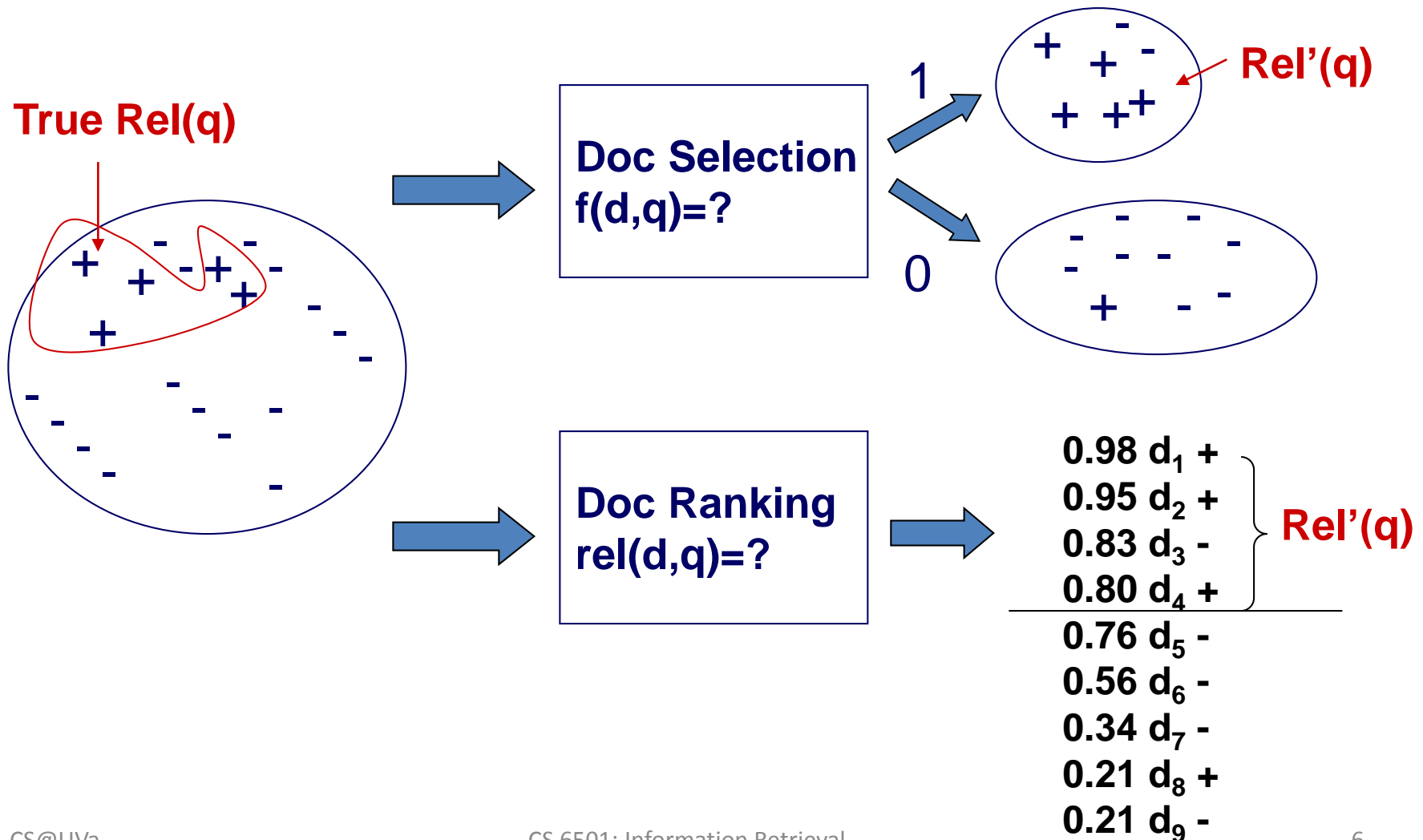


**Trick for speed-up:** when performing multi-way join, starts from lowest frequency term to highest frequency ones

# Deficiency of Boolean model

- The query is unlikely precise
  - “Over-constrained” query (terms are too specific): no relevant documents found
  - “Under-constrained” query (terms are too general): over delivery
  - It is hard to find the right position between these two extremes (hard for users to specify constraints)
- Even if it is accurate
  - Not all users would like to use such queries
  - All relevant documents are not equally relevant
    - No one would go through all the matched results
- Relevance is a matter of degree!

# Document Selection vs. Ranking



# Ranking is often preferred

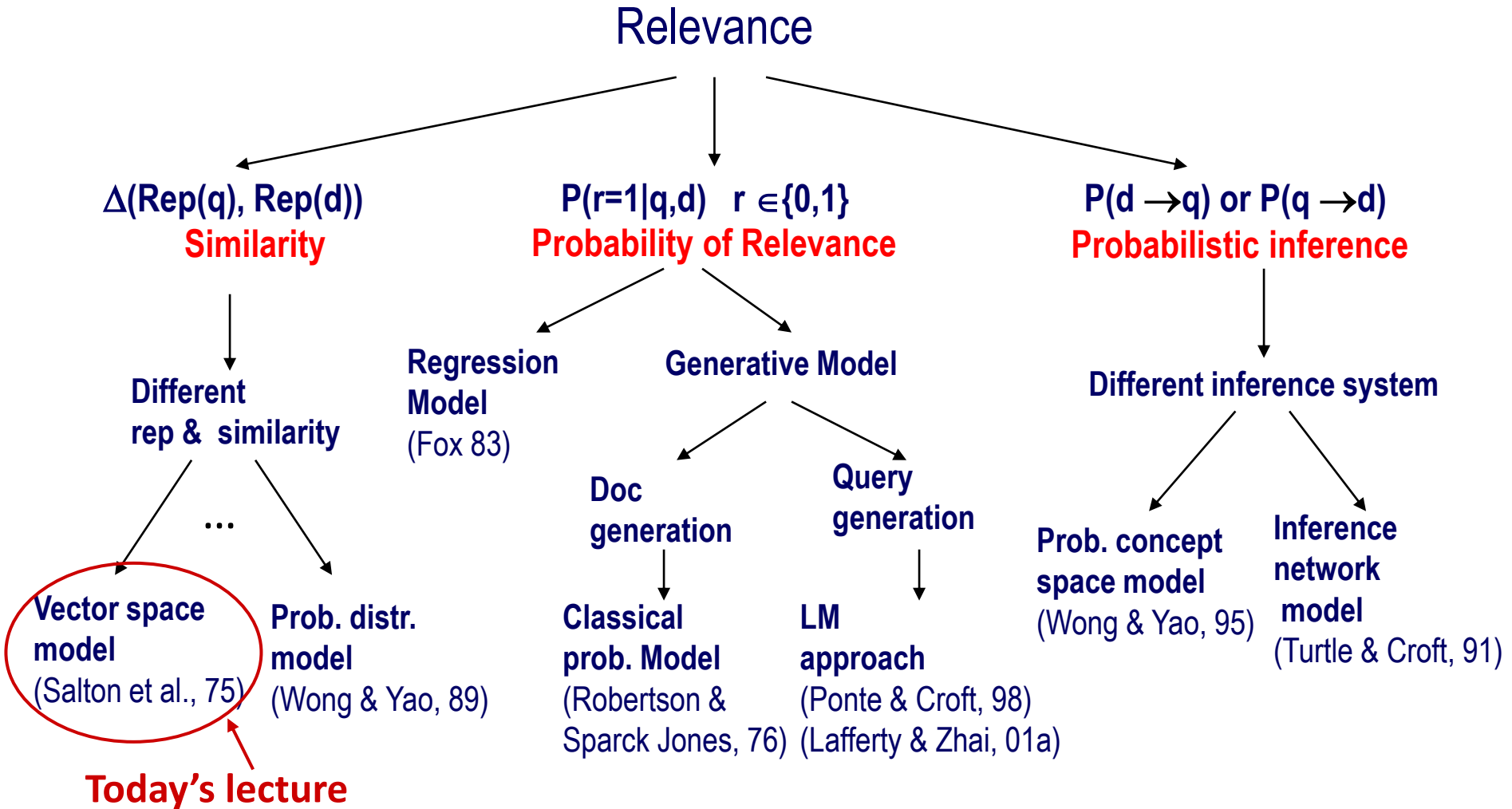
- Relevance is a matter of degree
  - Easier for users to find appropriate queries
- A user can stop browsing anywhere, so the boundary is controlled by the user
  - Users prefer coverage would view more items
  - Users prefer precision would view only a few
- Theoretical justification: Probability Ranking Principle

# Retrieval procedure in modern IR

- Boolean model provides all the ranking candidates
  - Locate documents satisfying Boolean condition
    - E.g., “obama healthcare” -> “obama” OR “healthcare”
- Rank candidates by relevance
  - Important: the notation of relevance
- Efficiency consideration
  - Top-k retrieval ([Google](#))



# Notion of relevance



# Intuitive understanding of relevance

- Fill in magic numbers to describe the relation between documents and words

	information	retrieval	retrieved	is	helpful	for	you	everyone
Doc1	1	1	0	1	1	1	0	1
Doc2	1	0	1	1	1	1	1	0



*E.g., 0/1 for Boolean models,  
probabilities for probabilistic models*

# Some notations

- Vocabulary  $V = \{w_1, w_2, \dots, w_N\}$  of language
- Query  $q = t_1, \dots, t_m$ , where  $t_i \in V$
- Document  $d_i = t_{i1}, \dots, t_{in}$ , where  $t_{ij} \in V$
- Collection  $C = \{d_1, \dots, d_k\}$
- $\text{Rel}(q, d)$ : relevance of doc  $d$  to query  $q$
- $\text{Rep}(d)$ : representation of document  $d$
- $\text{Rep}(q)$ : representation of query  $q$

# Vector Space Model

Hongning Wang

CS@UVa

# Relevance = Similarity

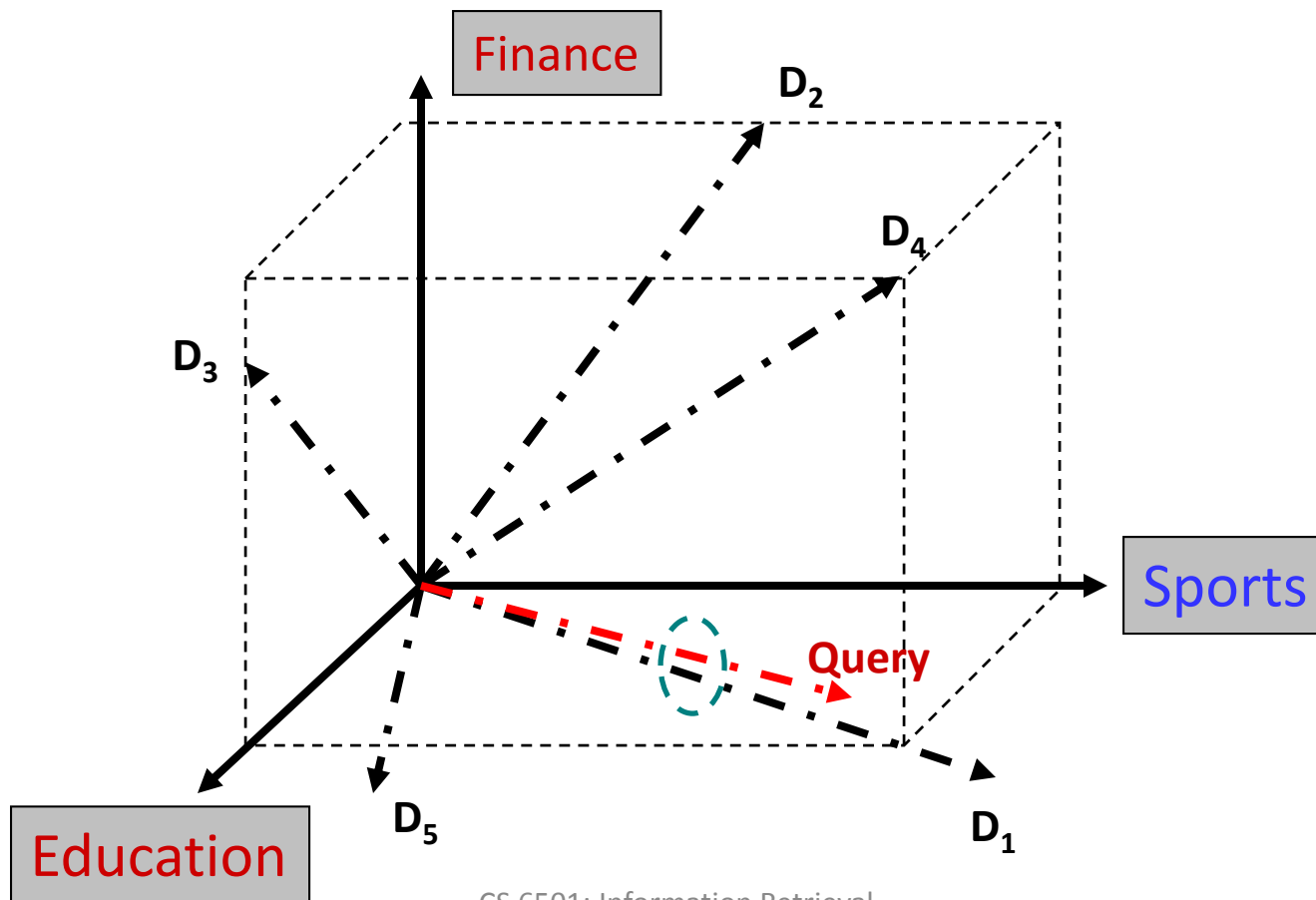
- Assumptions
  - Query and documents are represented in the same form
    - A query can be regarded as a “document”
  - $\text{Relevance}(d,q) \propto \text{similarity}(d,q)$
- $R(q) = \{d \in C \mid \text{rel}(d,q) > \theta\}$ ,  $\text{rel}(q,d) = \Delta(\text{Rep}(q), \text{Rep}(d))$
- Key issues
  - How to represent query/document?
  - How to define the similarity measure  $\Delta(x,y)$ ?

# Vector space model

- Represent both doc and query by concept vectors
  - Each concept defines one dimension
  - $K$  concepts define a high-dimensional space
  - Element of vector corresponds to concept weight
    - E.g.,  $d=(x_1, \dots, x_k)$ ,  $x_i$  is “importance” of concept  $i$
- Measure relevance
  - Distance between the query vector and document vector in this concept space

# VS Model: an illustration

- Which document is closer to the query?



# What the VS model doesn't say

- How to define/select the “basic concept”
  - Concepts are assumed to be orthogonal
- How to assign weights
  - Weight in query indicates importance of the concept
  - Weight in doc indicates how well the concept characterizes the doc
- How to define the similarity/distance measure



# What is a good “basic concept”?

- Orthogonal
  - Linearly independent basis vectors
    - “Non-overlapping” in meaning
    - No ambiguity
- Weights can be assigned automatically and accurately
- Existing solutions
  - Terms or N-grams, i.e., bag-of-words
  - Topics, i.e., topic model ← We will come back to this later

# How to assign weights?

- Important!
- Why?
  - Query side: not all terms are equally important
  - Doc side: some terms carry more information about the content
- How?
  - Two basic heuristics
    - TF (Term Frequency) = Within-doc-frequency
    - IDF (Inverse Document Frequency)

# TF weighting

- Idea: a term is more important if it occurs more frequently in a document
- TF Formulas
  - Let  $f(t, d)$  be the frequency count of term  $t$  in doc  $d$
  - Raw TF:  $tf(t, d) = f(t, d)$

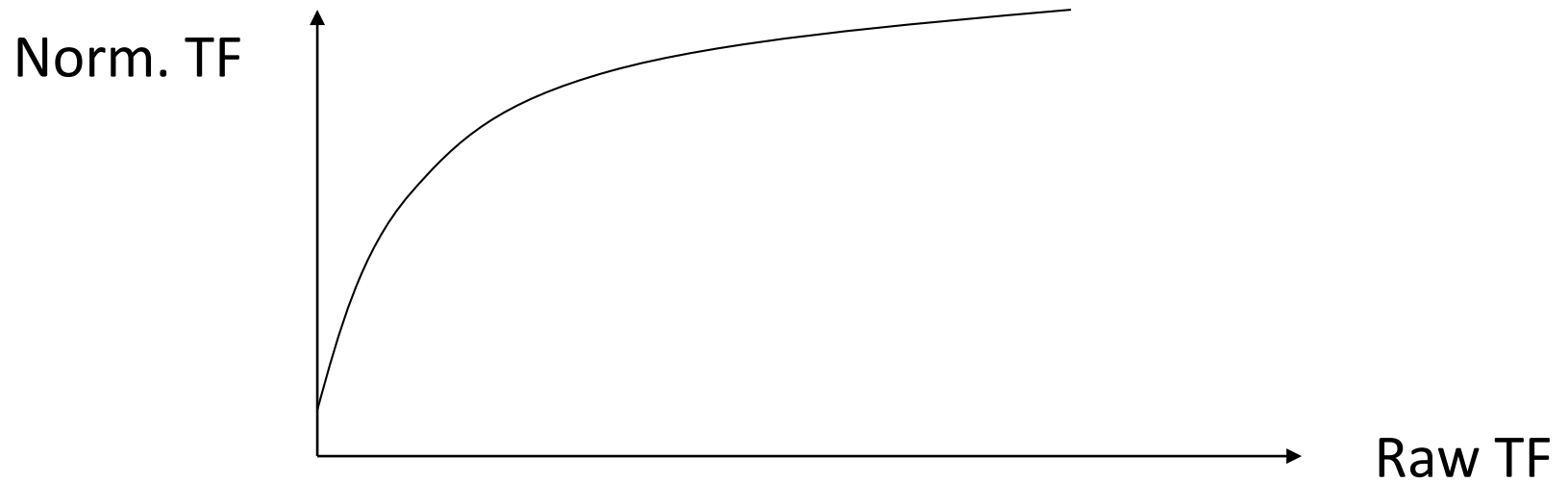
# TF normalization

- Two views of document length
  - A doc is long because it is verbose
  - A doc is long because it has more content
- Raw TF is inaccurate
  - Document length variation
  - “Repeated occurrences” are less informative than the “first occurrence”
  - Relevance does not increase proportionally with number of term occurrence
- Generally penalize long doc, but avoid over-penalizing
  - Pivoted length normalization

# TF normalization

- Sublinear TF scaling

$$- \text{tf}(t, d) = \begin{cases} 1 + \log f(t, d), & \text{if } f(t, d) > 0 \\ 0, & \text{otherwise} \end{cases}$$

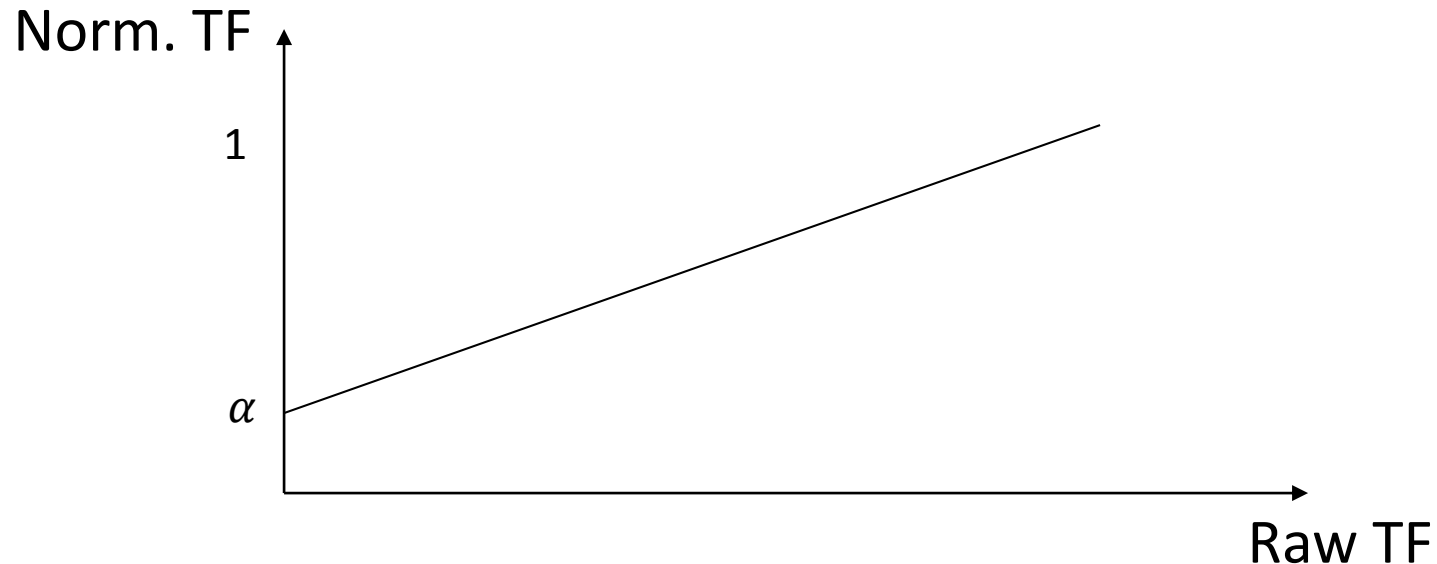


# TF normalization

- Maximum TF scaling

- $tf(t, d) = \alpha + (1 - \alpha) \frac{f(t, d)}{\max_t f(t, d)}$

- Normalize by the most frequent word in this doc



# Document frequency

- Idea: a term is more discriminative if it occurs only in fewer documents

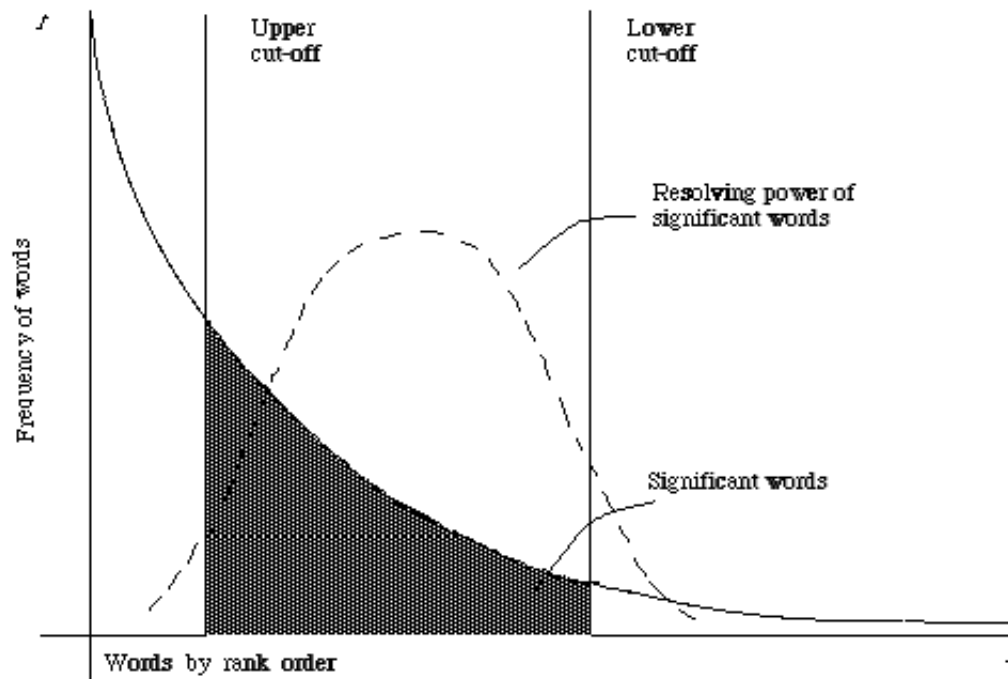


Figure 2.1. A plot of the hyperbolic curve relating  $f$ , the frequency of occurrence and  $r$ , the rank order (Adapted from Schultz<sup>44</sup>, page 120).

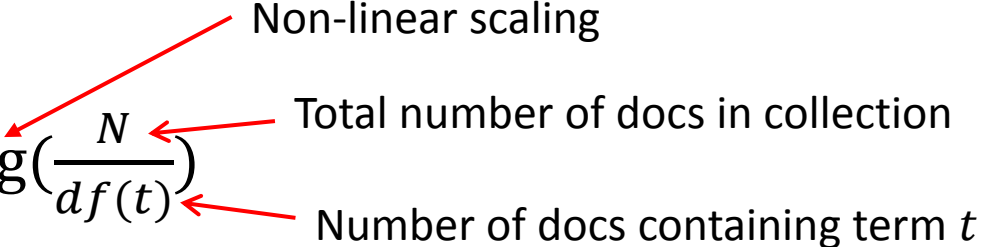
# IDF weighting

- Solution

- Assign higher weights to the rare terms

- Formula

- $IDF(t) = 1 + \log\left(\frac{N}{df(t)}\right)$



Non-linear scaling

Total number of docs in collection

Number of docs containing term  $t$

- A corpus-specific property

- Independent of a single document



# Why document frequency

- How about total term frequency?

- $ttf(t) = \sum_d f(t, d)$

Table 1. Example total term frequency v.s. document frequency in Reuters-RCV1 collection.

Word	ttf	df
try	10422	8760
insurance	10440	3997

- Cannot recognize words frequently occurring in a subset of documents

# TF-IDF weighting

- Combining TF and IDF
  - Common in doc  $\rightarrow$  high tf  $\rightarrow$  high weight
  - Rare in collection  $\rightarrow$  high idf  $\rightarrow$  high weight
  - $w(t, d) = TF(t, d) \times IDF(t)$
- Most well-known document representation schema in IR! (G Salton et al. 1983)



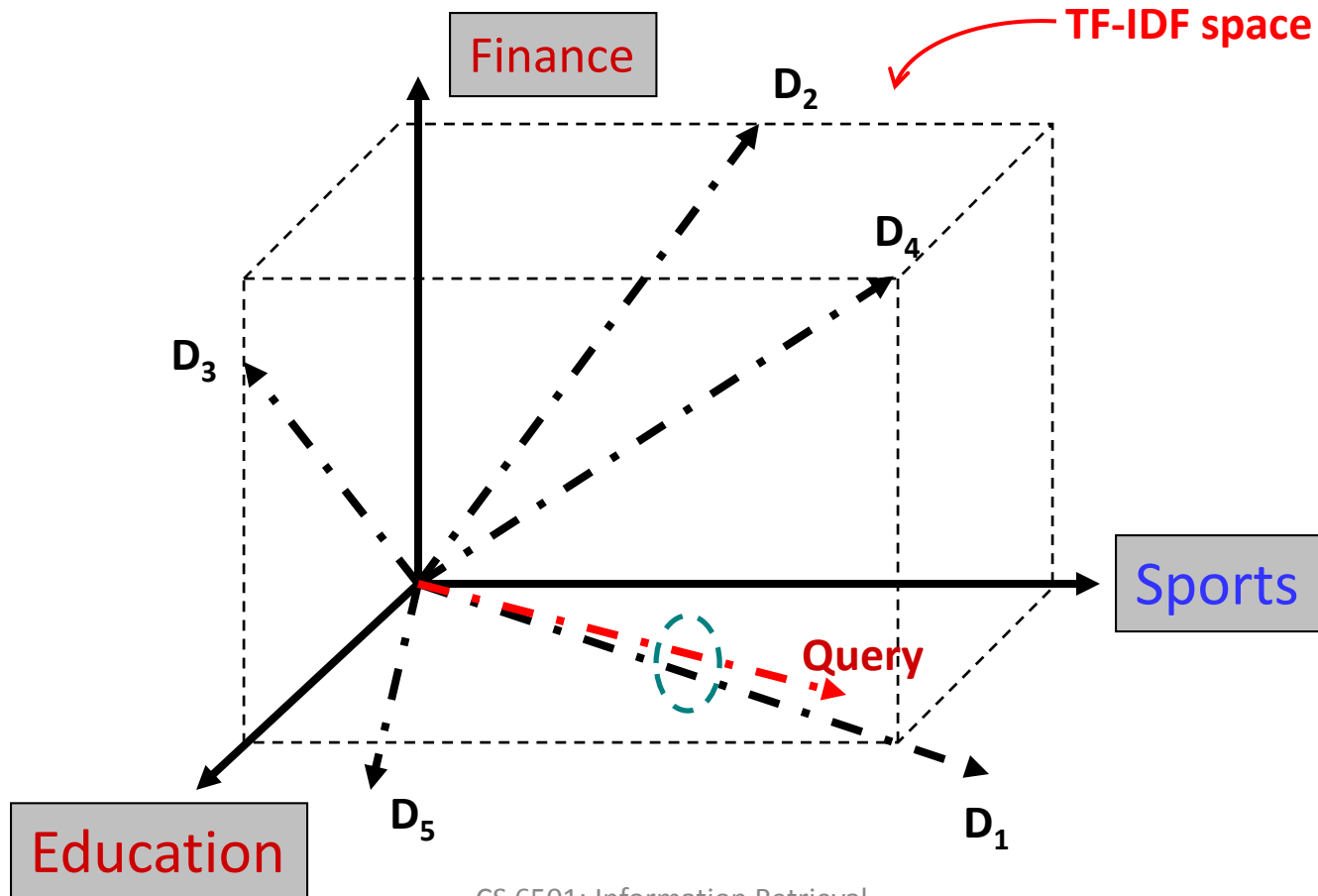
*"Salton was perhaps the leading computer scientist working in the field of information retrieval during his time." - wikipedia*

[Gerard Salton Award](#)

– highest achievement award in IR

# How to define a good similarity measure?

- Euclidean distance?



# How to define a good similarity measure?

- Euclidean distance

- $dist(q, d) =$

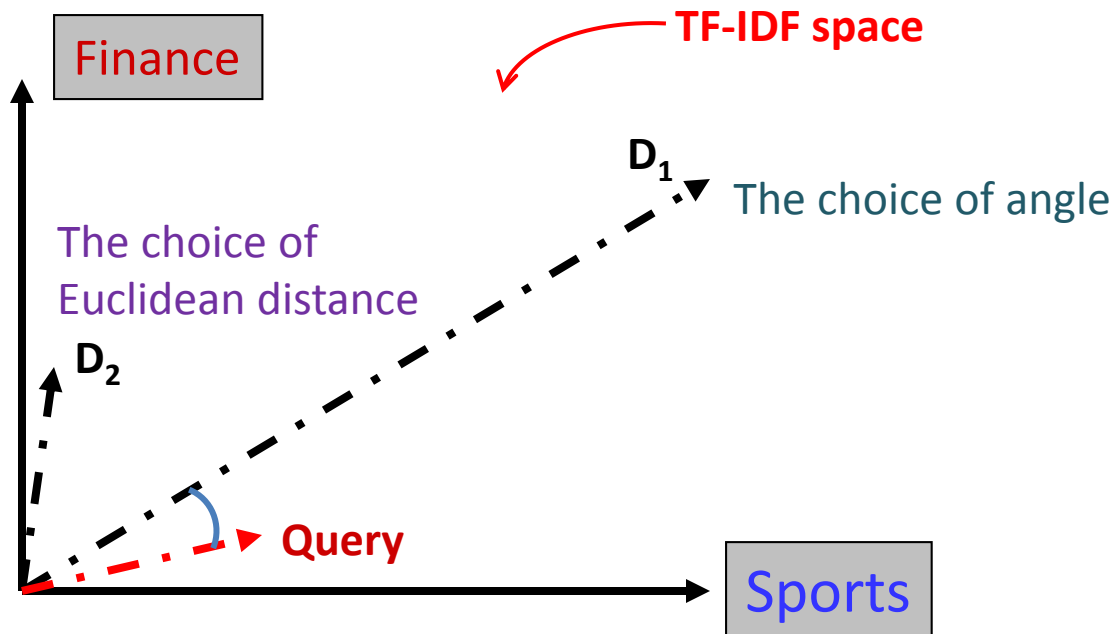
- $$\sqrt{\sum_{t \in V} [tf(t, q)idf(t) - tf(t, d)idf(t)]^2}$$

- Longer documents will be penalized by the extra words

- We care more about how these two vectors are overlapped

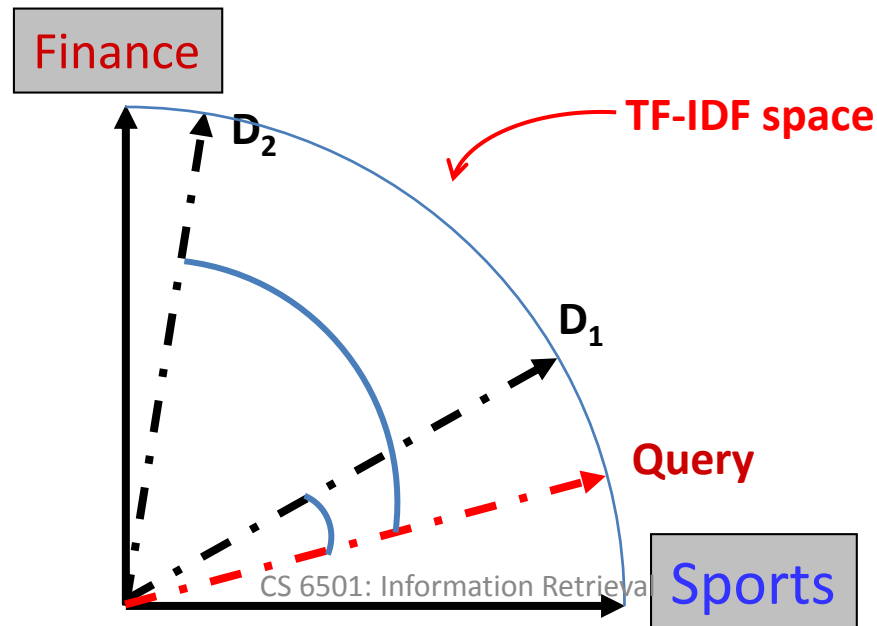
# From distance to angle

- Angle: how vectors are overlapped
  - Cosine similarity – projection of one vector onto another



# Cosine similarity

- Angle between two vectors
    - $\text{cosine}(V_q, V_d) = \frac{V_q \times V_d}{|V_q|_2 \times |V_d|_2} = \boxed{\frac{V_q}{|V_q|_2}} \times \frac{V_d}{|V_d|_2}$ 
      - Document length normalized
- TF-IDF vector
- Unit vector



# Fast computation of cosine in retrieval

- $\text{cosine}(V_q, V_d) = V_q \times \frac{V_d}{|V_d|_2}$ 
  - $|V_q|_2$  would be the same for all candidate docs
  - Normalization of  $V_d$  can be done in index time
  - Only count  $t \in q \cap d$
  - Score accumulator for each query term when intersecting postings from inverted index

# Fast computation of cosine in retrieval

- Maintain a score accumulator for each doc when scanning the postings

Query = “info security”

$S(d,q) = g(t_1) + \dots + g(t_n)$  [sum of TF of matched terms]

Info: (d1, 3), (d2, 4), (d3, 1), (d4, 5)

Security: (d2, 3), (d4, 1), (d5, 3)

Can be easily applied to TF-IDF weighting!

Accumulators:		d1	d2	d3	d4	d5
info	(d1,3) =>	<b>3</b>	0	0	0	0
	(d2,4) =>	3	4	0	0	0
	(d3,1) =>	3	4	<b>1</b>	0	0
	(d4,5) =>	3	4	1	5	0
security	(d2,3) =>	3	<b>7</b>	1	5	0
	(d4,1) =>	3	7	1	<b>6</b>	0
	(d5,3) =>	3	7	1	6	<b>3</b>

Keep only the most promising accumulators for top K retrieval



# Advantages of VS Model

- Empirically effective! (Top TREC performance)
- Intuitive
- Easy to implement
- Well-studied/Mostly evaluated
- The Smart system
  - Developed at Cornell: 1960-1999
  - Still widely used
- **Warning: Many variants of TF-IDF!**

# Disadvantages of VS Model

- Assume term independence
- Assume query and document to be the same
- Lack of “predictive adequacy”
  - Arbitrary term weighting
  - Arbitrary similarity measure
- Lots of parameter tuning!

# What you should know

- Document ranking v.s. selection
- Basic idea of vector space model
- Two important heuristics in VS model
  - TF
  - IDF
- Similarity measure for VS model